

Exercise 11

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Exercise 11.1

Let E_t be white noise with mean 0 and variance σ^2 . Regard the following three processes:

i) $X_t = t + E_t$

ii) $Y_t = X_t X_{t-1}$

iii) $Z_t = X_t t$

a) Which of these three processes are stationary, and which are not? Why?

$X_t = t + E_t$ is not constant, because there will be an upwards trend, as white noise will be added to the current time step.

$Y_t = X_t - X_{t-1} = t + E_t - (t-1 + E_{t-1}) = 1 + E_t - E_{t-1}$ is stationary, as 1 is a constant and E_t and E_{t-1} are both white noise.

$Z_t = X_t - t$ is stationary because $Z_t = X_t - t = t + E_t - t = E_t$ and E_t is white noise.

b) Optional: Compute the theoretical autocorrelation of the processes Y_t and Z_t and the cross-correlation between the two.

c) Simulate both Y_t and Z_t . To this end, assume that E_t follows a standard normal distribution $\mathcal{N}(0; 1)$. Simulate time series of length $n = 200$, and compare your empirical results to the theoretical ones of part b).

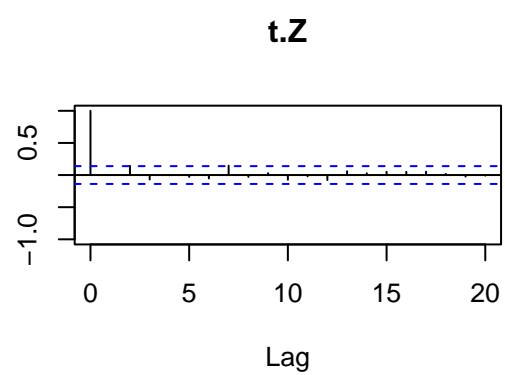
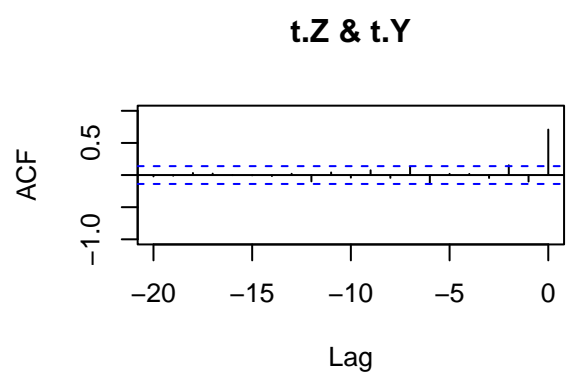
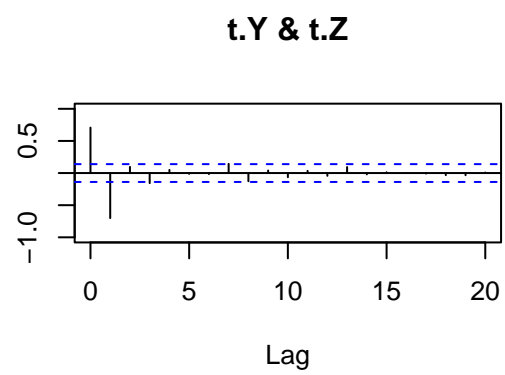
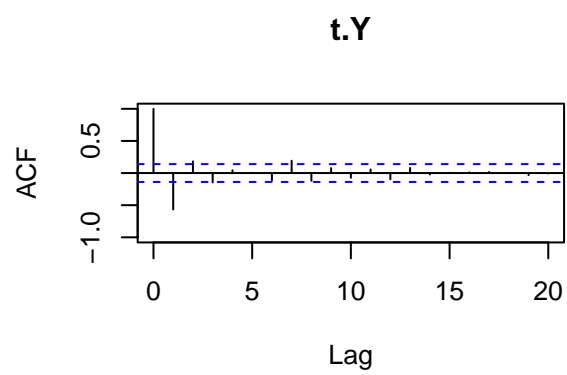
```
t.E <- ts(rnorm(201))
```

```
t.X <- (1:201) + t.E
```

```
t.Y <- diff(t.X)
```

```
t.Z <- t.E
```

```
acf(ts.intersect(t.Y, t.Z), ylim = c(-1, 1))
```



`ccf(t.Y, t.Z)`

t.Y & t.Z

