# Exercise 9

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Exercise 9.1

In this exercise, we look at the time series `prodn`, which is available in the package `astsa`. It contains monthly data about the Federal Reserve Board Production Index from 1948-1978, in total the time series contains data for n = 372 months.
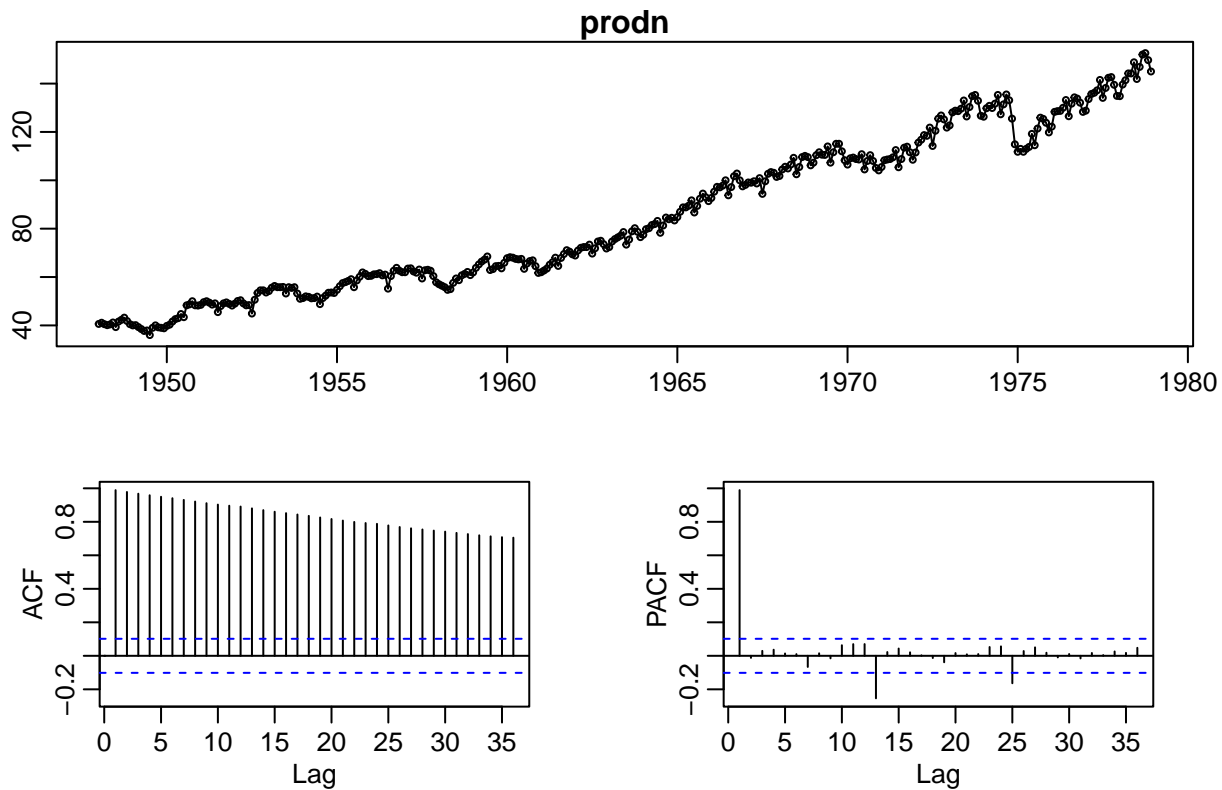
a) Plot the time series. What kind of non-stationarity is evident?

```
library(astsa, quietly = T)
library(forecast, quietly = T)

tsplot(prodn)
```
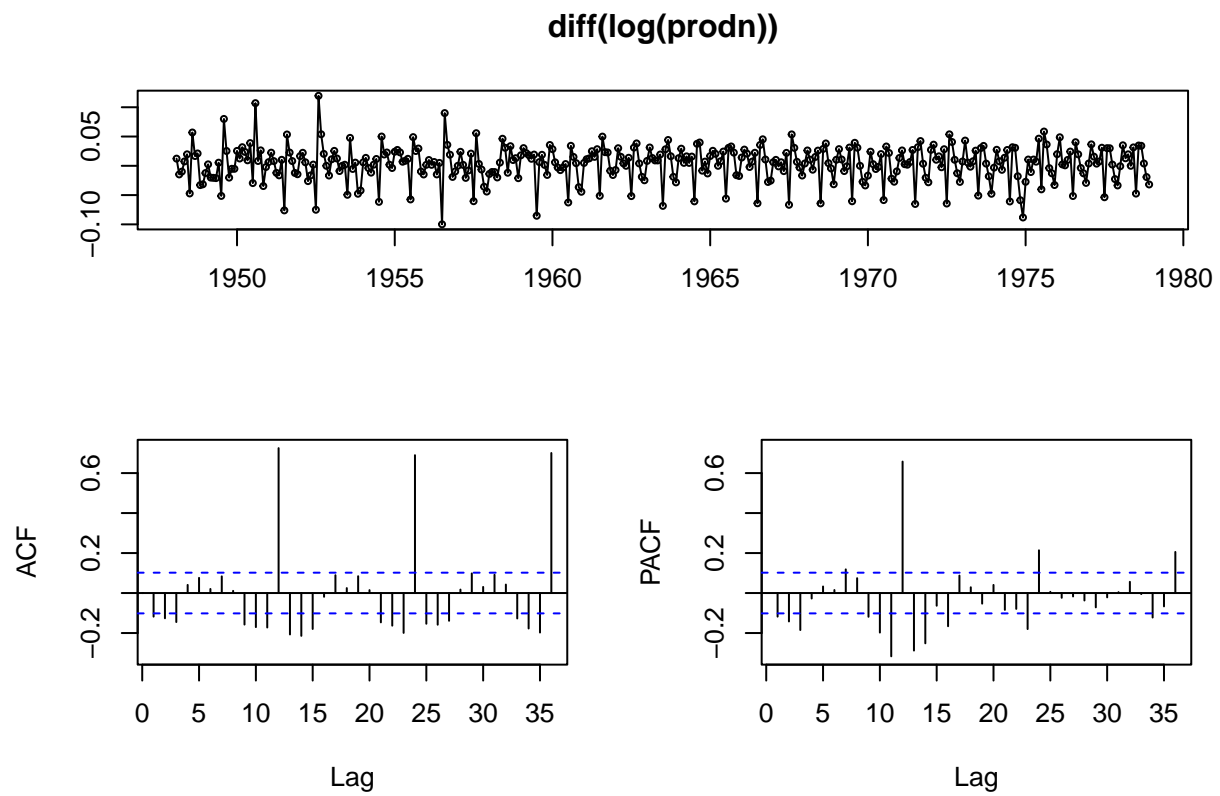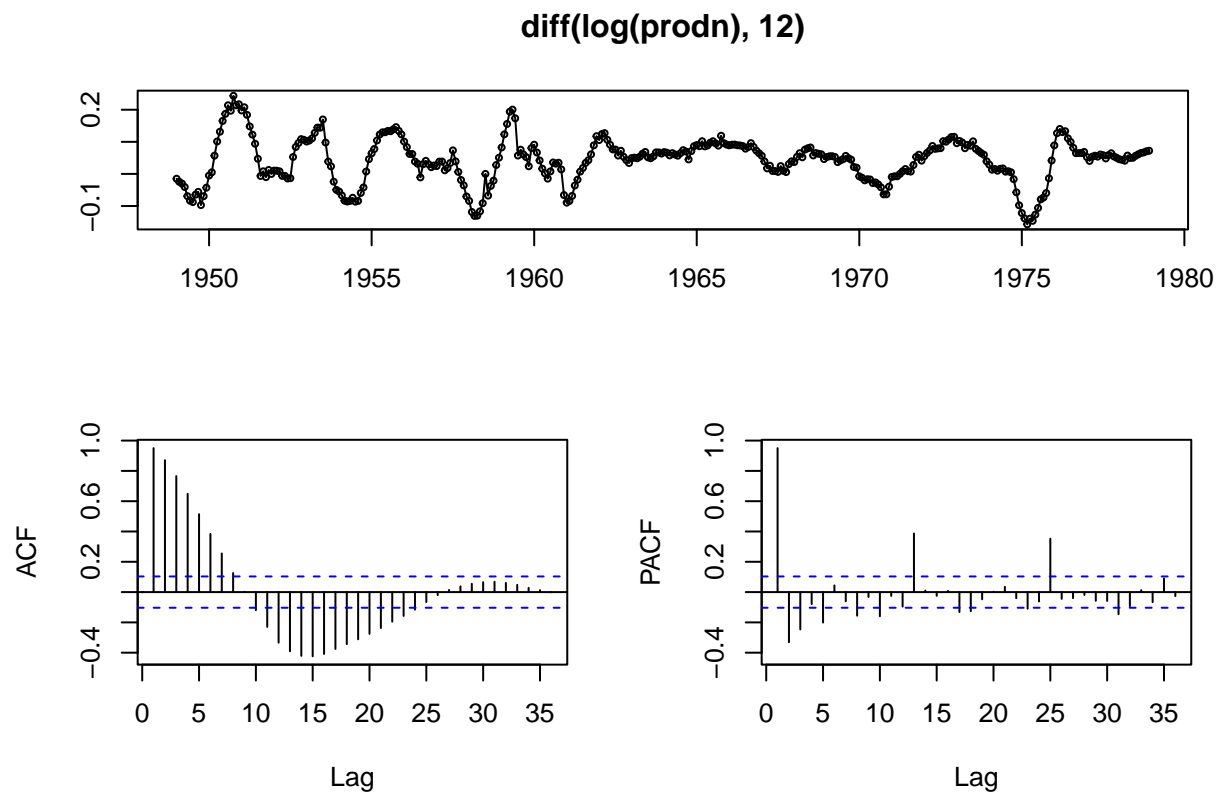


```
tsdisplay(prodn)
```
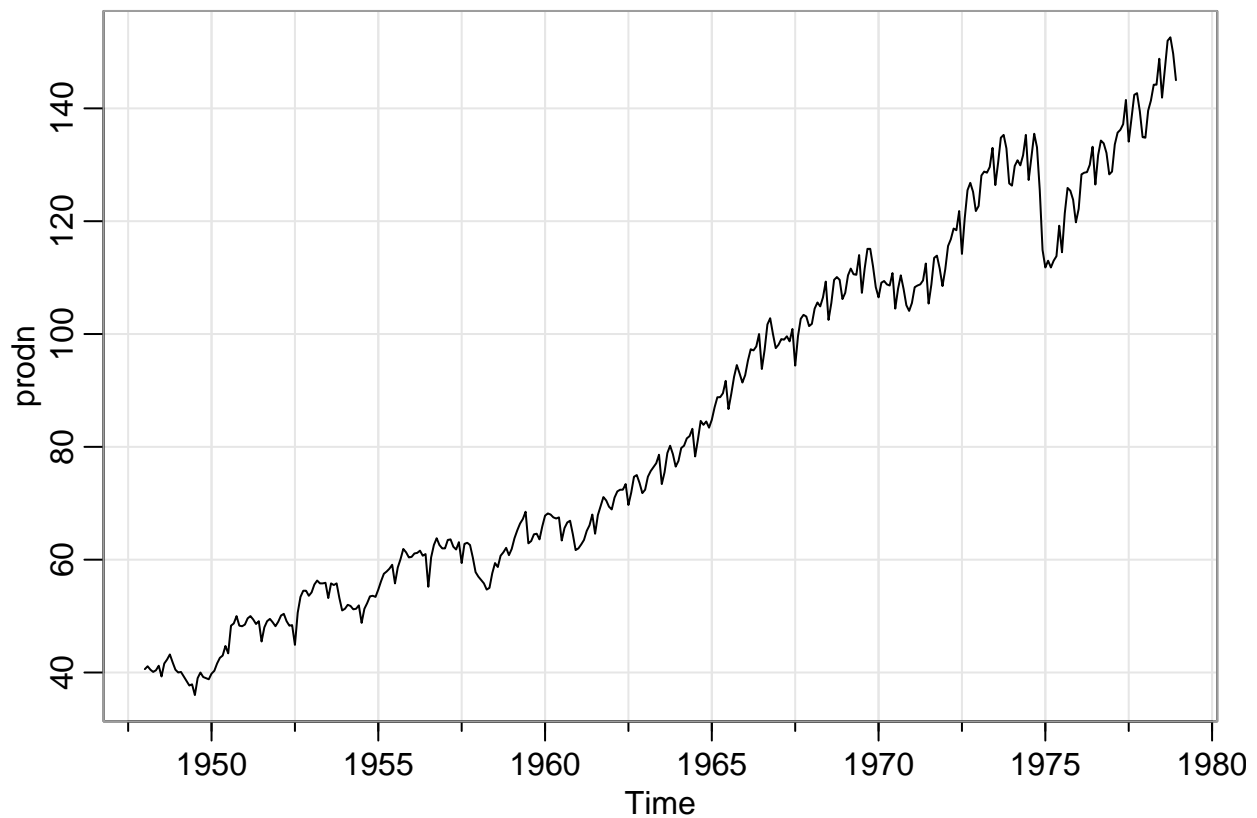
prodn

The ACF shows a slow decay, which implies that there is a trend. If there was also seasonality, the ACF would show an oscillatory behavior, which it does not.
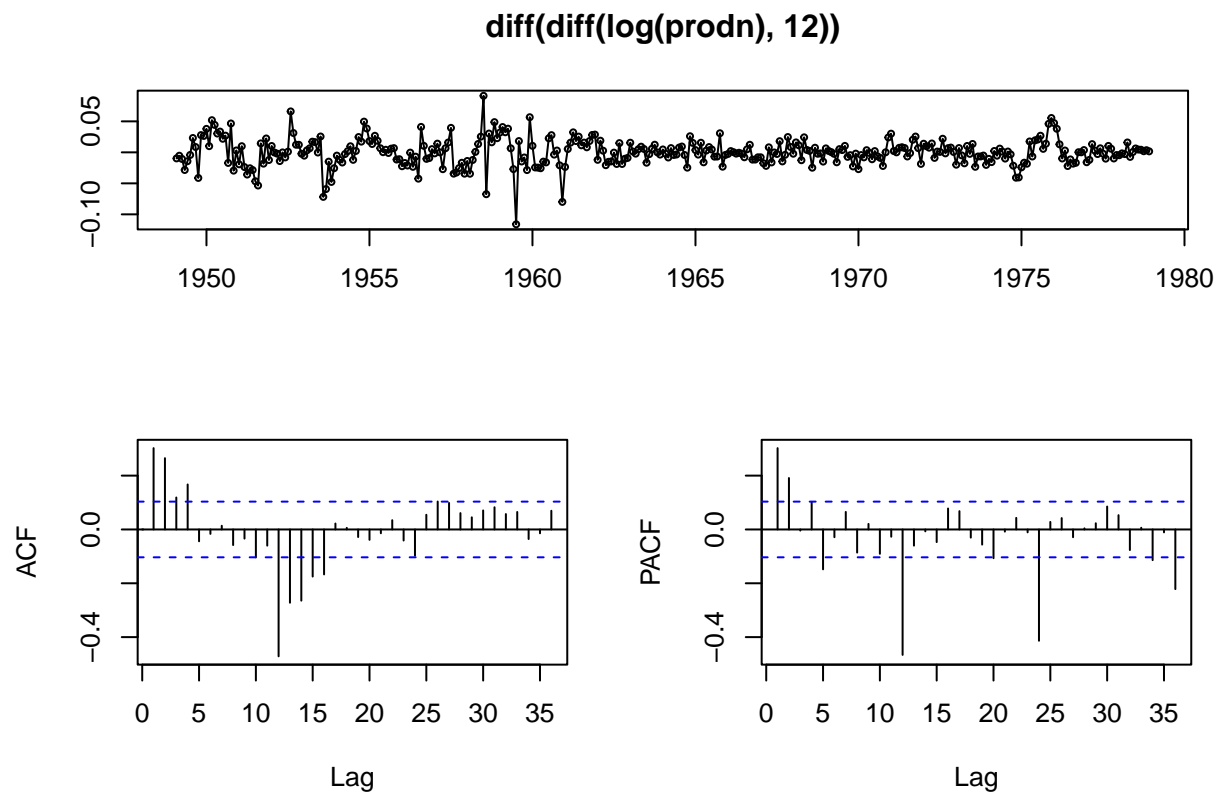
```
tsdisplay(diff(log(prodn)))
```

diff(log(prodn))


```
tsdisplay(diff(log(prodn), 12)) # remove seasonality
```
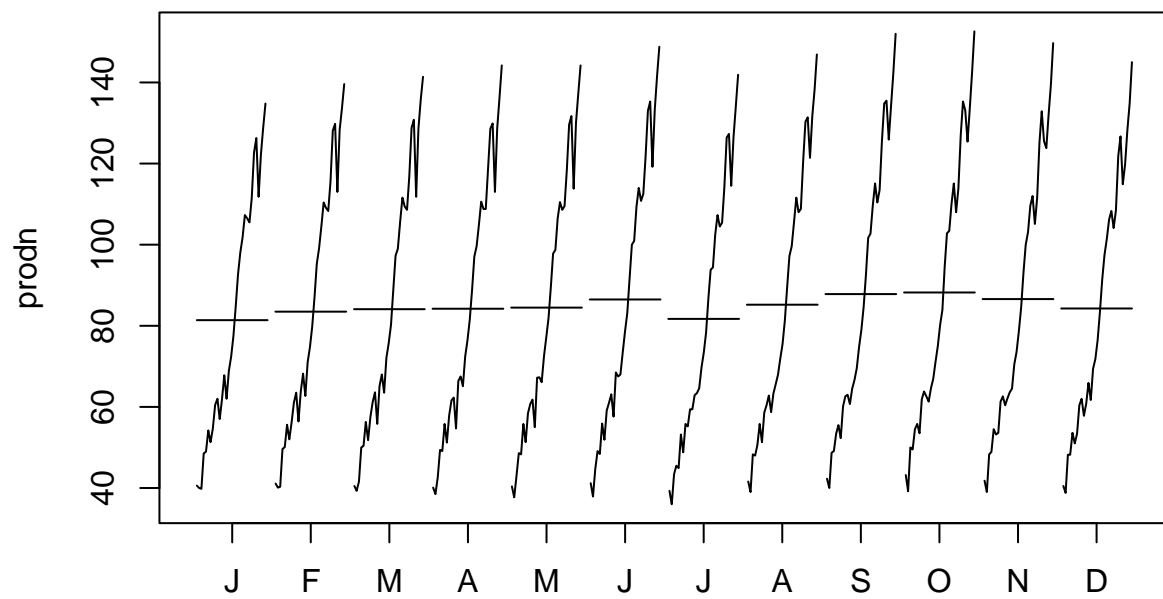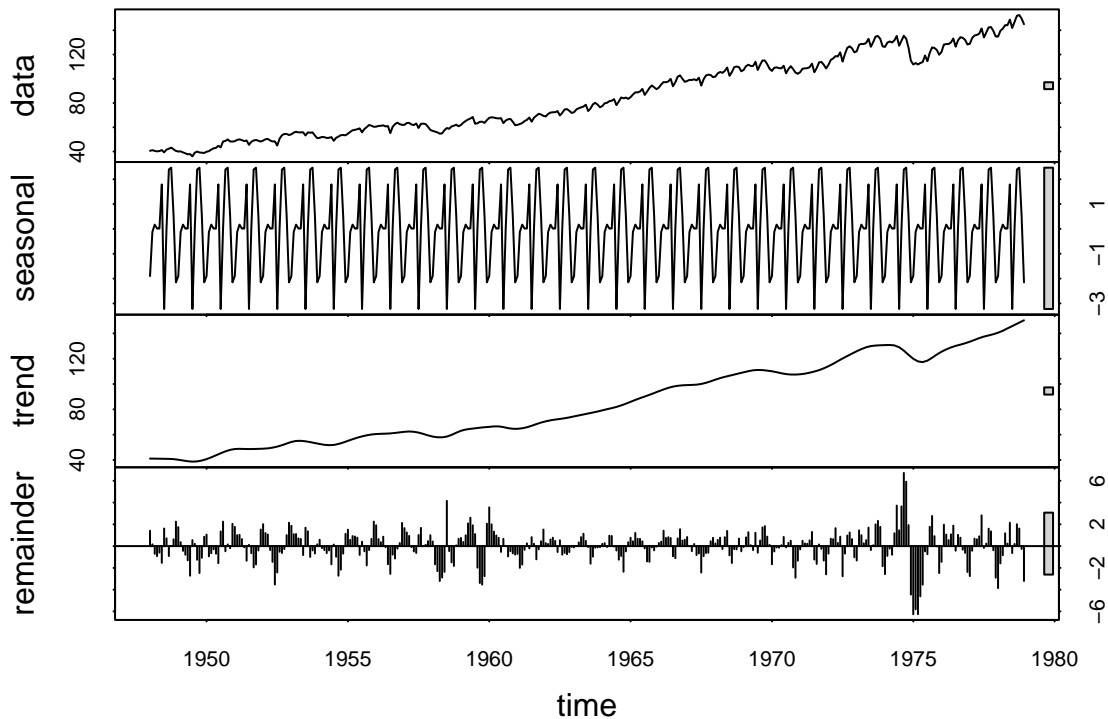
3

diff(log(prodn), 12)


```
tsdisplay(diff(diff(log(prodn), 12))) # remove trend
```

diff(diff(log(prodn), 12))

monthplot(prodn)

```
plot(stl(prodn, s.window = 'periodic'))
```

b) How can the time series be made stationary?

By applying a SARIMA$(1,1,0)(1,1,0)^{12}$ model.

c) Based on your considerations in b), what kind of model would you fit to the original time series `prodn`? Try different fits and choose your favorite.
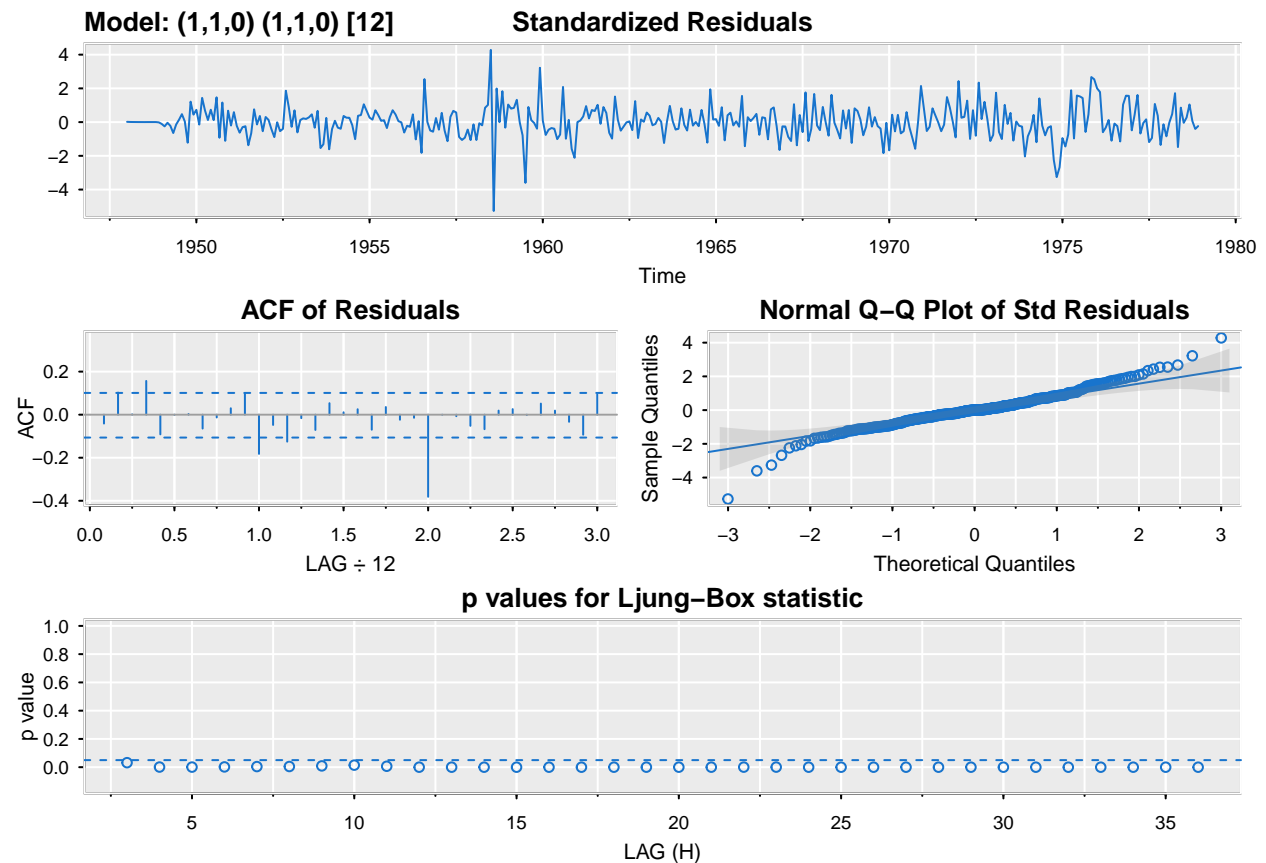
```
fit.110 <- arima(prodn, order = c(1, 1, 0), seasonal = c(1, 1, 0))
fit.110
```

```
##
## Call:
## arima(x = prodn, order = c(1, 1, 0), seasonal = c(1, 1, 0))
##
## Coefficients:
##           ar1      sar1
##        0.3530   -0.4147
## s.e.   0.0493    0.0474
##
## sigma^2 estimated as 1.813:  log likelihood = -617.36,  aic = 1240.72
```

```
sarima(prodn, 1, 1, 0, 1, 1, 0, 12, gg = TRUE, col = 4)
```

```
## initial  value 0.474717
## iter   2 value 0.309458
## iter   3 value 0.309455
## iter   4 value 0.309454
## iter   4 value 0.309454
## iter   4 value 0.309454
```

```
## final  value 0.309454
## converged
## initial  value 0.300777
## iter   2 value 0.300729
## iter   3 value 0.300727
## iter   3 value 0.300727
## iter   3 value 0.300727
## final  value 0.300727
## converged
```


Model: (1,1,0) (1,1,0) [12]   Standardized Residuals

ACF of Residuals

Normal Q–Q Plot of Std Residuals

p values for Ljung–Box statistic

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     include.mean = !no.constant, transform.pars = trans, fixed = fixed, optim.control = list(trace =
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1      sar1
##       0.3530   -0.4147
## s.e.  0.0493    0.0474
##
## sigma^2 estimated as 1.813:  log likelihood = -617.36,  aic = 1240.72
##
## $degrees_of_freedom
## [1] 357
```

```
##
## $ttable
##      Estimate     SE t.value p.value
## ar1    0.3530 0.0493  7.1651       0
## sar1  -0.4147 0.0474 -8.7553       0
##
## $AIC
## [1] 3.456045
##
## $AICc
## [1] 3.456139
##
## $BIC
## [1] 3.488496
```

```r
set.seed(3)
par(mfrow = c(3, 3))
summary(fit.110)
```

```
##
## Call:
## arima(x = prodn, order = c(1, 1, 0), seasonal = c(1, 1, 0))
##
## Coefficients:
##          ar1     sar1
##       0.3530  -0.4147
## s.e.  0.0493   0.0474
##
## sigma^2 estimated as 1.813:  log likelihood = -617.36,  aic = 1240.72
##
## Training set error measures:
##                      ME     RMSE       MAE        MPE     MAPE      MASE
## Training set 0.02312018 1.322627 0.9462347 0.03040184 1.206129 0.4810264
##                     ACF1
## Training set -0.04127911
```
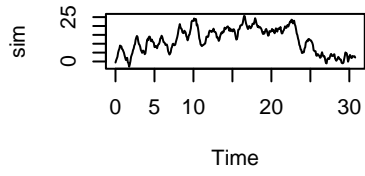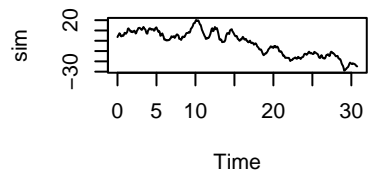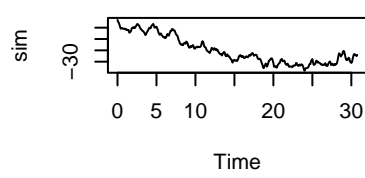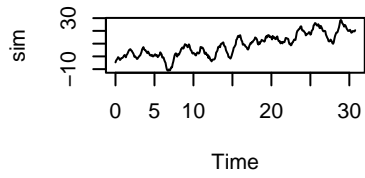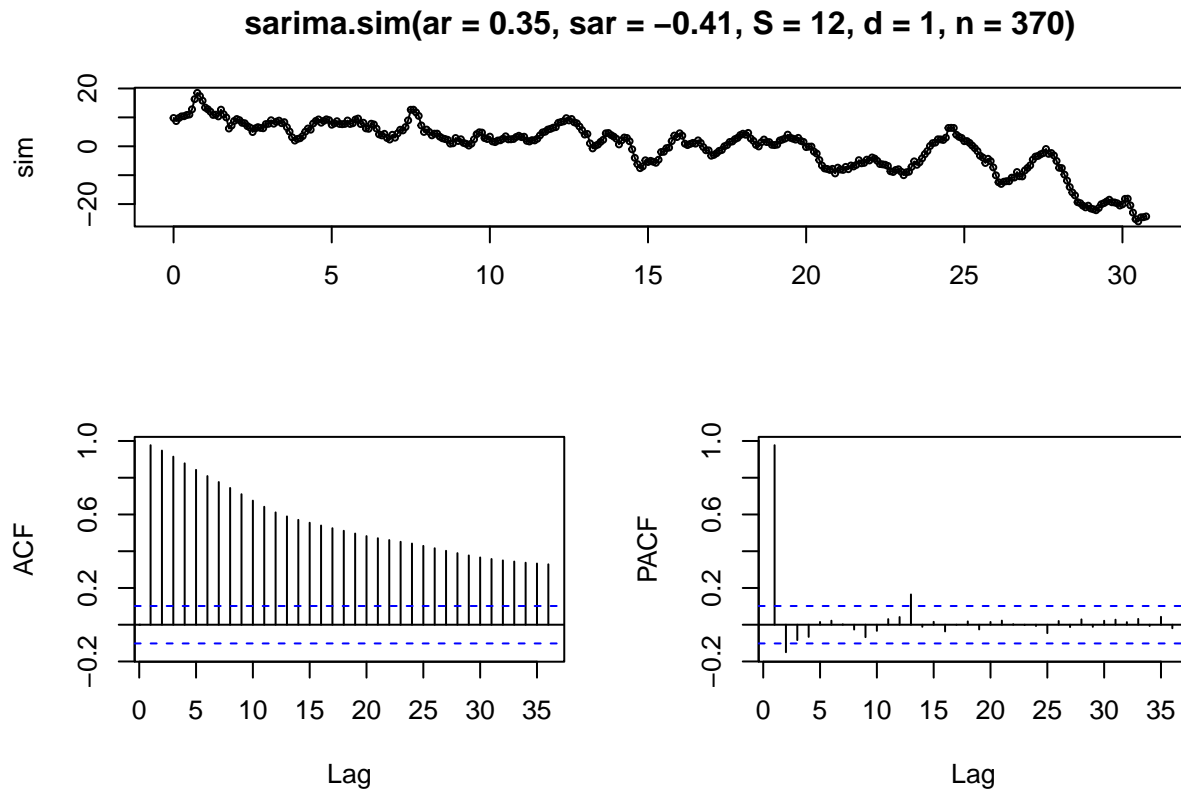
```r
replicate(9, plot(sarima.sim(ar = 0.35, sar = -0.41, S = 12, d = 1, n = 370) , ylab = 'sim'))
```
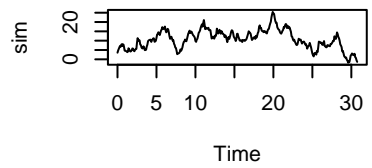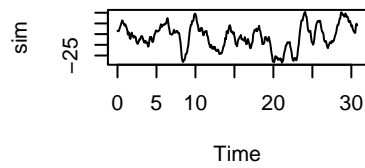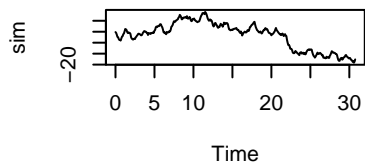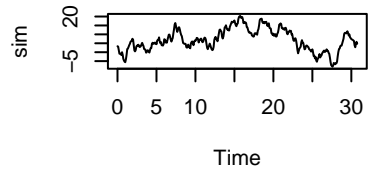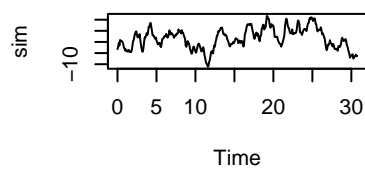
```
## [[1]]
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## NULL
```

```
tsdisplay(sarima.sim(ar = 0.35, sar = -0.41, S = 12, d = 1, n = 370) , ylab = 'sim')
```

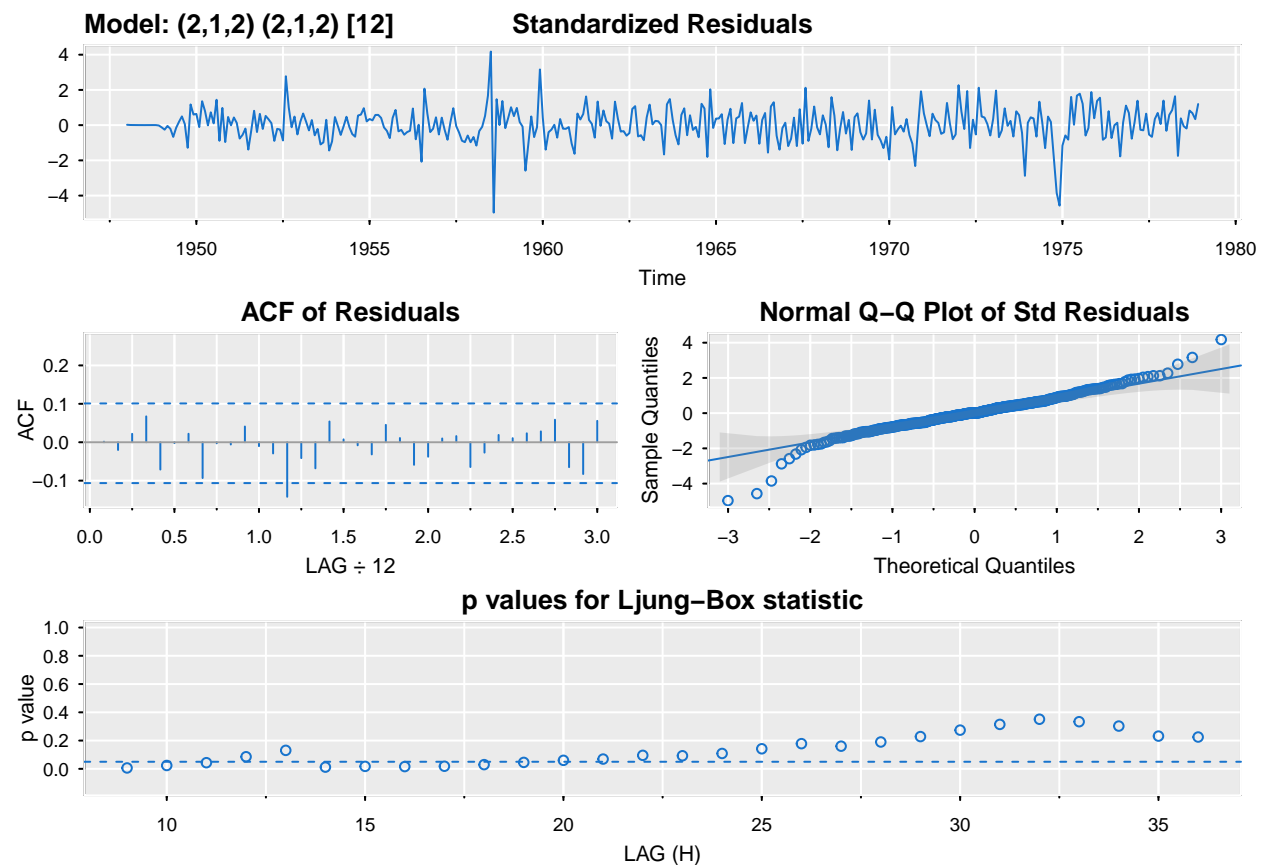**sarima.sim(ar = 0.35, sar = −0.41, S = 12, d = 1, n = 370)**

In most of the simulations, the ACF is clearly different to the original.

```
fit.212 <- sarima(prodn, 2, 1, 2, 2, 1, 2, 12, gg = TRUE, col = 4)
```

```
## initial  value 0.480262
## iter   2 value 0.320770
## iter   3 value 0.201735
## iter   4 value 0.184543
## iter   5 value 0.174271
## iter   6 value 0.173883
## iter   7 value 0.172984
## iter   8 value 0.172916
## iter   9 value 0.172828
## iter  10 value 0.172722
## iter  11 value 0.172533
## iter  12 value 0.172392
## iter  13 value 0.172303
## iter  14 value 0.172242
## iter  15 value 0.172138
## iter  16 value 0.171959
## iter  17 value 0.171758
## iter  18 value 0.171597
## iter  19 value 0.171557
## iter  20 value 0.171491
## iter  21 value 0.171293
```

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```
## iter   22 value 0.171073
## iter   23 value 0.170838
## iter   24 value 0.170713
## iter   25 value 0.170647
## iter   26 value 0.170613
## iter   27 value 0.170547
## iter   28 value 0.170364
## iter   29 value 0.170295
## iter   30 value 0.170278
## iter   31 value 0.170273
## iter   32 value 0.170272
## iter   33 value 0.170271
## iter   34 value 0.170270
## iter   35 value 0.170269
## iter   36 value 0.170269
## iter   37 value 0.170269
## iter   38 value 0.170269
## iter   39 value 0.170269
## iter   40 value 0.170269
## iter   41 value 0.170269
## iter   42 value 0.170269
## iter   43 value 0.170269
## iter   44 value 0.170269
## iter   45 value 0.170269
## iter   46 value 0.170269
## iter   46 value 0.170269
## iter   46 value 0.170269
## final   value 0.170269
## converged
## initial  value 0.163790
## iter    2 value 0.163559
## iter    3 value 0.163436
## iter    4 value 0.163329
## iter    5 value 0.163260
## iter    6 value 0.163041
## iter    7 value 0.162797
## iter    8 value 0.162053
## iter    9 value 0.160725
## iter   10 value 0.159828
## iter   11 value 0.159063
## iter   12 value 0.157449
## iter   13 value 0.156532
## iter   14 value 0.156065
## iter   15 value 0.155528
## iter   16 value 0.155173
## iter   17 value 0.153590
## iter   18 value 0.153470
## iter   19 value 0.153460
## iter   20 value 0.153434
## iter   21 value 0.153426
## iter   22 value 0.153422
## iter   23 value 0.153418
## iter   24 value 0.153412
## iter   25 value 0.153364
```

```
## iter   26 value 0.153353
## iter   27 value 0.153348
## iter   28 value 0.153347
## iter   29 value 0.153340
## iter   30 value 0.153335
## iter   31 value 0.153333
## iter   32 value 0.153333
## iter   33 value 0.153333
## iter   33 value 0.153333
## final  value 0.153333
## converged
```
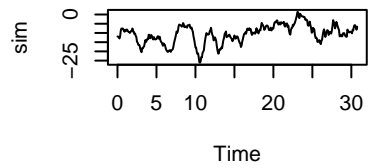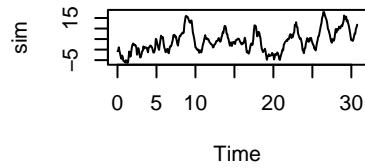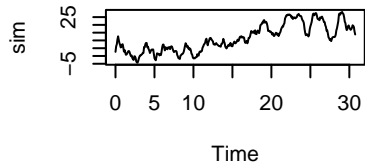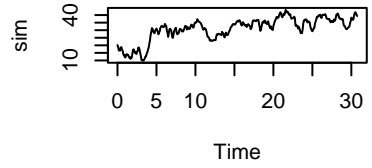

Model: (2,1,2) (2,1,2) [12]   Standardized Residuals

ACF of Residuals

Normal Q–Q Plot of Std Residuals

p values for Ljung–Box statistic

```r
set.seed(2)
par(mfrow = c(3, 3))
fit.212
```

```
## $fit
##
## Call:
## arima(x = xdata, order = c(p, d, q), seasonal = list(order = c(P, D, Q), period = S),
##     include.mean = !no.constant, transform.pars = trans, fixed = fixed, optim.control = list(trace =
##         REPORT = 1, reltol = tol))
##
## Coefficients:
##          ar1     ar2     ma1      ma2    sar1     sar2     sma1    sma2
##      -0.2786  0.3396  0.5851  -0.0522  0.3822  -0.2970  -1.1301  0.4891
```
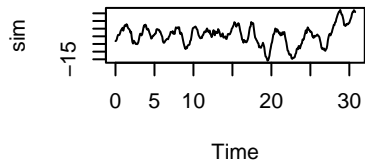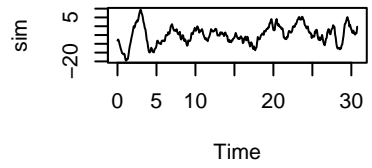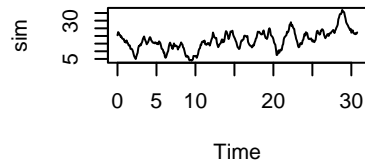
13

```
## s.e.    0.2202   0.1644   0.2331    0.1752   0.1604    0.0748    0.1631   0.1146
##
## sigma^2 estimated as 1.317:  log likelihood = -564.45,  aic = 1146.89
##
## $degrees_of_freedom
## [1] 351
##
## $ttable
##       Estimate      SE t.value p.value
## ar1   -0.2786 0.2202 -1.2649  0.2067
## ar2    0.3396 0.1644  2.0659  0.0396
## ma1    0.5851 0.2331  2.5103  0.0125
## ma2   -0.0522 0.1752 -0.2978  0.7660
## sar1   0.3822 0.1604  2.3823  0.0177
## sar2  -0.2970 0.0748 -3.9698  0.0001
## sma1  -1.1301 0.1631 -6.9299  0.0000
## sma2   0.4891 0.1146  4.2676  0.0000
##
## $AIC
## [1] 3.194682
##
## $AICc
## [1] 3.195828
##
## $BIC
## [1] 3.292036
```

```r
replicate(9, plot(sarima.sim(ar = c(-0.3, 0.3),
                             ma = c(0.6, 0),
                             sar = c(0.4, -0.3),
                             sma = c(-1.1, 0.5),
                             S = 12, d = 1, n = 370) , ylab = 'sim'))
```

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```
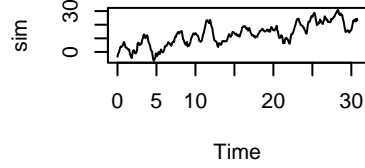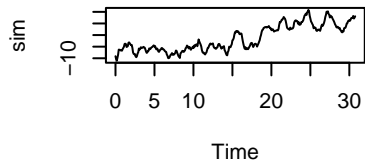
```
tsdisplay(sarima.sim(ar = c(-0.3, 0.3),
                     ma = c(0.6, 0),
                     sar = c(0.4, -0.3),
                     sma = c(-1.1, 0.5),
                     S = 12, d = 1, n = 370) , ylab = 'sim')
```

sarima.sim(ar = c(−0.3, 0.3), ma = c(0.6, 0), sar = c(0.4, −0.3),
sma = c(−1.1, 0.5), S = 12, d = 1, n = 370)