Exercise 5

Philipp Drebes

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Exercise 5.1

Simulations are key to validate models. Thus, we would like to use this exercise to simulate several time series by means of an ARMA model.

The innovation E_t shall follow a standard normal distribution $\mathcal{N}(0; 1)$ in every model.

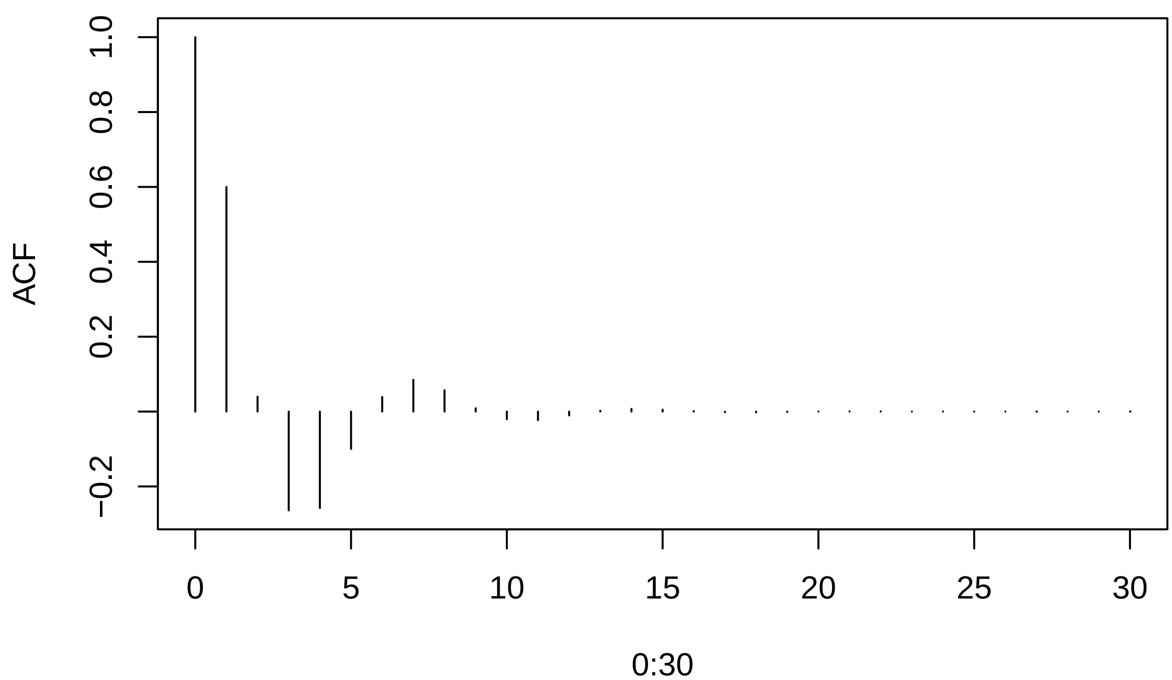
a. **AR(2) model with coefficients $\alpha_1 = 0.9$ and $\alpha_2 = -0.5$.**

First think of how the autocorrelations should behave theoretically.

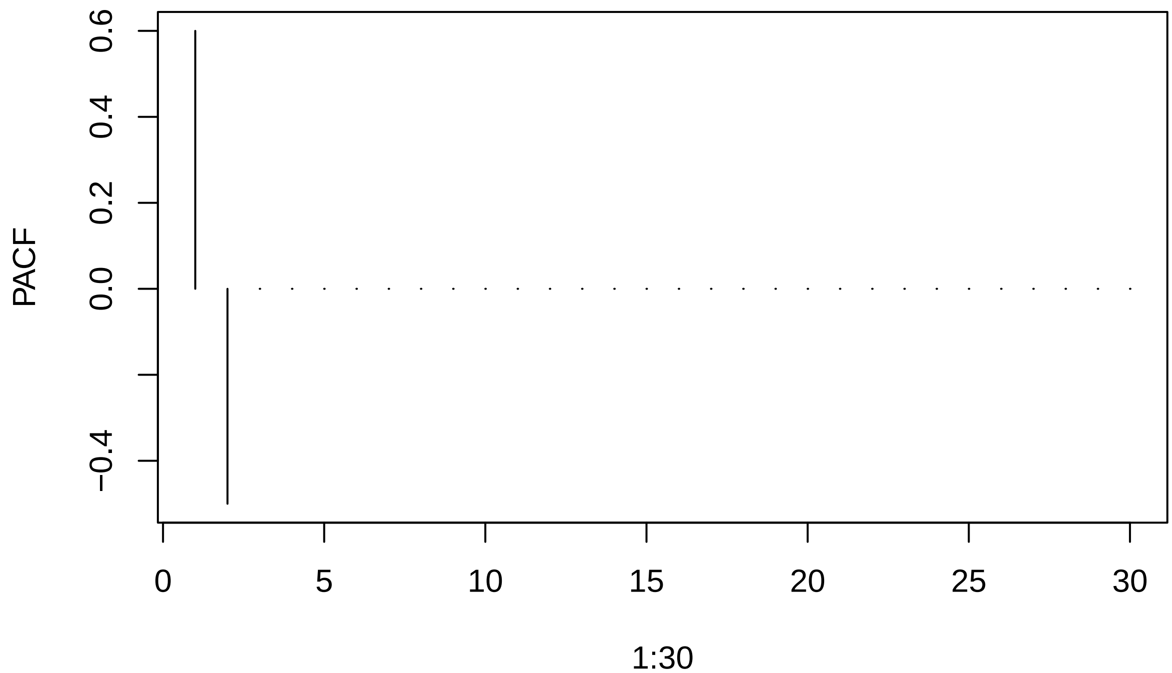
The partial autocorrelation should have no significant values after lag 2.

Use the procedure ARMAacf() to compute the theoretical autocorrelations of the models and plot them.

```
plot(0:30, ARMAacf(ar = c(0.9, -0.5), lag.max = 30),type = "h", ylab = "ACF")
```



```r
plot(1:30, ARMAacf(ar = c(0.9, -0.5), lag.max = 30, pacf = TRUE), type = "h", ylab = "PACF")
```

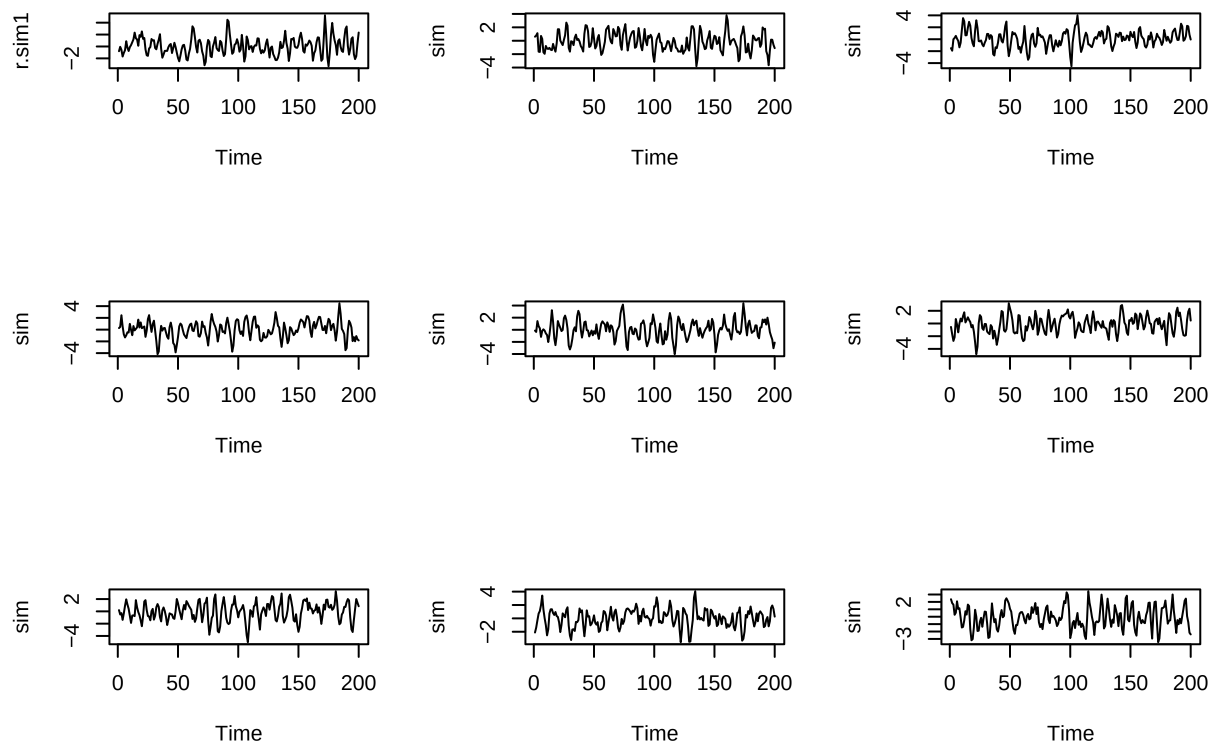


Now simulate a realization of length n = 200 for the models. Repeat each simulation several times to develop some intuition on what occurs by chance and what is structure.

```r
set.seed(989898)
par(mfrow=c(3,3))

r.sim1 <- arima.sim(n = 200, model = list(ar = c(0.9, -0.5)))
plot(r.sim1)

replicate(8, plot(arima.sim(n = 200, model = list(ar = c(0.9, -0.5))), ylab = 'sim'))
```




```
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```
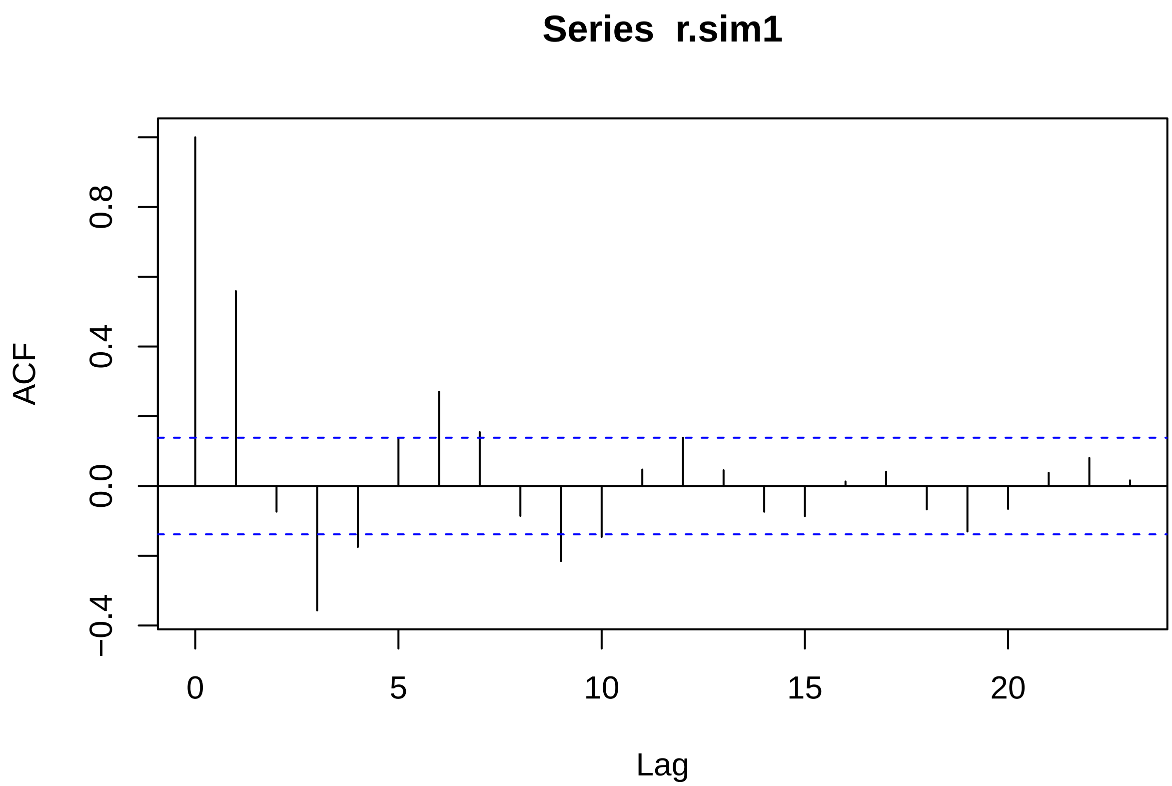
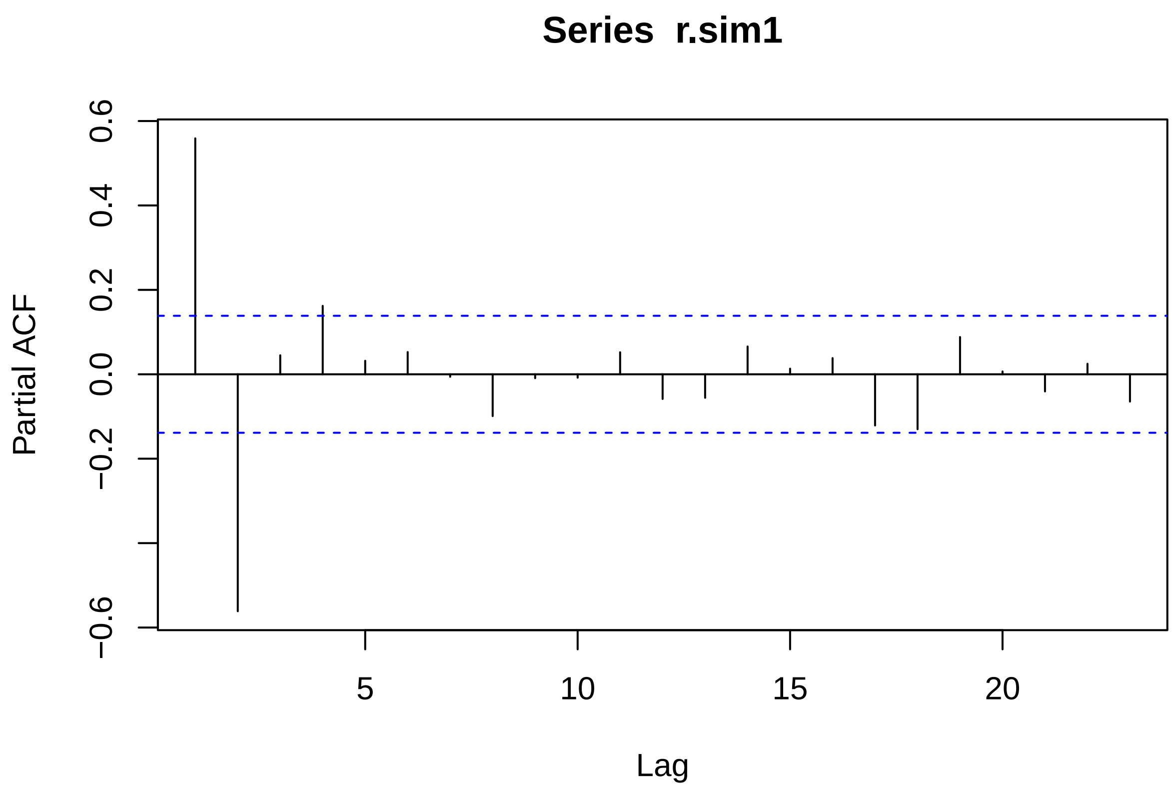

Inspect the time series plot and the correlograms with the ordinary and partial autocorrelations.

```
acf(r.sim1)
```




```
pacf(r.sim1)
```



The autocorrelation function hints at a seasonality. As expected the PACF has no more significant values after lag 2.

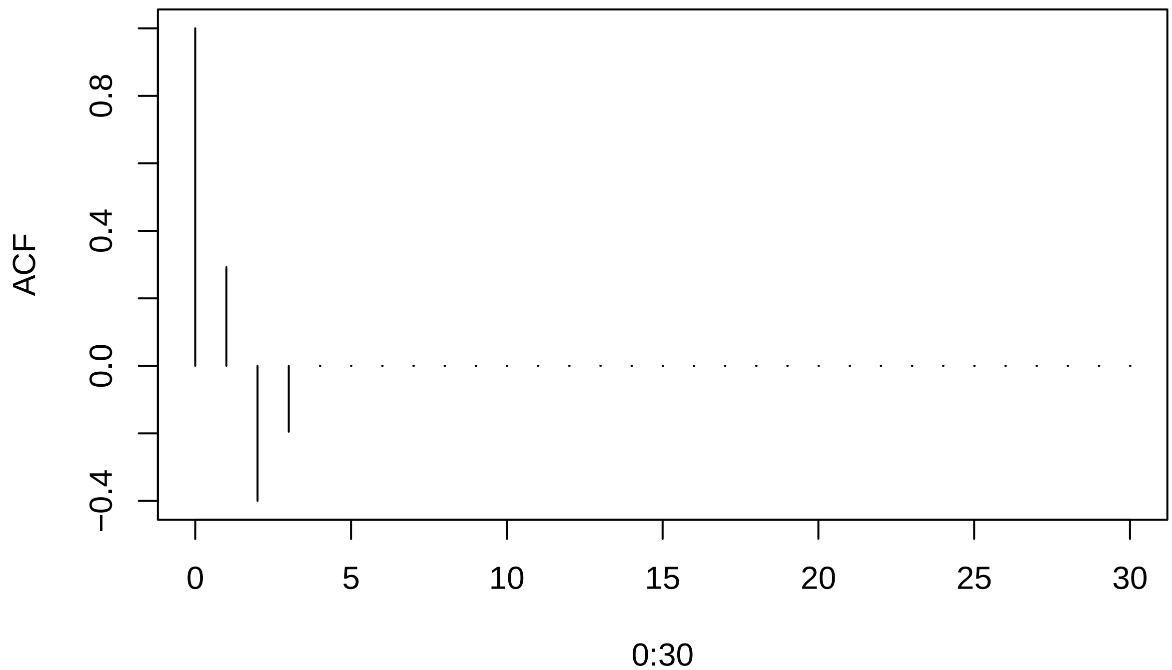
b. **MA(3) model with coefficients** $\beta_1 = 0.8$**,** $\beta_2 = -0.5$ **and** $\beta_3 = -0.4$**.**

First think of how the autocorrelations should behave theoretically.

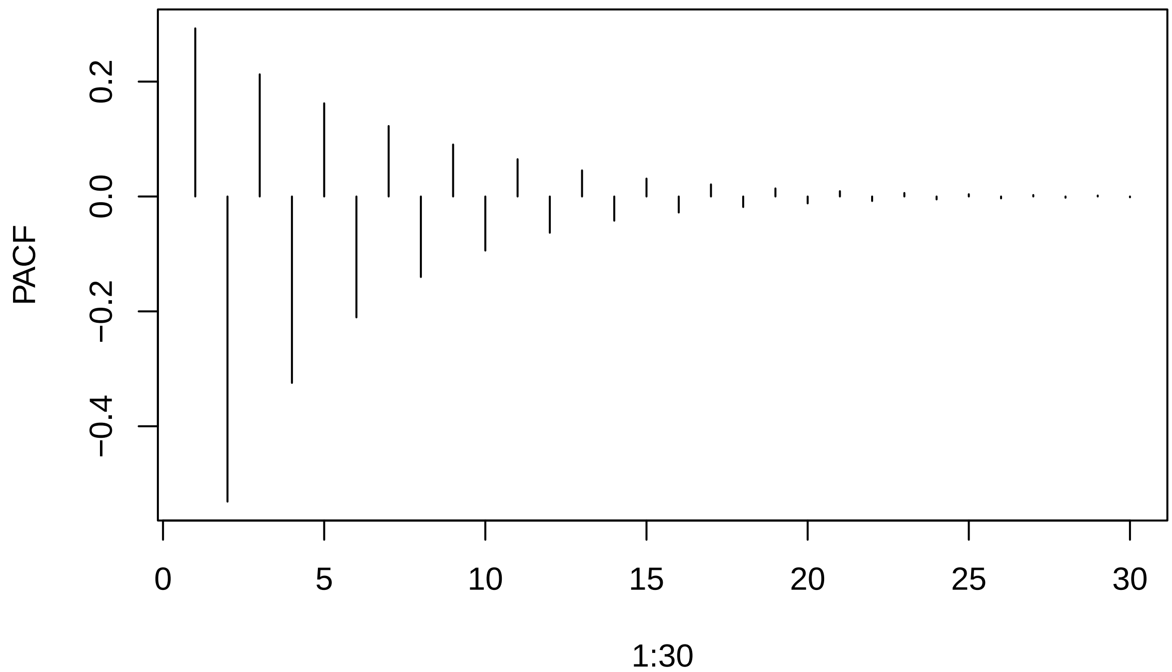
In the ACF, there should be 3 statistically significant "spikes" at lags 1, 2 and 3 followed by non-significant values for other lags.

Use the procedure ARMAacf() to compute the theoretical autocorrelations of the models and plot them.

```
plot(0:30, ARMAacf(ma = c(0.8, -0.5, -0.4), lag.max = 30),type = "h", ylab = "ACF")
```




```r
plot(1:30, ARMAacf(ma = c(0.8, -0.5, -0.4), lag.max = 30, pacf = TRUE), type = "h", ylab = "PACF")
```

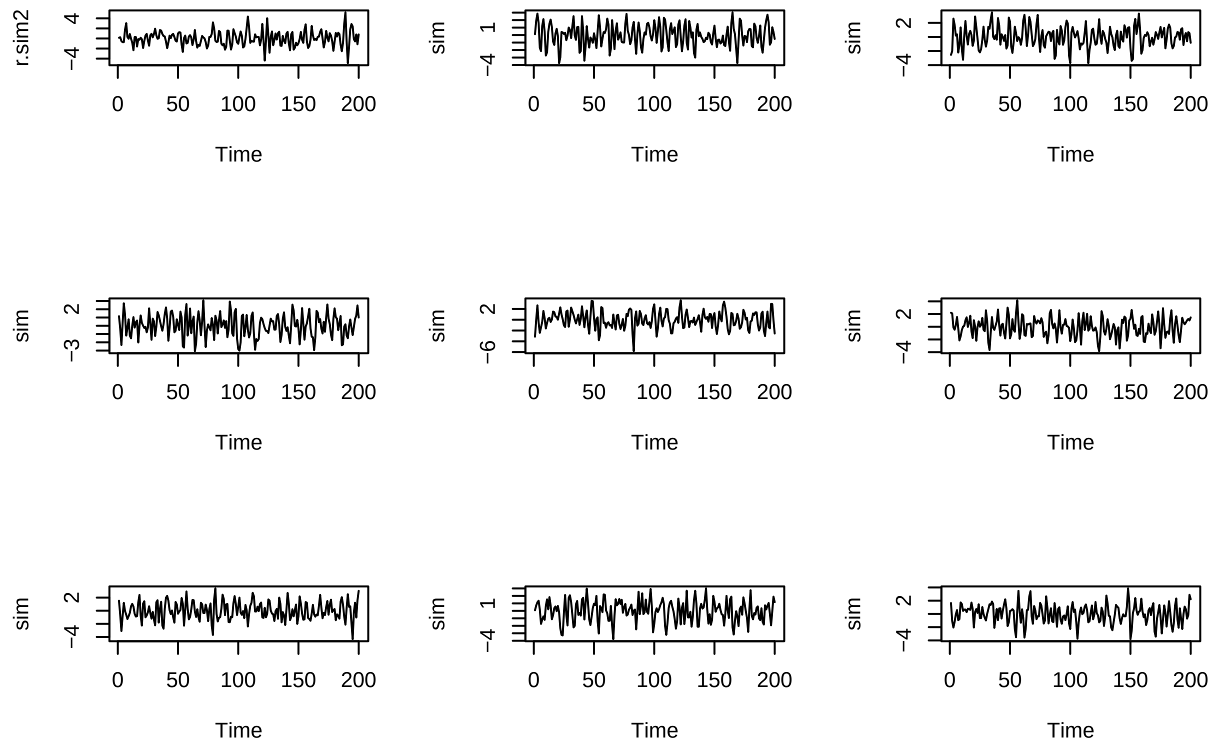


Now simulate a realisation of length n = 200 for the models. Repeat each simulation several times to develop some intuition on what occurs by chance and what is structure.

```
set.seed(989898)
par(mfrow=c(3,3))

r.sim2 <- arima.sim(n = 200, model = list(ma = c(0.8, -0.5, -0.4)))
plot(r.sim2)

replicate(8, plot(arima.sim(n = 200, model = list(ma = c(0.8, -0.5, -0.4))), ylab = 'sim'))
```

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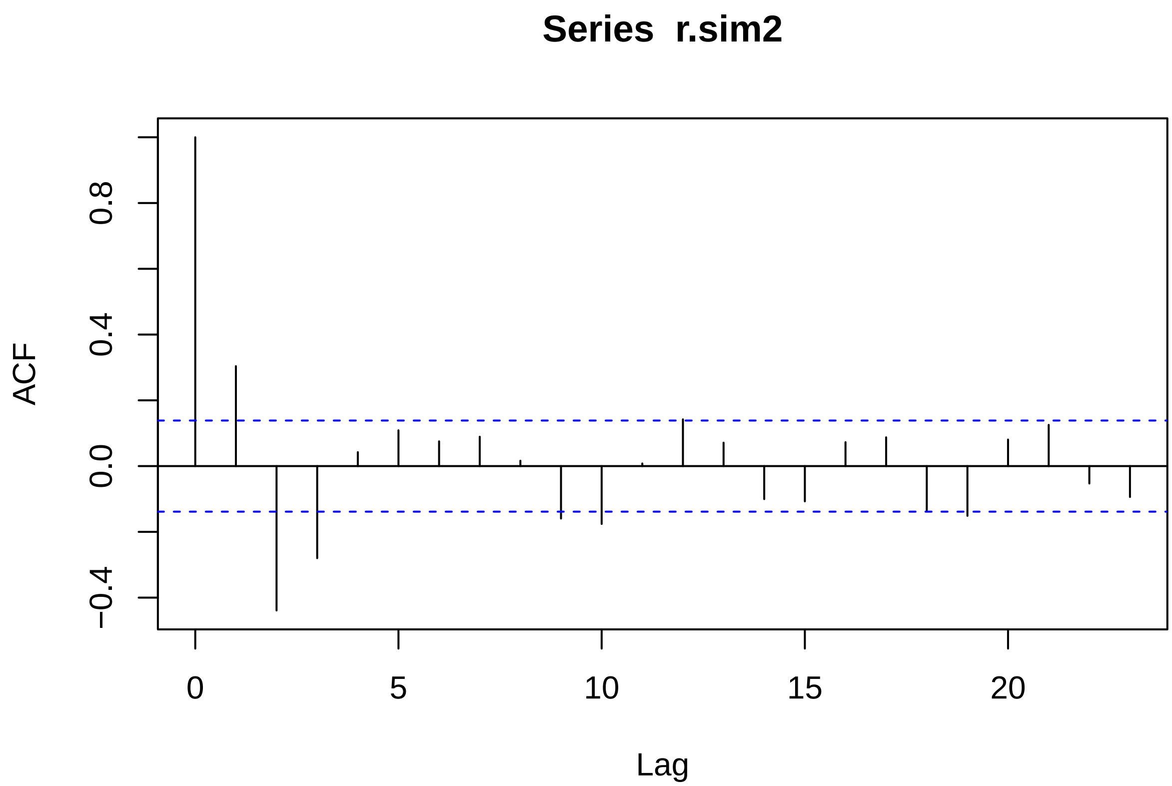
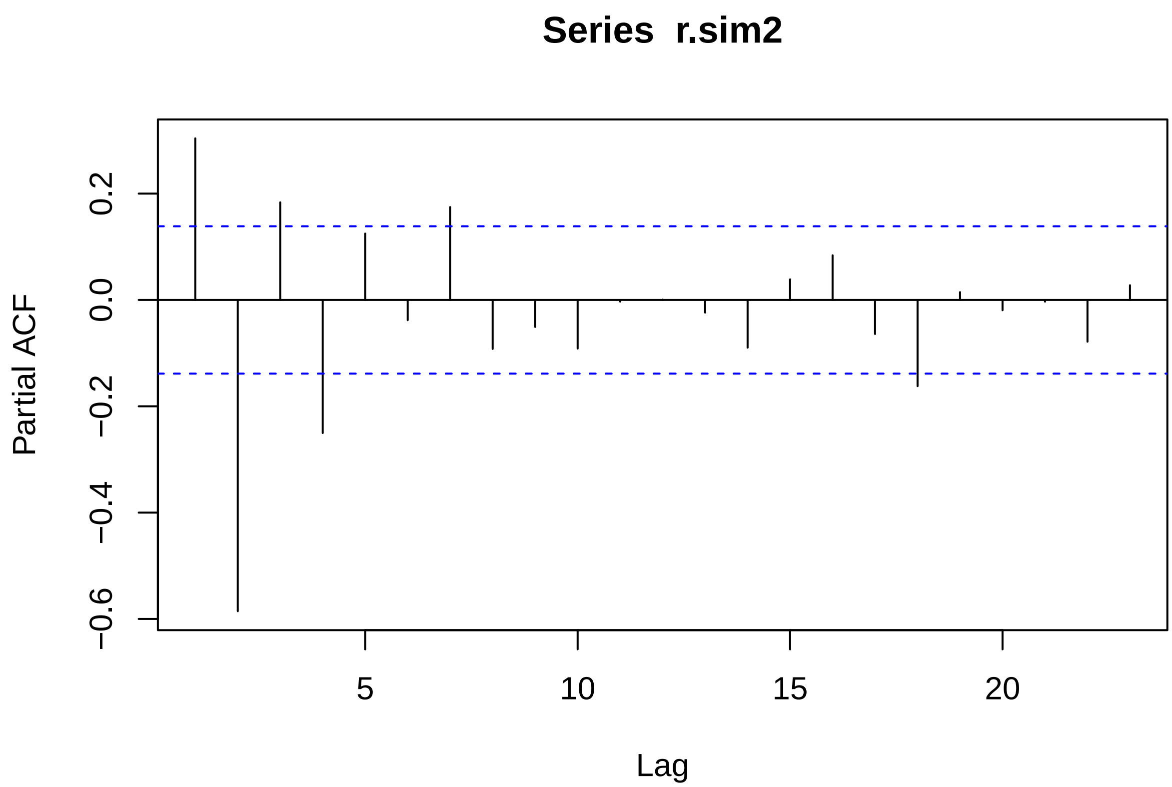
## [[8]]
## NULL
```

Inspect the time series plot and the correlograms with the ordinary and partial autocorrelations.

```
acf(r.sim2)
```




```
pacf(r.sim2)
```



The autocorrelation function of the simulated data is very similar to the theoretical. The partial autocorrelation function shows a similar pattern as the theoretical, however with not as many significant spikes.

Exercise 5.2

In this exercise we consider some examples of AR(p) models and check their stationarity.

a. **Test the models with the innovation E_t on stationarity with the help of the R function polyroot.**

i) $X_t = 0.5X_{t-1} + 2X_{t-2} + E_t$

```
polyroot(c(1, -0.5, -2))
```

```
## [1]  0.5930703-0i -0.8430703+0i
```

As the first value has an absolute value below 1, the time series is not stationary.

ii) $Y_t = Y_{t-1} + E_t$

```
polyroot(c(1, -1))
```

```
## [1] 1+0i
```

As the first value has an absolute value of 1, the time series is stationary.

b. **For which value of the coefficient α_2 of X_{t-2} is the model $X_t = 0.5 \cdot X_{t-1} + \alpha_2 \cdot X_{t-2} + E_t$ stationary?**

```r
for (i in seq(from = -10, to = 10, by = 0.01)) {
  root <- Re(polyroot(c(1, -0.5, i))[1])
  if (root >= 1) {
    print(i)
    break
  }
}
```

```
## [1] -0.5
```

Check:

```r
polyroot(c(1, -0.5, -0.5))
```

```
## [1]  1-0i -2+0i
```

For $\alpha_2 = 0.5$, the time series is stationary.

c. **Why is the model $Y_t = \alpha \cdot Y_{t-1} + E_t$ not stationary for $|\alpha| \geq 1$? Calculate the characteristic function and determine its roots to confirm this observation.**

```r
for (i in seq(from = -0.5, to = -1.5, by = -0.1)) {
  print(polyroot(c(1, i)))
}
```

```
## [1] 2+0i
## [1] 1.666667+0i
## [1] 1.428571+0i
## [1] 1.25+0i
## [1] 1.111111+0i
## [1] 1+0i
## [1] 0.9090909+0i
## [1] 0.8333333+0i
## [1] 0.7692308+0i
## [1] 0.7142857+0i
## [1] 0.6666667+0i
```