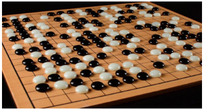
Reinforcement learning

Philippe Preux
philippe.preux@univ-lille.fr
SCOOL

This presentation:
`https://philippe-preux.github.io/talks/AISS-Insa-Rouen/AISS.pdf`

Based on my *Reinforcement Learning lecture notes*, in French only:
`https://philippe-preux.github.io/Documents/digest-ar.pdf`.

Reinforcement learning



The roots of reinforcement learning

Roots of RL in psychology: the "scientific study of behavior":

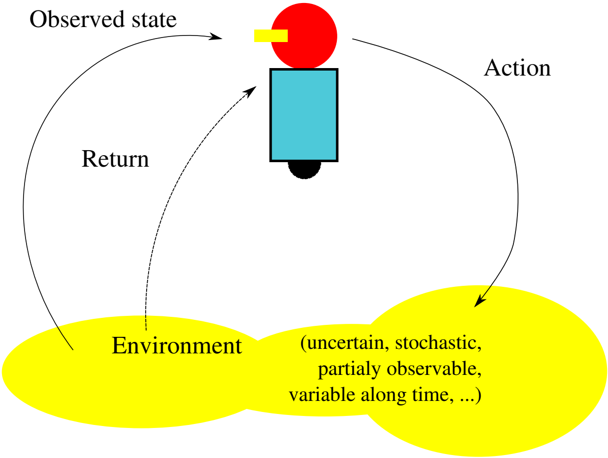
- Law of effect: Thorndike, 1898.
- Classical conditioning: Pavlov, 1903, 1927
- Operant conditioning: Skinner *et al.* from 1931 on.
  Key idea: behaviors are selected by their consequences.
  (selection of behavior akin selection of species.)
- Rescorla-Wagner law: 1972.

- Sutton's Ph.D. defended in Feb. 1984.

Outline

- Introduction
- Markov decision processes and Markov decision problems
- Reinforcement learning: definition and algorithms
- RL in practice

Markov decision problems



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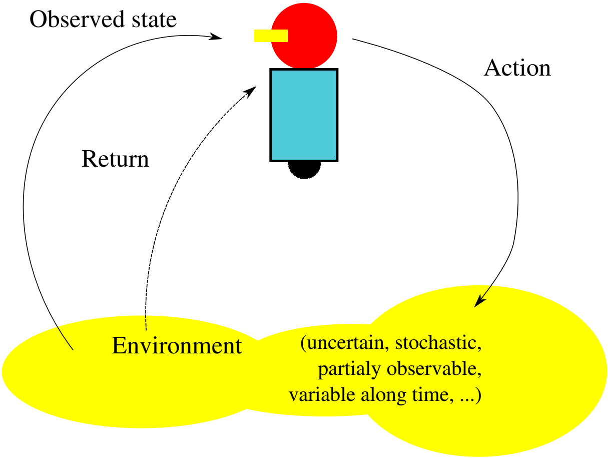

Learn an optimal behavior.

Markov decision process

Markov decision process

describes a dynamical decision system

Definition

A Markov decision process is defined by the tuple $(\mathcal{T}, \mathcal{X}, \mathcal{X}_0, \mathcal{X}_f, \mathcal{A}, \mathcal{P}, \mathcal{R})$ where:

- $\mathcal{T}$ is the set of instants of decision, $t \in \mathcal{T}$.
  For the sake of simplicity, $\mathcal{T}$ is usually the sequence of positive integers: $0, 1, 2, ...$

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- $\mathcal{A}$ is a finite set of actions, $a \in \mathcal{A}$.
- $\mathcal{P}$ is the transition function.
  Assume at time t, the environment is in state $x_t = x$ and performs action $a_t = a$, then $\mathcal{P}(x, a, x')$ is the probability that the environment will be in state x' at time of decision $t + 1$.
  That is: $\mathcal{P}(x, a, x') = Pr[x_{t+1} = x' | x_t = x, a_t = a]$.

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- $\mathcal{P}$ is the transition function.
- $\mathcal{R}$ is the return function, $r \in \mathbb{R}$.
  With the same assumption as for $\mathcal{P}$: $\mathcal{R}(x, a, x')$ is the expected return for the transition from state x to x' following action a.
  That is: $\mathcal{R}(x, a, x') = \mathbb{E}[r_t | x_{t+1} = x', x_t = x, a_t = a]$.

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1. items depend on t and do not depend on $t-1$, $t-2$, ...: Markov property.

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2. None of these items depend on $\mathcal{T}$: stationary system ($=$ non autonomous).

Markov decision process
The decision loop

$t \leftarrow 0$
loop
  observe state x_t
  take action a_t
  observe the immediate return r_t
  $t \leftarrow t + 1$
end loop

Why would we take an action or an other? What's the point?

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An MD process only specifies the dynamics of a system that takes decision: it describes the "how", not the "why".

Markov decision process
Example

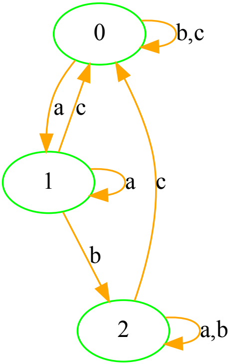
Let's play "21 with a dice".

Rules:

- you need 1 "standard" dice with 6 faces numbered from 1 to 6.
- You roll the dice, and note the value on its upper face: that's your initial score.
- Now repeatedly, you decide whether you roll it again or you stop the game.
- If you roll the dice, you add the value on its upper face to your current score.

Define a Markov decision process that models this dynamical system.

Running example: my little MDP



a, b, and c are actions.
Transitions are deterministic.
All transitions return 0 except the transition from 2 to 0 that returns 1.

Markov decision problem

The “why”.

Markov decision problem

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- Solution of an MD problem: a policy π that specifies the probability to perform any action a in any state x in order to optimize ζ.
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  $\pi(x, a) = Pr[a_t = a | x_t = x], \forall a \in \mathcal{A}$.
- Blackwell's theorem tells us that an optimal policy (for this ζ) is determinitic: $\pi(x)$.
  More than 1 action may be optimal is a state.

Markov decision problem
Remarks about the definition

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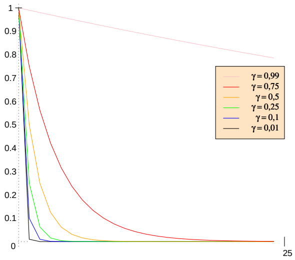

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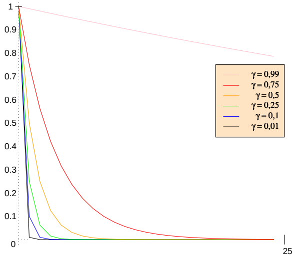

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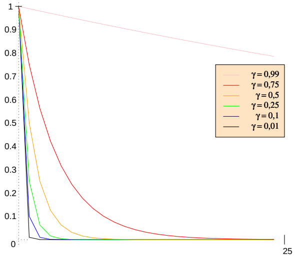

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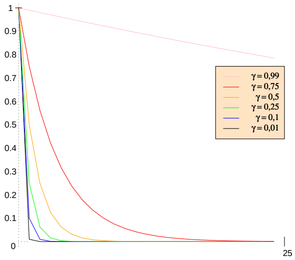

- Nice property: If we assume $\mathcal{R}$ is bounded, then ζ converges.
- This definition makes the maths easier.
- Short term consequences vs. longer consequences of actions.

Markov decision problem

A Markov decision process defines the "how": how does the system evolve in time?

A Markov decision problem defines the "why": why would the agent choose one or an other action?
Answer: to maximize ζ.

Pending questions:

- Does this problem have a solution?
- Can we compute it? In practice?
- How?

Markov decision problem

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- Let $x_0 \in \mathcal{X}_0$.
- Two episodes starting in x_0 will usually follow different trajectories.
- $\zeta(x), x \in \mathcal{X}_0$ is a random variable.
- We define $V^\pi(x)$, the value of a state x according to a policy π by:

  $$V^\pi(x) \stackrel{\text{def}}{=} \mathbb{E}[\zeta(x) | x_0 = x, a_{t\geq 0} \sim \pi]$$

  the actions being chosen according to policy π.

Markov decision problem

The value function: intuition

$$V^\pi(x) \stackrel{\text{def}}{=} \mathbb{E}[\zeta(x)|x_0 = x, a_{t\geq 0} \sim \pi]$$

$V^\pi(x)$ simply quantifies how good it is to be in state x while behaving according to policy π in order to optimize ζ.

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We will soon see that given V^π, we can improve the policy and obtain a policy which value is better.

Markov decision problem

Evaluation of the value of a policy

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express the expectation
and you get:

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$$V^\pi(x) = \sum_{a\in\mathcal{A}} \pi(x, a) \sum_{x'\in\mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^\pi(x')]$$

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This may look complicated but all terms are known except the vector V.

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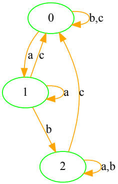
Development $\rightarrow$ a system of N linear equations with N unknowns, the $V^\pi(x), \forall x \in \mathcal{X}$.

Markov decision problem

Example (continued)



For this MDP , compute the value function of the uniformly random policy, $\pi(x, a) = 1/3$, for $\gamma = 0.9$.

System of linear equations?

Reminder:

$V^{\pi}(x) = \sum_{a \in \mathcal{A}} \pi(x, a) \sum_{x' \in \mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^{\pi}(x')]$

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$$
\begin{aligned}
V^\pi(0) = \quad & \pi(0, a) \sum_{x'} \mathcal{P}(0, a, x')[\mathcal{R}(0, a, x') + \gamma V^\pi(x')] + \\
& \pi(0, b) \sum_{x'} \mathcal{P}(0, b, x')[\mathcal{R}(0, b, x') + \gamma V^\pi(x')] + \\
& \pi(0, c) \sum_{x'} \mathcal{P}(0, c, x')[\mathcal{R}(0, c, x') + \gamma V^\pi(x')] \\
V^\pi(1) = \quad & \pi(1, a) \sum_{x'} \mathcal{P}(1, a, x')[\mathcal{R}(1, a, x') + \gamma V^\pi(x')] + \ldots \\
V^\pi(2) = \quad & \pi(2, a) \sum_{x'} \mathcal{P}(2, a, x')[\mathcal{R}(2, a, x') + \gamma V^\pi(x')] + \ldots
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Markov decision problem

Example (continued)

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$\rightarrow$

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& \mathcal{P}(0,a,2)[\mathcal{R}(0,a,2) + \gamma V^\pi(2)]) + \\
& \pi(0,b)(\mathcal{P}(0,b,0)[\mathcal{R}(0,b,0) + \gamma V^\pi(0)] + \\
& \mathcal{P}(0,b,1)[\mathcal{R}(0,b,1) + \gamma V^\pi(1)] + \\
& \mathcal{P}(0,b,2)[\mathcal{R}(0,b,2) + \gamma V^\pi(2)]) + \\
& \pi(0,c)(\mathcal{P}(0,c,0)[\mathcal{R}(0,c,0) + \gamma V^\pi(0)] + \\
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V^\pi(1) = \quad & \ldots \\
V^\pi(2) = \quad & \ldots
\end{aligned}
$$

$\rightarrow$

$$
\begin{aligned}
V^\pi(0) = \quad & 1/3(0 \times [\ldots] + \\
& 1 \times [0 + \gamma V^\pi(1)] + \\
& 0 \times [\ldots]) + \\
& 1/3(1 \times [0 + \gamma V^\pi(0)] + \\
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Markov decision problem

Example (continued)

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$$
\begin{aligned}
V^\pi(0) = \quad & 1/3(\gamma V^\pi(1) + \gamma V^\pi(0) + \gamma V^\pi(0)) \\
V^\pi(0) = \quad & 1/3(0.9 V^\pi(1) + 0.9 V^\pi(0) + 0.9 V^\pi(0)) \\
\rightarrow \quad & 0.4 V^\pi(0) - 0.3 V^\pi(1) = 0
\end{aligned}
$$

$$
\begin{aligned}
V^\pi(1) = \quad & \ldots \\
V^\pi(2) = \quad & \ldots
\end{aligned}
$$

Markov decision problem
Example (continued)

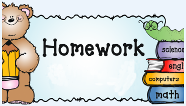
We obtain the system of linear equations:

$$\begin{pmatrix} 0.4 & -0.3 & 0 \\ -0.3 & 0.7 & -0.3 \\ -0.3 & 0 & 0.4 \end{pmatrix} V^\pi = \begin{pmatrix} 0 \\ 0 \\ 1/3 \end{pmatrix}$$

$\rightarrow$

$$V^\pi = \begin{pmatrix} 0{,}61 \\ 0{,}82 \\ 1{,}29 \end{pmatrix}$$

Markov decision problem

Evaluation of the value of a policy: 

Implement this approach in a generic way to solve any MDP.

Check your implementation on the little example.

Apply it to the "21 with a dice" problem.

Markov decision problem

Evaluation of the value of a policy: the dynamic programming approach

$V^\pi(x) = \sum_{a \in \mathcal{A}} \pi(x, a) \sum_{x' \in \mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^\pi(x')]$

Development $\rightarrow$ a system of N linear equations with N unknowns, $V^\pi(x), \forall x \in \mathcal{X}$.

In principle, an easy problem.
In practice, when N is large (*e.g.* 10^9), this is a challenging problem.

$V^\pi = \mathbf{A} V^\pi + \mathbf{B}$, where $\mathbf{A} \in \mathbb{R}^{N \times N}$ is an $N \times N$ matrix, and $\mathbf{B} \in \mathbb{R}^N$.
$\rightarrow (Id - \mathbf{A}) V^\pi = \mathbf{B}$

Thanks to contraction properties of **A**, this can be solved iteratively.

express **A** and **B** in terms of $\pi, \mathcal{P}, \mathcal{R}, \gamma$.

Markov decision problem

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Thanks to contraction properties of **A**, this can be solved iteratively.

$k \leftarrow 0$
$V_k \leftarrow 0, \forall x \in \mathcal{X}$
$\epsilon \leftarrow 10^{-6}$ // some small value
repeat
  $V_{k+1} \leftarrow \ldots$
  $\Delta \leftarrow ||V_k - V_{k+1}||_\infty$
  $k \leftarrow k+1$
until $\Delta < \epsilon \frac{1-\gamma}{2\gamma}$

This algorithm provides an estimation of V^π at most ϵ away from its true value.

It is very easy to implement and very fast, even when $N = 10^9$.

Markov decision problem

Evaluation of the value of a policy: the dynamic programming approach

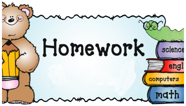
$$V^\pi(x) = \sum_{a \in \mathcal{A}} \pi(x, a) \sum_{x' \in \mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^\pi(x')]$$

is known as a "Bellman equation".

[Bellman, *Dynamic Programming*, Princeton U. Press, 1957]

Markov decision problem

Example (continued): the dynamic programming approach



Implement this dynamic programming approach for policy value evaluation and run it on the two examples.

Check that both methods give the same result.

Markov decision problem

Evaluation of the value of a policy: the Monte Carlo approach

Markov decision problem

Evaluation of the value of a policy: the Monte Carlo approach

Very simple approach: consists in simulating the Markov chain and estimating V^π by computing ζ.

To estimate $V^\pi(x)$:

$cr \leftarrow 0$
$cnt \leftarrow 0$
for $k \in \{0, ..., K-1\}$ **do**
  $t \leftarrow 0$
  $x_t \leftarrow x$
  repeat
    sample $a_t \sim \pi(x_t, .)$
    sample the next state $x_{t+1} \sim \mathcal{P}(x_t, a_t, .)$
    sample $r_t \sim \mathcal{R}(x_t, a_t, .)$
    $cr \leftarrow cr + \gamma^t r_t$
    $cnt \leftarrow cnt + 1$
  until γ^t is very small
end for $V^\pi(x) \leftarrow \frac{cr}{cnt}$

Markov decision problem

Evaluation of the value of a policy: the Monte Carlo approach

- When the MDP is deterministic, the double loop becomes a single loop.

  Wrt. traditionnal tree search algorithms, it is competitive in some cases: very wide tree, or very deep tree, or stochastic dynamics.

Markov decision problem

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Implement a Monte Carlo algorithm for policy evaluation. Apply it to the same examples as above and check that the 3 approaches provide the same results.

Markov decision problem
Policy improvement

- Once the value of a policy has been estimated, the policy can be improved.

Markov decision problem

Policy improvement

- Once the value of a policy has been estimated, the policy can be improved.
- Let us assume we estimated V^π. Then, we compute:

$$\pi'(x) \leftarrow \text{arg max}_{a \in \mathcal{A}} \sum_{x'} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^\pi(x')]$$

for each state $x \in \mathcal{X}$.

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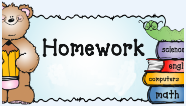
  for each state $x \in \mathcal{X}$.
- Then π' is either better than π, or they are equivalent, that is: $V^{\pi'} \geq V^\pi$.

Markov decision problem

Example (continued) 

Implement the policy improvement algorithm.

Apply it on the little MDP. How is the uniformly random policy improved?

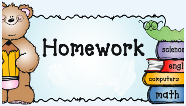
Markov decision problem
Policy iteration

- Start with a random policy.
- Then, alternate:
  - estimate the value of the current policy
  - improve the current policy

  until the value of the policy no longer improves.

This is the "policy iteration" algorithm [Howard, 1958].

Markov decision problem

Example (continued) 

Implement policy iteration and test it on the little MDP and on the "21 with a dice" problem.

Markov decision problem
Value iteration

- The value V^* of the optimal policy π^* is:
  $V^*(x) \stackrel{\text{def}}{=} \max_\pi V^\pi(x), \forall x \in \mathcal{X}$

Markov decision problem

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  $V^*(x) = \max_{a \in \mathcal{A}} \sum_{x' \in \mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^*(x')]$
  The "Bellman optimality equation".

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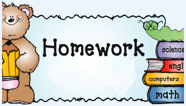
  The "Bellman optimality equation".
- and an algorithm to obtain V^* directly:
  - $k \leftarrow 0$
  - initialize V_k
    - compute $V_{k+1} \leftarrow \max_{a \in \mathcal{A}} \sum_{x' \in \mathcal{X}} \mathcal{P}(x, a, x')[\mathcal{R}(x, a, x') + \gamma V^*(x')]$ for all x
    - $k \leftarrow k + 1$
  - repeat
  - until $||V_k - V_{k-1}||_\infty \leq \frac{\epsilon(1-\gamma)}{2\gamma}$

  named "value iteration".

Markov decision problem

Example (continued)



Implement value iteration and test it on the little MDP and on the "21 with a dice" problem.

Markov decision problem

As a linear program

We can frame a Markov decision problem as a linear program:

$$\begin{array}{l} min \sum_x V[x] \\ \text{s.t. } V[x] - \sum_{x'} \mathcal{P}(x,a,x')(\mathcal{R}(x,a,x')) + \gamma V[x']) \geq 0 \ \forall (x,a) \in \mathcal{X} \times \mathcal{A} \end{array}$$

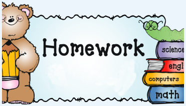
There is one constraint for each (state, action) pair and one variable per state.
The dual is:

$$\begin{array}{l} max \sum_{(x,a)} \sum_{x'} \mathcal{P}(x,a,x')\mathcal{R}(x,a,x')\xi(x,a) \\ \text{s.t. } \sum_a \xi(x',a) - \sum_{x,a} \gamma \mathcal{P}(x,a,x')\xi(x,a) \leq 1, \forall x' \in \mathcal{X} \\ \text{and } \xi(x,a) \geq 0, \forall (x,a) \in \mathcal{X} \times \mathcal{A} \end{array}$$

Once solved, $\xi(x,a)$ is non zero if action a is optimal in x.

Very interesting theoretically speaking.
In practice, policy iteration is (usually) the best way to go.

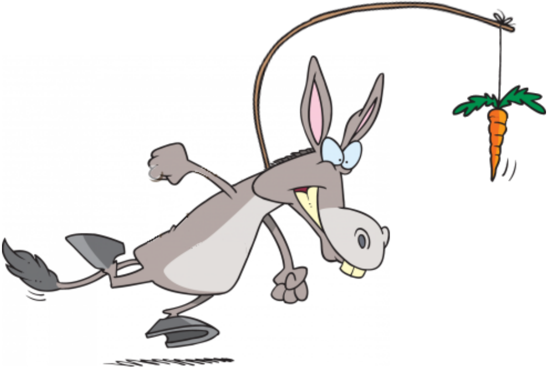
Markov decision problem

Example (continued) 

Implement the LP approach on the little MDP and on the “21 with a dice” problem.

Hint: in Python, use the PULP package to solve an LP.

From MDPs to Reinforcement Learning



From MDPs to Reinforcement Learning

What if $\mathcal{P}$ and $\mathcal{R}$ are unknown?

From MDPs to Reinforcement Learning

The quality function

- Reminder: $V^\pi \stackrel{\text{def}}{=} \mathbb{E}[\zeta(x)|x_0 = x, a_{t>0} \sim \pi]$
  Let us define $Q^\pi(x, a) \stackrel{\text{def}}{=} \mathbb{E}[\zeta(x)|x_0 = x, a_0 = a, a_{t>0} \sim \pi]$

  named the "quality" of the (state, action) pair (x, a).

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  Reminder: This quantifies what will happen to the agent in its future if it behaves according to π.

Reinforcement Learning
Bellman equation and the TD error [Sutton, 1988]

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  sum of what will happen immediately $+ \gamma \times$ what will happen then.
- at time t, $r_t + \gamma(V^\pi(x_{t+1})) - V^\pi(x_t)$

  is an estimation of the error of estimation of V: the difference between what we expected before performing a_t ($= V^\pi(x_t)$) and what we can expect now that we performed a_t and observed the x_{t+1} and r_t ($= r_t + \gamma(V^\pi(x_{t+1}))$). TD-error

  This TD-error may be used to learn the optimal behavior.

- computing V by gradient descent:

  $V(x_{t+1}) \leftarrow V(x_t) - \eta[r_t + \gamma(V_t(x_{t+1})) - V(x_t)]$
  where η is a learning rate, adequately decreasing along time.

Reinforcement Learning

The temporal difference

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Reinforcement Learning

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  $\rightsquigarrow$

  $Q(x_t, a_t) \leftarrow Q(x_t, a_t) + \eta[r_t + \gamma \max_b Q(x_{t+1}, b) - Q(x_t, a_t)]$

Reinforcement Learning
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- $\rightsquigarrow$ learning π^* algorithm.

The goal of this algorithm is to estimate Q^* by interacting with the environment and correcting its estimate of Q^*.

1. initialize Q.
2. set the agent in some random initial state $\in \mathcal{X}_0$.
3. run the agent in the environment:
   at each step, record the state x_t, the action performed a_t, the reward collected r_t, and the next state x_{t+1}.
4. correct Q.
5. continue until a terminal state is reached.
6. do it again and again (re-starting at step 2).

Reinforcement Learning

Q-Learning [Watkins, 1989]

$Q(x, a) \leftarrow$ some value (0, random, ...)
repeat
  $t \leftarrow 0$
  Initialize the state of the agent x_t
  while episode not completed **do**
    choose an action to perform in state x_t: a_t
    perform this action and observe r_t and x_{t+1}
    update:
    $Q(x_t, a_t) \leftarrow Q(x_t, a_t) + \alpha[r_t + \max_{b \in \mathcal{A}} Q(x_{t+1}, b) - Q(x_t, a_t)]$
    $t++$
  end while
until some condition is met.

At completion (if you looped enough): $\pi^*(x) = \arg\max_a Q(x, a), \forall x$

Reinforcement Learning
About Q-Learning

At completion (if you looped enough): $\pi^*(x) = \arg\max_a Q(x,a), \forall x$

- Remark: you never know if you looped enough!
- Asymptotic convergence to Q^*.

- α depends on x_t and a_t.
- for each (x,a), we should have:
  $\sum_{\{t \text{ at which } (x,a) \text{ is visited}\}} \alpha_t(s, n(s)) = +\infty$
  and
  $\sum_{\{t \text{ at which } (x,a) \text{ is visited}\}} \alpha_t^2(s, n(s)) < +\infty$
  e.g. $\alpha(x,a) \equiv \frac{1}{n(x,a)+1}$

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Reinforcement Learning

About Q-Learning

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Reinforcement Learning
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Reinforcement Learning
About Q-Learning

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Reinforcement Learning

About Q-Learning

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Reinforcement Learning
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Reinforcement Learning

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Reinforcement Learning

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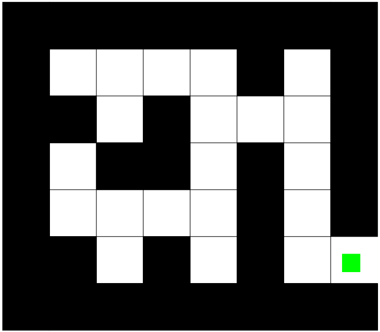
    - draw an action at random according to p.
  - Slowly decrease τ along the episodes.
  - Rationale: when τ is large, uniformly random selection. τ close to 0, greedy selection.



Goal: reach the green cell from any location.

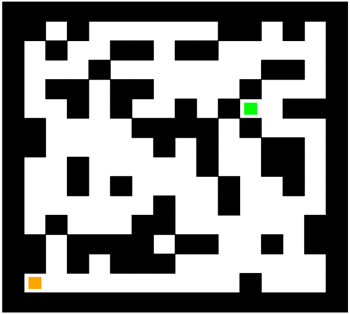
Question 1: propose a Markov decision process: what are $(\mathcal{T}, \mathcal{X}, \mathcal{X}_0, \mathcal{X}_f, \mathcal{A}, \mathcal{P}, \mathcal{R})$?

Question 2: model this task as a Markov decision problem: what is ζ?

Reinforcement Learning

Q-Learning in action: escaping a maze

We use an extremely basic Q-Learning.
Has a very local perception: sees only the 4 neighboring cells.
Gets $r = 100$ when reaching the goal, 0 otherwise.



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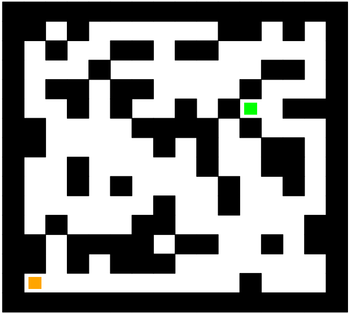
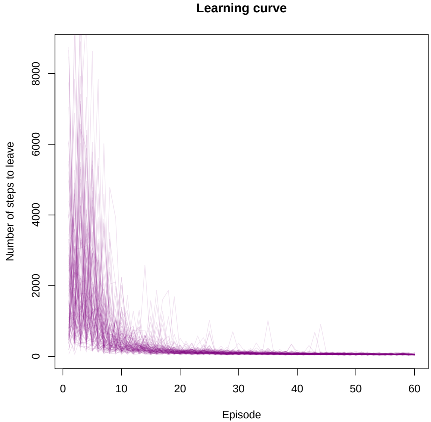
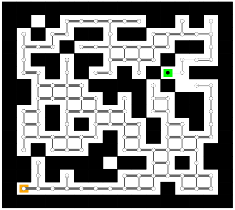




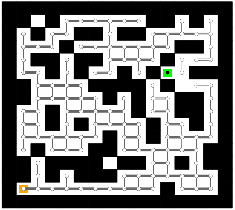
1st reach



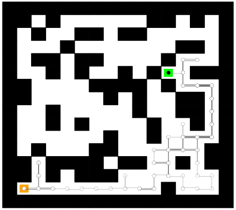
Reinforcement Learning

Q-Learning in action

1st reach



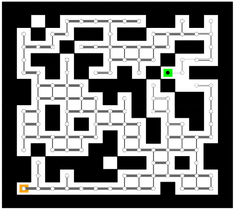
10th reach



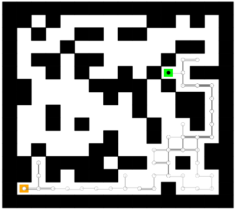
Reinforcement Learning

Q-Learning in action

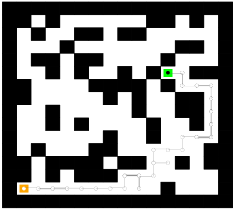
1st reach



10th reach



60th reach



Reinforcement Learning

Not only the last step matters: Eligibility traces

- When reaching the goal for the first time, only the last (x, a) pair is modified = rewarded.
- Previous pairs also had a role since they led to the last pair.
- They may be rewarded too.
- Less and less as they are further away the last pair.

[1] very weird name.

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$\rightarrow$ Eligibility traces[1]:

- each (x, a) pair has an associated eligibility $e(x, a)$.
- $Q(x, a)$ is updated for each (x, a) pair that is visited.
- $e(x, a)$ is set to 1 when this pair is visited.
- Then, $e(x, a)$ is decaying along time: $e(x, a) \leftarrow \lambda e(x, a)$.

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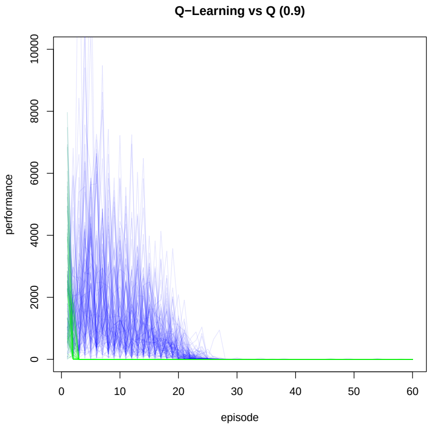
Reinforcement Learning

Not only the last step matters: Eligibility traces



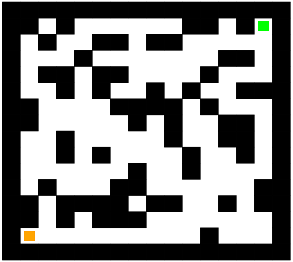
100 runs of Q-Learning vs. 100 runs of Q $(\lambda = 0.9)$.

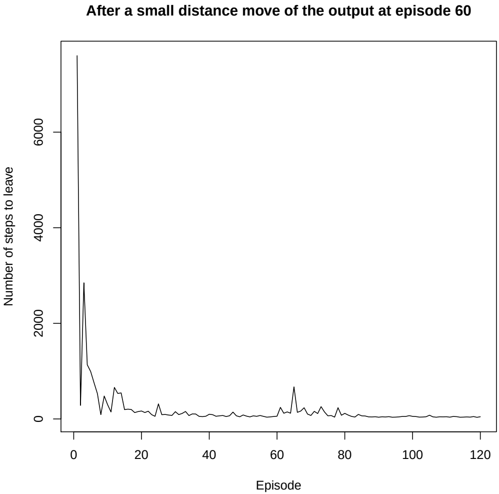
Q (λ)

initialize Q
repeat
  $e(x, a) \leftarrow 0, \forall (x, a) \in \mathcal{X} \times \mathcal{A}$
  $t \leftarrow 0$
  initialize x_0 and choose action a_0
  repeat
    emit a_t and observe r_t and x_{t+1}
    choose action a_{t+1} to be emitted in x_{t+1}
    $a_{t+1} \leftarrow \arg\max_{a \in \mathcal{A}} Q(x_{t+1}, a)$
    $\delta \leftarrow r_t + \gamma \widehat{Q}(x_{t+1}, a_{t+1}) - Q(x_t, a_t)$
    $e(x_t, a_t) \leftarrow e(x_t, a_t) + 1$
    for $(x, a) \in \mathcal{X} \times \mathcal{A}$ **do**
      $Q(x, a) \leftarrow Q(x, a) + \alpha \delta e(x, a)$
      $e(x, a) \leftarrow \gamma \lambda e(x, a)$
    end for
    $t \leftarrow t + 1$
  until end of episode
until ...

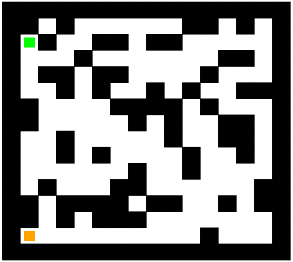
Reinforcement Learning
Q-Learning continuously adapts to its environment

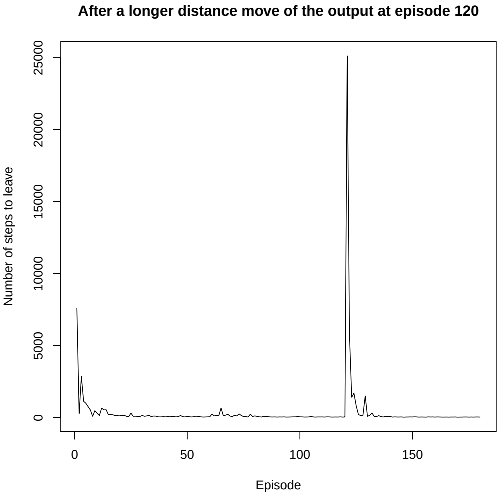
The goal state moves nearby:





The goal state moves farther away:

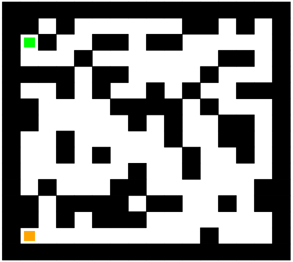


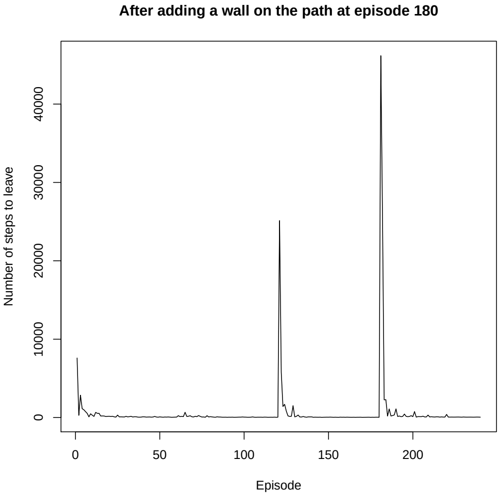


Reinforcement Learning

Q-Learning continuously adapts to its environment

Blocking the path:





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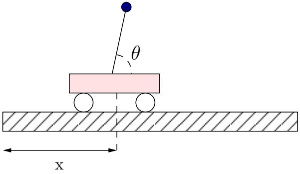
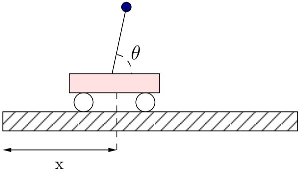
- This estimate may be updated/improved by learning.

Reinforcement Learning

Value function



State is $(\theta, \dot{\theta})$
Action is $\ddot{\theta}$



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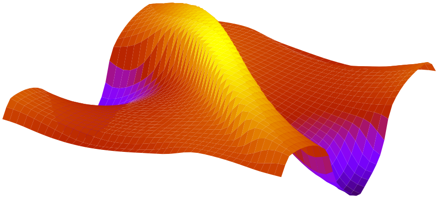


$(\theta, \dot{\theta})$ plane
z is $V(x)$
Maximize value $\rightsquigarrow$ reach the top of V

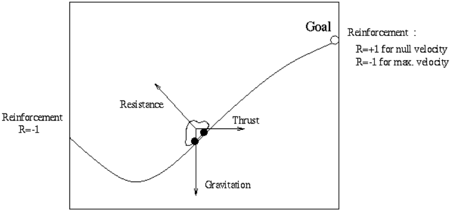
Reinforcement Learning

Handling large $\mathcal{X}$: the function approximator zoo

- neural network [Lin, 1991; Riedmiller, 2005; ...],
- random forest [Geurts *et al.*, 2006],
- SVM and kernels,
- and many other ideas from statistical learning (supervised learning).
- Tabular with progressive and adaptive state partitioning.

Reinforcement Learning

Progressive and adaptive state partitioning [Munos, Moore, MLJ, 2001]



Reinforcement Learning

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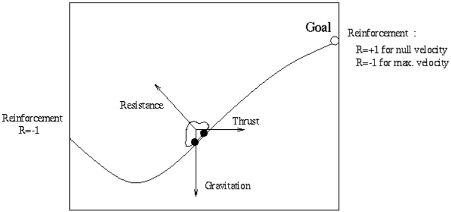

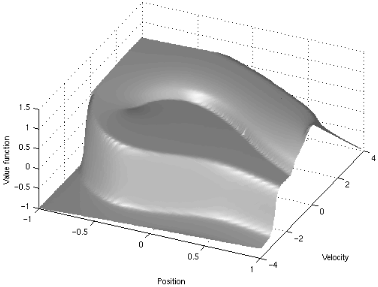

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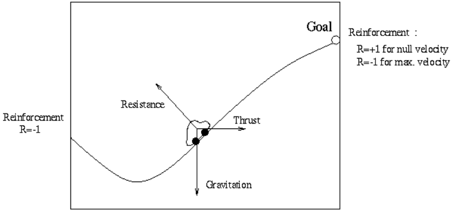

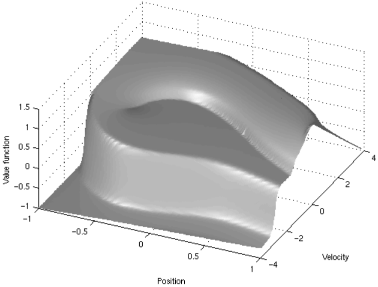

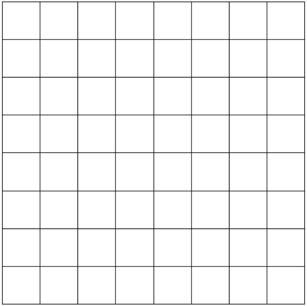

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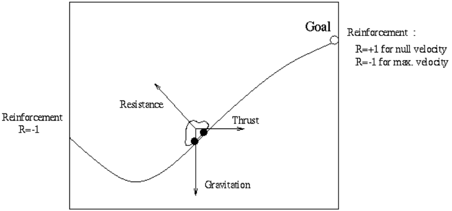

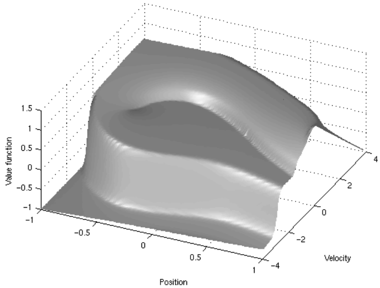

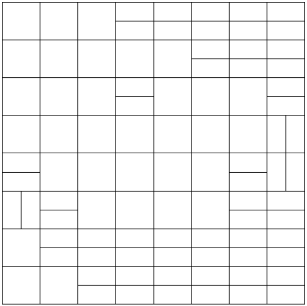

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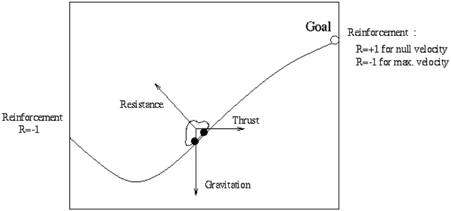

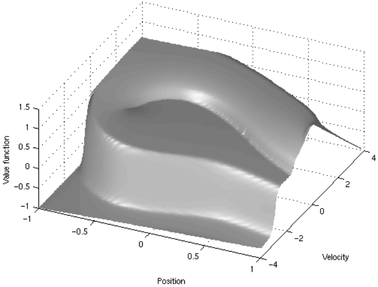

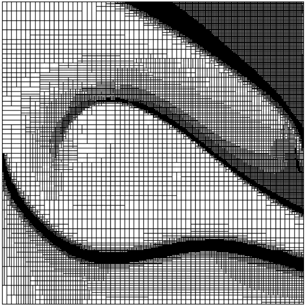

Trendy people call that “deep RL” though there is only shallow networks most of the times.

- Use of a neural network to represent Q.
- Input $=$ the current state
- Output $=$ either an estimate of $Q(x, a), \forall a$, or an estimate of $\pi(x)$.

Reminder: function approximation

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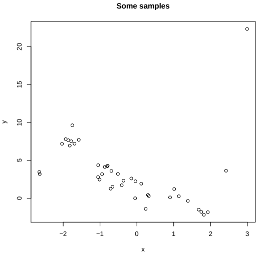
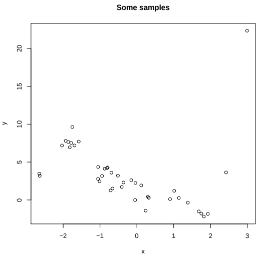
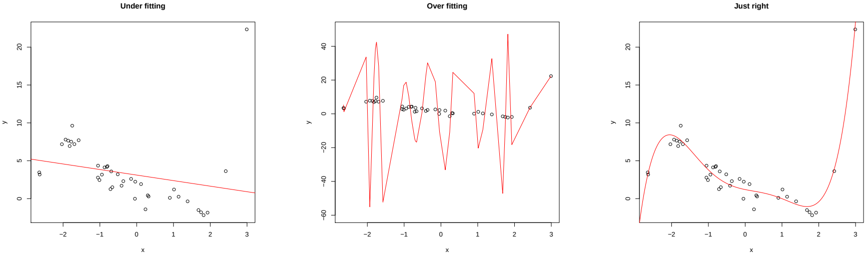


A few samples.

Reminder: function approximation





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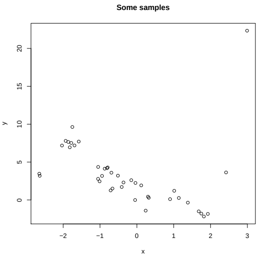

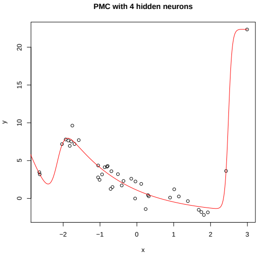

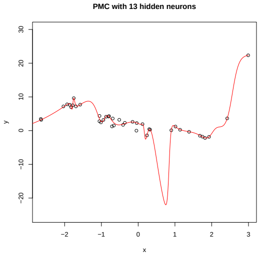

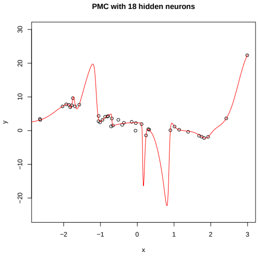

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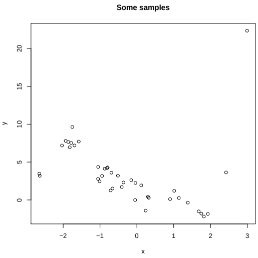

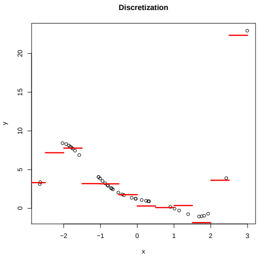

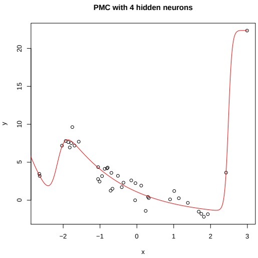

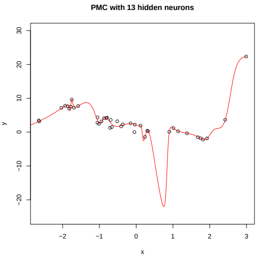

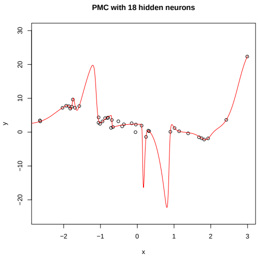

Tabular Q-Learning	DQN

Initialize the Q-table Q.
repeat
set initial state x_0
$t \leftarrow 0$
while episode not complete **do**
• select a_t and emit it
• observe r_t and x_{t+1}
• update $Q(x_t, a_t)$
• $t \leftarrow t + 1$
end while
until ...
	Initialize parameters θ Q
	repeat
	set initial state x_0
	$t \leftarrow 0$
	while episode not complete **do**
	• select a_t and emit it
	• observe r_t and x_{t+1}
	• update θ to minimise the prediction error (temporal difference) for (x_t, a_t)
	• $t \leftarrow t + 1$
	end while
	until ...

There are many things that can go wrong with this algorithm.
May be painful to debug.

- Design of the state space: the state is a vector; neighboring vectors should be neighbor states.

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Reinforcement Learning
Neural Q-Learning

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  - and more.

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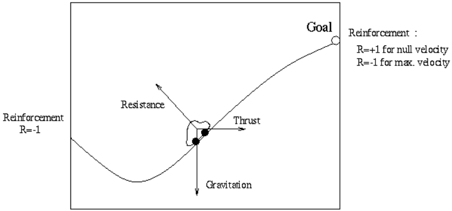
Neural networks may be tricky to train in supervised learning.
In RL, NN are used in a feedback loop: feedback loops are well-known to be very sensible, prone to very quickly (exponential rate) turn tiny errors into catastrophes.

When coding DQN:

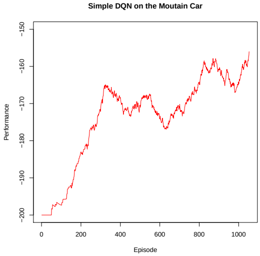
- Each part of the algorithm should be tested.
- One should check that the NN is able to regress the Q function.

Reinforcement Learning

Neural Q-Learning/DQN

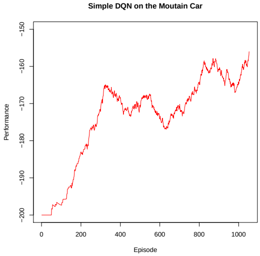


On the mountain-car:



red: 128 neurons

On the mountain-car:



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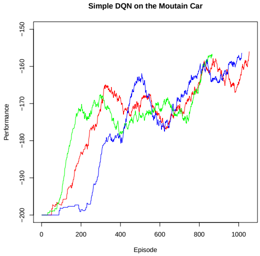

green: 32 neurons,
blue: 64 neurons

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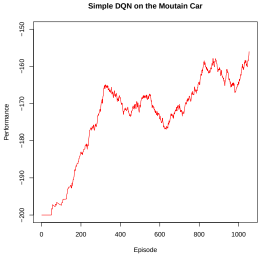

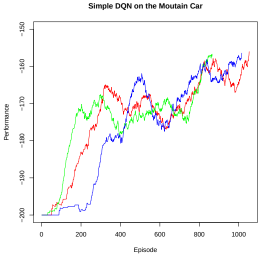

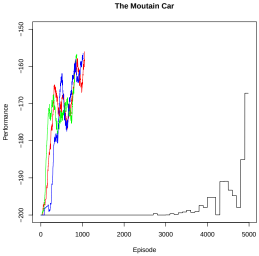

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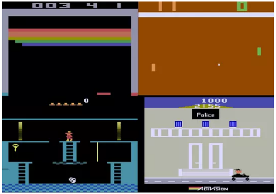
+ tabular QL

The Atari game suite:



- Input: image
  $\rightarrow$ CNN
  $\rightarrow$ fully connected layers
  $\rightarrow$ P outputs, one per action.
- DQN learns to play 49 games.
- The CNN part learns an embedding of the state (image) that is used to learn the value of each action.

An other family of RL algorithms

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- Idea:
  - Policy represented by a NN with parameters θ.
  - Run one or more trajectories with this policy.
  - We can show that maximizing ζ is equivalent to minimizing $\mathcal{L}(\theta) \equiv -Q \log(\pi^\theta)$.

initialize θ
repeat
  perform K episodes and collect transitions $(x_{k,t}, a_{k,t}, r_{k,t}, x_{k,t+1})$
  for for each episode k **do**
    for for each step t in episode k **do**
      $Q_{k,t} \leftarrow \sum_{j=t}^{j=|\tau_k|-1} \gamma^{j-t} r_{k,j}$
      $\theta \leftarrow \theta + \alpha\gamma^t Q_{k,t} \nabla_\theta \log\left(\pi^\theta(s_{k,t}, a_{k,t})\right)$
    end for
  end for
until ...

REINFORCE suffers from important drawbacks:

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- REINFORCE is unstable because the update have a large variance.
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Reinforcement Learning

Policy gradient approach

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  The trouble comes from the degenerescence of $\pi(x,.)$.
  We can add an extra term in $\mathcal{L}$ to force the non degenerescence of $\pi(x,.)$: maximize the entropy of $\pi(x,.) \rightarrow$
  $\mathcal{L}(\theta) \equiv -Q \log(\pi^\theta) + \lambda H(\pi^\theta)$.
  Regularization is a key ingredient in optimization.

One may use other types of optimization algorithm to optimize ζ.

Some researchers have investigated the use of evolutionary algorithms.

The combination of the goods of value based and policy gradient approaches.

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Interaction between 2 agents:

- an actor: a policy π that acts
- a critic: a value function that assesses the actions taken by the actor.

$\rightarrow$ actor-critic algorithms.

- $\mathcal{L}(\theta) \equiv -Q \log(\pi^\theta)$
  $\rightarrow$
  $\mathcal{L}(\theta) \equiv -(Q - b) \log(\pi^\theta)$.
  where b is a function of the (current) state.

Reinforcement Learning
Actor-critic

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  $\mathcal{L}_{AC}(\theta) \equiv -(Q - V) \log(\pi^\theta)$.
- We represent V with an other NN, and we update it with the temporal difference (as in DQN).

initialize θ_V and θ the weights for V and π
repeat
  perform K episodes and collect the transitions in T.
  update θ_V by minimizing the MSE of the TD on T
  for each episode k **do**
    for each time-step t of episode k **do**
      $\widehat{Q}_{k,t} \leftarrow \sum_{j=t}^{j=|\tau_k|-1} \gamma^{j-t} r_{k,j}$
      $\theta \leftarrow \theta + \alpha\gamma^t(\widehat{Q}_{k,t} - \widehat{V}_{k,t})\nabla_\theta \log(\pi^\theta(x_{k,t}, a_{k,t}))$
    end for
  end for
until ...

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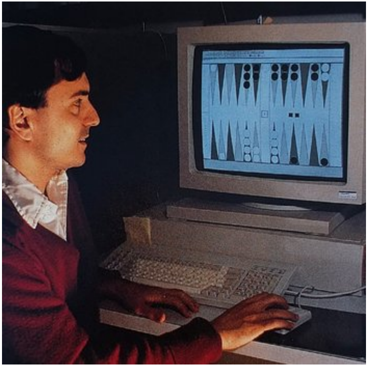
  - *etc*

Currently, state-of-the-art algorithms are:

- DQN when the number of possible actions is small.
- actor-critics like PPO, SAC.



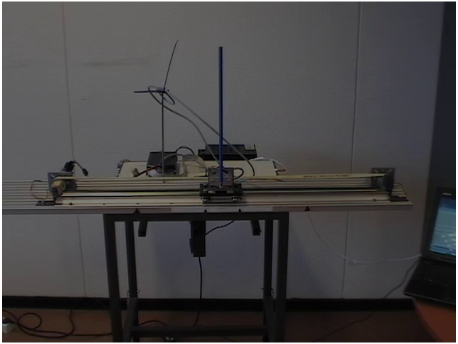
Early 1990's, a real tour de force.

- Backgammon is studied at least since 1974
- Branching factor: 800
- TD-Gammon: "successor of Neurogammon, trained by supervised learning. NeuroGammon won the 1st Computer Olympiad in London in 1989, handily defeating all opponents. Its level of play was that of an intermediate-level human player." (Source: wikipedia)
- raw representation of the board position
- trained with TD(λ) algorithm
- no knowledge, self-play
- hand-crafted features
- 3-plies in v3

Tesauro, Temporal Difference Learning and TD-Gammon, *Communications of the ACM*, 1995

Reinforcement Learning

Application to robotics: a real cartpole

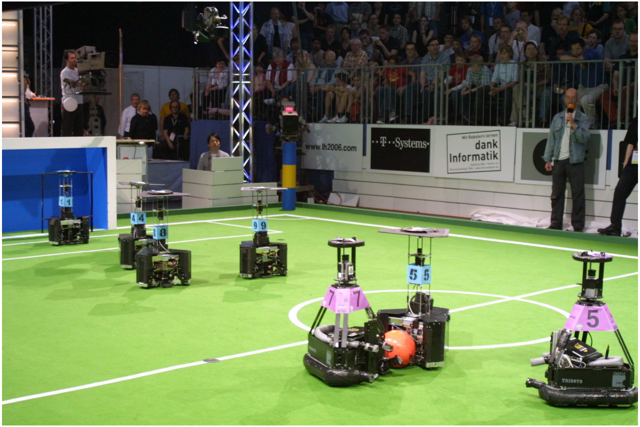


Neural Q-Learning. 2 hidden layers each made of 5 neurons with sigmoid activation function.

Riedmiller, Neural reinforcement learning to swing-up and balance a real pole, *Proc. 2005 IEEE International Conference on Systems, Man and Cybernetics*

Reinforcement Learning

Application to robotics



RL is used only to control the ball and the speed of the robot.

Lauer et al., Cognitive Concepts in Autonomous Soccer Playing Robots, *Cognitive Systems Research*, **11**(3), 287:309, September, 2010

(No Deep Learning! only shallow multi-layer perceptron)

Reinforcement Learning

Application: board games

- Learning to play board games using only the rules of the game.
- Alpha Go learned to play Go by using games played by humans.
- Alpha Zero learned to play even better by itself by RL.
- then other board games (chess, draughts, reversi, ...).
- then Starcraft II.

Reinforcement Learning

Alpha Zero type of algorithms

- RL
- $+$ various tricks to stabilize learning and make it more efficient
- MCTS as a key component
- moderately deep network as function approximator

Outro

Some big questions:

- learning representation
- generalization in RL
- time varying environments
- transfer learning
- life-long learning
- risk-aware RL
- explaination/accountability of the learned behavior
- hierarchical RL
- combination of symbolic AI with RL
- *etc*

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- *etc*

Advice: there are many many topics to study in RL. RL is not yet mature, a lot is yet to be discovered.
However, to have any chance of success, you need to understand what you are doing, not just run something and hope it will do something: in general it won't.

Reinforcement learning resources

The RL community is open source.
Lots of excellent codes available online.
Some nice RL libraries:

- rlberry: our home-brewed RL library: `https://github.com/rlberry-py/rlberry`
- stable-baselines3: `https://stable-baselines3.readthedocs.io/en/master/`
- clean-RL: `https://github.com/vwxyzjn/cleanrl`
- spinning-up RL: `https://spinningup.openai.com/en/latest/`
- mushroom RL: `https://mushroomrl.readthedocs.io/en/latest/`
- ...

Bibliography

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- Lapan, *Practical Deep Reinforcement Learning*, Packt, 2018
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- Tesauro, Temporal Difference Learning and TD-Gammon, *Communications of the ACM*, 1995

Thanks for your attention!

and congratulations!

and have fun with reinforcement learning!

Credits

All figures are mine, except:

- slide 3, the backgammon image comes from https://en.wikipedia.org/wiki/Backgammon, the go image comes from ?, the chatGPT logo comes from https://upload.wikimedia.org/wikipedia/commons/0/04/ChatGPT_logo.svg, and the data center image comes from https://www.edgeir.com/energy-monitoring-for-edge-data-centers-20220902.
- Slide 30, image comes from https://www.liveabout.com/craps-dice-control-537459
- Slide **??**, the figures come from Rémi Munos' website http://researchers.lille.inria.fr/munos/variable/index.html.
- Slide 41, the donkey was designed and made by Sertan Girgin in 2008 while he was a post-doc in my research group, SequeL.
- Slide **??**, the Atari image comes from https://deepai.org/publication/playing-atari-games-with-deep-reinforcement-learning-and-human-checkpoint-replay.
- Slide **??**, the Backgammon game image comes from https://en.wikipedia.org/wiki/Backgammon.
- Slide **??**, the cartpole image is one frame from a video of M. Riedmiller's NeuroInformatics group at U. Osnabrueck, https://www.riedmiller.me/subprojects/data-efficient-rl.
- Slide **??**, the image is one frame from a video of M. Riedmiller's NeuroInformatics group at U. Osnabrueck, https://www.youtube.com/watch?v=0vNXLVmExyI.
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image comes from https://newquayjunior.net/homework/.