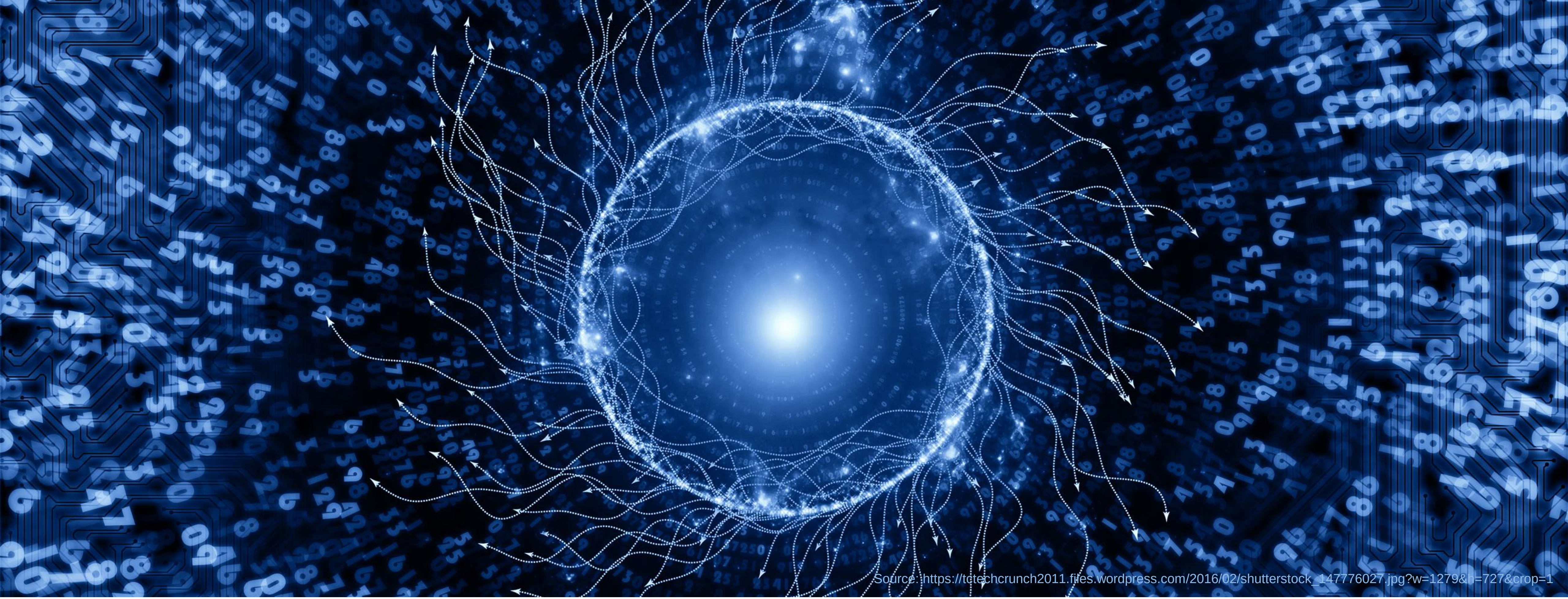

Source: https://tctechcrunch2011.files.wordpress.com/2016/02/shutterstock_147776027.jpg?w=1279&h=727&crop=1

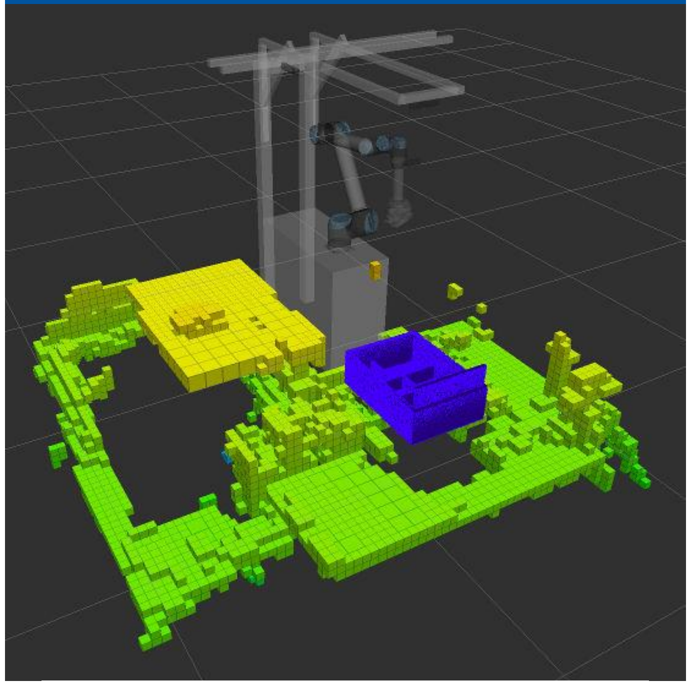
Intelligent Robotics

Modelbased Reinforcement Learning

Philipp Ennen, M.Sc.

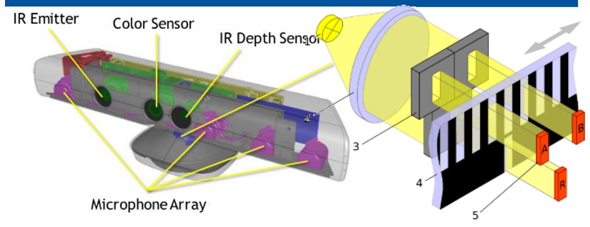
How do I (the robot) go there?

Model of the Robot and Environment



$$\begin{bmatrix}\dot{x}\\ \ddot{x}\\ \dot{\phi}\\ \ddot{\phi}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0\\ 0 & 0 & 0 & 1\\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0\end{bmatrix}\begin{bmatrix}x\\ \dot{x}\\ \phi\\ \dot{\phi}\end{bmatrix}+\begin{bmatrix}0\\ \frac{I+ml^2}{I(M+m)+Mml^2}\\ 0\\ \frac{ml}{I(M+m)+Mml^2}\end{bmatrix}u$$

Sensing



Motion Planning with iLQR

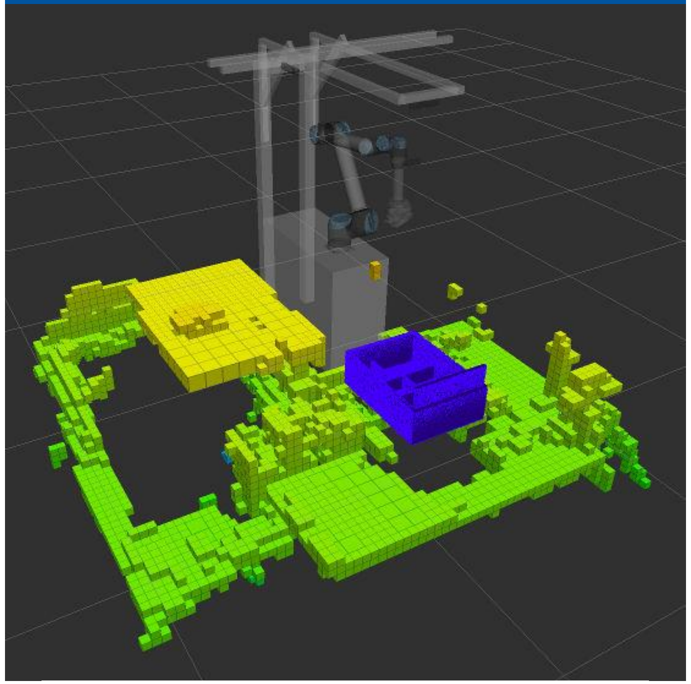
Requires Goalstate:
- i.e. hand-engineered
- i.e. via a cost function

Motion Control and Execution

Introduction

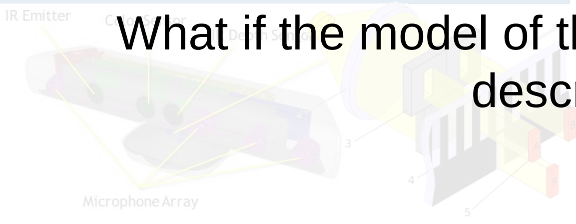
How do I (the robot) go there?

Model of the Robot and Environment



$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\phi} \\ \ddot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-(I+ml^2)b}{I(M+m)+Mml^2} & \frac{m^2gl^2}{I(M+m)+Mml^2} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-mlb}{I(M+m)+Mml^2} & \frac{mgl(M+m)}{I(M+m)+Mml^2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I+ml^2}{I(M+m)+Mml^2} \\ 0 \\ \frac{ml}{I(M+m)+Mml^2} \end{bmatrix} u$$

Sensing

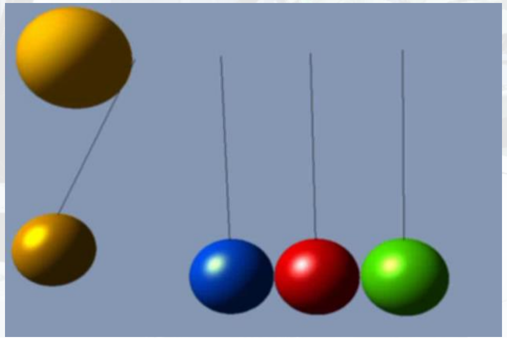


Motion Control and Execution

What if the model of the robot and environment is hard to describe (or unknown)?

Think about flexible objects!

Think about contact-situations!



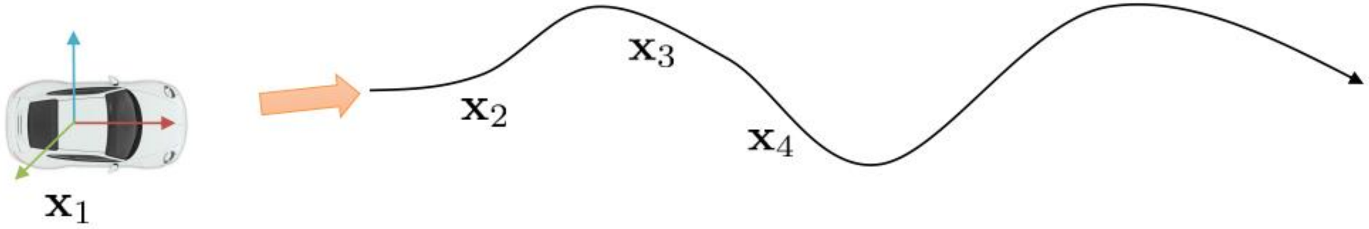
Requires Goalstate:
- i.e. hand-engineered
- i.e. via a cost function

Dynamics of unstructured environments

Motivation

Real-World Dynamics are Complex!

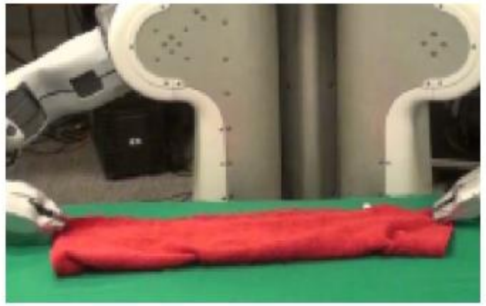
Dynamics of
a car



Often dynamic models exist

Dynamic models usually do not exist

Dynamics of
contacts



Dynamics of
flexible objects

Dynamics of
unstructured environments

Trajectory Optimization

- Trajectory Optimization: Calculates optimal sequence of actions using cost-function and dynamics

$$\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

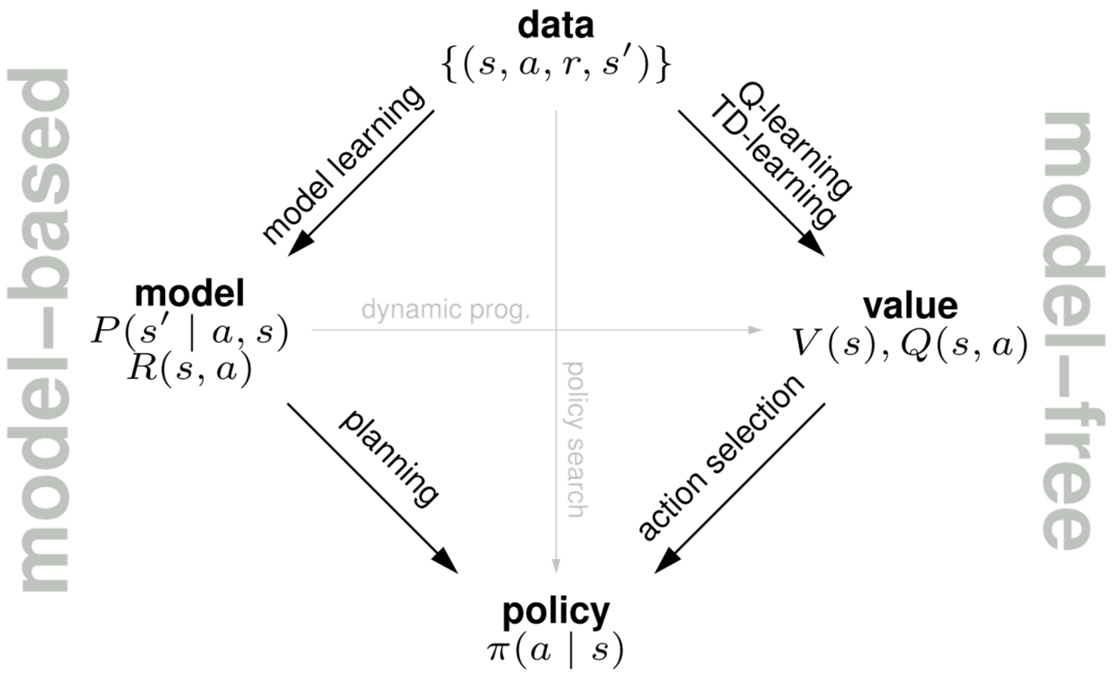
- Today: How can optimal action sequences be calculated if a dynamic model does not exist?

- Today we learn an algorithm based on
  - Learning a global dynamic model (“model-based reinforcement learning”)
  - Learning a local dynamic model

What if the model of the robot and environment is hard to describe (or unknown)?

This weeks topic!

Model-based RL:
- Learn to predict next state (using a dynamic model): $P(s'|s,a)$
- ~~Learn to predict immediate reward~~ $\cancel{P(r'|s,a)}$ (we assume to have this information)



Model-free RL:
- Learn to predict value: $\mathrm{V(s)}$ or $\mathrm{Q(s,a)}$

s: state
a: action
r: reward

Content

Motivation

Why do we want to learn the dynamics?

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1}, \mathbf{u}_{t-1})$$

$$\Updownarrow$$

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\dots)\dots), \mathbf{u}_T)$$

Usual procedure: Differentiate via Backpropagation and optimize (i.e. iLQR)

Requires: $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}, \frac{\partial c}{\partial \mathbf{x}_t}, \frac{\partial c}{\partial \mathbf{u}_t}$

Motivation

Why do we want to learn the dynamics?

- If $\mathbf{x}_{t+1} = f(\mathbf{x}_{t,}\mathbf{u}_t)$ is known, we can do trajectory optimization
  - In the stochastic case $\mathrm{p}(\mathbf{x}_{t+1}|\mathbf{x}_{t,}\mathbf{u}_t)$

Learn $f(\mathbf{x}_{t,}\mathbf{u}_t)$ with subsequent backpropagation (i.e. iLQR)

Modelbased Reinforcement Learning Version 0.5
1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}_, \mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_{i,}\mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}_, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)

Does Version 0.5 work?

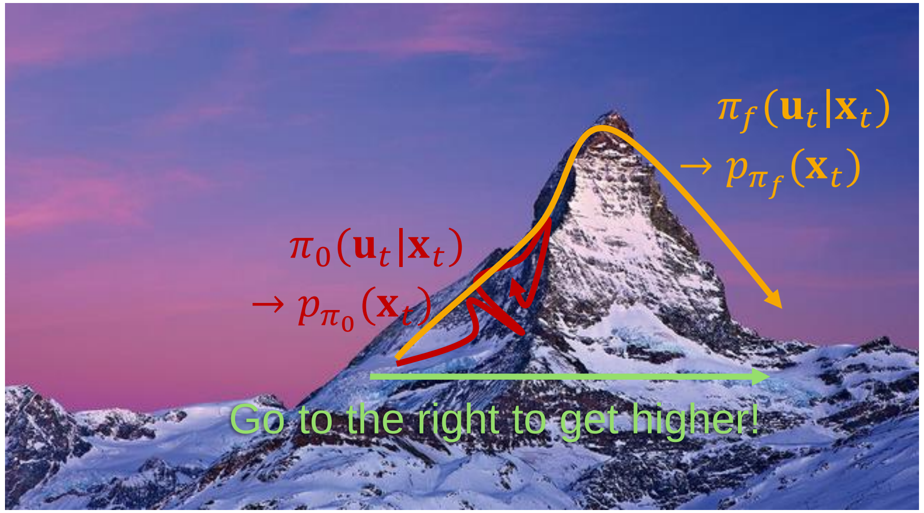
(often) **YES!**

- Traditional system identification uses this method (control theory)
- Initial policy must be chosen with caution
- Version 0.5 is very effective
  - If a representation of the dynamics based on physical laws exists
  - If only a few parameters must be learned

Learning Dynamic Models

Does Version 0.5 work?

(in general) **NO!**



1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \|f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i\|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR) → $\pi_f(\mathrm{u}_t|\mathrm{x}_t)$

$$p_{\pi_0}(\mathbf{x}_t) \neq p_{\pi_f}(\mathbf{x}_t)$$

(Distribution Mismatch Problem)

Distribution Mistmatch Problem increases if expressive classes of models are used (i.e. neural networks)

Can we do better?

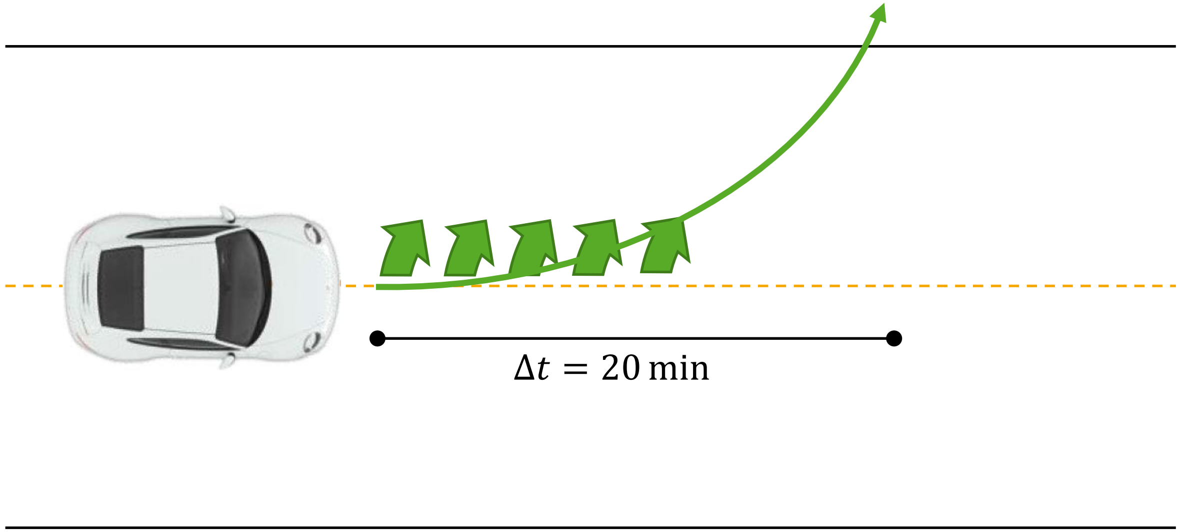
Can we make $p_{\pi_0}(\mathbf{x}_t) = p_{\pi_f}(\mathbf{x}_t)$?

Need to collect data from $p_{\pi_f}(\mathbf{x}_t)$!

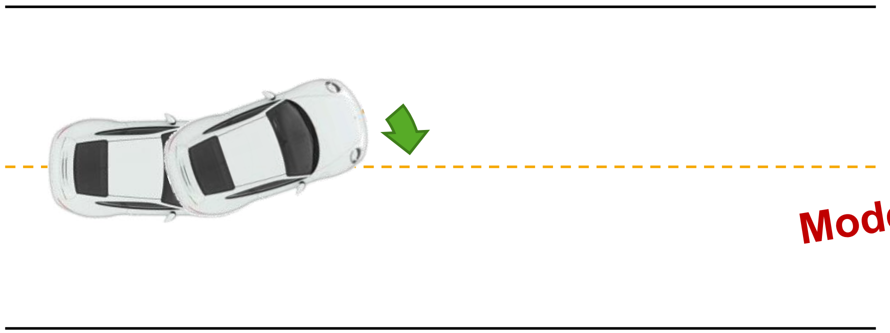
Modellbasiertes Reinforcement Learning Version 1.0

1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizesLearn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute those actions and add the resulting data $\{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$ to $\mathcal{D}$

What happens if the dynamic models contains little error?



Can we do better?



Modelpredictive control
helps with error

Modellbasiertes Reinforcement Learning Version 1.5

every N steps

1. Execute initial policy $\pi_0(\mathbf{u}_t|\mathbf{x}_t)$ (i.e. a random policy) and collect data $\mathcal{D} = \{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$
2. Learn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizesLearn dynamics $f(\mathbf{x}, \mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_i, \mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute the first planned action, observe resulting state $\mathbf{x}'$(MPC)
5. Append $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$ to dataset $\mathcal{D}$

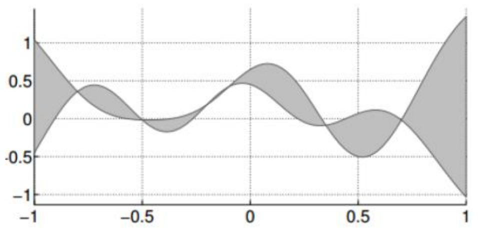
Summary

- Version 0.5: collect random samples, train dynamics, plan
  - Pro: simple, no iterative procedure
  - Con: distribution mismatch problem
- Version 1.0: iteratively collect data, replan, collect data
  - Pro: simple, solves distribution mismatch
  - Con: open loop plan might perform poorly, exp. in stochastic domains
- Version 1.5: iteratively collect data using MPC (replan in each step)
  - Pro: robust to small model errors
  - Con: comoputationally expensive, but have planning algorithm available

Content

What kind of models can we use?

Gaussian process



GP with input $(\mathbf{x}, \mathbf{u})$ and output $\mathbf{x}$'

Pro: very data-efficient

Con: not great with non-smooth dynamics

Con: very slow when dataset is big

Neural Network

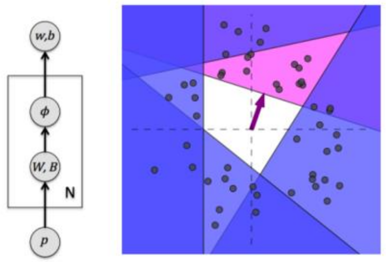


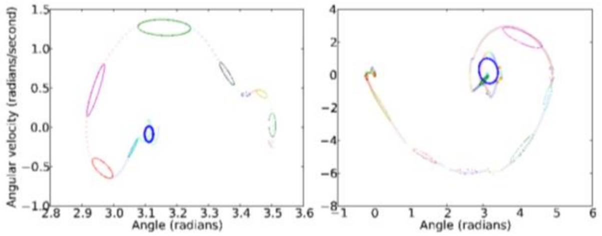
image: Punjani & Abbeel '14

Input is $(\mathbf{x}, \mathbf{u})$, output ist $\mathbf{x}$'

Pro: very expressive, can use lots of data

Con: not so great I low data regimes

Gaussian Mixture Model



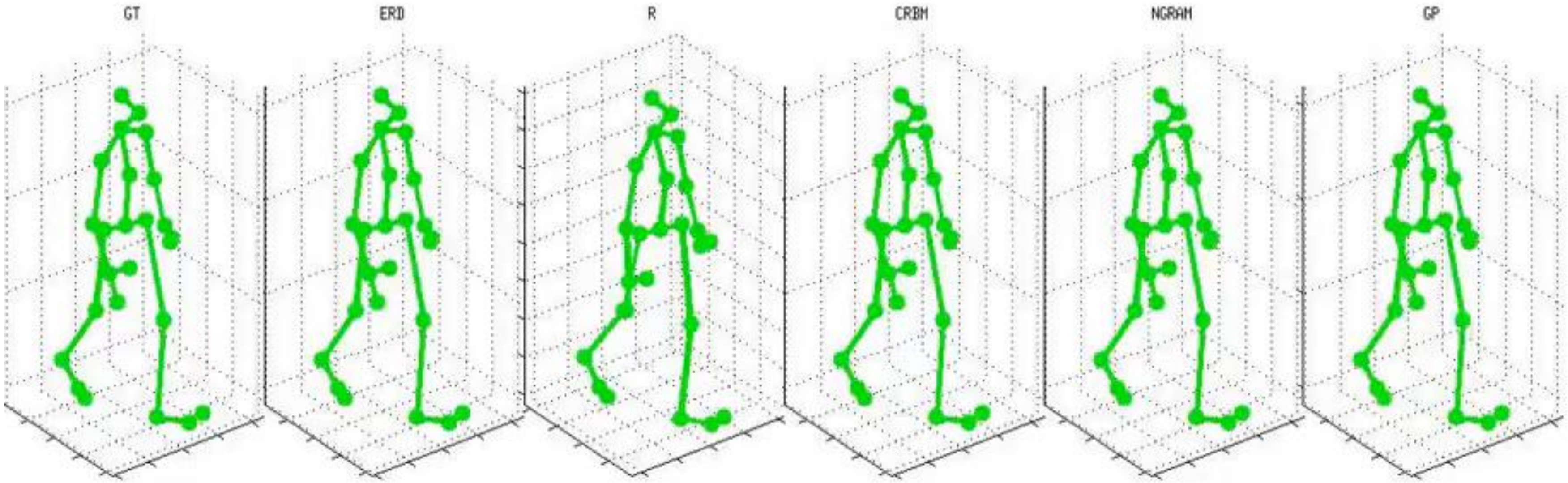
GMM over $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$ tuples

Train on $(\mathbf{x}, \mathbf{u}, \mathbf{x}')$, condition to get $p(\mathbf{x}'|\mathbf{x}, \mathbf{u})$

For i'th mixture element, $p_i(\mathbf{x}, \mathbf{u})$ gives region where the mode $p_i(\mathbf{x}'|\mathbf{x}, \mathbf{u})$ holds

Pro: very expressive, if the dynamics can be assumed as piecewise linear

Representation of Dynamic Models



Content

Global Dynamic Models

Challenges

Example: Global dynamic model $f(\mathbf{x}_t, \mathbf{u}_t)$ is represented by a neural network

Modellbasiertes Reinforcement Learning Version 1.0
1. Execute initial policy π_0 ($\mathbf{u}$_t |$\mathbf{x}$_t) (i.e. a random policy) and collect data $\mathcal{D}$={($\mathbf{x}$,$\mathbf{u}$,$\mathbf{x}$^′)_i}
2. Lean dynamics $f(\mathbf{x}_, \mathbf{u})$ that minimizes $\sum_i \left\| f(\mathbf{x}_{i,} \mathbf{u}_i) - \mathbf{x}'_i \right\|^2$
3. Backpropagate $f(\mathbf{x}_, \mathbf{u})$ and calculate sequence of actions (i.e. iLQR)
4. Execute those actions and add the resulting data $\{(\mathbf{x}, \mathbf{u}, \mathbf{x}')_i\}$ to $\mathcal{D}$

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution

Global Dynamic Models

The trouble with global models

- Planner will seek out regions where the model is erroneously optimistic
- Need to find a very good model in most of the state space to converge on a good solution
- In some tasks, the model is much more complex than the policy

Local models

Motivation

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t) \qquad \text{s.t.} \quad \mathbf{x}_t = f(\mathbf{x}_{t-1,} \mathbf{u}_{t-1})$$

$$\Updownarrow$$

$$\min_{\mathbf{u}_1,\dots,\mathbf{u}_T} c(\mathbf{x}_1, \mathbf{u}_1) + c(f(\mathbf{x}_1, \mathbf{u}_1), \mathbf{u}_2) + \cdots + c(f(f(\dots)\dots), \mathbf{u}_T)$$

Usual story: differentiate via backpropagation and optimize (i.e. iLQR)

need: $\dfrac{\partial f}{\partial \mathbf{x}_t}, \dfrac{\partial f}{\partial \mathbf{u}_t} \; \dfrac{\partial c}{\partial \mathbf{x}_t}, \dfrac{\partial c}{\partial \mathbf{u}_t}$

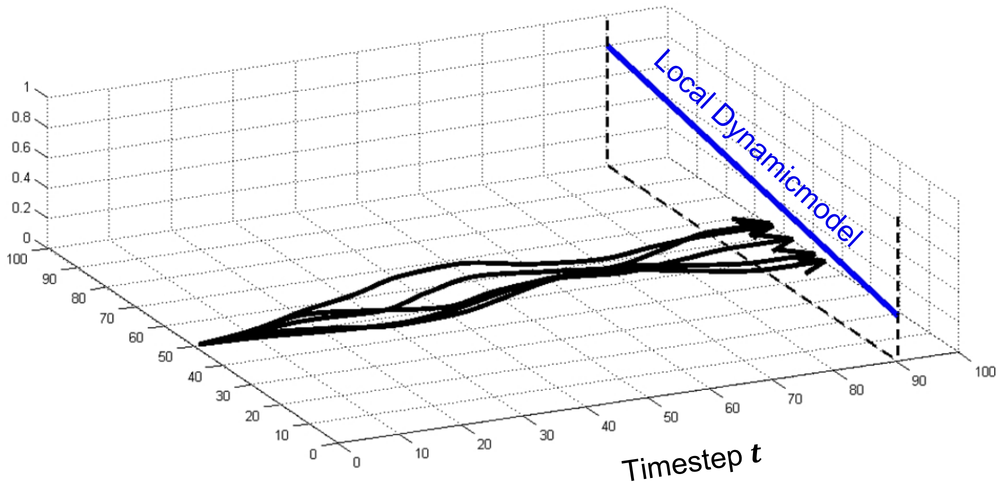
Local models

Approach

need: $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}$ $\frac{\partial c}{\partial \mathbf{x}_t}, \frac{\partial c}{\partial \mathbf{u}_t}$

idea: just fit $\frac{\partial f}{\partial \mathbf{x}_t}, \frac{\partial f}{\partial \mathbf{u}_t}$ around current trajectory or policy

$p(\mathbf{u}_t|\mathbf{x}_t)$ – time-varying linear-Gaussian controller – can **execute** on the robot and produces **trajectory distribution**

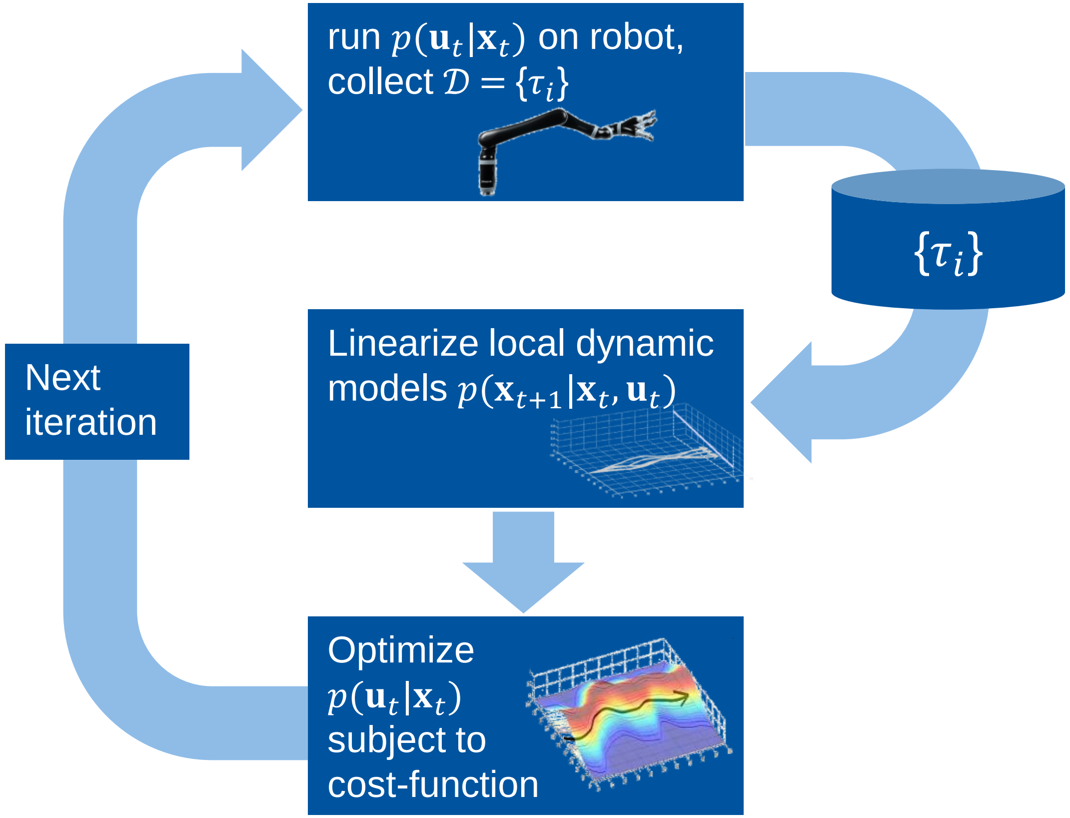


Learning a policy

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t) = \mathcal{N}(f(\mathbf{x}_t, \mathbf{u}_t), \Sigma)$$

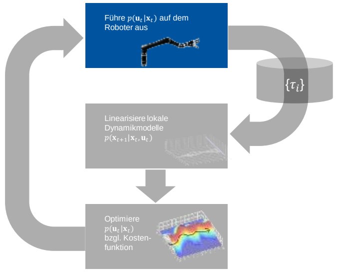
$$f(\mathbf{x}_t, \mathbf{u}_t) \approx \mathbf{A}_t \mathbf{x}_t + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{A}_t = \frac{\partial f}{\partial \mathbf{x}_t} \qquad \mathbf{B}_t = \frac{\partial f}{\partial \mathbf{u}_t}$$



Local models

Timedependent Linear Gaussian Controller



run $p(\mathbf{u}_t|\mathbf{x}_t)$ on robot, collect $\mathcal{D} = \{\tau_i\}$

iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \boldsymbol{\Sigma}_t)$$

Set $\boldsymbol{\Sigma}_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

$Q(\mathbf{x}_t, \mathbf{u}_t)$ is the cost to go: total cost we get after taking an action $\mathbf{u}_t$

$$Q(\mathbf{x}_t, \mathbf{u}_t) = \text{const} + \frac{1}{2}\begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{Q}_t \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix} + \begin{bmatrix}\mathbf{x}_t \\ \mathbf{u}_t\end{bmatrix}^T \mathbf{q}_t$$

$\mathbf{Q}_{\mathbf{u}_t,\mathbf{u}_t}$ is big, if changing $\mathbf{u}_t$ changes the Q-value a lot!

If $\mathbf{u}_t$ changes Q-value a lot, don't vary $\mathbf{u}_t$ so much. Exploration noise $\boldsymbol{\Sigma}_t$ must be low

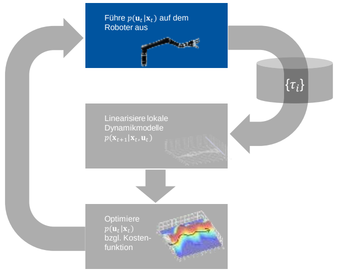
Local models

Timedependent Linear Gaussian Controller

run $p(\mathbf{u}_t|\mathbf{x}_t)$ on robot,
collect $\mathcal{D} = \{\tau_i\}$

iLQR produces: $\hat{\mathbf{x}}_t$, $\hat{\mathbf{u}}_t$, $\mathbf{K}_t$, $\mathbf{k}_t$

$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \boldsymbol{\Sigma}_t)$$



Set $\boldsymbol{\Sigma}_t = \mathbf{Q}^{-1}_{\mathbf{u}_t,\mathbf{u}_t}$

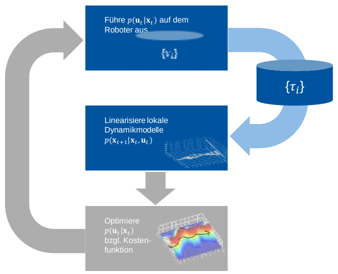
Standard LQR solves $\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} c(\mathbf{x}_t, \mathbf{u}_t)$

Linear-Gaussian solution solves $\min_{\mathbf{u}_1,\ldots,\mathbf{u}_T} \sum_{t=1}^{T} E[c(\mathbf{x}_t, \mathbf{u}_t) - \mathcal{H}(p(\mathbf{u}_t|\mathbf{x}_t))]$

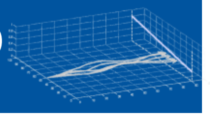
Maximum Entropy: act as randomly as possible while minimizing cost
- Entropy: A measure for the average information content

Local models

Linearize local dynamics



Linearize local dynamic models
$p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$



$\{(\mathbf{x}_t,\mathbf{u}_t,\mathbf{x}_{t+1})_i\}$

Version 1.0: Linearize $p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$ at each time step using linear regression

$$p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t) = \mathcal{N}(\mathbf{A}_t\mathbf{x}_t + \mathbf{B}_t\mathbf{u}_t + \mathbf{c}_t, \mathbf{N}_t) \qquad \mathbf{A}_t \approx \frac{\partial f}{\partial \mathbf{x}_t} \qquad \mathbf{B}_t \approx \frac{\partial f}{\partial \mathbf{u}_t}$$

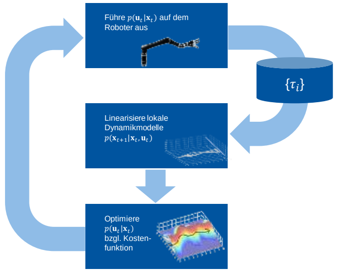
Can we do better?

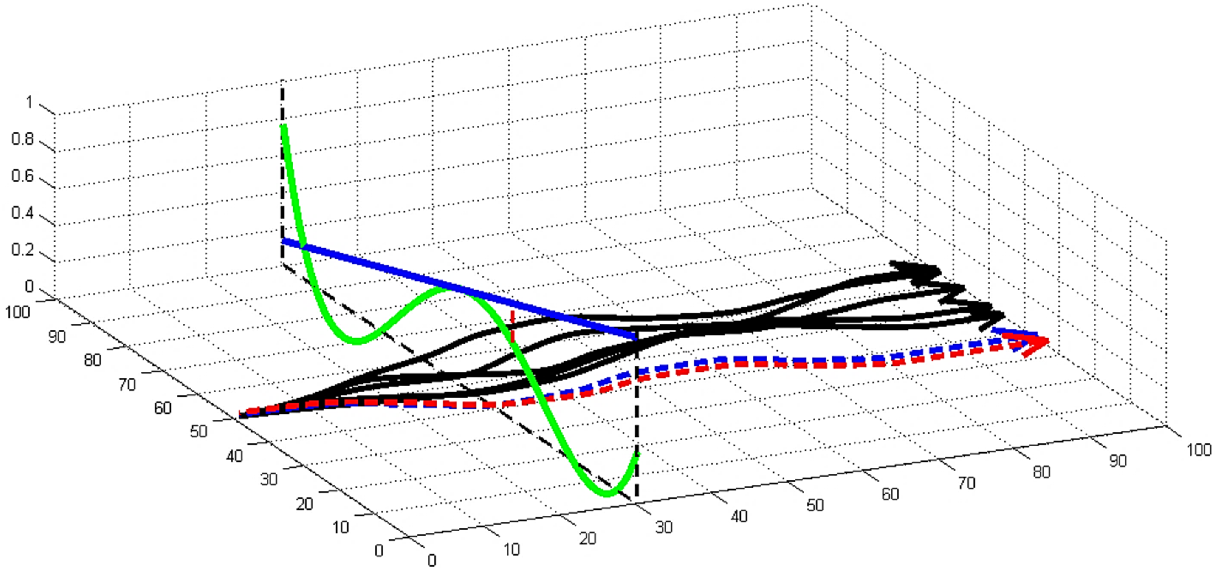
Version 2.0: Linearize $p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t)$ using *Bayesian* linear regression

- Bayesian linear regression uses prior: $p(\mathbf{x}_t,\mathbf{u}_t)p(\mathbf{x}_{t+1}|\mathbf{x}_t,\mathbf{u}_t) = p(\mathbf{x}_t,\mathbf{u}_t,\mathbf{x}_{t+1})$
- Use your favourite global model as a prior (GP, deep net, GMM)

Local models

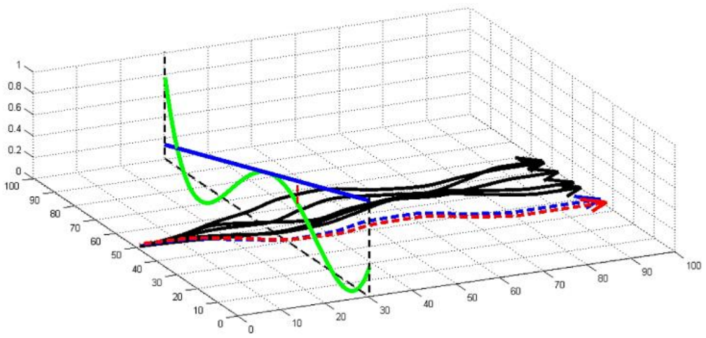
How to stay close to old controller?



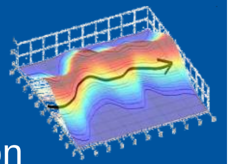


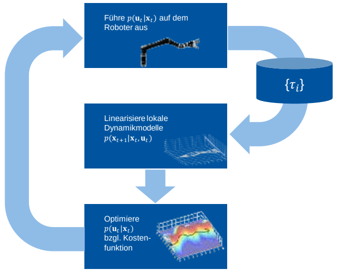
Local models

How to stay close to old controller?



Optimize $p(\mathbf{u}_t|\mathbf{x}_t)$ subject to cost-function





$$p(\mathbf{u}_t|\mathbf{x}_t) = \mathcal{N}(\mathbf{K}_t(\mathbf{x}_t - \hat{\mathbf{x}}_t) + \mathbf{k}_t + \hat{\mathbf{u}}_t, \mathbf{\Sigma}_t)$$

$$p(\tau) = p(\mathbf{x_1}) \prod_{t=1}^{T} p(\mathbf{u}_t|\mathbf{x}_t) p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)$$

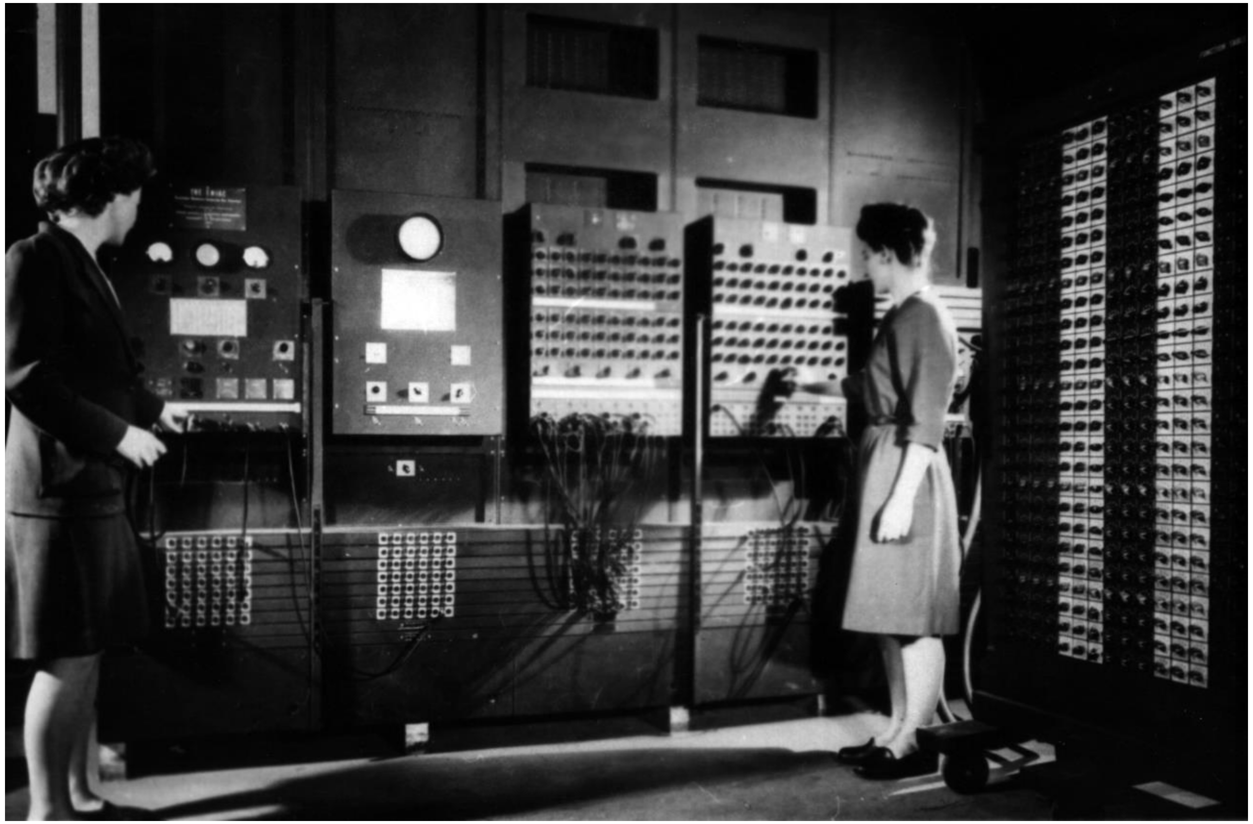
New trajectory distribution $p(\tau)$ must be similar to the old one $\bar{p}(\tau)$

If trajectory distribution is close, the dynamics will be close too!

What does “close” mean?
Kullback-Leibler divergence: $D_{KL}(p(\tau)||\bar{p}(\tau)) < \varepsilon$ **From here a lot of mathematics would follow!**

Its your turn!

Visit the website and implement it!



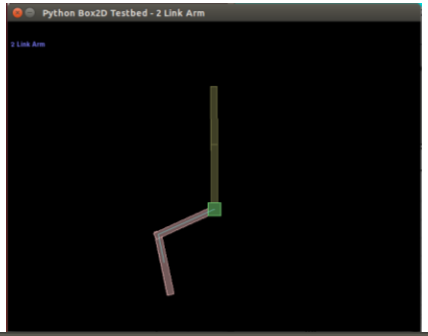
Introduction to the tasks

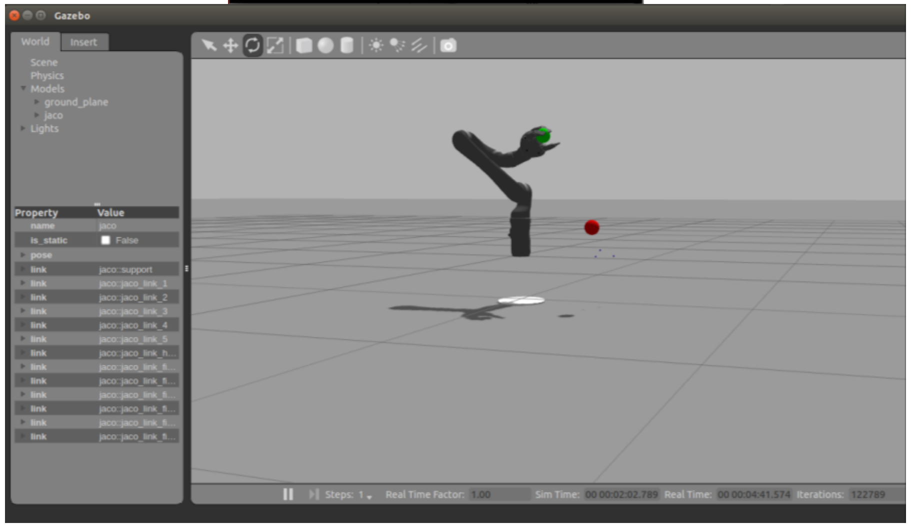
Tasks for today and tomorrow

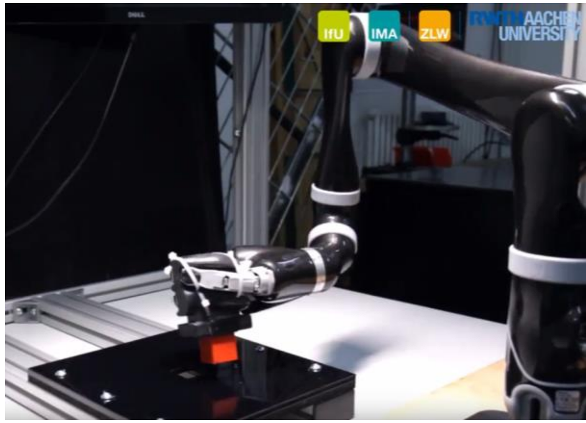
- Task 1:
  – Implement an LQR Backward and Forward pass
  – Try to understand it!
  – Test it with our test method

- Task 2:
  – Implement linearization of the dynamic model
  – Try to understand it!
  – Test it with our test-method
  – Test it on the Box2D Scenario

- Task 3:
  – Test it with Kinova Jaco 2 in simulation
  – Adjust cost function







Task 1 – Installation procedure

Download source code (do it in your home directory: `cd ~`):

```
git clone https://github.com/philippente/task1_lqr.git
```

Edit .bashrc to set environment variables:

```
gedit ~/.bashrc
```

At the end of file, the lines should look like this:

```
source /opt/ros/indigo/setup.bash
source /home/<USERNAME>/catkin_ws/devel/setup.bash
export
ROS_PACKAGE_PATH=$ROS_PACKAGE_PATH:/opt/ros/indigo/share:/opt/ros/indigo/stacks:/home//<USERNAME>/task1_lqr:/home/<USERNAME>/task1_lqr/src/gps_agent_pkg
```

Check if the blue part of the source folder and ROS_PACKAGE_PATH is correct!

Then save it and close it. Source the .bashrc (load the environment variables):

```
source ~/.bashrc
```

Now, compile some stuff:

```
cd task1_lqr
sh compile_proto.sh
catkin_make
```

Task 1 – Installation procedure

- Open PyCharm
- Import the folder *task1_lqr* as a new project
- Open within PyCharm: python/gps/algorithm/algorithm_traj_opt.py
- **Task: Implement the forward and backward pass of an LQR! Look at the website for advices:**
  https://philippente.github.io/irobotics.html

- You can test your implementation with a little test program
  - using a terminal, open the directory *task1_lqr*
  - Start the program with: `python python/gps/lqr_test.py`
  - Was it successful?

Thanks for your attention!

Gaussians

- Univariate Gaussian
- Multivariate Gaussian
- Law of Total Probability
- Conditioning (Bayes' rule)

- Disclaimer: lots of lienar algebra in next few lectures. In fact, pretty much all computations with Gaussians will be reduced to linear algebra!