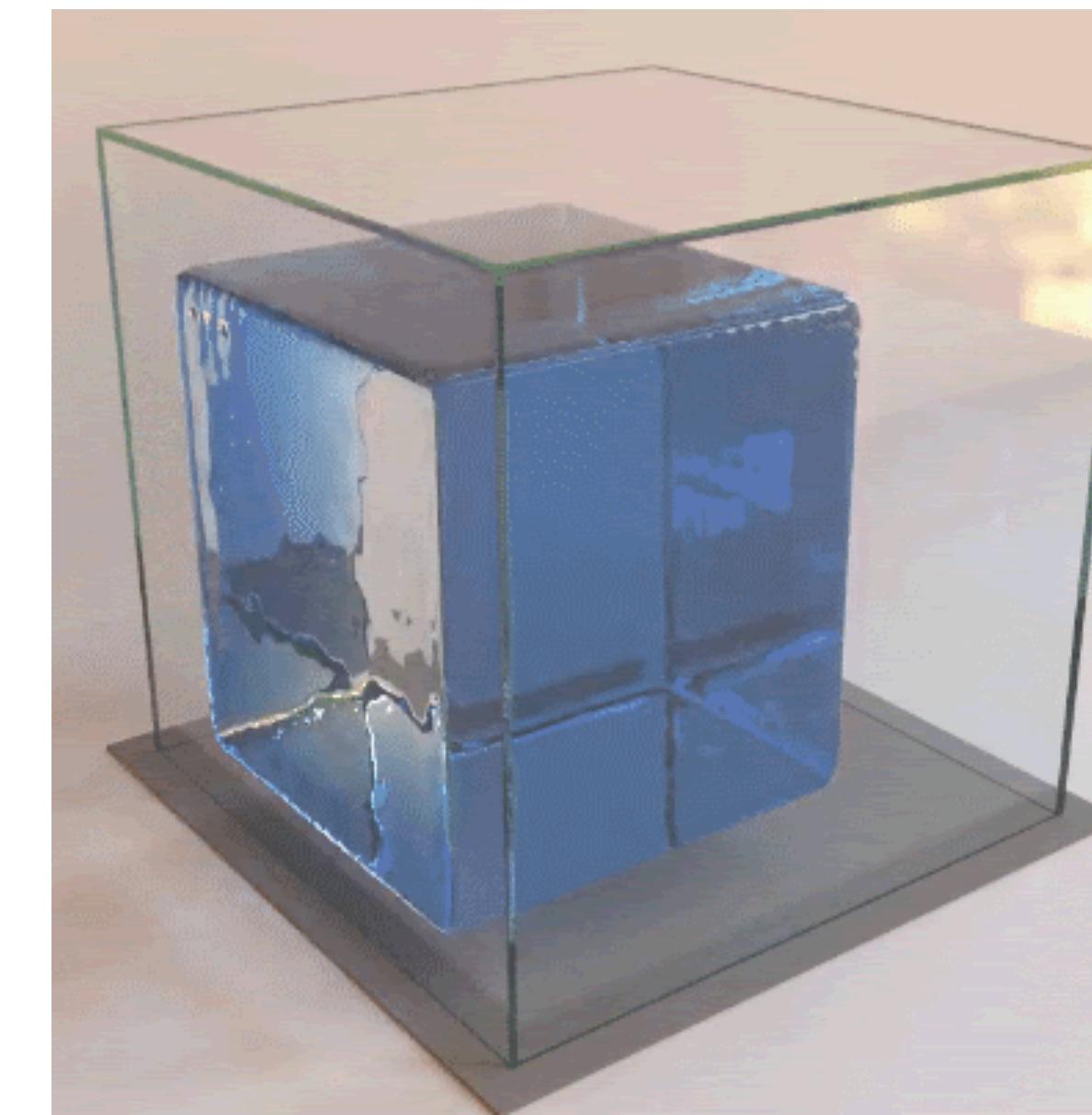
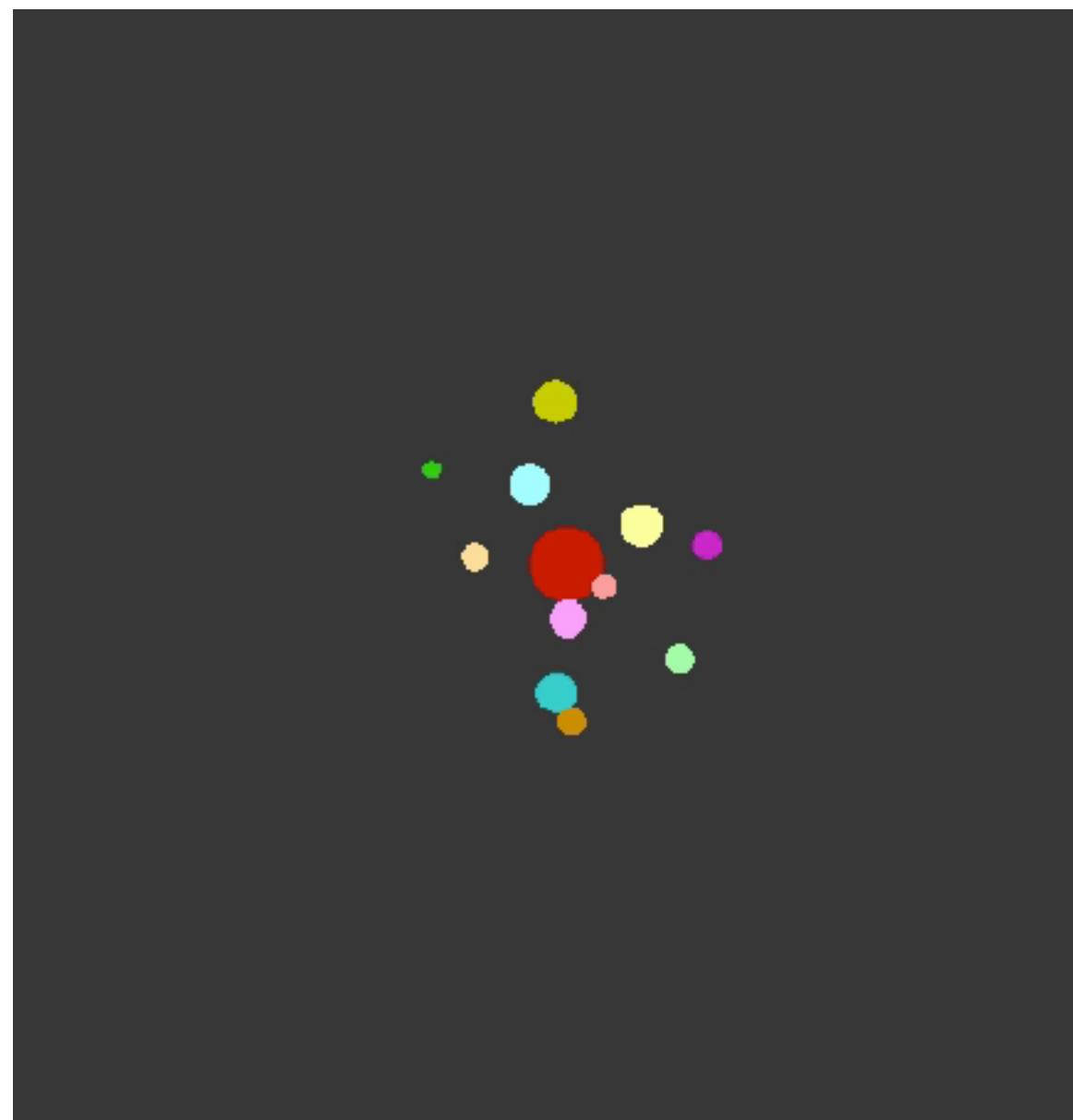


Structured models of objects, relations, and physics



Peter Battaglia



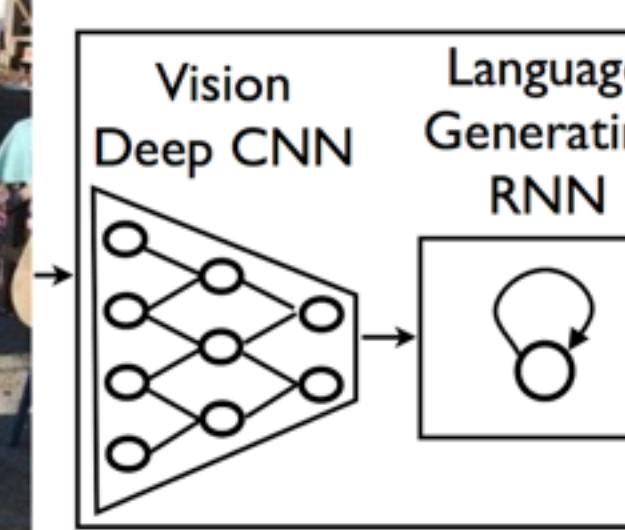
DeepMind

Interexperimental Machine Learning Workshop / Data Science Seminar
CERN (remote) - October 20, 2020

What is deep learning good at?

What is deep learning good at?

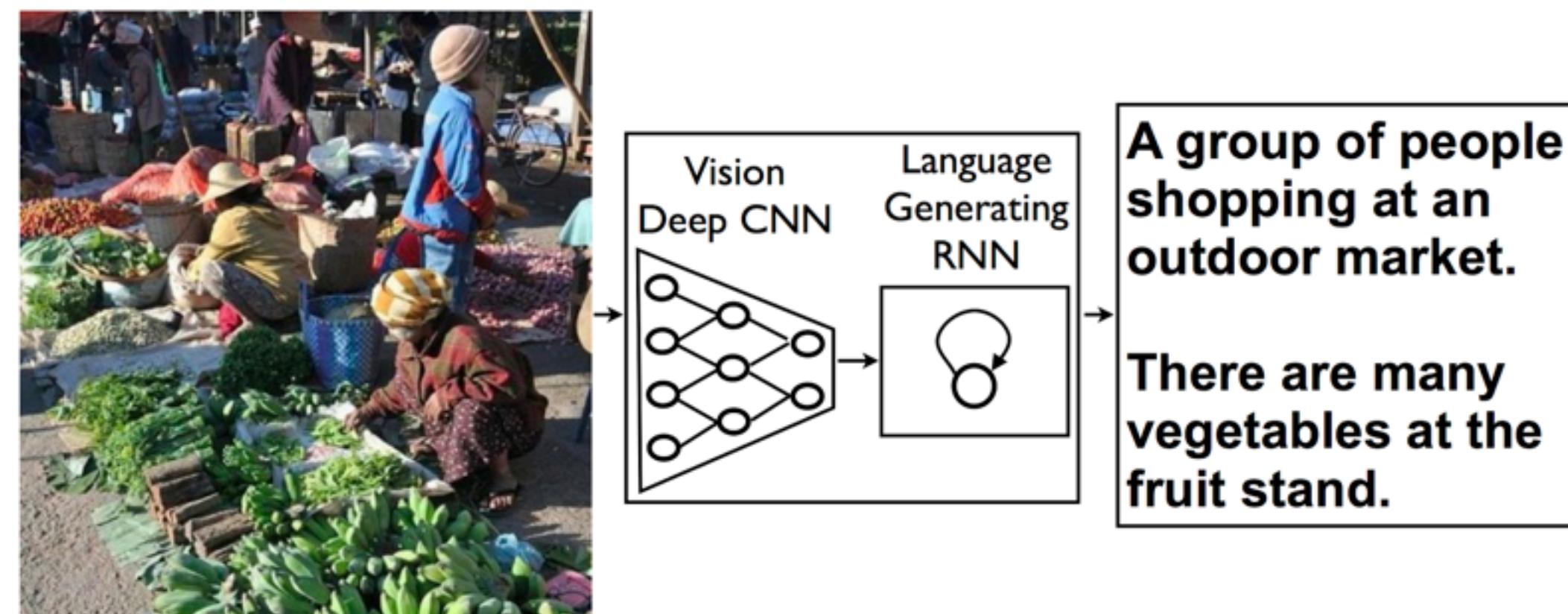
Image and language processing



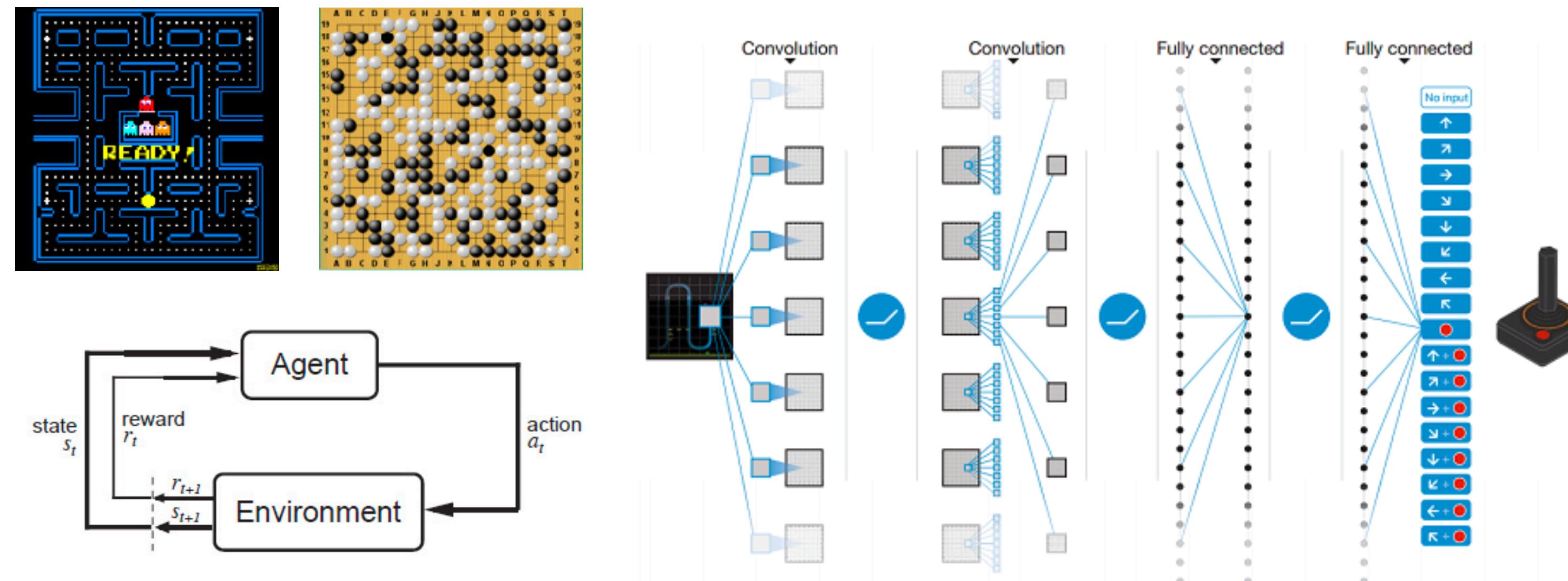
A group of people shopping at an outdoor market.
There are many vegetables at the fruit stand.

What is deep learning good at?

Image and language processing



Games (via deep reinforcement learning)



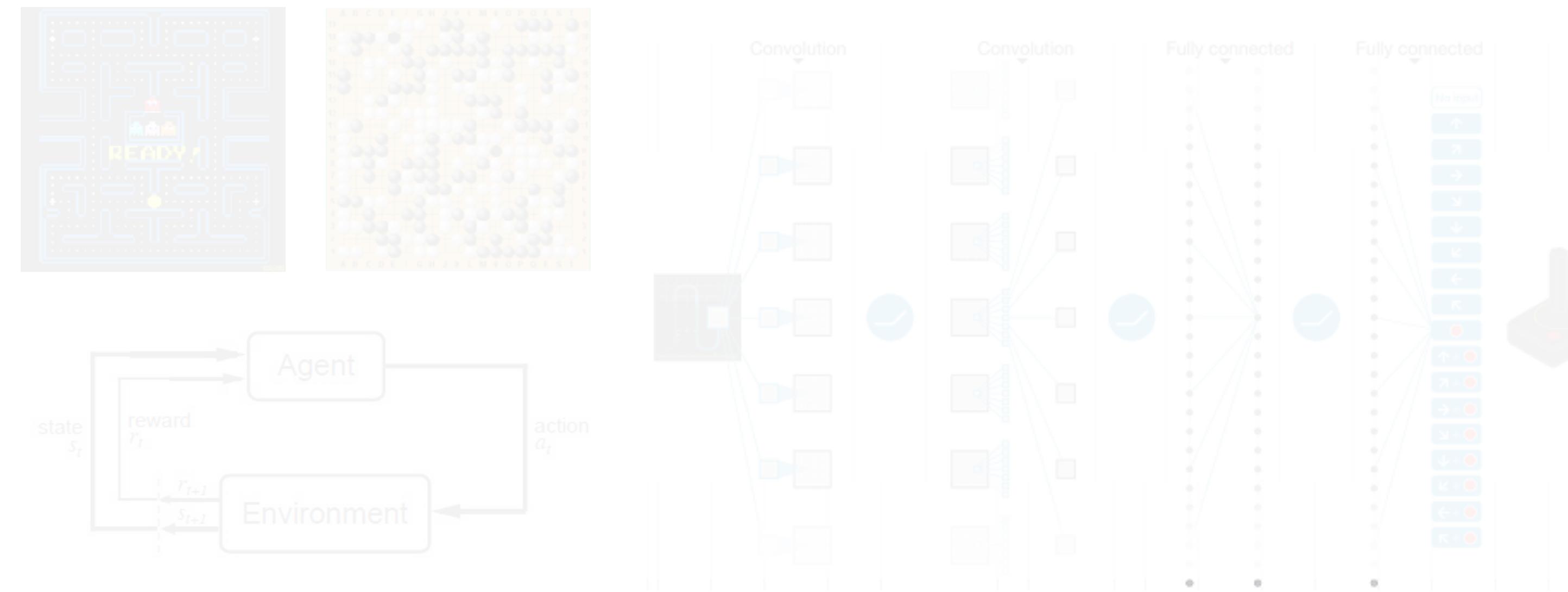
What is deep learning good at?

Image and language processing

What do many of deep learning's successes have in common?



Games (via deep reinforcement learning)



What is deep learning good at?

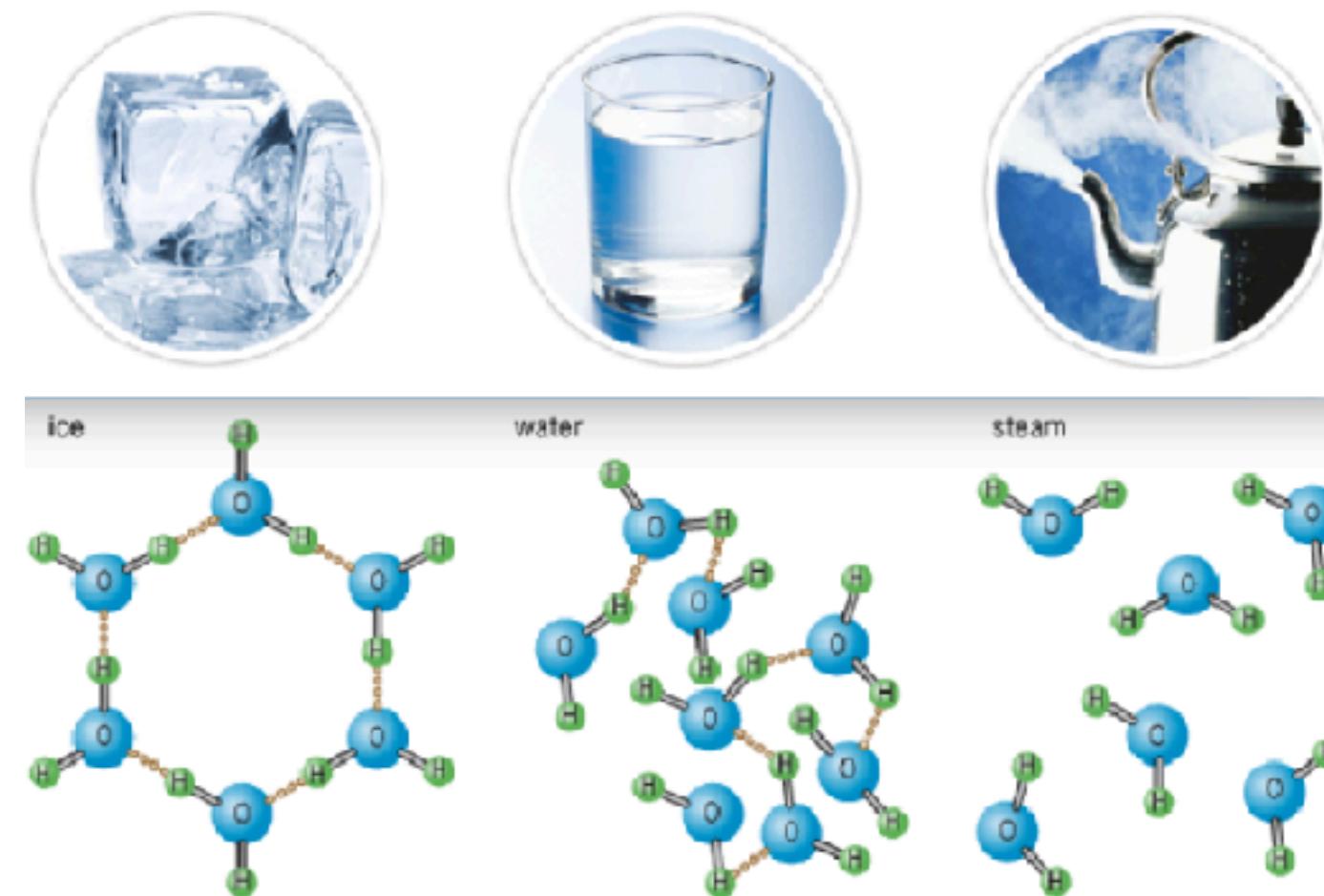
Image and language processing

What do many of deep learning's successes have in common?

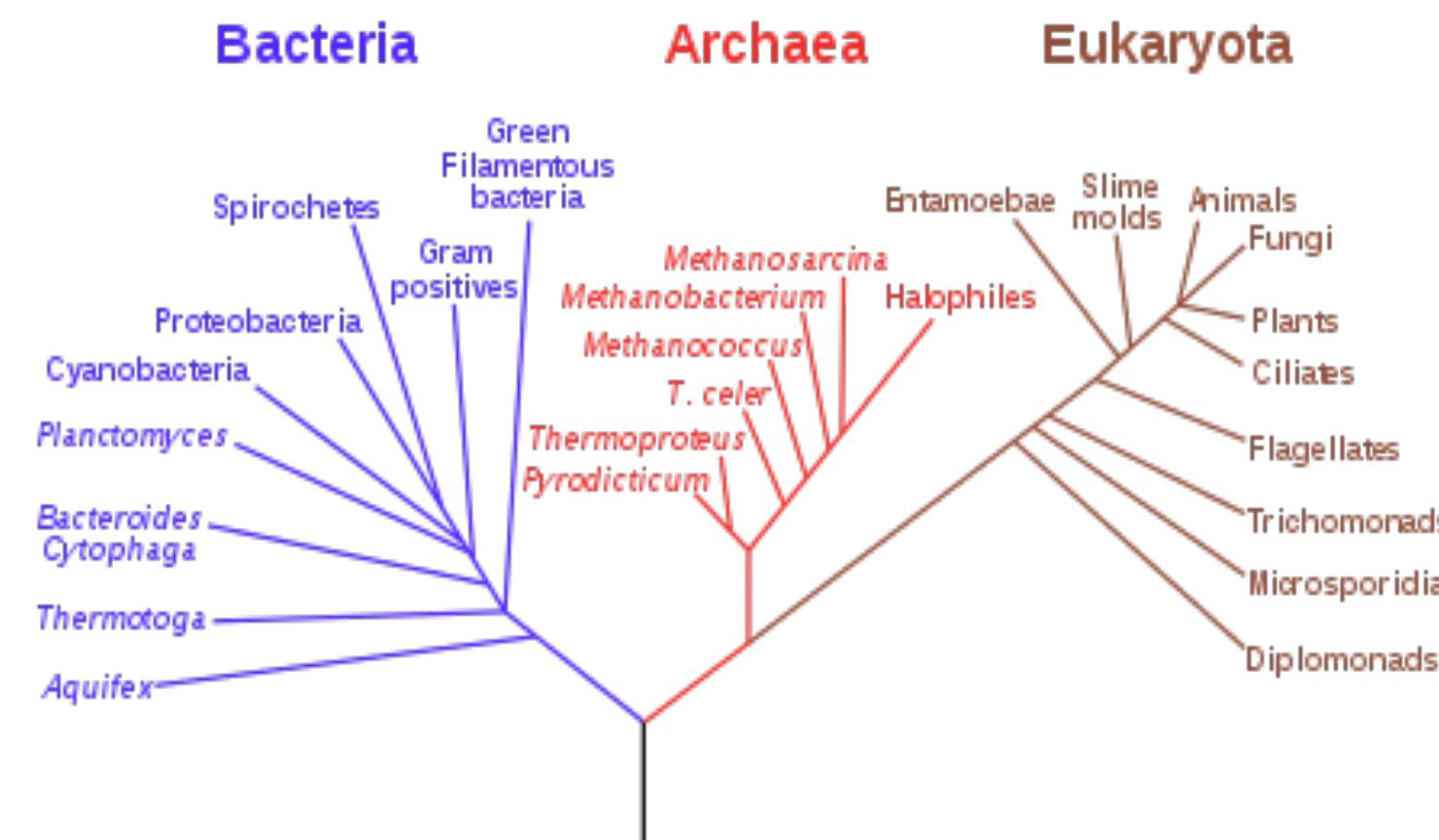
- * **Vectors**
- * **Grids**
- * **Sequences**

But many important domains are richly structured

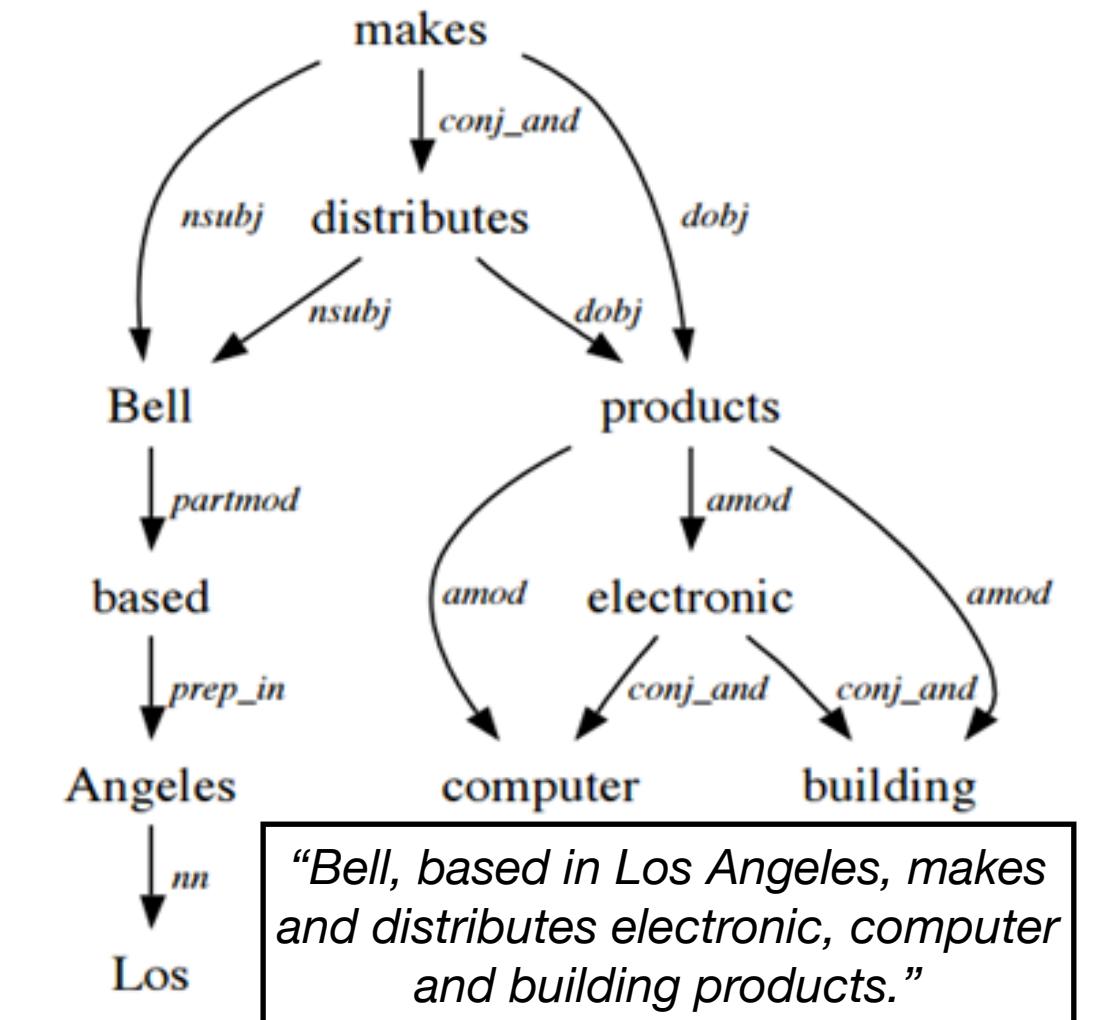
Molecules



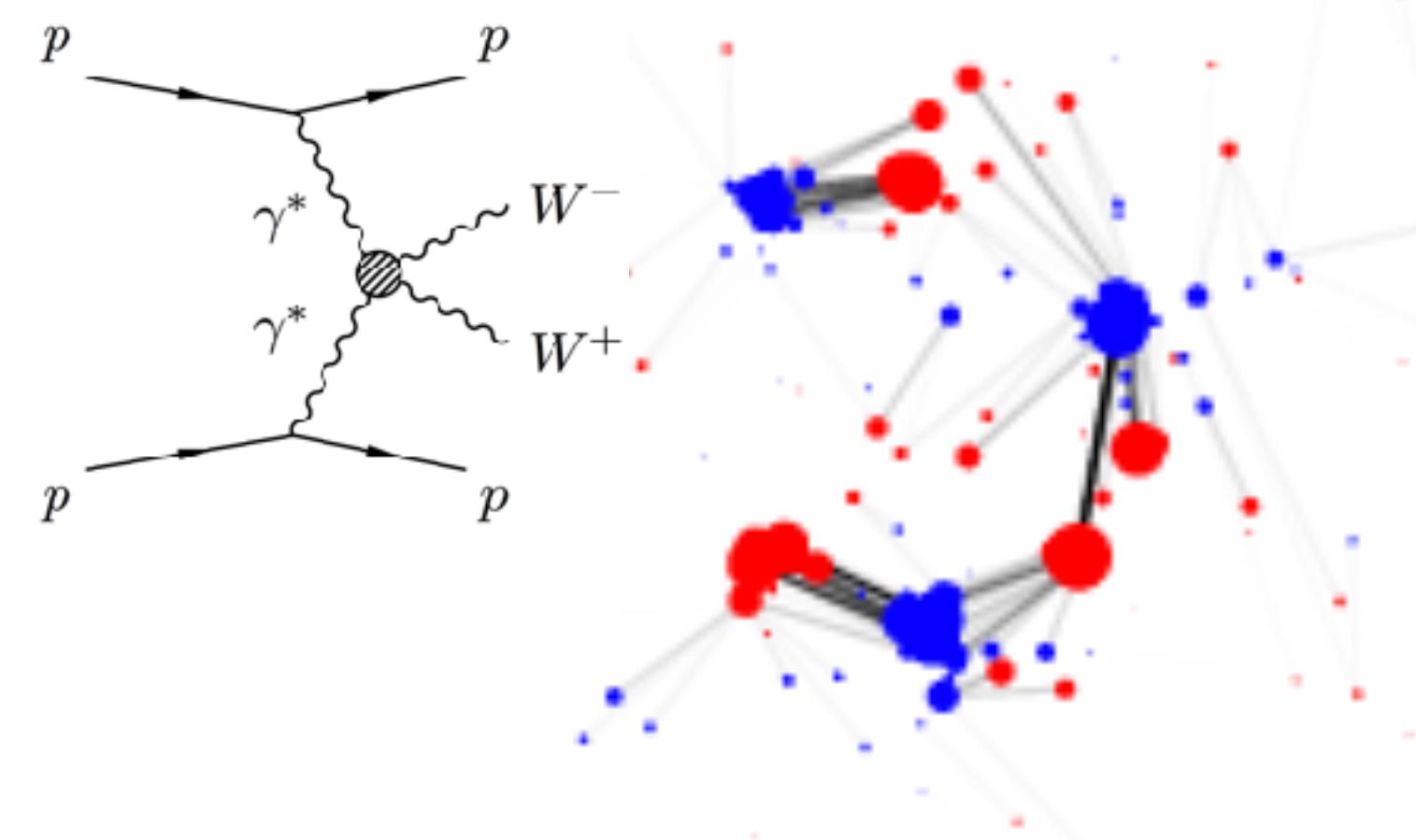
Biological species



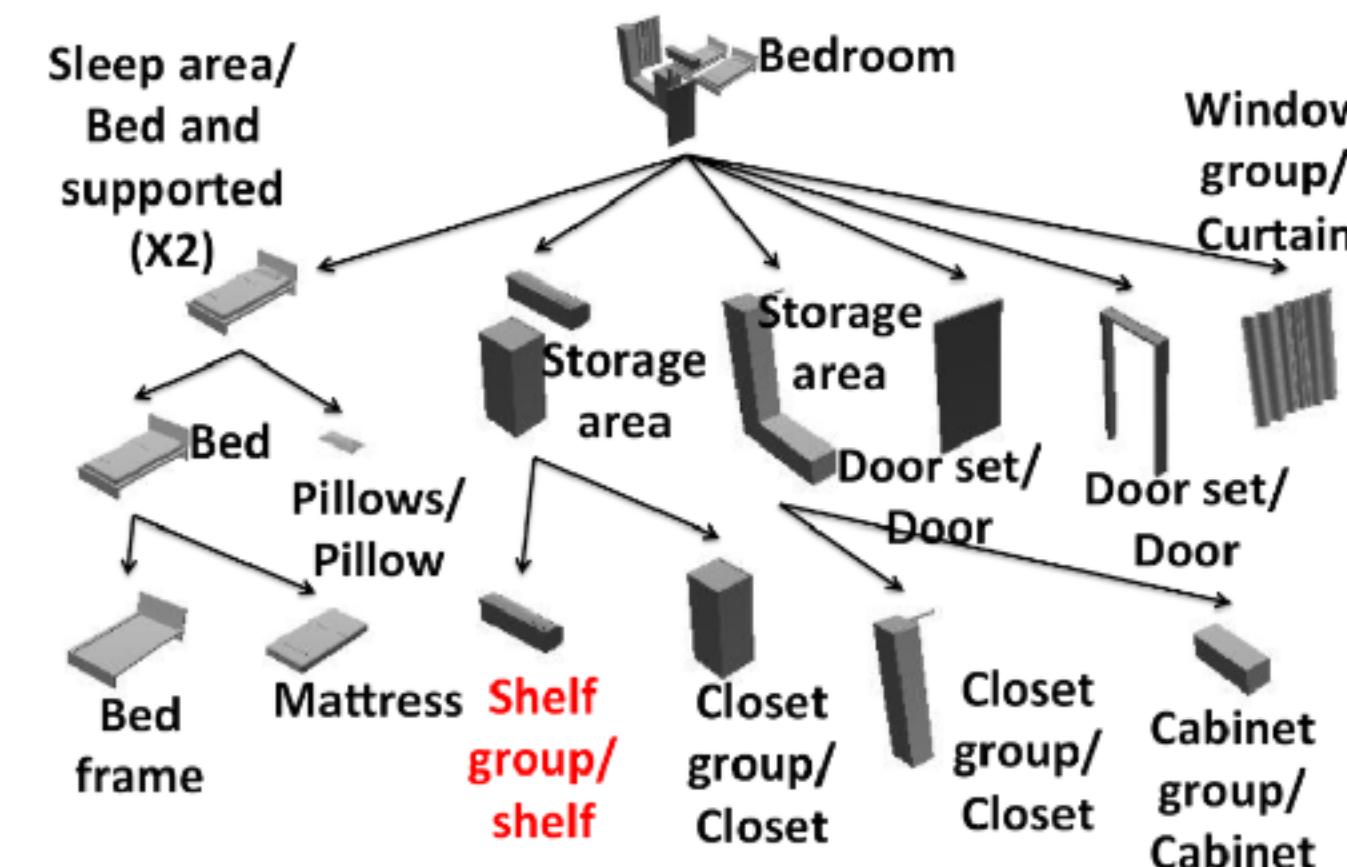
Natural language



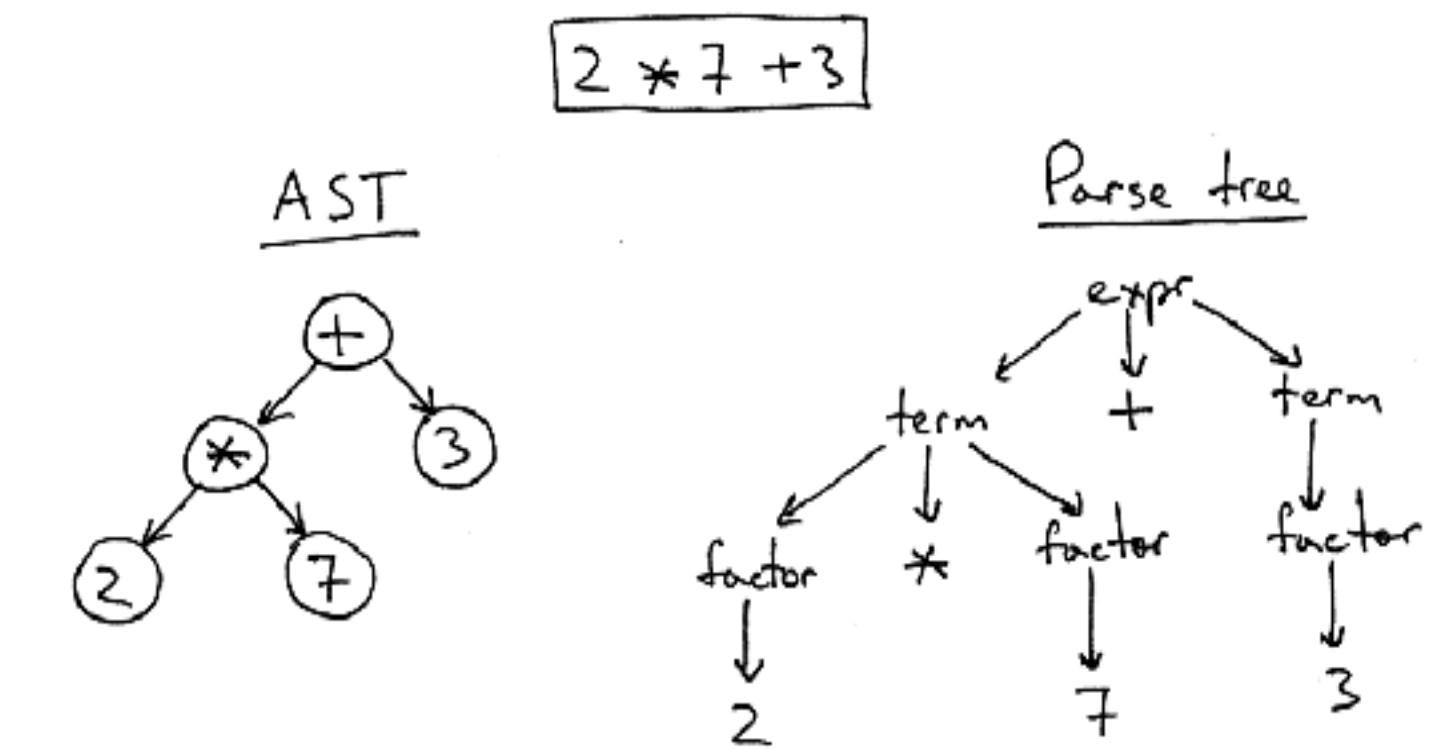
Sub-atomic particles



Everyday scenes



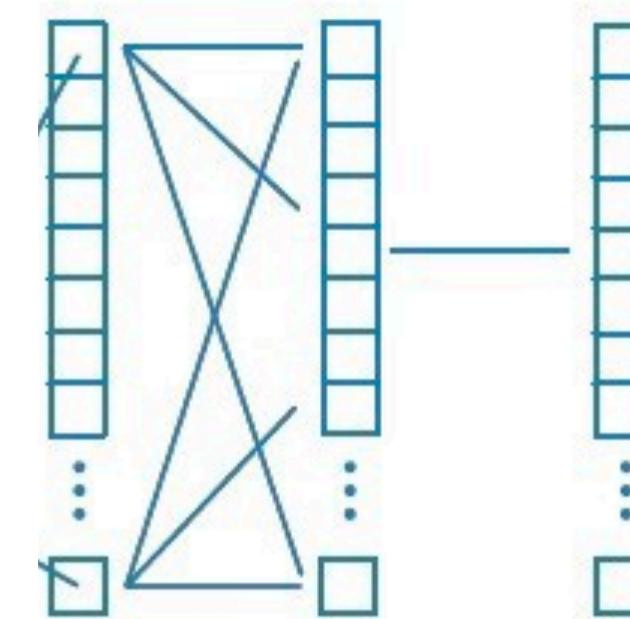
Code



What tool do I need?

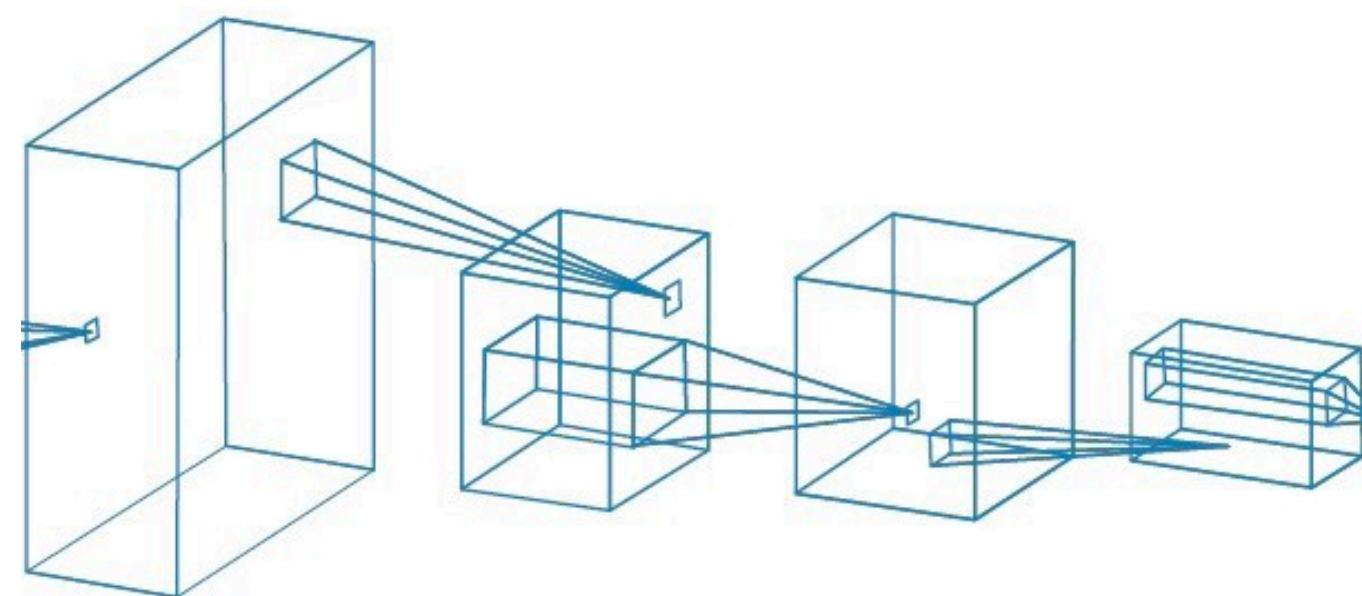
“My data is **vectors**”:

Multi-layer perceptron (MLP)



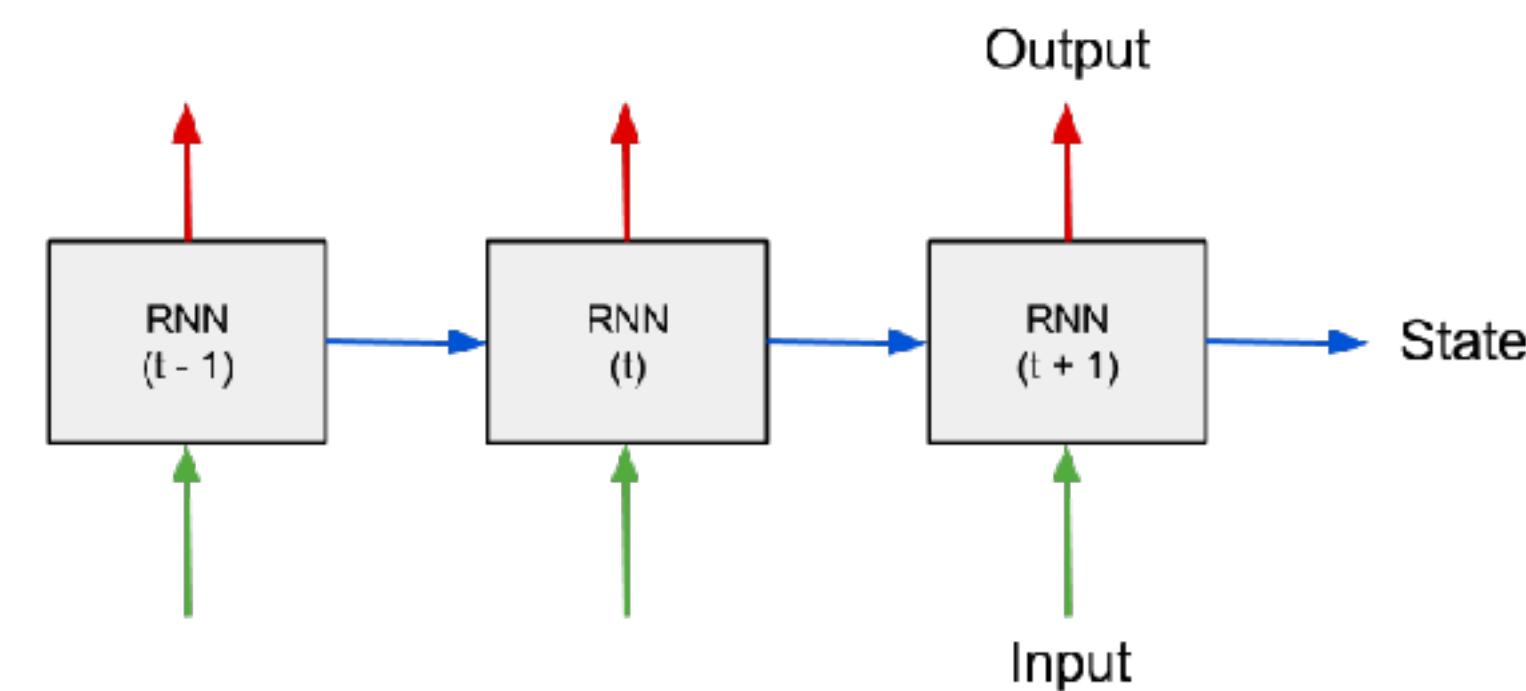
“My data is **grids**”:

Convolutional neural network (CNN)



“My data is **sequences**”:

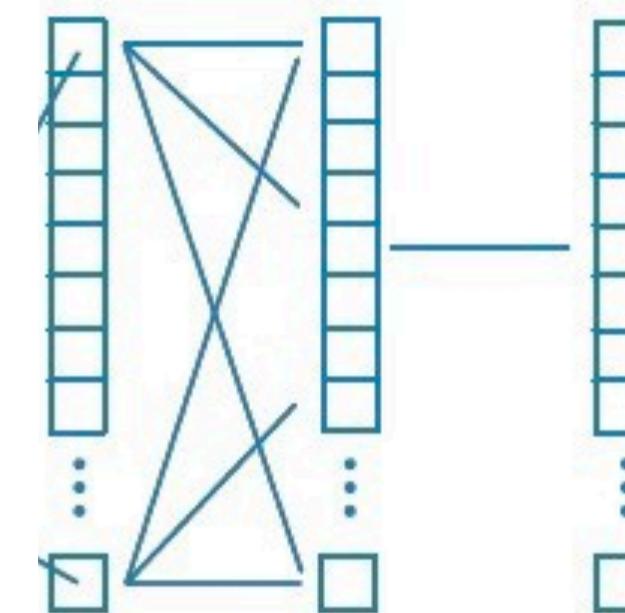
Recurrent neural network (RNN)



What tool do I need?

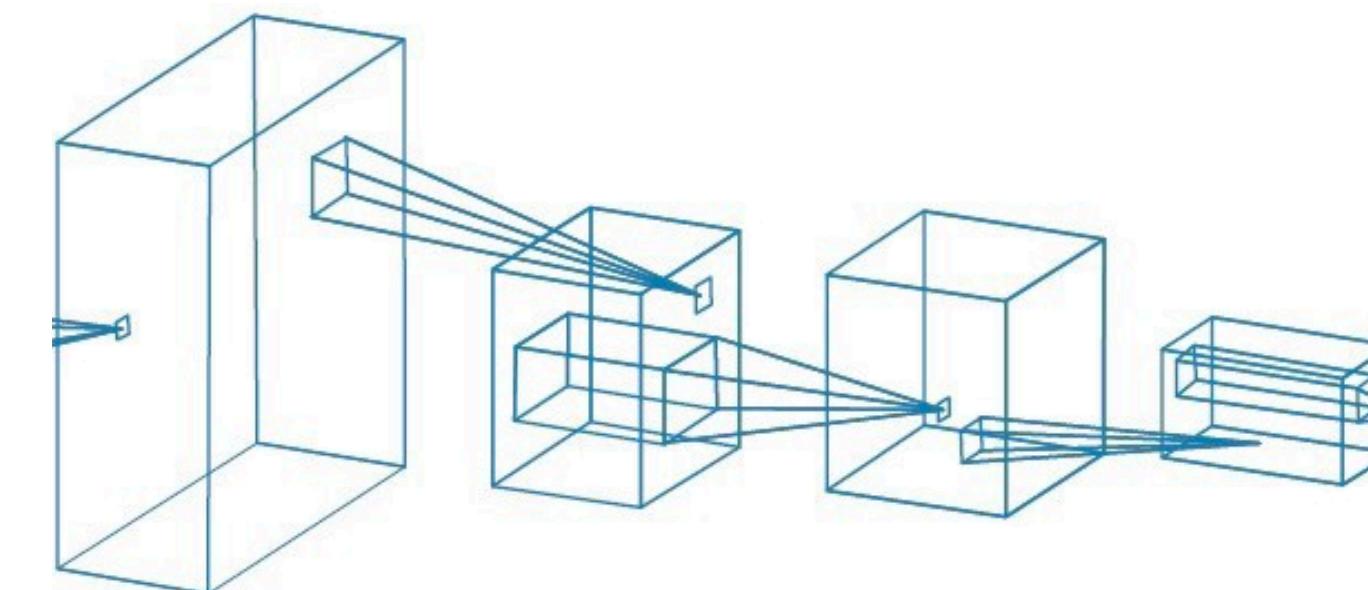
“My data is **vectors**”:

Multi-layer perceptron (MLP)



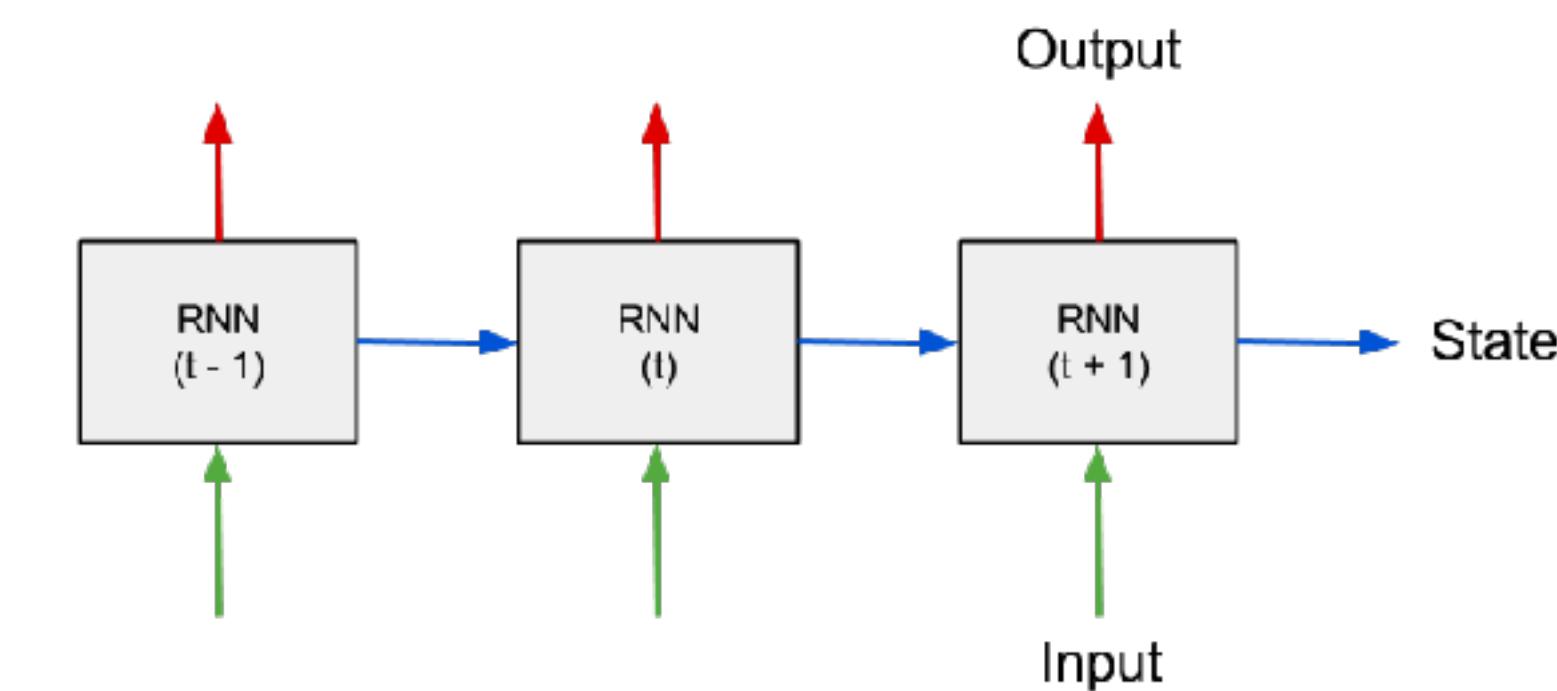
“My data is **grids**”:

Convolutional neural network (CNN)

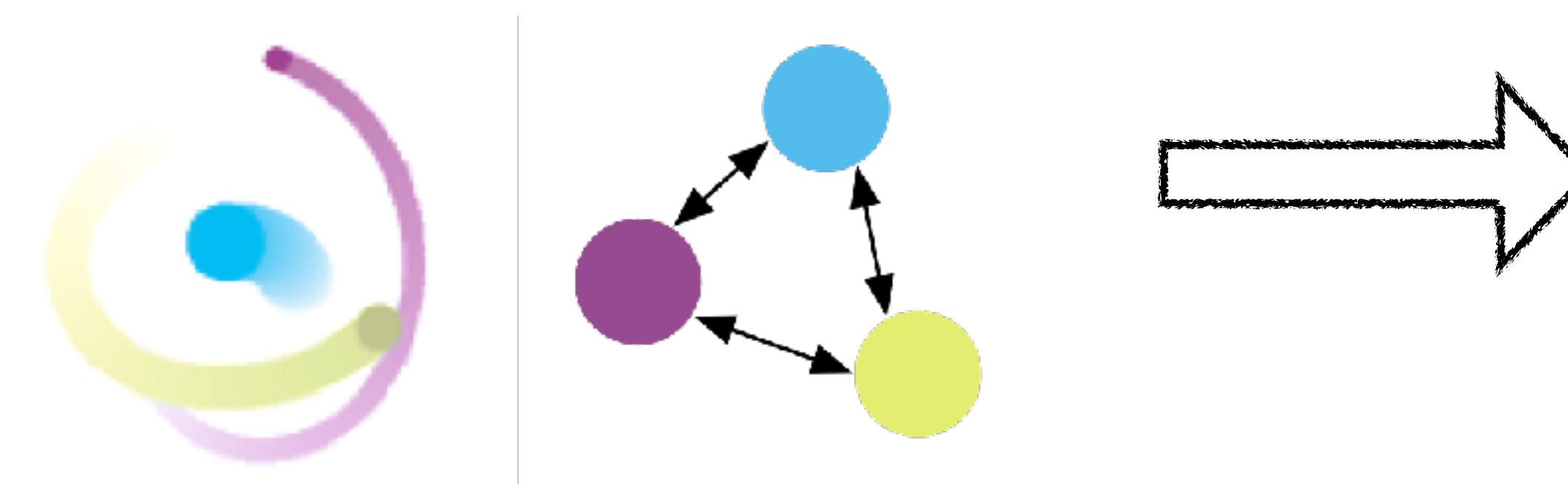


“My data is **sequences**”:

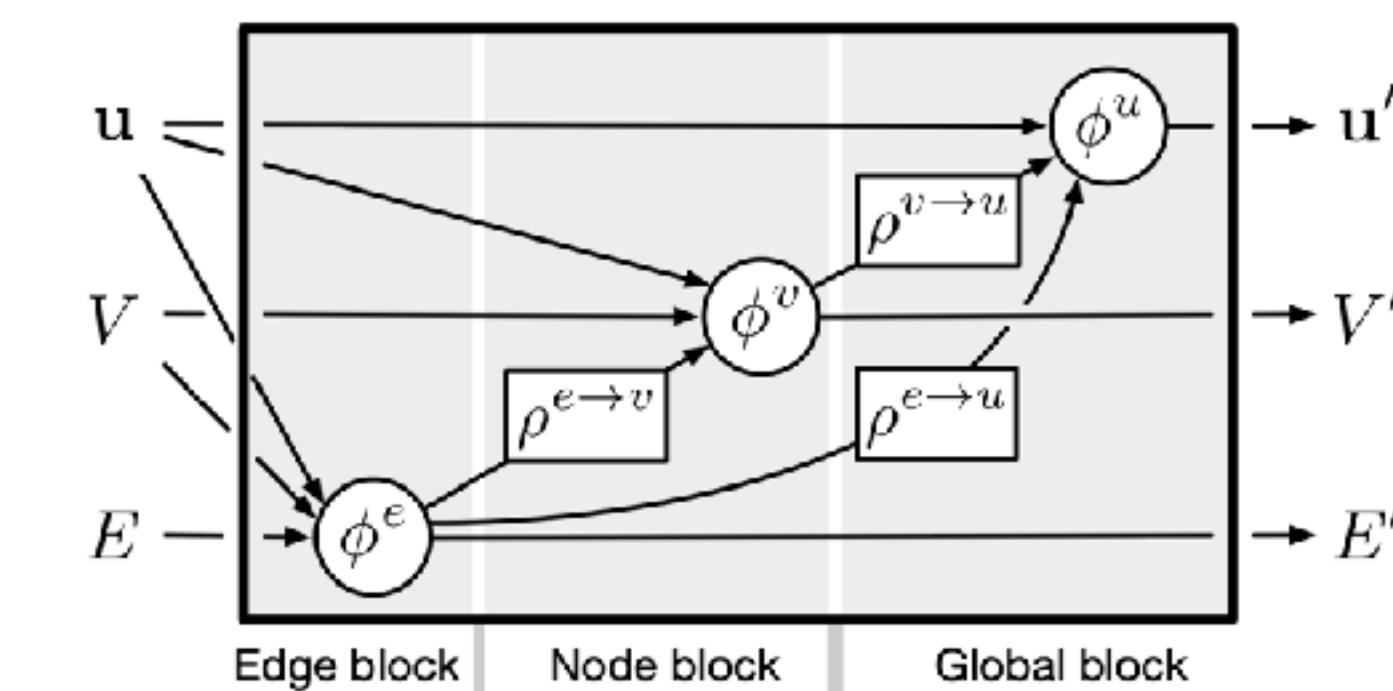
Recurrent neural network (RNN)



“My data is **structured**”:



Graph neural network (GNN)



Background: Graph Neural Networks

General idea

- Analogous to a convolutional network, but over arbitrary graphs (rather than just grids)
- Can learn to reason about entities and their relations

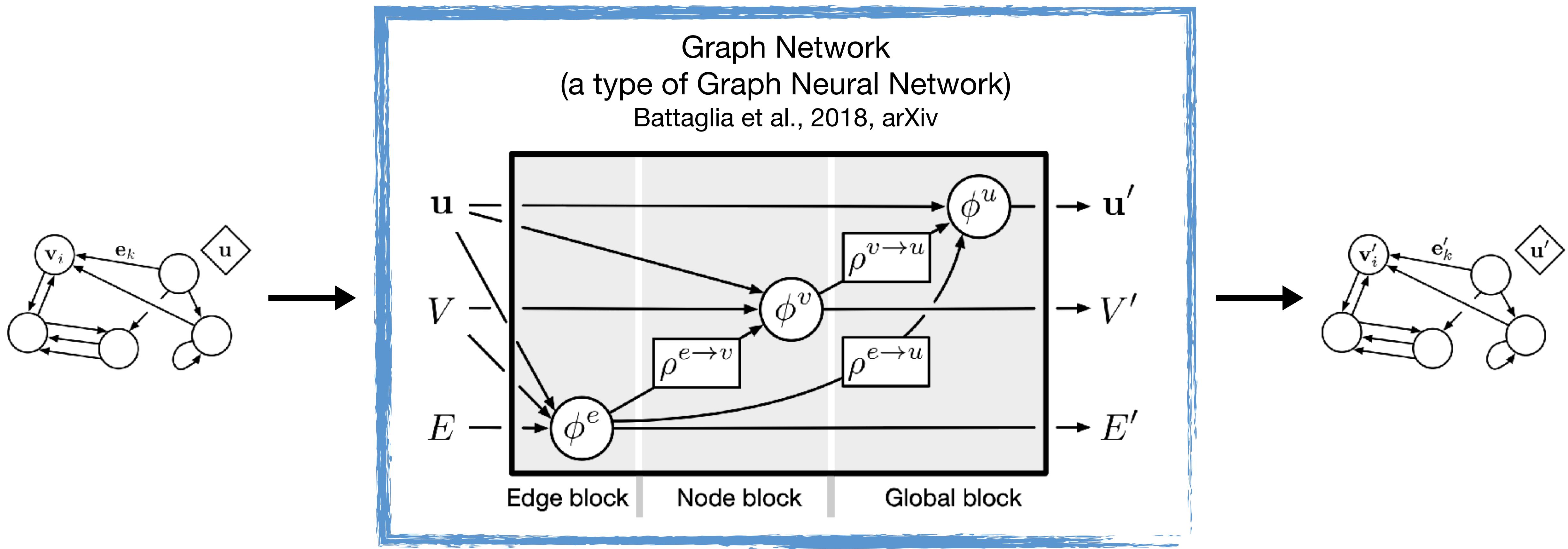
Key literature surveys

- [Scarselli et al. \(2009\) "The Graph Neural Network Model".](#)
Summarizes the initial papers on the topic from ~2005-2009. Original innovation, general formalism.
- [Li et al. \(2015\) “Gated graph sequence neural networks”.](#)
Simplified the formalism, trained via backprop, used RNNs for sharing update steps across time.
- [Bronstein et al. \(2016\) “Geometric deep learning: going beyond Euclidean data”.](#)
Survey of spectral and spatial approaches for deep learning on graphs.
- [Gilmer et al. \(2017\) “Neural Message Passing for Quantum Chemistry”.](#)
Introduced “message-passing neural network” (MPNNs) formalism, unifying various approaches such as graph convolutional networks.
- [Battaglia et al. \(2018\). “Relational inductive biases, deep learning, and graph networks”.](#)
Introduced the “graph network” (GN) formalism, extends MPNNs, unifies non-local neural networks/self-attention/Transformer.

Graph Networks (GNs)

Why do we need another graph neural network variant?

- We designed GNs to be both expressive, and easy to implement
- A GN block is a “graph-to-graph” function approximator
 - The output graph’s structure (number of nodes and edge connectivity) matches the input graph’s
 - The output graph-, node-, and edge-level attributes will be functions of the input graph’s

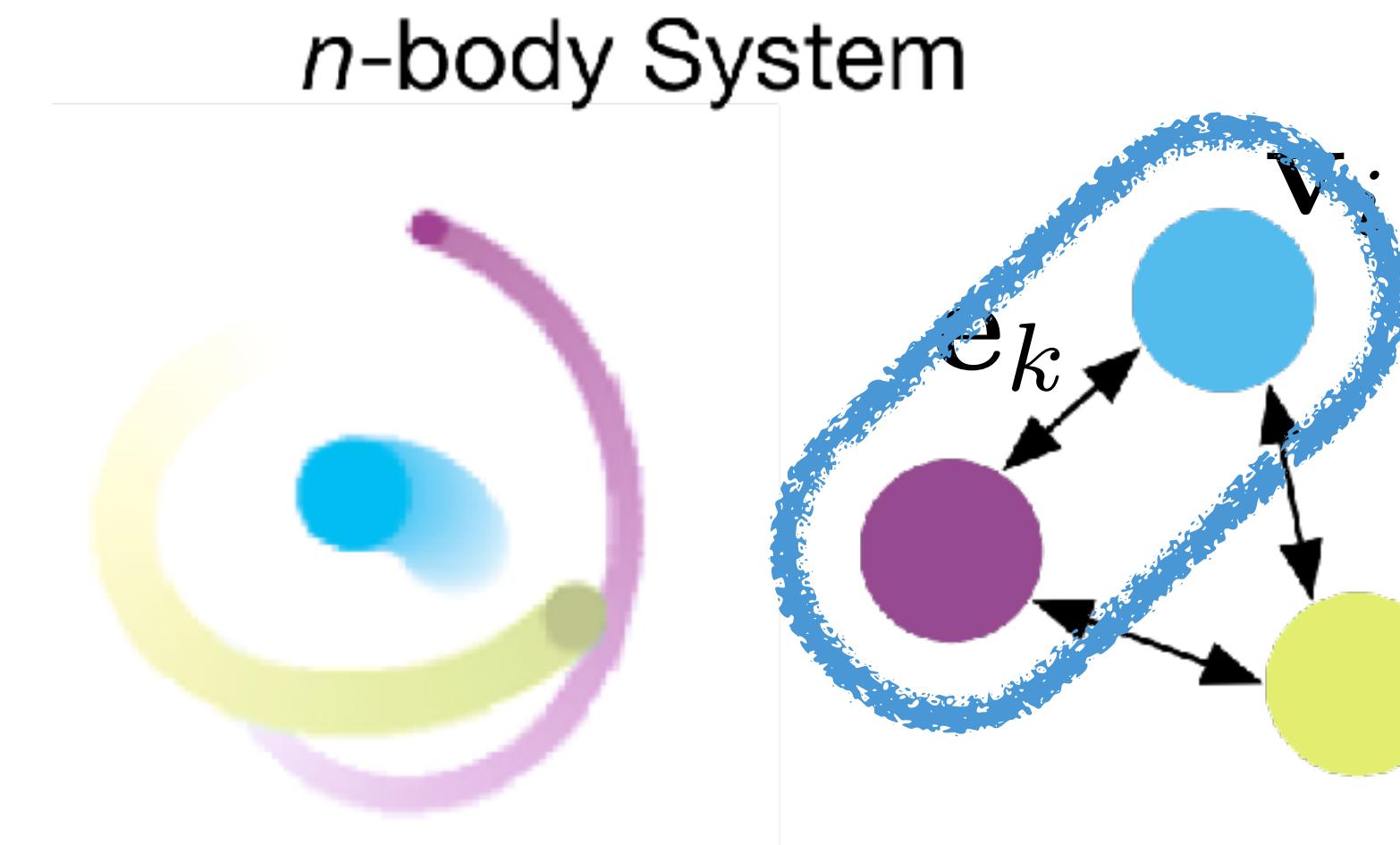


Interaction Network: Learning simulation as message-passing

n-body System



Interaction Network: Learning simulation as message-passing

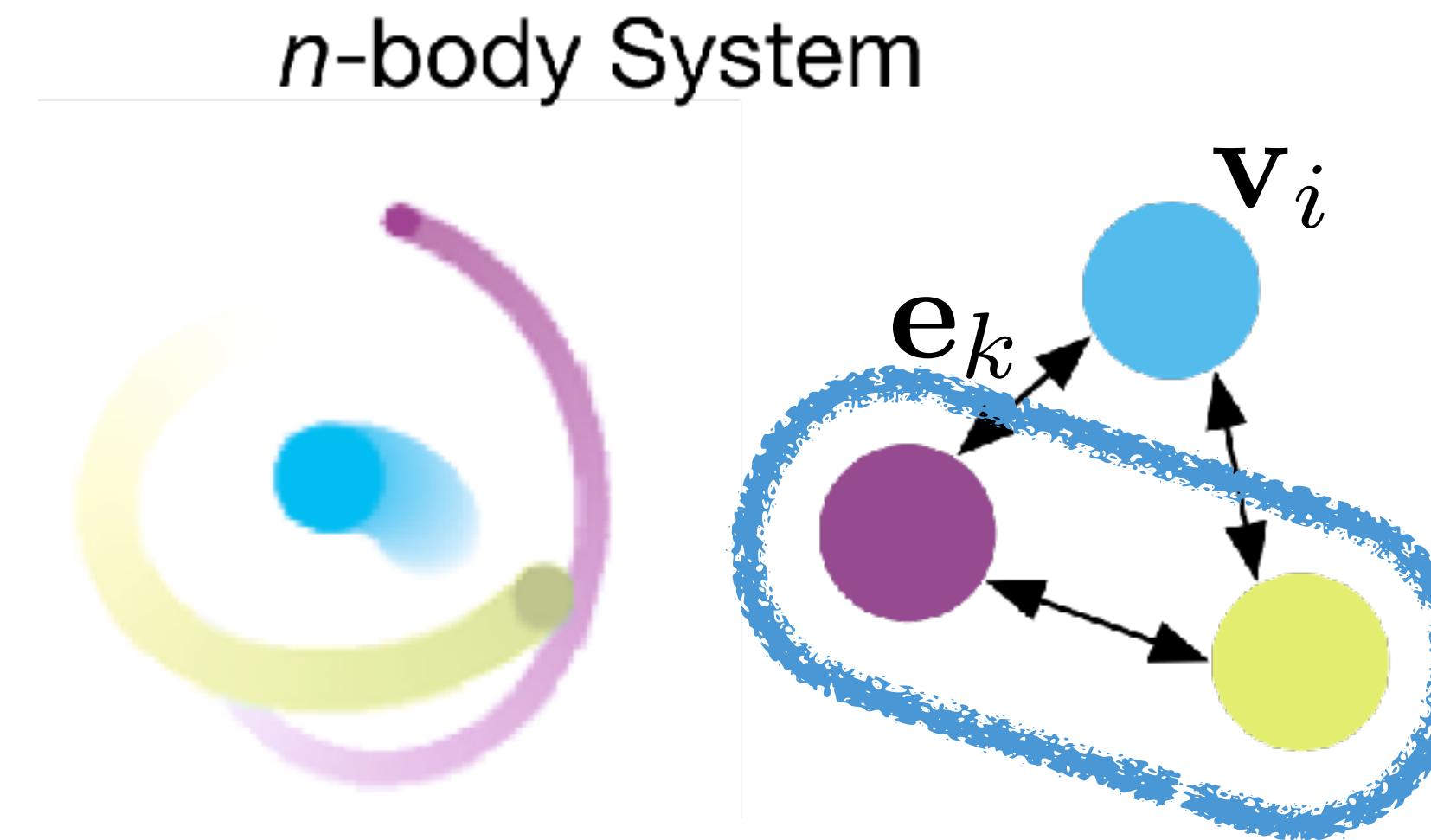


Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

- Compute “message” from node and edge attributes associated with an edge

Interaction Network: Learning simulation as message-passing

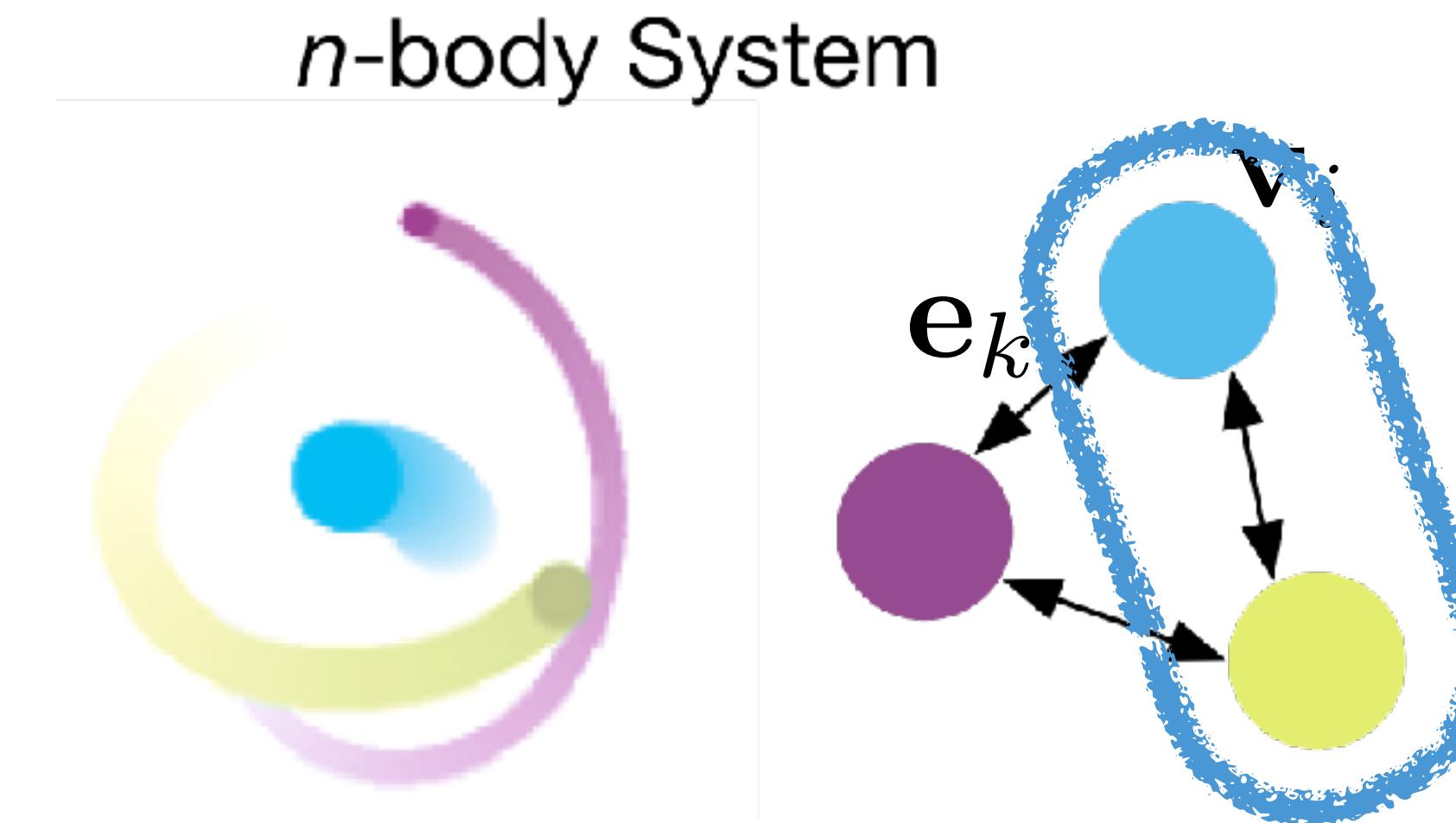


Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

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Interaction Network: Learning simulation as message-passing

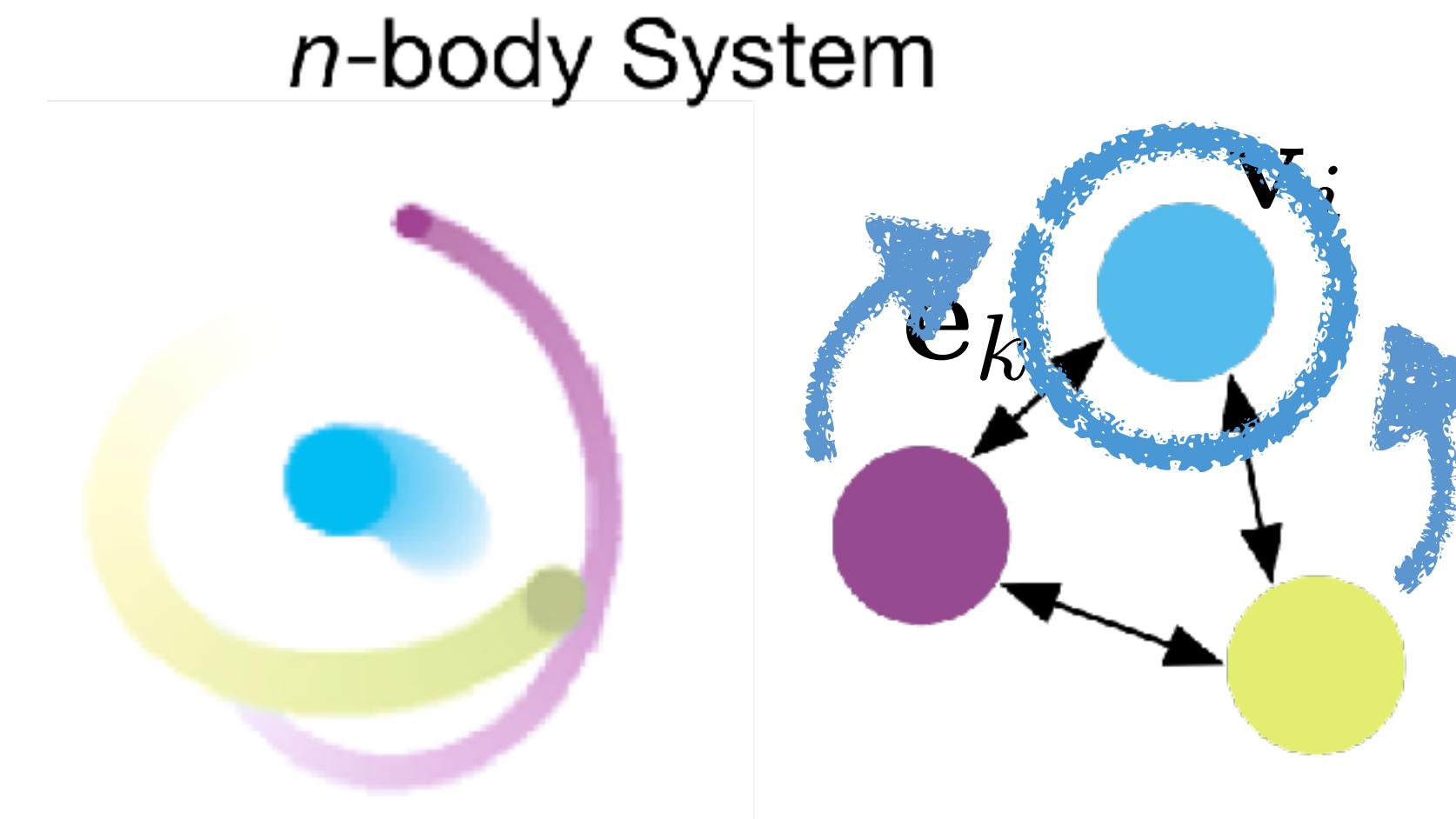


Edge function

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- Compute “message” from node and edge attributes associated with an edge

Interaction Network: Learning simulation as message-passing



Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

- Compute “message” from node and edge attributes associated with an edge

Message aggregation

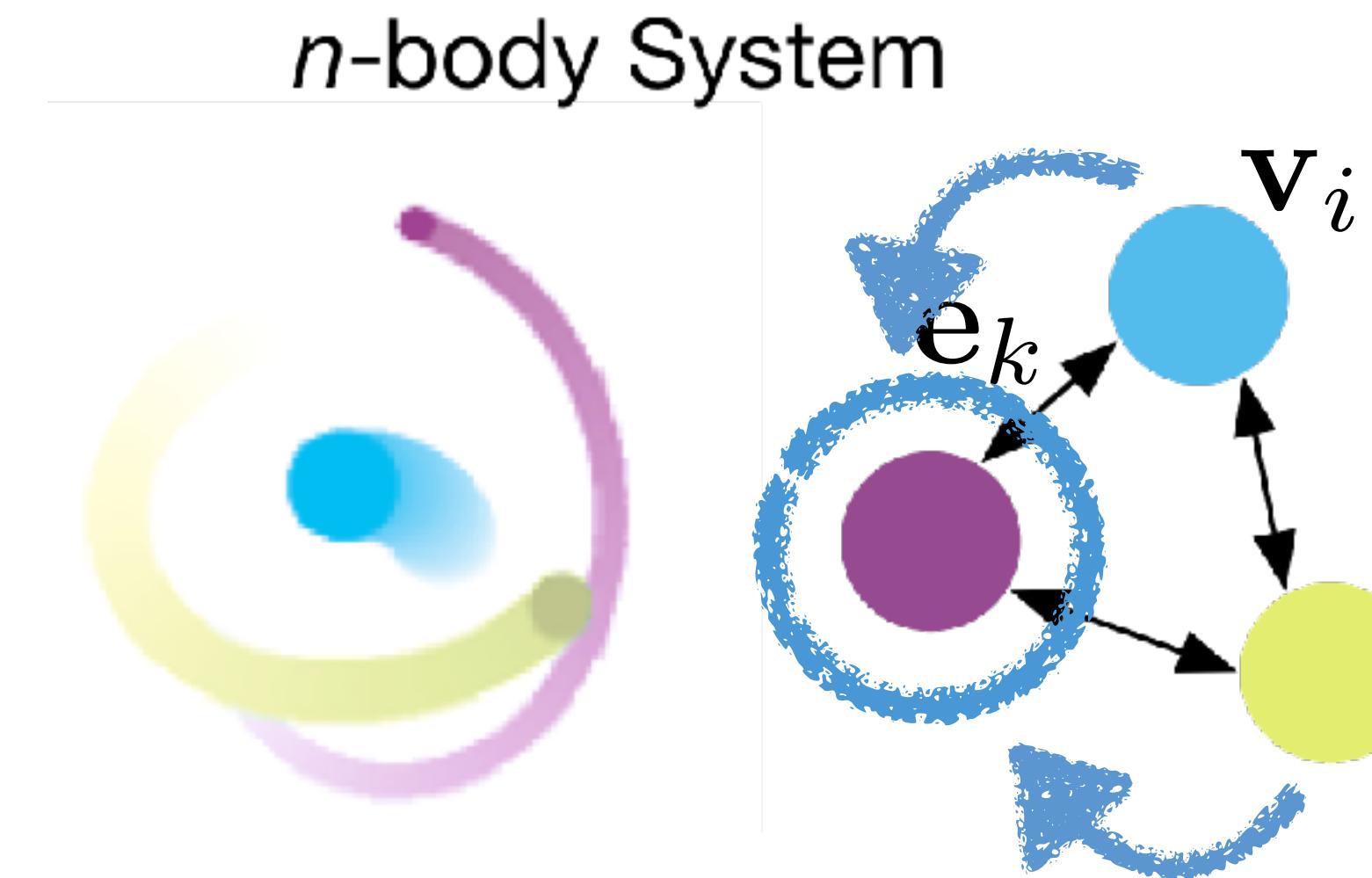
$$\bar{\mathbf{e}}'_i \leftarrow \sum_{r_k=i} \mathbf{e}'_k$$

Node function

$$\mathbf{v}'_i \leftarrow \phi^v(\bar{\mathbf{e}}'_i, \mathbf{v}_i, \mathbf{u})$$

- Update node info from previous node state and aggregated “messages”

Interaction Network: Learning simulation as message-passing



Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

- Compute “message” from node and edge attributes associated with an edge

Message aggregation

$$\bar{\mathbf{e}}'_i \leftarrow \sum_{r_k=i} \mathbf{e}'_k$$

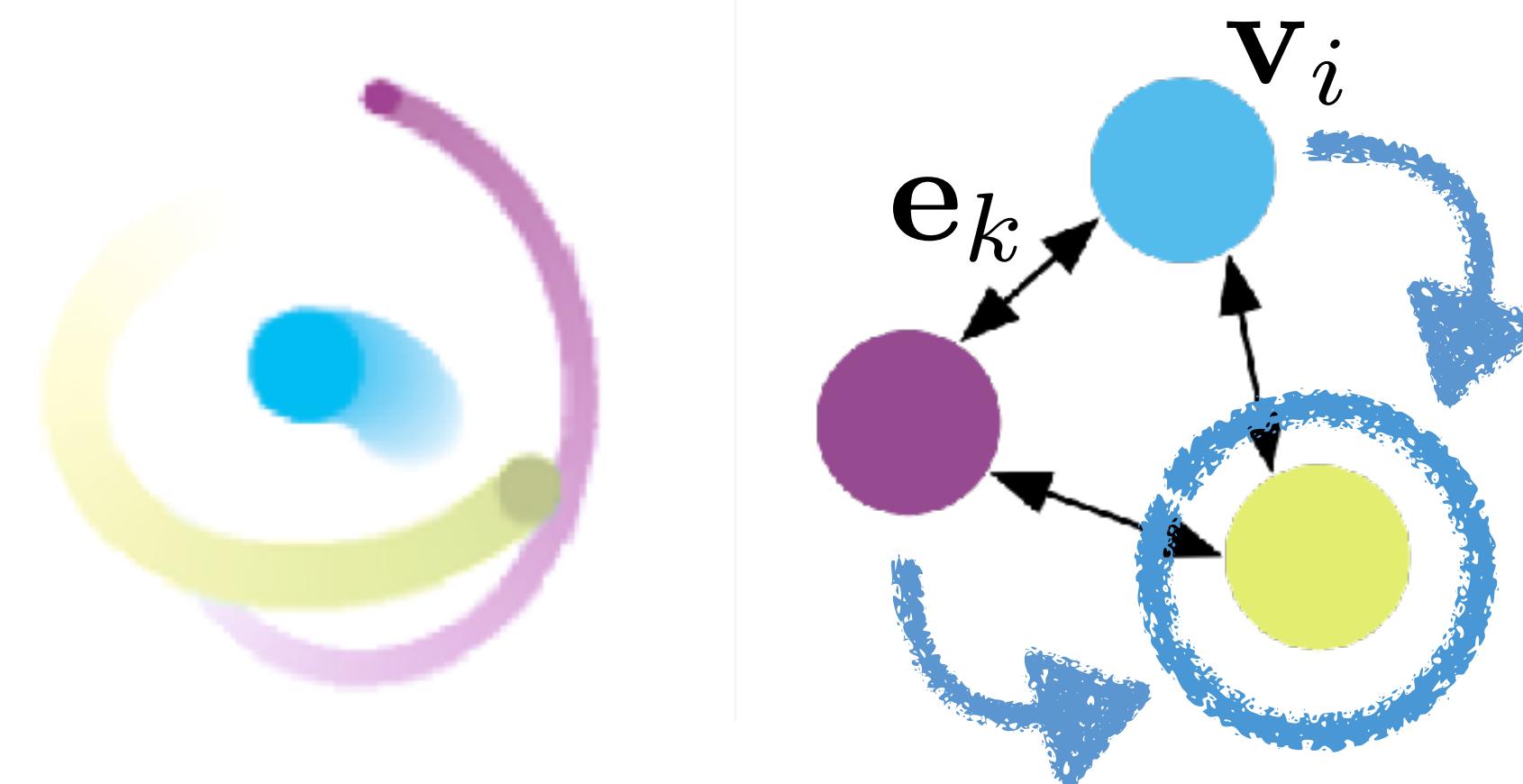
Node function

$$\mathbf{v}'_i \leftarrow \phi^v(\bar{\mathbf{e}}'_i, \mathbf{v}_i, \mathbf{u})$$

- Update node info from previous node state and aggregated “messages”

Interaction Network: Learning simulation as message-passing

n-body System



Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

- Compute “message” from node and edge attributes associated with an edge

Message aggregation

$$\bar{\mathbf{e}}'_i \leftarrow \sum_{r_k=i} \mathbf{e}'_k$$

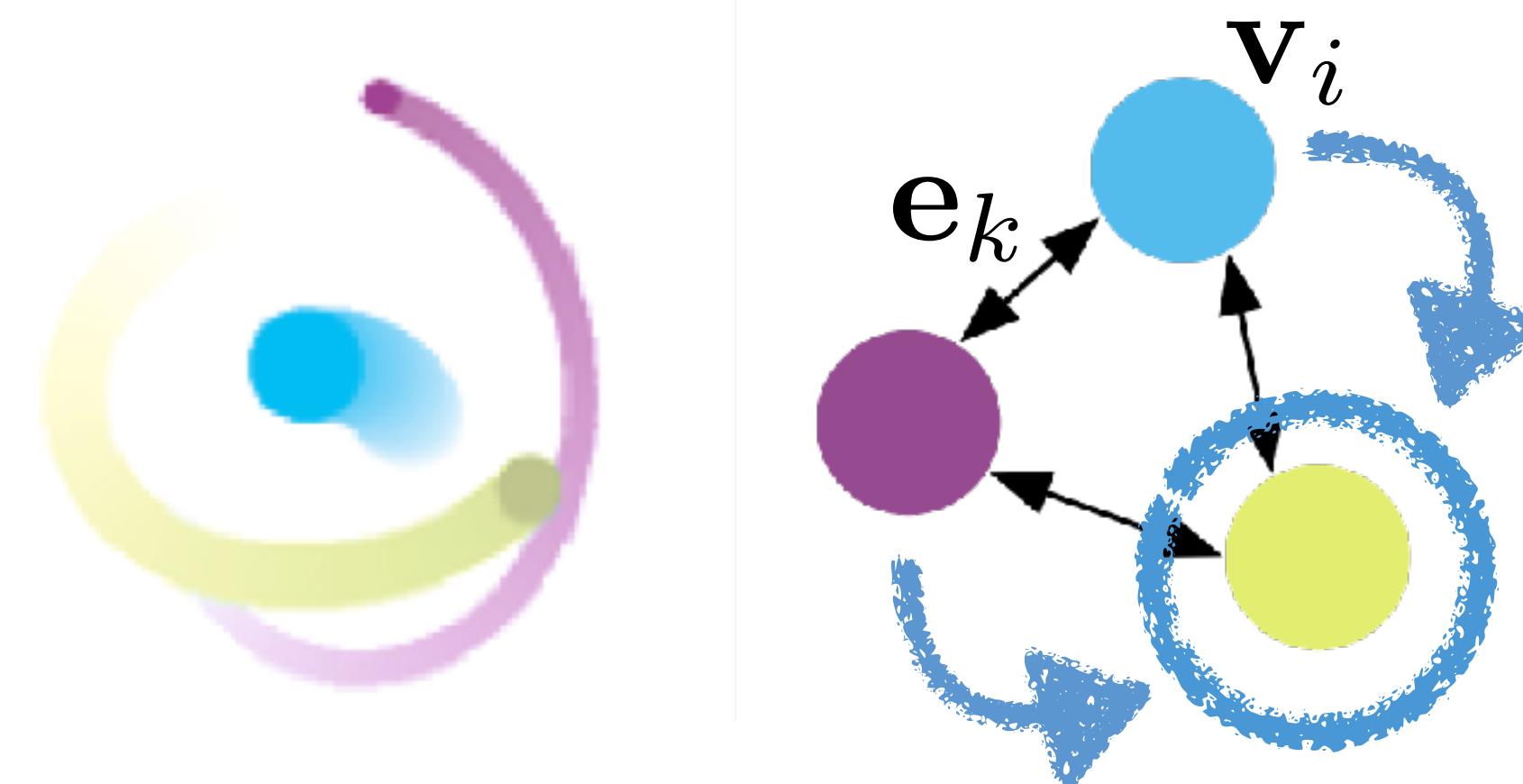
Node function

$$\mathbf{v}'_i \leftarrow \phi^v(\bar{\mathbf{e}}'_i, \mathbf{v}_i, \mathbf{u})$$

- Update node info from previous node state and aggregated “messages”

Interaction Network: Learning simulation as message-passing

n-body System



Edge function

$$\mathbf{e}'_k \leftarrow \phi^e(\mathbf{e}_k, \mathbf{v}_{r_k}, \mathbf{v}_{s_k})$$

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Message aggregation

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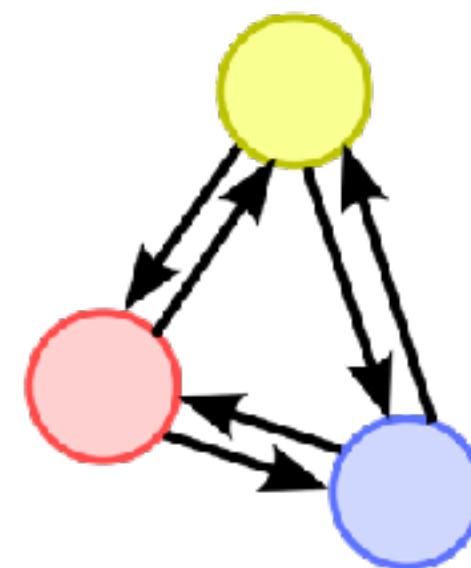
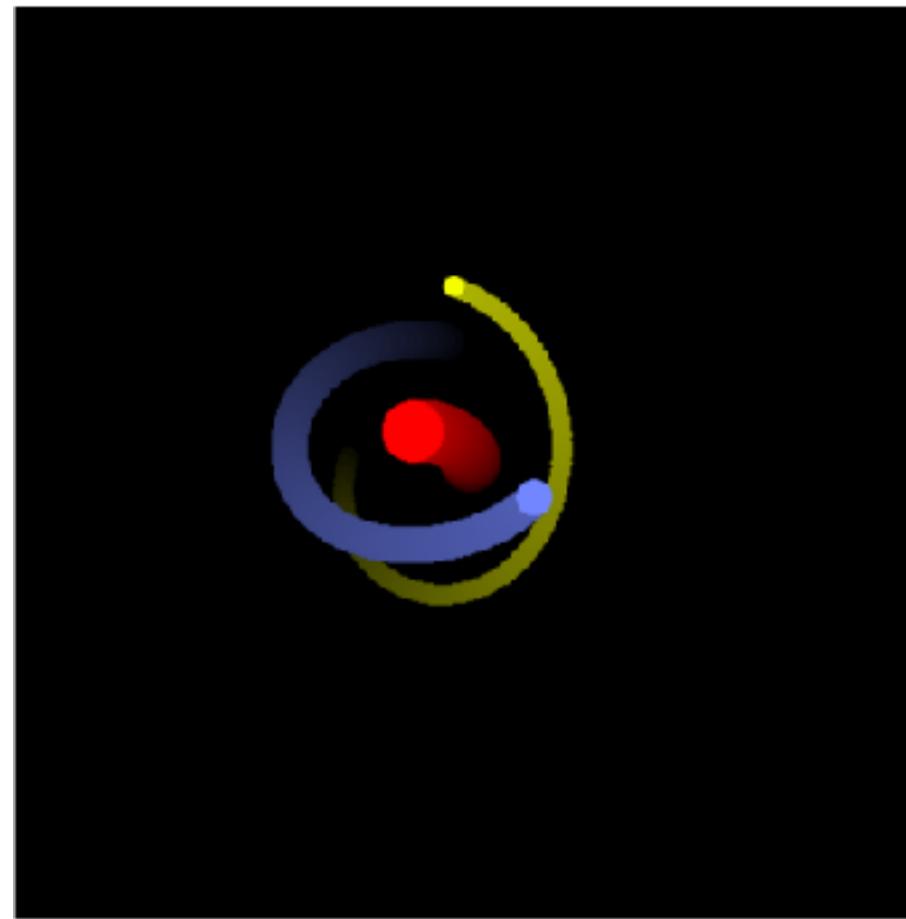
- Update node info from previous node state and aggregated “messages”

Trained to predict body coordinates



Physical systems as graphs

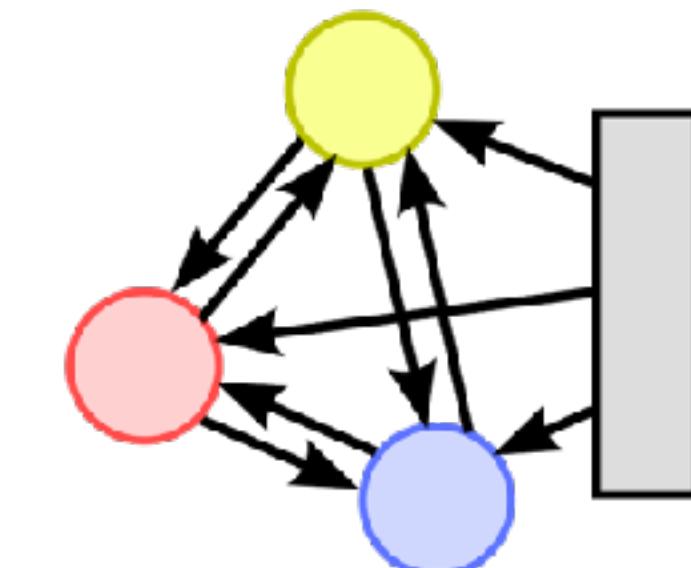
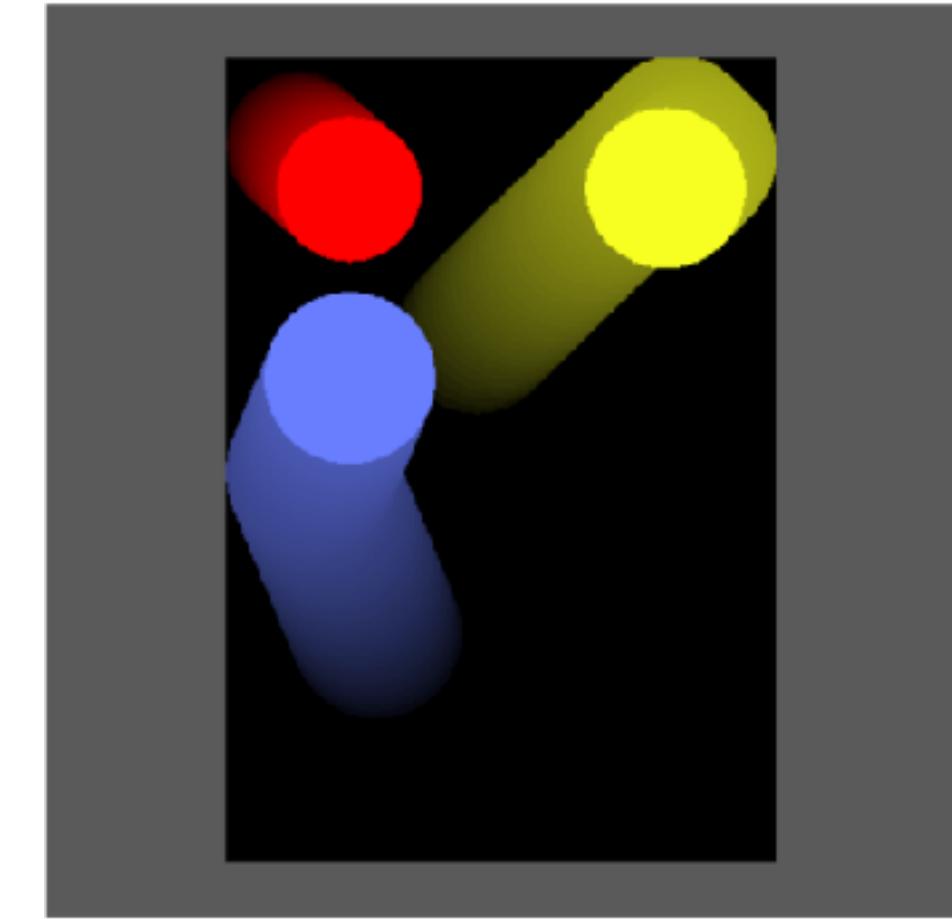
n-body



Nodes: bodies

Edges: gravitational forces

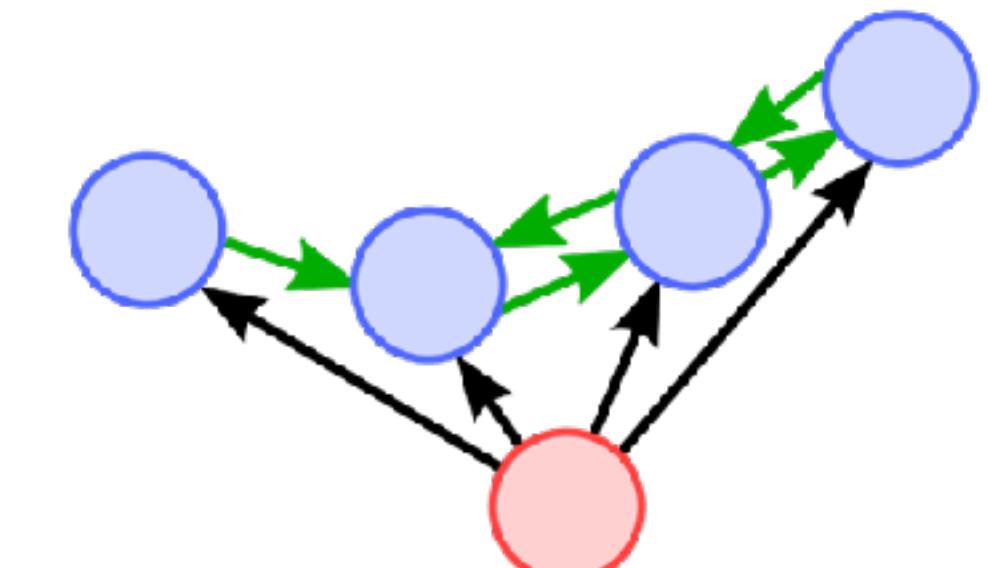
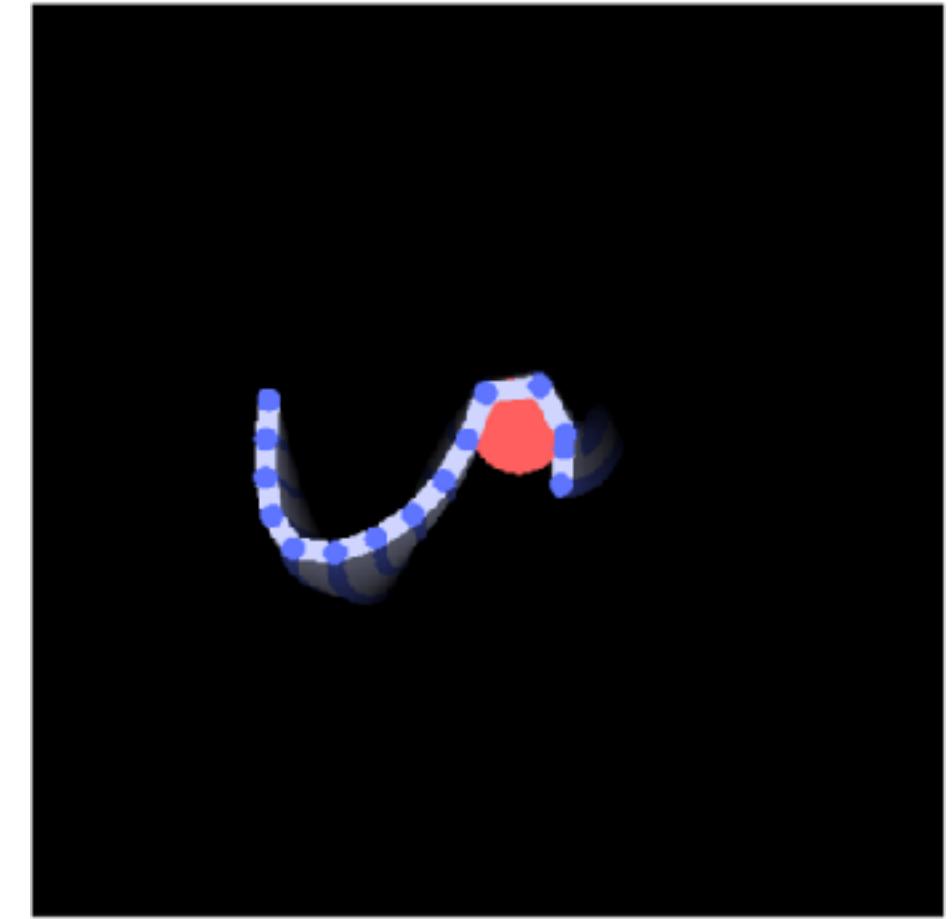
Balls



Nodes: balls

Edges: rigid collisions between
balls, and walls

String



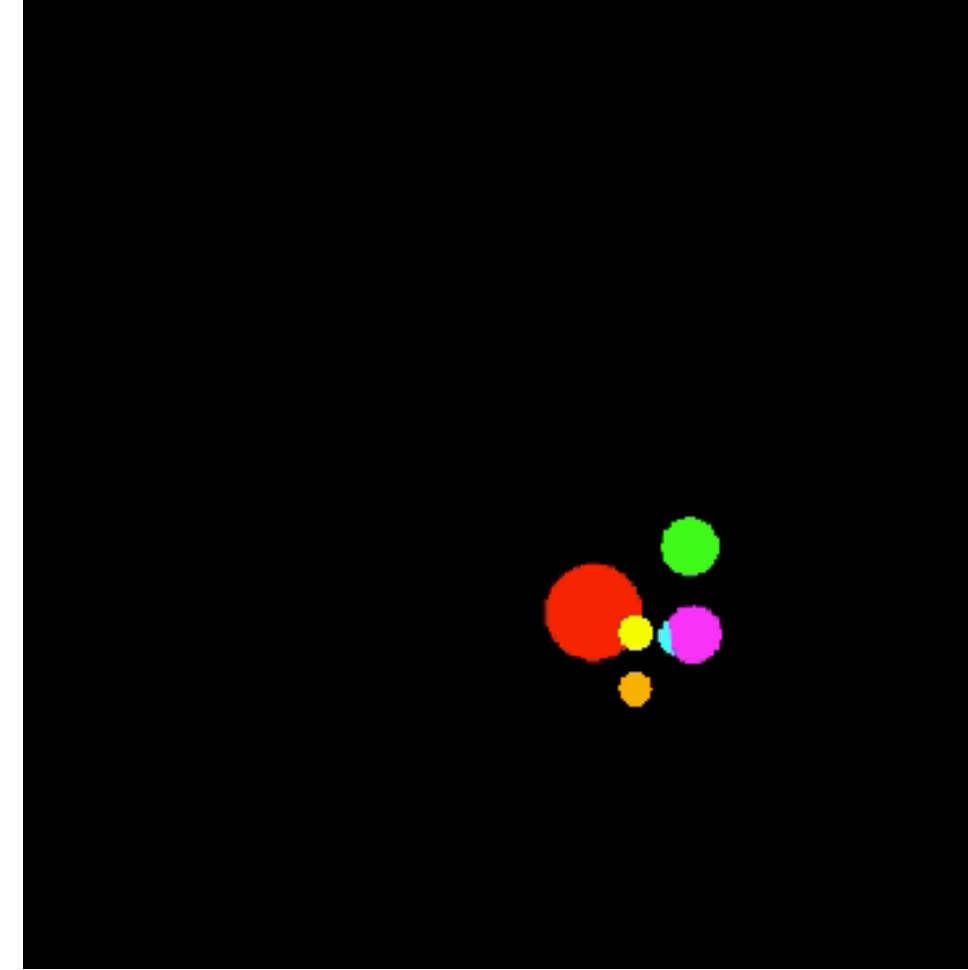
Nodes: masses

Edges: springs and rigid
collisions

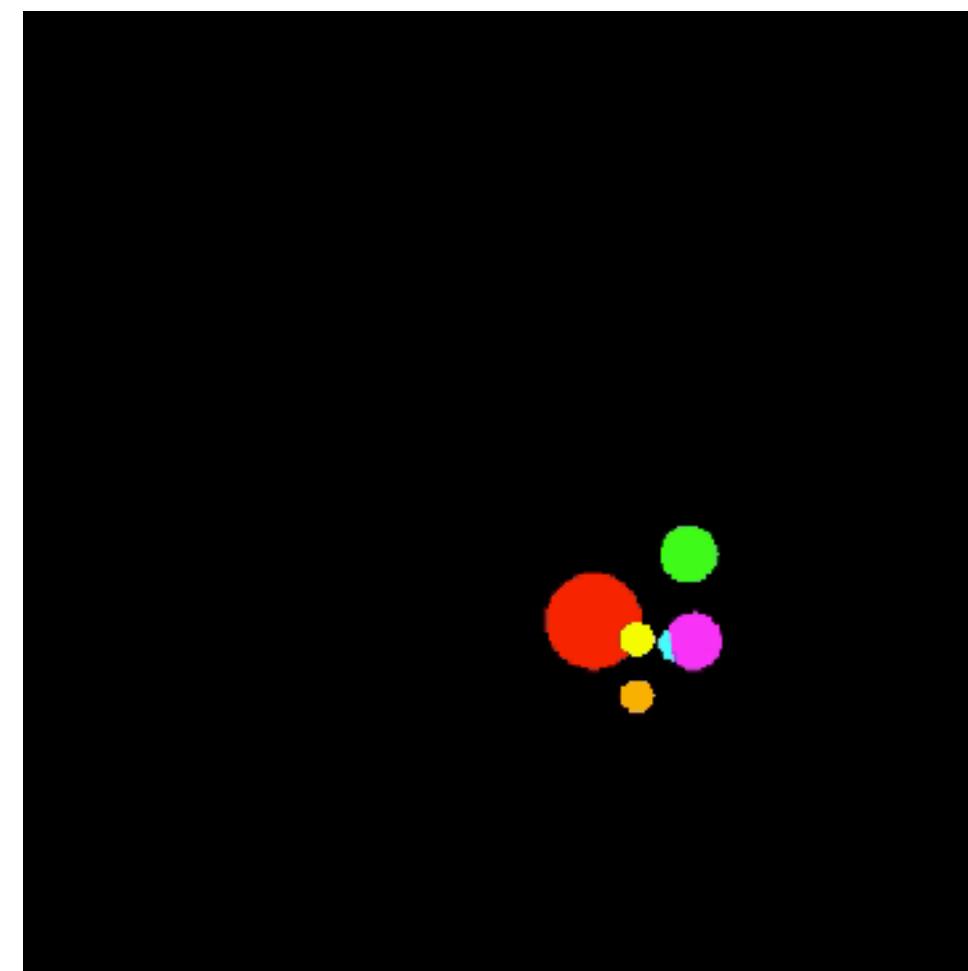
1000-step rollouts of true (top row) vs predicted (bottom row)

True

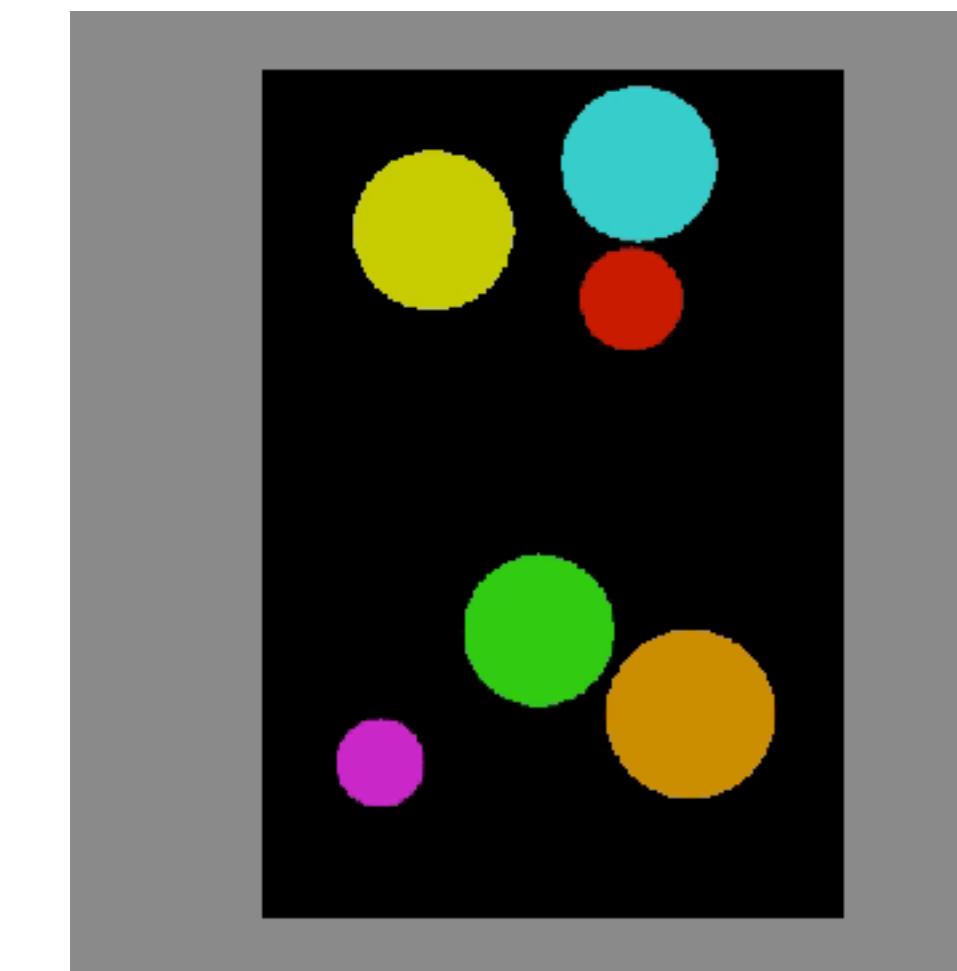
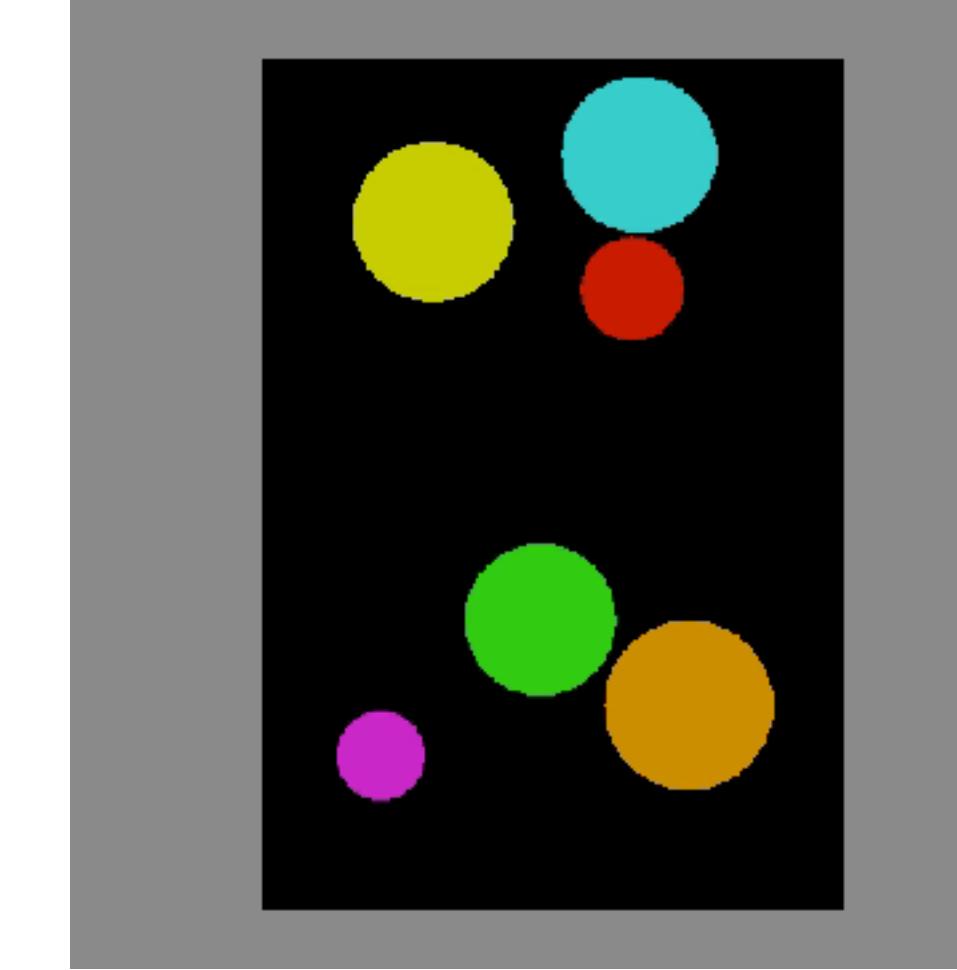
n-body



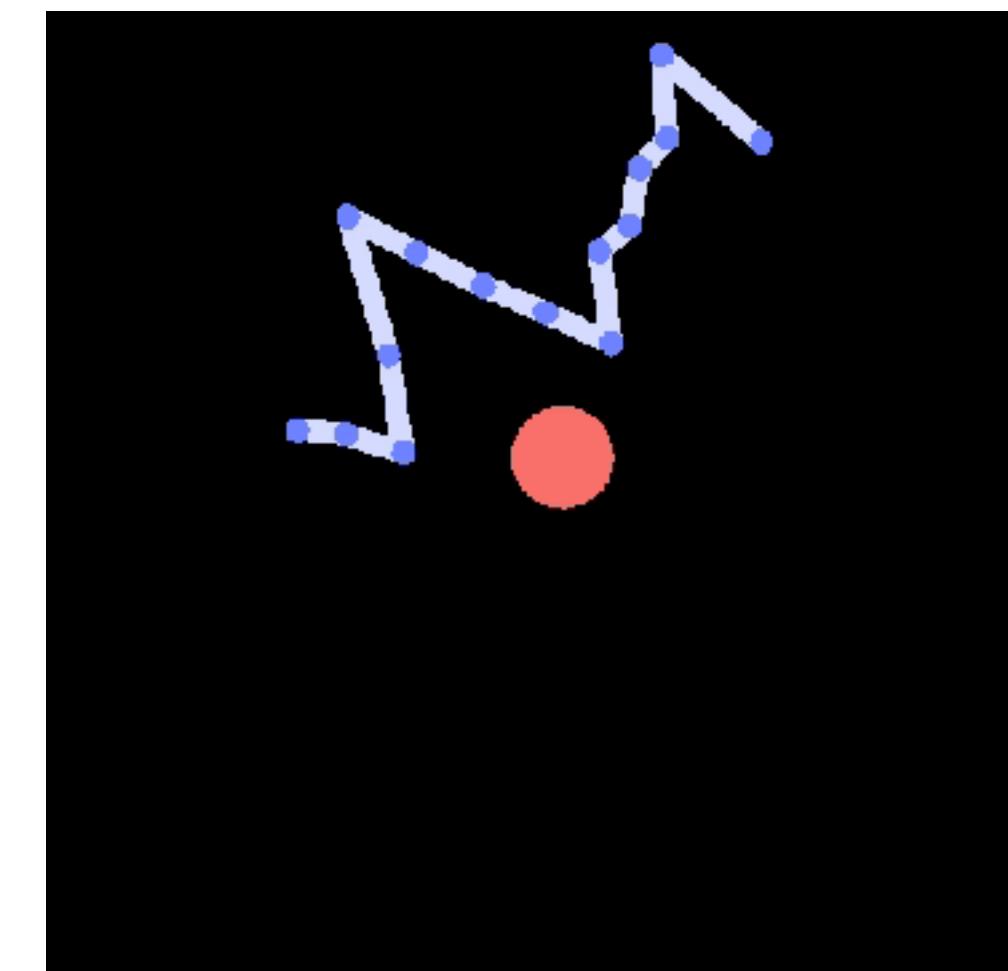
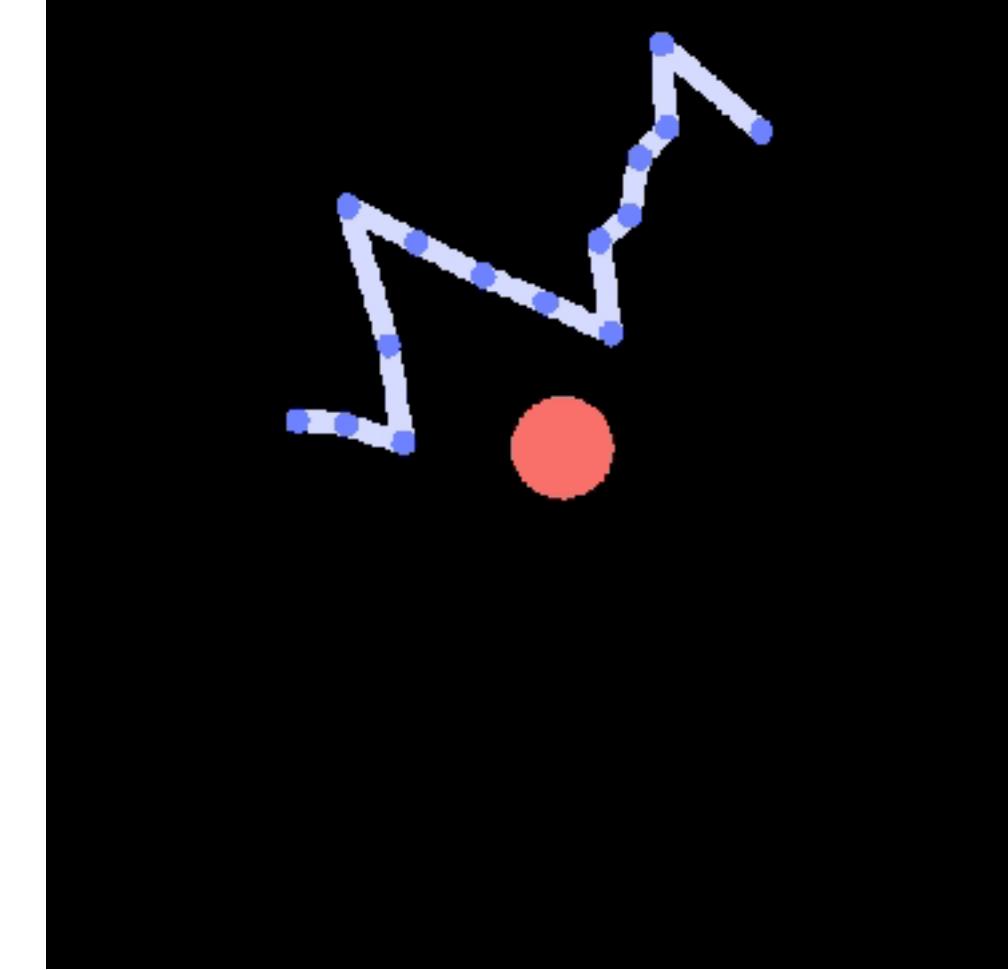
Model



Balls



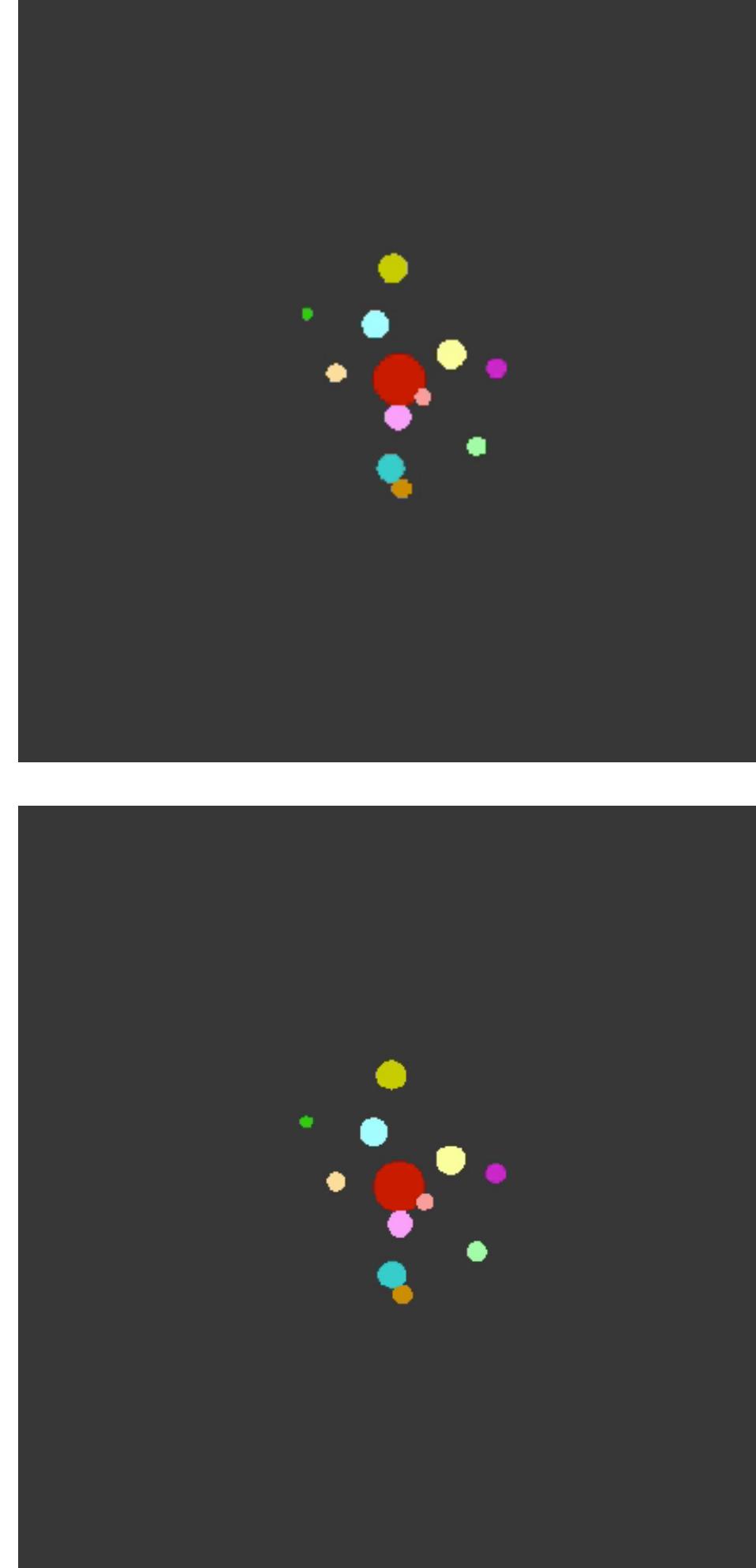
String



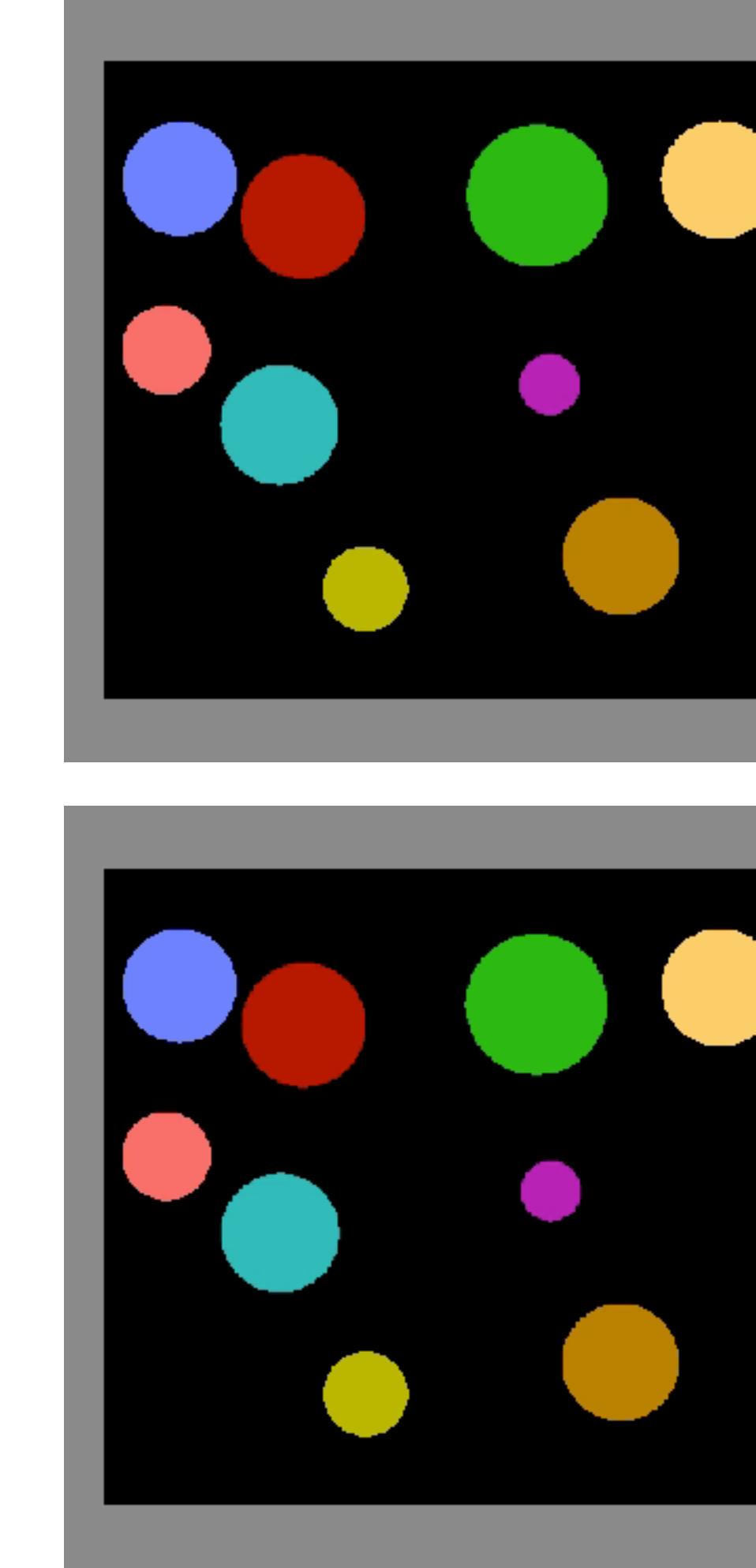
Zero-shot generalisation to larger systems

True
Model

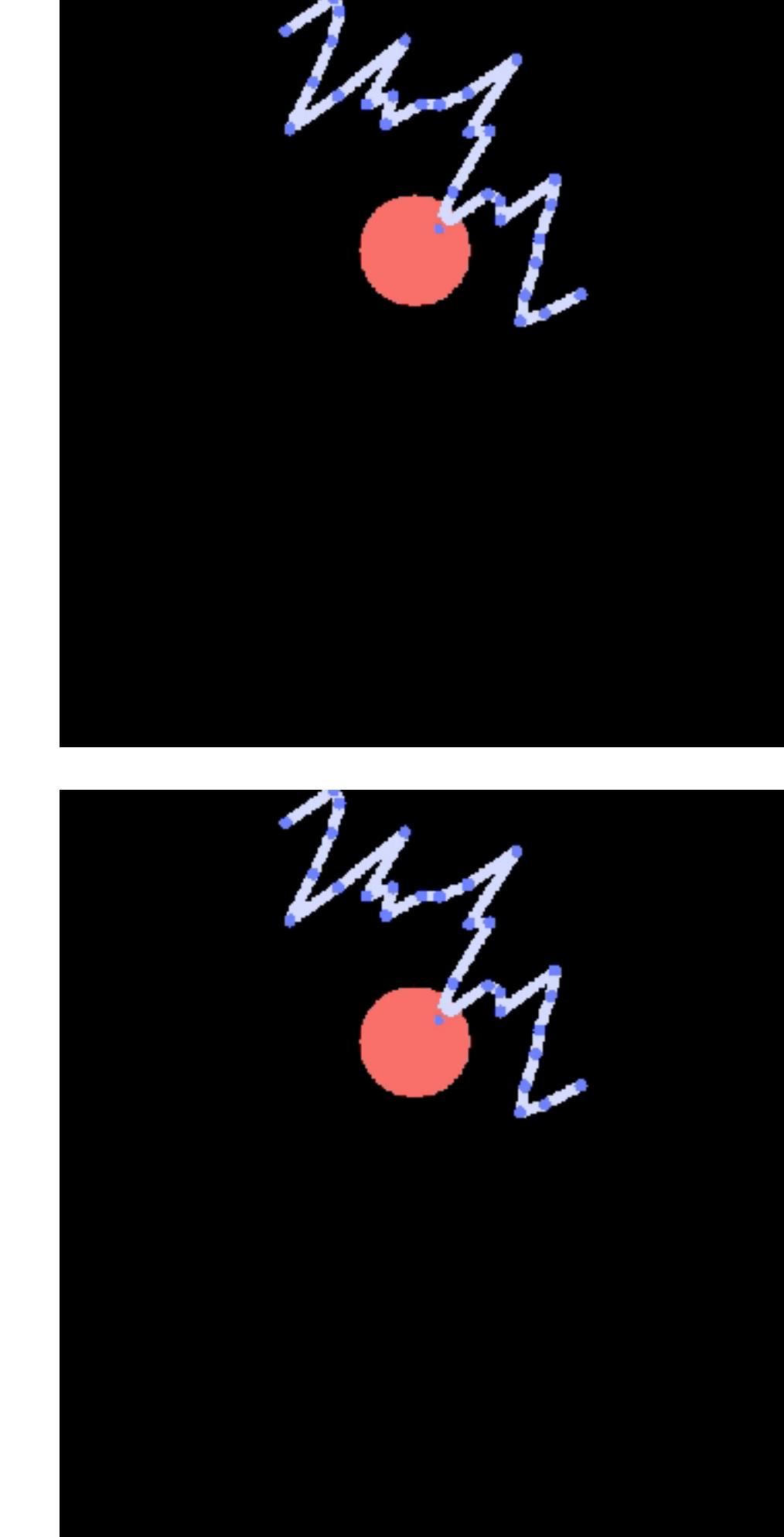
n-body



Balls



String



Interaction Network: Predicting potential energy

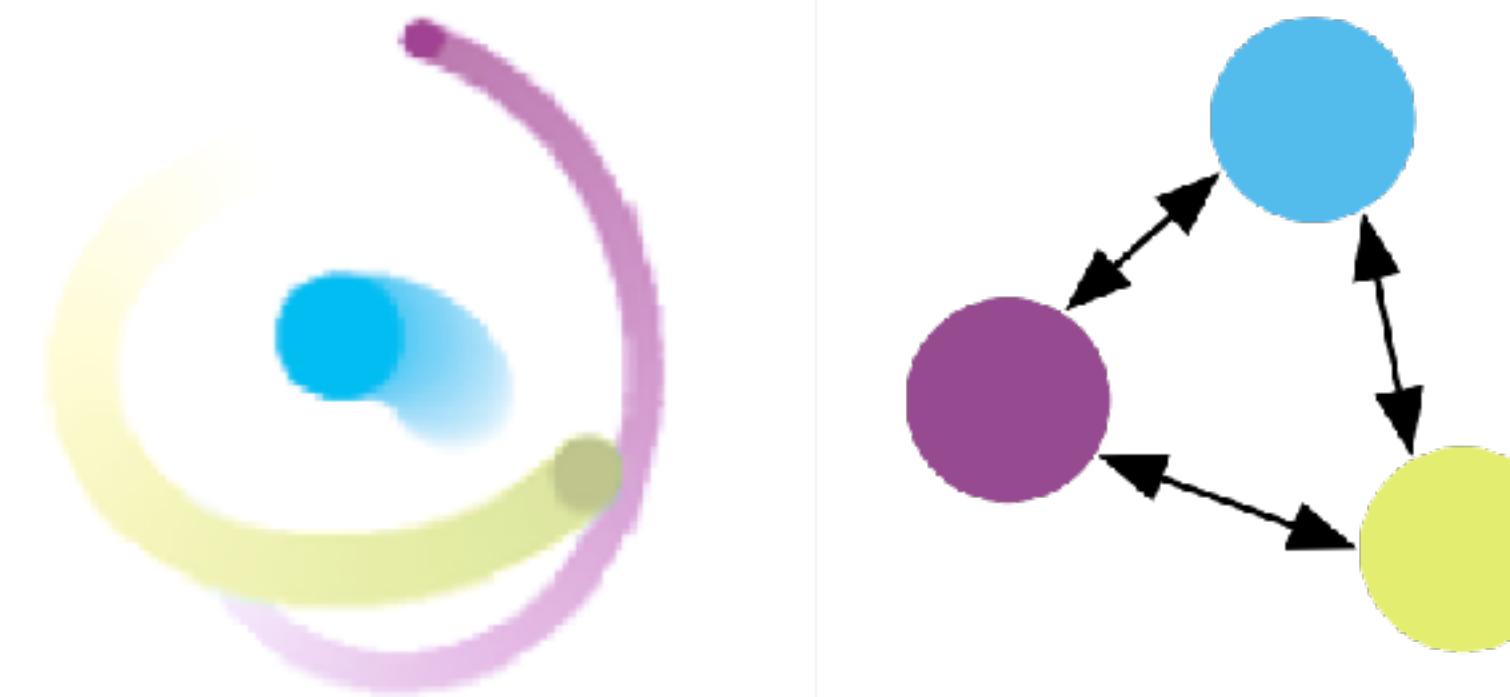
Node aggregation and global function

$$\bar{\mathbf{v}}' \leftarrow \sum_i \mathbf{v}'_i$$

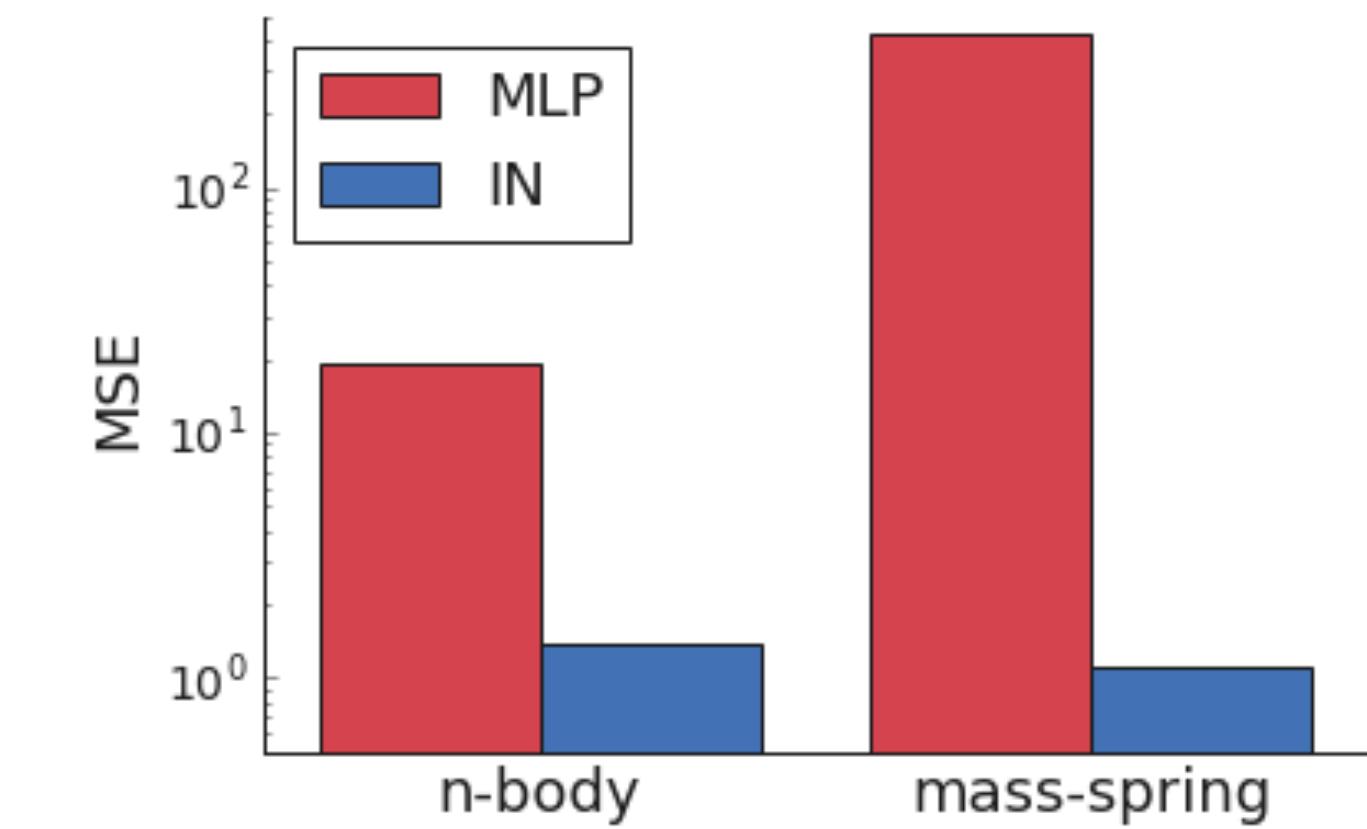
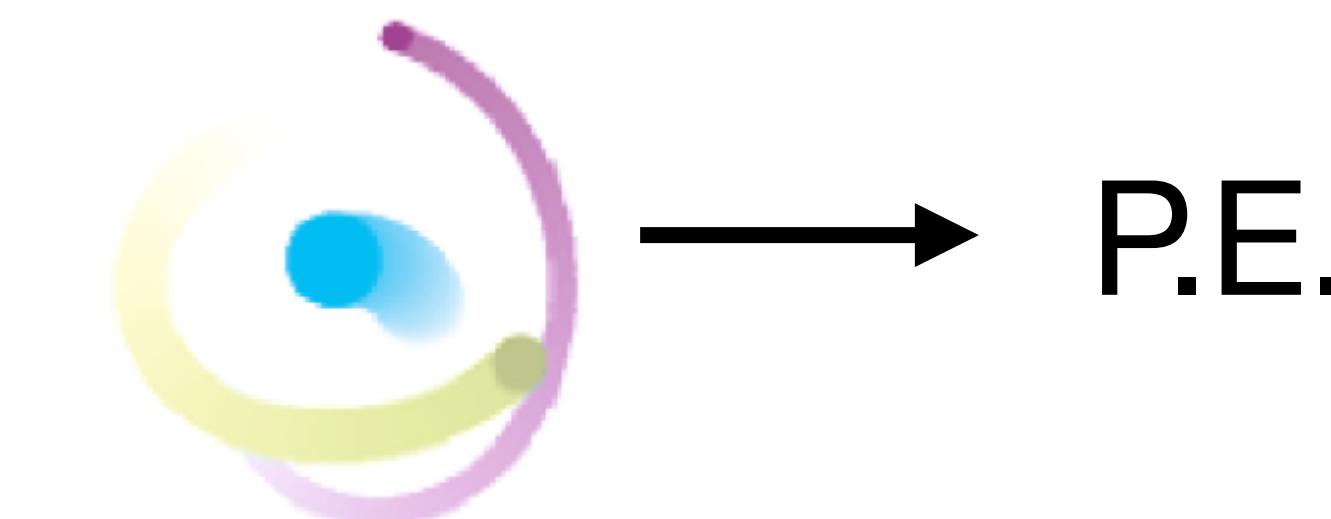
$$\mathbf{u}' \leftarrow \phi^u(\bar{\mathbf{v}}')$$

- Rather than making node-wise predictions, node updates can be used to make global predictions.

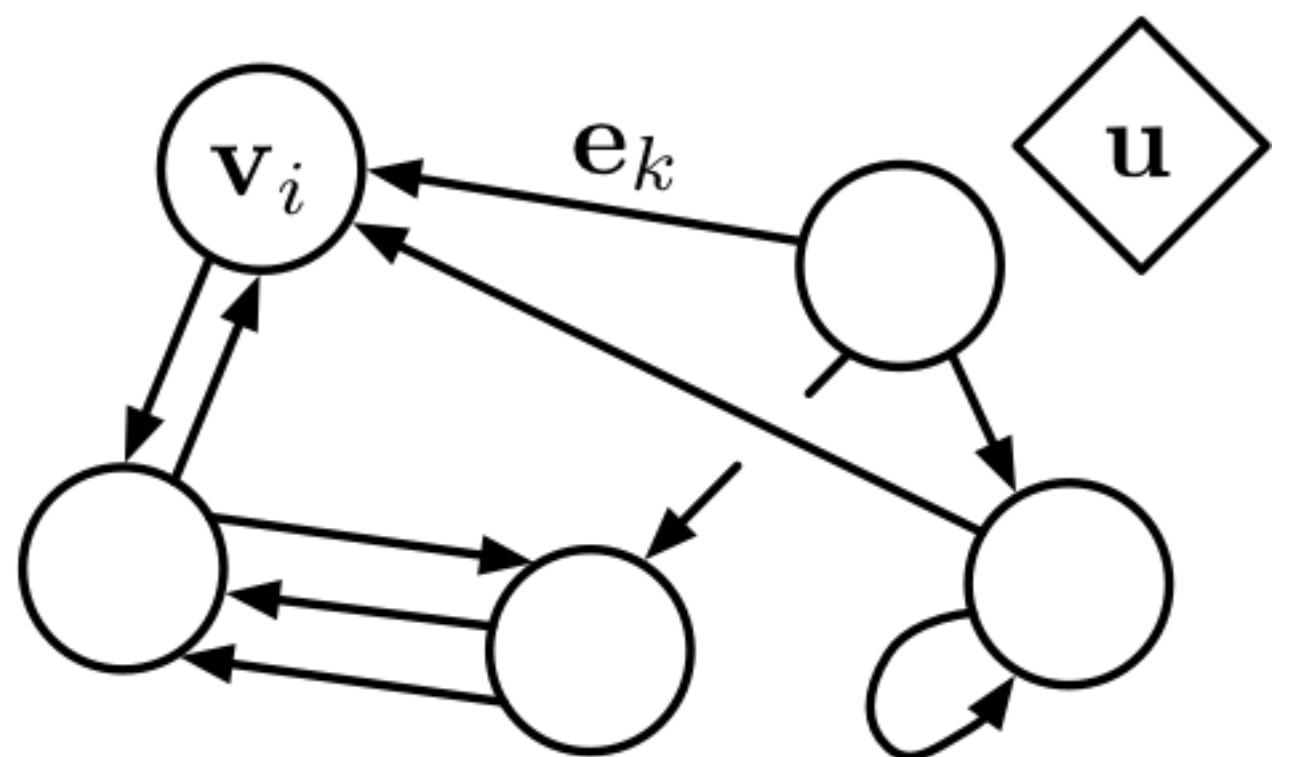
n-body System



Trained to predict system's potential energy



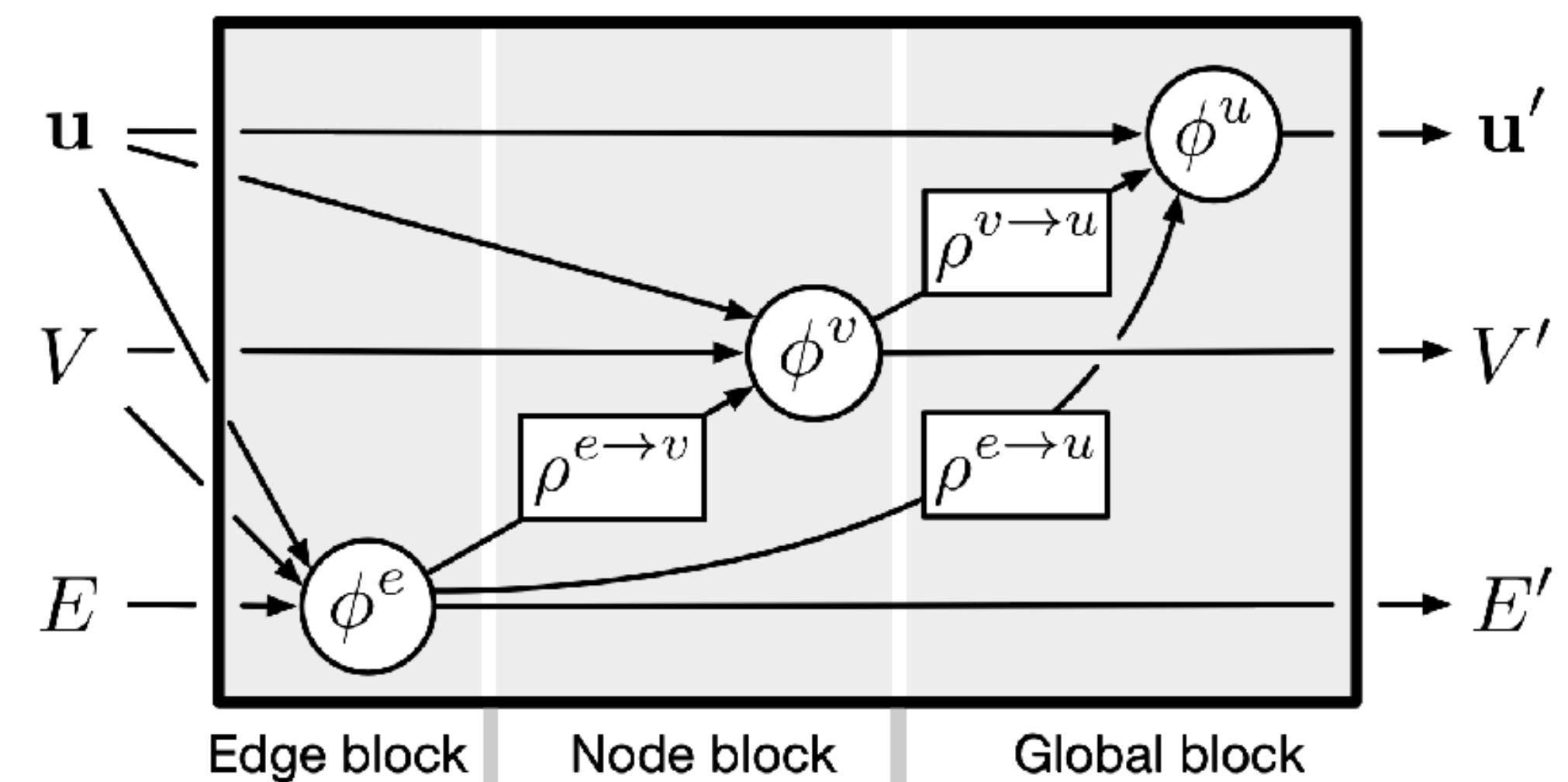
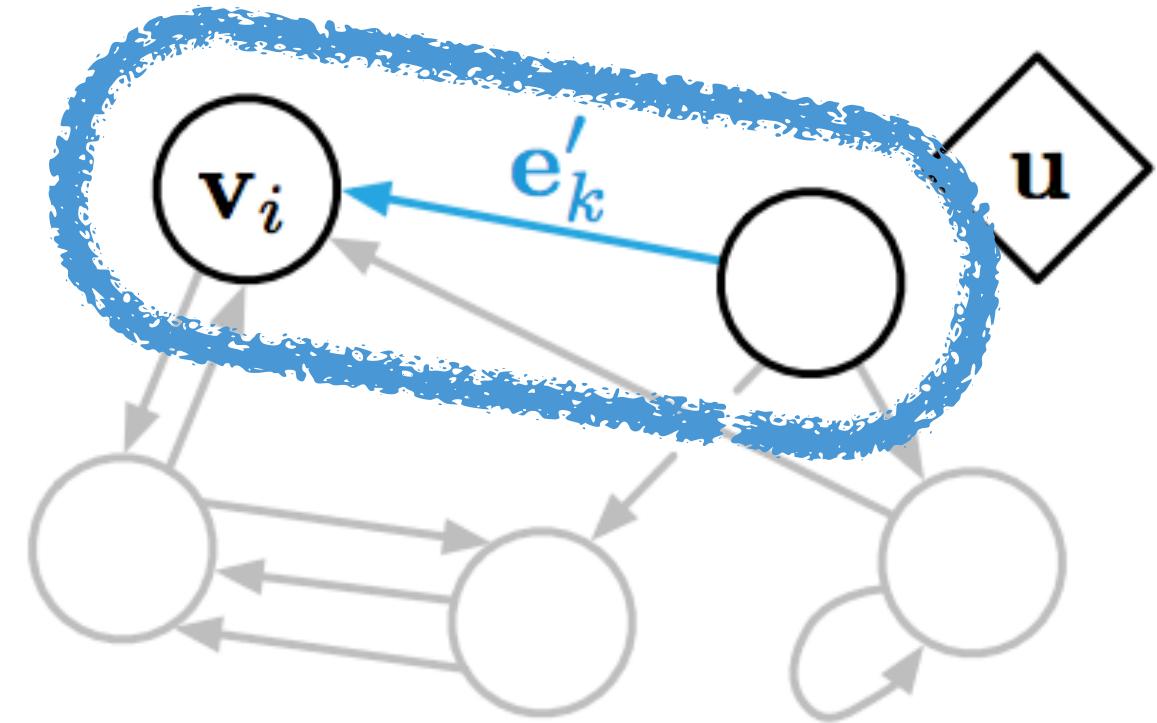
Our “Graph Network” generalizes/extends “Interaction Network”



Edge block

For each edge, e_k, v_{s_k}, v_{r_k}, u , are passed to an “edge-wise function”:

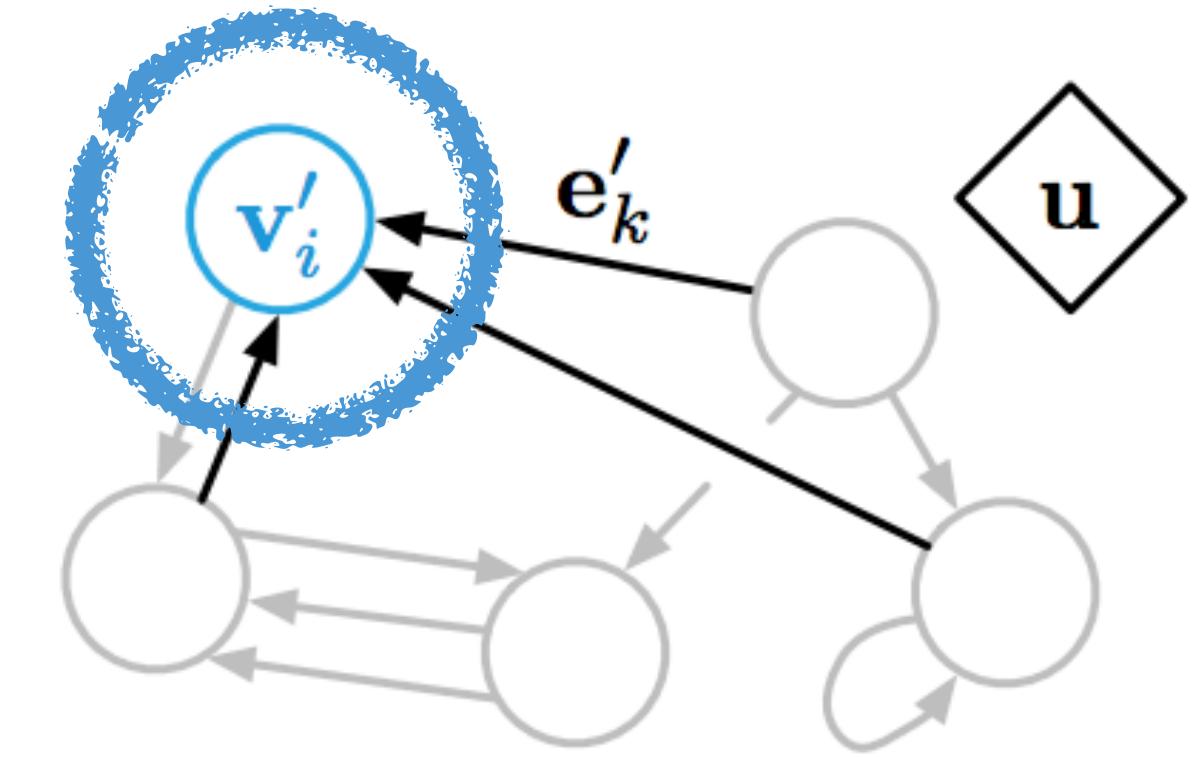
$$e'_k \leftarrow \phi^e(e_k, v_{r_k}, v_{s_k}, u)$$



Node block

For each node, \bar{e}'_i, v_i, u , are passed to a “node-wise function”:

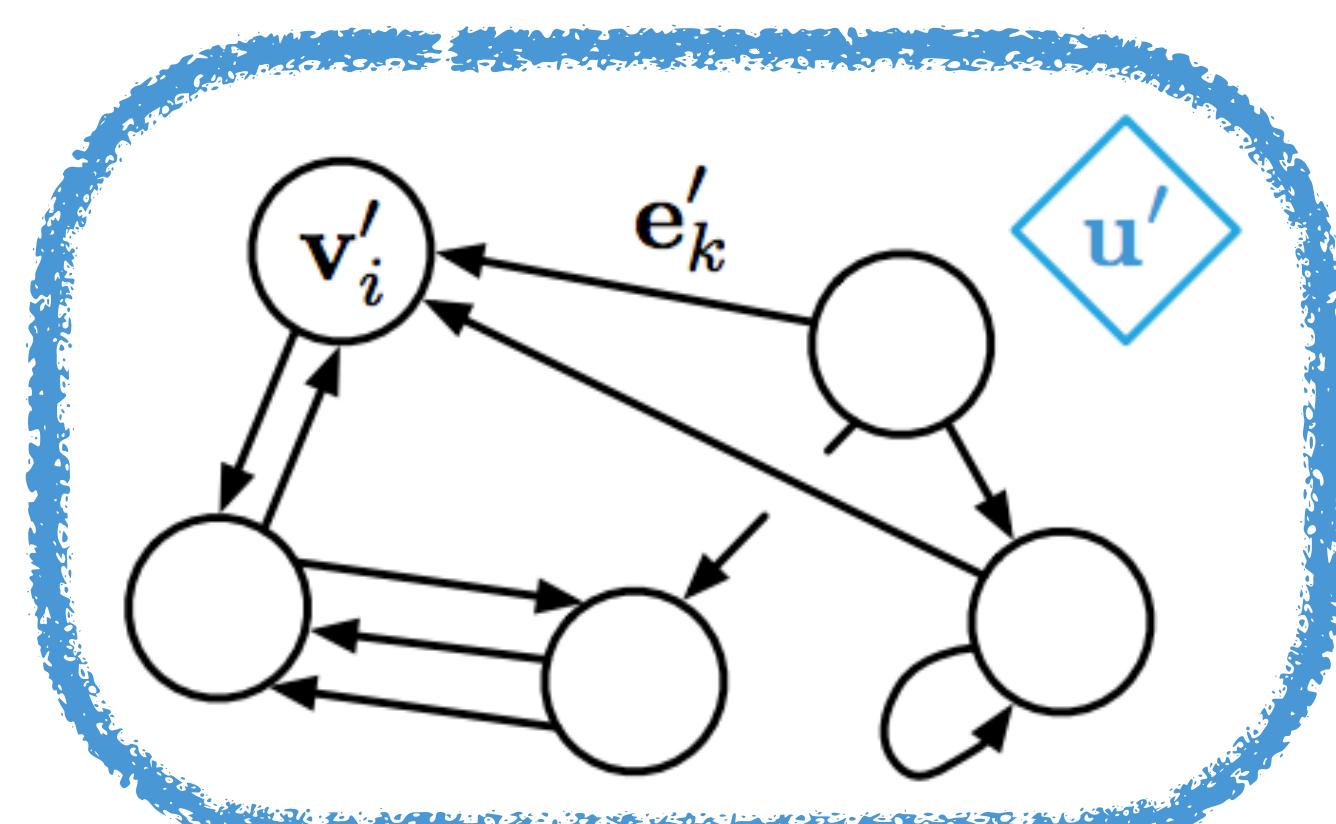
$$v'_i \leftarrow \phi^v(\bar{e}'_i, v_i, u)$$



Global block

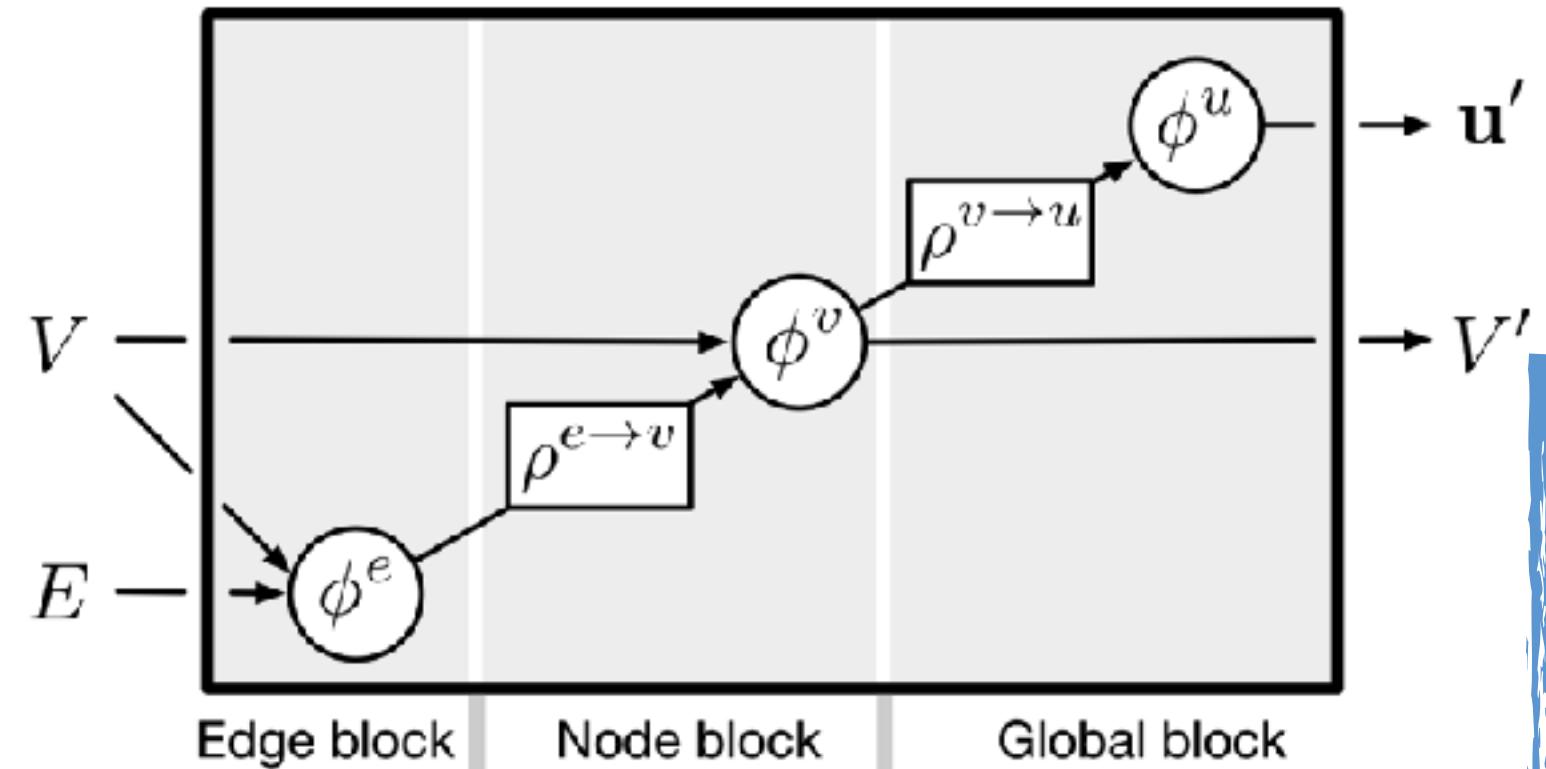
Across the graph, \bar{e}', \bar{v}', u , are passed to a “global function”:

$$u' \leftarrow \phi^u(\bar{e}', \bar{v}', u)$$

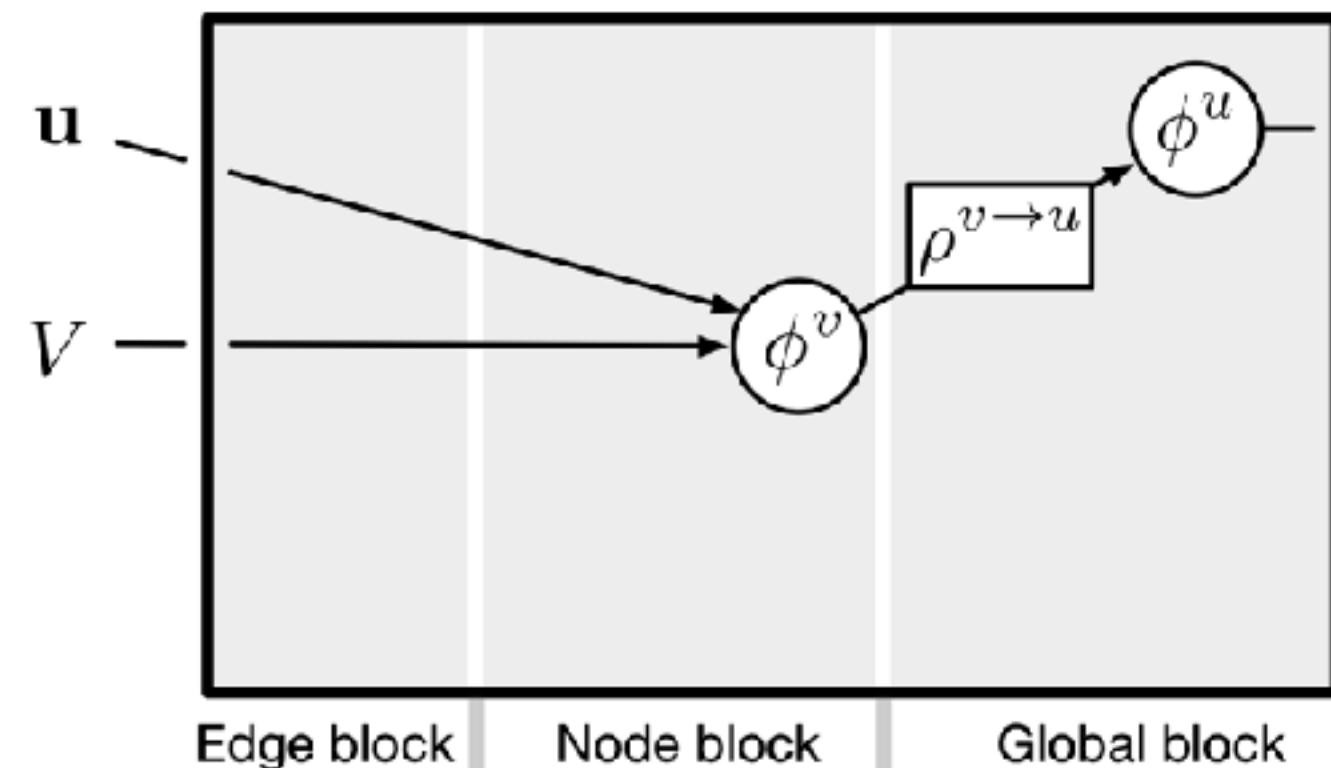


Message-Passing NN (eg. Interaction Net, GCN)

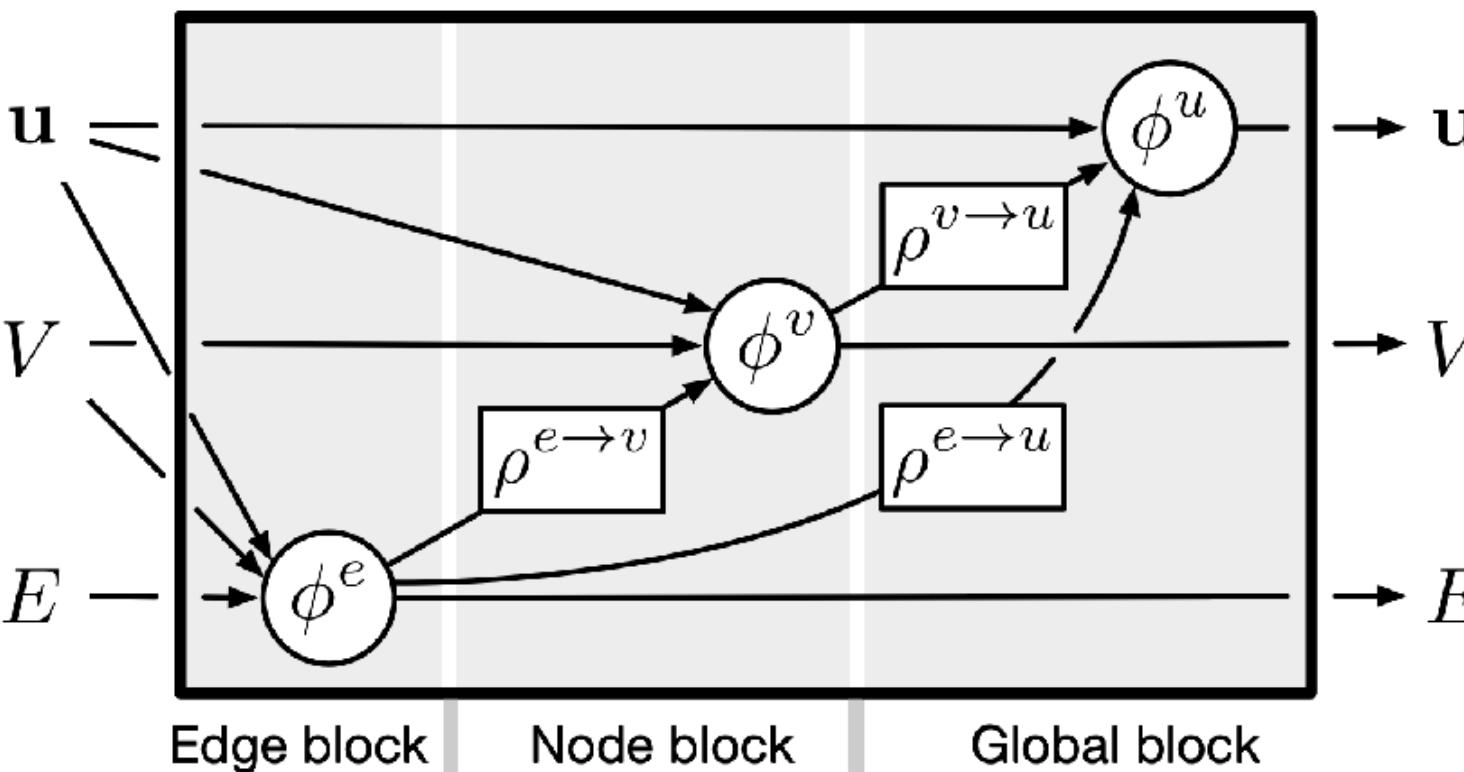
Gilmer et al. 2017



Deep Sets
Zhang et al. 2017

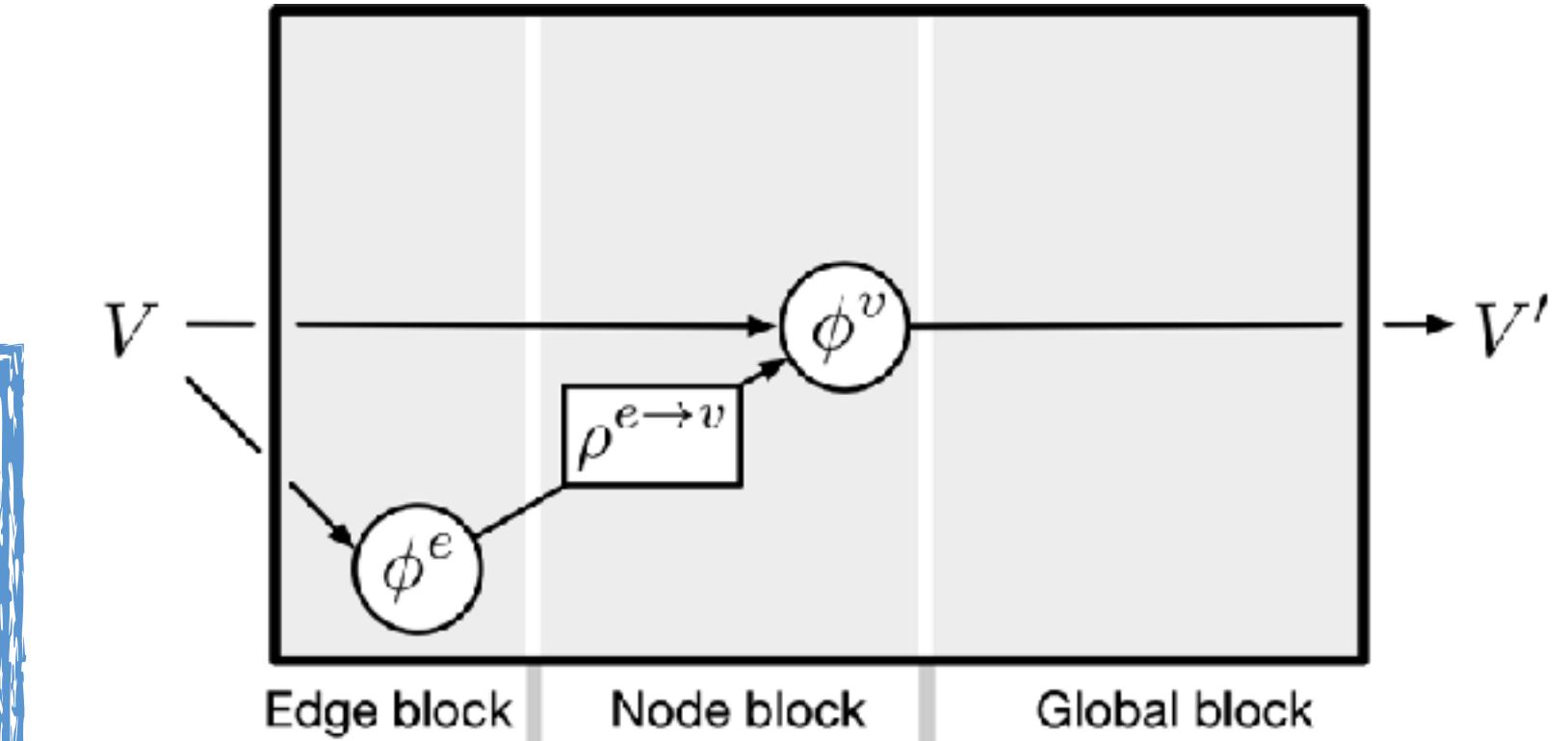


Graph Network
(a type of Graph Neural Network)
Battaglia et al. 2018

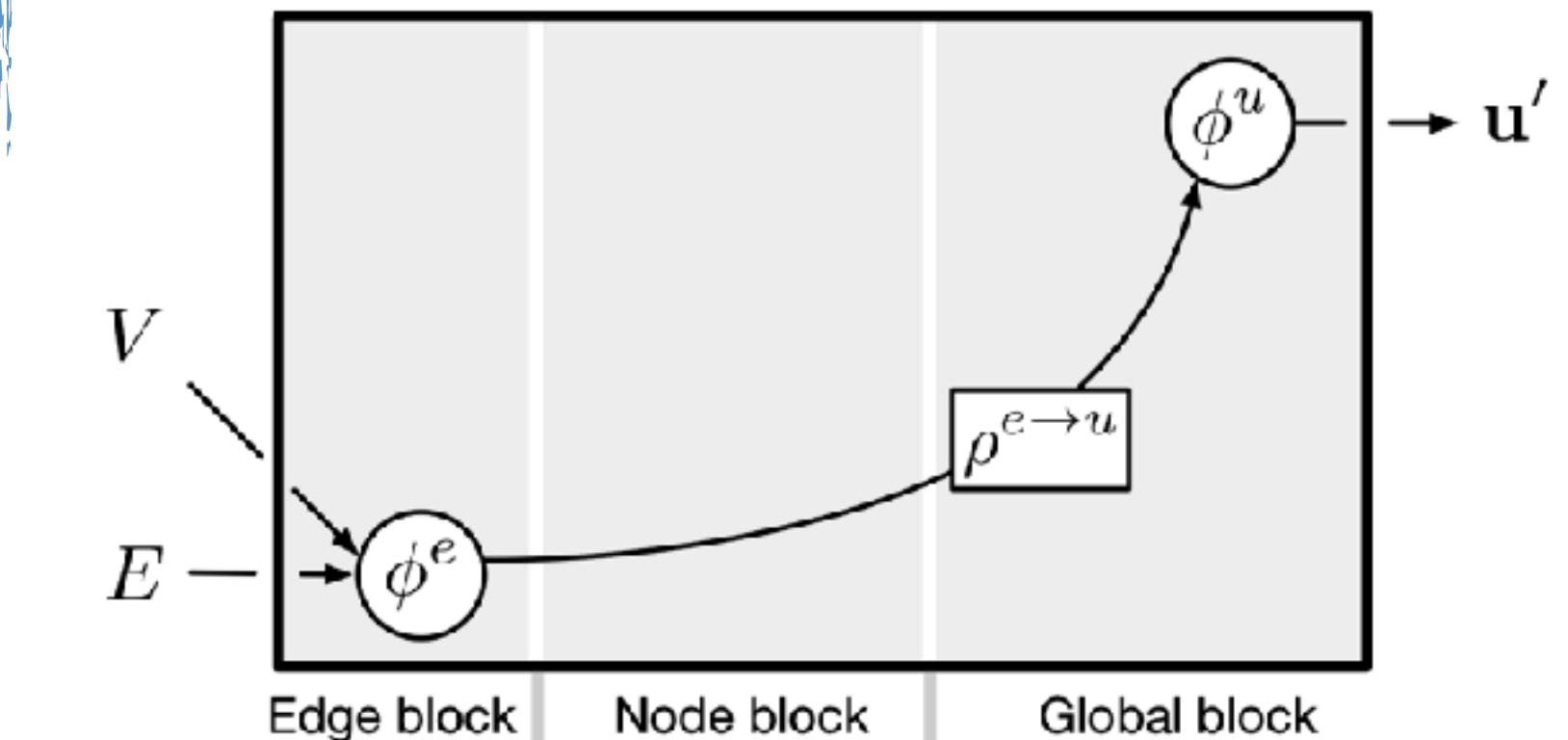


Non-Local NN (eg. Transformer)

Vaswani et al. 2017; Wang et al. 2017



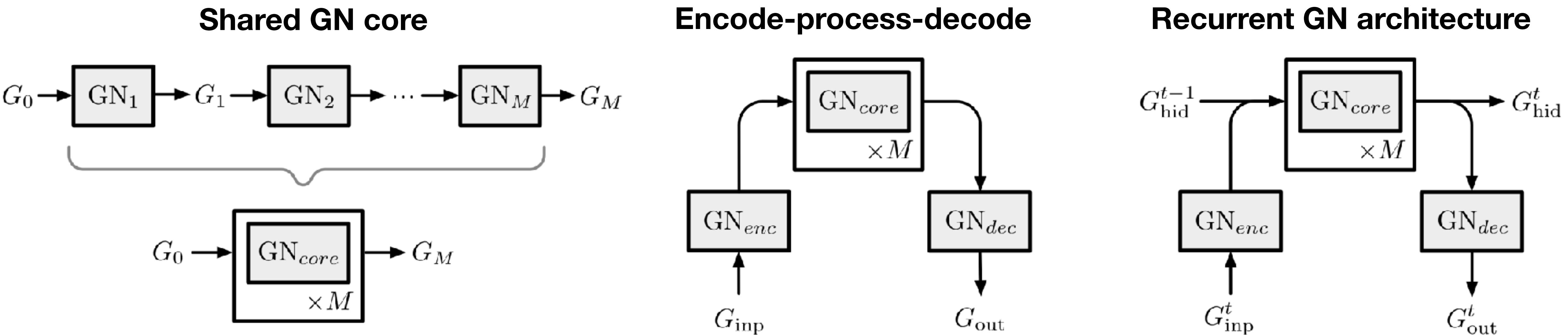
Relation Network
Raposo et al. 2017; Santoro et al. 2017



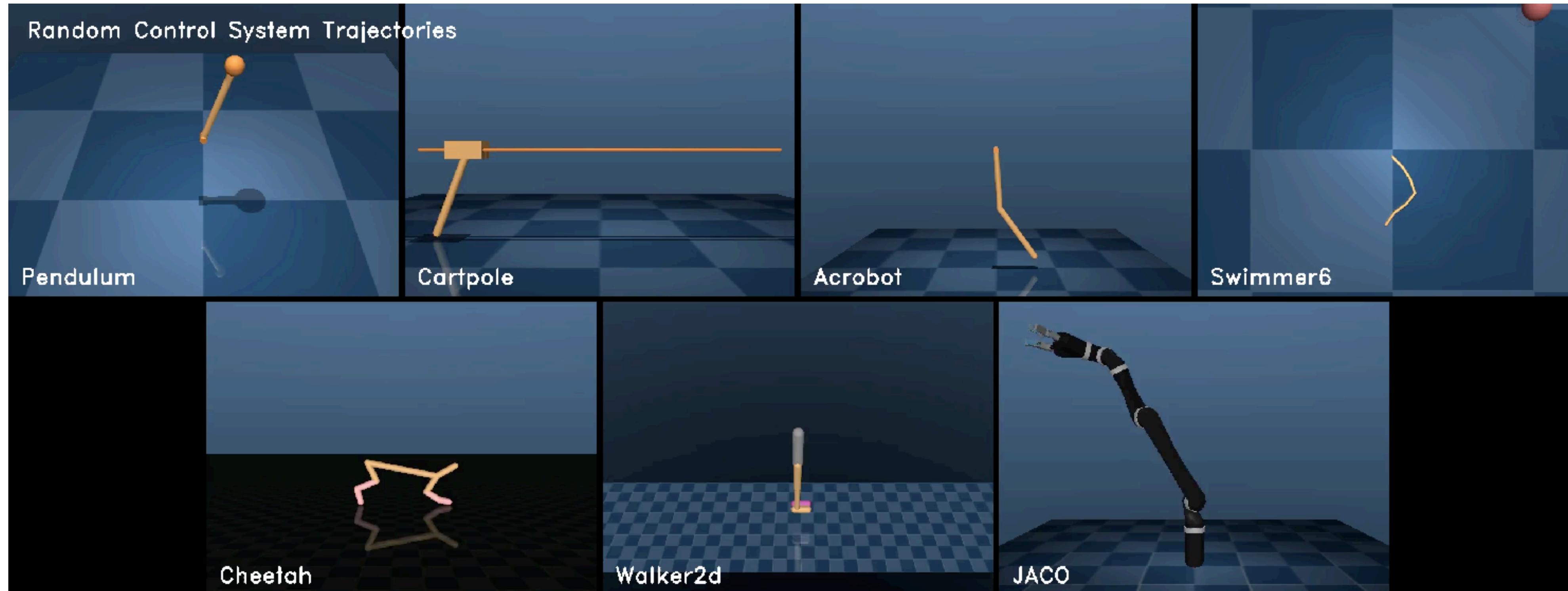
Battaglia et al., 2018, arXiv

Composing GN blocks

The GN's graph-to-graph interface promotes stacking GN blocks,
passing one GN's output to another GN as input



Systems: "DeepMind Control Suite" (Mujoco) & real JACO



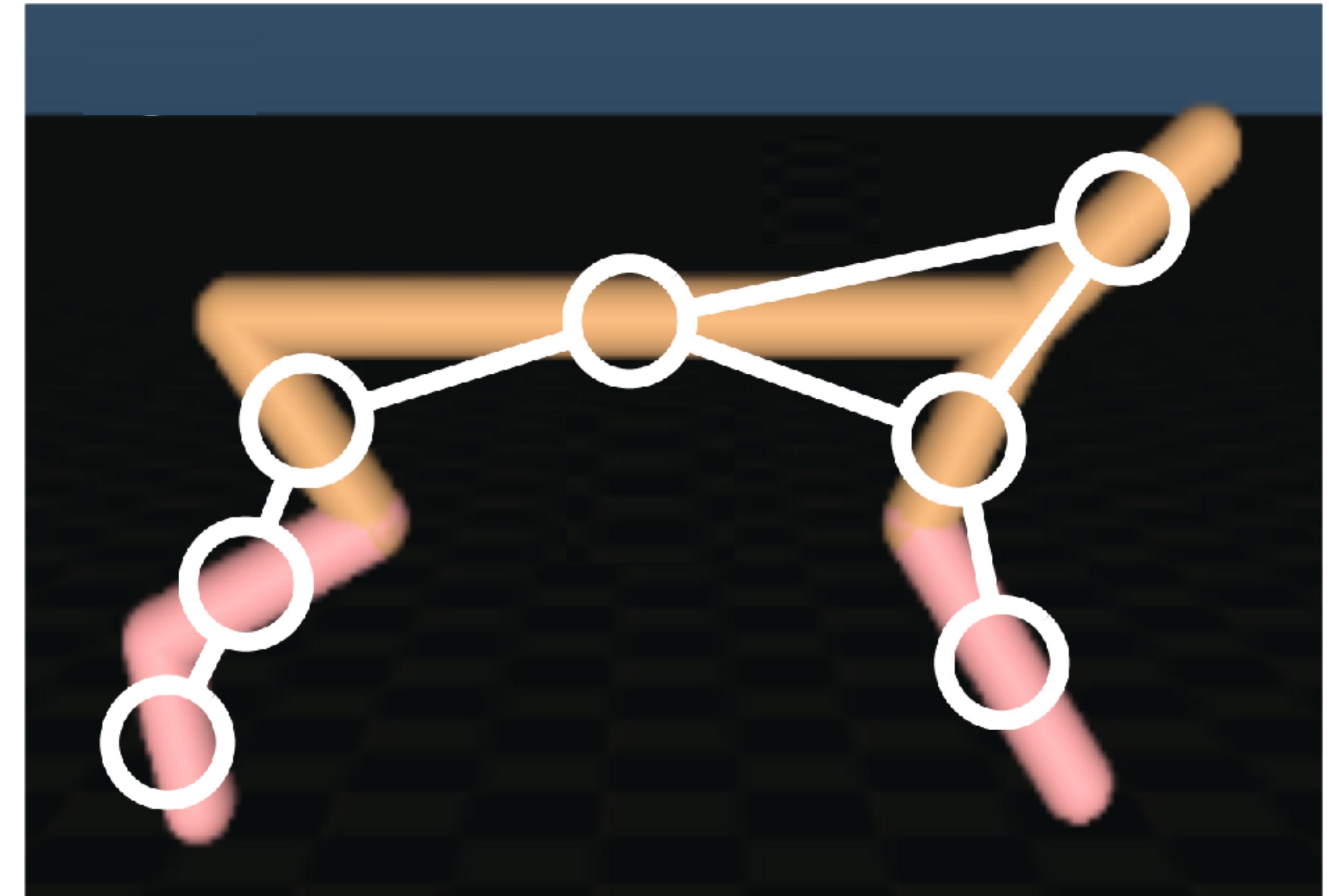
JACO Arm

DeepMind Control Suite (Tassa et al., 2018)

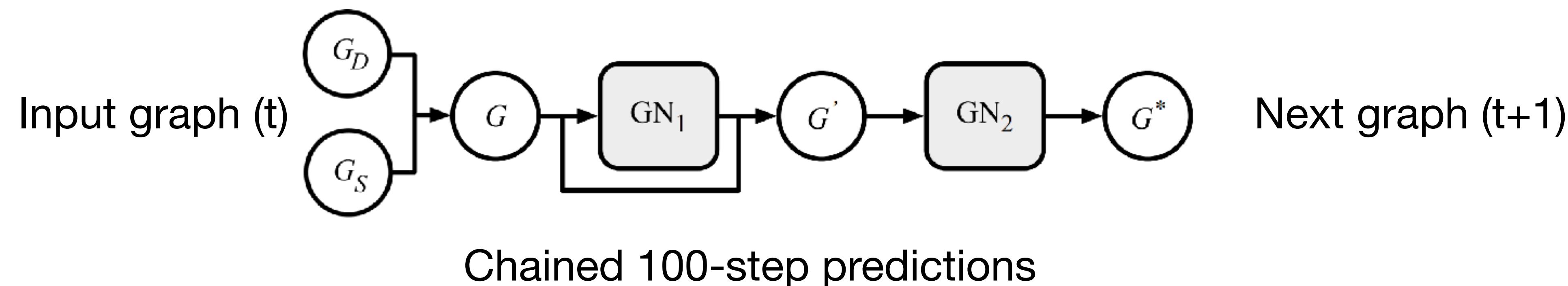
Kinematic tree of the actuated system as a graph

Controllable physical system as a graph:

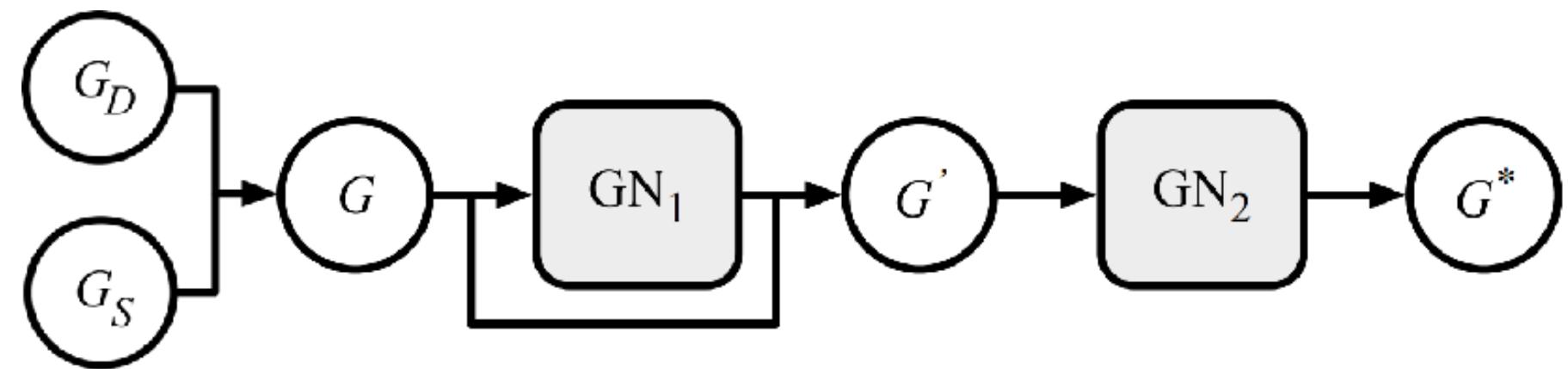
- Bodies → Nodes
- Joints → Edges
- Global properties



Forward model: supervised, 1-step training w/ random control inputs

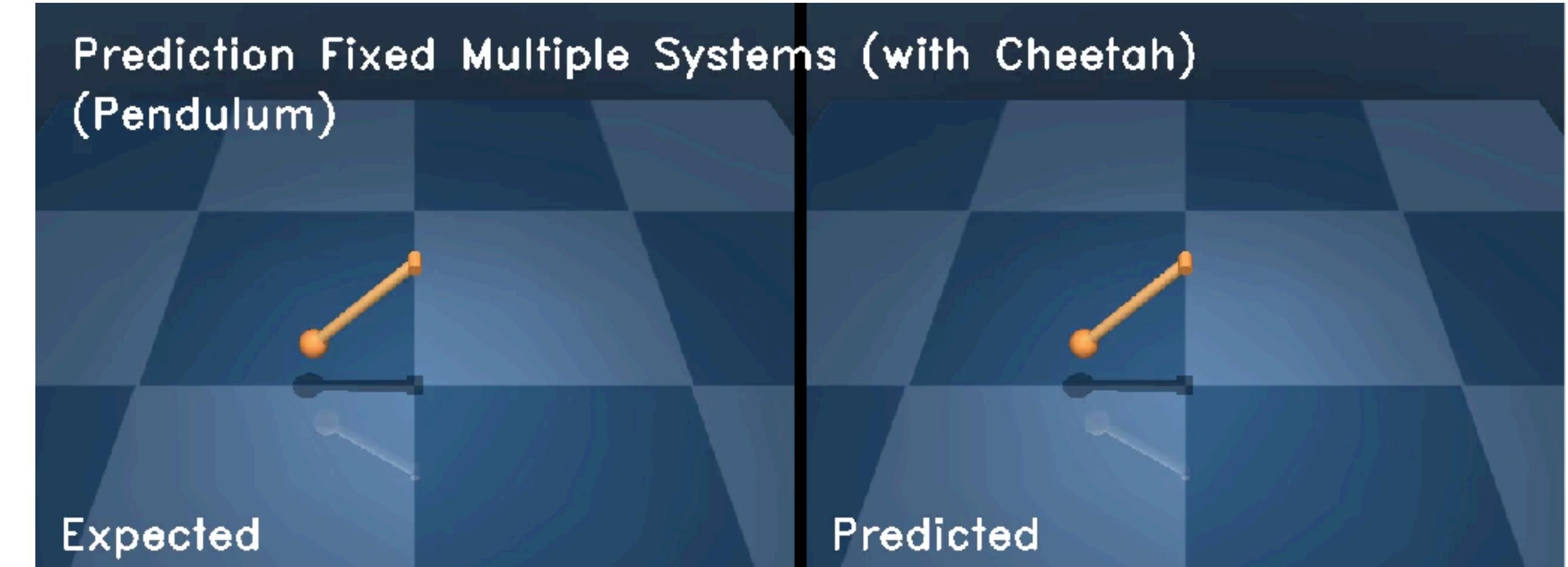


Forward model: Multiple systems & zero-shot generalization



Single model trained:

- Pendulum, Cartpole, Acrobot, Swimmer6 & Cheetah

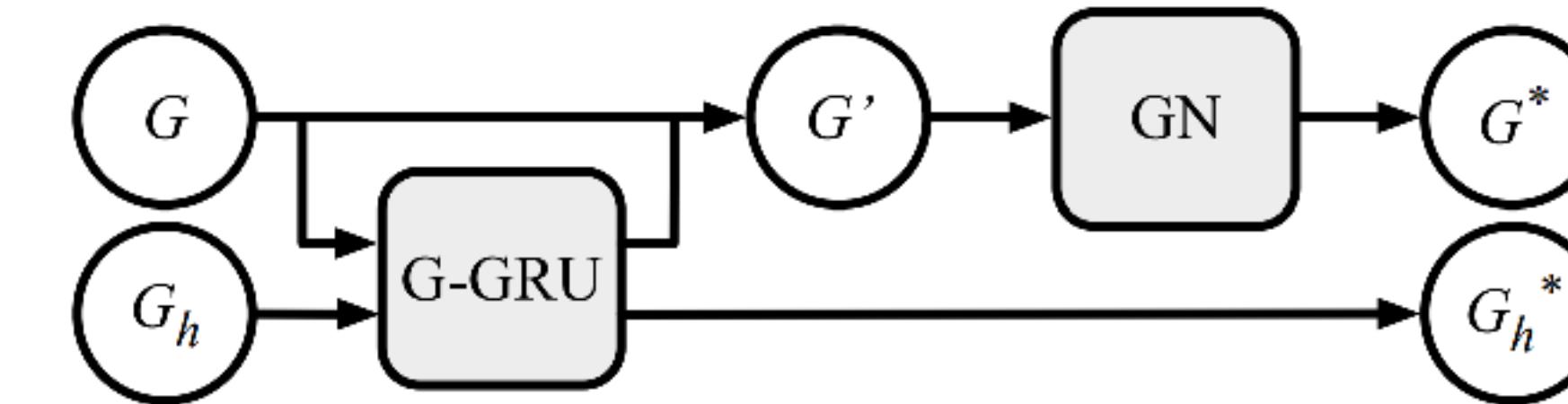


Zero-shot generalization: Swimmer

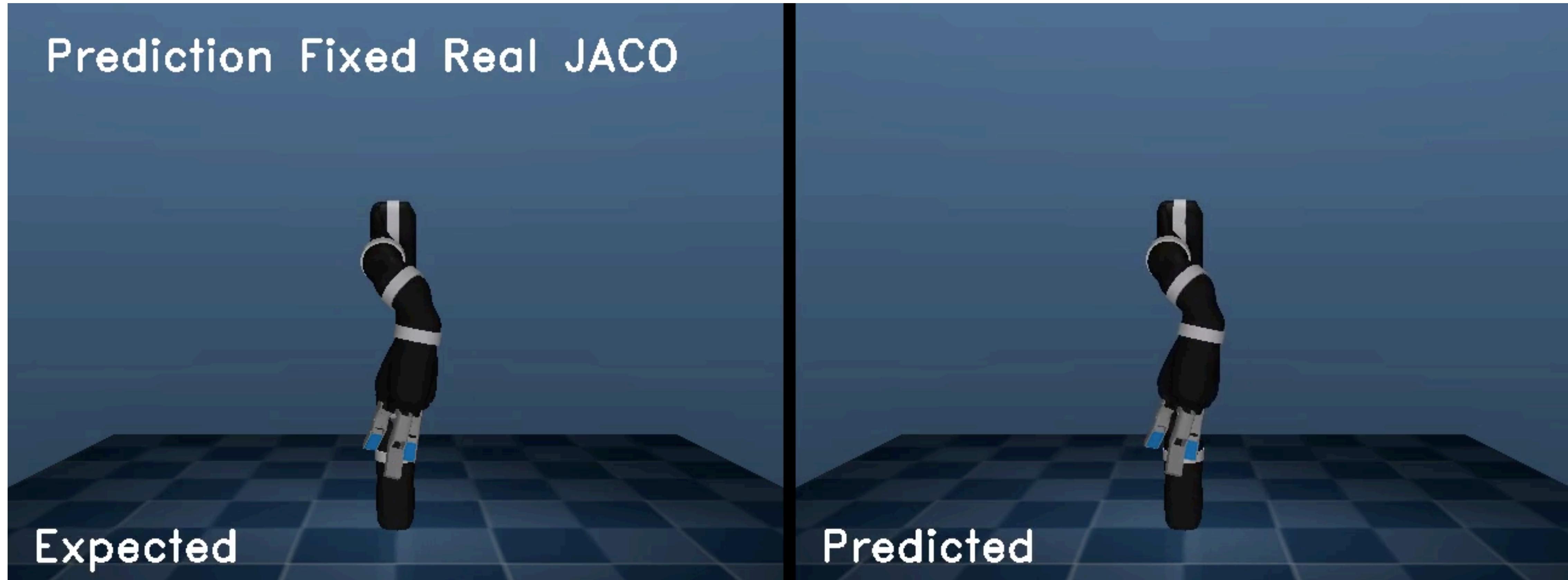
- # training links: {3, 4, 5, 6, -, 8, 9, -, -, ...}
- # testing links: {-, -, -, -, 7, -, -, 10-14}

Forward model: Real JACO data

Recurrent GN

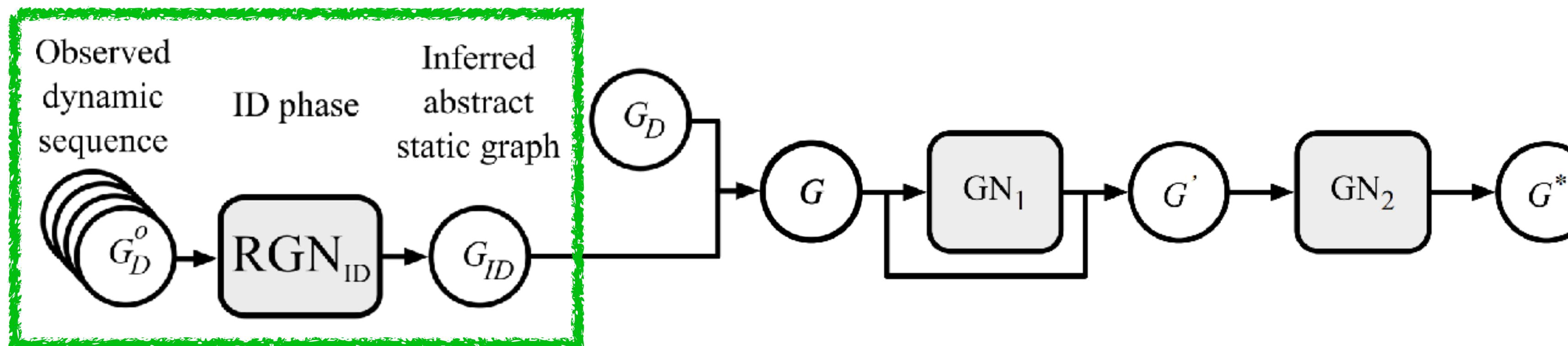


Prediction Fixed Real JACO

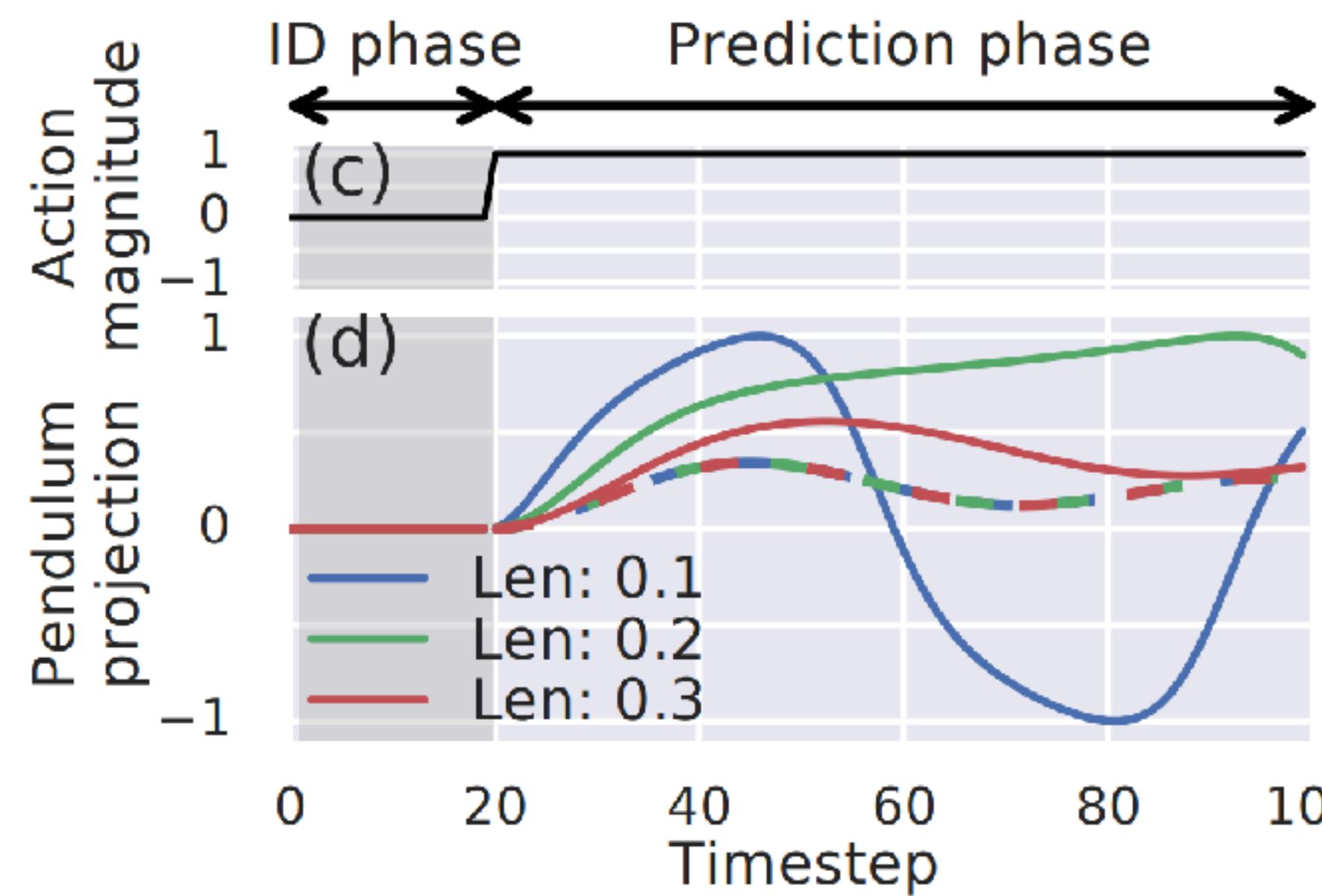


Inference: GN-based system identification

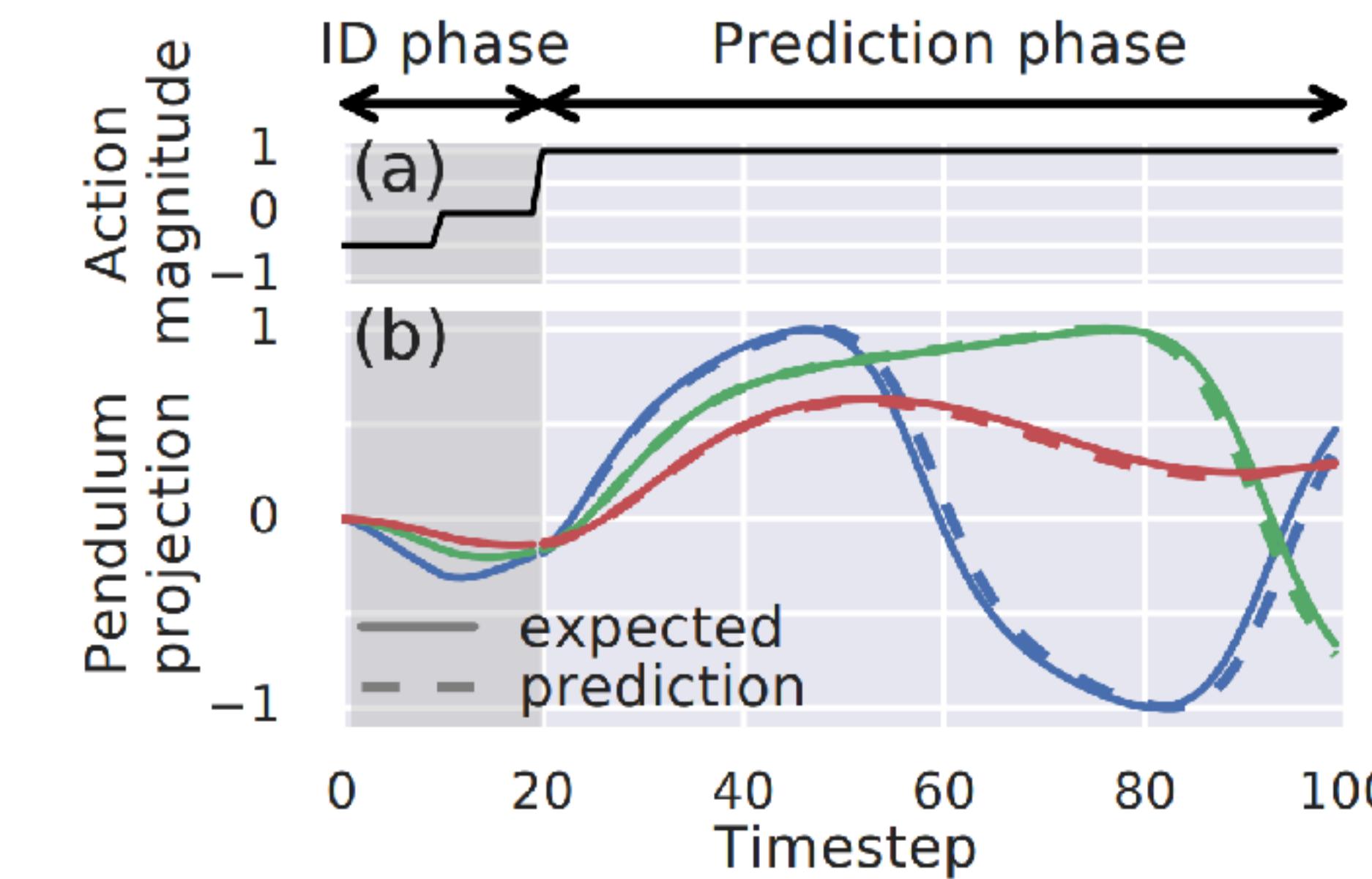
Unobserved system parameters (e.g. mass, length) are implicitly inferred



Unidentifiable condition

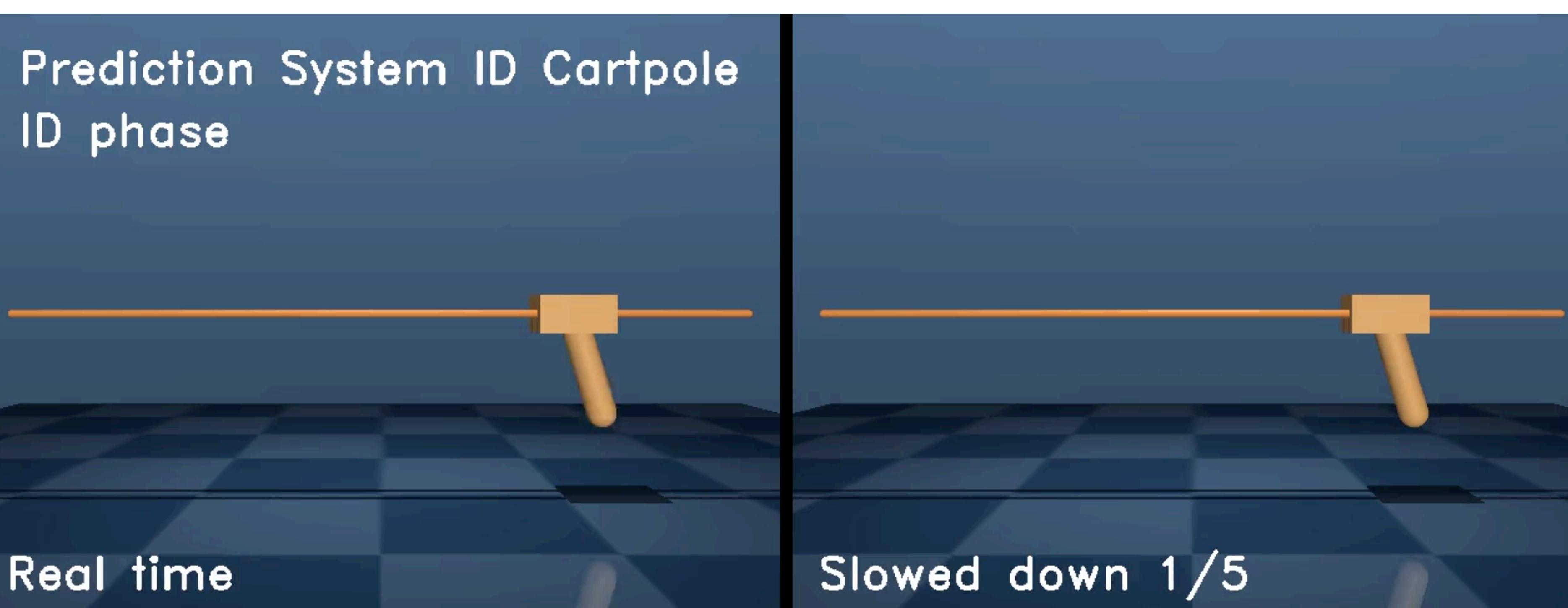
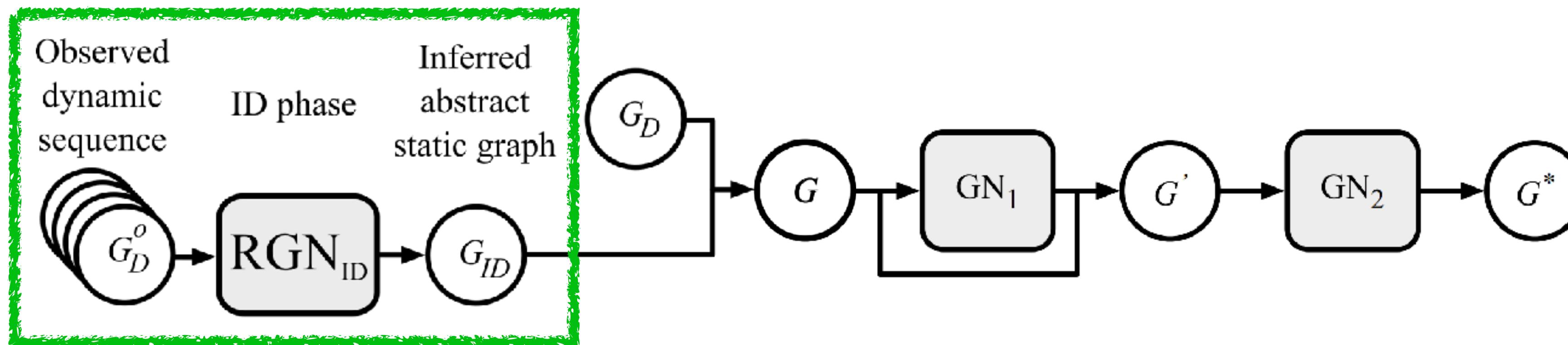


Identifiable condition



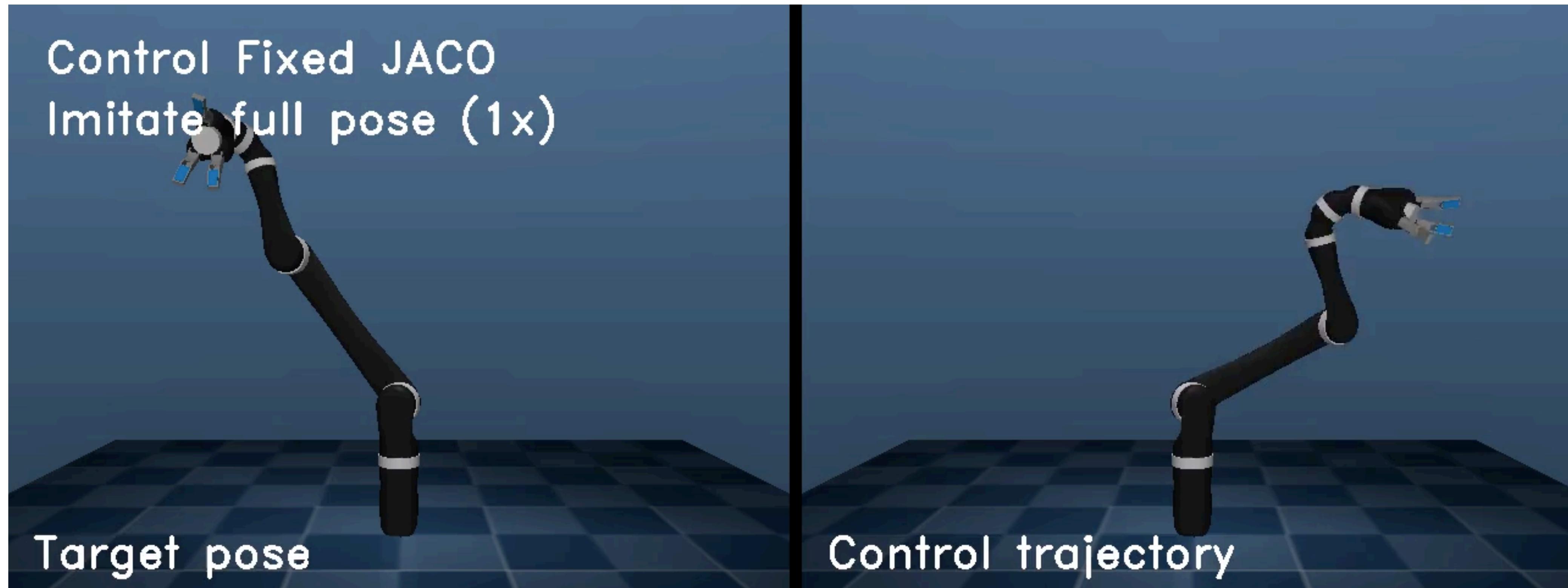
Inference: GN-based system identification

Unobserved system parameters (e.g. mass, length) are implicitly inferred

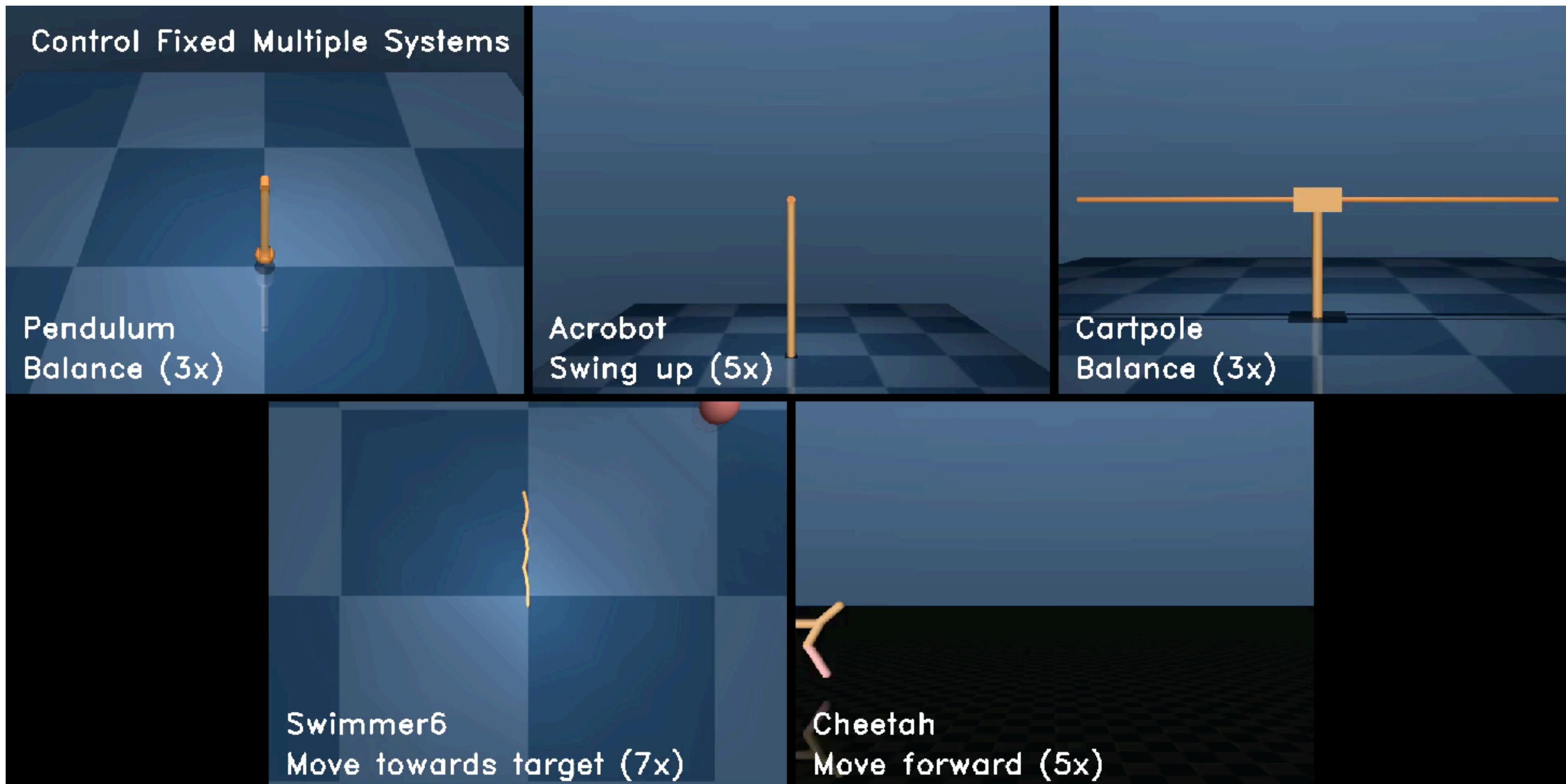


Control: Model-based planning

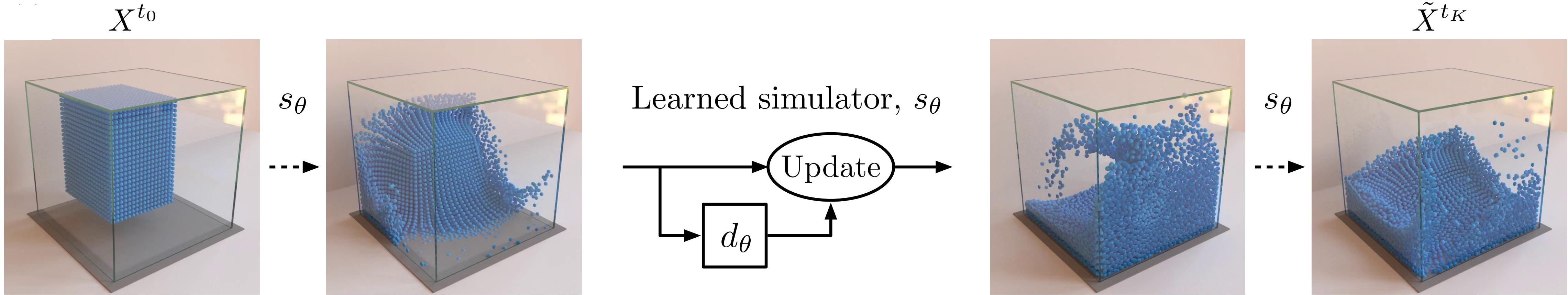
The GN-based forward model is differentiable, so we can backpropagate through it to search for a sequence of actions that maximize reward.



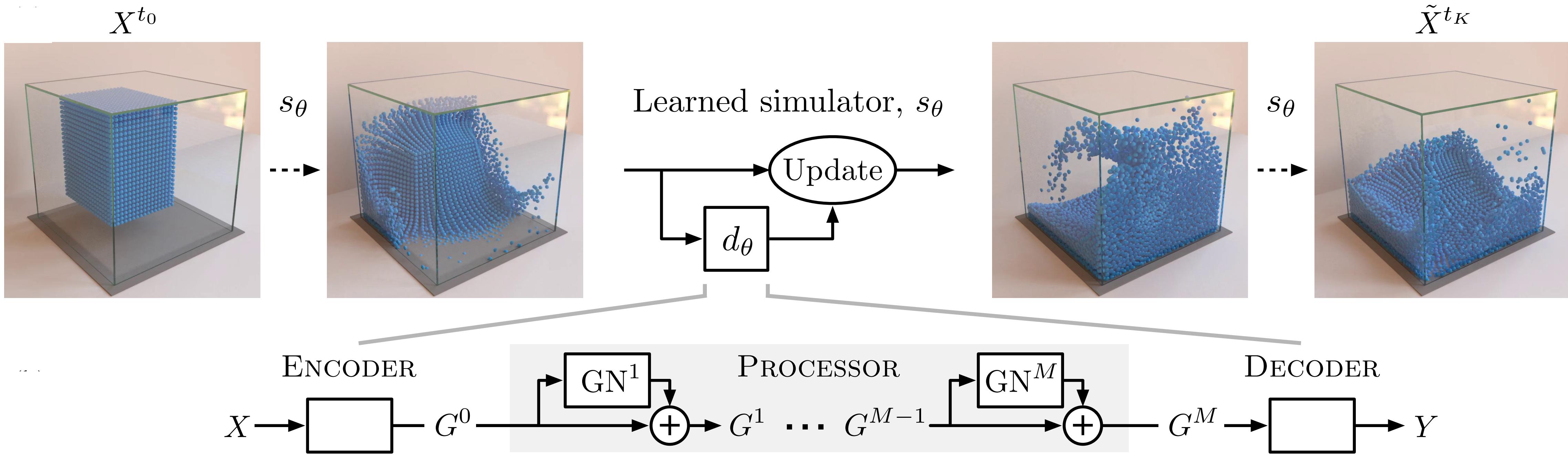
Control: Multiple systems via a single model



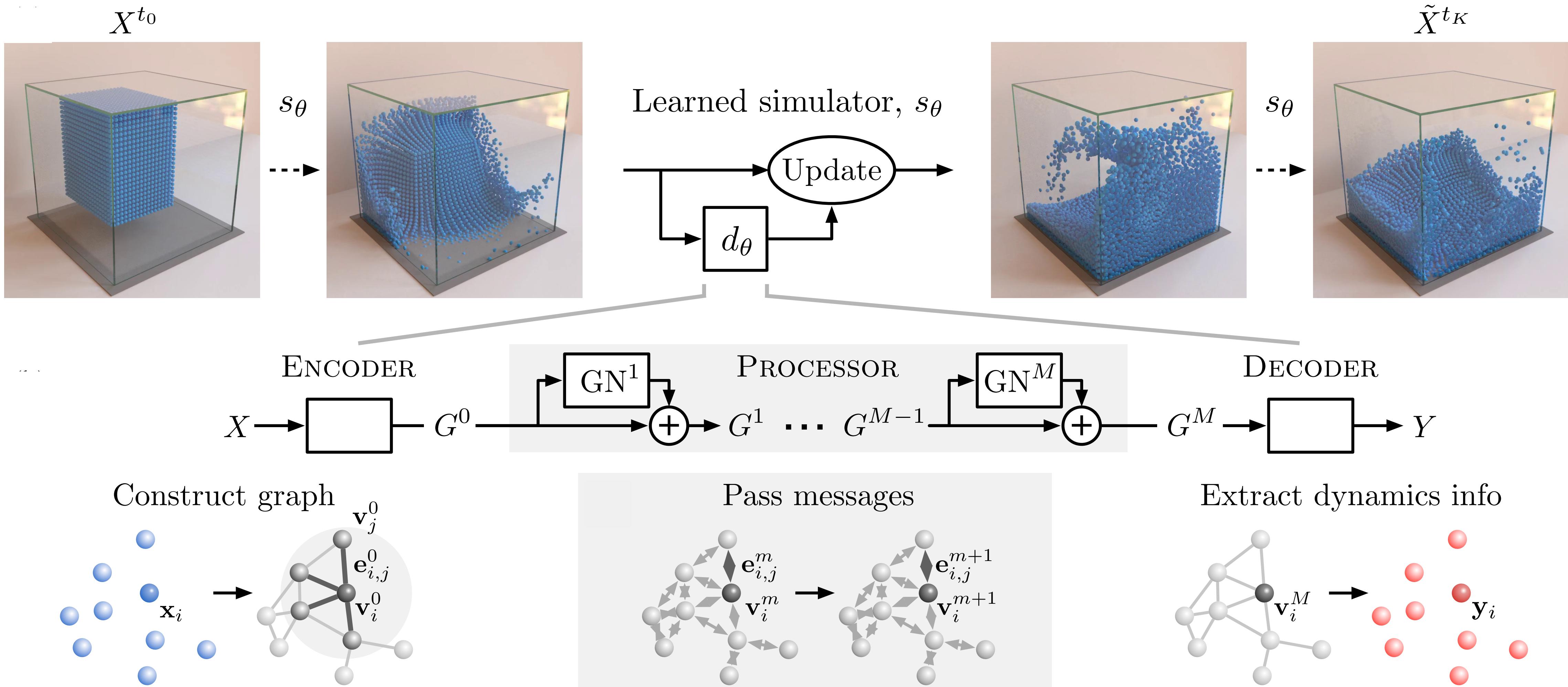
Learning to simulate fluids and complex materials



Learning to simulate fluids and complex materials

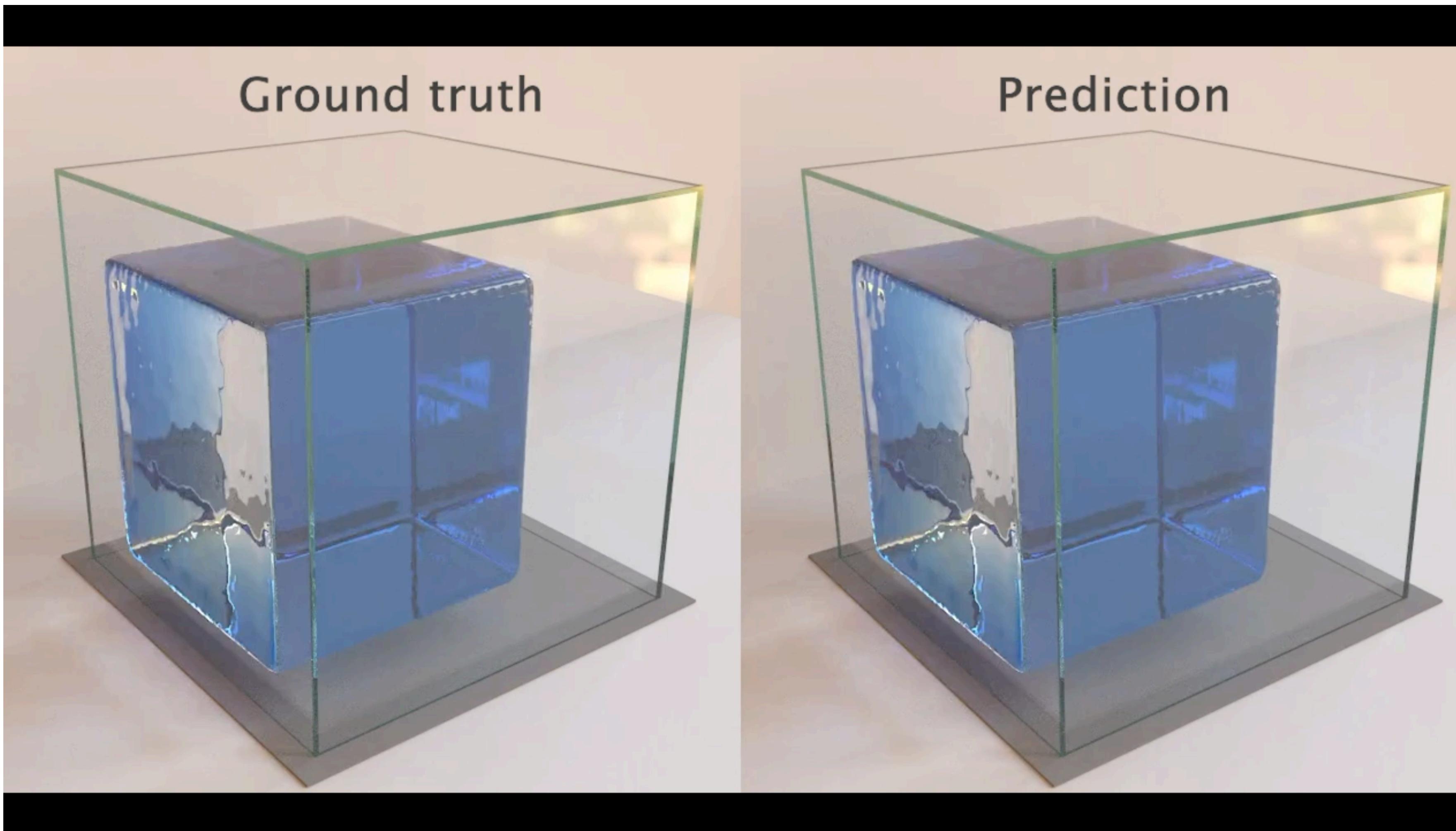


Learning to simulate fluids and complex materials



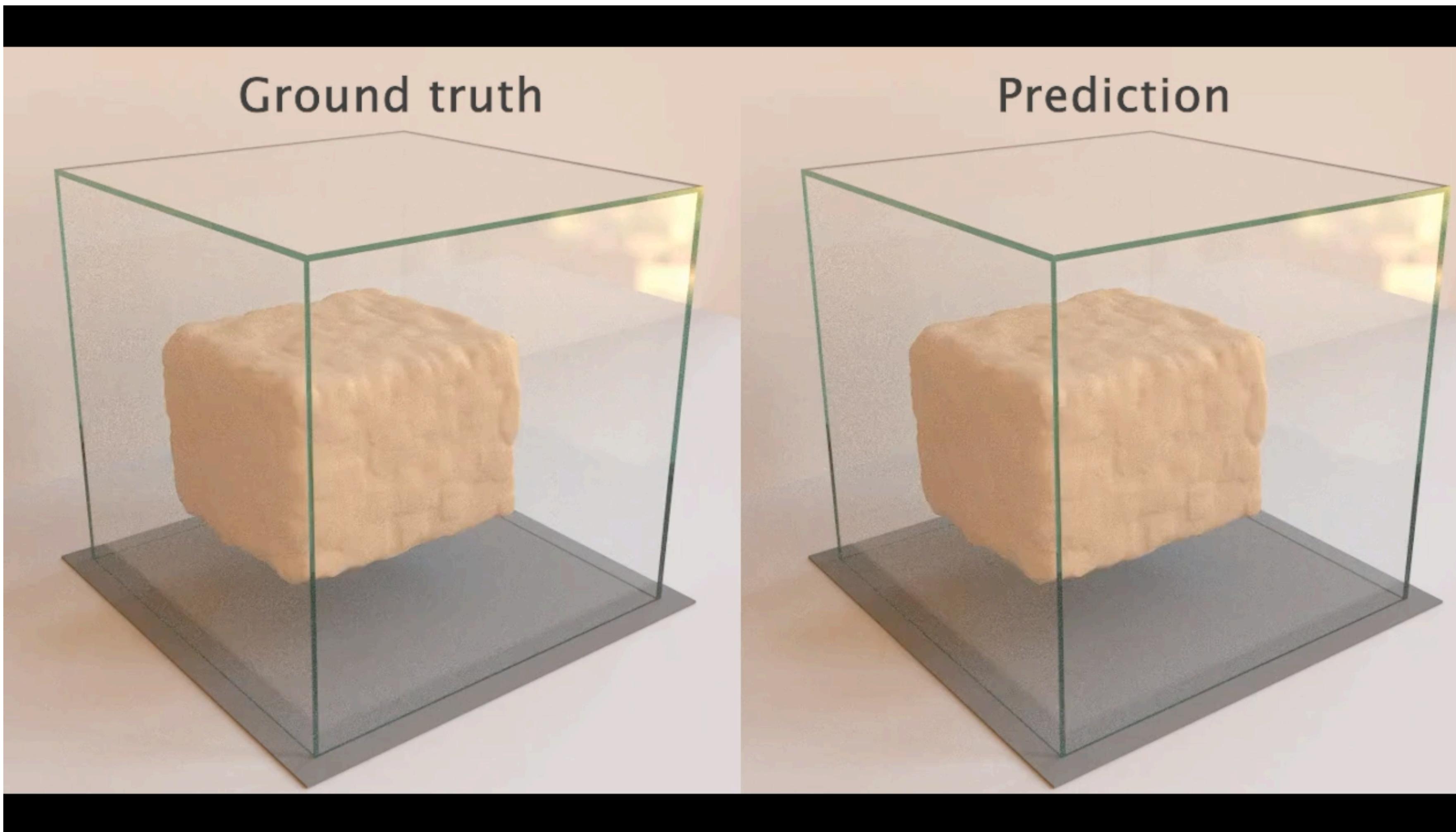
Learning to simulate fluids and complex materials

Water-3D (14k particles, SPH)



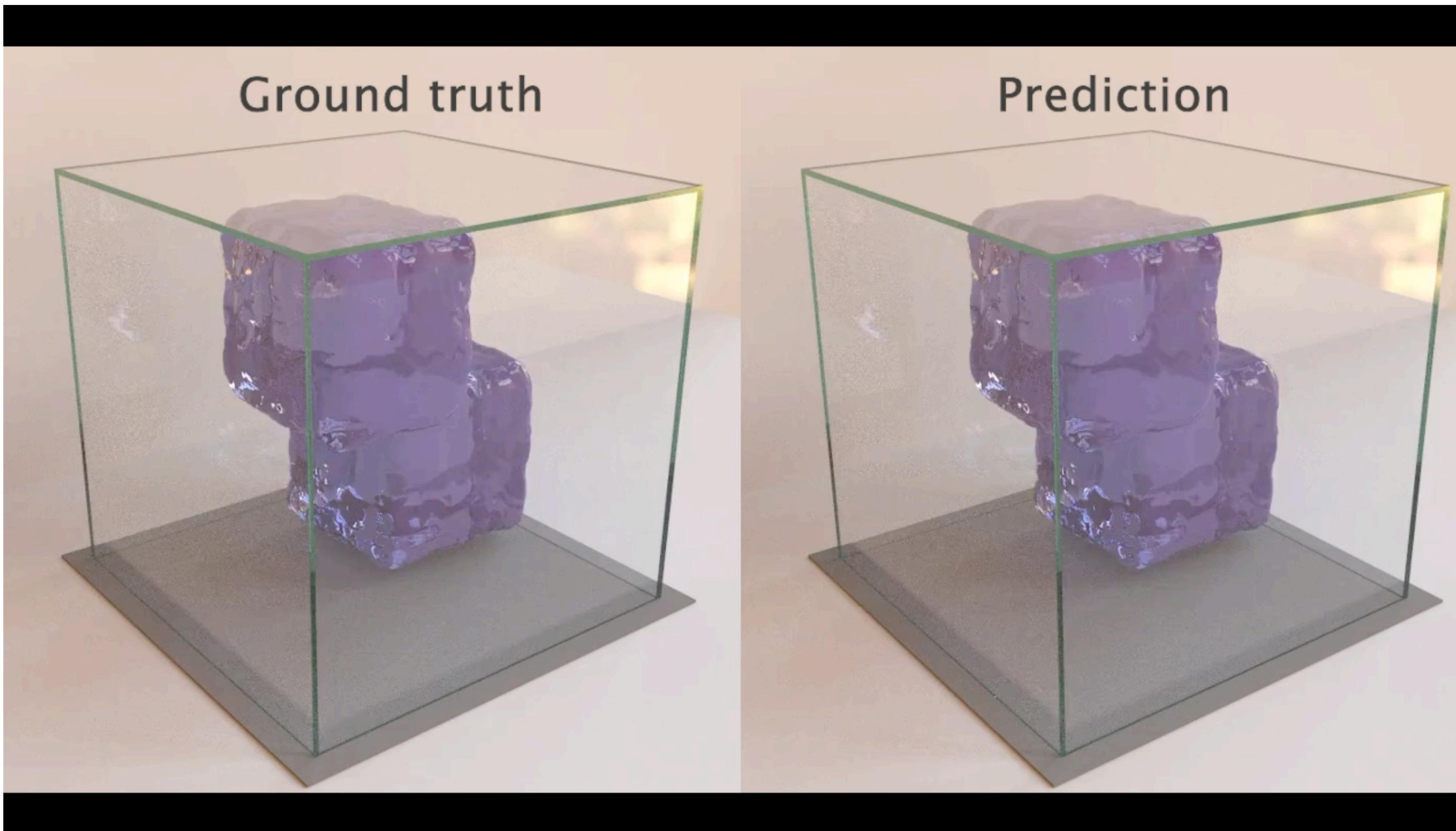
Learning to simulate fluids and complex materials

Sand-3D (19k particles, MPM)



Learning to simulate fluids and complex materials

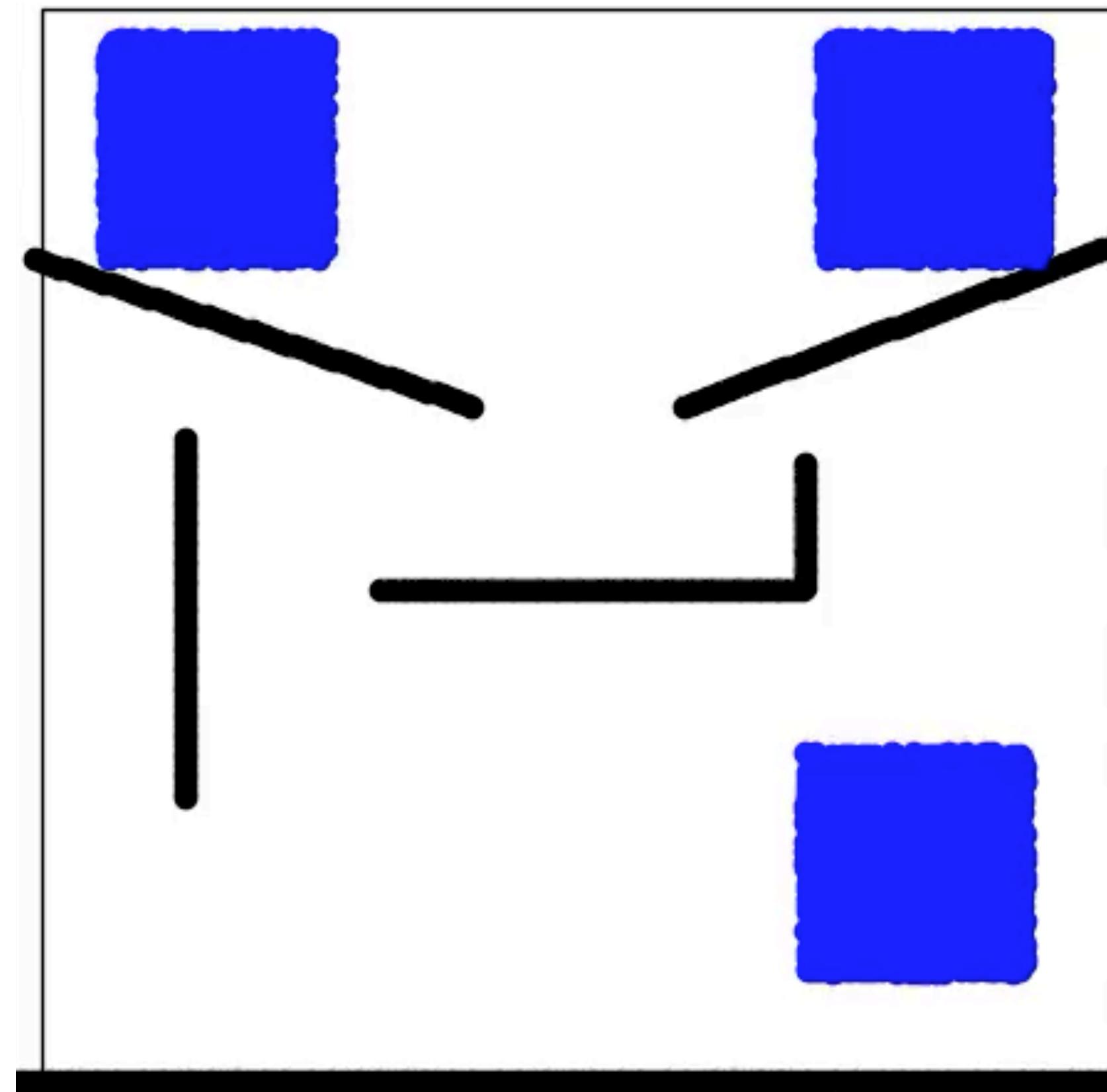
Goop-3D (19k particles, MPM)



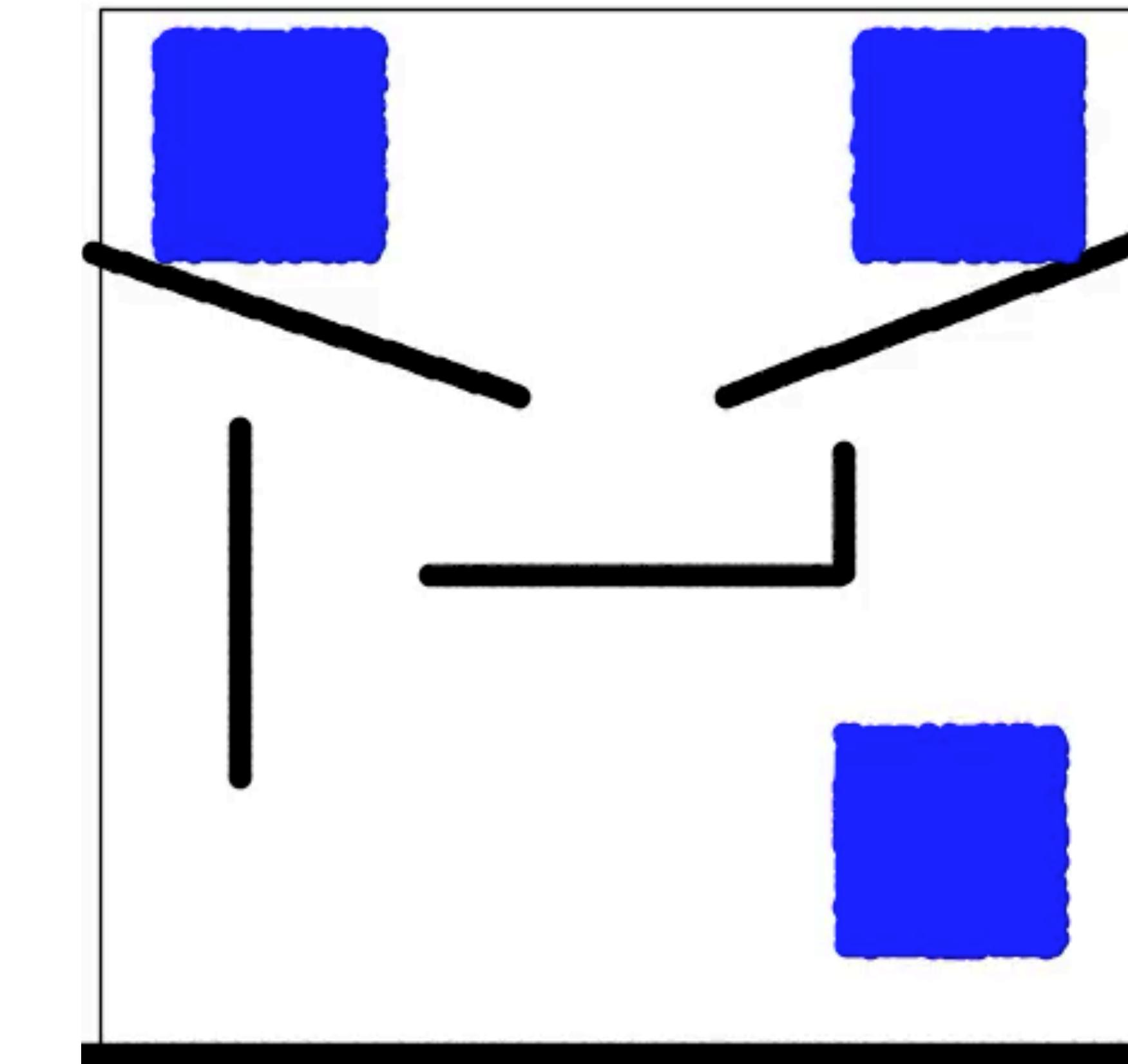
Learning to simulate fluids and complex materials

Water ramps

Ground truth (4943 parts)

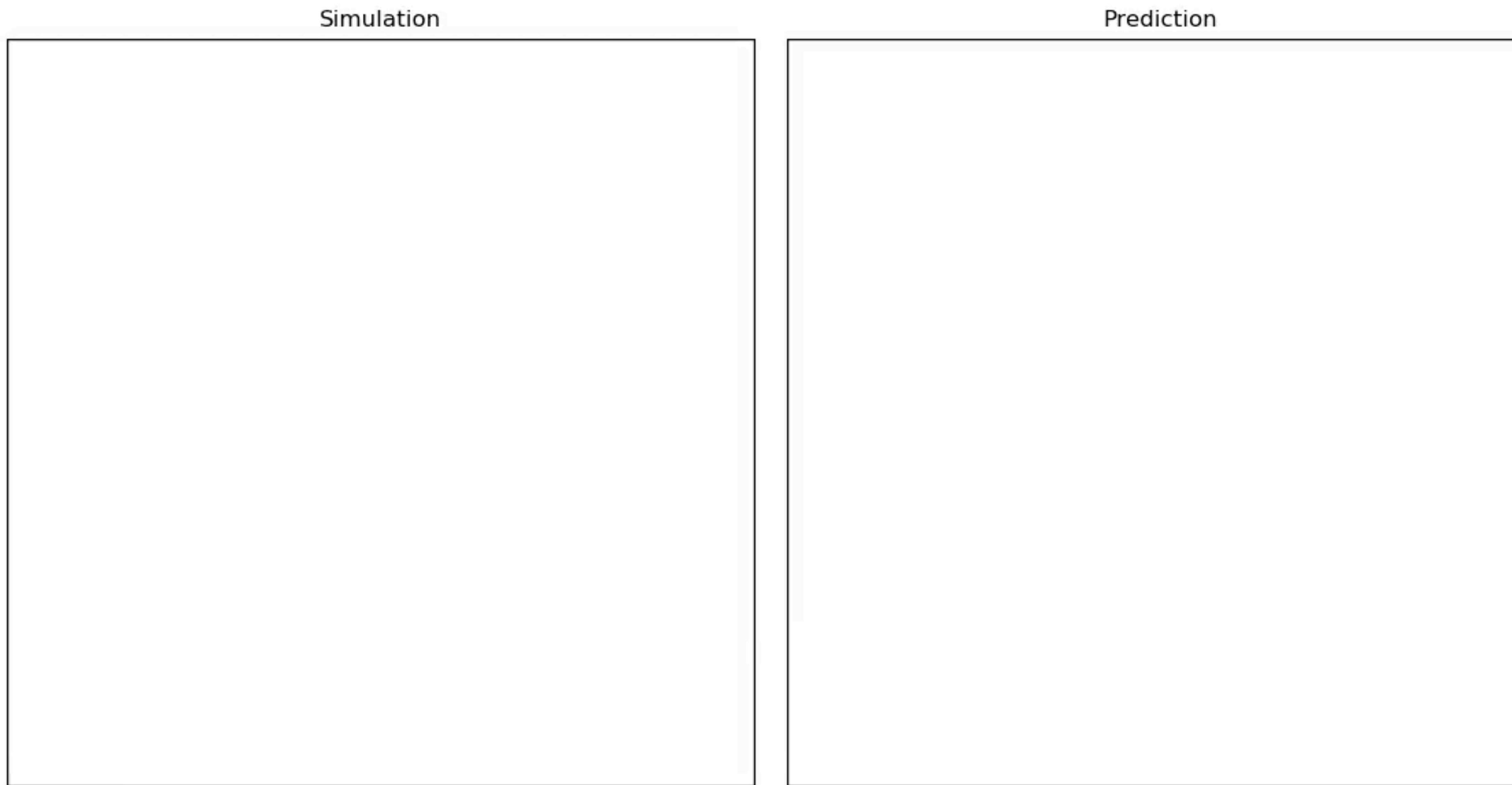


Predictions (4943 parts)



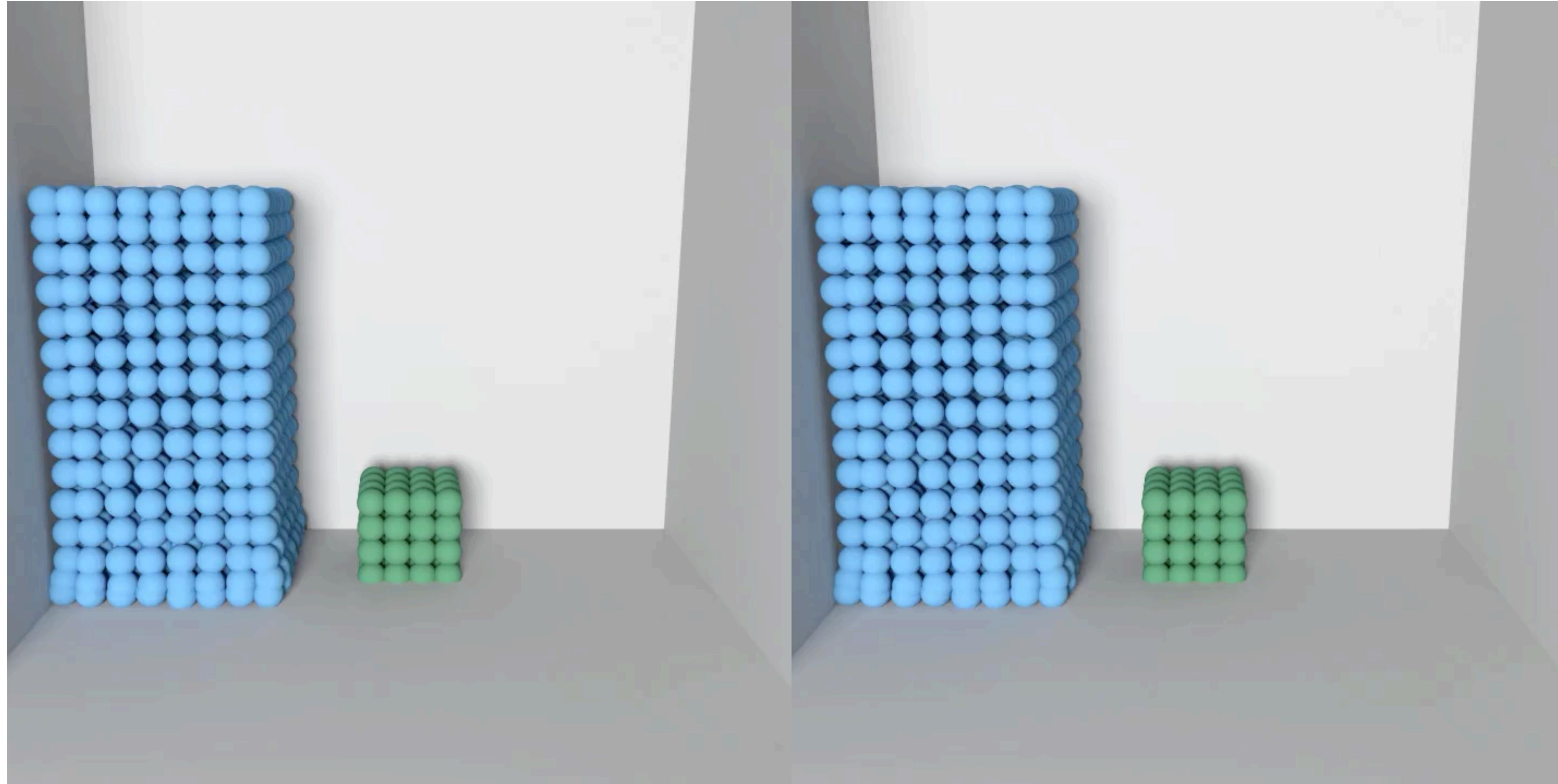
Learning to simulate fluids and complex materials

Multiple materials, generalization



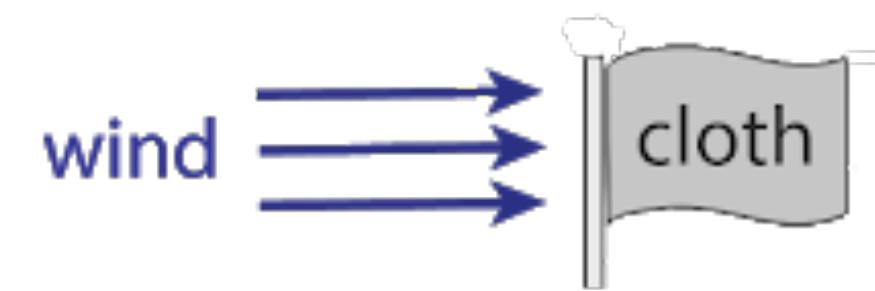
Learning to simulate fluids and complex materials

BoxBath

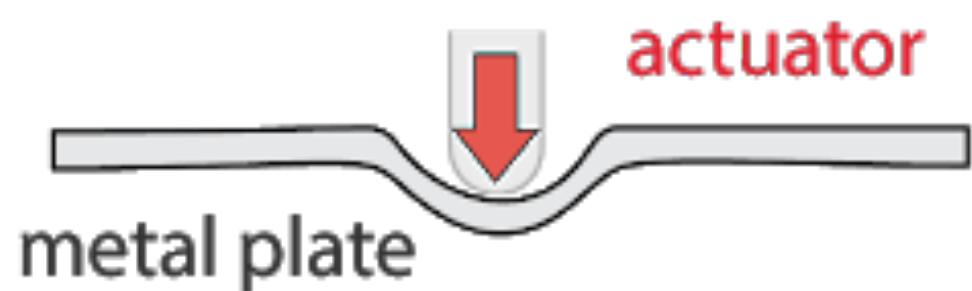


Learning to simulate meshes

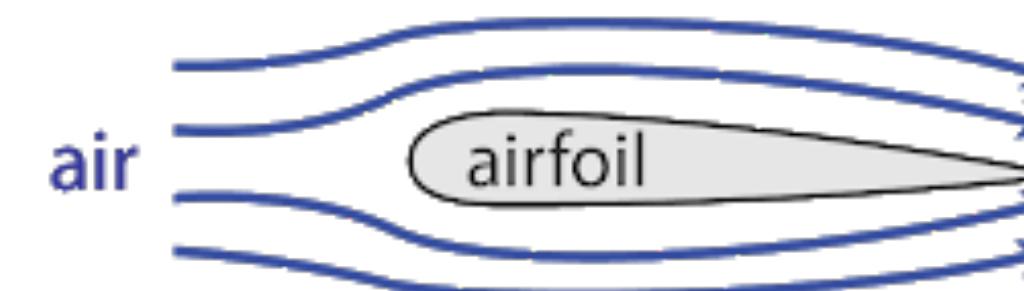
Cloth



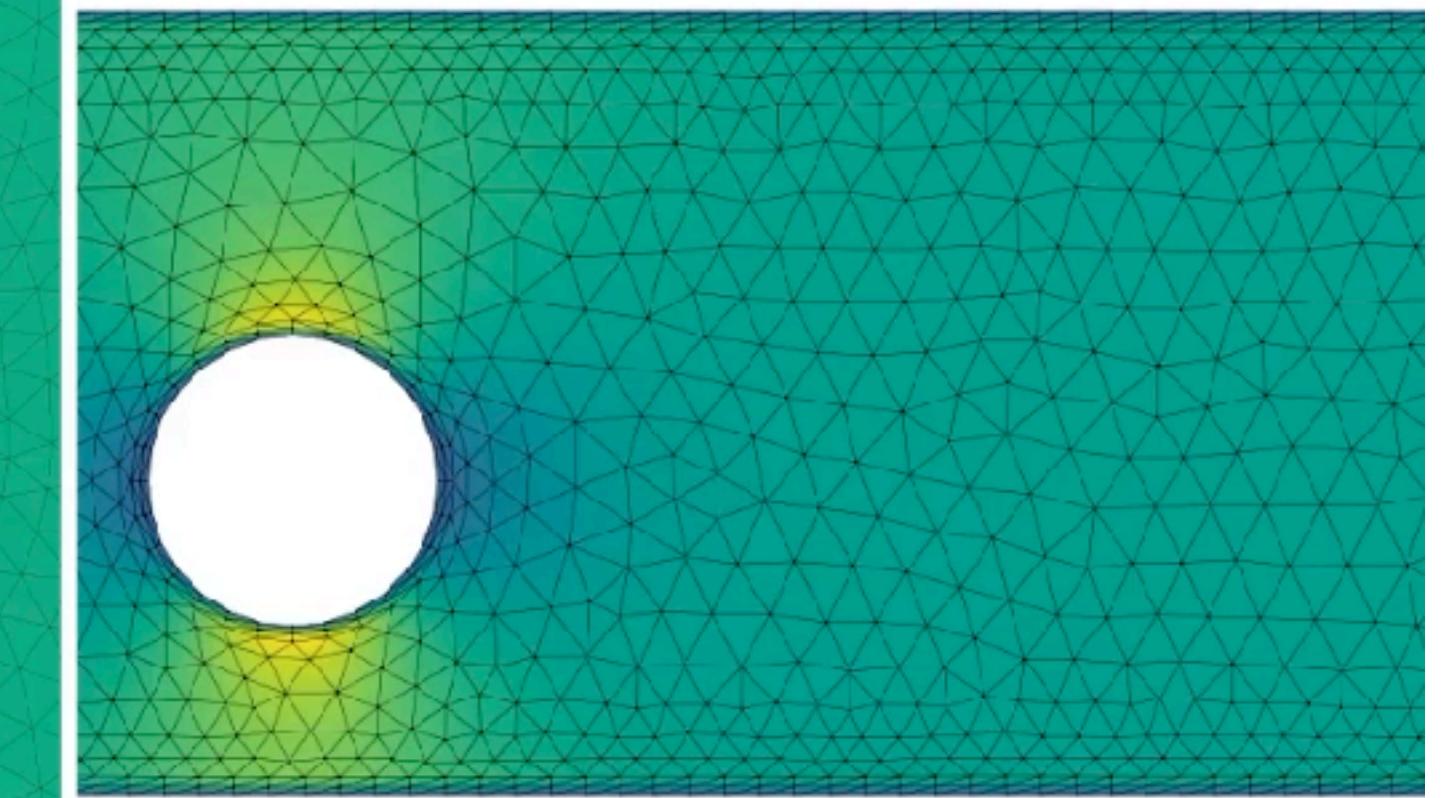
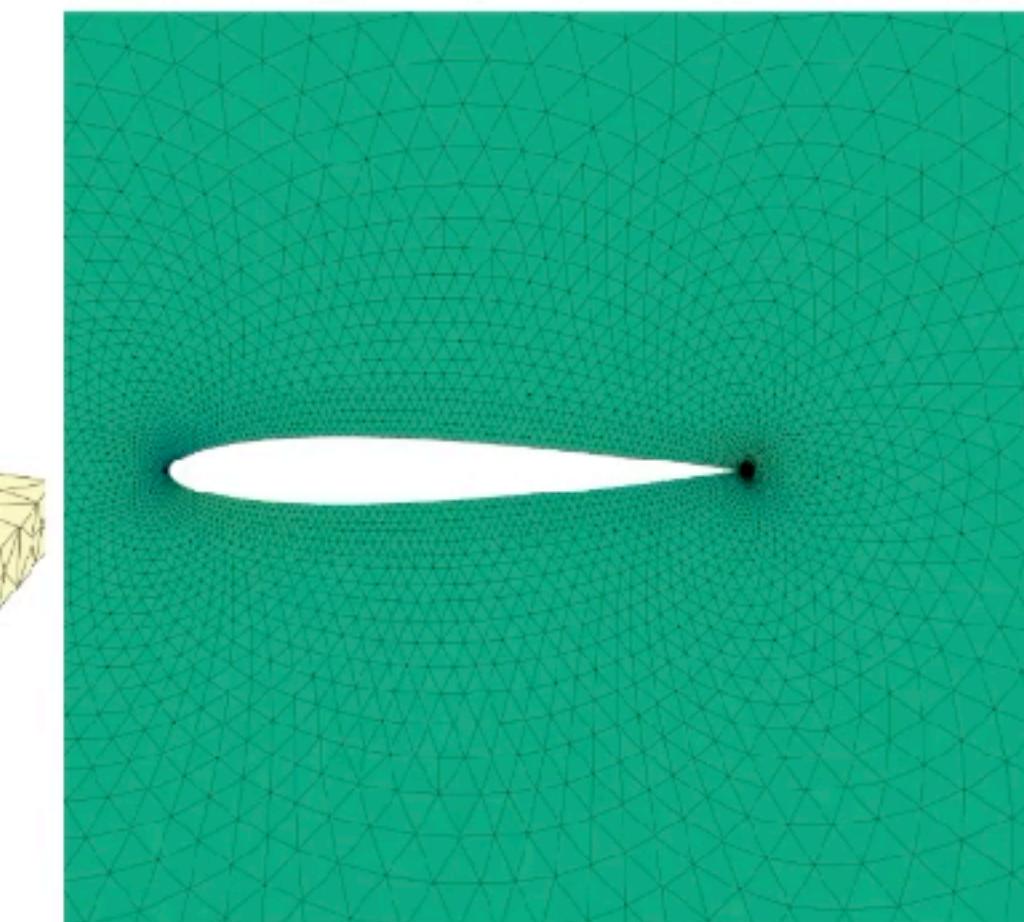
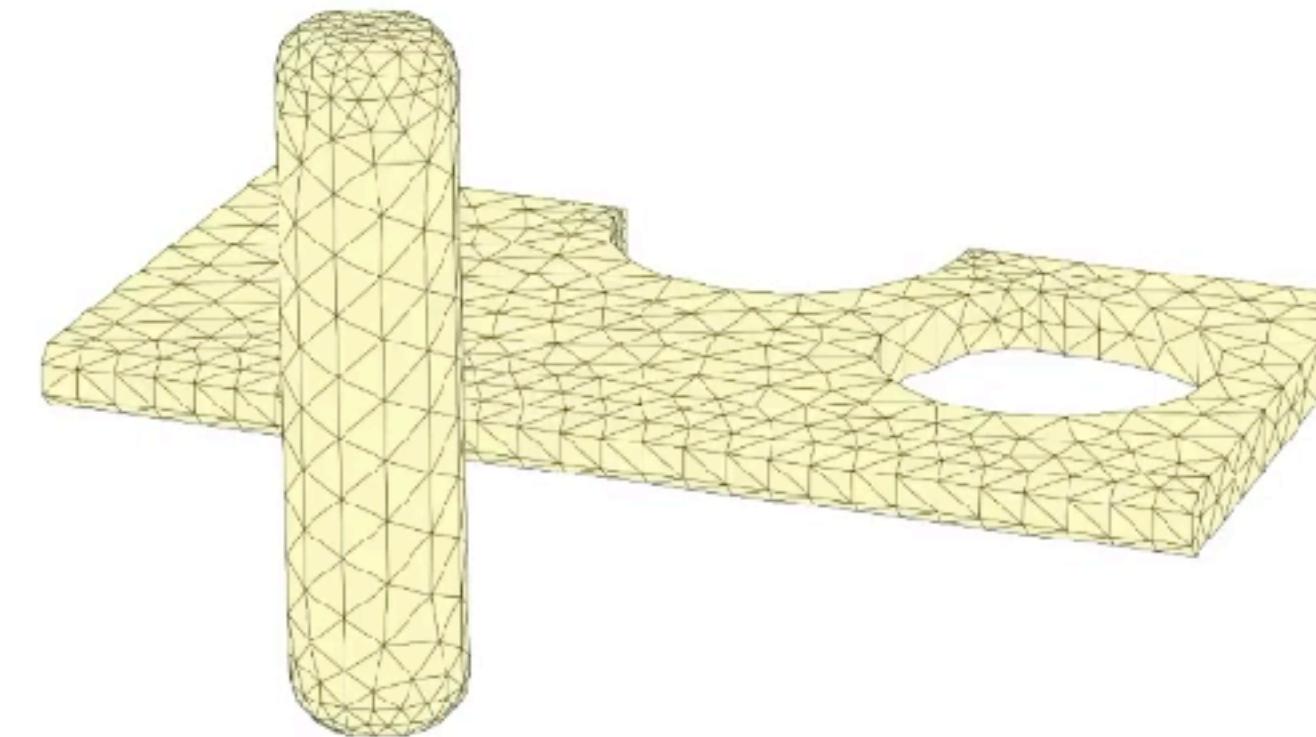
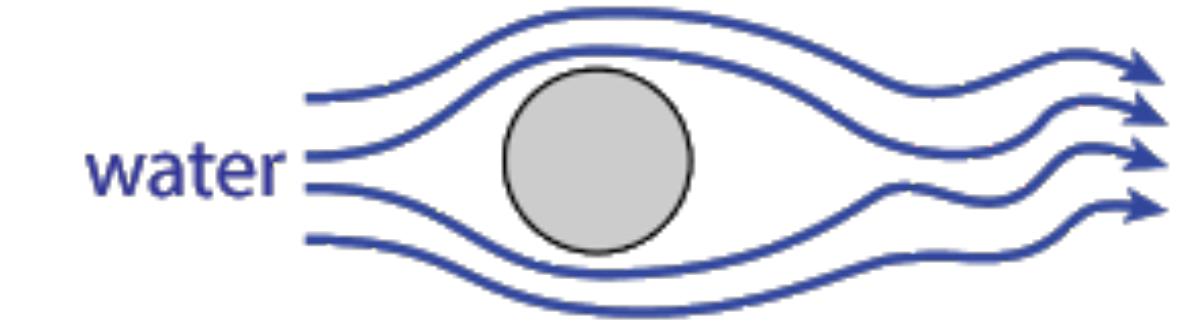
Structural mechanics



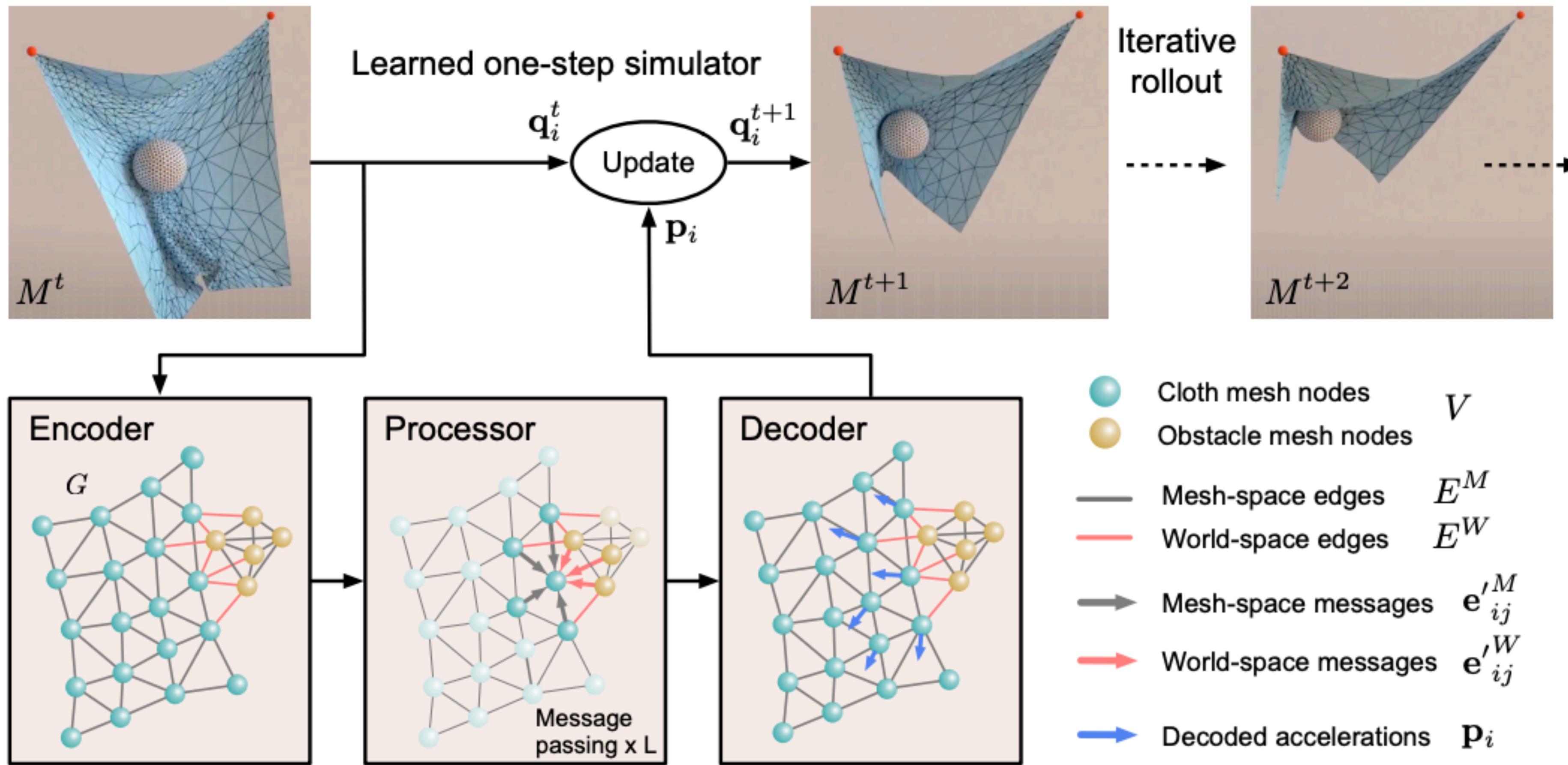
Aerodynamics (compressible)



Fluids (incompressible)

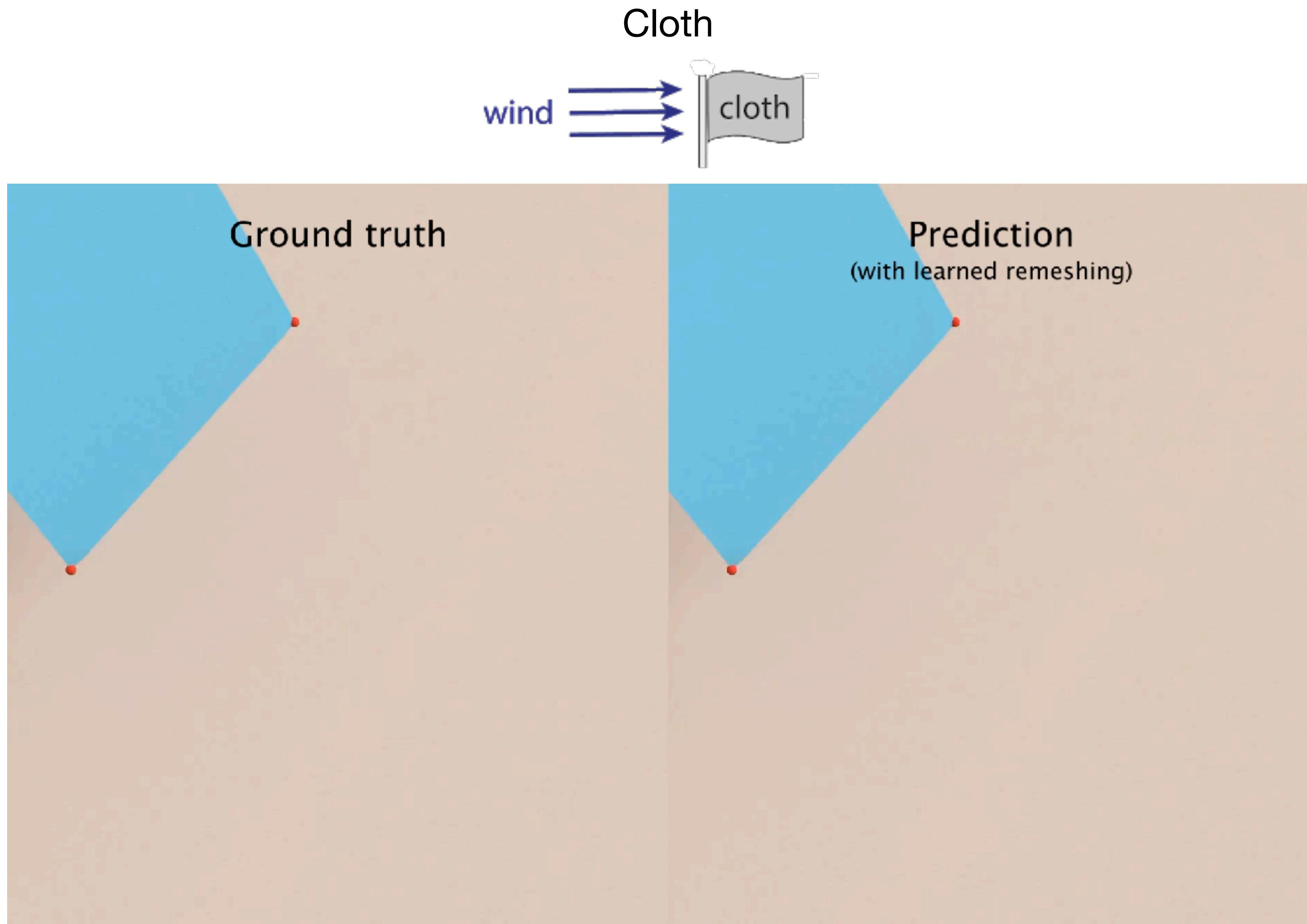


Learning to simulate meshes



10-100x faster than the simulator on which it was trained

Learning to simulate meshes

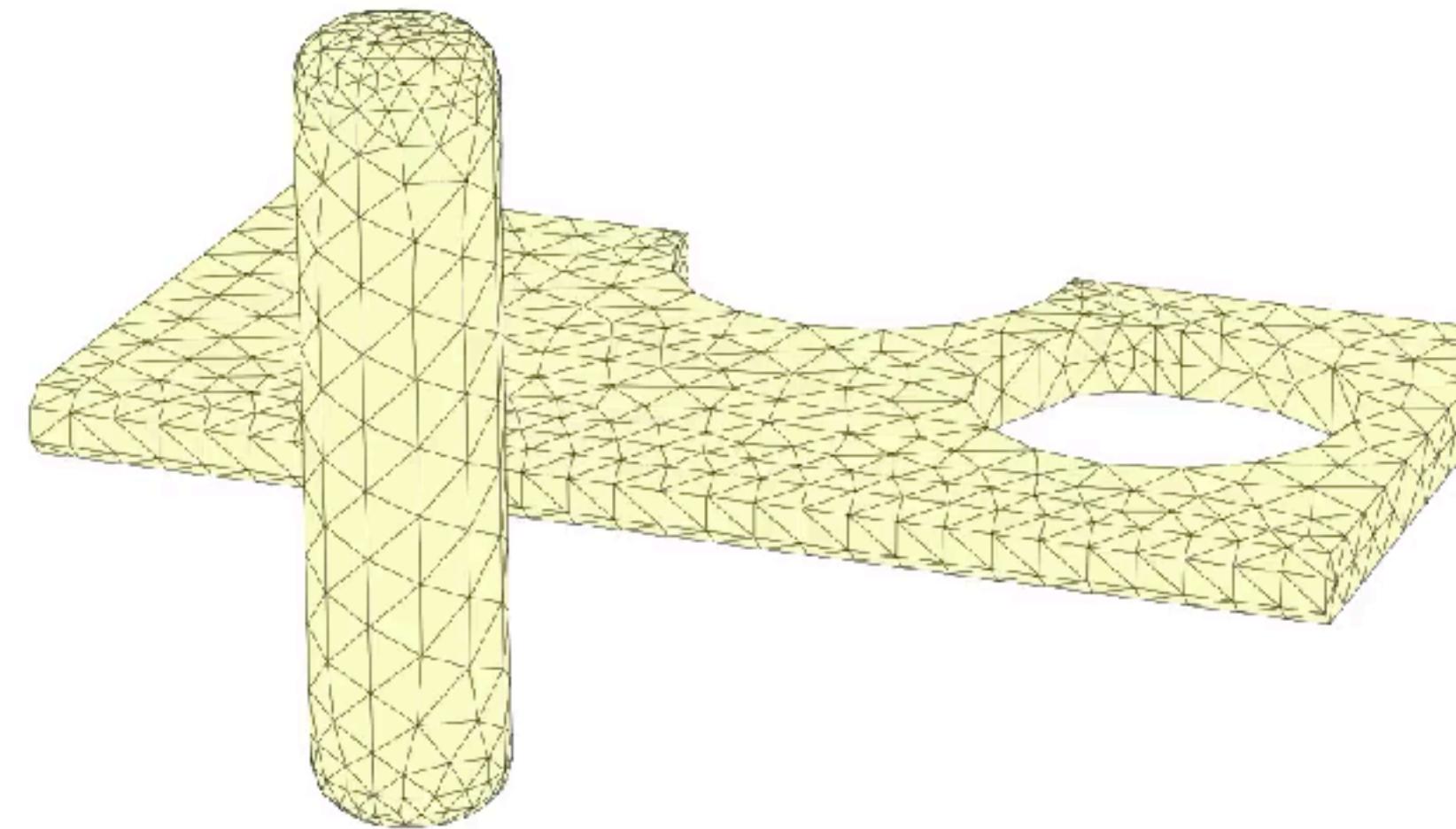


Learning to simulate meshes

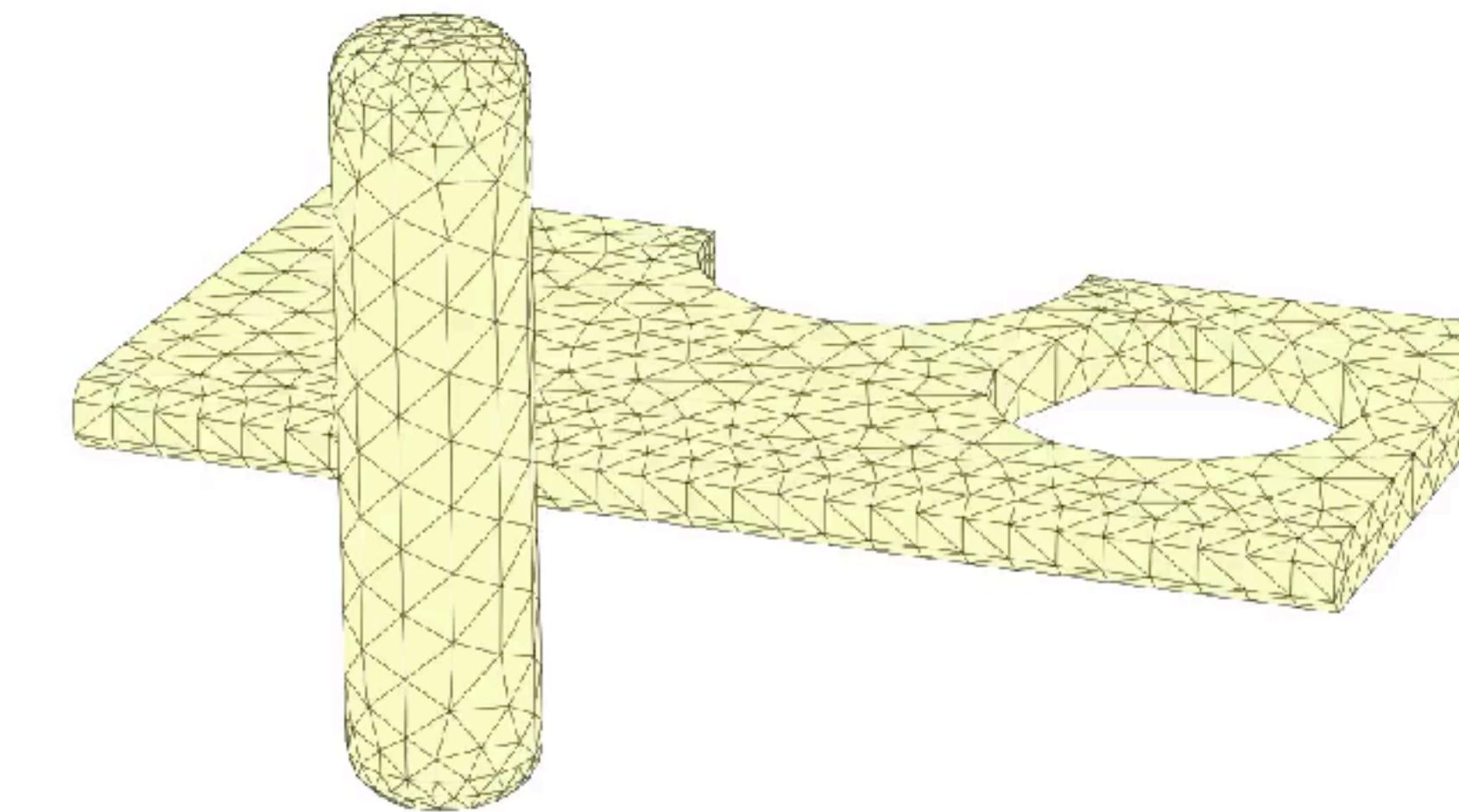
Structural mechanics



Ground truth

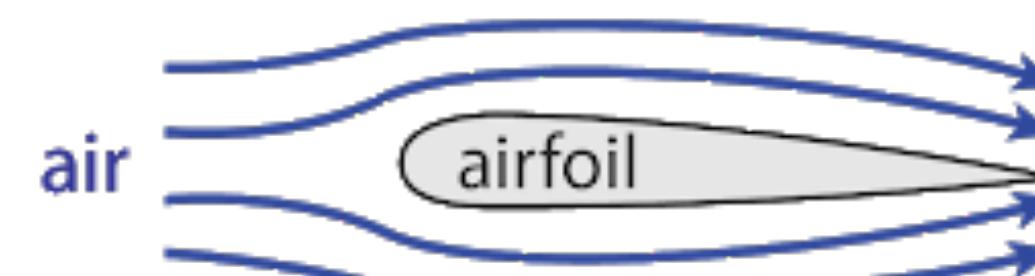


Prediction



Learning to simulate meshes

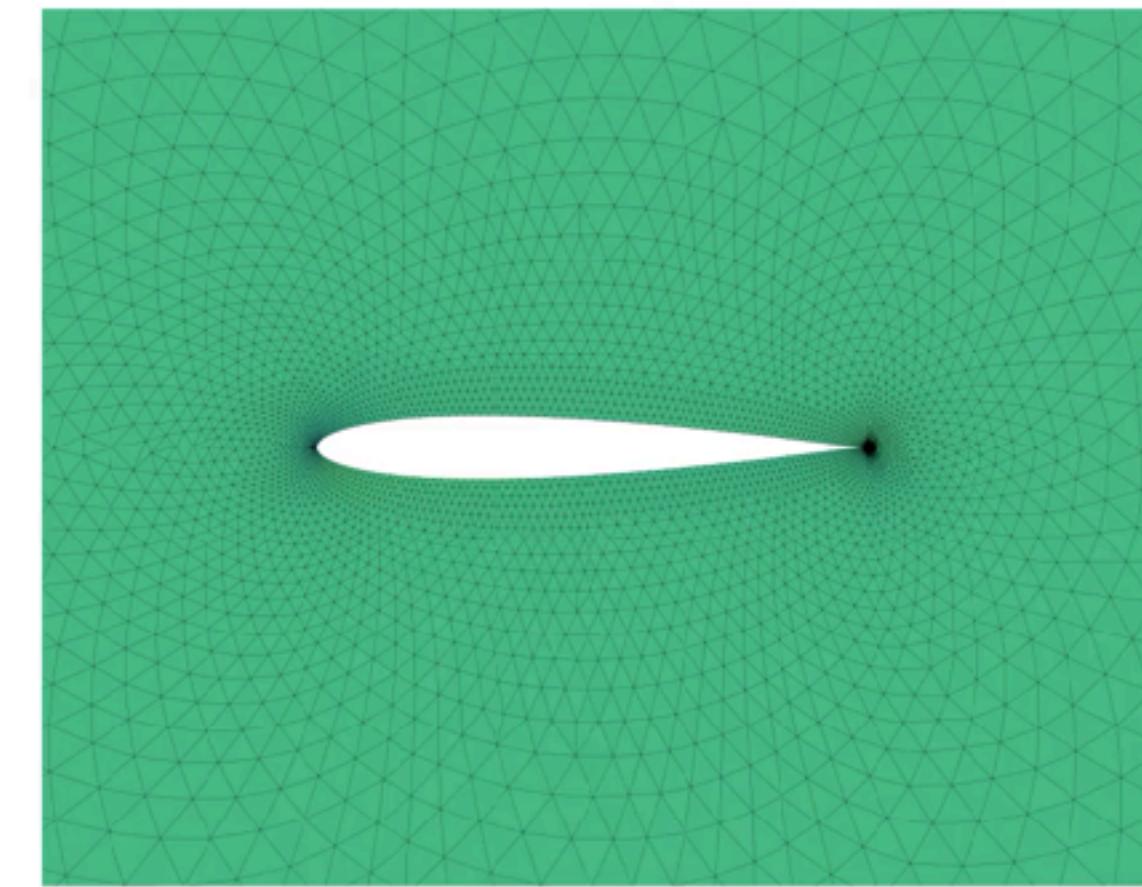
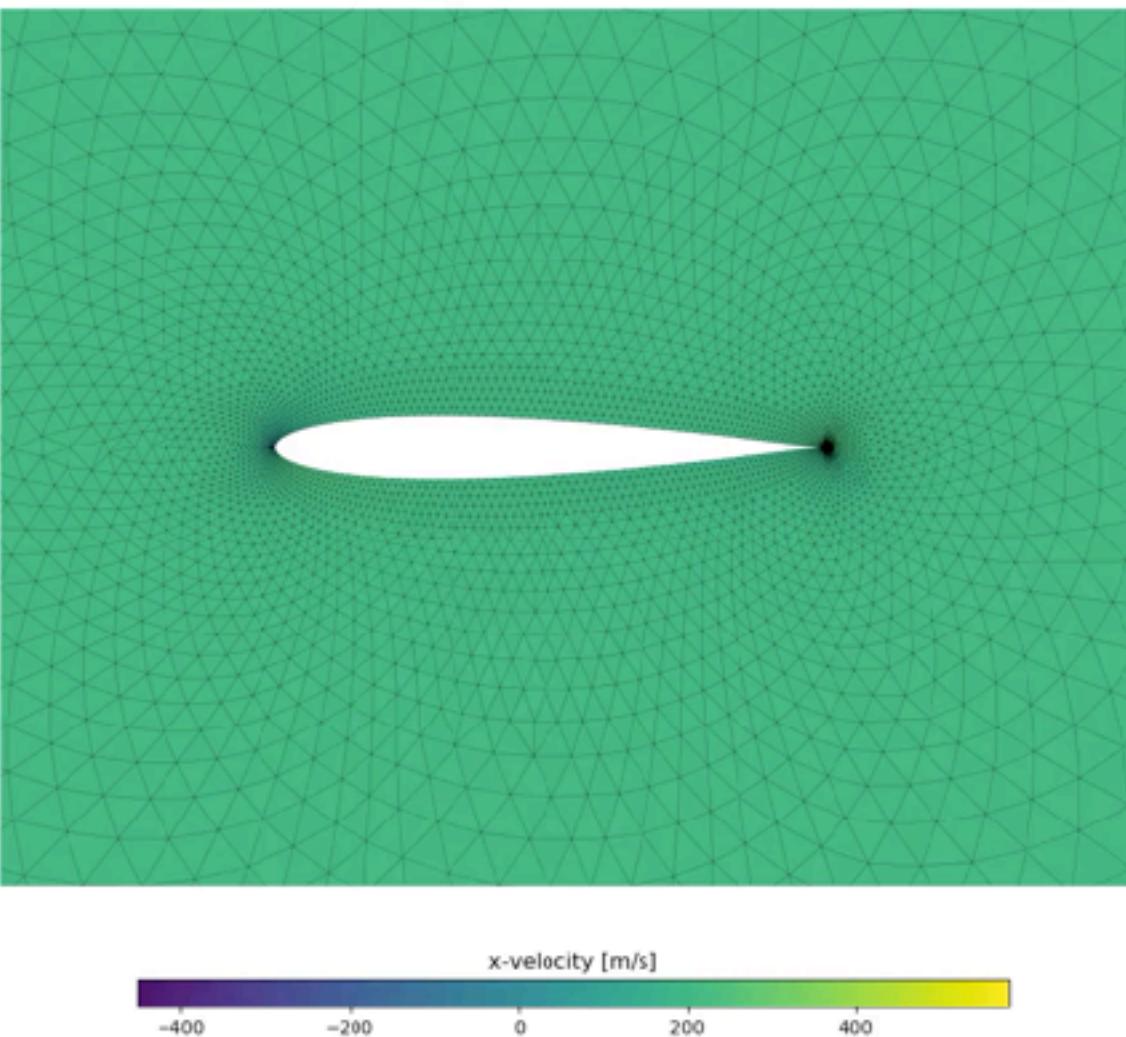
Aerodynamics (compressible)



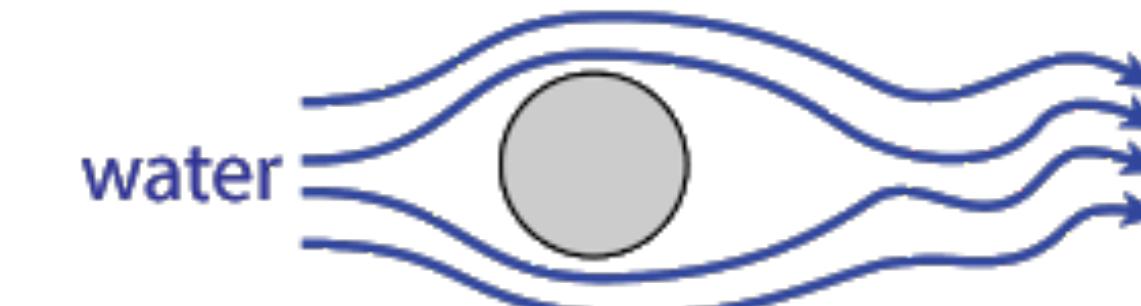
Ground truth

Prediction

mach number 0.66
angle of attack -22.3

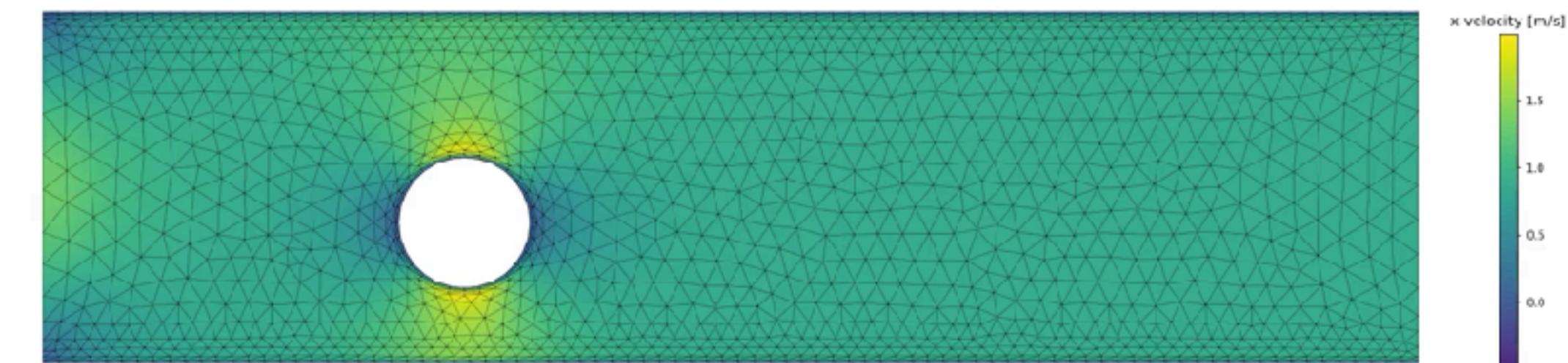


Fluids (incompressible)

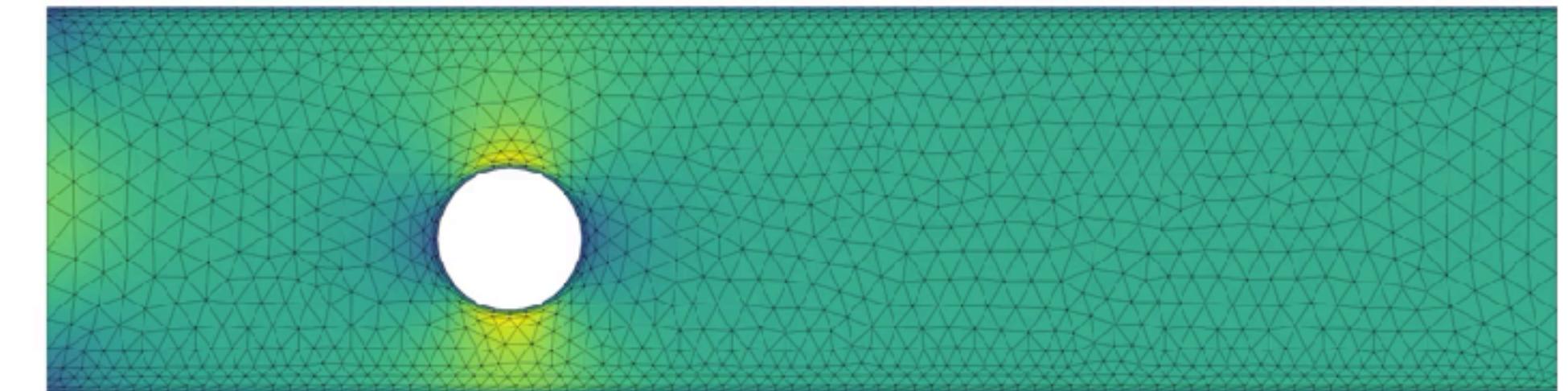


water

Ground truth

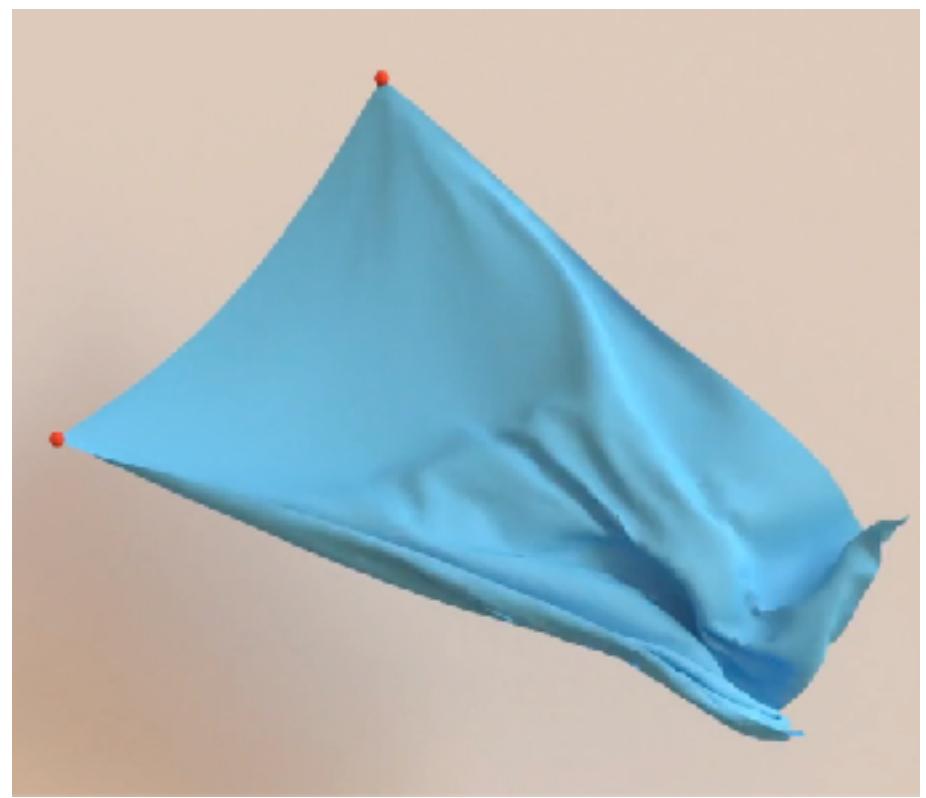


Prediction

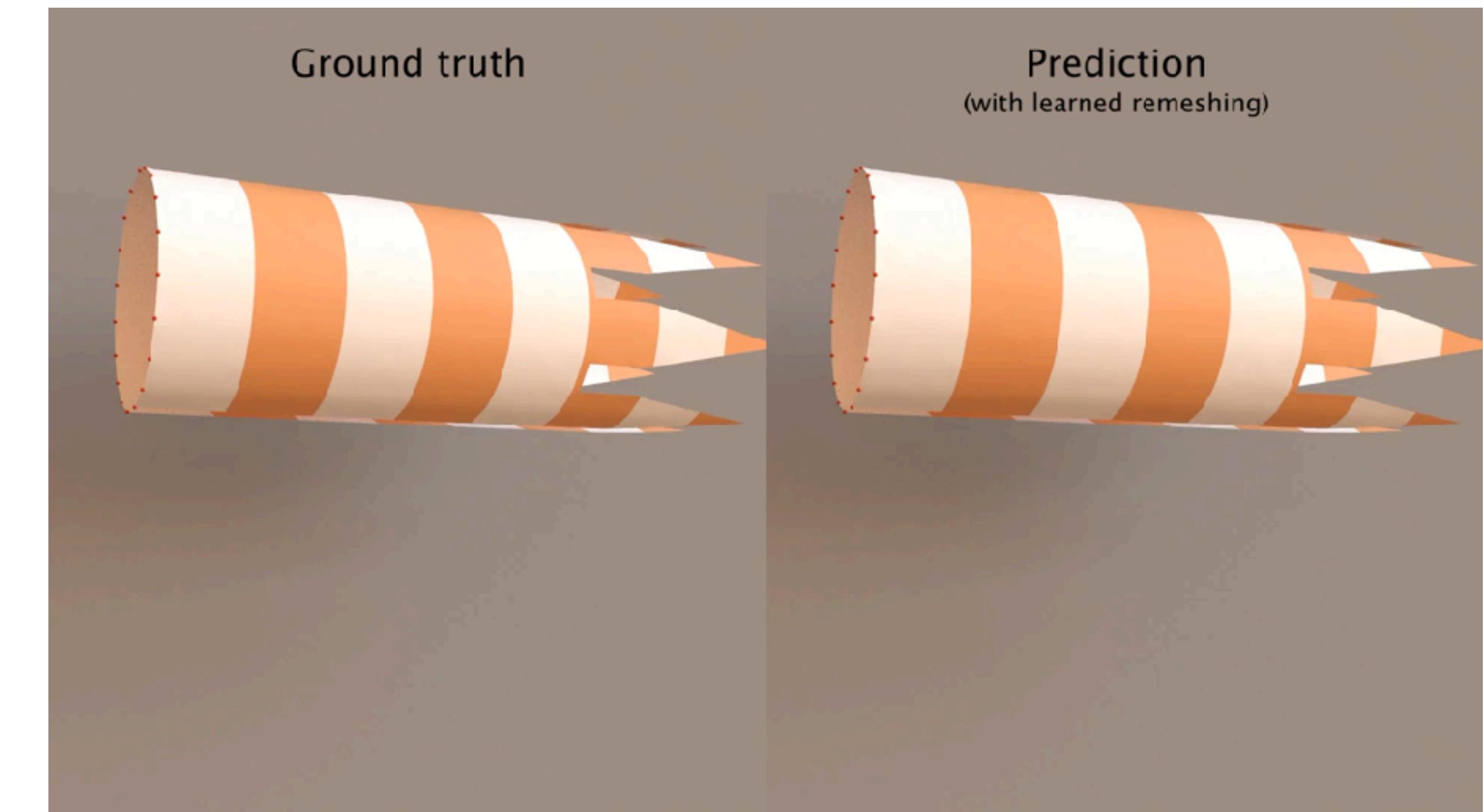
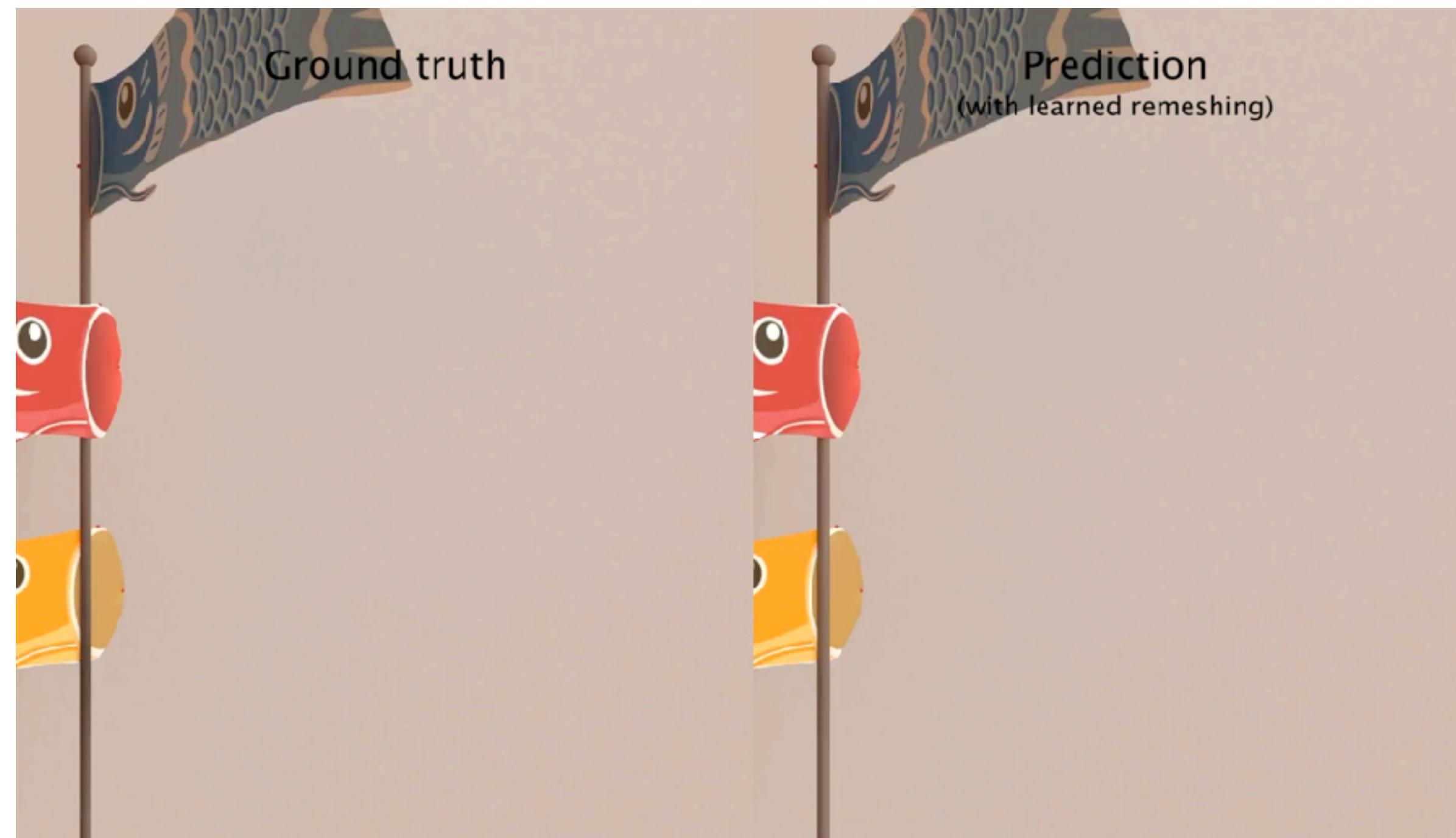


Learning to simulate meshes

Training



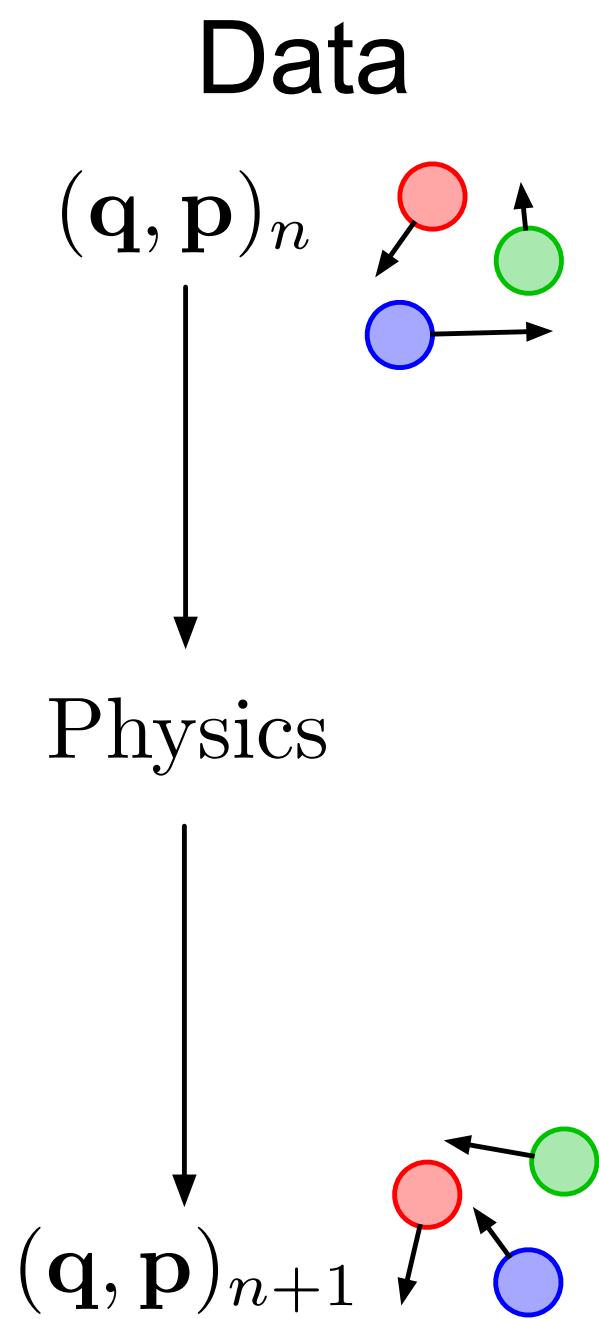
Generalization to different shapes



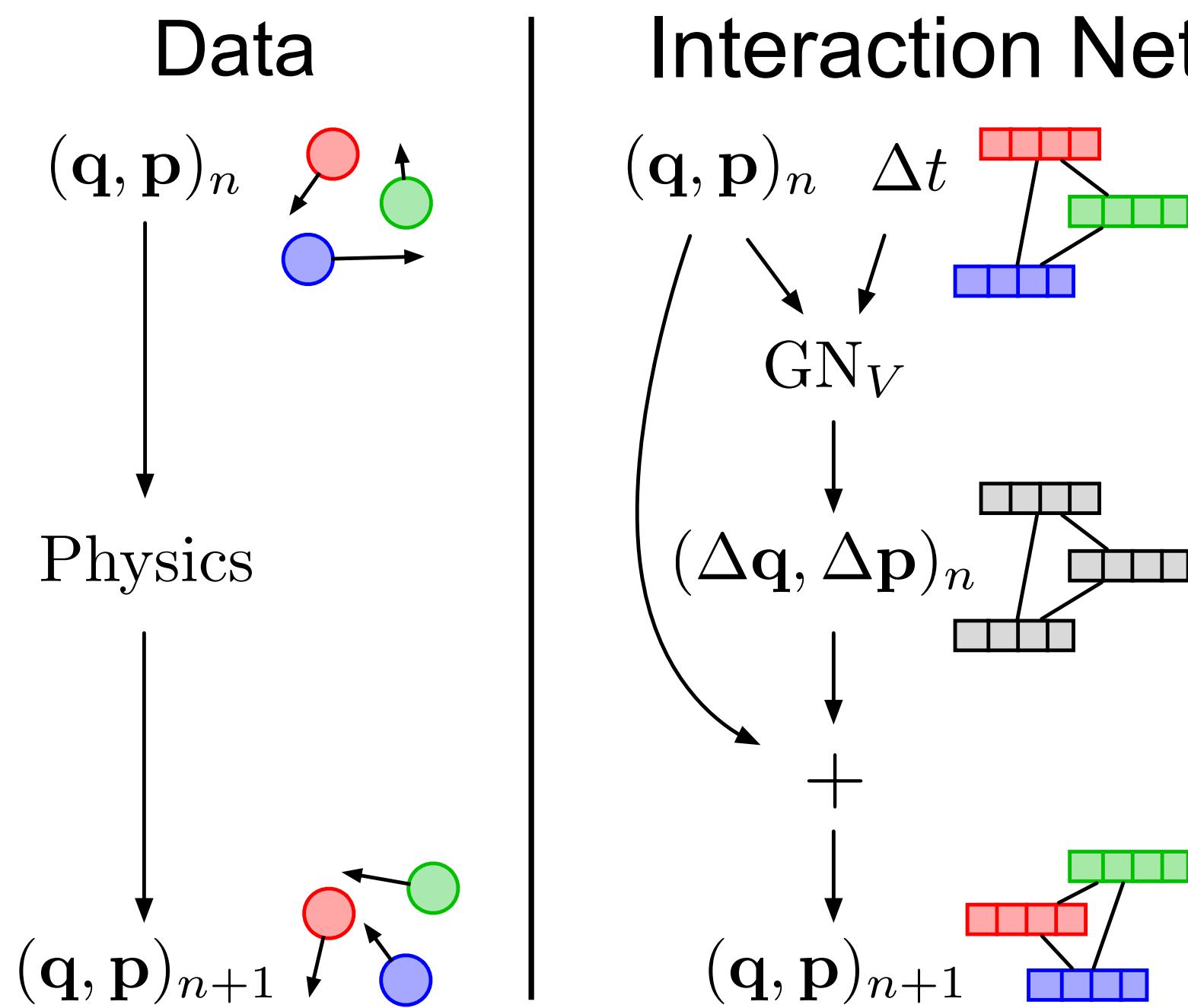
For a deep dive on using graph networks for learning complex simulations, see Alvaro Sanchez-Gonzalez's presentation:

Thursday @ 4-5pm

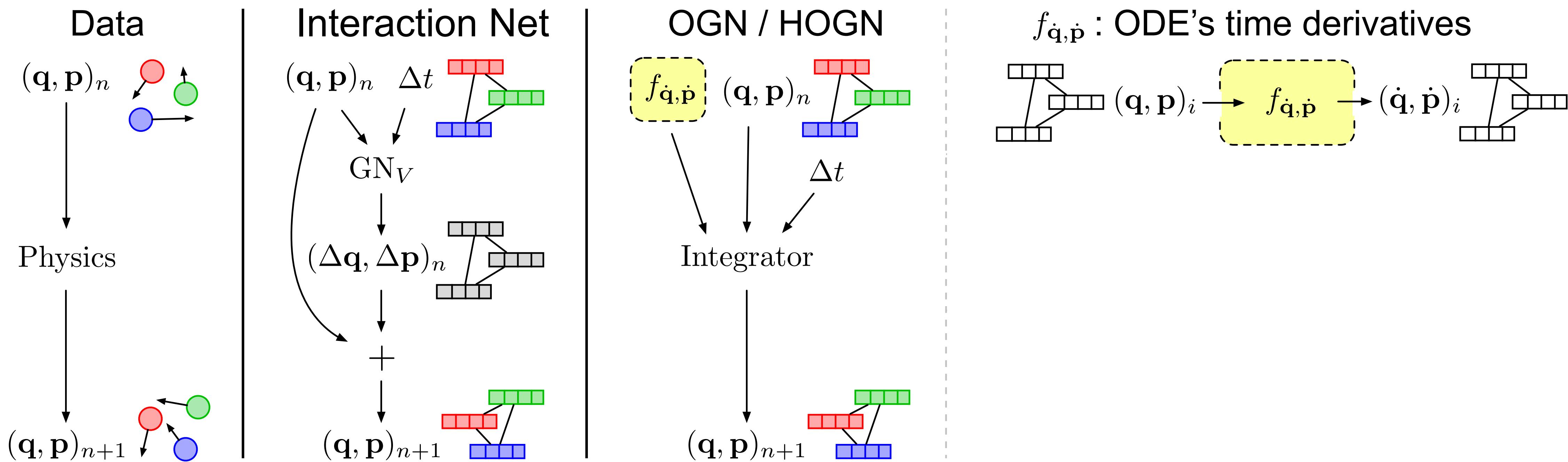
Hamiltonian ODE Graph Network



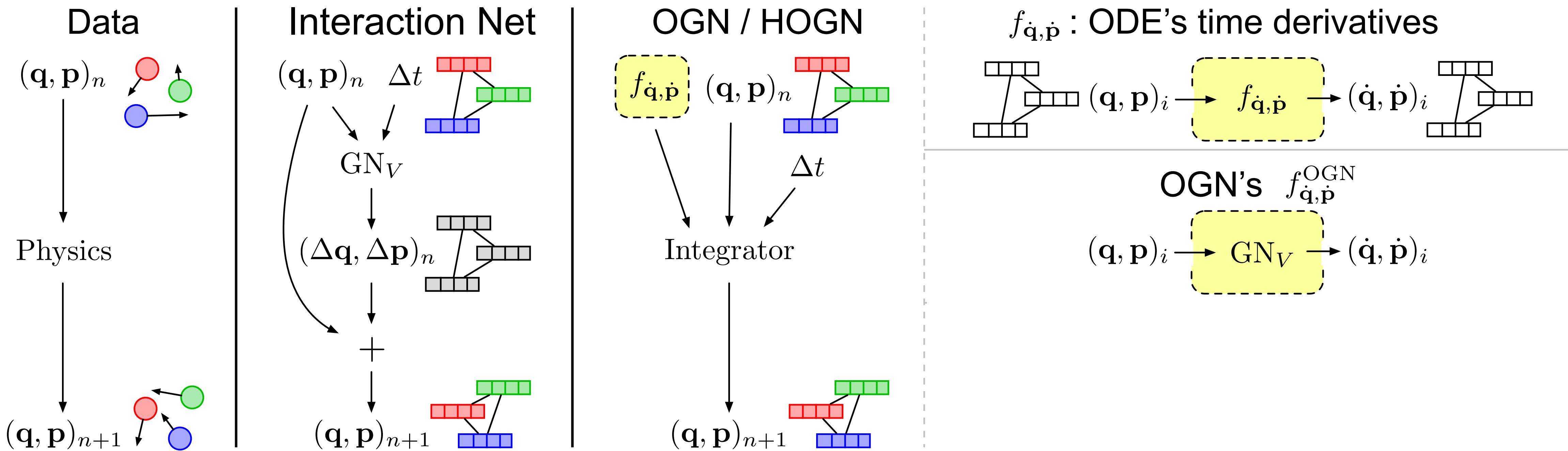
Hamiltonian ODE Graph Network



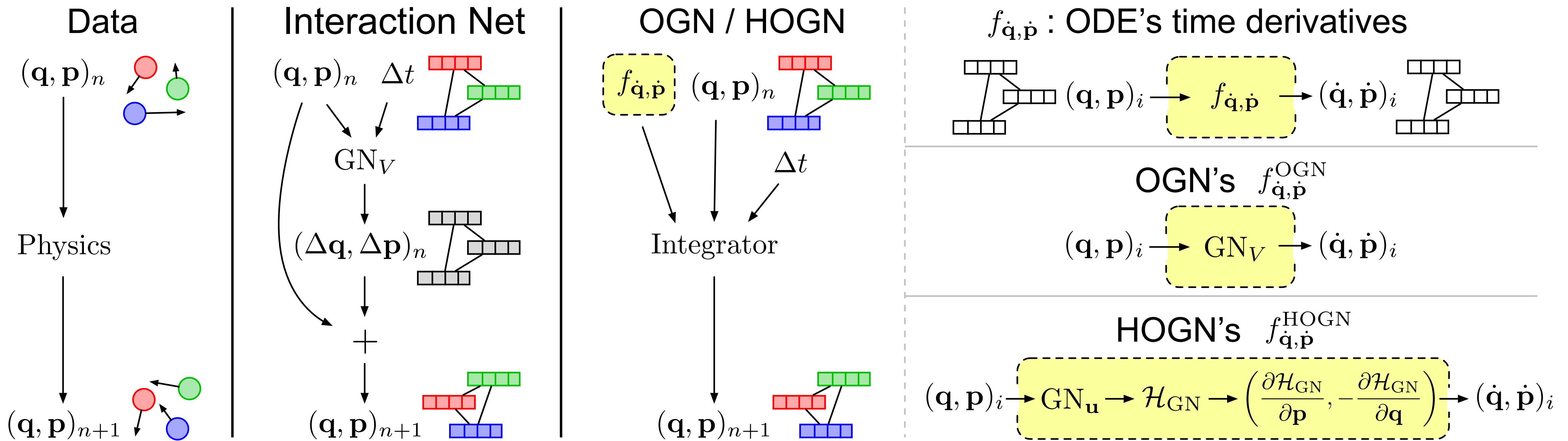
Hamiltonian ODE Graph Network



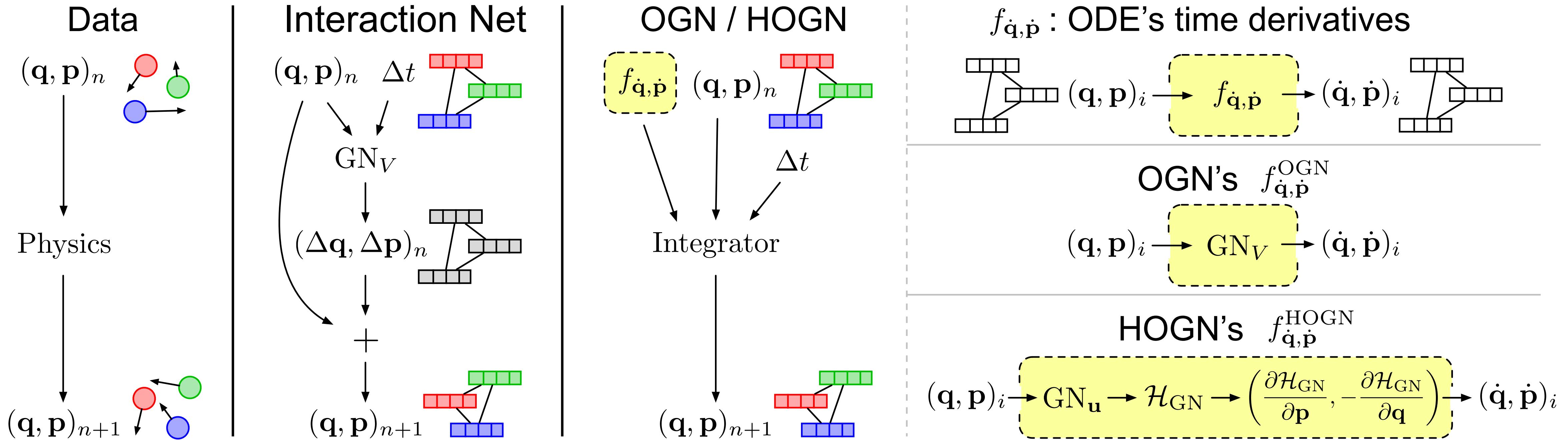
Hamiltonian ODE Graph Network



Hamiltonian ODE Graph Network

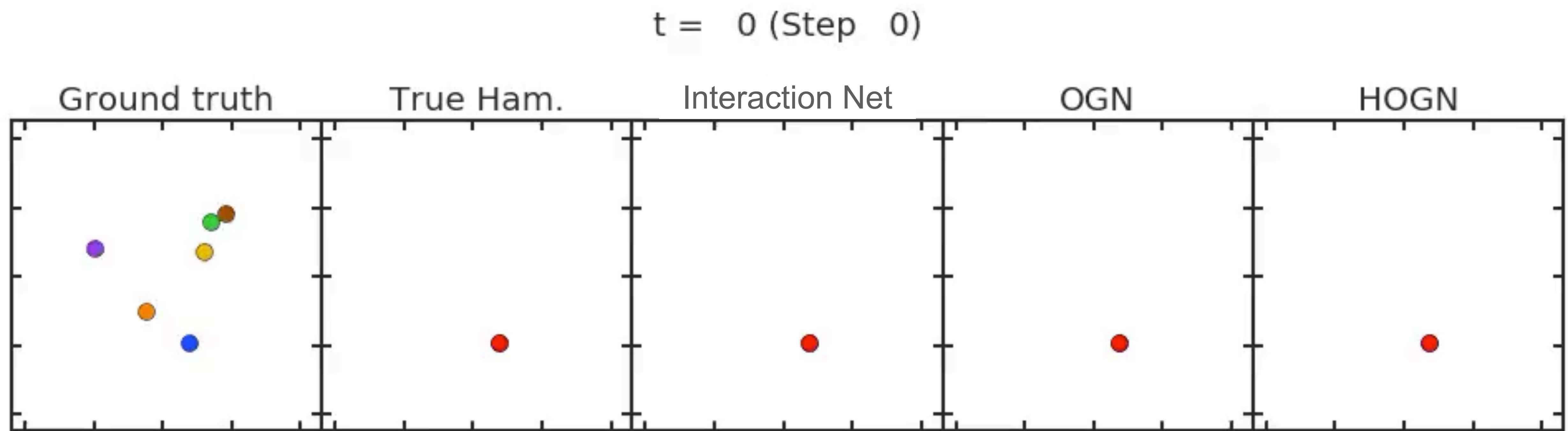


Hamiltonian ODE Graph Network



- * The general idea came from Kyle Cranmer, who was a co-author on this.
- * Also related to Greydanus et al. 2020 “Hamiltonian Neural Networks”.

Hamiltonian ODE Graph Network

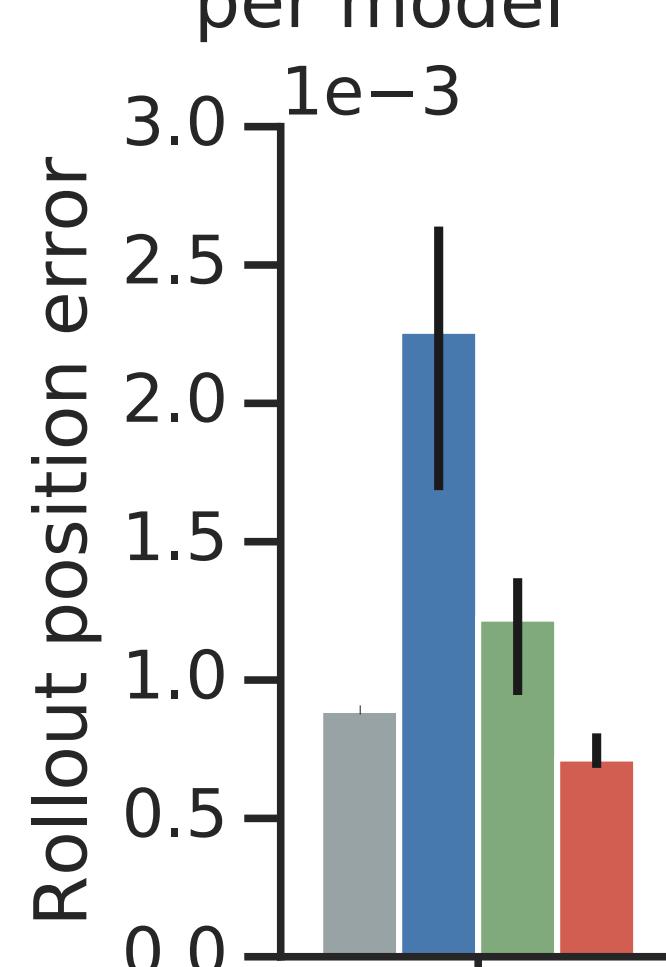


Sanchez-Gonzalez et al., 2019, arXiv/NeurIPS 2019 workshop

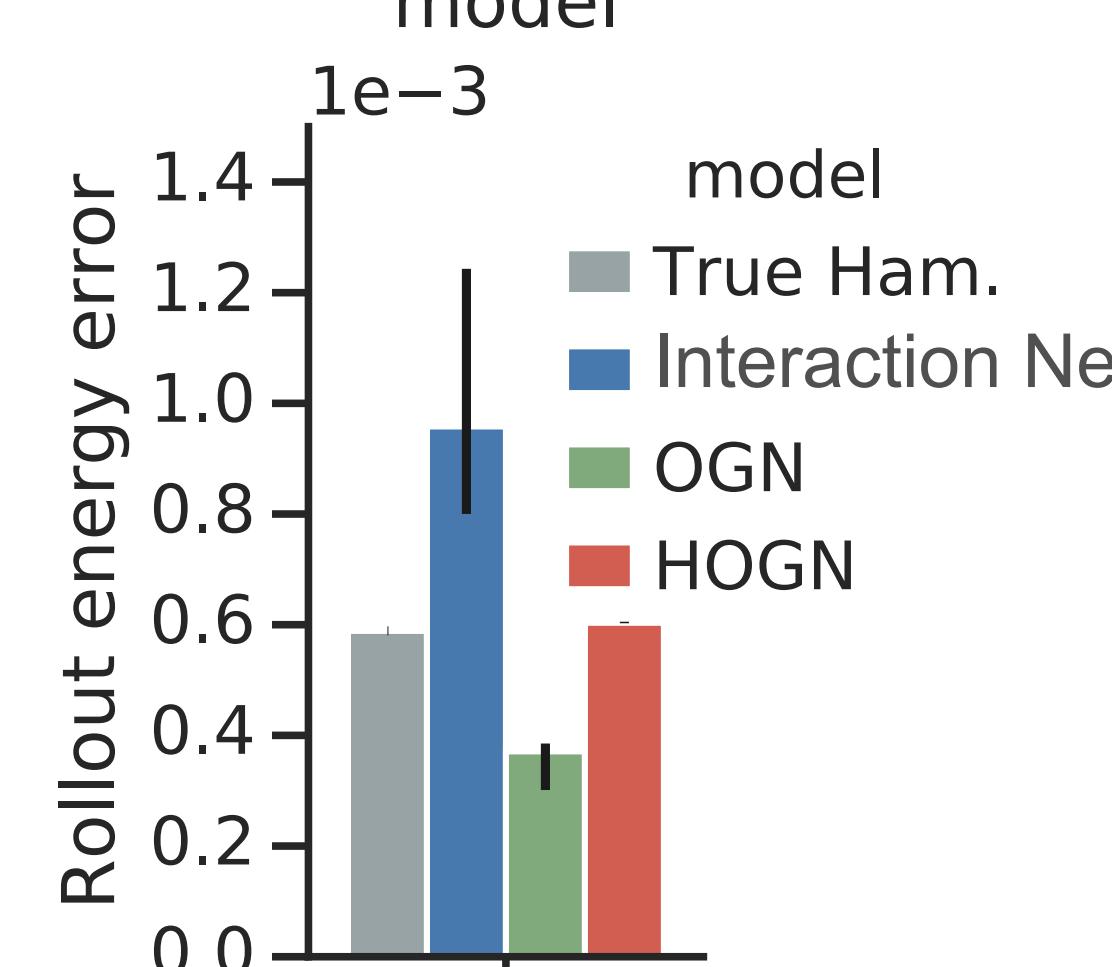
Hamiltonian ODE Graph Network

Performance

Predictive accuracy per model

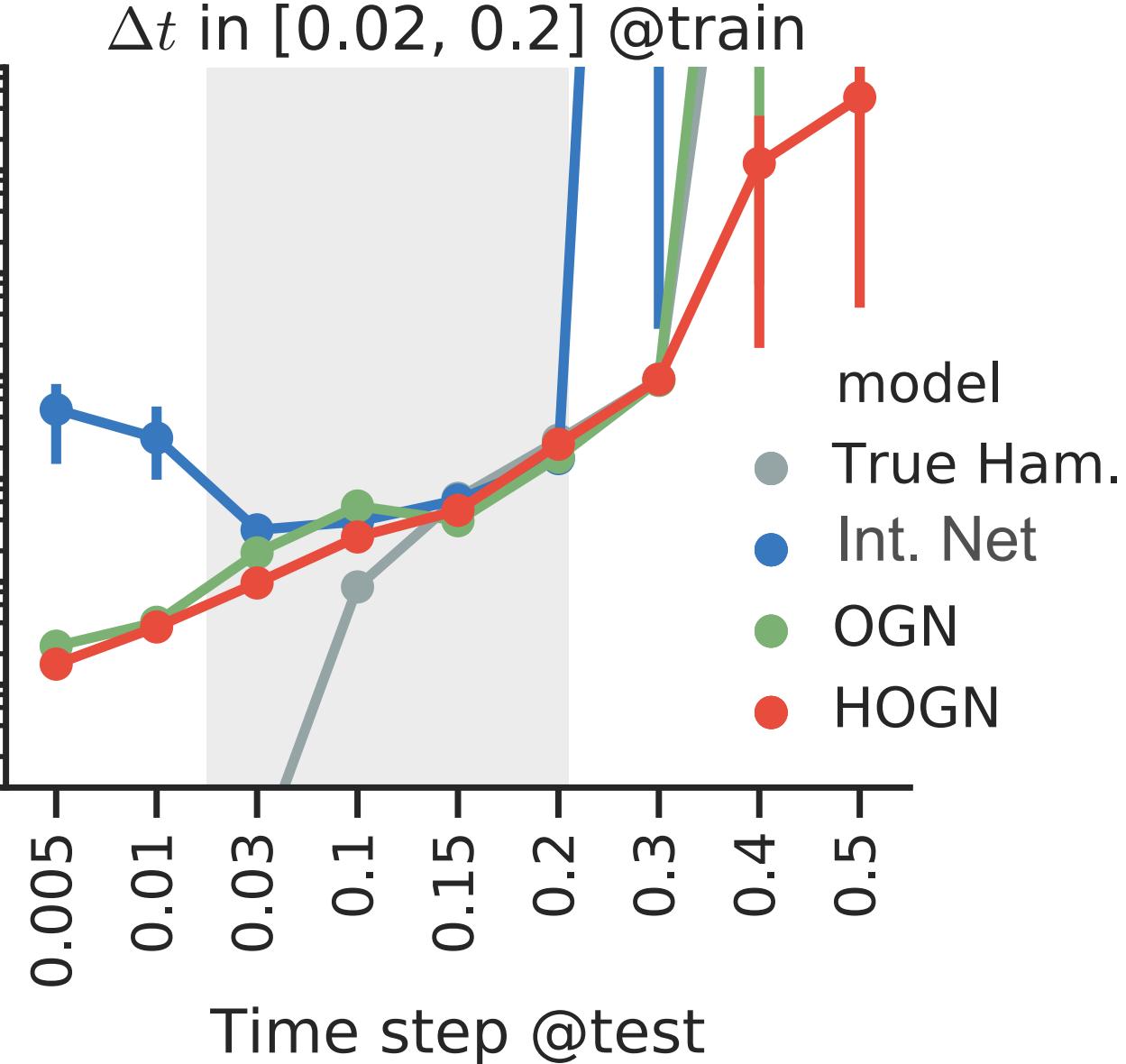
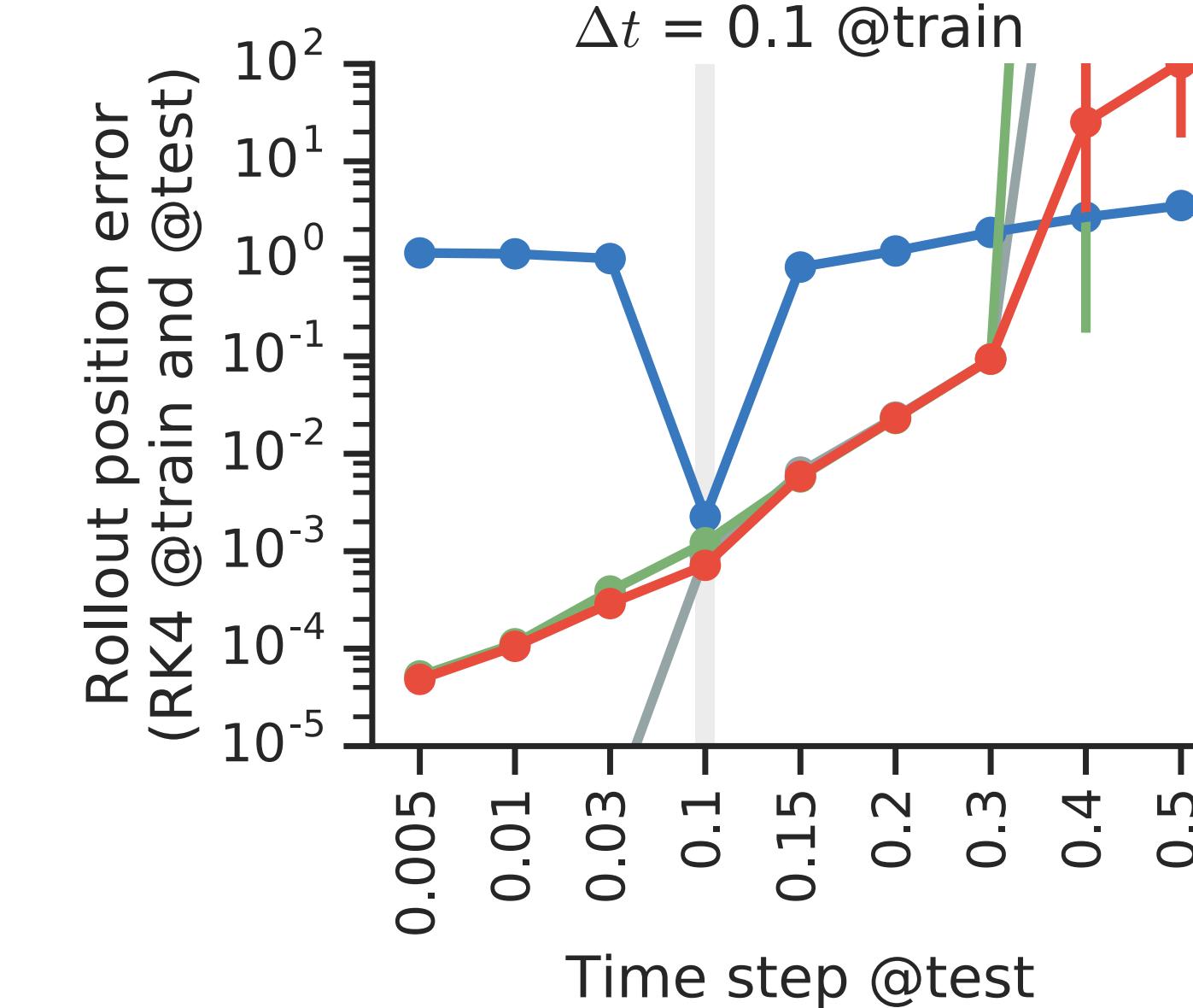


Energy accuracy per model



Generalization to untrained time steps

Predictive accuracy vs test Δt
 Δt in [0.02, 0.2] @train



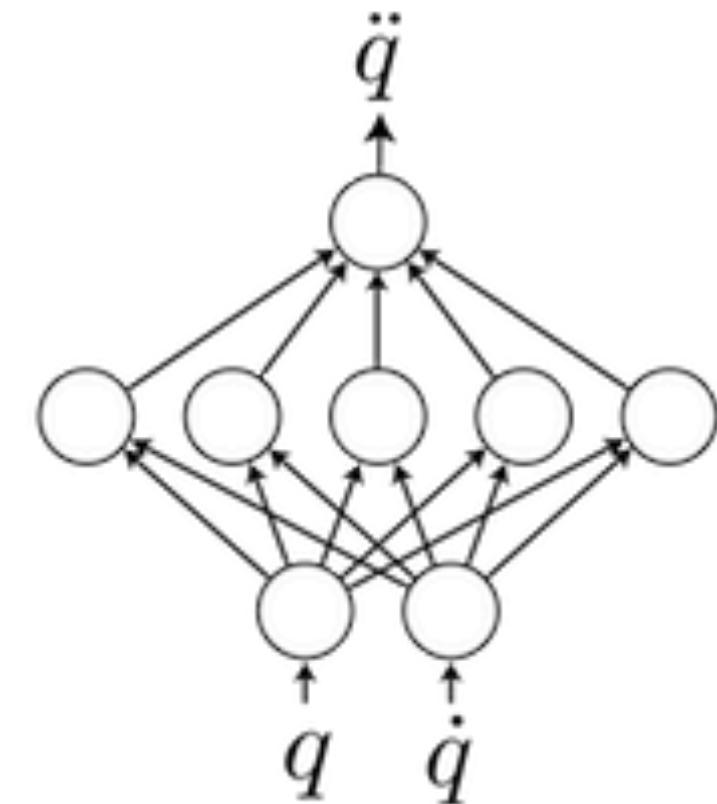
- OGN and HOGN used RK4 integrator (we also tested lower order RK integrators)
- We also tested symplectic integrators, and found HOGN has better energy accuracy/conservation

Lagrangian Neural Network

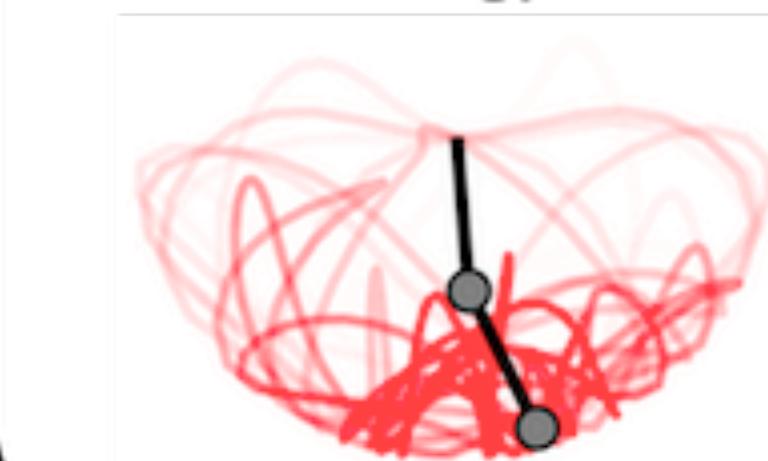
Double Pendulum

$$\begin{aligned} & \frac{1}{2}(m_1 + m_2)l_1^2\dot{\theta}_1^2 + \frac{1}{2}m_2l_2^2\dot{\theta}_2^2 \\ & + m_2l_1l_2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2) \\ & + (m_1 + m_2)gl_1 \cos \theta_1 \\ & + m_2gl_2 \cos \theta_2 \end{aligned}$$

Baseline NN

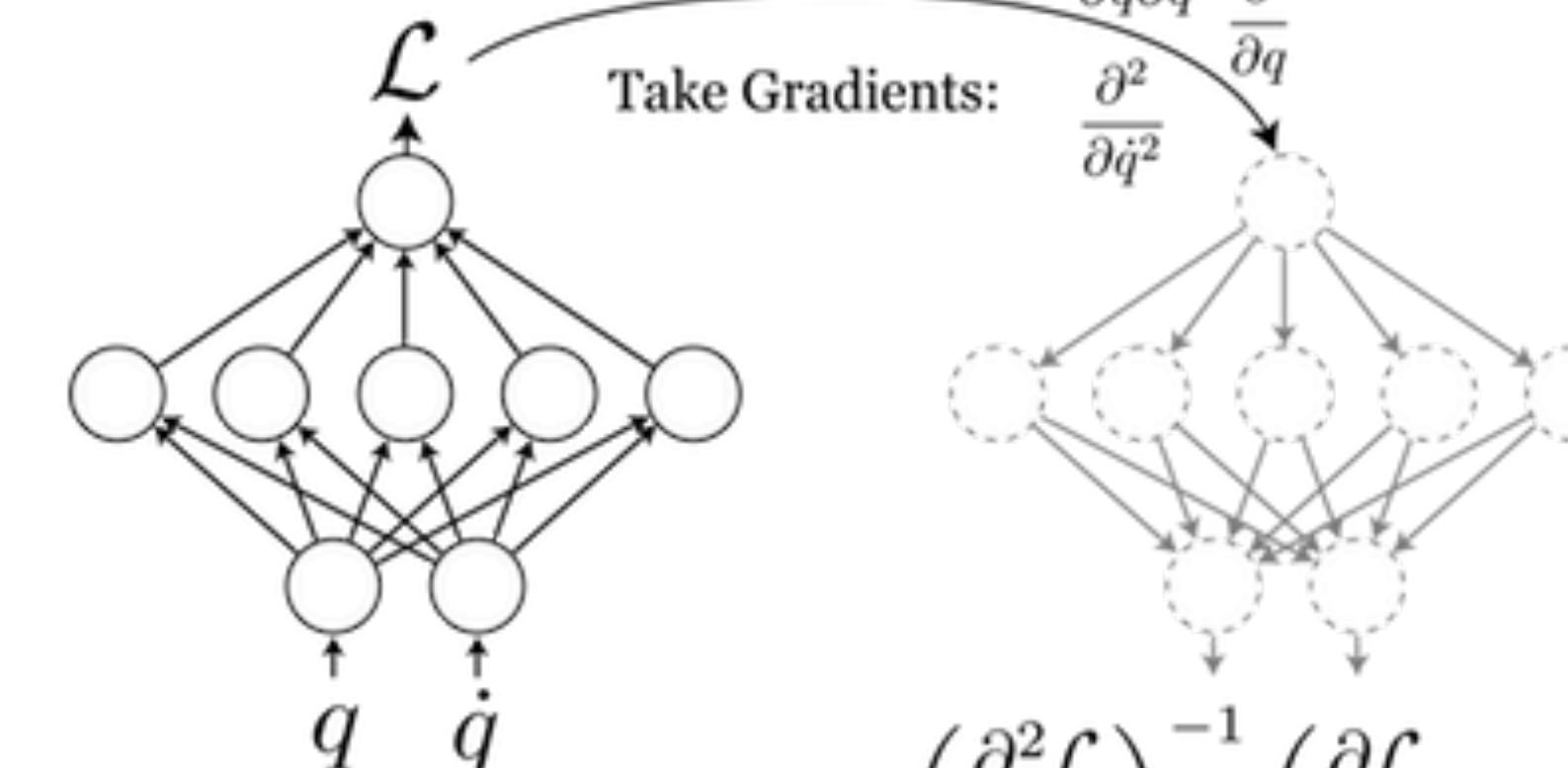


Loss of Energy

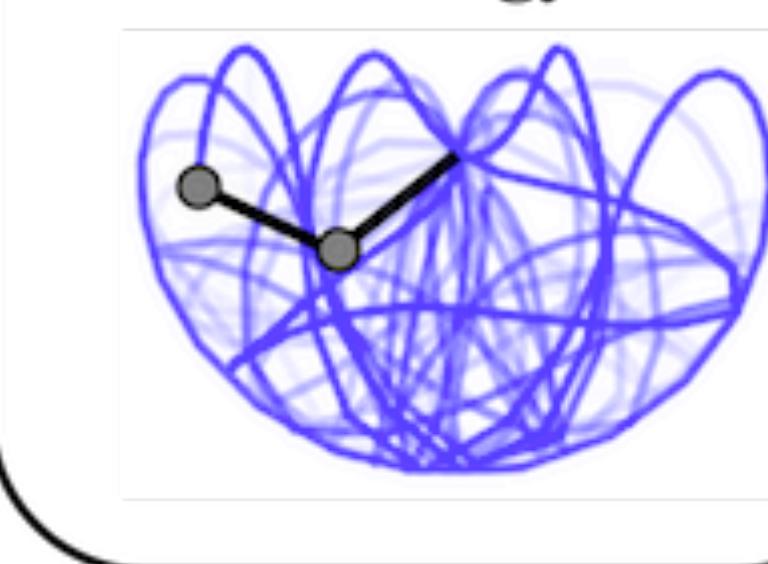


Observe State over Time

Lagrangian NN



Conservation of Energy



Lagrangian Neural Network

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_j} = \frac{\partial \mathcal{L}}{\partial q_j}$$

Euler-Lagrange equation

$$\frac{d}{dt} \nabla_{\dot{q}} \mathcal{L} = \nabla_q \mathcal{L}$$

Euler-Lagrange equation (vector form)

$$(\nabla_{\dot{q}} \nabla_{\dot{q}}^\top \mathcal{L}) \ddot{q} + (\nabla_q \nabla_{\dot{q}}^\top \mathcal{L}) \dot{q} = \nabla_q \mathcal{L}$$

Apply chain rule to get \dot{q} and \ddot{q}

$$\ddot{q} = (\nabla_{\dot{q}} \nabla_{\dot{q}}^\top \mathcal{L})^{-1} [\nabla_q \mathcal{L} - (\nabla_q \nabla_{\dot{q}}^\top \mathcal{L}) \dot{q}]$$

Solve for \ddot{q}

\mathcal{L} is represented by a trainable neural network, and JAX's auto-differentiation features let us compute \ddot{q} in a few lines of code, which is then passed to an integrator

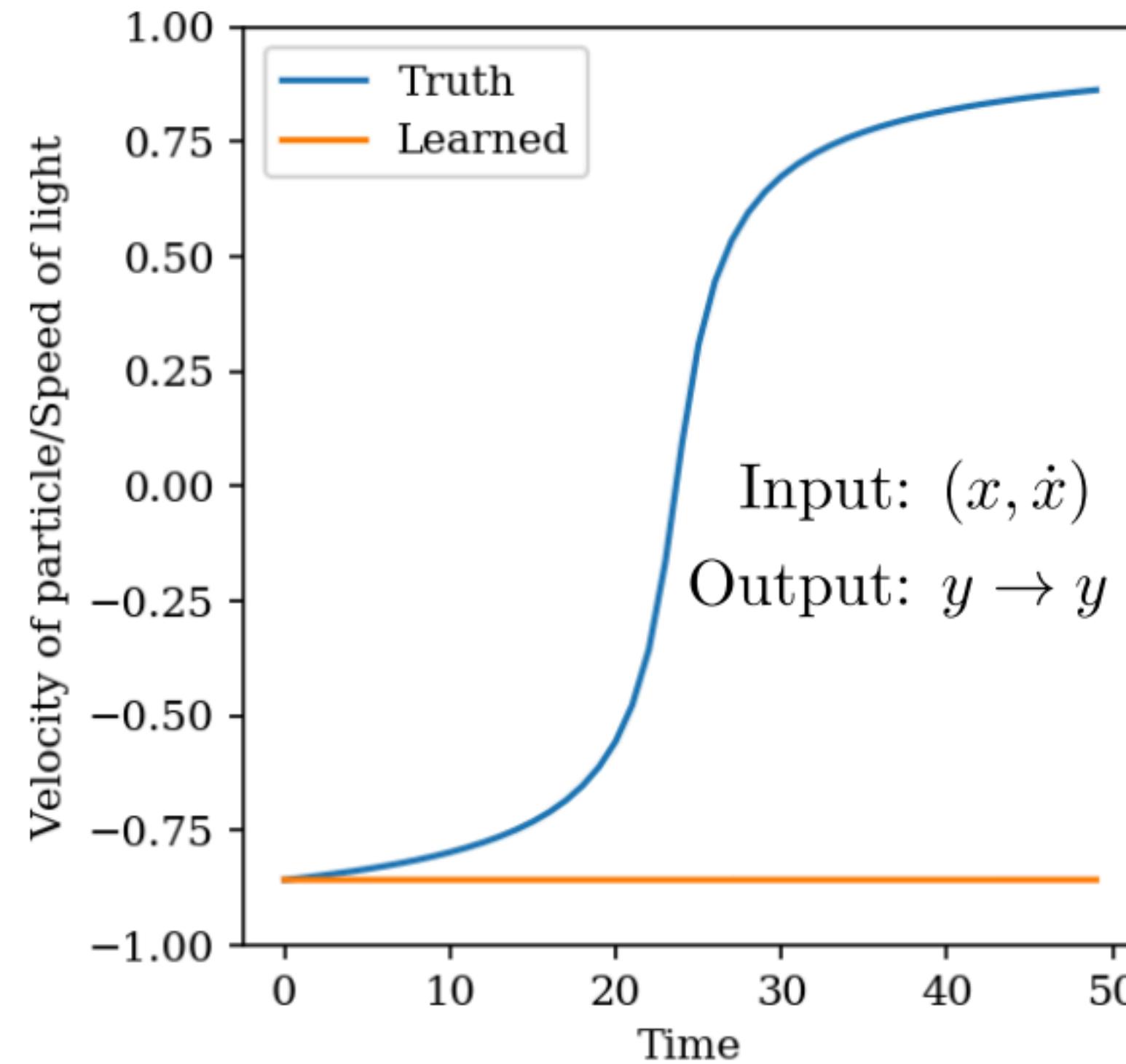
Lagrangian Neural Network

Advantage: You don't need canonical position and momentum coordinates

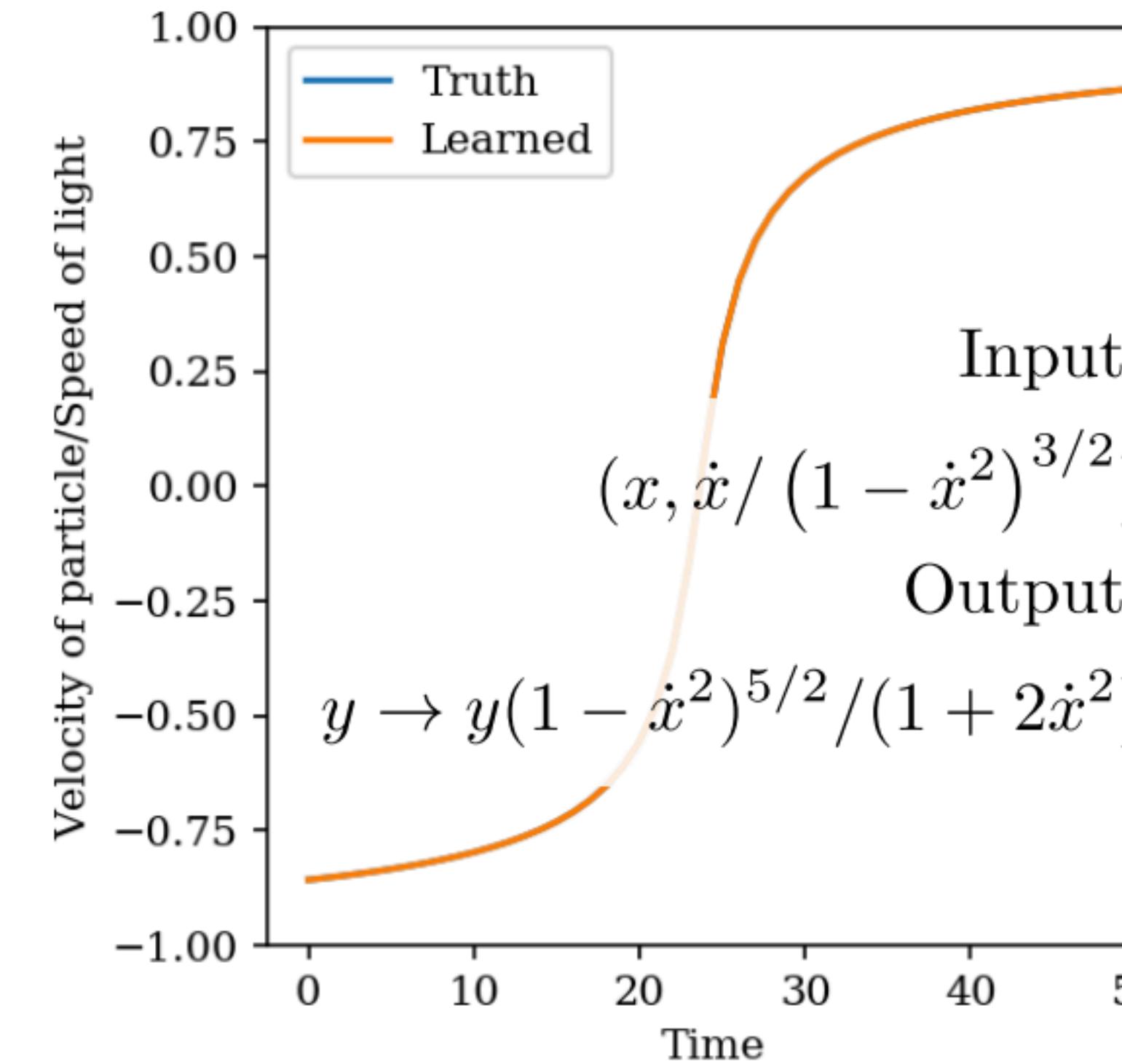
– remember the Hamiltonian approach required \mathbf{p} to compute $\dot{\mathbf{q}} = \frac{\partial \mathcal{H}}{\partial \mathbf{p}}$

Relativistic particle task, where canonical momentum is: $\dot{\mathbf{q}}(1 - \dot{\mathbf{q}}^2)^{-3/2}$

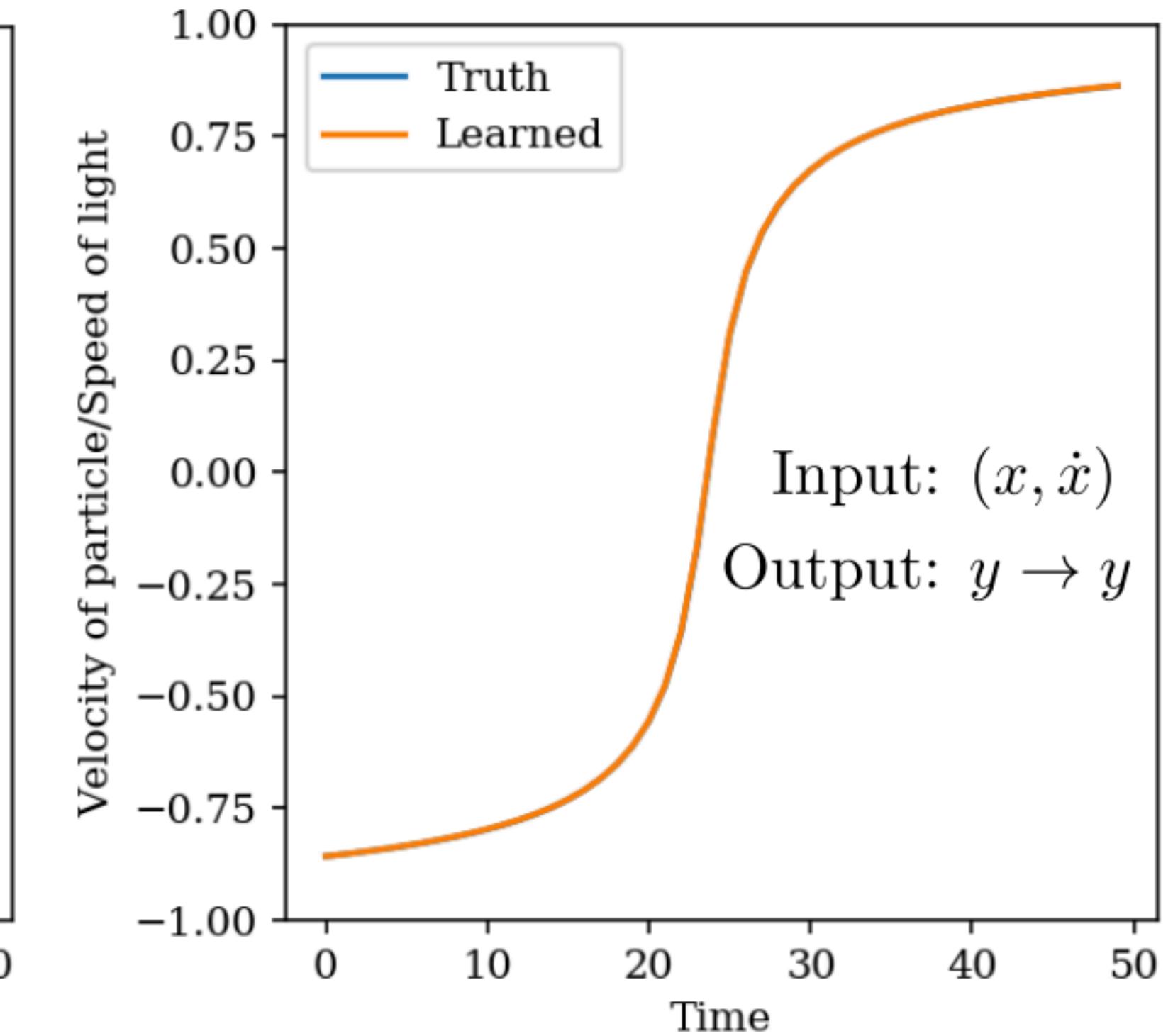
Hamiltonian NN (arbitrary coordinates)



Hamiltonian NN (canonical coordinates)



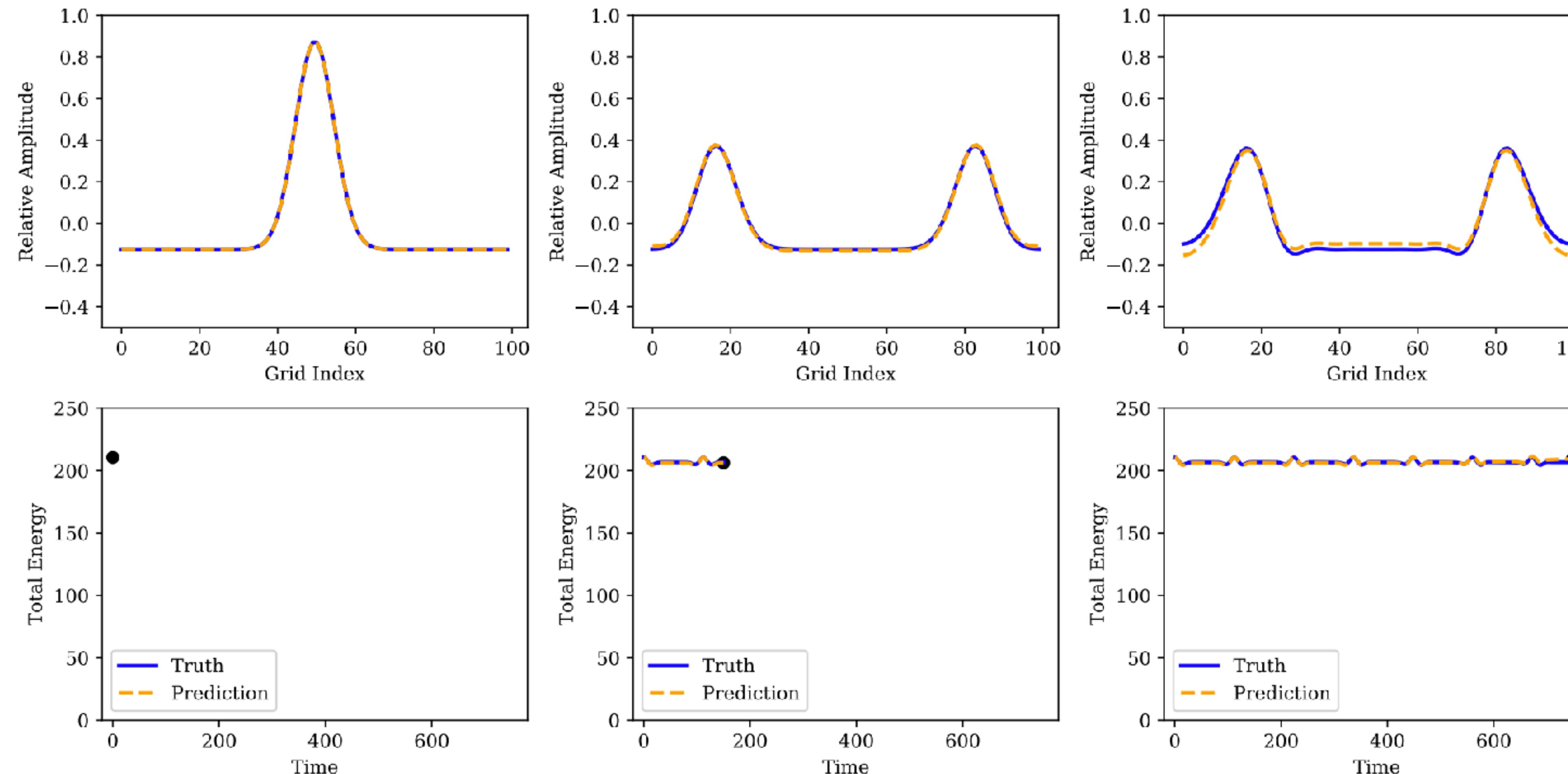
Lagrangian NN (arbitrary coordinates)



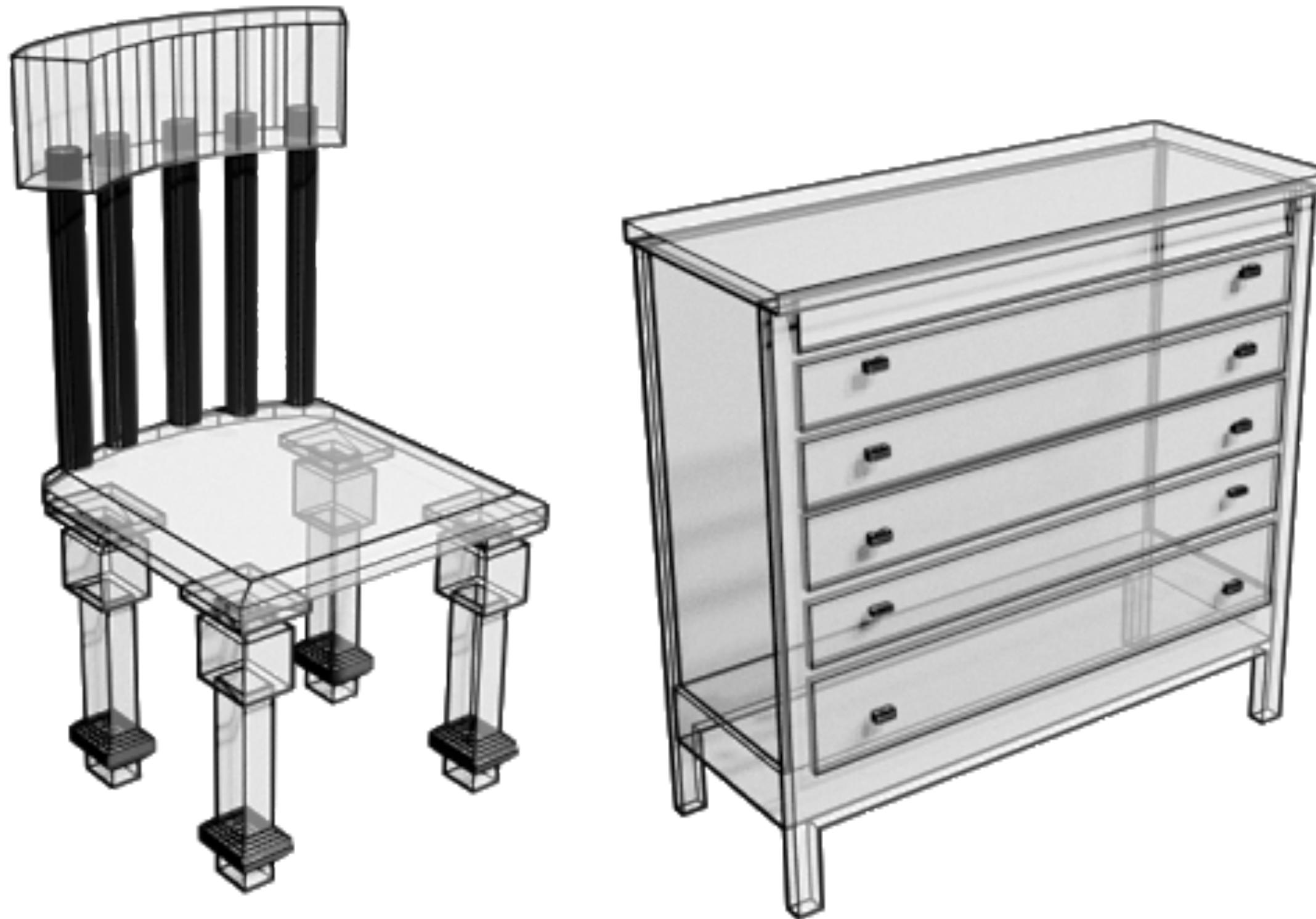
Lagrangian Graph Network

Can also use extend the LNN to apply to graph structures

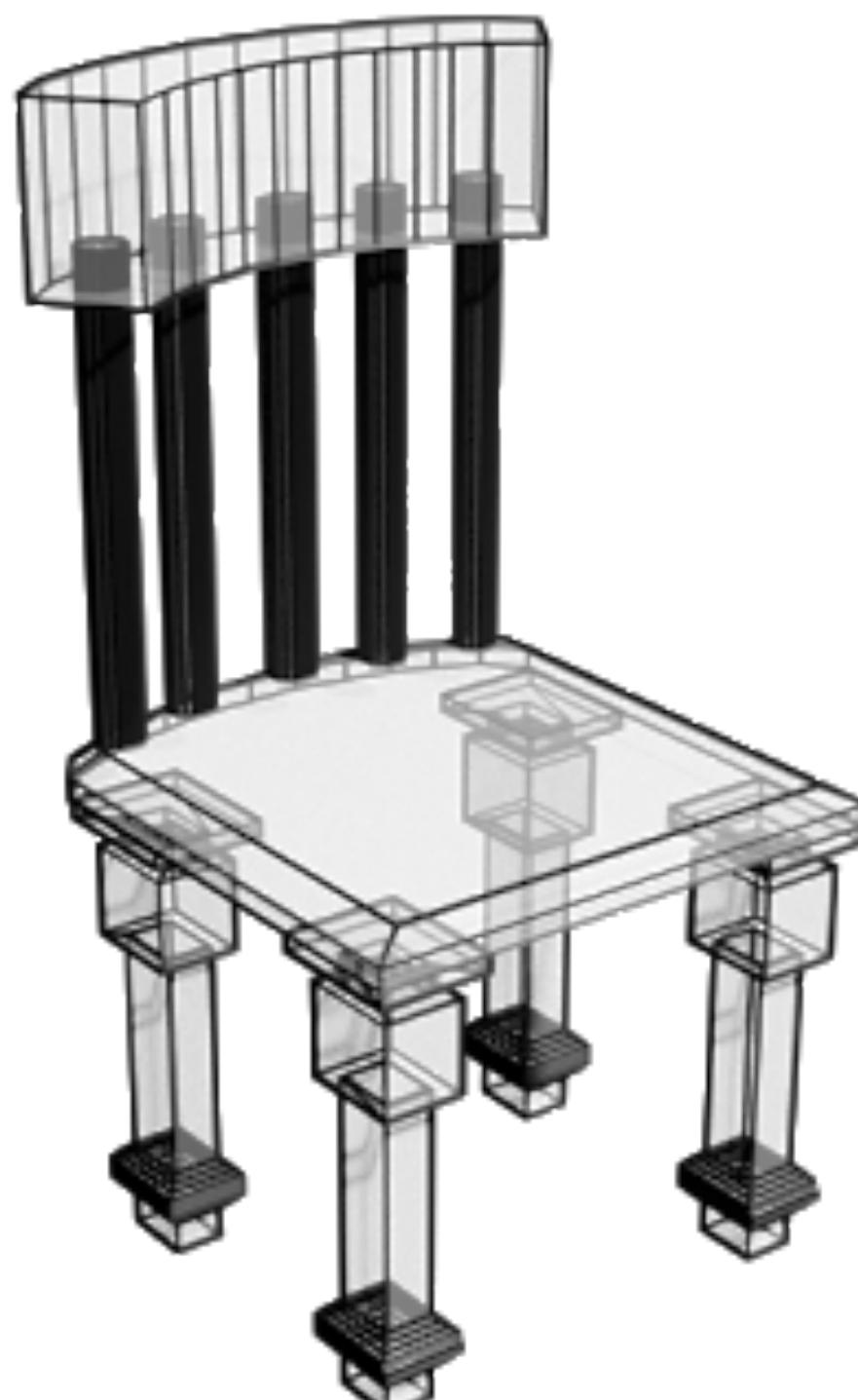
1D wave equation (periodic boundary conditions) - nodes are vertices on the mesh



PolyGen: Autoregressive generative model of 3D meshes



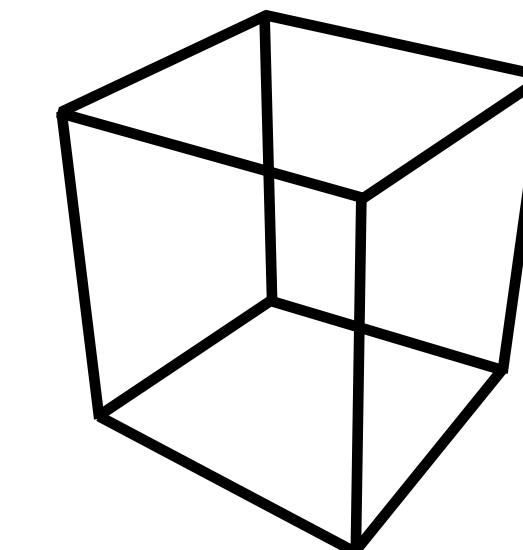
PolyGen: Autoregressive generative model of 3D meshes



.OBJ file: cube

```
v 0.000000 2.000000 2.000000
v 0.000000 0.000000 2.000000
v 2.000000 0.000000 2.000000
v 2.000000 2.000000 2.000000
v 0.000000 2.000000 0.000000
v 0.000000 0.000000 0.000000
v 2.000000 0.000000 0.000000
v 2.000000 2.000000 0.000000

f 1 2 3 4
f 8 7 6 5
f 4 3 7 8
f 5 1 4 8
f 5 6 2 1
f 2 6 7 3
```

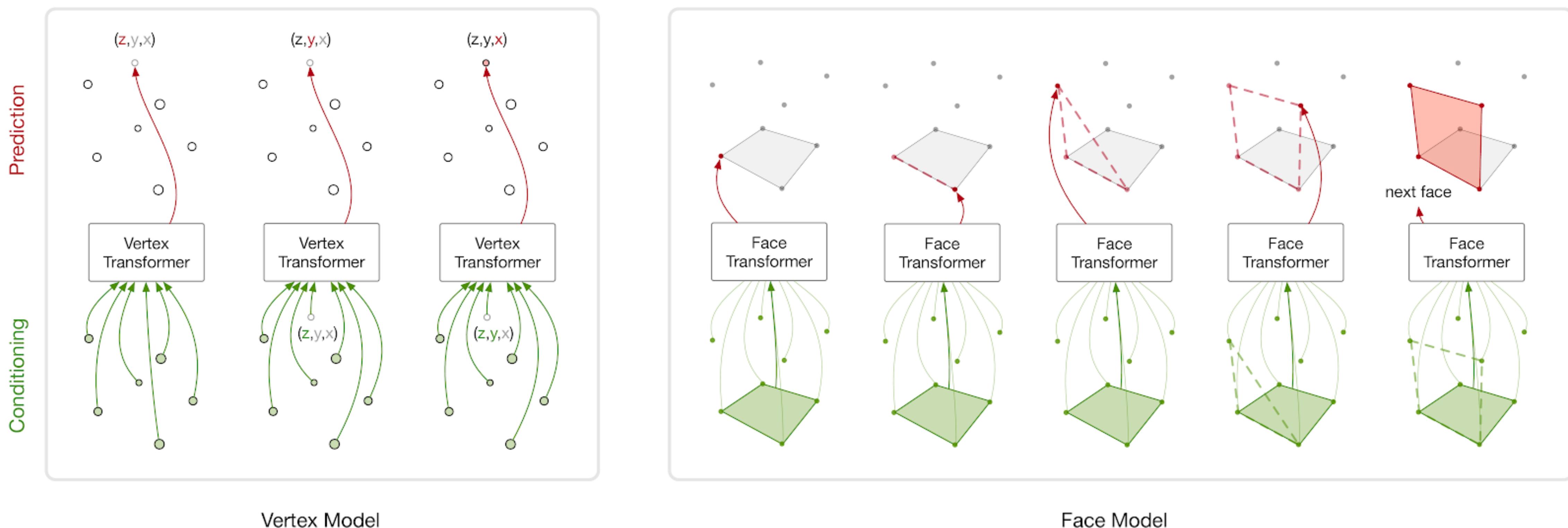


PolyGen: Autoregressive generative model of 3D meshes

Architecture: Transformer-based

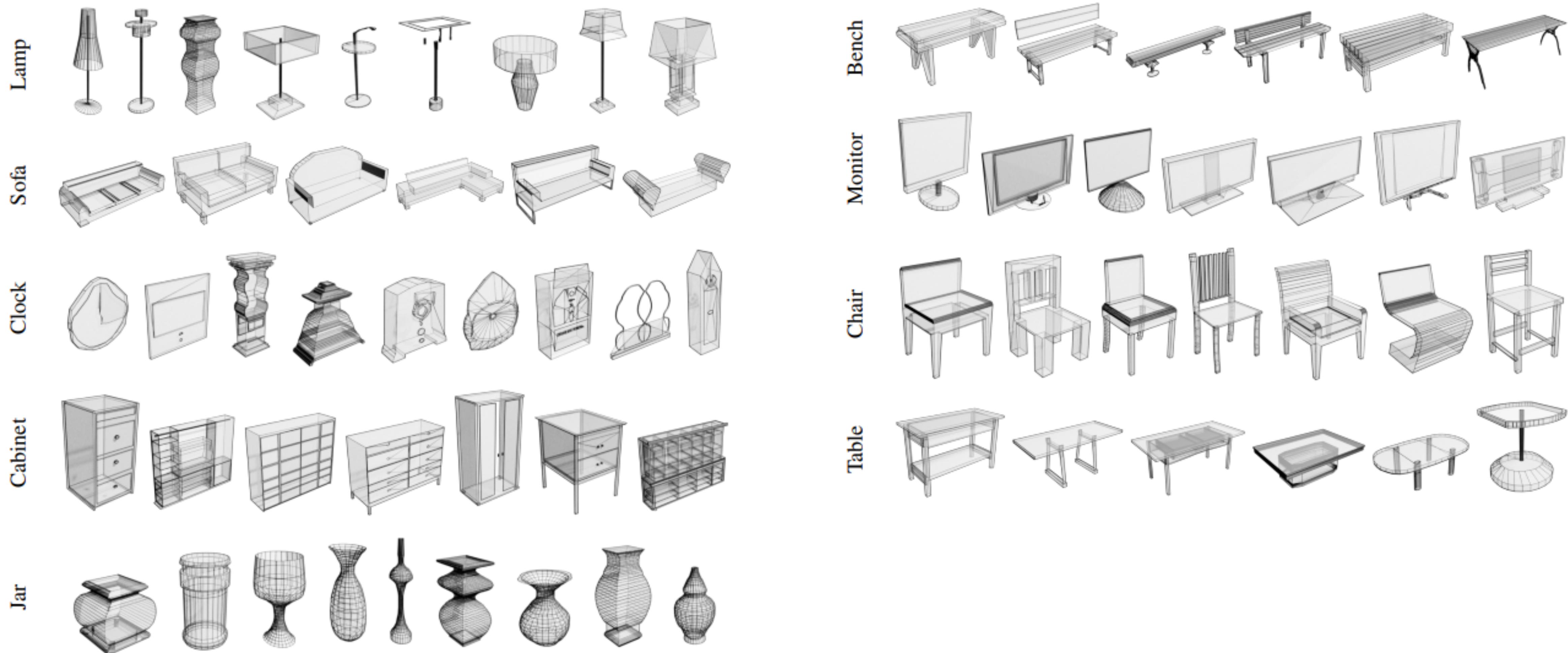
Two phases:

1. Vertex model
2. Face model



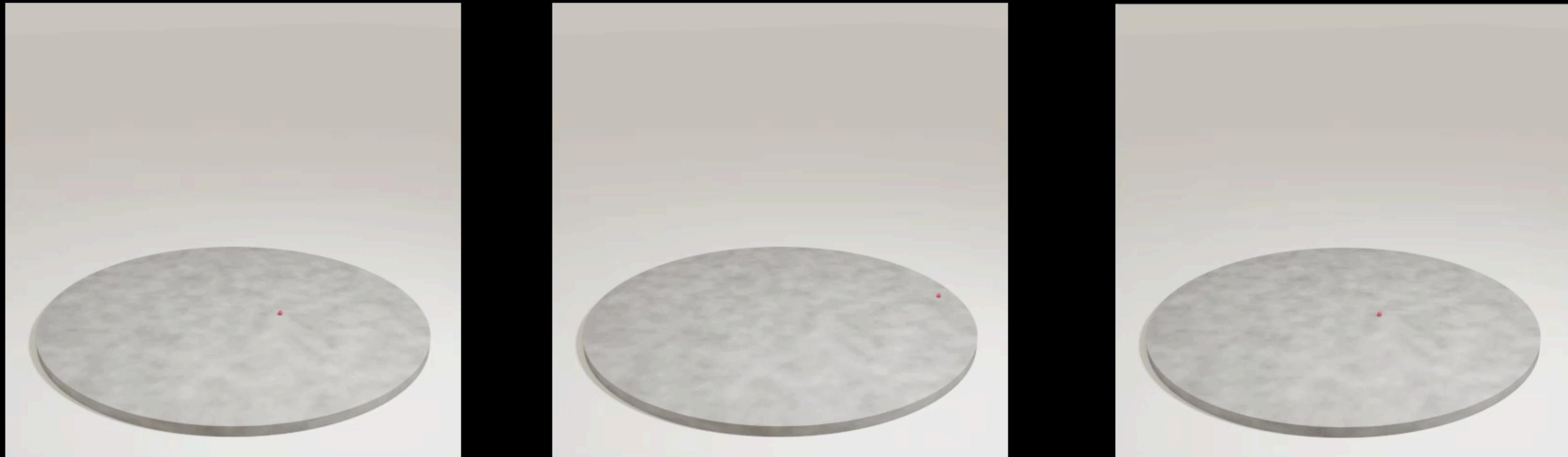
PolyGen: Autoregressive generative model of 3D meshes

Class-conditional samples



PolyGen

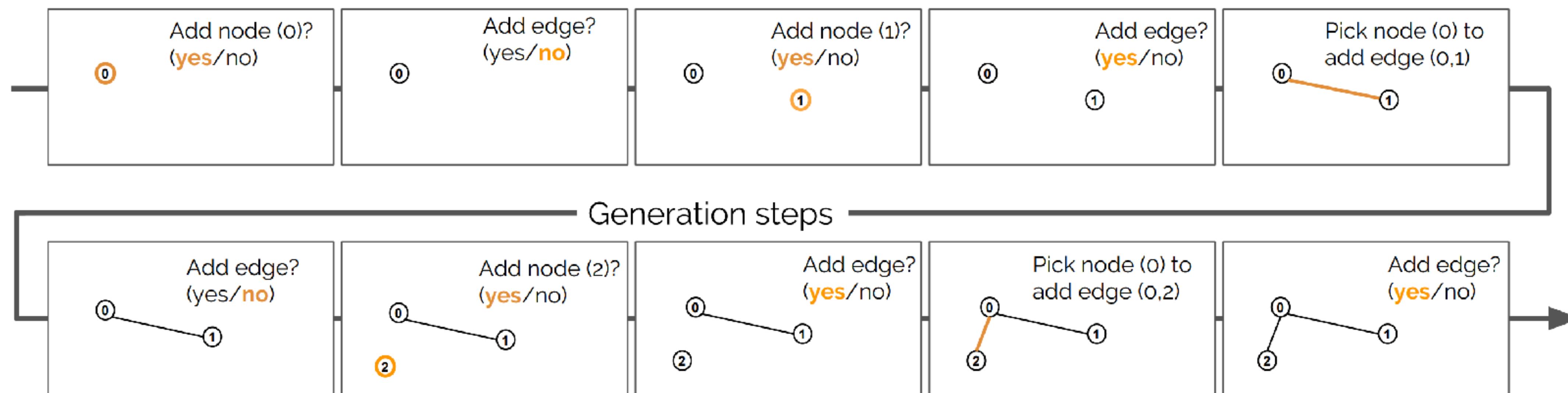
An Autoregressive Generative Model of 3D Meshes



Class conditional samples

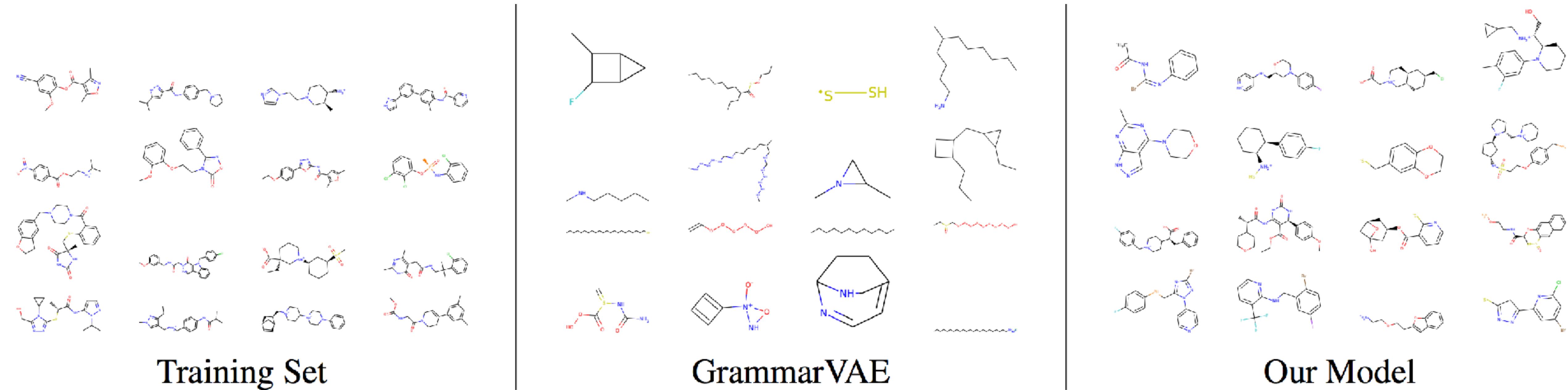
Learning deep generative models of chemical graphs

- Generative model defines joint distribution over graph-generating decisions (structure and order).
- Analogous to a decision tree, where decisions are selected by a GNN:
 1. *Add node?* If NO, terminate.
 2. If YES, *Add edge?* If NO, goto (1).
 3. If YES, *Pick node to add edge to*. Goto (2).
- Training optimizes the joint log-likelihood of structure and order, with Monte Carlo integration over permutations.



Learning deep generative models of chemical graphs

- GrammarVAE (Kusner et al., 2017) has qualitatively poorer samples from the prior.



- Our model learns a more accurate model than LSTMs, and can generate more novel molecules.

Arch	Grammar	Ordering	N	NLL	%valid	%novel
LSTM	Graph	Fixed	1	22.06	85.16	80.14
LSTM	Graph	Random	$O(n!)$	63.25	91.44	91.26
Graph	Graph	Fixed	1	20.55	97.52	90.01
Graph	Graph	Random	$O(n!)$	58.36	95.98	95.54

Build Graph Nets in Tensorflow

github.com/deepmind/graph_nets

```
# Provide your own functions to generate graph-structured data.  
input_graphs = get_graphs()  
  
# Create the graph network.  
graph_net_module = gn.modules.GraphNetwork(  
    edge_model_fn=lambda: snt.nets.MLP([32, 32]),  
    node_model_fn=lambda: snt.nets.MLP([32, 32]),  
    global_model_fn=lambda: snt.nets.MLP([32, 32]))  
  
# Pass the input graphs to the graph network, and return the output graphs.  
output_graphs = graph_net_module(input_graphs)
```

For GNN libraries in PyTorch, check out:

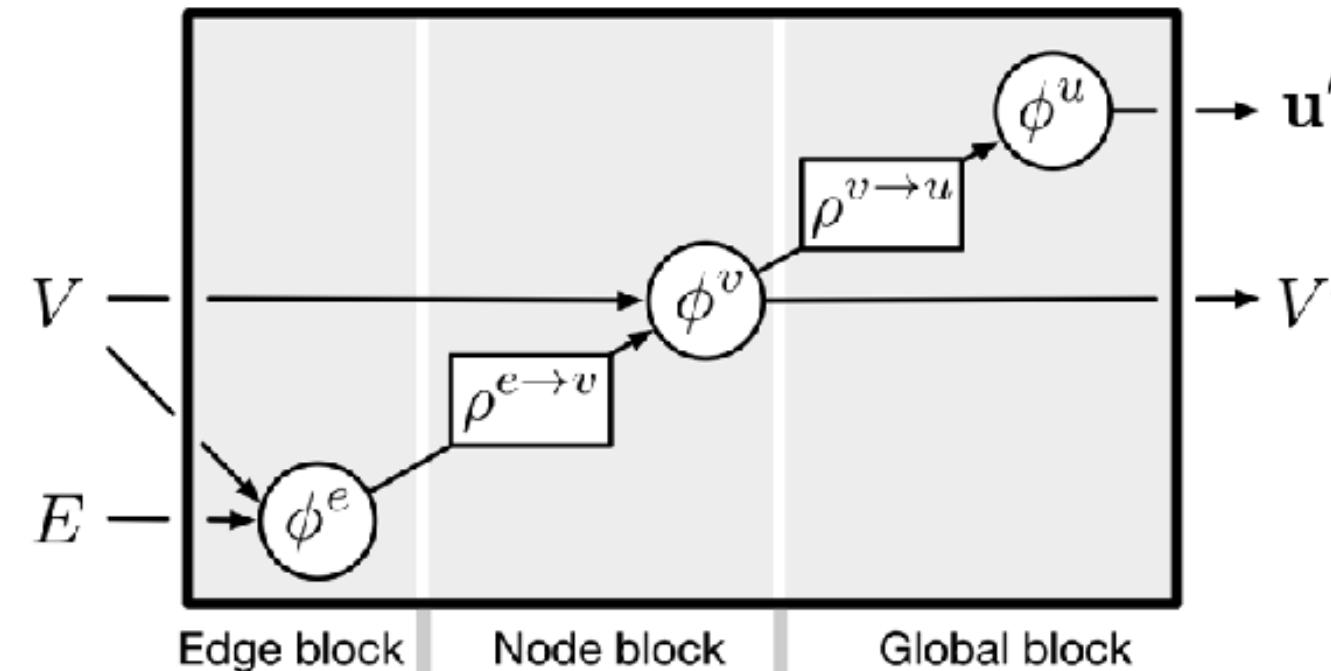
- pytorch_geometric: github.com/rusty1s/pytorch_geometric (for a GN analog, see MetaLayer)
- Deep Graph Library: github.com/dmlc/dgl

Build Graph Nets in Tensorflow

github.com/deepmind/graph_nets

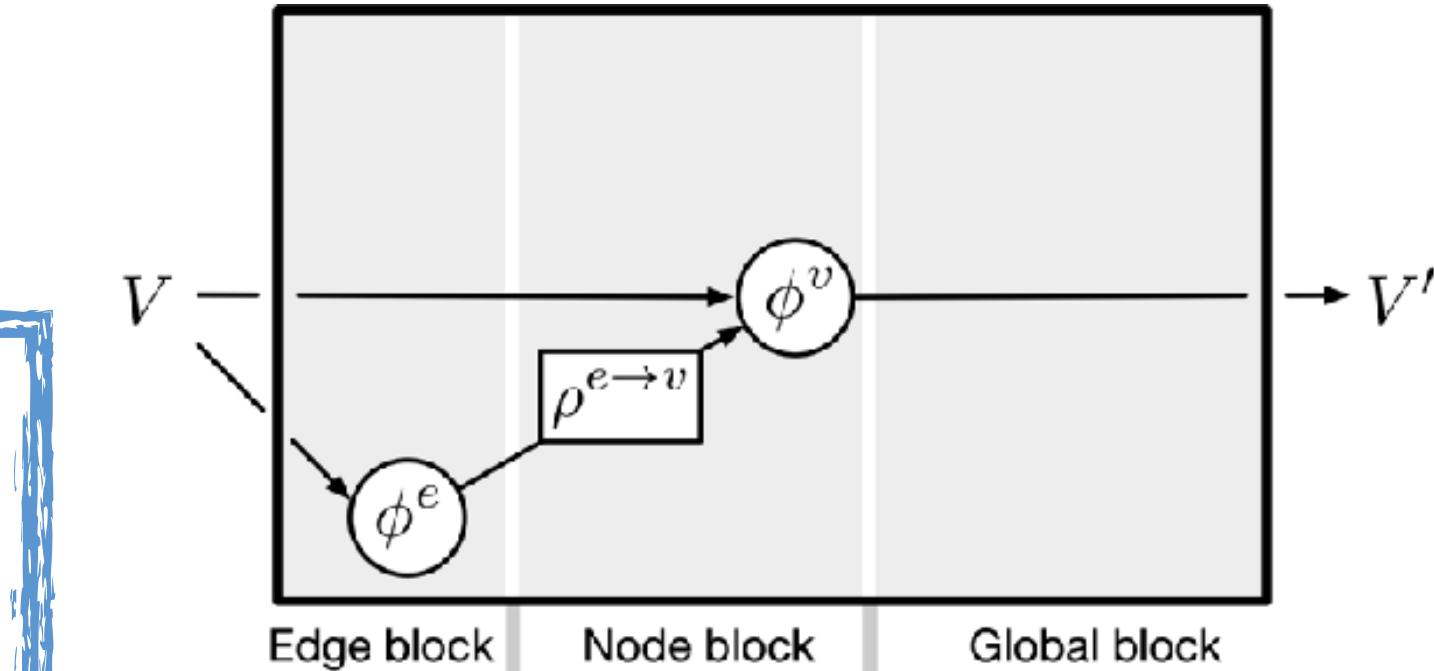
Message-Passing NN (eg. Interaction Net)

Gilmer et al. 2017



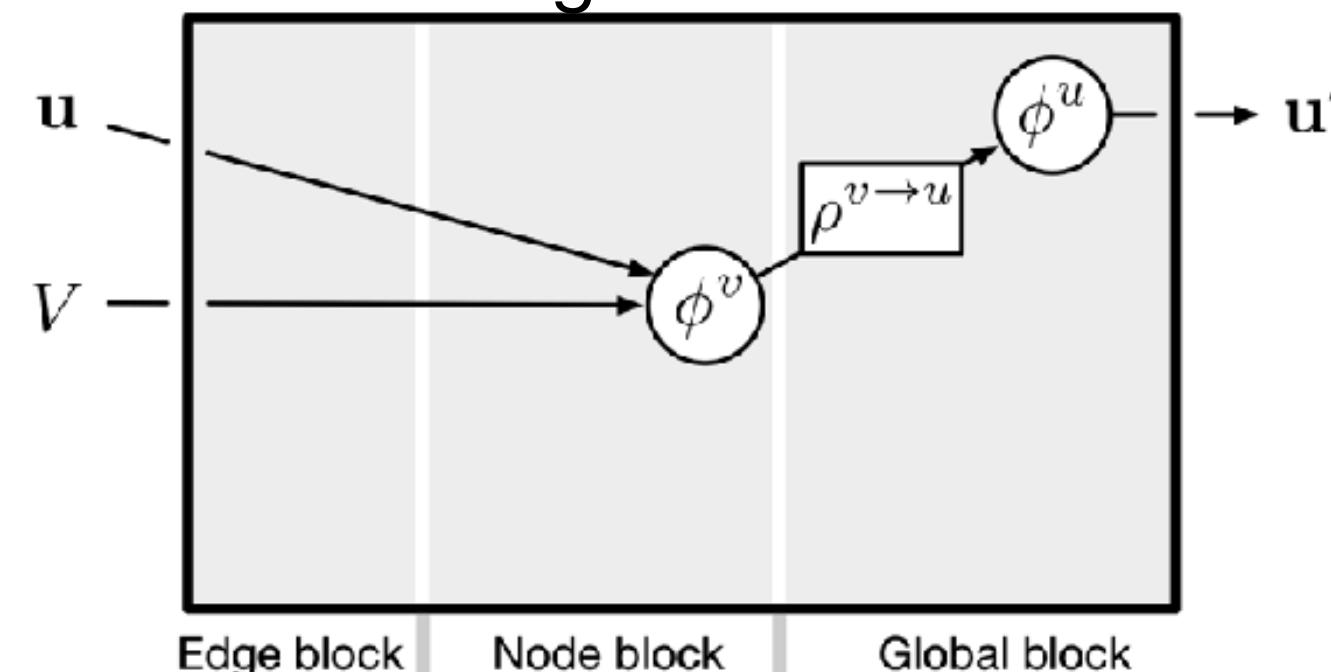
Non-Local NN (eg. Transformer)

Vaswani et al. 2017; Wang et al. 2017

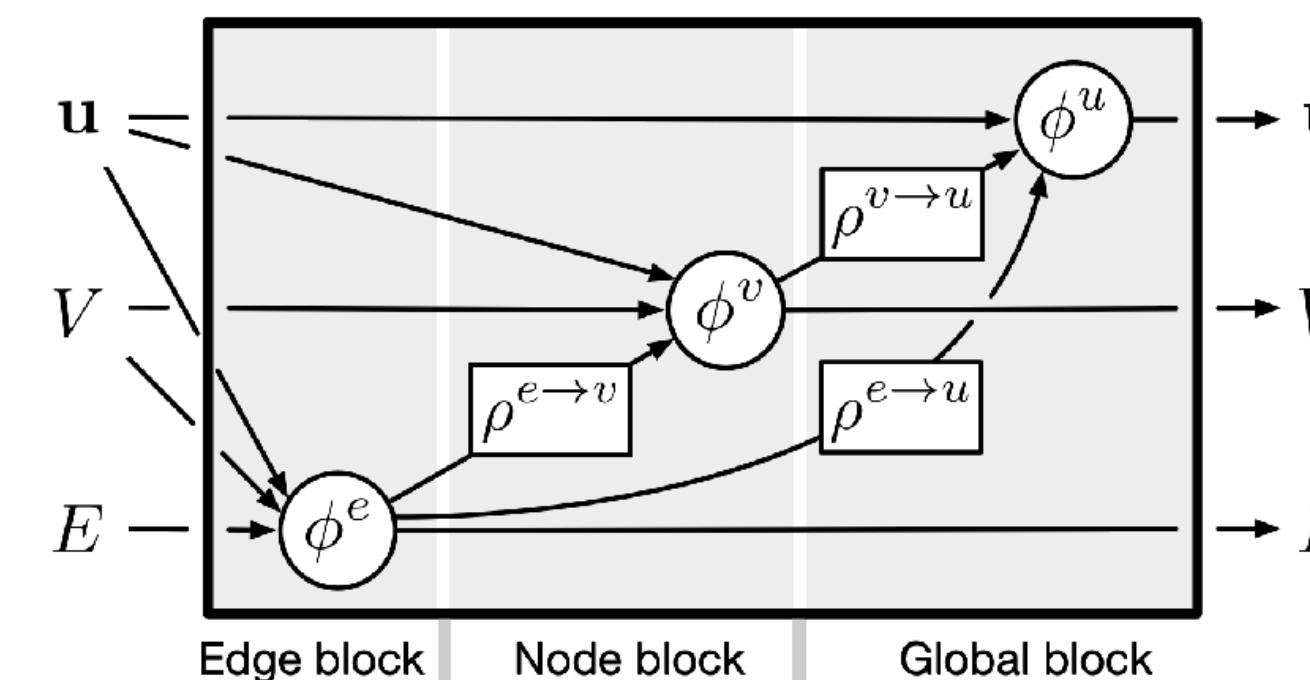


Deep Sets

Zhang et al. 2017

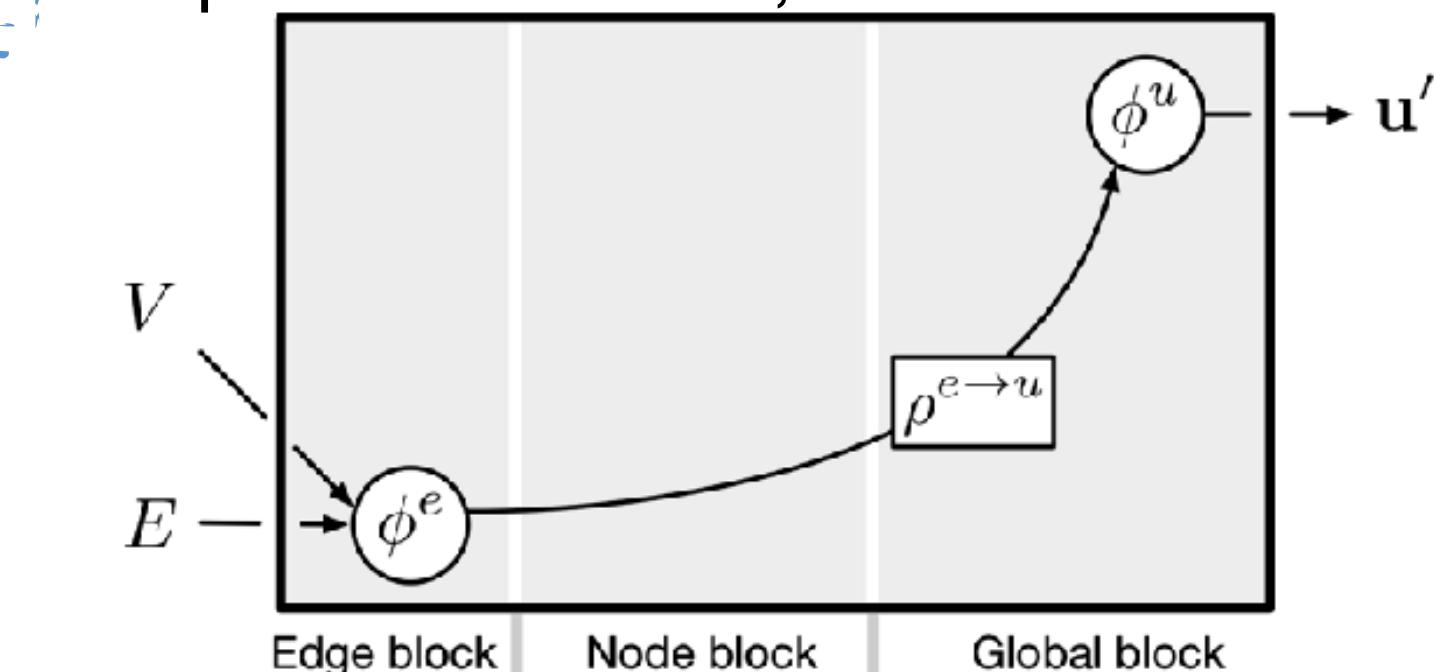


Graph Network
(a type of Graph Neural Network)



Relation Network

Raposo et al. 2017; Santoro et al. 2017

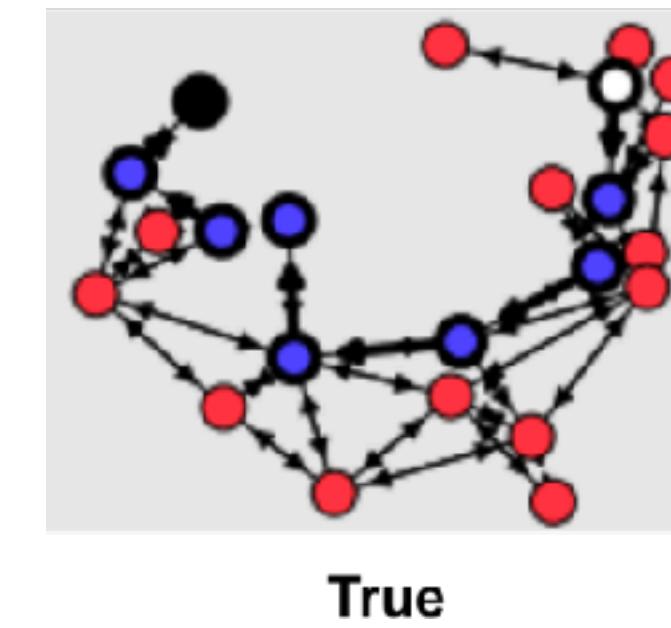


Build Graph Nets in Tensorflow

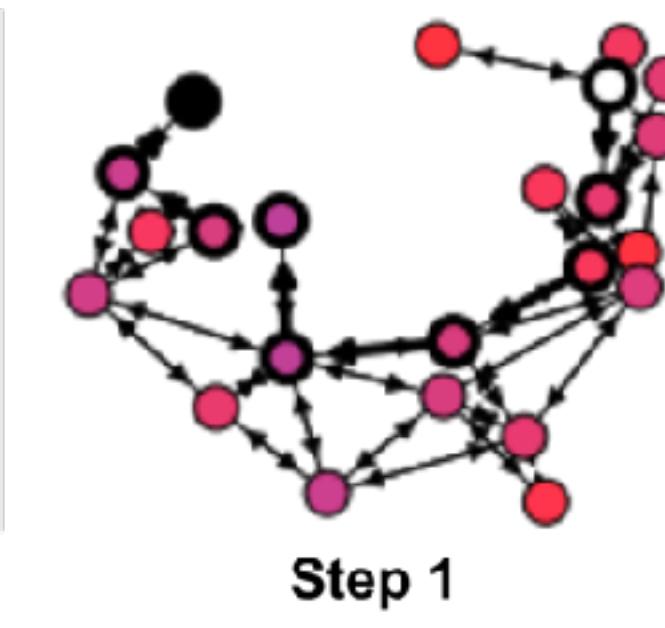
github.com/deepmind/graph_nets

IPython Notebook demos
(All use same architecture)

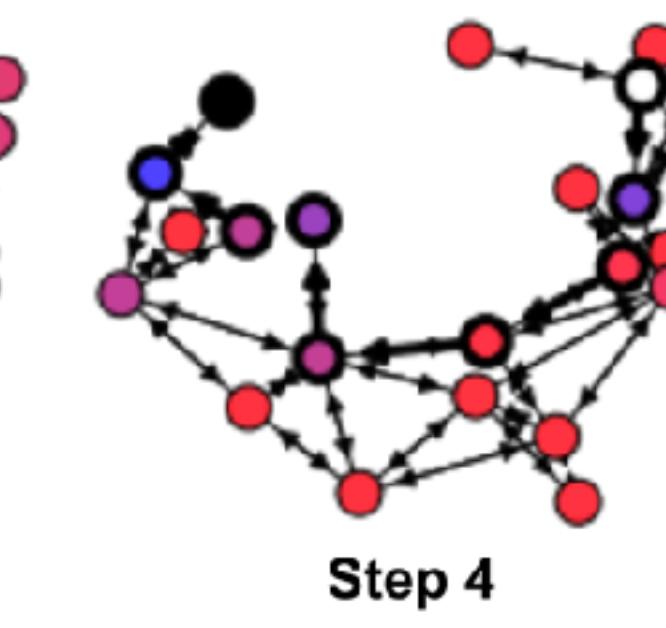
Shortest path:



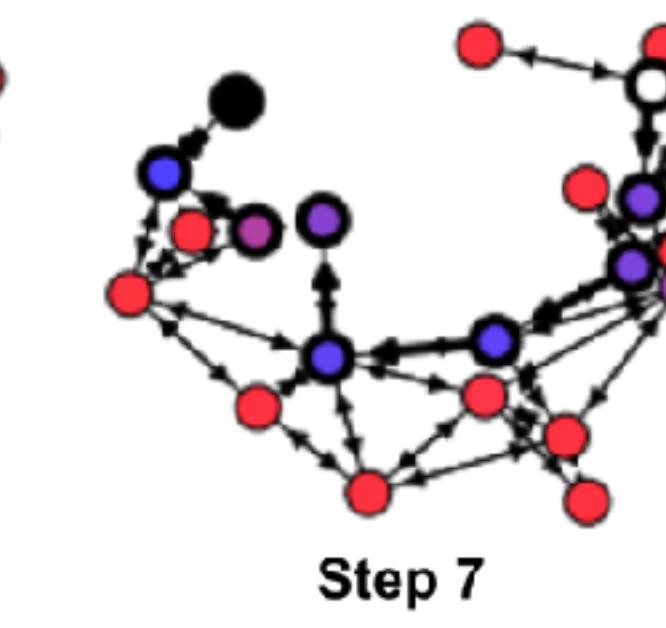
True



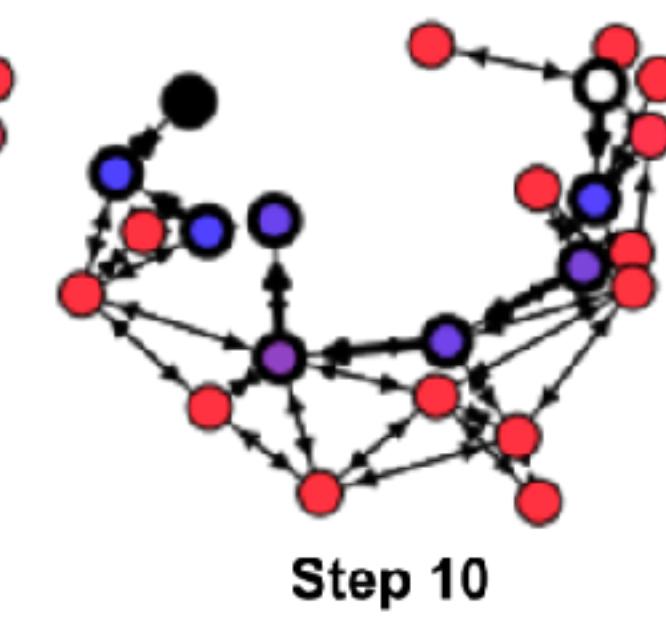
Step 1



Step 4



Step 7



Step 10

Sorting:

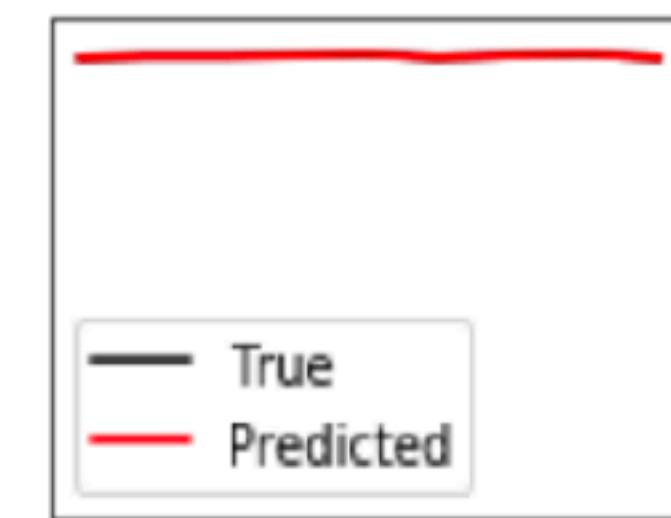


True

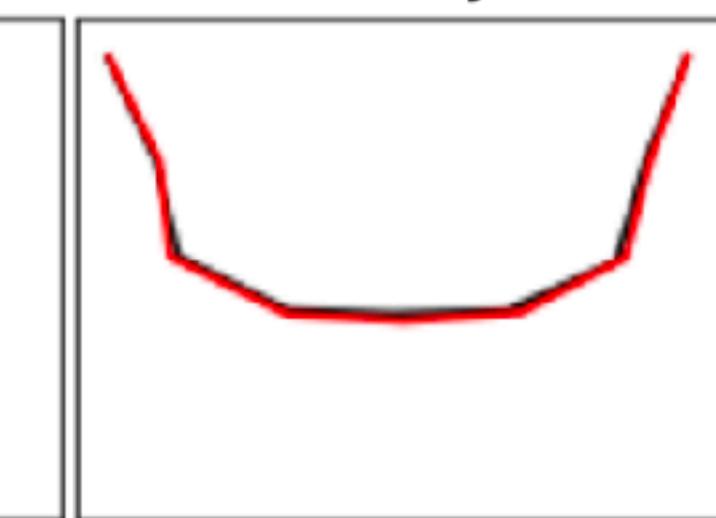


Predicted

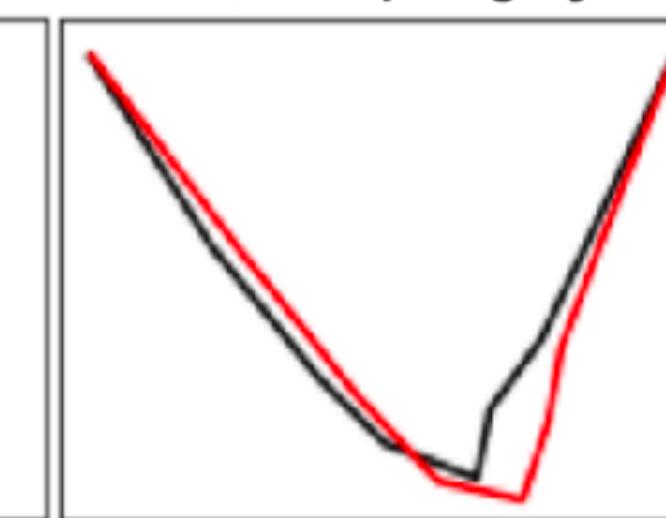
Predicting physics:



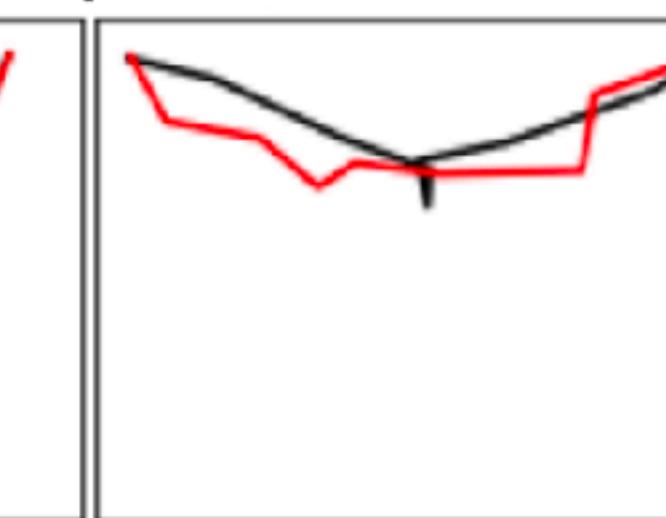
Time 0



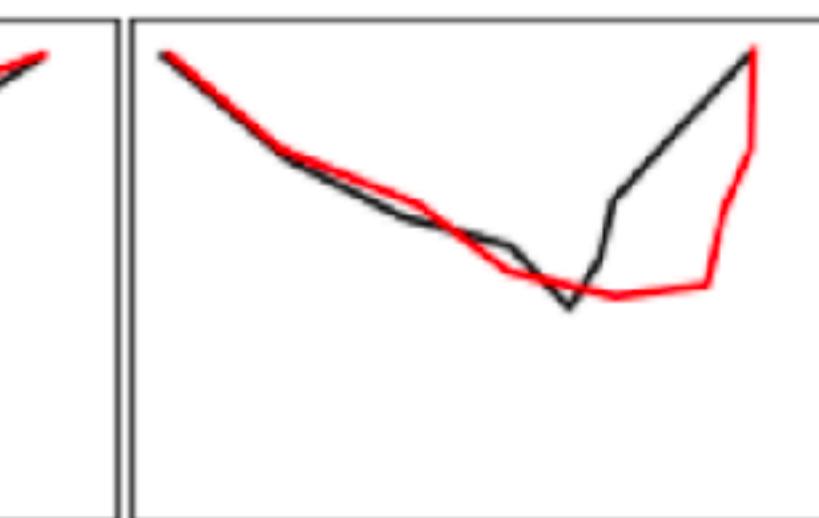
Time 8



Time 16



Time 32



Time 48

Conclusions

- Graph neural networks: a first-class member of the deep learning toolkit.
- Learned message-passing on graphs can capture complex physical knowledge.
- “Graph Nets” support learning simulation, as well as other forms of structured reasoning and decision-making.
- Build Graph Nets in Tensorflow: github.com/deepmind/graph_nets.
- For a recent review of GNNs in HEP, please check out:
Shlomi, Battaglia, Vlimant (2020) "Graph Neural Networks in Particle Physics".
[arXiv 2007.13681](https://arxiv.org/abs/2007.13681) and 2 weeks ago in [Machine Learning: Science and Technology](https://arxiv.org/abs/2307.13681)

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[Nash et al., 2020, ICML](#)