

Skewed Business Cycles*

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Abstract

Using firm-level panel data from the US Census and more than forty other countries, we show that the skewness of the growth rate of employment and sales is procyclical. In particular, during recessions, they display a large left tail of negative growth rates (and during booms, a large right tail of positive growth rates). These results are robust to different selection criteria, across countries, industries, and measures. We find similar results at the industry level: industries with falling growth rates see more left-skewed growth rates of firm sales. We then build a heterogeneous-agent model in which entrepreneurs face shocks with time-varying skewness that matches the firm-level distributions we document for the United States. Our quantitative results show that a negative shock to the skewness of firms' productivity growth (keeping the mean and variance constant) generates a significant and persistent drop in output, investment, hiring, and consumption.

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1 Introduction

This paper studies the cyclicalities of the distribution of the growth rate of firm-level outcomes. In the prior literature, recessions have been characterized as a combination of a negative first-moment (mean) shock and a positive second-moment (uncertainty) shock (Bloom, 2014). In this paper we argue that recessions are also accompanied by negative third-moment (skewness) shocks implying that, during economic downturns, a subset of firms does extremely badly, leading to a left tail of large negative outcomes. Consequently, the skewness of the growth rates is procyclical.

Using panel data on US publicly traded firms from Compustat, panel data on US firms including privately held ones from the Census Bureau, and panel data on firms from thirty-nine other countries, we show that the cross-sectional skewness of the distribution of several firm-level outcomes, such as sales growth, employment growth, and stock returns, is strongly procyclical, declining sharply during recessions. As an illustration of our main empirical result, the top panel of Figure 1 displays the distribution of firms' employment growth from the Census Longitudinal Business Dynamics dataset (LBD). The solid line shows the empirical density of annual employment growth pooling observations from the most recent two recession years, 2001–02 and 2008–09. The dashed line instead shows the density for the expansion years, in this case, years 2003 to 2006 and 2010 to 2014. One can clearly see that, relative to expansion periods, the distribution of employment growth during recessions has a thicker left tail, whereas the right exhibits little change, indicating an increase in dispersion that is mostly due to a widening left tail.

This asymmetric change in the distribution of employment growth from expansion to recession years can be quantified using the Kelley skewness (Kelley, 1947), which is defined as the difference between the log 90th-to-50th percentiles spread (a measure of dispersion in the right tail) and the log 50th-to-10th percentiles spread (a measure of dispersion in the left tail) divided by the log 90th-to-10th percentiles spread (a measure of the total dispersion of the distribution). For a distribution with a compressed upper half and a dispersed lower half (i.e., a left skew), the Kelley skewness is negative. In the case of the top panel of Figure 1, we find a decline of the dispersion of employment growth above the median from 0.22 to 0.20 from expansion to recession years whereas the dispersion below the median increases from 0.17 to 0.25. This asymmetric change

in the tails generates a decline in the Kelley skewness from 0.10 to -0.12 .¹ The bottom panel of Figure 1 shows a similar pattern for the distribution of sales growth in a sample of publicly traded firms in the United States. As in the case of employment growth, here we also find that recessions are characterized by a widening left tail, which gives rise to both an increase in dispersion and a decline in the skewness of the sales growth distribution.

The same empirical pattern is also clearly seen at the industry level. That is, for all narrowly defined (i.e., 2-digit) industries in the United States, the within-industry skewness of firm-level employment growth, sales growth, and stock returns are positively correlated with the industry economic cycle. Furthermore, the same pattern is also seen globally: in a panel of firms spanning thirty-nine countries that are both geographically and economically diverse, the skewness of the same firm-level variables within each country are robustly procyclical with respect to that country's business cycle.

Motivated by this robust empirical evidence, in the second part of the paper we build a heterogeneous-agents model where the key feature is the presence of a large number of entrepreneurs that face shocks with time-varying risk featuring both, time-varying variance and time-varying skewness. In order to capture the potentially non-linear response of firms to shocks, we assume that entrepreneurs are risk-averse, face a combination of convex and non-convex adjustment costs to capital, and can invest in capital and in a risk-free asset. We numerically solve the model and choose the parameters of the firm's productivity process so that our modeled economy matches the average skewness of the sales growth distribution we observe among US firms during expansionary periods and the large decline in skewness observed during a typical recession. Our results suggest that first-moment shocks combined with risk-aversion and capital adjustment costs, both of which generate asymmetries in the response of firms to shocks, are not sufficient to generate the large swings in the skewness of firm outcomes we document. Hence, in order to match the changes in skewness we observe in the data, we consider time-varying skewness in the firms' productivity process.

In our main quantitative exercise, we study the aggregate effects of a pure skewness shock—that is, a decline in the skewness of firms' productivity shocks while keeping the mean and variance constant. Our model predicts that a change in the skewness of

¹Put in a different way, a Kelley skewness of 0.10 indicates that during expansion, 45% of all the dispersion is accounted for by firms with employment growth below the median, whereas during recessions, this share increases to 56%.

the distribution of firm-level shocks that matches the decline in the skewness of sales growth we observe among US firms would reduce gross domestic output (GDP) by 1.7%. The decline in aggregate economic activity is quite persistent as GDP stays below its pre-shock level several quarters after the shock. This is in contrast to the standard uncertainty shock analyzed in the literature that typically generates a sharp drop and rapid rebound of GDP. This significant and persistent drop in output is driven by a decline in capital investment, which is the result of three forces. First, the presence of a fixed cost to capital adjustment creates a real options effect that reduces the incentives of firms to invest when skewness declines. Second, the drop in skewness makes capital riskier, inducing an increase in investment in the risk-free asset. Finally, relative to the standard uncertainty shock (a symmetric increase in dispersion), in our model a decline in skewness results in a widening left tail of the firm productivity distribution without a corresponding widening of the right tail (an asymmetric increase in dispersion). This change in skewness cancels out the increase in output generated by an uncertainty shock in models without adjustment costs. Hence, our results indicate that a negative shock to the skewness of firms' productivity (that keeps the mean and variance constant) can generate a recession by itself.

This paper is related to several strands of literature. First and foremost, our paper relates directly to the study of the effects of uncertainty on firms' decisions. Several papers have shown that an increase in uncertainty can have important macroeconomic implications in the presence of adjustment costs or financial frictions.²

Second, several authors have suggested that rare disasters—presumably arising from an asymmetric distribution of shocks—can generate large fluctuations in economic activity, such as the Great Recession. Reviving the ideas introduced first by [Rietz \(1988\)](#), [Barro \(2006\)](#) uses a panel of countries to estimate the probability of large disasters and argue that these low-probability events can have substantial implications for aggregate economic activity and asset pricing.³ The results of our paper can be seen as evidence that rare disasters also occur at the microeconomic level.

Finally, our paper contributes to a growing literature that focuses directly on the skewness of firms and workers outcomes such as firm productivity ([Kehrig \(2011\)](#)), em-

²See, for example, [Arellano *et al.* \(2010\)](#), [Fernandez-Villaverde *et al.* \(2011\)](#), [Bachmann and Bayer \(2013\)](#), [Bachmann and Bayer \(2014\)](#), [Gilchrist *et al.* \(2014\)](#), [Jurado *et al.* \(2015\)](#), [Leduc and Liu \(2016\)](#), [Basu and Bundick \(2017\)](#), [Berger *et al.* \(2017\)](#), [Alfaro *et al.* \(2018\)](#) and [Bloom *et al.* \(2018\)](#).

³See for instance [Gabaix \(2008, 2012\)](#), [Gourio \(2008, 2013, 2012\)](#), [Wachter \(2013\)](#), [Kilic and Wachter \(2015\)](#), among others.

ployment growth (e.g. [Ilut *et al.* \(2018\)](#) and [Decker *et al.* \(2015\)](#)), stock returns (e.g. [Harvey and Siddique \(2000\)](#), [Kapadia \(2006\)](#), and [Schmidt \(2016\)](#), and [Oh and Wachter \(2018\)](#)), and labor earnings (e.g. [Guvenen *et al.* \(2014\)](#)).

The rest of the paper is organized as follows. Section 2 describes the data we use and the basics statistics discussed in the empirical section. Section 3 shows the main empirical results of our paper, that is, that the skewness of several firm-level outcomes is procyclical. Section 4 shows the model and Section 5 shows our quantitative results. Section 6 concludes.

2 Data and Measurement

2.1 Data and Sample Selection

Our analysis is based on three large datasets. First, we extract panel data on employment at the firm-level from the Census Bureau’s Longitudinal Business Data Base (LBD). The LBD provides high quality measures of employment, wage bill, industry, and firm age for the entire US nonfarm private sector linked over time at the establishment-level from 1976 to 2015. From the LBD we construct employment at the firm and establishment-levels and use it to calculate cross-sectional moments of the distribution of employment growth at narrow firm population groups.

Second, we draw panel data information of publicly traded firms from Compustat, which contains information on sales, employment, stock prices, and other firm-level outcomes. We use data on quarterly sales, daily stock prices, annual sales, and annual employment from 1970 to 2017, and we restrict attention to a sample of firms with more than ten years of data to minimize the types of compositional issues identified in [Davis *et al.* \(2006\)](#).

Third, we study whether the patterns we document for the United States are also observed in other countries, both developed and developing. To that end, we use cross-country firm-level panel data containing sales and employment information between 1986 and 2016 from the Bureau van Dijk’s Osiris dataset. To ensure that changes in the sample of firms do not bias our results, we focus on firms that are present in the sample for ten years or more. Additionally, we restrict our sample to country/year bins with more than one hundred firms, countries with at least ten years of data, and years with five countries or more. Our main results are based on an unbalanced panel of firms spanning

forty countries from 1991 to 2015. We complement this dataset with information on firm-level stock prices obtained from the Global Compustat dataset. Applying similar selection criteria, we obtain a sample of daily stock price information for firms in 29 countries from 1985 to 2017.

Table I summarizes the data sources and the main sample characteristics. Table 2 shows a list of the countries we consider in our analysis and the data available for each of them. Additional details on data construction, selection criteria, and moment calculation can be found in Appendix A.⁴

2.2 Measuring Skewness

For most of our results, we measure the growth rate of a firm-level outcome as the log-difference between period t and $t + k$ where t is a quarter for stock returns, and a year in the case of employment and sales. For both dispersion and skewness, we use quantile-based measures that are robust to outliers, which are common in micro data sets. As we shall see, they also have magnitudes that are easy to interpret. Our measure of dispersion is the differential between the 90th and 10th percentiles, denoted by $P9010_t$, where t is a quarter or a year depending on the dataset. Additionally, we use the differentials between the 90th and 50th percentiles, $P9050_t$, and between the 50th and 10th percentiles, as measures of dispersion in the right and left tails respectively. Finally, our preferred measure of skewness is the Kelley skewness (Kelley, 1947), which is defined as

$$KSK_t = \underbrace{\frac{P90_t - P50_t}{P90_t - P10_t}}_{\text{Right Tail Share}} - \underbrace{\frac{P50_t - P10_t}{P90_t - P10_t}}_{\text{Left Tail Share}} \in [-1, 1]. \quad (1)$$

As seen here, the Kelley measure provides a simple decomposition of the share of total dispersion that is accounted for by the left and the right tails of a distribution.⁵ A negative value of Kelley skewness indicates that the left tail accounts for more than

⁴The online appendix—available at the authors’ websites—provides additional details about the underlying data and describes the material to replicate the results presented in the empirical section of the paper.

⁵An important drawback of this measure of skewness is that it is invariant to 20% of the observations in the sample (the top and bottom 10% of the distribution). In principle, Kelley skewness can be computed using any two symmetric percentiles, such as the 95th and 5th or 98th and 2nd. We have explored some of these alternative choices and did not find them to matter for our results (see section 3.4). Additional measures of skewness can be found in Kim and White (2004).

one-half of the total dispersion and the distribution is negatively skewed, and vice versa for a positive value.

3 Skewness over the Business Cycle

In this section, we show that the distribution of firm-level growth rates has a longer left tail in recessions in both the United States (Section 3.1) and across countries (Section 3.2), and then confirm that our results hold within industries (Section 3.3).

3.1 US Evidence

The first contribution of our paper is to show that the skewness of the growth rates of firm-level outcomes varies over time and is strongly procyclical, declining substantially during recessions and rising in booms. We start by considering the evolution of the Kelley skewness of the distribution of the growth rate of firm employment for a sample of firms from the Census' LBD which is displayed in the top panel of Figure 2. To calculate Kelley skewness we weight observations by firm's employment so that our measure reflects the underlying firm-size distribution.⁶ Figure 2 shows, first, that the skewness of employment growth, on average, is positive and around 10% for most the sample period. Second, the skewness of employment growth is strongly procyclical, declining from an average of 11% at the peak of the typical recession to around -10% at the trough, that is, a drop of 21 percentage points. Similarly, the bottom of Figure 2 shows the cross-sectional skewness of annual sales growth for a sample of publicly traded firms from Compustat. Relative to our sample from LBD, this is a more selective set of mostly large firms. Still, we find that the skewness of the distribution of sales growth is positive on average, and declines around 20 percentage points during a recession.

The decline in the skewness of firm growth occurring during recessions is driven by a rapid change in the relative weight of the tails of the distribution. This can be observed in the top panel of Figure 3 where we plot $P5010_t$ (black line with circles) and $P9050_t$ (blue line with squares) of employment growth. The bottom panel of Figure 3 shows the same set of statistics for sales growth. Two important aspects are worth noticing. First, during expansionary periods, the right tail outweighs the left tail ($P9050_t$ is most

⁶In particular, we weight the employment growth of firm i in period t by the average employment in periods t and $t + 1$, that is $\bar{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$. The results for publicly traded firms are unweighted since most of the firms are large.

of the times above the $P5010_t$), generating a distribution of firm’s outcomes that is positively skewed. Second, both for employment and sales growth, recessions are episodes in which the $P5010_t$ expands, indicating a left tail that stretches out, whereas the right tail contracts. This asymmetric change of the tails drives a drop in the skewness of firms’ employment and sales growth.⁷

To have a better sense of the magnitude of the change in skewness and its relation with the cycle, the left panel of Table II shows a set of time-series regressions of the form

$$KSK_t = \alpha + \beta \Delta GDP_t + \delta t + \epsilon_t, \quad (2)$$

where the dependent variable is the Kelley skewness of the cross-sectional distribution of different firm-level outcomes. In all regressions, the independent variable is the growth rate of real GDP per capita, which we have normalized to have unitary variance so the coefficients are comparable across columns, and t is a linear trend. The estimated coefficients are positive and large for all three variables—employment and sales growth, and stock returns—and also strongly statistically significant (at 1% level for the first two and 5% level for the third). For example, the estimated coefficient of 4.6 in column (1) implies that a three standard deviation—or about 6%—drop in GDP per capita growth is associated with a 0.14 fall in the Kelley skewness of firm employment growth distribution. Columns 2 shows a similar result for sales growth with a larger coefficient (5.37) and column 3 shows a smaller coefficient for stock returns (2.1) that is still highly significant.⁸

3.2 Cross-Country Evidence

Is the procyclical skewness we have documented so far a pattern specific to the United States or is it also observed in other countries? The second contribution of our paper is to shed light on this question using firm-level panel data covering almost forty countries that are both geographically and economically diverse, spreading over five continents

⁷This asymmetric change is also observed at higher frequencies. For instance, Figure A.1 shows a similar set of results for the annual change of real sales at the quarterly level.

⁸Table A.4 in Appendix B shows that the skewness of firm-level outcomes remains strongly procyclical if we residualize the firms’ outcomes by firm’s observable characteristics and fixed unobserved heterogeneity, if we consider the growth rate of sales-per-worker (more closely related to firm productivity), or if we look at the three years growth rate of firms’ outcomes (Appendix Table A.2). We also confirm that the dispersion of firms outcomes is countercyclical (Appendix Table A.3). We do not find significant business cycle variation in the Kurtosis of firms’ outcomes (right panel of Table A.4).

including developing countries (such as the United States, Germany, Japan, and others) and developed countries (such as Peru, Egypt, Thailand, and others).

The top panel of Figure 4 displays the empirical density of the distribution of the growth rate of annual real sales (in US dollars as of 2005) for a panel of firms spanning across thirty-nine countries from 1991 to 2015. The solid red line is the density of the growth rate of sales during recession periods, where a recession is defined as a year in which the growth rate of GDP is in the first decile of the country-specific GDP growth distribution. The dashed black line is the density of sales growth during expansion periods defined as years in which GDP growth is above the first decile of the country-specific distribution of GDP growth. Similar to the results presented in Figure 1, the dispersion of sales growth increases little during recession years, with $P9010_t$ raising slightly from 0.82 to 0.85. However, this modest increase masks larger changes in each tail: The left tail stretches out, with $P5010_t$ rising from 0.36 to 0.43, and the right tail shrinks, although by a smaller amount, with $P9050_t$ falling from 0.46 to 0.43. The opposite moves of each tail dispersion partially cancel out each other, leading to the smaller rise in $P9010_t$ just mentioned. In contrast, for skewness, the contraction of upper tail and expansion of lower tail inequality reinforce each other to generate a larger decline in Kelley skewness, which falls from 0.12 to 0.0.

To have a clearer picture of how skewness changes over the business cycle, the bottom left panel of Figure 4 shows a bin scatter plot in which the x-axis is the average of firm-employment growth within a country-year bin whereas the y-axis is the Kelley skewness of the same firm-level outcome. The data points align nicely along a straight line over a wide range of average employment growth rates (ranging from -15% to $20+\%$), confirming the strong positive relationship between skewness and the cycle. Further, the slope of the relationship is equal to 1.64 and is both statistically and economically significant. For example, when average firm employment growth is -15% (typically during a big recession) the Kelley skewness is -30% , implying that two-thirds of the mass of the distribution of employment growth is accounted for by the left tail. In contrast, when the average employment growth is 10% , skewness is 30% , indicating the opposite split, with two-thirds of total dispersion now being accounted for by the right tail. The bottom right panel of Figure 4 shows a similar result for sales growth. Importantly, to construct these figures we have controlled for country- and time-fixed effects, so these results are not driven by fixed characteristics of the countries considered in the sample or by global

shocks—such as the Great Recession—that can affect all countries at the same time.⁹

The center panel of Table II repeats the cyclical regression discussed above for the United States but this time exploiting the panel dimension of the cross-country data set to assess the cyclical nature of skewness in international data. This time, the dependent variable is the skewness of employment growth, sales growth, or stock returns, within a given country each year. The business cycle is captured by the growth rate of GDP per capita in the respective country. The regressions also include a full set of time- and country-fixed effects to control for aggregate economic conditions that might affect all countries simultaneously or fixed differences across countries. The regression results confirm our previous findings of procyclical skewness for all three variables with similar levels of statistical significance. Compared with the United States, the estimated coefficient is slightly higher for employment (5.30 across countries vs. 4.64 for the United States), somewhat lower for sales (3.19 vs. 5.37), and nearly identical for stock returns. These results further confirm the procyclical nature of skewness in firm-level outcomes.

3.3 Industry-Level Evidence

We finally turn to industry-level data from the United States and investigate the extent to which the skewness results are found in different industries. To this end, using LBD data, the top panel of Figure 5 shows a bin scattered plot of the skewness of employment growth within each industry against the average employment growth for the same industry in that year. In this case, a positive correlation indicates that periods of low economic activity at the *industry-level* are associated with a negative shift in skewness within that industry, and vice versa for periods of high economic activity. In terms of magnitudes, the top panel of Figure 5 shows that when the industry employment growth is -8%, the Kelley skewness is around 20%, indicating that 60% of the total dispersion of employment growth is accounted for by the left tail of the distribution. When the average employment growth is 8% instead, the Kelley skewness is 20%, indicating that the right tail accounts for 60% of the total dispersion. Similarly, the bottom panel of Figure 5 shows that the within-industry skewness of sales growth is higher when the average sales growth for that industry is higher. Hence, sectors that grow faster are also

⁹One important concern is that our cross country results are based on exclusively on publicly traded firms. Interestingly, we also find remarkably similar results if we consider an unbalanced panel of firms (private and publicly traded) drawn from the BvD Amadeus dataset as Figure A.2 shows. The BvD Amadeus dataset covers a shorter period of time (2000 to 2015 for most countries) over a smaller sample of European countries.

sectors in which the skewness of firm-level outcomes is higher.¹⁰

We then use firm-level data from Compustat to examine the relation of the industry cycle and the skewness of sales growth, employment growth, and quarterly returns within NAIC 2-digit industry-period bins. Columns 7 to 9 of Table II display a series of industry panel regression in which the dependent variable is the Kelley skewness of the growth rate of different firm-level outcomes across all firms within an industry-period bin. In this case, we capture the within-industry business conditions by the average growth rate in a industry-year bin and we have rescaled the real sales growth within each sector to have a variance of one so that the regression coefficient can be interpreted as the effect of a change in the within-industry sales growth of one standard deviation and can be easily compared to the coefficients of columns 1 to 2 of Table II. Importantly, we also include a full set of time and industry fixed-effects, so that the results are driven by within-industry rather than aggregate changes in growth rates.¹¹

Column 7 of Table II shows that the skewness of employment growth is significantly lower during industry slowdowns. Specifically, a one standard deviation decline in the within industry average sales growth is associated with a decline in the skewness of employment growth of 7 percentage points. This is almost two times larger than the effect of a change in one standard deviation in GDP growth on the skewness of employment growth across all firms in the economy. Similarly, a one standard deviation decline in average sales growth is correlated with a decline of 13 percentage points in the skewness of sales growth, and a decline of 1.6 percentage points in the skewness stock returns.

3.4 Robustness

In this section, we perform several robustness checks using the large LBD data set. First, we examine the robustness of procyclical skewness for firms in different age or size categories, as well as for establishments as opposed to firms. We then consider alternative measures of skewness. We also allow for firm entry and exit.

Figure 6 reports the results of these different analyses. The two figures (A, B) in the top panel plot the skewness of employment growth for firms within various size categories,

¹⁰Appendix Figure A.3 shows remarkably similar results for other firm’s outcomes, such as three-years sales growth, three-year employment growth, and stock returns.

¹¹We find a similar positive and statistically significant relationship between industry cycles and skewness when we consider each industry separately. Appendix Figure A.4 shows the coefficient of a set of within-industry time-series regressions of the Kelley skewness of firms’ growth on the within-industry average firm growth. Notice that there is substantial heterogeneity across industries and for all of them the coefficient on the average firm growth is economically and statistically significant.

ranging from firms with 1 to 19 employees at the low end to firms with more than 1000+ employee at the high end. The skewness of employment growth is procyclical for all groups. Second, panel C splits the sample by age and shows that despite level differences in skewness across groups (in particular younger firms have more positive skewness than average as could be expected—see [Haltiwanger *et al.* \(2016\)](#)), fluctuations in skewness are procyclical for all firm age categories. Panel D shows that skewness fluctuations are very similar for establishments and firms showing that our baseline results are not driven by a small number of large firms.¹²

Second, we explore the effect of the particular percentiles the Kelley skewness is based on because by using the 90th and the 10th percentiles we are effectively dropping 20 percent of the distribution, which is all in the tails. Because our results hinge on the differential response of the tails to business cycles, truncating these tails could matter. As noted earlier, the Kelley measure can be constructed for any two symmetric quantiles, so we compute two additional versions of the Kelley skewness: one that uses the 95th and 5th percentiles and another that uses 97.5th and 2.5th percentiles. The bottom left panel (E) of Figure 6 shows that these measures behave qualitatively similar to the standard Kelley measure, although the level of skewness in expansions is lower (more negative) before the 2000s with the latter measures that include a larger fraction of the observations at the tails of the distribution.

Third, our main results are based log growth rates in employment and sales, which required us to exclude firms that either enter or exit in one of the two years during which the growth rate is calculated. Since entry and exit have a clear cyclical nature, this could potentially matter for the cyclicity. In particular, if a firm exits the market due to a change in aggregate economic conditions or a new firm enters, our measure of growth rate, and consequently, the skewness of the distribution, will not take them into account. To address this issue, we calculate the skewness of the employment growth distribution measured as arc-percent change of employment which is defined as $2(x_{i,t+k} - x_{i,t}) / (x_{i,t+k} + x_{i,t})$. This measure has been popularized in the firm dynamics literature by [Davis and Haltiwanger \(1992\)](#) and has the advantage that, while it is similar to a percentage change, it allows for entry/exit by including both time t and $t + k$ measures in the denominator, one of which is allowed to be zero.¹³ The bottom

¹²Furthermore, Appendix Figure A.5 shows that the skewness of employment growth is also procyclical within even finer categories—establishment groups defined by size and age.

¹³Notice that, for a firm with a positive value of $x_{i,t}$ which is inactive in period $t + k$, and henceforth

right panel of Figure 6 shows that the cyclical properties of the skewness of employment growth do not change substantially when accounting for the entry and exit of firms.¹⁴

In summary, we have shown that the skewness of firm-level outcomes declines sharply during recessions, both at the aggregate and at the industry level. Motivated by this robust evidence, in the next section we study a heterogeneous agents model that we use to evaluate the macroeconomic importance of the large swings in the skewness we observe in the data.

4 Model

Given the robust evidence presented in the previous section, a natural question is to ask whether fluctuations in skewness at the firm level have aggregate implications. To answer this question, we build a heterogeneous-agent model with time-variation in the distribution of firm-level shocks. Specifically, we consider an economy populated by a large number of infinitely-lived households/entrepreneurs who combine capital and labor using a technology subject to idiosyncratic productivity shocks to produce a homogeneous good. An important point to stress is that the empirical evidence on skewness fluctuations we documented in the previous section pertained to firm *outcome* variables, and not to the shocks faced by firms. In the model, we will both specify aggregate and idiosyncratic shocks in a flexible manner and also incorporate various features of firms' problems (adjustment costs, constraints, risk aversion, etc.) to let the data sort out whether the observed business-cycle variations are inherited from the properties of shocks, from these other features, or emerge from the interaction of the two. Furthermore, entrepreneurs are able to save for tomorrow both in the form of the capital good or in a bond with a risk-free return. We now describe each component of the model in more detail.

has a value of $x_{i,t+k}$ equal to 0, the arc-percent change takes the value of -2. Similarly, for an entering firm (that is, $x_{i,t}$ is equal to 0 but $x_{i,t+k}$ is positive) the arc-percent change takes the value of 2.

¹⁴Our results are also robust when we look at firms of different size and age, and for the growth of establishments rather than firms. All these results are based on a sample of firms from Census' LBD and are under disclosure process.

4.1 Entrepreneurs

4.1.1 Production Technology

The production function of entrepreneur j is given by

$$y_{j,t} = A_t e_{j,t} k_{j,t}^\alpha n_{j,t}^\nu, \text{ with } \alpha + \nu < 1,$$

so it displays decreasing return to scale. The aggregate productivity shock, A_t , follows a first-order autoregressive process:

$$\log A_t = \rho_A \log A_{t-1} + \sigma_\eta \eta_t,$$

where η_t is a Gaussian innovation with zero mean and unitary variance. The idiosyncratic productivity process $e_{j,t}$ is given by

$$e_{j,t} = \rho e_{j,t-1} + \epsilon_{j,t}, \tag{3}$$

where $\epsilon_{j,t}$ has zero mean, time-varying variance, denoted by $\sigma_{\epsilon,t-1}$, and time-varying skewness, denoted by $\gamma_{\epsilon,t-1}$. Notice that we have assumed that the distribution of innovations in period t depends on the values of the variance and skewness observed in period $t - 1$. This timing captures the “news shock” aspect of firm-level risks in the model: an increase in dispersion or left-skewness of firms’ shocks represents news about the characteristics of the distribution of innovations in the future but not a change in the distribution from which the current realizations of $\epsilon_{j,t}$ are drawn.

4.1.2 Capital Adjustment Costs

As just discussed, the distribution of firm outcomes, such as sales growth or employment growth, can change asymmetrically because of the endogenous response of firms to symmetric shocks.¹⁵ To allow for this possibility, we consider a combination of convex and non-convex adjustment costs to capital. To this end, let $i_{j,t}$ denote *net* investment in capital:

$$i_{j,t} = k_{j,t+1} - (1 - \delta) k_{j,t}, \tag{4}$$

where δ is the depreciation rate of capital. Capital adjustment costs are given by the sum of a fixed *disruption cost*, ϕ_1 , paid by the entrepreneur for any net investment or

¹⁵Iltut *et al.* (2018), for instance, argues that a left skewed distribution of employment growth is the results of an asymmetric response of firms to positive and negative shocks.

disinvestment, a quadratic adjustment cost, ϕ_2 , and a *resale cost* for net disinvestment (partial irreversibility), ϕ_3 . Therefore, the total adjustment cost function for capital input is

$$\phi(k_{j,t+1}, k_{j,t}) = \phi_1 \mathbb{I}_{|i_{j,t}| > 0} y_{j,t} + \frac{\phi_2}{2} \left(\frac{i_{j,t}}{k_{j,t-1}} \right)^2 + (1 - \phi_3) |i_{j,t}| \mathbb{I}_{i_{j,t} < 0}, \quad (5)$$

where \mathbb{I} is an indicator function.

4.1.3 The Problem of the Entrepreneur

Entrepreneurs do not value leisure and value consumption streams according to an Epstein-Zin utility function as specified below. Entrepreneurs supply labor to their own firm (they cannot work for someone else's firm). They can save in capital and in a risk-free asset that pays an interest rate r_t . Denote the entrepreneur's value function by $V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$ where $k_{j,t}$ is the entrepreneur's stock of physical capital, $a_{j,t}$ is the beginning-of-the-period holdings in the risk-free asset, and $e_{j,t}$ is her idiosyncratic productivity. For notational simplicity, define the vector of aggregate states as $\Omega_t \equiv (A_t, \sigma_{\epsilon,t-1}, \gamma_{\epsilon,t-1}, \mu_t)$ where A_t is the aggregate productivity level, $\sigma_{\epsilon,t-1}$ and $\gamma_{\epsilon,t-1}$ are the variance and the skewness of the distribution of idiosyncratic shock, respectively, and μ_t is the distribution of entrepreneurs over idiosyncratic states. Then, we can write the dynamic problem of the entrepreneur with Epstein-Zin preferences as

$$V(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t) = \max_{\{c_{j,t}, k_{j,t+1}, a_{j,t+1}, n_{j,t}\}} \left(c_{j,t}^{1-\lambda} + \beta \mathbb{E} \left[V(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}; \Omega_{t+1})^{1-\xi} \right]^{\frac{1-\lambda}{1-\xi}} \right)^{\frac{1}{1-\lambda}}, \quad (6)$$

$$\begin{aligned} \text{s.t. } c_{j,t} + i_{j,t} + a_{j,t+1} &\leq y_{j,t} - w_t n_{j,t} - \phi(k_{j,t+1}, k_{j,t}) + (1 + r_t) a_{j,t}, \\ i_{j,t} &= k_{j,t+1} - (1 - \delta) k_{j,t}, \\ \mu_{t+1}(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}) &= \Gamma(\Omega_t), \\ k_{j,t} &> 0, a_{j,t} \geq 0, n_{j,t} > 0, \end{aligned}$$

given the laws of motion for A_t , $\sigma_{\epsilon,t}$, and $\gamma_{\epsilon,t}$. In this specification, ξ risk aversion, λ is inversely related to the elasticity of inter-temporal substitution. The term $w_t \equiv w(\Omega_t)$ denotes the wage rate in the economy. In what follows, we assume the interest rate

on the risk-free asset is fixed, that is $r_t = r(\Omega_t) = r$.¹⁶ Let $C^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, $K^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, $N^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, and $A^e(k_{j,t}, a_{j,t}, e_{j,t}; \Omega_t)$, denote the policy rules of consumption, next's period capital, current period labor, and risk-free asset for the entrepreneurs.

4.2 Non-Entrepreneurial Households

The economy is populated by a large number of identical hand-to-mouth household that consume C_t units of the homogeneous good and supply labor elastically which we denote by N_t . In concrete, we assume that the non-entrepreneurial households solve the static problem,

$$U(C_t, N_t) = \max_{C_t, N_t} \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \psi \frac{N_t^{1-\gamma}}{1-\gamma} \right\}, \quad (7)$$

$$C_t \leq w_t N_t,$$

given the law of motion of the aggregate state, Ω_t . Denote by $C(\Omega_t)$ and $N(\Omega_t)$ the optimal choices of consumption and labor for the non entrepreneurial household.

4.3 Recursive Competitive Equilibrium

Given the exogenous process for the aggregate productivity, A , the exogenous process of the variance and skewness of e_j , an interest rate of the risk-free asset, r , and the evolution of the idiosyncratic productivity processes for the entrepreneurs, $\{e_j\}_{j \in J}$, a recursive competitive equilibrium for this economy is a set of policy functions $\left\{ \{C_j^e, K_j^e, N_j^e, A_j^e\}_{j \in J}, C, N \right\}_{t=0}^{\infty}$, a wage function $\{w\}$, and value functions $\{V, U\}$ such that i) the policy and value functions solve (6) and (7) respectively, ii) labor market clears, that is

$$\int N^e(k_j, a_j, e_j; \Omega) d\mu(k_j, a_j, e_j) = N(\Omega),$$

and iv) the mapping $\Gamma(\omega)$ that determines the evolution of the joint distribution of e_j , k_j , and a_j is consistent with the policy functions, the evolution of the aggregate productivity process, and the evolution of the process of σ_ϵ and γ_ϵ .

¹⁶This implies that we will not solve the interest rate in equilibrium. The wage rate, however, is such that the labor market clears.

4.4 Parameters, Estimation, and Model Fit

In this section, we describe the quantitative specification of our modeled economy. To solve the entrepreneurs' problem we employ non-linear methods similar to [Krusell and Smith \(1998\)](#). Most of our parameters are standard in the macro literature and we take them from the existing estimates when possible. However, the parameters governing the stochastic process of productivity are novel to our analysis and we use a simulated method of moments approach to estimate them.

Frequency and Preferences

We set the time period to a quarter. For the entrepreneurs, we set ξ , the risk aversion coefficient, equal to 6.0 and $1/\lambda$, the elasticity of substitution, to $1/\lambda = 0.2$, which are in the midpoint of the values used in [Guvenen \(2009\)](#). The household's discount rate, β , is set to $0.95^{0.25}$, whereas the interest rate on the risk-free asset is set to match an annual return of 2%. For the non-entrepreneurial sector, we set σ to 2. For the labor supply of the non-entrepreneurial households, we fix a value of γ to 1.5 and we choose ψ so that they spend an average of 33% of their time working.

Production Technology and Adjustment Costs

The exponents of the capital and labor inputs in the entrepreneur's technology are set to $\alpha = 0.25$ and $\nu = 0.5$. The capital depreciation rate, δ , is set to match a 14% of annual depreciation. As for the adjustment cost parameters, we set the fixed adjustment cost of capital, ϕ_1 , equal to 1.5%, a quadratic adjustment cost, ϕ_2 , equal to 7.0, and a resale cost, ϕ_3 , equal to 34.0%.

Aggregate Productivity

We assume that the aggregate productivity follows a standard first-order autoregressive process with autocorrelation of 0.95 and normally distributed innovations with mean 0 and standard deviation of 0.75%, similar to the quarterly values used in other papers in the literature. [Table III](#) summarizes the set of calibrated parameters.

Idiosyncratic Productivity

To capture time-varying risk, we assume that the economy transitions between in two states. The first, which we denote as low risk state, corresponds to periods where the variance of the innovations of the idiosyncratic shocks is low, $\sigma_{\epsilon,t} = \sigma_L$, and the skewness is positive, $\gamma_{\epsilon,t} = \gamma_H$, as we observe in non-recession periods. The second state, or high

risk state, corresponds to periods of high dispersion, $\sigma_{\epsilon,t} = \sigma_H$, and negative skewness, $\gamma_{\epsilon,t} = \gamma_L$, as we observe during a typical recession. Low and high risk states alternate following a first-order Markov process. To capture the potential non-gaussian nature of the idiosyncratic shocks we assume that, conditional on the values of $\sigma_{\epsilon,t}$ and $\gamma_{\epsilon,t}$, the innovations in [3](#) are drawn from a mixture of two normally distributed random variables, that is,

$$\epsilon_{j,t} \sim \begin{cases} N(\mu^s, \sigma_1^s) & \text{with prob } p^s, \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1 - p^s, \end{cases} \quad (8)$$

where s can be a high or low risk state. Hence, in order to fully characterize the stochastic process faced by firms we need to find ten parameters, namely, $\{\mu^s, \sigma_1^s, \sigma_2^s, p^s\}$ with $s \in \{H, S\}$, and the parameters governing the transition probabilities between low and high risk periods, denoted by π_L and π_H respectively.

Since we do not directly observe the productivity process faced by the firms, we choose the parameters of the stochastic process of firm's productivity of our model to match the main features of the US data described in the empirical section of the paper. In particular, we take data of quarterly sales growth from a sample of publicly traded firms, and we search for parameters of the stochastic process so that the cross sectional distribution of sales growth derived from the model reproduces the observed average values of the 90th-to-50th percentiles spread, the 50th-to-10th percentiles spread, the Kelley Skewness, and the 90th-to-10th percentiles spread during expansion periods and the same set of moments for recession periods for a total of eight moments of the quarterly sales growth distribution.¹⁷ The probability of being in the high risk state in the next period conditional on being in the high risk state in this period, π_H , is set to be equal to the fraction recession quarters that are followed from another recession quarter in the data, $\pi_H = 0.84$, whereas the transition probability of the low risk state, π_L , is set so that the share of expansion quarters following another expansion quarter is 0.95. Recession and expansion periods in the data correspond to the recession quarters defined by the NBER from 1970 to 2014.

Based on our estimations, we find that in periods of low risk, the variance of the idiosyncratic productivity shocks, η , is equal to 4.85% whereas the skewness is equal to 0.85. In contrast, in periods of high risk, the variance of the productivity shocks is

¹⁷Appendix Figure [A.1](#) displays the evolution of the cross-sectional dispersion and skewness of the sales growth distribution for our sample of publicly traded firms from Compustat at the quarterly frequency.

equal to 6.85% and the skewness is negative and equal to -1.14. Table VI displays our estimates for the different parameters of the idiosyncratic productivity process whereas Table IV shows the targeted and model-simulated moments.¹⁸

5 Quantitative Results

In this section, we study the quantitative implications of our model. We first analyze standard business cycle statistics. Then, in our main quantitative exercise, we evaluate the response of our modeled economy to a shock that increases risk by reducing the skewness of idiosyncratic productivity while keeping the mean and variance constant. Finally, we compare the response of our model after a variance shock (i.e., a standard uncertainty shock which implies a symmetric increase in dispersion) to a negative skewness shocks (i.e., an asymmetric increase in dispersion), and then to a combined shock of dispersion and skewness (i.e., a change in risk that resembles what happens in a typical recession).

5.1 Business Cycle Statistics

Table VII shows a set of standard business cycle statistics generated from our modeled economy. To obtain these statistics we simulate our economy for 5,000 periods and we discard the first 500. We then calculate the standard deviation and correlation with aggregate output for several aggregate time series. All statistics are in the neighborhood of what is observed in the data: investment is more volatile than output whereas consumption is less volatile. Additionally, our model generates an average annual risk premium of 5.3%, which is in line with the empirical estimates based on US data. We conclude that our model is consistent with the standard business cycle statistics found in the literature.

5.2 Idiosyncratic Shocks and Model Fit

To evaluate the effects of a decrease in the skewness of firm-level shocks, we independently simulate 1,000 economies, each of 300 quarters length. For the first 150 periods,

¹⁸The variance of a random variable η which is distributed as a mixture of two normally distributed random variables is given by $Var(\eta) = \mathbb{E}(\eta^2) - \mathbb{E}(\eta)^2$ whereas the skewness is given by $Skew(\eta) = (\mathbb{E}(\eta^3) - 3\mathbb{E}(\eta)Var(\eta) - \mathbb{E}(\eta)^3) / Var(\eta)^{3/2}$. Here $\mathbb{E}(\eta)$ is the first moment of the η given by $\mathbb{E}(\eta) = p_1\mu_1 + p_2\mu_2$. Similarly, $\mathbb{E}(\eta)^2 = p_1(\mu_1^2 + \sigma_1^2) + p_2(\mu_2^2 + \sigma_2^2)$ and $\mathbb{E}(\eta^3) = p_1(\mu_1^3 + 3\mu_1\sigma_1^2) + p_2(\mu_2^3 + 3\mu_2\sigma_2^2)$ are the second and third moments.

the economy remains in the low-risk state, then all economies are hit by a change in the level of risk (i.e. a decrease in the skewness of firm-level shocks, an increase in dispersion of firm-level shocks, or both at the same time). From that period on, all economies evolve normally. We then average different macroeconomic outcomes across all simulated economies and we calculate the impact of the change in risk as the percentage deviation of a given macro variable relative to its value in the period previous the shock.

Comparing the impact of a change in risk that combines dispersion and skewness to a case in which either skewness or dispersion change while keeping the mean of the productivity shocks constant is key for the quantitative analysis we perform in the next section. Hence, before analyzing the effect on the macroeconomic aggregates it is informative to study the evolution of the distribution of idiosyncratic shocks experienced by the firms after a risk shock. In particular, we must make sure that our model can separate a change in dispersion from a change in the skewness of shocks without impacting the average productivity of the firms, so that our results are not driven by changes in the first moment of the productivity distribution, but only by changes in either the dispersion and or the skewness.

The left row of Figure 7 displays moments of the distribution of firm's idiosyncratic productivity growth, $\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$, for three cases. In the first, the economy moves from the low-risk state to the high-risk state leading to an increase in the variance and a decrease in the skewness of idiosyncratic shocks (blue line with circles) which corresponds to what is observed during a typical recession. In the second case, the increase in risk leads only to a decrease in the skewness of idiosyncratic shocks (black line with diamonds), and finally, in the third case, the increase in risk leads to an increase in the variance of idiosyncratic shocks only which is the typical uncertainty shock studied in the literature (red line with triangles).¹⁹ The top left panel of Figure 7 shows that the average firm in our model does not experience a change in firm-level productivity when risk changes. This ensures that our results are not driven by a change in average firm productivity. Then, comparing the black line in the middle and bottom left panels one can see that our model is able to generate a pure change in the skewness, that is, a change in the productivity distribution that reflects only a decrease in the skewness but a muted change in the mean and the variance of the firm-level productivity distribution.²⁰

¹⁹To make this comparison, we reestimate the parameters of the stochastic process in 8 to separate the changes in dispersion (a symmetric increase in risk) from changes in dispersion and skewness (an asymmetric increase in risk). Table V shows the estimation targets for each case.

²⁰The median firm, however, experiences an increase in productivity after a decline in the skewness

Similarly, our model can generate a pure uncertainty shock (the red line with triangles in the middle panel of Figure 7).

It is also important to analyze the impact of the change in risk on the sales growth distribution. The right row of Figure 7 shows the average, the dispersion, and the skewness of the annual change in quarterly sales implied by the model calculated as $\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$. It is not surprising that a change in risk that combines a simultaneous increase in the variance and a decrease in the skewness of firm-level productivity shocks generates an increase in the cross-sectional dispersion of sales growth and a large decrease in skewness (blue line with circles in the middle and bottom right panels). Comparing the case in which only dispersion changes—which is the typical uncertainty shock—to the case in which only the skewness changes—the baseline case we discuss in the following section—one can see that by considering a shock with time-varying skewness the model is able to capture the asymmetric response of the tails of the sales growth distribution (compare the red line with triangles to the blue line with circles in the bottom right panel). Moreover, the model generates a drop in Kelley skewness of around 20 percentage points which is in line with the drop observed during recession periods in the United States. This is the first results of our quantitative analysis: in the context of a model with adjustment cost to capital and risk-averse entrepreneurs, a pure uncertainty shock does not generate the large asymmetric changes in the sales growth distribution that we document in Section 3.²¹ Notice also that the average sales growth greatly responds to a change in the risk conditions in the economy (left bottom panel) but this response is only driven by the endogenous capital and hiring response of firms to a change in the risk conditions as the average productivity growth is unaltered.

5.3 The Macroeconomic Effect of a Skewness Shock

In this section, we analyze the macroeconomic effect of a decrease in the skewness of firm-level productivity. For doing that, we shock the economy with a change in the skewness of the innovations of $e_{j,t}$ and we calculate the response of different macroeconomic aggregates as the percentage change relative to their value prior the shock. In

that keeps the mean and variance constant. This increase in productivity goes against our results as our model predicts a negative aggregate response of the economy to a drop in skewness.

²¹Figure A.7c in the appendix shows that the dispersion and skewness of sales growth do not respond to a shock to aggregate productivity, A_t , neither. Furthermore, as it is shown in Figure A.7a, a change in the skewness of firm's shocks generates a persistent decline in the skewness of employment growth and a decline in the skewness of three-years sales growth.

our exercise, when the economy receives a skewness shock that drives the skewness from γ_H to γ_L , we keep the mean and variance of the idiosyncratic productivity constant at their low-risk level so our results reflect a pure change in the skewness of the distribution. Moreover, our timing assumption implies that in the period the shock arrives, the change in the skewness only represents news about the future economic conditions as the realizations of the productivity process that firms experience are drawn from a distribution with skewness equal to its pre-recession values.

Figure 8 shows that output declines by 1.4% four quarters after a skewness shock and 1.7% after eight quarters. This is a significant decline in aggregate economic activity considering that only the shape of the distribution of firm-level shocks has changed. Moreover, the decline in output is quite persistent, staying below its pre-shock level even after twelve periods after the shock. This is in contrast with the typical uncertainty shock that generates a decrease in output and a rapid rebound few quarters after the shock. In our model, the drop in output is generated by the rapid and persistent decline in capital investment after a change in skewness. The top right panel of Figure 9 shows that capital investment drops around 15% during the first quarter after the shock and stays below its pre-shock level for several quarters. Labor does not drop in the first period after the shock because labor is fully flexible and news about the future conditions of risk do not change firms' hiring decisions.²² In contrast, consumption declines rapidly in response to the decrease in the skewness of firm-level shocks, dropping around 1% relative to its pre-shock level, whereas the accumulation of risk-free asset increases because capital is now riskier.

Importantly, in the first quarter after the shock, the response of investment and consumption is not driven by a change in the skewness of the realizations of $e_{j,t}$ received by the firms—recall our timing assumption in equation 3—but by a change in the perception about the risk in the economy: at the moment of the shock, entrepreneurs receive news that in the future the distribution of $e_{j,t}$ will be left skewed and their endogenous responses drive a decline in investment and consumption. A decrease in skewness triggers a precautionary increase on entrepreneur's savings, but since capital is riskier, investment in the risk-free asset surges as it is shown in the bottom right panel of Figure 9. We conclude that a decline in the skewness of the distribution of idiosyncratic shocks can by itself generate a persistent drop in aggregate economic activity.

²²Adding labor adjustment costs will trigger an automatic response of labor to changes in risk, increasing the aggregate impact of a change in variance and skewness.

5.4 Variance and Skewness Shocks

Our empirical evidence indicates that a typical recession is characterized by an asymmetric increase in the dispersion of firm growth, which leads to a decline in the skewness. Hence, in this section, we evaluate the response of our modeled economy to a pure change in the variance of firm-level shocks—the typical uncertainty shock considered in the previous literature—and to a change in risk that combines both, an increase in the variance and a decrease in the skewness of firm-level shocks. This is displayed in Figure 10, where we plot the evolution of several economic aggregates after a shock that combines variance and skewness (blue line with circles), a pure skewness shock (black line with diamond), and pure variance shock (red line with triangles).

Starting with the effects of a pure uncertainty shock, we see that an increase in the variance of idiosyncratic productivity generates an increase in output and consumption. The difference with respect to a skewness shock is mainly due to the Oi (1961), Hartman (1972), and Abel (1983) effect: a symmetric increase in dispersion pushes up the productivity of some firms at the top of the distribution, which, in the absence of labor adjustment costs, increases labor demand and output for these firms.²³ This increase in productivity of firms at the top of the distribution more than compensates the decrease in productivity and labor demand from firms at the bottom, increasing aggregate output.

Capital investment responds negatively to an uncertainty shock in the first period (top right panel of Figure 10) first, because of a real-options effect generated by the fixed adjustment cost, and second, because of a change in the composition of assets in the economy as the accumulation of risk free assets increases. After the first few periods, however, these two reverse as investment jumps and the accumulation of risk-free assets declines. In contrast, a pure skewness shock does not generate an increase in aggregate output: a decrease in the skewness implies that the left tail of the productivity distribution widens, generating a decrease in investment due to the real-options effect, an increase in precautionary savings in the risk-free asset, and a muted Oi-Hartman-Abel effect as the right tail of the productivity distribution does not change.

The overall effect of an increase in risk that combines a variance and a skewness shock depends on the relative strengths the real-options channel, the risk-aversion, and the Oi-Hartman-Abel effect. For our model to match the asymmetric increase in the dispersion of the sales growth distribution that we observe in the data, an increase in

²³We omit the evolution of labor in Figure 10 since it follows the same pattern of aggregate output.

dispersion is mostly due to widening of the left tail of the distribution of firm-level shocks without a parallel widening of the right tail, which commands a decline in the skewness in productivity and sales growth.²⁴ The combined effect of dispersion and skewness is followed by a significant and persistent decline in aggregate economic activity shown by the blue line with circles in Figure 10.²⁵ In this case, output declines almost 2.0% in the first four quarters, the same as consumption, whereas investment drops almost 40% relative to its pre-shock level.

5.5 Robustness

In this section, we discuss the robustness of our findings to different parameterizations. Recall that in our baseline results, in the period in which a change in risk occurs, firms do not experience a change in the actual realizations of shocks but only receive news that in the next period the skewness of productivity shocks, for instance, will be lower. In the next period, however, the firm’s productivity distribution changes as the shocks are drawn from a left-skewed distribution. We compare this baseline case to one in which we keep the underlying distribution of firms shocks fixed so that we can evaluate the pure effect of a change in news about the future risk conditions.²⁶ This exercise is similar to an increase in the probability of a *disaster*, although in our case it represents an increase of *disasters* at the microeconomic level. The blue line with circles in Figure 11 shows that the overall effect of a skewness shock combines the impact of a change in the perceptions about future risk conditions and the actual change in the realizations of idiosyncratic shocks. In fact, a shock that only represents news about the future risk generates a decline in output of about 0.5%, which is around one-third to the overall

²⁴The asymmetric increase in dispersion generated by the model can be appreciated by comparing the response of the 90th-to-50th and the 50th-to-10th percentiles spreads generated by the model. Appendix Figure A.7b displays the evolution of these moments for the three cases we have discussed. In the case of a pure variance shock—red line with triangles—both tails of the distribution expand symmetrically (compare the 50th-to-10th percentile spread to the 90th-to-50th percentiles spread), but in the case of an increase in dispersion that is accompanied by a decrease in the skewness it is only the left tail that expands (measured by the 50th), whereas the dispersion of the right tail almost does not change—blue line with circles. This is exactly what we observe in the data when we compare periods of high and low risk in our model (see Table IV) and in the data during recessions periods (see Figure 2).

²⁵It is worth noticing that an aggregate mean shock (a decline in A_t) does not generate a sizable change neither in the dispersion nor the skewness of the distribution of sales growth as we show in Figure A.7c.

²⁶In particular, we simulate our model using the same realizations of the aggregate risk process used in our baseline analysis. In period T all economies receive a skewness shock, however, in this case, we keep the parameters determining the underlying idiosyncratic productivity process fixed at their pre-shock low-risk level.

decline in our baseline results.

We then study how our results change with the degree of risk aversion of the entrepreneurs and their elasticity of inter-temporal substitution while keeping the rest of the parameters at their values in Table VI. The red line with triangles in Figure 11 shows that decreasing entrepreneur’s risk aversion, ξ , from 6 to 2, does not impact our main results substantially in terms of aggregate output and consumption, although alters the effect of skewness on the accumulation of capital and the risk-free asset. An increase of the elasticity of intertemporal substitution, $1/\lambda$, from 0.2 to 0.5, does reduce the impact of skewness shocks on output and consumption (green line with squares) although the overall effect is still significant. Investment, in this case, changes much less relative to the benchmark. These differences highlight the importance of separating the effect of risk-aversion from inter-temporal substitution when evaluating the impact of risk shocks.

6 Conclusions

This paper studies how the distribution of the growth rate of firm-level variables changes over the business cycle. Using firm-level panel for the United States from Census and non-Census datasets, and firm-level panel data for over thirty other countries we reach three main conclusions. First, recessions are characterized by a large drop of the skewness of firm-level outcomes such as employment growth, sales growth, and stock returns. Hence, the skewness of firms’ outcomes is strongly procyclical. This decline in the skewness is driven by an asymmetric change in the dispersion of the distribution of firms’ outcomes. In particular, we find that most of the increase observed during the typical recession is accounted for by a left tail that stretches out. Second, the decline in the skewness of firms’ outcomes is not only a phenomenon observed in the United States but also in other countries, both developed and developing. Finally, we find strong procyclicality of the skewness at the industry level.

In the second part of our paper, we analyze the impact of a change in the skewness of firms’ idiosyncratic productivity in the context of a heterogeneous agents model. We assume that the exogenous idiosyncratic productivity process faced by entrepreneurs is subject to time-varying variance and time-varying skewness and we choose the parameters of this model to match the evolution of the dispersion and the skewness of the sales growth distribution in the United States. Our results suggest that a change in the skewness of the firm-level productivity distribution can by itself generate a significant decline

in aggregate economic activity even though the mean and variance of firms' shocks are held constant. In fact, in our modeled economy, a decline in the skewness of firm's shocks of the magnitude observed in the typical US recession generates a drop in GDP of 1.7%. The combined impact of a variance and skewness shock generates an even large decline in output (2.0%), consumption (2.0%), and investment (40.0%).

References

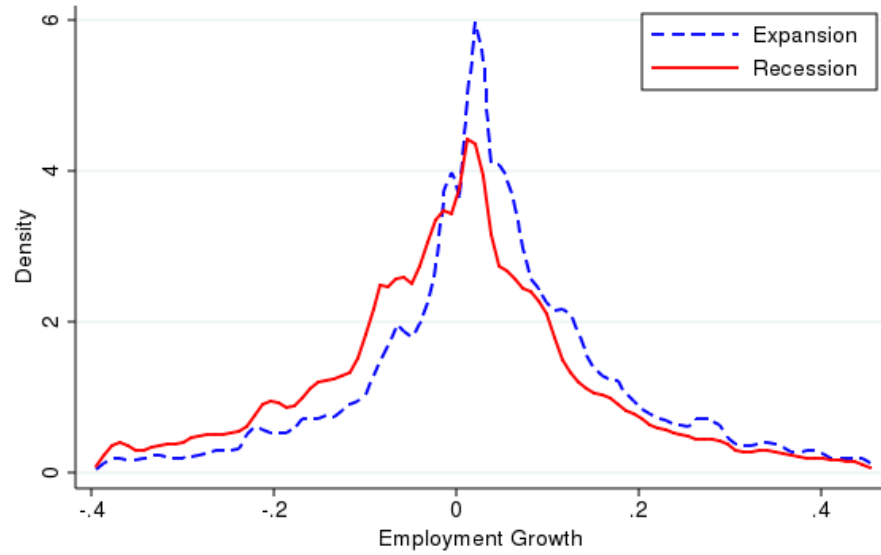
- ABEL, A. B. (1983). Optimal investment under uncertainty. *The American Economic Review*, **73** (1), 228–233.
- ALFARO, I., BLOOM, N. and LIN, X. (2018). *The Finance Uncertainty Multiplier*. Tech. rep., National Bureau of Economic Research.
- ARELLANO, C., BAI, Y. and KEHOE, P. (2010). Financial markets and fluctuations in uncertainty. *Federal Reserve Bank of Minneapolis Staff Report*.
- BACHMANN, R. and BAYER, C. (2013). ‘wait-and-see’ business cycles? *Journal of Monetary Economics*, **60** (6), 704–719.
- and — (2014). Investment Dispersion and the Business Cycle. *American Economic Review*, **104** (4), 1392–1416.
- BARRO, R. J. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, **121** (3), 823–866.
- BASU, S. and BUNDICK, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, **85** (3), 937–958.
- BERGER, D., DEW-BECKER, I. and GIGLIO, S. (2017). *Uncertainty shocks as second-moment news shocks*. Tech. rep., National Bureau of Economic Research.
- BLOOM, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives*, **28** (2), 153–76.
- , FLOETOTTO, M., JAIMOVICH, N., SAPORTA-EKSTEN, I. and TERRY, S. J. (2018). Really uncertain business cycles. *Econometrica*, **86** (3), 1031–1065.
- DAVIS, S. J. and HALTIWANGER, J. (1992). Gross job creation, gross job destruction, and employment reallocation. *Quarterly Journal of Economics*, **107** (3), 819–863.
- , —, JARMIN, R., MIRANDA, J., FOOTE, C. and NAGYPAL, E. (2006). Volatility and dispersion in business growth rates: Publicly traded versus privately held firms [with comments and discussion]. *NBER macroeconomics annual*, **21**, 107–179.
- DECKER, R., HALTIWANGER, J., JARMIN, R. and MIRANDA, J. (2015). Where has all the skewness gone? the decline in highgrowth (young) firms in the u.s. *Working Paper*.
- FERNANDEZ-VILLAVARDE, J., GUERRON-QUINTANA, P., RUBIO-RAMIREZ, J. F. and URIBE, M. (2011). Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, **101** (6), 2530–61.
- GABAIX, X. (2008). Variable Rare Disasters: A Tractable Theory of Ten Puzzles in Macro-finance. *American Economic Review*, **98** (2), 64–67.
- (2012). Variable rare disasters: An exactly solved framework for ten puzzles in macro-finance. *The Quarterly journal of economics*, **127** (2), 645–700.

- GILCHRIST, S., SIM, J. W. and ZAKRAJŠEK, E. (2014). *Uncertainty, financial frictions, and investment dynamics*. Tech. rep., National Bureau of Economic Research.
- GOURIO, F. (2008). Disasters and recoveries. *American Economic Review*, **98** (2), 68–73.
- (2012). Disaster risk and business cycle. *American Economic Review*, **102** (6), 2734–2766.
- (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, **5** (3), 1–34.
- GUVENEN, F. (2009). A Parsimonious Macroeconomic Model for Asset Pricing. *Econometrica*, **77** (6), 1711–1750.
- , OZKAN, S. and SONG, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy*.
- HALTIWANGER, J., JARMIN, R. S., KULICK, R. B. and MIRANDA, J. (2016). High growth young firms: Contribution to job, output and productivity growth. *US Census Bureau Center for Economic Studies Paper No. CES-WP-16-49*.
- HARTMAN, R. (1972). The effects of price and cost uncertainty on investment. *Journal of economic theory*, **5** (2), 258–266.
- HARVEY, C. R. and SIDDIQUE, A. (2000). Conditional skewness in asset pricing tests. *The Journal of Finance*, **55** (3), 1263–1295.
- ILUT, C., KEHRIG, M. and SCHNEIDER, M. (2018). Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news. *Journal of Political Economy*, **126** (5), 000–000.
- JURADO, K., LUDVIGSON, S. C. and NG, S. (2015). Measuring uncertainty. *American Economic Review*, **105** (3), 1177–1216.
- KAPADIA, N. (2006). The next microsoft? skewness, idiosyncratic volatility, and expected returns.
- KEHRIG, M. (2011). *The Cyclicalities of Productivity Dispersion*. Working Papers 11-15, Center for Economic Studies, U.S. Census Bureau.
- KELLEY, T. L. (1947). *Fundamentals of Statistics*. Harvard University Press.
- KILIC, M. and WACHTER, J. A. (2015). *Risk, Unemployment, and the Stock Market: A Rare-Event-Based Explanation of Labor Market Volatility*. NBER Working Paper 21575.
- KIM, T.-H. and WHITE, H. (2004). On more robust estimation of skewness and kurtosis. *Finance Research Letters*, **1** (1), 56–73.
- KRUSELL, P. and SMITH, A. A. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, **106** (5), 867–896.
- LEDUC, S. and LIU, Z. (2016). Uncertainty shocks are aggregate demand shocks. *Journal of Monetary Economics*, **82**, 20–35.

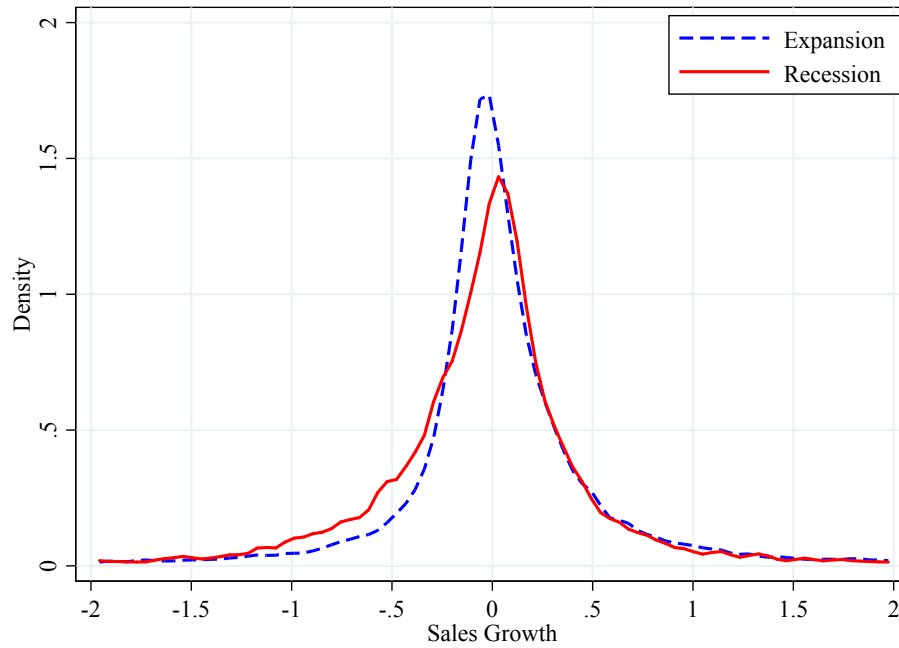
- OH, S. and WACHTER, J. A. (2018). *Cross-sectional Skewness*. Working paper, Mimeo, Wharton School at University of Pennsylvania.
- OI, W. Y. (1961). The desirability of price instability under perfect competition. *Econometrica: journal of the Econometric Society*, pp. 58–64.
- RIETZ, T. A. (1988). The equity risk premium: a solution. *Journal of Monetary Economics*, **22** (1), 117–131.
- SCHMIDT, L. (2016). Climbing and falling off the ladder: Asset pricing implications of labor market event risk. *Available at SSRN 2471342*.
- WACHTER, J. A. (2013). Can time-varying risk of rare disasters explain aggregate stock market volatility? *The Journal of Finance*, **68** (3), 987–1035.

FIGURE 1 – THE SKEWNESS OF FIRM OUTCOMES IS LOWER DURING RECESSIONS

(A) Census LBD: Employment Growth Distribution



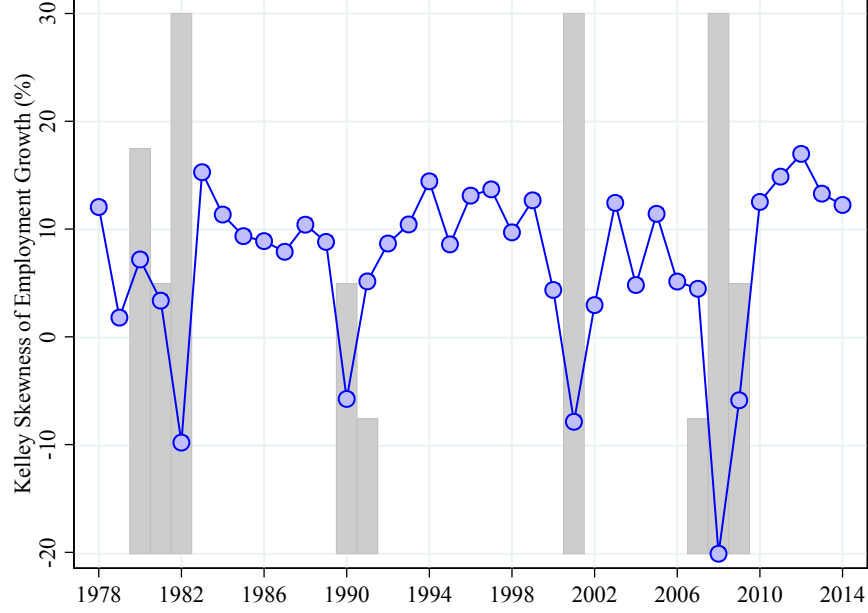
(B) Compustat: Sales Growth Distribution



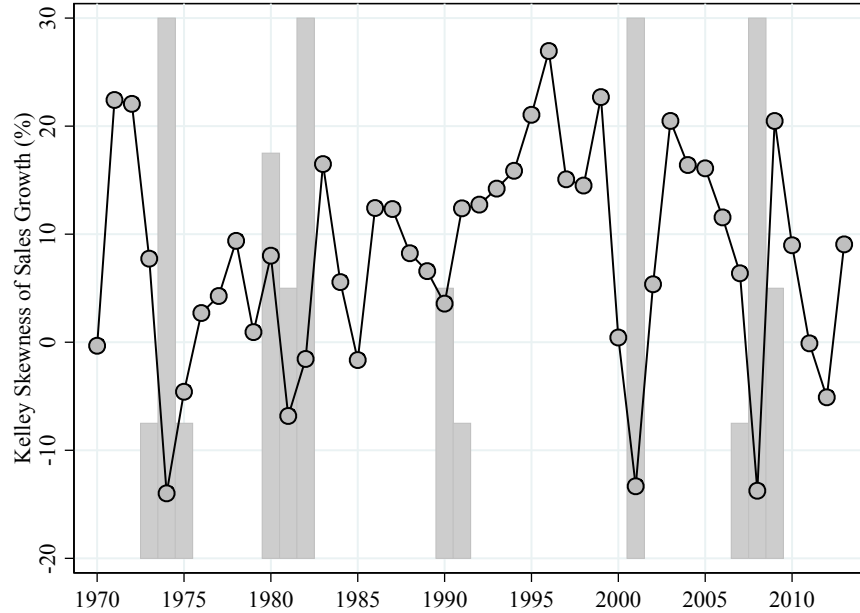
Note: The top panel of Figure 1 shows the employment-weighted empirical density of the distribution of employment growth for a sample of firms from LBD. The lower panel shows the empirical density of the distribution of sales growth from a sample of publicly traded firms from Compustat. Each density has been rescaled to have a median of zero and unitary variance. The blue-dashed line shows the density of a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) whereas the red-solid line shows the density of a pooled sample of recession years (2001 and 2008). In the top panel, the unscaled 10th percentile of the employment growth distribution during expansion (recession) periods is -16.5% (-26.9%), the 50th is 1.3% (-1.8%), and the 90th is 23.3% (18.0%). In the bottom panel, the corresponding moments are -21.7% (-47.4%), 5.3% (-3.0%), and 44.6% (33.0%). See Appendix A for additional details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 2 – THE SKEWNESS OF FIRM OUTCOMES IS STRONGLY PROCYCLICAL

(A) Census LBD: Skewness of Employment Growth Distribution



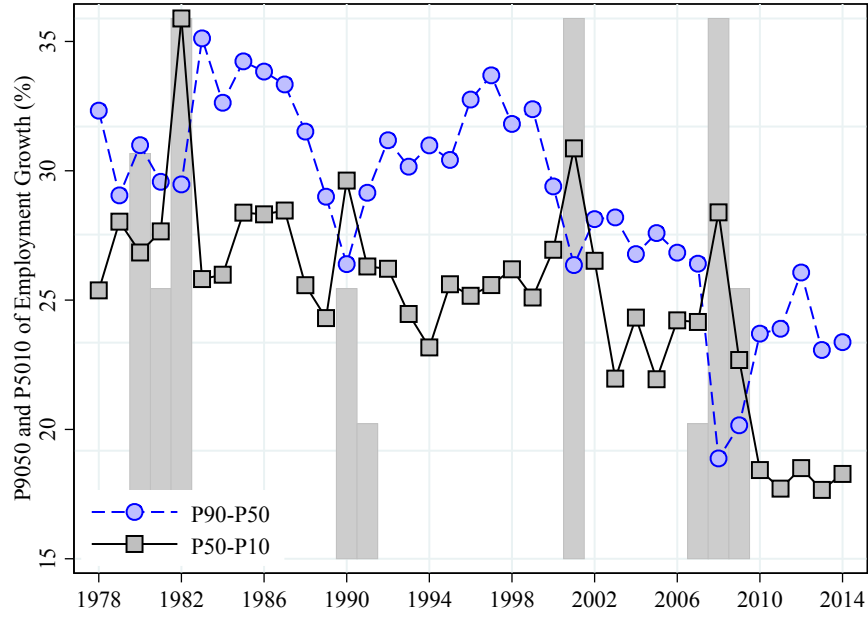
(B) Compustat: Skewness of Sales Growth Distribution



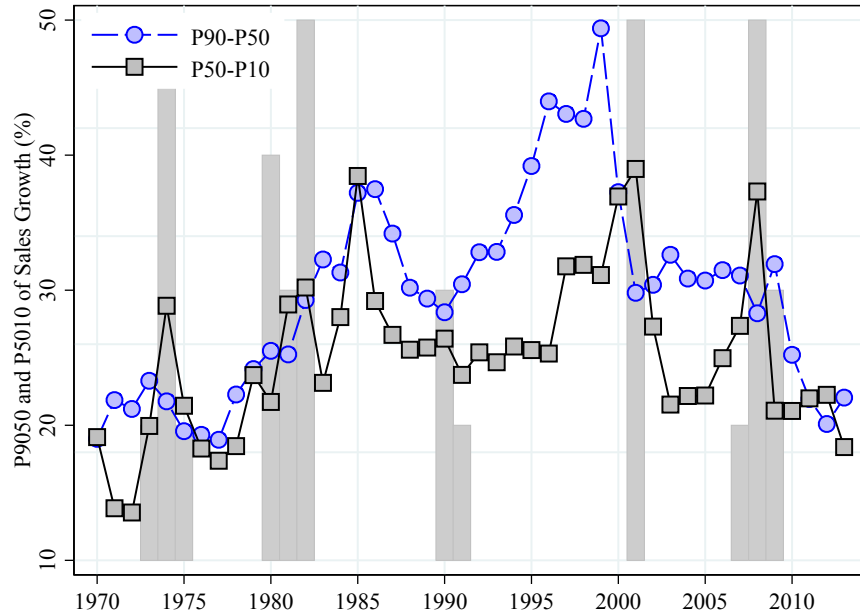
Note: The top panel of Figure 2 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment between years t and $t + 1$. The bottom panel shows the time-series of the cross-sectional Kelley skewness of the distribution of firm sales growth for a sample of publicly traded firms from Compustat. Compustat data shows a large decline in skewness in 2014 which is not found in the rest of the datasets. We are currently investigating the source of this anomaly. The shaded bars represent NBER recession periods. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 3 – DISPERSION OF LEFT TAIL OF FIRMS OUTCOMES IS COUNTERCYCLICAL

(A) Census LBD: Dispersion of Employment Growth



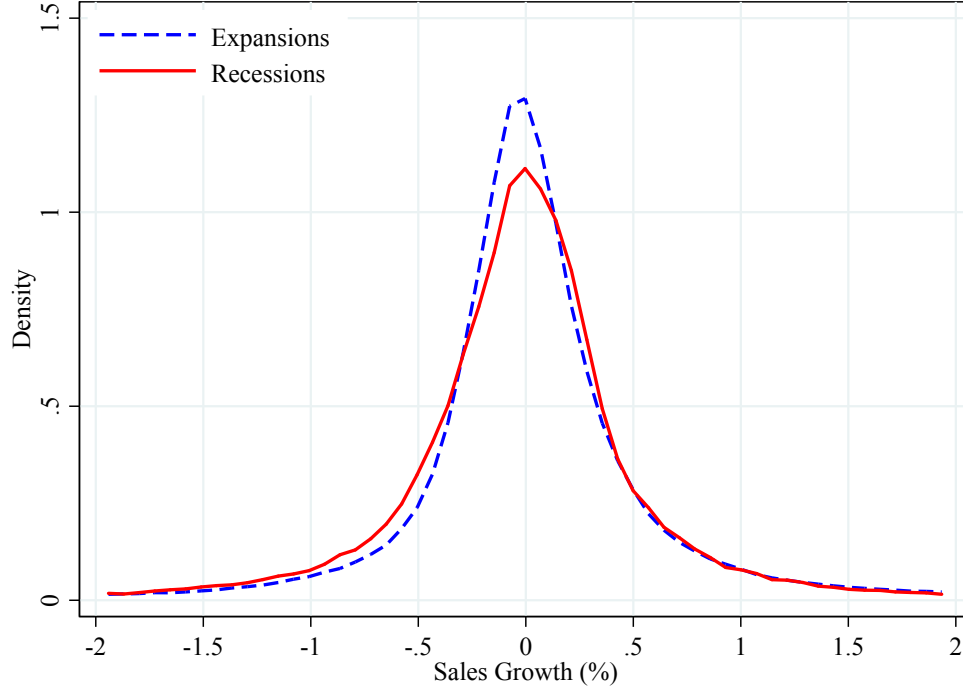
(B) Compustat: Dispersion of Sales Growth



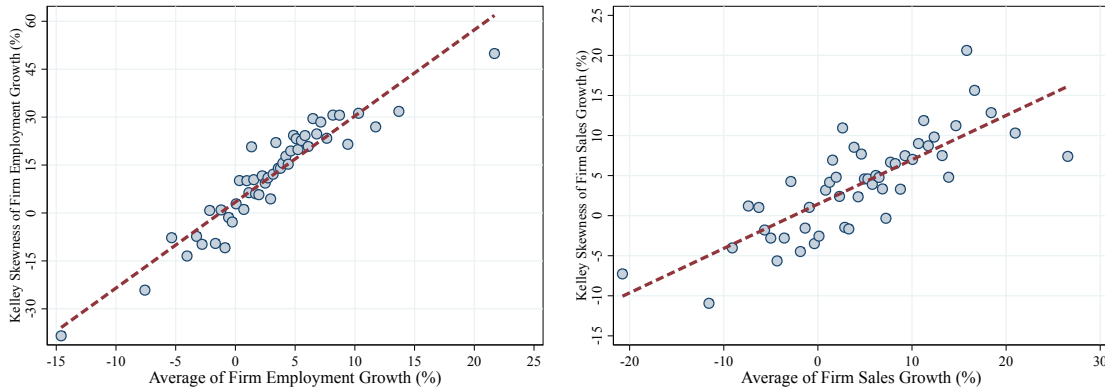
Note: The top panel of Figure 3 shows the time-series of the cross-sectional dispersion of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment between years t and $t + 1$. The bottom panel shows the time-series of the cross-sectional dispersion of the distribution of firm sales growth for a sample of publicly traded firms from Compustat. Compustat data shows a large jump in dispersion in 2014 which not found in the rest of the datasets. We are currently investigating the source of this anomaly. The shaded bars represent NBER recession periods. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 4 – THE SKEWNESS OF FIRM OUTCOMES IS CYCLICAL: CROSS COUNTRY EVIDENCE

(A) Cross-Country: Sales Growth Distribution



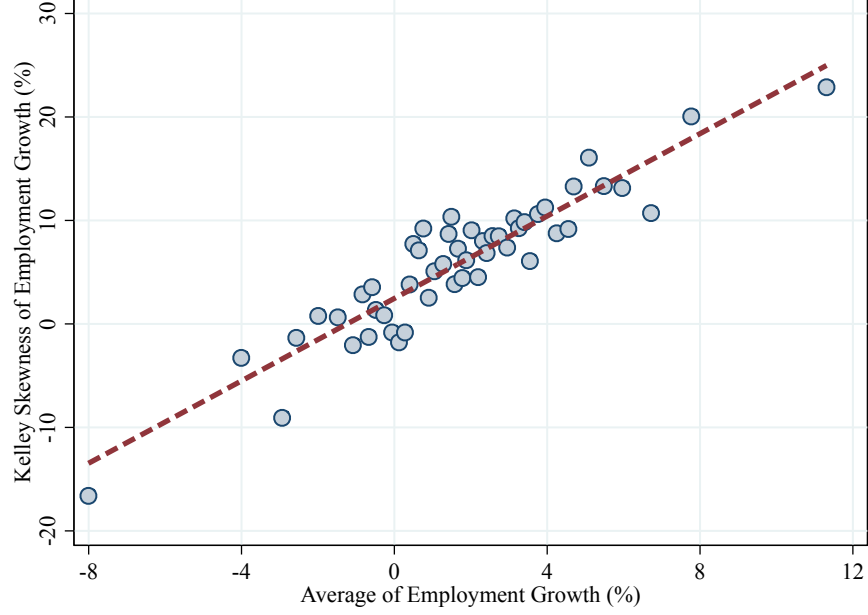
(B) Cross-Country: Firm-Level Employment and Sales Growth



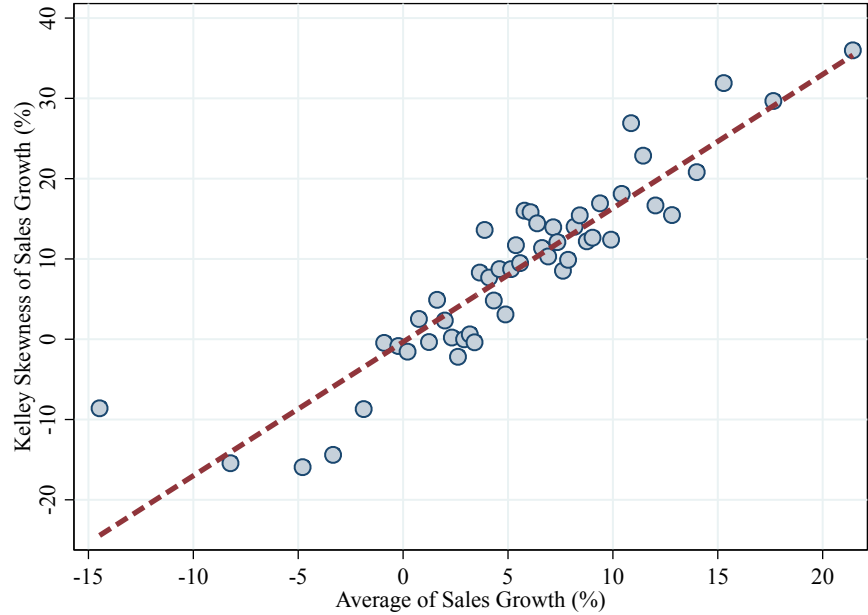
Note: The top panel of Figure 4 shows the empirical density of the growth rate of annual sales in US dollars for a sample of publicly traded firms from BvD Osiris dataset. Each density has been rescaled to have a median of zero and unitary variance. The red solid line is the empirical density over all the observations of firms during recession years, defined as years in which the country is in the first decile of the country-specific distribution of the growth rate of GDP per capita (74,009 observations). The blue dashed line is the empirical density over all the observations of firms during expansion periods (523,655 observations) which are years not classified as recessions. The unscaled 10th percentile of the sales growth distribution during expansion (recession) periods is -30.5% (-42.4%), the 50th percentile is 5.6% (0.0%), and the 90th percentile is 52.5% (43.6%). The bottom left (right) panel displays a binscatter plot showing the relation between the within-country average firm employment (sales) growth and the within-country Kelley skewness of firm employment (sales) growth for a sample of publicly traded firms from BvD Osiris dataset. The regression slope is equal to 1.64 (0.50). Binscatter plots control for time and country fixed effects. See Appendix A for details on the sample construction and moment calculations.

FIGURE 5 – THE SKEWNESS FIRM OUTCOMES IS CYCLICAL AT THE INDUSTRY LEVEL

(A) Census LBD: Industry Employment Growth Distribution

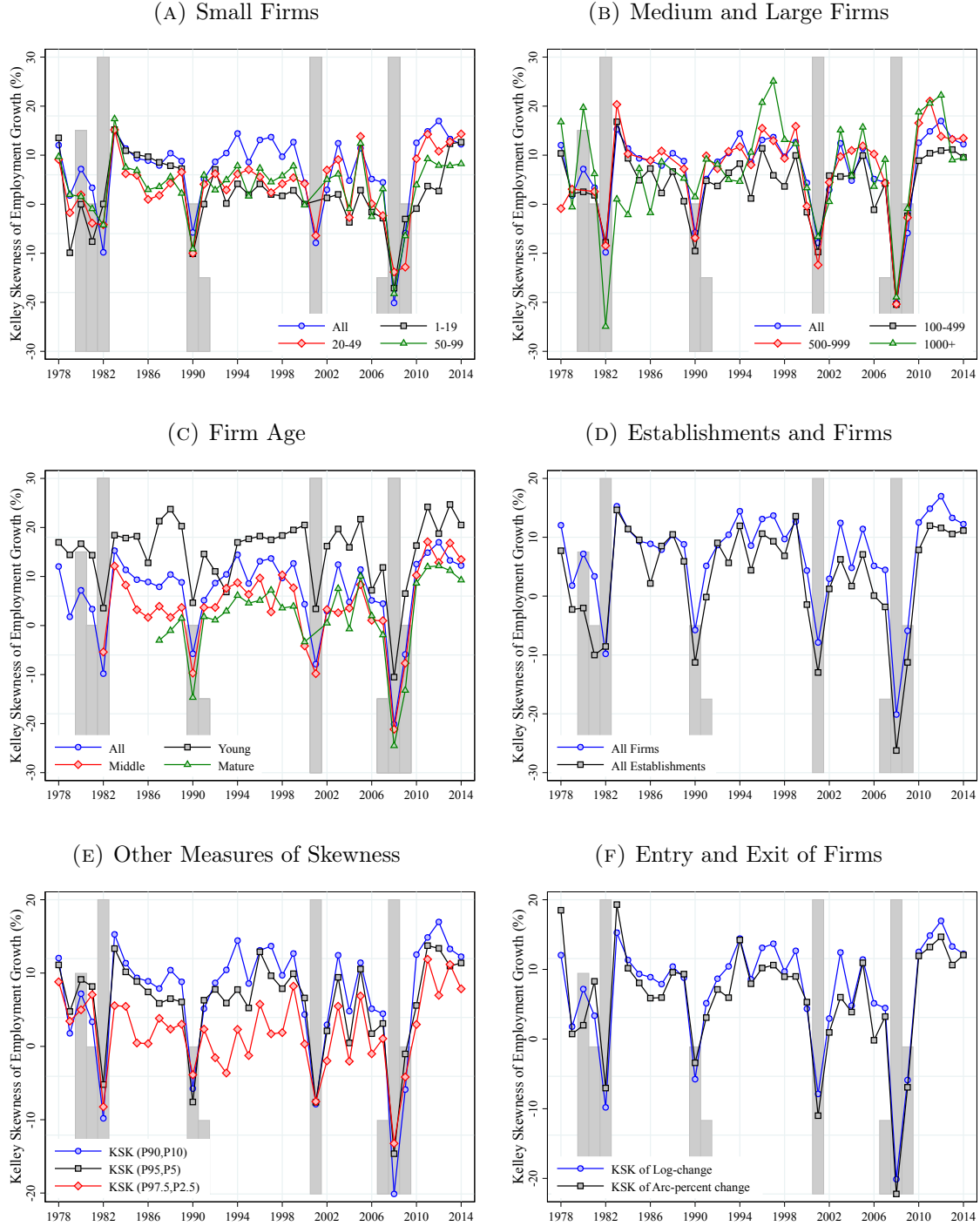


(B) Compustat: Industry Sales Growth Distribution



Note: The top panel of Figure 5 displays a binscattered plot showing the relation between the within-industry business cycle, measured by the average growth rate of employment, and the within-industry skewness, measured by the Kelley skewness of firm employment growth for a sample of firms from LBD. Each dot is a quantile of the industry-year distribution of average employment growth where an industry is defined by a 2-digits NAICS group. Moments are weighted by the average firm employment. Binscatter plots control for industry and time fixed effect. The slope coefficient is equal to 1.99 and is statistically significant at the 1%. The bottom panel shows the same statistics for sales growth distribution for a sample of publicly traded firms from Compustat. The slope coefficient is 1.33. See Appendix A for details on the sample construction and moment calculations in the LBD and Compustat.

FIGURE 6 – ROBUSTNESS USING CENSUS DATA



Note: Figure 6 is based on a sample of firms from LBD. The top panels show the Kelley skewness of the distribution of firm employment growth within different firm size groups. The center-left panel shows the skewness of the distribution of firm-employment growth within different firm age groups. Young firms are those of less than five years, Middle-aged firms are those between six and ten years old, and Mature firms are those of more than ten years old. Firms already in the sample in 1976 are not considered in any of these groups. Shaded bars represent the share of the year (in quarters) declared as recession years by the NBER. All moments weighted by average employment at the firm or establishment level. See Appendix A for details on the sample construction and moment calculations in the LBD.

TABLE I – DATA AND SAMPLE CHARACTERISTICS

Source	Country	Sample Period	Frequency	Comments
Census	United States	1978-2015	Annual	Employment data for entire nonfarm private sector
Compustat	United States	1970-2017	Quarterly	Employment, Sales, and Stock Prices for publicly traded firms
BvD Osiris	Several countries	1986-2015	Annual	Employment and Sales for publicly traded firms across 44 countries
Global Compustat	Several Countries	1970-2017	Daily	Stock Prices for publicly traded firms across 29 countries

TABLE II – THE SKEWNESS OF FIRMS OUTCOMES IS LOWER DURING RECESSIONS

Dependent Variable:		Kelley Skewness of the Growth Rate of Firms' Outcomes								
Sample:	Outcome:	United States			Cross-Country			Cross-Industry		
		(1) Emp.	(2) Sales	(3) Stock Price	(4) Emp.	(5) Sales	(6) Stock Price	(7) Emp.	(8) Sales	(9) Stock Price
$\Delta GDP_{i,t}$		4.64*** (1.45)	5.37*** (1.07)	2.09** (1.03)	5.39*** (1.47)	3.19*** (1.05)	2.11** (0.88)			
$\Delta S_{j,t}$								6.62*** (1.18)	13.24*** (1.39)	1.35** (0.51)
R^2		0.32	0.23	0.07	0.27	0.38	0.41	0.28	0.40	0.24
N		39	47	184	701	720	2,428	1,045	1,046	4,133
Period		1976-2014	1970-2017	1970-2016	1991-2015	1991-2015	1970-2017	1970-2017	1970-2017	1970-2016
Freq.		Yr	Yr	Qtr	Yr	Yr	Qtr	Yr	Yr	Qtr
F.E.		-	-	-	Yr/Ctry	Yr/Ctry	Qtr/Ctry	Yr/Ind	Yr/Ind	Qtr/Ind
Source		LBD	CSTAT	CSTAT	BvD	BvD	GCSTAT	CSTAT	CSTAT	CSTAT
Sample		-	231K	650K	357K	633K	5,800K	231K	231K	733K

Note: The left panel of Table II shows a set of time-series regressions for the United States in which the dependent variable is the Kelley Skewness of the distribution of one-year firm employment growth for a sample of firms from LBD (column 1), one-year sales growth distribution (column 2), and one-year stock returns (column 3), for a sample of firms from Compustat (CSTAT). In each regression, the independent variable is the annual growth rate of GDP per capita. LBD moments are weighted by firm size measured by the average employment of the firm between years t and $t + 1$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. Columns (4) to (6) show a set of country-panel regressions in which the dependent variable is the Kelley skewness of firm-level outcomes. Employment and sales data come from Bureau Van Dijk's Osiris (BvD) and stock returns are from Global Compustat (GCSTAT). In each regression, the independent variable is the growth rate of annual GDP per capita. Regressions include year and country fixed effects. Columns (7) to (9) show a series of industry-panel regression in which the dependent variable is the Kelley skewness of the within-industry distribution of firm's outcomes. In each regression, the independent variable is the average sales growth within the industry. The raw labeled Sample shows the underlying sample of firms used to calculate the cross-sectional moments. Sample size in LBD not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

TABLE III – CALIBRATED PARAMETERS

Preferences and Technology		
γ	0.45	Frisch elasticity of labor supply
ψ	2.5	Leisure preference, non entrepreneurs spend 1/3 time working
σ	2.0	Risk aversion, non entrepreneurial sector
$1/\lambda$	1/5	Elasticity of inter-temporal Substitution
ξ	6.0	Risk aversion
β	0.95 ^{0.25}	Annual discount factor of 95%
r	0.005	Annual return of risk-free asset of 2%
α	0.25	CRS production, markup of 33%
ν	0.50	CRS labor share of 2/3, capital share of 1/3
δ	3.8%	Annual depreciation of capital stock fo 14.4%
ρ_a	0.95	Quarterly persistent of aggregate productivity
σ_a	0.75%	Standard deviation of Innovation of aggregate productivity
ρ	0.95	Quarterly persistence of idiosyncratic productivity
Adjustment costs		
ϕ_1	1.50%	Fixed cost of changing capital stock
ϕ_2	7.0	Quadratic cost of changing capital stock
ϕ_3	34.0%	Resale loss of capital

Note: Table III shows the calibrated parameters referring to preferences, technology, and adjustment costs.

TABLE IV – RISK PROCESS MOMENTS

	$P90 - P10$	$P90 - P50$	$P50 - P10$	KSK	Yrs
Data					
Low Risk	0.54	0.30	0.24	0.10	03-06;10-14
High Risk	0.70	0.31	0.39	-0.11	01,08
$\Delta(H - L)$	0.16	0.01	0.15	-0.20	-
Model					
Low Risk	0.48	0.27	0.20	0.15	-
High Risk	0.58	0.26	0.32	-0.10	-
$\Delta(H - L)$	0.10	-0.01	0.12	-0.25	-

Note: The top panel of Table IV shows cross-sectional moments of the annual growth rate of quarterly sales from Compustat for low risk periods—quarters in the years 2003 to 2006 and quarters in the years 2010 to 2014—and high risk periods—quarters in years 2001 and 2008. Quarters in years 2002 and 2009 are discarded for not representing full recession years. The model moments, shown in the lower panel of Table IV, are calculated from a 5,000-quarters simulation with the first 500 periods discarded.

TABLE V – TARGETED MOMENTS FOR NUMERICAL COMPARISON

	$P9010$	$P9050$	$P5010$	KSK
Low Risk	0.54	0.30	0.24	0.10
High Risk	0.70	0.31	0.39	-0.10
Only Skewness	0.54	0.243	0.297	-0.10
Only Variance	0.70	0.39	0.31	0.10

Note: Table V shows the target used in the estimation of the firm-level productivity process. Rows labeled “Low Risk” and “High Risk” are used in the baseline estimation. The values for “Only Skewness” are used to estimate the parameters when the economy is shocked with a change in the skewness only. Similarly, the values for “Only Variance” are used to estimate the parameters when the economy is assumed to be shocked only by a change in the variance of firms’ shocks while keeping the skewness constant.

TABLE VI – PARAMETERS OF THE STOCHASTIC PROCESS

Parameter of Idiosyncratic Stochastic Process		
σ_1^L	1.45	Standard deviation of first mixture in low risk periods (%)
σ_2^L	7.55	Standard deviation of second mixture in low risk periods (%)
μ^L	-0.92	Mean of first mixture in low risk periods (%)
p^L	63.67	Probability of first mixture in low risk periods (%)
σ_1^H	4.37	Standard deviation of first mixture in high risk periods (%)
σ_2^H	9.06	Standard deviation of second mixture in high risk periods (%)
μ^H	1.98	Mean of first mixture in high risk periods (%)
p^H	78.28	Probability of first mixture in high risk periods (%)
Transition Probabilities of Risk States		
π_L	0.97	Quarterly probability of remaining in low risk state
π_H	0.84	Quarterly probability of remaining in high risk state

Note: The top panel of Table VI shows the parameters of the stochastic process of firm-level productivity. We target moments of the annual change of quarterly sales in Compustat. The parameters for low-risk periods (denoted by an upper script L) are obtained by targeting the P90-P10, P90-P50, P50-P10, and Kelley Skewness of the sales growth distribution for the all the full expansion years between 2000 and 2014. The parameters for high-risk periods (denoted by an upper script H) are obtained by targeting the same set of moments for years 2001 and 2008 (full recession years). The transition probability π_L is calculated as the share of expansion quarters that were followed by another expansion quarter whereas π_H is calculated as the share of recession quarters that were followed by another recession quarter using data from 1970 to 2014.

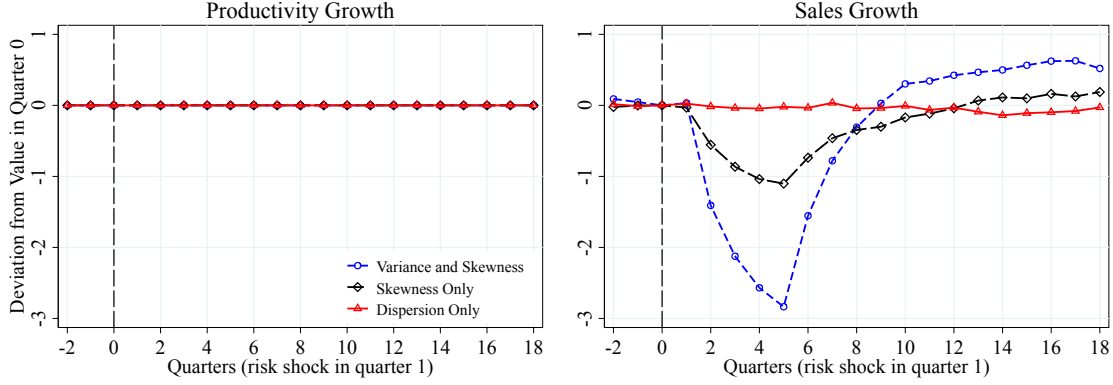
TABLE VII – BUSINESS CYCLE STATISTICS

	Data			Model		
	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x, y)$	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x, y)$
Output	1.47	1.00	1.00	2.00	1.00	1.00
Capital Investment	6.86	4.64	0.91	9.38	4.69	0.30
Consumption	1.21	0.82	0.87	1.81	0.91	0.65
Hours	1.89	1.28	0.87	2.00	1.00	1.00

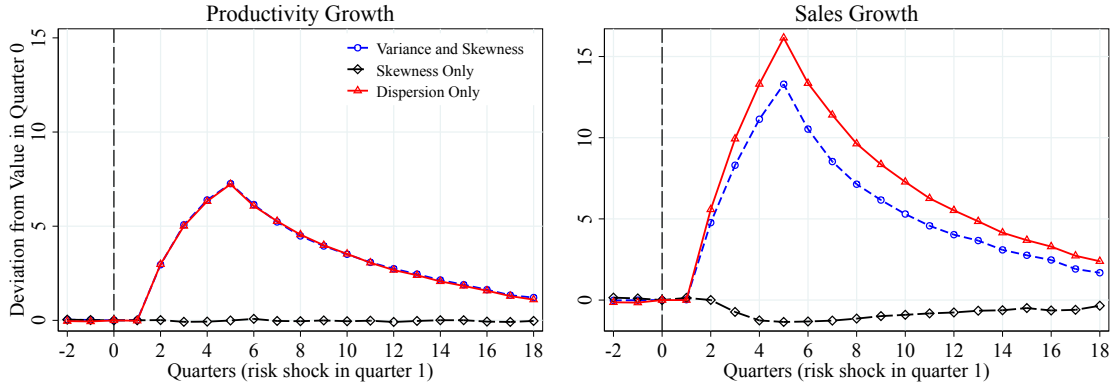
Note: The left panel of Table VII displays busyness cycles statistics for quarterly US data covering 1970Q1 to 2017Q4. The column $\sigma(x)$ is the standard deviation of the log variable in the first column. The column $\sigma(y)/\sigma(x)$ is the standard deviation of the variable relative to the standard deviation of log output. All business cycle data are current as of February 03, 2019. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (FRED GPDIC1), consumption is real personal consumption expenditures (FRED PCECC96), and hours is total non-farm business sector hours (FRED HOANBS). The second panel contains business cycle statistics computed from a simulation of the model of 5000-quarter with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1,600, in logs expressed as percentages.

FIGURE 7 – PRODUCTIVITY AND SALES GROWTH AFTER AN INCREASE IN RISK

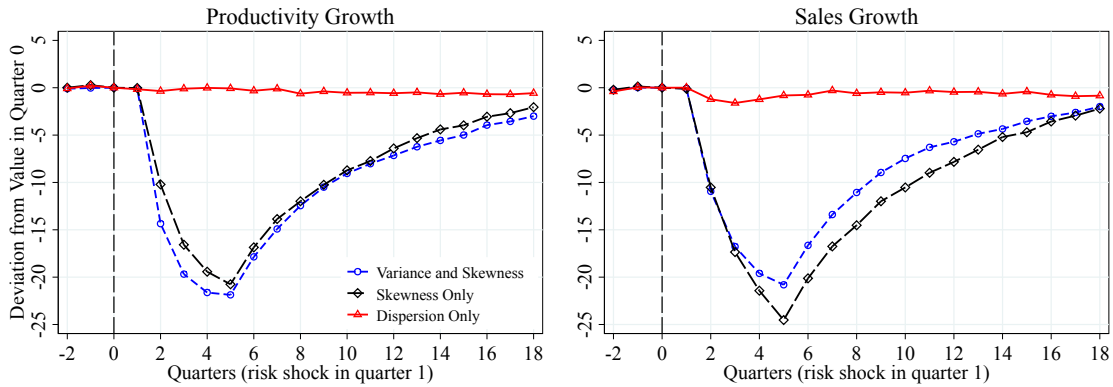
(A) Average



(B) P90-P10

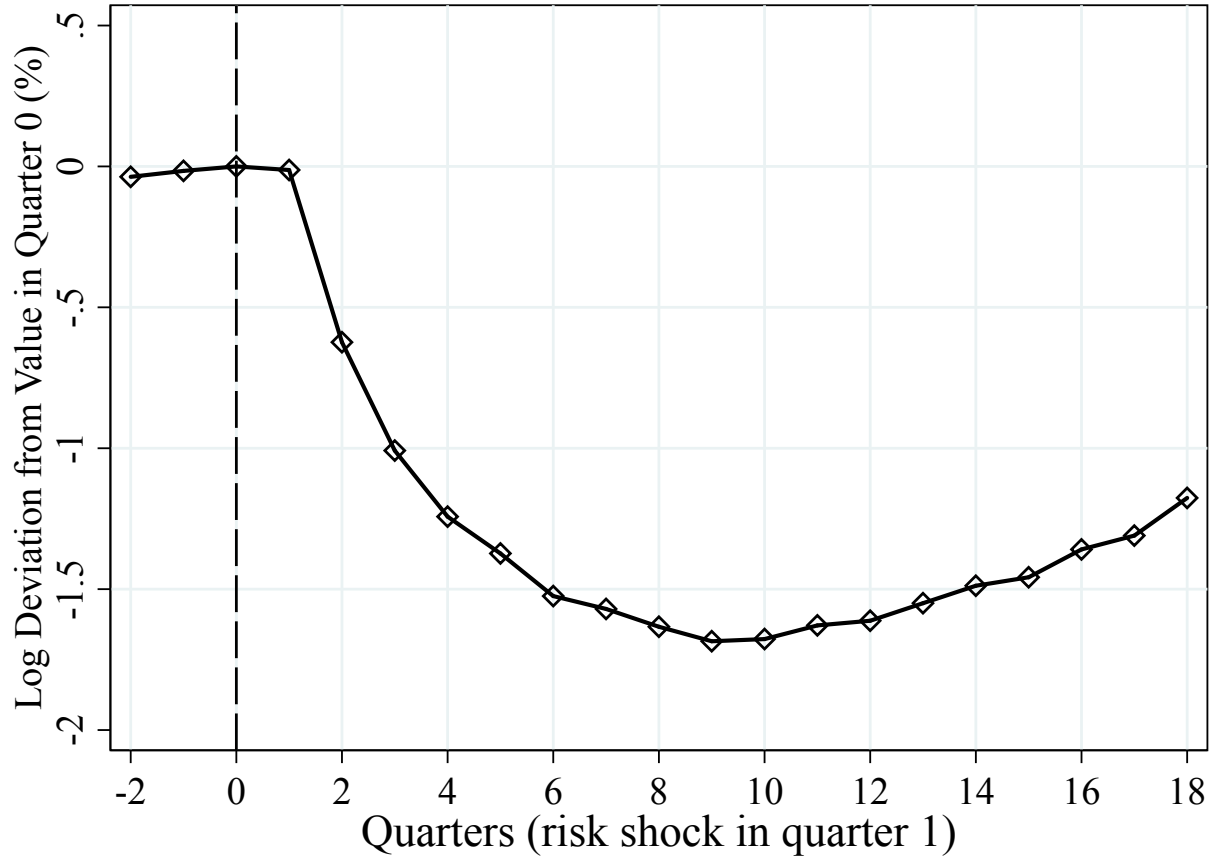


(c) Kelley Skewness



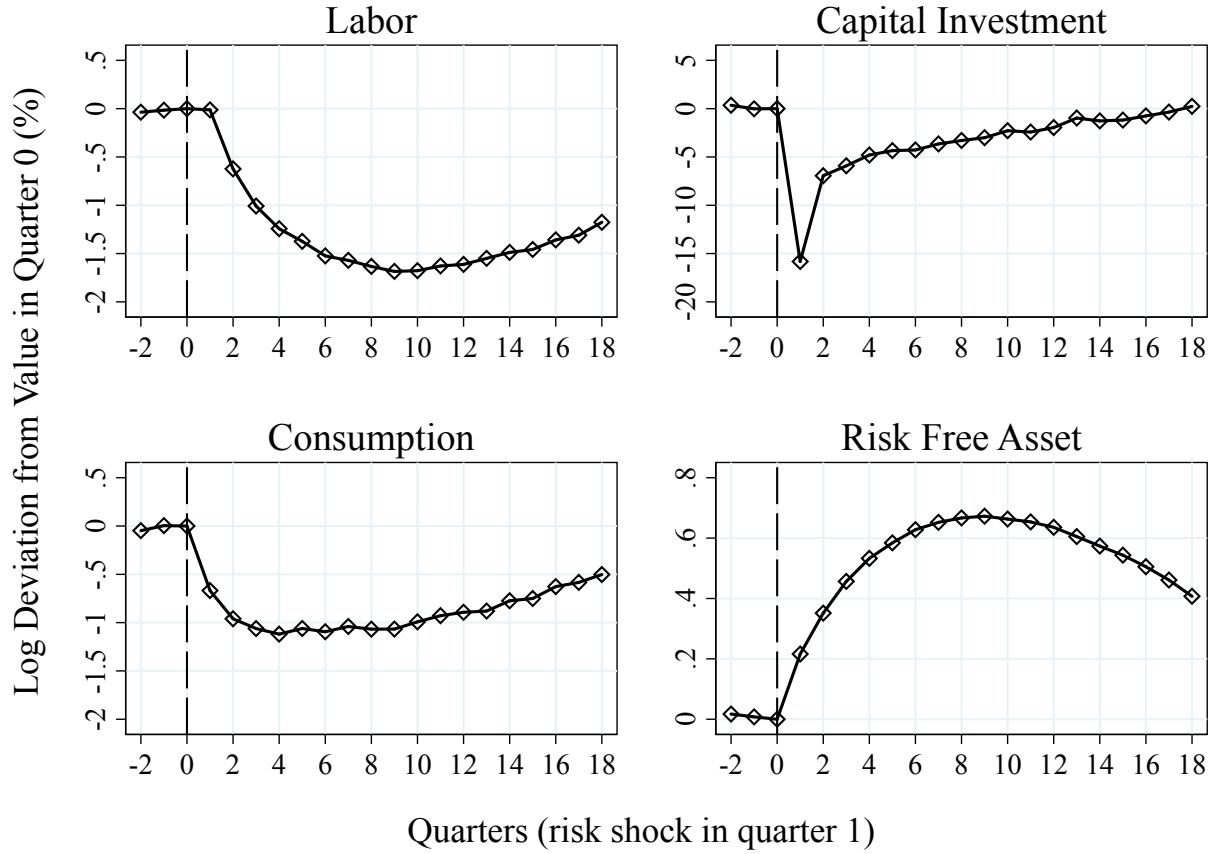
Note: The top-left panel of Figure 7 shows the average of the one-year productivity growth distribution ($\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$) whereas the top-right shows the average of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$) for different risk shocks. The middle and bottom panels show the dispersion and skewness. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation relative to the moment value in quarter 0.

FIGURE 8 – EFFECT OF SKEWNESS SHOCK IN OUTPUT



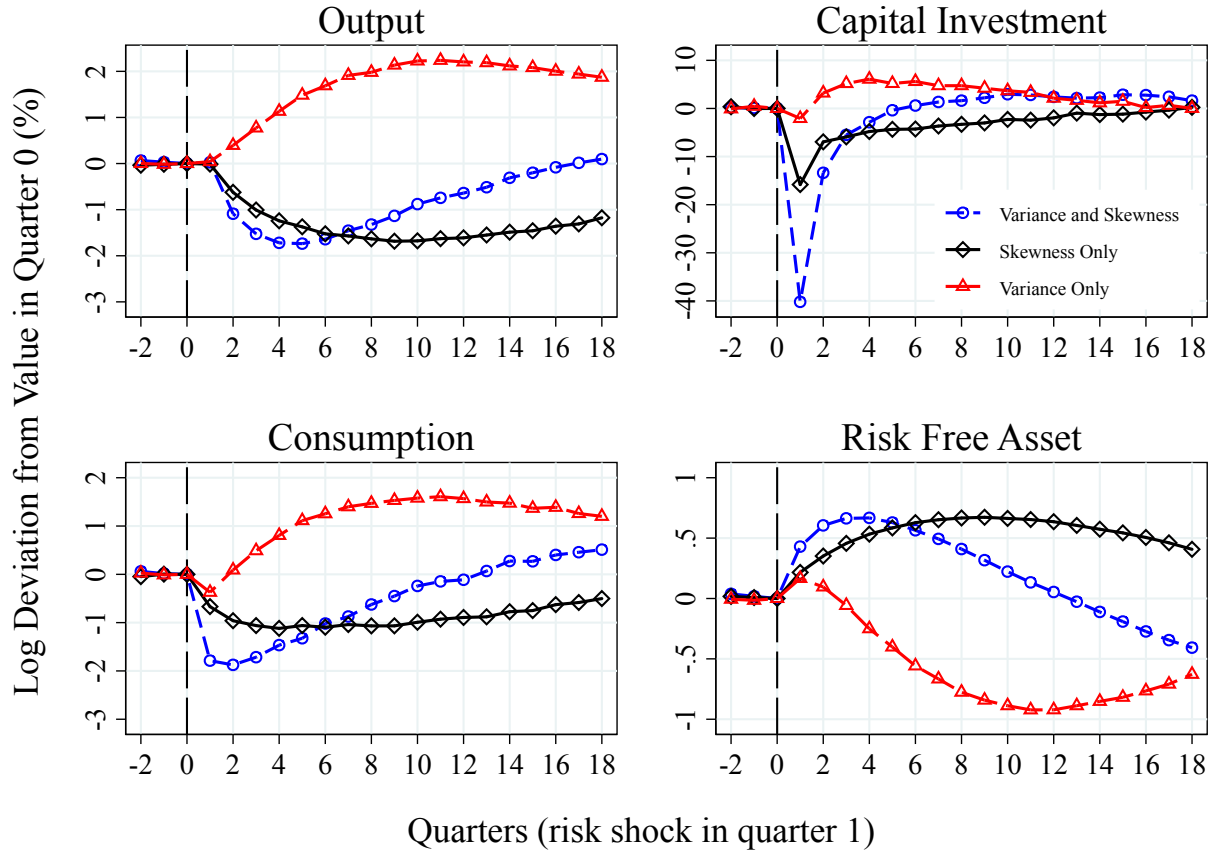
Note: Figure 8 shows the effect of a decline in the skewness of firm idiosyncratic productivity. The plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of Output from its value in quarter 0.

FIGURE 9 – EFFECT OF SKEWNESS SHOCK ON MACRO AGGREGATES



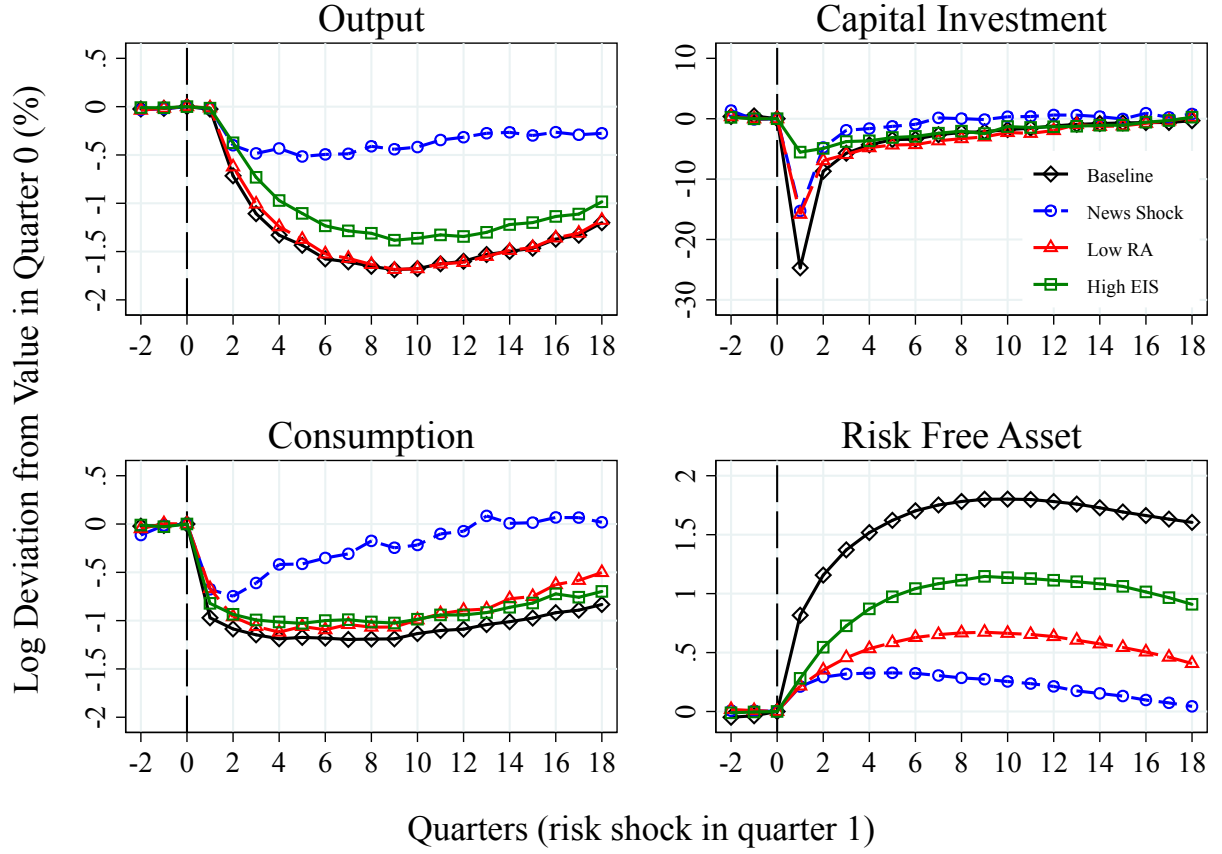
Note: Figure 9 shows the effect of a decline in the skewness of firm idiosyncratic productivity. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0.

FIGURE 10 – EFFECT OF SKEWNESS AND VARIANCE SHOCK ON MACRO AGGREGATES



Note: Figure 10 shows the effect of a decline in skewness of idiosyncratic shocks (black line with diamonds), an increase in variance of idiosyncratic shocks (red line with squares), and a decrease in skewness paired with an increase in the variance of idiosyncratic shocks (blue line with circles) for different macroeconomic outcomes. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness, increase in variance, or both, in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0. Labor is omitted since it follows the same pattern of Output.

FIGURE 11 – EFFECT OF SKEWNESS SHOCKS UNDER DIFFERENT PARAMETERIZATION



Note: Figure 11 shows the effect of a skewness shock (black line with diamonds) and the effect of a shock that only represents news about the future conditions of skewness without a change in the realizations of the idiosyncratic shocks. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0. Labor is omitted since it follows the same pattern of Output.

A Appendix: Data Sources and Variable Construction

This appendix describes the data sources and sample selection. Firm-level data for the United States comes from the Census Bureau’s Longitudinal Business Statistics (LBD) and Compustat. For the cross-country comparisons, we use firm-level data available in the Bureau van Dijk’s Osiris database and Global Compustat. The online appendix and replication packet—available on the author’s websites—contains further details of the construction of the sample and moments calculation.

A.1 United States: Longitudinal Business Database

We construct measures of employment growth at the firm-level using the Census Bureau’s Longitudinal Business Database (LBD). The LBD covers the universe of establishment in the nonfarm private sector in the United States from 1976 to 2015. It provides detailed establishment and firm-level information on employment, payroll, location, firm age, industry, legal form of organization, etc.. Crucially, firm and establishment identifiers in the LBD allow us to construct measures of employment growth at different time horizons. From the LBD, we select a sample of establishments that, in a given year, have nonnegative, non-missing employment and payroll and have valid industry data. We then sum up the employment within the same firm to construct an annual measure of employment. We measure the growth rate of employment of firm j in period t as the log-difference between periods t and $t + k$, $g_{j,t}^e = \log E_{j,t+k} - \log E_{j,t}$ where $k \in \{1, 3, 5\}$ and by the arc-percent change between the same periods.

Calculating the Kelley skewness requires the computation of specific different percentiles of the distribution sales growth distribution. Notice that a percentile provides information of a particular firm, which violates the disclosure criteria imposed by the Census Bureau. Hence, to avoid the disclosure of any sensitive information, we calculate the p th percentile of the employment growth distribution as the employment-weighted average on a band of $+1$ and -1 percent centered around p th. For instance, the 90th percentile of the distribution is the weighted average of the employment growth across all observations between the 89th and 91st percentiles of the distribution, both ends included. We proceed in the same way to construct the 10th and 50th percentiles of the distribution and use these values to calculate the Kelley skewness. All measures are weighted by the average employment of the firm between periods t and $t + k$, that is $\bar{E}_{j,t} = 0.5 \times (E_{j,t+k} + E_{j,t})$. The massive sample size of the LBD ensures that the sample used to calculate each of the percentiles is large enough to have an accurate approximation to the actual quantiles of the distribution.

We also use the LBD to compare the distribution of employment growth between recessions and expansions years using kernel density estimation. The sample selection is the same used in the rest of our results, however, the Census Bureau requires to drop the bottom and top 5% of the distribution. The kernel densities presented in Figure 3 were calculated over the remaining sample.

A.2 United States: Compustat

For the United States, we construct time-series of the cross-sectional dispersion and skewness of the sales growth distribution and the distribution of stock returns. To construct the time-series of the sales growth distribution we proceed as follows. We begin by retrieving firm-level

data of net sales, and other variables at a quarterly frequency, and employment at an annual frequency, from Compustat from 1964q1 to 2017q4 available at WRDS database.

The raw dataset of sales (Compustat variable `saleq`) and stock prices (Compustat variable `prccq`) contains more than 1.7 million quarter-firm observations with an average of approximately 4,660 firms per quarter. We drop all observations with negative sales, repeated observations, and incorporated outside the United States (we keep observation with Compustat variable `fic` equal to “USA”). We also drop all observations that do not have a SIC classification or with a classification above 90. Then, we deflate nominal sales by the CPI (FRED series `CPIAUCSL`), and we calculate the growth rate of sales as the log difference and the arc percentage change between quarter t and $t + k$ with $k \in \{4, 12, 20\}$. This leaves us with around 1 million sales growth (log difference) observations. For our main results, we consider firms with at least 10 years of data on quarterly sales (40 quarters, not necessarily continuous), which further reduces the sample to 819,977 observations between 1970q4 and 2017q2, with an average of 5,359 firms per quarter. Finally, in each quarter we calculate different cross-sectional moments discussed in the main body of this document. Our main sample considers firms with at 10 years of data (40 quarter) although our results remain robust if we drop this restriction or if we consider firms with 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before entering and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

For our result at the annual frequency, we follow the same sample selection. The raw annual dataset contains 500,004 year/firm observations. We drop all observations with negative sales and duplicated entries, with missing SIC classification or two digit SIC above 90. We deflate nominal variables using CPI (FRED series `CPIAUCSL`) and we calculate the growth rate of sales (Compustat variable `sale`) and employment (Compustat variable `emp`) as the log change between year t and $t + k$ with $k \in \{1, 3, 5\}$. This leaves us with 266,192 firm/year observation (sales growth) between 1970 and 2016, with an average of 5,663 firms per year. Our main sample consider only firms with at the least 10 years data (not necessarily continuous) but our results remain robust if we drop this restriction or if we consider firms with at least 25 years of data. When accounting for entry and exit of firms using the arc-percentage change, for each firm we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before and after exit, the firm has a value of sales or employment equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other). We complement this data with macroeconomic series from FRED (real gross domestic product per capita, FRED series `A939RX0Q048SBEA`).

A.3 Cross-Country: BvD Osiris and Global Compustat

Cross country firm-level panel data on sales and employment come from the Bureau van Dijk’s Osiris database. Osiris is a database of listed public companies, commodity producing firms, banks, and insurance companies from over 190 countries. The combined industrial company dataset contains financial information for up to 20 years and 80,000 companies. In our analysis, we focus on the industrial dataset.

The raw dataset contains 977,412 country/firm/year observations from 1982 to 2018. We then drop all observations with missing or negative sales, we clean all duplicated entries, and firms with missing NAIC classification. We transform all observations into US dollars using the exchange rate reported in the same database. Then, we deflate nominal sales using US annual CPI and calculate the growth rate of real sales as the log change and arc percentage change between years t and $t + k$ with $k \in \{1, 3\}$. This leaves us with 748,574 observations (log change of sales). We further restrict the sample to firms with more than 10 years of data; country/year cells with more than 100 observations; countries with more than 10 years of data; and years with more than 5 countries. This sample selection reduces the dataset to an unbalanced panel of 678,563 observations in 45 countries between 1989 and 2015. We complement this data with real GDP in US dollars from the World Bank’s World Development Indicators database.

The data on daily stock prices come from the Global Compustat database, which provides standardized information on publicly traded firms for several countries at annual, quarterly, and daily frequencies. The raw data contain firm-level observations of daily stock prices between 1985 and 2018 for 48 countries. We drop all duplicated observations and drop all firms with less than 2000 observations (firms with approximately 10 years of data). Then we calculate daily price returns as the log difference of the stock price between two consecutive trading days. We apply a similar sample selection, keeping firms with at least 10 years of daily price data. The total sample contains an unbalanced panel of 44 countries from 1985 to 2017 from which we drop all country quarter with less than 100 firms. The final data contains a total of 29 countries from 1985 to 2017. Then, within each quarter, we calculate the cross-sectional moments of the daily stock price distribution. We complement this dataset with per capita GDP growth from World Bank’s World Developing Indicators and quarterly GDP growth from the OECD Stats. Table A.1 shows the list of countries available in our dataset and the data available for each country.

TABLE A.1 – DATA AVAILABILITY BY COUNTRY

Source:	Osiris		Global Compustat		Amadeus		Osiris		Global Compustat		Amadeus	
	Sales	Emp	Stock		Sales	Emp	Sales	Emp	Stock		Sales	Emp
ARG	x	x			IRN	x	x					
AUS	x	x	x		ISL						x	x
AUT					ISR	x	x		x			
BEL	x	x	x		ITA	x	x		x		x	x
BLR					JPN	x	x		x			
BMU	x	x			KOR	x	x		x			
BRA	x	x	x		MEX	x	x					
CAN	x	x			MYS	x	x					
CHE	x	x	x		NLD	x	x		x		x	x
CHL	x	x	x		NOR	x	x		x		x	x
CHN	x	x			NZL				x			
DEU	x	x	x		PAK	x	x					
DNK	x	x	x		PER	x	x					
EGY	x	x			PHL	x	x					
ESP	x	x	x		POL				x		x	x
FIN	x	x	x		PRT						x	x
FRA	x	x	x		RUS	x	x		x			
GBR	x	x	x		SGP	x	x					
GRC	x	x	x		SWE	x	x		x		x	x
HKG	x	x			THA	x	x					
HUN					TUR	x	x		x			
IDN	x	x	x		UKR						x	x
IND	x	x	x		USA*	x	x		x			
IRL			x		ZAF	x	x		x			

Note: Table A.1 shows data available for each country (identified by it's iso-code). GCSTAT data refers to Global Compustat. Notice the data on global compustat contains stock price data for almost 10-0 countries. We keep only those countries with at least 100 firm-level observations per year with data of Quarterly GDP growth, which reduces the number of countries to 28. *We obtain data for the United States from Compustat and the Longitudinal Business Dataset.

B Appendix: Additional Robustness Results

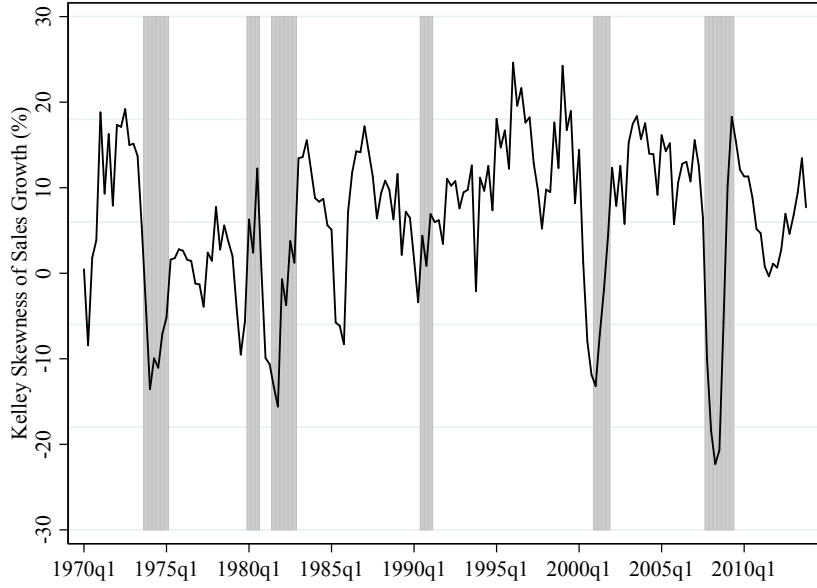
TABLE A.2 – SKEWNESS IS LOWER DURING INDUSTRY RECESSIONS

Kelley Skewness of Three-Years Growth Rate of Firms' Outcomes						
Sample:	United States			Cross-Industry		
	(1)	(2)	(3)	(7)	(8)	(9)
Source	CSTAT	LBD	CSTAT	CSTAT	CSTAT	CSTAT
	Sales	Emp.	Stock Price	Sales	Emp.	Stock Price
$\Delta GDP_{i,t}$	2.89** (1.32)	1.14** (0.50)	3.96*** (1.12)			
Industry Growth				5.92*** (1.60)	6.02*** (1.67)	0.70 (1.17)
R^2	0.01	0.15	0.10	0.12	0.11	0.09
N	182	35	180	3,652	930	3,602
Freq.	Qtr	Yr	Qtr	Qtr/	Yr	Qtr
F.E.	N	N	N	Qtr/Ind	Yr/Ind	Qtr/Ind
Sample	640K	-	650K	780K	193K	651K

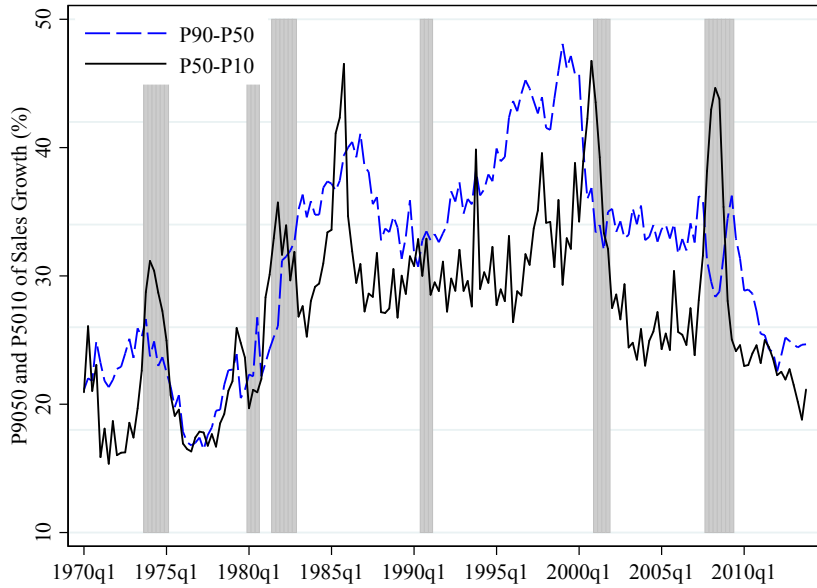
Note: Table A.2 shows a series of industry-level panel regressions. In each column, the dependent variable is the cross sectional Kelley skewness of the growth rates of real quarterly sales, annual employment growth, and quarterly stock returns distribution within period-industry cells defined by 2-digit NAICS (total of 22 industries) for a sample of publicly traded firms from the Compustat dataset. The independent variable, $\Delta \bar{S}_{j,t}$, is the average of the sales growth distribution within the period-industry cell. LBD moments were calculated weighting by firm-size. In all regressions, the sample period is 1970 to 2017 and consider a full set of period and industry fixed effects. Row labeled Sample corresponds to the total firm-period observations used to calculate the cross sectional moments. N corresponds to the number of period-industry observations used in the regressions. Standard errors in parentheses below the point estimates are clustered at the NAIC-2 industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

FIGURE A.1 – THE SKEWNESS OF FIRM-LEVEL QUARTERLY SALES GROWTH IS PROCYCLICAL

(A) Compustat: Skewness of Sales Growth Distribution



(B) Compustat: Upper and Lower Tail Dispersion of Sales Growth



Note: The top panel of Figure A.1 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm employment growth for a sample of firms from LBD. Moments are weighted by the average firm employment size between years t and $t + 1$. The bottom panel of Figure A.1 shows the time-series of the cross-sectional Kelley skewness of the distribution of firm quarterly sales growth for a sample of publicly traded firms from Compustat. The shaded bars represent NBER recession periods. See Appendix A.1 for additional details on the sample construction and moment calculations in the LBD and Compustat.

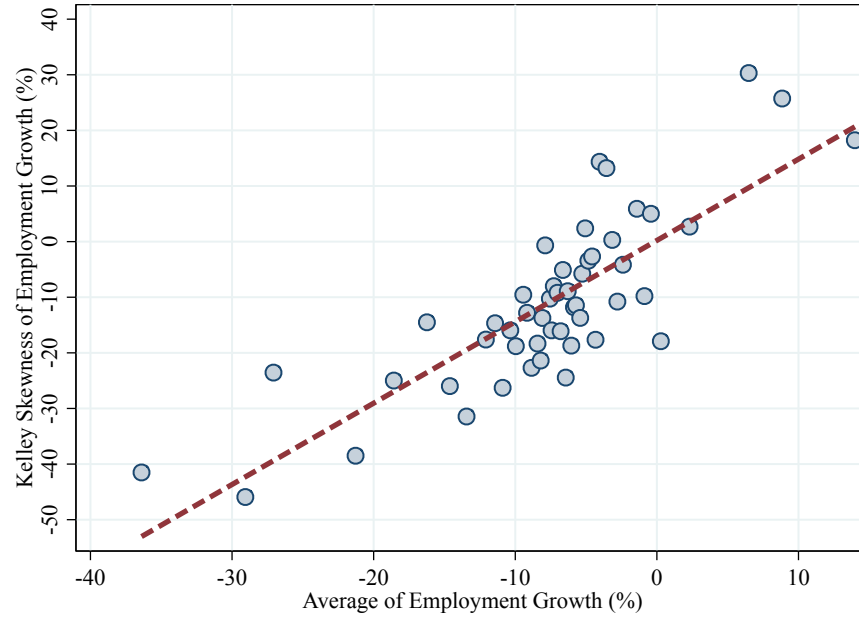
TABLE A.4 – HIGHER ORDER MOMENTS OF FIRM'S OUTCOMES

	Kelley Skewness				Crow-Siddiqui Kurtosis				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Residual Sales		Sales per Employee		Sales Deviation	Firm Sales		Stock Returns	
	One Year	Three Years	One Year	Three Years		One Year	Three Years	One Year	Three Years
$\Delta GDP_{i,t}$	3.80*** (1.42)	1.480 (0.91)	3.01** (1.23)	4.17*** (0.97)	1.46** (0.58)	0.36*** (0.08)	-0.19 (0.12)	0.16*** (0.06)	-0.21* (0.13)
N	178	178	174	166	47	184	182	180	180
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Qtr	Qtr	Qtr	Qtr
Sample	500K	500K	500K	500K	113K	640K	640K	650K	650K
Source	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT

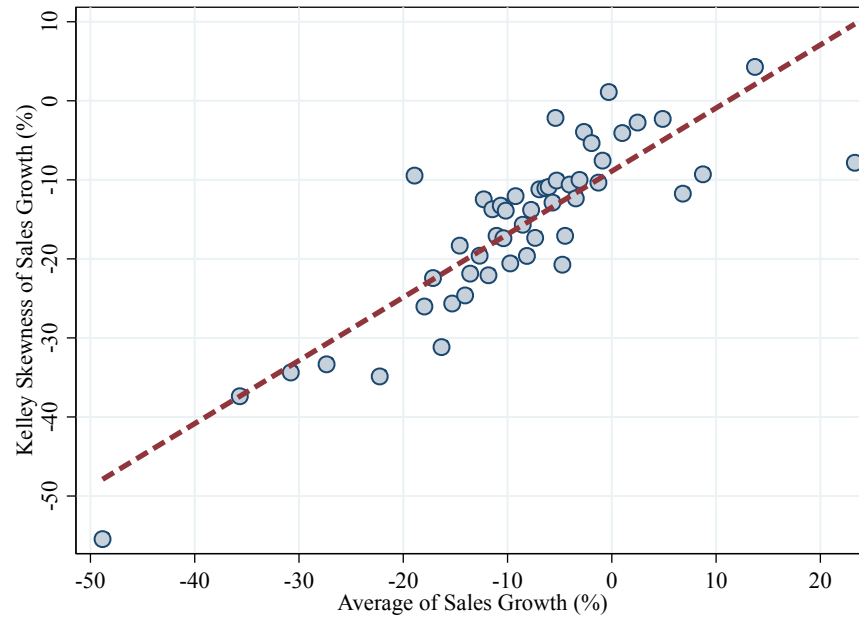
Note: The left panel of Table A.4 shows a series of time-series regressions for the United States in which the dependent variable is the Kelley Skewness of the one-year and three-years growth rate of residualized sales growth (columns 1 and 2) and the growth rate of sales-per-employee (columns 3 and 4) for a sample of firms from Compustat. In columns (1) and (2) we have orthogonalized the growth rates of sales from time fixed-effects, firm-fixed effect, size, and other firm-level observable characteristics. Column (5) shows the correlation of GDP growth and the cross-sectional skewness of the deviation of annual firms' sales from a HP trend. Compustat data covers the period 1970 to 2017. The dependent variable in columns (6) to (9) is the Crow-Siddiqui measure of Kurtosis defined as $CKU_i = \frac{P_{97.5t} - P_{25t}}{P_{75t} - P_{25t}}$. In each regression, the independent variable is the annual growth rate of quarterly GDP per capita. All firm-level moments were calculated weighting growth rate observations by firm size measured by the average sales of the firm between periods t and $t + k$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

FIGURE A.2 – PROCYCLICAL SKEWNESS IS ROBUST TO THE INCLUSION OF PRIVATE FIRMS (BvD AMADEUS DATABASE)

(A) Cross-Country: Firm-Level Employment Growth

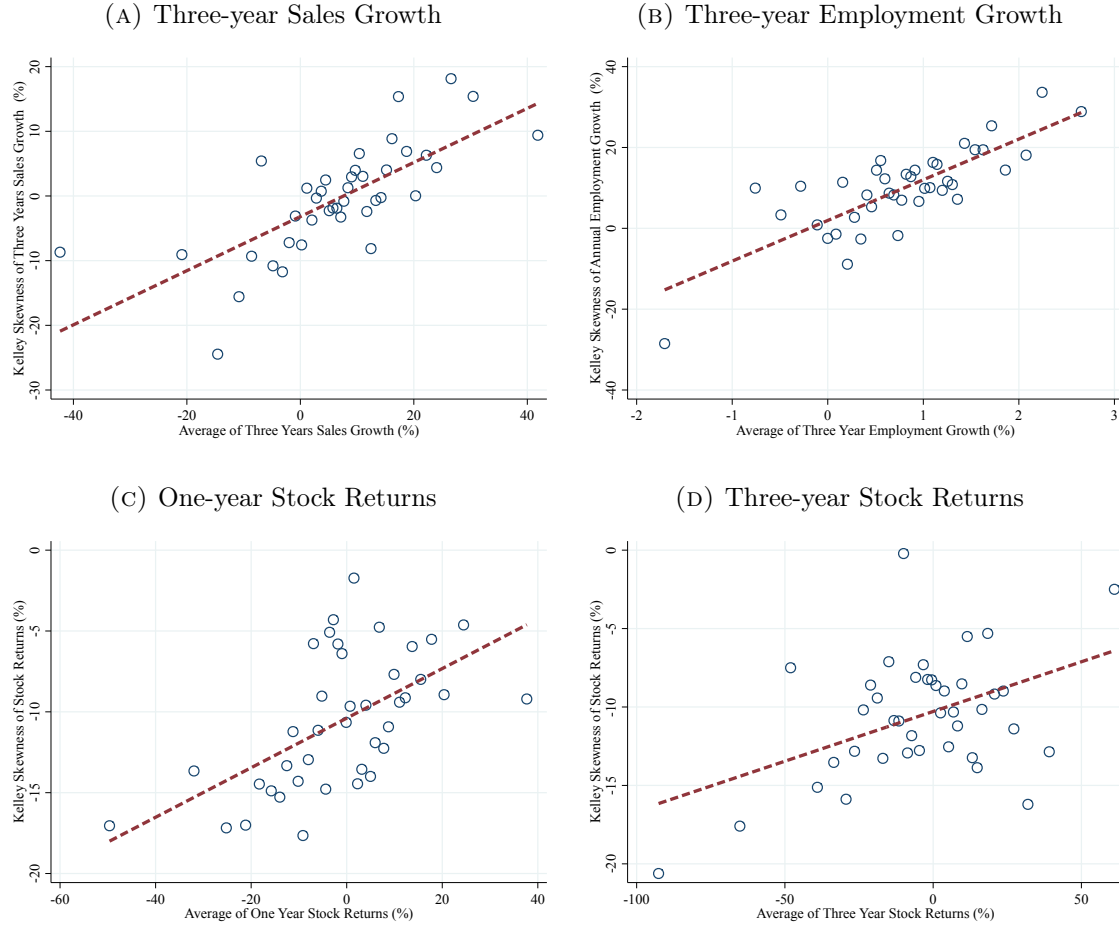


(B) Cross-Country: Firm-Level Sales Growth



Note: Figure A.2 shows bin scatter plots of the Kelley skewness and average employment and sales growth. The Figure is based on an unbalanced panel of firms from the BvD Amadeus database in the following European countries: AUT, BEL, BLR, CHE, DEU, DNK, ESP, FIN, FRA, GBR, GRC, HUN, IRL, ISL, ITA, NLD, NOR, POL, PRT, SWE, UKR. The data covers years 2000 to 2015. BvD Amadeus contains private and publicly traded firms. We apply the same selection criteria we use for the rest of the data.

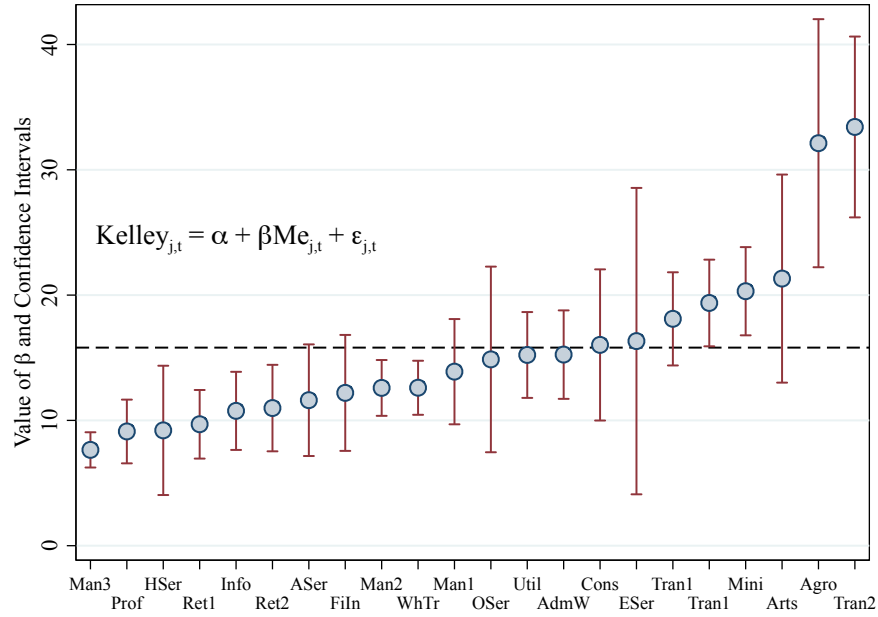
FIGURE A.3 – THE SKEWNESS OF SEVERAL FIRMS’ OUTCOMES IS LOWER DURING INDUSTRY CYCLES



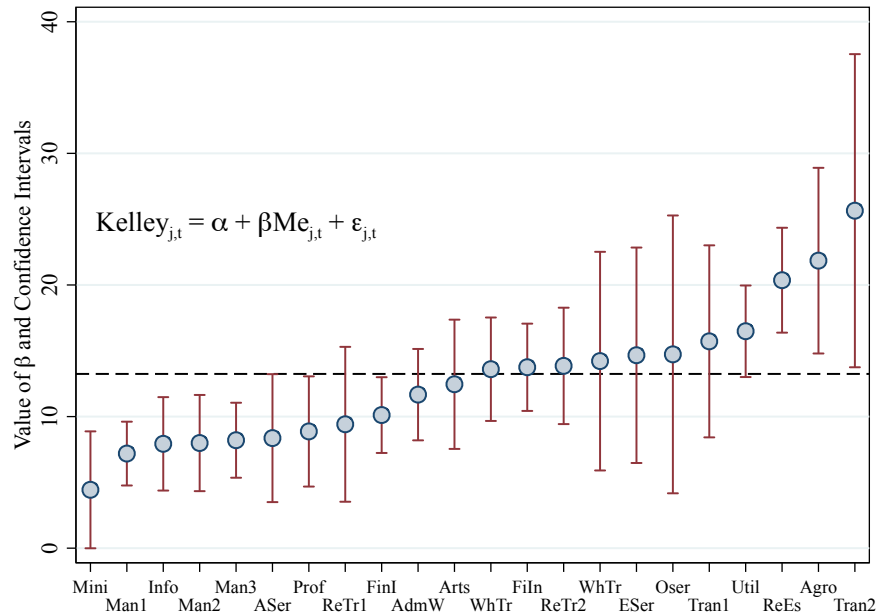
Note: The top left panel of Figure A.3 displays a bin scattered plot showing the relation between the within-industry business cycle, measured by the average growth rate of sales, and the within-industry skewness, measured by the Kelley skewness of sales growth for a sample of Compustat firms. Each dot is a quantile of the industry-year distribution of average sales growth. The rest of the plots show similar statistics for employment growth and stock returns.

FIGURE A.4 – THE SKEWNESS FIRMS’ OUTCOMES IS LOWER DURING WITHIN-INDUSTRY CYCLES

(A) Industry: Firm-Level Employment Growth

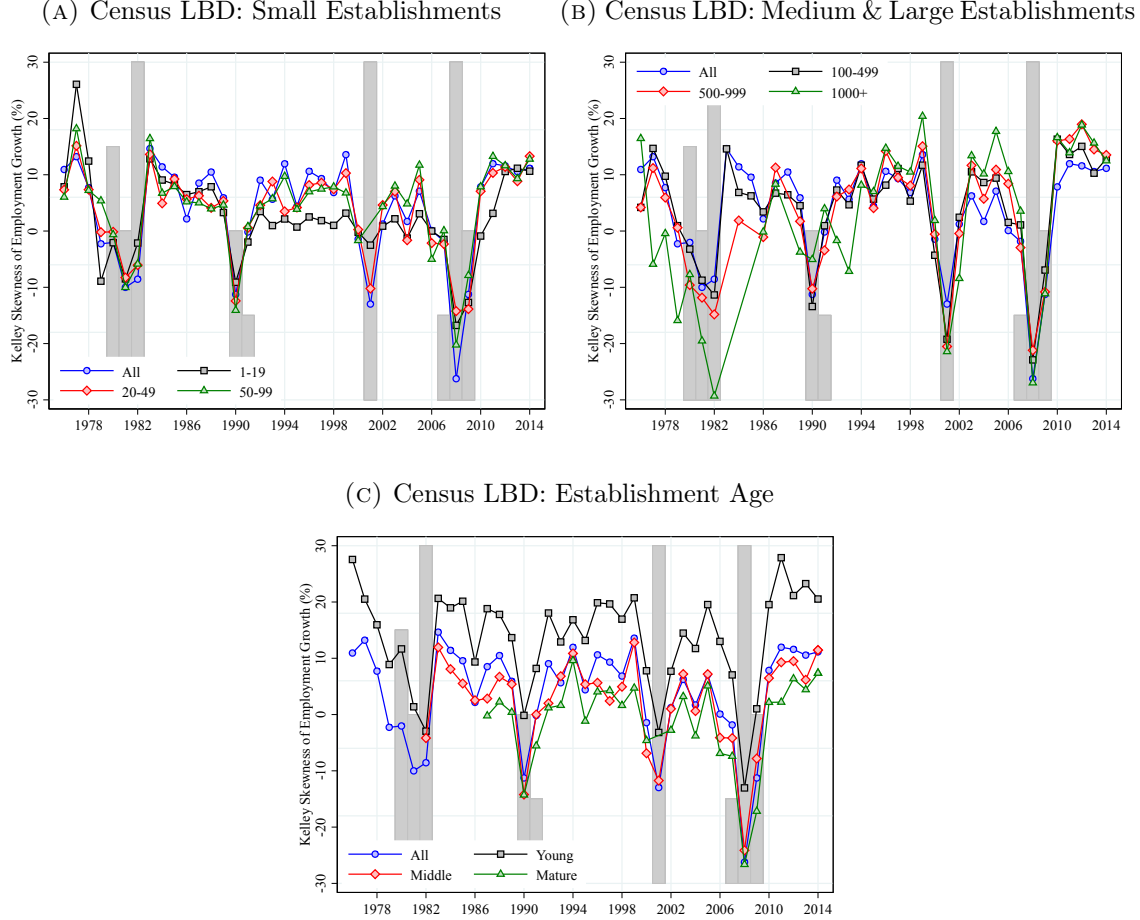


(B) Industry: Firm-Level Sales Growth



Note: Figure A.4 shows the coefficients and confidence intervals for within-industry regression of the cross-sectional Kelley skewness on the average growth of employment (top panel) and sales (bottom panel) for a sample of publicly traded firms from Compustat. Each industry regression includes a linear trend. Confidence intervals are calculated at 95% of significance. Industries are defined as 2-digit NAIC. In each plot, the dashed line is the coefficient of a panel regression of within industry skewness and average firm growth controlling form time and fixed effect. See Appendix A.1 for additional details on the sample construction and moment calculations in Compustat.

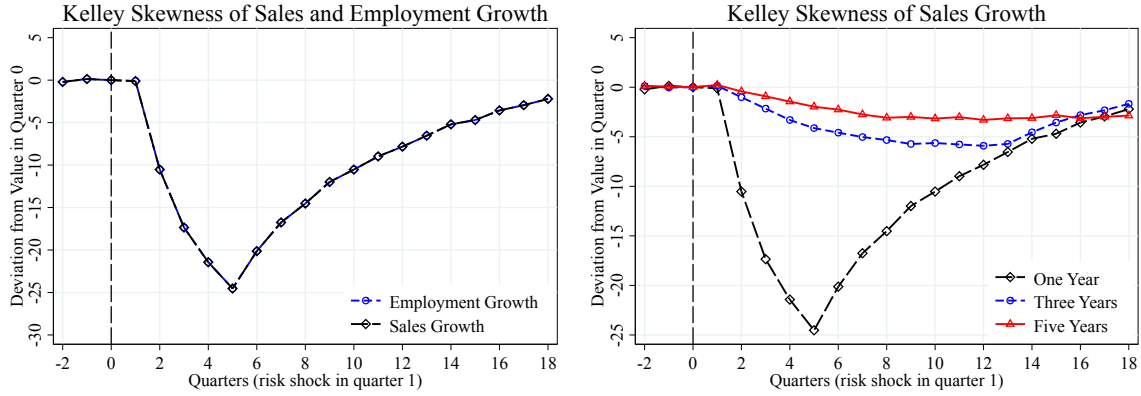
FIGURE A.5 – SKEWNESS OF EMPLOYMENT GROWTH DISTRIBUTION WITHIN ESTABLISHMENT GROUPS



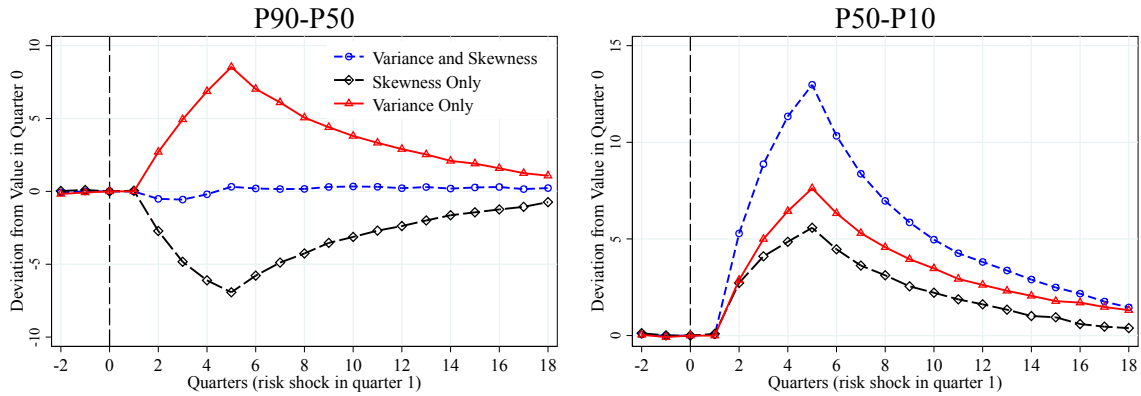
Note: Figure A.5 is based on the Longitudinal Business Database, LBD. The top left and right panels show the skewness of the distribution of establishment-employment growth within different establishment size groups defined by establishment average employment calculated for each establishment i as $\bar{E}_{i,t} = 0.5 \times (E_{i,t} + E_{i,t+1})$; The bottom panel shows the skewness of the distribution of establishment-employment growth within different establishment age groups. Young establishments are those of less than five years, Middle-aged establishments are those between six and ten years, whereas Mature establishments are those of more than ten years old. Establishments already in the sample in 1976 were not considered in any of these groups. All moments weighted by establishment size defined by $\bar{E}_{i,t}$. In all plots, the blue line with circles is the skewness of employment growth for all establishments in the sample.

FIGURE A.6 – MODEL GENERATED MOMENTS

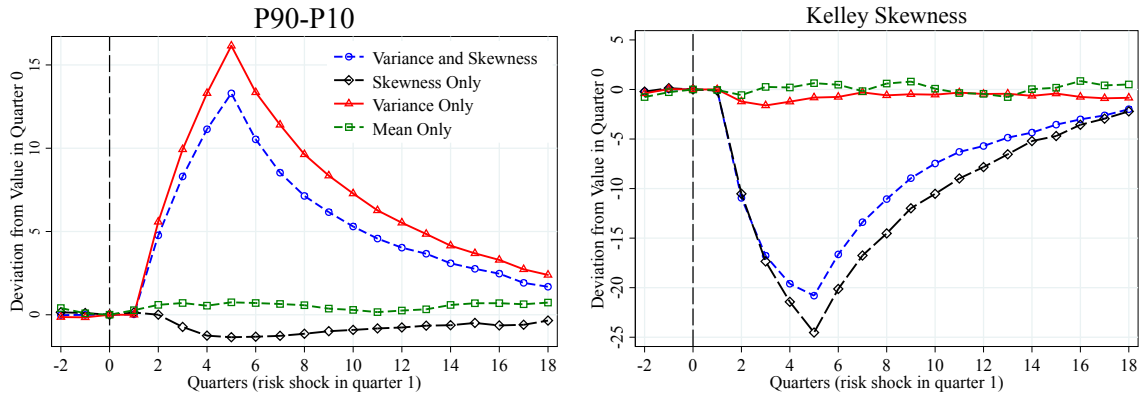
(A) Skewness of Employment and Sales Growth



(B) Right and Left Tail Dispersion of Sales Growth



(C) Aggregate Productivity Shock does not Affect Dispersion or Skewness of Sales Growth



Note: Figure A.6 shows different model generated moments of the sales growth and employment growth distribution. Each plot is based on independent simulations of 1,000 economies of 300-quarter length. We impose a drop in the skewness, increase in variance, or both, in quarter 1, allowing normal evolution of the economy afterward. We plot the deviation of each macroeconomic aggregate from its value in quarter 0.