



# Learning to Rank

Zoekmachines 2023 - IR0

---

Philipp Hager

September 26, 2023

University of Amsterdam  
p.k.hager@uva.nl



- PhD student with Maarten de Rijke
- IRLab at UvA and the Mercury ML Lab at Booking.com
- **Previously:**
  - Recommender systems at Blinkist, Berlin
  - M.Sc. Hasso-Plattner Institute, Potsdam
- **Research interests:** Ranking with user interactions, user simulation, and evaluating IR systems offline.

# Motivation





TripAdvisor

<https://www.tripadvisor.com> › Attractions-g188590-A... ⋮

## THE 15 BEST Things to Do in Amsterdam

Top **Attractions in Amsterdam** · 1. Anne Frank House · 2. Van Gogh Museum · 3. Rijksmuseum · 4. Vondelpark · 5. The Jordaan · 6. Centraal Station · 7. Heineken ...

Attractions: 3,115

Attraction Photos: 309,790

Attraction Reviews: 645,083



iamsterdam.com

<https://www.iamsterdam.com> › see-and-do › attraction... ⋮

## Attractions and sights | I amsterdam

Most popular **attractions** · Heineken Experience · ARTIS · Koninklijk Paleis (Royal Palace) · Anne Frank House · **Amsterdam** Canal Cruise - 100 highlights · Johan Cruijff ...

[Free things to do in Amsterdam](#) · [Rembrandts Amsterdam...](#) · [This is holland](#)



PlanetWare

<https://www.planetware.com> › amsterdam-nl-nh-amst ⋮

## 24 Top-Rated Tourist Attractions in Amsterdam



## Textual Signals:

- **Query content:** text
- **Document content:** title, page content

How well does the query text match the document text? [6]

- BM-25
- TF-IDF / vector space models
- Language models

**But there are many signals beyond text:**

- **Query:** type, language
- **Document:** urls, images
- **User context:** location, date, device, search history
- **Metadata:** popularity, recency, page quality, spam, adult content, ...
- **External stakeholders:** advertisers, auctions, content creators, ...

Modern search engines use a lot of features:

- **Airbnb** [10]: > 195 features
- **Bing** [15]: > 136 features
- **Istella** [7]: > 220 features
- **Yahoo** [2]: > 700 features

How do we combine all of these signals?

# Learning to Rank

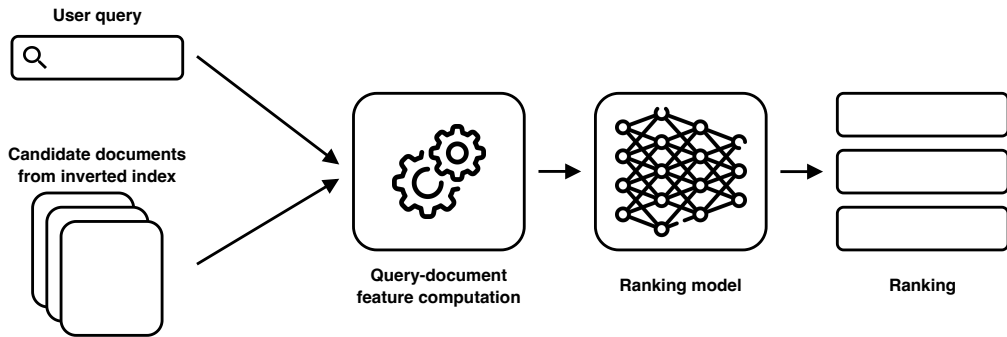
---



## Learning to Rank (LTR) is

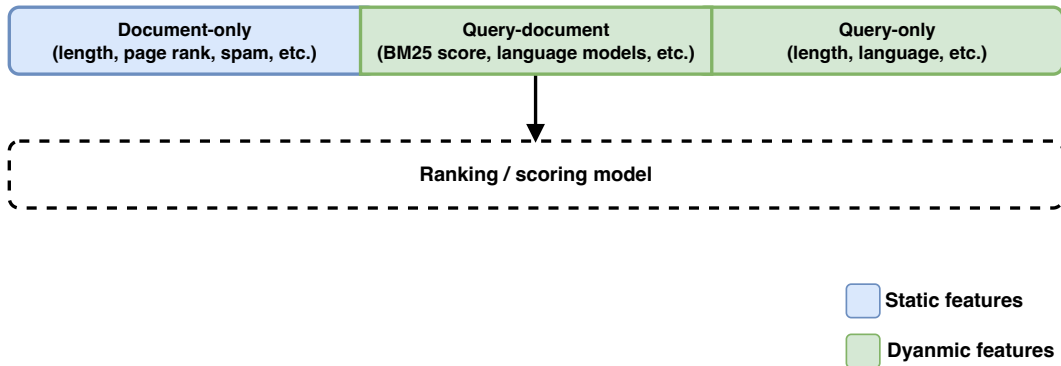
“...a task to automatically construct a ranking model using training data, such that the model can sort new objects according to their degrees of relevance, preference, or importance.” – Liu [13]

# Learning to Rank

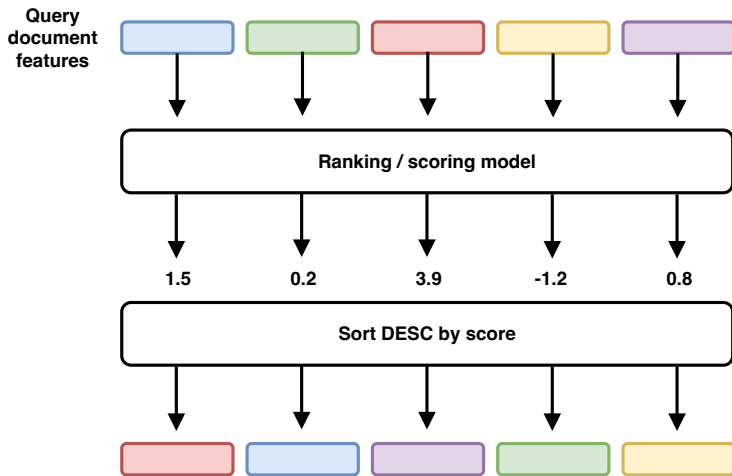


# Features

Traditionally LTR used *hand-crafted* numerical features, nowadays, we also use *deep learned* features. We can categorize features into:



# Problem Formulation



# The goal of learning to rank

Thus, we have:

- a **feature vector** for each query-document pair:  $\vec{x}_{q,d} \in \mathbb{R}^m$
- a **relevance judgment** for each query-document pair, e.g.:  $y_{q,d} \in \{0, 1, 2, 3, 4\}$

A ranking model  $f : \vec{x} \rightarrow \mathbb{R}$  scores each query-document pair to optimize the order of items when sorting descendingly by  $f(\vec{x}_{q,d}) = s_{q,d}$ .

**How can we learn a ranking model  $f$ ?**

## Pointwise Methods

---

**Regression:** Relevance as a real-valued score [4, 9]

For example, we can use linear regression for our ranking model:

$$f(x) = w_0 + w_1x_1 + w_2x_2 + \cdots + w_nx_n = s$$

We assign a **weight**  $w$  to each feature in  $x$  to minimize the error between the predicted scores and true relevance.

Usually we quantify how far off our predictions are using the **mean squared error** loss:

$$\mathcal{L}_{mse} = \frac{1}{n} \sum_{i=1}^n (y_i - s_i)^2$$

## Pointwise LTR - MSE

Relevance labels $y$	Predicted scores $s$	Squared Error $(y - s)^2$
Q	Q	
1	0.9	$(1 - 0.9)^2 = 0.01$
0	0.7	$(0 - 0.7)^2 = 0.49$
1	0.6	$(1 - 0.6)^2 = 0.16$
0	0.2	$(0 - 0.2)^2 = 0.04$
0	0.1	$(0 - 0.1)^2 = 0.01$
		<hr/> <b>MSE = 0.142</b>



**Regression:** Relevance as a real-valued score [4, 9]

**Classification:** Relevance as *unordered* categories [3, 14]

**Ordinal regression:** Relevance as *ordered* categories [5, 16]

**What are challenges with these approaches?**

Some (solvable) challenges include:

- **Class imbalance:** We have way more irrelevant than relevant documents
- **Feature normalization:** Feature distributions can differ greatly between queries

### **A more fundamental problem:**

Pointwise methods predict a score for each query-document independently, but document scores are fundamentally dependent on each other in a ranking.

**Meaning, minimizing a pointwise loss does not always lead to a better ranking.**

## Pointwise LTR: A lower loss does not imply a better ranking

Relevance labels	Predicted scores
	
1	0.6
0	0.5
0	0.5
0	0.5
0	0.5

What is the loss?

$$\mathcal{L}_{mse} = \frac{1}{n} \sum_{i=1}^n (y_i - s_i)^2$$

## Pointwise LTR: A lower loss does not imply a better ranking

Relevance labels	Predicted scores
	
1	0.6
0	0.5
0	0.5
0	0.5
0	0.5

Loss  $\mathcal{L}_{mse} = 1.16$   
MRR = 1, nDCG = 1

## Pointwise LTR: A lower loss does not imply a better ranking

Relevance labels	Predicted scores
<input type="text" value="Q"/>	<input type="text" value="Q"/>
1	0.2
0	0.2
0	0.2
0	0.2
0	0.1

Loss  $\mathcal{L}_{mse} = 0.97$   
MRR = 0.2, nDCG = 0.39

## Pairwise Methods

---

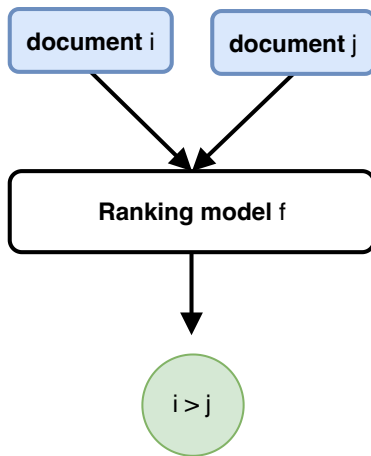
**Observation:** For a good ranking, we only require relative relevance levels:

$$y_i > y_j \rightarrow s_i > s_j$$

How can we optimize a model with pairs of items?



## Naive Pairwise Model



$$P(i > j) = f(x_i, x_j)$$

## Naive Pairwise Model

Let's (naively) change the ranking model to take **document pairs as input**:

$$P(i > j) = f(x_i, x_j)$$

**But pairwise document inputs are not a good idea:**

- This method has quadratic complexity  $O(N^2)$  during **training** and **inference** and thus does not scale to many documents.
- Pair-wise preferences have to be aggregated and can lead to paradoxical situations:

$$s_1 > s_2$$

$$s_2 > s_3$$

$$s_3 > s_1$$

**A better idea:** Let's keep the ranking model **unchanged**:

$$f(\vec{x}_i) = s_i$$

But the **loss function** is based on pairs of documents:

$$\mathcal{L}_{pairwise}(s, y) = \sum_{y_i > y_j} \phi(s_i - s_j)$$

We still score one document at-a-time and can sort according to scores, but the model is optimized over item pairs.

Pairwise loss functions minimize the number of incorrectly ranked pairs:

where  $y_i > y_j$ , but our model falsely predicts  $s_i < s_j$ .

Pairwise loss functions generally have the following form:

$$\mathcal{L}_{pairwise}(s, y) = \sum_{y_i > y_j} \phi(s_i - s_j)$$

where  $\phi$  can be the:

- **Hinge** function in RankingSVM [11, 12]:  $\phi(z) = \max(0, 1 - z)$
- **Exponential** function in RankBoost [8]:  $\phi(z) = e^{-z}$
- **Logistic / Sigmoid** function in RankNet [1]:  $\phi(z) = \log(1 + e^{-z})$

## RankNet

---

Introduced by Burges et al. [1] in 2005 to train neural ranking models.  
Popular in industry applications and won the ICML 2015 test of time award.<sup>1</sup>

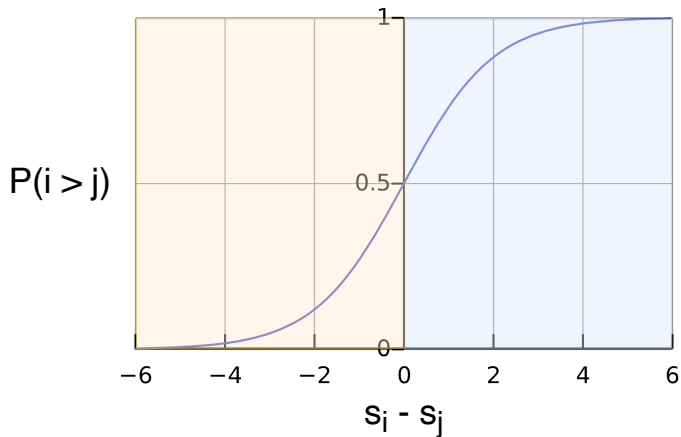
RankNet defines the probability that document  $i$  should be ranked over document  $j$  as:

$$P(i > j) = \text{sigmoid}(s_i - s_j)$$

RankNet then uses the log loss between the predicted probabilities for each pair and their true/target probability:  $\bar{P}(i > j)$ .

---

<sup>1</sup><https://icml.cc/2015/index.html%3Fp=51.html>



Mapping the difference in scores between to items to the **predicted probability**  $P(i > j)$  using the sigmoid function.

Now that we have a model prediction for each item pair,  
where do we get the **target probability** of  $\bar{P}(i > j)$  from?

1. Ask annotators to judge pairs of items (infeasible, it requires  $O(N^2)$  annotations).
2. We make up probabilities based on relevance judgments:
  - If  $y_i > y_j$ , set  $\bar{P}(i > j) = 1$
  - If  $y_i = y_j$ , set  $\bar{P}(i > j) = 0.5$
  - If  $y_i < y_j$ , set  $\bar{P}(i > j) = 0$

These *made-up/virtual* target probabilities were chosen mainly for convenience as they simplify the final loss to:

$$\mathcal{L}_{RankNet}(s, y) = \sum_{y_i > y_j} \log(1 + e^{-(s_i - s_j)})$$



## What are problems of pairwise methods?

The made-up target probabilities  $\bar{P} \in \{0, 0.5, 1\}$  are quite crude, since any difference in relevance labels is treated equally. For example:

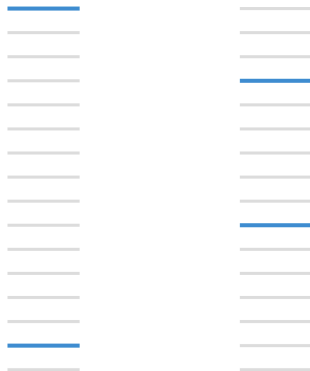
$$P(4 > 1) = 1.0$$

$$P(4 > 3) = 1.0$$

Not very elegant, but works well in practice. . .

**A more important limitation: We treat all item pairs as equally important, but are they?**

## Pairwise LTR: Minimizing pairwise errors



Reducing pairwise errors from 13 (left) to 11 (right),  
while top-heavy measures like MRR and nDCG degrade [1, Figure 1].

## Pairwise LTR: Minimizing pairwise errors



The **black** arrows denote the RankNet gradients, while what we'd arguably want are the **red** arrows [1, Figure 1].

## Listwise Methods

---

**Motivation:** Can we directly optimize IR metrics such as nDCG, Precision, and MRR?

**Reciprocal Rank:** Reciprocal of the rank of the first relevant item after sorting by our scores  $s$ :

$$RR = \frac{1}{\text{rank}_i}$$

**Discounted Cumulative Gain:**

$$DCG = \frac{1}{n} \sum_{i=1}^n \frac{2^{y_i} - 1}{\log(i + 1)}$$

**Non-smooth and discontinuous**

- Ranking metrics typically only **depend on the rank of an item, not on its score**
- Model **scores change smoothly**, the **ranks of documents change abruptly**

### Non-differentiable

Ranking metrics rely on a sorting operation that is non-smooth and discontinuous w.r.t. to model parameters  $\theta$ :

$$\frac{\partial \text{RR}}{\partial \theta} = ???$$

$$\frac{\partial \text{DCG}}{\partial \theta} = ???$$

Thus, ranking metrics are either **flat** (with zero gradient) or **discontinuous**

**Holy grail of LTR:** Finding methods that (indirectly) optimize listwise IR metrics

# LambdaRank

---



## Observations:

- I. To train a model, we don't need the costs just the gradients (of the costs w.r.t model scores)
- II. Gradients should be larger for pairs that have a greater impact on our metric



**Idea:** Scale the RankNet gradients for each document pair based on the change in nDCG we would observe after swapping the two items:

$$\mathcal{L}_{\text{LambdaRank}}(s, y) = \sum_{y_i > y_j} \Delta \text{nDCG}(i, j) \log(\text{sigmoid}(s_i - s_j))$$

When implementing LambdaRank using multiple additive regression trees (MART), it's called **LambdaMART** and is a very strong baseline methods for learning to rank.

## Conclusion

---

## To recap:

- Today's search and recommender systems use many signals for ranking.
- These signals can be computed beforehand (static features) or have to be computed when the user submits their query (dynamic features).
- Learning to rank is a method to learn models that automatically combine these query and document features into a single ranking.

## Pointwise

- Predict **relevance independently of other items**
- Ignores that the predicted scores are used to sort items

## Pairwise

- **Loss** is based on **item pairs** and minimizes the number of incorrectly ranked pairs
- Ignores that **not all document pairs have the same impact** on ranking metrics

## Listwise

- Optimize a list of items based on **non-differentiable ranking metrics**
- **Approximations** by heuristics, bounding, or probabilistic ranking methods
- Typically considered the strongest method out of the three

In this lecture, we discussed:

- the motivation behind **learning to rank**
- the **pointwise, pairwise, and listwise approaches** to LTR, their pros and cons
- the intuition behind the **most important algorithms**: RankNet and LambdaRank

**And with that, thanks for listening. Are there any remaining questions?**

## References

---

- [1] Chris Burges, Tal Shaked, Erin Renshaw, Ari Lazier, Matt Deeds, Nicole Hamilton, and Greg Hullender. Learning to rank using gradient descent. In *Proceedings of the International Conference on Machine Learning (ICML)*, pages 89–96, 2005. doi: 10.1145/1102351.1102363. URL <https://doi.org/10.1145/1102351.1102363>.
- [2] Olivier Chapelle and Yi Chang. Yahoo! learning to rank challenge overview. In *Proceedings of the Learning to Rank Challenge*, volume 14 of *Proceedings of Machine Learning Research (PMLR)*, pages 1–24, 6 2011. URL <https://proceedings.mlr.press/v14/chapelle11a.html>.

- [3] William S. Cooper, Fredric C. Gey, and Daniel P. Dabney. Probabilistic retrieval based on staged logistic regression. In *Proceedings of the Annual International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR)*, pages 198–210, 1992. doi: 10.1145/133160.133199. URL <https://doi.org/10.1145/133160.133199>.
- [4] David Cossock and Tong Zhang. Subset ranking using regression. In *Proceedings of the Annual Conference on Learning Theory (COLT)*, pages 605–619, 2006. doi: 10.1007/11776420\_44. URL [https://doi.org/10.1007/11776420\\_44](https://doi.org/10.1007/11776420_44).
- [5] Koby Crammer and Yoram Singer. Pranking with ranking. In *Advances in Neural Information Processing Systems (NIPS)*, volume 14, 2001. URL [https://proceedings.neurips.cc/paper\\_files/paper/2001/file/5531a5834816222280f20d1ef9e95f69-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2001/file/5531a5834816222280f20d1ef9e95f69-Paper.pdf).
- [6] W Bruce Croft, Donald Metzler, and Trevor Strohman. *Search engines: Information retrieval in practice*, volume 520. Addison-Wesley Reading, 2010.

- [7] Domenico Dato, Sean MacAvaney, Franco Maria Nardini, Raffaele Perego, and Nicola Tonellotto. The istella22 dataset: Bridging traditional and neural learning to rank evaluation. In *Proceedings of the International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR)*, pages 3099–3107, 2022. doi: 10.1145/3477495.3531740. URL <https://doi.org/10.1145/3477495.3531740>.
- [8] Yoav Freund, Raj Iyer, Robert E. Schapire, and Yoram Singer. An efficient boosting algorithm for combining preferences. *Journal of Machine Learning Research (JMLR)*, 4:933–969, 2003. ISSN 1532-4435.
- [9] Norbert Fuhr. Optimum polynomial retrieval functions based on the probability ranking principle. *ACM Transactions on Information Systems (TOIS)*, 7(3):183–204, 1989. ISSN 1046-8188. doi: 10.1145/65943.65944. URL <https://doi.org/10.1145/65943.65944>.



- [10] Malay Haldar, Mustafa Abdool, Prashant Ramanathan, Tao Xu, Shulin Yang, Huizhong Duan, Qing Zhang, Nick Barrow-Williams, Bradley C. Turnbull, Brendan M. Collins, and Thomas Legrand. Applying deep learning to Airbnb search. In *Proceedings of the ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*, pages 1927–1935, 2019. doi: 10.1145/3292500.3330658. URL <https://doi.org/10.1145/3292500.3330658>.
- [11] Ralf Herbrich, Thore Graepel, and Klaus Obermayer. Large margin rank boundaries for ordinal regression. In *Advances in Large Margin Classifiers*, chapter 7, pages 115–132. The MIT Press, 1999. URL [http://www.herbrich.me/papers/nips98\\_ordinal.pdf](http://www.herbrich.me/papers/nips98_ordinal.pdf).
- [12] Thorsten Joachims. Optimizing search engines using clickthrough data. In *Proceedings of the ACM SIGKDD International Conference on Knowledge Discovery and Data Mining (KDD)*, pages 133–142, 2002. doi: 10.1145/775047.775067. URL <https://doi.org/10.1145/775047.775067>.
- [13] Tie-Yan Liu. Learning to rank for information retrieval. *Foundations and Trends in Information Retrieval*, 3(3):225–331, 2009. doi: 10.1561/15000000016. URL <https://doi.org/10.1561/15000000016>.

- [14] Ramesh Nallapati. Discriminative models for information retrieval. In *Proceedings of the Annual International ACM SIGIR Conference on Research and Development in Information Retrieval (SIGIR)*, pages 64–71, 2004. doi: 10.1145/1008992.1009006. URL <https://doi.org/10.1145/1008992.1009006>.
- [15] Tao Qin and Tie-Yan Liu. Introducing LETOR 4.0 datasets. *CoRR*, abs/1306.2597, 2013. URL <http://arxiv.org/abs/1306.2597>.
- [16] Amnon Shashua and Anat Levin. Ranking with large margin principle: Two approaches. In *Advances in Neural Information Processing Systems (NIPS)*, volume 15, 2002. URL [https://proceedings.neurips.cc/paper\\_files/paper/2002/file/51de85ddd068f0bc787691d356176df9-Paper.pdf](https://proceedings.neurips.cc/paper_files/paper/2002/file/51de85ddd068f0bc787691d356176df9-Paper.pdf).