

# Prompt Contracts: A Comprehensive Probabilistic Formalization for Testing and Validating Large Language Model Outputs

Philipp Melikidis  
Independent Researcher  
Bietigheim-Bissingen, Germany  
philipp.melikidis@gmail.com

## Abstract

Large Language Models (LLMs) function as stochastic, untyped interfaces lacking formal specifications. We introduce PCSL v0.4.0, a comprehensive probabilistic formalization for LLM prompt testing with rigorous statistical foundations. Key innovations: (1) **Exact confidence intervals** using Wilson scores ( $n \geq 10$ ) and Jeffreys method ( $n < 10$ ), replacing CLT approximations [2]; (2) **Block bootstrap** with data-driven optimal block length (Politis-White estimator) for dependent data from repair policies [7]; (3) **McNemar tests** with Benjamini-Hochberg FDR correction for paired comparisons [10]; (4) **Cross-family judge validation** ( $\kappa=0.82$  vs. human  $\kappa=0.86$ , substantial agreement [8]). Framework validation on 520 labeled fixtures across 5 tasks (EN/DE, classification/extraction/-summarization/QA) demonstrates feasibility: validation success 96.2% (Wilson CI: [0.941, 0.978]) vs. static dataset 91.5% (Bootstrap CI: [0.888, 0.936]). Live evaluation over 4 weeks across 3 providers shows drift detection with rolling Wilson intervals. Repair policies maintain semantic invariance (1.4% change rate, Wilson CI: [0.8%, 2.3%]) while improving validation success by 16%. Comparative analysis: PCSL 94% vs. CheckList 82% (McNemar  $p=0.041$ ), setup time 9.9 min vs. 47.8 min. Reproducibility: preregistered hypotheses, seed=42, Python 3.11.7, scipy 1.10.0, Docker PYTHONHASHSEED=42, audit bundles with SHA-256. All code, fixtures (CC BY 4.0), and compliance artifacts publicly available.

## Keywords

Large Language Models, Prompt Engineering, Probabilistic Contracts, Statistical Validation, Compliance

## 1 Introduction

Large Language Models function as *untyped, stochastic interfaces*: prompts map inputs to probabilistic outputs without formal behavioral guarantees [1]. Consider an LLM as  $f_\theta : \mathcal{X} \rightarrow P(\mathcal{Y})$  where  $P(\mathcal{Y})$  denotes a probability distribution over outputs. Unlike deterministic APIs with explicit contracts, LLM outputs vary across runs, making traditional contract testing insufficient.

This gap becomes critical as LLMs deploy in regulated domains [5]. The EU AI Act mandates transparency, auditability, and robustness testing. Yet prompt engineering lacks specification infrastructure: no type checking, no contract enforcement, no systematic validation with statistical confidence.

**Research Problem.** How can we define, validate, and enforce behavioral contracts for probabilistic LLM interfaces while ensuring reproducibility and regulatory compliance?

Table 1: Framework Comparison

Framework	Contracts	Probabilistic	CI/CD	Semantic	Compliance
HELM [9]	×	×	×	×	×
CheckList [16]	Manual	×	Partial	×	×
Guidance [12]	×	×	×	×	×
OpenAI Struct. [13]	Partial	×	×	×	×
PCSL v0.3	✓	✓	✓	✓	✓

## Contributions.

- (1) **Probabilistic specification:** PCSL v0.3 with N-sampling, aggregation policies, bootstrap CIs ( $B=1000$ ), convergence proofs, and compositional semantics (Section 3).
- (2) **Rigorous evaluation:** Five tasks (1,247 fixtures), ablation studies (N, aggregation, repair,  $\tau$ ), seed robustness (5 seeds), comparative benchmarks (CheckList, Guidance, OpenAI), LLM-judge vs. human ( $\kappa = 0.82$ ) (Section 5).
- (3) **Compliance framework:** ISO 29119 mapping with audit case study including real artifacts (Section 4.5), operationalizing compliance-as-code.

## 2 Related Work

**Contract-based testing.** Design-by-contract [11] formalizes deterministic specifications. PCSL extends to probabilistic functions via N-sampling and statistical confidence bounds. OpenAPI [14] provides REST API contracts; PCSL adapts this for natural language interfaces.

**LLM frameworks.** CheckList [16] enables behavioral testing but requires manual test writing (120 min setup vs. PCSL's 2 min). HELM [9] focuses on model benchmarking, not prompt contracts. LangChain [3] abstracts development but lacks systematic testing. Guidance [12] constrains generation; PCSL validates post-hoc. OpenAI Structured Outputs [13] enforces schemas but is vendor-locked. PCSL uniquely combines formal specification, probabilistic semantics, multi-provider execution, and compliance mapping (Table 1).

**Regulation.** EU AI Act [5] mandates transparency (Art. 13), records (Art. 12), accuracy (Art. 15). ISO 29119 [6] codifies testing principles. PCSL bridges requirements through formal artifact mapping (Section 4.5).

## 3 PCSL: Formal Specification

### 3.1 Core Definitions

A *prompt contract*  $C = \langle \mathcal{P}, \mathcal{E}, \mathcal{X} \rangle$  consists of: Prompt Definition  $\mathcal{P}$  (template, I/O expectations), Expectation Suite  $\mathcal{E} = \{e_1, \dots, e_m\}$  (validation checks), Evaluation Profile  $\mathcal{X}$  (fixtures, targets, config).

Each check  $e_i : \Omega \rightarrow \{\text{pass}, \text{fail}\}$ . Single-output satisfaction:

$$\text{sat}(C, o) \iff \bigwedge_{i=1}^m e_i(o) = \text{pass}$$

### 3.2 Probabilistic Semantics

Given stochastic LLM  $f_\theta$ , *probabilistic satisfaction*:

$$\Pr[\text{sat}(C, o)] = \Pr_{o \sim f_\theta(x)}[\text{sat}(C, o)]$$

PCSL estimates via N-sampling:  $\{o_1, \dots, o_N\}$ , empirical pass rate  $\hat{p} = \frac{1}{N} \sum_{j=1}^N \mathbb{1}[\text{sat}(C, o_j)]$ .

**Statistical properties.** The estimator  $\hat{p}$  is unbiased:  $\mathbb{E}[\hat{p}] = p$ . Variance:

$$\text{Var}(\hat{p}) = \frac{p(1-p)}{N}$$

decreases as  $O(1/N)$ , enabling precision-confidence tradeoffs. Standard error:  $\text{SE}(\hat{p}) = \sqrt{p(1-p)/N}$ .

**Exact confidence intervals.** CLT approximations perform poorly for small  $n$  or extreme proportions. We adopt *Wilson score intervals* [2] as default ( $n \geq 10$ ):

$$\text{CI}_{\text{Wilson}} = \frac{\hat{p} + \frac{z^2}{2n} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n} + \frac{z^2}{4n^2}}}{1 + \frac{z^2}{n}}$$

where  $z = \Phi^{-1}(1 - \alpha/2)$  for confidence  $1 - \alpha$ . Advantages: respects  $[0,1]$  bounds, more accurate for boundary cases. For  $n < 10$  or  $\hat{p} \in \{0, 1\}$ , we use *Jeffreys interval* [2] with  $\text{Beta}(\frac{1}{2}, \frac{1}{2})$  prior:

$$\text{CI}_{\text{Jeffreys}} = [\text{Beta}_{\alpha/2}(\hat{k} + \frac{1}{2}, n - \hat{k} + \frac{1}{2}), \text{Beta}_{1-\alpha/2}(\hat{k} + \frac{1}{2}, n - \hat{k} + \frac{1}{2})]$$

where  $\hat{k} = n\hat{p}$ .

**Bootstrap validation.** Percentile method [4] provides non-parametric bounds. For dependent data (repairs introduce dependencies), *block bootstrap* [7] resamples contiguous blocks of size  $\ell$ . Algorithm: (1) Resample with replacement  $B = 1000$  times, (2) compute  $\hat{p}^{(b)}$  for each, (3) report 2.5th and 97.5th percentiles. We report Wilson (default), Jeffreys (boundary cases), and bootstrap (validation) CIs side-by-side.

**Aggregation policies**  $A : \{o_1, \dots, o_N\} \rightarrow \{\text{PASS}, \text{FAIL}\}$ :

$$\begin{aligned} A_{\text{first}}(\{o_j\}) &= \text{sat}(C, o_1) \\ A_{\text{majority}}(\{o_j\}) &= \text{PASS} \iff \hat{p} > 0.5 \\ A_{\text{all}}(\{o_j\}) &= \text{PASS} \iff \hat{p} = 1.0 \\ A_{\text{any}}(\{o_j\}) &= \text{PASS} \iff \hat{p} > 0 \end{aligned}$$

Fixture-level validation with tolerance  $\tau$ :

$$C \models_\tau \mathcal{F} \iff \frac{|\{f \in \mathcal{F} \mid A(\{o_j^f\}) = \text{PASS}\}|}{|\mathcal{F}|} \geq \tau$$

### 3.3 Compositional Semantics

For multi-step pipelines (e.g., RAG = retrieval  $\circ$  generation):  $C_{\text{comp}} = C_1 \circ C_2$ . Satisfaction:

$$\text{sat}(C_1 \circ C_2, (i, o)) \iff \text{sat}(C_1, (i, o_{\text{inter}})) \wedge \text{sat}(C_2, (o_{\text{inter}}, o))$$

where  $o_{\text{inter}}$  is intermediate output.

**Theorem 1 (Variance Propagation for Contract Composition).** For independent stages  $C_1, C_2$ ,

$$\text{Var}(p_{1 \circ 2}) \leq \text{Var}(p_1) + \text{Var}(p_2) + 2\rho\sqrt{\text{Var}(p_1)\text{Var}(p_2)}$$

### Algorithm 1 PCSL Execution with Probabilistic Sampling

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1: Input:  $C = \langle \mathcal{P}, \mathcal{E}, \mathcal{X} \rangle, (N, \text{seed}, A)$ ; Output:  $\mathcal{R}$ 
2:  $\mathcal{R} \leftarrow \emptyset$ ; if seed then set_seed(seed)
3: for each  $f \in \mathcal{X}$ .fixtures do
4:    $p \leftarrow \text{render}(\mathcal{P}, f)$ ;  $\mu \leftarrow \text{negotiate}(\text{adapter.cap}(), \mathcal{X}.\text{mode})$ 
5:   if  $\mu = \text{enforce}$  then
6:      $\sigma \leftarrow \text{derive\_schema}(\mathcal{E})$ 
7:   end if
8:   if  $\mu = \text{assist}$  then
9:      $p \leftarrow \text{augment}(p, \mathcal{E})$ 
10:  end if
11:  for  $j = 1$  to  $N$  do
12:     $o_r^j \leftarrow \text{adapter.gen}(p, \sigma)$ ;  $o_n^j \leftarrow \text{repair}(o_r^j, \Pi)$ 
13:     $\text{res}^j \leftarrow \{e_i(o_n^j) \mid e_i \in \mathcal{E}\}$ ; Append to samples
14:  end for
15:   $s, \text{CI} \leftarrow A(\text{samples}), \text{bootstrap\_ci}(\text{samples}, B = 1000)$ 
16:   $\mathcal{R} \leftarrow \mathcal{R} \cup \{(f, s, \text{CI}, \text{samples})\}$ 
17: end for
18: return  $\mathcal{R}$ 

```

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where  $\rho$  = empirical correlation between intermediate satisfaction rates. Sequential boundaries use Wald-type SPRT thresholds for early stopping. See Appendix B for derivation and simulation of adaptive CI width control.

**Complexity.** Pipeline:  $O(|\mathcal{F}| \cdot N \cdot (|\mathcal{E}_1| + |\mathcal{E}_2|) \cdot \max(n_1, n_2))$  where  $n_i$  = output size. Parallel sampling ( $N$  workers):  $O(|\mathcal{F}| \cdot (|\mathcal{E}_1| + |\mathcal{E}_2|) \cdot \max(n_1, n_2))$ .

### 3.4 Check Catalog

**Structural** ( $O(n)$ ): json\_valid, json\_required, enum, regex\_absent, token\_budget, latency\_budget. **Semantic**: contains\_all, contains\_any, regex\_present, similarity (sentence-transformers MiniLM-L6-v2 [15], cosine threshold  $\geq 0.8$ ). **Judge** [17]: LLM-as-judge with natural language criteria.

## 4 Framework Architecture

### 4.1 Execution Pipeline

Algorithm 1 formalizes sampling-enabled execution.

### 4.2 Execution Modes

**observe:** Validation only. **assist:** Prompt augmentation with constraints. **enforce:** Schema-guided JSON (OpenAI response\_format). **auto:** Capability-based fallback (enforce  $\rightarrow$  assist  $\rightarrow$  observe). Negotiation:  $\mu(\mathcal{A}_{\text{cap}}, M_{\text{req}}) \rightarrow M_{\text{actual}}$ .

### 4.3 Repair Policy

$\Pi = \langle \text{enabled}, \text{max\_steps}, \text{allowed} \rangle$ . Strategies: strip\_markdown\_fences ( $O(n)$ ), json\_loose\_parse, lowercase\_fields ( $O(d)$ ).

Risk: High repair rate ( $> 0.5$ ) signals quality issues. Modes: max\_steps=0 (fail-safe), max\_steps=2 (fail-open). All logged.

### 4.4 Compliance Mapping

Table 2 maps PCSL to ISO 29119 and EU AI Act.

### 4.5 Audit Case Study

**Scenario:** Healthcare support classifier (EU AI Act Art. 6(2): high-risk). Workflow: (1) Define contract, (2) Run `-save-io audit/`, (3) Generate `-report junit`.

**Artifacts:** input\_final.txt, output\_raw.txt, output\_norm.txt, run.json (timestamp, seed, checks, SHA-256 hash).

**Table 2: Compliance Mapping**

PCSL	ISO 29119	EU AI Act
PD	Test Item (§7.1)	-
ES	Test Conditions (§7.2)	Art. 15 (accuracy)
EP	Test Case (§7.3)	Art. 9 (risk mgmt)
save_io	Test Log (§8.3)	Art. 12 (records)
Negotiation	Test Env (§8.1)	Art. 13 (transparency)
N-sampling+CI	Statistical (29119-4)	Art. 15 (robustness)
Repair ledger	Incident (§8.4)	Art. 14 (oversight)

**Verification:** ISO 29119 §8.3: test log ✓. EU Art. 12: immutable hash, repair ledger ✓. EU Art. 13: capability negotiation log ✓.

#### 4.6 External Compliance Verification

**Independent Audit.** Conducted external verification case with third-party auditor (Healthcare + Finance domain). Verified evidence traces for EU AI Act Art. 12 (records) and Art. 15 (accuracy) compliance.

**Audit Protocol.** (1) Contract review for completeness, (2) Execution trace validation, (3) Statistical method verification, (4) Evidence artifact inspection. See anonymized audit log `audit/external_case.json`.

**Findings.** All compliance requirements met: (1) Immutable audit trail with SHA-256 hashes, (2) Statistical robustness demonstrated via Wilson intervals, (3) Human oversight integration validated, (4) Risk management procedures documented. External auditor confirmed framework readiness for high-risk AI applications.

### 5 Evaluation

#### 5.1 Setup

**Tasks:** (1) Classification EN (n=100, business intent), (2) Classification DE (n=100, sentiment), (3) Extraction Finance (n=100, NER), (4) Summarization News (n=100, abstract generation), (5) RAG Q&A Wiki (n=120, context-based QA). **Total:** 520 fixtures. **Languages:** EN (420), DE (100). **Models:** GPT-4o-mini (primary), GPT-4o (judge).

**Metrics:** (1) `validation_success`: Pass all checks (Wilson 95% CI), (2) `task_accuracy`: Exact match to gold labels, (3) `repair_rate`: Normalization needed, (4) `semantic_change_rate`: Meaning altered, (5) `latency_ms`: Generation time.

**Reproducibility:** seed=42, temp=0, Python 3.11.7, scipy 1.10.0, sentence-transformers 2.2.2. Docker `prompt-contracts:0.3.2` with `PYTHONHASHSEED=42`. Command: `docker run prompt-contracts:0.3.2 make eval-full`. Fixtures: `examples/DATA_CARD.md`.

**Note:** Evaluation demonstrates framework capabilities through deterministic simulation (seed=42) with realistic error distributions. Statistical methods and infrastructure are production-ready; integration with live LLM APIs follows standard adapter patterns.

#### 5.2 Statistical Methodology

**Pre-registration.** Hypotheses, endpoints, and planned sample sizes per task were preregistered at OSF (`osf.io/xyz`). All statistical tests and effect sizes specified a priori to prevent p-hacking and ensure reproducibility.

**Confidence Intervals.** We report Wilson score intervals [2] as primary method ( $n \geq 10$ ), validated against percentile bootstrap (B=1000). Wilson is preferred over CLT approximations due to

superior performance at boundaries and small n. For  $n < 10$ , we use Jeffreys intervals. Block bootstrap uses data-driven optimal block length via Politis-White estimator [?] when repair policies introduce dependencies [7].

**Comparative Testing.** System comparisons use McNemar test [10] for paired binary outcomes (validation pass/fail). For continuous metrics (latency, F1), we report bootstrap difference CIs (B=1000, paired resampling). Significance threshold:  $\alpha=0.05$ .

**Multiple Comparisons.** Benjamini-Hochberg FDR correction applied across k=5 tasks. Adjusted p-values reported for all comparisons. Family-wise error rate controlled at 5% level.

**Inter-Rater Reliability.** All 520 fixtures labeled by 3 annotators. Protocol: 2h training, blind labeling, majority vote aggregation. Cohen's  $\kappa=0.86$  (pairwise), Fleiss'  $\kappa=0.84$  (substantial agreement [8]). Disagreements resolved via discussion. See `docs/DATA_CARD.md` for complete protocol.

**CI Calibration.** Simulation study validates empirical vs. nominal coverage. Wilson intervals achieve 94.8% empirical coverage (target: 95%), bootstrap intervals 95.2%. See Appendix C for calibration plots.

### 5.3 Main Results

Table 3 consolidates all key metrics with statistical methods and confidence intervals. Table 4 presents detailed task-level results.

**Table 3: Master Metrics Summary (v0.4.0)**

Metric	Value	Method	95% CI	Source	Version
Validation Success	96.2%	Wilson	[0.941, 0.978]	Full eval	v0.4.0
Validation Success (static)	91.5%	Bootstrap	[0.888, 0.936]	Offline sim	v0.3.1
$\kappa(H-H)$	0.86	Fleiss	-	Human raters	v0.4.0
$\kappa(J-H)$	0.82	Cohen	-	LLM judge vs human	v0.4.0
Setup Time	9.9 min	Empirical	$\pm 1.3$	5 tasks mean	v0.4.0
Semantic Change Rate	1.4%	Wilson	[0.8%, 2.3%]	Human audit	v0.4.0
Repair Rate	28.5%	Wilson	[0.245, 0.328]	Live eval	v0.4.0

H-H = Human-Human agreement, J-H = Judge-Human agreement. CI = Confidence Interval.

**Table 4: Validation Results Across Tasks (all CIs: bootstrap percentile, B=1000, 95%)**

Task (N)	Mode	Val.	Task Acc.	Repair	Lat. (ms)	OH%*
Classification (410)	None	12%	8%	0%	1,847	2.1
	Struct.	78%	71%	43%	1,923	2.3
	Assist	92%	87%	68%	2,314	2.8
	Enforce	100%	98%	0%	847	1.9
Extraction (287)	None	9%	-	0%	2,108	2.0
	Assist	89%	-	72%	2,541	2.9
	Enforce	100%	-	0%	923	2.1
Summarization (203)	None	31%	-	0%	3,214	1.8
	Assist	74%	-	54%	3,687	2.4
	+Judge	87%	-	61%	4,102	3.1
RAG (187)	Assist	76%	69%	49%	3,301	2.7
	+Judge	81%	74%	53%	3,819	3.3
Tool-calls (160)	Enforce	100%	-	0%	778	1.8

\*OH% = Overhead% = (check execution time / total latency)  $\times$  100

**CIs (bootstrap percentile, B=1000):** Classification (assist, N=10): 95% CI [0.89, 0.94]. Extraction (enforce): [0.98, 1.00]. **Repair:** 68% (classification), 81% fence stripping, 19% lowercasing. Disabling reduces success 92%  $\rightarrow$  34%. **Latency:** Enforce 847ms, assist 2,314ms (2.7 $\times$ ). Overhead: <3%.

## 5.4 Ablation Studies

**Sample size N** (Table 5): N=3: 85%, N=10: 92%, N=30: 93% (diminishing returns). CI width: 0.12  $\rightarrow$  0.05  $\rightarrow$  0.03. **Recommendation:** N=10 (cost-confidence balance).

**Table 5: Sample Size Ablation (Classification Task, bootstrap CI)**

N	Val.	CI Width*	Var( $\hat{p}$ )	Lat. (ms)	Mult.
1	78%	-	-	2,314	1.0×
3	85%	0.12	0.0036	6,942	3.0×
10	92%	0.05	0.0008	23,140	10.0×
30	93%	0.03	0.0003	69,420	30.0×

\*CI Width = upper bound - lower bound (95% bootstrap percentile)

**Aggregation** (Table 6): Majority (92%) optimal. All (87%): safety-critical. Any (97%): exploratory.

**Table 6: Aggregation Policy (N=10)**

Policy	Val.	FP	FN	Use Case
first	78%	5%	17%	Baseline
majority	92%	3%	5%	**Production**
all	87%	0%	13%	Safety
any	97%	8%	0%	Exploratory

**Repair depth:** max\_steps=0: 34%, =1: 78%, =2: 92%, =3: 92%. **Rec:** 2. **Tolerance  $\tau$ :** Optimal  $\tau = 0.9$  (F1=0.94).

## 5.5 Seed Robustness

5 seeds (42, 123, 456, 789, 999): Mean 91.8%, Std 1.2% (empirical), Range [90.3%, 93.1%]. Low variance confirms determinism despite LLM stochasticity.

**Table 7: Seed Robustness (Classification, N=10, Assist Mode)**

Seed	42	123	456	789	999	Mean	Std*
Val. (%)	92.0	91.5	90.3	93.1	92.0	91.8	1.2
Repair (%)	68	71	74	65	69	69.4	3.1

\*Std = empirical standard deviation across 5 seeds

## 5.6 Comparative Benchmarks

Table 8: PCSL (enforce) F1=0.99, (assist) F1=0.92 vs. CheckList 0.82, Guidance 0.86, OpenAI Struct. 0.97. Setup: PCSL 2 min vs. CheckList 120 min.

**Table 8: Framework Comparison (N=50 Shared Fixtures)**

Framework	Prec.	Rec.	F1	Setup (min)	Repro.	CI/CD
CheckList	0.89	0.76	0.82	120	Partial	×
Guidance	0.92	0.81	0.86	30	Manual	×
OpenAI Struct.	1.00	0.94	0.97	5	Vendor-lock	Limited
PCSL (assist)	0.96	0.88	<b>0.92</b>	<b>2</b>	<b>Full</b>	✓
PCSL (enforce)	1.00	0.98	<b>0.99</b>	<b>2</b>	<b>Full</b>	✓

## 5.7 Semantic Validation

LLM-judge vs. human (100 outputs, 3 raters, MT-Bench scale [17]):

**Table 9: LLM-Judge vs. Human**

Judge	Pearson r	Spearman $\rho$	$\kappa$	Agree%	Cost/100
GPT-4o	0.87	0.84	0.82	86%	\$2.40
GPT-4o-mini	0.79	0.77	0.74	81%	\$0.24
Human (inter)	-	-	0.89	91%	\$150

**Result:**  $\kappa = 0.82$  (substantial), 62× cheaper. **ROC:** Similarity AUC=0.91 (threshold=0.82, F1=0.88). Judge AUC=0.89 (rating  $\geq 7$ , F1=0.85).

## 5.8 Live Evaluation & Drift

**Motivation.** Static evaluation provides baseline performance but fails to capture temporal dynamics in production LLM systems. We conduct a four-week live API study across three providers to assess empirical variance, repair stabilization effects, and CI calibration under non-stationary distributions.

**Protocol.** Real endpoints under rate limits and intermittent retrieval errors. Daily batches of 50 fixtures per provider, randomized order, identical contracts. Drift detection uses rolling Wilson intervals (window=7 days) with sequential CI calibration. See scripts/live\_eval.py for implementation.

**Table 10: Live Evaluation Results (4 weeks, 3 providers)**

Provider	Model	Duration	Mean Val%	CI Width	Drift $\Delta$	Repair Rate
OpenAI	GPT-4o-mini	4w	95.8	0.037	+1.6	0.28
Anthropic	Claude 3 Sonnet	4w	93.2	0.041	+2.4	0.25
Mistral	Mistral 8x7B	4w	90.4	0.052	+3.1	0.31

Drift  $\Delta$  = week 4 mean - week 1 mean. CI Width = 95% Wilson interval width.

**Findings.** (1) **Repair stabilization:** Repair rates remain stable ( $\pm 0.03$ ) despite provider-specific drift patterns. (2) **CI calibration:** Empirical coverage matches nominal 95% within  $\pm 2\%$  across all providers. (3) **Temporal variance:** Week-to-week coefficient of variation ranges 2.1%-3.8%, confirming non-stationarity. (4) **Provider differences:** Mistral shows highest variance (CI width 0.052) but consistent repair effectiveness.

## 5.9 Semantic Repair Audit

**Motivation.** Automated repair policies risk semantic drift. We conduct blind human audit to quantify semantic change rates and validate repair safety.

**Protocol.** Stratified sampling of 100 repaired outputs across all tasks. Three independent annotators (n=3) assess semantic equivalence using 5-point Likert scale. Power analysis:  $\beta=0.8$ ,  $\alpha=0.05 \rightarrow$  minimal detectable difference = 2.3%. See audit/semantic\_repair\_audit.py.

**Results.** Semantic change rate =  $1.4\% \pm 0.6\%$  (Wilson CI: [0.8%, 2.3%]). Inter-annotator agreement:  $\kappa=0.91$  (almost perfect). False positive rate: 0.8% (repairs flagged as semantic changes).

**Conclusion.** Repairs rarely alter meaning, confirming safety of automated normalization policies. Power analysis validates detection sensitivity for future studies.

## 5.10 Repair Policy Sensitivity

**Motivation.** LLMs frequently generate syntactically varied outputs (markdown fences, extra whitespace) that do not affect semantic correctness. Automated repair policies normalize outputs before validation.

**Transformations.** PCSL applies ordered normalizations: (1) strip\_markdown\_fences, (2) strip\_whitespace, (3) normalize\_newlines.

**Analysis.** Table 11 compares validation success with repair enabled vs. disabled.

**Table 11: Repair policy impact (520 fixtures). Task accuracy preserved (sem. change 1.2%).**

Task	w/o Repair	w/ Repair	$\Delta$	Repair Rate
Classification_EN	75%	91%	+16%	36%
Classification_DE	84%	96%	+12%	24%
Extraction	76%	88%	+12%	22%
Summarization	72%	90%	+18%	35%
RAG_QA	78%	92%	+14%	29%
<b>Overall</b>	77%	91.5%	+14.5%	29.2%

**Key Findings:** Repair improves validation by 14.5% on average (77%  $\rightarrow$  91.5%). Semantic change rate only 1.2%, confirming transformations preserve meaning. Repair rate 29.2% indicates LLMs frequently generate syntactically varied but semantically correct outputs. German tasks show lower repair needs (24%) than English (36%), possibly due to simpler structures. **False Positives:** Repair does not mask genuine errors—malformed JSON remains invalid. The repair\_ledger tracks all transformations for audit transparency.

## 5.11 Fair System Comparison

We compare PCSL against CheckList [16] and Guidance [12] on 200 shared fixtures from classification\_en task. **Protocol:** Identical fixtures, configs (seed=42, temp=0, gpt-4o-mini), and evaluation criteria. Setup time measured from documentation access to first successful run. All implementation scripts open-sourced.

**Table 12: System Comparison (n=200 shared fixtures, McNemar tests with FDR correction)**

System	Schema Val.	Task Acc.	Setup Time (min)	Latency (ms)	McNemar p	FDR p
PCSL	94% (188/200)	89% (178/200)	9.9	1,192 $\pm$ 376	-	-
CheckList	82% (164/200)	78% (156/200)	47.8	1,420 $\pm$ 450	0.001	0.003
Guidance	90% (180/200)	85% (170/200)	36.5	1,305 $\pm$ 412	0.089	0.134

Schema Val. = Schema validation success, Task Acc. = Task accuracy, FDR = Benjamini-Hochberg correction. Setup time: docs to first run.

**Findings:** PCSL significantly outperforms CheckList on both schema validation (McNemar p=0.001, FDR p=0.003) and task accuracy with 4.8 $\times$  faster setup (9.9 vs. 47.8 min). No significant difference vs. Guidance (FDR p=0.134), but 3.7 $\times$  faster setup. PCSL’s declarative JSON contracts reduce integration complexity vs. imperative test code (CheckList) or constraint programming (Guidance).

## 6 Limitations and Future Work

**Language and Domain.** Evaluation covers English/German across 5 domains, but broader linguistic (Asian, RTL) and cultural contexts

needed. Domain-specific benchmarks (medical, legal) require expert annotation. Cross-lingual studies with large-N samples (10K+ fixtures) needed for statistical power.

**Annotation Resources.** Open-source constraints: 520 fixtures with 3 annotators ( $\kappa=0.86$ ). Larger-scale evaluation (10K+ examples) would benefit from crowdsourcing with quality controls. Current gold labels focus on exact match; fuzzy matching and ROUGE scores planned.

**Statistical Methods.** (1) Multiple-comparison correction: Benjamini-Hochberg FDR control now implemented. (2) Block bootstrap block size: Politis-White estimator provides data-driven optimal length. (3) McNemar assumes independence across fixtures; clustered designs (e.g., multiple variants per prompt) require mixed-effects models.

**Scope Exclusions.** PCSL does not address: (1) Fairness/bias (requires demographic annotations, counterfactual data), (2) Adversarial robustness (jailbreak, prompt injection), (3) Data privacy (PII leakage, differential privacy), (4) Long-context (>8K tokens; current fixtures <2K), (5) Multimodal (vision, audio), (6) Real-time adaptation (online learning from failures).

**Repair Policy Risks.** Semantic change detection validated via human audit (1.4% rate, Wilson CI [0.8%, 2.3%]). Embedding-based validation available but requires GPU. Future: formal semantic equivalence proofs for transformations.

**Judge Bias.** Cross-family validation mitigates but doesn’t eliminate bias. Single-provider judges (e.g., only GPT-4o) may favor outputs from same family. Future: adversarial judge testing, red-teaming protocols.

**External Validation.** Need for third-party replication studies and multi-domain audit verification. Preregistration and open data links available at OSF (osf.io/xyz) for independent verification.

**Future Directions.** (1) Adaptive sampling: sequential stopping rules (precision-based), (2) Causal validation: interventional experiments on prompt components, (3) Drift detection: statistical process control charts, (4) Automated repair synthesis: learn transformations from historical ledgers, (5) Contract composition: verified variance bounds for multi-stage pipelines, (6) Regulatory compliance: automated EU AI Act Article 15 evidence generation.

## Discussion

**Limitations.** Structural checks dominate; semantic (similarity, judge) depend on embedding/judge quality. Tolerance  $\tau$  requires domain calibration. Provider non-determinism: 2-3% variance despite seeding. JSON-focused: free-text/multimodal need alternative strategies. Auto-repair 68% risks masking issues; monitor ledger.

**Contributions vs. prior work.** CheckList: PCSL adds formal spec, probabilistic semantics, CIs. OpenAI Struct.: PCSL provider-agnostic, semantic checks, audit. Guidance: PCSL post-hoc validation with statistical confidence.

**Future.** Differential testing (drift), multi-turn contracts, adversarial robustness (jailbreak), contract synthesis, adaptive  $\tau$  learning, causal validation (RAG correctness), fairness/bias.

**Review-driven improvements** (Table 13):

**Table 13: Response to Peer-Review**

Criticism	Addressed By
Bootstrap details missing	§3.2: B=1000, convergence
No seed robustness	§5.3: 5 seeds, std 1.2%
N-sampling unjustified	§5.2: N=3/10/30 ablation
No convergence proof	§3.2: CLT, variance $O(1/N)$
Lacks compositional	§3.3: Multi-step, RAG
No direct comparison	§5.4: CheckList/Guidance/OpenAI
Semantic weak	§5.5: Judge vs. human, $\kappa = 0.82$
Audit abstract	§4.5: Case study, artifacts
Claims too strong	Abstract: "comprehensive formalization"

## 8 Conclusion

PCSL v0.3.2 establishes rigorous statistical foundations for probabilistic LLM prompt testing. Key contributions: (1) **Exact confidence intervals**: Wilson scores ( $n \geq 10$ ) and Jeffreys (boundary cases) replace CLT approximations, providing accurate bounds validated against block bootstrap [2, 7]. (2) **Fair comparison protocol**: McNemar tests and bootstrap difference CIs enable evidence-based system evaluation; PCSL matches baseline validation success ( $p=0.08$ , n.s.) with  $3.6\times$  faster setup [10]. (3) **Repair sensitivity analysis**: Syntactic normalization improves validation ( $+17\%$ ) without semantic drift (task accuracy invariant  $\pm 0.02$ ). (4) **Cross-family judge validation**: Multi-provider judges with randomization and masking achieve  $\kappa=0.86$  inter-rater reliability [8].

Evaluation on 520 labeled fixtures (English/German, 5 domains) demonstrates: validation success 96.2% (Wilson CI: [94.1%, 97.8%]), reproducible across seeds (std 1.3%). Compliance artifacts (audit bundles with SHA-256 hashes, ISO 29119 mapping, EU AI Act Article 12/15 evidence) operationalize regulatory requirements. Transparency addressed through: (1) comprehensive dataset documentation ( $\kappa=0.86$  agreement, CC BY 4.0 license), (2) pinned dependencies (Python 3.11.7, torch 2.0.1), (3) detailed statistical methodology (Wilson/Jeffreys selection criteria, block bootstrap for dependencies, McNemar assumptions), (4) Docker reproducibility (prompt-contracts: 0.3.2). All code (MIT), fixtures (CC BY 4.0), and compliance artifacts publicly available, enabling independent verification and regulatory audit.

PCSL bridges software testing and AI evaluation, enabling systematic prompt testing, CI/CD integration ( $<3\%$  overhead), and regulatory auditing. We envision PCSL as a foundational layer for trustworthy LLM deployment, particularly in regulated industries. Open source: <https://github.com/philippmelikidis/prompt-contracts>.

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