Aufg. 1 b)

$$\left(\sum_{i=1}^{n} i\right)^2 \overset{!}{=} \sum_{i=1}^{n} i^3$$

Bekannt aus Vorlesung:

$$\sum_{i=1}^{n} i = \frac{n\cdot(n+1)}{2}$$

$$\Rightarrow \sum_{i=1}^{n} i^3 \overset{!}{=} \left(\frac{n\cdot(n+1)}{2}\right)^2$$

IA: $\sum_{i=1}^{1} i^3 = \left(\frac{1\cdot 2}{2}\right)^2$

$1 = 1$ ✓

IV: $\sum_{i=1}^{n} i^3 = \left(\frac{n\cdot(n+1)}{2}\right)^2$

IS: $n \rightsquigarrow n+1$

$$\sum_{i=1}^{n+1} i^3 \overset{!}{=} \left(\frac{(n+1)\cdot(n+2)}{2}\right)^2$$

$$= (n+1)^3 + \sum_{i=1}^{n} i^3 \overset{!}{=} \left(\frac{(n+1)\cdot(n+2)}{2}\right)^2$$

$$\overset{IV}{=} (n+1)^3 + \frac{n^2(n+1)^2}{4}$$

$$= (n+1)^2\left((n+1) + \frac{n^2}{4}\right)$$

$$= \frac{(n+1)^2}{4}\left(4n+4+n^2\right)$$

$$= \frac{(n+1)^2}{4}(n+2)^2 \Rightarrow ✓ \quad \blacksquare$$

1 a) $T(1) = 1 \qquad T(n) = T(n-1) + n$ für $n \geq 2$

Behaupt: $T(n) \leq n^2 \ \forall n \in \mathbb{N}$

IA: $n = 2$

$T(2) \leq 2^2$

$3 \leq 4$ ✓

IV: $T(n) = T(n-1) + n \leq n^2$

IS: $n \rightsquigarrow n+1$

$$T(n+1) \overset{!}{\leq} (n+1)^2$$

$$\overset{\text{Formel}}{\Rightarrow} T(n) + n + 1 \leq (n+1)\cdot(n+1)$$

$$\overset{\text{Formel}}{\Rightarrow} \underbrace{(T(n-1)+n)}_{\leq n^2 \text{ nach IV}} + n + 1 \leq (n+1)\cdot(n+1)$$

$$\Rightarrow n^2 + n + 1 \leq n^2 + 2n + 1$$

$\blacksquare$