1) a)

Z: $\forall n \in \mathbb{N} | n \geq 2 : \prod_{k=2}^{n} \left(1 - \frac{1}{k}\right) = \frac{1}{n}$

IA: $n = 2 \Rightarrow \left(1 - \frac{1}{2}\right) = \frac{1}{2} \Leftrightarrow \frac{1}{2} = \frac{1}{2}$ ✓

IS: $n \leadsto n+1$

$$\prod_{k=2}^{n+1} \left(1 - \frac{1}{k}\right) \overset{!}{=} \frac{1}{n+1}$$

$$\Leftrightarrow \left(\prod_{k=2}^{n} \left(1 - \frac{1}{k}\right)\right) \cdot \left(1 - \frac{1}{n+1}\right) \overset{!}{=} \frac{1}{n+1}$$

$$\overset{IV}{\Leftrightarrow} \frac{1}{n} \cdot \left(1 - \frac{1}{n+1}\right) = \frac{1}{n+1}$$

$$\Leftrightarrow \frac{1}{n} - \frac{1}{n^2+n} = \frac{1}{n+1}$$

$$\Leftrightarrow \frac{n+1}{n^2+n} - \frac{1}{n^2+n} = \frac{1}{n+1}$$

$$\Leftrightarrow \frac{n}{n^2+n} = \frac{1}{n+1} \quad \Leftrightarrow \frac{1}{n+1} = \frac{1}{n+1}$$ ✓ □

1 b)

Z: $\forall n \in \mathbb{N} : \prod_{k=1}^{n} \left(1 + \frac{2}{k}\right) = \sum_{i=1}^{n+1} i$

IA: $n = 1 \rightarrow \left(1 + \frac{2}{1}\right) = 1 + 2 \Leftrightarrow 3 = 3$ ✓

IS: $n \leadsto n+1 \quad \prod_{k=1}^{n+1} \left(1 + \frac{2}{k}\right) \overset{!}{=} \sum_{i=1}^{n+2} i$

Summenformel: $\sum_{i=1}^{n} i = \frac{n \cdot (n+1)}{2}$

$$\Rightarrow \sum_{i=1}^{n+2} i = \frac{n \cdot (n+1)}{2} + (n+2) + (n+1) = \frac{n \cdot (n+1)}{2} + 2n + 3$$

$$\Rightarrow \left(\prod_{k=1}^{n} \left(1 + \frac{2}{k}\right)\right) \cdot \left(1 + \frac{2}{n+1}\right) \overset{!}{=} \frac{n \cdot (n+1)}{2} + 2n + 3$$

$\overset{N}{\iff}$ $\sum_{i=1}^{n+1} i \cdot \left(1+\frac{2}{n+1}\right) = \frac{n\cdot(n+1)}{2} + 2n+3$

$\overset{\text{Summenf.}}{\iff}$ $\left(\frac{n\cdot(n+1)}{2} + (n+1)\right)\cdot\left(1+\frac{2}{n+1}\right) = \frac{n\cdot(n+1)}{2} + 2n+3$

$\iff \frac{n\cdot(n+1)}{2} + n+1 + \frac{2\cdot(n\cdot(n+1))}{2n+2} + \frac{2n+2}{n+1} = \frac{n\cdot(n+1)}{2} + 2n+3$

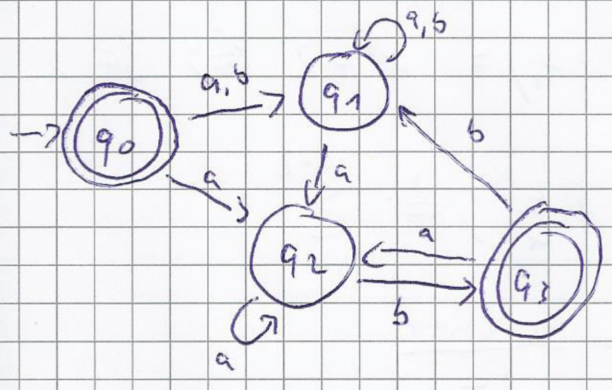
$\iff \frac{2\cdot(n\cdot(n+1))}{2n+2} + 2 = n+2$

$\iff \frac{2\cdot(n^2+n)}{2n+2} = n$

$\iff \frac{2n^2+2n}{2n+2} = n$

$\iff \frac{n^2+n}{n+1} = n \iff n = n$ ✓ □

2) a) NFA $M = (\{q_0, q_1, q_2, q_3\}, \{a, b\}, \delta, q_0, \{q_0, q_3\})$

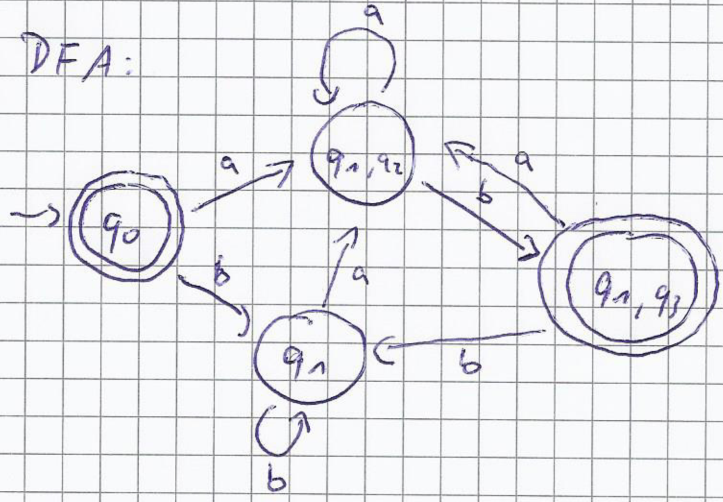


M': $Q' = P\{q_0, q_1, q_2, q_3\}$ $\Sigma = \{a, b\}$

$\delta': Q' \times \Sigma \to Q'$ $q_0' = \{q_0\}$

$F' = \{q \in Q' \mid q \cap F \neq \emptyset\}$

DFA:



2) b)

$$L(G) = \{ (b^n a^m b)^p \mid n, m, p \in \mathbb{N}_0 \}$$

n und m sind für jede der p Schleifen frei wählbar (können variieren).

$$\Rightarrow L(G) = \{ (b^{n_0} a^{m_0} b)^x \dots (b^{n_i} a^{m_i} b)^x \mid n_0 \dots n_i, m_0 \dots m_i, i \in \mathbb{N}_0 \; x \in \mathbb{N}_0 \}$$

Alle Wörter mit b am Ende sind darstellbar in $L(G)$.
Da q_0 Endzustand ist ist auch ε in $L(G)$ enthalten.