

Mathematical Entity Linking Methods and Applications

Doctoral Defense Philipp Scharpf







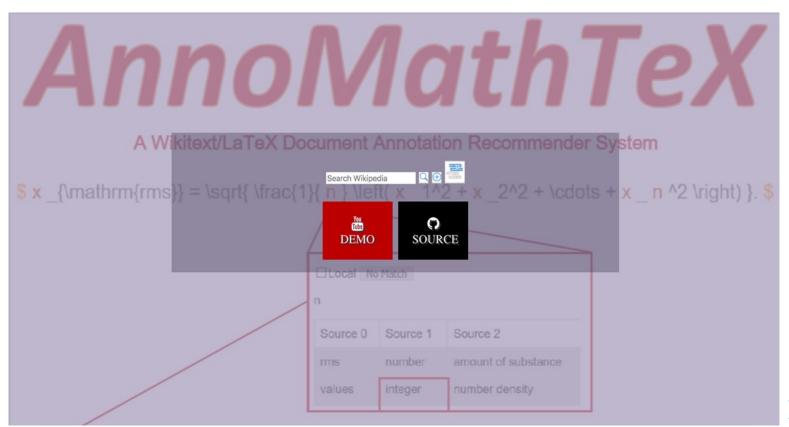






AnnoMathTeX Demovideo





https://annomathtex. wmflabs.org

Introduction

Research problem

Research objective

Research questions





Knowledge Graph Wikidata





https://wikidata.org



Launched 2012 to connect multilingual Wikipedia articles

in language-independent concept items

Free, **open** access, collaborative **semantic** knowledge-base



create, read, accept, decline or edit content



Wikidata Entity Linking









https://www.wikidata.org/wiki/Q2992084





in





Paris (Q90)

https://www.wikidata.org/wiki/Q90



Problem: Classical Entity Linking (EL)





representations

- Washington
- President Washington
- George Washington
- G. Washington
- Father of the United States
- The American Fabius
- The First President



concept

Item

George Washington (Q23)



Problem: Mathematical EL (MathEL)





$$\frac{1}{c^2}\frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi + \left(\frac{m_0 c}{\hbar}\right)^2 \psi = 0 \qquad u_{tt} + Au + f(u) = 0$$

$$\partial_{ct}^{2}h_{n}\left(z,t\right)-\partial_{z}^{2}h_{n}\left(z,t\right)+\nu_{n}^{2}\,h_{n}\left(z,t\right)=0\quad \nabla^{a}\nabla_{a}\psi=\mu^{2}\psi$$

$$\frac{\hbar^2}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{\hbar^2 \partial^2 \psi}{\partial x^2} = -2i \hbar \frac{\partial \psi}{\partial \tau} \qquad -\hbar^2 \frac{\partial^2 \psi}{\partial t^2} + c^2 \hbar^2 \nabla^2 \psi = m_0^2 c^4 \psi$$

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \frac{2\alpha + a}{c^2} \frac{\partial \phi}{\partial t} - \frac{\alpha^2 + a\alpha}{c^2} \phi = 0 \quad u_{tt} - \Delta u + m^2 u + G'(u) = 0$$

$$\frac{\left(\eta^{\mu\nu}\frac{\partial}{\partial x^{\mu}}\frac{\partial}{\partial x^{\nu}}-\left(\frac{mc}{\hbar}\right)^{2}\right)\phi=0}{\left(-\frac{1}{c^{2}}\frac{\partial^{2}}{\partial t^{2}}+\sum_{i=1}^{p}\frac{\partial}{\partial x^{i}}\frac{\partial}{\partial x^{i}}-\left(\frac{mc}{\hbar}\right)^{2}\right)\phi=0}$$

$$u_{tt}-\Delta u+mu+\mathcal{P}'(u)=0$$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \sum_{i=1}^p \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^i} - \left(\frac{mc}{\hbar}\right)^2\right) \phi = 0$$

$$u_{tt}-\Delta u+m\ u+\mathcal{P}'(u)=0$$



concept

Klein–Gordon equation (Q868967)



representations

Mathematical Entity Linking Challenges



Different symbols for constants or variables are used

$$\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi + \left(\frac{m_0 c}{\hbar}\right)^2 \psi = 0$$

Constants appear in different terms

Additional terms and functions are introduced

$$\frac{\hbar^2}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{\hbar^2 \partial^2 \psi}{\partial x^2} = -\frac{2i\hbar}{\partial \tau} \frac{\partial \psi}{\partial \tau}$$

Additional (index or semantic) **sub- or superscripts** are introduced $\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\sum_{i=1}^p\frac{\partial}{x^i}\frac{\partial}{x^i}\right)$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\sum_{i=1}^p \frac{\partial}{x^i}\frac{\partial}{x^i} - \left(\frac{mc}{\hbar}\right)^2\right)\varphi = 0$$

Substitutions, i.e., identifiers are subsumed into others and then appear implicitly

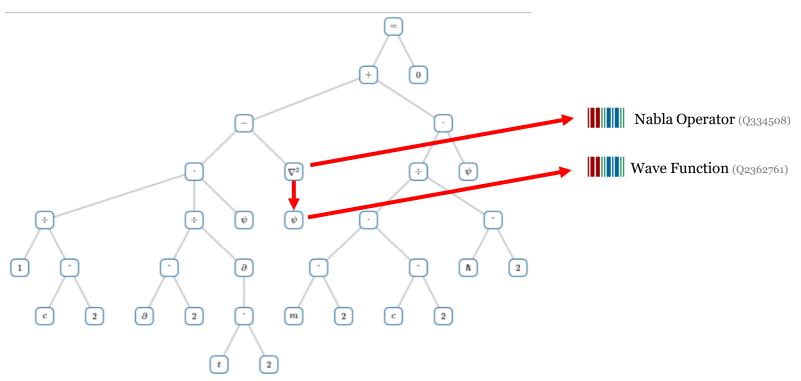


Mathematical Entity Linking Challenges





$$rac{1}{c^2}rac{\partial^2}{\partial\,t^2}\psi-
abla^2\,\psi+rac{m^2c^2}{\hbar^2}\psi=0$$



Research problem



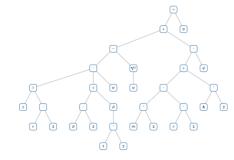
Resolving mathematical ambiguities is difficult

1. many different formula representations $\frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \nabla^2 \psi + \left(\frac{m_0 c}{\hbar}\right)^2 \psi = 0$ $u_{tt} + Au + f(u) = 0$

2. many involved symbols

\nabla: "del" (Q334508), m: "mass" (Q11423) , \hbar: "Planck constant" (Q122894) , ...

3. very complex semantics



Research questions (high-level)



- How to discover a math entity?
- How to recognize a math entity?
- How to represent a math entity in a KG?
- How to populate the KG with math entities?
- How to query the KG for math entities?
- How to apply linking math entities?

Publications overview (topics)



Reviews ("Where")

- Entity Linking with Wikidata (Scharpf24b?)
- Mathematics in Wikidata (Scharpf21a)

Methods ("How")

- Formula Classification
 & Clustering
 (Scharpf19a, Scharpf23a)
- Formula Annotation Recommendation (Scharpf19b, Scharpf21b)
- Formula Benchmarking (Schubotz18a, Scharpf18)

Applications ("Why")

- Mathematical Question Answering (Schubotz18b, Scharpf20, Scharpf22)
- Mathematical (Physics) Question Generation (Scharpf22, Scharpf23b)
- STEM Document Classification Explainability (Schubotz20, Scharpf20, Scharpf24a?)

13 publications first author + 3 co-outhor

Publications overview (focus)



Reviews ("Where")

- Entity Linking with Wikidata (Scharpf24b?)
- Mathematics in Wikidata (Scharpf21a)

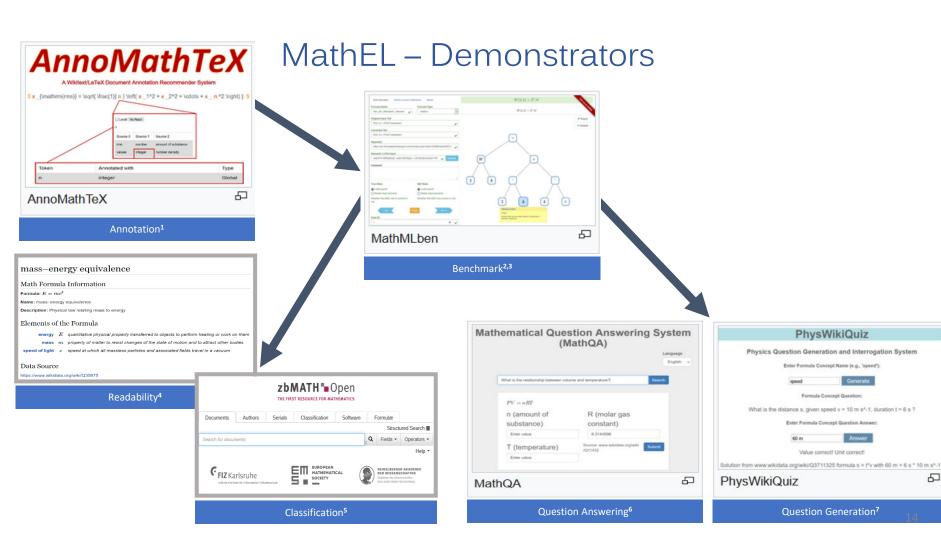
Methods ("How")

- Formula Classification
 & Clustering
 (Scharpf19a,
 Scharpf23a)
- Formula Annotation Recommendation (Scharpf19b, Scharpf21b)
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Applications ("Why")

- Mathematical Question Answering (Schubotz18b, Scharpf20, Scharpf22)
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- STEM Document Classification Explainability (Schubotz20, Scharpf20, Scharpf24a?)

Presented in depth in this talk



Reviews ("Where?")



Entity Linking with Wikidata

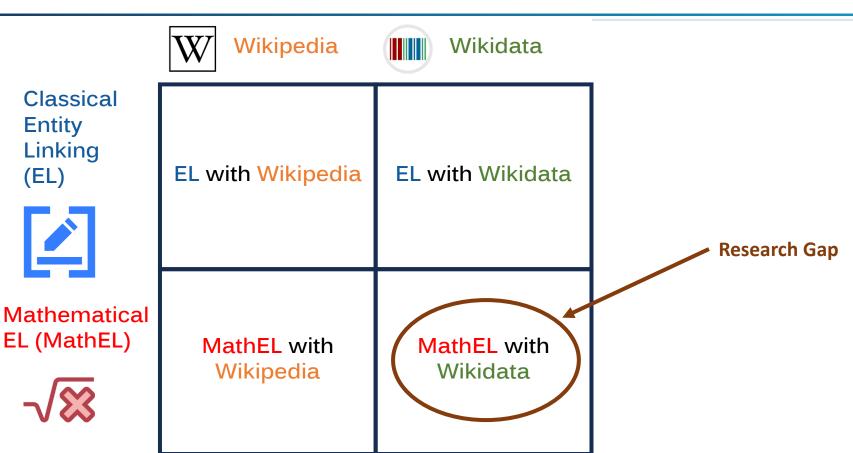
Mathematics in Wikidata





Research Gap





Why Wikidata?



Linking (mathematical) entities to Wikidata instead of Wikipedia, allows for

modeling formula expression substructure



structured queries

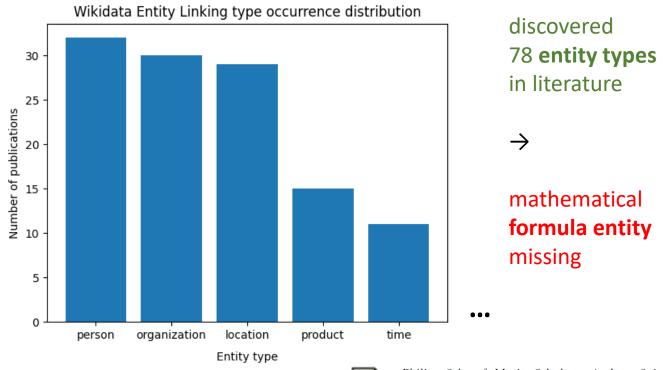


multilingual entity representations



Entity Linking Types in Literature





Philipp Scharpf, Moritz Schubotz, Andreas Spitz, Norman Meuschke, André Greiner-Petter, and Bela Gipp. Entity linking with wikidata: A systematic literature review. In *CSUR*. Submitted to ACM Comput. Surv. (CSUR), Feb. 2024.

Methods ("How?)



Formula Concept Retrieval

Formula Annotation Recommendation





Research question



How to discover mathematical concept entities?



From arXiv to Wikidata



Transfer mathematical knowledge

from arXiv (unstructured)

to Wikidata (structured)



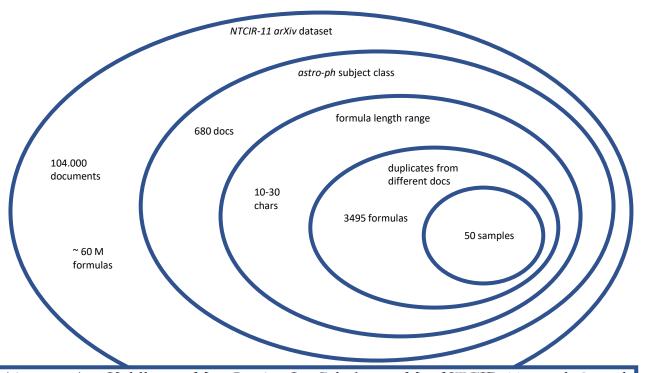
arXiv.org



Formula Concept Discovery: Dataset









Aizawa, A., Kohlhase, M., Ounis, I., Schubotz, M.: NTCIR-11 math-2 task overview. In: NTCIR. National Institute of Informatics (NII) (2014)

Formula Concept Discovery: Candidates





Formula string candidates



Formula vectors k-nearest neighbors

Hubble's law

$$p = \omega \rho$$

$$p = \kappa \rho$$

$$\omega = p/\rho$$

$$p_d = \omega \rho_d$$

Formula name candidates



Surrounding text Word window

Name (QID)

Hubble parameter (Q179916)

Equation of state (Q214967)

Accelerating universe (Q1049613)

Dark fluid (Q5223514)

Dark energy (Q18343)

...



Formula Concept Discovery: Evaluation





Retrieving equivalent representations using the k-nearest neighbor algorithm

# I	Formula	Name (QID)		$s_{e_m}, s_{\hat{e}_m}, s_{e_s}, s_{\hat{e}_s}$	Encoding: sample formula
1 1	$H = \dot{a}/a$	hubble parameter (Q179916)		0.0, 0.1, 0.0, 0.9	\hat{e}_s : $H_i = \dot{R}/R$
2 1	$p = \omega \rho$	equation of state (Q214967)		0.3, 0.0, 0.1, 0.6	e_s : $p_d = w \rho_d$
3 0	$\omega = p/\rho$	accelerating universe (Q1049613)	4 / 3	0.7, 0.0, 0.0, 0.3	e_m : $p = \omega \rho$
4 1	$p = -A/\rho^{\alpha}$	dark fluid (Q5223514)	4 / 4	0.7, 0.0, 0.3, 0.0	e_m : $p = -\frac{A}{\rho^{\alpha}}$
5 <i>j</i>	$p_d = w \rho_d$	dark energy (Q18343)	4 / 3	0.3, 0.0, 0.3, 0.3	e_s : $p_X = \omega_X \rho_X$

...

rel. success

(c_m)	math2vec	0.70]	tormulas
(\hat{e}_m)	semantics tf-idf	0.15		text
(e_s)	semantics2vec	0.11	_	frequenc
(\hat{e}_s)	math tf-idf	0.04		sequenc

We achieve best results for encoding the formula constituents with sequence (math2vec)



Formula Concept Discovery: Evaluation





Retrieving concept name candidates from the surrounding text

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ormula	Encoding: sample formula	$s_{e_m}, s_{\hat{e}_m}, s_{e_s}, s_{\hat{e}_s}$	d / \hat{d}	Name (QID)	Formula	#
3 $\omega = p/\rho$ accelerating universe (Q1049613) 4 / 3 0.7, 0.0, 0.0, 0.3 e_m : $p = \omega \rho$		\hat{e}_s : $H_i = \dot{R}/R$	0.0, 0.1, 0.0, 0.9	32 / 32	hubble parameter (Q179916)	$H = \dot{a}/a$	1
		e_s : $p_d = w \rho_d$	0.3, 0.0, 0.1, 0.6	6 / 5	equation of state (Q214967)	$p = \omega \rho$	2
4 $p = -A/\rho^{\alpha}$ dark fluid (Q5223514) 4 / 4 0.7, 0.0, 0.3, 0.0 e_m : $p = -\frac{A}{\rho^{\alpha}}$		e_m : $p = \omega \rho$	0.7, 0.0, 0.0, 0.3	4 / 3	accelerating universe (Q1049613)	$\omega = p/\rho$	3
		e_m : $p = -\frac{A}{\rho^{\alpha}}$	0.7, 0.0, 0.3, 0.0	4 / 4	dark fluid (Q5223514)	$p = -A/\rho^{\alpha}$	4
5 $p_d = w\rho_d$ dark energy (Q18343) 4 / 3 0.3, 0.0, 0.3, 0.3 e_s : $p_X = \omega_X \rho_X$		e_s : $p_X = \omega_X \rho_X$	0.3, 0.0, 0.3, 0.3	4 / 3	dark energy (Q18343)	$p_d = w \rho_d$	5

surrounding text window

For 72%, we retrieve useful formula name candidates (recommendations)

For 82% of the retrieved name candidates, there is a Wikidata QID available



Research question

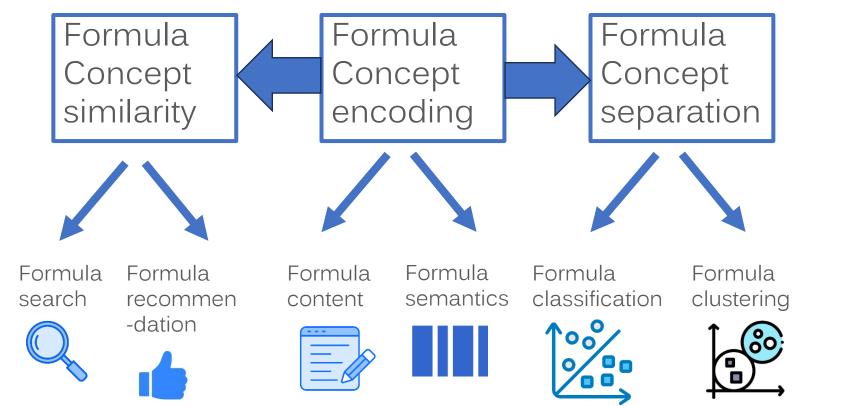


How to recognize mathematical concept entities?



Formula Concept Retrieval Workflow



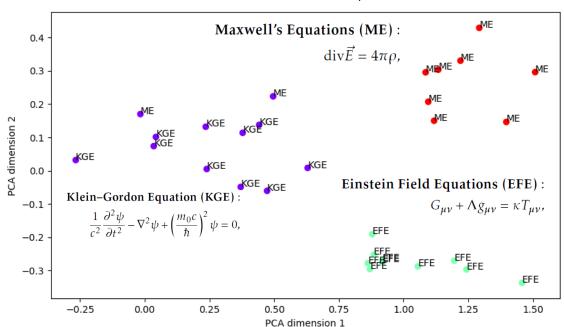


Formula Concept Clustering and Similarity



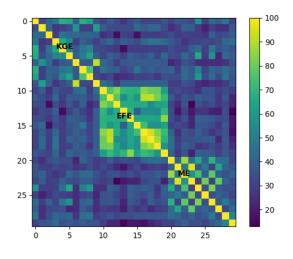


Formula Content Space



Philipp Scharpf, Moritz Schubotz, Howard S. Cohl, Corinna Breitinger, and Bela Gipp. Discovery and recognition of formula concepts using machine learning. Springer Scientometrics Journal, Feb. 2023.

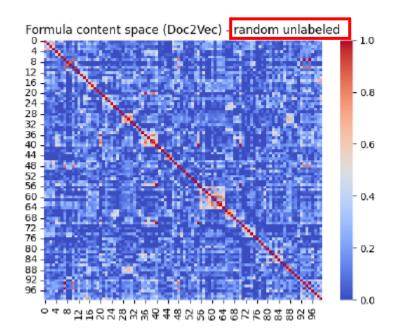
- Classification accuracy up to 0.95
- Cluster purity up to 0.87

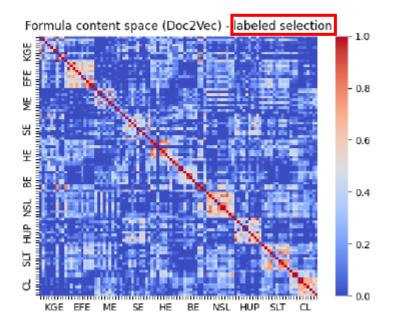


Fuzzy string similarity percentages heatmap

Formula Concept Similarity







High intra-class similarity / coherence (red) and low inter-class similarity / separability (blue)



Methods ("How?)



Formula Concept Retrieval

Formula Annotation Recommendation





Research question



How to populate the (Wikidata) Knowledge Graph

with mathematical concept entities?



From Wikipedia to Wikidata





Transfer mathematical knowledge

from Wikipedia (semi-structured)

to Wikidata (structured)

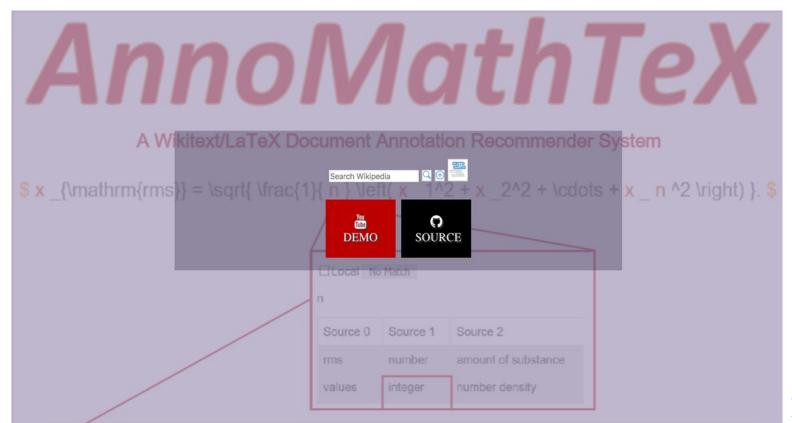






Formula Annotation Recommender





https://annomathtex. wmflabs.org

Formula Annotation Recommendation



ACM / IW3C2, April 2021.

FORMULA ANNOTATION

	_ocal	
No	match:	

Submit

Formula: E=mc^2

Not a formula				
Wikidata1	Wikidata2	WordWindow	FormulaConceptDB	Manual
mass-energy equivalence (Q35875)	mass-energy equivalence (Q35875)	formula (N/A)	mass-energy equivalence (Q35875)	
	time-independent Schrödinger equation (Q25829357)	einstein (N/A)	Philipp Scharpf, Moritz Schubot mathematical wikidata entities in recommendation. In <i>Proceedings</i>	n wikipedia articles us



Identifier Annotation Recommendation



Local No mato	h:	ATION	\$ E =	= m c ^2 \$		×
Not an id	lentifier ed Identifiers: 61/9	96				
Source	1	Source 2	Source 3	Source 4	Source 5	
mass (Q11423)	hypothesis tests (N/A)	N/A (N/A)	motion (N/A)	exhaust gas mass flow (Q320176)	
field (Q	185674)	edges (N/A)		law (N/A)	mass (Q11423)	
integer	(Q12503)	mass (Q11423)		newton (N/A)		
numbei	r (N/A)	message (N/A)		acceleration (Q11376)		
particle	(N/A)	rest mass (N/A)		mass (Q11423)		



Philipp Scharpf, Moritz Schubotz, and Bela Gipp. Fast linking of mathematical wikidata entities in wikipedia articles using annotation recommendation. In *Proceedings of the Web Conference (WWW) 2021*. ACM / IW3C2, April 2021.

Evaluation of Time Savings (Speed)



Identifiers	Time (seconds)
Recommendation	2.6
Manual	6.3
Formulae	Time (seconds)
Recommendation	2.8
Manual	4.0

Speedup
by a factor of
2.4 and 1.4
compared to
manual labeling



Philipp Scharpf, Moritz Schubotz, and Bela Gipp. Fast linking of mathematical wikidata entities in wikipedia articles using annotation recommendation. In *Proceedings of the Web Conference (WWW) 2021*. ACM / IW3C2, April 2021.

Applications ("Why?")



Mathematical
Question Answering

Physics

Question Generation





Research question



How to apply and evaluate

linking mathematical entities?



Applications ("Why?")



Mathematical Question Answering

Physics

Question Generation





MathQA Framework





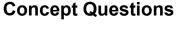


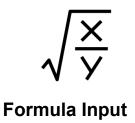


Geometry Questions

WIKIDATA SPARQL













Relationship Questions





MathQA Demo



Home Examples Video Github

Mathematical Question Answering System (MathQA)

Language

English

Search

https://mathqa. wmflabs.org



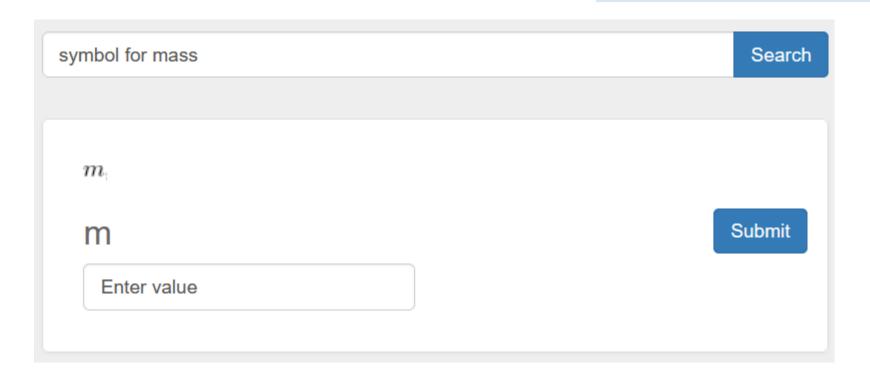
MathQA – Formula Concept Query



What is the formula for an id	eal gas?		Search	
PV = nRT				
T (temperature)	R (gas constant)			
293	8.3144598			
n (amount of		Submit		
substance)				
2				
Solution				
P*V = 4.87e+03		https://mathqa.	P. Scharpf, M. Schubotz, and B. Gipp. "Mining Mathe Answering via Unsupervised Formula Labeling". In: J	

MathQA – Identifier Query







MathQA – Relationship Query



what is the relationship between mass and energy?		
$E = mc^2$		
c (speed of light)	m (mass)	
299792458	Enter value	
Source: www.wikidata.org/wiki/Q35875		Submit



MathQA – Direct Formula Input



T = \frac{{\hbar c^3 }}{{8\pi kGM}}	Search
$T = \frac{\hbar c^3}{8\pi kGM}$	
G	pi
Enter value	Enter value
С	k
Enter value	Enter value
M	hbar
Enter value	Enter value
	Submit

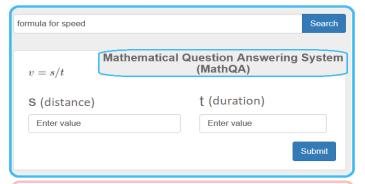
https://mathqa. wmflabs.org



MathQA – Competitors

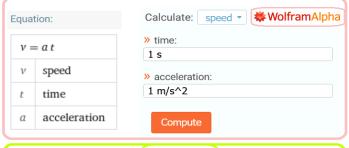


MathQA



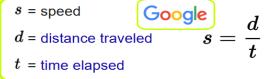
https://mathqa.wmflabs.org





https://www.wolframalpha.com

Google



https://www.google.com



MathQA – Competitors



Query Concept Name	MathQA Formula	Wolfram Alpha Formula	Google Formula
acceleration	a = dv/dt	v = at	$\bar{a} = \Delta v / \Delta t$
angular acceleration	$\alpha = d\omega/dt$	$\omega = \alpha t$	$\alpha = \Delta\omega/\Delta t$
angular frequency	$\omega = 2\pi f$	$\nu = \omega/(2\pi)$	$\omega = 2\pi/t$
angular momentum	$L = r \times p$	$L = I\omega, \omega = 2\pi n$	L = mvr
angular velocity	$\omega = d\varphi/dt \cdot u$	$\omega = \alpha t$	$\omega = \Delta \theta / \Delta t$

Relevant	52%	48%	68% (58% boxed)
Calculation	80%	81%	2%

https://mathqa. wmflabs.org



Applications ("Why?")



Mathematical Question Answering

Physics

Question Generation





Physics Question Generation





PhysWikiQuiz

https://physwikiquiz.wmflabs.org

Physics Question Generation and Test System

Enter Formula Concept Name (e.g., 'speed'):



Formula Concept Question:

What is the distance s, given speed $v = 10 \text{ m s}^{-1}$, duration t = 6 s?

Enter Formula Concept Question Answer:



Value correct! Unit correct!

Solution from www.wikidata.org/wiki/Q3711325 formula s = t*v with 60 m = 6 s * 10 m s^1 .

https://physwikiquiz. wmflabs.org



Philipp Scharpf, Moritz Schubotz, Andreas Spitz, André Greiner-Petter, and Bela Gipp. Collaborative and ai-aided exam question generation using wikidata in education. In Wikidata@ISWC

Conclusion & Outlook Conclusion

Contributions

Future Work





Conclusion



• MathEL *improves* the **representation**, **retrieval**, **and application** of math entities in a KG

- MathEL methods and applications for
 - defining mathematical concepts
 - recognizing entities in documents
 - structuring data in the KG
 - populating the KG
 - querying math entities
 - applying MathEL in various IR domains

retrieval & analysis (examples)
classification, clustering, matrices
high community agreement
annotation recommendation speedup
formula search & question answering
e.g., reliable question generation

energy

Contributions



- 1. Formula classification with up to 95% accuracy
- 2. Formula search outperforming the search-engine *Google* by 24% in MRR
- 3. Formula annotation acceleration by a factor of up to 2.4 x compared to human
- 4. Formula question answering outperforming the knowledge-engine *Wolfram Alpha* with 8% more relevant answers
- 5. Formula question generation (reliable AND scalable) using a KG and a CAS with with 13 x more coverage than its best-performing competitor

Practical Impact: Wikipedia





Mass-energy equivalence

ARTICLE

From Wikipedia, the free encyclopedia

In physics, mass-energy equivalence is the principle that mass is a form of energy and that in the rest frame, mass and energy are equivalent and differ only by a constant. [1][2] The principle is fundamental to many fields of physics, including nuclear and particle physics and is described by Albert Einstein's famous formula: [3]

Mass-energy relation

$$E = m c^2$$

Math Formula Information

Formula: $E=mc^2$

Name: mass-energy equivalence

Description: Physical law relating mass to energy

FORMULA DETAIL PAGE

Elements of the Formula

 ${f energy}\ E$ quantitative physical property transferred to objects to perform heating or work on them

mass m property of matter to resist changes of the state of motion and to attract other bodies

speed of light c speed at which all massless particles and associated fields travel in a vacuum

Data Source

https://www.wikidata.org/wiki/Q35875

https://en.wikipedia.org /wiki/Mass%E2%80%93 energy equivalence

Practical Impact: zbMATH



Fine-Grained MSC Prediction API



Example Queries

Text to Keywords Text to MSCs Keywords to MSCs

Public API (https://automscpredictor.wmcloud.org) to be used in production

by the mathematical library **zbMATH Open** (currently accessed around 40 Million times per year*)

Future Work (AnnoMathTeX)

equation

(Q25829357)



FORMULA ANNOTATION

■ Local	
No match:	Submit

 $$E = m c^2 $$

+ unsupervised auto-annotation of full document

Formula: E=mc^2

Not a formula				
(Open)LLaMA	Falcon	BERT & Co.	GPT-J/NeoX	Manual
mass-energy equivalence (Q35875)	mass-energy equivalence (Q35875)	formula (N/A)	mass-energy equivalence (Q35875)	
	time-independent Schrödinger	einstein		

(N/A)



Philipp Scharpf, Moritz Schubotz, and Bela Gipp. Fast linking of mathematical wikidata entities in wikipedia articles using annotation recommendation. In *Proceedings of the Web Conference (WWW) 2021*. ACM / IW3C2, April 2021.

Future Work (PhysWikiQuiz)



PhysWikiQuiz (rule-based)

- Knowledge Graph (Wikidata)
- Computer Algebra System (Sympy)



GPT (learning-based)

- Large Language Model
- = verified creative question

PhysWikiQuiz

https://physwikiquiz.wmflabs.org

Physics Question Generation and Test System

Enter Formula Concept Name (e.g., 'speed'):



Formula Concept Question:

What is the distance s, given speed $v = 10 \text{ m s}^{-1}$, duration t = 6 s?

Enter Formula Concept Question Answer:



Value correct! Unit correct!

Solution from www.wikidata.org/wiki/Q3711325 formula s = t*v with 60 m = 6 s * 10 m s^-1 .

https://physwikiquiz.wmflabs.org



Philipp Scharpf, Moritz Schubotz, Andreas Spitz, André Greiner-Petter, and Bela Gipp. Collaborative and ai-aided exam question generation using wikidata in education. In Wikidata@ISWC

Future Work (PhysWikiQuiz)



PhysWikiQuiz API

physwikiquiz.wmflabs.org/api/v1?name=speed

{"explanation_text": "Solution from www.wikidata.org/wiki/Q3711325 formula v = s/t with 2.5 m s^-1 = 10 m / 4 s .", "formula_unit_dimension": "m s^-1", "identifier_values": [2.5,10,4], "question_text": "What is the speed v, given distance s = 10 m, duration t = 4 s ?"}

GPT-4 (API)



ME

create a story question from this: What is the distance s, given speed $v = 4 \text{ m s}^{-1}$, duration t = 9 s?



In the adventure story "Race Against Time," the protagonist has a magical pair of boots that allow him to run at a constant speed of 4 m/s without getting tired. At a crucial point in the story, he must reach a portal to save his friends, but the portal is only open for 9 seconds. How far can the protagonist travel during this limited time?

Enter Answer:

36 m

Answer

Value correct! Unit correct!

GPT-4 provided

wrong answers

in almost 40%

of our test cases!

https://physwikiquiz. wmflabs.org



Philipp Scharpf, Moritz Schubotz, Andreas Spitz, André Greiner-Petter, and Bela Gipp. Collaborative and ai-aided exam question generation using wikidata in education. In Wikidata@ISWC