A question for the Fifth Workshop on Generalised Baire Spaces

Rosario Mennuni

Disclaimer. The answer to this question might be easy or already known.

The most basic version of the Omitting Type Theorem states:

Theorem 1. Let T be a consistent first-order theory in a countable language, $x = (x_0, \ldots, x_{n-1})$ an n-tuple of variables, and $\pi(x)$ a partial n-type over $\emptyset$ such that for no formula $\varphi(x)$ consistent with T we have $T \cup \{\varphi(x)\} \vdash \pi(x)$. Then there is a countable $M \models T$ such that no element of M^n realises $\pi(x)$.

A better version states (see [2, Theorem 10.3]):

Theorem 2. Let T be a consistent first-order theory in a countable language, and for every $n \in \omega$ let A_n be a meagre subset of the space $S_n(T)$ of n-types over $\emptyset$. Then there is a countable $M \models T$ such that, for every $n \in \omega$ and $p(x) \in A_n$, no element of M^n realises $p(x)$.

The basic version can be generalised to what is called the *κ-omitting types theorem* (see [1, Theorem 2.2.19]).

Theorem 3. Let T be a consistent first-order theory in a language of size κ, $x = (x_0, \ldots, x_{n-1})$ an n-tuple of variables, and $\pi(x)$ a partial n-type over $\emptyset$ such that for no set of formulas $\Phi(x)$ of size $< \kappa$ consistent with T we have $T \cup \Phi(x) \vdash \pi(x)$. Then there is $M \models T$ with $|M| \leq \kappa$ and such that no element of M^n realises $\pi(x)$.

Question. Let L be a language of size κ, and let T be a consistent L-theory. Equip each $S_n(T)$ with the topology generated by declaring partial types of size $< \kappa$ to induce open sets, and replace “meagre” with “κ-meagre”. For which cardinals κ does Theorem 3 generalise in the fashion of Theorem 2?

References

[1] C.C. Chang, H.J. Keisler. *Model Theory*, Third edition, Studies in Logic and the Foundations of Mathematics 73. North-Holland Publishing Co., Amsterdam, (1990).

[2] B. Poizat. *A Course in Model Theory.* Universitext. Springer (2000).