

A Cohen real out of nowhere

Let $\mathbb{M}_\kappa^{\mathcal{U}}$ denote the κ -Miller forcing with respect to some uniform $<\kappa$ -complete normal ultrafilter \mathcal{U} on κ , i.e. extending the co-bounded filter.

Let $\mathbb{P} := \ast_{n<\omega} \mathbb{M}_\kappa^{\dot{\mathcal{U}}_n}$ be an ω -iteration with full support, and $\dot{\mathcal{U}}_n$ are names for uniform $<\kappa$ -complete normal ultrafilters. Note that any initial segment \mathbb{P}_n satisfies the Laver property, i.e.

$$\forall f \in \kappa^\kappa \cap V^{\mathbb{P}_n}: \exists g \in \kappa^\kappa \cap V \ f \leq g \Rightarrow \exists \text{slalom } (s_i)_{i<\kappa} \in V \ \forall i < \kappa \ f \restriction i \in s_i.$$

We shall show that \mathbb{P} adds a Cohen real over the ground model V . This answers a question of V. Fischer and D. Montoya, whether the Laver property is preserved under κ -support iterations, in the negative.