RESEARCH AND TEACHING STATEMENT

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1. Research

My background is in mathematical logic, in particular in the field of set theory. I am particularly interested in descriptive set theory, large cardinals and connections between set theory and computability. The current focus of my work is on generalised Baire spaces and a mix of applications of descriptive set theory, forcing and large cardinals. The main directions of my past research are described in the next 4 sections.

1.1. **Descriptive set theory, large cardinals and forcing.**

Descriptive set theory studies definable subsets of and definable relations on the real numbers and other Polish spaces, i.e. separable complete metric spaces. The field originates from the work of Borel, Lebesgue and others as a systematic study of definable sets, for instance Borel sets and analytic sets, the continuous images of Borel sets. These sets have desirable properties for the analyst, for instance they are Lebesgue measurable.

Beginning with my dissertation, I have been interested in applications of large cardinals to projective sets. The reason for the use of large cardinals here is that beyond analytic sets, many natural questions become undecidable in the standard axioms of sets theory. For instance, Lebesgue measurability of sets on the next level of the projective hierarchy beyond analytic.

Borel equivalence relations have been at the centre of descriptive set theoretic research for decades. Harrington and Hjorth started to work on projective equivalence relations using large cardinal assumptions [Hjo93]. In my Ph.D. thesis under the supervision of Prof. Ralf Schindler and the papers [Sch14, SS11], I studied a class of projective equivalence relations on the reals called thin, which do not have a perfect set of pairwise inequalent reals. A perfect set is a closed set without isolated points. From determinacy or large cardinal assumptions, I proved that such equivalence relations remain essentially the same in all sufficiently nice generic extensions, by proper forcing, in the sense that they do not gain new equivalence classes. Rather than considering extensions, a natural question is what happens when one looks at inner models of set theory, i.e. (transitive) submodels of the universe of set theory with the same ordinals. I found a characterisation via descriptive set theory when equivalence relations as above are essentially the same in such models. The proofs used inner model theory in an essential way. A recent joint paper with Fabiana Castiblanco [CS21] shows that a number of well known forcing adding reals preserve thin equivalence relations as above as well by means of inner model theory. In a recent joint paper with Juan Aguilera and Sandra Müller [AMS21], we explore new ideas between determinacy for games of length ω^2 and games of length ω.

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In large cardinal theory, Peter Holy and me in [HS17] recently introduced a hierarchy of Ramsey-like cardinals in strength between weakly compact and measurable cardinals, naturally extending work of Victoria Gitman. These large cardinals form a strict hierarchy of consistency strength, with the weakest having the same consistency strength as Schindler's remarkable cardinals. These large cardinals can be characterised via elementary embedding and via games that are essentially versions of the cut-and-choose game where the opponent plays families of subset of κ and the other player chooses a filter that measures these sets. The ideas in this paper led to papers by Welch and Nielsen [NW19] and very recently Foreman, Magidor and Zeman [FMZ20].

Regarding forcing, in the last few years we obtained a sequence of results in joint work with the Bonn group [HKL$^+$16, HKS18, HKS19] and with the New York group [GHH$^+$17], which begin a systematic study of class forcing and its properties that diverge from set forcing. Class forcing is essential for changing the universe globally. In [HKL$^+$16], we give the first proof of the existence of a class forcing for which the forcing theorem fails. In the recent [GHH$^+$17], we then determine the consistency strength of the forcing theorem in second order set theory with classes and show that it is among others equivalent to the theory $\mathsf{ETR}_{\mathrm{Ord}}$ that consists of second order set theory with the scheme of class recursion of length Ord.

Regarding forcing in the setting of models of set theory without choice, a recent joint paper with Asaf Karagila [KS20] proves surprising properties of collapse forcing over such models and provides general combinatorial and topological characterisations when adding a Cohen subset preserves Dedekind finiteness.

I am further very interested in applications of descriptive set theory in various directions:

In a recent paper with Sandra Müller, David Schrittesser and Thilo Weinert accepted for the Israel Journal of Mathematics [MSSW], we prove analogues to Lebegue's density theorem for other ideals on the Borel sets instead of the Lebesgue null sets, but also prove that there is no analogue for the ideal of countable sets. Incidentally, the proof uses forcing, although the theorem is a rather simple statement about Borel sets and ideals.

In a recent paper with Andre Nies and Katrin Tent [NST] in revision for the Journal of Mathematical Logic, we study an application of descriptive set theory to the isomorphism problem for automorphism groups of ω-categorical structures. Partially answering a question of Kechris, Nies and Tent, and contrary to their expectation, we show that this isomorphism problem is rather low in the Borel reducibility hierarchy by proving that it is Borel below an equivalence relation induced by a Borel action of a countable group.

1.2. **Generalised descriptive set theory.** In the last decades, generalised descriptive set theory, the study of definable subsets of generalized Baire spaces, emerged as a field of independent interest. A generalised Baire space consists of the set κ^κ of functions $f\colon \kappa \to \kappa$, where κ is an uncountable regular cardinal with $\kappa^{<\kappa} = \kappa$, equipped with the topology generated by basic open sets of the form $N_t = \{x \in \kappa^\kappa \mid t \subseteq x\}$, with t ranging over the set $\kappa^{<\kappa}$ of sequences with values in κ and length $<\kappa$.

While some of the first results in this field were proved in the 1990s [Vää91, MV93], the interest grew with connections to model theory [FHK14]. An introduction to this field and a current and future research project is described in detail in my research proposal.

In a joint 2015 paper with Philipp Lücke [LS15], we studied analogues in this setting to classical results for analytic sets in Polish spaces. We showed that in contrast to the countable case, here a nonempty analytic set, defined as a projection of a closed subset of $\kappa^\kappa \times \kappa^\kappa$, is not necessarily a continuous image of the generalised Baire space κ^κ. Again in contract to the classical setting, an injective continuous image of κ^κ is not necessarily a κ-Borel set. The proofs of these results use combinatorial constructions. For the second result, we found an interesting example that connects this problem to the combinatorics of coherent ladder systems of closed subsets of ordinals $\alpha < \kappa$. For the special case of a successor cardinal $\kappa = \nu^+$, one can show that the set of threadable ladder systems on κ with an appropriate topology is a set as claimed in the result.

Many results in descriptive set theory take the form of dichotomies. Such a result states that an object is either small in a prescribed sense or contains a large object from a given list. For instance, the perfect set property of a set A states that A is either countable or contains a perfect set, i.e. this set is nonempty and closed without isolated points. In the early days of descriptive set theory, this was understood as a version of the continuum hypothesis for definable sets. For a while, it remained open whether the perfect set property can hold for all definable subsets of κ^κ, or even coanalytic subsets of κ^κ. I solved this in [Sch17] 24 years after this problem was published in one of the first papers in the field [MV93].

In a joint paper with Luca Motto Ros and Philipp Lücke [LRS16] in a similar direction, we studied a different notion of largeness given by superperfect sets. While perfect sets can be understood as the sets of branches through $<\kappa$-closed subtrees of $\kappa^{<\kappa}$, superperfect sets are the set of branches of such trees where every splitting node has κ many direct successors. A set is small with respect to the superperfect set property, also called the Hurewicz dichotomy, if it can be covered by a K_κ set, which is a union of κ many κ-compact sets. A set satisfies the Hurewicz dichotomy if it is either small or contains a superperfect subset. We showed that the superperfect set property can hold for all analytic subsets of κ^κ, using an inaccessible cardinal for weakly compact κ and using a forcing that can make a ground model set into a K_κ set for the remaining cardinals. We improved this to all definable sets in unpublished work with Luca Mottor Ros and Philipp Lücke, and to more general results in recent joint work in preparation with Dorottya Sziraki.

A recent paper [LS] with Philipp Lücke connects the study of subsets of generalised Baire spaces with Kurepa trees.

1.3. **Connections between set theory and computability.** Infinite time Turing machines were invented by Hamkins and Kidder in 2000 [HL00] as a natural machine model beyond the Turing barrier and are by now well studied. These machines, and the sets that they define, link computability and descriptive set theory: In higher recursion theory, Π^1_1 sets were characterised by transfinite processes bounded by ω_1^{ck} in time, and one can formalise this correspondence via infinite time Turing machines. The connection with descriptive set theory comes from Hamkins' and Lewis' result showed that the sets of reals decided by infinite time Turing machines fall strictly between Π^1_1 and Δ^1_2.

Building on their work and that of Welch [Wel09], I analysed properties of these (and related) notions of computability together with their connections with descriptive set theory in joint work [SS12, CS17b, SRC20, CSW20].

This is where I first came upon the notion of a recognisable set, determined by a program that tests if the input is the given set. In contrast to finite computation, this is a new phenomenon in the setting of infinite computation and descriptive set theory. Surprisingly, large cardinals have a strong effect on these sets, as we proved in joint work with Philip Welch and Merlin Carl [CSW18]. We solved some of the main questions on this topic and related questions of Hamkins in an ongoing project with Philip Welch.

Two further projects [CS17a, CS18] were on algorithmic randomness in this setting. This field uses tools from computability theory to give a formal definition of the notion of an infinite binary sequence that we would expect to be the result of independent coin tosses. It is connected with theoretical computer science via Kolmogorov complexity, a measure of the information content of finite strings. By varying the tools, a hierarchy of randomness notions is obtained. The stronger the tools, the easier it is to detect irregular behaviour, and so the harder it is for a sequence to be random.

I am interested in extensions of results of Hjorth and Nies for Π^1_1-randomness in their landmark paper [HN07]. This theme is motivated by Martin-Löf's critique that the usual randomness notions are too weak. We proved version of some of their results in the setting of infinite time Turing machines in [CS18]. The main result of characterise randomness with respect to infinite time Turing machines via set theory. Another result shows that mutual random sequences share only computable information.

In a quite different direction that I explored several years ago, I would like to mention my joint papers on Wadge theory between descriptive set theory, computability and theoretical computer science [RSS15, IST17, Sch18].

1.4. **Automatic structures.** Finite automata play a crucial role in many areas of computer science. In particular, finite automata have been used to represent certain infinite structures. The basic notion of this branch of research is the class of automatic structures, introduced by Khoussainov and Nerode [KN94] (see also [KM07]). A structure is automatic if its domain as well as its relations are recognised by (synchronous multi-tape) finite automata processing finite words. This can easily be extended to automata on ordinals. This class has the remarkable property that the first-order theory of any automatic structure is decidable.

I am interested in the important goal in this field to classify those structures that are automatic, or α-automatic for an ordinal α.

In my first paper with Frank Stephan [SS13], we determined the supremum of α-automatic ordinals, building on work of Delhomme. In joint work with Alexander Kartzow and Martin Huschenbett [HKS17], we obtain a complete classification of the ω^n-automatic Boolean algebras. This answers a question of Finkel and Todorcevic [FT$^+$12] and extends work of Khoussainov, Nies, Rubin and Stephan [KNRS04]. My technical contribution was to prove pumping lemmas and growth lemmas for ordinal automata.

Besides finite automata reading finite or infinite words, there are also finite automata reading finite or infinite trees. In joint work with Khoussainov, Jain and Stephan [JKSS16], we found a connection between ω^n-automatic and tree automatic structures, and separated these classes in [HKS17].

More recently, I became interested in the open problem whether isomorphism of tree-automatic ordinals is decidable. We prove a partial result in [JKSS19], assuming that the ordinals are given with addition.

In a popular science application of automatic structures, we show in joint work with Hamkins and Brumleve [BHS12] that the "mate in n moves" problem for chess on an infinite 2-dimensional board with finitely many pieces in arbitrary positions is decidable. Decidability of the "mate in any finite number of moves" problem remains open.

2. Teaching

2.1. **Teaching experience.** I have had the opportunity to teach 8 lecture courses in mathematics and a number of seminars for bachelor and master students (some courses and seminar were taught jointly with colleagues) as a fixed term assistant professor at the University of Bonn and as an associate professor (substituting for an unfilled position) at the University of Münster, Germany.

I have taught lecture courses on introduction to mathematical logic, set theory, descriptive set theory and model theory before. The material overlaps with some courses for M1 and M2 Master Logique et Fondements de l'Informatique. I further assisted in introductions to linear algebra and calculus and would be willing to teach those, or other courses for the License as required.

I have experience teaching in English and German from lectures at the University of Bonn, since the master's program is in English and the bachelor's program in German. The number of students ranged from around 50 for the introductory courses to around 10 for the more advanced ones. I further prepared problem sets and was responsible for organising tutorials given by students for a number of lecture courses, for instance calculus and linear algebra for more than 100 students. Moreover, I taught a number of advanced seminars in mathematical logic and introductory seminars in other mathematical fields jointly with colleagues.

At the University of Münster, I taught the cycle of two introductory courses in mathematical logic over one year in German. The first course was an introduction to logic for around 30 students that covers the completeness and incompleteness theorems and a short introduction to model theory, the second an introduction to set theory for around 15 students. I have developed my own teaching materials for all my courses.

I am happy to supervise students at all levels. I supervised bachelor students in Bonn and Bristol; in Germany this usually requires a habilitation which I only obtained in 2017. I further co-supervised several bachelor and master students in Bonn. Two master students with whom I worked for their thesis now work as postdoctoral researchers at the Universities of Amsterdam and Vienna, and a bachelor student that I supervised last year in Bristol is now studying in the master's program of the University of Oxford. I also helped supervise several Ph.D. students in Bonn, Münster and Bristol and have written joint papers with them.

2.2. **Approach to teaching.** I am a friendly and approachable teacher. During a lecture course, I try to present mathematics as something that grows out of natural questions that students might ask. I encourage students to interact and ask questions during and after lectures and in discussions by email or otherwise. My main goal is that students get into thinking about and doing math on their own, encouraged by the lecture or dicussion.

I have only given lectures with blackboards, but hope to improve my experience with online teaching this summer, when I teach the introduction to mathematical logic course at the University of Bonn.

The background of students is important to me when planning a course. For instance, in my introduction to model theory at the University of Bonn, I put a strong focus on connections with algebra throughout the course, since the students had a stronger background in algebra than in logic and this was useful to explain abstract concepts in way that was more familiar to them.

I have successfully tried some teaching concepts, for instance longer projects than the usual weekly problem sets, and questions and answer sessions.

When supervising projects, I meet with students every one or two weeks to keep updated about their progress. Working on projects with students at all levels is the most rewarding part of teaching for me.

3. Administration

I have experience in organising students' tutorial sessions for logic and mathematics courses in Bonn for several years. I have further managed my Marie Curie grant in Bristol and co-organised a number of international conferences and workshops in Bonn, Bristol and Auckland. As the webmaster of the European Set Theory Society ESTS for several years, I am responsible for taking care of their website ests.wordpress.com. I further have some insight in international university systems from working and studying in Germany, the UK, the US, Austria and New Zealand.

4. Popular Science

I participated in several events with the public engagement group of the School of Mathematics of the University of Bristol in 2018-2020:

- Futures: European Researcher's Night 2019 in Bristol: This festival allows everyone at the University to present or engage with the public. I started the project of doing math experiments with a group of researchers from other fields. The presentations used geometric shapes to explain shortest paths and minimal surfaces, creating fractal-like shapes with colours on paper to explain fractals, and puzzles. This was set up in a museum within a science fair with more than 100 participants, both classes of school children and the general public.
- Futures: European Researcher's Night 2020 in Bristol: I participated again with different experiments and additionally in a speed dating event that allowed 5-10 minutes to explain research to participants.
- Cheltenham Science Festival 2020: We organised a table of math puzzles on the topic of mathematics in everyday life, some based on the above experiments.

I gave a general audience talk: Large infinities in mathematics, in the Basic Notions Seminar, Bonn 2017.

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