GENERALISED BAIRE SPACES AND LARGE CARDINALS
8 - 10 FEBRUARY 2024

Event Programme

THURSDAY 8TH FEBRUARY

- 9:30-10:20 Miguel Moreno: The Borel reducibility main gap
- 10:20-10:50 BREAK
- 10:50-11:40 Dorottya Sziraki: Covering with closed sets

  This talk focuses on very general dichotomy theorems which yield several known and new regularity properties as special cases.One example of such a principle is a covering property for sigma-ideals generated by families of closed subsets of Polish spaces which was introduced by Alain Louveau. We show that a variant of this property for definable subsets of the kappa-Baire space is consistent relative to a Mahlo cardinal and discuss applications. For example, we derive the determinacy of an abstract class of long games due to Alexander Kechris that encompasses several games characterizing well-known regularity properties. This is joint work in progress with Philipp Schlicht.

- 11:40-12:30 Farmer Schlutzenberg: Sigma_1 and Pi_1(X) definability above large cardinals

  I will talk about some results on the Sigma_1(X) and Pi_1(X) definability over H_{kappa^+} of pathological subsets of P(kappa), when there are large cardinals in V_kappa. In particular, if kappa is a limit of measurable cardinals then letting X=H_\kappa\cup OR, there is no Sigma_1(X) wellorder of a subset of P(\kappa) of length \geq\kappa^+, and there is no Sigma_1(X) mad family of cardinality >kappa. However, the existence of regular kappa with large cardinal properties together with Pi_1({kappa}) mad families and maximal independent families are consistent relative to large cardinals. And in M_1, the minimal proper class mouse with a Woodin cardinal, for every uncountable cardinal kappa which is not a limit of measurable cardinals, there is a good Sigma_1(H_kappa\cup {kappa}) wellorder of H_{kappa^+}, where X is as before. Some of the results address questions of Lücke and Müller. Reference: "Low level definability above large cardinals".

- 12:30-14:00 LUNCH
- 14:00-15:30 Discussion Time
- 15:30-16:00 Beatrice Pitton: The SLO principle for Borel subsets of the generalized Cantor space

  The Wadge hierarchy establishes a hierarchy of complexity through the comparison of sets via continuous reductions. The Semi-Linear Ordering principle (SLO) asserts that, for any two subsets A and B of a space X, either A can be continuously reduced to B or the complement of B can be continuously reduced to A. While classical descriptive set theory primarily focuses on studying subsets of the space of all countable binary sequences, generalized descriptive set theory aims at developing a higher analogue in which ω is replaced with an uncountable cardinal κ satisfying the condition κ<κ = κ. Motivated by understanding the Wadge structure for (various classes of) generalized Borel sets, in this talk we will first discuss the consistency of the failure of the SLO principle for Σ0(2^(κ+)) sets and then, starting from the bottom of the Wadge hierarchy, we will analyze the validity of the semi-linear ordering principle as we ascend through the difference hierarchy. This is joint work with Luca Motto Ros and Philipp Schlicht.

- 16:00-16:30 David Chodounský: Some Wadge classes on ω1

- 16:30-17:00 BREAK

- 17:00-17:30 Luca Motto Ros: Uniformization results and Feldman-Moore theorem in generalized descriptive set theory

  Whether Borel sets can have a Borel uniformization is a central theme in classical descriptive set theory. A crucial application of such uniformization results is the so-called Feldman-Moore theorem, a cornerstone in the theory of countable Borel equivalence relations. In this talk I will explore which of these results remain (consistently) true when moving to the context of generalized descriptive set theory.

- 17:30-18:00 Philipp Lücke: Separating rank-into-rank axioms through their descriptive consequences

  In my talk, I want to present results from an ongoing project with Vincenzo Dimonte (Udine) that show that very strong large cardinal axioms can be distinguished through their provable influence on the structural properties of simply definable subsets of generalized Baire spaces. These results deal with sets defined by Σ_1-formulas with different sets of parameters and generalizations of the Perfect Set Property to higher function spaces.

FRIDAY 9TH FEBRUARY

- 9:30-10:20 Jouko Väänänen: Uncountable models and generalized Baire spaces
- 10:20-10:50 BREAK
- 10:50-11:40 Šárka Stejskalová: Subtrees of weak Kurepa trees
- 11:40-12:30 Chris Lambie-Hanson: More subtrees of weak Kurepa trees
- 12:30-14:00 LUNCH
- 14:00-16:00 Discussion Time
- 16:00-16:20 Nick Chapman: Strong Measure Zero Sets on the Higher Cantor Space

  As introduced by Borel in the early 20th century, a set of reals is strong measure zero if it can be covered by a sequence of intervals whose lengths shrink arbitrarily fast. This notion admits a natural generalisation in the context of the higher Cantor space 2^kappa. However, contrasting the situation on 2^omega, much about the behaviour of strong measure zero sets on 2^kappa is unknown; in particular, the consistency of Borel's Conjecture in this context ("A set is strong measure zero iff it has size at most kappa") is still open. After touching on the current state of our knowledge about strong measure zero sets on kappa, focusing on some of the difficulties one runs into when generalising proof strategies from the countable case, I will for kappa inaccessible sketch the construction of a model of |2^kappa| = kappa^{++} and "forall X \subseteq 2^kappa: X is strong measure zero iff |X| \leq kappa^+". Time permitting, we will also briefly discuss a strengthened variant of strong measure zero, where it is demanded that each point be covered stationarily often.

- 16:20-16:40 Calliope Ryan-Smith: VC dimension in generalised Baire space
- 16:40-17:00 BREAK
- 17:00-17:30 Claudio Agostini: Changes of topologies and the kappa-Borel hierarchy
- 17:30-18:00 Jonathan Schilhan: Strong almost disjointness and complex analysis

  A Wetzel family is a family of complex entire functions that attains at each point less values than members of the family. Recently, we have shown together with T. Weinert, that the existence of such a family is consistent with arbitrary values of the continuum. It turns out that the key to answering this is a question about strong almost disjointness. Two functions f,g \colon kappa^+ \to kappa are strongly almost disjoint if f \cap g is finite. Are arbitrarily large strongly almost disjoint families consistent? In the case of kappa = aleph_omega this was a question left open by Zapletal.

- 19:00 Conference dinner at Your Kitchen, Queens Road

SATURDAY 10TH FEBRUARY

- 9:30-10:20 Grigor Sargsyan: HOM representations for sets of sets of reals

  We discuss the problem of forcing Martin's Maximum over models of determinacy. The most promising approach to this problems involves generalizing the concept of homogenously Suslin sets of reals to homogenously Suslin sets of sets of reals. We will discuss a recent progress towards finding such representations.

- 10:20-10:50 BREAK

- 10:50-11:40 Christopher Henney-Turner: Löwenheim-Skolem-Tarski numbers for regularity quantifiers

  The LST number of a generalised quantifier is the least cardinal for which a Löwenheim-Skolem type theorem holds with respect to that quantifier. In this talk, we will examine the possible LST numbers of a family of quantifiers related to regular cardinals, including the Härtig quantifier I and the equal cofinality quantifier $Q^{e.c.}$. We will find upper and lower bounds for the LST numbers of these quantifiers, and characterise which values the LST numbers can take between these bounds.

- 11:40-12:30 Peter Holy: Large cardinal compactness

  The compactness of first order logic can be seen as a property of omega with respect to this logic: Every first order theory T for which every subset of T of size less than omega (i.e., every finite subset of T) has a model has a model itself. Stronger logics usually fail to have this compactness property for omega. However, classical results of Magidor closely connect compactness properties of second order logic with large cardinals: Generalizing the principle of compactness by replacing omega with an arbitrary cardinal kappa in the above, Magidor showed in 1971 that the least such compactness cardinal kappa for second order logic is exactly the least extendible cardinal, and in 1985, Makowsky showed that Vopenka's principle holds if and only if every abstract logic has such a compactness cardinal.
  In this talk, I will present a new type of compactness principle that we call outward compactness: The basic idea is to only consider theories T with the property that all small (i.e., <kappa-sized) subsets of T are consistent not only in our set theoretic universe V, but also in suitable outer models of V. This principle allows us to characterize a number of large cardinals, including measurable cardinals, strong cardinals and supercompact cardinals, using compactness properties of second order logic. It also allows us to characterize when Ord is Woodin, using compactness properties of arbitrary abstract logics.
  This is joint work with Philipp Lücke and Sandra Müller.

- 12:30-14:00 LUNCH

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