OPEN QUESTIONS ON GENERALISED BAIRE SPACES

Abstract. Open questions collected at the seventh workshop on generalised Baire spaces that took place at the University of Bristol in February 2024.

1. Open questions

(1) (Miguel) Do all analytic strong measure 0 subsets of ${}^{\kappa}2$ have size $\leq\kappa$?
(2) (Miguel) Is it consistent that there is a stable unsuperstable theory T such that $\cong_T$ is Δ^1_1?
(3) (Miguel) Are filter reflection and $=^{\kappa}_{X}\hookrightarrow_B=^{2}_{Y}$ equivalent?
(4) (Dorottya) Does the κ-perfect set property for κ-analytic sets (equivalently for closed sets) imply the open graph dichotomy for κ-analytic sets?
(5) (Dorottya) Does the PSP for closed sets already imply it?
(6) (Dorottya) Do any of these statements imply $CCP_\kappa(\Sigma^1_1, D_\alpha)$, i.e. the version for definable families of closed sets.
(7) (Dorottya) $CCP_\kappa(X)$: For any κ-ideal $\mathcal{I}$ on X generated by a family of closed sets, either $X\in\mathcal{I}$ or there exists a continuous function $f\colon {}^{\kappa}\kappa\to X$ such that $f(N_t)\in\mathcal{I}^+$ for all $t\in {}^{<\kappa}\kappa$.
(8) (Dorottya) $CCP_\kappa(\mathsf{D}_\kappa)$: $CCP_\kappa(X)$ holds for all subsets X of ${}^{\kappa}\kappa$ definable from a κ-sequence of ordinals.
(9) (Dorottya) Does $CCP_\kappa(\mathsf{D}_\kappa)$ have at least the consistency strength of a Mahlo cardinal?
(10) (Miguel) Define $=^{\theta}_{S}\subseteq \theta^\kappa\times\theta^\kappa$, $2\leq\theta\leq\kappa$, as $\eta=^{\theta}_{S}\xi$ if $\{\alpha<\kappa\mid\eta(\alpha)\neq\xi(\alpha)\}\cap S$ is not stationary.
(11) (Miguel) For which values of θ and $S\subseteq\kappa$ stationary, does $=^{\theta}_{X}\hookrightarrow=^{2}_{S}$ hold?
(12) (Miguel) Can the cardinal arithmetic assumptions of the Borel reducibility main gap be relaxed?
(13) (Miguel) Is isomorphism of graphs of size κ Σ^1_1-complete?
(14) (Miguel) Is the an analytic κ-MAD family in ${}^{\kappa}2$?
(15) (Miguel) Is it consistent that the definable sets in $M_{\kappa^+,\kappa}$ are precisely the Borel* sets under isomorphism?
(16) (Philipp S.) Does the version of the Lusin-Novikov uniformisation theorem for ${}^{\kappa}\kappa$ always fail, i.e., can one prove that exists a κ-Borel relation with sections of size $\leq\kappa$ that does not admit a κ-Borel measurable uniformisation?
(17) (Philipp S.) Is the consistency strength of (any version of) the Hurewicz dichotomy for Π^1_1 or projective subset of ${}^{\omega}2$ or ${}^{\kappa}2$ an inaccessible or less? (A flawed proof by Stern claims that ZFC suffices to force this for ${}^{\omega}2$.)
(18) (Claudio) Is there a κ-Kurepa subtree T of ${}^{<\kappa}\kappa$ such that the κ^+-Borel hierarchy for the space $[T]$ collapses? (This means there exists some $\alpha<\kappa^+$ such that every κ-Borel set is $\mathbf{\Sigma}^0_\alpha$.)
(19) (Yurii) Is there a $<\kappa$-closed or $<\kappa$-distributive forcing adding a dominating κ-real without adding κ-Cohen reals?
(20) (Nick) Is the Borel conjecture consistent for κ?

School of Mathematics, University of Bristol, Fry Building, Woodland Road, Bristol, BS8 1UG, UK
Email address: philipp.schlicht@bristol.ac.uk