

# OPEN QUESTIONS ON GENERALISED BAIRE SPACES

ABSTRACT. Open questions collected at the seventh workshop on generalised Baire spaces that took place at the University of Bristol in February 2024.

## 1. OPEN QUESTIONS

- (1) Do all analytic strong measure 0 subsets of  ${}^\kappa 2$  have size  $\leq \kappa$ ?
- (2) Is it consistent that there is a stable unsuperstable theory  $T$  such that  $\cong_T$  is  $\Delta_1^1$ ?
- (3) Are filter reflection and  $=_X^\kappa \hookrightarrow_B =_Y^2$  equivalent?
- (4) (Dorottya) Does the  $\kappa$ -perfect set property for  $\kappa$ -analytic sets (equivalently for closed sets) imply the open graph dichotomy for  $\kappa$ -analytic sets?
- (5) (Dorottya) Does the PSP for closed sets already imply it?
- (6) (Dorottya) Do any of these statements imply  $CCP_\kappa(\Sigma_1^1, D_\alpha)$ , i.e. the version for definable families of closed sets.
- (7) (Dorottya)  $CCP_\kappa(X)$ : For any  $\kappa$ -ideal  $\mathcal{I}$  on  $X$  generated by a family of closed sets, either  $X \in \mathcal{I}$  or there exists a continuous function  $f: {}^\kappa \kappa \rightarrow X$  such that  $f(N_t) \in \mathcal{I}^+$  for all  $t \in {}^{<\kappa} \kappa$ .
- (8) (Dorottya)  $CCP_\kappa(D_\kappa)$ :  $CCP_\kappa(X)$  holds for all subsets  $X$  of  ${}^\kappa \kappa$  definable from a  $\kappa$ -sequence of ordinals.
- (9) (Dorottya) Does  $CCP_\kappa(D_\kappa)$  have at least the consistency strength of a Mahlo cardinal?
- (10) Define  $=_S^\theta \subseteq \theta^\kappa \times \theta^\kappa$ ,  $2 \leq \theta \leq \kappa$ , as  $\eta =_S^\theta \xi$  if  $\{\alpha < \kappa \mid \eta(\alpha) \neq \xi(\alpha)\} \cap S$  is not stationary.
- (11) For which values of  $\theta$  and  $S \subseteq \kappa$  stationary, does  $=_X^\theta \hookrightarrow =_S^2$  hold?
- (12) Can the cardinal arithmetic assumptions of the Borel reducibility main gap be relaxed?
- (13) Is isomorphism of graphs of size  $\kappa$   $\Sigma_1^1$ -complete?
- (14) Is the an analytic  $\kappa$ -MAD family in  ${}^\kappa 2$ ?
- (15) Is it consistent that the definable sets in  $M_{\kappa^+, \kappa}$  are precisely the Borel\* sets under isomorphism?
- (16) (Philipp S.) Does the version of the Lusin-Novikov uniformisation theorem for  ${}^\kappa \kappa$  always fail, i.e., can one prove that exists a  $\kappa$ -Borel relation with sections of size  $\leq \kappa$  that does not admit a  $\kappa$ -Borel measurable uniformisation?
- (17) (Claudio) Is there a  $\kappa$ -Kurepa subtree  $T$  of  ${}^{<\kappa} \kappa$  such that the  $\kappa^+$ -Borel hierarchy for the space  $[T]$  collapses? (This means there exists some  $\alpha < \kappa^+$  such that every  $\kappa$ -Borel set is  $\Sigma_\alpha^0$ .)
- (18) (Yurii) Is there a  $<\kappa$ -closed or  $<\kappa$ -distributive forcing adding a dominating  $\kappa$ -real without adding  $\kappa$ -Cohen reals?
- (19) (Nick) Is the Borel conjecture consistent for  $\kappa$ ?

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