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# Assignment Nr. 1

## 1.1 Primary Ray-Generation for a Perspective Camera Model

To get to the center of image plane from the origin, we have to move in direction of the viewing direction $d$: $o + f \cdot d$. To move in $y$ direction in the image plane, the vector is already given as the up-verctor $u$. As the vector in $x$ direction is perpendicular to both $d$ and $u$, we can construct it by $v = d \times u$. Because $u$ and $d$ are normalized, $v$ is also normalized. To computed how far we have to move in $x$ direction from the center of our image plane for a given coordinate $x$, we compute $x_{diff} = x - \frac{resX}{2}$. Similar for $y$ direction. In combination, we end up with $d_r = o + f \cdot d + u \cdot (y - \frac{resY}{2}) + v \cdot (x - \frac{resX}{2})$

TODO I believe it is

$$d_r = f \cdot d + (x - \frac{resX}{2}) \cdot (d \times u) + (y - \frac{resY}{2}) \cdot u$$

## 1.2 Ray-Surface Intersection

### a)

As the plane is defined by $(p - n) \cdot a = 0$, in order to check if $r(t)$ is in the plane, we have to find a $t$ such that $(r(t) - n) \cdot a = 0$.

$$\begin{aligned}
&(r(t) - a) \cdot n = 0 \\
&((o + t \cdot d) - a) \cdot n = 0 \\
&(o + t \cdot d) \cdot n - a \cdot n = 0 \\
&(o + t \cdot d) \cdot n = a \cdot n \\
&o \cdot n + t \cdot d \cdot n = a \cdot n \\
&t \cdot d \cdot n = a \cdot n - o \cdot n \\
&t \cdot d \cdot n = a \cdot n - o \cdot n \\
&t = \frac{a \cdot n - o \cdot n}{d \cdot n}
\end{aligned}$$

This fraction exists if $d \cdot n \neq 0$, or in other words the vector $d$ of the ray is not parallel to the plane.

### b)

In order for $r(t)$ to intersect a bounding box, it has to exist an $t$, such that all cordinates of $r(t)$ are within the boundaries defined by $min$ and $max$. Looking at the $min$ value of $x$ direction first:

$$\begin{aligned}
o_x + t_{min_x} \cdot d_x &= x_{min} \\
\Rightarrow t_{min_x} &= \frac{x_{min} - o_x}{d_x}
\end{aligned}$$

We see that $t_{min} \geq \frac{x_{min} - o_x}{d_x}$, so that the x coordinate of $r(t)$ can be in our box. If we proceed similar for $y, z$ coordinates, we end up with $t_{min} = \frac{min - o}{d}$, where the division is the element wise division. The same applied to $max$ results in $t_{max} = \frac{max - o}{d}$.
If $max(t_{min}) > min(t_{max})$, then $r(t)$ doesn't pass through our box, since before the last of our coordinates is within our box, the first of the other coordinates is out of the box again. Otherwise, $r(t)$ passes throught our box. The first $t$ where it does so is $max(t_{min})$.

TODO I belive this does not hold for $d_i$ ¡ 0, then the $t_{min_i} and t_{max_i}$ have to be switched.

## c)

There are several ways to do this, here we show a parametical check:
$r(t)$ can only pass through the triangle, if it passes through the plane we get if we extend the triangle. We get the normal of the plane by computing $n = (p_2 - p_1) \times (p_3 - p_1)$.
If we set $a = p_3$, we can compute the intersection point $p$ of $r(t)$ and the plane according to 1.2 a). In order to check if $p$ is on the plane, we check there exist $\alpha, \beta$ such that $p = p_1 + \alpha \cdot (p_2 - p_1) + \beta(p_3 - p_1)$, with $\alpha, \beta \geq 0, \alpha + \beta \leq 1$.

TODO How do you compute $\alpha, \beta$?