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Assignment Nr. 1

1.1 Primary Ray-Generation for a Perspective Camera Model

To get to the center of the image plane from the origin, we have to move in direction of the viewing direction d: $f \cdot d$. To move in y direction in the image plane, the vector is already given as the up-vector u. As the vector in x direction is perpendicular to both d and u, we can construct it by $v = d \times u$. Because u and d are normalized, v is also normalized. To computed how far we have to move in x direction from the center of our image plane for a given coordinate x, we compute $x_{diff} = x - \frac{resX}{2}$. Similar for y direction. In combination, we end up with

$$d_r = f \cdot d + (x - \frac{resX}{2}) \cdot (d \times u) + (y - \frac{resY}{2}) \cdot u$$

With this definition, the origin of the image plane (pixel coordinates 0,0) is at the lower left.

1.2 Ray-Surface Intersection

a)

As the plane is defined by $(p - n) \cdot a = 0$, in order to check if $r(t)$ is in the plane, we have to find a t such that $(r(t) - n) \cdot a = 0$.

$$\begin{aligned}
&(r(t) - a) \cdot n = 0\\
&((o + t \cdot d) - a) \cdot n = 0\\
&(o + t \cdot d) \cdot n - a \cdot n = 0\\
&(o + t \cdot d) \cdot n = a \cdot n\\
&o \cdot n + t \cdot d \cdot n = a \cdot n\\
&t \cdot d \cdot n = a \cdot n - o \cdot n\\
&t \cdot d \cdot n = a \cdot n - o \cdot n\\
&t = \frac{a \cdot n - o \cdot n}{d \cdot n}
\end{aligned}$$

This fraction exists if $d \cdot n \neq 0$, or in other words the vector d of the ray is not parallel to the plane.

b)

For the ray to be possibly in the bounding box for the x-,y-, or z-Direction it needs to hold:

$$\begin{aligned}
min_i \leq r(t)_i \leq max_i\\
min_i \leq o_i + td_i \leq max_i
\end{aligned}$$

for $d_i > 0$ this is:

$$\frac{min_i - o_i}{d_i} \leq t \leq \frac{max_i - o_i}{d_i}$$

for $d_i < 0$ this is:

$$\frac{max_i - o_i}{d_i} \leq t \leq \frac{min_i - o_i}{d_i}$$

If $d_i = 0$ we check if $min_i < o_i < max_i$. If this is the case we set $t_{low\,i} = -\infty$ and $t_{high_i} = \infty$. Else there are no intersections with the bounding box

In this way we have defined ranges for t for each of the dimensions. To find the total range of t we have to take the maximum of the lower ends of the ranges and the minimum of the upper ends of the ranges.

We calculate $t_{low} = \frac{min-o}{d}$ and $t_{high} = \frac{max-o}{d}$ for each dimension (x,y,z) we swap $t_{low\,i}$ and t_{high_i} if $d_i < 0$ to get t'_{low} and t'_{high}.

The ray is in the bounding box for $\max(t'_{low}) \leq t \leq \min(t'_{high})$. If $\max(t'_{low}) > \min(t'_{high})$, then there are no intersections of the ray with the bounding box. The first intersection of the ray with the bounding box is for $\max(t'_{low})$

c)

There are several ways to do this, here we show a parametical check:
$r(t)$ can only pass through the triangle, if it passes through the plane we get if we extend the triangle. We get the normal of the plane by computing $n = (p_2 - p_1) \times (p_3 - p_1)$.
If we set $a = p_3$, we can compute the intersection point p of $r(t)$ and the plane according to 1.2 a). In order to check if p is on the plane, we check there exist α, β such that $p = p_1 + \alpha \cdot (p_2 - p_1) + \beta(p_3 - p_1)$, with $\alpha, \beta \geq 0, \alpha + \beta \leq 1$.

TODO How do you compute α, β?