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Assignment Nr. 1

1.1 Primary Ray-Generation for a Perspective Camera Model

1.2 Ray-Surface Intersection

a)

As the plane is defined by $(p - n) \cdot a = 0$, in order to check if $r(t)$ is in the plane, we have to find a t such that $(r(t) - n) \cdot a = 0$.

$$
\begin{aligned}
&(r(t) - n) \cdot a = 0 \\
&((o + t \cdot d) - a) \cdot n = 0 \\
&(o + t \cdot d) \cdot n - a \cdot n = 0 \\
&(o + t \cdot d) \cdot n = a \cdot n \\
&o \cdot n + t \cdot d \cdot n = a \cdot n \\
&t \cdot d \cdot n = a \cdot n - o \cdot n \\
&t \cdot d \cdot n = a \cdot n - o \cdot n \\
&t = \frac{a \cdot n - o \cdot n}{d \cdot n}
\end{aligned}
$$

This fraction exists if $d \cdot n \neq 0$, or in other words the vector d of the ray is not parallel to the plane.

b)

In order for $r(t)$ to intersect a bounding box, it has to exist an t, such that all cordinates of $r(t)$ are within the boundaries defined by min and max. Looking at the min value of x direction first:

$$
\begin{aligned}
o_x + t_{min_x} \cdot d_x &= x_{min} \\
\Rightarrow t_{min_x} &= \frac{x_{min} - o_x}{d_x}
\end{aligned}
$$

We see that $t_{min} \geq \frac{x_{min} - o_x}{d_x}$, so that the x coordinate of $r(t)$ can be in our box. If we proceed similar for y, z coordinates, we end up with $t_{min} = \frac{min - o}{d}$, where the division is the element wise division. The same applied to max results in $t_{max} = \frac{max - o}{d}$.
If $max(t_{min}) > min(t_{max})$, then $r(t)$ doesn't pass through our box, since before the last of our coordinates is within our box, the first of the other coordinates is out of the box again. Otherwise, $r(t)$ passes throught our box. The first t where it does so is $max(t_{min})$.

c)

There are several ways to do this, here we show a parametical check:
$r(t)$ can only pass through the triangle, if it passes through the plane we get if we extend the triangle. We get the normal of the plane by computing $n = (p_2 - p_1) \times (p_3 - p_1)$.
If we set $a = p_3$, we can compute the intersection point p of $r(t)$ and the plane according to 1.2 a). In order to check if p is on the plane, we check there exist α, β such that $p = p_1 + \alpha \cdot (p_2 - p_1) + \beta(p_3 - p_1)$, with $\alpha, \beta \geq 0, \alpha + \beta \leq 1$.