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Assignment Nr. 4

4.1 Radiometry

Let w' be the direction from x to d

$$\begin{aligned}
 E &= \int_{\omega} L(x, w) \cos(\theta) d\omega \\
 &= L(x, w') \cos(\theta) \\
 &= \frac{d^2 \Phi \cos(\theta)}{dA \cos(\theta) d\omega}
 \end{aligned}$$

as a point is perpendicular to another $\cos = 1$

$$= \frac{d^2 \Phi \cos(\theta)}{dA d\omega}$$

Assume $dA = 1$, then $d\omega = 4\pi r^2$

$$= \frac{d^2 \Phi \cos(\theta)}{4\pi r^2}$$

The radius is given by $r = \sqrt{r^2 + d^2}$

$$= \frac{d^2 \Phi \cos(\theta)}{4\pi r^2 + d^2}$$

4.2 Analytical solution of the rendering equation in 2D

a)

$$\begin{aligned}
 L(x, \omega_o) &= L_e(x, \omega_o) + \int_0^\pi f_r(\omega_i, x, \omega_o) L(x, \omega_i) \cos(\theta_i) d\omega_i \\
 &= 0 + \int_0^\pi \frac{1}{2} L(x, \omega_i) \cos(\theta_i) d\omega_i \\
 &= \frac{1}{2} \int_0^\pi L(x, \omega_i) \cos(\theta_i) d\omega_i \\
 &= \frac{1}{2} \int_0^\pi L(x, \omega_i) \cos(\theta_i) d\omega_i \\
 &= \frac{1}{2} \left(\int_0^{\tan^{-1}(\frac{1}{p+1})} L(x, \omega_i) \cos(\theta_i) d\omega_i + \int_{\tan^{-1}(\frac{1}{p+1})}^{\pi - \tan^{-1}(\frac{1}{1-p})} L(x, \omega_i) \cos(\theta_i) d\omega_i + \int_{\pi - \tan^{-1}(\frac{1}{1-p})}^\pi L(x, \omega_i) \cos(\theta_i) d\omega_i \right)
 \end{aligned}$$

The first and the third integral refer to incident angles that point past the light source, therefore they are zero.

$$\begin{aligned}
 &= \frac{1}{2} \int_{\tan^{-1}(\frac{1}{p+1})}^{\pi - \tan^{-1}(\frac{1}{1-p})} 1 \cdot \cos(\theta_i) d\omega_i \\
 &= \frac{1}{2} \int_{\tan^{-1}(\frac{1}{p+1})}^{\pi - \tan^{-1}(\frac{1}{1-p})} \cos(|\omega_i - \frac{\pi}{2}|) d\omega_i \\
 &= \frac{1}{2} \left(\int_{\tan^{-1}(\frac{1}{p+1})}^{\frac{\pi}{2}} \cos(\frac{\pi}{2} - \omega_i) d\omega_i + \int_{\frac{\pi}{2}}^{\pi - \tan^{-1}(\frac{1}{1-p})} \cos(\omega_i - \frac{\pi}{2}) d\omega_i \right)
 \end{aligned}$$

4.3 Simple Path Tracer