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Assignment Nr. 12

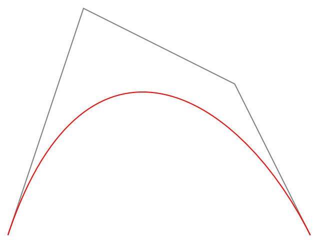
1 De Casteljau

- $u_0 = 0.5$

$$\begin{aligned}
b_0^3(0.5) &= 0.5 \cdot b_1^2(t) + 0.5 \cdot b_0^2 \\
&= 0.5 \cdot (0.5 \cdot b_1^1(t) + 0.5 \cdot b_2^1) + 0.5 \cdot (0.5 \cdot b_0^1(t) + 0.5 \cdot b_1^1) \\
&= 0.5 \cdot (0.5 \cdot (0.5 \cdot b_1^0(t) + 0.5 \cdot b_0^0) + 0.5 \cdot (0.5 \cdot b_2^0(t) + 0.5 \cdot b_1^0)) \\
&+ 0.5 \cdot (0.5 \cdot (0.5 \cdot b_2^0(t) + 0.5 \cdot b_1^0) + 0.5 \cdot (0.5 \cdot b_3^0(t) + 0.5 \cdot b_2^0)) \\
&= 0.5 \cdot (0.5 \cdot (0.5 \cdot \begin{pmatrix}1\\3\end{pmatrix} + 0.5 \cdot \begin{pmatrix}0\\0\end{pmatrix}) + 0.5 \cdot (0.5 \cdot \begin{pmatrix}3\\2\end{pmatrix} + 0.5 \cdot \begin{pmatrix}1\\3\end{pmatrix})) \\
&+ 0.5 \cdot (0.5 \cdot (0.5 \cdot \begin{pmatrix}3\\2\end{pmatrix} + 0.5 \cdot \begin{pmatrix}1\\3\end{pmatrix}) + 0.5 \cdot (0.5 \cdot \begin{pmatrix}4\\0\end{pmatrix} + 0.5 \cdot \begin{pmatrix}3\\2\end{pmatrix})) \\
&= 0.5 \cdot (0.5 \cdot \begin{pmatrix}0.5\\1.5\end{pmatrix} + 0.5 \cdot \begin{pmatrix}2\\2.5\end{pmatrix}) + 0.5 \cdot (0.5 \cdot \begin{pmatrix}2\\2.5\end{pmatrix} + 0.5 \cdot \begin{pmatrix}3.5\\1\end{pmatrix}) \\
&= 0.5 \cdot \begin{pmatrix}1.25\\2\end{pmatrix} + 0.5 \cdot \begin{pmatrix}2.25\\1.75\end{pmatrix} \\
&= \begin{pmatrix}1.75\\1.875\end{pmatrix}
\end{aligned}$$

- $u_1 = 0.75$

$$\begin{aligned}
b_0^1 &= 0.75 \begin{pmatrix}1\\3\end{pmatrix} + 0.25 \begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0.75\\2.25\end{pmatrix} \\
b_1^1 &= 0.75 \begin{pmatrix}3\\2\end{pmatrix} + 0.25 \begin{pmatrix}1\\3\end{pmatrix} = \begin{pmatrix}2.5\\2.25\end{pmatrix} \\
b_2^1 &= 0.75 \begin{pmatrix}4\\0\end{pmatrix} + 0.25 \begin{pmatrix}3\\2\end{pmatrix} = \begin{pmatrix}3.75\\0.5\end{pmatrix} \\
b_0^2 &= 0.75 \begin{pmatrix}2.5\\2.25\end{pmatrix} + 0.25 \begin{pmatrix}0.75\\2.25\end{pmatrix} = \begin{pmatrix}2.05\\2.25\end{pmatrix} \\
b_1^2 &= 0.75 \begin{pmatrix}3.75\\0.5\end{pmatrix} + 0.25 \begin{pmatrix}2.5\\2.25\end{pmatrix} = \begin{pmatrix}3.4375\\0.9375\end{pmatrix} \\
b_0^3 &= 0.75 \begin{pmatrix}3.4375\\0.9375\end{pmatrix} + 0.25 \begin{pmatrix}2.05\\2.25\end{pmatrix} = \begin{pmatrix}3.090625\\1.265625\end{pmatrix}
\end{aligned}$$

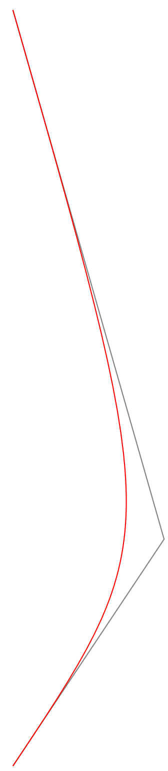


2 Derivative of a Bezier Curve

2.1 a)

$$
\begin{aligned}
\frac{\partial P(t)}{\partial t} &= \frac{\partial \sum_{i=0}^{n} B_i^n(t) b_i}{\partial t} \\
&= \sum_{i=0}^{n} \frac{\partial B_i^n(t) b_i}{\partial t} \\
&= \sum_{i=0}^{n} b_i \frac{\partial B_i^n(t)}{\partial t} \\
&= n \sum_{i=0}^{n} B_i^{n-1}(t)(b_{i+1} - b_i)
\end{aligned}
$$

b)



Points are given as: $q_0 = 4(b_1 - b_0) = (4, 12), q_1 = (8, -2), q_2 = (4, -8)$.

c)

C^∞

3 Hermite curves

a)

The points b_0, b_3 a simply given by P_0, P_1. In the lecture, we saw that we can convert from Bezier to Hermite via: $P_0' = 3(b_1 - b_0)$. Therefore, to get b_1, we can solve for it:

$$\begin{aligned} P_0' &= 3(b_1 - b_0) \\ P_0'/3 &= b_1 - b_0 \\ P_0'/3 + b_0 &= b_1 \\ P_0'/3 + P_0 &= b_1 \end{aligned}$$

Similar works for b_2. Therefore:

$$\begin{aligned} G_B &= \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/3 \end{pmatrix} \cdot \begin{pmatrix} P_0 \\ P_1 \\ P_0' \\ P_1' \end{pmatrix} \end{aligned}$$

b)

$$\begin{aligned} G_H &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -3 & 0 & 0 \\ 0 & 0 & -3 & -3 \end{pmatrix} \cdot \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & 3 & 0 & 0 \\ 0 & 0 & -3 & 3 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ 1 & 3 \\ 3 & 2 \\ 4 & 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 \\ 1 & 3 \\ 3 & 9 \\ 3 & -6 \end{pmatrix} \end{aligned}$$

4 Splines

It is not possible to draw a circle with a uniform B-Spline, but it is possible to draw a very good apprximation of a circle, which is almost indistinguishable.

This is the case, because B-Splines are defined as piecewise polynomial functions, which can not exactly represent the arc of a circle, which would require the capability to represent a square root.

5 Random questions

a)

As high frequencies represent the sharp edges in an image, applying a high pass filter will sharpen the edges.

b)

The Nyquist frequency is the highest frequency that can be represented. It is the half of the sampling frequency. As soon as frequencies higher than Nyquist are present, an aliasing effect occurs.