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Assignment Nr. 4

4.1 Radiometry

a)

As the point light source emits Φ_S Watts in every direction, its radiance is $L_o(d,\omega) = \frac{\Phi_S}{4\pi}$ in every direction. Because it is the only light source in the scene which emits light to x from the direction ω_i, the integral for the irradiance of x $I_x = \int_\omega L(x,w)cos(\theta)d\omega$ collapses to $I_x = L_o(d,\omega_i)cos(\theta)$. This would assume however that the light source sits on the unit circle around x. Therefore, we have to normalize by the distance between d and x, resulting in $I_x = \frac{L_o(d,\omega_i)cos(\theta)}{|x-d|} = \frac{L_o(d,\omega_i)cos(\theta)}{r^2+d^2} = \frac{\Phi_S cos(\theta)}{4\pi(r^2+d^2)}$

Let w' be the direction from x to d

$$
\begin{aligned}
E &= \int_\omega L(x,w)cos(\theta)d\omega \\
&= L(x,w')cos(\theta) \\
&= \frac{d^2\Phi cos(\theta)}{dAcos(\theta)d\omega} \\
\text{as a point is perpendicular to another cos } =1 \\
&= \frac{d^2\Phi cos(\theta)}{dA1d\omega} \\
\text{Assume } dA = 1\text{, then } dw = 4\pi r^2 \\
&= \frac{d^2\Phi cos(\theta)}{4\pi r^2} \\
\text{The radius is given by } r = \sqrt{r^2+d^2} \\
&= \frac{d^2\Phi cos(\theta)}{4\pi r^2 + d^2}
\end{aligned}
$$

a)

$\omega = (\theta, \phi)$

$$
\begin{aligned}
E &= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} L(x,\omega)\cos\theta\sin\theta d\theta d\phi \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{d^2\Phi}{dA\cos(\theta)d\omega}\cos\theta\sin\theta d\theta d\phi \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{d^2\Phi}{\sin(\theta)d\theta d\phi \cdot r^2\cos(\theta)d\omega}\cos\theta\sin\theta d\theta d\phi \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{d^2\Phi}{r^2 d\omega} \\
&= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \frac{d^2\Phi}{r^2 d\theta d\phi} \\
&= \\
&= \frac{\Phi_S\cos(\theta)}{4\pi(r^2+d^2)}
\end{aligned}
$$

4.2 Analytical solution of the rendering equation in 2D

a)

$$
\begin{aligned}
L(x,\omega_o) &= L_e(x,\omega_o) + \int_0^{\pi} f_r(\omega_i,x,\omega_o)L(x,\omega_i)cos(\theta_i)d\omega_i \\
&= 0 + \int_0^{\pi}\frac{1}{2}L(x,\omega_i)cos(\theta_i)d\omega_i \\
&= \frac{1}{2}\int_0^{\pi} L(x,\omega_i)cos(\theta_i)d\omega_i \\
&= \frac{1}{2}\int_0^{\pi} L(x,\omega_i)cos(\theta_i)d\omega_i \\
&= \frac{1}{2}(\int_0^{tan^{-1}(\frac{1}{p+1})} L(x,\omega_i)cos(\theta_i)d\omega_i + \int_{tan^{-1}(\frac{1}{p+1})}^{\pi-tan^{-1}(\frac{1}{1-p})} L(x,\omega_i)cos(\theta_i)d\omega_i + \int_{\pi-tan^{-1}(\frac{1}{1-p})}^{\pi} L(x,\omega_i)cos(
\end{aligned}
$$

The first and the third integral refer to incident angles that point past the light source, therefore t

$$
\begin{aligned}
&= \frac{1}{2}\int_{tan^{-1}(\frac{1}{p+1})}^{\pi-tan^{-1}(\frac{1}{1-p})} 1\cdot cos(\theta_i)d\omega_i \\
&= \frac{1}{2}\int_{tan^{-1}(\frac{1}{p+1})}^{\pi-tan^{-1}(\frac{1}{1-p})} cos(|\omega_i-\frac{\pi}{2}|)d\omega_i \\
&= \frac{1}{2}\left(\int_{tan^{-1}(\frac{1}{p+1})}^{\frac{\pi}{2}} cos(\frac{\pi}{2}-\omega_i)d\omega_i + \int_{\frac{\pi}{2}}^{\pi-tan^{-1}(\frac{1}{1-p})} cos(\omega_i-\frac{\pi}{2})d\omega_i\right)
\end{aligned}
$$

b)

$$
\begin{aligned}
L(x,\omega_o) &= L_e(x,\omega_o) + \int_{y\in S} f_r(\omega_i,x,\omega_o)\cdot L(y,-\omega_i(x,y))\cdot \frac{\cos\phi_i\cos\phi_y}{|x-y|}\cdot dy_x \\
&= L_e(x,\omega_o) + \int_{-1}^{1} f_r(\omega_i,x,\omega_o)\cdot L(y,-\omega_i(x,y))\cdot \frac{\cos\phi_i\cos\phi_y}{|x-y|}\cdot dy_x \\
&\text{If we are not in a light source this is:} \\
&= \int_{-1}^{1} f_r(\omega_i,x,\omega_o)\cdot L(y,-\omega_i(x,y))\cdot \frac{\cos\phi_i\cos\phi_y}{|x-y|}\cdot dy_x \\
&= \int_{-1}^{1} f_r(\omega_i,x,\omega_o)\cdot L(y,-\omega_i(x,y))\cdot \frac{\cos\phi_i\cos\phi_y}{\sqrt{(x_x-y_x)^2+(x_y-y_y)^2}}\cdot dy_x \\
&= \int_{-1}^{1} f_r(\omega_i,x,\omega_o)\cdot 1\cdot \frac{\cos\phi_i\cos\phi_y}{\sqrt{(x_x-y_x)^2+(x_y-y_y)^2}}\cdot dy_x \\
&=
\end{aligned}
$$

4.3 Simple Path Tracer