Probabilistic Machine Learning

Exercise Sheet #1

due on Tuesday 27 April 2021, 10am sharp

1. **EXAMple Question** The lecture introduced the two foundational rules of probabilistic reasoning, called

   (a) the Sum Rule and

   (b) the Product Rule, as well as their direct consquence,

   (c) Bayes' Theorem

   Formally state all three of them.

2. **Theory Question** Assume that, if A is true, B becomes more plausible. That is,

   $$P(B|A) \geq P(B).$$

   Using the rules of probability stated above, show the following relationships (stated in the lecture without proof)

   (a) $P(B|\neg A) \leq P(B)$ ("If A is false, B becomes *less* plausible")

   (b) $P(A|B) \geq P(A)$ ("If B is true, A becomes *more* plausible")

   (c) $P(A|\neg B) \leq P(A)$ ("If B is false, A becomes *less* plausible")

   Additionally show that probabilistic reasoning includes Boolean logic as a special case, by showing that if $A \Rightarrow B$ is interpreted as equivalent to $P(B \mid A) = 1$, then the following two statements hold:

   (d) $P(\neg A \mid \neg B) = 1$ ("modus tollens")

   (e) $P(B \mid \neg A) \leq P(B)$ ("If A is false, B becomes less plausible")

   (f) $P(A \mid B) \geq P(A)$ ("if B is true, A becomes more plausible")

3. **Practical Question** In previous lecture courses you have become accustomed with various models of *Deep Learning*. In popular texts it can sometimes sound as if deep learning has made all other concepts of machine learning obsolete. The point of this exercise is to reflect on this sentiment. On Ilias you can find a jupyter notebook that loads the famous "Keeling curve", a time series of atmospheric CO_2 concentrations collected at the Mauna Loa observatory in Hawaii. Your task is, using only a deep learning framework (like TensorFlow, pyTorch, etc.) to produce an *extrapolation* of this dataset from today 40 years into the future, until 2060. You are free to use whichever architecture you like, but try to use only 'deep neural networks' (that's a vague term, of course).