

Final Report - Research Internship

Philipp Weder

1 Introduction

2 Modelling of the swimmer as a control problem

We restrict ourselves to considering the swimmer **SPr4** proposed in [1]. Let (S_1, S_2, S_3, S_4) be a regular reference tetrahedron centered at $c \in \mathbb{R}^3$ such that $\text{dist}(c, S_i) = 1$ for all $i \in \mathbb{N}_4$. Then the swimmer consists of four balls $(B_i)_{i \in \mathbb{N}_4}$ of \mathbb{R}^3 centered at $b_i \in \mathbb{R}^3$, all of radius $a > 0$, such that the ball B_i can move along the ray starting at c and passing through S_i . This reflects the situation where the balls are linked together by think jacks that are able to elongate. However, the viscous resistance of these jacks is neglected and therefore the fluid is assumed to permeate the entire open set $\mathbb{R}^3 \setminus \bigcup_{i=1}^4 \overline{B}_i$. The balls do not rotate around their arms which implies that the shape of the swimmer is completely determined by the four lengths $\xi_1, \xi_2, \xi_3, \xi_4$ of its arms, measured from the c to the center of each ball b_i . However, there are no restrictions for the rotation of the swimmer around the center c , i.e. for fixed arm lengths, the swimmer is considered to be a rigid body in a Stokesian fluid. Hence, the geometrical configuration of the swimmer can be described by two sets of variables:

- (i) The vector of *shape variables* $\xi := (\xi_1, \xi_2, \xi_3, \xi_4) \in \mathcal{M} := (\sqrt{\frac{3}{2}}a, +\infty)^4 \subseteq \mathbb{R}_+^4$, where the lower bound in the open intervals is chosen such that the balls cannot overlap.
- (ii) The vector of *position variables* $p = (c, R) \in \mathcal{P} := \mathbb{R}^3 \times SO(3)$.

To be more precise, we consider the reference tetrahedron convexly spanned by the four unit vectors $z_1 := (2\sqrt{2}/3, 0, -1/3)$, $z_2 := (-\sqrt{2}/3, -\sqrt{2}/3, -1/3)$, $z_3 := (-\sqrt{2}/3, \sqrt{2}/3, -1/3)$ and $z_4 := (0, 0, 1)$. Position and orientation in \mathbb{R}^3 are then described by the coordinates of the center $c \in \mathbb{R}^3$ and the rotation $R \in SO(3)$ of the swimmer with respect to the reference orientation induced by the reference tetrahedron. Thus, we set $b_i := c + \xi_i R z_i$ for the center of the ball B_i .

The swimmer is completely described by the parameters $(\xi, p) \in \mathcal{M} \times \mathcal{P}$. Indeed, if we denote by B_a the ball in \mathbb{R}^3 of radius a centered at the origin, then for any $r \in \partial B_a$, the position of the current point on the i -th sphere of the swimmer in the state (ξ, p) is given, for any $(\xi, p, r) \in \mathcal{M} \times \mathcal{P} \times \partial B_a$, by the function

$$r_i(\xi, p, r) := c + R(\xi_i z_i + r). \quad (1)$$

Note that the functions $(r_i)_{i \in \mathbb{N}_4}$ are analytic in $\mathcal{M} \times \mathcal{P}$ and thus we can use them to calculate the instantaneous velocity on the i -th sphere B_i , which for any $(\xi, p, r) \in \mathcal{M} \times \mathcal{P} \times \partial B_a$ and every $i \in \mathbb{N}_4$ is given by

$$u_i(\xi, p, r) = \dot{c} + \omega \times (\xi_i z_i + r) + R z_i \dot{\xi}_i, \quad (2)$$

where ω is the axial vector associated with the skew matrix $\dot{R}R$.

In [1] it is shown that the system **SPr4**, i.e. both the shape ξ and the position p , is controllable only using the rate of change $\dot{\xi}$ of the shape. To do so, we have to understand how p changes when we vary $\dot{\xi}$. To that and, the assumptions of *self-propulsion* and negligible inertia of the swimmer are made. They imply that the total viscous force and torque exerted by the surrounding fluid on the swimmer must vanish. More precisely, the system can be written as

$$\dot{p} = F(R, \xi) \dot{\xi} \text{ with } F(R, \xi) := [f_1(R, \xi) | f_1(R, \xi) | f_1(R, \xi) | f_1(R, \xi) | f_1(R, \xi) | f_1(R, \xi)]^T \quad (3)$$

3 Symmetries

4 The control problem in the regime of small strokes

5 Energy optimizing strokes

References

- [1] F. Alouges, A. DeSimone, L. Heltai, A. Lefebvre-Lepot, and B. M. and, “Optimally swimming stokesian robots,” *Discrete & Continuous Dynamical Systems - B*, vol. 18, no. 5, pp. 1189–1215, 2013. DOI: 10.3934/dcdsb.2013.18.1189.