STIR and Tensorflow

Philipp Windischhofer

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The big picture

- STIR
- iterative reconstruction algorithms
- find the optimum of a *cost function*

$$P(\text{global LOR response}|\text{image}) = \prod_{\text{all LORs}} P(\text{LOR response}|\text{image})$$

$$P(\text{LOR response}|\text{image}) = \int_{\text{LOR}} \text{image density}$$

- Image (estimate) $\rightsquigarrow$ LOR response: *forward projection*

Ray tracing

- in the following: how to compute / approximate this integral?
- represent *image* as a discretized array of voxels

$$\int_{\text{LOR}} \text{image density} \rightsquigarrow \sum_{\text{traversed voxels}} \text{LOI} \cdot \text{voxel brightness}$$

- LOI = length of intersection of the LOR through a specific voxel

(Standard) STIR spends $> 90\%$ of the reconstruction time for *forward projection*

- is there a way to make it faster?

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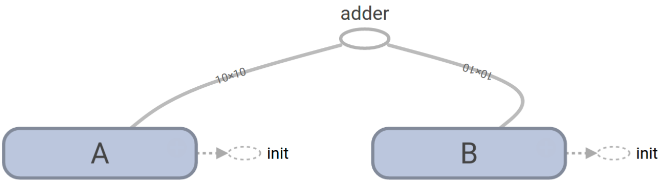
Tensorflow

- *Google*: "An open-source software library for machine intelligence"
  - heavily used in the machine-learning community
- Much more than that:
  - a software package for numerical calculations using data-flow *graphs*



  - Design the graph *now*, save it to a file, run it *later*...
  - Can run it (almost) everywhere: CPU, GPU, a cluster, ...

How to do ray tracing on a CPU?

Siddon's algorithm (2D)

- a (very) sequential algorithm

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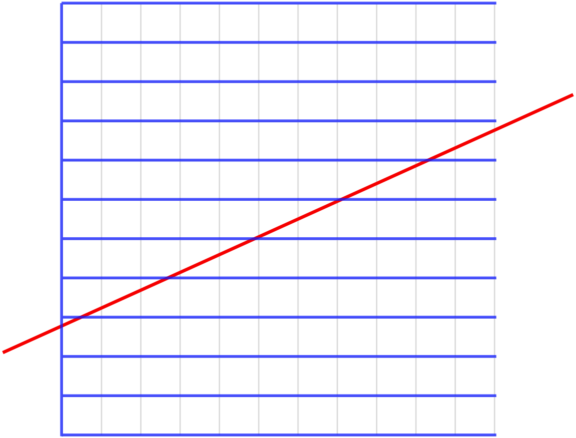

- get intersections of LOI with all horizontal planes

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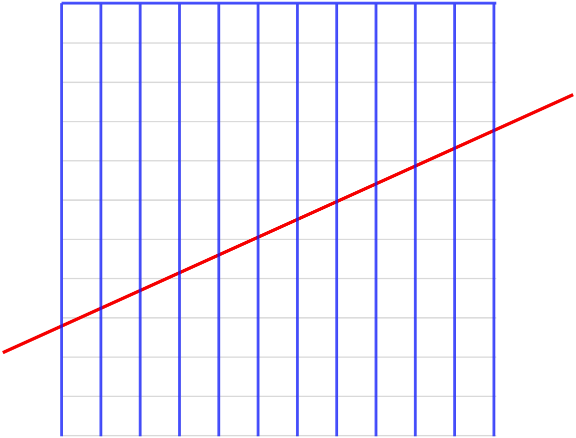

- get intersections of LOI with all vertical planes

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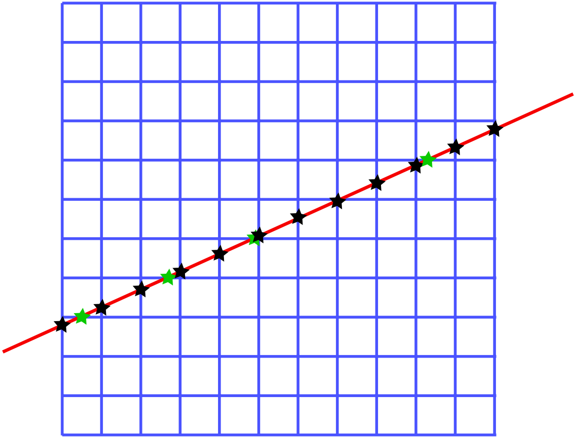


- combine them and get the individual LOIs

How to do ray tracing on a CPU?

Siddon's algorithm (2D)

- parametrize the LOR

$$\mathbf{x} = (\mathbf{x}_{\mathbf{end}} - \mathbf{x}_{\mathbf{start}}) \cdot \alpha + \mathbf{x}_{\mathbf{start}}$$

$$\alpha \in [0, 1]$$

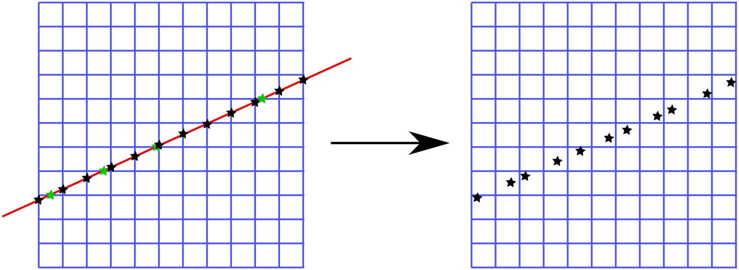
- get intersections with horizontal and vertical planes in terms of the line parameter α
- merge and sort them
- *differences* between neighbouring α values in the merged list describe the LOIs!

Siddon and Tensorflow

- Siddon: $\mathcal{O}(N)$ for a LOR of length N (this means a voxel array of size N^3!!)
- many branches, a loop over the LOR, ...
- very inefficient when running with Tensorflow (GPU $\neq$ CPU architecture)
  - especially problematic when trying to trace multiple LORs "in parallel"
- for Tensorflow: need an algorithm that ...
  - uses the same algorithmic steps on *all* voxels of the LOR
  - uses the same algorithmic steps on *all* LORs

Alternatives to Siddon

- naive approach: for every LOR, compute the LOI through *every* voxel, then add them all up
  - works fine with Tensorflow, but scales as $\mathcal{O}(N^3)$
  - CPU & Siddon beat it again for real-world array sizes
- alternative approach: think of (intersection) points *without* the notion of an LOR



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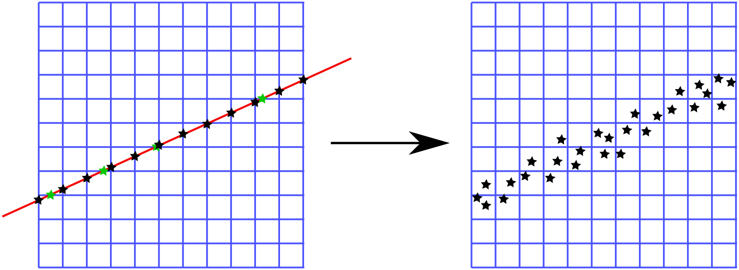
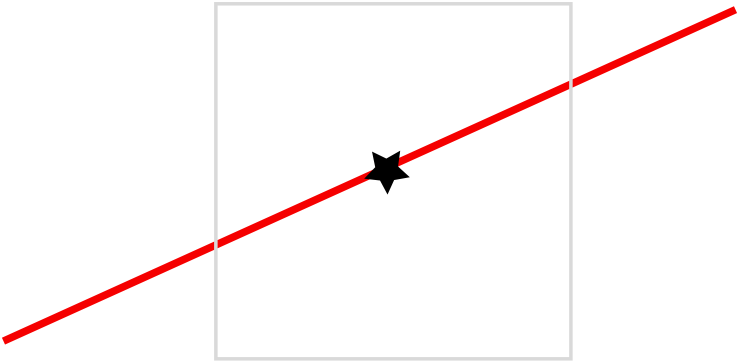


Voxel-by-voxel LOI calculation

- still not much easier than the full problem!
- a cube is very discontinuous, hard to do it analytically without branches
- what if content with an *approximate* solution?



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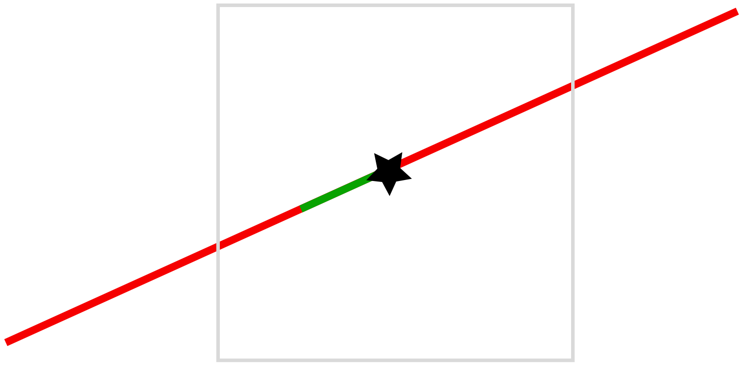

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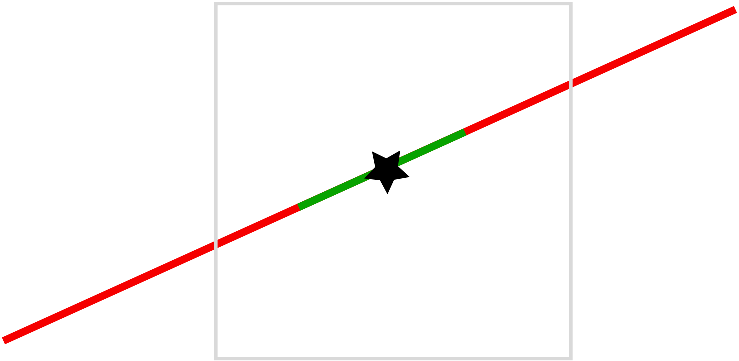

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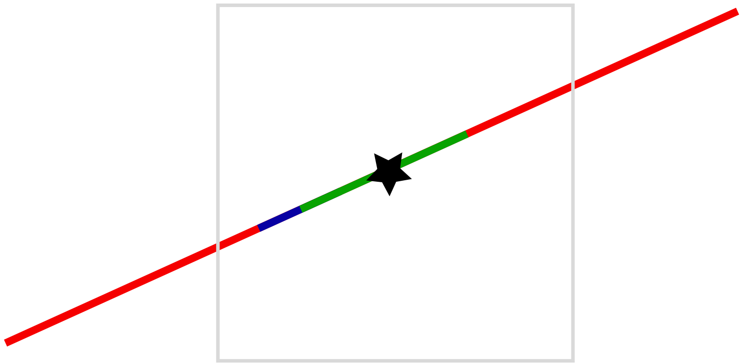

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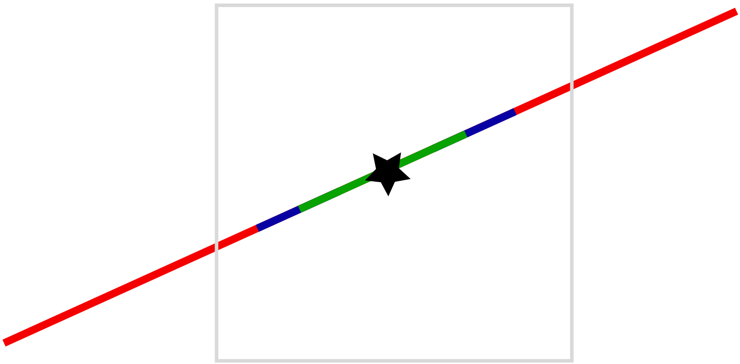


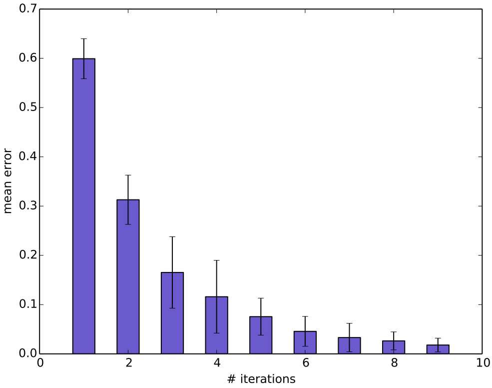
Ray marching: an iterative algorithm

- Use *signed distance function* of a cube to cut down the number of iterations that are needed
  - a heuristic that produces an *underestimation* of the distance to the voxel boundary
- need to assess the residual error that remains

Ray marching: an iterative algorithm

Accuracy

- assume: have "enough" points along the LOR



- with 6 iterations, are already at $\sim 5\%$ level!

Ray marching: an iterative algorithm

Accuracy

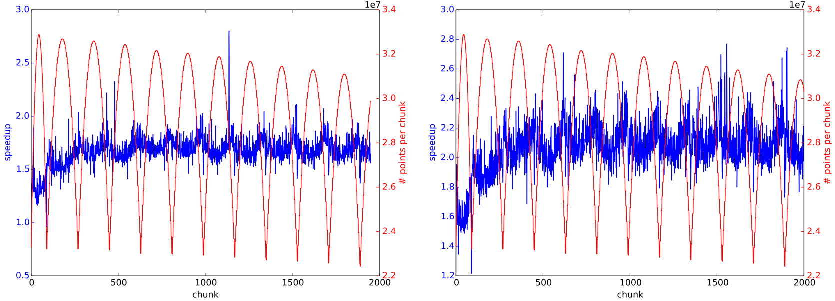
- are there ever enough points?
- does not matter for STIR!

Bringing together STIR and Tensorflow

The infrastructure

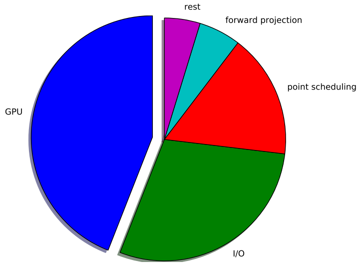
- Build the Tensorflow graph in a Python script
  - much easier and faster to prototype
  - Python API much better documented and more stable
- <u>Save</u> the complete graph into a `ProtoBuf` file
- <u>Load</u> the file using the C++ API and make use of it in STIR
  - link STIR against the Tensorflow (shared) library

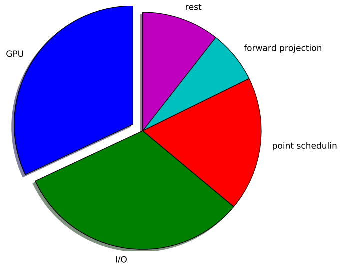
Speedup without caching



left: 6 iterations, **right**: 2 iterations

- average speedup is similar





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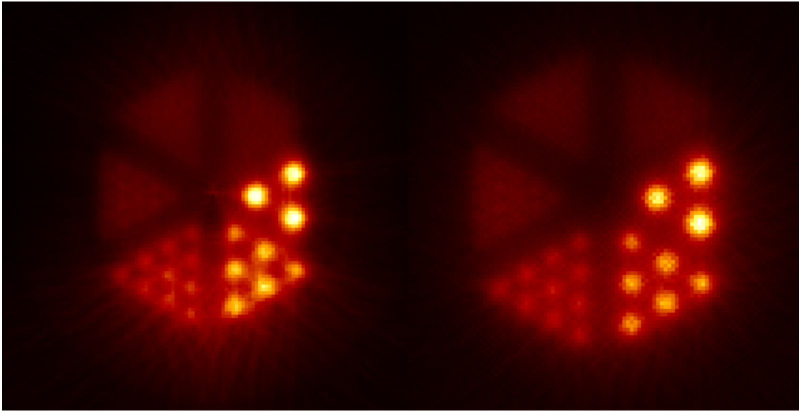
- *I/O*: converting from `ProjMatrixElemsForOneBin` to `Tensor` and back
- *point scheduling*: choose points to sample the TOR / LOR

Images



left: 2 iterations, 20LORs per matrix element, **right**: original STIR

- some artefacts: too few points

How to proceed?

- demonstrated a proof-of-concept of integrating Tensorflow into STIR
- whole toolchain is in place
  - separation of graph generation (Python) and usage (C++) works very well
- Speedup of ~ 2 observed

Not too bad, but still some limitations:

- how to improve utilization of GPU (conversion between Tensors and STIR objects)?
- is there a better way to estimate the matrix elements?
- so far, use only a single thread (no `openMP` support)
  - Tensorflow sessions tend to allocate all of the available GPU memory to avoid fragmentation...

Where to find the code?

- STIR-TF: `https://github.com/philippwindischhofer/STIR/tree/stir-tf`
  - in particular: `https://github.com/philippwindischhofer/STIR/blob/stir-tf/documentation/statistics/doc/main.pdf`
- ray tracing scripts: `https://gitlab.phys.ethz.ch/luster/tf-raytracing`

Any comments and contributions are welcome!