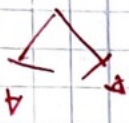
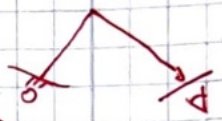
Lecture 1: Basics

Capture Devices

Passive:
- + work indoors + outdoors
- − Need Compute
- − Features needed
- → RGB
- → Stereo and Multi-View



Active:
- + No features needed
- − Fails outdoors
- − Sensitiv to scattering.
- → ToF (time of flight) − slow + very precise
- → Structured light
- → LiDAR.



↳ Same as ToF but needs precise calibration between projector and sensor

Stereo Matching: Sparse vs. Dense Matching

Wider dist. between cameras: ↑ distortion, ↑ occlussions
Smaller dist between cameras: ↑ error

Feature extraction vs. Not.

Search window: smaller = more noise
larger = less details

Correspondence Failures: 1) Non-Lambertian surfaces ⇒ Specular reflection (= not diffuse)
2) Occlussions/Repititions
3) Textureless Surface

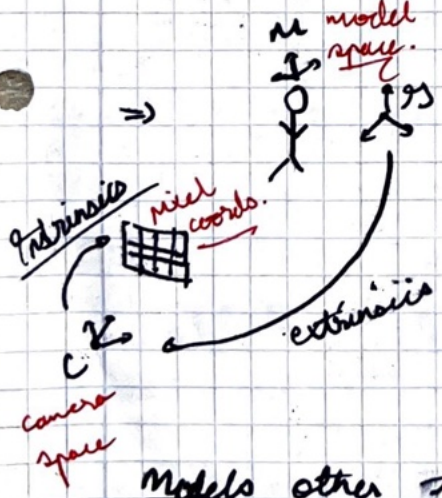
Camera Concepts

→ Intrinsics:
- → Focal Length
- → Principal Point.
- → (Skew)
- → (Distortion)

$$K = \begin{pmatrix} f_x & \gamma & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{pmatrix}$$

γ: shear coefficient
c_x, c_y: principal point or optical center.
$f_x f_y$: focal length.

→ Extrinsics: R, t.



$\Rightarrow K\vec{x_c} = z_c\vec{u}$

Models other than Pinhole

→ Radial + Tangential Distortion: use approximate values using camera calibration.

Fish eye lens.

↳ image plane not parallel to lens

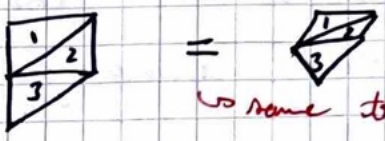
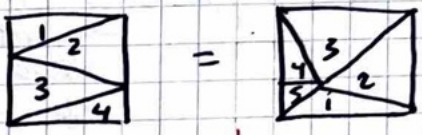
Lecture 2: Surface Representations

Polygonal Meshes:

obviously efficient rendering

- Error in shape representation: $O(\frac{1}{k})$ ↝ i.e. doubling (2x) the # of edges gives $k(\frac{1}{2})^2$ reduction in error.
- Allows for recursive subdivisions
  ↳ i.e. higher resolution

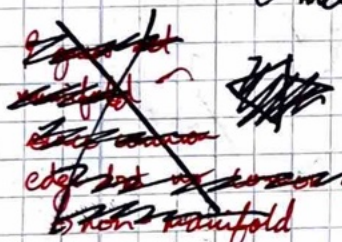
Topology (triangulation) vs. Geometry

Topologically equivalent ⇕ Geometrically Different	 ↝ same triangulation
Geometrically equivalent ⇕ Topologically Different	 ↳ different triangulation but same rectangle.

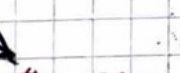
Manifolds

* Triangle mesh is called manifold if:

(i) Intersection of two triangles are (+ have):
- * empty ↝ i.e. no area.
- * common vertex
- * common edge.



here intersection spans the area of small triangle ⇒ not empty. ⇒ Non-manifold



(ii) Edges have:
- * One adjacent triangle → border edge
- * Two adjacent triangles → inner edge

edge ab has three adjacent triangle ⇒ Non-manifold

(iii) Vertices:
- * Build a single open fan → border vertex
- * Build a single closed fan → inner vertex.

vertex connected to two open fans ⇒ non-manifold

vertex is connected to TWO closed fans ⇒ Non-manifold

Mesh Data Structures

(i) Shared Vertex
- * OFF Obj STL etc. ↝ PLY allows for binary storage

Shared Vertex data structure:
- * list v → list f.
  - — — — } vertices
  - 3 — — — 3 pos.

v ...
v ...
f ...

* (ii) Half-Edge Data Structure ↝ only accepts manifold representations (since stores adjacent opp. edges)
  ↳ store vertices + faces + edges (i.e. opposite edge adjacent edge etc.)

Useful for geometric computations ☺

Surfaces

(i) Explicit surfaces: $f: \mathbb{R}^2 \to \mathbb{R}$ ↝ for terrain in games
i.e. give me x, y and I give height

(ii) Parametric surfaces:

Constructed from Bezier Curves → Bezier patches

Used in CAD. ✓ design + construction etc.

Bezier curve ✓

↝ bezier patch:
$f(u,v): \mathbb{R}^2 \to \mathbb{R}^3$
$\in C$

(iii) Constructive Solid Geometry:

Boundaries created from boolean operations:

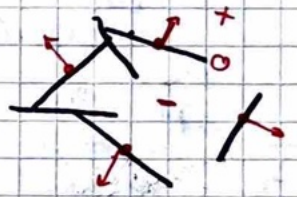
(iv) Implicit Surfaces: → SDF
→ Heat
→ Density

Defining Implicit Surfaces:

(i) Hoppe method: $f(\vec{x}) = (\vec{x} - \vec{p}) \cdot \vec{n}_p$ ↝ normal of point p
for any x ✓ ↳ nearest p. (from point cloud e.g.)

→ local method ✓
→ bad with outliers

→ Creates Voronoi diagram which ✓ defines closest points
→ Piecewise linear
→ Discontinuous along edges of voronoi



(ii) RBF: Radial Basis Functions:

→ global method ✓
→ Better at handling outliers

We assume smoothness ⇒ $f(\vec{x}) = \sum_i^n \alpha_i \underbrace{\varphi_i(\vec{x})}_{\text{some } C^2 \text{ function}} + b \cdot x + d$

To solve: $f(\vec{p}_i) = 0$
$f(\vec{p}_i + \varepsilon \cdot \vec{n}_p) = \varepsilon$
$f(\vec{p}_i - \varepsilon \vec{n}_p) = -\varepsilon$
} our constraints

Unknowns: $\begin{matrix} \alpha_1 \\ \vdots \\ \alpha_n \end{matrix} \Big| \begin{matrix} b_1 \\ b_2 \\ b_3 \end{matrix} \Big| d$ ✓ $n + 3 + 1 = n+4$

Linear ⇒ SVD to solve for α

(iii) Poisson Reconstruction

$\nabla \cdot \nabla f(\vec{x}) = \nabla \vec{N}$ ↝ normal vector field
is basically divergence of all our normals

divergence of our $\nabla f(\vec{x})$ ↳ gradient field

for poisson we don't use ✓ SDF

Rendering Implicit Surfaces

(i) Ray Casting

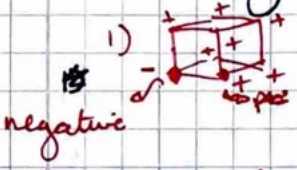
1) Ray Marching : • Apply fixed step length.
• Linearly Interpolate if zero crossing.

2) Sphere Tracing : • Dynamic Steps
• Stop if distance below threshold.

↳ Go-to method usually.

(ii) Marching Cubes :

1) Determine zero crossings for all grids cells.



2) If zero-crossing use Lookup table to triangulate

3) Linearly interpolate triangulation

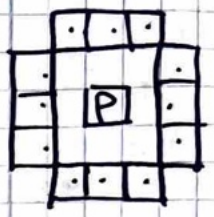
3) Find Triangulation

4) Linearly interpolate triangulation

Lecture 3: 3D reconstruction Methods

Feature Detection

(i) **FAST Detector**: for pixel $p(x,y)$ if ≥ 12 pixels are brighter (in circular kernel for example) ⇒ feature point



partial derivatives wrt. x, y: $I(u,v) - I(u+1,v)$ etc.

(ii) **Harris Corner Detector**: if we have I_x, I_y:

$$H = \begin{pmatrix} \sum_{c \in W} I_x^2, & \sum_{c \in W} I_x I_y \\ \sum_{c \in W} I_{x,y}, & \sum_{c \in W} I_y^2 \end{pmatrix} \rightsquigarrow \text{Hessian of some image patch } W. \text{ basically}$$

& we can do SSD for e.g. Harris but not robust

we find λ_1, λ_2 of H: $\lambda_1 \sim \lambda_2$ + small ⇒ flat region
$\lambda_1 >> \lambda_2$ OR $\lambda_2 >> \lambda_1$ ⇒ Edge region
$\lambda_1 \sim \lambda_2$ + large ⇒ corner. region.

Trick to find λ_1, λ_2 w/o SVD: $R = \det(M) - \alpha (\text{trace}(M))^2$
$= \lambda_1 \lambda_2 - \alpha (\lambda_1 + \lambda_2)^2$ ⇒ properties of det + trace

if $\lambda_1 \sim \lambda_2 \sim 0$ ⇒ $R \sim 0$ i.e. flat region
if $\lambda_1 >> \lambda_2$ ⇒ $\lambda_1 \lambda_2 \sim 0$ ⇒ $R = -\alpha \lambda_1^2$ ⇒ edge
if $\lambda_1 \sim \lambda_2 \neq 0$ ⇒ $\lambda_1^2 + \lambda_2^2 - \alpha \lambda_2^2$ ⇒ corner.
(α is usually 0.04 - 0.06)

if: $|R|$ small → flat region
$R < 0$ → edge region
$R > 0$ → corner region
} cool trick actually :)

(iii) **SIFT Detector**: finds features that are scale invariant (+filter (usually HARRIS) + rotation invariant) ⇒ i.e. applies multiple scalings + blur and finds features not affected + finds dominant rotation direction to compare features.

Correspondence Matching

(i) **Simple SSD per feature**: shitty + not robust

(ii) **SIFT Descriptor**: after SIFT filters optimal features: (i) takes 16-sub blocks neighbourhood around feature point, (ii) rotates neighbourhood in preferential direction, (iii) Calculates 8-bin orientation histogram per sub-block ⇒ 8·16 = 128 dimensional feature vector ⇒ per feature I guess.

(iii) **ConvNet**: Basically siamese (shared weights) network inputting some image/volume patch from two views and outputs 1/0 depending on match or not.

(iv) SURF, ORB etc.

Main 3D recon methods:

(i) **Visual Hull Carving**: If we have: → n cameras
→ we know each camera's intrinsics
→ we know each camera's extrinsics
→ we segment subject in each camera

Carve out voxel grid based on each camera segmentation ⇐

(ii) Structure from Motion (SfM): Given: → Set of images
→ Features + correspondences for each image

Bundle Adjustment

Bundle Adjustment ⇐ { Goal: Find 3D locations X_j + Find Camera poses T_i.

Re-projection energy. optimizes over T_i + X_j

$$E_{re-proj}(T, X) = \sum_{i=1}^{m} \sum_{j=1}^{n} \| x_{ij} - \pi_i(T_i X_j) \|_2^2$$

images i | feature points j | 2D coords of feature point j in image i | 3D X_j transformed to camera frame i: $T_i X_j$ then projected to camera i.

* Unknowns: → $6 \cdot (m-1)$ ↝ Extrinsics of all cameras except first (since identity)
→ $3 \cdot n$ ↝ Feature points in 3D space.
→ k (optional:) ↝ Camera Intrinsics per camera.

* Constraints: → $2 \cdot m \cdot n$ ↝ all feature points in all images in 2D space.

⇒ $2 \cdot m \cdot n \geq 6(m-1) + 3n + k$.

(iii) Multiple View Stereo (MVS): Given: → Set of images
→ Poses of images T_i
→ 3D locations of features X_j

* We can do frame pair matching + vote

Goal: → Dense Point Cloud

* Global optimization is NP-Hard

* Plug point clouds to surface fitting ↝ e.g. poisson surface fitting

↳ Usually Pipeline: 2D images → SfM → MVS → Reconstruction → 3D mesh.

(iv) SLAM: ↝ smaller compute than SfM + MVS.

Frame to Frame OR Frame to Model

* Basically real-time SfM + MVS

* Different than Bundle Adjustment since no simultaneous offline global optimization of all poses + 3D locations.

* Types: RGB { 1) ORB SLAM, 2) LSD SLAM. } can also either use feature points for alignment OR directly image

RGB-D { 3) KinectFusion

SLAM usually has Loop-Closure Problems

RGB-D Bundling

$$E_{bundle}(T) = \sum_{i,j} \sum_{k} \| T_i p_{ik} - T_j p_{jk} \|_2^2$$

k: corr. in frames i & j / p_{ik}: backprojected corr. k in i in global CS. (same but for j)

1) Take back projected corr. in i, 2) Transform to WS w/ T_i, 3) to j camera frame, 4) Project to 2D

$$E_{depth}(T) = \sum_{i,j} \sum_{k} \| (p_{ik} - T_i^{-1} T_j \pi_j^{-1}(D_j(\pi_j(T_j^{-1} T_i p_{ik})))) \cdot n_{ik} \|_2$$

5) Depth Value in j, 6) backproject in j, 7) transform to WS w/ T_j, 8) to original i camera frame T_i^{-1}

$$E_{color}(T) = \sum_{i,j} \sum_{k} \| \nabla I(\pi_i(p_{ik})) - \nabla I(\pi_j(T_j^{-1} T_i p_{ik})) \|_2^2$$

Color gradient of original pixel in frame i | color gradient of projected pixel in frame j

Lecture 4: Optimization Methods:

Linear Least Squares

* Means the model is linear in parameters

⇒ $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \ldots$

↳ here X_2 can be x^2, $\log x$, $\frac{1}{x}$ doesn't matter still linear since linearly multiplying by β_2.

* Simplified to: $Ax = b$

↳ overdetermined usually

* Usually solved as normal equation: $A^T A x = A^T b$

Solvers: ↳ of linear least squares

→ Iterative Solvers: conjugate

1) Preconditioned Conjugate Gradient Descent. ↳ most common iterative solver.
2) Gauss-Seidel Iteration
3) Jacobi Iteration

→ Direct Solvers:

related to each other I think:

1) QR, LU - Decomposition
2) SVD
3) Cholesky

Non-Linear Least Squares

* No longer linear in params: e.g. $Y = \beta_0 X^{\beta_1}$ ↳ β_1 not linearly multiplied

Solvers: we're trying to solve $\underset{x}{\operatorname{argmin}} f(x) = \|F(x)\|_2^2$

→ First Order Solvers: Gradient Descent ofc.

⇒ $x_{k+1} = x_k - t \cdot \nabla f(x_k)$ ↳ momentum / line search standard BS.

→ Second Order Solvers:

(i) Newton's Method: $x_{k+1} = x_k - H_f(x_k)^{-1} \nabla f(x_k)$ ↳ comes from taylor btw: $f(x+t) \approx f(x) + \nabla f \cdot t + \frac{1}{2} H_f \cdot t^2$

⇒ $f'(x+t) = \nabla f + H_f \cdot t = 0$

⇒ $t = H_f^{-1} \nabla f$.

so: $x_{k+1} = x_k - t$.

(ii) Gauss-Newton: Approximation of Newton:

if applied to linear system converges in one step

Assume $x_k \in \mathbb{R}^n$

$H_f \approx 2 J_F^T J_F$ ⇒ $x_{k+1} = x_k - [2 J_F^T J_F]^{-1} \nabla f(x_k)$

H_f: $n \times n$; J_F: $n \times n$; F: residuals where $F \in \mathbb{R}^n$

linear system btw. ??

$\Rightarrow 2 \underbrace{J_F^T J_F}_{A} \underbrace{[x_k - x_{k+1}]}_{x} = \underbrace{\nabla f(x_k)}_{b}$

⇒ solve with iterative linear solvers. like preconditioned conjugate gradient descent (PCG)

(iii) Levenberg: $x_{k+1} = x_k - [2 J_F^T J_F + \lambda I] \nabla f(x_k)$

↳ Tikhonov regularization i.e. damping factor for each term in J s.t. $\lambda \uparrow$ if $f(x_k) < f(x_{k-1})$

↳ $\lambda \downarrow \rightarrow$ GN

$\lambda \uparrow \rightarrow$ Gradient Descent.

(iv) Levenberg-Marquardt (LM): faster convergence this way than Levenberg

$$x_{k+1} = x_k - \left[2J_F^T J_F + \lambda \cdot \operatorname{diag}(J_F^T J_F)\right]^{-1} \nabla f(x_k)$$

more sophisticated Levenberg since each λ scaled by curviture of (diagonal is independent parts of curve)

(v) BFGS / LBFGS:

$$x_{k+1} = x_k - B_k \nabla f(x_k)$$ Basically approximation of Hessian.

where $B_{k+1} = B_k + \alpha \cdot u u^T + \beta \cdot v v^T$

In Practice we use Limited Memory BFGS (LBFGS)

Outlier Handling:

? How do we convexify our problems? course to fine over residuals

(i) RANSAC: basically trial & error

(ii) Lifting Schemes:

given: $f(x) = \sum r_i(x)^2 \Rightarrow f_{robust}(x) = \sum w_i^2 r_i(x)^2 + \text{reg} \sum (1-w^2)^2$

our lifting term that penalizes $w \to 0$ (inliers are $w = 1$)

(iii) Robust Norms: usually hard to optimise

$\Rightarrow$ Iteratively Reweighted Least Squares

$f_{robust}(x) = \sum w_i r_i(x)^2$ and $w_i = |r_i(x)|^{p-2}$ for some p-norm.

assuming we wanted $\arg\min_x f(x) = \arg\min_x \|F(x)\|_p^p$

Derivative Types:

(i) Numeric derivatives: $\frac{df(x)}{dx} \approx \frac{f(x+h) - f(x)}{h}$ ~ l.c. take very small h.

→ Easy to implement ⇒ good for debugging

→ Slow and numerically unstable

(ii) Automatic Differentiation: Use dual numbers: $e \neq 0$, $e^2 = 0$

Example: $f(x) = x^2$ $\left.\frac{df(x)}{\partial x}\right|_{x=10}$? $\Rightarrow f(10+e) = 100 + 20e + e^2$ ($e^2 = 0$)

$$\Rightarrow \left.\frac{df}{dx}\right|_{x=0} = \frac{20}{e}$$

(iii) Symbolic Differentiation:

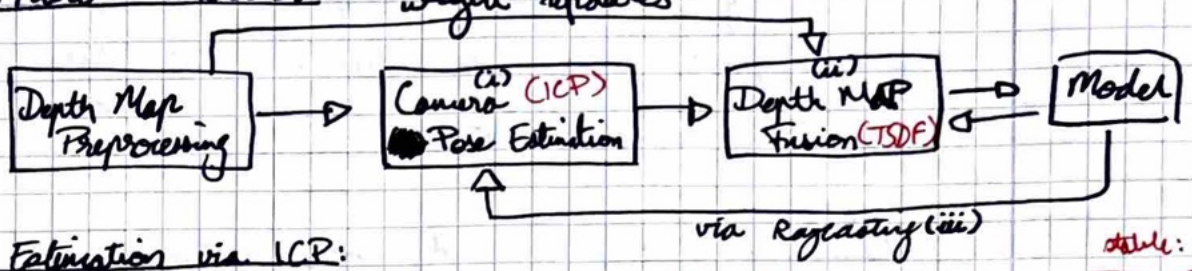
Basically no numerical solutions (Maple :))

Other Solvers: 1) Inequality Constraints: Lagrange multipliers etc.

2) Gradient-Free: Monte Carlo etc.

Lecture 5: Rigid Surface Tracking + Recon (i.e. KinectFusion revision)

KinectFusion Reminder



(i) Pose Estimation via ICP:

Basic Steps of ICP: 1) Selecting source points
2) Find Correspondences
3) Weigh Correspondences
4) Reject Outliers
5) Assign Errors metric
6) Minimize errors for pose estimate

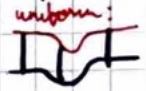
(iterate)

Of course to find registration for correspondences of problem.

Filters: → Use all points
→ Uniform subsampling
→ Random sampling
→ Stable sampling

stable:

uniform:



Finding Correspondences in ICP:

a) Nearest neighbour → FLANN etc.
b) Projective correspondences → did in da project i.e.

only works if we have depth values

* We can improve matching via only checking compatible points:

$p_s \rightarrow \pi(T_d T_s (\pi^{-1}(D(p_s))))$ — my correspondence in other frame.

a) Color Compatibility
b) Normal Compatibility
c) Pruning — e.g. at border.
d) Reject angle and distance outliers

Errors in ICP:

had closed form linear solution!?

a) Point to Point: $\sum_i \|(Tp_i - q_i)\|_2^2 = E(T)$

b) Point to Plane: $\sum_i \|(Tp_i - q_i) \cdot n_i\|_2^2 = E(T)$

idea here is energy lowest if $p_i \in q_i$ on the same surface w/ normal n_i

→ usually solved w/ LM if not linearized

→ can be linearized in $\theta \approx 0$ assuming small rotations ($\cos\theta \approx 1$) ⇒ Cholesky / SVD

* Symmetry ICP: $(p - q) \cdot (n_p + n_q)$

(ii) SDF / TSDF: (Implicit Representations):

- Basically representation of distance to zero-crossing
- TSDF: just applies truncation band. i.e. if >1 from zero-crossing / <-1 ⇒ 1 / -1 everywhere

Volumetric Fusion: also exists → Surfel representation: point + normal + radius

Running average concept: $D_{i+1}(x) = \frac{W_i(x) D_i(x) + w_{i+1}(x) d_{i+1}(x)}{W_i(x) + w_{i+1}(x)}$

tsdf value at voxel i currently (i.e. distance to iso surface)

$W_{i+1} = W_i(x) + w_{i+1}(x)$

weights can either be 1 everywhere OR closer to camera T weight, away ⊥ weights (uncertainty weighting)

(iii) Surface Rendering

a) Raycasting: Throw multiple rays from camera.
→ Surface found from sign change + interpolate to find intersection
→ Normal found from TSDF

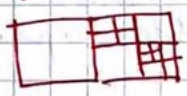
(important to note on SDFs:
=0 → surface
<0 → unknown (could have another surface behind)
>0 → free space.)

b) Marching Cubes: Basically use Marching Cubes table based on zero-crossing.

Follow Ups to KinectFusion:

1) Extended KinectFusion: + easy to implement
+ unlimited spatial volume.
− cannot reintegrate previous regions.

2) Hierarchical Fusion: + fast lookups
+ efficient storage (high local res)
− costly updates (insertions/removals)
(basically uses sparse voxel octrees →)



3) Voxel Hashing: + extremely fast updates (insertions/removals) O(1)
+ efficient storage
− lookup slower ⇒ raycasting not as fast.
+ cheap streaming

Loop Closure Follow Ups:

1) Elastic Fusion: → Surfel based
→ Detects loop closure by localization
→ Non-rigidly warp to close loop.

2) Bundle Fusion: → Added sparse energy term for pose estimation
→ Includes surface de-integration if pose had wrong estimate

Lecture 6: Deformation of Non-rigid Surface Tracking

Mesh Deformations (standard)

- $E = \lambda_{fit} E_{fit}(V) + \lambda_{reg} E_{reg}(V, X)$ → we pick non-rigid regularizes which affects behaviours of our mesh.

$E_{fit}(V) = \sum_{i \in C} \| c_i - v_i \|_2^2$ → minimize distance between vertex and target
(target pos.)

Regularizes Types

→ Our goal for regularizes:
- → Smooth Global shape
- → Global changes preserve local detail
- → Should be zero in undeformed state.

1-ring fan:

i) Laplacian Surface Editing:

$E_{reg}(V, X) = \sum_i \sum_{j \in N} \| (v_i' - v_i) - (v_j' - v_j) \|_2^2$

(N: our fan neighborhood (1-ring))

→ i.e. deformation of v_j' should be very similar to 1-ring neighbour v_i'

ii) As Rigid as Possible (ARAP):

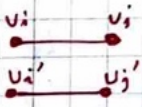
- $E_{reg}(R, V) = \sum_i \sum_{j \in N} \| (v_i - v_j) - R_i (v_i' - v_j') \|_2^2$

→ rotation with euler α, β, γ

means that after some R edge

done per fan over all vertices.

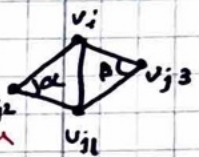
$v_i' v_j' \doteq v_i v_j$



after some Rotation.

- we could add weight to give bigger priority to closer neighbours: $w_{ij} = \frac{1}{2}(\cot \alpha_{ij} + \cot \beta_{ij})$

as $\alpha_{ij} \uparrow$ $\cot \alpha_{ij} \downarrow$
⇒ if v_{j2} closer than v_{j1} means lower weight for v_{j1}



- Solving: Unknowns: $3n + 3n$ (3 positions per vertex; 3 euler angles per vertex)

Constraints: $3n$ (per position) valence + $3c$ (from E_{fit})

* average of fan neighbours

→ We can do Levenberg Marquadt (non-linear)
OR
→ Flip-flop optimization: 1) Assume $v_i' v_j'$ constant and solve for R
2) Assume R constant and solve for $v_j' v_i'$
⇒ linear problem can be solved with SVD / Procrustes.

iii) Embedded Deformation (ED): relaxed version of ARAP

$E_{reg}(M, V) = \sum_i \sum_{j \in N} \| (v_i - v_j) - M_i (v_i' - v_j') \|_2^2$

$E_{rot} = \sum_i Rot(M_i)$, $Rot(M) = (c_1 \cdot c_2)^2 + (c_1 \cdot c_3)^2 \dots + (c_{c_1} \cdot c_{c_1 - 1})^2 + \dots$

(i.e. we want M to resemble R but allow for more DoF in our deformation (shear etc.))

- Solving: Unknowns: $12n_i$

⇒ quartic problem to optimize

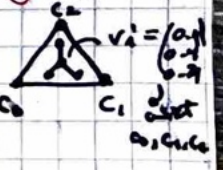
Main goal: dimensionality reduction.

Deformation Proxies

→ instead of deforming mesh ⇒ computationally expensive

deform proxy

i) Cages: linear combination of vertices wrt. cage.
(we use barycentric coordinates for that.



we can apply ARAP/ED/Laplacian here.

⇒ we deform cages and by proxy our vertices using barycentric coordinates.

ii) 3D Grids : [illegible]

iii) Skeletons : → hierarchical : Body → left shoulder → hand, → head.

e.g. blend or skinning

Called linear blend skinning or

- Each vertex where is a linear combination of bone matrices : $v_i = \sum_u w_{iu} M_u v_i'$
  - w_{iu} = weight for each bone
  - M_u = each bone has a matrix

iv) Deformation Graphs : assign a region/neighborhood a control node. → similar to cage but instead we have deformation gone.

Non-Rigid Tracking → first correspondences then ICP.

needs predefined mesh.

- Non-Rigid ICP: → more degrees of freedom

$$E = \lambda_{point} E_{point} + \lambda_{plane} E_{plane} + \lambda_{colos} E_{colos} + \lambda_{reg} E_{reg}.$$

e.g. ARAP / ED.

$\lambda_{plane} E_{plane}$: from regular point to plane ICP

$\lambda_{colos} E_{colos}$: i.e. lower energy if colors match

E_{point}: $\sum_i \| v_i - T v_i' \|_2^2$

E_{plane}: $\sum_i [(v_i - T v_i') \cdot n_{v_i}]^2$ → forces v_i' to stay on the plane of v_i

$n_i \left[v_i' - v_i \right] \Rightarrow E = 0$

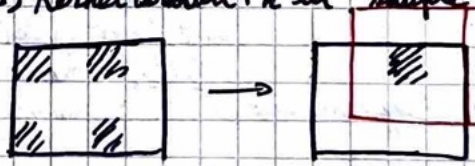
E_{colos}: $\sum_i [I(\pi(v_i)) - I'(\pi(T v_i))]^2$ → lower energy if colors match.

E_{reg}: ARAP.

* Important hyperparameter-thresholds for non-rigid ICP:
  1) distance
  2) angle
  3) Occlussion
  4) Viewing angle

Improves with more cameras.

* Correspondence Association: → for speed.
  - e.g.: Projective correspondences every k-the element
  - 2) Kernel around k-th sample for minimum reduction

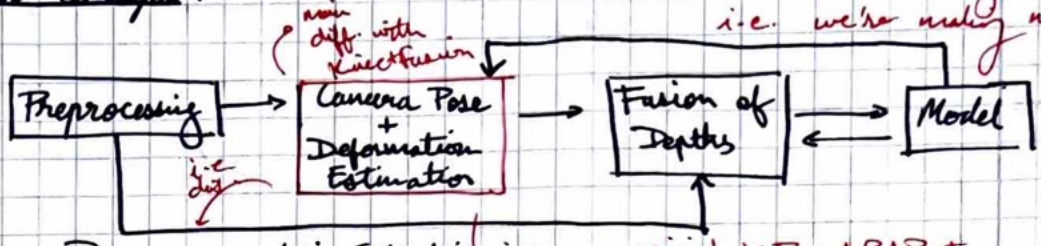


→ find best match within kernel.

Lecture 7: Non-Rigid Tracking and Reconstructions

→ here we're tracking AND fusion (non-rigid) i.e. we're making mesh.

General Concepts:



* For Camera Pose + Deformation Estimation: i.e. non-rigid ICP + ARAP etc.
- iterate:
  1) Deform SDF
  2) Find isosurface → usually with marching cubes since raycasting hard for non-rigids
  3) Find Correspondences
  4) Optimize deformations → non-rigid ICP with some non-rigid regularizers
  5) Integrate RGB-D data to model

→ we use marching cubes here.

* Main Methods discussed:

(i) Dynamic Fusion: → Uses Deformation Graphs (ED)
→ Non-rigid ICP
→ Volumetric Fusion.

(ii) Volume Deform: → Uses regular deformation grids + ARAP
→ Non-rigid ICP using sparse SIFT matches.
→ Volumetric Fusion
(texture) → We can increase resolution based on grid. This means higher stability compared to dynamic fusion.

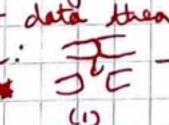
More stable + Better Texture than this.

(iii) Real-time Geometry & Motion Reconstruction:
→ Deformation Graphs (ED)
→ Non-rigid ICP (dense depth + dense color) instead of SIFT.
→ illumination correction

(iv) Fusion 4D: → way more complex.
→ 8 Depth Maps (16 IR cameras)
→ Key volumes: multiple poses → i.e. allows for topological changes guy removing coat etc.

(v) Motion 2 Fusion:
→ Basically faster Fusion 4D: 200FPS
→ ED Graphs
→ Key Volumes
→ Multiple Depth Cameras.

* Motion 2 Fusion had 2 novelties: → i.e. solving topology problem.
1) 2 way backward and forward non-rigid alignment
solves it by working reference to data then data to reference:



(1) close to open: reference to data
(2) open to close: data to reference.

2) Detail Layer: they increased resolution of grid at surface (viewing ray)

(vi) Looking Good: → increase / improve reconstructions using deep architectures

(vii) DeepDeform: → Find better correspondences than SIFT for best tracking
→ Use Siamese network that shares weights for two RGBs to give you correspondences.

The two RGBs are the source query points and target where correspondences need to be found.

Fast motion ICP messes with reconstruction: course is

surface moving or skin deforming?

+ ⇒ For non-rigid reconstructions, it accumulates drift if tracking is wrong.

Tracking with a pre-defined mesh mitigates drift wrt. base mesh.

Lecture 8: Parametric Face Models

Parameters

- Face Model
  - Identity
    - (i) Geometry
    - (ii) Albedo
  - (iii) Expression
- (iv) Illumination Model
- Pose

(i) Geometry (ii) Albedo → diffuse reflection of skin (dark → light skin etc.)

(i) Geometry — done using PCA in morphable model paper

(iii) Expression → is controlled by linear interpolation in blendshapes paper.

(iv) Illumination Model → i.e. spherical harmonics + environment maps.

Pose → just transformation of face + intrinsics of camera.

(i) Identity: Geometry

* Here by geometry we mean structure of face (can include expressions)
* Steps: (According to face warehouse + morphable model)
  1) Scan a bunch of faces
  2) Fit topologically consistent template non-rigidly to faces
  3) Since all registration from some model we have 1 to 1 correspondences
  4) Create one massive vector of all positions of vertices per face
  5) Find PCA-basis for all the vectors (i.e. all faces)

! usually PCA means no semantic meaning

$\Rightarrow M_{geo}(\alpha) = a_{id} + E_{id}\alpha$ → linear parameter.

a_{id} (average shape) → this also means we get average face like this.

E_{id} → PCA basis that we found.

PCA sidenote:

[redacted] eigenvectors of the

* The principle components are the n largest eigenvalues of covariance matrix

↳ also means direction of largest variance in our data or line with minimal dist. to data.

Covariance:

$$cov(X,X) = \frac{1}{N-1}\sum_i \bar{X}_i \bar{X}_i^T$$

where $\bar{X}_i = X_i - \bar{X}$ is average $\frac{X_i}{c}$

(ii) Identity: Albedo

* Here we mean light/dark skin
* Steps exactly the same as Geometry but done independently of Geometry.

[redacted]

$M_{alb}(\beta) = a_{alb} + E_{alb}\beta$ → linear parameter.

a_{alb} (average albedo) → PCA basis we found

(iii) Expression:

* We can parametrize expressions by:

done usually to compress data. ~ 1) PCA ~ no semantic meaning though :/
2) BlendShapes ↳ has semantic meaning

* BlendShapes: Steps:

1) Artist makes n number of target expressions (e.g. happy sad etc.)

2) We give a weight (blendshape weights) to each expression then sum/interpolate all expressions to get final expression
↳ e.g. 0.5 happy + 0.5 sad.

(iv) Illumination Model:

Steps
↳ Environment Maps
↳ Spherical Harmonics ~ parametrization of environment maps

*) Environment maps place the object a cube/sphere map of some panorama scene

2) Project lighting unto object (which is in center of sphere/cube)
↳ Assumptions: → Distant light
→ No scattering

* Spherical harmonics parametrize / reduce dimensionality of environment maps. (orthogonal basis)

⇒ we can linearly interpolate basis spheres for different lighting

↳ Assumptions: → Lambertian Surface:
→ Distant Smooth lighting

↳ i.e. diffuse lighting only depends on $\vec{n}$

Lecture 9: Face Tracking and Reconstruction

* Goal: find parameters p^* of parametric face model to fit RGB 2D image.

* We make energy term: (for energy minimization)

$$E(P) = E_{dense}(P) + E_{sparse}(P) + E_{reg}(P)$$

E_{dense}: Energy of color and geometry of mask wrt. P

E_{sparse}: Energy Correspondences terms wst. P

E_{reg}: regularization of P

$$E_{dense}(P) = E_{geom}(P) + E_{col}(P)$$

point to point & point to plane ; E_{col}: color.

$$E_{geom}(P) = \sum_i \| M_i(P) - D(\pi(M_i(P))) \|_2^2 + \left[(M_i(P) - D(\pi(M_i(P)))) \cdot n_i \right]^2$$

point to point: $\| M_i(P) - D(\pi(M_i(P))) \|_2^2$; point to plane: $\left[(M_i(P) - D(\pi(M_i(P)))) \cdot n_i \right]^2$

- $\sum_i$: summed over all vertices
- $M_i(P)$: mask vertex
- $D(\pi(M_i(P)))$: correspondent depth value at projected mask vertex in camera frame
- n_i: dot product of mask mesh normal

$$E_{col}(P) = \sum_i \| M_i(P)_c - I(\pi(M_i(P))) \|_2^2$$

- $M_i(P)_c$: color of mask at vertex i (in mesh)
- $I(\pi(M_i(P)))$: color of pixel at projected mask vertex in camera frame.

$$E_{reg}(P) = \sum_k \left(\frac{\alpha_k}{\sigma_{id,k}} \right)^2 + \sum_l \left(\frac{\beta_l}{\sigma_{alb,l}} \right)^2 + \sum_m \left(\frac{\delta_m}{\sigma_{exp,m}} \right)^2$$

α, β, δ are parameter vectors of id, alb, exp ! centered at zero ofc. The summands done for each dimension

done for all correspondences

$$E_{sparse}(P) = \sum_j \| \pi(M_j(P)_m) - C_j \|_2^2$$

- $\pi(M_j(P)_m)$: 2D coordinates of projected mask in camera frame
- C_j: 2D coordinates of correspondences in 2D image

$\Rightarrow P^* = \arg\min_P E(P)$, solved with non-linear least squares.

↳ Gauss Newton or LM solver.

$\Rightarrow P_{n+1} = P_n - (J_F^T J_F)^{-1} J_F^T \cdot F$ ↳ Usually done Rough → Dense pyramid

Partial Derivatives of 2D image wst. P:

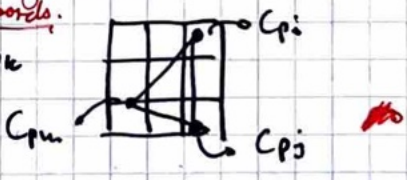
↳ needed for E_{color} + E_{reg}

↳ probably the same but not sure :/

1) For each color pixel:

$$C_{pixel} = \alpha C_{p_i} + \beta C_{p_j} + \gamma C_{p_k}$$

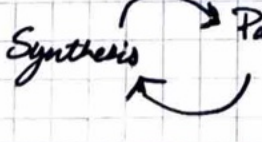
- C_{p_i}: colors at p_i etc.
- α, β, γ: Barycentric coords.
- linear interpolation of vertex color.



2) record triangle vertex index, barycentric coords α, β, γ.

3) $\Rightarrow \frac{\partial C_{pixel}(P)}{\partial P_j} = \alpha \frac{\partial C_{p_i}(P)}{\partial P_j} + \beta \frac{\partial C_{p_j}}{\partial P_j} + \gamma \frac{\partial C_{p_m}}{\partial P_j}(P)$

* This parameter update done:



* Can be done without depth via multiple views
  ↳ energy jointly optimized for 6 images

* Energy optimization dependent on:
  → Input Data (RGB vs. RGB-D)
  → Face Model + Illumination (course vs. dense)
  → Run-time requirements, hardware
  → Application

** Two main papers using this:
  → Real-time expression transfer for facial reenactment
    ↳ for two live videos
  → Face2Face
    ↳ expression to RGB video

Lecture 10: Body & Hand Tracking

→ Parameterization for consistent topology of hand + body

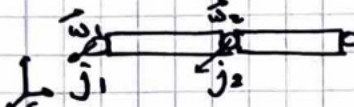
→ Parameters of body: $M(\vec{\theta}, \vec{\beta}, \vec{u} \mid \phi)$
- $\vec{\theta}$: pose of body
- $\vec{\beta}$: shape
- $\vec{u}$: texture
- ϕ: hyperparameters.

CAESAR Dataset
→ 125 M 125 F
→ 74 markers.
→ Various ethnicities, age, weight.

→ Same idea as before: 1) average mesh + Non-rigid ICP
2) PCA of all mesh vectors.

SCAPE: → joint shape + pose learning.
→ blend shapes used first for poses

Skeletons Sidenote:



: each joint has a transformation matrix related to rotation by rodrigues formula

$$\Rightarrow \boxed{p_s = g_1(\vec{\omega}_1, j_1)\, g_2(\vec{\omega}_2, j_2)\, p_b}$$

→ poses are given by $\theta = (\vec{\omega}_1, \dots \vec{\omega}_k)$
where $\mathcal{J} = (\vec{j}_1, \dots \vec{j}_k)$

Linear Blend Skinning: For any vertex $\vec{t_i}$ on mesh:

$$\boxed{t_i' = \sum_{k=1} w_{k,i}\, g_k'(\theta, \mathcal{J})\, \bar{t_i}}$$

vertices are linear combinations of all joints ←
- $w_{k,i}$: weight given to each joint
- $g_k'(\theta, \mathcal{J})$: transformation of each joint

Corrective Blend Shapes:

Basically displace each vertex by $P = \begin{bmatrix} \Delta x_1 & \Delta y_1 & \Delta z_1 \\ \vdots & & \\ \Delta x_n & \Delta y_n & \Delta z_n \end{bmatrix}$ to simulate muscle compression :)

Non-rigid registration: $E = w_d E_d + w_s E_s + w_m E_m$

$E_d = \sum_i w_i \, \text{dist}^2(T_i v_i, D)$ — Data errors.

$E_s = \sum \| T_i - T_j \|_F^2$ — Transformation Smoothness

$E_m = \sum \| T_{k,i} v_{k,i} - m_i \|^2$ — marker Errors.

Hand Models:

Taylor: Uses subdivision model → recursive coarser mesh to get:
→ C_2 continuity
⇒ Fast LM convergence

Issues with Model Generation + Fitting:

- → Expensive Scanning setup.
- → Neutral Pose similar but not identical
- → Ambiguities in fitting w change shape or expression?
- → Drift in non-rigid registration