theory_gaussian

May 21, 2020

1 Theory: Bessel-Gauss beam

[4]:
```python
# from beams package ./beams/test_gaussian.py
from test_gauss import plot_pupil_function, plot_gaussian
```

1.0.1 References:

wiki

1.1 The Gaussian beam

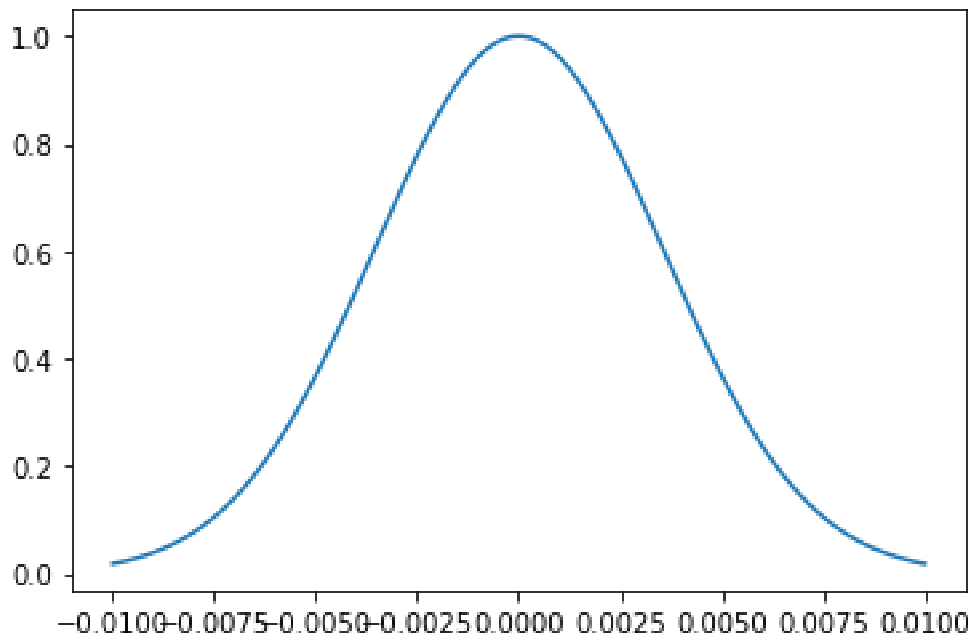
The Gaussian beam is described at the focus by the Gaussian function:

$$E(x) = \exp\left(\frac{-x^2}{w_0^2}\right) ,$$

where w_0 is the beam waist radius.

Note, at $x = w_0$, $E(w_0) = \exp(-1) = 1/e$; therefore, w_0 describes the radius from centre where the amplitude drops to $1/e$ of its maximum. Also note that intensity (which we measure on the camera) $I = |E|^2$; thus, $I(w_0) = 1/e^2$, and w_0 describes the $1/e^2$ radius. Finally, $2w_0$ is the $1/e^2$ diameter, and $2w_0 = FWHM \times \sqrt{2}/\ln(2)$.

[2]:
```python
plot_pupil_function()
```



1.1.1 Gaussian beam in the pupil plane

Let's assume that the Gaussian beam in the pupil plane is given as:

$$E_2(x) = \exp\left(\frac{-x^2}{w_0^2}\right) .$$

1.1.2 Gaussian beam in the image plane

Using the Fourier transforming property of the lens, we can recover the amplitude at the image plane. Ignoring the amplitude component:

$$E_1(u) = \exp(-\pi^2 w_0^2 u^2) ,$$

which is evaluated at $u = x_1/(f\lambda)$. Thus, the field at the focus becomes

$$E_1(x_1) = \exp(-\pi^2 w_0^2 x_1^2/(f\lambda)^2) = \exp(-x_1^2/w_1^2) ,$$

where $w_1 = f\lambda/(\pi w_0)$.

1.1.3 Gaussian beam propagation at the sample

Importantly, we know that the beam waist at focus ($1/e$ radius of amplitude) is $w_1 = f\lambda/(\pi w_0)$.

We can figure out beam propagation from the Fresnel diffraction solution to the Helmholtz equation, but this is done for us and is well-known for a Gaussian beam. The Gaussian beam amplitude is given as:

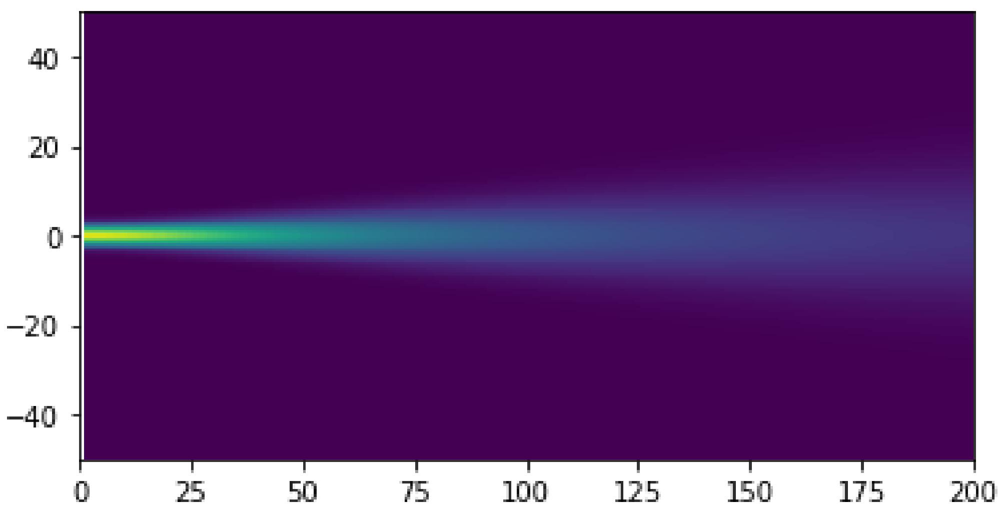
$$E_1(x, z) = \frac{w_1}{w(z)} \exp\left(\frac{-x^2}{w(z)^2}\right) \exp\left(-i(kz + k\frac{x^2}{2R(z)} - \psi(z))\right) ,$$

where $w(z) = w_1\sqrt{1+(z/z_R)^2}$, $z_R = \pi w_1^2 n/\lambda$ is the Reyleigh range, $R(z) = z(1+(z_R/z)^2)$ is the radius of curvature, and $\psi(z) = \arctan(z/z_R)$ is the Guoy phase.

[5]:
```
plot_gaussian()
```

```
2.174693142407658e-06

F:\Work\Projects\deep-learning\deep-learning-lsm\beams\test_gauss.py:60:
RuntimeWarning: divide by zero encountered in double_scalars
  rz = zv*(1+(zr/zv)**2)
F:\Work\Projects\deep-learning\deep-learning-lsm\beams\test_gauss.py:60:
RuntimeWarning: invalid value encountered in double_scalars
  rz = zv*(1+(zr/zv)**2)
```



1.1.4 Implementation note

Note that in the formula for E_1, if $z = 0$, $R(z)$ is a division by 0.

We make a modified parameter $zR(z) = (z^2 + z_R^2)$, and modify the last exponent to include $kx^2z/(2(zR_z(z)))$ which evaluates to 0 when $z = 0$.