theory_bessel_gauss

April 16, 2020

1 Theory: Bessel-Gauss beam

[1]:
```
# from beams package ./beams/test_airy.py
from test_bessel_gauss import plot_bessel_function, plot_bessel_gauss_function,␣
↪plot_bg_pupil_full, plot_bg_propagation_pupil
```

1.0.1 References:

1. Gori, F., Guattari, G. & Padovani, C. Bessel-Gauss beams. Optics Communications 64, 491–495 (1987).
2. Vaity, P. & Rusch, L. Perfect vortex beam: Fourier transformation of a Bessel beam. Opt. Lett., OL 40, 597–600 (2015).

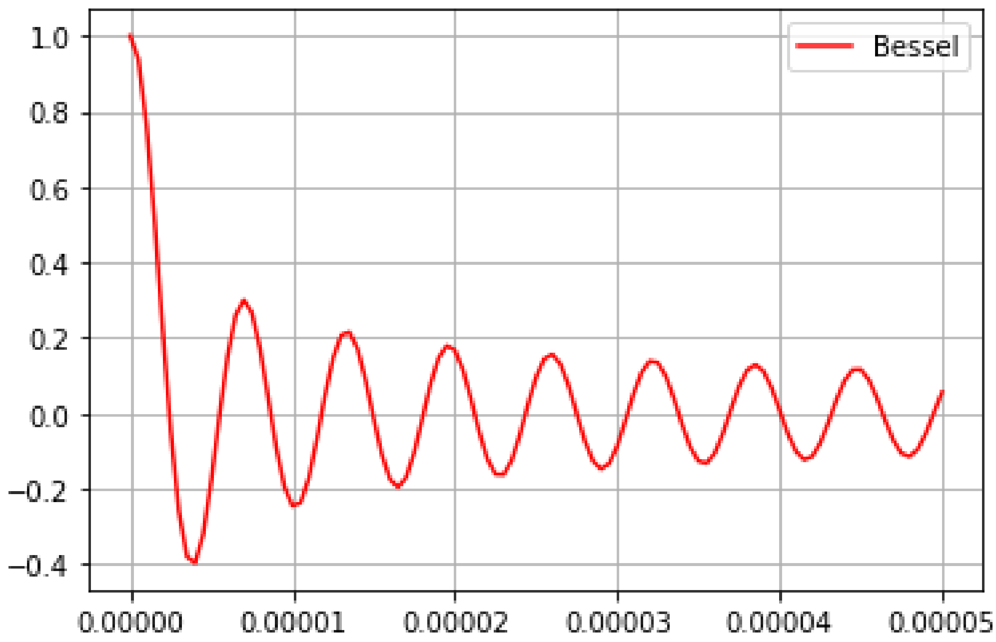
1.1 The Bessel beam

The Bessel beam, described by the Bessel function along the radial coordinate, is a non-diffracting beam solution to the wave equation. Here, we only consider a zero-order Bessel beam, which is described by:

$$E(r, z) = J_0(k_r r)\exp(ik_z z)\,,$$

where k_r and k_z are radial and longitudinal wave vectors: $k = \sqrt{k_r^2 + k_z^2} = 2\pi/\lambda$, and J_0 is the zero-order Bessel function, visualised below.

[2]:
```
plot_bessel_function()
```

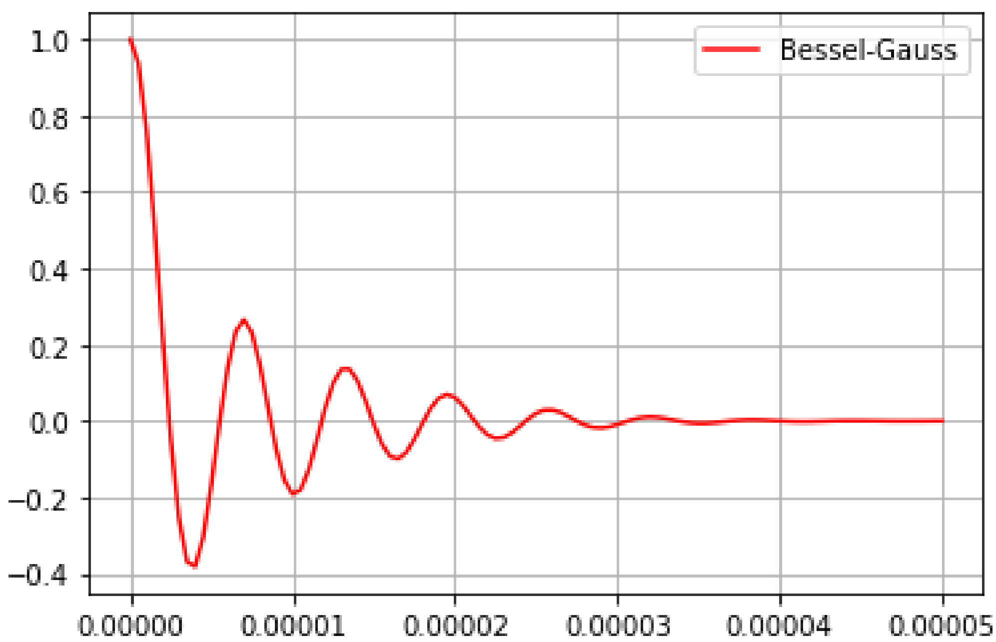


Each peak described a circular intensity that has equal energy. A finite energy beam can be described by a Bessel modulated by a Gaussian envelope - a Bessel-Gauss beam, described in the focus ($z = 0$) as:

$$E(r) = J_0(k_r r)\exp(-r^2/w_g^2)\,,$$

where w_g is the waist of the Gaussian modulation beam.

[3]:
```
plot_bessel_gauss_function()
```



The goal of this work is to describe the Bessel-Gauss beam in the pupil plane, relate it to experimental configurations, and to describe the propagation close to the focal plane

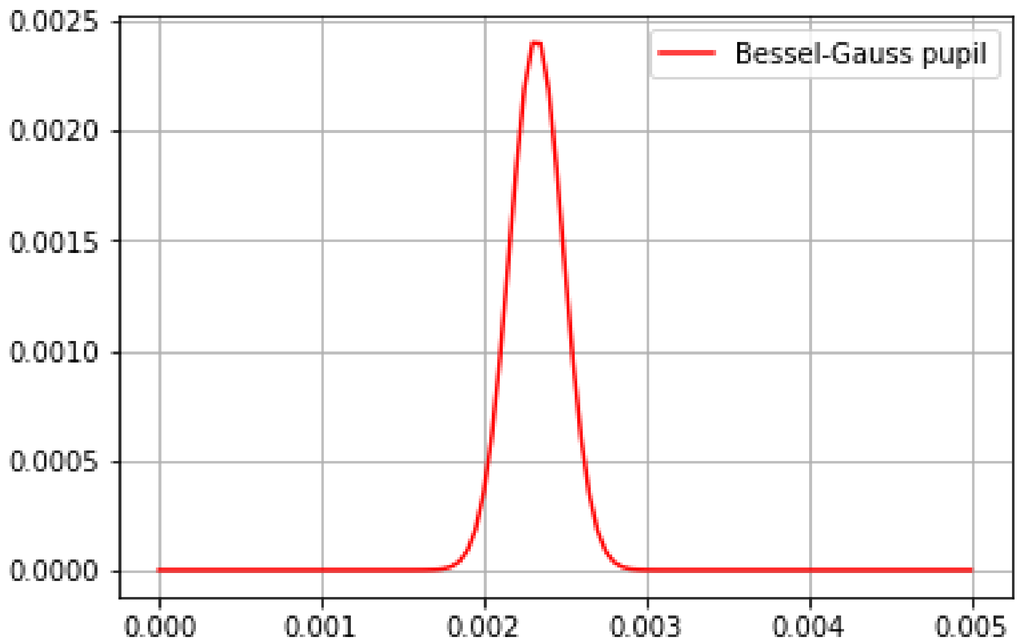
1.2 Bessel-Gauss beam in the pupil plane

The pupil function can be described by a Fourier transform of the beam at focus. Following the derivation in [2], using the Bessel function identity, the pupil function is:

$$E_1(r_1) = \frac{w_g}{w_0} \exp\left(-\frac{r_1^2 + r_r^2}{w_0^2}\right) I_0\left(\frac{2r_r r_1}{w_0^2}\right) \; ,$$

where $w_0 = 2f/kw_g$, $r_r = k_r f/k$ and I_0 is the modified Bessel function of zero order and of first kind.

[4]:
```
plot_bg_pupil_full()
```



Typically, r_r is much larger than w_0. In this case, $I_0(x) \sim \exp(x)$, reducing the above equation to:

$$E_1(r_1) = \frac{w_g}{w_0} \exp\left(-\frac{(r_1 - r_r)^2}{w_0^2}\right) \; .$$

This describes a ring with a Gaussian thickness.

If we consider the refocussing of this beam in 2D (x,z), it describes two off-axis Gaussian beams that cross at focus, creating a Bessel beam via interference. In 3D, this can be described by vectors following a conical shape.

1.3 Bessel-Gauss beam propagation at the focus

Let's consider again the field of the Bessel-Gauss at focus ($z = 0$), which we name $E_2(r_2)$. Here, we follow the work of [1]. As the field propagates in z, it is subject to diffraction, which can be described by the Fresnel diffraction integral:

$$\begin{aligned} E(r, z) &= (-ik/z) \exp(i(kz + kr^2/(2z))) \\ &\quad \times \int_0^\infty E(r_0, 0) \exp(ikr_0^2/(2z)) J_0(kr_0 r/z) r_0 dr_0 \end{aligned}$$

According to [1], this can transformed into the following form:

$$\begin{aligned} E(r, z) &= (Aw_g/w(z)) \\ &\quad \times \exp\{i[(k - k_r^2/(2k))z - \Phi(z)]\} J_0(k_r r/(1 + iz/L)) \\ &\quad \times \exp\{[-1/w(z)^2 + ik/(2R(z))](r^2 + k_r^2 z^2/k^2)\}, \end{aligned}$$

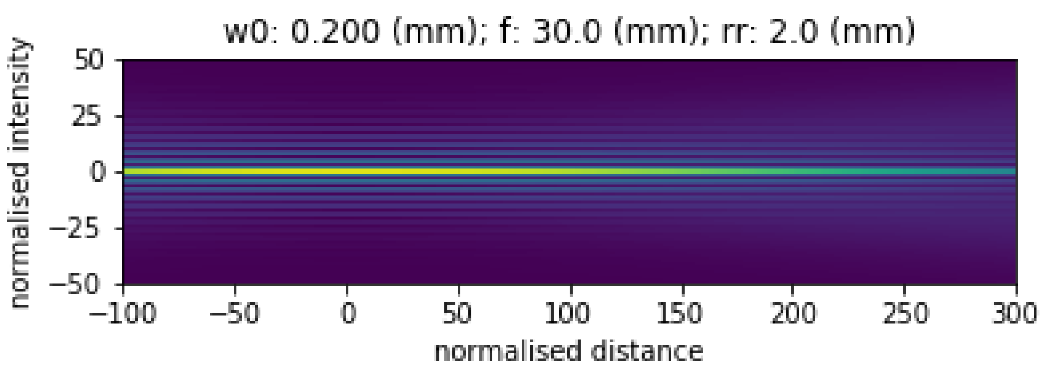
where

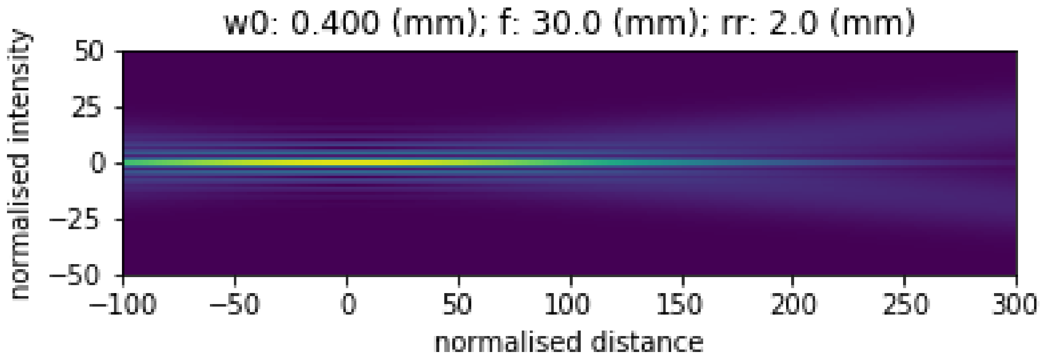
$$\begin{aligned} L &= kw_g^2/2 \\ w(z) &= w_g[1 + (z/L)^2]^{1/2} \\ \Phi(z) &= \arctan(z/L) \\ R(z) &= z + L^2/z \end{aligned}$$

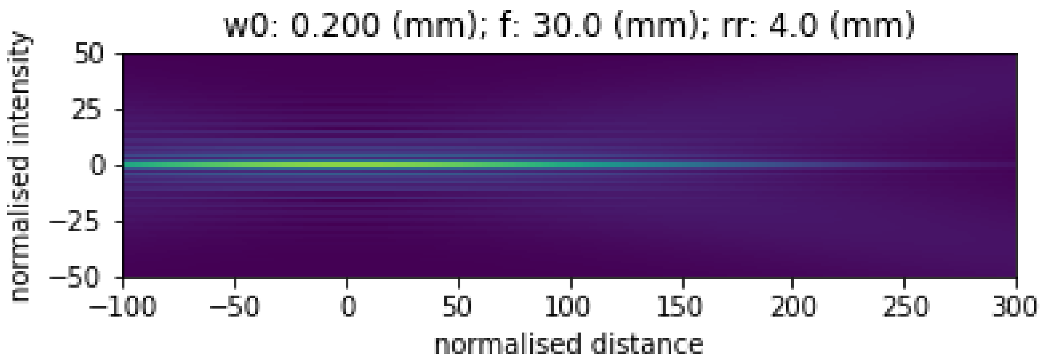
Note the similarities to a Gaussian beam propagation, where $R(z)$ is the radius of curvature, $\Phi(z)$ is the Gouy phase, $w(z)$ is the spot size parameter, and L is an effective analog of the Rayleigh range.

We plot the Bessel-Gauss propagation using the pupil parameters, which can be trivially substituted into the above as $w_g = 2f/(w_0 k)$ and $k_r = r_r k/f$.

[7]:
```
plot_bg_propagation_pupil(rr = 2e-3, w0 = 2e-4, f = 30e-3)
plot_bg_propagation_pupil(rr = 2e-3, w0 = 4e-4, f = 30e-3)
plot_bg_propagation_pupil(rr = 4e-3, w0 = 2e-4, f = 30e-3)
```







Note, the plots are of the absolute electric field. Intensity can be expressed as $I = |E|^2$.