FINM 32000: Homework 1

Philip Lee (UCID #12129240)

January 27, 2022

1. (a) How should the integer j be chosen?
Using $N = 5\text{e}+4$, $\Delta t = \frac{T}{N} = 5\text{e}-6$.
$\Delta x = \sigma\sqrt{3\Delta t} = 1.549\text{e}-3$.
Therefore, $j \approx \log(\frac{114}{100})\Delta x - 0.5 = 84.078 \implies 84$

Using barrier_trinom_pricer, the barrier put price is \$5.30.

(b) $(K - S_T)^+ = (K - S_T)^+ \mathbb{1}_{S_t<H} + (K - S_T)^+ \mathbb{1}_{S_t \geq H}$
Therefore, we can price the up-and-in put using the up-and-out put and a vanilla European put.
$P - P_{\mathbb{1}_{S_t<H}} = P_{\mathbb{1}_{S_t \geq H}} = \$0.22$

(c) i. The continuously-monitored barrier option will be priced lower than or equal to the discretely-monitored. This is because the discrete monitoring has a chance of missing a crossing of the H threshold and not triggering the barrier whereas the cross will trigger for the continuously monitored.

ii.
$$\begin{aligned} S_t &= 114 \\ \alpha &= \frac{C_t^{BS}(S_t, K = 136.8)}{P_t^{BS}(S_t, K = 95)} = \frac{5}{6} \\ S_0 &= 100 \\ P_0^{\text{cont}} &= P_0^{BS}(S_0, K = 95) - \alpha C_0^{BS}(S_0, K = 136.8) \\ &= 5.03 \end{aligned}$$

2. (a)
$$\begin{aligned} S = 100, rGrow = r = 0, K = 100 \\ IV(C^{BS} = 11.25, T = 0.5) = 0.4001 \\ IV(C^{BS} = 12.00, T = 1.0) = 0.3019 \end{aligned}$$

(b)
$$\begin{aligned} IV_{T=0.75} &= \frac{IV_{T=0.50} + IV_{T=1.00}}{2} \\ &= 0.3510 \\ sigma &= 0.3510, S = 100, \\ r_{grow} &= 0, r = 0, K = 100, T = 0.75 \\ C^{BS} &= 12.08 \end{aligned}$$

(c) $V = -C(T = 0.75) + C(T = 1.00) = -12.08 + 12.00 = 0.08$. Get Paid \$0.08 initially, non-negative payoff in the future: Type II arb. At $t = 0.75$, if the options are ITM, exercise $C(T = 1.00) = S_0.75 - K$ to pay off $-C(T = 0.75) = K - S_0.75$, whereas if the options are OTM, either sell $C(T = 1.00)$ or hold it for longer to speculate. If cash settled, exercise for money to balance the account, if physically settled, exercise for shares to transfer to the buyer of $C(T = 0.75)$.