36702 TA Session 4

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Part I. Homework 3 Review

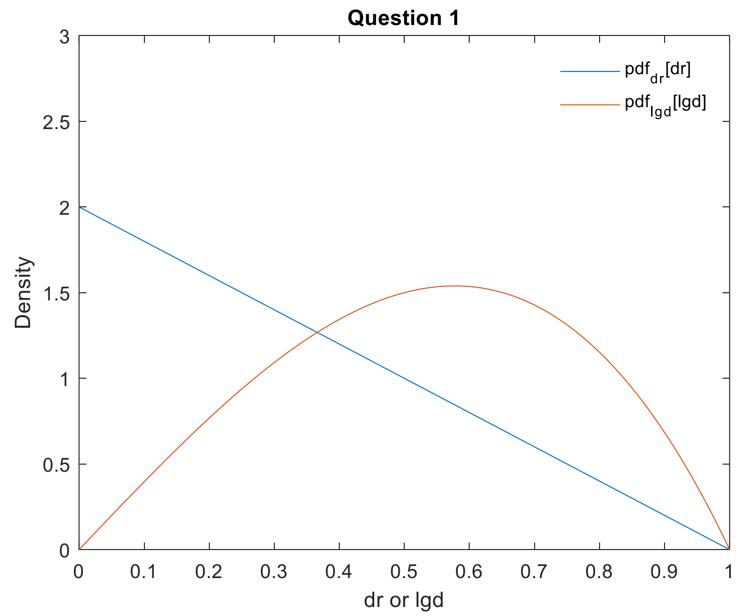
Q1. LGD PDF Derivation & Plotting

- Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio lgd is a function of dr: $lgd[dr] = dr^{1/2}$.

- Part 1. Derive and state the function $pdf_{lgd}[lgd]$, L2.S38–39, change of variable.

  - Step 1: Since $lgd = dr^{1/2}$, we have $dr = lgd^2$
  - Step 2: Then $\frac{\partial dr}{\partial lgd} = \frac{\partial lgd^2}{\partial lgd} = 2lgd$
  - Step 3: Apply change of variable (or the chain rule) to derive PDF of lgd based on PDF of dr, $pdf_{lgd}[lgd] = \left|\frac{\partial dr}{\partial lgd}\right| pdf_{dr}[dr] = 2lgd(2 - 2dr) = 2lgd(2 - 2lgd^2) = 4lgd - 4lgd^3$
  - Sanity check (not required): The area under the PDF of LGD, $pdf_{lgd}[lgd] = \int_0^1 (4u - 4u^3)du = 1$.

- Part 2. Plot two functions: $pdf_{dr}[dr]$ *and* $pdf_{lgd}[lgd]$ on [0, 1]

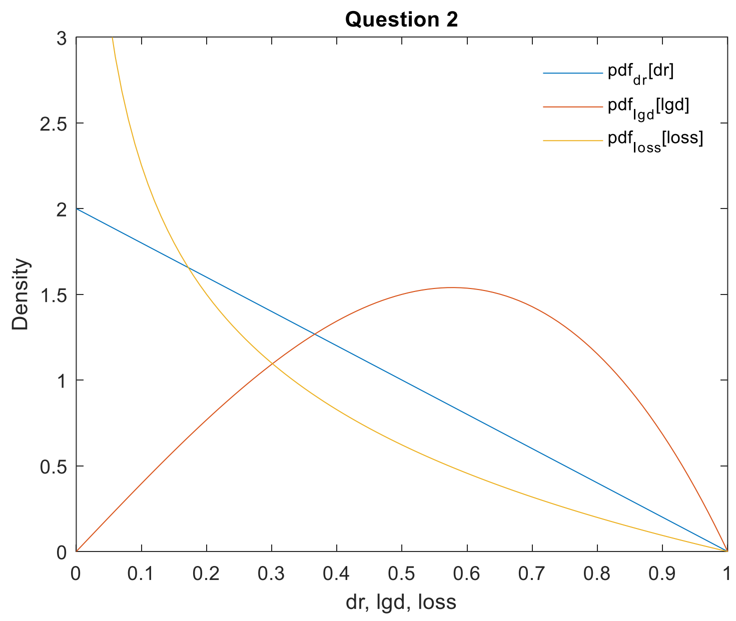


Q2. Loss PDF Derivation & Plotting

- Same as Q1. Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio lgd is a function of dr: $lgd[dr] = dr^{1/2}$.

- Part 1. Derive and state the function $pdf_{loss}[loss]$, L2.S38–39, change of variable.
    - Since $loss = dr \cdot ldg = dr \cdot dr^{1/2} = dr^{3/2}$, we have $dr = loss^{2/3}$
    - Therefore, $\frac{\partial dr}{\partial loss} = \frac{\partial loss^{2/3}}{\partial loss} = \frac{2}{3} loss^{-1/3}$
    - Apply the chain rule to derive PDF of loss from PDF of dr, $pdf_{loss}[loss] = \left|\frac{\partial dr}{\partial loss}\right| pdf_{dr}[dr] = \frac{2}{3} loss^{-1/3}(2 - 2dr) = \frac{2}{3} loss^{-1/3}\left(2 - 2loss^{2/3}\right) = \frac{4}{3} loss^{-1/3} - \frac{4}{3} loss^{1/3}$
    - Sanity check (not required): the area under $pdf_{loss}[loss] = \int_0^1 \left(\frac{4}{3}u^{-1/3} - \frac{4}{3}u^{1/3}\right) du = 1$

- Part 2. Plot three functions: $pdf_{dr}[dr]$, $pdf_{lgd}[lgd]$ and $pdf_{loss}[loss]$ on [0, 1]



Q2 Part 3. Credit Identities of a Loan

- L4.S17-24

- Need to know: $PD = \mathbb{E}[dr] = \int_0^1 dr \cdot PDF_{dr} \cdot d[dr] = 0.3333$

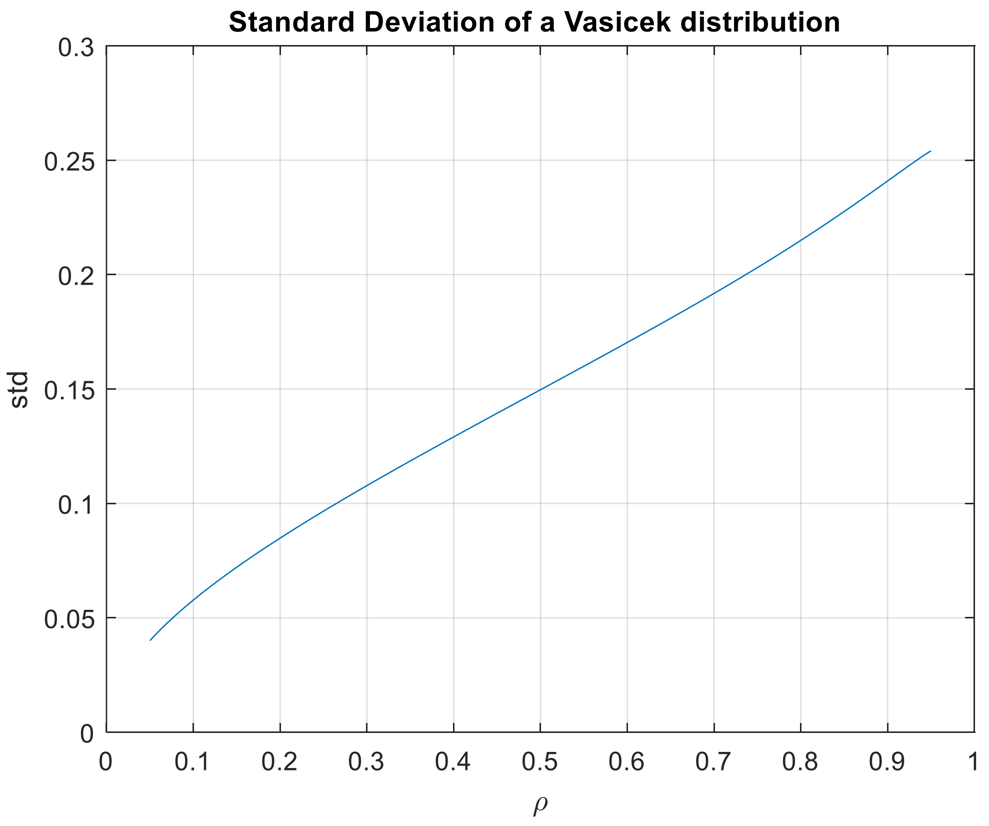
- State the values of
  - Expected loss, $EL = \mathbb{E}[loss] = \int_0^1 loss \cdot PDF_{loss} \cdot d[loss] = 0.2286$
  - Expected LGD, $ELGD = \frac{EL}{PD} = \frac{0.2286}{0.3333} = 0.6857$
  - "Time-weighted" LGD $= \mathbb{E}[lgd] = \int_0^1 lgd \cdot PDF_{lgd} \cdot d[lgd] = 0.5333$

Q3. Std. of a Vasicek distribution and Plotting

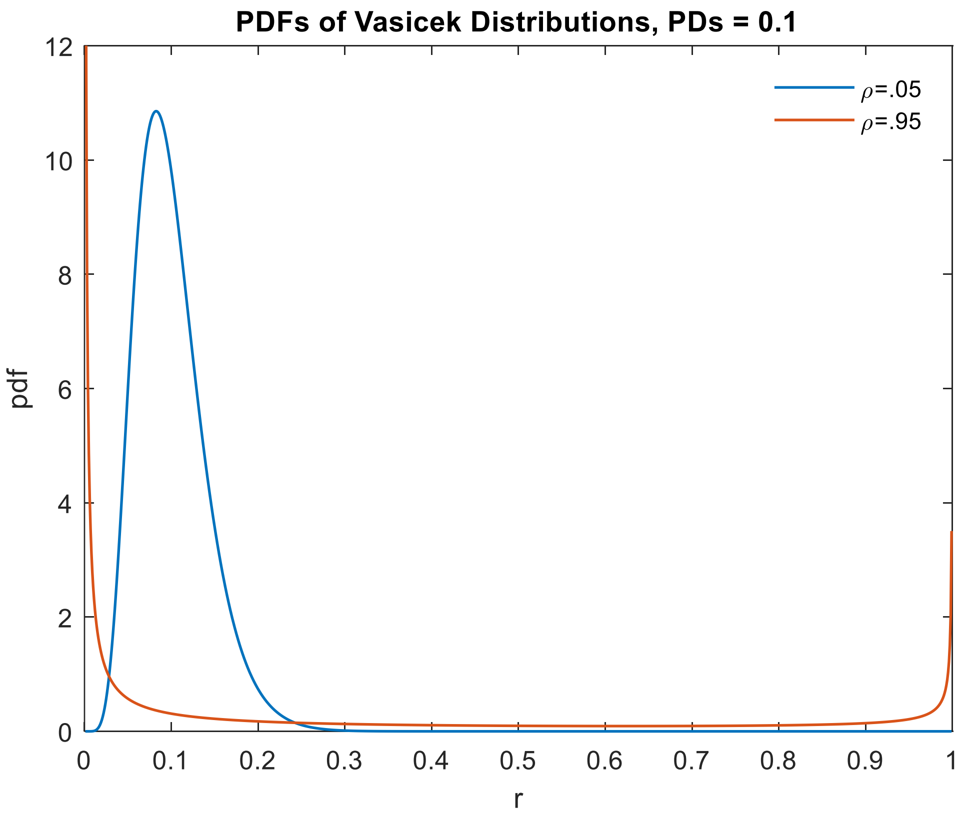
- Part 1. Express the standard deviation of a Vasicek distribution as an integral that involves the Vasicek PDF.
  - Let $f_{cPD}[r]$ denote PDF of Vasicek distribution, then
  - $\boldsymbol{f_{cPD}[r] = \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi[\Phi^{-1}[r]]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[r]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right]}$
  - Let $SD_{cPD}[r]$ denote the standard deviation then
  - $\boldsymbol{SD_{cPD}[r] = \sqrt{\mathbb{E}[(r-\mu)^2]} = \sqrt{\int_0^1 f_{cPD}[r]\cdot(r-\mu)^2 dr}}$, and $\boldsymbol{\mu = PD}$
  - Therefore, $\boldsymbol{SD_{cPD}[PD,\rho] = \sqrt{\int_0^1 \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi[\Phi^{-1}[r]]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[r]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right](r-PD)^2 dr}}$

Q3 Parts 2 & 3. Plotting

- Plot the standard deviation for $0.05 < \rho < 0.95$

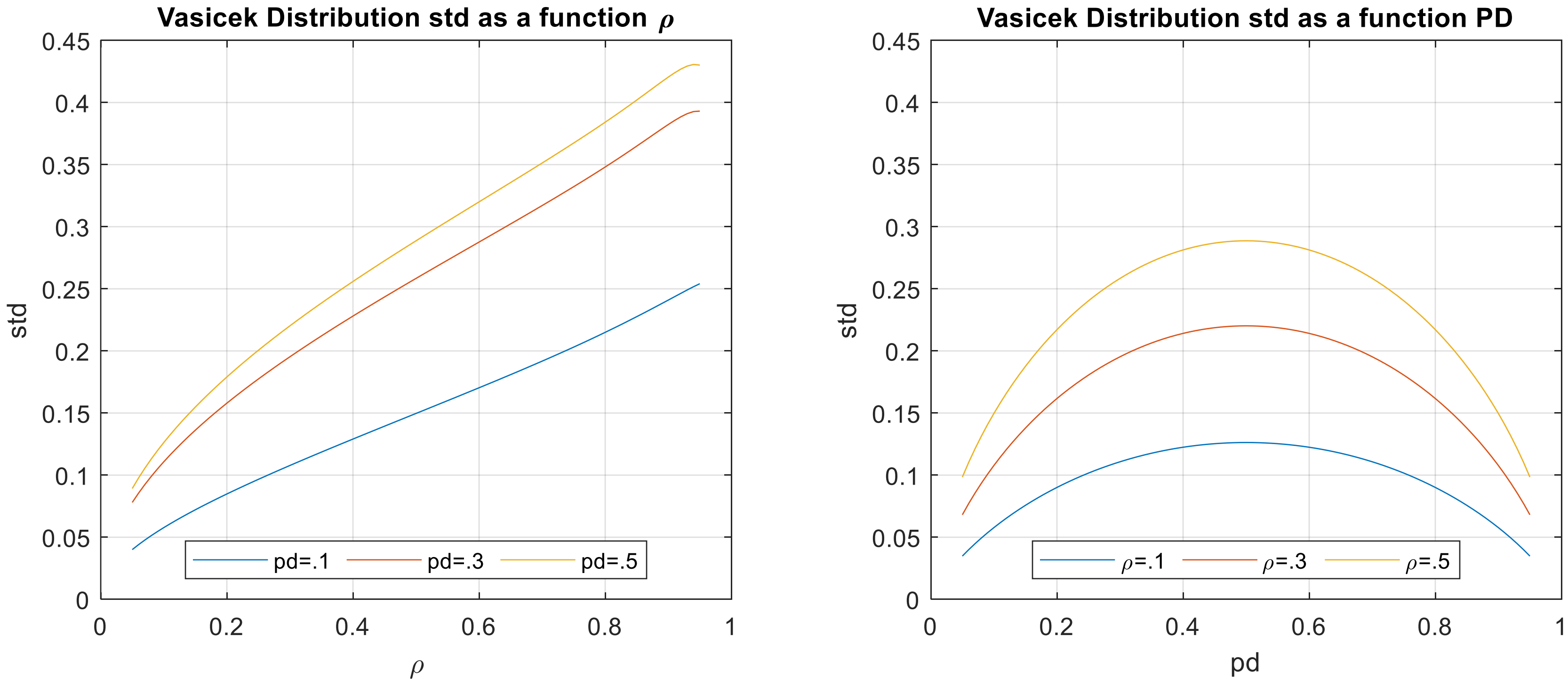


- Plot two Vasicek distributions: PD = 0.10, ρ = 0.05 and PD = 0.10, ρ = 0.95



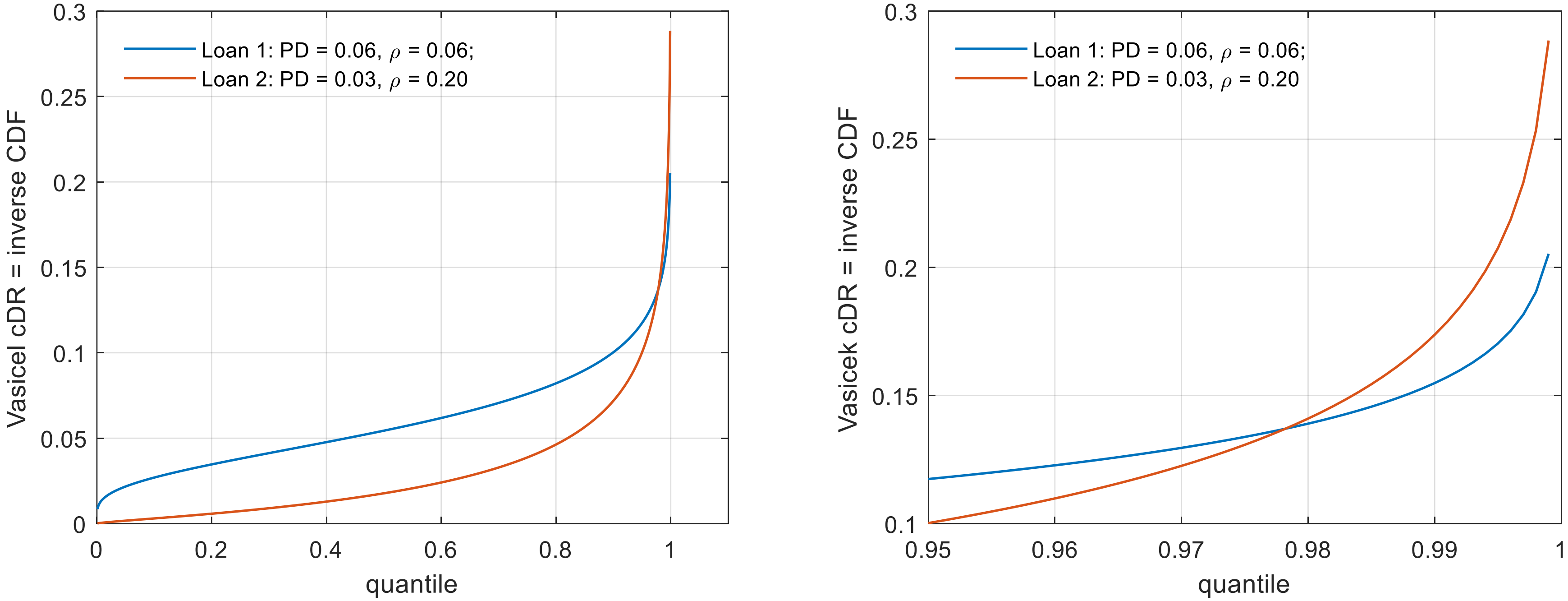
Modeling Thinking: Parameter Tests

- Sensitivity analysis: Which parameter is a larger driver of std, *PD* or ρ?



Q4. Unexpected Behaviors of cPD

- Two loans ~ Vasicek[PD, ρ]. $PD_1 = 0.06$, $\rho_1 = 0.06$; $PD_2 = 0.03$, $\rho_2 = 0.20$. Part A: Plot and compare the two inverse CDF's.



… The first loan has twice the PD, and in 98% of the years, it has a greater default rate. Only in the ~2% (=1 - 0.9782) of worst years does the other loan have greater conditional default since its ρ value is much higher.

Part II. Perspectives and Hints for Homework 4

Q1. Loan Identities from Data

- A loan can take one of four states as follows:

State	A	B	C	D
Probability of state	0.40	0.30	0.20	0.10
cDR	0.02	0.04	0.06	0.08
cLGD	0.10	0.30	0.50	0.70

- What is the value of
  - The expected loss of the loan (EL)?
  - The expected LGD of the loan (ELGD)?
  - The “time-weighted LGD” of the loan?
- L4.S17-24

Q2. Alternative Hypothesis for LGD Function

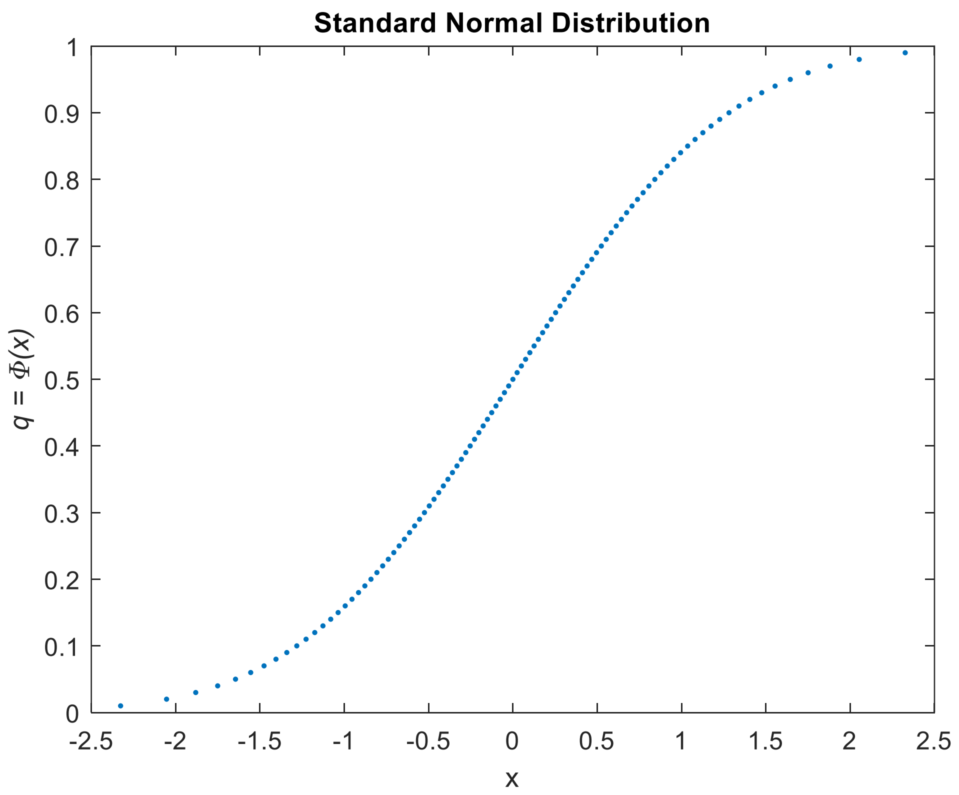
- Let PD = 5%, ELGD = 30%, and ρ = 15%.
- Assume the alternative LGD function (L4.S57 – 63), and plot the function within the unit square for four values of the "a" parameter: {-2, 0, 1, 2}
  - Hint: L4.S62

Q3. Parameters Testing

- Suppose that cPD ~ Vasicek [PD = 0.02, ρ = 0.10]. Assuming that cPD and cLoss are comonotonic.
- Part 1. Plot three LGD functions for three possible distributions of cLoss:
  a. cLoss ~ Vasicek [EL = 0.01, ρ = 0.05]
  b. cLoss ~ Vasicek [EL = 0.01, ρ = 0.1]
  c. cLoss ~ Vasicek [EL = 0.01, ρ = 0.15]
  - Limit the default axis to {0, 0.5} and limit the vertical axis to {0, 1.2}.
- Part 2. Comment on the usefulness of each possible LGD function.
  - Hint: You should be plotting three cLGD functions against cPD. X-axis must cover the range [0, 0.5].

Q3. Technical Notes

- Key formulas on L2.S32,S36
- What is a quantile, q?
  - q = 0 : .01 : 1, or
    $q \in \{0, 0.01, 0.02, \ldots, 0.99, 1\}$
- How to compute q?
  - Let $CDF: F_X(x) = P(X \leq x) = q$ and is monotonic
  - Then, $x = F_X^{-1}(q)$



Q4. ELGD = ?

- Using the assumptions of Question 3(b), what is the value of ELGD?
    - Hint: You should know the identities of a loan well and try not to over think!