Portfolio Credit Risk

University of Chicago Masters in Financial Mathematics 36702

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Lecture 2

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Simulation and non-simulation portfolio default modeling

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Lec 1: PDs and ρ's

PDs are hard to estimate.

- **For example, in 2006 the PD of a US mortgage was thought to be a few basis points.**
- **By 2008, defaults of US mortgages triggered a worldwide financial collapse from which we have not yet recovered.**

36702 mostly lets someone else perform the estimates.

It would be great if the distribution of the portfolio default rate depended *only* on PDs and ρ's.

- **But last week you saw that the copula also matters.**
- **A worst-case event was found to be more than three times as likely under the t_4-distribution than under the Gauss.**
  - **All PDs were 10%, and all correlations were zero.**
  - **By the way, the probability is 0.0034, not 0.034 as on Lec 1 Slide 54.**

Lec 1: The Gauss copula

The Gauss copula is implicitly defined by the vector central limit theorem.

- **The Gauss copula is part of the multinormal distribution.**
  - **A copula is a multivariate distribution that has uniform marginals.**
  - **The standard multinormal distribution essentially converts the uniform marginals to standard normal variates using $Z_i = \Phi^{-1}[u_i]$.**
  - **You could connect any kind of variables this way. For example, you could have *t*-distributed margins connected by the Gauss copula.**
  - **Simulations produce Bernoulli-distributed default indicators.**

We touched on the *t*-copula.

- **Uncorrelated variables with a *t*-copula are *not* independent,**
  - **because the multivariate *t* is a normal variance mixture distribution.**
- **Compared to Gauss, the t-copula produces more bad outcomes, such as that all three firms default.**

Lec 1: The model is complete

The model says D_i depends only on PD_i and Z_i.

- **It says Z_i is independent of the past.**
- **It says economic variables affect PD_i.**
  - **Once PD is set only a random number, Z_i, matters.**

The model says there can be no deeper analysis.

- **The model asserts that it is complete.**
  - **If someone says that a more-complicated model works better, it would not be asking too much to make them *show* it works better.**

Questions? Comments?

Any questions about Lecture 1?

Any comments? How’s everything?

Tonight's list of topics

The standard portfolio simulation

The approach of not simulating

The single risk factor (SRF) model

The Vasicek distribution

Basel minimum capital requirement

Multistate simulation models

Distribution of the default rate

There are two main approaches.

- **Simulations keep track of all the PDs and correlations, etc.**
  - **Easy to run, but it feels something like a black box.**
- **Non-simulation approaches simplify and provide insight.**
  - **A non-simulation approach is used outside the US to determine the minimum amount of capital that a bank must hold to support a loan.**

Either way, the Gauss copula is assumed.

- **There is not enough data to prove that this is wrong.**
- **We could work with the *t*-copula, but intuitions about portfolio credit risk would be harder to develop.**
  - **The more-complicated details get in the way.**
- **We assume good estimates of PDs and correlation matrix.**

Portfolio default simulation

Inputs: PD_i for i = 1, 2, …, N firms.

$\rho_{i,j}$ for Corr[Z_i,Z_j]. Let VCV = [[$\rho_{i,j}$]].

Output: Distribution of the default rate, DR.

Each run: Draw a vector of N correlated jointly standard normals, $\mathbf{z} = [z_1, z_2, \ldots, z_N,]$, that obey $Var[z] =$ VCV.

– "Jointly": the simulation assumes the Gauss copula.

For each firm i, $D_i = 1$ if $z_i < \Phi^{-1}[PD_i]$; otherwise $D_i = 0$.

DR = $\sum_i D_i / N$; repeat many times to estimate distribution.

One run with 4-firm portfolio

Firm	PD_i	Correlation Matrix $\rho_{i,j}$				Simulated Z_i	$\Phi^{-1}[PD_i]$	D_i
1	0.1	1	0.1	0.2	0.3	-1.3559	-1.2816	1
2	0.2	0.1	1	0.4	0.5	-0.6171	-0.8416	0
3	0.3	0.2	0.4	1	0.6	-0.4817	-0.5244	0
4	0.4	0.3	0.5	0.6	1	-0.0562	-0.2533	0
		Number of defaults in this simulation run =						1

Be confident that you can perform this type of simulation.

On Saturday, Lisheng will verify

- **The expected number of defaults**
- **The standard deviation of the number of defaults**
- **The standard deviation assuming all pairwise correlations = zero**

Monte Carlo simulation

Essentially, the portfolio simulation evaluates integrals of the multivariate normal distribution.

- **For example, the average rate at which three firms default estimates their joint default rate under the Gauss copula.**

One of the earliest uses of computers was to evaluate integrals using Monte Carlo simulation.

- **The object of study was the atomic nucleus.**

Time and risk

This is a one-period model.

- **Estimate the PDs and loadings.**
- **Get the distribution of the default rate.**
- **You’re done.**

But real life goes on, usually like it is today:

- **If the current quarter is weak, next quarter is likely to be weak.**
- **If the current quarter is not weak, next quarter is likely to be not weak.**

This model is not a good match for sluggish reality.

- **The serial dependence of credit loss data is a separate channel of inquiry.**
  - **We undertake this in Week 5.**

Portfolio loss simulation

Simulation can handle portfolio *loss* as well as default.

Two more inputs are needed:

- **The dollar exposure to each loan**
- **The distribution of LGD for each loan**

If a loan defaults in a simulation run,

- **Draw a value of LGD from its distribution.**
- **Multiply LGD by exposure to find dollar loss.**

Summing over all firms, $Loss = \sum_i \$Loss_i \,/\, \sum_i \$Exposure_i$

- **We assume that all exposures are equal; one loan per firm.**
  - **We're studying how the models work, not arithmetic!**
- **Tonight, we model only the default rate.**
  - **We get back to modeling loss in Week 4.**

Questions? Comments?

That is the simulation that is run by most big banks.

Non-simulation: Topics

Simplification and insight

The single factor risk model

The conditionally expected default rate

Change of variable and the Vasicek distribution

Simplification and insight

Suppose you want to know the effect of adding a loan.

- **You could compare the results of two simulations: one has the extra loan and the other does not have it.**
  - **You need the loan's correlation with every other loan in your portfolio.**
  - **To assign these, a practical person relies on patterns or rules of thumb.**
  - **The correlation matrix contains more richness than is often wanted.**

If simplification extends far enough, we arrive at some compact expressions Advantages:

- **The risk of a loan can be analyzed irrespective of portfolio.**
  - **Outside the US, bank supervisors require banks to hold capital greater than the incremental risk contributed by the loan.**
  - **Capital depends on the loan, not on the bank that makes the loan.**
- **The risks of different kinds of loans can be compared easily.**
  - **Should we lend to investment grade firms or to non-investment grade?**

The ASRF model

Asymptotic: The number of loans is so great that the Law of Large Numbers can operate.

- **The default rate that is observed equals to the default rate that would be expected, given economic conditions.**

Single risk factor: The correlations between firms could be produced by a single systematic risk factor.

- **This has nice effects, as you'll soon see:**
  - **All loans are essentially exposed to the same risk.**
  - **There is no opportunity for diversification.**
  - **Portfolio risk equals the sum of the risks of the loans in the portfolio.**

Single risk factor

In the single risk factor model:

- **All correlations arise from a single systematic risk factor.**
  - **The systematic risk factor affects *every* Z_i.**
  - **It is the *only* source of correlation between them.**

I prefer to define the systematic risk factor as *bad*.

- **If the factor is big, then firms are more likely to default.**

It is still the case that Z_i is *good*.

- **If Z_i is big, then Firm i is less likely to default.**

$$Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1-\rho_i}\, X_i$$

This defines the single risk factor model.

- **Z and X_i are assumed independent standard normal.**
  - **Therefore, Z and $\{X_i\}$ are *jointly* normal.**
  - **Therefore, $\{Z_i\}$ are *jointly* normal.**
- **Z is the "systematic" risk factor.**
  - **It affects every Z_i, $i \in \{1, 2, \ldots, N\}$.**
- **$-\sqrt{\rho_i}$ is Firm i's "loading" on the systematic risk factor.**
- **X_i is an "idiosyncratic" variable.**
  - **X_i is independent of Z and independent of every X_j, $j \neq i$.**
  - **It affects Z_i and it affects no other latent variables.**
  - **Its effect is idiosyncratic to Firm i.**

$$Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1 - \rho_i}\, X_i$$

Handy facts about the single risk factor model:

- $E[Z_i] = -\sqrt{\rho_i}\, E[Z] + \sqrt{1 - \rho_i}\, E[X_i] = 0$
- $Var[Z_i] = \rho_i\, Var[Z] + (1 - \rho_i)\, Var[X_i] = 1$
- $Cov\left[Z_i, Z_j\right] = \mathrm{E}\left[(Z_i - 0)\left(Z_j - 0\right)\right]$
- $= E\left[\left(-\sqrt{\rho_i}\, Z + \sqrt{1 - \rho_i}\, X_i\right)\left(-\sqrt{\rho_j}\, Z + \sqrt{1 - \rho_j}\, X_j\right)\right] = \sqrt{\rho_i\, \rho_j}$
- $= Corr\left[Z_i, Z_j\right]$
  - **The N values of ρ_i imply the $N(N-1)/2$ values of $\rho_{i,j}$.**
  - **There is a kind of pattern in the correlation matrix.**
  - **One possible pattern is that all correlations are equal.**

Questions? Comments?

Two ways to simulate DR

First way is just like the previous simulation.

- **Correlations between firms equal $\sqrt{\rho_i \rho_j}$.**
  - **Given the PD's and correlations, you simulate as before.**

Second way works like this:

- **Draw Z and N values X_i IID standard normal.**
- **Calculate the N values of $Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1-\rho_i}\, X_i$.**
- **Compare each to the respective default point, $\Phi^{-1}[PD_i]$.**
- **Add up the number of defaults and find DR. Then, repeat.**

The two ways give the same answer, eventually.

- **The second way is closer to the derivations that follow.**

Why/not a second risk factor?

You can write a two-factor model and simulate it.

- Each firm would have two systematic loadings.

What happens depends on the loadings.

- Maybe there are software firms depending on a software factor and hardware firms depending on a hardware factor.
  - Then, two industries can have downturns at different times.
- Maybe the ratio of the loadings is the same for every firm.
  - Then, the two-factor model degenerates to a one factor model.

As always, a model with more explanatory factors provides a better fit to historical data.

- Also as always, the forecasts of the richer model might be worse because it contains more things that can go wrong.
- We stick with the single factor model.

$Z_i = -\sqrt{\rho_i}\, Z + \sqrt{1-\rho_i}\, X_i$: Properties

- **When Z is elevated, each Z_i tends to be depressed.**
  - **Therefore, each firm is more likely to default than otherwise.**
- **A firm with a greater value of ρ_i is more affected by Z.**
  - **Such a firm is said to be a "cyclical" firm, such as an airline.**
  - **A firm with low ρ_i is less affected, like Proctor and Gamble.**
- **Most defaults are *idiosyncratic*.**
  - **If $\rho_i = 0.1$, then 10% of the variance of Z_i comes from Z.**
    - **The other 90% comes from X_i.**
  - **Even in a run where Z takes an elevated value like 2:**
    - $Z_i \approx -\sqrt{0.1} * 2 + \sqrt{0.9}\, X_i = -0.32 + 0.95\, X_i$.

Questions? Comments?

The single risk factor model is the main topic for today.

Conditional PD: cPD

Conditional PD is probability of default given conditions.

- **In the single factor model, this means the value of Z.**
- **Once you know Z, default depends only on $X_i \sim N[0,1]$:**

$$cPD = Pr\left[\, Z_i < \; \Phi^{-1}[PD_i] \mid Z = z\right]$$

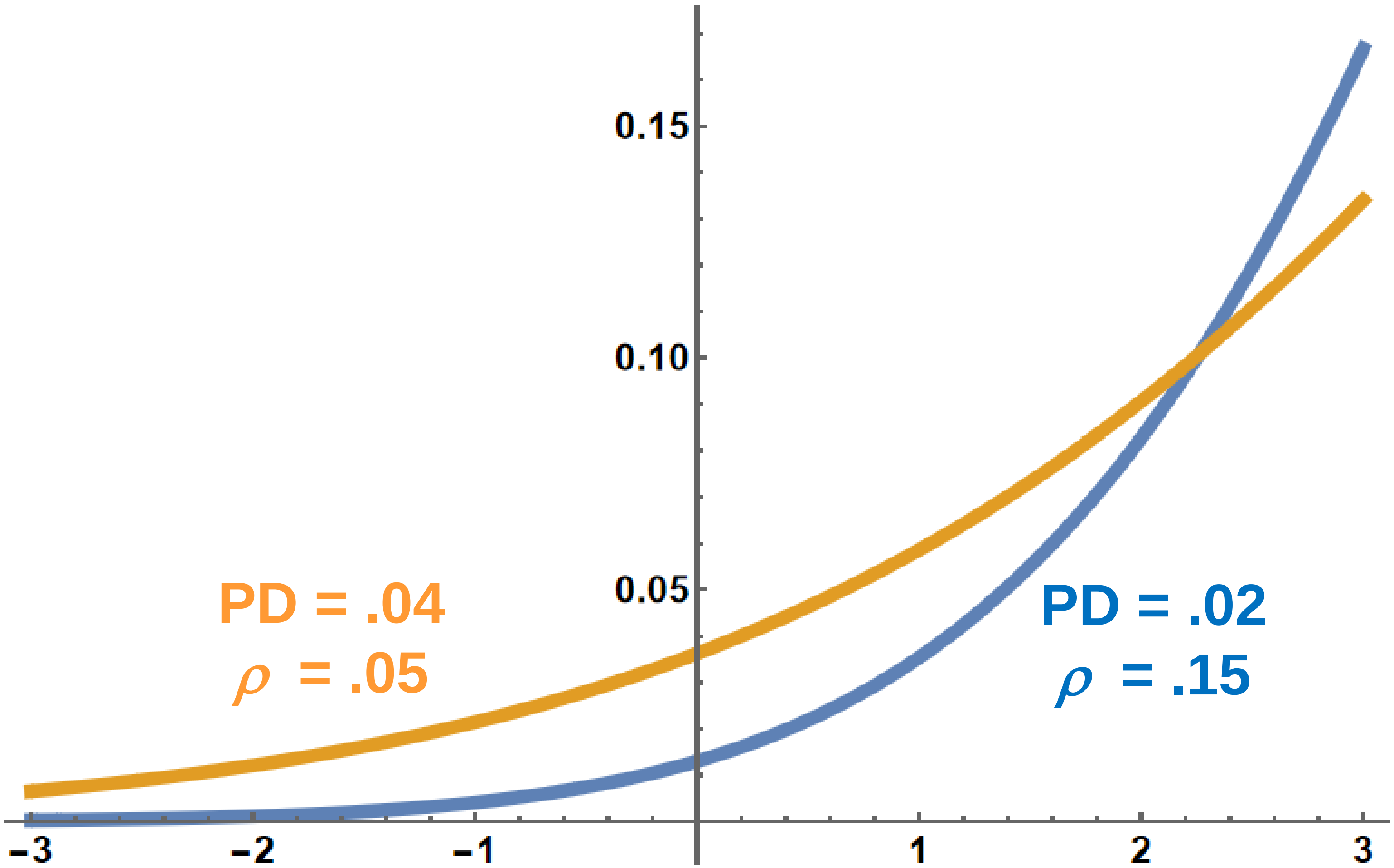
$$= Pr\,[\, -\sqrt{\rho_i}\, z + \sqrt{1-\rho_i}\, X_i < \; \Phi^{-1}\,[PD_i]$$

$$= Pr\left[X_i < \frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right] \; = \; \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\, z}{\sqrt{1-\rho_i}}\right]$$

This applies to each firm in a single risk factor model.

- **Conditional PD is monotonic in both Z and PD.**
- **This is sometimes called the Vasicek formula.**

cPD as a function of Z



Conditional independence

We don't know the conditions of the next year.

- **Even if every firm in a large portfolio has the same PD, we do not know the default rate in the next year.**
  - **Reason: next year might be a downturn with many defaults or an expansion year with few defaults. We don't know future conditions.**
- **We do not (yet) know the distribution of the default rate.**

But $Z = z$ fully specifies conditions in a SRF model.

- **Given that, a default depends an X_i, which is independent.**
  - **The defaults of firms are therefore *conditionally* *independent*.**
- **If there are N firms that default independently at rate cPD, then the number of defaults is distributed Binomial [N, cPD].**
- $(\Sigma D | Z = z) \sim Bin[N, cPD]$.

PMF of the number of defaults

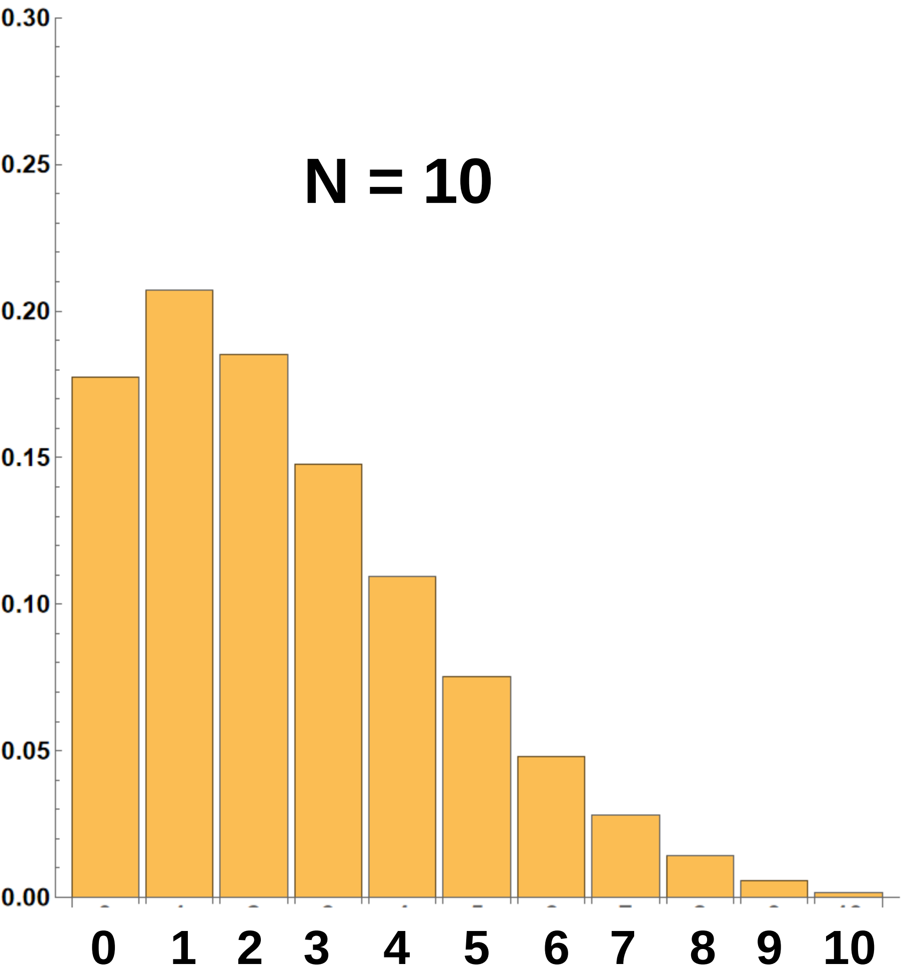
Suppose a portfolio with N statistically identical firms:

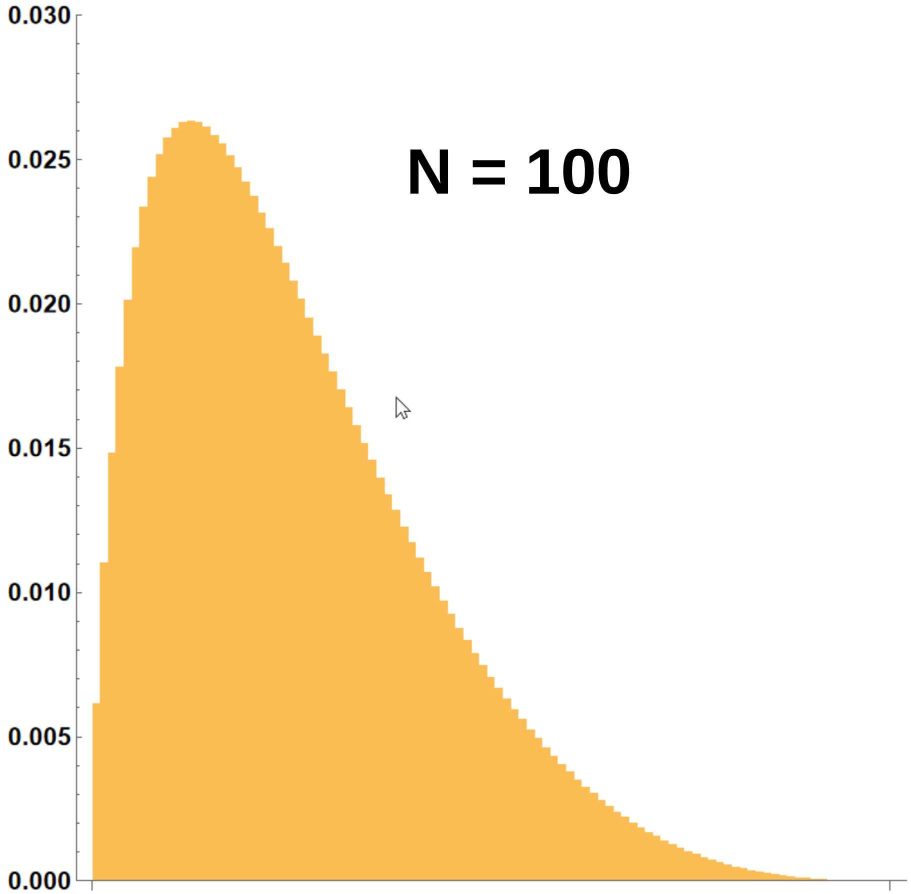
- **Each firm has the same value of PD.**
- **There is a single systematic risk factor.**
- **Each firm has the same value of ρ.**

$$PMF[\Sigma D] = \int_{-\infty}^{\infty} PMF[\Sigma D, z]dz = \int_{-\infty}^{\infty} PMF[\Sigma D|z]PDF[z]dz$$

$$= \int_{-\infty}^{\infty} \binom{N}{\Sigma D} \left(\Phi\left[\frac{\Phi^{-1}[PD] + \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right]\right)^{\Sigma D} \left(1 - \Phi\left[\frac{\Phi^{-1}[PD] + \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right]\right)^{N-\Sigma D} \phi[z]\ dz$$

PMFs: PD = 0.25, ρ = 0.25





Questions? Comments?

You should be able to derive the Vasicek formula for cPD and for the PMF.

Letting N $\rightarrow \infty$?

- **As the number of firms in the portfolio increases, the PMF apparently approaches a continuous distribution.**
  - **Instead, we find the distribution of the cPD of a *single firm*.**
    - **Its cPD is a monotonic function of the standard normal risk factor.**
    - **The default rate of a large portfolio of identical firms would have the *same* distribution; the math is easier if we use a single firm.**

- **To derive the distribution of cPD:**
  - **From the function cPD[Z], find the inverse CDF of cPD.**
  - **Invert the inverse CDF to find the CDF.**
  - **Differentiate the CDF to find the PDF.**

- **The resulting distribution is the Vasicek Distribution.**
  - **Same guy did the interest rate model that made him famous.**
    - **But the Vasicek Distribution stuff made him rich.**

From cPD[Z] to the inverse CDF

$$cPD_i[z] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\,z}{\sqrt{1-\rho_i}}\right]$$

Consider this a function of the quantile of Z: $q = \Phi[z]$:

$$cPD_i[q] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\;\Phi^{-1}[q]}{\sqrt{1-\rho_i}}\right]$$

Done! Put in the quantile of Z, and this gives you the value of the random variable cPD at that quantile.

- **Simply put, this is the inverse CDF of variable cPD.**

From inverse CDF to CDF

Start with the inverse CDF from the previous slide:

$$\boldsymbol{cPD_i[q] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\,\Phi^{-1}[q]}{\sqrt{1-\rho_i}}\right]}$$

To invert the function, solve for $\boldsymbol{q}$:

$$\boldsymbol{q = \Phi\left[\frac{\sqrt{1-\rho_i}\,\Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right] = CDF_i[cPD_i]}$$

Done!

From CDF to PDF

The derivative of the CDF is the PDF:

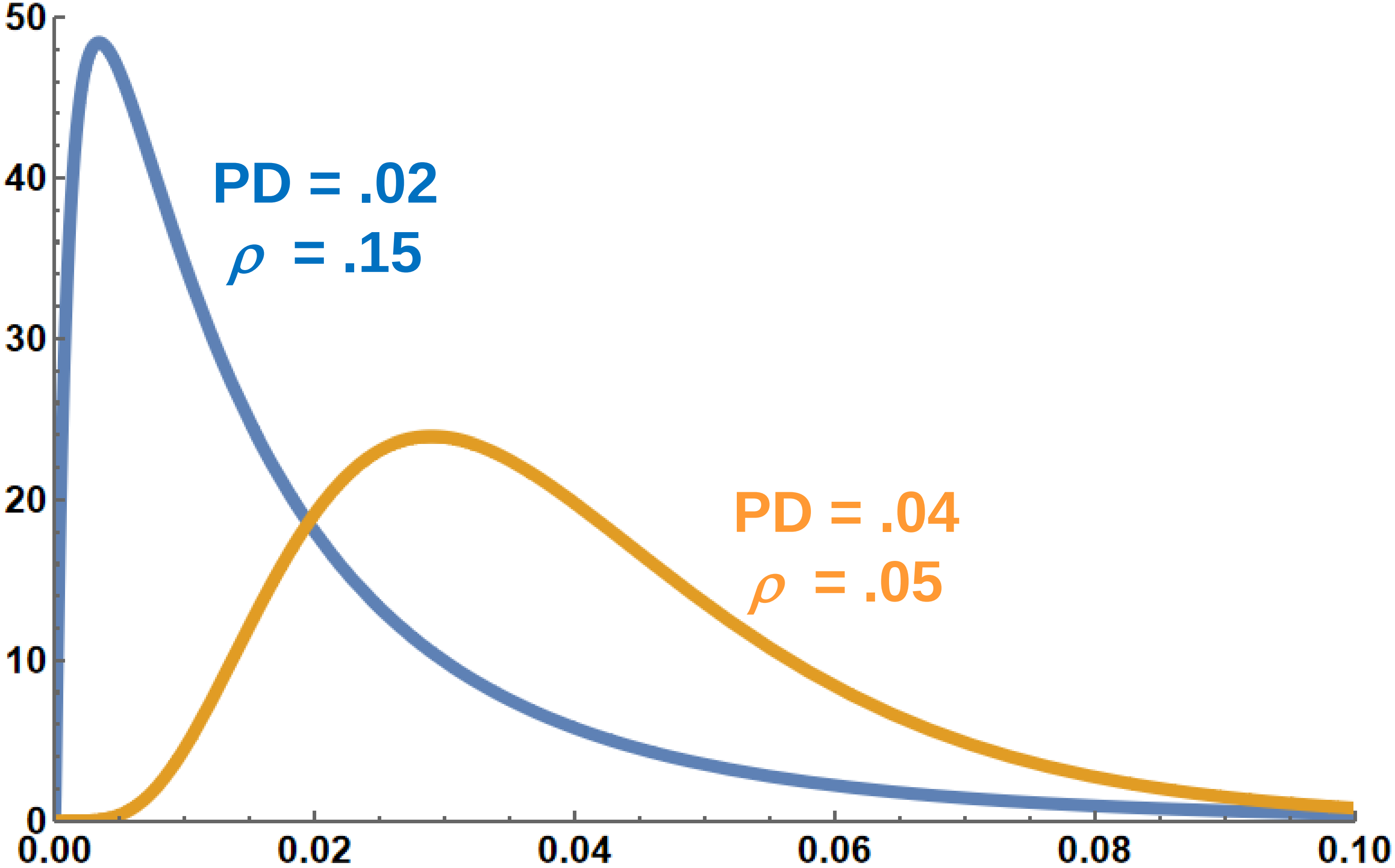
$$\boldsymbol{CDF_i[cPD_i] = q = \Phi\left[\frac{\sqrt{1-\rho_i}\,\Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right]}$$

$$\boldsymbol{PDF_i[cPD_i] = \frac{\sqrt{1-\rho_i}}{\sqrt{\rho_i}\,\phi[\Phi^{-1}[cPD_i]]}\,\phi\left[\frac{\sqrt{1-\rho_i}\,\Phi^{-1}[cPD_i] - \Phi^{-1}[PD_i]}{\sqrt{\rho_i}}\right]}$$

where $\boldsymbol{\phi}$ symbolizes the PDF of the standard normal

Done!

Vasicek PDFs



Vasicek Distributions

Inverse CDF: $\boldsymbol{CDF^{-1}[q] = \Phi\left[\frac{\Phi^{-1}[PD]+\sqrt{\rho}\,\Phi^{-1}[q]}{\sqrt{1-\rho}}\right]}$

CDF: $\boldsymbol{CDF[cPD] = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[cPD]-\,\Phi^{-1}[PD]}{\sqrt{\rho}}\right]}$

PDF: $\boldsymbol{PDF[cPD] = \frac{\sqrt{1-\rho}}{\sqrt{\rho}\,\phi\left[\Phi^{-1}[cPD]\right]}\,\phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}[cPD]-\Phi^{-1}[PD]}{\sqrt{\rho}}\right]}$

Questions? Comments?

You should be able to derive these formulas.

However, I don't test for it.

Shortcut to the PDF

$$cPD_i[z] = \Phi\left[\frac{\Phi^{-1}[PD_i] + \sqrt{\rho_i}\,z}{\sqrt{1-\rho_i}}\right]$$

The Vasicek formula allows this interpretation of Z:

- ***Z is whatever makes the expected default rate go up and down.***
  - **Z summarizes the effects of all observable macroeconomic factors plus the effects of unobservable factors ("the animal spirits of businessmen").**
  - **But Z is not the same thing as GDP or any particular real-world variable.**

cPD is a monotonic function of normally distributed Z.

- **This meets the conditions for the change-of-variable technique.**
  - **Change-of-variable is a compact redo of the steps we just completed.**

Change-of-variable technique

Suppose the distribution of Z is known, and we want the distribution of R = g[Z], where g is monotonic.

$$\boldsymbol{CDF_R[r] = \Pr[R < r] = \Pr[g[Z] < r] = \Pr\left[Z < g^{-1}[r]\right] = CDF_Z[g^{-1}[r]]}$$

$$\boldsymbol{PDF_R[r] = \frac{\partial\, CDF_Z[g^{-1}[r]]}{\partial\, r} = \left|\frac{\partial g^{-1}[r]}{\partial r}\right| PDF_z[g^{-1}[r]]}$$

So, "change-of-variable" is also called "the chain rule".

$$\boldsymbol{PDF_R[r] = \left|\frac{\partial g^{-1}[r]}{\partial r}\right| PDF_Z[g^{-1}[r]]}$$

Applied to cPD and the standard normal distribution of Z, it produces the PDF of the Vasicek distribution.

Nice properties of Vasicek dist.

- It has support on [0,1].
  - The other common distribution with support on [0,1] is Beta.
- The first parameter is the mean.
  - The mean conditionally expected default rate is PD.
  - Later we use the Vasicek Distribution to model cLoss.
    - Then the mean parameter is EL, expected loss.
- The second parameter takes a limited range of values.
  - Most estimated values of ρ seem to be between 5% and 15%.
    - The range reflects the difference between cyclical and non-cyclical.
- It has an explicit PDF, CDF, and $\text{CDF}^{-1}[\cdot]$, as you saw.

Bad properties of Vasicek dist.

It is the distribution of a variable that is unobservable,
- unless there is a portfolio with an infinite number of firms,
  - and that's impossible.

It has parameters that require estimates,
- and the estimates require data,
  - and the data requires uniform portfolios,
    which we do not have, and we don't have much data on non-uniform portfolios, either.

But like the CLT, it is a plank in a shipwreck.
- Any bit of help is helpful, since we are otherwise clueless.
  - Searching a data set for strong correlations−ad hoc specification search−produces notably poor forecasts, as we'll discuss in Week 5.

Vasicek summary

We chose the Gauss copula.

- **The simplest one is the independence copula.**
- **Next-simplest stems from the single risk factor model.**
  - **Simpler models have fewer things that can go wrong. That's good!**

If a *firm* responds to a single risk factor, then its cPD is a monotonic function of the risk factor.

- **Gaussian risk factor $\Rightarrow$ Vasicek distribution.**

If a *portfolio* contains statistically identical firms (uniform values of PD and ρ), each has the same cPD.

- **A small portfolio would have random default rates with expectations equal to cPD.**
- **The default rate of a uniform asymptotic portfolio equals cPD.**

Questions? Comments?

You must know the change of variable formula.

You should be able to derive it.

The Basel capital requirement

The Basel Committee

The Bank for International Settlements is in Basel, CH.

- **There is a Basel Committee on Bank Supervision, BCBS.**

The BCBS drafted legislation requiring banks to have minimum *capital*. “Basel II”, “Basel III”, etc.

- **A similar law was adopted by each developed country.**
  - **The US has other requirements that tend to be more binding than Basel.**

***Capital* is like net worth: assets less liabilities.**

- **But it is an accounting concept, not fully marked-to-market.**

The capital *requirement* is like a margin requirement.

- **To make a given loan, a bank must have minimum capital.**
  - **Capital lets the bank survive if it loses some money.**
  - **This protects bank depositors and the public.**

One rule to ring them all

The BCBS wanted a function that would set minimum required capital for any loan.

- **The characteristics of the loan would imply minimum capital.**
- **Minimum capital would be the same for every bank,**
  - **irrespective of which bank makes the loan.**
- **Minimum capital would be *additive*.**
  - **There is no risk offset through diversification.**
  - **Otherwise, different banks would have different requirements.**

The single risk factor model fills the bill.

- **The characteristics of the loan are its PD, ρ, and ELGD.**
  - **The bank estimates PD and ELGD; BCBS specifies the value of ρ.**
- **Minimum capital would be loss at quantile 0.999.**
- **Portfolio required capital is the sum of loan required capital.**

Basel formula and cPD

Per dollar of a "wholesale" loan, Basel requires a bank to have capital of this amount:

- **where "R" is correlation (ρ)**
- **"N" is the standard normal CDF (Φ), and**
- **"M" is the maturity of the loan in years ranging from 1 to 5.**

$$K = \left[LGD \times \underbrace{N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1-R}} \right)} - (LGD \times PD) \right] \times \left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b} \right)$$

This is cPD at q = 0.999.

Three main differences

1. **Capital is required for *loss*, not just for *default*.**
   - **The formula multiplies by LGD to take care of this.**

2. **Capital is required only for *unexpected* loss.**
   - **Reserves should handle expected loss.**
   - **Expected loss, LGD x PD, is subtracted from the risk.**

3. **Loans might deteriorate but not default.**
   - **Basel adjusts by a maturity adjustment factor.**
     - **Loans with longer maturity require perhaps 3 times more capital.**
     - **Don't try to make money trading off this idea!**

$$K = \underbrace{\left[LGD \times N\left(\frac{N^{-1}(PD) + \sqrt{R} \times N^{-1}(0.999)}{\sqrt{1-R}} \right)}_{\boxed{1}} - \underbrace{\left(LGD \times PD \right) \right]}_{\boxed{2}} \times \underbrace{\left(\frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b} \right)}_{\boxed{3}}$$

More Basel calibration

Basel specifies these parameters in the formula:

- R (correlation) = 0.12 + .12 * Exp[-50 PD]
  - Note: R [PD = 0] = .24, R [PD = 1] = .12, monotonic decreasing.
  - There is no evidence that correlation and PD are related this way.
- b (in the maturity adjustment) = $(0.11852 - 0.05478\ \mathrm{Log[PD]})^2$
  - Don't ask. Apparently, no one else has.

A bank might estimate Basel parameters like this:

- A loan's PD equals the average annual default rate
  - within the rating grade that the bank assigns to the loan.
- LGD is the average LGD in historical "downturn" conditions,
  - taken among loans with similar seniority and security.
- M is maturity in years, bounded between 1 and 5.
  - Simple calculations are sometimes not so simple.
- All estimates are subject to supervisory oversight.

Basel formula summary

Basel requires banks to have minimum capital.

- **Capital is expensive to banks, so there are games galore surrounding the input estimates.**
- **In addition to the credit capital that we've discussed, capital is required for other things like operational risk.**

Minimum capital is a high percentile of the cPD formula. Minimum capital for the portfolio is the sum of minimum capital for each loan, because there's a single risk factor.

The formula depends on estimates of PD and LGD.

- **These must be estimated by the bank.**
- **The estimation process is overseen by bank supervisors.**

Questions? Comments?

Multistate simulation models

Simulating rating transitions

So far, we have simulated a two-state model.

- A firm either defaults or it doesn't.

It is possible to model not just the transition to default but also transitions to other states.

- Usually, the other states are internal rating grades.

This requires the probability that a firm with a given rating experiences transition to a new rating…

A rating transition matrix

High grade

Rat'g	Rating at year end (%)							
	AAA	AA	A	BBB	BB	B	CCC	Default
AAA	87.74	10.93	0.45	0.63	0.12	0.10	0.02	0.02
AA	0.84	88.23	7.47	2.16	1.11	0.13	0.05	0.02
A	0.27	1.59	89.05	7.40	1.48	0.13	0.06	0.03
BBB	1.84	1.89	5.00	84.21	6.51	0.32	0.16	0.07
BB	0.08	2.91	3.29	5.53	74.68	8.05	4.14	1.32
B	0.21	0.36	9.25	8.29	2.31	63.89	10.13	5.58
CCC	0.06	0.25	1.85	2.06	12.34	24.86	39.97	18.60
D	0	0	0	0	0	0	0	100

High yield

The numbers are outdated, but this gives an idea.

The most likely thing is no change of rating.

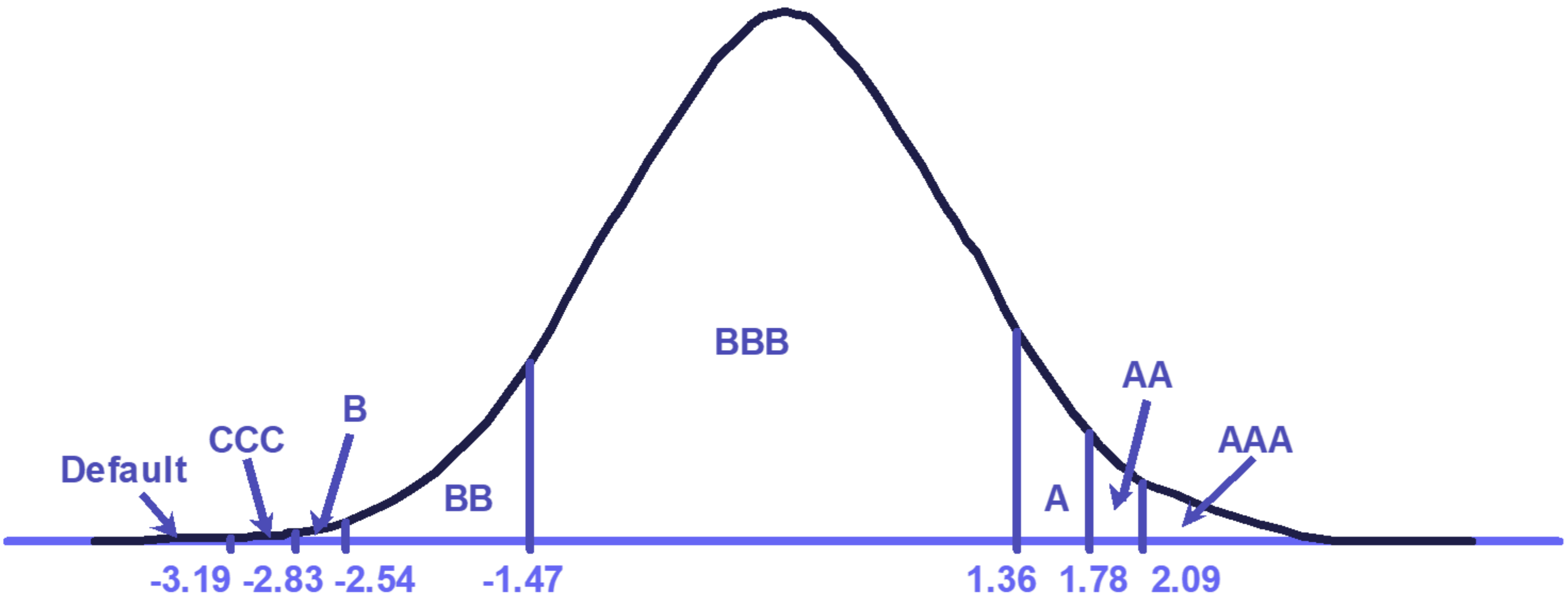
Consider Firm i, rated BBB

Let Z_i control the transition to *<u>any</u>* other grade.

- **According to the previous slide, a firm rated BBB defaults if Z_i is in the worst 0.07% of its range.**
- **The firm transitions from BBB to CCC if Z_i is very low but above the 0.07 percentile.**
  - **Specifically, the BBB is downgraded to CCC if its value of Z is between the 0.07% quantile and the 0.16% quantile.**
- **And so forth, right up through upgrades to AAA.**

Partition the range of Z_i according to the transition probabilities…

Transitions for a firm rated BBB



The latent variable Z_i controls all the transitions.

Transition matrix reflections

A transition matrix requires a cost matrix.

- **In a default-only model, the cost is LGD.**
- **Here you need the cost of transition from every initial state to every other state.**
  - **In practice, the states would be the bank's internal ratings.**
  - **In practice, the costs might be fixed amounts or distributions.**

The set-up is too rich for a non-simulation approach.

The rest of the course studies the default-only model.

- **You can always simulate the multi-state model if needed.**

Questions? Comments?

Don’t forget

Homework Set 2 is due by 6PM Wednesday April 13.

Lisheng’s TA session will be 6PM Sunday April 10.