FinMath 36702 Homework 3 Due 6pm 20 April 2022.

Lisheng will discuss strategies for solving in the TA session on April 17 and present full solutions on April 24. Please submit homeworks as detailed in "FINM36702 Assignment Submission Instructions" located on Canvas.

Question 1. Suppose that the default rate of a portfolio has the triangular distribution: $pdf_{dr}[dr] = 2 - 2dr$. Suppose that in this portfolio lgd is a function of dr: $lgd[dr] = dr^{1/2}$. Derive and state the function $pdf_{lgd}[lgd]$. Create a single diagram containing plots of $(pdf_{dr}[dr]$ and $pdf_{lgd}[lgd])$ for variables in the range between 0 and 1.

Question 2. Making the same assumptions as in Question 1, derive and state $pdf_{loss}[loss]$. Create a diagram containing the two plots from Question 1 along with the plot of $pdf_{loss}[loss]$ for variables in the range between 0 and 1; limit the vertical axis to the range from zero to 3. State the values of

- Expected loss, EL
- Expected LGD, ELGD
- "Time-weighted" LGD

3. Express the standard deviation of a Vasicek distribution as an integral that involves the Vasicek PDF. For distributions with PD = 0.10, numerically integrate and plot the standard deviation for $0.05 < \rho < 0.95$. On a separate diagram, plot two Vasicek distributions: PD = 0.10, $\rho = 0.05$ and PD = 0.10, $\rho = 0.95$, limiting the vertical axis to {0, 0.12}.

4. Suppose two loans have Vasicek distributions. One loan has PD = 0.06, $\rho = 0.06$, the second loan has PD = 0.03, $\rho = 0.20$, and both loans respond to the same systematic risk factor. Plot on a single diagram the two inverse CDFs. At the lower quantiles, the first loan has greater cPD than the second. The situation is reversed at very high quantiles. Estimate the quantile at which both loans have the same value of cPD.