

## Homework #6

Due on Monday, Nov 8, at 6:00pm.

Case: AQR's Momentum Funds (A) [9-211-025].

### 1 The Momentum Product

*This section is not graded, and you do not need to submit your answers. But you are expected to consider these issues and be ready to discuss them.*

1. What is novel about the AQR Momentum product under construction compared to the various momentum investment products already offered?
2. Name three reasons the momentum investment product will not exactly track the momentum index, (ie. why the strategy will have tracking error.)
3. When constructing the momentum portfolio, AQR ranks stocks on their returns from month  $t - 12$  through  $t - 2$ . Why don't they include the  $t - 1$  return in this ranking?

### 2 Investigating Momentum

In this section, we empirically investigate some concerns regarding AQR's new momentum product.

On Canvas, find the data file, “**momentum\_data.xlsx**”.

- The first tab contains the momentum factor as an excess return:  $\tilde{r}^{\text{mom}}$ .
- The second tab contains returns on portfolios corresponding to scored momentum deciles.
  - $r^{\text{mom}(1)}$  denotes the portfolio of stocks in the lowest momentum decile, the “losers” with the lowest past returns.
  - $r^{\text{mom}(10)}$  denotes the portfolio of stocks in the highest momentum decile.
- The third tab gives portfolios sorted by momentum and size.
  - $r^{\text{momSU}}$  denotes the portfolio of small stocks in the top 3 deciles of momentum scores.
  - $r^{\text{momBD}}$  denotes the portfolio of big-stocks in the bottom 3 deciles of momentum scores.

Note that the Fama-French momentum return,  $\tilde{r}^{\text{mom:FF}}$ , given in the first tab, is constructed by FF as,

$$\tilde{r}^{\text{mom:FF}} = \frac{1}{2} (r^{\text{momBU}} + r^{\text{momSU}}) - \frac{1}{2} (r^{\text{momBD}} + r^{\text{momSD}}) \quad (1)$$

## 1. Is momentum still profitable?

The excess returns of lucrative trading strategies often disappear once the strategy is well-known. The first widely-cited paper on momentum was published in 1993. Have momentum returns or risk changed since then? The AQR case takes place at the end of 2008. Have momentum returns changed in 2009-2021?

Investigate by filling out the summary statistics below for the full-sample and three sub-samples.

- (a) Using the data provided, fill in Table 1 with the appropriate stats for  $\tilde{r}^{\text{mom:FF}}$ .

Table 1: Momentum performance over time.

Subsample	mean	vol	Sharpe	skewness	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
1927-2021						
1927-1993						
1994-2008						
2009-2021						

- (b) Has momentum changed much over time, as seen through these subsample statistics?
- (c) Does this data support AQR's argument that momentum is an important piece of the ideal portfolio? What if mean returns to momentum are in actuality near zero due to transaction costs—would there still be evidence here that momentum is valuable?

## 2. The case is about whether a long-only implementation of momentum is valuable.

Construct your own long-only implementation:<sup>1</sup>,

$$\tilde{r}^{\text{momU:FF}} = \frac{1}{2} (r^{\text{momBU}} + r^{\text{momSU}}) - r^f$$

Note that this is following the FF approach of treating big and small stocks separately. This would be very similar to a scaled version of,

$$\tilde{r}^{\text{momU}} = \frac{1}{3} (r^{\text{mom}(8)} + r^{\text{mom}(9)} + r^{\text{mom}(10)}) - r^f$$

For the question below, use the FF-style,  $\tilde{r}_t^{\text{momU:FF}}$ .

- (a) Fill out Table 2 for the data **in the period 1994-2021**.
- (b) Is long-only momentum as attractive as long-short momentum with respect to mean, volatility, and Sharpe Ratio?

<sup>1</sup>Note that we must subtract the risk-free rate to get the excess return of this portfolio,  $\tilde{r}^{\text{momU}}$ .

Table 2: Long-only momentum performance.

1994-2021	mean	vol	Sharpe	skew	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
Long-and-short, $(\tilde{r}^{\text{mom:FF}})$						
Long-only $(\tilde{r}^{\text{momU:FF}})$						

- (c) Is long-only momentum as diversifying as long-short momentum with respect to market and value premia?
- (d) Show a plot of the cumulative product of  $1 + \tilde{r}^{\text{mom:FF}}$  and  $1 + \tilde{r}^{\text{momU:FF}}$  over the 1994-2021 subsample.<sup>2</sup>

### 3. Is momentum just data mining, or is it a robust strategy?

Assess how sensitive the threshold for the “winners” and “losers” is in the results. Specifically, we compare three constructions:

- long the top 1 decile and short the bottom 1 deciles

$$\tilde{r}^{\text{momD1}} = r^{\text{mom}(10)} - r^{\text{mom}(1)}$$

- long the top 3 deciles and short the bottom 3 deciles

$$\begin{aligned}\tilde{r}^{\text{momD3}} &= \frac{1}{3} (r^{\text{mom}(8)} + r^{\text{mom}(9)} + r^{\text{mom}(10)}) - \frac{1}{3} (r^{\text{mom}(1)} + r^{\text{mom}(2)} + r^{\text{mom}(3)}) \\ &= \frac{1}{3} \sum_{k=8}^{10} r^{\text{mom}(k)} - \frac{1}{3} \sum_{k=1}^3 r^{\text{mom}(k)}\end{aligned}$$

- long the top 5 deciles and short the bottom 5 deciles

$$\tilde{r}^{\text{momD5}} = \frac{1}{5} \sum_{k=6}^{10} r^{\text{mom}(k)} - \frac{1}{5} \sum_{k=1}^5 r^{\text{mom}(k)}$$

- (a) Compare all three constructions, (in the full-sample period,) by filling out the stats in the table below **for the period 1994-2021**.
- (b) Do the tradeoffs between the 1-decile, 3-decile, and 5-decile constructions line up with the theoretical tradeoffs we discussed in the lecture?
- (c) Should AQR’s retail product consider using a 1-decile or 5-decile construction?
- (d) Does  $\tilde{r}^{\text{momD3}}$  have similar stats to the Fama-French construction in (1). Recall that construction is also a 3-decile, long-short construction, but it is segmented for small and large stocks. Compare the middle row of Table 3 with the top row of Table 2.

<sup>2</sup>This is not quite a cumulative return given that these are **excess** returns, which is why we must add 1 before taking the cumulative product.

Table 3: Robustness of momentum construction.

1994-2021	mean	vol	Sharpe	skewness	corr. to $\tilde{r}^m$	corr. to $\tilde{r}^v$
$\tilde{r}^{\text{momD1}}$						
$\tilde{r}^{\text{momD3}}$						
$\tilde{r}^{\text{momD5}}$						

4. **Does implementing momentum require trading lots of small stocks—thus causing even larger trading costs?**

For regulatory and liquidity reasons, AQR is particularly interested in using larger stocks for their momentum baskets. (Though they will launch one product that focuses on medium-sized stocks.)

Use the data provided on both small-stock “winners”,  $r^{\text{momSU}}$ , and small-stock “losers”,  $r^{\text{momSD}}$ , to construct a small-stock momentum portfolio,

$$r_t^{\text{momS}} = r_t^{\text{momSU}} - r_t^{\text{momSD}}$$

Similarly, use the data provided to construct a big-stock momentum portfolio,

$$r_t^{\text{momB}} = r_t^{\text{momBU}} - r_t^{\text{momBD}}$$

(a) Fill out Table 4 over the sample 1994-2021.

Table 4: Momentum of small and big stocks.

1994-2021	mean	vol	Sharpe	skewness	corr. to $\tilde{r}[m]$
All stocks, $\tilde{r}^{\text{mom:FF}}$					
Small stocks $r_t^{\text{momS}}$					
Large stocks $r_t^{\text{momB}}$					

(b) Is the attractiveness of the momentum strategy mostly driven by the small stocks? That is, does a momentum strategy in large stocks still deliver excess returns at comparable risk?

5. **In conclusion**, what is your assessment of the AQR retail product? Is it capturing the important features of the Fama-French construction of momentum? Would you suggest any modifications?

### 3 Extensions

This section is not graded, and you do not need to submit your answers. We may discuss some of these extensions.

1. In Section 2 we analyzed whether Momentum changes substantially when we modify the construction of the factor. Let's examine that question for the Value factor.

Re-do Tables 2, 3, and 4 but for the decile and size portfolios of the Value factor. Get this data from Ken French's website.<sup>3</sup>

Based on these statistics,

- (a) Is the long-only version of Value substantially different?
- (b) Is the 1 or 5 decile version of Value substantially different?
- (c) Does the 3 decile version of Value look like the Fama-French version?
- (d) Do the Big Value and Small Value factors look similar?

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<sup>3</sup>[https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

Specifically, see the following data sets:

- Size-sorted portfolios for value. (For instance, “big stocks value”,  $r^{vB}$ ): “6 Portfolios Formed on Size and Book-to-Market (2 x 3)”
- Decile portfolios for value,  $r^{v(k)}$ : “Portfolios Formed on Book-to-Market”