Problem Set 4

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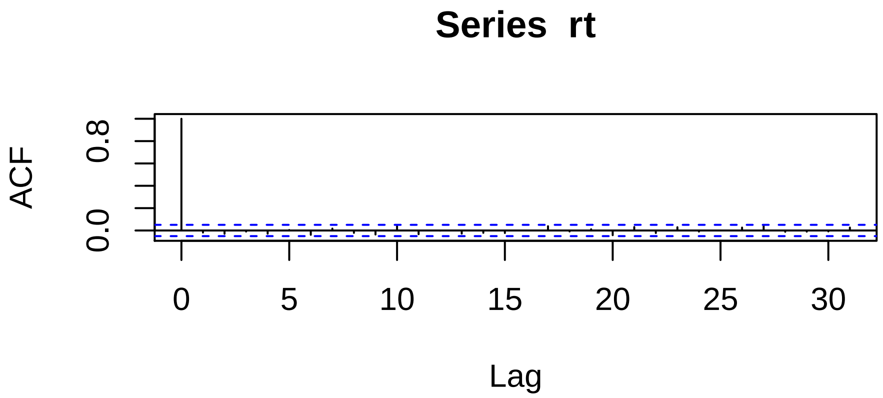
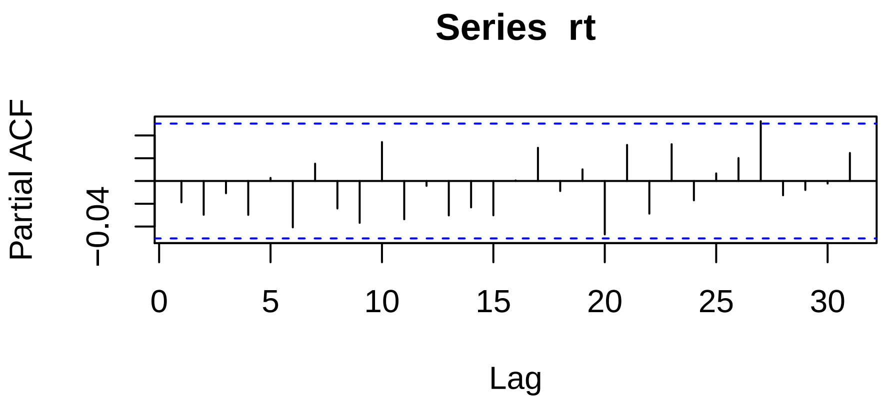
1

a. The expected value of r_t is not zero, since we reject the null hypothesis that the mean is 0. There are no serial correlations in r_t, evidenced by the high p-value from the Box-Ljung test.

```
da=read.table("d-amzn3dx0914.txt", header=T)
library(fGarch)

## Loading required package:  timeDate
## Loading required package:  timeSeries
## Loading required package:  fBasics
##
##
## Rmetrics Package fBasics
## Analysing Markets and calculating Basic Statistics
## Copyright (C) 2005-2014 Rmetrics Association Zurich
## Educational Software for Financial Engineering and Computational Science
## Rmetrics is free software and comes with ABSOLUTELY NO WARRANTY.
## https://www.rmetrics.org --- Mail to:  info@rmetrics.org

source("Igarch.R")
source("garchM.R")
source("Tgarch11.R")
source("Tsats.R")
rt=log(da$amzn+1)*100
t.test(rt)

##
##  One Sample t-test
##
## data:  rt
## t = 2.0296, df = 1509, p-value = 0.04257
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.003999691 0.234462533
## sample estimates:
## mean of x
## 0.1192311

par(mfcol=c(2,1))
acf(rt)
pacf(rt)
Box.test(rt,lag=10,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  rt
## X-squared = 10.9743, df = 10, p-value = 0.3595
```



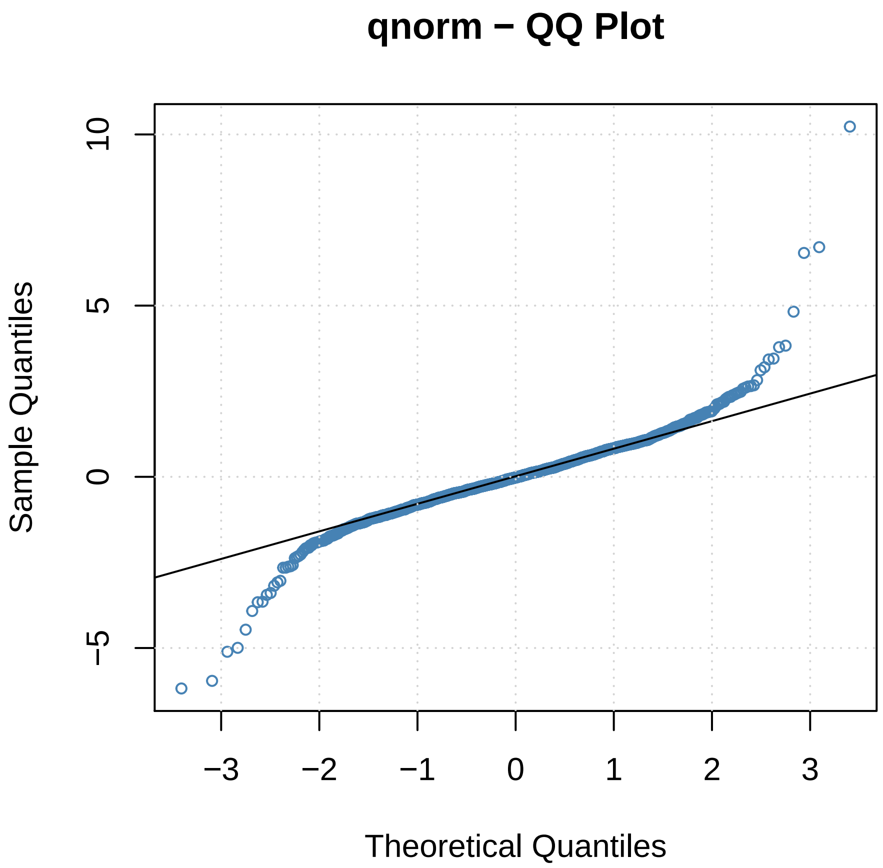


b. Fitted Model: $(1\text{-}.496L)r_t$=.0533+$(1\text{-}.532L)a_t$, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim$ N(0,1), σ_t^2=.0228+.00711$a_{\text{t-1}}$+.988$\sigma^2{}_{\text{t-1}}$
The model is not adequate because the QQ plot shows that the model does not satisfy the linearity assumptions.

```
arima(rt,order=c(0,0,1))$aic

## [1] 6782.306

arima(rt,order=c(1,0,0))$aic

## [1] 6782.34

arima(rt,order=c(1,0,1))$aic

## [1] 6779.944

m1=garchFit(~arma(1,1)+garch(1,1),data=rt,trace=F)
m1@fit$ics
```

```
##       AIC      BIC      SIC     HQIC
## 4.461512 4.482651 4.461481 4.469385
```

```
par(mfcol=c(1,1))
plot(m1,which=13)
```

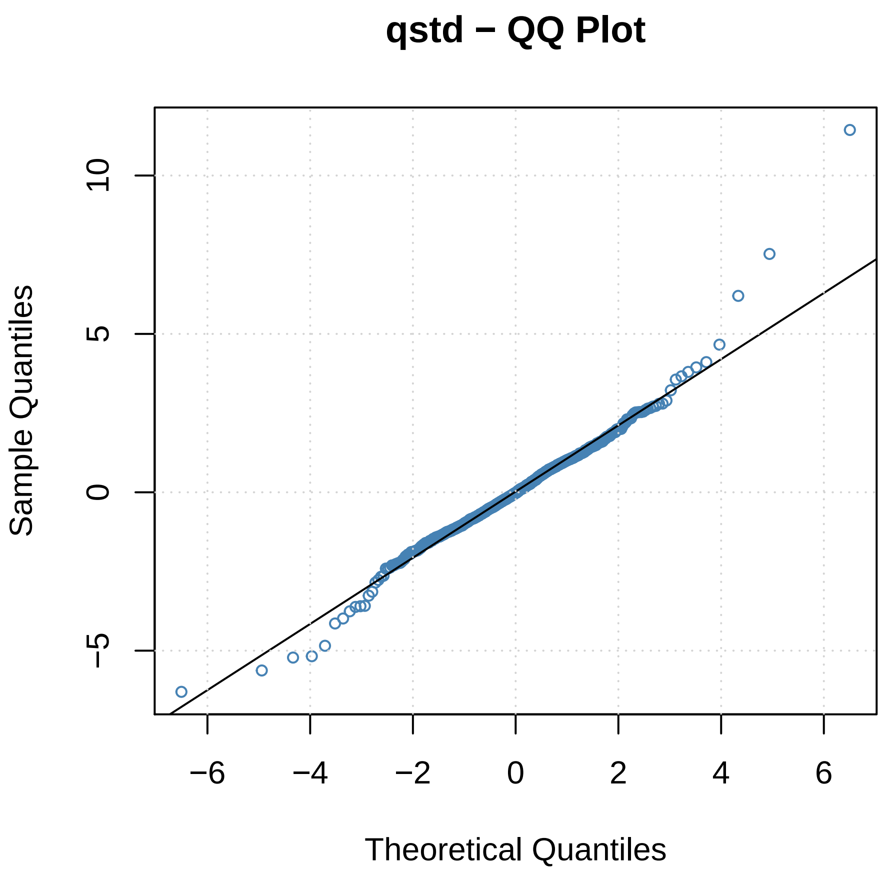


c. Fitted Model: $(1+.775\mathrm{L})r_t=.160+(1+.791\mathrm{L})a_t$, $a_t=\sigma_t\epsilon_t$, $\epsilon_t \sim t^*{}_{4.23}$, $\sigma_t^2=.0580+.0162a^2{}_{\text{t-1}}+.970\sigma^2{}_{\text{t-1}}$
The QQ-plot fit with the student-t innovations is better than the one with a normal distribution.

```
m2=garchFit(~arma(1,1)+garch(1,1),data=rt,trace=F,cond.dist="std")
m2@fit$ics
```

```
##       AIC      BIC      SIC     HQIC
## 4.235383 4.260044 4.235340 4.244567
```

```
plot(m2,which=13)
```



d.

```
pm3=predict(m2,5)
pm3


##   meanForecast meanError standardDeviation
## 1   0.07995075  1.886543          1.886543
## 2   0.09848840  1.889150          1.888886
## 3   0.08412987  1.891617          1.891194
## 4   0.09525142  1.893986          1.893468
## 5   0.08663710  1.896282          1.895707
```

2

a. Fitted Model: σ_t^2=.00144+.990$a^2$$_{\text{t-1}}$+(1-.990)$\sigma^2$$_{\text{t-1}}$

```
at=rt-mean(rt)
m3=Igarch(at,volcnt=T)


## Estimates:  0.001441415 0.9903561
## Maximized log-likehood:  3371.468
##
## Coefficient(s):
##          Estimate  Std. Error  t value Pr(>|t|)
## omega 0.00144142  0.00228797     0.63   0.5287
```

```
## beta  0.99035606  0.00270885   365.60   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b. No, the Ljung-Box statistic give Q(10) = 10.571 with a p-value of .3919

```
names(m3)

## [1] "par"        "volatility"

sresid=at/m3$volatility
Box.test(sresid,lag=10,type="Ljung")

##
##  Box-Ljung test
##
## data:  sresid
## X-squared = 10.571, df = 10, p-value = 0.3919
```

c. No, the Ljung-Box statistic give Q(10) = 3.879 with a p-value of .9527

```
Box.test(sresid^2,lag=10,type="Ljung")

##
##  Box-Ljung test
##
## data:  sresid^2
## X-squared = 3.8785, df = 10, p-value = 0.9527
```

d. Yes, the residuals are serially uncorrelated. The squared residuals are also serially uncorrelated.

```
v1=(1-0.990)*at[length(at)]^2+.990*m3$volatility[length(at)]^2
sqrt(v1)

## [1] 1.939348
```

3

a. The expected MCD log return is not zero, since a t-test tells us that we reject the null stating that the mean is 0. There is no serial correlation in the log returns: the Box-Ljung test gives a p-value of .3918. However, there is an ARCH effect in the log returns, as evidenced by the extremely small p-value in the Box-Ljung Test

```
db=read.table("m-mcd3dx6614.txt", header=T)
logret=log(db$mcd+1)
t.test(logret)
```

```
##
##  One Sample t-test
##
## data:  logret
## t = 4.3532, df = 580, p-value = 1.586e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.007509528 0.019856397
## sample estimates:
##  mean of x
## 0.01368296
```

```
Box.test(logret,lag=12,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  logret
## X-squared = 12.6927, df = 12, p-value = 0.3918
```

```
Box.test((logret-mean(logret))^2,lag=12,type="Ljung")
```

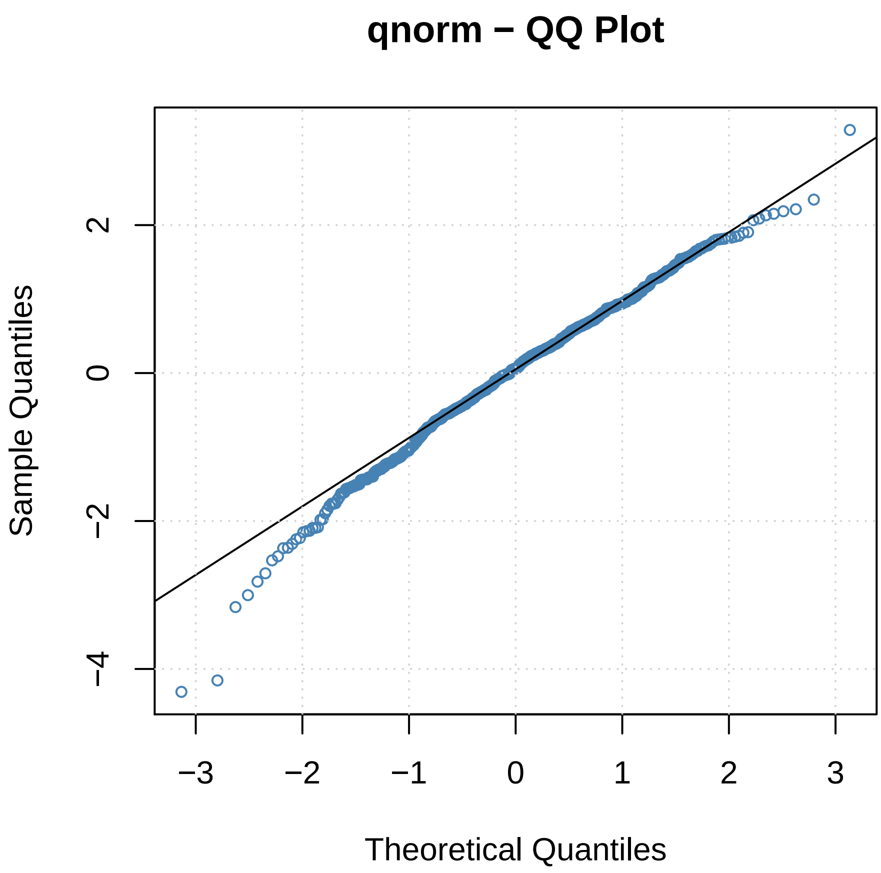
```
##
##  Box-Ljung test
##
## data:  (logret - mean(logret))^2
## X-squared = 157.9892, df = 12, p-value < 2.2e-16
```

b. Fitted Model: r_t=.0122+a_t, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim$ N(0,1), σ_t^2=4.960 $\times$ 10^{-5}+.0892a_{t-1}+.902$\sigma^2{}_{t-1}$
The QQ-plot shows that the linearity assumption is somewhat fulfilled. At lower quantiles the data tends to depart from the line.

```
m4=garchFit(~garch(1,1),data=logret,trace=F)
m4@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## -2.493689 -2.463639 -2.493783 -2.481975
```

```
plot(m4,which=13)
```



c. Fitted Model: σ_t^2=.926$a^2$$_{\text{t-1}}$+(1-.926)$\sigma^2$$_{\text{t-1}}$

```
m5=Igarch(logret)
```

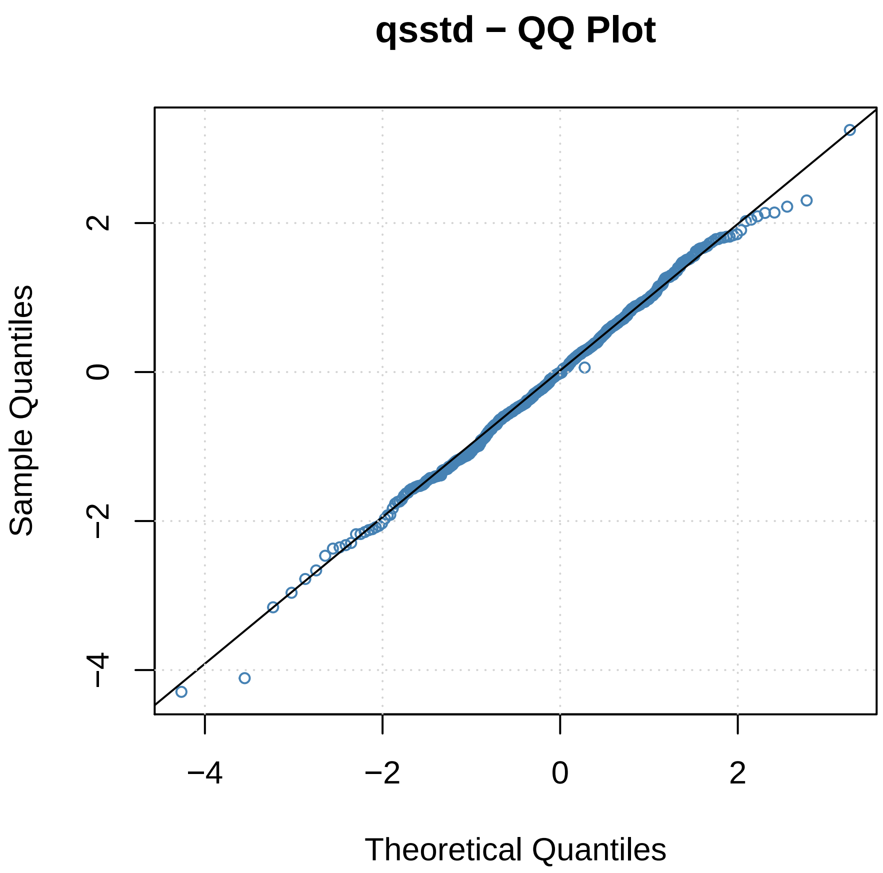
```
## Estimates:  0.926094
## Maximized log-likehood:  -716.6672
##
## Coefficient(s):
##       Estimate  Std. Error  t value   Pr(>|t|)
## beta 0.9260940   0.0161806  57.2348 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

d. Fitted Model: r_t=.0120+a_t, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim t^*$$_{10.00,\ .833}$, σ_t^2=7.486 × 10^{-5}+.0970$a^2$$_{\text{t-1}}$+.893$\sigma^2$$_{\text{t-1}}$
From the QQ-plot, we see that the data fit quite well, so the model is adequate.

```
m6=garchFit(~garch(1,1),data=logret,trace=F,cond.dist="sstd")
m6@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## -2.514099 -2.469024 -2.514309 -2.496527
```

```
plot(m6,which=13)
```



e. t = (.833-1)/.0507= -3.294, which is greater in absolute value than 1.96. Thus, we reject the null hypothesis that the log return has a symmetric distribution, the the log returns are skewed.

```
(.833-1)/.0507

## [1] -3.293886
```

f. Fitted Model: r_t=.004+1.662σ_t^2+a_t, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim$ N(0,1), σ_t^2=4.850 $\times$ 10^{-5}+.0892a_{t-1}+.902$\sigma^2{}_{t-1}$
The risk premium, γ, is statistically significant at the 5 percent level. The t-ratio is 1.98, which is barely above 1.96.

```
m7=garchM(logret)

## Maximized log-likehood:  729.8678
##
## Coefficient(s):
##           Estimate  Std. Error  t value   Pr(>|t|)
## mu    4.00381e-03 4.09801e-03  0.97701 0.32856258
## gamma 1.66190e+00 8.37012e-01  1.98551 0.04708747 *
## omega 4.84952e-05 4.93846e-05  0.98199 0.32610377
## alpha 8.91997e-02 2.43178e-02  3.66808 0.00024438 ***
## beta  9.02421e-01 2.69120e-02 33.53228 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

g. Fitted Model: r_t=.0117+a_t, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim$ N(0,1), σ_t^2=7.430 $\times$ 10^{-5}+(.0754+.0423)a_{t-1}+.890$\sigma^2{}_{t-1}$

```r
m8=Tgarch11(logret)
```

```
## Log likelihood at MLEs:
## [1] 729.2205
##
## Coefficient(s):
##          Estimate  Std. Error  t value   Pr(>|t|)
## mu    1.16630e-02 2.56895e-03  4.53999 5.6257e-06 ***
## omega 7.43046e-05 6.37978e-05  1.16469  0.2441444
## alpha 7.53745e-02 2.53634e-02  2.97178  0.0029607 **
## gam1  4.22521e-02 3.71873e-02  1.13620  0.2558738
## beta  8.90210e-01 3.24329e-02 27.44779 < 2.22e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4

a. We see that there are ARCH effects in the data, so a garch is fitted. We find that a garch(1,1) with skew-Student-t innovations fit the best, despite having a lower AIC when compared to the garch(1,1) with Gaussian and Student-t innovations. Addtionally, we find that the skew parameter is significant: (.7432-1)/.0486=-5.284 < -1.96

```r
logsimp=log(db$vwretd+1)
t.test(logsimp)
```

```
##
##  One Sample t-test
##
## data:  logsimp
## t = 4.2134, df = 580, p-value = 2.917e-05
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  0.004274232 0.011738564
## sample estimates:
##   mean of x
## 0.008006398
```

```r
Box.test(logsimp,lag=12,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  logsimp
## X-squared = 10.8736, df = 12, p-value = 0.5398
```

```r
Box.test((logsimp-mean(logsimp))^2,lag=12,type="Ljung") #ARCH Effects
```

```
##
##  Box-Ljung test
##
## data:  (logsimp - mean(logsimp))^2
## X-squared = 26.787, df = 12, p-value = 0.008291
```

```
m9=garchFit(~garch(1,1),data=logsimp,trace=F)
m9@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## -3.377737 -3.347687 -3.377831 -3.366022
```

```
plot(m9,which=13)
m10=garchFit(~garch(1,1),data=logsimp,trace=F,cond.dist="std")
m10@fit$ics
```

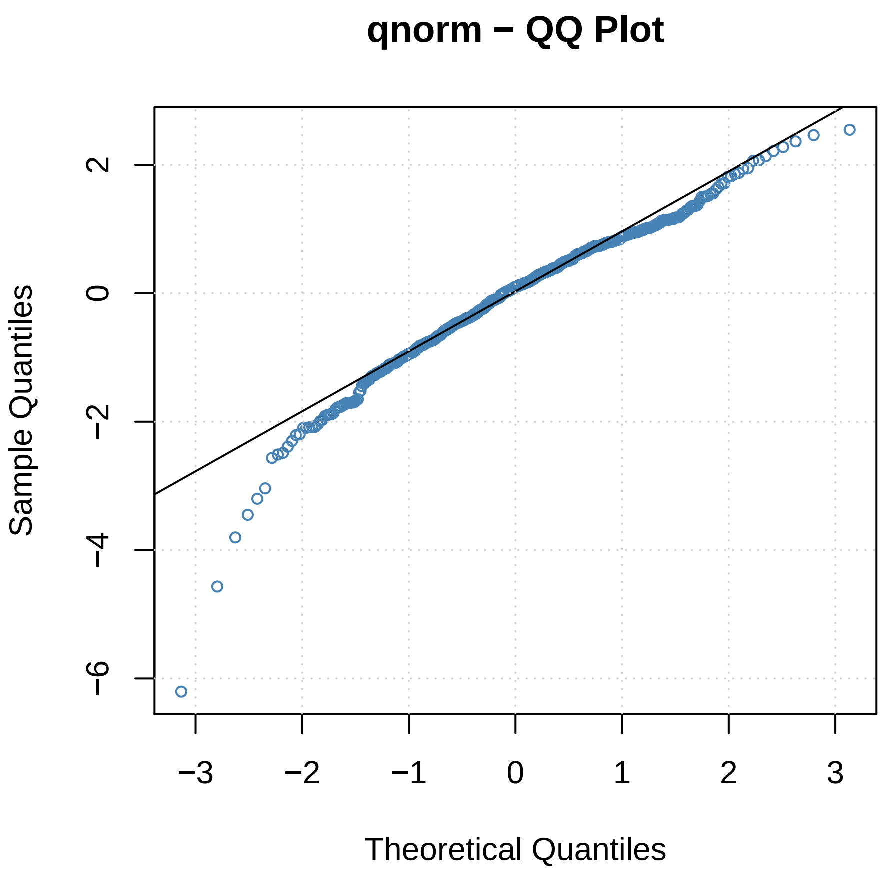
```
##       AIC       BIC       SIC      HQIC
## -3.438604 -3.401041 -3.438750 -3.423960
```

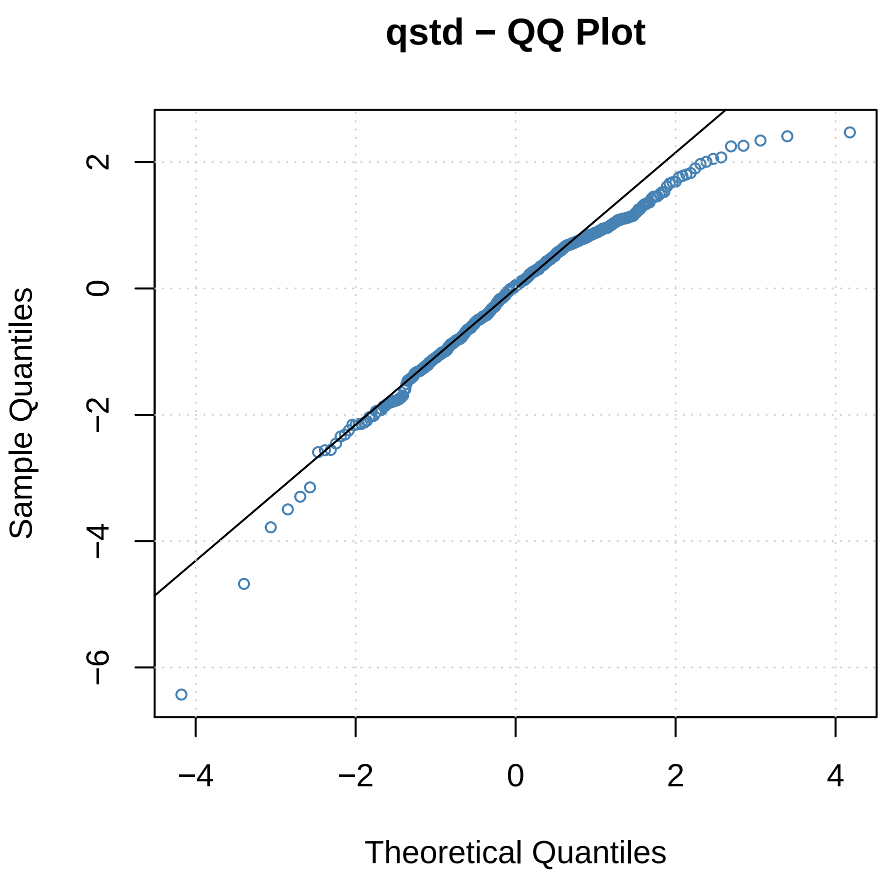
```
plot(m10,which=13)
m11=garchFit(~garch(1,1),data=logsimp,trace=F,cond.dist="sstd")
m11@fit$ics
```

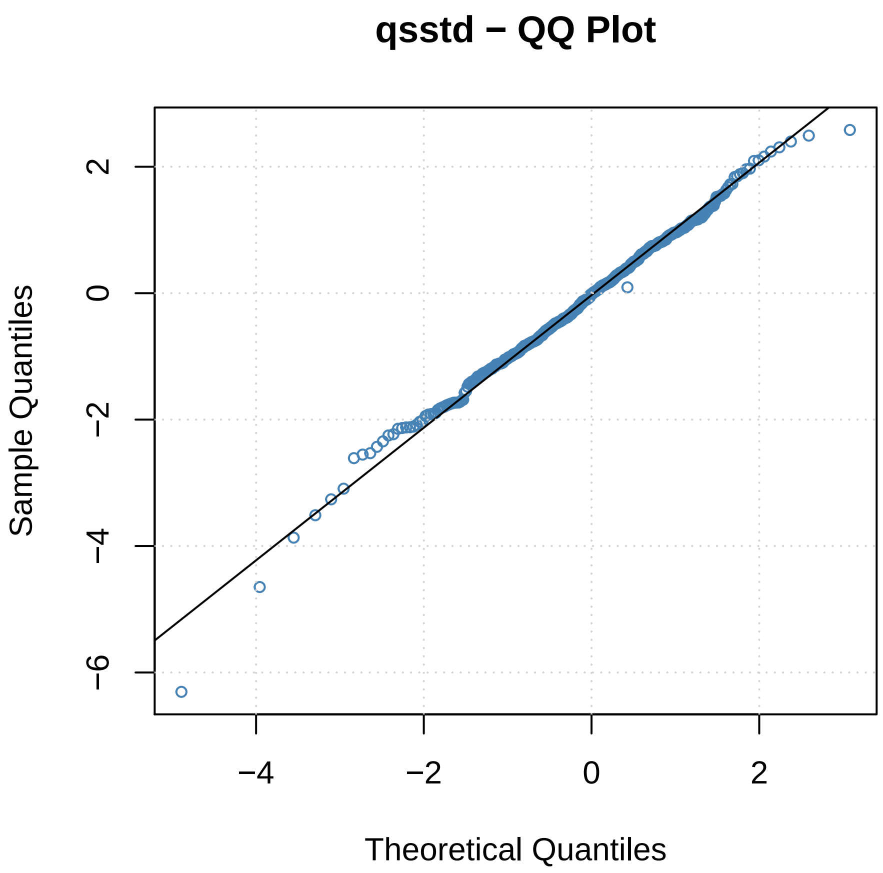
```
##       AIC       BIC       SIC      HQIC
## -3.471990 -3.426916 -3.472201 -3.454419
```

```
plot(m11,which=13)
(.7432-1)/.0486
```

```
## [1] -5.283951
```


qnorm − QQ Plot
Sample Quantiles
Theoretical Quantiles
2
0
−2
−4
−6
−3
−2
−1
0
1
2
3


qstd − QQ Plot
Sample Quantiles
Theoretical Quantiles
2
0
−2
−4
−6
−4
−2
0
2
4

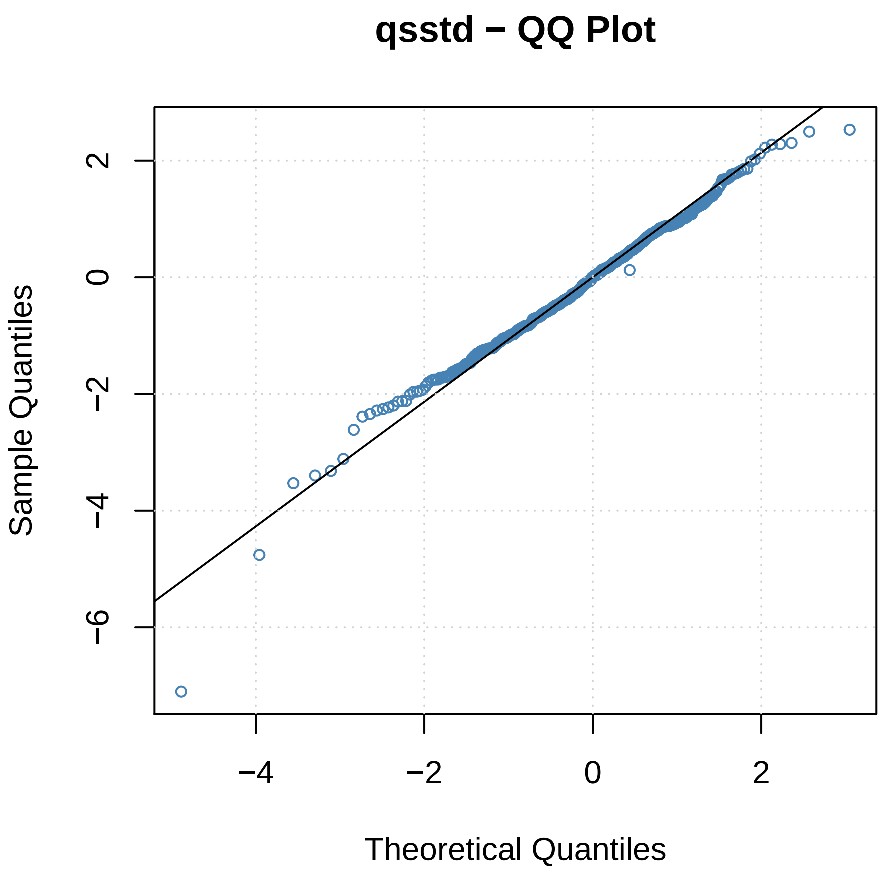


b.

```
predict(m11,5)


##   meanForecast  meanError standardDeviation
## 1  0.008977569 0.03178158        0.03178158
## 2  0.008977569 0.03265735        0.03265735
## 3  0.008977569 0.03346940        0.03346940
## 4  0.008977569 0.03422418        0.03422418
## 5  0.008977569 0.03492724        0.03492724
```

c. Fitted Model: r_t=.00811+a_t, a_t=$\sigma_t\epsilon_t$, $\epsilon_t \sim t^*{}_{7.641,\ .738}$, σ_t^2=1.926 × 10^{-4}+.0506($|a_{t\text{-}1}|$-.981$a_{t\text{-}1}$)^2+.802$\sigma^2{}_{t\text{-}1}$

```
m12=garchFit(~aparch(1,1),data=logsimp, trace=F,delta=2,include.delta=F, cond.dist="sstd")
m12@fit$ics


##       AIC       BIC       SIC      HQIC
## -3.492118 -3.439531 -3.492404 -3.471617


plot(m12,which=13)
```



5

a. Two models are tested. Since we have ARMA effects by looking at the acf and pacf, and ARMA(10,0)+GARCH(1,1) model is fitted. A GARCH(1,1) with Student-t innovations is also fitted. Looking at the QQ-plots, the latter model is the better fit. Additionally, looking at the AICs, the latter model also has a lower AIC.
Fitted Model: $r_t = a_t$, $a_t = \sigma_t \epsilon_t$, $\epsilon_t \sim t^*{}_{10.00}$, $\sigma_t^2 = 9.688 \times 10^{-8} + .0366 a^2{}_{t-1} + .961 \sigma^2{}_{t-1}$

```
dc=read.table("d-exusuk-0615.txt", header=T)
exch=log(dc$Value)
acf(exch) #Unit Root
dex=diff(exch)
par(mfcol=c(2,1))
acf(dex)
pacf(dex)
t.test(dex)

##
##  One Sample t-test
##
## data:  dex
## t = -0.5117, df = 2314, p-value = 0.6089
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  -0.0003210425  0.0001881731
## sample estimates:
##     mean of x
## -6.643474e-05
```

```
Box.test(dex,lag=20,type="Ljung")
```

```
##
##  Box-Ljung test
##
## data:  dex
## X-squared = 61.7564, df = 20, p-value = 3.796e-06
```

```
ar(dex,method="mle")$order
```

```
## [1] 10
```

```
m13=arima(dex,order=c(10,0,0),include.mean=F)
m13
```

```
##
## Call:
## arima(x = dex, order = c(10, 0, 0), include.mean = F)
##
## Coefficients:
##          ar1     ar2      ar3      ar4      ar5     ar6     ar7      ar8      ar9
##       0.0029  0.0181  -0.0183  -0.0147  -0.0654  0.0516  0.0239  -0.0390  -0.0067
## s.e.  0.0208  0.0207   0.0207   0.0207   0.0207  0.0207  0.0207   0.0207   0.0208
##          ar10
##       -0.0752
## s.e.   0.0208
##
## sigma^2 estimated as 3.842e-05:  log likelihood = 8483.37,  aic = -16944.74
```

```
tsdiag(m13,gof=20)
arima(dex,order=c(1,0,1),include.mean=F)
```

```
##
## Call:
## arima(x = dex, order = c(1, 0, 1), include.mean = F)
##
## Coefficients:
```

```
## Warning in sqrt(diag(x$var.coef)):  NaNs produced
```

```
##         ar1     ma1
##       8e-04  0.0011
## s.e.    NaN     NaN
##
## sigma^2 estimated as 3.901e-05:  log likelihood = 8465.67,  aic = -16925.34
```

```
c1=Tstats(m13)
m14=arima(dex,order=c(10,0,0),include.mean=F,fixed=c1)
```

```
## Warning in arima(dex, order = c(10, 0, 0), include.mean = F, fixed = c1):  some AR parameters
were fixed:  setting transform.pars = FALSE
```

```
m14
```

```
##
## Call:
## arima(x = dex, order = c(10, 0, 0), include.mean = F, fixed = c1)
##
## Coefficients:
##        ar1  ar2  ar3  ar4      ar5     ar6  ar7  ar8  ar9     ar10
##          0    0    0    0  -0.0650  0.0508    0    0    0  -0.0782
## s.e.     0    0    0    0   0.0207  0.0207    0    0    0   0.0207
##
## sigma^2 estimated as 3.853e-05:  log likelihood = 8480.1,  aic = -16952.2
```

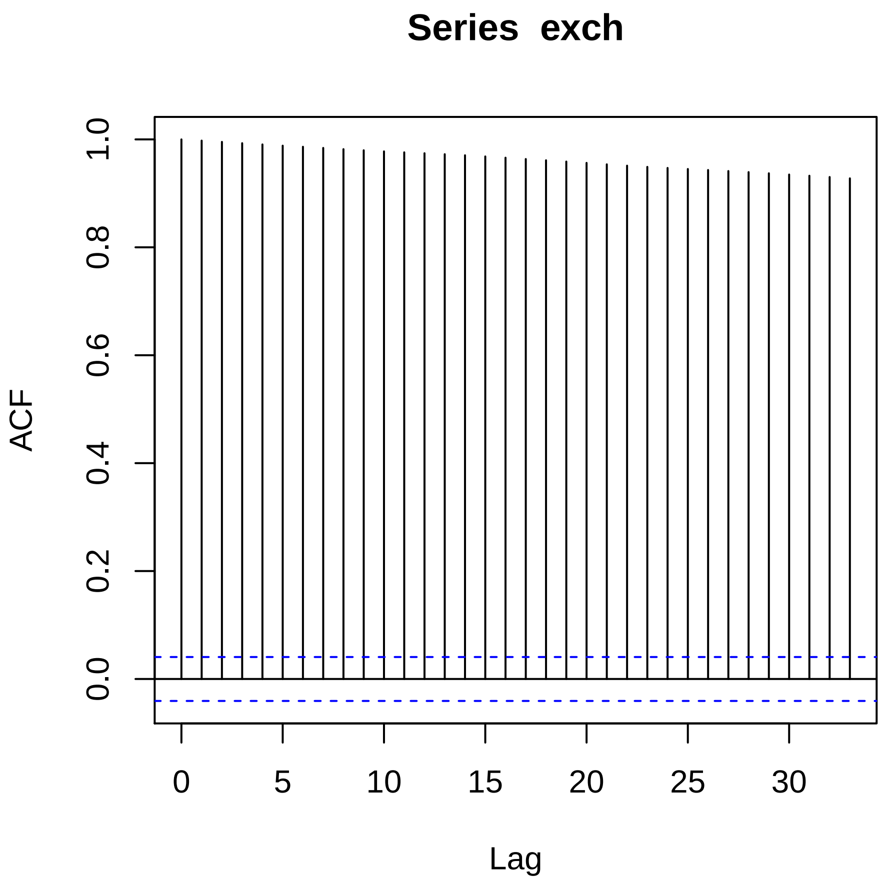
```
m15=garchFit(~arma(10,1)+garch(1,1),data=dex,trace=F)
m15@fit$ics
```

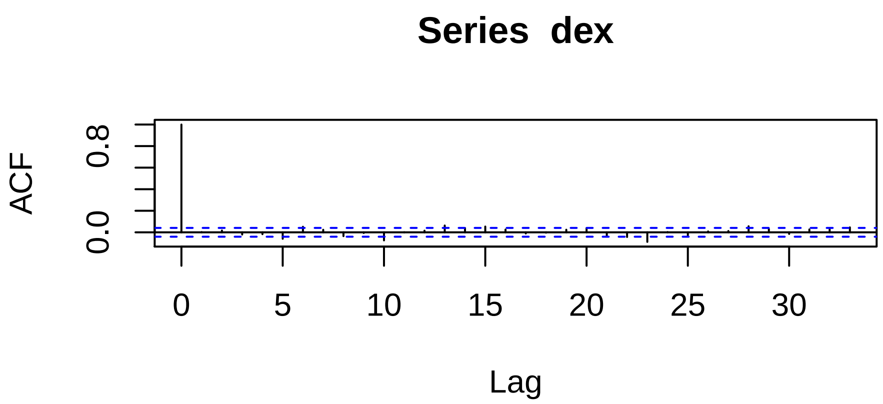
```
##       AIC       BIC       SIC      HQIC
## -7.619037 -7.581798 -7.619120 -7.605465
```

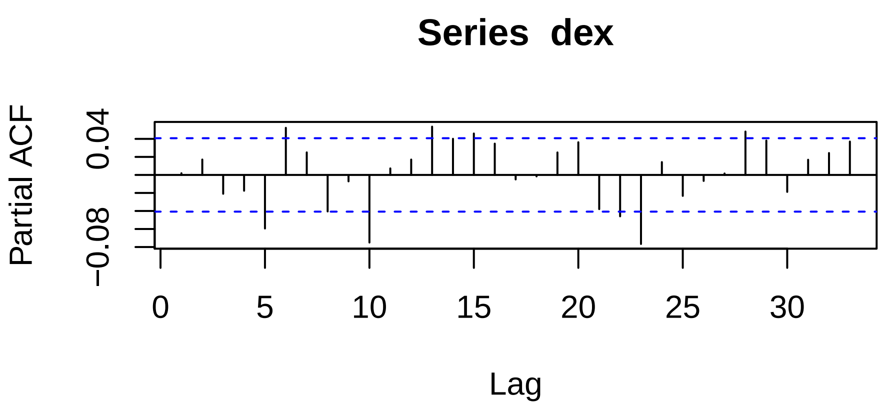
```
par(mfcol=c(1,1))
plot(m15,which=13)
m16=garchFit(~garch(1,1),data=dex, trace=F, cond.dist="std", include.mean=F)
m16@fit$ics
```

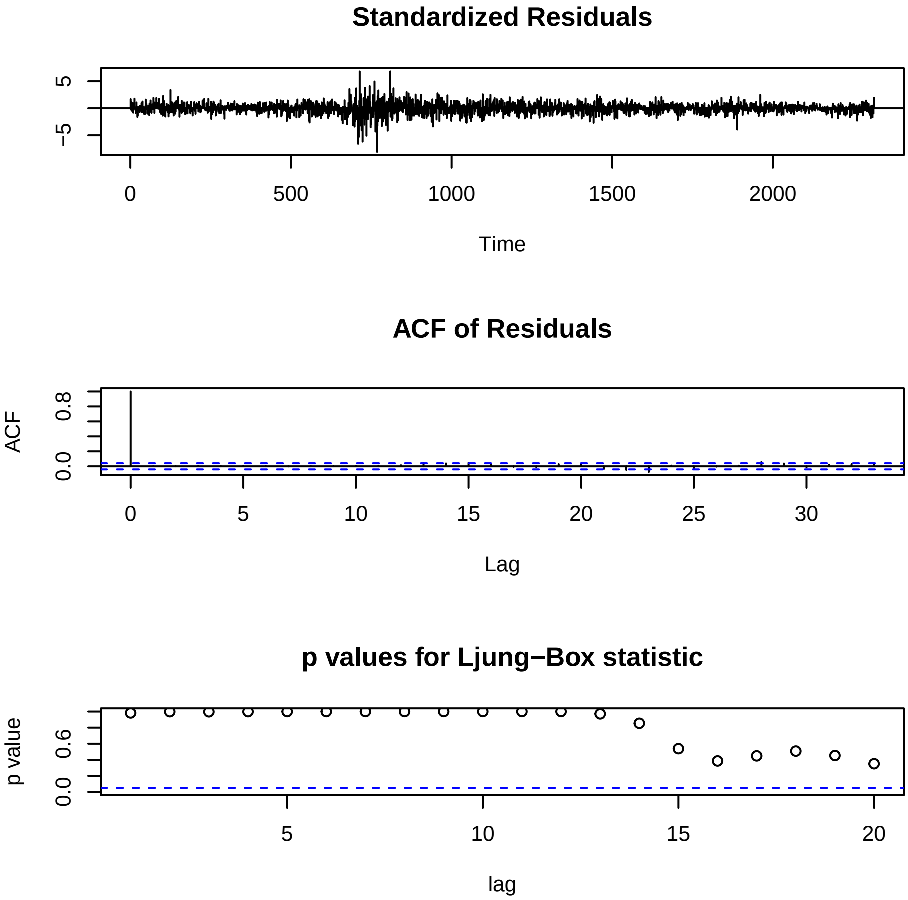
```
##       AIC       BIC       SIC      HQIC
## -7.629885 -7.619955 -7.629891 -7.626266
```

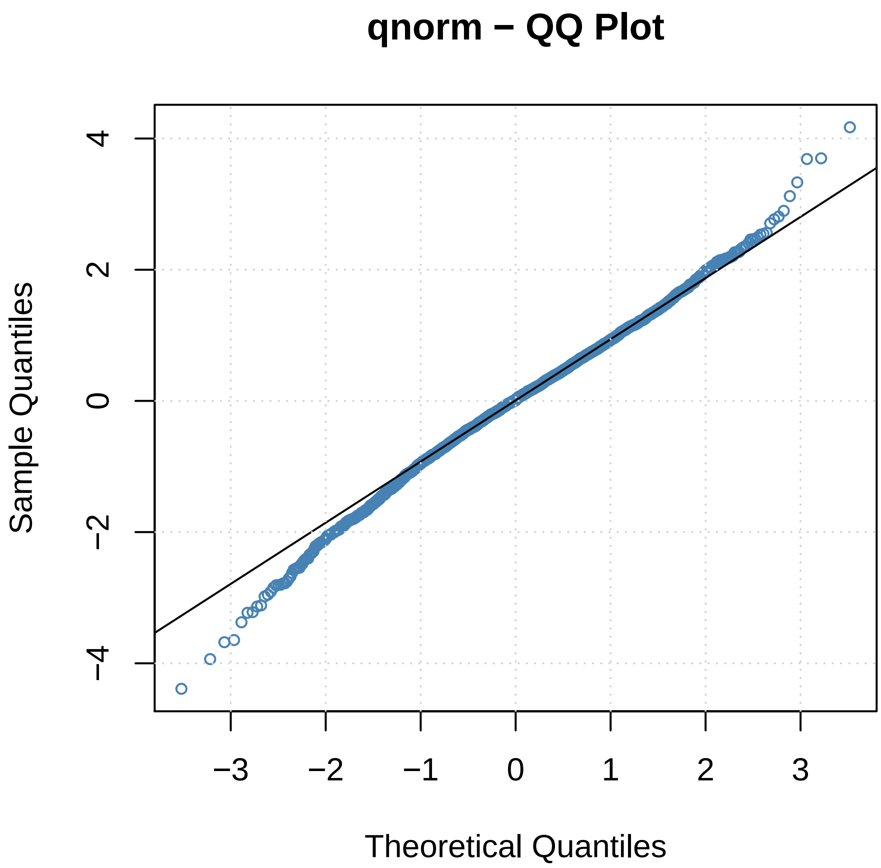
```
plot(m16,which=13)
```

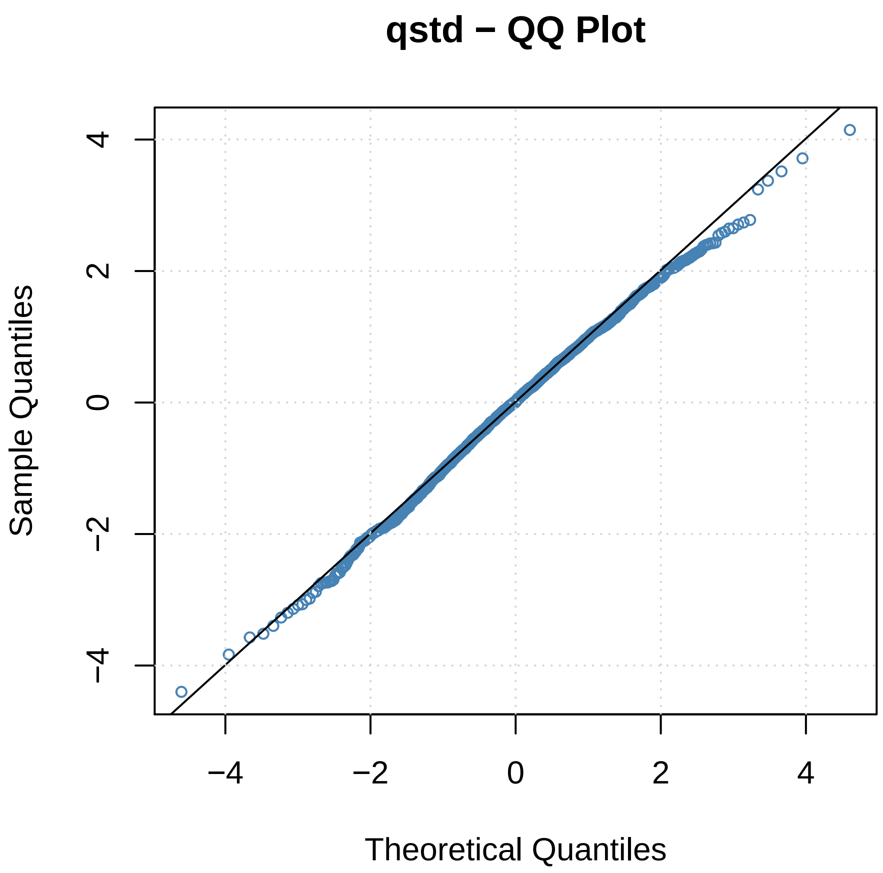

Series exch
ACF
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
Lag


Series dex
ACF
0.8
0.0
0
5
10
15
20
25
30
Lag


Series dex
Partial ACF
0.04
−0.08
0
5
10
15
20
25
30
Lag


Standardized Residuals
5
−5
0
500
1000
1500
2000
Time
ACF of Residuals
ACF
0.8
0.0
0
5
10
15
20
25
30
Lag
p values for Ljung−Box statistic
p value
0.6
0.0
5
10
15
20
lag


qnorm − QQ Plot
Sample Quantiles
4
2
0
−2
−4
−3
−2
−1
0
1
2
3
Theoretical Quantiles



b. Fitted Model: $r_t = a_t$, $a_t = \sigma_t \epsilon_t$, $\epsilon_t \sim t^*{}_{10.00,\ .908}$, $\sigma_t^2 = 5.703 \times 10^{-4} + .0213(|a_{t-1}| - .426 a_{t-1})^2 + .973\sigma^2{}_{t-1}$
The leverage parameter 0.426 is significantly different from zero so the leverage effect is statistically significant.

```
dex100=dex*100
m17=garchFit(~aparch(1,1),data=dex100, trace=F,delta=2,include.delta=F, cond.dist="sstd",include.m
m17@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## 1.571404 1.586300 1.571391 1.576833
```

```
(.908-1)/.0266
```

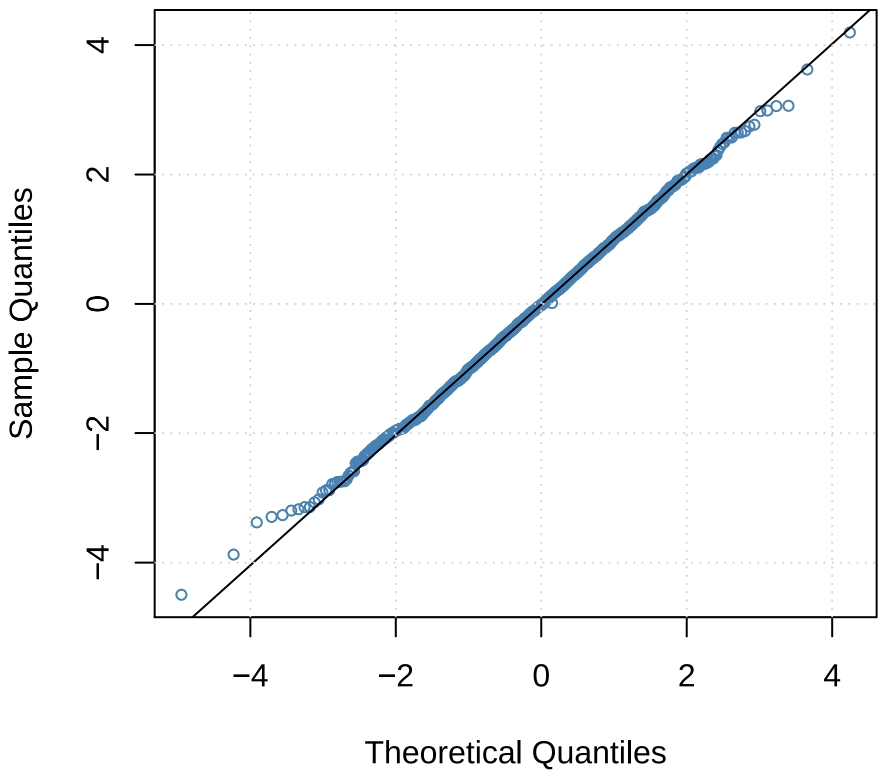
```
## [1] -3.458647
```

```
plot(m17,which=13)
m19=garchFit(~garch(1,1),data=dex100,trace=F,cond.dist="std",leverage=T,include.mean=F)
m19@fit$ics
```

```
##       AIC       BIC       SIC      HQIC
## 1.575246 1.587659 1.575237 1.579770
```

```
plot(m19,which=13)
```

qsstd − QQ Plot



qstd − QQ Plot