

1. Consider the following graph:

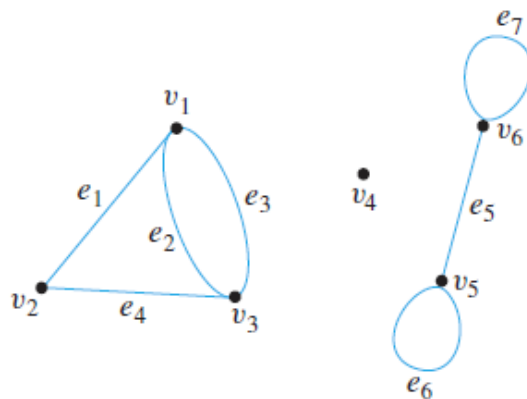


Figure 1: Question 1

- (a) Write the vertex set and the edge set, and give a table showing the edge-endpoint function.
- (b) Find all edges that are incident on  $v_1$ , all vertices that are adjacent to  $v_1$ , all edges that are adjacent to  $e_1$ , all loops, all parallel edges, all vertices that are adjacent to themselves, and all isolated vertices.
2. Draw the graph specified as follows (maybe more than one):

vertex set =  $\{v_1, v_2, v_3, v_4\}$

edge set =  $\{e_1, e_2, e_3, e_4\}$

edge-endpoint function:

Edge	Endpoints
$e_1$	$\{v_1, v_3\}$
$e_2$	$\{v_2, v_4\}$
$e_3$	$\{v_2, v_4\}$
$e_4$	$\{v_3\}$

3. Find the degree of each vertex of the graph  $G$  shown below. Then find the total degree of  $G$ .

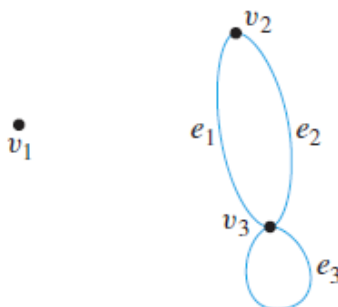


Figure 2: Question 3

4. Draw a graph with the specified properties or show that no such graph exists.
- (a) A graph with four vertices of degrees 1, 1, 2, and 3.
  - (b) A graph with four vertices of degrees 1, 1, 3, and 3.
  - (c) A simple graph with four vertices of degrees 1, 1, 3, and 3.
5. In the graph below, determine which of the following walks are trails, paths, circuits, or simple circuits.

- |  |                          |                                  |
|--|--------------------------|----------------------------------|
| a. $v_1 e_1 v_2 e_3 v_3 e_4 v_3 e_5 v_4$ | b. $e_1 e_3 e_5 e_5 e_6$ | c. $v_2 v_3 v_4 v_5 v_3 v_6 v_2$ |
| d. $v_2 v_3 v_4 v_5 v_6 v_2$             | e. $v_1 e_1 v_2 e_1 v_1$ | f. $v_1$                         |

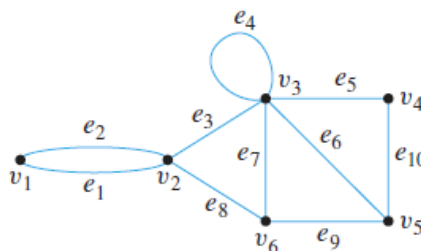


Figure 3: Question 5

6. Which of the following graphs are connected?
7. Find all connected components of the following graph  $G$ .

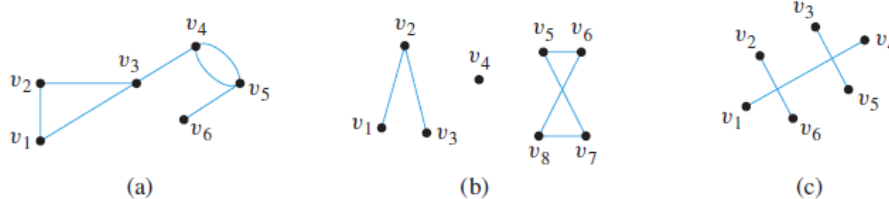


Figure 4: Question 6

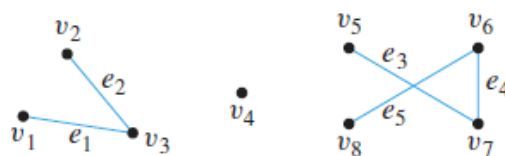


Figure 5: Question 7

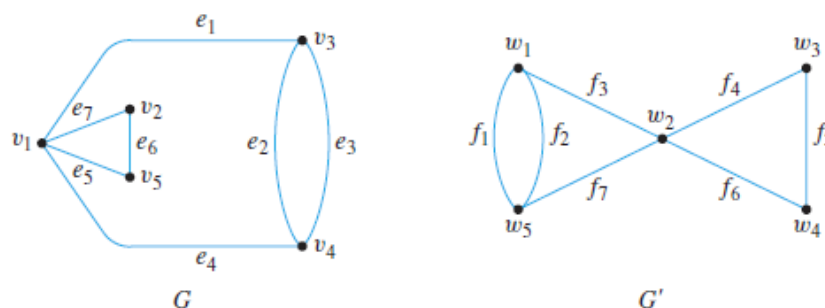


Figure 6: Question 8

8. Show that the following two graphs are isomorphic.
9. Give an example of a graph with five vertices and four edges that is not a tree.
10. (a) Prove that a tree with more than one vertex has at least two vertices of degree 1.  
(b) Find all nonisomorphic trees with four vertices.