1.

	0	1	2	3
x_i	4	6	8	10
y_i	6	3	9	0

$n=3$ (Intervalle)

① $a_i = y_i$

$a_0 = y_0 = 6$
$a_1 = y_1 = 3$
$a_2 = y_2 = 9$

$S_0(x) = 6 + b_0(x-4) + c_0(x-4)^2 + d_0(x-4)^3$
$S_1(x) = 3 + b_1(x-6) + c_1(x-6)^2 + d_1(x-6)^3$
$S_2(x) = 9 + b_2(x-8) + c_2(x-8)^2 + d_2(x-8)^3$

s.t. $S_0(x_0) = y_0$, $S_1(x_1) = y_1$, $S_2(x_2) = y_2$

② $h_i = h_{i+1} - h_i$

$h_0 = 2$
$h_1 = 2$
$h_2 = 2$

③ $c_0 = 0, c_n = c_3 = 0$ (Hilfsgrössen)

④ c_1: $2(h_0+h_1)c_1 + h_1c_2 = 3\frac{y_2-y_1}{h_1} - 3\frac{y_1-y_0}{h_0}$

$8c_1 + 2c_2 = 3\frac{9-3}{2} - 3\cdot\frac{3-6}{2} = 3\cdot 3 + \frac{3\cdot 3}{2} = 9 + 4.5 = 13.5$

c_{n-1}: $h_{n-2}c_{n-2} + 2(h_{n-2}+h_{n-1})c_{n-1} = 3\frac{y_n-y_{n-1}}{h_{n-1}} - 3\frac{y_{n-1}-y_{n-2}}{h_{n-2}}$

$2\cdot c_1 + 2\cdot 4\cdot c_2 = 3\left(\frac{0-9}{2} - \frac{9-3}{2}\right) = -22.5$

$8c_1 + 2c_2 = 13.5$
$2c_1 + 8c_2 = -22.5 \Rightarrow c_1 = 2.55 \quad c_2 = -3.45$

(falls $n \geq 4$ gilt für $i = 2, \ldots, n-2$:
$h_{i-1}c_{i-1} + 2(h_{i-1}+h_i)c_i + h_ic_{i+1} = 3\frac{y_{i+1}-y_i}{h_i} - 3\frac{y_i-y_{i-1}}{h_{i-1}}$)

⑤ $b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{3}(c_{i+1}+2c_i)$ $\quad c_0^0, c_1^{2.55}, c_2^{-3.45}, c_3^0$

$b_0 = \frac{3-6}{2} - \frac{2}{3}(c_1 + 2\cdot c_0) = -3.2$
$b_1 = \frac{9-3}{2} - \frac{2}{3}(c_2 + 2\cdot c_1) = 1.9$
$b_2 = \frac{0-9}{2} - \frac{2}{3}(c_3 + 2\cdot c_2) = 0.1$

⑥ $d_i = \frac{1}{3h_i}(c_{i+1} - c_i)$

$d_0 = \frac{1}{6}\cdot 2(c_1 - c_0) = 0.425$
$d_1 = \frac{1}{6}(c_2 - c_1) = -1$
$d_2 = \frac{1}{6}(c_3 - c_2) = 0.575$

$\Rightarrow$ (a, b, c, d)

$S_0(x) = 6 - 3.2(x-4) + 0\cdot(x-4)^2 + 0.425(x-4)^3$
$S_1(x) = 3 + 1.9(x-6) + 2.55(x-6)^2 - 1\cdot(x-6)^3$
$S_2(x) = 9 + 0.1(x-8) - 3.45(x-8)^2 + 0.575\cdot(x-8)^3$