

2. a1) $\frac{\partial^2 w}{\partial t^2}$ where $w(x, t) = \sin(x + ct)$ $\frac{d}{dx} \sin x = \cos x$

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

$$\begin{aligned} \frac{d}{dt} \left(\frac{d}{dt} \sin(x+ct) \right) &= \frac{d}{dt} \cos(x+ct) \cdot c = -\sin(x+ct) \cdot c \cdot c = -c^2 \sin(x+ct) \\ \frac{d}{dx} \left(\frac{d}{dx} \sin(x+ct) \right) &= \frac{d}{dx} \cos(x+ct) = -\sin(x+ct) \cdot 1 = -1^2 \sin(x+ct) \end{aligned} \quad \left. \vphantom{\frac{d}{dt}} \right\} \text{ where } c=1, -1$$

2 a2) $v(x, t) = \sin(x + ct) + \cos(2x + 2ct)$

$$\begin{aligned} \frac{d}{dt} \left(\frac{d}{dt} \sin(x+ct) + \cos(2x+2ct) \right) &= \frac{d}{dt} (c \cdot \cos(x+ct) - \sin(2x+2ct) 2c) = c \frac{d}{dt} \cos(x+ct) - 2c \frac{d}{dt} \sin(2x+2ct) \\ &= -c^2 (\sin(x+ct) + 4 \cos(2x+2ct)) \end{aligned} \quad \left. \vphantom{\frac{d}{dt}} \right\} \text{ for } c=1, -1$$

$$\frac{d}{dx} \left(\frac{d}{dx} \sin(x+ct) + \cos(2x+2ct) \right) = \frac{d}{dx} (\cos(x+ct) - 2 \sin(2x+2ct)) = -\sin(x+ct) - 4 \cos(2x+2ct)$$