CMPT 440 – Spring 2019: Quantum Finite Automata

Phil Kirwin
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Theoretical Background

Simply put, a Quantum Finite Automata (QFA) is an automaton that is capable of determining the probability that a string s is accepted by a quantum system. Such models are useful because, according to Ambainis et al. (1999), they may be simpler to implement in quantum computing than a model such as a quantum Turing machine. According to Kondacs and Watrous (1997), a QFA can be defined by the tuple

$$M = (Q, \Sigma, \delta, q_0, Q_{acc}, Q_{rej})$$

where Q is a finite set of states, Σ is a finite input alphabet, δ is a transition function, $q_0 \in Q$ is the initial state, $Q_{acc} \subset Q$ is the set of accepting states, and $Q_{rej} \subset Q$ is the set of rejecting states. Elements of $Q_{acc} \cup Q_{rej}$ are considered halting states, as reaching these states halts the processing of the input string, while the set $Q_{non} = Q - (Q_{acc} \cup Q_{rej})$ is the set of non-halting states, as reaching these states causes processing of the input string to continue.

An Example

This example is provided by Ambainis and Freivalds (1998). Let $M = (Q, \Sigma, \delta, q_0, Q_{acc}, Q_{rej})$ such that $Q = \{q_0, q_1, q_{acc}, q_{rej}\}$, $\Sigma = \{a\}$, the initial state is q_0, $Q_{acc} = \{q_{acc}\}$, and $Q_{rej} = \{q_{rej}\}$. The transition function δ will be defined by the linear transformation V_x where $\Gamma = \Sigma \cup \{\$\}$ (as \$ is the teminating symbol for a string) and $x \in \Gamma$. Let V_x be defined as

$$V_a(|q_0\rangle) = \frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle + \frac{1}{\sqrt{2}}|q_{rej}\rangle,$$

$$V_a(|q_1\rangle) = \frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle - \frac{1}{\sqrt{2}}|q_{rej}\rangle,$$

$$V_\$(|q_0\rangle) = |q_{rej}\rangle, V_\$(|q_1\rangle) = |q_{acc}\rangle.$$

Now consider the string aa. For the first input character, and given that we begin at q_0, we may apply $V_a(|q_0\rangle)$. This results in a rejecting state, q_{rej}, with a probability of $(\frac{1}{\sqrt{2}})^2$, or $\frac{1}{2}$. Conversely, a non-halting state is reached with a probability of $\frac{1}{2}$. In the latter case, V_a collapses to $\frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle$ and the second a is processed, leading to a non-halting state in every case. Finally, the input terminating symbol, \$, is processed via $V_\$$. Given the previous transformation resulting in $\frac{1}{2}|q_0\rangle + \frac{1}{2}|q_1\rangle$, We have that the probability of reaching an accepting state via $V_\$(|q_0\rangle)$ is $(\frac{1}{2})^2 = \frac{1}{4}$, while the probability of reaching a rejecting state via $V_\$(|q_1\rangle)$ is $(\frac{1}{2})^2 = \frac{1}{4}$. Thus the probability of the string aa being accepted by the QFA defined by M is $\frac{1}{4}$, while the probability of it being rejected is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

References

A. Ambainis and R. Freivalds. 1-way quantum finite automata: strengths, weaknesses and generalizations. In *Proceedings 39th Annual Symposium on Foundations of Computer Science (Cat. No. 98CB36280)*, pages 332–341. IEEE, 1998.

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