Introduction to Differential Geometry Notes

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1 What is a curve?

Def'n 1.1.1
Parameterized curve in $\mathcal{R}^n$ is map $\gamma(t) : (\alpha, \beta) \longrightarrow \mathcal{R}^n$ for some α, β with $-\infty \leq \alpha < \beta < \infty$

Level curves in $\mathcal{R}^n$
i.e. $y^2 - x^2 = 0 \leftarrow$ parabola in $\mathcal{R}^2$
The level curve above is at "level" 0. In general, we could have a level curve at level 'c' $f(x, y) = c$

smooth function $f : (\alpha, \beta) \longrightarrow \mathcal{R}^n$ is said to be smooth if derivative $\frac{d^n f}{dt^n}$ exists $\forall n \geq 1$ and $t \in (\alpha, \beta)$

Def'n 1.1.5
If γ is a parameterized curve, first derivative $\dot{\gamma}$ is called the tangent vector of γ at point $\gamma(t)$