a). $\sum_{i} \theta(1)$

$= \underbrace{k}_{\text{number of times looping}} \theta(1)$

$= \theta(k)$

to find k:

[illegible]

$$2^{2^k} = n$$

$$k = \log(\log n)$$

$$= \theta(\log(\log n))$$

b.) $\sum_{i=1}^{n} \theta(1) + O\left(\sum_{k=0}^{i^{3}} \theta(1)\right)$

$= \theta(n) + \sum_{i} \sum_{k=0}^{i^3} \theta(1)$

$= \theta(n) + \sum_{i} \theta(i^3)$

$i = \sqrt{n}, 2\sqrt{n}, \ldots, \sqrt{n}\cdot\sqrt{n}$

$= \theta(n) + \theta(\sqrt{n}^3 + (2\sqrt{n})^3 + \ldots + (\sqrt{n}\sqrt{n})^3)$

$= \theta(n^{\frac{7}{2}})$

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c. $\sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + O(\sum_{m} \theta(1))$

$= \sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + O(\theta(\log n))$

$= \sum_{i=1}^{n} \theta(n) + \sum_{k} \theta(\log n)$

$= \sum_{i=1}^{n} \theta(n) + \theta(\log n) \times 1$

$= \theta(n^2) + \theta(n \log n)$

$= \theta(n^2)$

d.) $\sum_{i=0}^{n} \theta(1) + O(\theta(i))$

$= \theta(n) + \sum_{i} \theta(i)$

$i = 10, \frac{3}{2}\cdot 10, \left(\frac{3}{2}\right)^2 \cdot 10, \ldots, \left(\frac{3}{2}\right)^x \cdot 10$

find x:

$\left(\frac{3}{2}\right)^x 10 = n$

$\frac{n}{10} = \left(\frac{3}{2}\right)^x$

$\log\left(\frac{n}{10}\right) = x \log\left(\frac{3}{2}\right)$

$x = \frac{\log\left(\frac{n}{10}\right)}{\log\left(\frac{3}{2}\right)} \Leftrightarrow x = {}^{\frac{3}{2}}\log\left(\frac{n}{10}\right)$

$= \theta(n) + \theta\left(10 + \frac{3}{2}\cdot 10 + \ldots + \left(\frac{3}{2}\right)^x \cdot 10\right)$

$= \theta(n) + \theta\left(\frac{10\left(\frac{3}{2}^x - 1\right)}{\frac{1}{2}}\right)$

$= \theta(n) + \theta\left(20\left(\frac{n}{10} - 1\right)\right)$

$= \theta(n) + \theta(n)$

$= \theta(n)$