Statistics 435/535/711 25 April 2022
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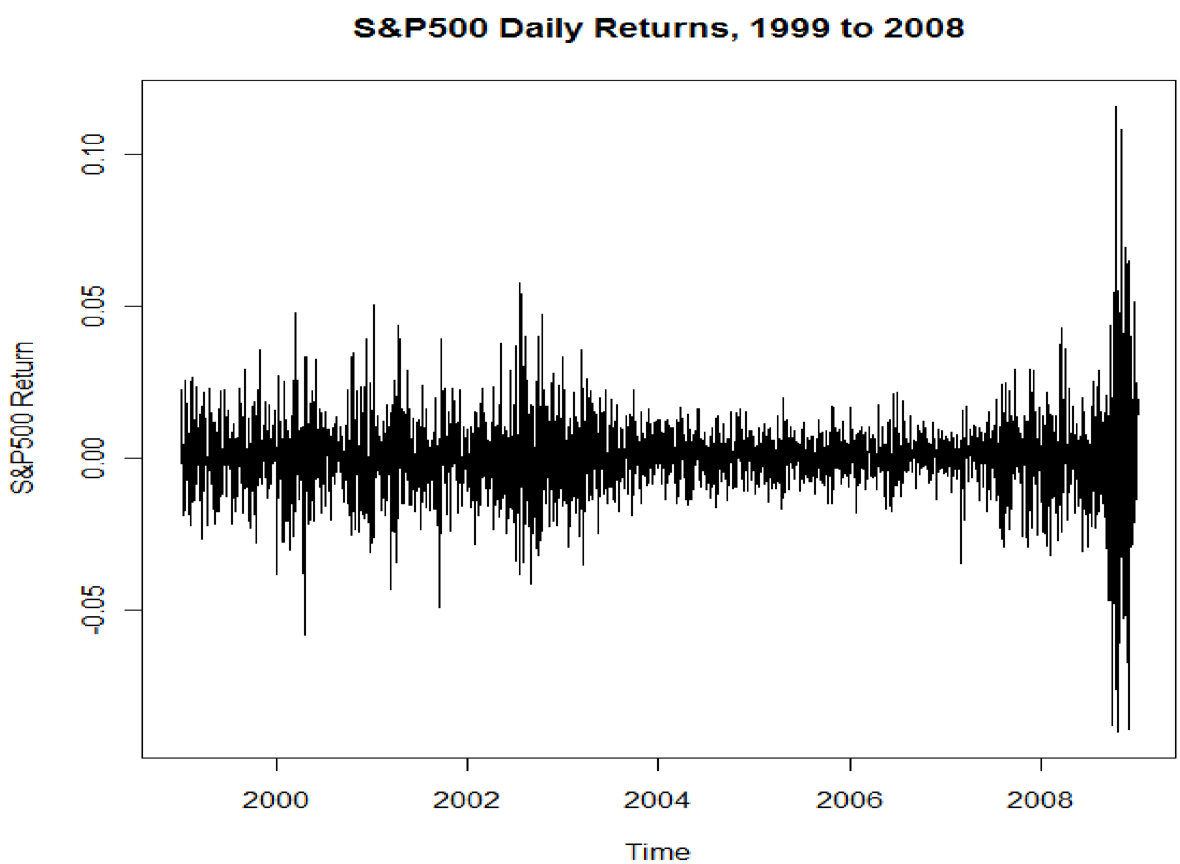
GARCH Modelling--continued

Next, we consider a financial return series with higher frequency of observation.

Example 2. Problem 3.11 in Tsay uses the data set d-gmsp9908.txt, which contains daily simple returns for the S&P500 from 1999 to 2008.

```
> gmsp<-read.csv("F:/Stat71121Fall/d-gmsp9908.txt")
> attach(gmsp)
> head(gmsp)
      date        gm        sp
1 19990104 -0.009607 -0.000919
2 19990105  0.052910  0.013582
3 19990106  0.046064  0.022140
4 19990107 -0.006405 -0.002051
5 19990108  0.032232  0.004221
6 19990111  0.074941 -0.008792

> plot(ts(sp,start=c(1999,1),freq=251),xlab="Time",ylab="S&P500
Return",main="S&P500 Daily Returns, 1999 to 2008")
```

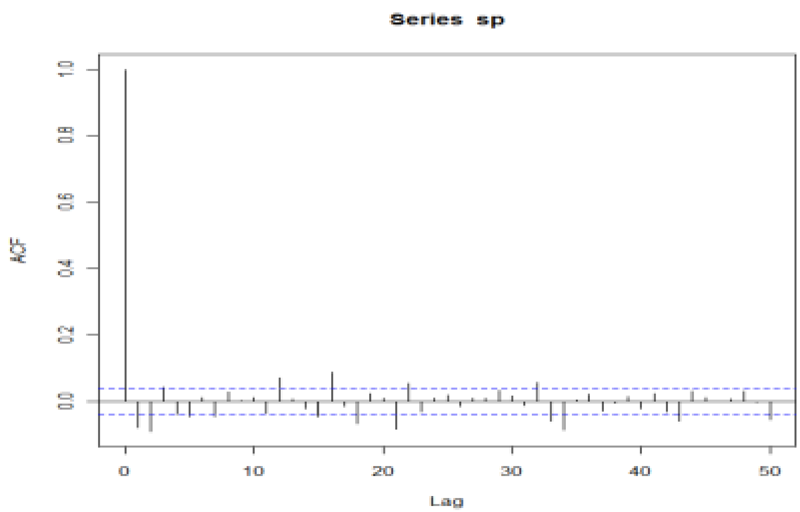


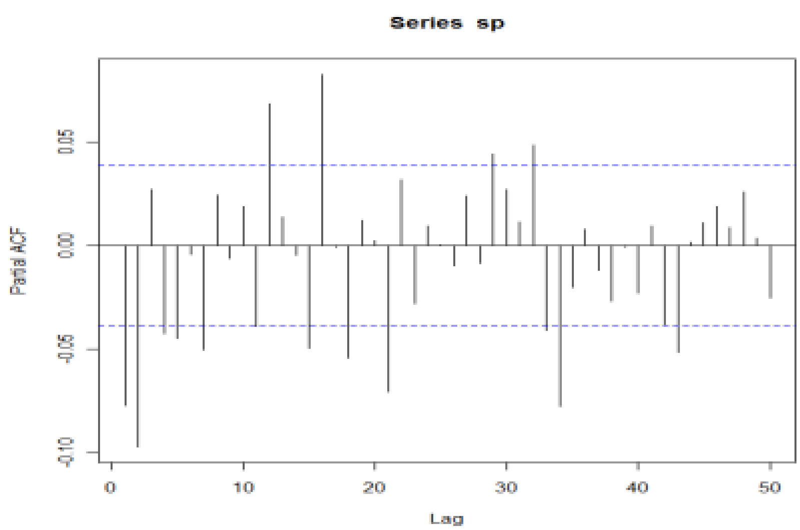
The high volatility at the end of the series occurred in 2008, of course. First, we calculate the skewness and kurtosis of the series.

```
> skewness(sp)
[1] 0.09406745
> kurtosis(sp)
[1] 11.76145
```

There is relatively high kurtosis, and essentially no skewness.

Let's first fit an ARIMA model to describe the level of the data. Here are the correlations and partial correlations of the series. The series is long, and these plots are carried out to lag 50.



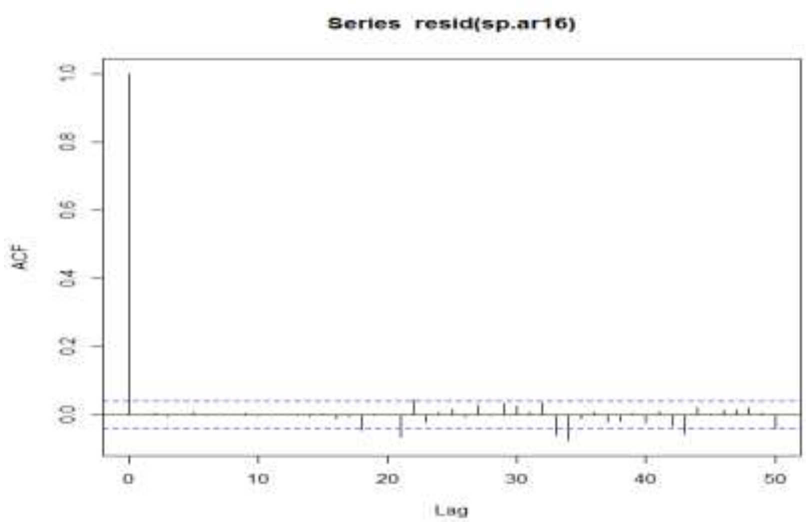


There are significant partial correlations at some high lags, 12, 15, 16, 18, 21, 29, 32, 34, and 43. Let's try fitting an AR(16)—the time series is very long and can tolerate such a fit.

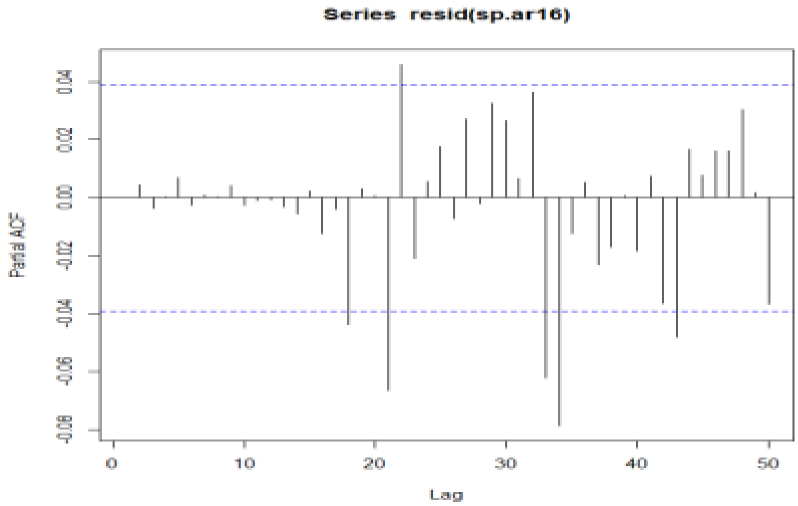
```
> sp.ar16<-arima(ts(sp),order=c(16,0,0))
> coeftest(sp.ar16)

z test of coefficients:

             Estimate  Std. Error z value  Pr(>|z|)
ar1       -7.5559e-02  1.9872e-02 -3.8023 0.0001434 ***
ar2       -1.0038e-01  1.9927e-02 -5.0376 4.714e-07 ***
ar3        2.3503e-02  2.0033e-02  1.1732 0.2407091
ar4       -5.6167e-02  2.0037e-02 -2.8031 0.0050611 **
ar5       -4.3324e-02  2.0005e-02 -2.1656 0.0303393 *
ar6       -8.3010e-03  2.0010e-02 -0.4148 0.6782524
ar7       -4.5206e-02  2.0025e-02 -2.2575 0.0239761 *
ar8        2.5209e-02  2.0035e-02  1.2583 0.2082907
ar9       -5.3251e-03  2.0050e-02 -0.2656 0.7905576
ar10       2.0740e-02  2.0040e-02  1.0349 0.3007023
ar11      -3.1203e-02  2.0094e-02 -1.5528 0.1204628
ar12       7.5194e-02  2.0087e-02  3.7435 0.0001815 ***
ar13       5.4904e-03  2.0121e-02  0.2729 0.7849553
ar14       1.2067e-04  2.0140e-02  0.0060 0.9952196
ar15      -4.4155e-02  2.0034e-02 -2.2040 0.0275268 *
ar16       8.4351e-02  2.0012e-02  4.2150 2.498e-05 ***
intercept -3.5695e-05  2.2365e-04 -0.1596 0.8731933
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Next, let's examine the residual diagnostics.

```
> acf(resid(sp.ar16),50)
```

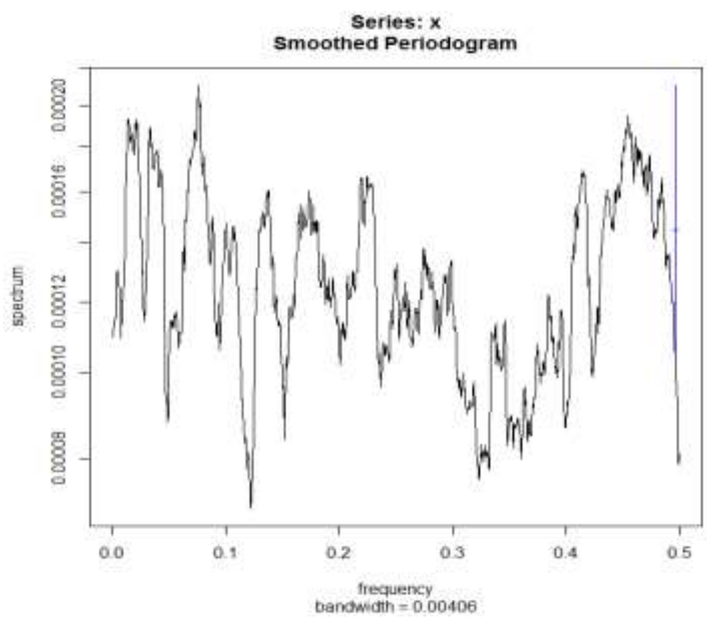


```
> pacf(resid(sp.ar16),50)
```



Although there a quite a few significant residual acf and pacf values, the effects are relatively small.

```
> spectrum(resid(sp.ar16),span=36)
```




```
> bartlettB.test(resid(sp.ar16))

        Bartlett B Test for white noise

data:
= 0.35696, p-value = 0.9996
```

The spectral plot does not appear to be sufficiently flat to be consistent with reduction to white noise. However, Bartlett's test does not reject the hypothesis of white noise. So let's continue.

```
> skewness(resid(sp.ar16))
[1] -0.0702149
> kurtosis(resid(sp.ar16))
[1] 9.785211
```

The kurtosis calculation for the AR(16) residuals gives 9.79, a reduction from 11.76 for the data series.

Now that we have determined that an AR(16) model is reasonable to model the level of the time series, we fit an AR(16)–GARCH(1,1) model using the **fGarch** package.

```
> sp.ts<-ts(sp)
> model1<-garchFit(~arma(16,0)+garch(1,1),data=sp.ts,trace=FALSE)
> summary(model1)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~arma(16, 0) + garch(1, 1), data = sp.ts,
    trace = FALSE)

Mean and Variance Equation:
 data ~ arma(16, 0) + garch(1, 1)
<environment: 0x0d181ce8>
 [data = sp.ts]

Conditional Distribution:
 norm

Coefficient(s):
         mu          ar1          ar2          ar3          ar4          ar5
 3.2801e-04  -6.3309e-02  -4.3710e-02  -1.2746e-02  -2.0900e-02  -5.9459e-02
        ar6          ar7          ar8          ar9         ar10         ar11
-2.6426e-02  -2.8588e-02  -9.6074e-03  -1.8068e-02   7.0066e-03  -4.2454e-03
       ar12         ar13         ar14         ar15         ar16        omega
 4.5484e-02   1.2443e-02  -4.6661e-03  -2.4251e-02   2.6447e-02   1.0349e-06
     alpha1        beta1
 7.3978e-02   9.2094e-01
```

```
Std. Errors:
 based on Hessian

Error Analysis:
         Estimate  Std. Error  t value Pr(>|t|)
mu      3.280e-04   1.837e-04    1.786 0.074125 .
ar1    -6.331e-02   2.103e-02   -3.011 0.002607 **
ar2    -4.371e-02   2.076e-02   -2.105 0.035281 *
ar3    -1.275e-02   2.075e-02   -0.614 0.539030
ar4    -2.090e-02   2.072e-02   -1.008 0.313226
ar5    -5.946e-02   2.074e-02   -2.867 0.004149 **
ar6    -2.643e-02   2.070e-02   -1.277 0.201693
ar7    -2.859e-02   2.064e-02   -1.385 0.165988
ar8    -9.607e-03   2.049e-02   -0.469 0.639198
ar9    -1.807e-02   2.041e-02   -0.885 0.376081
ar10    7.007e-03   2.031e-02    0.345 0.730104
ar11   -4.245e-03   2.039e-02   -0.208 0.835041
ar12    4.548e-02   2.029e-02    2.241 0.025012 *
ar13    1.244e-02   2.023e-02    0.615 0.538511
ar14   -4.666e-03   2.038e-02   -0.229 0.818924
ar15   -2.425e-02   2.049e-02   -1.184 0.236604
ar16    2.645e-02   2.026e-02    1.305 0.191752
omega   1.035e-06   3.129e-07    3.307 0.000942 ***
alpha1  7.398e-02   9.674e-03    7.647 2.07e-14 ***
beta1   9.209e-01   1.033e-02   89.144  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 7883.457    normalized:  3.134575
```

The GARCH(1,1) part of the model fit is

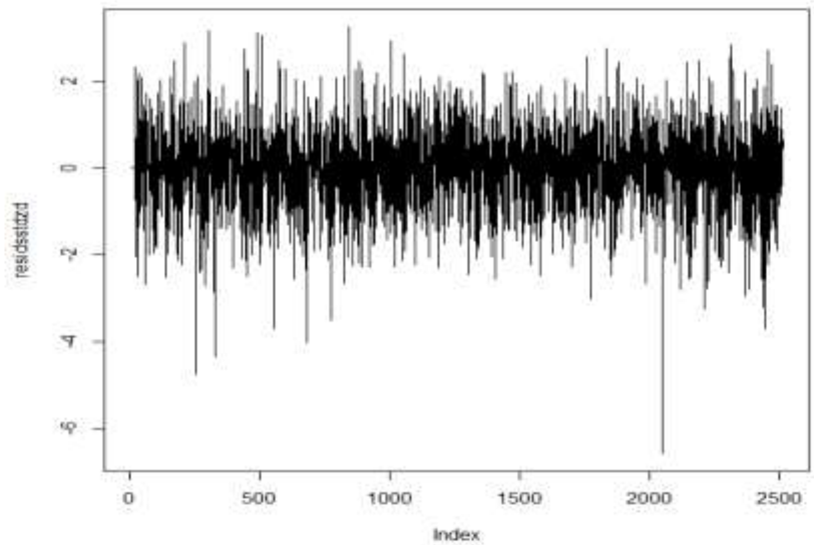
$$\sigma_t^2 = 0.000001 + 0.074u_{t-1}^2 + 0.921\sigma_{t-1}^2.$$

The two GARCH estimated coefficients add to 0.995, barely below the required limit 1.

```
Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  215.2082  0
 Shapiro-Wilk Test  R    W      0.9893745 1.014775e-12
 Ljung-Box Test     R    Q(10)  1.439284  0.9991123
 Ljung-Box Test     R    Q(15)  2.082002  0.9999613
 Ljung-Box Test     R    Q(20)  5.661904  0.9992831
 Ljung-Box Test     R^2  Q(10)  13.88708  0.1782048
 Ljung-Box Test     R^2  Q(15)  17.08818  0.3136206
 Ljung-Box Test     R^2  Q(20)  18.64555  0.5449612
 LM Arch Test       R    TR^2   14.73366  0.2563262

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-6.253246 -6.206884 -6.253371 -6.236420
```

The tests shown directly above confirm that the standardized residuals from the AR(16)–GARCH(1,1) fit are well described by a normal distribution and are consistent with a white noise hypothesis.
Next, we plot the standardized residuals.

```
> residsstdzd<-residuals(model1,standardize=TRUE)
> plot(residsstdzd,type='l')
```

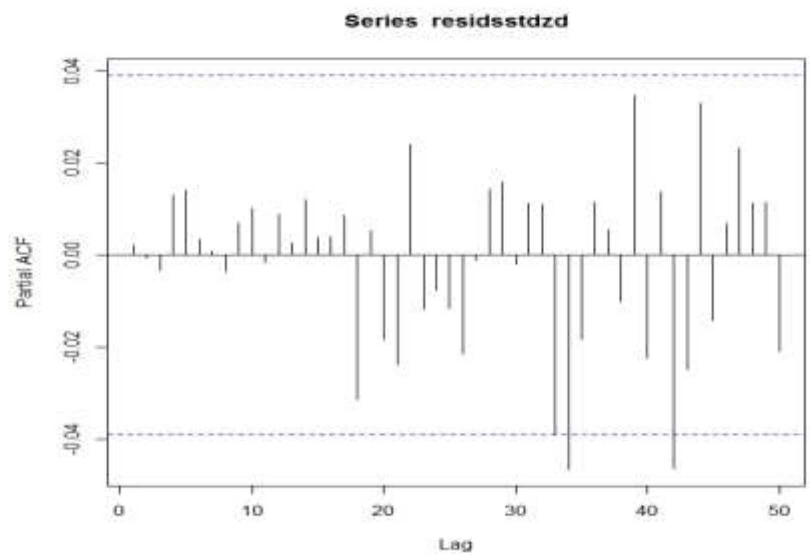



```
> skewness(residsstdzd)
[1] -0.3175428
attr(,"method")
[1] "moment"
> kurtosis(residsstdzd)
[1] 1.281063
attr(,"method")
[1] "excess"
```

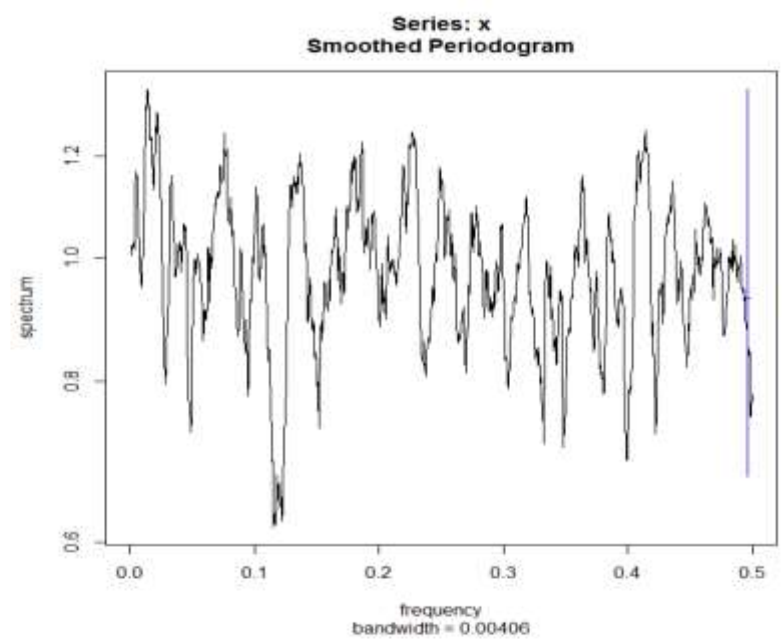
The kurtosis of the standardized residuals is 4.28 (the excess kurtosis is 1.28). Although this indicates that the standardized residuals have a distribution that has fatter tails than a normal distribution, the value 4.28 represents a substantial reduction from the value 11.76 for the data series.

Next, we examine some diagnostics for the standardized residuals.

```
> pacf(residsstdzd,50)
```




```
> spectrum(residsstdzd,span=36)
```




```
> bartlettB.test(residsstdzd)

        Bartlett B Test for white noise

data:
= 0.30879, p-value = 1
```

Thus, the standardized residuals have structure which is sufficiently consistent with a white noise hypothesis.