Statistics 435/535/711 27 April 2022
P. Shaman

ARCH and GARCH Models, continued

The file PGEmonthly7504.txt contain simple monthly returns for Pacific Gas and Electric (PG&E) common stock for the years 1975 through 2004. PG&E services northern California. During 2000 and 2001 the energy market in California experienced severe price increases which PG&E could not pass along to its customers, and the company was forced to file for bankruptcy. The following brief chronology and description are taken from Wikipedia, http://en.wikipedia.org/wiki/California_electricity_crisis.

Chronology[1][2][3]	
1996	California begins to modify controls on its energy market and takes measures ostensibly to increase competition.
September 23, 1996	The Electric Utility Industry Restructuring Act (Assembly Bill 1890) becomes law.[4]
April 1998	Spot market for energy begins operation.
May 2000	Significant rise in energy price.
June 14, 2000	Blackouts affect 97,000 customers in San Francisco Bay area during a heat wave.
August 2000	San Diego Gas & Electric Company files a complaint alleging manipulation of the markets.
January 17–18, 2001	Blackouts affect several hundred thousand customers.
January 17, 2001	Governor Davis declares a state of emergency.
March 19–20, 2001	Blackouts affect 1.5 million customers.
April 2001	Pacific Gas & Electric Co. files for bankruptcy.
May 7–8, 2001	Blackouts affect upwards of 167,000 customers.
September 2001	Energy prices normalize.
December 2001	Following the bankruptcy of Enron, it is alleged that energy prices were manipulated by Enron.
February 2002	Federal Energy Regulatory Commission begins investigation of Enron's involvement.
Winter 2002	The Enron Tapes scandal begins to surface.
November 13, 2003	Governor Davis ends the state of emergency.

The **California electricity crisis**, also known as the **Western U.S. Energy Crisis** of 2000 and 2001, was a situation in which California had a shortage of electricity caused by market manipulations, illegal[*citation needed*] shutdowns of pipelines by Texas energy consortiums, and capped retail electricity prices.[5] The state suffered from multiple large-scale blackouts, one of the state's largest energy companies collapsed, and the economic fall-out greatly harmed Governor Gray Davis's standing.

Drought, delays in approval of new power plants,[6] and market manipulation decreased supply. This caused 800% increase in wholesale prices from April 2000 to December 2000.[7] In addition, rolling blackouts adversely affected many businesses dependent upon a reliable supply of electricity, and inconvenienced a large number of retail consumers.

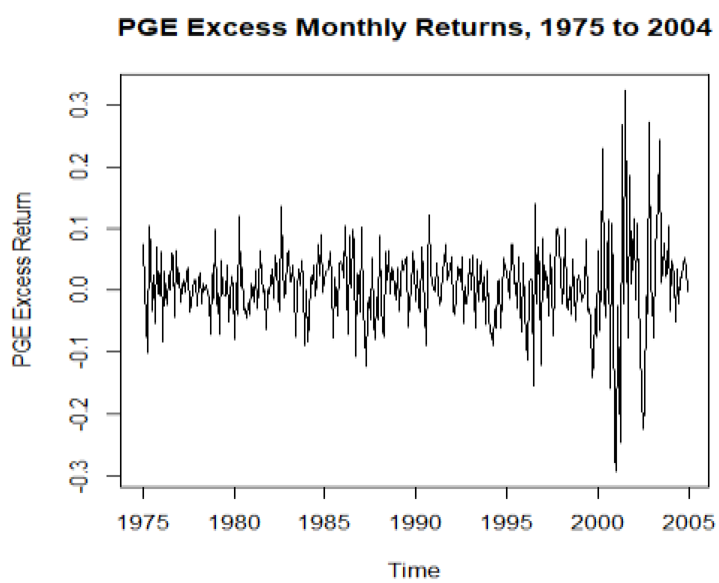
California had an installed generating capacity of 45GW. At the time of the blackouts, demand was 28GW. A demand supply gap was created by energy companies, mainly Enron, to create an artificial shortage. Energy traders took power plants offline for maintenance in days of peak demand to increase the price.[8][9] Traders were thus able to sell power at premium prices, sometimes up to a factor of 20 times its normal value. Because the state government had a cap on retail electricity charges, this market manipulation squeezed the industry's revenue margins, causing the bankruptcy of Pacific Gas and Electric Company (PG&E) and near bankruptcy of Southern California Edison in early 2001.[10]

The financial crisis was possible because of partial deregulation legislation instituted in 1996 by the California Legislature (AB 1890) and Governor Pete Wilson. Enron took advantage of this deregulation and was involved in economic withholding and inflated price bidding in California's spot markets.[11]

The crisis cost between $40 to $45 billion.[12]

```
> pge<-read.csv("F:/Stat71121Fall/PGEmonthly7504.txt")
> attach(pge)
> head(pge)
      DATE TBill30Return PGEPrice   PGEReturn PGEExcessReturn
1 19750131    0.00537288   21.750  0.08074534     0.075372463
2 19750228    0.00516055   22.000  0.01149425     0.006333705
3 19750331    0.00443025   20.500 -0.04681820    -0.051248429
4 19750430    0.00447929   18.375 -0.09756100    -0.102040267
5 19750530    0.00485688   20.375  0.10884354     0.103986658
6 19750630    0.00412427   21.250  0.06601227     0.061887997
```

A plot of the excess monthly returns follows.

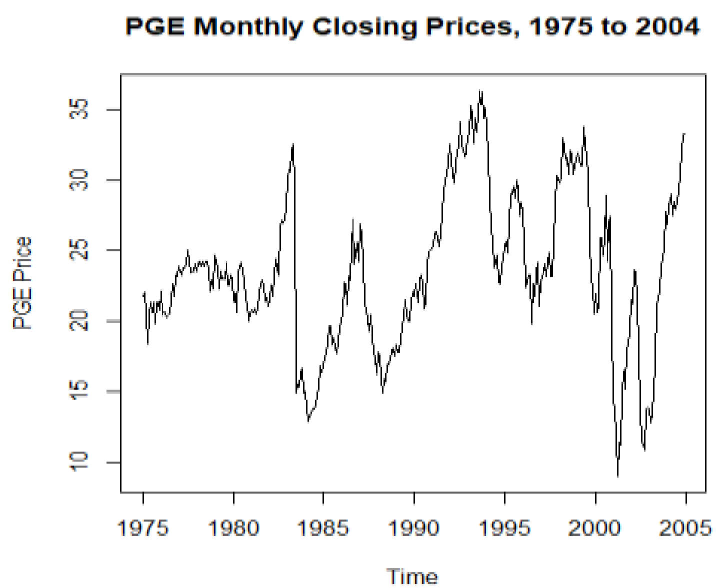


The following list of excess returns is for the months July 1999 through December 2003.

```
> PGEExcessReturn[295:348]
 [1] -0.029425375 -0.045647182 -0.140193475 -0.117842595 -0.028590120
 [6] -0.074148045  0.065369404 -0.064403471  0.028359604  0.229664141
[11] -0.004909041 -0.043647103  0.045517968  0.113367761 -0.159757508
[16]  0.108180572  0.013607786 -0.266004198 -0.292873857 -0.025111752
[21] -0.156448607 -0.246846881  0.267724795 -0.020847268  0.324497495
[26]  0.099899933 -0.076313626  0.185942748  0.011224695  0.049756507
[31]  0.116074154 -0.015108004  0.109135036 -0.004186393 -0.086547202
[36] -0.169526589 -0.224582435 -0.185129899 -0.009472103 -0.037745489
[41]  0.271449411  0.005383315 -0.008127903 -0.077154475  0.053788796
[46]  0.112785562  0.133768466  0.243073398  0.013510095  0.032722565
[51]  0.077128865  0.022312327  0.026550536  0.104630599
```

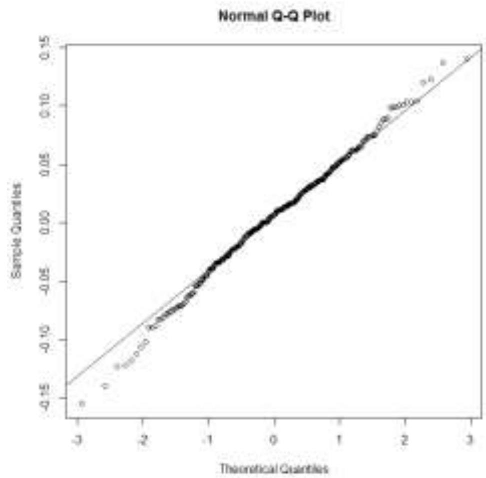
As the tabulation shows, there were many large monthly increases and decreases during the period from July 1999 to the end of 2003. There were four decreases of the excess returns which exceeded −0.20, and five increases which exceeded 0.20.

And here is a plot of the monthly closing prices:



The big drop from June to July in 1983 is for a two-for-one split.

Let’s begin by excluding data for the years 2000 through 2004. We work with the excess returns. As the plot on page 2 shows, there is relatively constant volatility for the years 1975 to 1999.

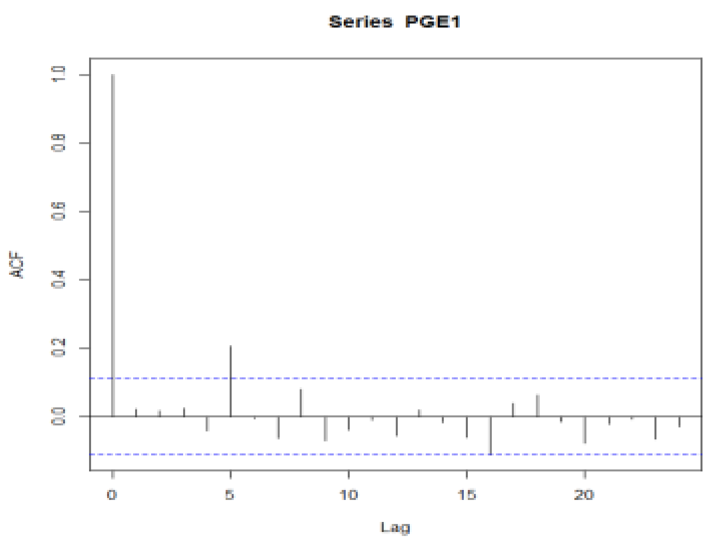


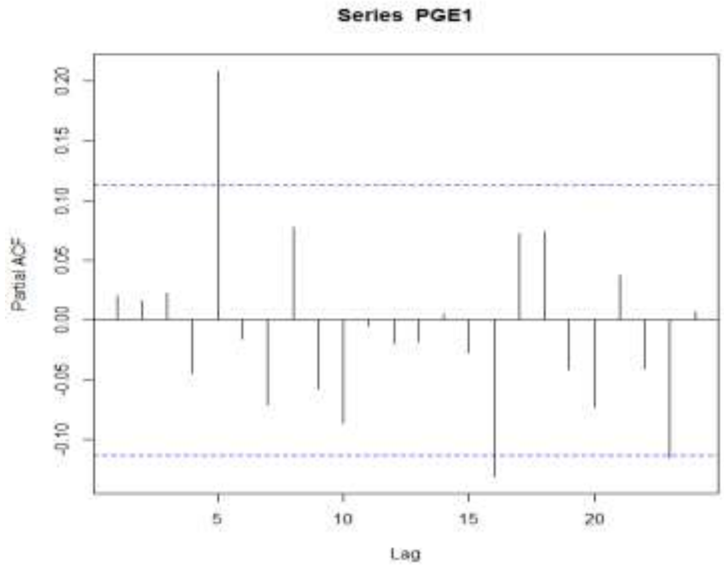
The lower tail of the distribution is somewhat long relative to normality. However, as the following calculation shows, there is very little excess kurtosis.

```
> library("moments")
> skewness(PGEExcessReturn[-sel])
[1] -0.1942407
> kurtosis(PGEExcessReturn[-sel])
[1] 3.318572
```

Let's begin by fitting an ARMA model to the data.

```
> PGE1<-PGEExcessReturn[-sel]
> acf(PGE1)
```



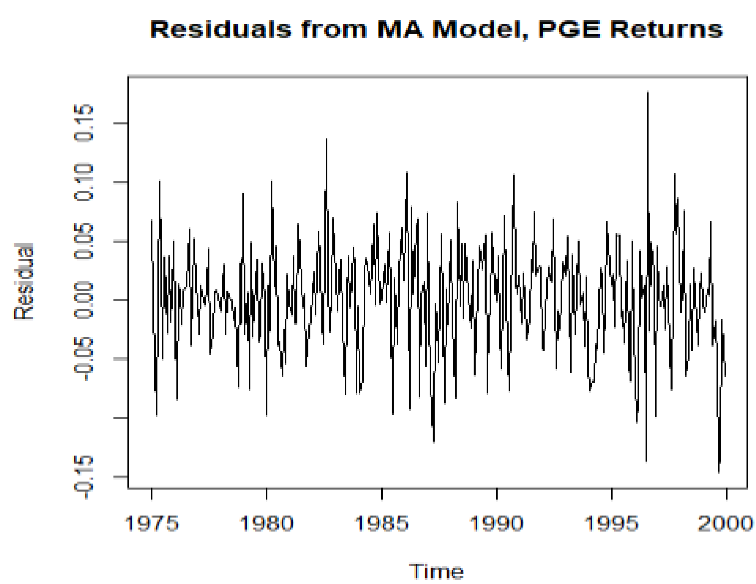


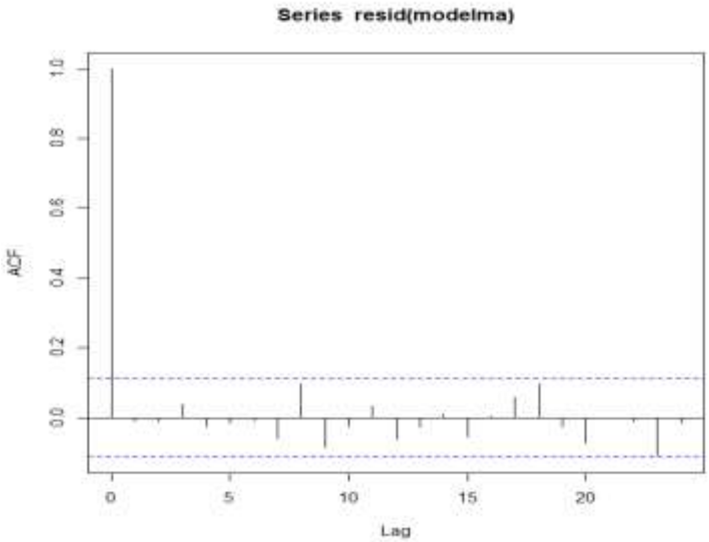
Let's try both ARMA(5,0,0)(1,0,0)$_{16}$ and ARMA(0,0,5)(0,0,1)$_{16}$ model fits.

```
> modelar<-
arima(ts(PGE1),order=c(5,0,0),seasonal=list(order=c(1,0,0),period=16))
> modelar

Call:
arima(x = ts(PGE1), order = c(5, 0, 0), seasonal = list(order = c(1, 0,
0),
    period = 16))

Coefficients:
         ar1     ar2     ar3      ar4     ar5     sar1  intercept
      0.0372  0.0183  0.0215  -0.0647  0.2284  -0.1361     0.0045
s.e.  0.0566  0.0565  0.0571   0.0592  0.0590   0.0609     0.0032

sigma^2 estimated as 0.002282:  log likelihood = 486.46,  aic = -956.92

> coeftest(modelar)

z test of coefficients:

            Estimate Std. Error z value  Pr(>|z|)
ar1        0.0371889  0.0566433  0.6565 0.5114727
ar2        0.0183429  0.0564825  0.3248 0.7453673
ar3        0.0214648  0.0570769  0.3761 0.7068667
ar4       -0.0647064  0.0591884 -1.0932 0.2742938
ar5        0.2284385  0.0590234  3.8703 0.0001087 ***
sar1      -0.1361072  0.0609150 -2.2344 0.0254582 *
intercept  0.0045088  0.0032112  1.4041 0.1602893
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

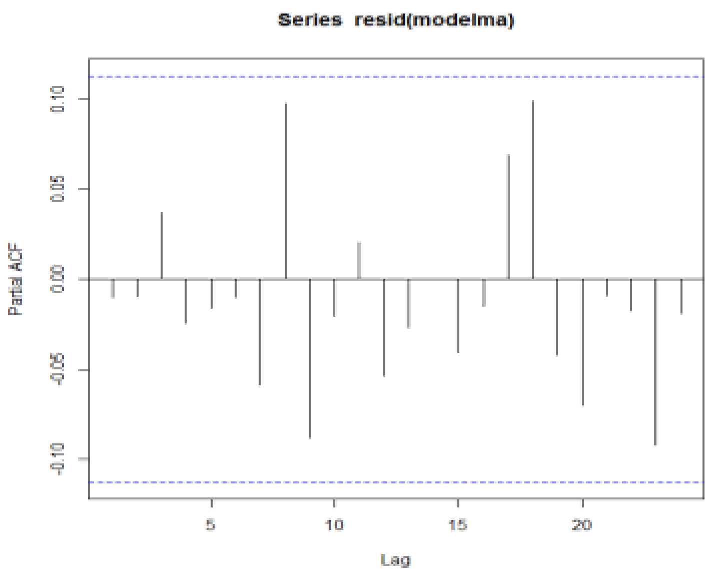
```
> modelma<-
arima(ts(PGE1),order=c(0,0,5),seasonal=list(order=c(0,0,1),period=16))
> modelma

Call:
arima(x = ts(PGE1), order = c(0, 0, 5), seasonal = list(order = c(0, 0,
1),
    period = 16))

Coefficients:
         ma1     ma2      ma3      ma4     ma5     sma1  intercept
      0.0434  0.0613  -0.0441  -0.0190  0.2648  -0.1485     0.0045
s.e.  0.0560  0.0556   0.0613   0.0537  0.0622   0.0644     0.0031

sigma^2 estimated as 0.002261:  log likelihood = 487.71,  aic = -959.43
> coeftest(modelma)

z test of coefficients:

            Estimate Std. Error z value  Pr(>|z|)
ma1        0.0433682  0.0559617  0.7750   0.43836
ma2        0.0613190  0.0556211  1.1024   0.27027
ma3       -0.0441127  0.0612585 -0.7201   0.47146
ma4       -0.0190216  0.0536582 -0.3545   0.72297
ma5        0.2648076  0.0622255  4.2556 2.085e-05 ***
sma1      -0.1484560  0.0644491 -2.3035   0.02125 *
intercept  0.0045495  0.0030775  1.4783   0.13933
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Let's select the ARMA(0,0,5)(0,0,1)$_{16}$ model. The residual analysis follows.

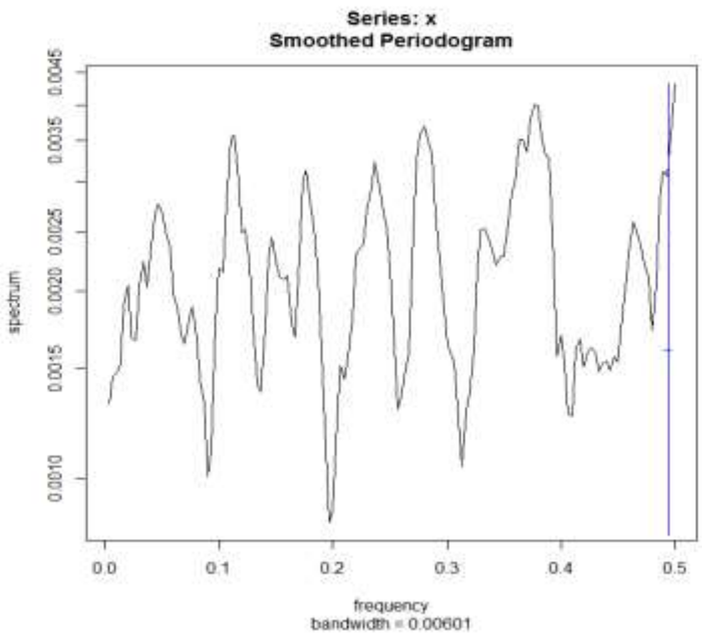




Series resid(modelma)
ACF
Lag

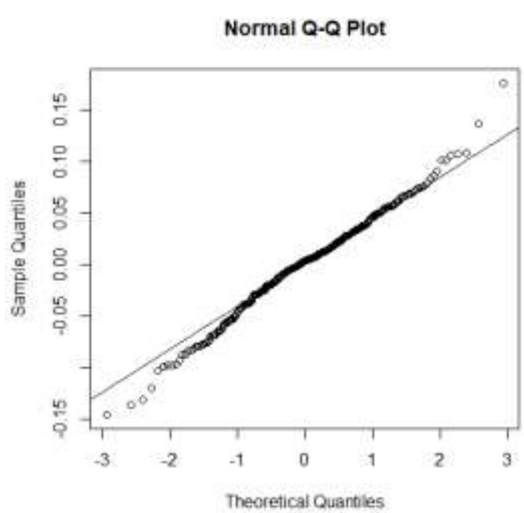


Series resid(modelma)
Partial ACF
Lag




```
> library("hwwntest")
> bartlettB.test(resid(modelma))

        Bartlett B Test for white noise

data:
= 0.40759, p-value = 0.9963
```

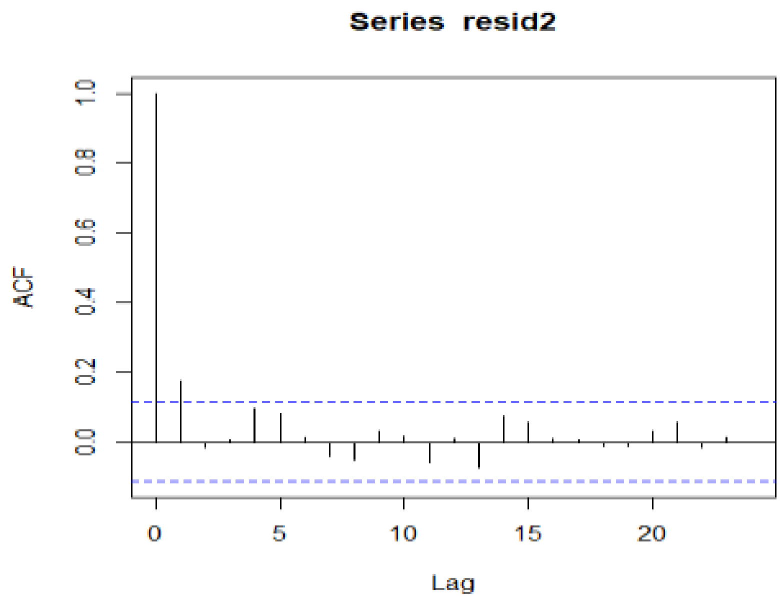
The model gives adequate reduction to white noise.

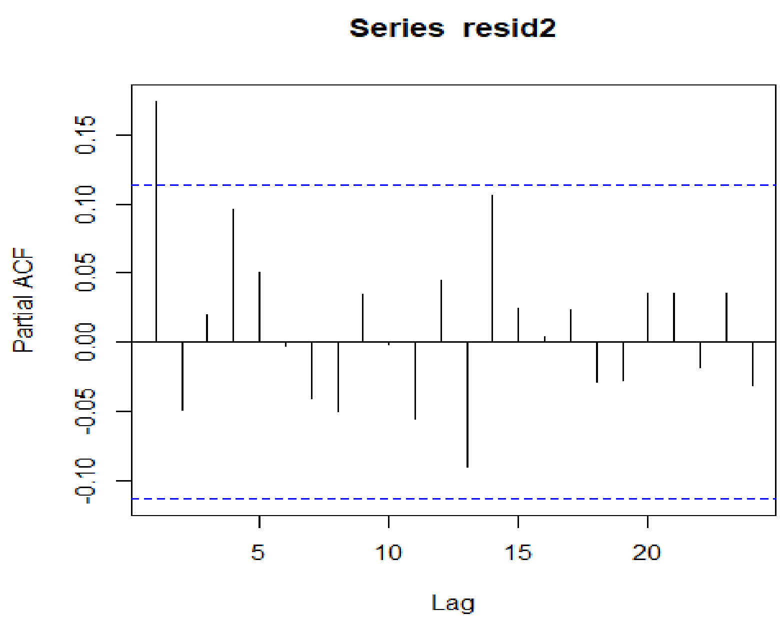


```
> skewness(resid(modelma))
[1] -0.1028527
> kurtosis(resid(modelma))
[1] 3.615201
```

There is very little change in the original skewness and kurtosis calculations.

Next, let's examine the acf and pacf of the squared ARMA(0,0,5)(0,0,1)$_{16}$ residuals.



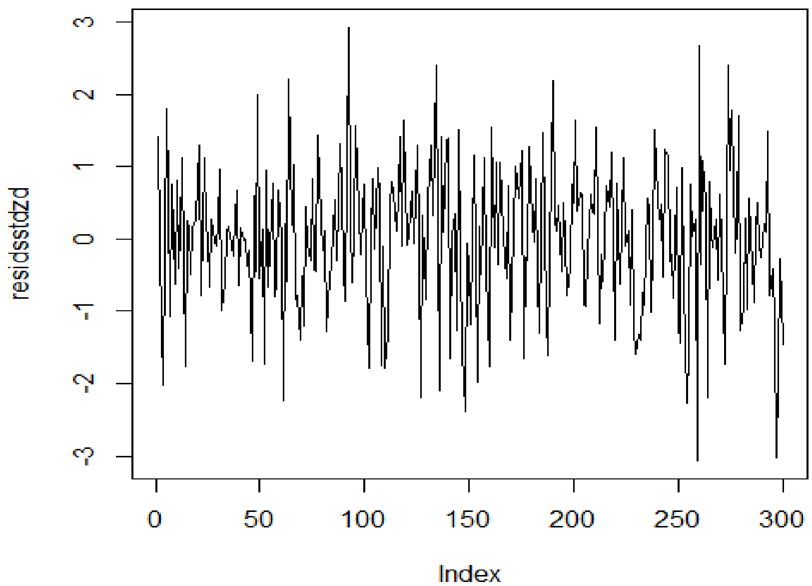


Perhaps there is some ARCH or GARCH structure. Let's fit an ARCH(1) model to the residuals from the ARIMA(0,0,5)(0,0,1)$_{16}$ fit.

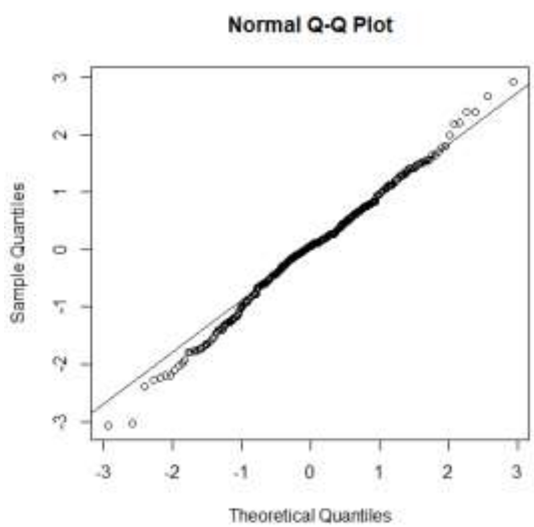
```
> library("fGarch")

> modelarch<-garchFit(~garch(1,0),data=ts(resid(modelma)),trace=FALSE)
> modelarch

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 0), data = ts(resid(modelma)), trace =
FALSE)

Mean and Variance Equation:
 data ~ garch(1, 0)
<environment: 0x0c0c1590>
 [data = ts(resid(modelma))]

Conditional Distribution:
 norm

Coefficient(s):
       mu      omega     alpha1
0.0001610  0.0019654  0.1256741

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0001610   0.0026899    0.060    0.952
omega  0.0019654   0.0001992    9.867   <2e-16 ***
alpha1 0.1256741   0.0670710    1.874    0.061 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 491.2353    normalized:  1.637451
```

The ARCH coefficient is marginally significant.

```
> residsstdzd<-residuals(modelarch,standardize=TRUE)
> plot(residsstdzd,type='l')
```



The acf and pacf values of the standardized residuals are not significant. The normal quantile plot follows.

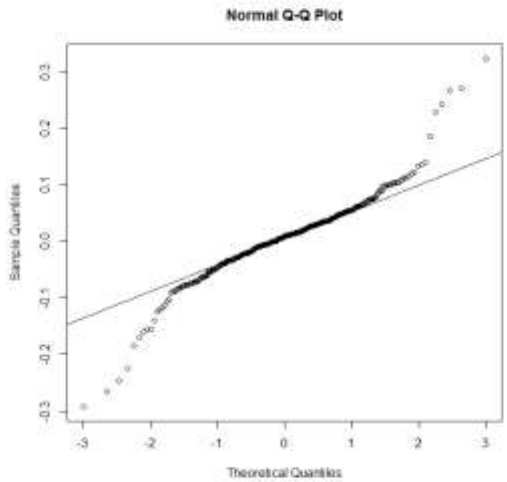



```
> skewness(residsstdzd)
[1] -0.1707349
attr(,"method")
[1] "moment"
> kurtosis(residsstdzd)
[1] 0.2117951
attr(,"method")
[1] "excess"
```

Compare the output on page 9. The results here show the ARCH estimation has lowered the excess kurtosis very slightly. However, the data does have very weak ARCH structure.

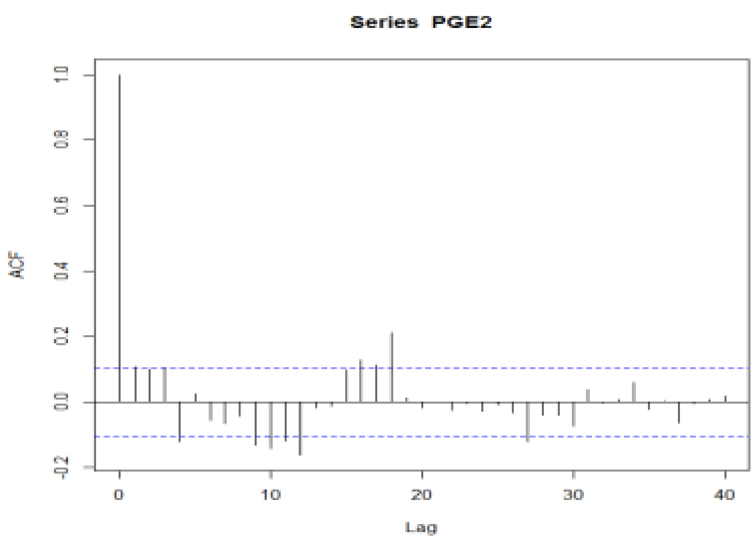
Next, let's treat the entire time span, 1975 through 2004.

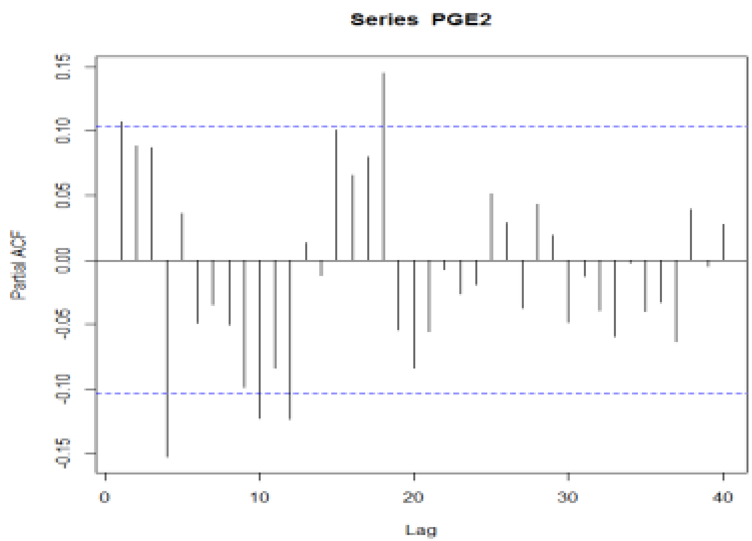
```
> PGE2<-PGEExcessReturn
```




```
> skewness(PGE2)
[1] -0.00307579
> kurtosis(PGE2)
[1] 7.497972
```

The distribution is decidedly nonnormal. There is no skewness, and the kurtosis is 7.50. Let's examine the correlations and partial correlations.

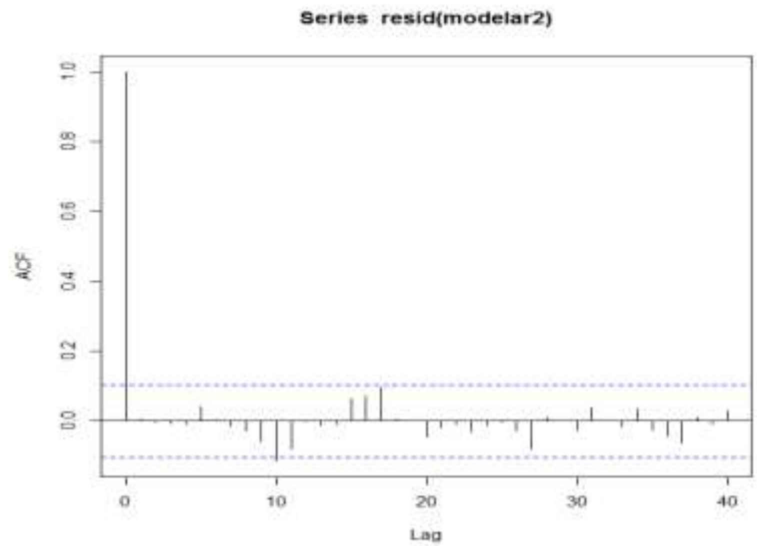


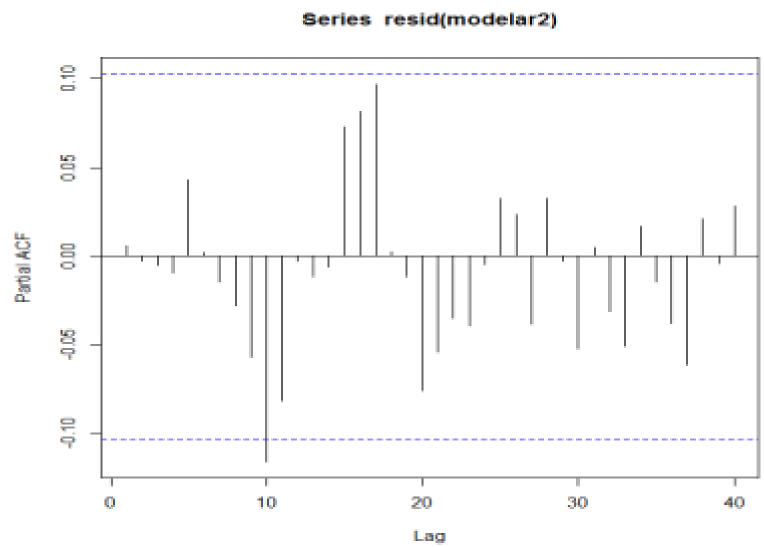


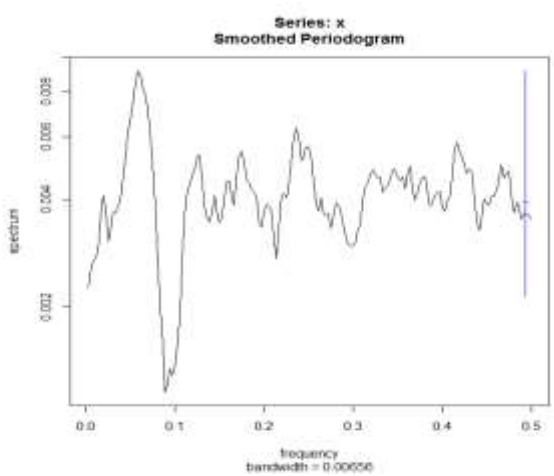
First we fit a model to the level of the time series. The following model fit, ARIMA(4,0,0)(3,0,0)$_6$, addresses the partial autocorrelations at lags 4, 12, and 18.

```
> modelar2<-
arima(ts(PGE2),order=c(4,0,0),seasonal=list(order=c(3,0,0),period=6))
> modelar2

Call:
arima(x = ts(PGE2), order = c(4, 0, 0), seasonal = list(order = c(3, 0,
0), period = 6))

Coefficients:
         ar1     ar2     ar3      ar4      sar1      sar2    sar3
intercept
      0.0802  0.0635  0.0930  -0.1378  -0.0269  -0.1332  0.1876
0.0064
s.e.  0.0526  0.0534  0.0522   0.0526   0.0531   0.0519  0.0520
0.0039

sigma^2 estimated as 0.004155:  log likelihood = 475.67,  aic = -933.35

> coeftest(modelar2)

z test of coefficients:

            Estimate Std. Error z value  Pr(>|z|)
ar1        0.0802481  0.0525713  1.5265 0.1268946
ar2        0.0635024  0.0533816  1.1896 0.2342065
ar3        0.0930161  0.0522366  1.7807 0.0749668 .
ar4       -0.1378454  0.0525701 -2.6221 0.0087383 **
sar1      -0.0268944  0.0531014 -0.5065 0.6125243
sar2      -0.1331550  0.0518822 -2.5665 0.0102734 *
sar3       0.1875925  0.0519687  3.6097 0.0003065 ***
intercept  0.0063909  0.0038614  1.6551 0.0979041 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Residual analysis follows.







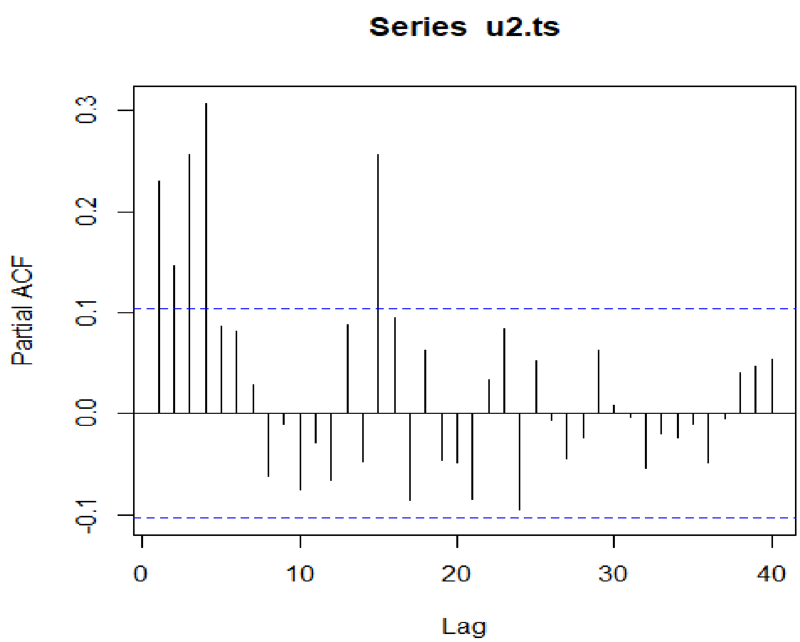
There is a significant lag 10 result in both the acf and the pacf residual plots, and the residual spectral density has a peak at a low frequency. However, the Bartlett test indicates adequate reduction to white noise.

```
> bartlettB.test(resid(modelar2))

        Bartlett B Test for white noise

data:
= 0.39819, p-value = 0.9974

> skewness(resid(modelar2))
[1] -0.03381328
> kurtosis(resid(modelar2))
[1] 6.514183
```

The ARIMA residuals have somewhat smaller kurtosis, down from the original value 7.50.

The squared ARIMA residuals will reveal if there is volatility structure which requires modelling. The pacf plot follows.

```
> u2.ts<-resid(modelar2)^2
> pacf(u2.ts,40)
```



There is certainly structure in the squared residuals. Next, we try fitting GARCH(1, 1), GARCH(1, 2), and GARCH(2, 1) models to capture changing volatility. The models are fit using residuals from the ARIMA fit.

GARCH(1, 1)

```
> u.ts<-ts(resid(modelar2))
> modelgarch11<-garchFit(~garch(1,1),data=u.ts,trace=FALSE)
> summary(modelgarch11)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 1), data = u.ts, trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 1)
<environment: 0x0b146d68>
 [data = u.ts]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1
0.00046300  0.00021629  0.16995865  0.77375239

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.630e-04   2.613e-03    0.177 0.859372
omega  2.163e-04   9.135e-05    2.368 0.017894 *
alpha1 1.700e-01   5.084e-02    3.343 0.000829 ***
beta1  7.738e-01   5.621e-02   13.765  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 525.6179    normalized:  1.46005

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  6.376332  0.04124745
 Shapiro-Wilk Test  R    W      0.9933682 0.1146099
 Ljung-Box Test     R    Q(10)  13.41336  0.2014698
 Ljung-Box Test     R    Q(15)  14.32062  0.5013648
 Ljung-Box Test     R    Q(20)  23.13873  0.2820342
 Ljung-Box Test     R^2  Q(10)  9.879078  0.4511656
 Ljung-Box Test     R^2  Q(15)  23.28878  0.07819802
 Ljung-Box Test     R^2  Q(20)  28.93448  0.08905749
 LM Arch Test       R    TR^2   9.833645  0.6305522

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.897877 -2.854698 -2.898120 -2.880708
```

GARCH(2, 1)

```
> modelgarch21<-garchFit(~garch(2,1),data=u.ts,trace=FALSE)
> summary(modelgarch21)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(2, 1), data = u.ts, trace = FALSE)

Mean and Variance Equation:
 data ~ garch(2, 1)
<environment: 0x088d9b30>
 [data = u.ts]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1      alpha2       beta1
0.00046300  0.00026922  0.10271542  0.11067405  0.71786912

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     0.0004630   0.0025866    0.179   0.8579
omega  0.0002692   0.0001184    2.274   0.0230 *
alpha1 0.1027154   0.0614125    1.673   0.0944 .
alpha2 0.1106740   0.0824071    1.343   0.1793
beta1  0.7178691   0.0816253    8.795   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 526.5419    normalized:  1.462616

Standardised Residuals Tests:
                                Statistic p-Value
 Jarque-Bera Test   R    Chi^2  5.260779  0.07205038
 Shapiro-Wilk Test  R    W      0.9937074 0.1405121
 Ljung-Box Test     R    Q(10)  13.01736  0.2227031
 Ljung-Box Test     R    Q(15)  13.97739  0.5272444
 Ljung-Box Test     R    Q(20)  23.10399  0.2837174
 Ljung-Box Test     R^2  Q(10)  10.24368  0.4193807
 Ljung-Box Test     R^2  Q(15)  23.2799   0.07837522
 Ljung-Box Test     R^2  Q(20)  29.49949  0.07837976
 LM Arch Test       R    TR^2   9.909098  0.6239352

Information Criterion Statistics:
      AIC       BIC       SIC      HQIC
-2.897455 -2.843482 -2.897834 -2.875994
```

GARCH(1, 2)

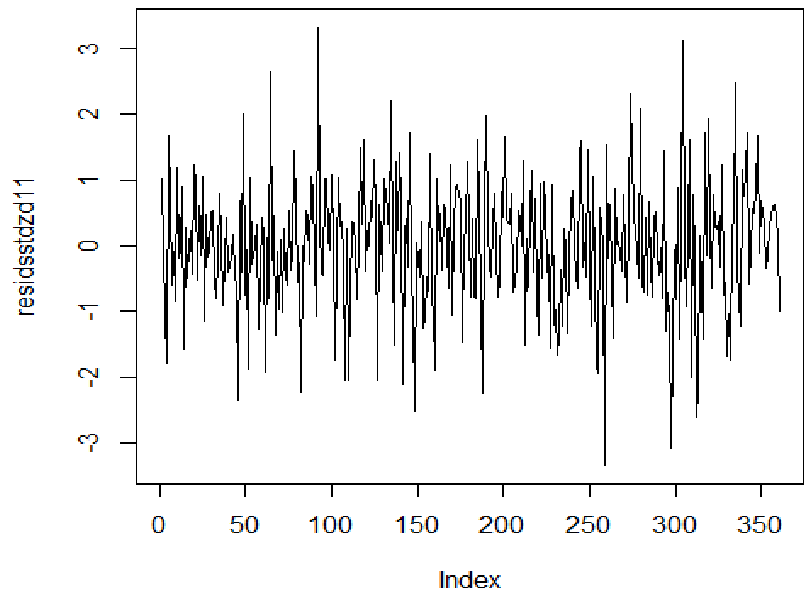
```
> modelgarch12<-garchFit(~garch(1,2),data=u.ts,trace=FALSE)
Warning message:
In sqrt(diag(fit$cvar)) : NaNs produced

> summary(modelgarch12)

Title:
 GARCH Modelling

Call:
 garchFit(formula = ~garch(1, 2), data = u.ts, trace = FALSE)

Mean and Variance Equation:
 data ~ garch(1, 2)
<environment: 0x0b54e730>
 [data = u.ts]

Conditional Distribution:
 norm

Coefficient(s):
        mu       omega      alpha1       beta1       beta2
0.00046300  0.00021635  0.16978010  0.77380388  0.00000001

Std. Errors:
 based on Hessian

Error Analysis:
        Estimate  Std. Error  t value Pr(>|t|)
mu     4.630e-04   2.615e-03    0.177  0.85946
omega  2.163e-04   7.915e-05    2.733  0.00627 **
alpha1 1.698e-01   3.690e-02    4.601 4.21e-06 ***
beta1  7.738e-01          NA       NA       NA
beta2  1.000e-08          NA       NA       NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:
 525.6176    normalized:  1.460049
```

The GARCH(1, 2) estimation has failed.

Next, we form the standardized residuals and calculate their skewness and kurtosis for the GARCH(1, 1) and GARCH(2, 1) models. We also describe the models.

GARCH(1, 1)

```
> residsstdzd11<-residuals(modelgarch11,standard=TRUE)
> plot(residsstdzd11,type='l')
```




```
> kurtosis(residsstdzd11)
[1] 0.6000038
attr(,"method")
[1] "excess"
```

The plot shows that the GARCH(1, 1) model has done a very good job of capturing volatility structure—the standardized residuals exhibit very little changing volatility, and their kurtosis is now estimated as 3.60, close to the value 3 for normality.

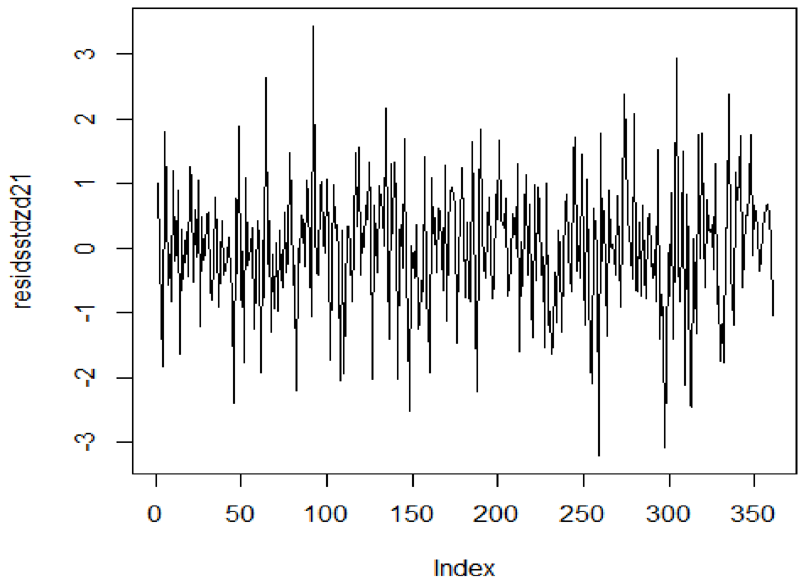
The GARCH(1, 1) fitted model is

$$\sigma_t^2 = 0.0002163 + 0.1700 u_{t-1}^2 + 0.7738 \sigma_{t-1}^2 .$$

All the estimated parameters are positive, and the sum of the last two is less than 1, as required. AIC for the model is −2.898.

GARCH(2, 1)

```
> residsstdzd21<-residuals(modelgarch21,standard=TRUE)
> plot(residsstdzd21,type='l')
```




```
> kurtosis(residsstdzd21)
[1] 0.5379694
attr(,"method")
[1] "excess"
```

The comments given above for the GARCH(1, 1) model fit are also appropriate here—the standardized residuals exhibit very little changing volatility, and their kurtosis is now estimated as 3.54, close to the value 3 for normality.

The GARCH(2, 1) fitted model is

$$\sigma_t^2 = 0.0002692 + 0.1027u_{t-1}^2 + 0.1107u_{t-2}^2 + 0.7179\sigma_{t-1}^2 \,.$$

All the estimated parameters are positive, and the sum of the last three is less than 1, as required. AIC for the model is −2.897.

In summary, both the GARCH(1, 1) and GARCH(2, 1) models have performed well, and there is essentially little difference between the results they have given.