

Assignment 3—due 7 April at 11:59 pm

1. This problem involves examination of monthly Lydia Pinkham data for the period 1938 through 1952. It addresses model construction and estimation of the 90 per cent duration interval. The data are in `Lydiamonthly.txt`.

(a) Construct a model relating sales and advertising with sales as the response. Explore the inclusion of lagged sales and lagged advertising variables. Include estimation of seasonal structure, dummies for outliers, and necessary calendar variables. Describe your fitted model.

(b) Analyze the residuals from your model. Give a normal quantile plot, plot the residuals vs. time, present the residual autocorrelations and partial autocorrelations, and produce the residual spectral density plot. Discuss the results carefully.

(c) Calculate the estimate of the 90 per cent duration interval and discuss the result.

2. The file `qconsumption2.txt` contains quarterly data measuring U.S. Real Personal Consumption Expenditures percentage changes for the period 1953.1 to 2019.4. The data are seasonally adjusted.

(a) Work with the data spanning the period 1953.1 to 2007.4.

(i) Plot the data vs. time and discuss features of the plot. Give a list of the economic downturns as determined by the Business Cycle Dating Committee of NBER. Can you relate these recessions to movements in the plot?

(ii) Begin by identifying outliers and form dummies for them. [Form the dummies to have length 220, so they cover the time span from 1953.1 to 2007.4.]

(iii) An ARX model has the form

$$y_t = \mu + \phi_1(y_{t-1} - \mu) + \cdots + \phi_p(y_{t-p} - \mu) + \beta_1 x_{1t} + \cdots + \beta_k x_{kt} + \varepsilon_t.$$

That is, it combines AR and regression structures. As an example, suppose we have identified two outliers and formed dummies for them, `d1` and `d2`. Then R commands to fit such a model to the percentage changes, if the chosen order of the AR structure is `p`, are as follows:

```
df<-data.frame(d1,d2)
arxmodel<-arima(Pctchange.ts[1:220],order=c(p,0,0),xreg=df)
```

Fit an ARX model to the data covering the period 1953.1 to 2007.4. Explain how you arrived at your model fit.

(iv) Examine the residuals to investigate whether your selected model has achieved reduction to white noise. For this purpose include in your discussion consideration of the residual autocorrelations and partial autocorrelations and the residual spectral density. Also examine and discuss the plot of the residuals vs. time. Perform Bartlett's test to determine if the fit has produced reduction to white noise. [Bartlett's test is described and its use is illustrated in the 28 March notes.]

(v) Find the zeros of the autoregressive polynomial for your model fit and interpret the results.

(b) Repeat the analysis in part (a) for the full time period in the data set, 1953.1 to 2019.4. Note that now the time series spans 268 quarters.

(c) The quarterly percentage changes for 2020.1 to 2021.4 were -6.9 , -33.4 , 41.4 , 3.4 , 11.4 , 12.0 , 2.0 , and 3.1 . Comment briefly on the implications of these numbers.