Documentation for the computer program that finds oriented matroids that are fixed under the action of a group

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1 Introduction

The symmetric group $\mathfrak{S}_n$ acts on the set of vertices of the oriented matroid Grassmannian $\mathrm{MacP}(r, n)$ (thus, oriented matroids) by permuting the elements of the oriented matroids. For given integers r and n, and a group $G \subseteq \mathfrak{S}_n$, this code finds all oriented of rank r on n elements, that are fixed under the action of the group G.

Consult the file *Documentation for finding all oriented matroids.pdf* in this repository for more details. The results of this program are presented in [1, Sect. 6.4].

2 The code

We first find ordinary matroids that are fixed under the action of G, and later check for every of its orientations, whether it yields an oriented matroid that is fixed under the group G. The process is explained on an example in [1, Example 6.4.5].

The code consists of the following files.

- `OMs.c`
- `short_l.c`
- `longer_l.c`
- `construct_fixed_OMs.c`

Again, the file `OMs.c` contains all necessary functions for dealing with chirotopes and oriented matroids, whereas the remaining files yield fixed oriented matroids.

In the code, we first find orbits of the action of the group G on the bases of matroids, thus on tuples in $\binom{[n]}{r}$. Since we are searching for fixed matroids M under the action of G, either all tuples in an orbit of G are bases of M, or none of them is a basis of M. Therefore, we try for the union of every subset of the orbits, whether it satisfies the bases exchange axioms, thus is the set of bases of an ordinary matroid.

This is done by short_l.c and longer_l.c, whereas construct_fixed_OMs.c finds all orientations of the fixed ordinary matroids, thus all oriented matroids whose underlying matroids are fixed under the action of the group G.

2.1 short_l.c and longer_l.c

These two files construct ordinary matroids of rank R on N elements that are fixed under the group action that is given in the code (most of the times that is a subgroup of the symmetric group $\mathfrak{S}_n$ that fixes a certain oriented matroid). They try for every subset of orbits, whether the union of that subset satisfies the basis exchange axioms, which is done in the function ispossible. More precisely, ispossible checks whether a given subset of the first z orbits (given by entries 1 in the tuple l) can be extended to the set of bases of a matroid, where the orbits that correspond to 0-entries in l cannot be taken into consideration.

The file short_l.c makes the first size entries of the tuple l (thus desides for the first size orbits, whether they contribute the set of bases of a matroid), and stores them in an output file. Further on, the file longer_l.c reads those partial choice of orbits, and tries to complete it. If a group action does not have many orbits, like for example the Sylow p-subgroups of $\mathfrak{S}_n$, the file short_l.c can construct complete matroids that are fixed under the action of that group (then the parameter size has to be set to the number of orbits). If that is not the case, computing first partial l's can significantly improve the running time, for which the file longer_l.c is needed.

The output are all ordinary matroids that are fixed under the action of the given group, where the output is formated in such a way, that 1 means that the corresponding orbit is a subset of the bases set, where 0 denotes that the corresponding orbit is not a subset of the bases set.

2.2 construct_fixed_OMs.c

This code takes the output of longer_l.c, which are all ordinary matroids that are fixed under the given group action, and tries to construct oriented matroids out of them. The function constructall tries all sign combination for the orbits that contain bases of the ordinary matroid, and calls construct to check which of them satisfies chirotope axioms. The output are all oriented matroids that are fixed under the given group action.

References

[1] N. Palić. *Grassmannians, measure partitions and waists of spheres.* PhD thesis, Freie Universität Berlin, 2018.