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Low-Complexity Methods for Magnetomer Calibration
Philomaths Technical Note - TN9

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1 Introduction

This technical note analyses two low-complexity magnetometer calibration methods. Such methods are useful when the magnetometer is being connected to a Micro-controller Unit (MCU) which has only a small amount of storage and a limited computational capability. An example of such an MCU is the Arduiono. The methods could be applied to any vector Magnetometer, but we will use an example the Ivensense MPU-9250 [1].

2 Model

In this paper, we will use the model described by Kok et al [2].

The model relates to a measurement y^b of the local magnetic field m_n in the navigation frame. Denoting the rotation from the navigation frame to the body frame as R^{bn} we can define the magnetic field in the body frame as

$$\boldsymbol{m^b} = R^{bn}\boldsymbol{m^n} \tag{1}$$

If there were no measurement errors or distortions, then then $y^b = \boldsymbol{m^b}$, but there will be distortions and noise. Kok analyses possible types of distortion/noise and concludes that

$$\boldsymbol{y^b} = C_{sc}C_{no}(C_{si}\boldsymbol{m^b} + \boldsymbol{o_{hi}}) + \boldsymbol{o_{zb}} + \boldsymbol{e^b} \tag{2}$$

where

- C_{no} is a 3 x 3 matrix representing the non-orthoganality of the magnetometer with respect to the accelerometer,
- $\boldsymbol{o_{zb}}$ is a vector representing the zero bias of the magnetometer,
- C_{sc} is a 3 x 3 diagonal matrix representing the difference in sensitivity of the three magnetometer axes,
- $\boldsymbol{e^b}$ is a vector representing the identically distributed (i.i.d) Gaussian noise,
- $\boldsymbol{o_{hi}}$ is a vector representing the additional hard iron magnetic field component, which is fixed in the body frame,
- C_{si} is a 3 x 3 matrix representing the soft iron effect.

C_{sc} is given by a diagonal matrix

$$C_{sc} = \begin{bmatrix} \frac{1}{f_0} & 0 & 0 \\ 0 & \frac{1}{f_1} & 0 \\ 0 & 0 & \frac{1}{f_0} \end{bmatrix} \tag{3}$$

where the f_i correspond to the sensitivity adjustment values. In the MPU-9250, these values are measured at the time of manufacture and stored in the Fuse ROM. For vectors, the indices $0, 1, 2$ correspond to the x, y, z axes respectively.

A Simplified model

We will start by using a simplified model that makes the following assumptions:

- there is no misalignment between the magnetometer and the accelerometer, so $C_{no} = I$,

- the magnetometer has no zero bias,
- the noise e_b, is negligible,
- the off-elements of C_{si} are negligible so that C_{si} is a diagonal matrix with diagonals (c_{11}, c_{22}, c_{33}).

With these assumptions, the measurement model becomes:

$$\boldsymbol{y^b} = C_{sc} C_{si} \boldsymbol{m^b} + C_{sc} \boldsymbol{o_{hi}} \tag{4}$$

As well, define $D = C_{sc} C_{si}$ so

$$\boldsymbol{y^b} = D \boldsymbol{m^b} + C_{sc} \boldsymbol{o_{hi}} \tag{5}$$

This gives the magnetic field measurement after hard/soft iron distortion and differing sensitivities of the magnetometer axes.

Given that C_{sc} and C_{si} are diagonal we have that D is given by

$$D = \begin{bmatrix} \frac{c_{00}}{f_0} & 0 & 0 \\ 0 & \frac{c_{11}}{f_1} & 0 \\ 0 & 0 & \frac{c_{22}}{f_0} \end{bmatrix} \tag{6}$$

B Flipping Calibration

An established simple technique for calibration is as follows:

- Make a measurement of $\boldsymbol{y^b}$ with the magnetometer,
- Flip (rotate by 180°) the magnetometer on the z-axis and make a measurement,
- flip it on the y-axis and make a measurement,
- flip it on the z-axis and make a measurement.

This first measurement will be:

$$\boldsymbol{y_1^b} = (d_{00} \boldsymbol{m}_0^{\boldsymbol{b}} + \frac{\boldsymbol{o_{hi}}}{f_0}, d_{11} \boldsymbol{m}_1^{\boldsymbol{b}} + \frac{\boldsymbol{o_{hi}}}{f_1}, d_{22} \boldsymbol{m}_2^{\boldsymbol{b}} + \frac{\boldsymbol{o_{hi}}}{f_2})^T \tag{7}$$

Now flipping round the i^{th} axis will leave $\boldsymbol{m_i^b}$ the same but multiply the value of the other two components by -1. So the sequence of flipping will produce the following:

$$\boldsymbol{y_2^b} = (-d_{00}\boldsymbol{m_0^b} + \frac{\boldsymbol{o_{hi}}}{f_0}, -d_{11}\boldsymbol{m_1^b} + \frac{\boldsymbol{o_{hi}}}{f_1}, d_{22}\boldsymbol{m_2^b} + \frac{\boldsymbol{o_{hi}}}{f_2})^T \tag{8}$$

$$\boldsymbol{y_3^b} = (d_{00}\boldsymbol{m_0^b} + \frac{\boldsymbol{o_{hi}}}{f_0}, -d_{11}\boldsymbol{m_1^b} + \frac{\boldsymbol{o_{hi}}}{f_1}, -d_{22}\boldsymbol{m_2^b} + \frac{\boldsymbol{o_{hi}}}{f_2})^T \tag{9}$$

$$\boldsymbol{y_4^b} = (-d_{00}\boldsymbol{m_0^b} + \frac{\boldsymbol{o_{hi}}}{f_0}, d_{11}\boldsymbol{m_1^b} + \frac{\boldsymbol{o_{hi}}}{f_1}, -d_{22}\boldsymbol{m_2^b} + \frac{\boldsymbol{o_{hi}}}{f_2})^T \tag{10}$$

We define an estimator $\boldsymbol{h}$ given by

$$\boldsymbol{h} = \frac{\boldsymbol{y_1^b} + \boldsymbol{y_2^b} + \boldsymbol{y_3^b} + \boldsymbol{y_4^b}}{4} \tag{11}$$

The expectation of h is given by

$$\boldsymbol{\hat{h}} = E(\boldsymbol{h}) = (\frac{\boldsymbol{o_{hi}}}{f_0}, \frac{\boldsymbol{o_{hi}}}{f_1}, \frac{\boldsymbol{o_{hi}}}{f_2})^T \tag{12}$$

giving an expected value of

$$\boldsymbol{\hat{h}} = C_{sc}\boldsymbol{o_{hi}} \tag{13}$$

so $\boldsymbol{h}$ can be subtracted from equation 5 to remove the hard iron bias term, $C_{sc}\boldsymbol{o_{hi}}$.

C Winer Calibration

Winer [3] describes a simple calibration method that consists of rotating the magnetometer through many angles, and finding the maximum and minimum for each axis of the magnetometer.

We can gather the maximum values from the three axes into a vector $\boldsymbol{u^b}$

$$\boldsymbol{u^b} = (d_{00}||\boldsymbol{m^n}|| + \frac{o_{hi_0}}{f_0}, d_{11}||\boldsymbol{m^n}|| + \frac{o_{hi_1}}{f_1}, d_{22}||\boldsymbol{m^n}|| + \frac{o_{hi_2}}{f_2})^T \tag{14}$$

,

and the minimum values are gathered into a vector $\boldsymbol{l^b}$

$$\boldsymbol{l^b} = (-d_{00}||\boldsymbol{m^n}|| + \frac{o_{hi_0}}{f_0}, -d_{11}||\boldsymbol{m^n}|| + \frac{o_{hi_1}}{f_1}, -d_{22}||\boldsymbol{m^n}|| + \frac{o_{hi_2}}{f_2})^T \tag{15}$$

This assumes the rotational sampling of the response surface catches all the extreme values.

If we add $\boldsymbol{u^b}$ to $\boldsymbol{l^b}$ we can see that the terms relating to the terrestrial magnetic field cancel out, so a suitable estimator for the hard iron bias is

$$\hat{h_w} = (\frac{\boldsymbol{u^b} + \boldsymbol{l^b}}{2}) \tag{16}$$

and the expectation of $\widehat{h_w}$ is

$$E(\hat{h_w}) = C_{sc}\boldsymbol{o_{hi}} \tag{17}$$

so $\hat{h_w}$ can be subtracted from equation 5 to remove the hard iron bias term.

Now consider the soft iron distortion. If we subtract $\boldsymbol{l^b}$ from $\boldsymbol{u^b}$, the hard iron bias terms cancel and we are left with

$$\boldsymbol{u^b} - \boldsymbol{l^b} = 2(d_{00}||\boldsymbol{m^n}||, \frac{c_{11}}{f_1}||\boldsymbol{m^n}||, \frac{c_{22}}{f_2}||\boldsymbol{m^n}||)^T \tag{18}$$

so consider a simple estimator E for the D to be

$$\begin{array}{rl} t_{00} = & \frac{u_0^b - l_0^b}{2} \\ t_{11} = & \frac{u_1^b - l_1^b}{2} \\ t_{22} = & \frac{u_2^b - l_2^b}{2} \end{array} \tag{19}$$

with the off-diagonal terms set to zero.

Consulting equation 18 this provides an estimate of the $\frac{c_{ii}}{f_i}$ up to a multiplicative constant i.e.

$$\begin{array}{rl} E(t_{00}) = & ||\boldsymbol{m^n}||d_{00} \\ E(t_{11}) = & ||\boldsymbol{m^n}||d_{11} \\ E(t_{22}) = & ||\boldsymbol{m^n}||d_{22} \end{array} \tag{20}$$

or more compactly $\hat{T} = ||\boldsymbol{m^n}||D$.

Magnetometers are often used in a Attitude Heading Reference System (AHRS), where the algorithm normally works with the norm of the magnetic field vector i.e. it does not use the absolute field strength. In such systems the value of $||\boldsymbol{m^n}||$ does not matter, so we can set $||\boldsymbol{m^n}|| = 1$, or some other arbitrary value. Winer removes $||\boldsymbol{m^n}||$ by dividing by

$$y^b_{avg} = \frac{f_0(u_0^b - l_0^b) + f_1(u_1^b - l_1^b) + f_2(u_2^b - l_2^b)}{6} \tag{21}$$

The expectation of this is

$$E(y^b_{avg}) = \frac{d_{00}||\boldsymbol{m^n}|| + d_{11}||\boldsymbol{m^n}|| + d_{22}||\boldsymbol{m^n}||}{3} = ||\boldsymbol{m^n}||\frac{\mathrm{Tr}(D)}{3} \tag{22}$$

so it is represents some measure of the average field strength in the body frame.

So if our estimator becomes

$$\begin{aligned} \widehat{s_{00}} &= \tfrac{u^b_0 - l^b_0}{2y^b_{avg}} \\ \widehat{s_{11}} &= \tfrac{u^b_1 - l^b_1}{2y^b_{avg}} \\ \widehat{s_{22}} &= \tfrac{u^b_2 - l^b_2}{2y^b_{avg}} \end{aligned} \tag{23}$$

with an expectation of

$$\begin{aligned} E(\widehat{s_{00}}) &= \tfrac{3d_{00}}{\mathrm{Tr}(D)} \\ E(\widehat{s_{11}}) &= \tfrac{3d_{11}}{\mathrm{Tr}(D)} \\ E(\widehat{s_{33}}) &= \tfrac{3d_{22}}{\mathrm{Tr}(D)} \end{aligned} \tag{24}$$

Noting our definition of D we can write equation 23 in matrix form

$$S = \begin{bmatrix} \frac{u^b_0 - l^b_0}{2y^b_{avg}} & 0 & 0 \\ 0 & \frac{u^b_1 - l^b_1}{2y^b_{avg}} & 0 \\ 0 & 0 & \frac{u^b_2 - l^b_2}{2y^b_{avg}} \end{bmatrix} \tag{25}$$

with an expectation of

$$\hat{S} = \frac{3}{\mathrm{Tr(D)}} \begin{bmatrix} d_{00} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{22} \end{bmatrix} = \frac{3}{\mathrm{Tr(D)}} D \tag{26}$$

Using this method of normalization has the advantage compared to setting $||\boldsymbol{m^n}|| = 1$ that the calibrated field strength will not be too different from the measured field strength. It has the disadantage that it may be difficult to compare the calibration of two different magnetometers because the normalization factor $\mathrm{Tr}(D)$ will differ between systems. It also makes it more difficult to full analyze the effects of measurement error.

The calibration is complete. Our calibration results can be applied to the model in equation 5

$$y^b = D\boldsymbol{m^b} + C_{sc}\boldsymbol{o_{hi}} \tag{27}$$

So first subtract out estimate $\hat{h_w}$ (equation 16 and multiply be $\hat{S}$ (equation 25 giving

$$y^b_{cal} = \hat{S}^{-1}(y^b - \widehat{h_w}) \tag{28}$$

Using equations 17 and 26, the expectation of y^b_{cal} is

$$E(y^b_{cal}) = \frac{||d||}{\sqrt{3}}(S^{-1}(D\boldsymbol{m^b} + C_{sc}\boldsymbol{o_{hi}} - C_{sc}\boldsymbol{o_{hi}}) \tag{29}$$

which becomes

$$E(y^b_{cal}) = \frac{3}{\mathrm{Tr}(D)}\boldsymbol{m^b} \tag{30}$$

i.e the magnetic field strength apart from a muliplicative constant that will be removed with normalization.

D Adding noise to the equation

If we include the Gaussian noise to our model, equation 5 becomes

$$\boldsymbol{y^b} = D\boldsymbol{m^b} + C_{sc}\boldsymbol{o_{hi}} + \boldsymbol{e^b} \tag{31}$$

Considering Flipping calibration, the estimator for the hard iron bias $C_{sc}\boldsymbol{o_{hi}}$ is h. Adding noise to equation 11 gives

$$\hat{h} = \frac{\boldsymbol{y^b_1} + \boldsymbol{o^b_1} + \boldsymbol{y^b_2} + \boldsymbol{o^b_2} + \boldsymbol{y^b_3} + \boldsymbol{o^b_3} + \boldsymbol{y^b_4} + \boldsymbol{o^b_4}}{4} \tag{32}$$

where the $\boldsymbol{o^b_j}$ is the gauusian noise generated by the j^{th}measurement of $\boldsymbol{y^b}$. We assume the noise is i.i.d Gaussian zero mean noise with a variance equal to σ^2. Accordingly the expectation of h remains equal to $C_{sc}\boldsymbol{o_{hi}}$ The Variance of $E(s)$ becomes

$$\mathrm{Var}(\boldsymbol{h}) = E(||\boldsymbol{h} - \hat{\boldsymbol{h}}||^2) = \frac{\sigma^2}{4} \tag{33}$$

Considering Winer calibration, the estimator for the hard iron bias $C_{sc}\boldsymbol{o_{hi}}$ is h_w. Adding noise to equation 16 gives

$$\hat{\boldsymbol{h_w}} = (\frac{\boldsymbol{u^b} + \boldsymbol{o_u^b} + \boldsymbol{l^b} + \boldsymbol{o_l^b}}{2}) \tag{34}$$

where $\boldsymbol{o_u^b}$ and $\boldsymbol{o_l^b}$ is the gauusian noise generated by the measurement of the $\boldsymbol{u_i^b}$ and $\boldsymbol{l_i^b}$ respectively. We assume the noise is i.i.d Gaussian zero mean noise with a variance equal to σ^2. Accordingly the expectation of h_w remains equal to $C_{sc}\boldsymbol{o_{hi}}$ The Variance of $E(h_w)$ becomes

$$\mathrm{Var}(\boldsymbol{h_w}) = E(||\boldsymbol{h_w} - \hat{\boldsymbol{h_w}}||^2) = \frac{\sigma^2}{2} \tag{35}$$

The estimator of the elements of the diagonal elements of D is of the form 23

$$\widehat{s_{ii}} = \frac{\boldsymbol{u_i^b} + o_u^b - l_i^b - o_l^b}{2y_{avg}^b} = \frac{\boldsymbol{u_i^b} - l_i^b}{2y_{avg}^b} + \frac{o_u^b - o_l^b}{2y_{avg}^b} \tag{36}$$

or using equation 26, 22, and our assumption the noise is zero mean, we have

$$\widehat{s_{ii}} = \frac{3}{\mathrm{Tr(D)}} 2d_{ii} \tag{37}$$

and the variance of the estimator will be

$$\mathrm{Var}(\widehat{d_{00}}) = \frac{2\sigma^2}{4||\boldsymbol{m^n}||^2}\frac{3}{\mathrm{Tr}(D)} = \frac{\sigma^2}{2||\boldsymbol{m^n}||^2}\frac{3}{\mathrm{Tr}(D)} \tag{38}$$

E Error due to Rotation of Principle Axes

The above assumes that D is diagonal. This represents an ellipse with the principal axes aligned along the x, y, and z axes. Suppose the soft iron effect also causes a rotation of the principle axes i.e

$$D^{''} = R_{\theta,\phi} D \tag{39}$$

where R is the matrix representing a rotation of the axes through an azimuth angle θ and and elevation angle ϕ. The rotation will create off-diagonal elements in $D^{''}$. For example, if $\phi = 0$ then we have

$$R_{\theta,\phi} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{40}$$

If we consider equation 6, but using D'' instead of D we have that

$$\boldsymbol{y}^{\boldsymbol{b}'} = R_{\theta,\phi} D'' R^{bn} \boldsymbol{m}^{\boldsymbol{n}} \tag{41}$$

Now if $R^{bn}\boldsymbol{m}^{\boldsymbol{n}}$ is oriented along the x-axes, this becomes

$$\boldsymbol{y}^{\boldsymbol{b}'} = R_{\theta,\phi}(||\boldsymbol{m}^{\boldsymbol{n}}||, 0, 0)^T \tag{42}$$

so if $\phi = 0$, using equation 40 we have

$$\boldsymbol{y}^{\boldsymbol{b}'} = (||\boldsymbol{m}^{\boldsymbol{n}}||cos\theta, ||\boldsymbol{m}^{\boldsymbol{n}}||sin\theta, 0) \tag{43}$$

From this we can see the maximum value of the x-axis reading will no longer be equal to $||\boldsymbol{m}^{\boldsymbol{n}}||$ but $||\boldsymbol{m}^{\boldsymbol{n}}||cos\theta$. If θ is small this will cause only a small amount of error, but if θ is large, then there will be a very significant error in the calibration. Accordingly, in its present form, the Winer algorithm appears only to be able to calibrate accurately soft iron effects where the off-diagonal elements are small.

It may be possible to correct for this by recording the value of x-axis, y-axis, and z-axis magnetometer readings at each extremum. From this information it may be possible deduce the rotation of the principle axes, but this would add additional complexity to an algorithm that is specifically designed to be low-complexity. Nevertheless the recording of all three axes at each extremum could provide an indication whether the off-diagonal terms are small. So for example at the maximum x-value the y-value and z-value are both small, that would support a view that the off-diagonal elements are small.

F Error Analysis of Flipping Calibration

From equation 33 the measurement noise will introduce a bias error of the order of $\frac{\sigma}{2}$, where σ is the standard deviation of the error on the individual measurements. This bias error is a systematic error which will affect the ultimate measurement results. It is a reasonable system design aim to make the bias error considerably less than the filtered measurement error. Without doing a detailed error analysis, a rule of thumb would be that the bias error should be ten times less than σ, implying that the variance should be 100 times less. With Flipping calibration, this could be approximated by repeating the entire flipping procedure 16 times. However, this could introduce additional errors due to incorrect alignment of the unit upon each flip. A simpler way would be for each orientation, average 16 individual measurements, to give $\overline{\boldsymbol{y_1}}, \overline{\boldsymbol{y_2}}, \overline{\boldsymbol{y_3}}, \overline{\boldsymbol{y_4}}$.

In order to reduce the possibilities for alignment errors, it is recommended that a piece of paper be taped to the measurement surface, and an outline of the unit be drawn on the paper. Any subsequent flips should ensure that the unit is still aligned with that outline.

G Error Analysis of Winer Calibration Maneuver

In order to carry out the calibration, Winer suggests maneuvering the magnetometer through a Figure 8 pattern. Whilst moving in this pattern the software records the readings from the x, y, and z axes of the magnetometer. The Figure 8 pattern is commonly used for calibrating magnetometers. If well executed, it will take many samples which are approximately uniformly distributed over the three dimensional response surface of the magnetometer. This appears well suited to a calibration method that fits all the points to ellipsoid. In this case, the fact that the points are widely distributed increases the accuracy of the points, and the fact that all the points are used dramatically reduces the effect of the measurement error for individual measurements.

A by-product of the low-complexity of the Winer method is that only 6 points are used are the maximum and minimum of the x, y, and z readings. One significant source of error for the Winer method is that the maximum or minimum reading for a particular axis may not actually correspond with the true minimum or maximum. Winer ameliorates this by taking many measurements (about 128) and also doing scatter plots of the measurements which can be used to check there is reasonable coverage at the extrema. However taking numerous measurements has a draw back, given each measurement is noisy.

Assume that the pattern is perfectly executed and each time the six extremum points are measured each time. For each extremum, there is a sampling of a random variable, and if that sample is more extreme than the last, then it is taken as the new estimate of the extremum. If the measurement error is Gaussian, then it is well known that such a sampling method will be biased. This is because if you keep sampling a Gaussian random variable and remembering the largest value, the over time that largest value will become larger and larger. In particular [4]

$$E(y) \approx \mu + \sigma \Phi^{-1} \left(\frac{n - \frac{\pi}{8}}{n - \frac{\pi}{4} + 1} \right) \tag{44}$$

where μ is the mean of the Gaussian variable, σ^2 is the variance, n is the number of samples and $\Phi(x)$ is the cumulative Gaussian distribution. This grows slowly without limit as n increases.

Another potentially significant source of error when using the Winer calibration method is the measurement noise, analyzed in section D.

From equation 35, the bias error is of the order of $\frac{\sigma}{\sqrt{2}}$. Again, it would be good to reduce this by factor of 10, which could be approximated by averaging 48 measurements. However with the MPU-9250 there is a difficulty with this approach. The fastest update rate of the magnetometer reading is 100Hz, so it would take about 5 seconds to gather 48 measurements, so blurring the result, unless the person carrying out the Figure 8 maneouvre was able to move agonizingly slowly.

Let us assume that the soft iron effect is small so the diagonal elements of D are near 1 and so $\frac{\mathrm{Tr}(D)}{3} \approx 1$ so equation 38 becomes

$$\mathrm{Var}(\widehat{s_{ii}}) \approx \frac{\sigma^2}{2||\boldsymbol{m^n}||^2} \tag{45}$$

and the expectation of the estimator from equation 23 is approximately d_{ii}. After calibration, measurements on the MPU-9250 indicate that at a maximum of 495 mG and standard deviation of 7.3 mG. Using these measurements implies that the standard deviation of the estimate of the d_{ii} is about 0.01, so the 95% Confidence Interval of the estimates of the $d_i i$ is + 0.02 or + 2%. Clearly if the magnetometer has a much high signal to noise ratio, the CI will be considerably reduced.

As was the case with the bias error, there is clearly a need to be able to average the measurements in order to be able to get an accurate estimate of D. Due to blurring, there will be limits to how much average can take place, but it might be possible by a trial and error approach to be able to average a few measurements together, so making a moderate improvement.

H Possible improvements to the Winer Method

It may be possible to significantly reduce the sources of error in the Winer by replacing the Figure 8 maneuver by a more targeted approach. Using a Figure 8 means that almost all the points gathered in the Figure 8 rare discarded, recording only the six extrema.

Instead, it is proposed the software be altered to print out for the operator, in a continuous fashion, the x, y, and z axis readings. Then the operator could take execute a manual gradient descent procedure [5]. The procedure assumes the operator has a compass that is able to indicate the direction of true north. Once he direction of true north is known, then the operator knows that local magnetic vector points in that direction but is tilted upwards or downward depending on whether they located in the northern or southern hemisphere. The idea of the algorithm is align and anti-align the x-axis with the magnetic vector, and do the same for the y-axis and the z-axis. Here are the step by step instructions for the operator:

- Orient the x-axis in the direction of true north
- Keeping the x-axis aligned with true north, rotate the box downwards towards the earth whilst observing the x-axis values. Depending in which hemisphere the procedure is being performed, the readings should either increase or decrease.
- If they are increasing keep rotating until a maximum value is found. Try moving a small amount to the left or right to see if you can increase the x-value further. Once you have found the maximum, keep the unit still at that orientation for a second or two. Then keep rotating the box in the same manner that you had started, while maintaining the x-axis pointing to true North. Now the readings should be decreasing. Keep rotating until you find the minimum value, then try to move to left or right to see if you can decrease the value further. Once you have found the minimum, keep the unit still at that orientation for a second or two.
- If they are decrease, the follow the procedure outlined in the above step, except look first for a minimum value and then a maximum value.
- Repeat above steps but with the y-axis and then do the same with the z-axis.

This procedure significantly reduces the two major sources of error associated with using the Figure 8.

- Instead of randomly sampling the angle space and hoping there is a sample near the six extrema, the gradient descent approach looks for the actual extrema so is much more likely to find them.
- Because the operator is able to move slowly in the gradient descent approach and is encouraged to pause for a second or two, it is possible to average every measurement fed into the calibration for a second or so, so greatly reducing the measurement error.
- Because the approach is more accurate, it is more repeatable.

I Conclusion

This paper was originally motivated to settle an issue [6] that had been raised on the Winer MPU-9250 Github repository. In essence, the issue was how or whether to apply the MPU-2950 fuse values calibration algorithm. The fuse values are the sensitivity of the magnetometer axes as measured at the time of manufacture. In the context of this paper, the fuse values correspond to the f_i. From the previous development, it can be seen that the magnetometer can be calibrated without knowledge of these values. This means that if the person doing the calibration is not confident that the fuse values are correct, then the values should be ignored, and the fuse values should not be applied to any subsequent measurements.

If the person doing the calibration is confident that that fuse values are correct, then they should be applied to every measurement, including the measurements made to do the calibration and any subsequent measurement. This can be seen to be true by considering the converse, a systems engineer would never multiply measurements by arbitrary factors prior to calibration on the basis that the calibration will remove the arbitrary factors. If you are confident of the fuse values and you do not apply them, then you are feeding incorrect measurements into the calibration.

J Resources

The resources for this technical note are available for access at https://github.com/philomaths-org/covid-19. The calibration folder contains the pdf for this paper and relevant code. You can access resources for earlier versions of this note on Github by clicking on the tag corresponding to the earlier technical note's version number.

References

[1] T. IvenSense, "Mpu - 9250." https://invensense.tdk.com/products/motion-tracking/9-axis/mpu-9250/, 2021.

[2] M. Kok, J. D. Hol, T. B. Schön, F. Gustafsson, and H. Luinge, "Calibration of a magnetometer in combination with inertial sensors," in *2012 15th International Conference on Information Fusion*, pp. 787–793, IEEE, 2012.

[3] K. Winer, "Simple and effective magnetometer calibration." https://github.com/kriswiner/MPU6050/wiki/Simple-and-Effective-Magnetometer-Calibration, 2017.

[4] soakley (https://stats.stackexchange.com/users/24073/soakley), "Expected value of maximum of samples from normal distribution." Cross Validated. URL:https://stats.stackexchange.com/q/344044 (version: 2018-05-02).

[5] Wikipedia contributors, "Gradient descent — Wikipedia, the free encyclopedia," 2021. [Online; accessed 9-April-2021].

[6] C. R. Drane, "The magcalibration does not appear to be applied consistently when calculating the magscale values." Github.