Simulation Results:
Two Plates Capacitor Analysis
and Capacitive Biosensor Design

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SECTION: 3

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Part 1: Parallel Plate Capacitor Analysis

Model Setup

A parallel plate capacitor configuration was simulated using COMSOL Multiphysics. The specifications were as follows:

- **Geometry:** Two square copper plates were modeled, each measuring 1 mm × 1 mm with a thickness of 0.05 mm. They were positioned parallel to the xy-plane, occupying $z = [0, 0.05\,\mathrm{mm}]$ and $z = [0.15\,\mathrm{mm}, 0.2\,\mathrm{mm}]$, creating a separation distance $d = 0.1\,\mathrm{mm}$. A dielectric block matching the plate area (1 mm × 1 mm) and filling the gap thickness (0.1 mm) was included. Plate area $A = 1\,\mathrm{mm}^2$.
- **Materials:** Plates were assigned the properties of Copper ($\sigma = 5.96 \times 10^7\,\mathrm{S\,m^{-1}}$, relative permittivity $\epsilon_r = 1$). The dielectric material had a relative permittivity ϵ_r varied parametrically from 1 to 10. The surrounding medium was Air ($\epsilon_r = 1$).
- **Domain:** The capacitor structure was enclosed within an air simulation box of dimensions 2 mm × 2 mm × 0.5 mm.
- **Physics:** The Electrostatics (es) physics interface was employed.
- **Boundary Conditions:** One plate was set as a Terminal with an applied voltage $V_0 = 5\,\mathrm{V}$. The other plate was set to Ground ($V = 0\,\mathrm{V}$). The outer boundaries of the air box were defined with Zero Charge condition, simulating an open environment.
- **Mesh:** A physics-controlled mesh was generated by COMSOL, with refinements automatically applied near the plate edges and within the dielectric gap to accurately capture field gradients.

Simulation Results

The simulation was performed using a Stationary study to find the steady-state electrostatic solution. Results for the baseline case ($\epsilon_r = 1$) and the parametric variation of ϵ_r are presented.

Electric Potential and Field ($\epsilon_r = 1$)

The simulation confirms the expected behavior: the electric potential decreases linearly across the dielectric gap from 5 Vto 0 V. Correspondingly, the electric field is predominantly uniform and directed along the z-axis (E_z) between the plates. For $\epsilon_r = 1$, the field magnitude is approximately $E_z \approx V_0/d = 5\,\mathrm{V}/0.1\,\mathrm{mm} = 5 \times 10^4\,\mathrm{V\,m^{-1}}$. Fringing fields are observed near the plate edges, as illustrated in Figure 1.

Stored Electrostatic Energy and Capacitance ($\epsilon_r = 1$)

The total stored electrostatic energy (W_e) within the simulation domain, predominantly localized in the dielectric, was calculated by COMSOL as $W_e = 1.09 \times 10^{-12}\,\mathrm{J}$

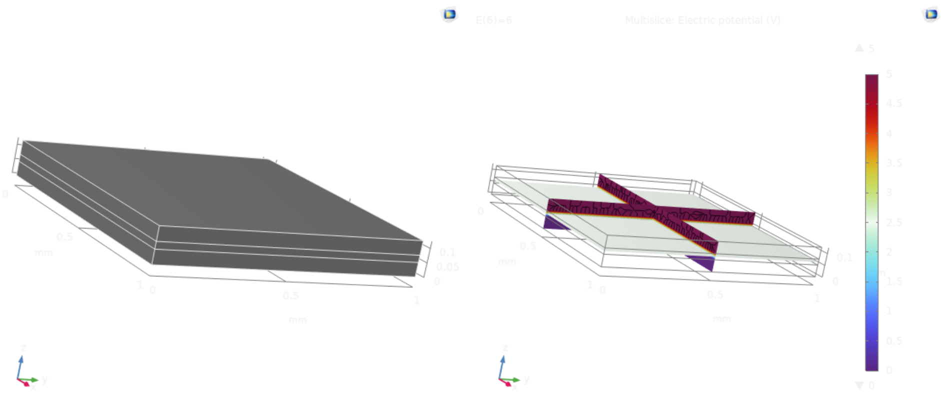

Figure 1: Electric potential distribution (V, color map) and electric field lines (streamlines) for the baseline case with dielectric $\epsilon_r = 1$. The plot clearly shows the linear potential gradient within the 0.1 mm gap and the uniform nature of the field away from the edges.

for the $\epsilon_r = 1$ case (Figure **??**). The capacitance (C) can be derived using the stored energy:

$$C = \frac{2W_e}{V_0^2} = \frac{2 \times 1.09 \times 10^{-12}\,\mathrm{J}}{(5\,\mathrm{V})^2} = 8.72 \times 10^{-14}\,\mathrm{F} = 0.0872\,\mathrm{pF}$$

This value compares well with the ideal parallel plate capacitance formula $C_{\text{ideal}} = \epsilon_0\epsilon_r A/d = (8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}) \times 1 \times (1 \times 10^{-6}\,\mathrm{m^2})/(0.1 \times 10^{-3}\,\mathrm{m}) = 0.0885\,\mathrm{pF}$. The small difference ($\approx 1.5\%$) is attributed to fringing field effects captured by the FEM simulation but neglected in the ideal formula. The corresponding surface charge density on the plates is approximately $\rho_s = \epsilon_0\epsilon_r E_z \approx (8.854 \times 10^{-12}\,\mathrm{F/m}) \times 1 \times (5 \times 10^4\,\mathrm{V/m}) = 4.427 \times 10^{-7}\,\mathrm{C\,m^{-2}}$.

Parametric Study: Varying Relative Permittivity

A parametric sweep was conducted, varying the relative permittivity (ϵ_r) of the dielectric material from 1 to 10. The capacitance was calculated for each value. As expected theoretically ($C \propto \epsilon_r$), the simulation results show a linear increase in capacitance with increasing ϵ_r. The computed values are listed in Table 1 and visualized in Figure 2.

Capacitance Calculation for Specific Materials

Using the simulated capacitance for vacuum ($\epsilon_r = 1$) as a baseline ($C_1 = 0.0872\,\mathrm{pF}$), the expected capacitance for the same parallel plate capacitor geometry filled with other common dielectric materials can be calculated using $C = \epsilon_r C_1$. Table 2 lists these values.

Table 1: Simulated capacitance as a function of dielectric relative permittivity (ϵ_r).

Relative Permittivity (ϵ_r)	Calculated Capacitance (pF)
1.0	0.0872
2.0	0.1745
3.0	0.2618
4.0	0.3492
5.0	0.4365
6.0	0.5239
7.0	0.6112
8.0	0.6986
9.0	0.7859
10.0	0.8733

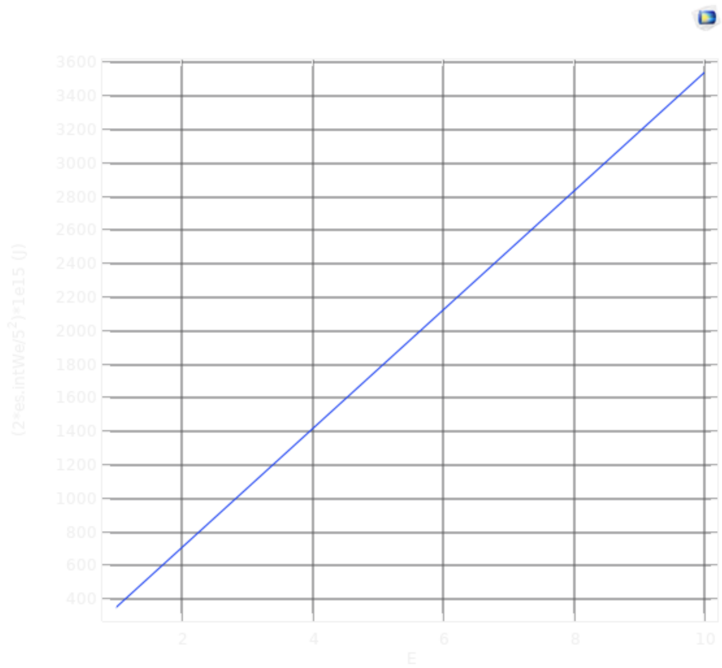

Figure 2: Simulated capacitance (pF) plotted against the relative permittivity (ϵ_r) of the dielectric material, demonstrating the expected linear relationship.

Table 2: Dielectric constants of various materials and the corresponding calculated capacitance for the simulated $1\,\text{mm}^2$ capacitor with $0.1\,\text{mm}$ separation, based on the baseline simulated value $C_1 = 0.0872\,\text{pF}$.

Material	Dielectric Constant (ϵ_r)	Calculated Capacitance (pF)
Vacuum	1	0.0872
Air	1.00054	0.0872
Paper	3.5	0.3052
Teflon	2.1	0.1831
Glass	4.6	0.4011
Water	80	6.9760

Part 2: Capacitive Biosensor for Tumor Detection

Model Setup - Initial Design

A capacitive biosensor was designed and simulated to evaluate its potential for detecting tumors within skin tissue, based on the differing dielectric properties between healthy and tumorous tissues.

- **Geometry:**
  - **Sensor Plates:** Two copper plates, each 10 mm × 10 mm × 2 mm. They were modeled coplanar (side-by-side in the xy-plane) positioned at $z = [12\,\text{mm}, 14\,\text{mm}]$ with a center-to-center separation defining a gap of 0.1 mm between their inner edges.
  - **Skin Tissue Model:** A rectangular block representing skin, with dimensions 20 mm (x) × 10 mm (y) × 10 mm(z), located from $z = 0$ to $z = 10\,\text{mm}$.
  - **Tumor Model (Case 2 only):** A region within the skin block representing a tumor, dimensions 20 mm (x) × 8 mm (y) × 10 mm(z), assumed to occupy the space from $y = 1\,\text{mm}$ to $y = 9\,\text{mm}$ within the skin's extent.
  - **Positioning:** The bottom surface of the sensor plates ($z = 12\,\text{mm}$) was positioned 2 mm above the top surface of the skin model ($z = 10\,\text{mm}$).
  - **Simulation Domain:** An encompassing air box with dimensions 30 mm × 20 mm × 20 mm.
- **Materials:** Sensor Plates: Copper. Healthy Skin: Assigned $\epsilon_r = 34$. Tumor Region: Assigned $\epsilon_r = 50$. Surroundings: Air ($\epsilon_r = 1$).
- **Physics & Boundary Conditions:** Electrostatics interface. One plate defined as Terminal ($V_0 = 5\,\text{V}$), the other as Ground. Outer domain boundaries set to Zero Charge.

Physical Principle of Tumor Impact on Capacitance

The detection principle relies on the contrast in relative permittivity between healthy skin tissue ($\epsilon_r = 34$) and tumorous tissue (modeled with $\epsilon_r = 50$). Tumors often exhibit higher permittivity due to factors like increased water content, cell density, and membrane permeability compared to surrounding healthy tissue. The coplanar sensor generates fringing electric fields that penetrate the underlying skin. When a tumor with higher ϵ_r is present within this field region, it enhances the local electric field storage capability (polarization). This increase in the effective permittivity experienced by the sensor's field lines leads to a measurable increase in the overall capacitance (C). The difference in capacitance, $\Delta C = C_{\text{tumor}} - C_{\text{healthy}}$, serves as the detection signal.

Simulation Results - Comparison of Healthy and Tumorous Skin Cases

Simulations were performed for two scenarios: the sensor interacting with only healthy skin, and the sensor interacting with skin containing the tumor region.

- **Healthy Skin Case:** The simulation yielded a total stored energy $W_{e,\text{healthy}} = 7.65 \times 10^{-11}$ J. The corresponding capacitance is $C_{\text{healthy}} = \frac{2W_{e,\text{healthy}}}{V_0^2} = 6.12\,\text{pF}$.
- **Tumorous Skin Case:** With the tumor ($\epsilon_r = 50$) present, the stored energy increased to $W_{e,\text{tumor}} = 8.26 \times 10^{-11}$ J. The capacitance is $C_{\text{tumor}} = \frac{2W_{e,\text{tumor}}}{V_0^2} = 6.61\,\text{pF}$.

The Sensor

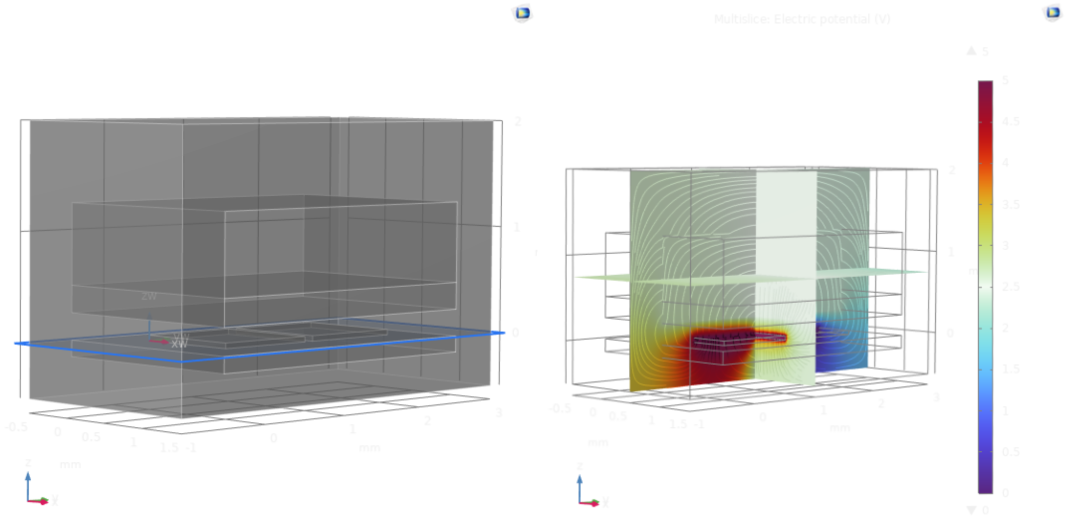

Figure 3: .

Capacitance Difference (ΔC) for Initial Design

The sensitivity of the initial sensor design is quantified by the capacitance difference between the two cases:

$$\Delta C = C_{\text{tumor}} - C_{\text{healthy}} = 6.61\,\text{pF} - 6.12\,\text{pF} = 0.49\,\text{pF}$$

This positive difference confirms the expected increase in capacitance due to the tumor.