Exploring Lagrangian Optimization

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Section 1: The Extreme Value Theorem in $\mathbb{R}^2$

Chapter 1

Hungry Joe

Our story begins with a random guy named "Joseph-Louis." Because his name is kinda long, we'll just refer to him as "Joe." Joe is pretty good at math, but he isn't really that good at making dietary choices. Despite this, Joe wants to optimize the satisfaction he gets from every meal he eats. Joe usually prefers vegetables and light snacks over meats.

Today, Joe is at Carl's Parlor (run by the arbitrarily named "Carl-Friedrich") in search for the maximum satisfaction he can get from the sweetness of icecream. Joe won't be satisfied enough if he has too little or too much icecream. In other words, he desires for a "Goldilocks" amount of sweetness today. If he's only considering sweetness (s) as a factor of his satisfaction, then his satisfaction S can be described as:

$$S(s) = 8e^{-\frac{(s-4)^2}{64}} \tag{1.1}$$

> **Example 1.** If Joe wants at least 1 unit of sweetness and at most 5 units, what is the maximum satisfaction that Joe can attain?

Chapter 2

Utilmaxxing

Theorem 1 († The Extreme Value Theorem in $\mathbb{R}^2$). Suppose that $f(x)$ is continuous on the interval $[a, b]$ then there are two numbers $a \leq c, d \leq b$ so that $f(c)$ is an absolute maximum for the function and $f(d)$ is an absolute minimum for the function.

Section 2: The Extreme Value Theorem in $\mathbb{R}^3$

Chapter 3

Hungrier Joe

Since Joe is a math aficionado, he had already mentally precomputed that he needed 4 units of sweetness in order to achieve his maximum satisfaction of 8 utils. Because of this, Joe was fixated on a far more troubling matter...

Like other icecream parlors, Carl's Parlor serves high-quality vegetable-based chicken strips as an icecream topping. Unfortunately, that is the ONLY topping at Carl's.

Joe ponders the most optimal combination of cotton candy icecream and chicken strips that will provide him with the maximum satisfaction. Joe's satisfaction S can now be represented in terms of sweetness (s) and umami (u) as:

$$S(s,u) = 8e^{-\frac{(s-4)^2+(u-4)^2}{64}} \tag{3.1}$$

> **Example 2.** Joe desires for at least 0 units of either taste and a total sum of tastes that does not exceed 16 units.
>
> What is the maximum satisfaction that Joe can achieve?

Chapter 4

Nerd Face Emoji

Section 3: The Method of Lagrange Multipliers

Chapter 5

Another Order

Delightful! Joe greatly enjoyed the addition of meat- the piquant umami was a new experience for his buds.

Despite already heightening his satisfaction twice, Joe was yet again deciding on another combination of a meat-topped icecream. This time, his only constraint is that he wants the umami flavor to be inversely proportional to half the sweetness felt.

Example 3. Given that his satisfaction can again be represented by eq. (3.1), Joe desires for a nonnegative amount of each flavor and wants to try a combination where the flavor of umami he attains is inversely proportional to half the sweetness.

With what combination of (u, s) can Joe attain his maximum satisfaction?

Chapter 6

A Joe Analysis

Chapter 7

Metonymization, Part 1

Now, for what this project was all about, let's apply Joe's strategy to a generalized example.

Section 4: The Cobb-Douglas Production Function

Chapter 8

Carl's Parlor

After Joe decided to buy everything from Carl's Parlor, Carl-Friedrich decided to not accept anymore customers until the icecream and chicken dispensers were refilled. Also, Carl was the only worker at his icecream shop, so he is interested in hiring more workers.

To financially plan these business plans, Carl viewed a couple YouTube videos on the **Cobb-Douglas production function**.

> **Remark.** The Cobb-Douglas production function is an economics concept that relates a firm's production output Y in terms of two inputs, labor (L) and capital (K).
>
> The production function can be written as:
>
> $$Y(L, K) = AL^{\alpha}K^{1-\alpha}$$
>
> Where..
>
> - A is a constant for ‡*total-factor productivity*,
> - α is the output elasticities of either capital

If there are any constraints on the variables L and K, like a budget, then the method of Lagrange multipliers can be utilized to find the specific quantities of capital and labor that maximizes output $Y(L, K)$ for constraint functions.

In this question, Carl's

Chapter 9

Money-Mouth Face Emoji

Chapter 10

Metonymization, Part 2

Section 5: The End Stuff

Chapter 11

Concluding Remarks

"Sorry if 'metonymization' is not a word." - **Aaron**