# Exploring Lagrangian Optimization


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## Section 1: The Extreme Value Theorem in $\mathbb{R}^2$

# Chapter 1

# Hungry Joe

Our story begins with a random guy named "Joseph-Louis." Because his name is kinda long, we'll just refer to him as "Joe." Joe is pretty good at math, but he isn't really that good at making dietary choices. Joe wants to optimize the satisfaction he gets from every meal he eats.

Today, Joe is at Carl's Parlor in search for the maximum satisfaction he can get from the sweetness of icecream. Joe won't be satisfied enough if he has too little or too much icecream. He desires for his "Goldilocks" amount of sweetness today. If he's only considering sweetness ($s$) as a factor of his satisfaction, then his satisfaction $S$ can be described as:

$$S(s) = 8e^{-\frac{(s-4)^2}{64}} \tag{1.1}$$

> **Example.** If Joe wants at least 1 unit of sweetness and at most 5 units, what is the maximum satisfaction that Joe can attain?

# Chapter 2

# Utilmaxxing

**Theorem 1** († The Extreme Value Theorem in $\mathbb{R}^2$)**.** Suppose that $f(x)$ is continuous on the interval $[a, b]$ then there are two numbers $a \leq c, d \leq b$ so that $f(c)$ is an absolute maximum for the function and $f(d)$ is an absolute minimum for the function.

## Section 2: The Extreme Value Theorem in $\mathbb{R}^3$

# Chapter 3

# Hungrier Joe

Since Joe is a math wizard, he already mentally precomputed that he needed 4 units of sweetness in order to achieve his maximum satisfaction of 8 utils. Because of this, Joe was more fixated on a far more troubling matter...

Like other icecream parlors, Carl's Parlor serves high-quality chicken strips as an icecream topping. Unfortunately, that is the ONLY icecream topping at Carl's.

Joe ponders the most optimal combination of cotton candy icecream and chicken strips that will provide him with the maximum satisfaction. Joe's satisfaction $S$ can now be represented in terms of sweetness ($s$) and umami ($u$) as:

$$S(s,u) = 8e^{-\frac{(s-4)^2+(u-4)^2}{64}} \tag{3.1}$$

> **Example.** Joe desires for: at least 0 units of either taste, and a sum of tastes that does not exceed 16 units. What is the maximum satisfaction that Joe can achieve?

# Chapter 4

# Nerd Emoji

## Section 3: The Method of Lagrange Multipliers

# Chapter 5

# Another Order

Delightful! Joe greatly enjoyed the addition of meat. Despite hightening his satisfaction, he was yet again deciding on another combination of meat-topped icecream. This time, his only constraint was that he wanted the umami flavor to be inversely proportional to half the sweetness.

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# Joe's Math

Chapter 7

# A Brief Generalization

## Section 4: The Cobb-Douglas Production Function

## Chapter 8

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# Another Brief Generalization

## Section 5: The Stuff I Thought I Should Put at the End but Wasn't Sure if It Was Necessary (and Also, I'm Really, Really Sorry if This Sentence Feels a Bit Like a Rambling Octopus Arm Draped Across the Page, I Was Just Feeling a Touch Existential and Wanted to Add One Last Little Footnote, Like a Confetti Cannon Shooting out "Oh Wells" into the Void, So Please Forgive My Overenthusiasm for Epilogue Extravaganzas)

Chapter 11

# Concluding Remarks