Bellwork 9/27

Find $f'(3)$ if $f(x) = x^3$.

Use $f'(a) = \lim_{x \to a} \left[\frac{f(x) - f(a)}{x - a} \right]$

Recall: $z^3 - y^3 = (z - y)(z^2 + zy + y^2)$

: : reset

Bellwork 9/27 - Solution

$$
\begin{aligned}
f'(3) &= \lim_{x \to 3} \left[\frac{f(x) - f(3)}{x - 3} \right] \\
&= \lim_{x \to 3} \left(\frac{x^3 - 3^3}{x - 3} \right) \\
&= \lim_{x \to 3} \left[\frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{x-3}} \right] \\
&= \lim_{x \to 3} \left(x^2 + 3x + 9 \right) = \boxed{27}
\end{aligned}
$$

Exercise 1

$$f(x) = x^2 + x$$

.

1. Find $f'(x)$
2. Where does $f'(x) = 0$?
3. What is the equation of the tangent line at that point?

Exercise 1 - Solutions, Part 1

$$f'(x) = \lim_{h \to 0} \left[\frac{(x+h)^2 + x + h - (x^2 + x)}{h} \right]$$

$$= \lim_{h \to 0} \left(\frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} \right)$$

$$= \lim_{h \to 0} \left(\frac{2xh + h^2 + h}{h} \right) = \lim_{h \to 0} (2x + h + 1)$$

$$\Longrightarrow \boxed{f'(x) = 2x + 1}$$

Exercise 1 - Solutions, Part 2

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$\boxed{x = -\frac{1}{2}} \text{ or } \boxed{\text{at the point } \left(-\frac{1}{2}, -\frac{1}{4}\right)}$$

Exercise 1 - Solutions, Part 3

Equation of a line g:

$g(x) = y = mx + b$ where m is the slope and b is the y-intercept.

$$f'(x) = 0 \implies m = 0$$

$$\implies y = b$$

Since we have that $\left(-\frac{1}{2}, -\frac{1}{4}\right)$ lies on $f(x)$,

$$b = -\frac{1}{4} \implies \boxed{y = -\frac{1}{4}} \text{ or } \boxed{g(x) = -\frac{1}{4}}$$

Exercise 2

Find $f'(x)$ if $f(x) = \sqrt{2x+1}$

Exercise 2 - Solution

$$f'(x) = \lim_{h\to 0}\left[\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{\sqrt{2(x+h)+1}-\sqrt{2x+1}}{h}\cdot\frac{\sqrt{2(x+h)+1}+\sqrt{2x+1}}{\sqrt{2(x+h)+1}+\sqrt{2x+1}}\right]$$

$$= \lim_{h\to 0}\left\{\frac{2x+2h+1-2x-1}{h[\sqrt{2(x+h)+1}+\sqrt{2x+1}]}\right\}$$

$$= \lim_{h\to 0}\left[\frac{2\cancel{h}}{\cancel{h}(\sqrt{2(x+h)+1}+\sqrt{2x+1})}\right] = \boxed{\frac{1}{\sqrt{2x+1}}}$$