Find the values a and b that make $f(x)$ continuous.

$$f(x) = \begin{cases} ax^2 + b \text{ if } x \leq 0 \\ bx^2 - x + a \text{ if } 0 < x \leq 2 \\ x + 3b \text{ if } x > 2 \end{cases}$$

reset

$$a(0) + b = b(0) - 0 + a \qquad 4b - 2 + a = 2 + 3b$$

$$b = a \qquad\qquad 4 = b + a$$

$$\implies 4 = 2b$$

$$b = 2$$

$$a = 2$$

Sketch a function with all of the following properties:

1. Continuous everywhere except at $x = -1$ and $x = 4$.

2. Vertical asymptote at $x = -1$.

3. Removable discontinuity at $x = 4$.

Sketch a function with all of the following properties:

1. Continuous on $x > -1$.

2. Jump discontinuity at $x = -3$.

3. Vertical asymptote at $x = -1$.

4. Always positive when $x < 0$.

Sketch a function with all of the following properties:

1. Neither left nor right continuous at $x = -1$.

2. Vertical asymptote at $x = 1$.

3. Oscillating discontinuity at $x = 2$.

4. Continuous everywhere except at $x = -1$, 1, 2.

Where is $h(x) = \ln[1 + \sin(x)]$ continuous?

$$h(x) = \ln[1 + \sin(x)]$$

In this case, $h(x)$ is continuous if the output of $\ln(...)$ is defined.

$$\implies 1 + \sin(x) > 0$$
$$\sin(x) > -1$$

Since the range of $\sin(x)$ is $[-1, 1]$, we only need to find x where $\sin(x) = -1$ in order to find where $h(x)$ is undefined.

$$\implies x \neq 2\pi n - \frac{\pi}{2} \text{ and } x \neq 2\pi n - \frac{3\pi}{2} \text{ where } n \in \mathbb{Z}$$

Therefore, $h(x)$ is continuous everywhere besides

$$x = 2\pi n - \frac{\pi}{2}, \ 2\pi n - \frac{3\pi}{2} \text{ where } n \in \mathbb{Z}$$

Evaluate:

$$\lim_{x \to 1} \left[\arcsin \left(\frac{1 - \sqrt{x}}{1 - x} \right) \right]$$

Because arcsin is a continuous function,

$$\lim_{x \to 1} \left[\arcsin \left(\frac{1 - \sqrt{x}}{1 - x} \right) \right] = \arcsin \left[\lim_{x \to 1} \left(\frac{1 - \sqrt{x}}{1 - x} \right) \right]$$

$$\arcsin \left[\lim_{x \to 1} \left(\frac{1 - \sqrt{x}}{1 - x} \right) \right] = \arcsin \left\{ \lim_{x \to 1} \left[\frac{1 - \sqrt{x}}{(1 - \sqrt{x})(1 + \sqrt{x})} \right] \right\}$$

$$= \arcsin \left[\lim_{x \to 1} \left(\frac{1}{1 + \sqrt{x}} \right) \right] = \arcsin \left(\frac{1}{2} \right) = \boxed{\frac{\pi}{6}}$$