Bellwork 10/19

$$f(x) = 3\cos(x) + x$$

Find the equation of a line tangent to f at $x = 0$.

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Bellwork 10/19 - Solution

$$f'(x) = -3\sin(x) + 1$$

Point-Slope Form: $y - f(0) = f'(0)(x)$

$$\implies \boxed{y = x + 3}$$

Exercise 1

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = \sin(3^x)$$

Exercise 1 - Solution

$$y = \sin(u) \implies \frac{\mathrm{d}y}{\mathrm{d}u} = \cos(u)$$

$$u = 3^x \implies \frac{\mathrm{d}u}{\mathrm{d}x} = 3^x \ln(3)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(3^x) \cdot 3^x \ln(3)}$$

Exercise 2

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = e^{\csc(x)}$$

Exercise 2 - Solution

$$y = e^u \implies \frac{\mathrm{d}y}{\mathrm{d}u} = e^u$$

$$u = \csc(x) \implies \frac{\mathrm{d}u}{\mathrm{d}x} = -\csc(x)\cot(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = e^{\csc(x)} \cdot [-\csc(x)\cot(x)]}$$

Exercise 3

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = \cot[\sec(x)]$$

Exercise 3 - Solution

$$y = \cot(u) \implies \frac{\mathrm{d}y}{\mathrm{d}u} = -\csc^2(u)$$

$$u = \sec(x) \implies \frac{\mathrm{d}u}{\mathrm{d}x} = \sec(x)\tan(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = -\csc^2[\sec(x)] \cdot \sec(x)\tan(x)}$$

Exercise 4

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = e^{\sin(x^2)}$$

Exercise 4 - Solution

$$y = e^u \implies \frac{\mathrm{d}y}{\mathrm{d}u} = e^u$$

$$u = \sin(v) \implies \frac{\mathrm{d}u}{\mathrm{d}v} = \cos(v)$$

$$v = x^2 \implies \frac{\mathrm{d}v}{\mathrm{d}x} = 2x$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}v} \cdot \frac{\mathrm{d}v}{\mathrm{d}x}$$

$$\implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = e^{\sin(x^2)} \cdot \cos(x^2) \cdot 2x}$$

Exercise 5

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = \tan\left[2^{\cot(x)}\right]$$

Exercise 5 - Solution

$$y = \tan(u) \implies \frac{\mathrm{d}y}{\mathrm{d}u} = \sec^2(u)$$

$$u = 2^v \implies \frac{\mathrm{d}u}{\mathrm{d}v} = 2^v \ln(2)$$

$$v = \cot(x) \implies \frac{\mathrm{d}v}{\mathrm{d}x} = -\csc^2(x)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}v} \cdot \frac{\mathrm{d}v}{\mathrm{d}x}$$

$$\implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = \sec^2(2^{\cot(x)}) \cdot 2^{\cot(x)} \ln(2) \cdot [-\csc^2(x)]}$$