Bellwork 11/1

Suppose $y = \sqrt{e^x}$, where x and y are functions of t.

1. If $\frac{\mathrm{d}y}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 0$.
2. If $\frac{\mathrm{d}x}{\mathrm{d}t} = 1$, find $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = \ln(4)$.

: : reset

Differentiate:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{e^x}{2\sqrt{e^x}}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) \implies \frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\sqrt{e^x}}{2}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

Substitute:

1. $1 = \frac{1}{2}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) \implies \boxed{\frac{\mathrm{d}x}{\mathrm{d}t} = 2}$
2. $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\sqrt{e^{\ln(4)}}}{2} = \frac{\sqrt{4}}{2} \implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}t} = 1}$

Exercise 1

Suppose $6x^2 + 5y^2 = 30$, where x and y are functions of t.

1. If $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{2}$, find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 5$ and $y = 12$.
2. If $\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{4}$, find $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 10$ and $y = 3$.

Exercise 1 - Solution

Implicitly Differentiate:

$$12x\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) + 10y\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{6x}{5y}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

Substitute:

1. $\frac{1}{2} = \left[-\frac{6(5)}{5(12)}\right]\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) \implies \boxed{\frac{\mathrm{d}x}{\mathrm{d}t} = -1}$
2. $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{6(10)}{5(3)}\left(-\frac{1}{4}\right) \implies \boxed{\frac{\mathrm{d}y}{\mathrm{d}t} = 1}$