Bellwork 10/30

$$f(x) = \ln\left(x^2 + 1\right)$$

Find the equation of the line tangent to f at $x = -1$.

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Bellwork 10/30 - Solution

$$f'(x) = \frac{2x}{x^2 + 1}$$

Point-Slope Form: $y - f(-1) = f'(-1)(x + 1)$

$$\implies \boxed{y = -x - 1 + \ln(2)}$$

Exercise 1

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y = \frac{(x^2 - 1)^x}{\sqrt{x+1}}$$

Exercise 1 - Solution, Part 1

Take the natural logarithm of both sides:

$$\ln(y) = \ln\left[\frac{(x^2-1)^x}{\sqrt{x+1}}\right]$$

$$\ln(y) = x\ln(x^2-1) - \frac{1}{2}\ln(x+1)$$

Implicitly differentiate:

$$\frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \ln(x^2-1) + \frac{2x^2}{x^2-1} - \frac{1}{2x+2}$$

Exercise 1 - Solution, Part 2

Solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$ and substitute for y:

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = \left[\frac{(x^2-1)^x}{\sqrt{x+1}}\right]\left[\ln(x^2-1) + \frac{2x^2}{x^2-1} - \frac{1}{2x+2}\right]}$$

Exercise 2

Find $\frac{\mathrm{d}u}{\mathrm{d}v}$:

$$u = \frac{v^{\ln(v)}}{\ln(v)}$$

Exercise 2 - Solution

$$\ln(u) = \ln(v)\ln(v) - \ln[\ln(v)]$$

$$\frac{1}{u}\left(\frac{\mathrm{d}u}{\mathrm{d}v}\right) = \frac{2\ln(v)}{v} - \frac{1}{v\ln(v)}$$

$$\boxed{\frac{\mathrm{d}u}{\mathrm{d}v} = \left[\frac{v^{\ln(v)}}{\ln(v)}\right]\left[\frac{2\ln(v)}{v} - \frac{1}{v\ln(v)}\right]}$$