Bellwork 11/3

Suppose $y = 3^{x \ln(x)}$, where x and y are functions of t.

If $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{\ln(3)}$, find $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 1$.

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Bellwork 11/3 - Solution, Part 1

Logarithmic Differentiation:

$$\ln(y) = x\ln(x)\ln(3)$$

$$\frac{1}{y}\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = \ln(3)[\ln(x)+1]\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = y\ln(3)[\ln(x)+1]\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \ln(3)\left[3^{x\ln(x)}\right][\ln(x)+1]\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

Bellwork 11/3 - Solution, Part 2

Substitute:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \ln(3)\left[3^{(1)\ln(1)}\right][\ln(1)+1]\left[\frac{1}{\ln(3)}\right]$$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}t} = 1}$$

Exercise 1

A particle's motion can be described by the following elliptical equation:

$$x^2 + xy + y^2 = 4$$

When the particle is at $(-2, 0)$, $\frac{\mathrm{d}x}{\mathrm{d}t} = -3$.

Find $\frac{\mathrm{d}y}{\mathrm{d}t}$.

Exercise 1 - Solution

Implicitly Differentiate:

$$2x\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)+y\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)+x\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)+2y\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)=0$$

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{-2x-y}{x+2y}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

Substitute:

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{4}{-2}(-3)$$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}t}=6}$$