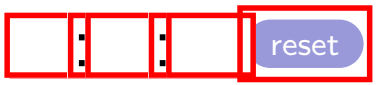
# Bellwork 10/3

A particle moves along the $x$-axis. Its position can be described by the equation:

$$x(t) = t^2 + t$$

Find $a(t)$ or $x''(t)$, the function that describes the acceleration of this particle.

$$\text{Recall: } x'(t) = \lim_{h \to 0} \left[ \frac{x(t+h) - x(t)}{h} \right]$$



# Bellwork 10/3 - Solution

$$v(t) = \lim_{h \to 0} \left[ \frac{(t+h)^2 + (t+h) - (x^2 + x)}{h} \right] = 2t+1$$

$$\implies a(t) = v'(t) = \lim_{h \to 0} \left[ \frac{2(t+h) + 1 - (2t+1)}{h} \right]$$

$$\implies \boxed{a(t) = 2}$$

# Exercise 1

A particle moves along the $x$-axis. Its position can be modeled by:

$$x(t) = t^3 - 3t^2 - 9t - 1$$

1. When is the particle moving to the right?
2. When is the particle moving to the left?

# Exercise 1 - Solutions

First, we find where the particle changes directions:

$$v(t) = 3t^2 - 6t - 9 = 0$$
$$\implies 3(t - 2t - 3) = 0$$
$$\implies t = -1, 3$$

Next, we test $t$ in $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$ to get:

1. The particle is $\boxed{\text{moving to the right when } t < -1 \text{ or } t > 3}$.
2. The particle is $\boxed{\text{moving to the left when } -1 < t < 3}$.

# Exercise 2

From Exercise 1:

$$x(t) = t^3 - 3t^2 - 9t - 1$$

$$v(t) = 3t^2 - 6t - 9$$

The particle is moving to the right when $t < -1$ or $t > 3$.
The particle is moving to the left when $-1 < t < 3$.

When is the particle..

1. Speeding up?
2. Slowing down?

# Exercise 2 - Solutions, Part 1

The particle is speeding up when its velocity and acceleration have the same sign. It is slowing down when the particle's velocity and acceleration have opposite signs.

# Exercise 2 - Solutions, Part 2

We have found:

$$v(t) > 1 \text{ if } t < -1 \text{ or } t > 3, \text{ and } v(t) < 1 \text{ if } -1 < t < 3$$

Now, we must find where $a(t)$ changes signs:

$$a(t) = 6t - 6 = 0$$

$$\implies 6(t-1) = 0 \implies t = 1$$

Then, we test values for $t < 1$ and $t > 1$, so:

$$a(t) > 1 \text{ if } t > 1, \text{ and } a(t) < 1 \text{ if } t < 1$$

The particle $\boxed{\text{is speeding up when } -1 < t < 1 \text{ or } t > 3}$ and $\boxed{\text{slowing down when } t < -1 \text{ or } 1 < t < 3}$.

# Exercise 3

Another particle moves along the $x$-axis. Its position can be described by the following:

$$x(t) = e^t - t$$

When is the particle moving to the right? Left?

# Exercise 3 - Solutions

Find $t$ where $v(t) = 0$:

$$v(t) = e^t - 1 = 0$$
$$\implies e^t = 1$$
$$\implies t = 0$$

Now, we find the sign of $v(t)$ for $t < 0$ and $t > 0$:

$$v(-1) = e^{-1} - 1 < 0$$

$$v(1) = e^1 - 1 > 0$$

$\therefore$ the particle moves $\boxed{\text{to the right when } t > 0}$, and $\boxed{\text{to the left when } t < 0}$.