Bellwork 10/3

A particle moves along the x-axis. Its position can be described by the equation:

$$x(t)=t^2+t$$

Find a(t) or x''(t), the function that describes the acceleration of this particle.

Recall:
$$x'(t) = \lim_{h \to 0} \left[\frac{x(t+h) - x(t)}{h} \right]$$

Bellwork 10/3 - Solution

$$v(t) = \lim_{h \to 0} \left[\frac{(t+h)^2 + (t+h) - (x^2 + x)}{h} \right] = 2t + 1$$

$$\implies a(t) = v'(t) = \lim_{h \to 0} \left[\frac{2(t+h) + 1 - (2t+1)}{h} \right]$$

$$\implies a(t) = 2$$

Exercise 1

A particle moves along the x-axis. Its position can be modeled by:

$$x(t) = t^3 - 3t^2 - 9t - 1$$

- When is the particle moving to the right?
- When is the particle moving to the left?

Exercise 1 - Solutions

First, we find where the particle changes directions:

$$v(t) = 3t^2 - 6t - 9 = 0$$

$$\implies 3(t - 2t - 3) = 0$$

$$\implies t = -1, 3$$

Next, we test t in $(-\infty, -1)$, (-1, 3), and $(3, \infty)$ to get:

- The particle is moving to the right when t < -1 or t > 3.
- ② The particle is moving to the left when -1 < t < 3.

Exercise 2

From Exercise 1:

$$x(t) = t^3 - 3t^2 - 9t - 1$$
$$v(t) = 3t^2 - 6t - 9$$

The particle is moving to the right when t < -1 or t > 3. The particle is moving to the left when -1 < t < 3.

When is the particle..

- Speeding up?
- Slowing down?

Exercise 2 - Solutions, Part 1

The particle is speeding up when its velocity and acceleration have the same sign. It is slowing down when the particle's velocity and acceleration have opposite signs.

Exercise 2 - Solutions, Part 2

We have found:

$$v(t) > 1$$
 if $t < -1$ or $t > 3$, and $v(t) < 1$ if $-1 < t < 3$

Now, we must find where a(t) changes signs:

$$a(t) = 6t - 6 = 0$$

$$\implies 6(t - 1) = 0 \implies t = 1$$

Then, we test values for t < 1 and t > 1, so:

$$a(t) > 1 \text{ if } t > 1, \text{ and } a(t) < 1 \text{ if } t < 1$$

The particle is speeding up when -1 < t < 1 or t > 3 and slowing down when t < -1 or 1 < t < 3.

Exercise 3

Another particle moves along the x-axis. Its position can be described by the following:

$$x(t) = e^t - t$$

When is the particle moving to the right? Left?

Exercise 3 - Solutions

Find t where v(t) = 0:

$$v(t) = e^t - 1 = 0$$
 $\implies e^t = 1$
 $\implies t = 0$

Now, we find the sign of v(t) for t < 0 and t > 0:

$$v(-1) = e^{-1} - 1 < 0$$

 $v(1) = e^{1} - 1 > 0$

... the particle moves to the right when t > 0, and to the left when t < 0.