Bellwork 9/18

Let g and h be the functions defined by:

$$g(x) = \sin\left[\frac{\pi}{2}(x+2)\right] + 3;\ h(x) = -\frac{1}{4}x^3 - \frac{3}{2}x^2 - \frac{9}{4}x + 3$$

If f is a function that satisfies

$$g(x) \leq f(x) \leq h(x) \text{ for } -2 < x < 0,$$

what is $\lim_{x \to -1} f(x)$?

1. 3
2. 3.5
3. 4
4. The limit cannot be determined from the information given.

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Bellwork 9/18 - Solution

$$g(x) \leq f(x) \leq h(x)$$

$$\lim_{x \to -1} g(x) \leq \lim_{x \to -1} f(x) \leq \lim_{x \to -1} h(x)$$

$$4 = \lim_{x \to -1} f(x) = 4$$

$$\implies \boxed{\lim_{x \to -1} f(x) = 4}$$

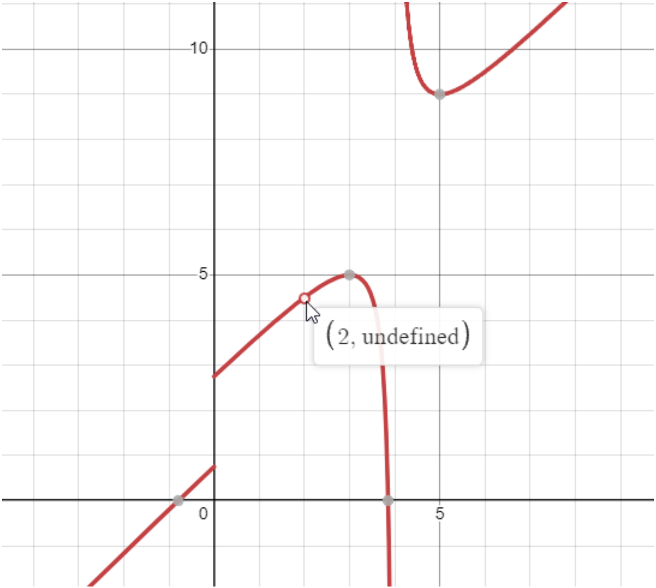
Exercise 1

$$f(x) = \frac{|x|}{x} + \frac{x^2 - 4}{x - 2} + \frac{1}{x - 4}$$

What type of discontinuity exists at,

1. $x = 0$
2. $x = 2$
3. $x = 4$

Exercise 1 - Solutions



Exercise 2

$$\text{Let } f(x) = \begin{cases} ax+1 & \text{if } x < 2 \\ ax^2 & \text{if } x \geq 2 \end{cases}$$

For what value a is $f(x)$ continuous?

Exercise 2 - Solutions

$$f(x) = \begin{cases} ax + 1 & \text{if } x < 2 \\ ax^2 & \text{if } x \geq 2 \end{cases}$$

For $f(x)$ to be continuous,

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x) = f(2)$$

$$\implies a(2) + 1 = a(2^2)$$

$$\implies 1 = 2a$$

$$\implies \boxed{a = \frac{1}{2}}$$

Exercise 3

$$\text{Let } g(x) = \begin{cases} be^{\frac{2x}{\pi}} - 2 & \text{if } x < \frac{\pi}{2} \\ b\sin(x) & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

For what value b is $g(x)$ continuous?

Exercise 3 - Solutions

$$g(x) = \begin{cases} be^{\frac{2x}{\pi}} - 2 & \text{if } x < \frac{\pi}{2} \\ b\sin(x) & \text{if } x \geq \frac{\pi}{2} \end{cases}$$

For $g(x)$ to be continuous,

$$\lim_{x \to \frac{\pi}{2}^-} g(x) = \lim_{x \to \frac{\pi}{2}^+} g(x) = g\left(\frac{\pi}{2}\right)$$

$$\implies be^{\frac{2}{\pi} \cdot \frac{\pi}{2}} - 2 = b\sin\left(\frac{\pi}{2}\right)$$

$$\implies be^1 - 2 = b$$

$$\implies b(e-1) = 2$$

$$\implies \boxed{b = \frac{2}{e-1}}$$