

Find the general indefinite integral:

$$\int (2 + \tan^2 \theta) d\theta$$

Hint: $1 + \tan^2 \theta = \sec^2 \theta$

reset

Bellwork 1/12 - Solution

$$\begin{aligned}\int (2 + \tan^2 \theta) d\theta &= \int (1 + \sec^2 \theta) d\theta \\ &= \int d\theta + \int \sec^2 \theta d\theta \\ &= \boxed{\theta + \tan \theta + C}\end{aligned}$$

Exercise 1

The velocity function (in $\frac{\text{m}}{\text{s}}$) is given for a particle moving along a line.

$$v(t) = 6t - 5; 0 \leq t \leq 5$$

For the given t interval, find the particle's:

- 1 Displacement
- 2 Distance Traveled

Exercise 1 - Solution: Displacement

$$\int_0^5 v(t) dt = s(5) - s(0) = \Delta s = \text{displacement}$$

$$\begin{aligned}\implies \Delta s &= \int_0^5 6t - 5 dt \\ &= 6 \int_0^5 t dt - 5 \int_0^5 dt \\ &= 3[t^2]_0^5 - 5[t]_0^5 \\ &= \boxed{50 \text{ meters}}\end{aligned}$$

Exercise 1 - Solution: Distance Traveled

Find where $v(t)$ changes sign:

$$\begin{aligned}v(t) &= 6t - 5 = 0 \\ \implies t &= \frac{5}{6}\end{aligned}$$

Create the integral:

$$\begin{aligned}\int_0^5 |v(t)| dt &= \left| \int_0^{\frac{5}{6}} 6t - 5 dt \right| + \left| \int_{\frac{5}{6}}^5 6t - 5 dt \right| \\ &= \left| [3t^2 - 5t]_0^{\frac{5}{6}} \right| + \left| [3t^2 - 5t]_{\frac{5}{6}}^5 \right| \\ &= \boxed{\frac{325}{6} = 54.1\bar{6} \text{ meters}}\end{aligned}$$

Exercise 2

The velocity function (in $\frac{\text{m}}{\text{s}}$) is given for a particle moving along a line.

$$v(t) = 6t^2 + 2t - 4; 0 \leq t \leq 4$$

For the given t interval, find the particle's:

- 1 Displacement
- 2 Distance Traveled

Exercise 2 - Solution: Displacement

$$\int_0^4 v(t) dt = s(4) - s(0) = \Delta s = \text{displacement}$$

$$\implies \Delta s = \int_0^4 6t^2 + 2t - 4 dt$$

$$= 6 \int_0^4 t^2 dt + 2 \int_0^4 t dt - 4 \int_0^4 dt$$

$$= 2[t^3]_0^4 + [t^2]_0^4 - 4[t]_0^4$$

$$= \boxed{128 \text{ meters}}$$

Exercise 2 - Solution: Distance Traveled

Find where $v(t)$ changes sign:

$$v(t) = 6t^2 + 2t - 4 = 0$$

$$\implies t = \cancel{1}, \frac{2}{3} \quad (t \in [0, 4])$$

Create the integral:

$$\begin{aligned} \int_0^5 |v(t)| dt &= \left| \int_0^{\frac{2}{3}} 6t^2 + 2t - 4 dt \right| + \left| \int_{\frac{2}{3}}^4 6t^2 + 2t - 4 dt \right| \\ &= \left| [2t^3 + t^2 - 4t]_0^{\frac{2}{3}} \right| + \left| [2t^3 + t^2 - 4t]_{\frac{2}{3}}^4 \right| \\ &= \boxed{\frac{3544}{27} \approx 131.259 \text{ meters}} \end{aligned}$$