Bellwork 9/26

Find the limits:

$$\lim_{x\to\infty} \frac{\sqrt{1+4x^6}}{2-x^3} \text{ and } \lim_{x\to-\infty} \frac{\sqrt{1+4x^6}}{2-x^3}$$

: : reset

Bellwork 9/26 - Solution, Part 1

For $x \to \infty$, we use $x^3 = +\sqrt{x^6}$:

$$\lim_{x\to\infty}\left(\frac{\sqrt{1+4x^6}}{2-\sqrt{x^6}}\right) = \lim_{x\to\infty}\left(\frac{\frac{\sqrt{1+4x^6}}{x^3}}{\frac{2}{x^3}-1}\right) = \lim_{x\to\infty}\left(\frac{\sqrt{\frac{1}{x^6}+\frac{4\cancel{x^6}}{\cancel{x^6}}}}{\frac{2}{x^3}-1}\right)$$

$$= \boxed{-2}$$

Bellwork 9/26 - Solution, Part 2

For $x \to -\infty$, we use $x^3 = -\sqrt{x^6}$:

$$\lim_{x\to\infty}\left(\frac{\sqrt{1+4x^6}}{2+\sqrt{x^6}}\right) = \lim_{x\to\infty}\left(\frac{\frac{\sqrt{1+4x^6}}{x^3}}{\frac{2}{x^3}+1}\right) = \lim_{x\to\infty}\left(\frac{\sqrt{\frac{1}{x^6}+\frac{4\cancel{x^6}}{\cancel{x^6}}}}{\frac{2}{x^3}+1}\right)$$

$$= \boxed{2}$$

Exercise 1

$$f(x) = \frac{1}{x+1}$$

$$f'(2) = ?$$

Exercise 1 - Solution

$$f'(2) = \lim_{x \to 2} \left[\frac{f(x) - f(2)}{x - 2} \right]$$

$$\implies f'(2) = \lim_{x \to 2} \left(\frac{\frac{1}{x+1} - \frac{1}{2+1}}{x - 2} \right) = \lim_{x \to 2} \left[\frac{\frac{3-(x+1)}{3(x+1)}}{x - 2} \right]$$

$$= \lim_{x \to 2} \left[\frac{-\cancel{(x-2)}}{3(x+1)\cancel{(x-2)}} \right] = \lim_{x \to 2} \left[\frac{-1}{3(x+1)} \right] = \boxed{-\frac{1}{9}}$$

Exercise 2

$$f(x) = \sqrt{2x+1}$$

$$f'(4) = ?$$

Exercise 2 - Solution

$$f'(4) = \lim_{x\to 4}\left[\frac{f(x)-f(4)}{x-4}\right]$$

$$\implies f'(4) = \lim_{x\to 4}\left(\frac{\sqrt{2x+1}-\sqrt{2\cdot 4+1}}{x-4}\right)$$

$$= \lim_{x\to 4}\left(\frac{\sqrt{2x+1}-3}{x-4}\right) = \lim_{x\to 4}\left[\frac{(\sqrt{2x+1}-3)}{(x-4)}\cdot\frac{(\sqrt{2x+1}+3)}{(\sqrt{2x+1}+3)}\right]$$

$$= \lim_{x\to 4}\left[\frac{2x+1-9}{(x-4)\sqrt{2x+1}+3(x-4)}\right] = \lim_{x\to 4}\left[\frac{2\cancel{(x-4)}}{\cancel{(x-4)}\sqrt{2x+1}+3\cancel{(x-4)}}\right]$$

$$= \lim_{x\to 4}\left[\frac{2}{\sqrt{2x+1}+3}\right] = \lim_{x\to 4}\left[\frac{2}{\sqrt{2x+1}+3}\right] = \boxed{\frac{1}{3}}$$