Bellwork 9/19

Find the values a and b that make $f(x)$ continuous.

$$f(x) = \begin{cases} ax^2 + b \text{ if } x \leq 0 \\ bx^2 - x + a \text{ if } 0 < x \leq 2 \\ x + 3b \text{ if } x > 2 \end{cases}$$

: : reset

Bellwork 9/19 - Solution

$$a(0) + b = b(0) - 0 + a \qquad 4b - 2 + a = 2 + 3b$$

$$b = a \qquad 4 = b + a$$

$$\implies 4 = 2b$$

$$b = 2$$

$$a = 2$$

Exercise 1

Sketch a function with all of the following properties:

1. Continuous everywhere except at $x = -1$ and $x = 4$.
2. Vertical asymptote at $x = -1$.
3. Removable discontinuity at $x = 4$.

Exercise 2

Sketch a function with all of the following properties:

1. Continuous on $x > -1$.
2. Jump discontinuity at $x = -3$.
3. Vertical asymptote at $x = -1$.
4. Always positive when $x < 0$.

Exercise 3

Sketch a function with all of the following properties:

1. Neither left nor right continuous at $x = -1$.
2. Vertical asymptote at $x = 1$.
3. Oscillating discontinuity at $x = 2$.
4. Continuous everywhere except at $x = -1, 1, 2$.

Exercise 4

Where is $h(x) = \ln[1 + \sin(x)]$ continuous?

Exercise 4 - Solution

$$h(x) = \ln[1 + \sin(x)]$$

In this case, $h(x)$ is continuous if the output of ln(...) is defined.

$$\begin{aligned} \implies 1 + \sin(x) &> 0 \\ \sin(x) &> -1 \end{aligned}$$

Since the range of $\sin(x)$ is $[-1, 1]$, we only need to find x where $\sin(x) \neq -1$ in order to find where $h(x)$ is undefined.

$$\implies x \neq 2\pi n - \frac{\pi}{2} \text{ and } x \neq 2\pi n - \frac{3\pi}{2} \text{ where } n \in \mathbb{Z}$$

Therefore, $h(x)$ is continuous everywhere besides

$$x = 2\pi n - \frac{\pi}{2},\ 2\pi n - \frac{3\pi}{2} \text{ where } n \in \mathbb{Z}$$

Exercise 5

Evaluate:

$$\lim_{x \to 1} \left[\arcsin \left(\frac{1 - \sqrt{x}}{1 - x} \right) \right]$$

Exercise 5 - Solution

Because arcsin is a continuous function,

$$\lim_{x\to 1}\left[\arcsin\left(\frac{1-\sqrt{x}}{1-x}\right)\right] = \arcsin\left[\lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)\right]$$

$$\arcsin\left[\lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)\right] = \arcsin\left\{\lim_{x\to 1}\left[\frac{\cancel{1-\sqrt{x}}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})}\right]\right\}$$

$$= \arcsin\left[\lim_{x\to 1}\left(\frac{1}{1+\sqrt{x}}\right)\right] = \arcsin\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$$