Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ of the implicitly defined curve:

$$x^3 + xy^2 + y = x^2 y$$

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Bellwork 10/25 - Solution

$$3x^2 + y^2 + 2xy\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) + \frac{\mathrm{d}y}{\mathrm{d}x} = 2xy + x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)$$

$$(2xy + 1 - x^2)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 2xy - 3x^2 - y^2$$

$$\boxed{\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2xy - 3x^2 - y^2}{2xy + 1 - x^2}}$$

Exercise 1

Find the equations of the lines tangent to the ellipse below, at $x = -2$:

$$\frac{x^2}{8} + \frac{y^2}{4} = 1$$

Exercise 1 - Solution, Part 1

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$\frac{x}{4} + \frac{y}{2}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x}{2y}$$

Find the y_0 values (when $x = -2$):

$$y_0 = \pm\sqrt{4\left[1 - \frac{(-2)^2}{8}\right]} = \pm\sqrt{2}$$

Exercise 1 - Solution, Part 2

$$\text{Point-Slope Form: } y \pm \sqrt{2} = \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{-2} (x+2)$$

$$\Longrightarrow \boxed{y = \frac{1}{\sqrt{2}}(x+2) + \sqrt{2}}; \boxed{y = -\frac{1}{\sqrt{2}}(x+2) - \sqrt{2}}$$

Exercise 2

Find the equation of the line tangent to the hyperbola below, at $(2, 2)$:

$$\frac{y^2}{2} - \frac{x^2}{4} = 1$$

Exercise 2 - Solution

Find $\frac{\mathrm{d}y}{\mathrm{d}x}$:

$$y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) - \frac{x}{2} = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x}{2y}$$

$$\text{Point-Slope Form: } y - 2 = \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_2 (x-2)$$

$$\Longrightarrow \boxed{y = \frac{1}{2}(x-2) + 2}$$