Bellwork 9/22

$$\text{Let } f(x) = \begin{cases} 2x - 3 & \text{if } x < 1 \\ \cos(\pi x) & \text{if } x \geq 1 \end{cases}$$

Does the Intermediate Value Theorem guarantee a solution to $f(x) = -0.25$ in the interval $(0, 2)$?

Why or why not?

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Bellwork 9/22 - Solution, Part 1

$\boxed{\text{Yes!..}}$

In order to use the IVT here, f must be continuous on $[0, 2]$.

Since $2x - 3$ and $\cos(\pi x)$ are continuous for all x, we just need to test for the continuity of f at $x = 1$:

$$\lim_{x \to 1^-} f(x) \stackrel{?}{=} \lim_{x \to 1^+} f(x) \stackrel{?}{=} f(1)$$

$$2(1) - 3 = \cos[\pi(1)] \implies -1 = -1$$

$$\therefore f \text{ is continuous at } x = 1$$

Bellwork 9/22 - Solution, Part 2

Since f is continuous on $[0, 2]$, we now check if -0.25 is between $f(0)$ and $f(2)$:

$$f(0) = -3 \text{ and } f(2) = 1$$

Because $f(0) < -0.25 < f(2)$, the IVT guarantees a value x in $(0, 2)$ such that $f(x) = -0.25$.

Exercise 1

Find the limits:

1. $$\lim_{x\to\infty}\left(\frac{3x^3-7}{2x^3-x+1}\right)$$

2. $$\lim_{x\to\infty}\left(\frac{3x^2-7}{2x^3-x+1}\right)$$

Exercise 1 - Solutions

1. $$\lim_{x\to\infty}\left(\frac{3x^3-7}{2x^3-x+1}\right)=\lim_{x\to\infty}\left(\frac{\frac{3\cancel{x^3}}{\cancel{x^3}}-\frac{7}{x^3}}{\frac{2\cancel{x^3}}{\cancel{x^3}}-\frac{x}{x^3}+\frac{1}{x^3}}\right)$$
$$=\lim_{x\to\infty}\left(\frac{3-\frac{7}{x^3}}{2-\frac{1}{x^2}+\frac{1}{x^3}}\right)=\frac{3-0}{2-0+0}=\boxed{\frac{3}{2}}$$

2. $$\lim_{x\to\infty}\left(\frac{3x^2-7}{2x^3-x+1}\right)=\lim_{x\to\infty}\left(\frac{\frac{3x^2}{x^3}-\frac{7}{x^3}}{\frac{2\cancel{x^3}}{\cancel{x^3}}-\frac{x}{x^3}+\frac{1}{x^3}}\right)$$
$$=\lim_{x\to\infty}\left(\frac{\frac{3}{x}-\frac{7}{x^3}}{2-\frac{1}{x^2}+\frac{1}{x^3}}\right)=\frac{0-0}{2-0+0}=\boxed{0}$$

Exercise 2

Find the limit:

$$\lim_{x \to -\infty} \left[\frac{1 + e^x \sin(x)}{e^{x-1} - 1} \right]$$

Exercise 2 - Solution

$$\lim_{x\to-\infty}\left[\frac{1+e^{x}\sin(x)}{e^{x-1}-1}\right]=\frac{1+0}{0-1}$$

$$=\frac{1}{-1}=\boxed{-1}$$

Exercise 3

Find the limit:

$$\lim_{x\to\infty}\left(\frac{3^x+2}{e^{2x}+1}\right)$$

Exercise 3 - Solution

$$\lim_{x\to\infty}\left(\frac{3^x+2}{e^{2x}+1}\right)=\lim_{x\to\infty}\left(\frac{\frac{3^x}{e^{2x}}+\frac{2}{e^{2x}}}{\frac{\cancel{e^{2x}}}{\cancel{e^{2x}}}+\frac{1}{e^{2x}}}\right)$$

$$=\lim_{x\to\infty}\left[\frac{\left(\frac{3}{e^2}\right)^x+\frac{2}{e^{2x}}}{1+\frac{1}{e^{2x}}}\right]=\frac{0+0}{1+0}=\boxed{0}$$