Bellwork 11/6

A particle's motion can be described by the following elliptical equation:

$$2x^2 + xy + 2y^2 = 8$$

When the particle is at $(-1, 0)$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 4$.

Find $\frac{\mathrm{d}y}{\mathrm{d}t}$.

: : reset

Bellwork 11/6 - Solution

Implicitly Differentiate:

$$4x\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) + y\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right) + x\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) + 4y\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right) = 0$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{-4x - y}{x + 4y}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)$$

Substitute:

$$\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{-1} = \left(\frac{4}{-1}\right)(4)$$

$$\boxed{\left.\frac{\mathrm{d}x}{\mathrm{d}t}\right|_{-1} = -16}$$

Exercise 1

Find the linear approximation of the function $f(x) = \sqrt{1+x}$ at $x = 3$ and use it to approximate the numbers $\sqrt{3}$ and $\sqrt{5}$.

Exercise 1 - Solution, Part 1

$$f(x) = \sqrt{1+x} \implies \sqrt{3} = \sqrt{1+2} \text{ and } \sqrt{5} = \sqrt{1+4}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}$$

Use the equation of the line tangent to f at $x = 3$:

$$g(x) = f'(3)(x-3) + f(3)$$

$$g(x) = \frac{1}{4}(x-3) + 2$$

Exercise 1 - Solution, Part 2

Since $f(x) \approx g(x)$, we find $g(2)$ and $g(4)$:

$$f(2) \approx g(2) \implies f(2) \approx \frac{1}{4}(-1)+2 \implies \boxed{f(2) \approx 1.75}$$

$$f(4) \approx g(4) \implies f(4) \approx \frac{1}{4}(1)+2 \implies \boxed{f(4) \approx 2.25}$$

Exercise 2

Estimate $e^{0.75}$ using linear approximation.

Exercise 2 - Solution

$$f(x) = e^{1-x^2} \implies e^{0.75} = e^{1-0.5^2} = f(0.5)$$

$$f'(x) = -2e^{1-x^2}x$$

Use the equation of the line tangent to f at $x = 1$:

$$g(x) = f'(1)(x-1) + f(1)$$

$$g(x) = -2(x-1) + 1$$

Since $f(0.5) \approx g(0.5)$:

$$f(0.5) \approx -2(0.5-1) + 1$$

$$\boxed{f(0.5) \approx 2}$$