

Bellwork 9/27

Find $f'(3)$ if $f(x) = x^3$.

Use $f'(a) = \lim_{x \rightarrow a} \left[\frac{f(x) - f(a)}{x - a} \right]$

Recall: $z^3 - y^3 = (z - y)(z^2 + zy + y^2)$

reset

Bellwork 9/27 - Solution

$$\begin{aligned}f'(3) &= \lim_{x \rightarrow 3} \left[\frac{f(x) - f(3)}{x - 3} \right] \\&= \lim_{x \rightarrow 3} \left(\frac{x^3 - 3^3}{x - 3} \right) \\&= \lim_{x \rightarrow 3} \left[\frac{\cancel{(x-3)}(x^2 + 3x + 9)}{\cancel{x-3}} \right] \\&= \lim_{x \rightarrow 3} (x^2 + 3x + 9) = \boxed{27}\end{aligned}$$

Exercise 1

$$f(x) = x^2 + x$$

- 1 Find $f'(x)$
- 2 Where does $f'(x) = 0$?
- 3 What is the equation of the tangent line at that point?

Exercise 1 - Solutions, Part 1

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 + x + h - (x^2 + x)}{h} \right] \\&= \lim_{h \rightarrow 0} \left(\frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h} \right) \\&= \lim_{h \rightarrow 0} \left(\frac{2xh + h^2 + h}{h} \right) = \lim_{h \rightarrow 0} (2x + h + 1) \\&\implies \boxed{f'(x) = 2x + 1}\end{aligned}$$

Exercise 1 - Solutions, Part 2

$$f'(x) = 0 \implies 2x + 1 = 0$$

$$x = -\frac{1}{2} \quad \text{or} \quad \text{at the point } \left(-\frac{1}{2}, -\frac{1}{4}\right)$$

Exercise 1 - Solutions, Part 3

Equation of a line g :

$g(x) = y = mx + b$ where m is the slope and b is the y-intercept.

$$f'(x) = 0 \implies m = 0$$

$$\implies y = b$$

Since we have that $(-\frac{1}{2}, -\frac{1}{4})$ lies on $f(x)$,

$$b = -\frac{1}{4} \implies \boxed{y = -\frac{1}{4}} \text{ or } \boxed{g(x) = -\frac{1}{4}}$$

Exercise 2

Find $f'(x)$ if $f(x) = \sqrt{2x + 1}$

Exercise 2 - Solution

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \left[\frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \right] \\&= \lim_{h \rightarrow 0} \left[\frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{\sqrt{2(x+h)+1} + \sqrt{2x+1}}{\sqrt{2(x+h)+1} + \sqrt{2x+1}} \right] \\&= \lim_{h \rightarrow 0} \left\{ \frac{2x + 2h + 1 - 2x - 1}{h[\sqrt{2(x+h)+1} + \sqrt{2x+1}]} \right\} \\&= \lim_{h \rightarrow 0} \left[\frac{2\cancel{h}}{\cancel{h}(\sqrt{2(x+h)+1} + \sqrt{2x+1})} \right] = \boxed{\frac{1}{\sqrt{2x+1}}}\end{aligned}$$