Bellwork 1/12

Find the general indefinite integral:

$$\int (2 + \tan^2\theta) d\theta$$

Hint: $1 + \tan^2\theta = \sec^2\theta$

: : reset

Bellwork 1/12 - Solution

$$\begin{aligned}\int (2 + \tan^2\theta)d\theta &= \int (1 + \sec^2\theta)d\theta \\ &= \int d\theta + \int \sec^2\theta\, d\theta \\ &= \boxed{\theta + \tan\theta + C}\end{aligned}$$

Exercise 1

The velocity function $\left(\text{in } \frac{\text{m}}{\text{s}}\right)$ is given for a particle moving along a line.

$$v(t) = 6t - 5;\ 0 \leq t \leq 5$$

For the given t interval, find the particle's:

1. Displacement
2. Distance Traveled

Exercise 1 - Solution: Displacement

$$\int_0^5 v(t)dt = s(5) - s(0) = \Delta s = \text{displacement}$$

$$\begin{aligned}
\implies \Delta s &= \int_0^5 6t - 5dt \\
&= 6\int_0^5 tdt - 5\int_0^5 dt \\
&= 3[t^2]_0^5 - 5[t]_0^5 \\
&= \boxed{50 \text{ meters}}
\end{aligned}$$

Exercise 1 - Solution: Distance Traveled

Find where $v(t)$ changes sign:

$$v(t) = 6t - 5 = 0$$
$$\implies t = \frac{5}{6}$$

Create the integral:

$$\int_0^5 |v(t)|dt = \left|\int_0^{\frac{5}{6}} 6t - 5dt\right| + \left|\int_{\frac{5}{6}}^5 6t - 5dt\right|$$
$$= \left|[3t^2 - 5t]_0^{\frac{5}{6}}\right| + \left|[3t^2 - 5t]_{\frac{5}{6}}^5\right|$$
$$= \boxed{\frac{325}{6} = 54.1\overline{6} \text{ meters}}$$

Exercise 2

The velocity function $\left(\text{in } \frac{\text{m}}{\text{s}}\right)$ is given for a particle moving along a line.

$$v(t) = 6t^2 + 2t - 4;\ 0 \leq t \leq 4$$

For the given t interval, find the particle's:

1. Displacement
2. Distance Traveled

Exercise 2 - Solution: Displacement

$$\int_0^4 v(t)dt = s(4) - s(0) = \Delta s = \text{displacement}$$

$$\begin{aligned}
\implies \Delta s &= \int_0^4 6t^2 + 2t - 4dt \\
&= 6\int_0^4 t^2 dt + 2\int_0^4 tdt - 4\int_0^4 dt \\
&= 2[t^3]_0^4 + [t^2]_0^4 - 4[t]_0^4 \\
&= \boxed{128 \text{ meters}}
\end{aligned}$$

Exercise 2 - Solution: Distance Traveled, Part 1

Find where $v(t)$ changes sign:

$$v(t) = 6t^2 + 2t - 4 = 0$$
$$\implies t = \cancel{-1}, \frac{2}{3} \ (t \in [0, 4])$$

Create the integral:

$$\int_0^5 |v(t)| dt = \left| \int_0^{\frac{2}{3}} 6t^2 + 2t - 4 dt \right| + \left| \int_{\frac{2}{3}}^4 6t^2 + 2t - 4 dt \right|$$
$$= \left| [2t^3 + t^2 - 4t]_0^{\frac{2}{3}} \right| + \left| [2t^3 + t^2 - 4t]_{\frac{2}{3}}^4 \right|$$
$$= \boxed{\frac{3544}{27} \approx 131.259 \text{ meters}}$$