Calculus I

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Contents

Lecture 1: Introduction

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- First part: fundamental groups. We follow Munkres:
  - Chap 9 'Fundamental group'
  - Chap 11 'The Seifert-Van Kampen thm' and
  - Chap 12/13 'Classification covering spaces'
- Second part: Homology groups, via cudi, chapter of a book.
- 10 problems (solve them during the semester) No feedback during the semester, but asking questions is allowed. Working together is allowed.
- Exam (completely open book) First 4 questions: 1h30 prep. Last one 30 min, no prep
  1. Theoretical question (open book, explain the proof, . . .) /4
  2. New problem (comparable to one of the 10 problems) /4
  3. Explain your solution n-th problem solved at home /4
  4. Explain your solution m-th problem solved at home /4
  5. 4 small questions /4

  After the exam, hand in your solutions of the other problems. After a quick look, the points of 3. and 4. can be ± 1 (in extreme cases ± 2)
- No exercise classes for this course!

Chapter 1

Introduction

1.1 What is algebraic topology?

Functor from category of topological spaces to the category of groups.

- Category: set of spaces and morphisms.
- Functor: $X \rightsquigarrow G_X$ and $f : X \to Y \rightsquigarrow f_* : G_X \to G_Y$ such that
  - $(f \circ g)_* = f_* \circ g_*$
  - $(1_X)_* = 1_{G_X}$

Two systems we'll discuss:

- fundamental groups
- homology groups

Example. Suppose we have a functor. If $G_X \ncong G_Y$, then X and Y are not homeomorphic. If 'shadows' are different, then objects themselves are different too.

Proof. Suppose X and Y are homeomorphic. Then $\exists f : X \to Y$ and $g : Y \to X$, maps (maps are always continuous in this course), such that $g \circ f = 1_X$ and $f \circ g = 1_Y$. Then $f_* : G_X \to G_Y$ and $g_* : G_Y \to G_X$ such that $(g \circ f)_* = (1_X)_*$ and $(f \circ g)_* = (1_Y)_*$. Using the rules discussed previously, we get

$$g_* \circ f_* = 1_{G_X} \quad f_* \circ g_* = 1_{G_Y},$$

which means that $f_* : G_X \to G_Y$ is an isomorphism.