Rules List

You'll get this rules list on the exam along with the probability and combinatorics formulas on the next page. You may also use the book, what information is generally accessible on the internet, and the lectures. You may not ask anyone about, share information about, or receive information from anyone regarding the exam questions.

$$\begin{array}{r|ll}
m & \mathscr{A} & \\
n & \mathscr{B} & \\
 & \mathscr{A} \wedge \mathscr{B} & \wedge\text{I } m, n
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} \wedge \mathscr{B} & \\
 & \mathscr{A} & \wedge\text{E } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} \wedge \mathscr{B} & \\
 & \mathscr{B} & \wedge\text{E } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} & \\
 & \mathscr{A} \vee \mathscr{B} & \vee\text{I } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} & \\
 & \mathscr{B} \vee \mathscr{A} & \vee\text{I } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} \vee \mathscr{B} & \\
i & \quad \vert\ \underline{\mathscr{A}} & \\
j & \quad \vert\ \mathscr{C} & \\
k & \quad \vert\ \underline{\mathscr{B}} & \\
l & \quad \vert\ \mathscr{C} & \\
 & \mathscr{C} & \vee\text{E } m, i\text{–}j, k\text{–}l
\end{array}$$

$$\begin{array}{r|ll}
i & \quad \vert\ \underline{\neg\mathscr{A}} & \\
j & \quad \vert\ \bot & \\
 & \mathscr{A} & \text{IP } i\text{–}j
\end{array}$$

$$\begin{array}{r|ll}
i & \quad \vert\ \underline{\mathscr{A}} & \\
j & \quad \vert\ \mathscr{B} & \\
 & \mathscr{A} \rightarrow \mathscr{B} & \rightarrow\text{I } i\text{–}j
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A} \rightarrow \mathscr{B} & \\
n & \mathscr{A} & \\
 & \mathscr{B} & \rightarrow\text{E } m, n
\end{array}$$

$$\begin{array}{r|ll}
i & \quad \vert\ \underline{\mathscr{A}} & \\
j & \quad \vert\ \bot & \\
 & \neg\mathscr{A} & \neg\text{I } i\text{–}j
\end{array}$$

$$\begin{array}{r|ll}
m & \neg\mathscr{A} & \\
n & \mathscr{A} & \\
 & \bot & \neg\text{E } m, n
\end{array}$$

$$\begin{array}{r|ll}
m & \bot & \\
 & \mathscr{A} & \text{X } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A}(\ldots c \ldots c \ldots) & \\
 & \forall x \mathscr{A}(\ldots x \ldots x \ldots) & \forall\text{I } m
\end{array}$$

$$\begin{array}{r|ll}
m & \forall x \mathscr{A}(\ldots x \ldots x \ldots) & \\
 & \mathscr{A}(\ldots c \ldots c \ldots) & \forall\text{E } m
\end{array}$$

$$\begin{array}{r|ll}
m & \mathscr{A}(\ldots c \ldots c \ldots) & \\
 & \exists x \mathscr{A}(\ldots x \ldots c \ldots) & \exists\text{I } m
\end{array}$$

$$\begin{array}{r|ll}
m & \exists x \mathscr{A}(\ldots x \ldots x \ldots) & \\
i & \quad \vert\ \underline{\mathscr{A}(\ldots c \ldots c \ldots)} & \\
j & \quad \vert\ \mathscr{B} & \\
 & \mathscr{B} & \exists\text{E } m, i\text{–}j
\end{array}$$

$$\begin{array}{r|ll}
 & c = c & =\text{I}
\end{array}$$

$$\begin{array}{r|ll}
m & a = b & \\
n & \mathscr{A}(\ldots a \ldots a \ldots) & \\
 & \mathscr{A}(\ldots b \ldots a \ldots) & =\text{E } m, n
\end{array}$$

Probability and Combinatorics Formulas

Inclusion–exclusion principle:

$$|A \cup B| = |A| + |B| - |A \cap B|$$

I.E. the number things that are As or Bs is the same as the number of As plus the number of Bs minus the number of things that are both A and B.

Generalized pigeonhole principle:

If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

Permutations:

$$P(n,r) = n(n-1)(n-2)...(n-r+1) = \frac{n!}{(n-r)!}$$

Combinations:

$$C(n,r) = \frac{n(n-1)(n-2)...(n-r+1)}{r(r-1)(r-2)...} = \frac{n!}{r!(n-r)!}$$

Restricted conjunction rule:

P(A and B) = P(A) × P(B)
when A and B are independent

General conjunction rule:

P(A and B) = P(A) × P(B|A)

Restricted disjunction rule:

P(A or B) = P(A) + P(B)
when A and B are mutually exclusive

General disjunction rule:

P(A or B) = P(A) + P(B) – P(A and B)

Negation rule:

P(A) = 1 – P(not-A)

Conditional Probability:

P(A|B) = P(A and B)/P(B)

Bayes' Theorem (with denominator replaced by law of total probability):

$$P(A_1 \mid B) = \frac{\mathrm{P(A_1) \times P(B \mid A_1)}}{\mathrm{[P(A_1) \times P(B \mid A_1)] + [P(A_2) \times P(B \mid A_2)]}}$$

When A_1 and A_2 are mutually exclusive and jointly exhaustive

Law of Total Probability (and Conditionalized form):

$$\mathrm{P(A) = P(A \mid B_1)P(B_1) + P(A \mid B_2)P(B_2) + \cdots + P(A \mid B_k)P(B_k)}$$
$$\mathrm{P(A \mid C) = P(A \mid B_1 \,\&\, C)P(B_1 \mid C) + \cdots + P(A \mid B_k \,\&\, C)P(B_k \mid C)}$$

When B_1 … B_k are mutually exclusive and exhaustive

PHIL 005 Final Exam Topics:

From first midterm (big ideas might reoccur on the final, but you won't need to make any truth tables):

- Arguments vs. Non-arguments
- Deductive vs. Inductive Arguments
- Goodness for Inductive and Deductive Arguments
  - Inductive: Strong/Weak; Cogent/Uncogent
  - Deductive: Valid/Invalid; Sound/Unsound
- What these things are:
  - Conditional (and antecedent/consequent), Explanation (and explanans/explanandum), Necessary and sufficient conditions
  - Truth Function
  - For sentences: tautology, self-contradictory, contingent
  - For pairs of sentences: equivalent, contradictory
  - For sets of sentences: consistent, inconsistent (book: jointly im/possible)
- How to translate simple sentences and arguments into sentential logic
- The connection between argument form and validity
  - Valid form => argument is valid (but not necessarily vice versa)
- Ways of proving argument validity/invalidity
  - Counterexample method
  - Truth Tables
    - TTs can do more, like prove tautologies, consistency, etc

The rest of deductive formal logic:

- Proofs in Truth Functional Logic (Do Carnap assignments again to practice!)
- Translations into First-Order logic with relations and identity. (See Carnap again.)
- Proofs in First-Order logic, including with relations, multiple quantifiers, and identity (Carnap!)
- Derived Rules (Not using them in proofs but the idea)
- Showing FOL arguments are invalid
  - Finite Domain Method
  - Finding a model that's a counterexample to validity (which is a generalization of the finite domain method)
- Basic Model Theory (Remember: the book says "interpretation" instead of "model")
  - What a model is (domain + extension of predicates, relations)
  - What an isomorphism is (a predicate- and relation- preserving bijective map) and why they're important (since they're used to individuate models, i.e. two models are the same model iff there is a bijection between them)
  - Models make sentences true or false
  - We can understand validity in terms of models (i.e. an arg is valid when all of the models of the premises are models of the conclusion)
- Metatheoretic Stuff including Expressive Adequacy, Soundness, and Completeness
  - The basic ideas: you might be asked basic questions about what these are, what it means for a proof system to have them, and whether the proof systems we've studied have them.
  - On the proofs: You won't be asked to reproduce the proofs, but you might be asked about the very basic ideas of them, including that:
    - Soundness works by mathematical induction on the length of proofs (by going through each rule to prove that it preserves soundness)
    - Completeness works by translating into claim about sets of sentences then constructing a model to satisfy the set
- The basic idea of other logics (i.e. what the most basic idea of higher order logic is and what the most basic idea of modal logic is)

Formalization of inductive reasoning and other kinds of formal reasoning:

- Mathematical Induction
    - You won't be asked to give a proof by mathematical induction, but you will be expected to understand them, including their parts (base case and inductive step) and how they work in general (so look at examples, including the textbook linked in the homework problem set)
- Simple combinatorics (problems including the product rule, sum rule, combinations, permutations, inclusion-exclusion principle, pigeonhole principle)
- Probability stuff (All the rules on the rule list)
    - Admissions Bias example ("Simpson's Paradox")
    - Venn's (Betrand's) Paradox (about the square factories)
- Statistics: What is Statistics?
    - Quantitative vs Categorical Data
    - Descriptive vs Inferential Statistics
        - Descriptive Statistics
            - The idea of Pearson's Correlation
            - How descriptive statistics can be misleading (e.g. the datasaurus)
        - Inferential Statistics
            - Null Hypothesis Significance Testing (the idea)
            - P-Values (What they are)
- Infinite Combinatorics: Infinite Cardinals (including Cantor's diagonal argument and theorem)

PHIL 005 Final Exam Practice/Sample Problems

(The first few are more like questions from the first exam, but they're good warm ups.)

_______ 1) Which of the following is a necessary condition for being a cat?

a. Having black fur.
b. Weighing at least 8 pounds.
c. Clawing the furniture.
d. Being a calico.
e. Being an animal.

Some entertainers are not magicians, for some comedians are not magicians and some magicians that are not comedians are entertainers.

_______ 2) Which of the following correctly expresses the form of this argument?

a. Some C are not M.
<u>Some M are E.</u>
Some E are not M.

b. Some M that are not C are E.
<u>Some E are not M.</u>
Some C are not M.

c. Some C are not M.
<u>Some M that are not C are E.</u>
Some E are not M.

d. Some E are not M.
<u>Some C are not M.</u>
Some M that are not C are E.

e. Some E are not M.
<u>Some C are not M.</u>
All M that are not C are E.

______ 3) Which of the following substitutions proves the argument invalid?

a. M = mammals, C = animals, E = dogs.
b. M = cats, C = trees, E = animals.
c. M = mammals, C = cats, E = animals.
d. M = animals, C = trees, E = cats.
e. M = fish, C = dogs, E = mammals.

True/False:

______ 4) Since the there are infinitely-many rational numbers and infinitely-many real numbers, there are the same number of rational numbers as there are real numbers.

______ 5) If two events are probabilistically independent, they must be mutually exclusive.

______ 6) If P(A) = .6, P(B) = .4, and P(A|B) = .5, then P(A&B) = P(A) x P(B)

Give the exact number answer. Remember that Google can act as a calculator for you.

______ 7) Of the 9 paintings that I own, I want to hang three of them in a row on the wall. How many different ways are there to hang the pictures (order matters)?

______ 8) There are 50 states in the US, and everybody has exactly one favorite one. How many people do I need to ask to be certain I asked at least three people with the same favorite?

______ 9) Suppose there are 4 bags of marbles on my desk. 3 of the bags contain 40 red marbles and 60 blue marbles. The other one contains 70 red marbles and 10 blue marbles. If I draw a marble at random from one of the bags, what's the chance I drew from one of the bags with 40 reds in it, given that I got a blue marble?

For each of the following, if the argument is valid, prove it using truth functional logic or first order logic. If the argument is invalid, provide a line of a truth table showing it's invalid (if it's in truth functional logic) or provide a model counterexample (if it's in first order logic).

______ 10) $A \rightarrow B \therefore (A \wedge B) \vee (\neg A \wedge \neg B)$

______ 11) $C \rightarrow (E \wedge G), \neg C \rightarrow G \therefore G$

______ 12) $\exists x Mx \rightarrow \exists x Nx, \neg \exists x Nx \therefore \forall x \neg Mx$

______ 13) $\exists x(Px \wedge \neg Qx) \therefore \forall x(Px \rightarrow \neg Qx)$

Remember to look for more practice problems in the book! There are tons of problems at the ends of different sections and there is a solutions manual on their website. Also practice with the old Carnap assignments!