Compositional Areal Data

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Compositional areal data

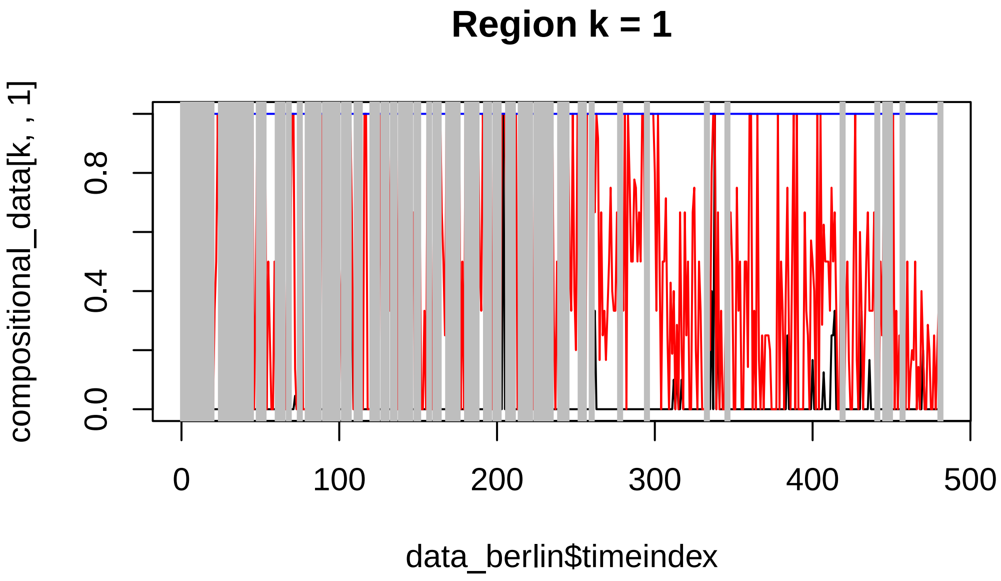
Real-world example

Berlin real-estate transactions in three categories

```
load("DATA_BERLIN.Rdata")

total_transactions <- data_berlin$cases$unbebaute_flaeche + data_berlin$cases$bebaute_flae

compositional_data <- array(, dim = c(length(data_berlin$spatial_index), length(data_berli

compositional_data[,,1] <- data_berlin$cases$unbebaute_flaeche / ifelse(total_transactions
compositional_data[,,2] <- data_berlin$cases$bebaute_flaeche / ifelse(total_transactions =
compositional_data[,,3] <- data_berlin$cases$eigentumswohnung / ifelse(total_transactions
```

Bi-monthly data, each postcode region

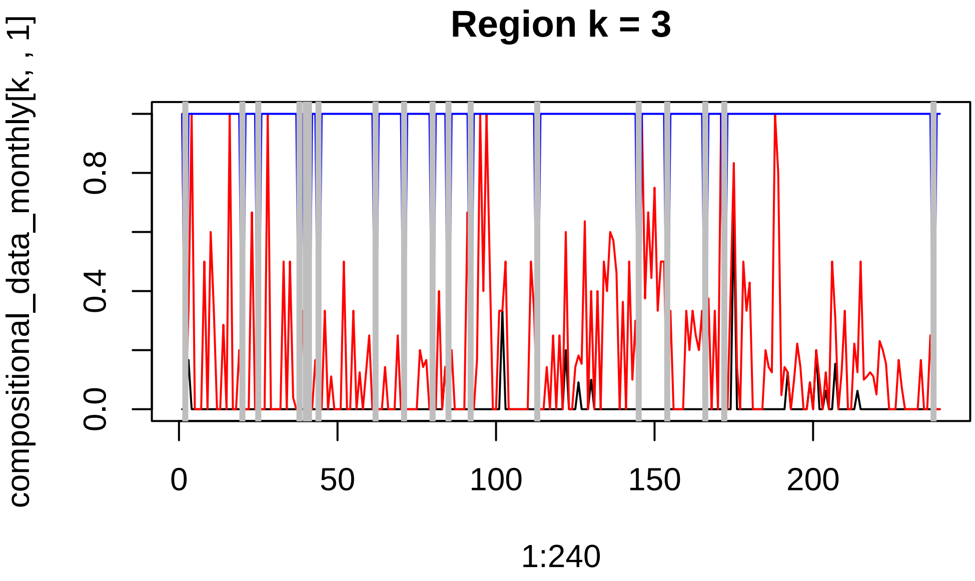
```
k <- 1
plot(data_berlin$timeindex, compositional_data[k,,1], type = "l", ylim = c(0, 1), main = p
lines(data_berlin$timeindex, apply(compositional_data[k,,1:2], 1, sum), col = "red")
lines(data_berlin$timeindex, apply(compositional_data[k,,1:3], 1, sum), col = "blue")
abline(v = which(apply(compositional_data[k,,1:3], 1, sum) == 0), lwd = 3, col = "grey")
```



Aggregate to monthly data

```
total_transactions_monthly <- total_transactions[, which(data_berlin$day == 1)] + total_tr

compositional_data_monthly <- array(, dim = c(length(data_berlin$spatial_index), (length(d

compositional_data_monthly[,,1] <- (data_berlin$cases$unbebaute_flaeche[, which(data_berli
compositional_data_monthly[,,2] <- (data_berlin$cases$bebaute_flaeche[, which(data_berlin$
compositional_data_monthly[,,3] <- (data_berlin$cases$eigentumswohnung[, which(data_berlin
```

```
k <- 3
plot(1:240, compositional_data_monthly[k,,1], type = "l", ylim = c(0, 1), main = paste("Re
lines(1:240, apply(compositional_data_monthly[k,,1:2], 1, sum), col = "red")
lines(1:240, apply(compositional_data_monthly[k,,1:3], 1, sum), col = "blue")
abline(v = which(apply(compositional_data_monthly[k,,1:3], 1, sum) == 0), lwd = 3, col = "
```



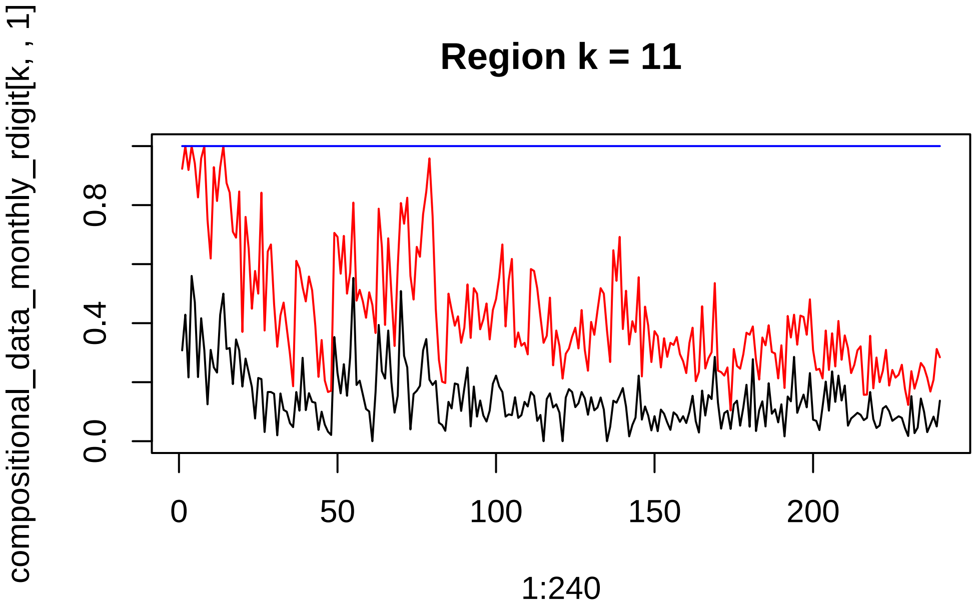
Aggregate to 3-digit postcode regions

```
r_digits <- 3
aggregate_spatial_IDs <- floor(data_berlin$spatial_index/(10^(r_digits-1)))
total_transactions_monthly_rdigit <- t(sapply(unique(aggregate_spatial_IDs), function(x) a

length(unique(aggregate_spatial_IDs))
```

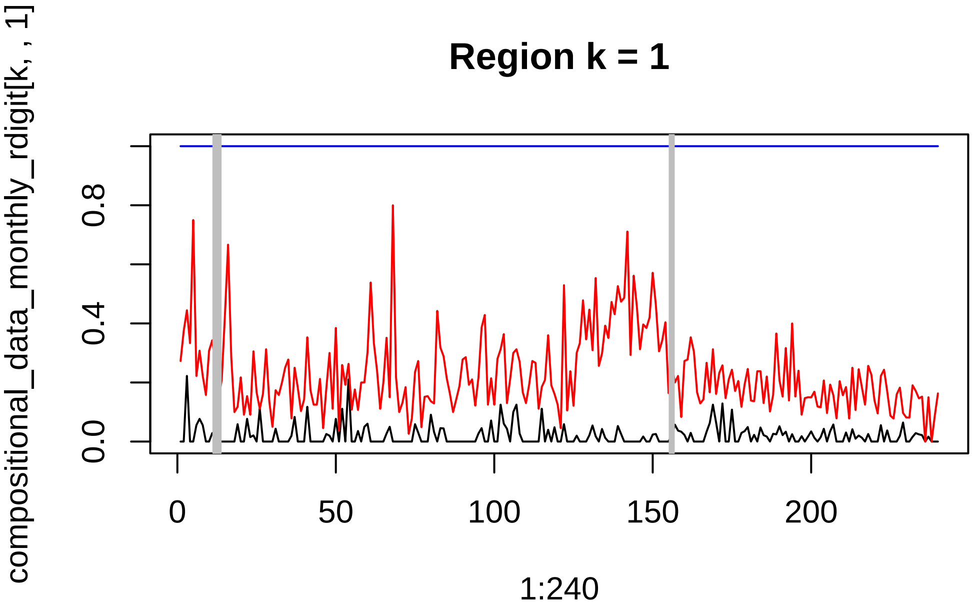
```
[1] 24
```

```
compositional_data_monthly_rdigit <- array(, dim = c(length(unique(aggregate_spatial_IDs))

compositional_data_monthly_rdigit[,,1] <- t(sapply(unique(aggregate_spatial_IDs), function
compositional_data_monthly_rdigit[,,2] <- t(sapply(unique(aggregate_spatial_IDs), function
compositional_data_monthly_rdigit[,,3] <- t(sapply(unique(aggregate_spatial_IDs), function
```

```
k <- 11
plot(1:240, compositional_data_monthly_rdigit[k,,1], type = "l", ylim = c(0, 1), main = pa
lines(1:240, apply(compositional_data_monthly_rdigit[k,,1:2], 1, sum), col = "red")
lines(1:240, apply(compositional_data_monthly_rdigit[k,,1:3], 1, sum), col = "blue")
abline(v = which(apply(compositional_data_monthly_rdigit[k,,1:3], 1, sum) == 0), lwd = 3,
```



```
k <- 1
  plot(1:240, compositional_data_monthly_rdigit[k,,1], type = "l", ylim = c(0, 1), main =
lines(1:240, apply(compositional_data_monthly_rdigit[k,,1:2], 1, sum), col = "red")
lines(1:240, apply(compositional_data_monthly_rdigit[k,,1:3], 1, sum), col = "blue")
for(k in 1:length(unique(aggregate_spatial_IDs))){
  abline(v = which(apply(compositional_data_monthly_rdigit[k,,1:3], 1, sum) == 0), lwd = 3
}
```



Code to share dataset

```
library("spdep")
```

```
Loading required package: sp
```

```
Loading required package: spData
```

```
To access larger datasets in this package, install the spDataLarge
package with: `install.packages('spDataLarge',
repos='https://nowosad.github.io/drat/', type='source')`
```

```
Loading required package: sf
```

```
Linking to GEOS 3.11.0, GDAL 3.5.3, PROJ 9.1.0; sf_use_s2() is TRUE
```

```
library("maptools")
```

```
Checking rgeos availability: FALSE
Please note that 'maptools' will be retired during 2023,
plan transition at your earliest convenience;
some functionality will be moved to 'sp'.
    Note: when rgeos is not available, polygon geometry     computations in maptools depend
    which has a restricted licence. It is disabled by default;
    to enable gpclib, type gpclibPermit()
```

```
gpclibPermit()
```

```
Warning in gpclibPermit(): support for gpclib will be withdrawn from maptools
at the next major release
```

```
[1] TRUE
```

```
year <- data_berlin$year[which(data_berlin$day == 1)]
month <- data_berlin$month[which(data_berlin$day == 1)]
postcode <- unique(floor(data_berlin$spatial_index/(10^(r_digits-1))))
map <- unionSpatialPolygons(data_berlin$map, aggregate_spatial_IDs)
```

```
Warning in showSRID(uprojargs, format = "PROJ", multiline = "NO", prefer_proj =
prefer_proj): Discarded datum Unknown based on WGS84 ellipsoid in Proj4
definition
```

```
save(compositional_data_monthly_rdigit, year, month, postcode, map, file = "Compositional_

# load("Compositional_Data_Berlin.Rda")
```

ILR Transformation

```
library("compositions")
```

```
Welcome to compositions, a package for compositional data analysis.
Find an intro with "? compositions"
```

```
Attaching package: 'compositions'

The following objects are masked from 'package:stats':

    anova, cor, cov, dist, var

The following objects are masked from 'package:base':

    %*%, norm, scale, scale.default
```

```r
dim(compositional_data_monthly_rdigit)
```

```
[1]  24 240   3
```

```r
sum(is.na(compositional_data_monthly_rdigit))
```

```
[1] 0
```

```r
sum(compositional_data_monthly_rdigit == 0)
```

```
[1] 1469
```

```r
ilr_compositional_data_monthly_rdigit <- apply(compositional_data_monthly_rdigit, c(1,2),
sum(is.na(ilr_compositional_data_monthly_rdigit))
```

```
[1] 0
```

```r
dim(ilr_compositional_data_monthly_rdigit)
```

```
[1]   2  24 240
```

QML

Functions needed for estimation

```r
qml_spatiotemporal_compositions_p <- function(Y, W){

  dimY <- dim(Y)
  n    <- dimY[1]
  p    <- dimY[2]
  t    <- dimY[3]

  dimW <- dim(W)
  if(dimW[1] != n | dimW[2] != n){
    stop("Dimension of W is wrong")
  }

  vec <- function(x){
    return(as.vector(x))
  }

  LogLikelihood <- function(pars, Y, W){

    dimY <- dim(Y)
    n    <- dimY[1]
    p    <- dimY[2]
    t    <- dimY[3]

    A_tilde <- matrix(rep(pars[1:p], n), n, p, byrow = TRUE)
    Psi     <- matrix(pars[(p+1):(p^2+p)], p, p)
    Pi      <- matrix(pars[(p^2+p+1):(2*p^2+p)], p, p)
    sig_u   <- pars[2*p^2+p + 1]

    S         <- diag(n*p) - t(Psi) %x% W
    log_det_S <- determinant(S, logarithm = TRUE)$modulus

    sum_eps_2 <- 0
    for(i.t in 2:t){
      vec_u_t   <- S %*% vec(Y[,,i.t]) - vec(A_tilde) - (diag(p) %x% Y[,,i.t-1]) %*% vec(P
      sum_eps_2 <- sum_eps_2 + sum(vec_u_t^2)
    }

    LL <- -1/2 * log(2*pi) - ((t-1)*n * sig_u^2) / (2*p) + (t-1)/(n*p) * log_det_S - 1/(2*

    return((-1) * LL) # ((t-1)*n*p) *
  }
```

```
residuals <- function(pars, Y, W){

  dimY <- dim(Y)
  n    <- dimY[1]
  p    <- dimY[2]
  t    <- dimY[3]

  A_tilde <- matrix(rep(pars[1:p], n), n, p, byrow = TRUE)
  Psi     <- matrix(pars[(p+1):(p^2+p)], p, p)
  Pi      <- matrix(pars[(p^2+p+1):(2*p^2+p)], p, p)
  sig_u   <- pars[2*p^2+p + 1]

  S         <- diag(n*p) - t(Psi) %x% W
  log_det_S <- determinant(S, logarithm = TRUE)$modulus

  U_t <- array(, dim = c(n*p, t))
  for(i.t in 2:t){
    vec_u_t   <- S %*% vec(Y[,,i.t]) - vec(A_tilde) - (diag(p) %x% Y[,,i.t-1]) %*% vec(P
    U_t[,i.t] <- vec_u_t
  }

  return(U_t)
}


start_a   <- rep(1, p)
start_Psi <- matrix(rep(0.2, p^2), p, p)
start_Pi  <- matrix(rep(0.2, p^2), p, p)

start_pars <- c(start_a, vec(start_Psi), vec(start_Pi), 1)

LB_a   <- rep(-1000, p)
LB_Psi <- matrix(rep(0, p^2), p, p)
LB_Pi  <- matrix(rep(-1, p^2), p, p)

LB <- c(LB_a, vec(LB_Psi), vec(LB_Pi), 0)

UB_a   <- rep(1000, p)
UB_Psi <- matrix(rep(1, p^2), p, p)
UB_Pi  <- matrix(rep(1, p^2), p, p)
```

```
  UB <- c(UB_a, vec(UB_Psi), vec(UB_Pi), 10000)

  # LogLikelihood(start_pars, Y, W)

  out <- solnp(start_pars, fun = LogLikelihood, Y = Y, W = W, control = list(trace = TRUE)
               LB = LB, UB = UB)

  res <- residuals(out$pars, Y = Y, W = W)

  A_tilde_est   <- matrix(rep(out$pars[1:p], n), n, p, byrow = TRUE) #  - E_logsq_errors #
  Psi_est       <- matrix(out$pars[(p+1):(p^2+p)], p, p)
  Pi_est        <- matrix(out$pars[(p^2+p+1):(2*p^2+p)], p, p)
  sigu_est      <- out$pars[2*p^2+p + 1]

  return(list(A_tilde_est = A_tilde_est, Psi_est = Psi_est, Pi_est = Pi_est, sigu_est = si

}
```

Estimation

```
library("Rsolnp")

dim(ilr_compositional_data_monthly_rdigit)
```

```
[1]   2  24 240
```

```
Y_ilr <- aperm(ilr_compositional_data_monthly_rdigit, c(2,1,3)) # change order of dimensio
dim(Y_ilr)
```

```
[1]  24   2 240
```

```
W <- nb2mat(poly2nb(map), style = "W")

output_qml <- qml_spatiotemporal_compositions_p(Y_ilr, W) # needs n x p x t
```

```
Iter: 1 fn: 38.3848  Pars:   0.1220840164  0.2697704525  0.0219953022  0.1236713684  0.000000
Iter: 2 fn: 38.3848  Pars:   0.12208358041  0.26977048367  0.02199528216  0.12367175429  0.00
solnp--> Completed in 2 iterations
```

```
output_qml$A_tilde_est[1,]
```

```
[1] 0.1220836 0.2697705
```

```
ilrInv(output_qml$A_tilde_est[1,])
```

```
          1           2           3 
"0.2698735" "0.3207319" "0.4093946" 
attr(,"class")
[1] "acomp"
```

```
output_qml$Psi_est
```

```
           [,1]        [,2]
[1,] 0.02199528 2.63690e-08
[2,] 0.12367175 7.77219e-02
```

```
ilrInv(diag(output_qml$Psi_est)) # ???
```

```
          1           2           3 
"0.3175912" "0.3276254" "0.3547834" 
attr(,"class")
[1] "acomp"
```

```
ilrInv(output_qml$Psi_est[1,]) # ??
```

```
          1           2           3 
"0.3281626" "0.3385309" "0.3333065" 
attr(,"class")
[1] "acomp"
```

```r
ilrInv(output_qml$Psi_est[2,]) # ??
```

```
          1           2           3 
"0.2948543" "0.3512083" "0.3539374" 
attr(,"class")
[1] "acomp"
```

```r
ilrInv(output_qml$Psi_est[,1]) # ??
```

```
          1           2           3 
"0.3111996" "0.3210319" "0.3677685" 
attr(,"class")
[1] "acomp"
```

```r
ilrInv(output_qml$Psi_est[,2]) # ??
```

```
          1           2           3 
"0.3225945" "0.3225945" "0.3548111" 
attr(,"class")
[1] "acomp"
```

```r
output_qml$Pi_est
```

```
            [,1]       [,2]
[1,]  0.39331146 -0.1342512
[2,] -0.00452918  0.7492386
```

```r
output_qml$sigu_est
```

```
[1] 0.1142352
```