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New Acquisition strategies for Compressed Sensing in Magnetic Resonance Imaging

Philippe Ciuciu

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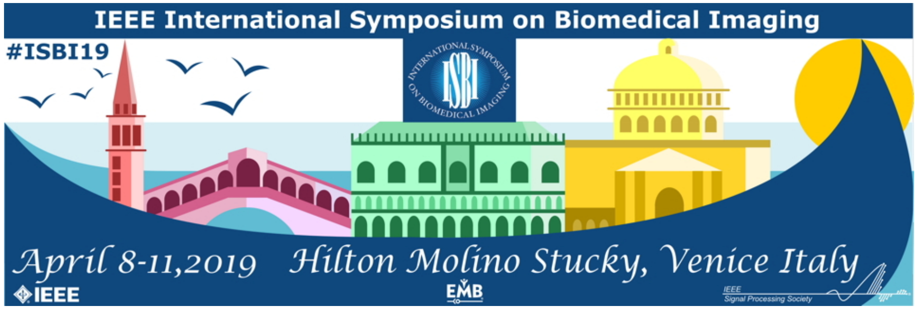

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CEA/NeuroSpin

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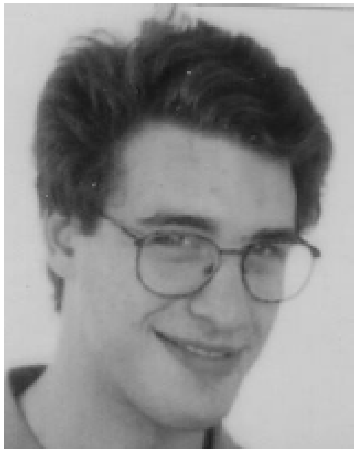
IMT, U. Toulouse

Jonas Kahn
CNRS/IMT

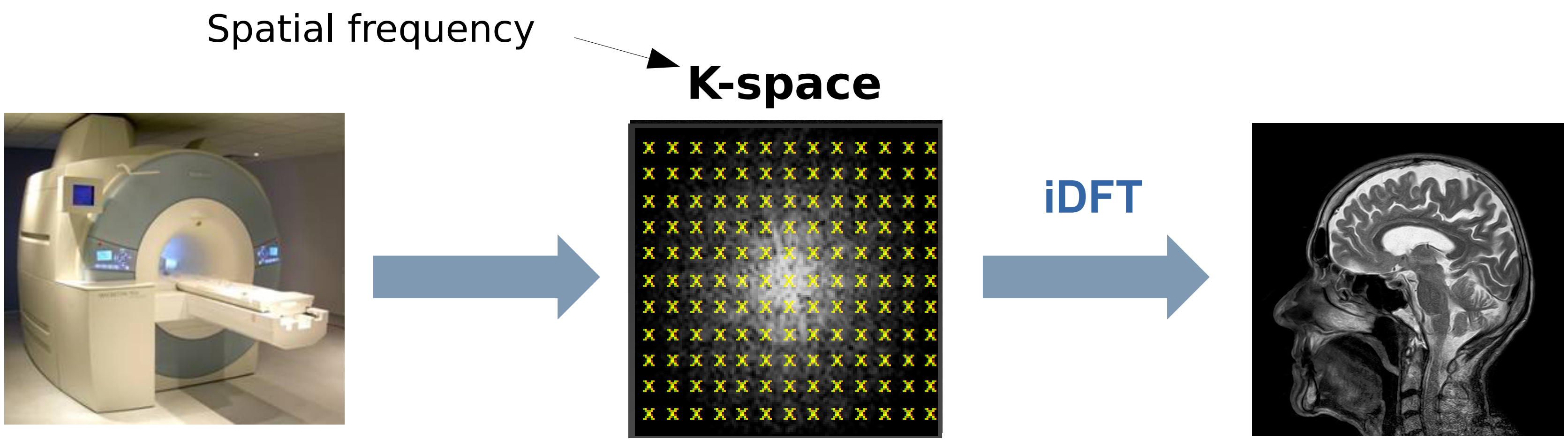


Outline

Part I: Background in Magnetic Resonance Imaging

- Sampling k-space & Cartesian reconstruction
- Trajectories and acquisition strategies
- Image reconstruction strategies (Jeff)

Sampling in MRI



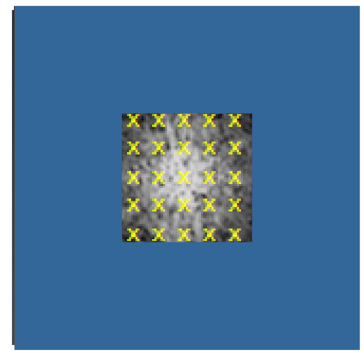
Perfect reconstruction of an object would require measurement of *all* locations
In k-space (infinite!)

Data is acquired **point-by-point** in k-space (sampling) along **curves**

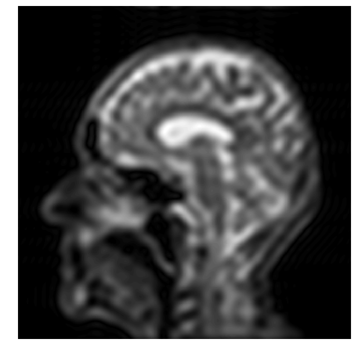
Sampling in MRI

Spatial frequency

K-space



iDFT

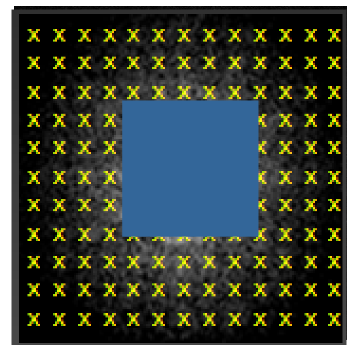


Low frequencies = Contrast

Sampling in MRI

Spatial frequency

K-space



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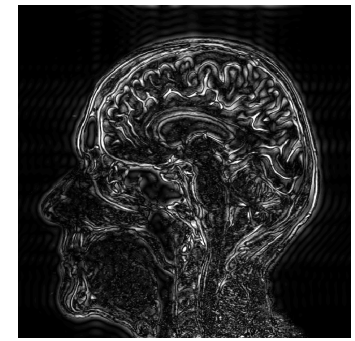
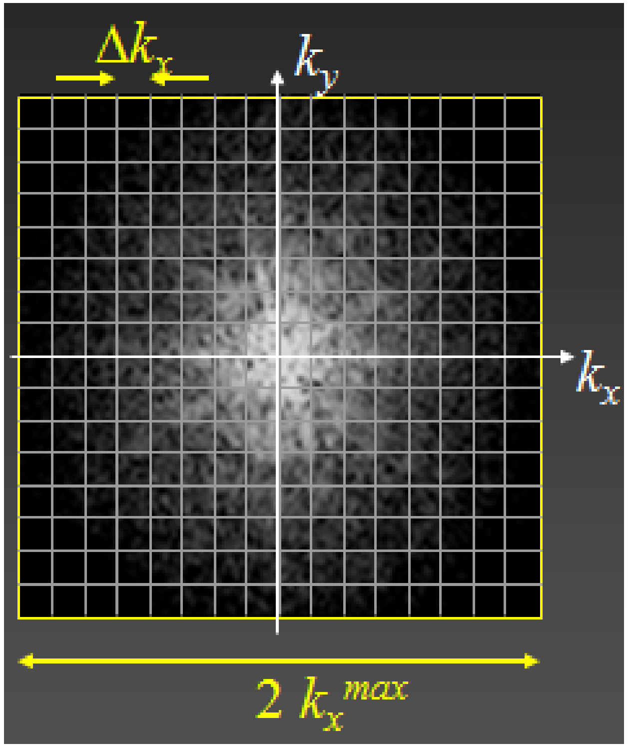


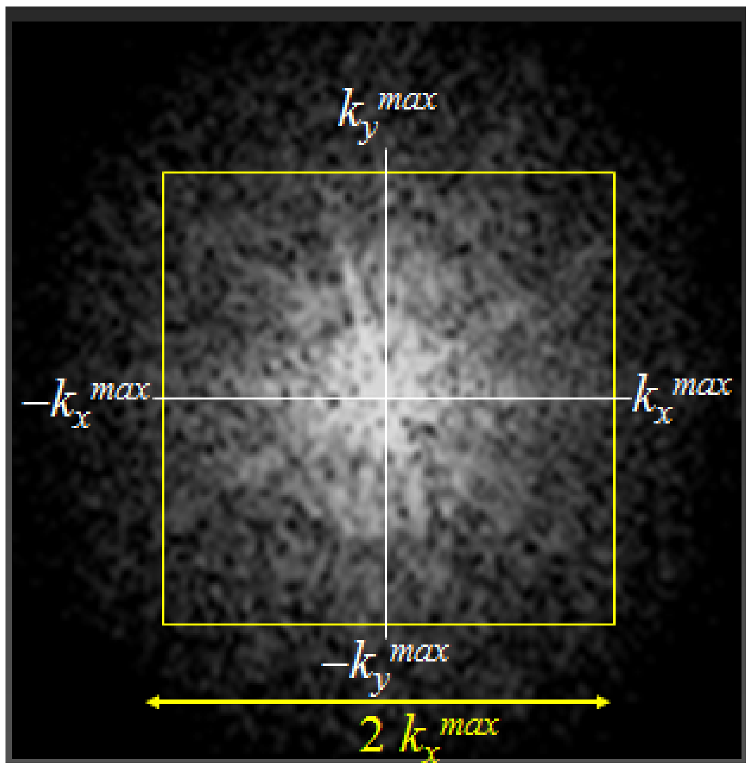
High frequencies = boundaries/edges

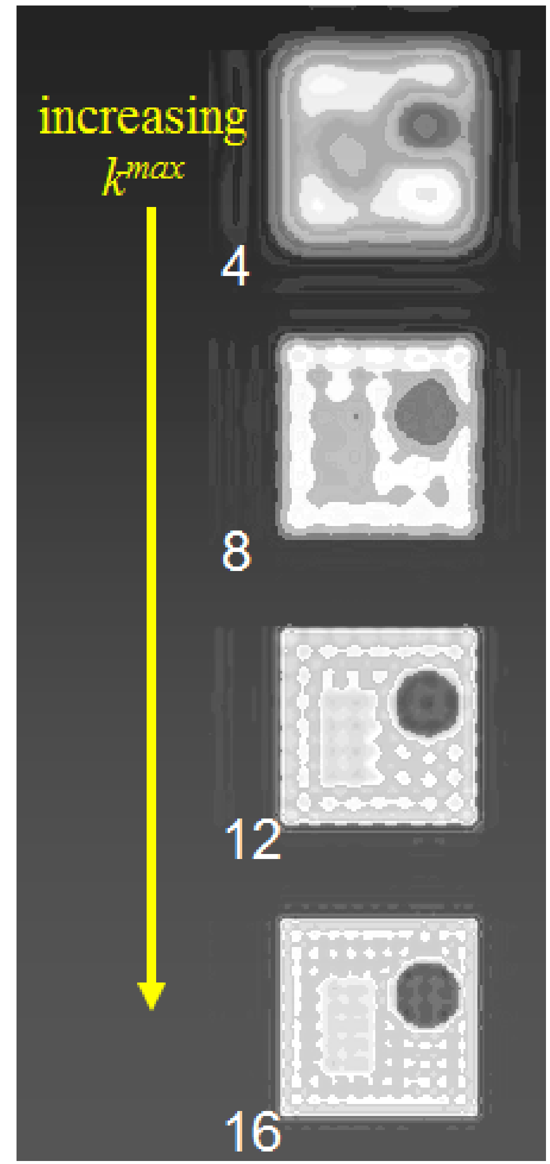
Sampling the k-space



1. What is the highest frequency we need to sample in *k*-space (k^{max})?
2. How close should the samples be in *k*-space (Δk)?

Choosing the maximal frequency

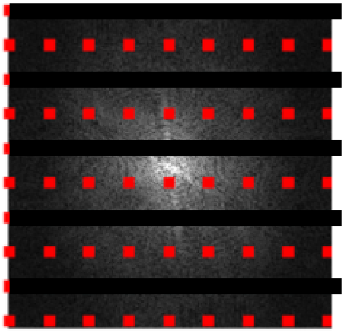




k^{max} determines image resolution
Large k^{max} means high resolution!

Nyquist Sampling Theorem

K-space



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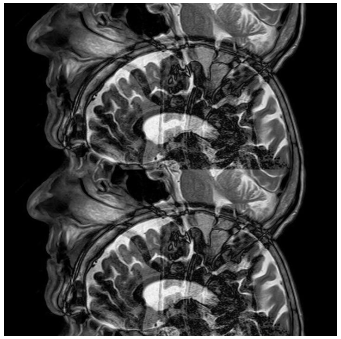


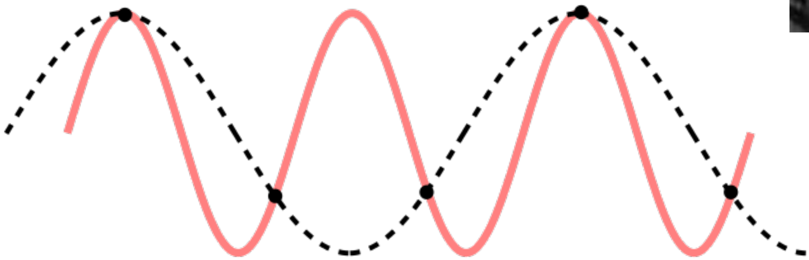
Nyquist-Shannon theory

↑ resolution ⇒ ↑ #samples

Long acquisition times

The sampling frequency should be at least twice the highest frequency contained in the signal

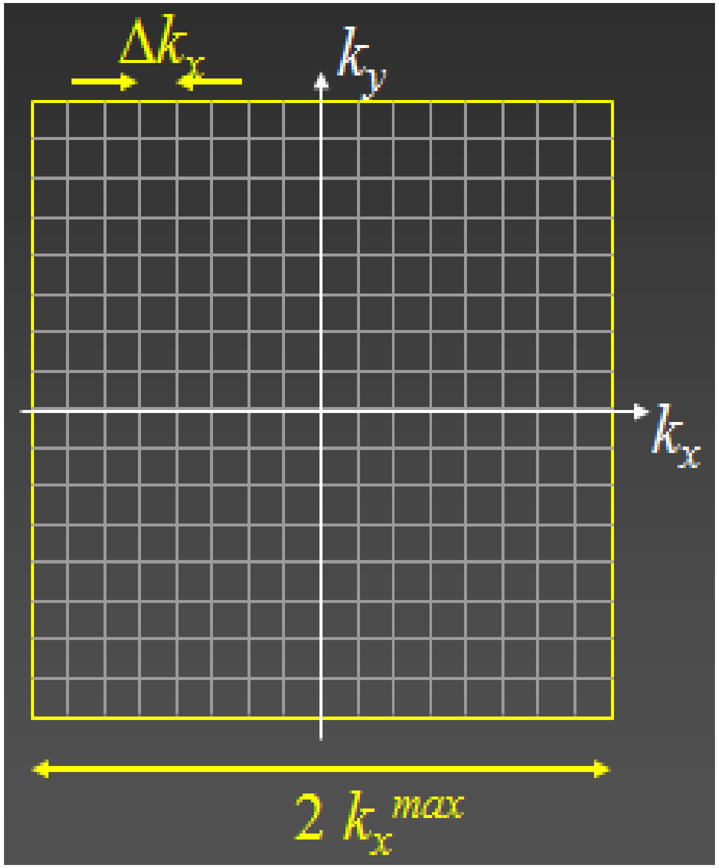
Harry Nyquist



Nyquist Sampling Theorem

$$\mathrm{FOV}_x = \frac{1}{\Delta k_x}$$

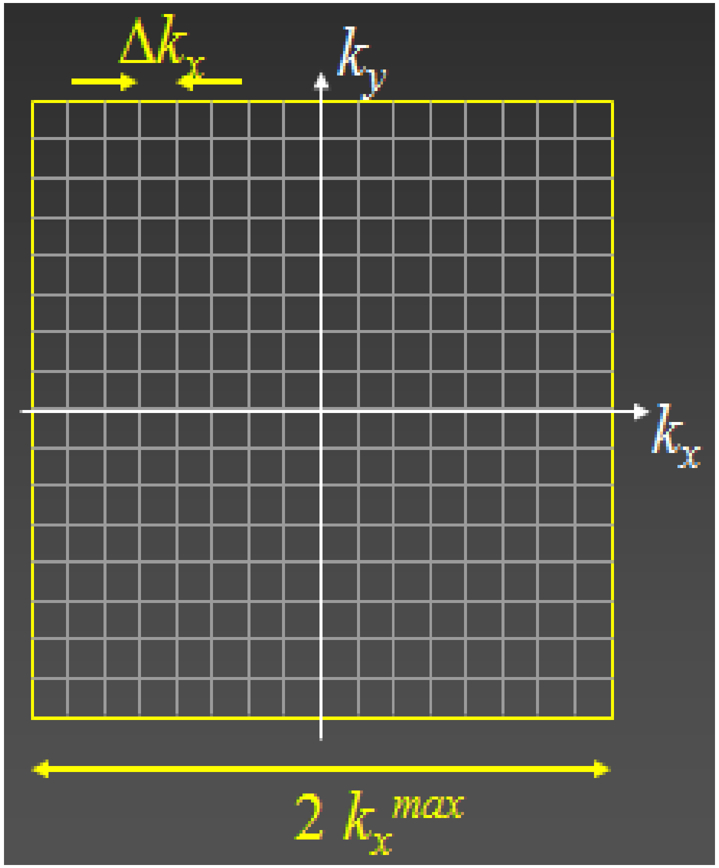
$$\Delta x = \frac{1}{2k_x^{\max}}$$



Nyquist Sampling Theorem

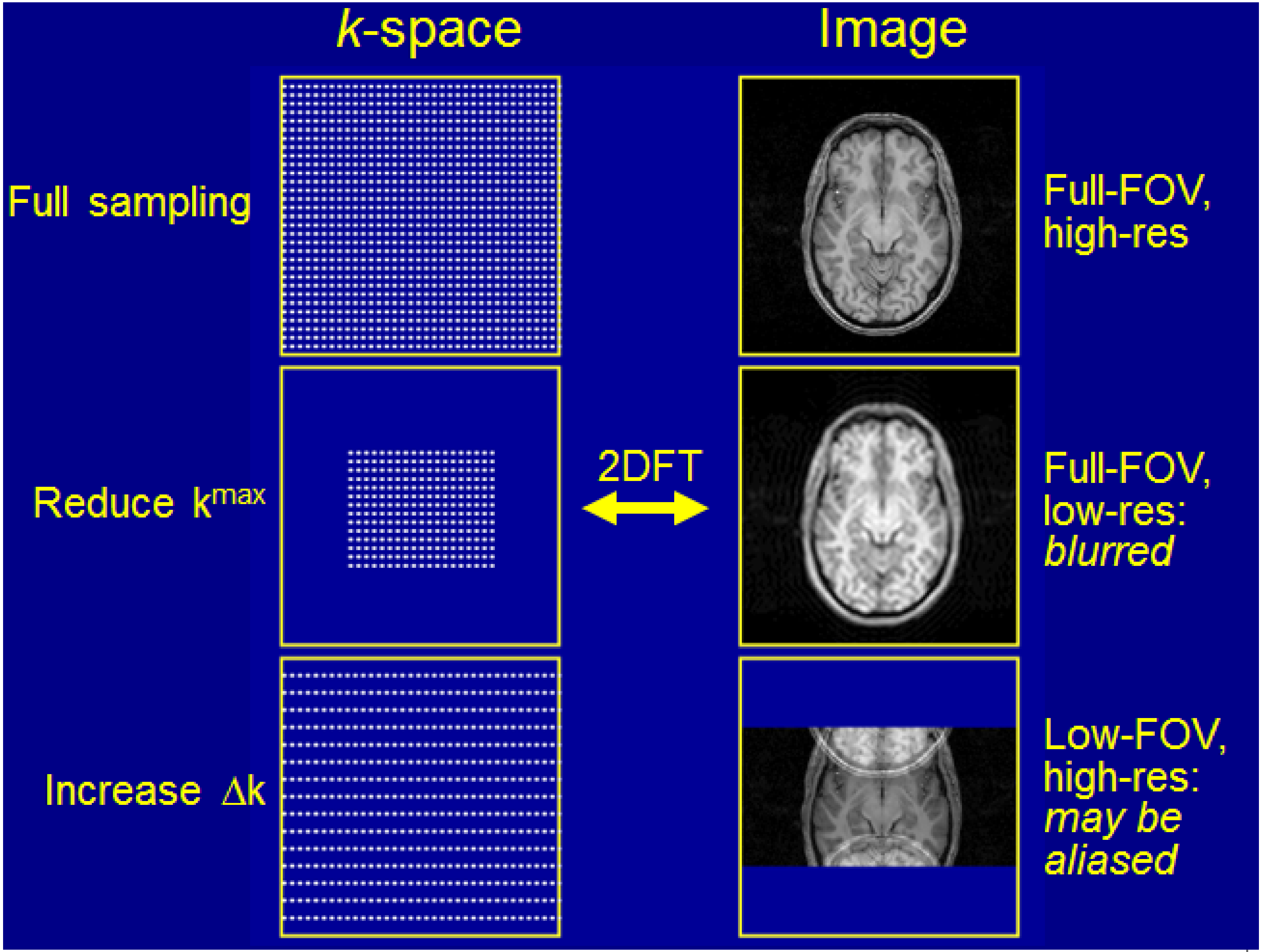
$$x_{\max} = \frac{1}{\Delta k_x}$$

$$2k_x^{\max} = \frac{1}{\Delta x}$$



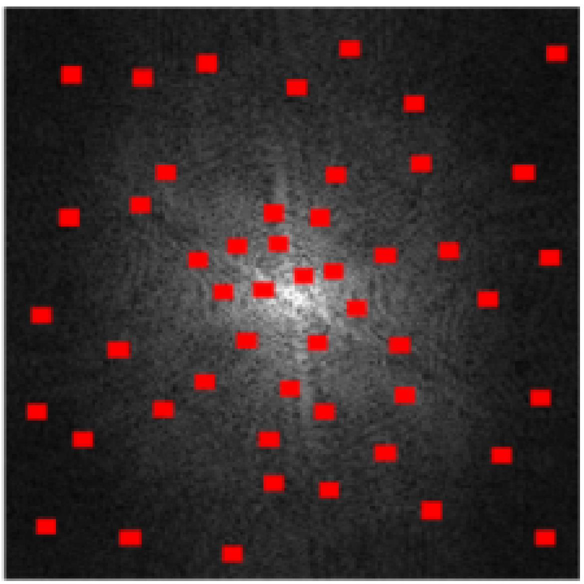
k-space and image resolution are inversely related:
resolution in one domain determines extent in other.

Nyquist Sampling Theorem



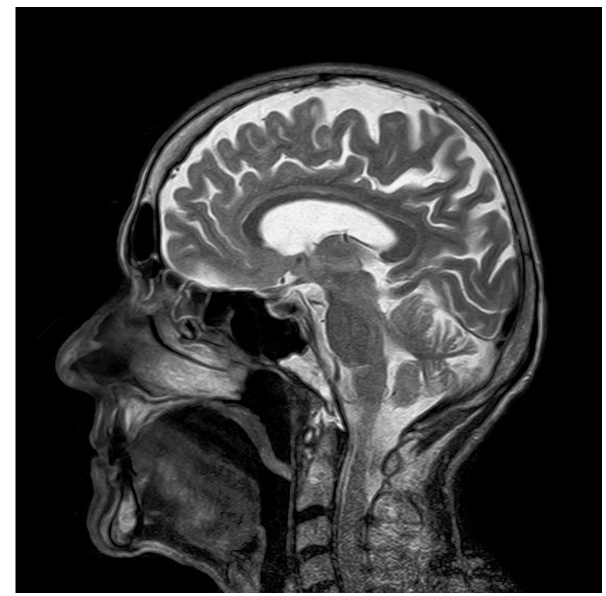
Beyond Nyquist Sampling Theorem

k-space



Nonlinear reconstructions

Image



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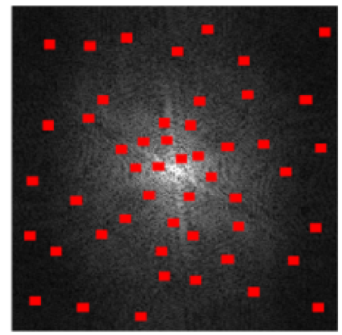
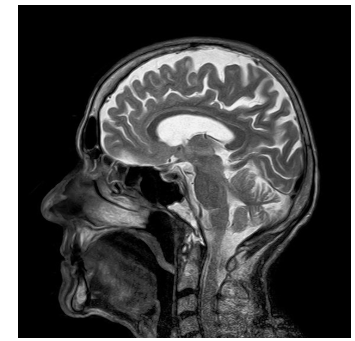
Sampling Trajectories

k-space

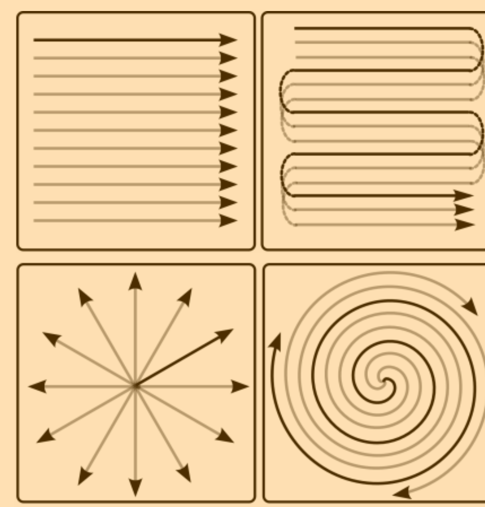


Nonlinear reconstructions

Image



Sampling in MRI: $k(t) = k(0) + \gamma \int_0^t G(\tau) d\tau$



- **Segmented acquisition**:
  Scan time proportional to number of shots

- **Hardware constraints on gradients**:
  $G_{max} < 40$ mT/m ; $S_{max} < 200$ T/m/s
  → bounded velocity and acceleration

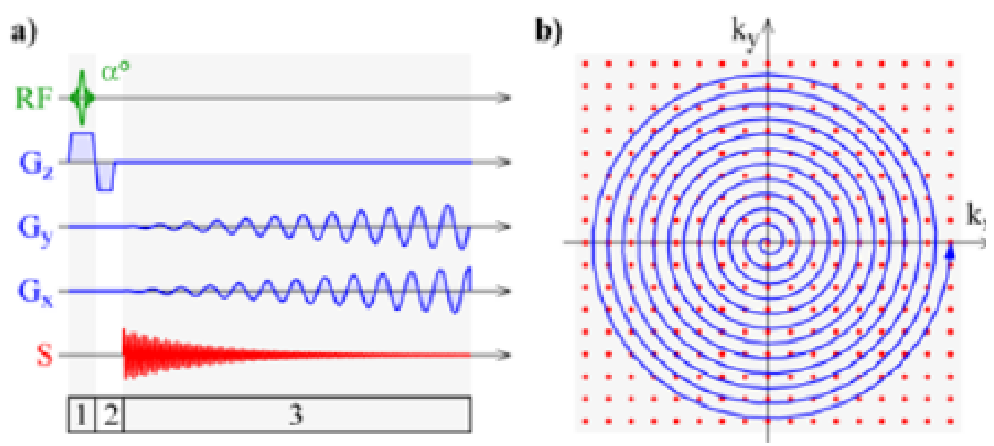
Modeling of k-space Trajectories

Let $\boldsymbol{k} : [0, T] \to \mathbb{R}^d, (d = 2, 3)$denote the sampling curve:

$$\boldsymbol{k}(t) = \boldsymbol{k}(0) + \gamma \int_0^t \boldsymbol{G}(\tau) d\tau \qquad d = 2 \to \boldsymbol{k} = (k_x, k_y),\ \boldsymbol{G} = (G_x, G_y)$$

$$\mathcal{S} = \begin{cases} \|\dot{\boldsymbol{k}}\|_{2,\infty} & < \quad \gamma G_{\max} & \text{(bounded speed)} \\ \|\ddot{\boldsymbol{k}}\|_{2,\infty} & < \quad \gamma S_{\max} & \text{(bounded acceleration)} \end{cases}$$

$$\text{where } \|\boldsymbol{c}\|_{2,\infty} = \sup_{1 \le i \le p} \left(|\boldsymbol{c}_x[i]|^2 + |\boldsymbol{c}_y[i]|^2 \right)^{1/2}$$



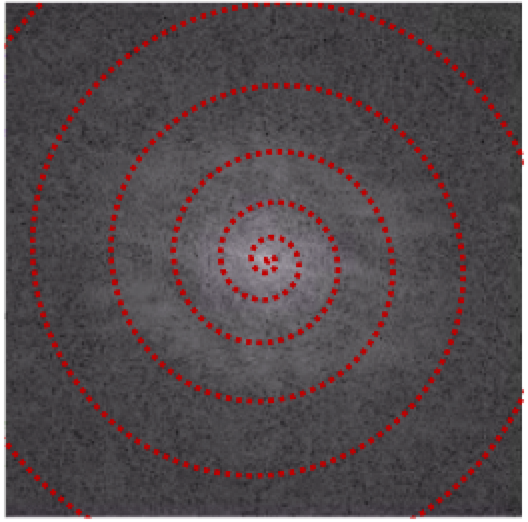
K-space location is proportional to accumulated area under gradient waveforms

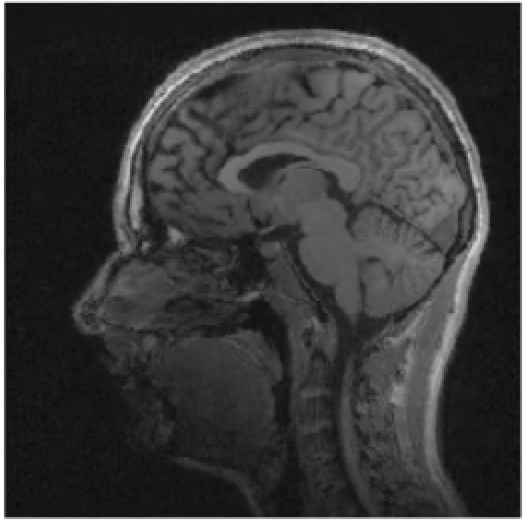
Modeling of k-space Trajectories

Sampling the curve $\boldsymbol{k}(t)$ generates a set of measurements:

$$E(\boldsymbol{k}) = \{\hat{x}(\boldsymbol{k}(p\Delta t))\}_{p\in\{0,\ldots,T/(\Delta t)\}}$$

with $\hat{x}$ the Fourier transform of x





Examples of k-space Trajectories

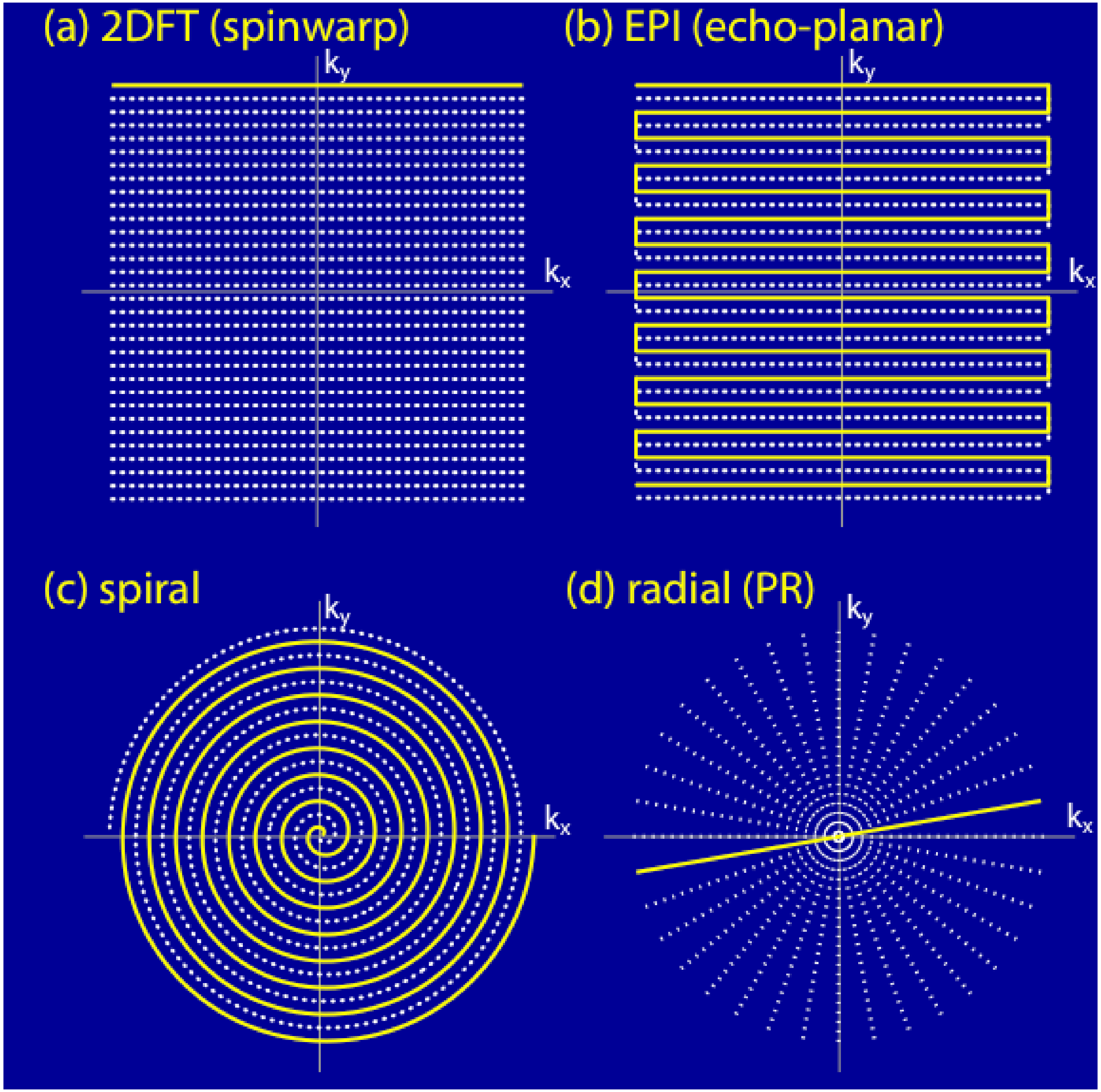
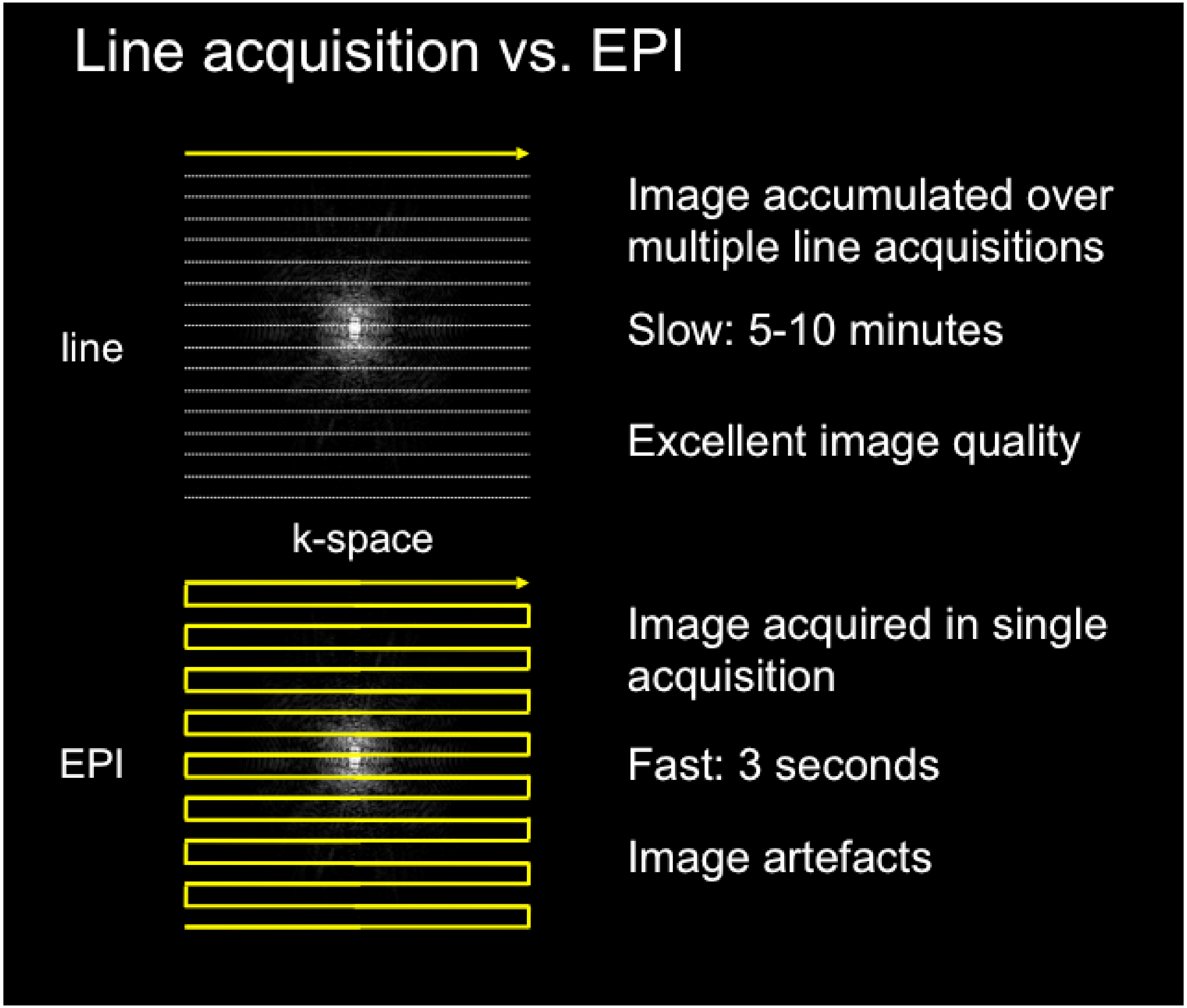


Image quality vs Acquisition Times



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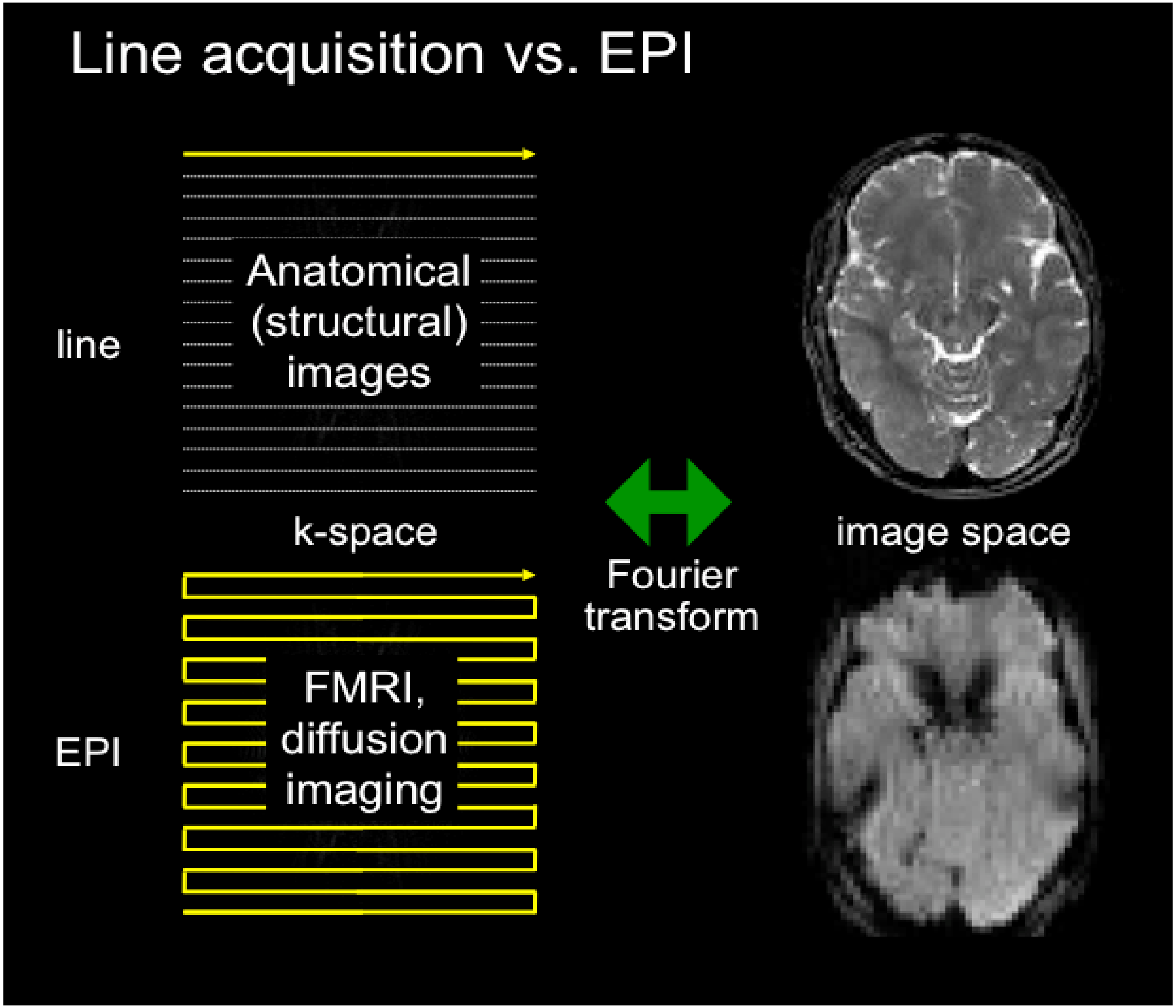

Outline

Part I: Background in Magnetic Resonance Imaging

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- Image reconstruction strategies (see Jeff)

Outline

Part II: Compressed Sensing in MRI

- Evolution of Compressed Sensing
- CS with blocks of measurements

Challenges

- **Questions in sampling theory**
  - How to choose the set of measurements E?
  - How to generate a set of segmented trajectories $\boldsymbol{k}$?
  - How to reconstruct x given E?

- **Practical consequences**
  - Shorten acquisition times
  - Increase image resolution
  - Improve medical diagnostics and reduce exam costs

A fundational result

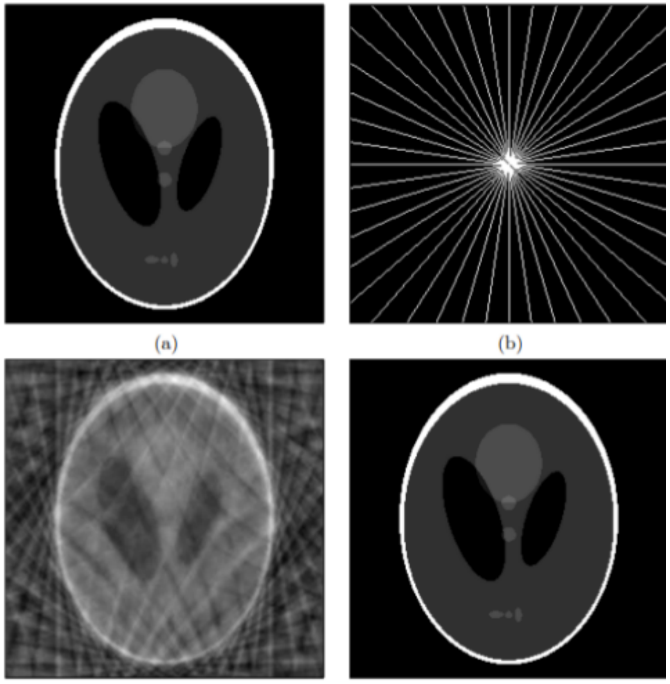
Supporting Thorem [Candes, Romberg and Tao, IEEE IT 2006]

Let $\boldsymbol{x} \in \mathbb{C}^n$ denote an s-sparse signal. Samples m values $y_k = \hat{\boldsymbol{x}}(J_k)$ where indexes $(J_k)_{1\leq k\leq m}$ are **i.i.d. and uniformly distributed**.
If

$$m \geq C \cdot s \cdot \log(n)$$

$\boldsymbol{x}$ can be recovered **exactly**, with high probability by solving:

$$\min_{\boldsymbol{x}\in\mathbb{C}^n, \hat{\boldsymbol{x}}(J_k)=y_k} \|\boldsymbol{x}\|_1$$



As the measurements are not *independent*, this theorem does not explain this result!

Preliminaries

Wavelet expansions

- Let $\boldsymbol{\Psi} = (\Psi, \ldots, \Psi_n) \in \mathbb{C}^{n\times n}$denote an orthogonal transform. The image to reconstruct writes:

$$\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{z} = \sum_{i=1}^{n} z_i \Psi_i \quad \text{where} \quad z_i = \langle \boldsymbol{x}, \Psi_i \rangle$$

- Objective: retrieve $\boldsymbol{z}$ (not $\boldsymbol{x}$)

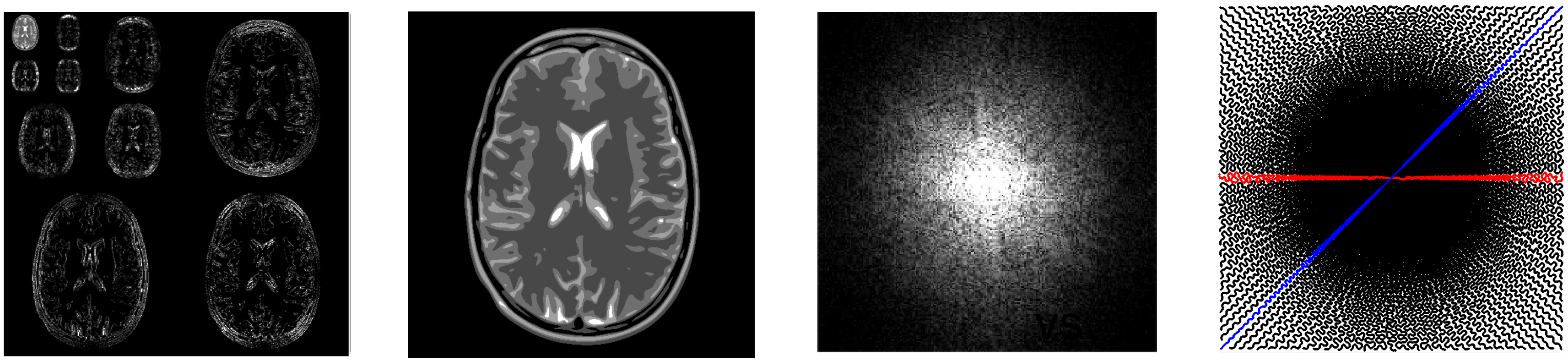
Fourier sampling

- Sensing matrix: $\boldsymbol{A}_0 = \boldsymbol{F}^*\boldsymbol{\Psi} = \begin{pmatrix} \boldsymbol{a}_1^* \\ \boldsymbol{a}_2^* \\ \vdots \\ \boldsymbol{a}_n^* \end{pmatrix}$. One measurement reads $y_k = \langle \boldsymbol{a}_k, \boldsymbol{z} \rangle$
- Let $\Omega \subseteq \{1, \cdots, n\}$ and $\boldsymbol{A}_\Omega = (\boldsymbol{a}_i^*)_{i\in\Omega}$ the sampling matrix. We get:

$$\boldsymbol{y} = \boldsymbol{A}_\Omega \boldsymbol{z} \ (+\boldsymbol{b})$$

Preliminaries

$\boldsymbol{z}$ | $\boldsymbol{x} = \boldsymbol{\Psi}\boldsymbol{z}$ | $\boldsymbol{F}^*\boldsymbol{\Psi}\boldsymbol{z} = \boldsymbol{A}\boldsymbol{z}$ | $\boldsymbol{A}_\Omega\boldsymbol{z}$



Sparsity: Let $S = \{i, z_i \neq 0\}$ denote the support of $\boldsymbol{z}$.

We assume that: $|S| = s \ll n$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noiseless case):

$$\widehat{\boldsymbol{z}} = \operatorname*{arg\,min}_{\boldsymbol{z}\in\mathbb{C}^n, \boldsymbol{A}_\Omega\boldsymbol{z}=\boldsymbol{y}} \|\boldsymbol{z}\|_1$$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noisy case):

$$\widehat{\boldsymbol{z}} = \operatorname*{arg\,min}_{\boldsymbol{z}\in\mathbb{C}^n} \frac{1}{2}\|\boldsymbol{y} - \boldsymbol{A}_\Omega\boldsymbol{z}\|_2^2 + \lambda\|\boldsymbol{z}\|_1$$

$$\widehat{\boldsymbol{x}} = \boldsymbol{\Psi}\widehat{\boldsymbol{z}}$$

Evolution of Compressed Sensing

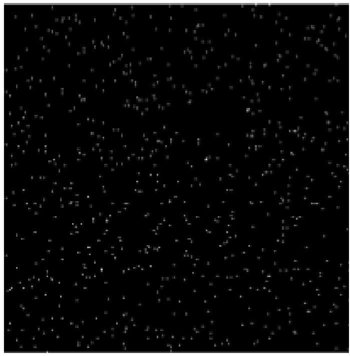
Uniform drawings: the early results

Coherence: $\kappa(\boldsymbol{A}_0) = n \max_{1 \leq k \leq n} \|\boldsymbol{a}_k\|_\infty^2$

Assume that $\boldsymbol{x}$ is s-sparse.

Draw m samples uniformly at random in $\{1, \ldots, n\}$.

$m \geq C \cdot \kappa(\boldsymbol{A}_0) \cdot s \cdot \log(\frac{n}{\epsilon}) \Rightarrow$ exact recovery with probability $1 - \epsilon$



Uniform sampling

[1] **Emmanuel Candes and Terence Tao.**
Near-optimal signal recovery from random projections: Universal encoding strategies?
Information Theory, IEEE Transactions on, 52(12):5406–5425, 2006.

[2] **Emmanuel Candes and Yaniv Plan.**
A probabilistic and ripless theory of compressed sensing.
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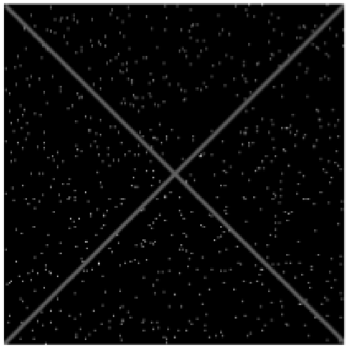

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Fourier-Wavelet system satisfies $\kappa(\boldsymbol{A}_0) \propto n$!

$\Rightarrow m \gg n$

Coherence barrier!

Evolution of Compressed Sensing

Variable density sampling

Local coherence: $\mu(\boldsymbol{A}_0) = \sum_{k=1}^{n} \|\boldsymbol{a}_k\|_\infty^2.$

Assume that $\boldsymbol{x}$ is s-sparse.

Set $\boldsymbol{A}_\Omega = \begin{pmatrix} \boldsymbol{a}_{J_1} \\ \vdots \\ \boldsymbol{a}_{J_m} \end{pmatrix}$ with $\mathbb{P}(J_k = \ell) = \pi_\ell \propto \|\pi_\ell\|_\infty^2$

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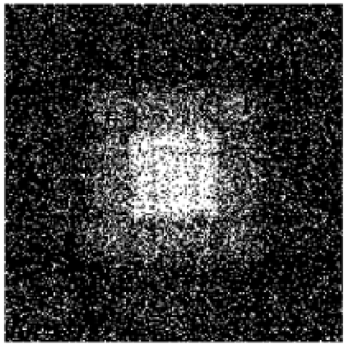
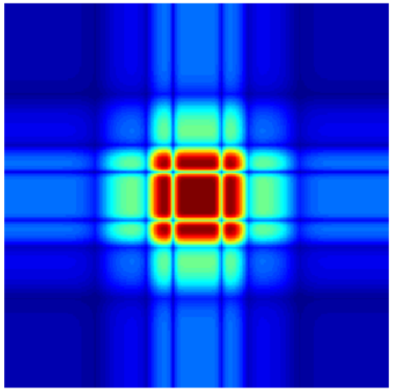


Variable density sampling



π in 2D

[1] **Gilles Puy, Pierre Vandergheynst, and Yves Wiaux.**
On variable density compressive sampling.
Signal Processing Letters, IEEE, 18(10):595–598, 2011.

[2] **Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.**
Variable density sampling with continous sampling trajectories.
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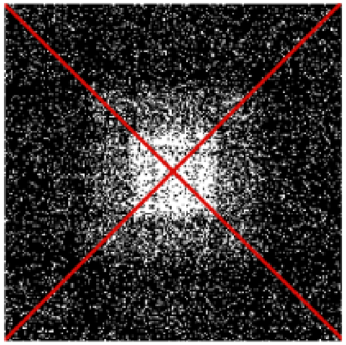

Variable density sampling

Fourier-Wavelet system satisfies $\mu(\boldsymbol{A}_0) \propto \log(n)$!

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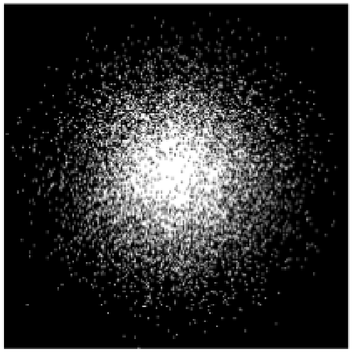
Evolution of Compressed Sensing

Variable density sampling with structure

Assume that $\boldsymbol{x}$ has a certain "sparsity structure".

Set $\boldsymbol{A}_\Omega = \begin{pmatrix} \boldsymbol{a}_{J_1} \\ \vdots \\ \boldsymbol{a}_{J_m} \end{pmatrix}$ with $\mathbb{P}(J_k = \ell) = \tilde{\pi}_\ell$

$m \geq C \cdot s \cdot \log(\frac{n}{\epsilon})$ $\Rightarrow$ exact recovery with probability $1 - \epsilon$



Variable density sampling + structure

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Generalized sampling and infinite-dimensional compressed sensing.
Foundations of Computational Mathematics, to appear, 2015.

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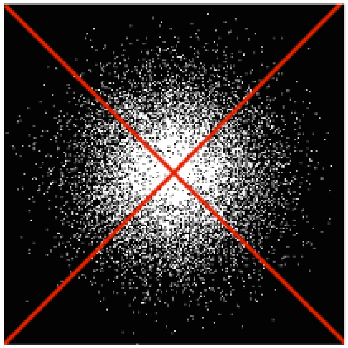

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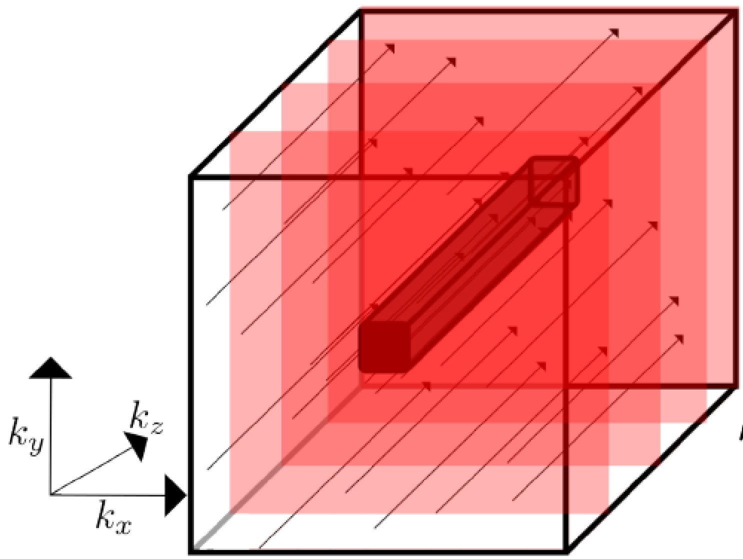
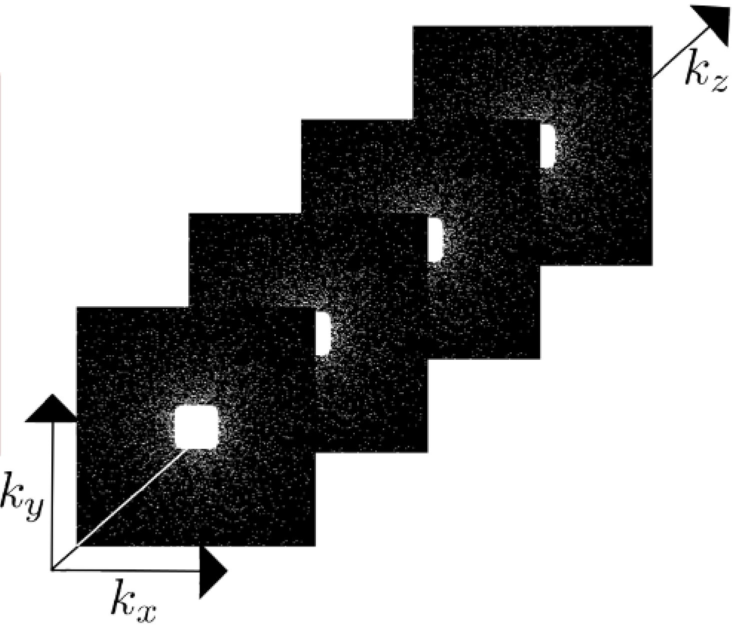
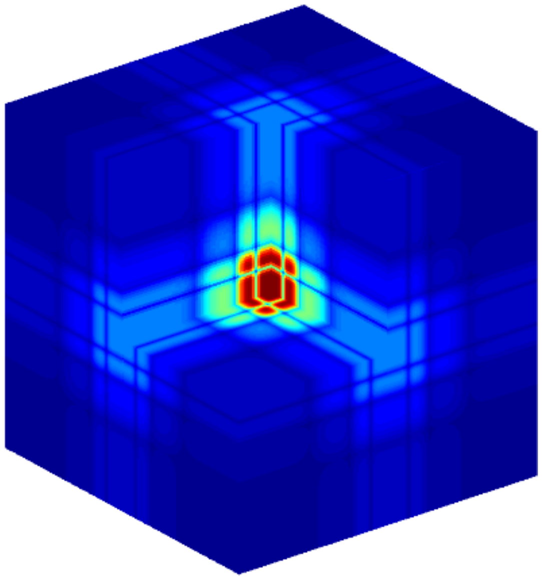
Only works for isolated measurements!

Current compressed sensing theories are partially disconnected from applications!

Possible implementation







π in 3D

Perform the readout along the third dimension!
Suboptimal as compared to π in 3D.

Outline

Part II: Compressed Sensing in MRI

- Evolution of Compressed Sensing
- CS with blocks of measurements

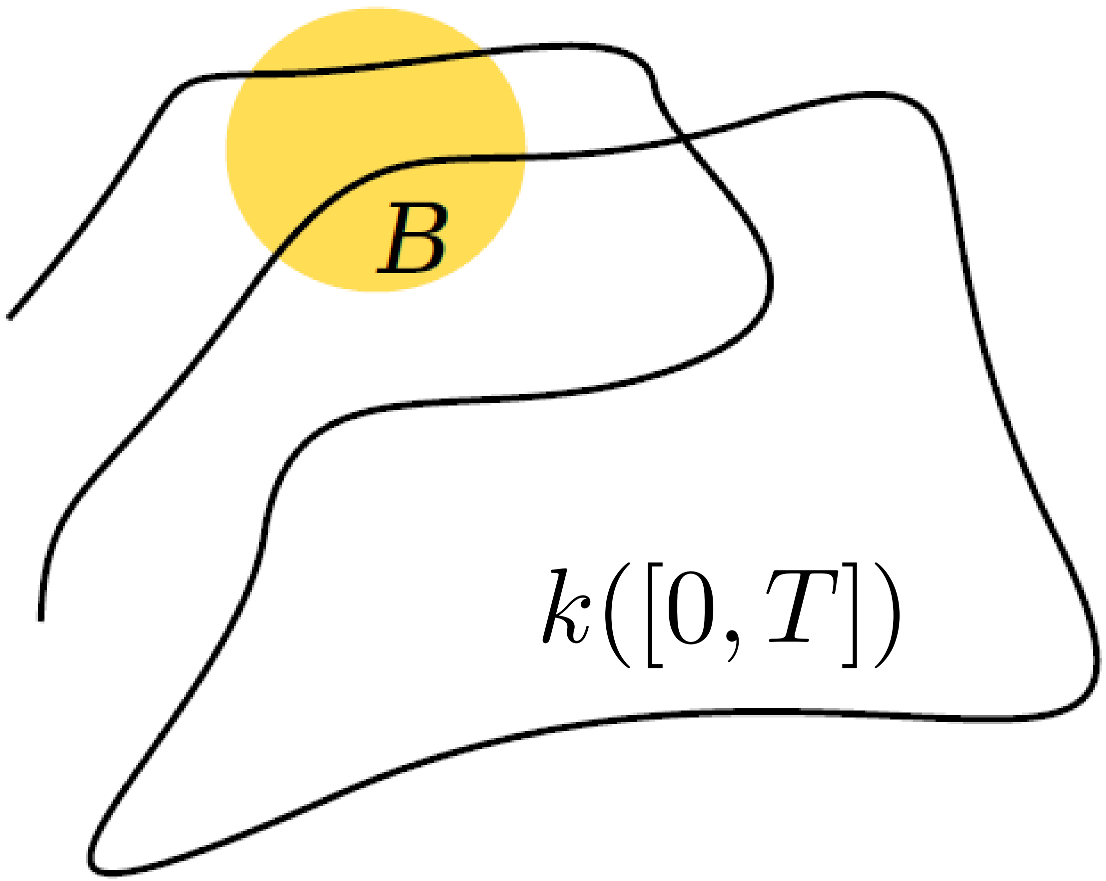
Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

Continuous Variable Density Sampler

Pushforward measure - illustration



Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.
Variable density sampling with continous sampling trajectories.
SIAM Journal on Imaging Science, 2014.

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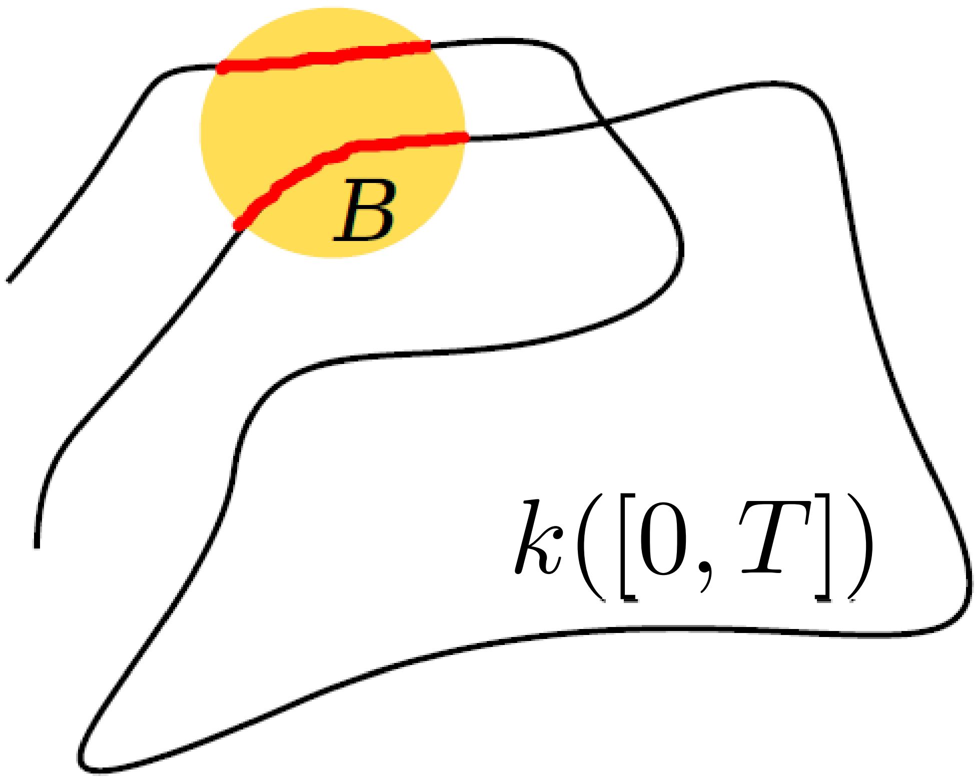
Pushforward measure - illustration



$$\nu(B) = k_* \lambda_T(B) = \lambda_T(k^{-1}(B))$$

λ_T is the (normalized) Lebesgue measure.

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Push Forward Measure - Definition

Pushforward measure

Let $\Omega = [0,1]^d$, where $d = 2$ or 3 denote the space dimension. We equip Ω with the Borel algebra $\mathcal{B}$. Let (X, Σ) be a measurable space and $k : X \mapsto \Omega$ be a measurable mapping. $\mu : X \mapsto [0; +\infty)$ denote a measure. The *pushforward measure* ν of μ is defined by:

$$\nu(B) = k_* \mu(B) = \mu(k^{-1}(B)), \quad \forall \in \mathcal{B}$$

Ex. 1: Measures supported by curves

Ex. 2: Atomic measures

$k : \{1, \ldots, m\} \mapsto \Omega$ where $k[i] = p_i$ denotes the i-th point. Set μ as the *counting measure* defined for any set $I \subseteq \{1, \ldots, m\}$ by $\mu(I) = \frac{|I|}{m}$. Then ν is defined by

$$\nu = \frac{1}{m} \sum_{i=1}^{m} \delta_{p_i}$$

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Push Forward Measure - Theorem

Weak convergence

A sequence of measures (μ_n) is said to weakly converge to μ, if for any bounded continuous function ϕ,

$$\int_\Omega \phi(x)d\mu_n(x) \underset{n\to+\infty}{\longrightarrow} \int_\Omega \phi(x)d\mu(x)$$

Shorthand notation: $\mu_n \rightharpoonup \mu$.

Variable density sampler

A sequence of (random) trajectories $k_n : X_n \mapsto \Omega$ is said to be a π-Variable Density Sampler if

$$k_{n*}\mu \rightharpoonup \pi \quad \text{almost surely}$$

Examples

i.i.d. drawing, random walks ...

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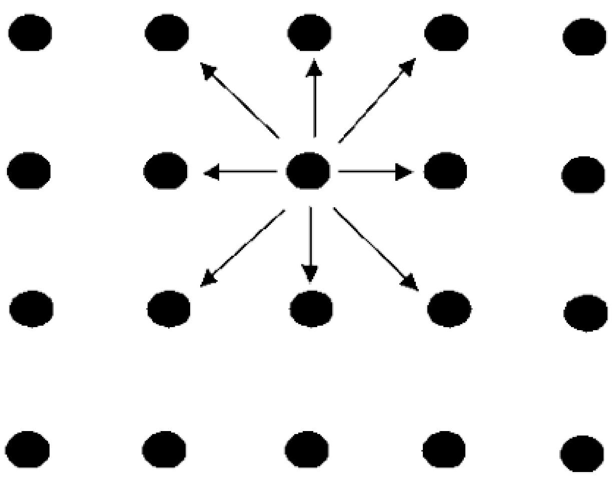
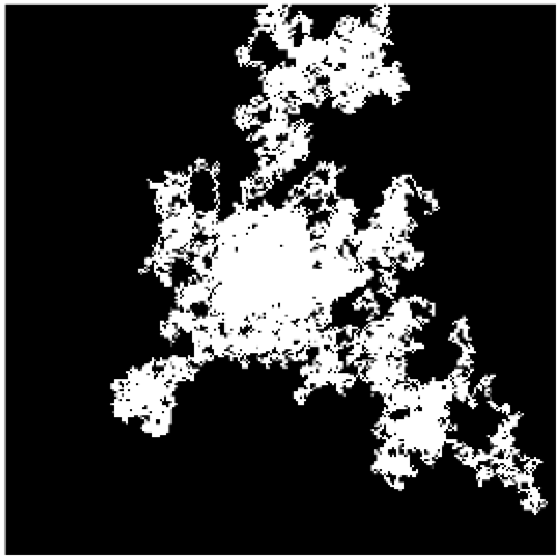
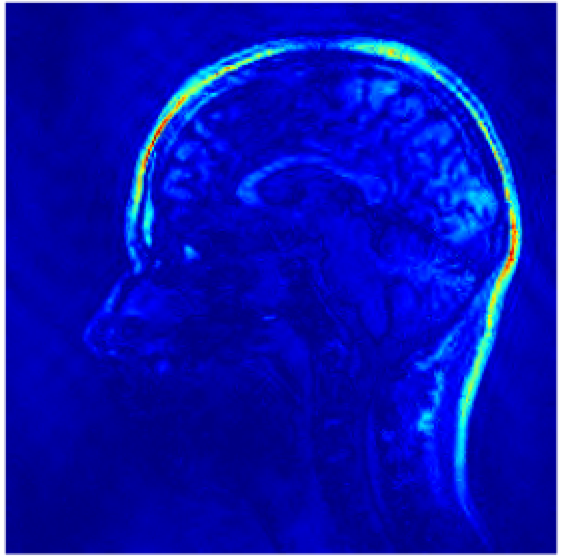
Sampling along Random Walks

Construction of a discrete Markov chain

Given a target probability distribution $\pi \in \mathbb{R}^n$.
Define a Markov chain $\boldsymbol{K} = (k_i)_{i \in \mathbb{N}}$ on the set $\{1, \ldots, n\}$. Use the Metropolis algorithm to construct a stochastic transition matrix $\boldsymbol{P} \in \mathbb{R}^{n \times n}$ such that π is the stationary distribution of $\boldsymbol{K}$.







- Time to cover the *k*-space is slow
- Local approach

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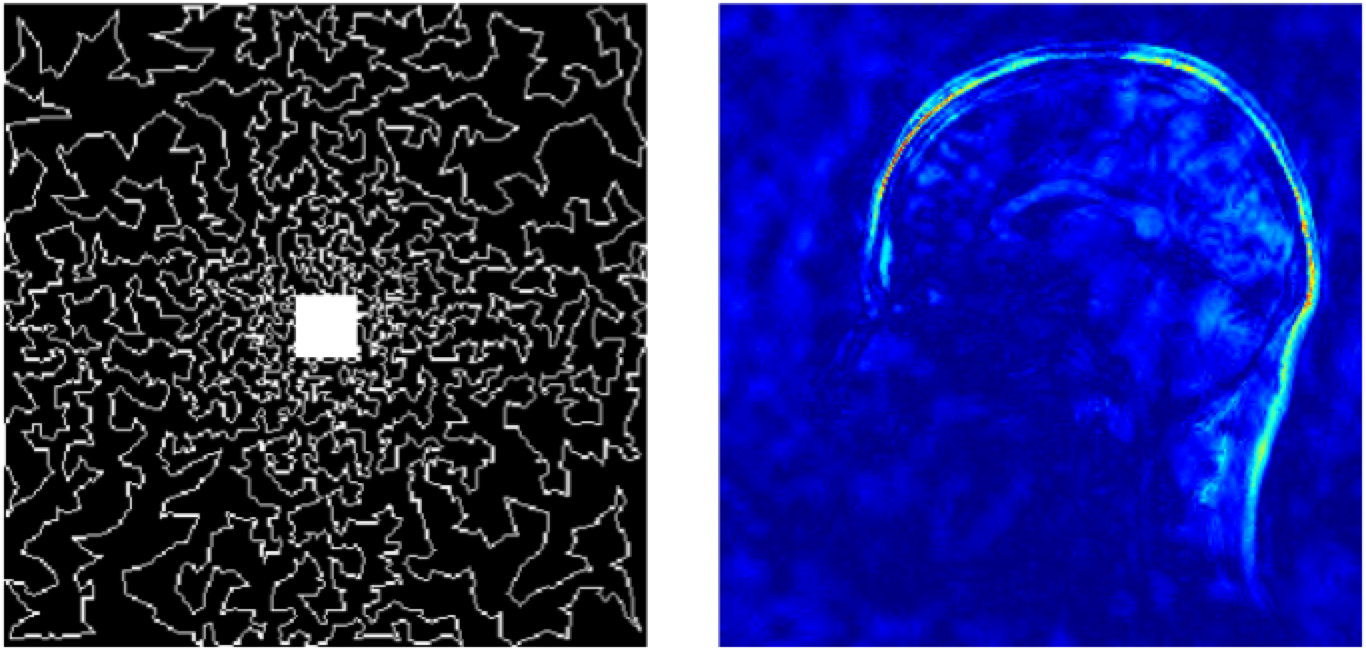
Solving the Travelling Salesman Problem

Idea : cover the k-space more quickly with a global approach.

(a) (b) PSNR=24.1dB



- Pushforward measure far from π. From which distribution should we sample the initial points to reach a given target distribution?

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TSP Sampler

- Let

$$q = \frac{\pi^{d/(d-1)}}{\int_\Omega \pi^{d/(d-1)}}$$

- $(k[i])_{i\in\mathbb{N}^*}$ a sequence of points in Ω, *i.i.d.* drawn $\sim q$.
- $\boldsymbol{k}_N = (k[i])_{i\leq N}$
- Denote $T(\boldsymbol{k}_N)$ the length of the TSP amongst $\boldsymbol{k}_N$.
- $\gamma_N : [0, T(\boldsymbol{k}_N)] \mapsto \Omega$ denotes the parametrization of the curve at speed 1.

Theorem

Almost surely w.r.t. the law $q^{\otimes\mathbb{N}}$ of the sequence $(k[i])_{i\in\mathbb{N}^*}$ of random points in the hypercube, $(\gamma_N)_{N_i n\mathbb{N}}$ is a π-variable density sampler, i.e.,

$$\gamma_{N*}\lambda_{T(\boldsymbol{k}_N)} \rightharpoonup \pi$$

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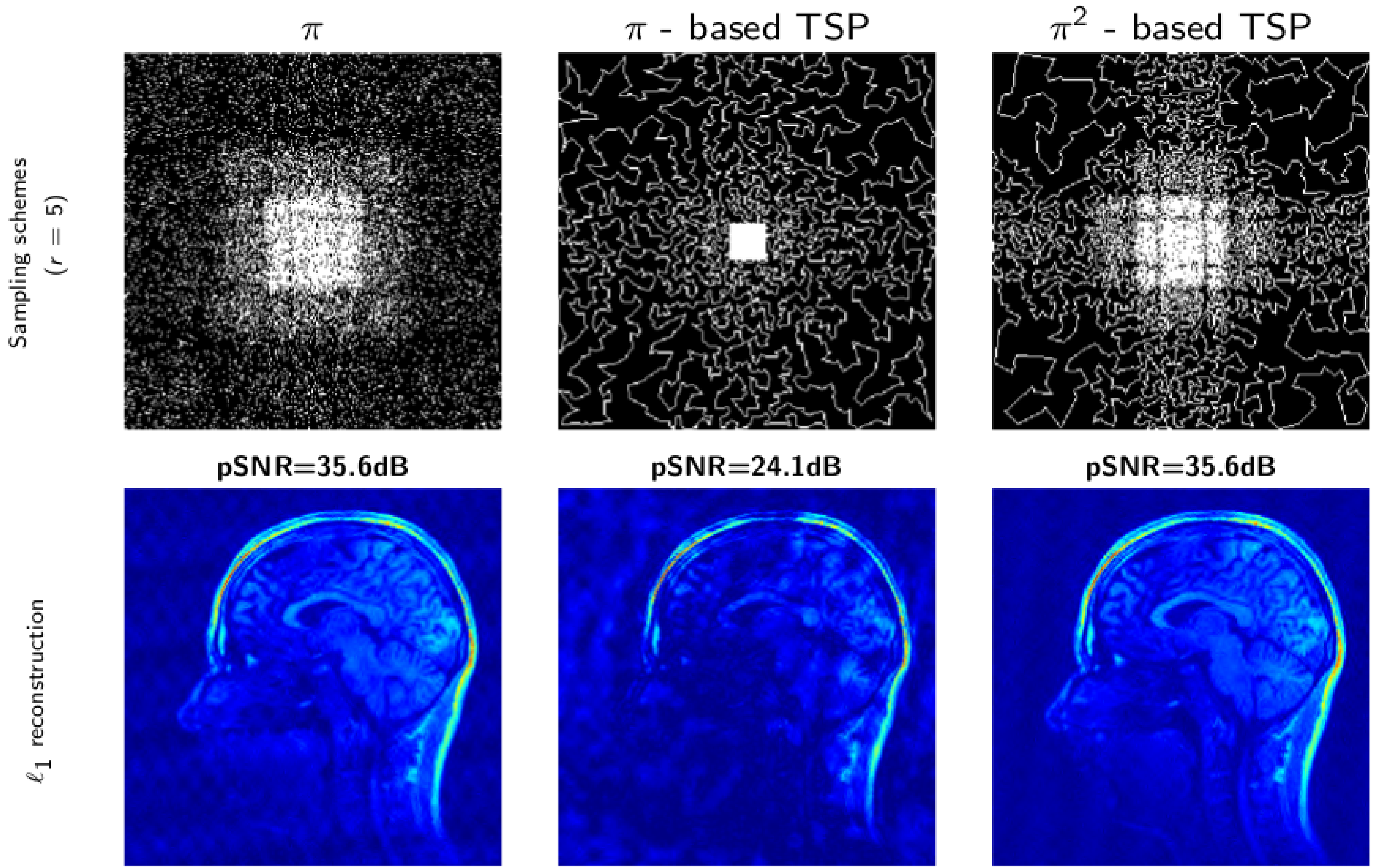
The Traveling Salesman Sampler in 2D



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The Traveling Salesman Sampler in 3D

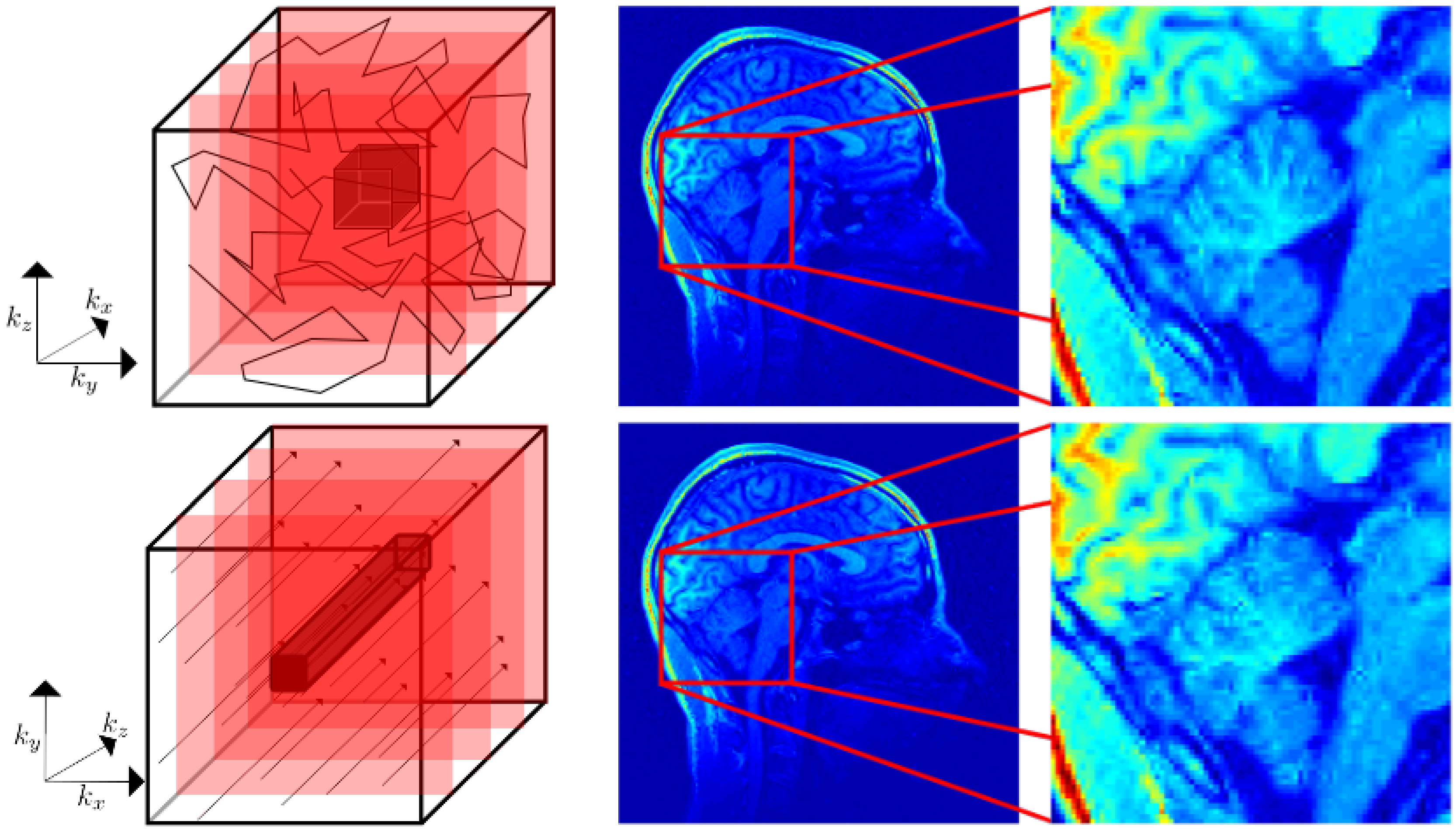

Figure: 3D reconstruction results for $r = 8.8$ for various sampling strategies. **Top row:** TSP-based sampling schemes (PSNR=42.1 dB). **Bottom row:** 2D random drawing and acquisitions along parallel lines [Lustig et al., 2007] (PSNR=40.1 dB).

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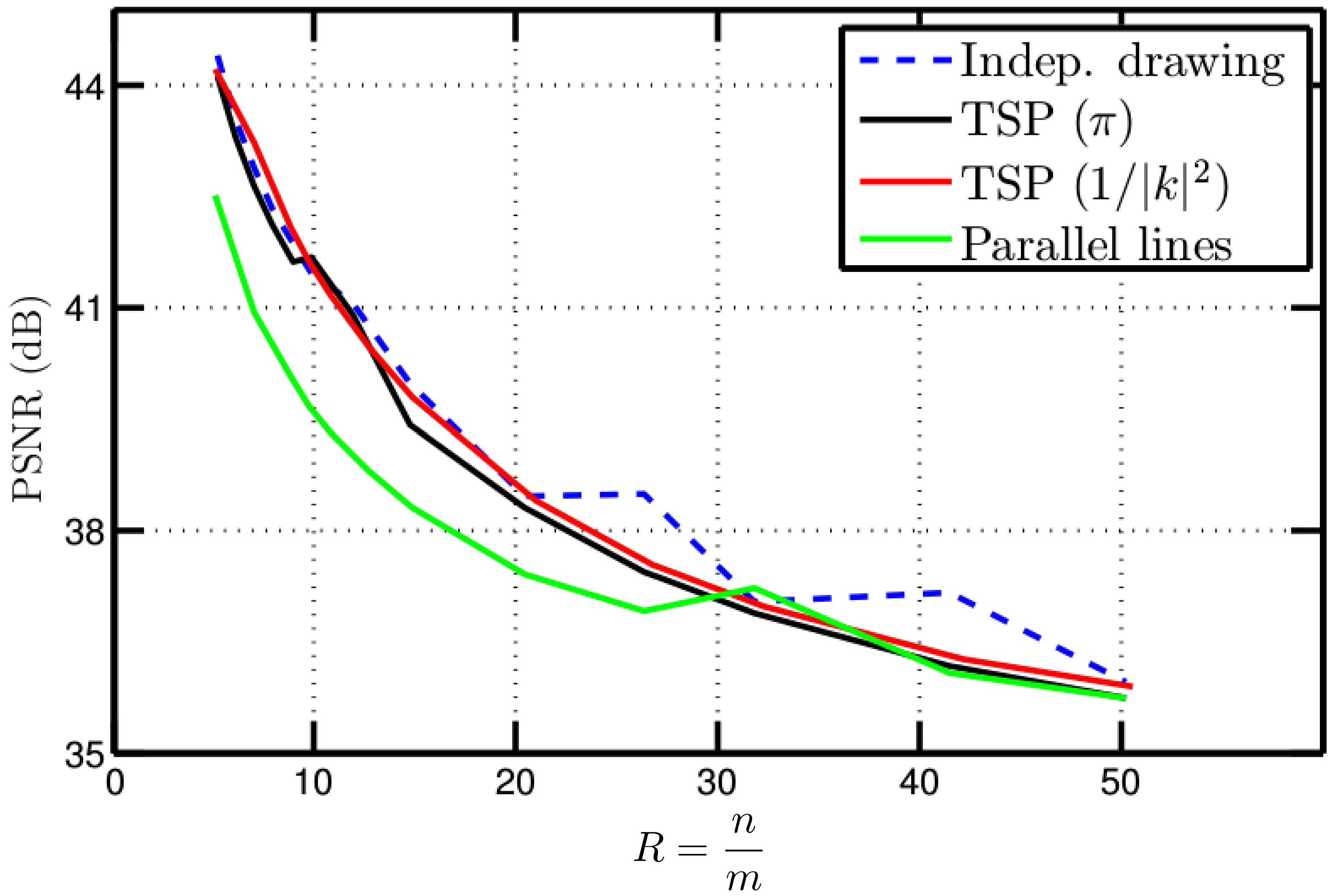
The Traveling Salesman Sampler in 3D



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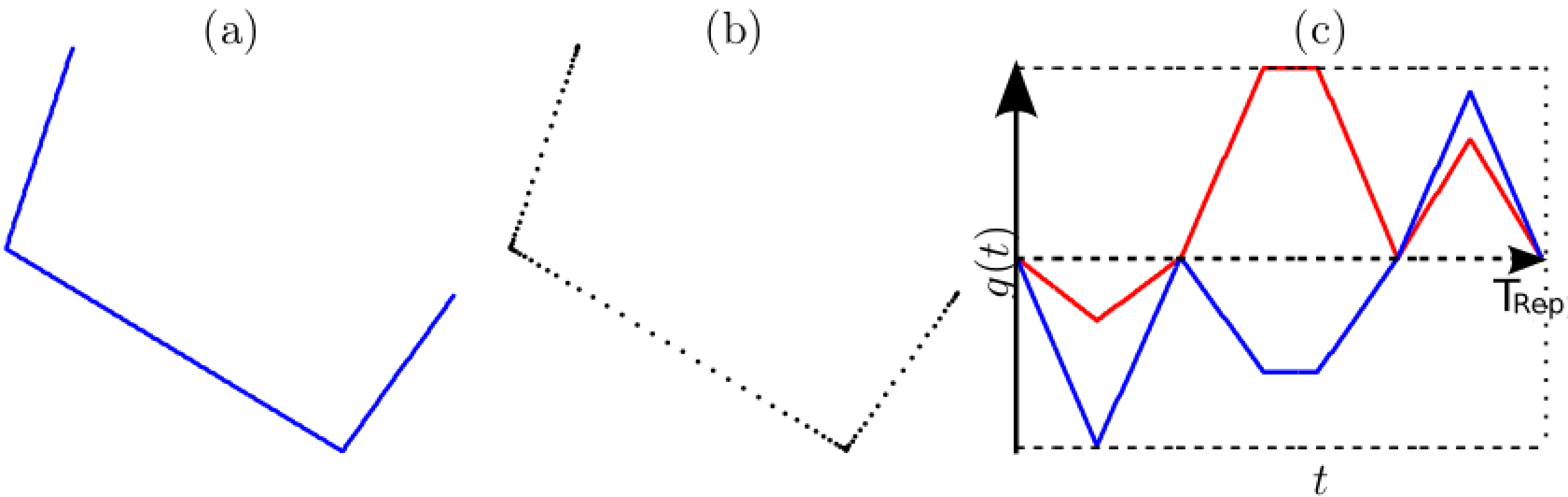
The Parameterization Problem

Finding a parameterization in $\mathcal{S}$ corresponding to a curve support is not easy !

- Classical approach, find an admissible parameterization [Hargreaves et al., 2004, Lustig et al., 2008]:

$$T_{\text{rep}} = \min T' \text{ such that } \exists r : [0, T'] \mapsto [0, T], c \circ r \in \mathcal{S}$$

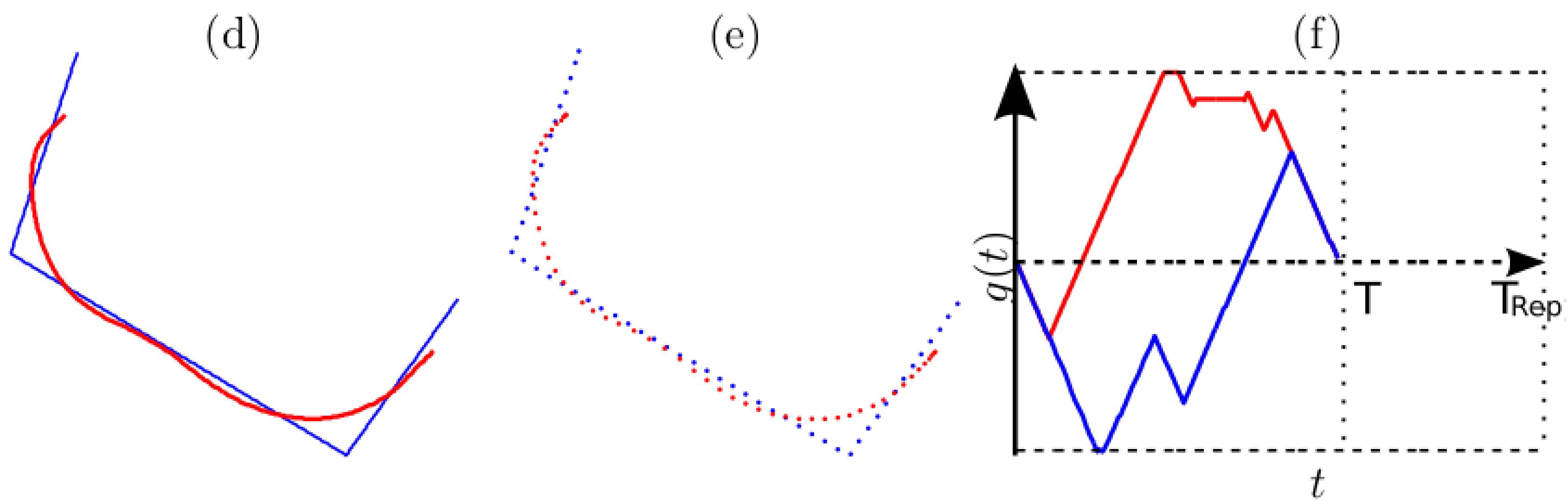


➡ Stick to the support of c

[Chauffert et al, IEEE TMI 2016]

Projection onto the MRI constraints

- Projection onto $\mathcal{S}$ [Chauffert et al., 2014b]



Leave the support of $\mathcal{C}$ but keep control on the sampling density

[Chauffert et al, IEEE TMI 2016]

The Projection Algorithm

For an input parameterized curve $c : [0,T] \mapsto \Omega$, define:

$$P_{\mathcal{S}}(c) = \arg\min_{k \in \mathcal{S}} \int_0^T (k(t) - c(t))^2 \, dt$$

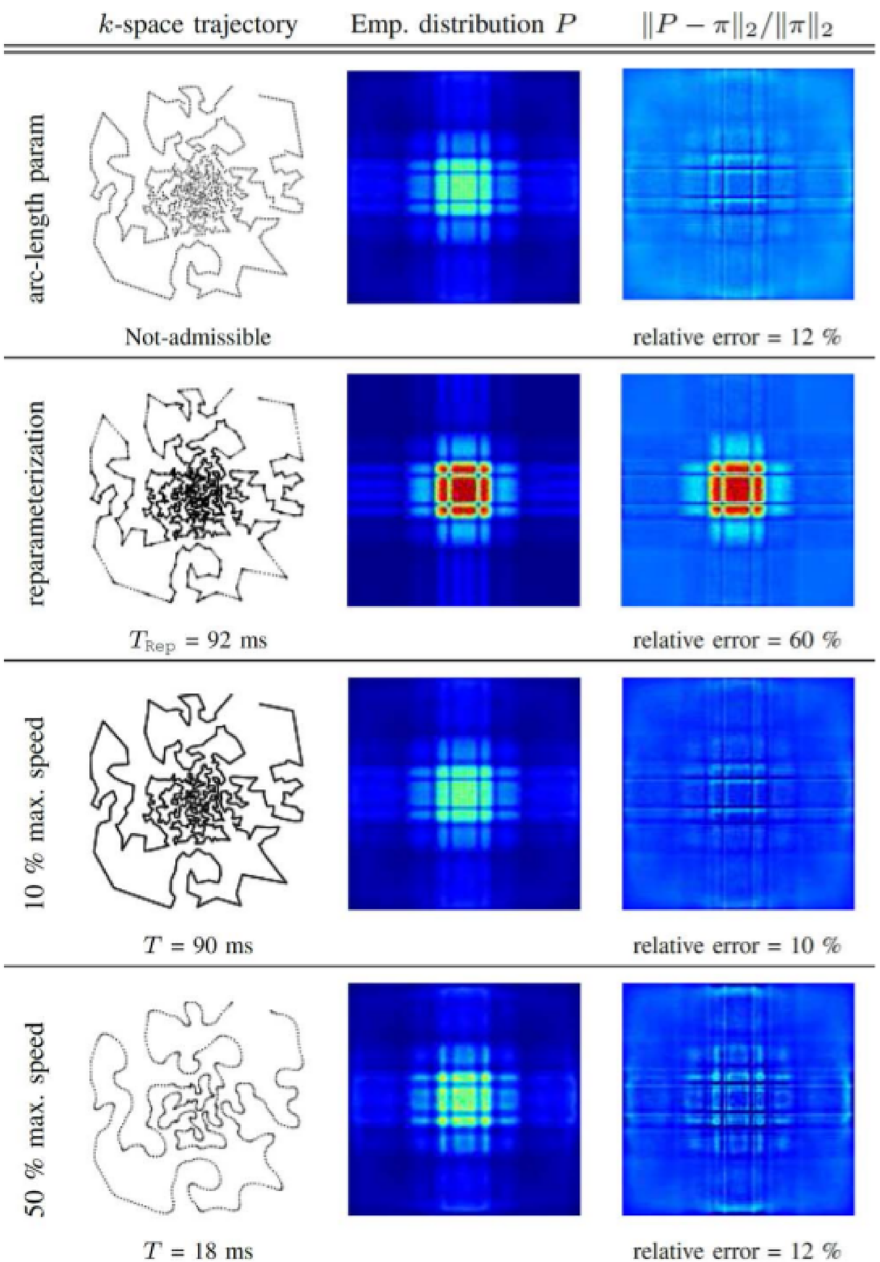
Main properties [Chauffert et al., 2014b]

- Fast resolution using accelerated proximal gradient descent on the dual.
- The sampling time is fixed (equal to T).
- The sampling distribution is well preserved (approximation of Wasserstein distance $\mathcal{W}_2$).

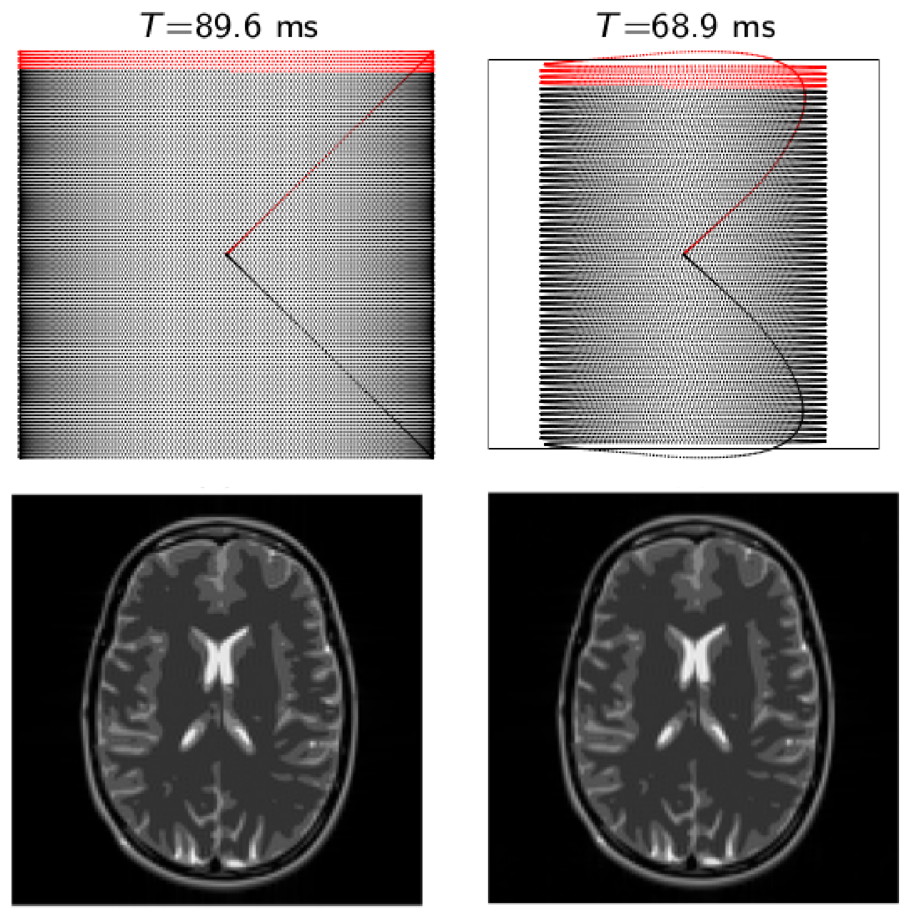
⇒ Leave the support of c but keep control on the sampling density

[Chauffert et al, IEEE TMI 2016]

Illustration & Results



EPI trajectories



[Chauffert et al, IEEE TMI 2016]

Interim Summary

- **Heuristic conconclusions**

  The main features characterizing the efficiency of a given sampler are :

  - Its empirical (pushforward) measure/density
  - Its mixing time, i.e. its ability to cover k-space rapidly

- **The new quest …**

  Design a variable density sampler with:

  1) Prescribed density
  2) Good mixing properties

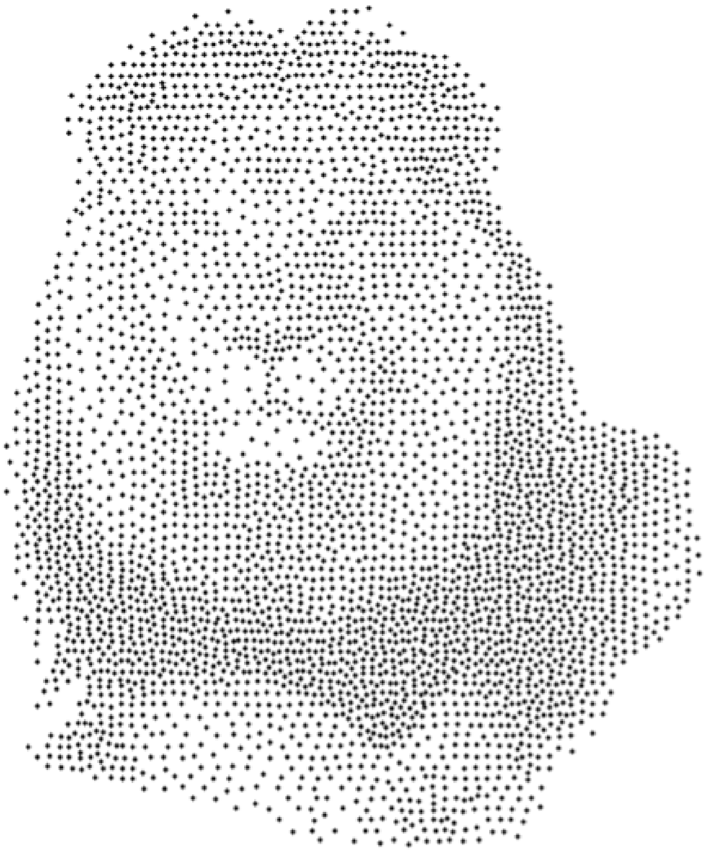
  - Belonging to a certain class, i.e. being compliant with MR hardware constraints and a given imaging contrast (e.g. T_1, T_2, etc)

Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

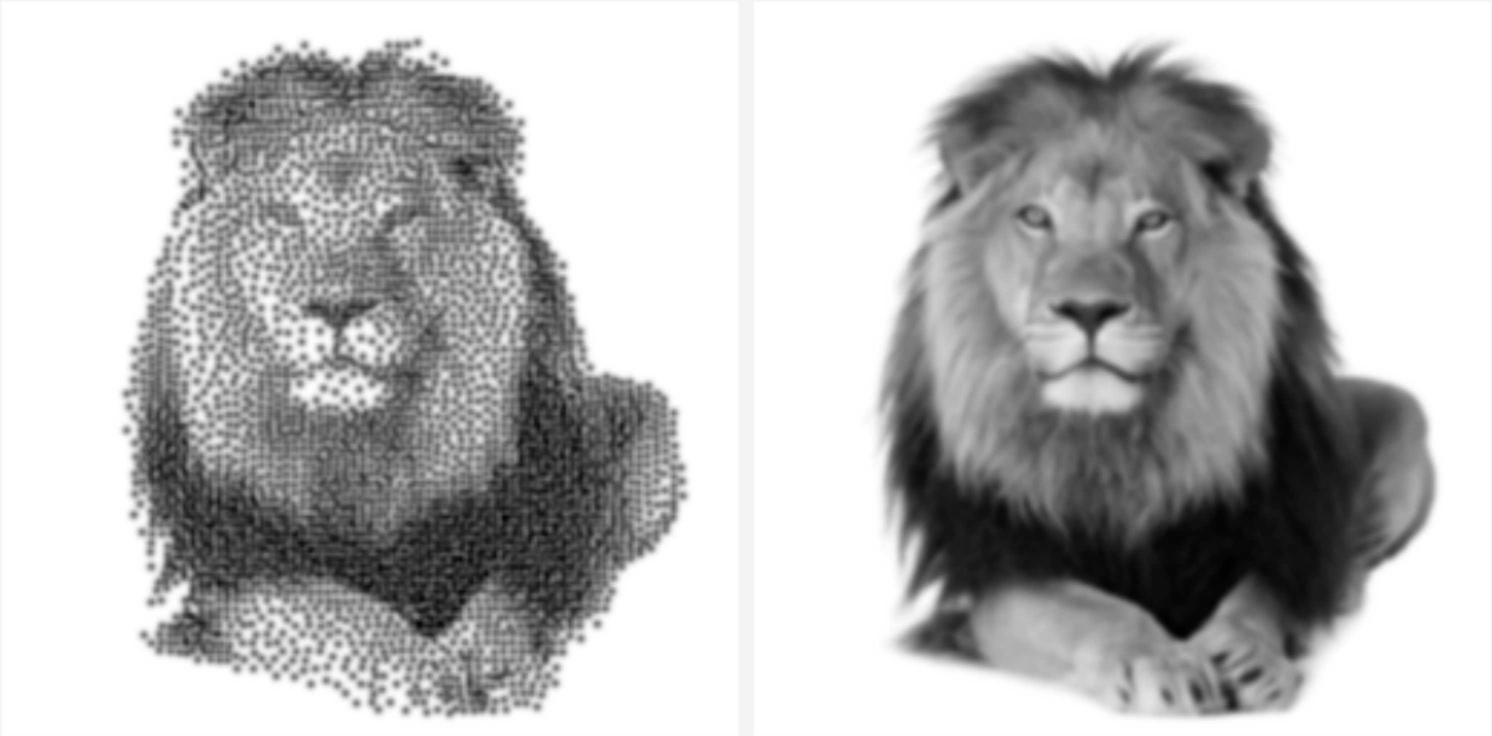
How to measure similarity between images?



Halftoned and reference images

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 1$.

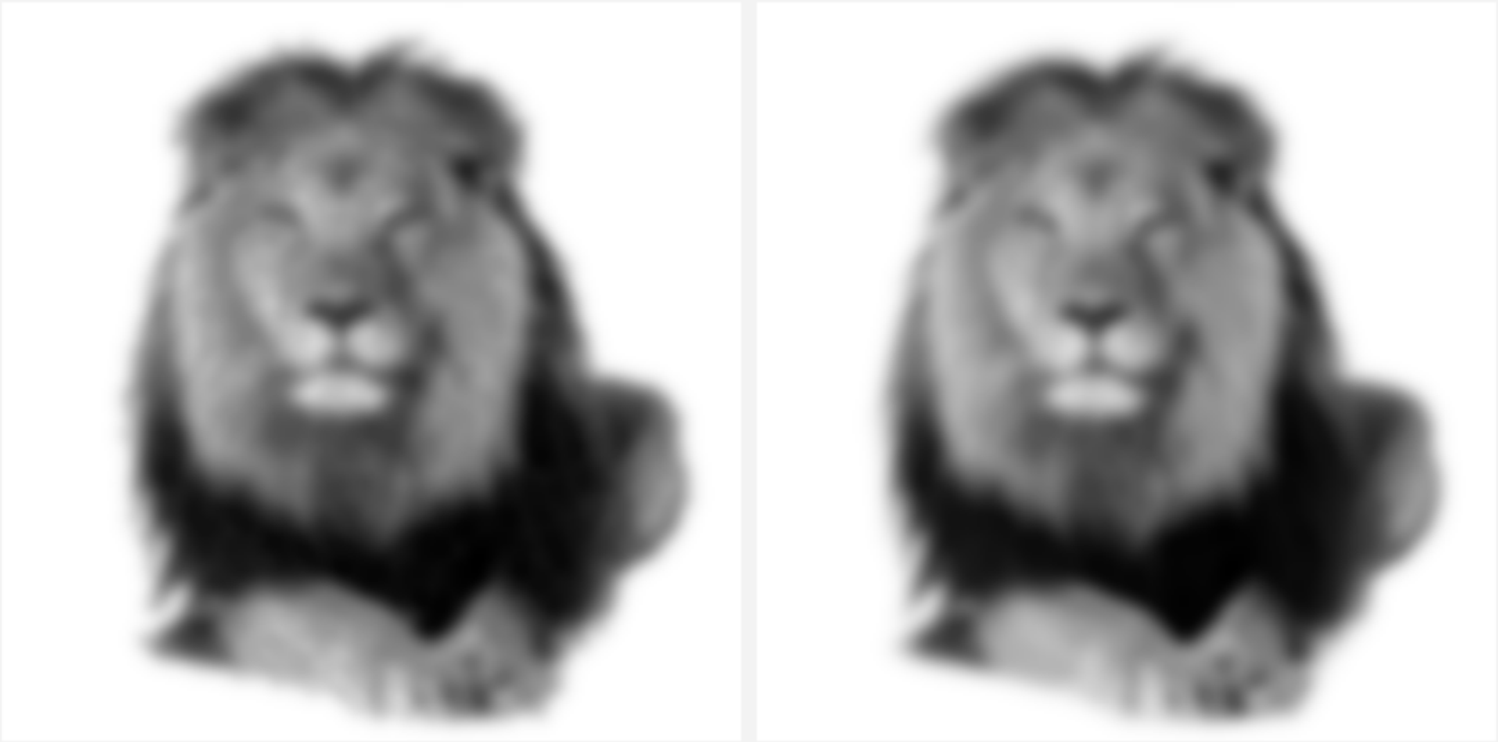
Why are the images alike?

The multiresolution feature of the human visual system.

Convolution with a Gaussian $\sigma = 2$.

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 3$.

The images are similar only at **low resolution**

Halftoning

Constructing a metric

Let π denote the target image (the lion).
Let ν denote the atomic measure (the dots).

A natural distance to quantify image similarity reads:

$$\mathcal{D}_h(\nu - \pi) = \|h \star \nu - h \star \pi\|_2 \,.$$

The kernel h can be chosen as the impulse response of the visual system.

[Chauffert et al, Construct Approx, 2017]

Halftoninig as a projection problem

How to find the point locations?

Let $\Omega \subseteq \mathbb{R}^d$ denote the image domain.
Let $\mathcal{M}(\Omega^N)$ denote the set of atomic measures with N points.

$$\mathcal{M}(\Omega^N) = \left\{ \nu = \frac{1}{N} \sum_{i=1}^{N} \delta_{p_i}, \, p_i \in \Omega \right\}$$

The halftoning problem can be formulated as a projection problem:

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \mathcal{D}_h(\nu - \pi) \, .$$

[Chauffert et al, Construct Approx, 2017]

Link with Attraction-Repulsion

Proposition

The projection problem

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \frac{1}{2} \|h \star \nu - h \star \pi\|_2^2$$

can be rewritten

$$\min_{(p_1,\ldots,p_N)\in\Omega^N} \underbrace{\frac{1}{N}\sum_{i=1}^{N}\int_\Omega H(p_i - x)\pi(x)\,dx}_{\text{Attraction potential}} - \underbrace{\frac{1}{2N^2}\sum_{1\le i,j\le N} H(p_i - p_j)}_{\text{Repulsion potential}}$$

where $\hat{H} = |\hat{h}|^2$.

[Chauffert et al, Construct Approx, 2017]

Modeling MRI kinematic constraints

$$\boxed{\min_{\boldsymbol{k}\in\mathcal{Q}_N} \frac{1}{2}\|h\star\nu(\boldsymbol{k}) - h\star\pi\|_2^2 \qquad \text{with } \nu(\boldsymbol{k}) = \frac{1}{N}\sum_{i=1}^{N}\delta_{k[i]},\ \ \boldsymbol{k} = (k[i])_{1\le i\le N}\in\mathcal{Q}_N}$$

$$\text{where } \mathcal{Q}_N = \{\|\dot{\boldsymbol{k}}\|_{2,\infty} \le \gamma G_{\max},\ \|\ddot{\boldsymbol{k}}\|_{2,\infty} \le \gamma S_{\max}\}$$

$$\min_{\boldsymbol{k}\in\mathcal{Q}_N} \underbrace{\frac{1}{N}\sum_{i=1}^{N}\int_\Omega H(x-k[i])\pi(x)\,dx}_{F_{\mathrm{a}}(\boldsymbol{k})} - \underbrace{\frac{1}{2N^2}\sum_{1\le i,j\le N} H(k[i]-k[j])}_{F_{\mathrm{r}}(\boldsymbol{k})}$$

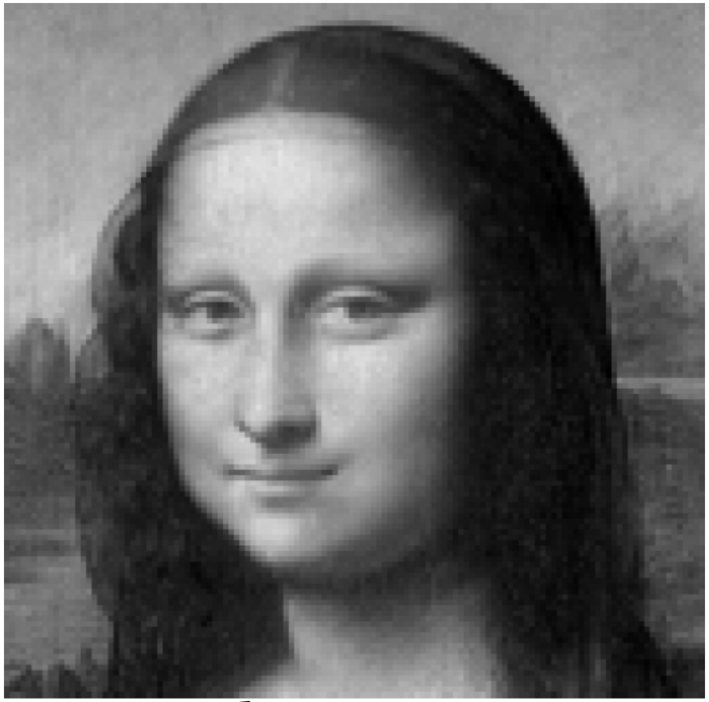
attraction *repulsion*

Projected gradient descent: $\boldsymbol{k}^{(t+1)} = P_{\mathcal{Q}_N}(\boldsymbol{k}^{(t)} - \tau^{(k)}\nabla(F_a - F_r)(\boldsymbol{k}^{(t)}))$

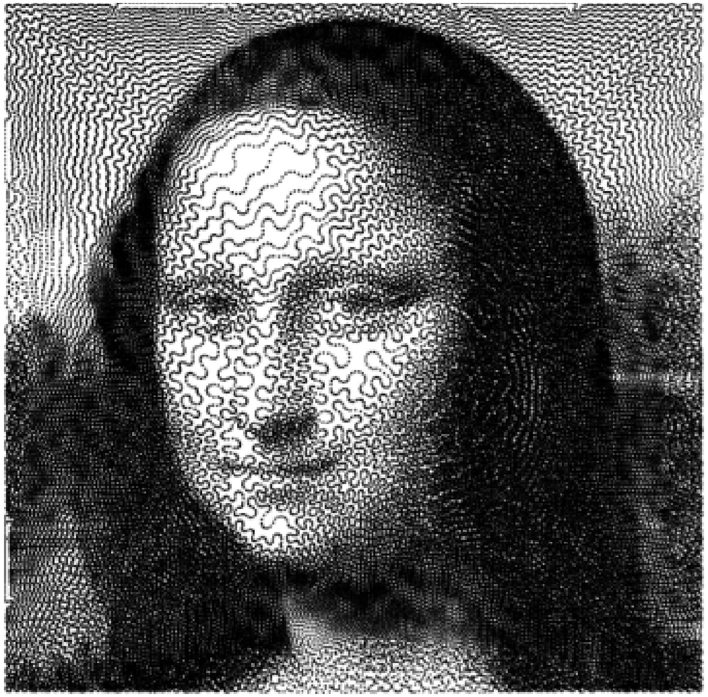
[Chauffert et al, IEEE TMI 2016]

Inertial adaptive step-size (BB update)

Image stippling with projection onto MRI kinematic constraints



$h_\sigma \star \pi$



$h_\sigma \star \nu$

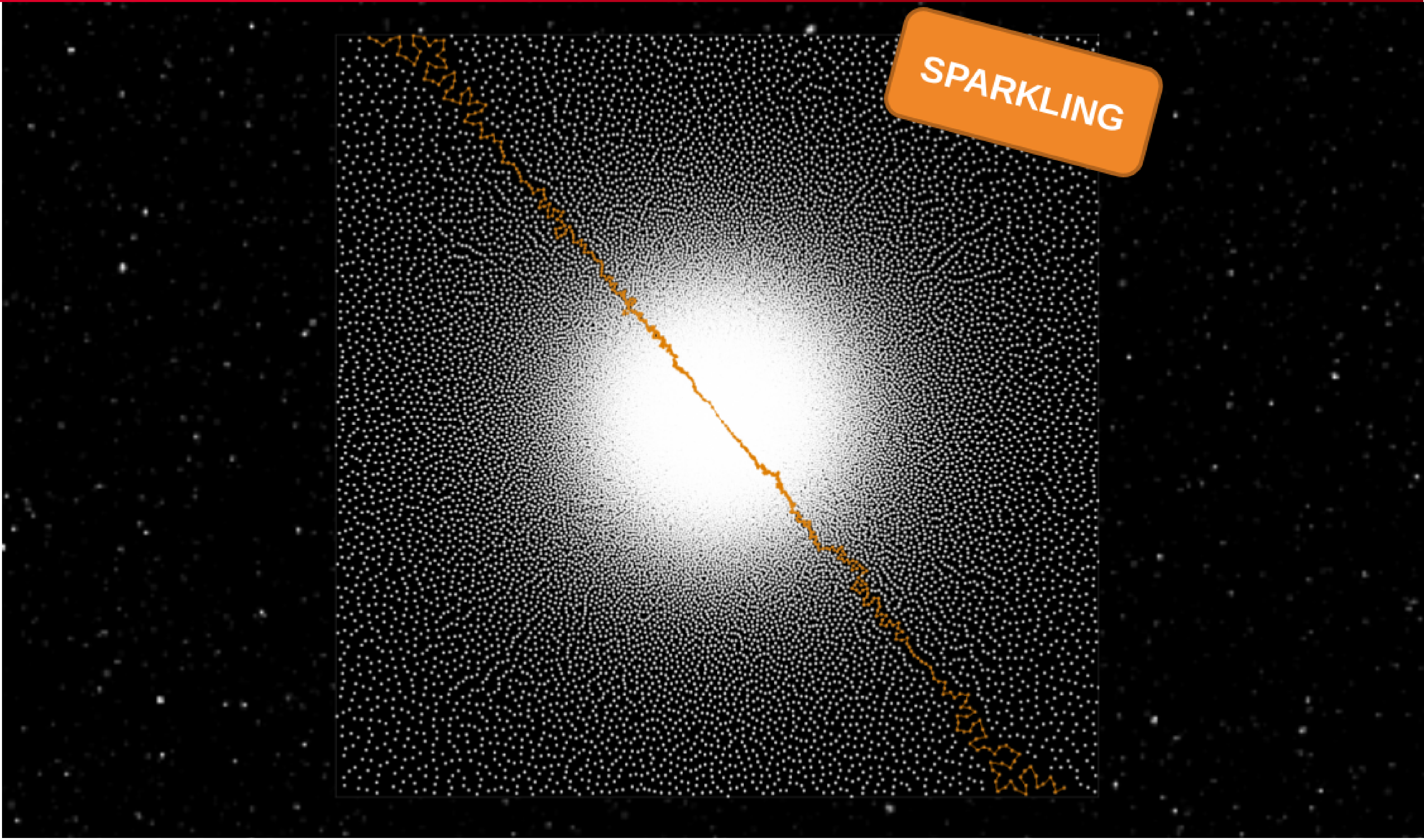
[Chauffert et al, Construct Approx, 2017]

Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

SPARKLING
Spreading Projection Algorithm for Rapid K-space sampLING



Sparkling recipe

Controlled Density

Locally uniform coverage

Hardware constraints

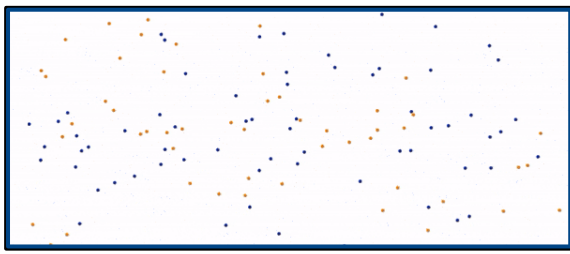
$$\begin{aligned} \|\dot{\boldsymbol{k}}\| &\leq \alpha \\ \|\ddot{\boldsymbol{k}}\| &\leq \beta \end{aligned}$$

SPARKLING

Spreading Projection Algorithm for Rapid K-space sampLING

Echo time constraints

Sparkling acceleration



Reduce #shots → Reduce Acquisition time

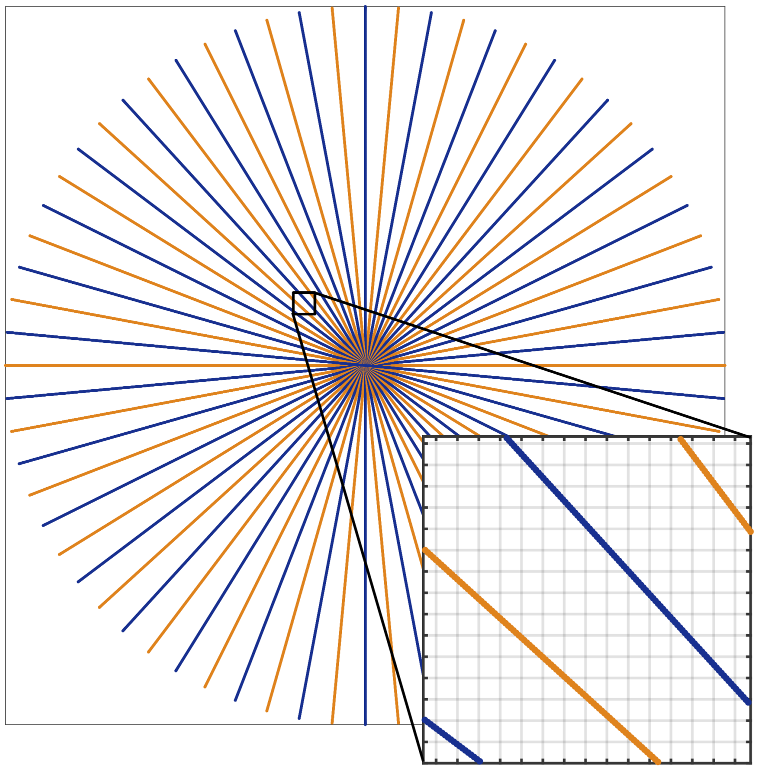
Acceleration Factor AF = N/ns
Subsampling Factor R = n/m
(with n=NxN=#pixels, ns=#shots and m=#samples)

$$\mathbf{AF \geq R}$$

Radial VDS Sparkling

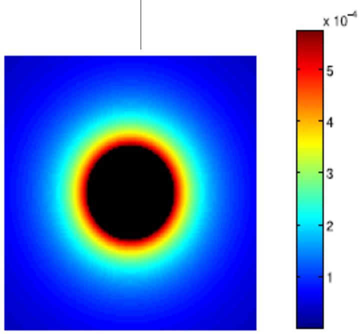
Tobs = 30 ms

Input



Gmax = 40 mT/m
Smax = 200 T/m/s
Δt = 10μs

SPARKLING

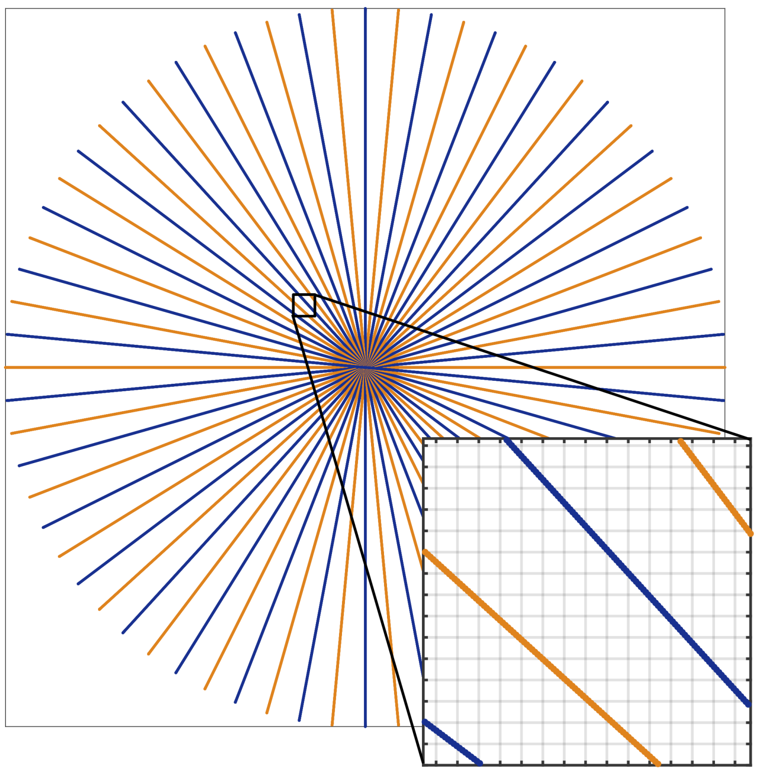


Acceleration factor in time:
AF = N/nc = 512/34 = 15

Radial VDS Sparkling

Tobs = 30 ms

Input



Gmax = 40 mT/m
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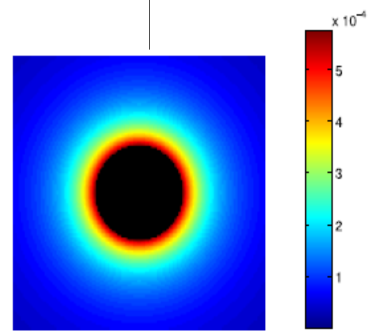

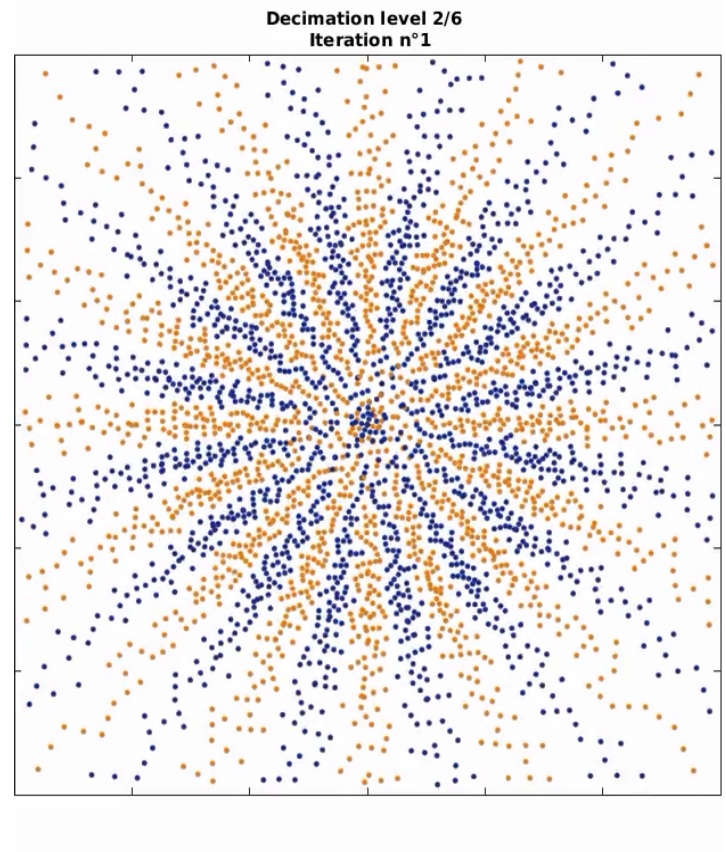

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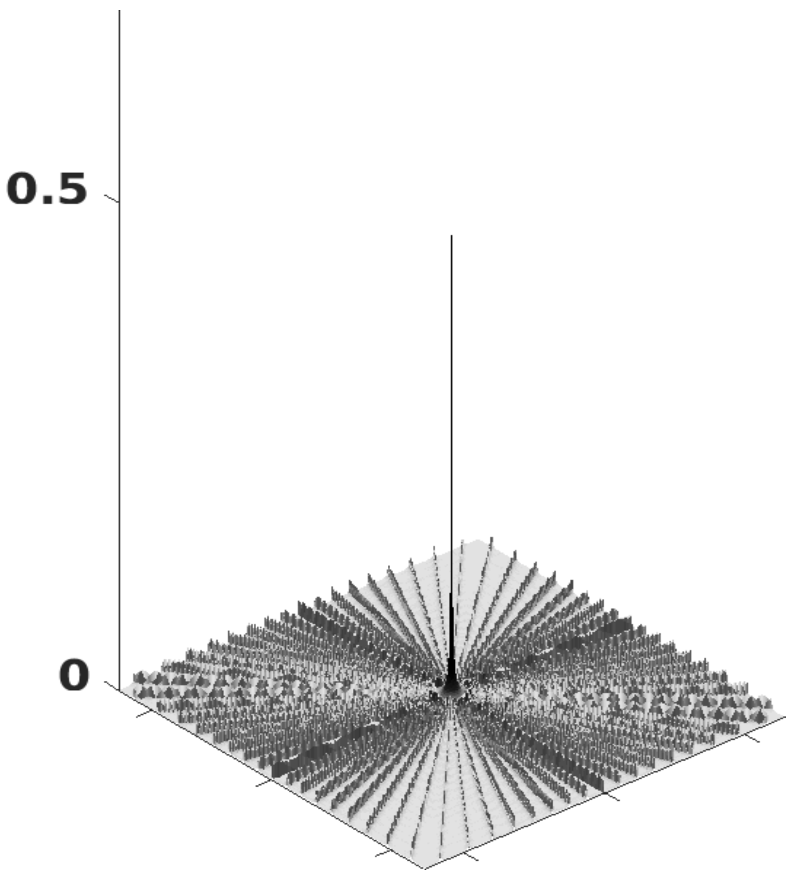

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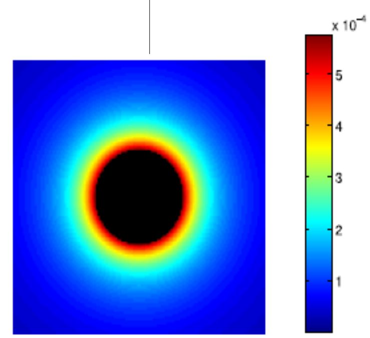
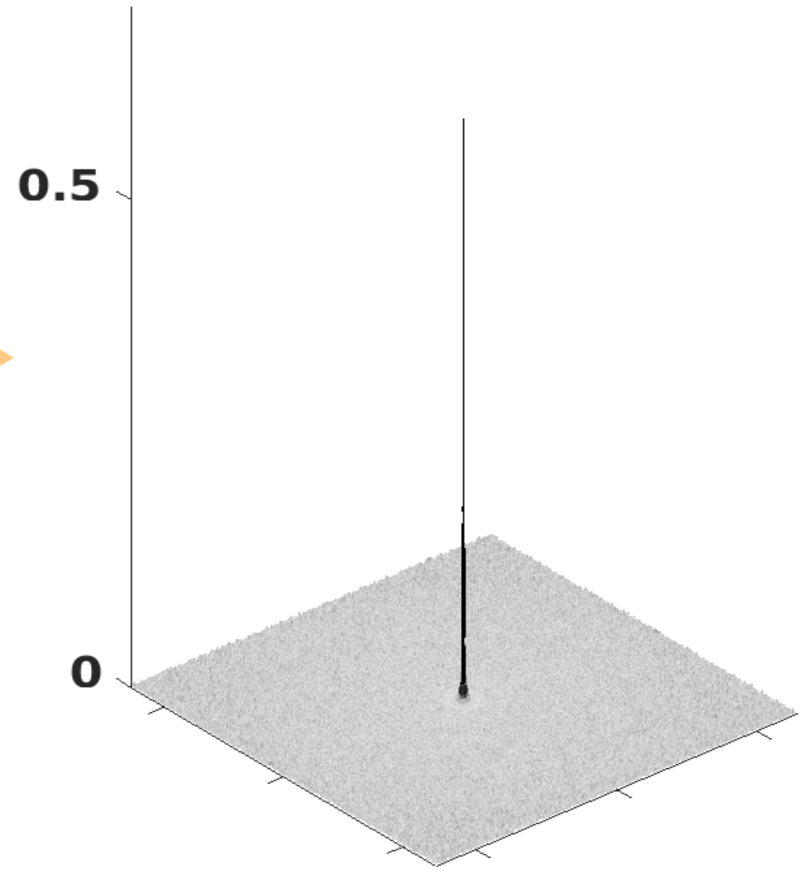


Output

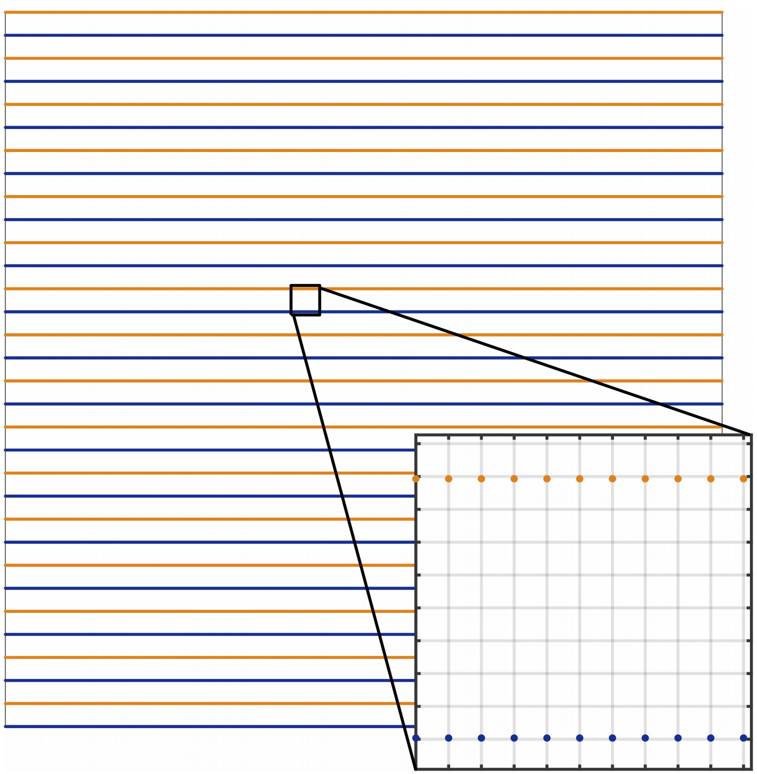


Acceleration factor in time:
AF = N/nc = 512/34 = 15

Uniform Sparkling

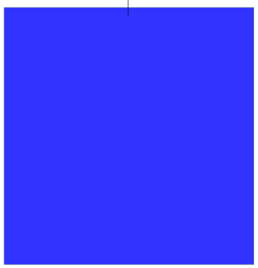
Tobs = 20 ms

Input



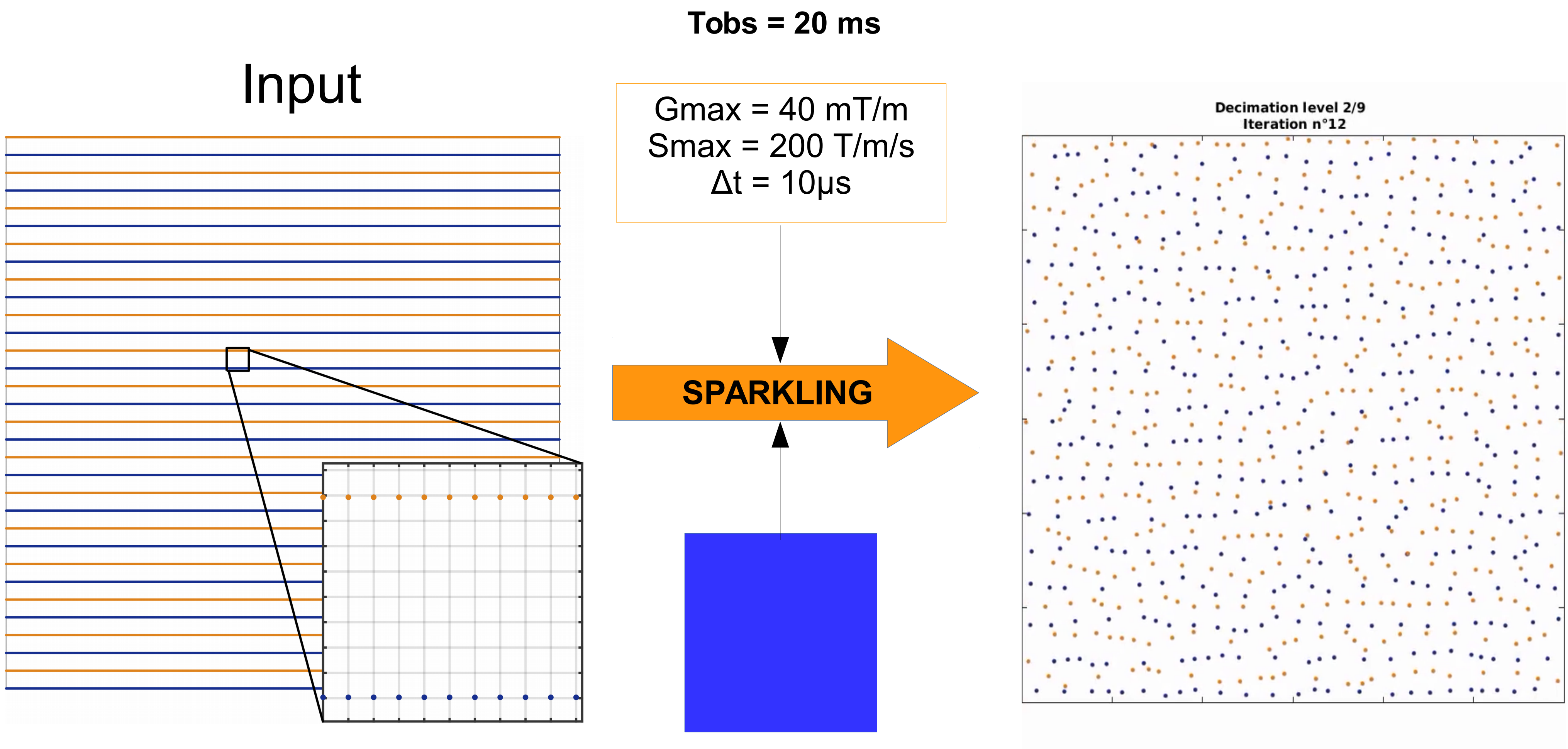
Gmax = 40 mT/m
Smax = 200 T/m/s
Δt = 10µs

SPARKLING



Acceleration factor in time:
AF = N/nc = 256/32 = 8

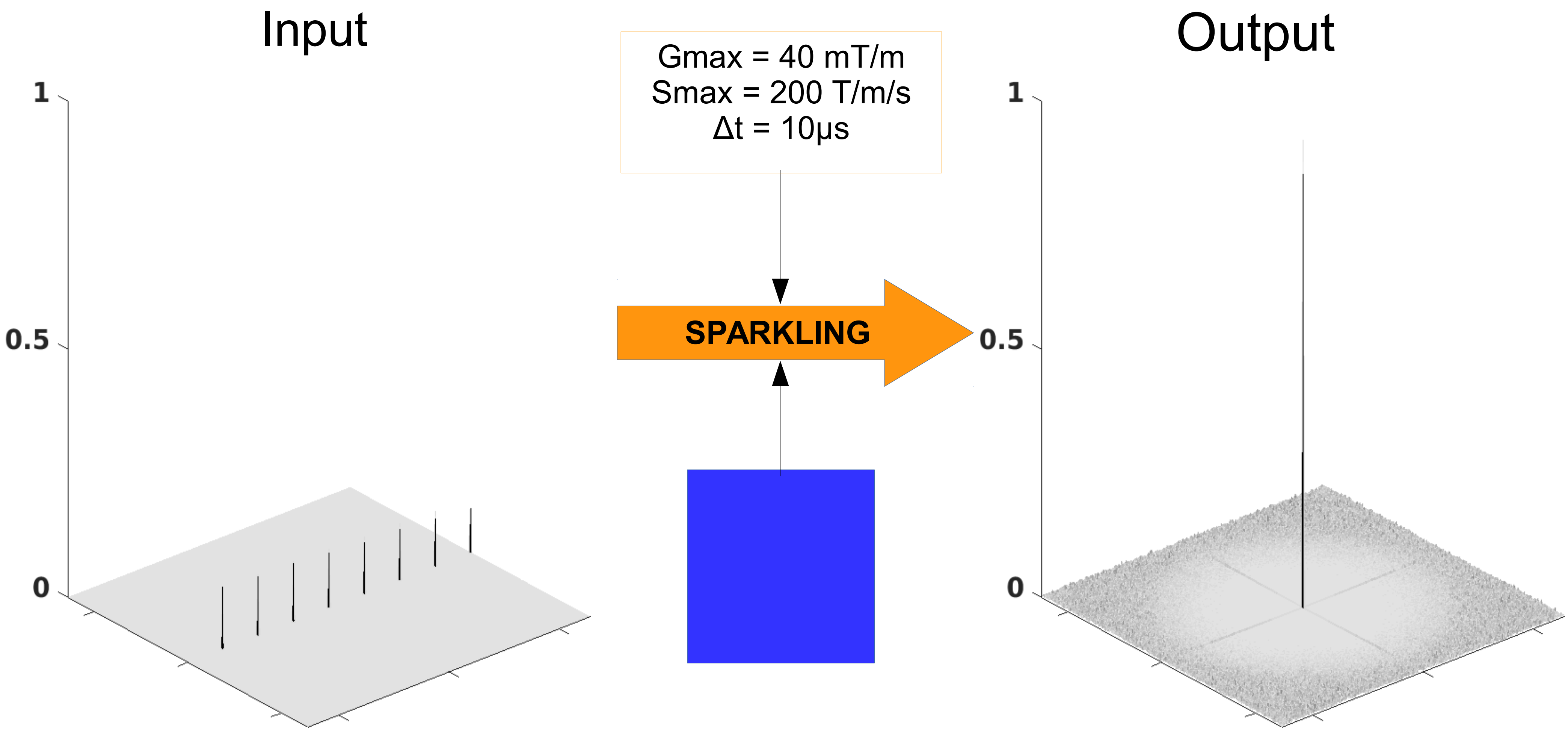
Uniform Sparkling



Acceleration factor in time:
AF = N/nc = 256/32 = 8

Uniform Sparkling

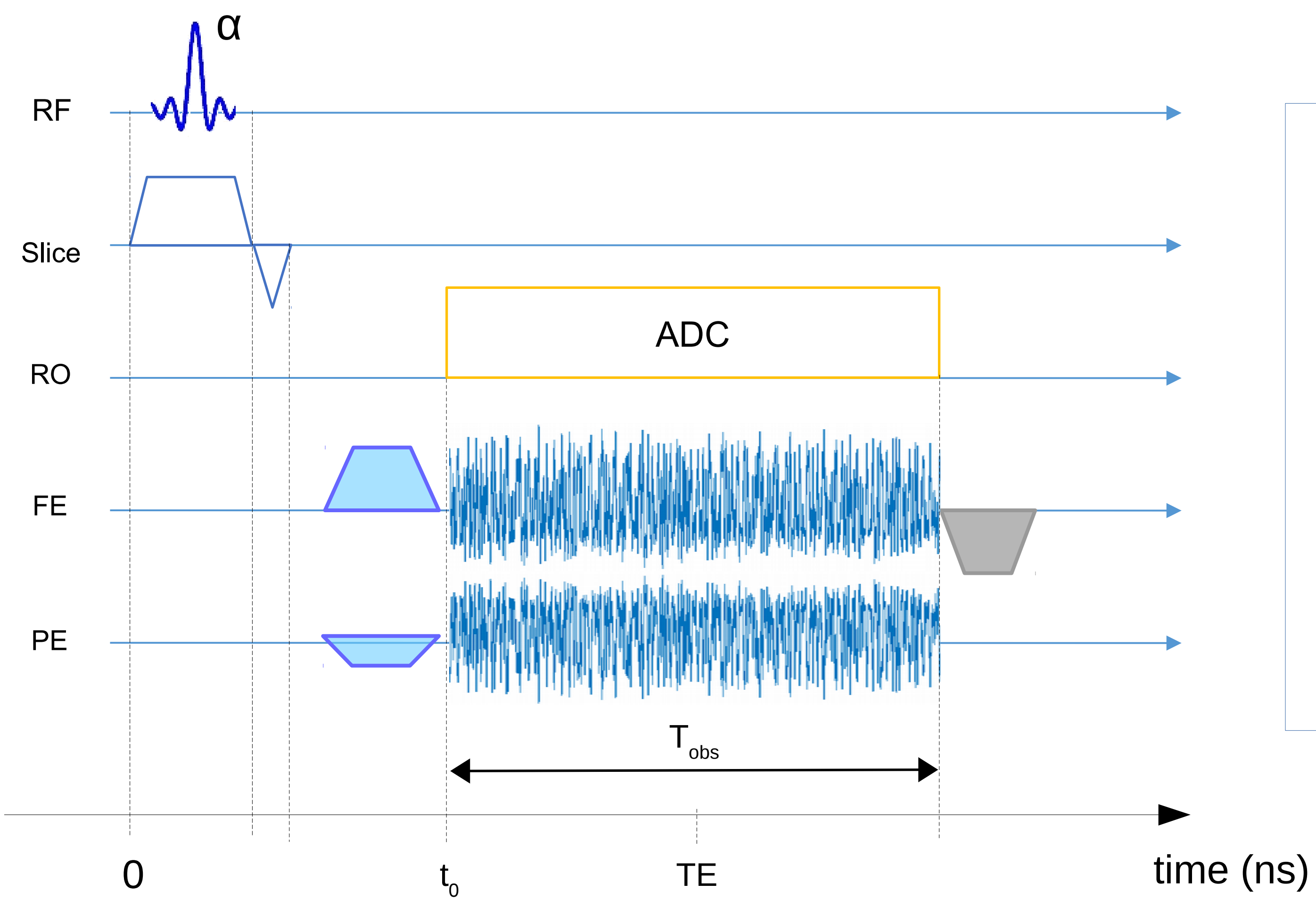
Tobs = 20 ms



Acceleration factor in time:
AF = N/nc = 256/32 = 8

Sequence Implementation

- T_2*-weighted Gradient-recalled echo sequence (GRE)
- Based on Siemens FLASH sequence



Sequence parameters:

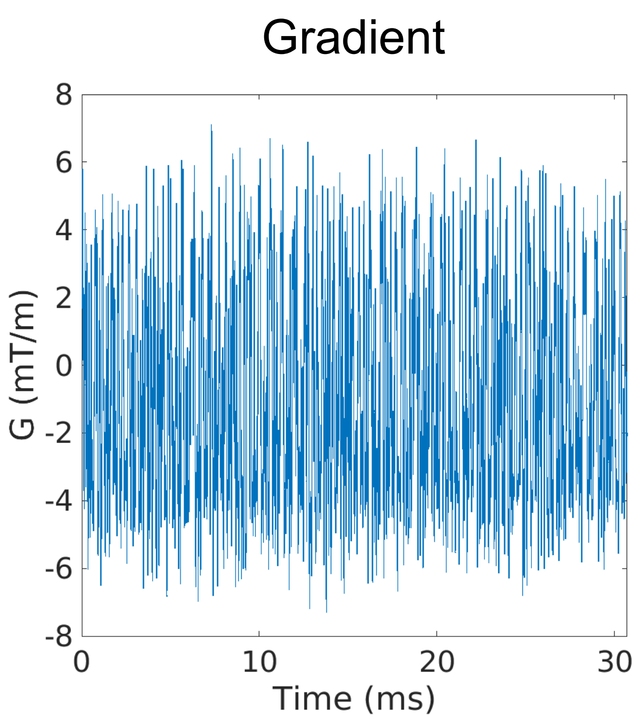
- N=512
- FOV=200x200 mm²
- **TR=550 ms → interleaved**
- TE=30 ms
- α=25°
- BW=32.55 Hz/px
- Tobs=30.72 ms
- Slice thickness: 3 mm
- 32-channel receiver coil

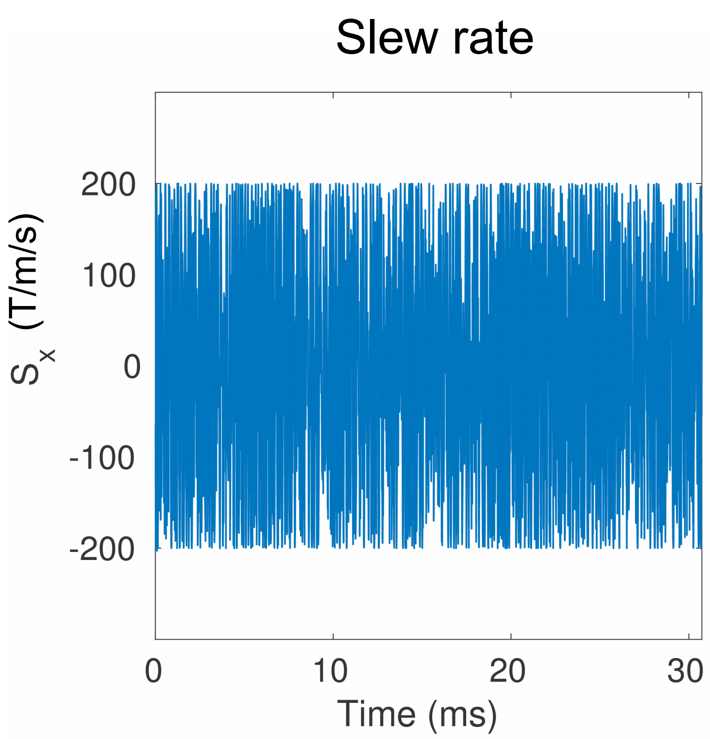
- Gradient raster time = 10 µs

Kasper et al. 2018. NeuroImage.

Sequence Implementation

Siemens 7 Tesla scanner





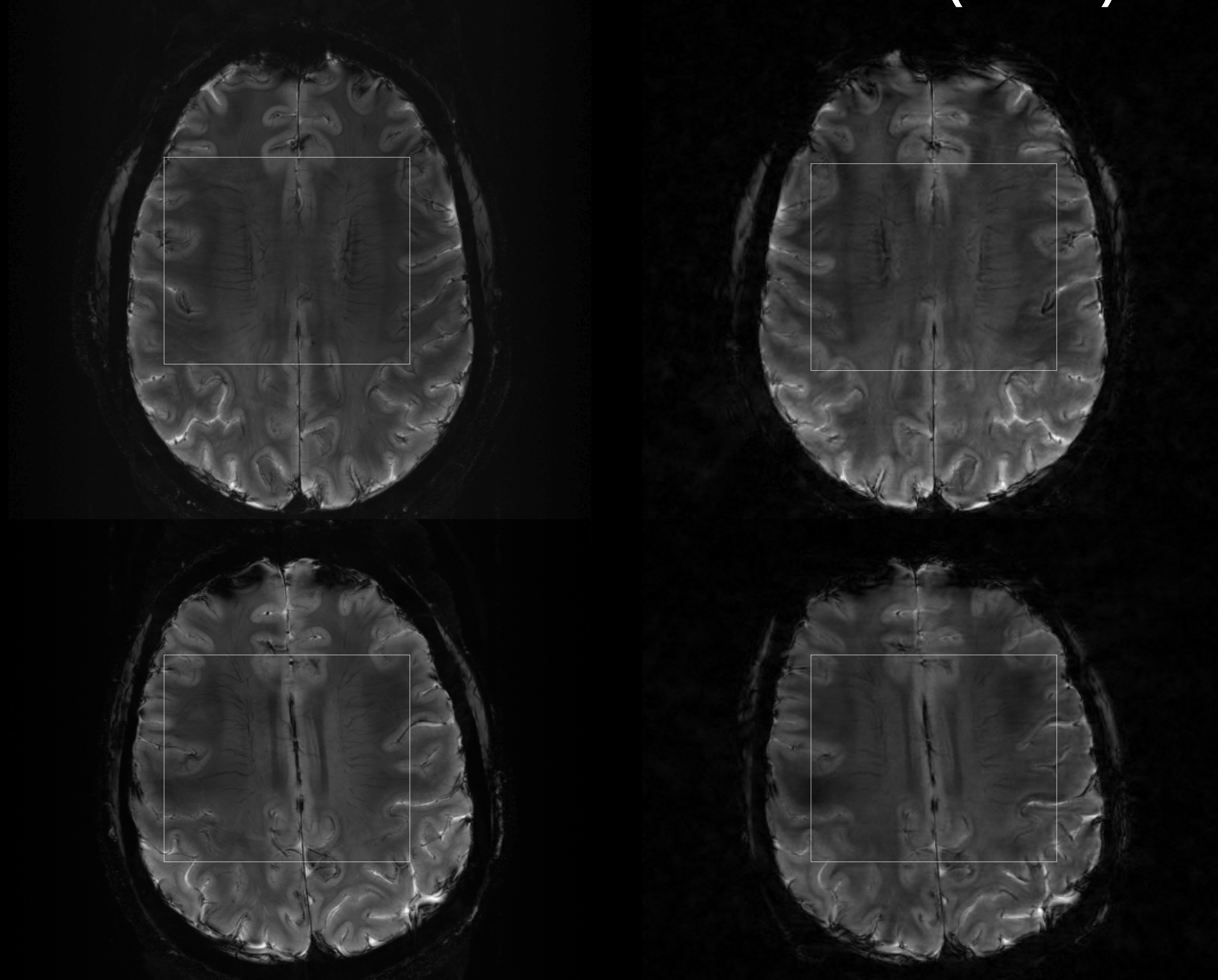
In vivo results: 0.39 mm – 11 slices of 3-mm thickness

REFERENCE
TA = 4min42s

In-out SPARKLING
TA = 18s (AF=15)

Subject 1

Subject 2



Lazarus et al ,in revision to MRM 2018

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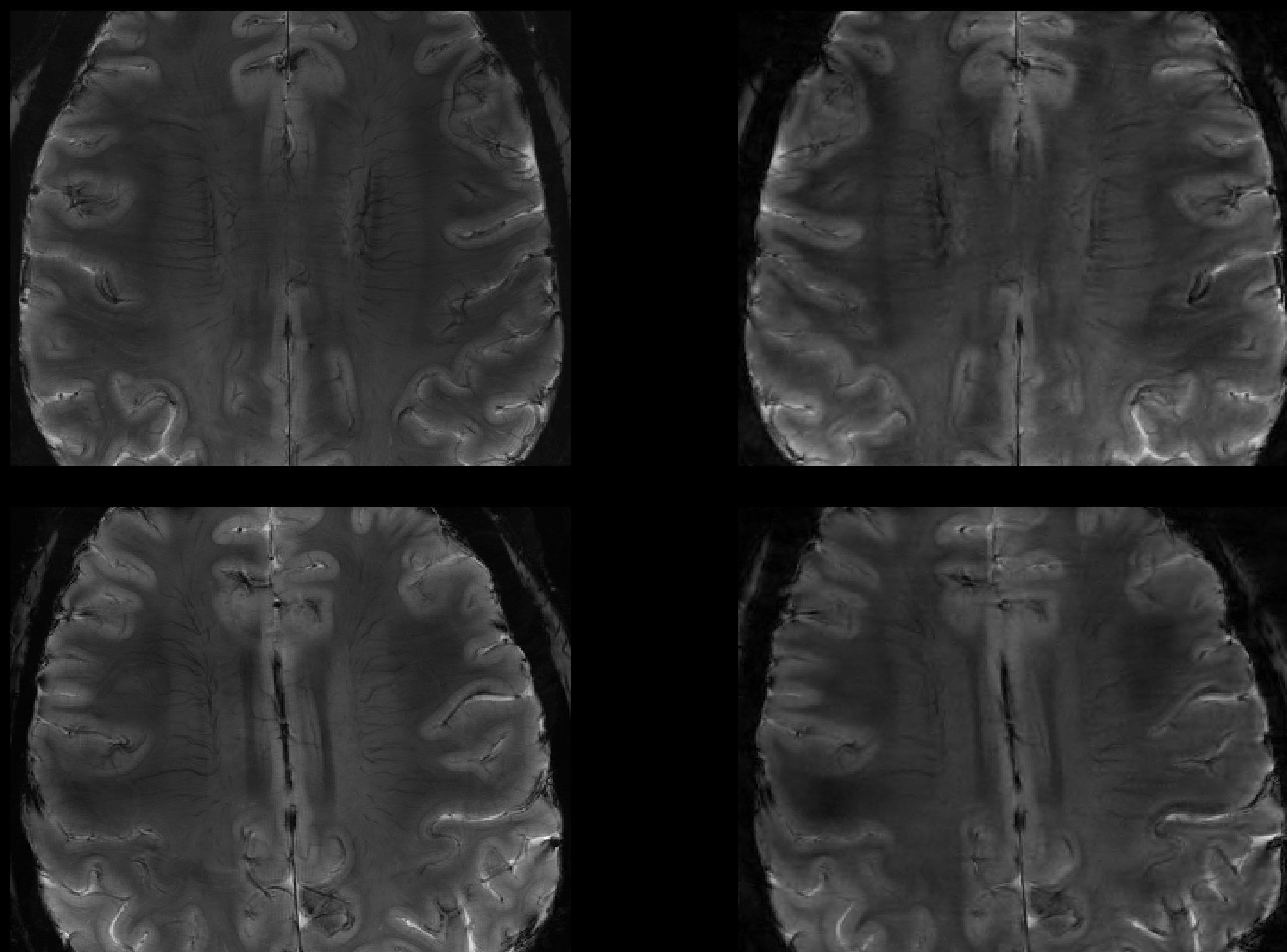
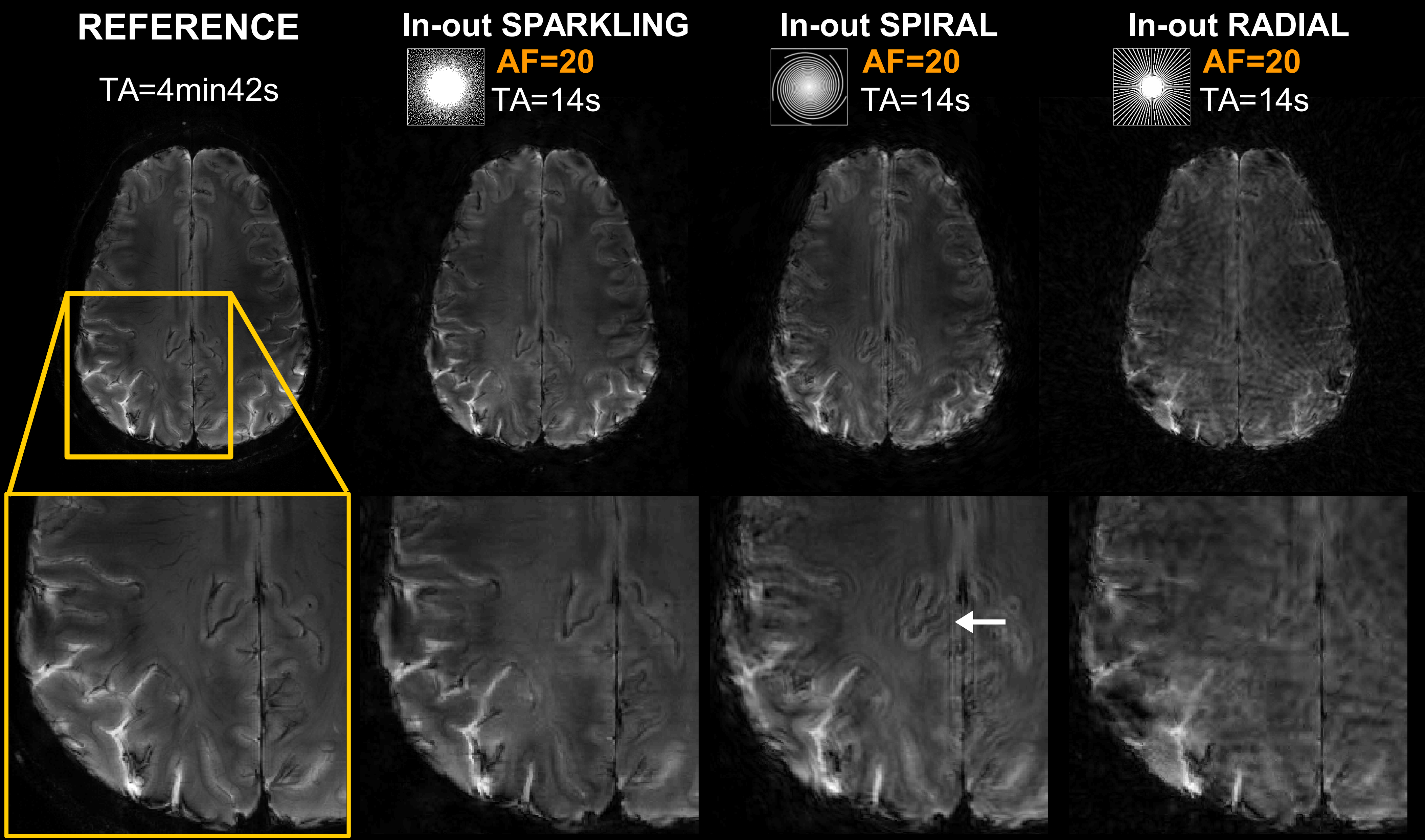


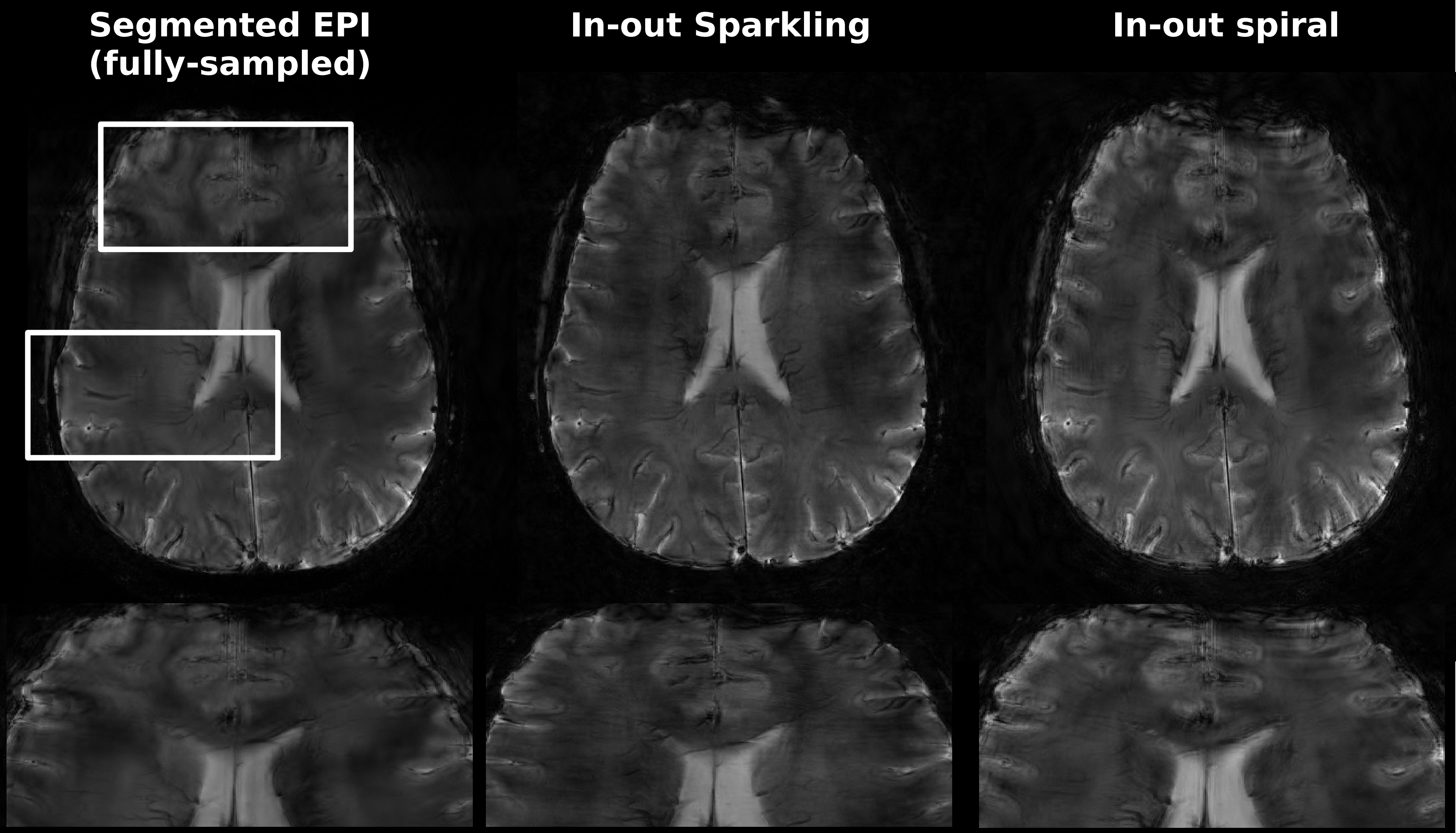
Lazarus et al ,in revision to MRM 2018

In vivo results at 0.39mm resolution
26 shots – 11 slices



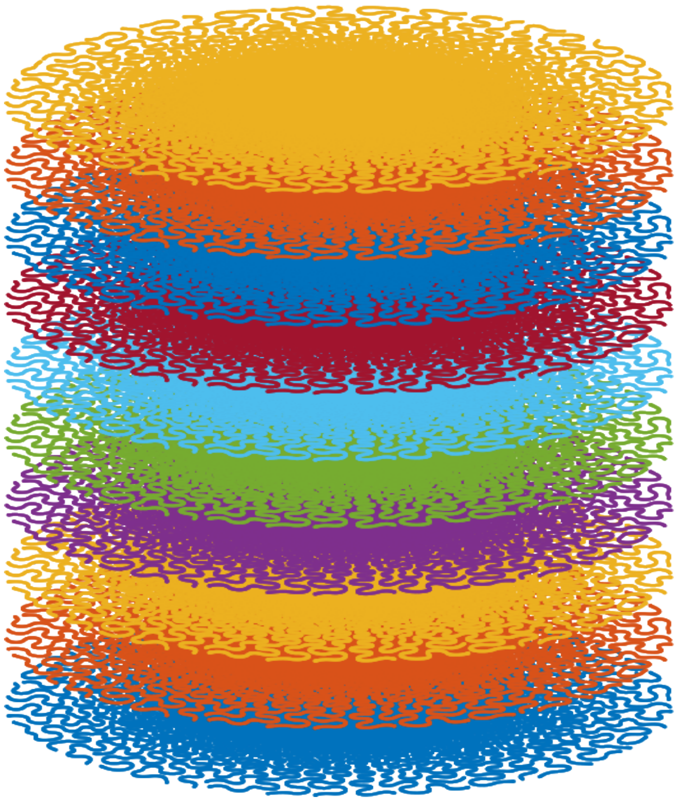
Lazarus et al ,in revision to MRM 2018

In vivo results at 0.39mm – 43 shots – 11 slices – BW=400kHz – TA=23s



Stack-of-Sparkling (SOS)

Regular SOS

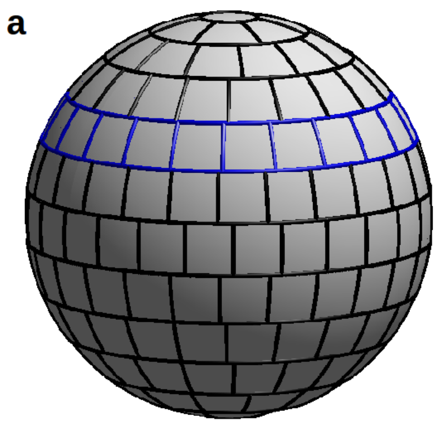


Z-vd SOS

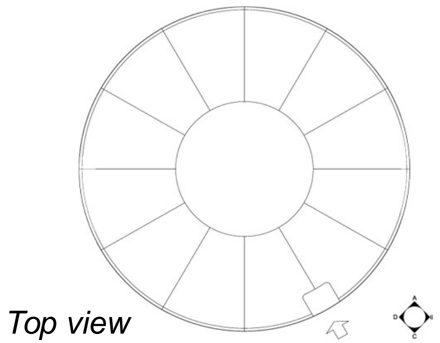
$$n(k_z) = n(0)\frac{\int \pi(:,:,k_z)}{\int \pi(:,:,0)},$$

Fully 3D Sparkling

- Shot-by-shot generation



Equal area tessellation



From 1 year to 20 min of computation time

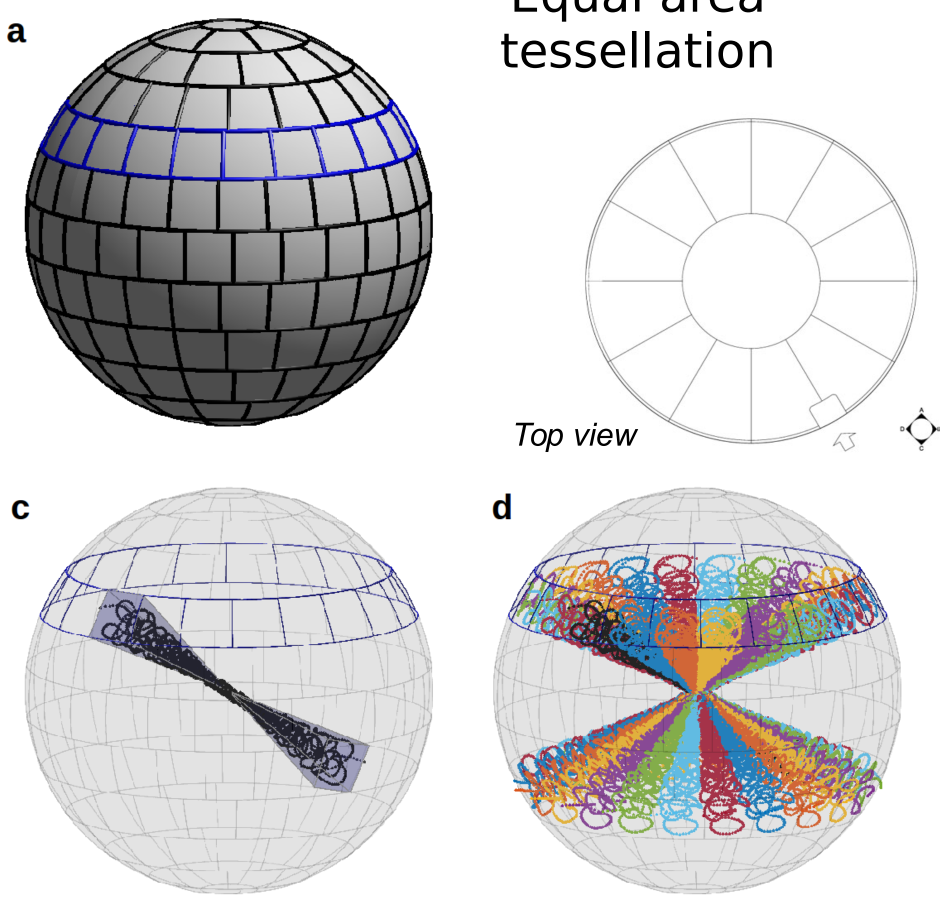
Leopardi et al. 2016. Electronic Transactions on Numerical Analysis

Fully 3D Sparkling

- Shot-by-shot generation

Equal area tessellation

From 1 year to 20 min of computation time

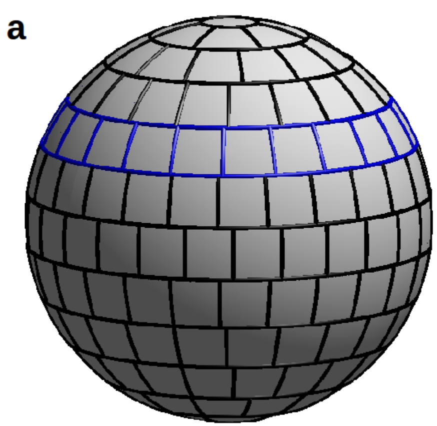


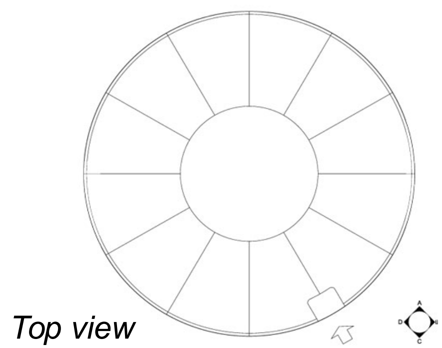
Leopardi et al. 2016. Electronic Transactions on Numerical Analysis

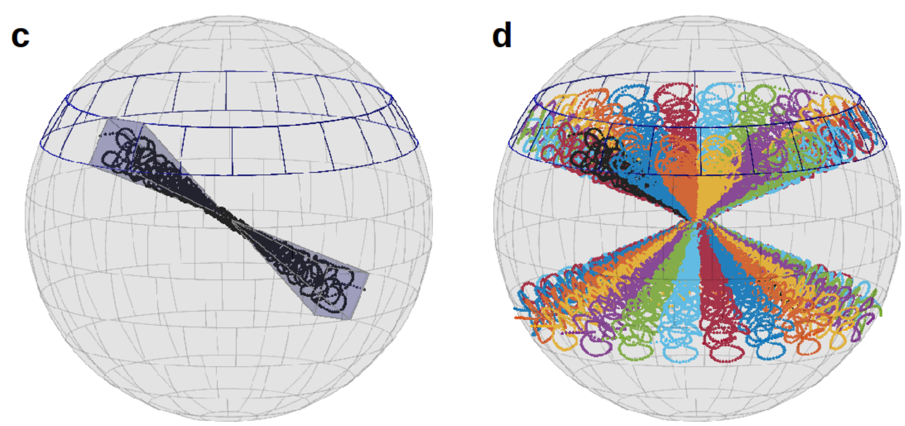
Fully 3D Sparkling

Shot-by-shot generation

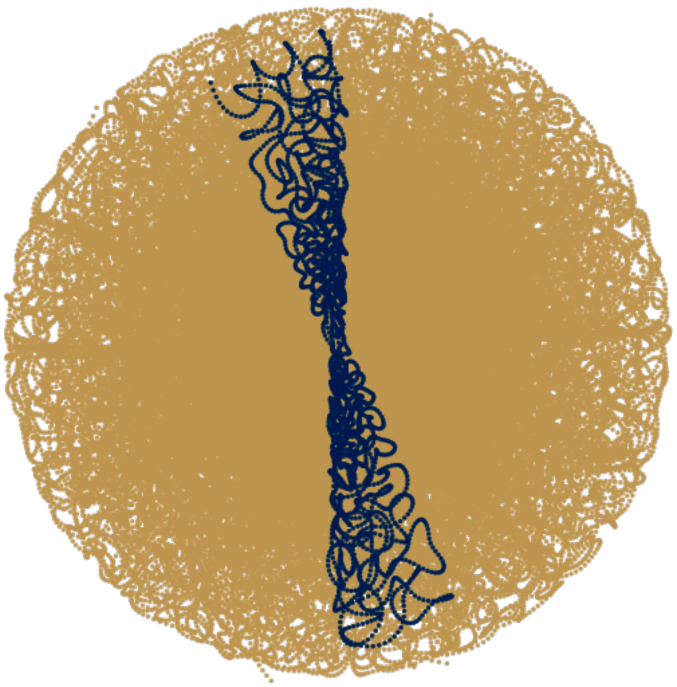
Equal area
tessellation







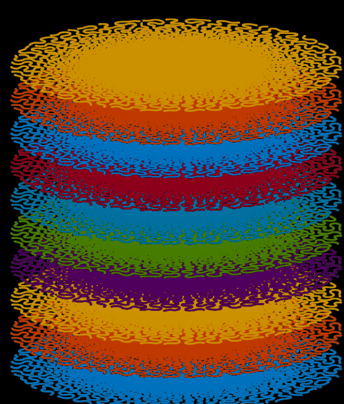
From 1 year to 20 min of
computation time



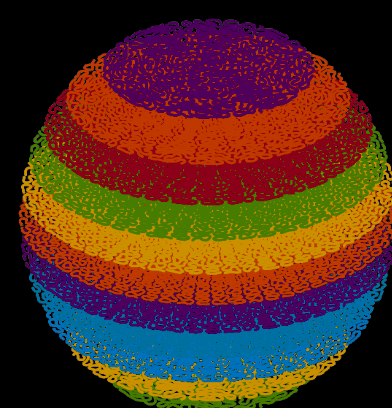
Leopardi et al. 2016. Electronic Transactions on Numerical Analysis

Ex vivo comparison of different Sparkling strategies

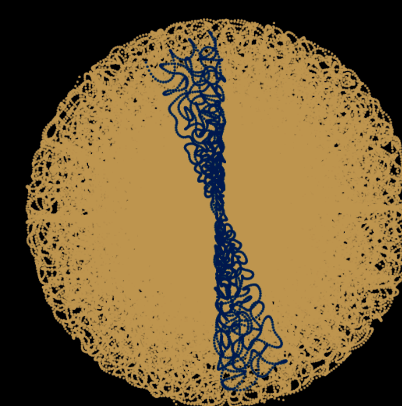
T2* contrast
Isotropic resolution of 0.6 mm
Tobs=15 ms – FOV=20x20x14 cm^3



VS.



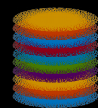
VS.



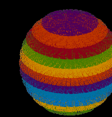
Regular SOS | **z-vd SOS** | **Fully 3D**

Comparison of different Sparkling methods 4000 shots – AF=20

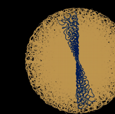
IPAT 4
TA=14min31s



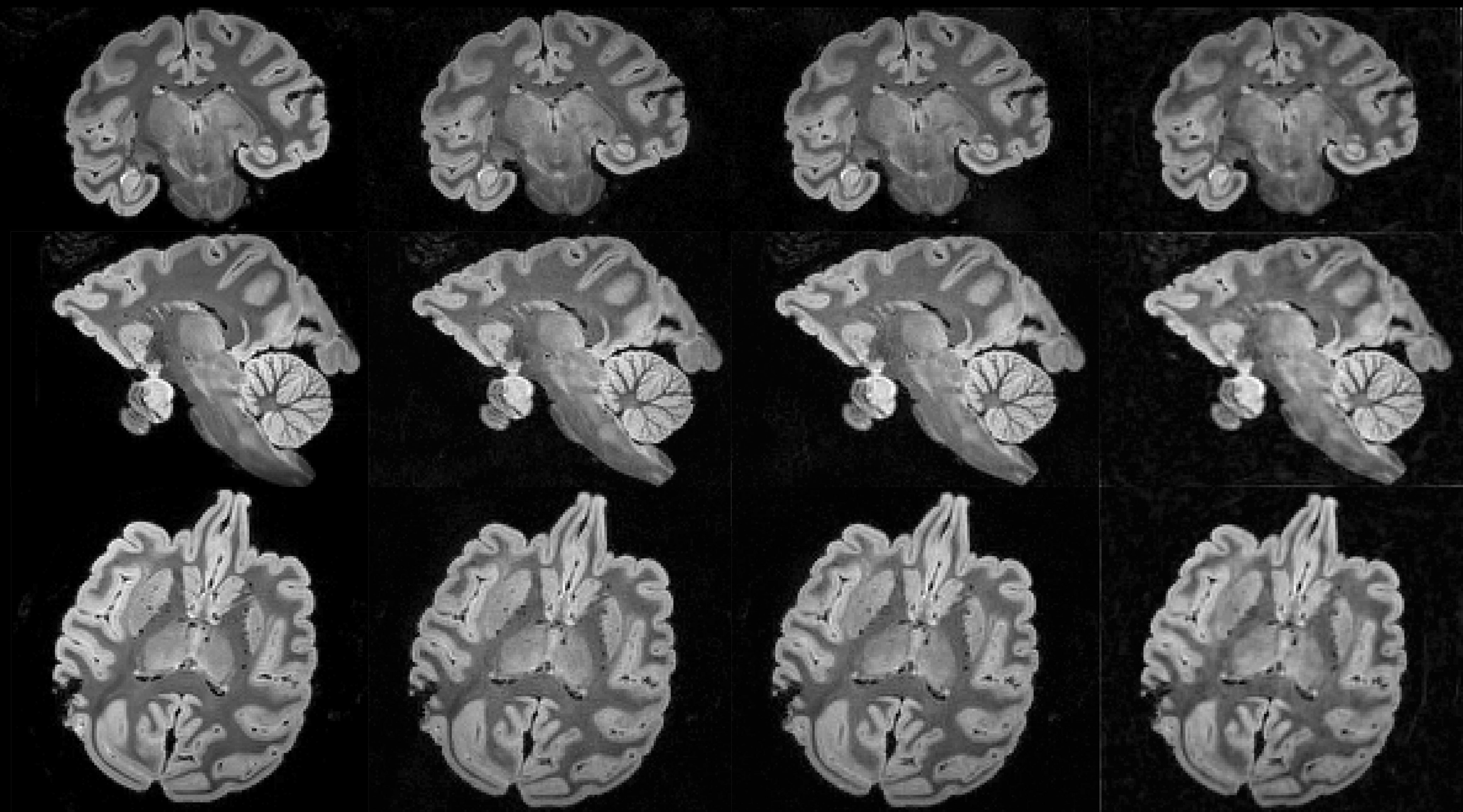
Regular SOS
TA=2min40s
SSIM=0.87



z-vd SOS
TA=2min40s
SSIM=0.88

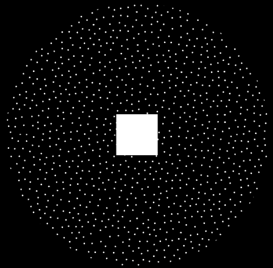


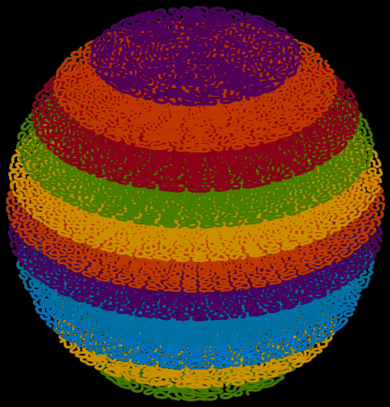
Fully 3D
TA=2min40s
SSIM=0.79



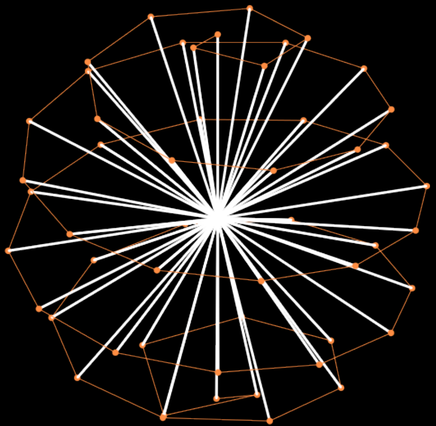
Ex vivo comparison with other strategies

T2* contrast
Isotropic resolution of 0.6 mm
TA=45s



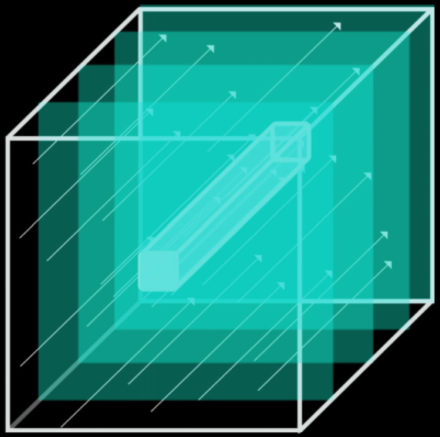


VS.



Larson et al. 2007

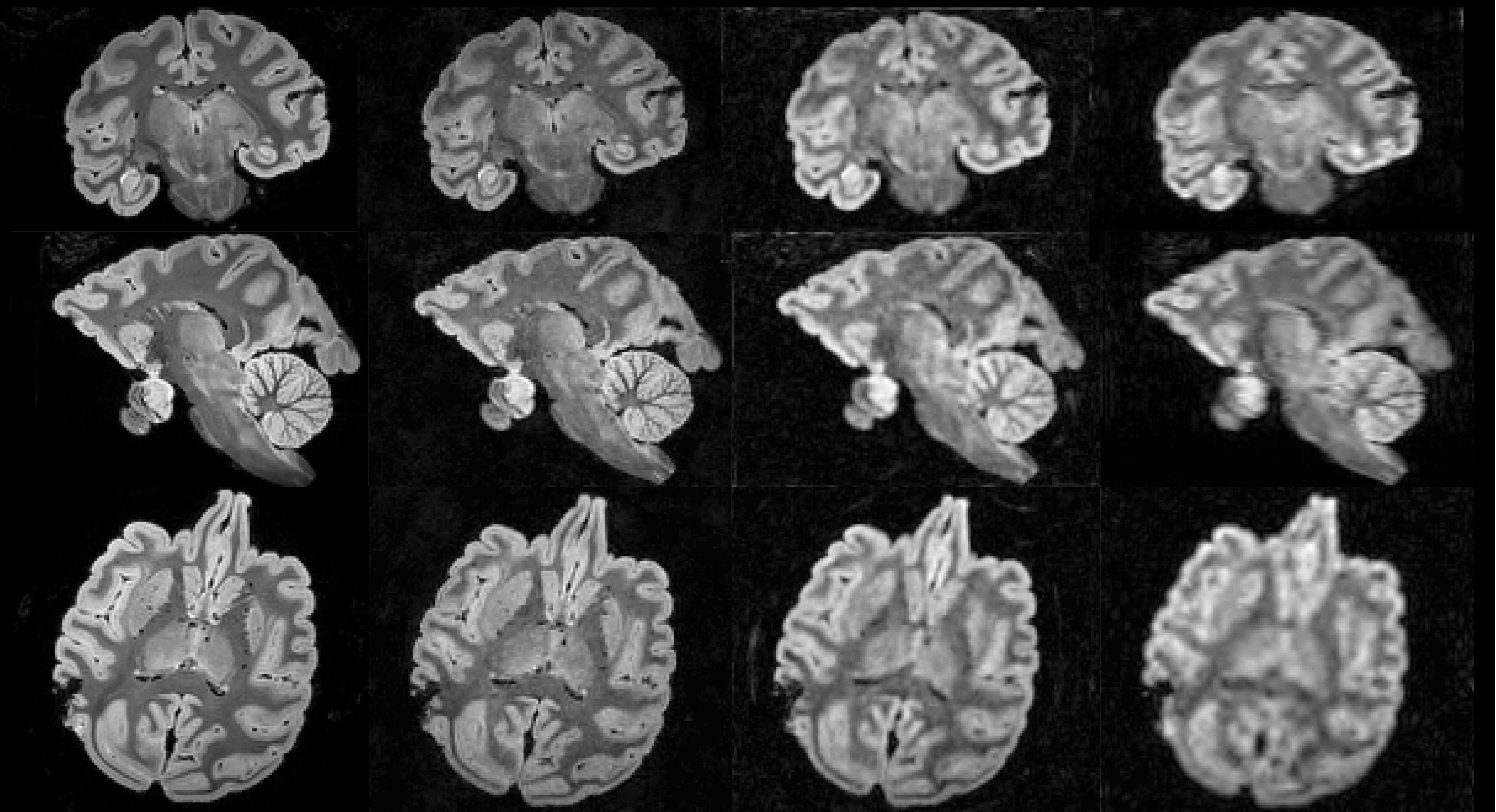
VS.



Lustig et al. 2008

SPARKLING vs. other strategies
1140 shots - AF=69

IPAT 4	SPARKLING	RADIAL	Poisson disk lines
TA=14min31s	TA=45s	TA=45s	TA=45s
	SSIM=0.86	SSIM=0.72	SSIM=0.58



Conclusion

- **SPARKLING 2D**
  - *Ex vivo* and *in vivo* validation of SPARKLING 2D:

    Demonstration of its **superiority** over both radial and spiral trajectories
  - Sampling **efficiency**, robustness to subsampling
  - Adaptability to hardware constraints and various densities (e.g. anisotropic)
- **SPARKLING 3D**
  - Higher acceleration factors
  - *Ex vivo* validation of SPARKLING 3D
- **MR image reconstruction**
  - 2D **offline/online** multichannel image recon with sensitivity maps extraction
  - 3D **offline** single/multi-channel image recon
  - **2D/3D calibrationless** image recon with group sparsity promotion
  - **Analysis-based** regularization using redundant decompositions (e.g. curvelets) and primal/dual recon algorithms

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