

DE LA RECHERCHE À L'INDUSTRIE



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New Acquisition strategies for Compressed Sensing in Magnetic Resonance Imaging

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Acknowledgements

Nicolas Chauffert
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Jonas Kahn
CNRS/IMT



Outline

Part I: Background in Magnetic Resonance Imaging

- Sampling k-space & Cartesian reconstruction
- Trajectories and acquisition strategies

Part II: Compressed Sensing in MRI

- Evolution of Compressed Sensing
- (CS with blocks of measurements)

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

Online Material

Code for MRI sampling available for reproducible research

- 7 Jupyter notebooks (Python 3.5) for Part I-II
- Matlab code for Part II (TSP+ Markov)
- Sparkling code for Part III *not yet* provided

Code for image reconstruction in Fourier imaging

- PySAP python package developed at CEA
- Off-line discussion for PR contributions, installation issues

Test data

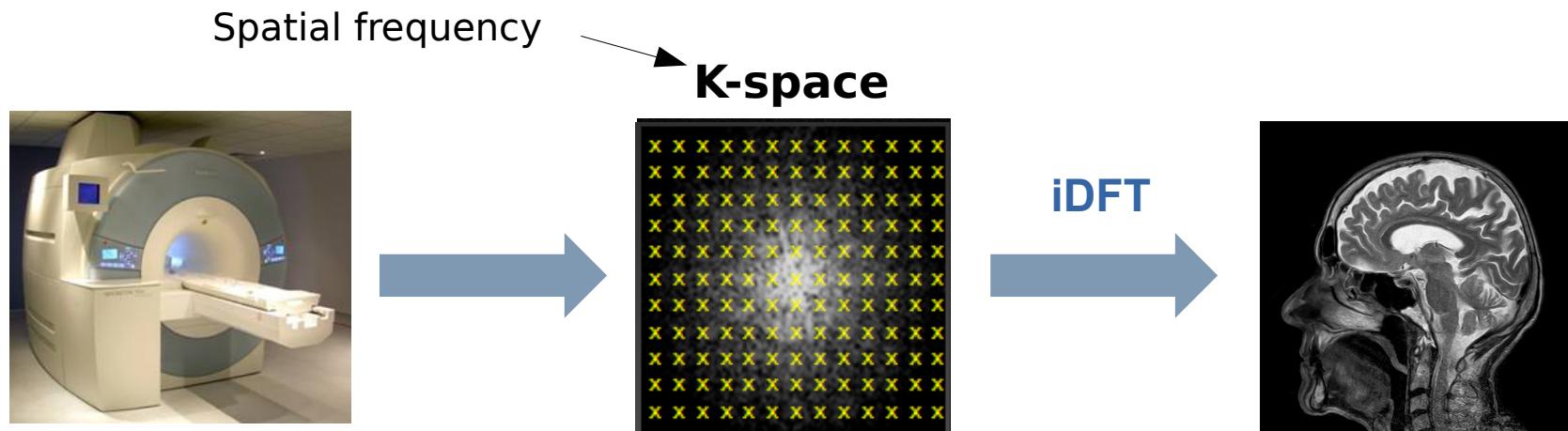
- Numerical phantoms at different image resolution
- Real T2*-w images collected at 7 Tesla at NeuroSpin
- Sparkling trajectories provided for demos

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Part I: Background in Magnetic Resonance Imaging

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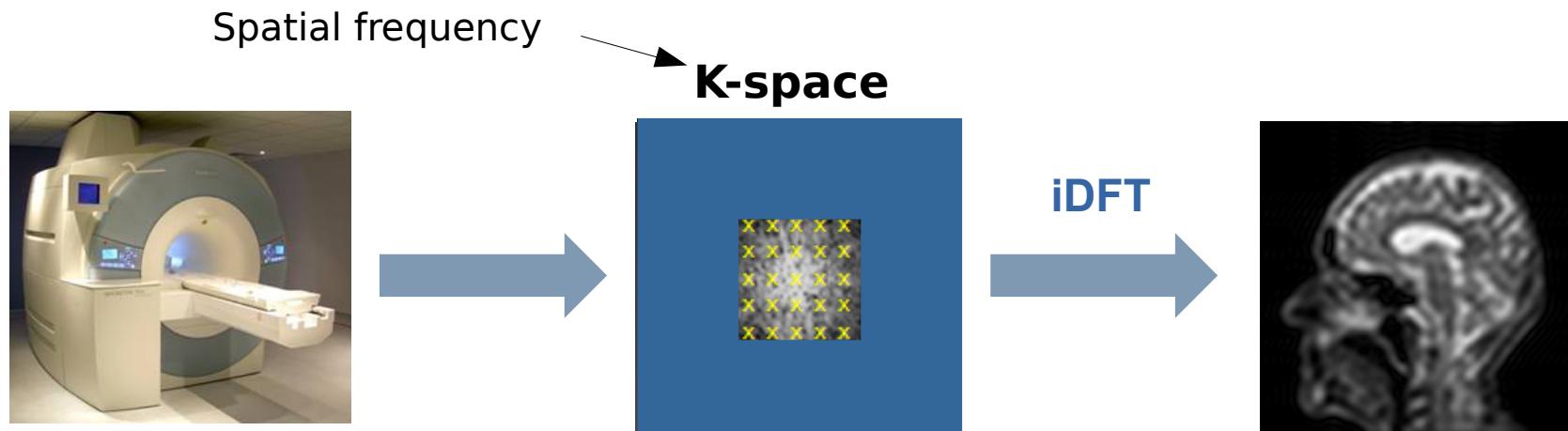
Sampling in MRI



Perfect reconstruction of an object would require measurement of *all* locations
In k-space (infinite!)

Data is acquired **point-by-point** in k-space (sampling) along **curves**

Sampling in MRI

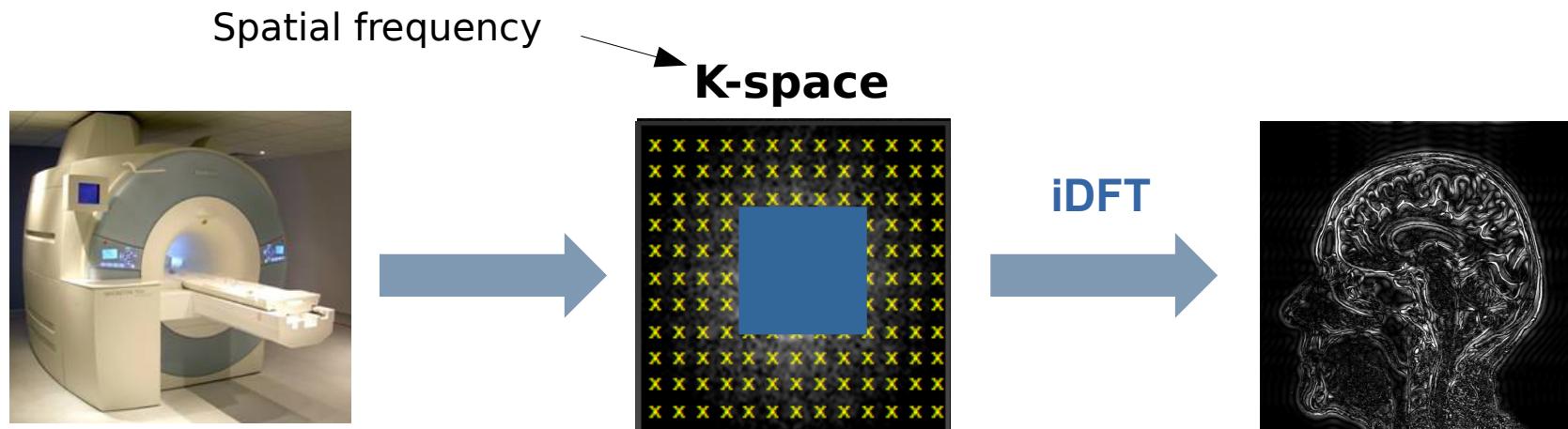


Low frequencies = Contrast



Go to 1st demo/notebook

Sampling in MRI

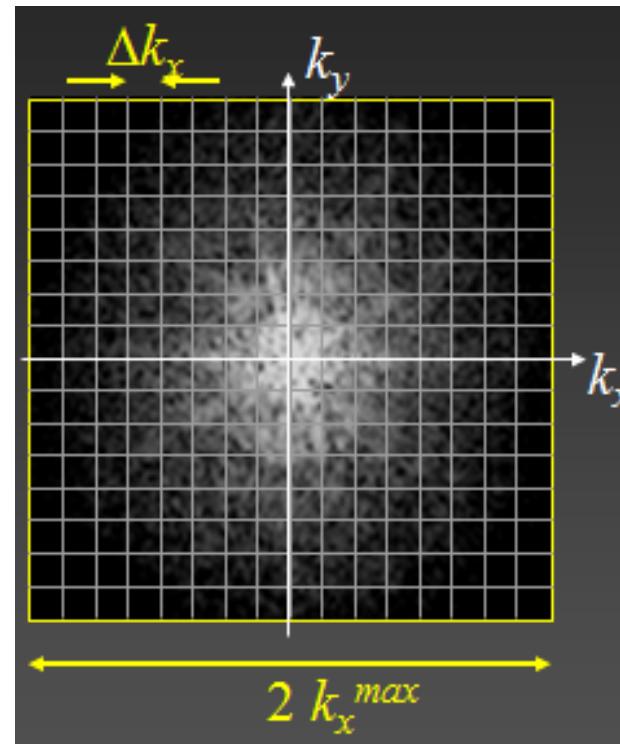


High frequencies = boundaries/edges



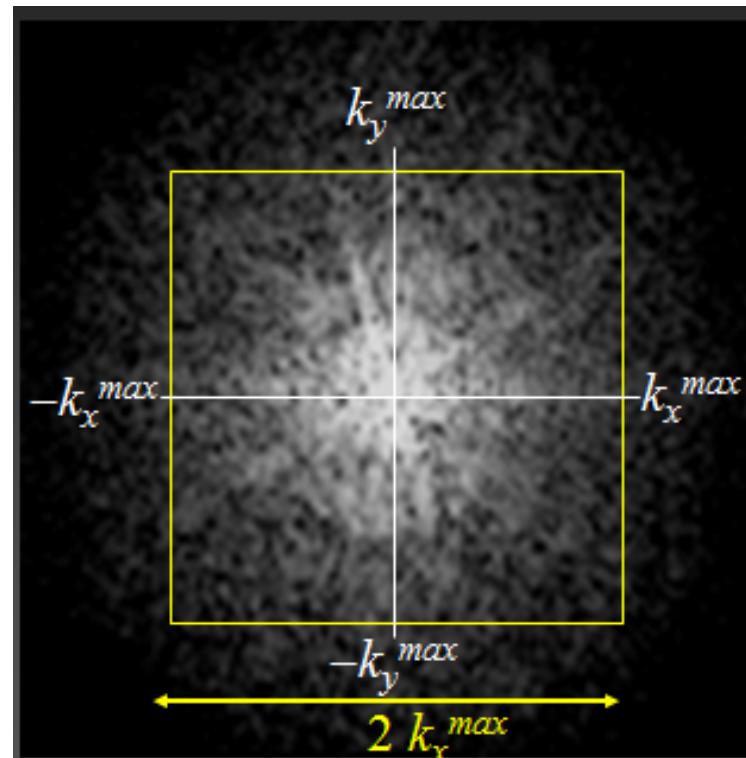
Go to 1st demo/notebook

Sampling the k-space



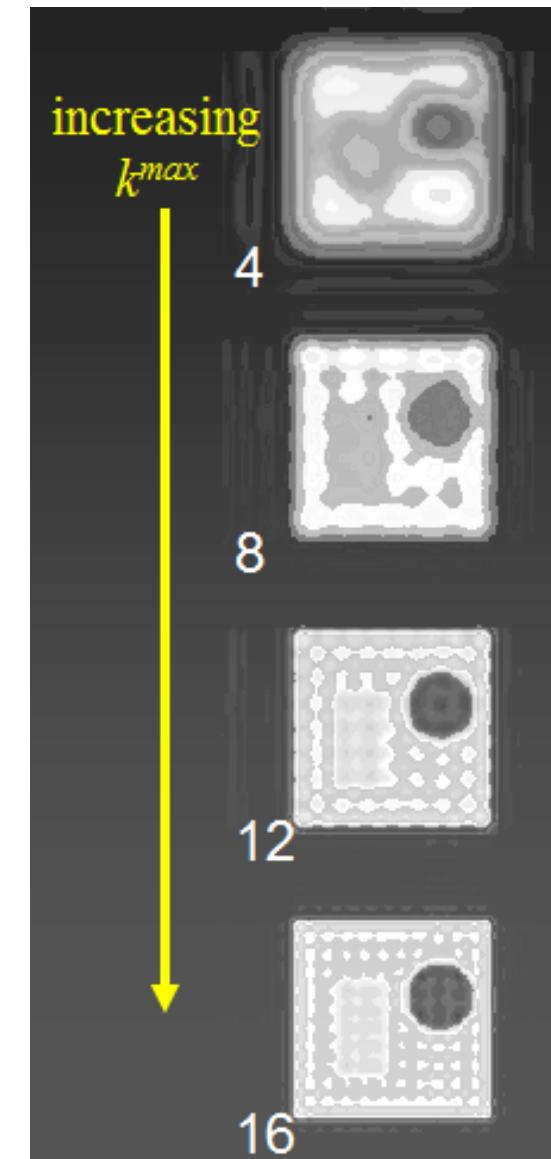
1. What is the highest frequency we need to sample in k -space (k^{\max})?
2. How close should the samples be in k -space (Δk)?

Choosing the maximal frequency



k^{\max} determines image resolution

Large k^{\max} means high resolution!



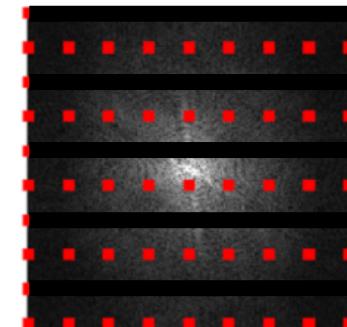
Go to 1st demo/notebook

| PAGE 10

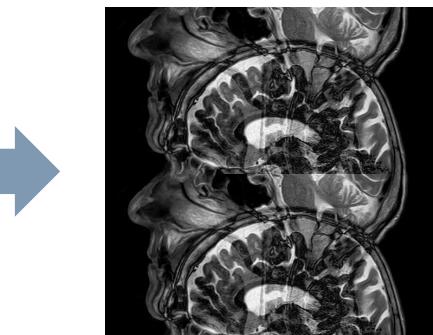
Nyquist Sampling Theorem



K-space



iDFT



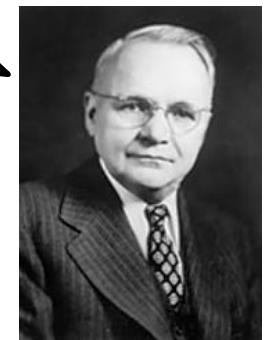
Nyquist-Shannon theory

↑ resolution \Rightarrow ↑ #samples

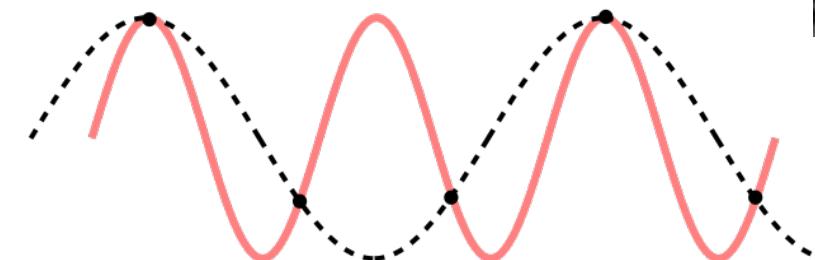


Long acquisition times

The sampling frequency should be at least twice the highest frequency contained in the signal



Harry Nyquist

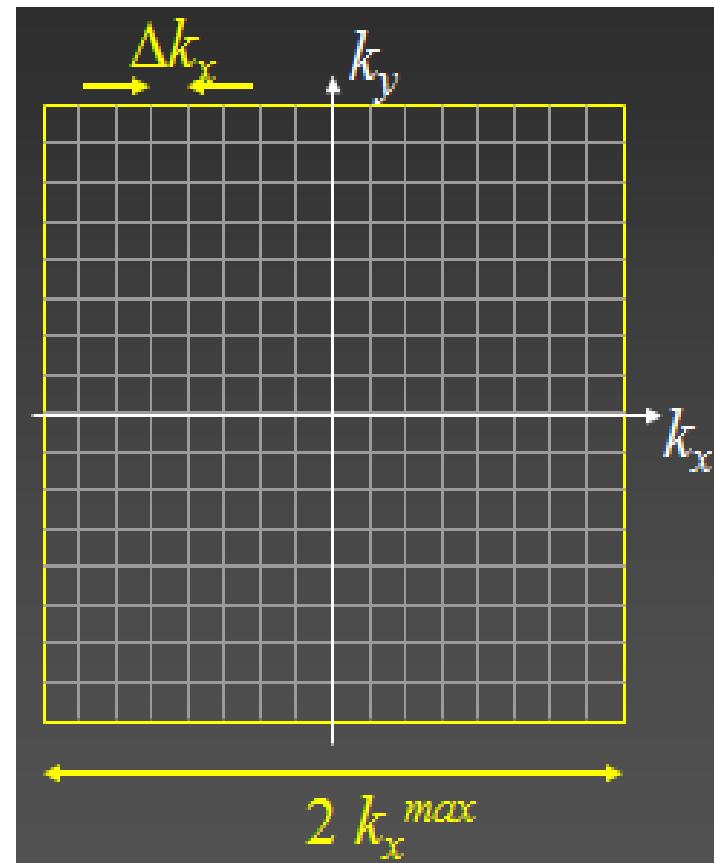


Go to 3rd notebook

Nyquist Sampling Theorem

$$\text{FOV}_x = \frac{1}{\Delta k_x}$$

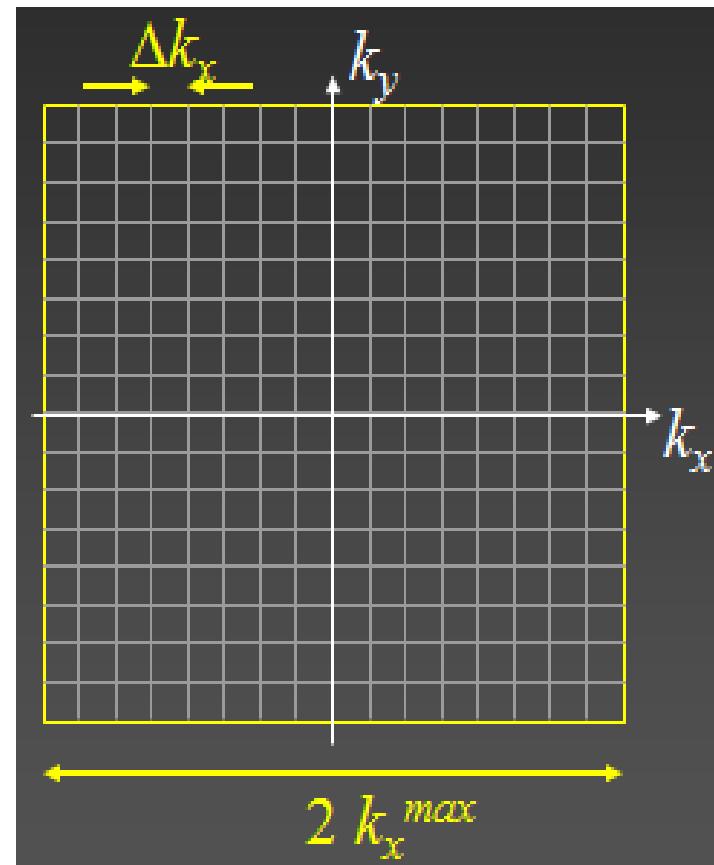
$$\Delta x = \frac{1}{2k_x^{\max}}$$



Nyquist Sampling Theorem

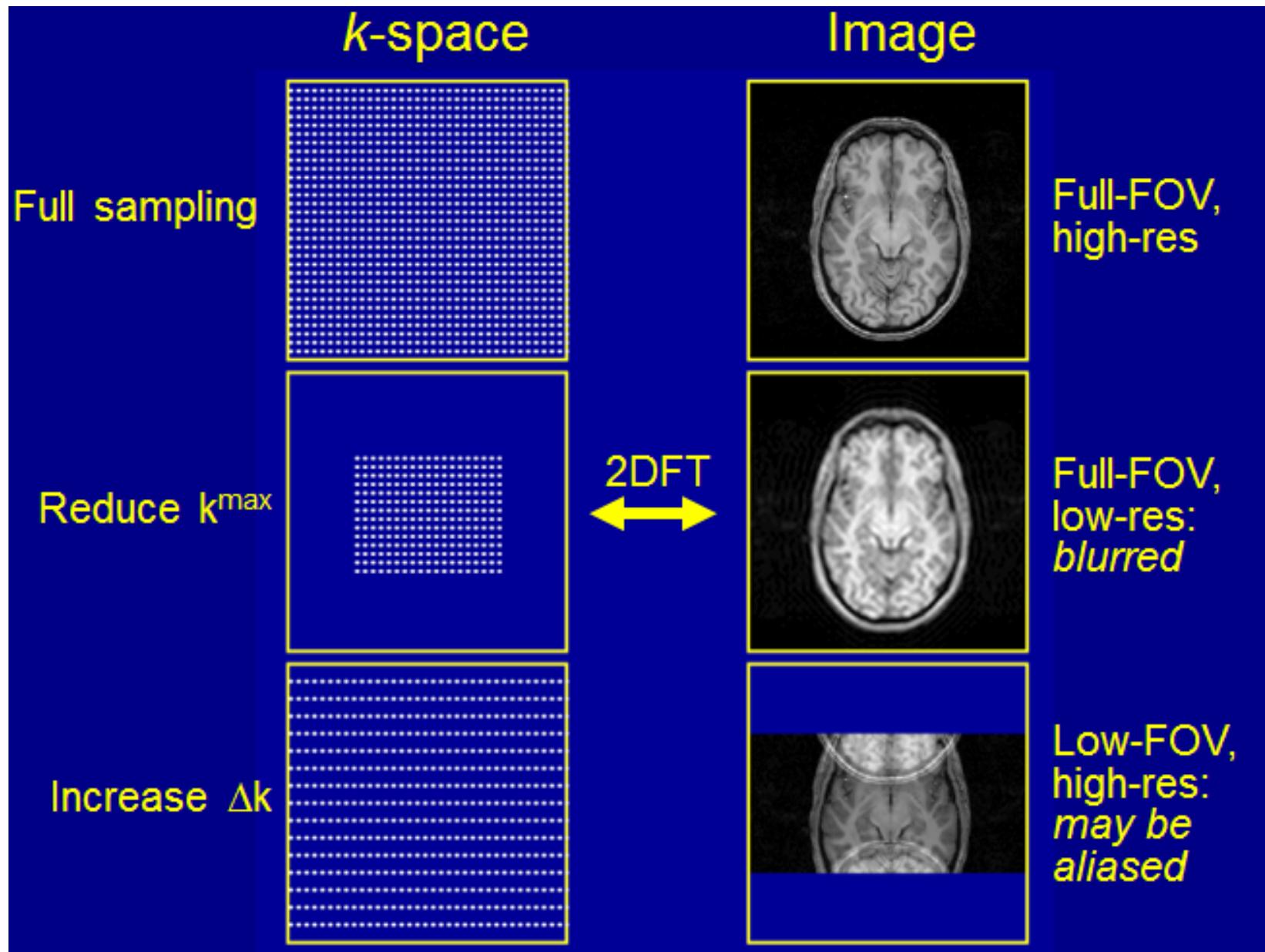
$$x_{\max} = \frac{1}{\Delta k_x}$$

$$2k_x^{\max} = \frac{1}{\Delta x}$$



k-space and image resolution are inversely related:
resolution in one domain determines extent in other.

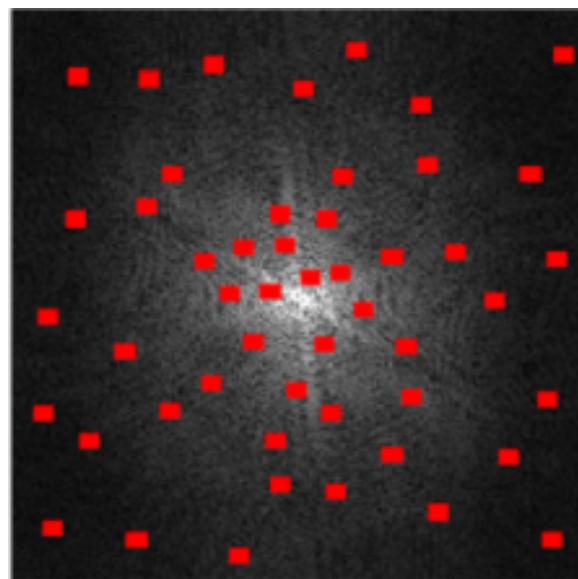
Nyquist Sampling Theorem



Go to 3rd notebook

Beyond Nyquist Sampling Theorem

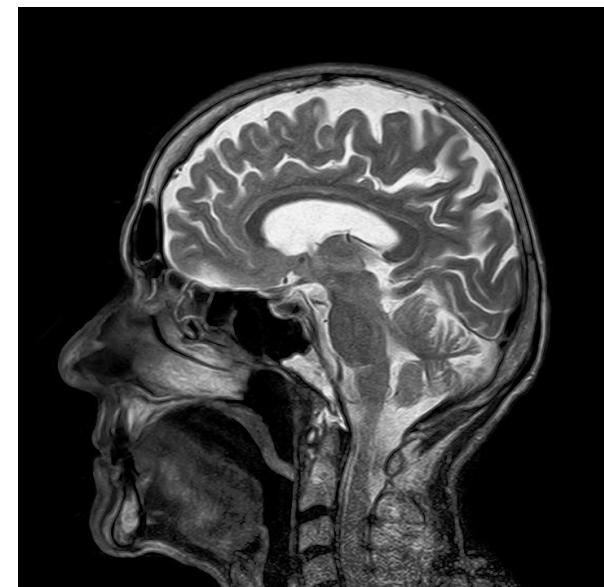
k-space



Nonlinear
reconstructions



Image



Go to 2nd notebook

[Lustig et al, Magn Reson Med 2007]

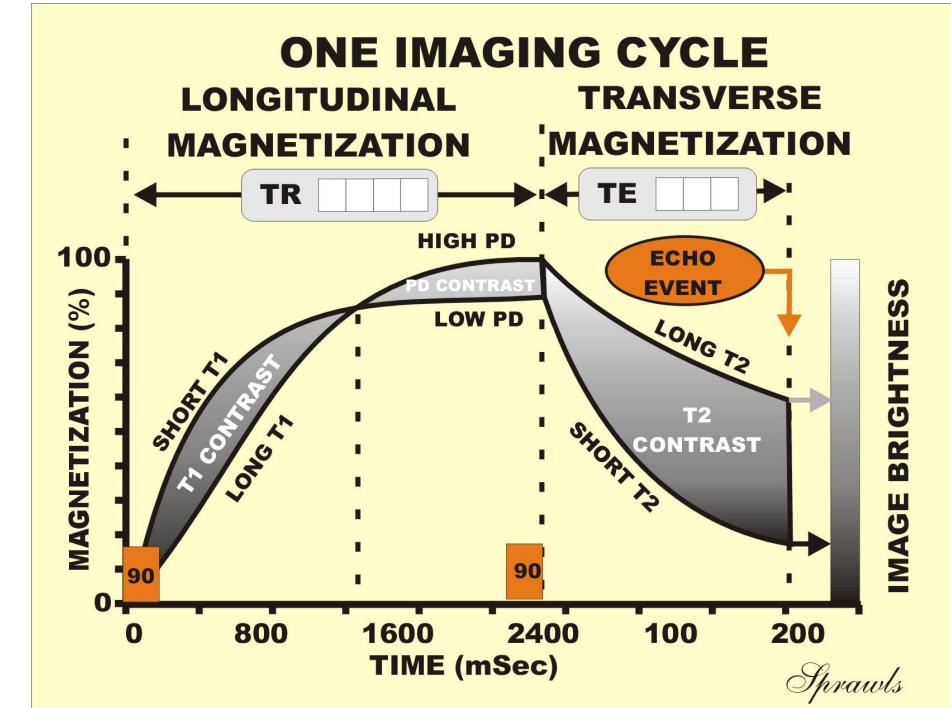
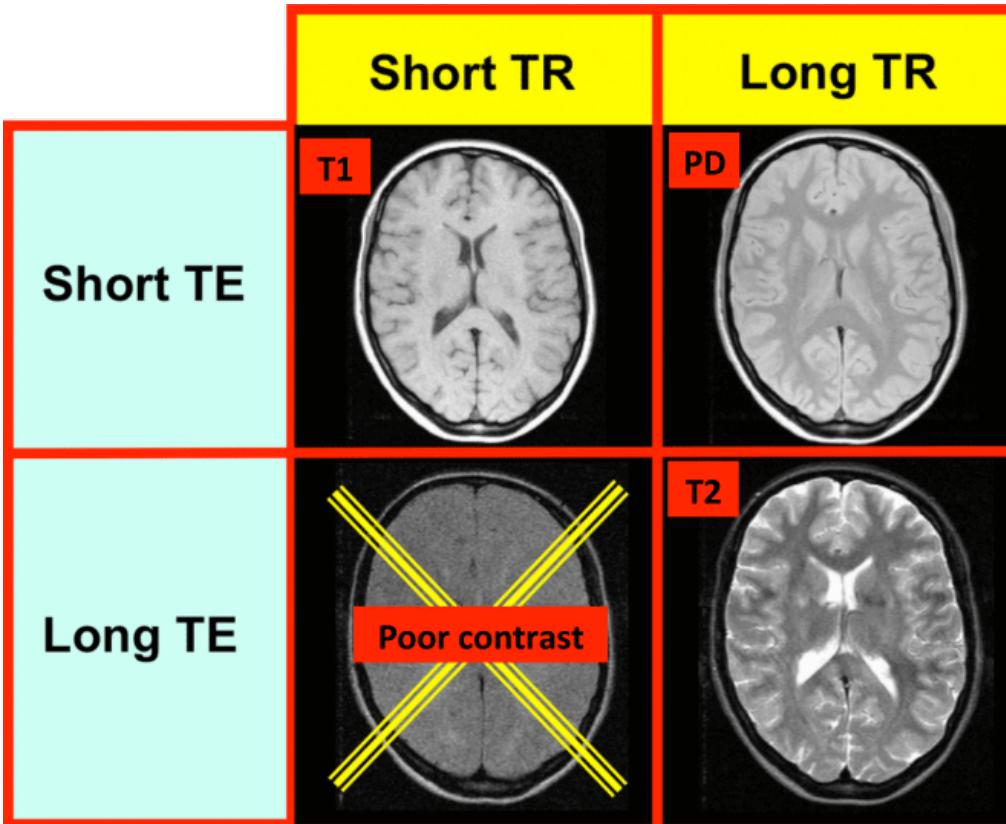
Outline

Part I: Background in Magnetic Resonance Imaging

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Short background in MRI

MR is versatile: multiple imaging contrasts



Simplified Bloch equations:

- Longitudinal magnetization:
- $$M_z(t) \simeq M_0(1 - e^{-t/T_1})$$
- Transverse magnetization:

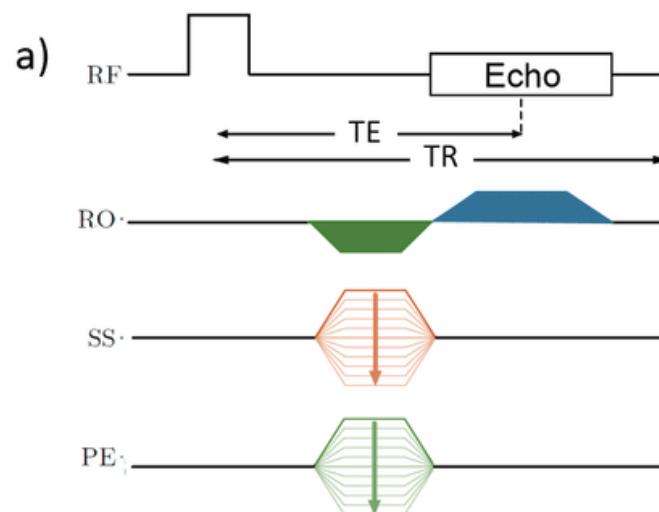
$$M_{xy}(t) = M_0 e^{-t/T_2}$$



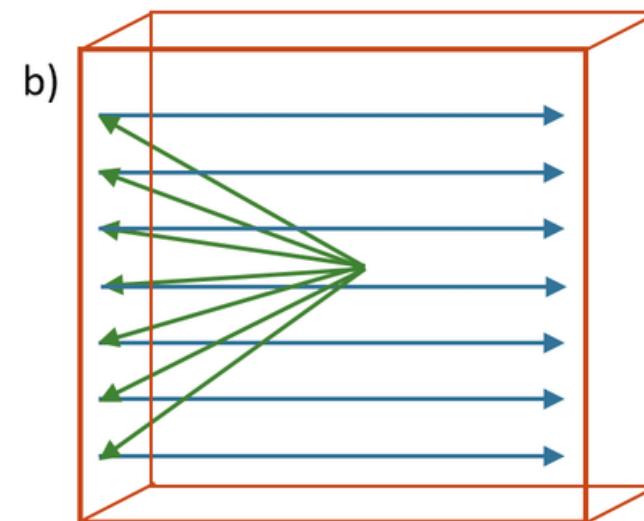
The MR signal has a short lifespan

Segmented Acquisition in MRI

Chronogram pulse sequence



K-space sampling



TR: Time between 2 successive RF pulses

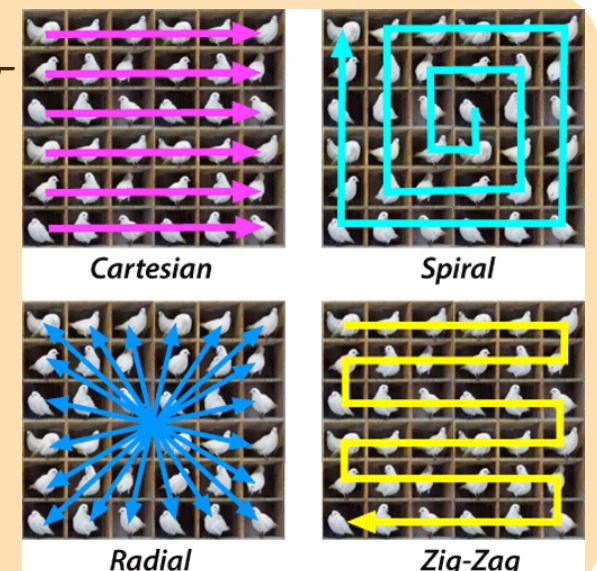
TE: time point associated with the center
of echo: $k_x(TE) = 0$

T_{obs} or readout: length of each segment/shot

Sampling Trajectories

Sampling in MRI: $k(t) = k(0) + \gamma \int_0^t G(\tau) d\tau$

- **Segmented acquisition:**
Scan time proportional to number of shots
- **Hardware constraints on gradients:**
 $G_{\max} < 40 \text{ mT/m}$; $S_{\max} < 200 \text{ T/m/s}$
 → bounded velocity and acceleration



Let $\mathbf{k} : [0, T] \rightarrow \mathbb{R}^d$, ($d = 2, 3$) denote the sampling curve:

$$\mathbf{k}(t) = \mathbf{k}(0) + \gamma \int_0^t \mathbf{G}(\tau) d\tau \quad d = 2 \rightarrow \mathbf{k} = (k_x, k_y), \mathbf{G} = (G_x, G_y)$$

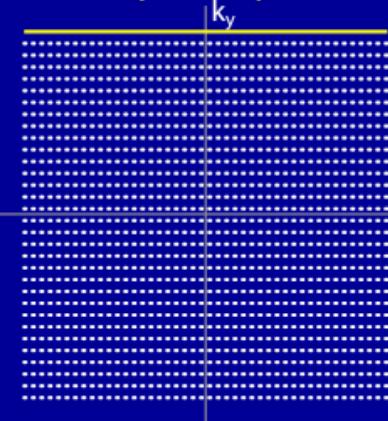
$$\mathcal{S} = \begin{cases} \|\dot{\mathbf{k}}\|_{2,\infty} &< \gamma G_{\max} & \text{(bounded speed)} \\ \|\ddot{\mathbf{k}}\|_{2,\infty} &< \gamma S_{\max} & \text{(bounded acceleration)} \end{cases}$$

where $\|\mathbf{c}\|_{2,\infty} = \sup_{1 \leq i \leq p} (|\mathbf{c}_x[i]|^2 + |\mathbf{c}_y[i]|^2)^{1/2}$

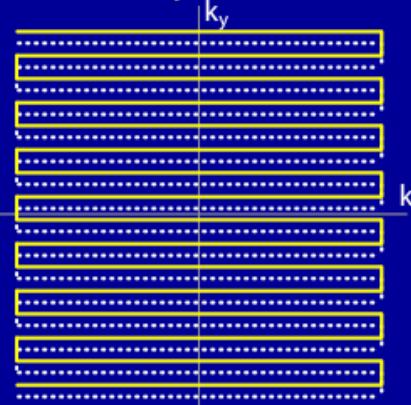
K-space location is proportional to accumulated area under gradient waveforms

Examples of k-space Trajectories

(a) 2DFT (spinwarp)



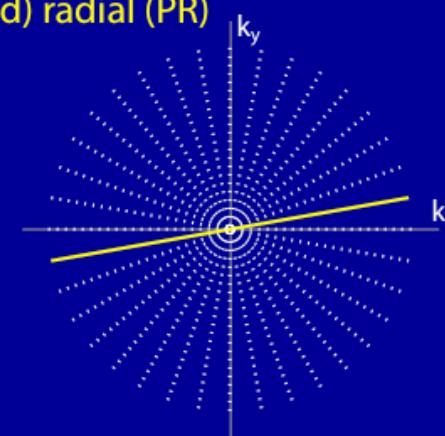
(b) EPI (echo-planar)



(c) spiral



(d) radial (PR)



Line acquisition vs. EPI

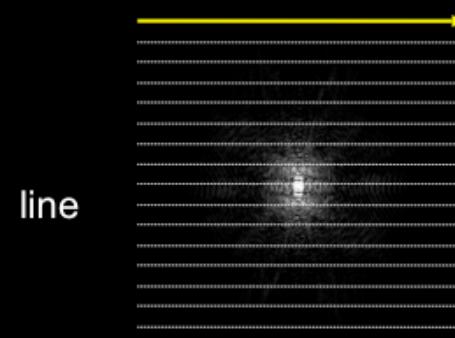


Image accumulated over
multiple line acquisitions

Slow: 5-10 minutes

Excellent image quality

k-space

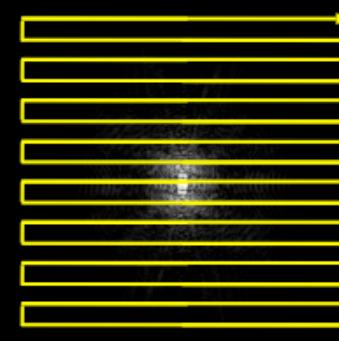


Image acquired in single
acquisition

Fast: 3 seconds

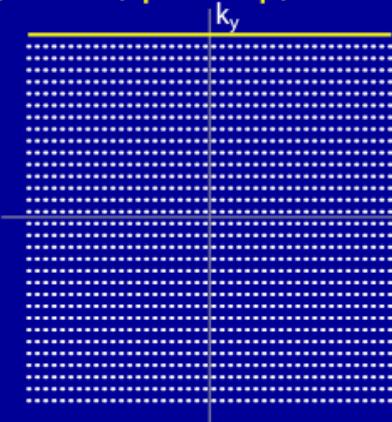
Image artefacts



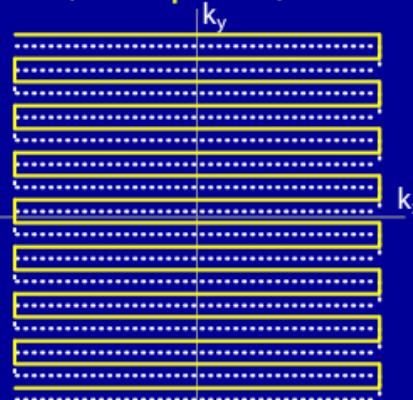
Go to the 5th and 6th demos

Examples of k-space Trajectories

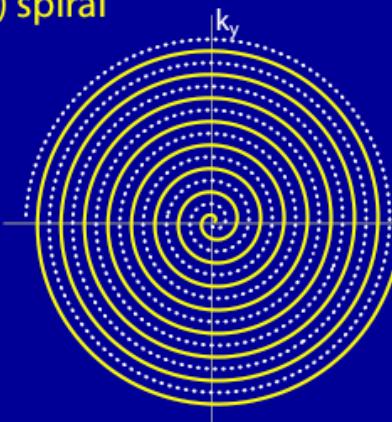
(a) 2DFT (spinwarp)



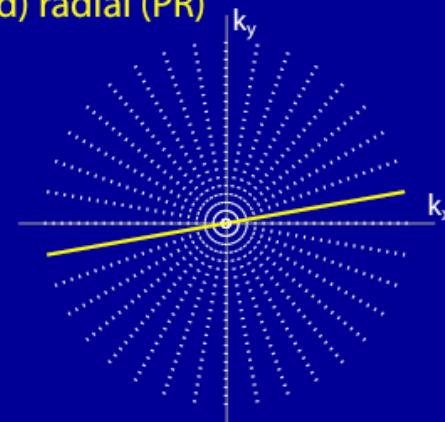
(b) EPI (echo-planar)



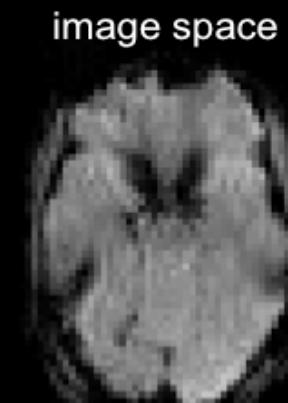
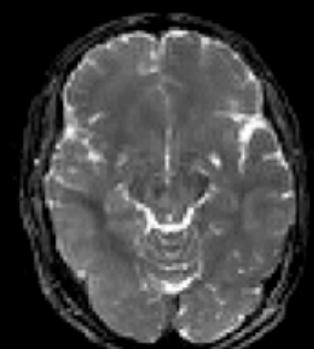
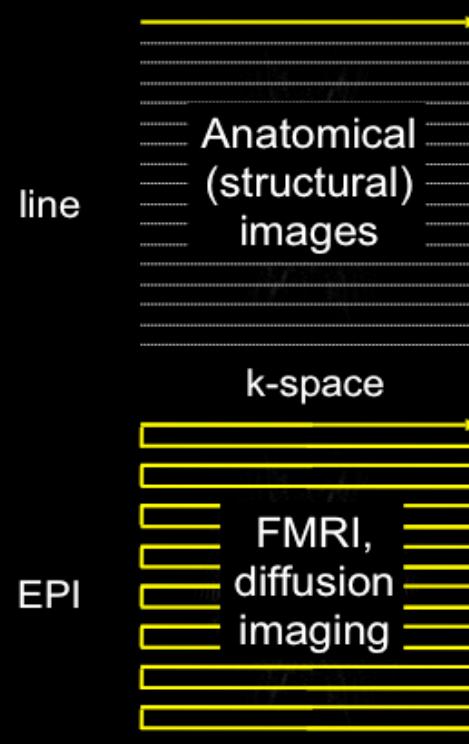
(c) spiral



(d) radial (PR)



Line acquisition vs. EPI



Interim Summary

- **Cartesian vs non-Cartesian trajectories**
 - **Cartesian**: easier to implement less prone to gradient errors, play one gradient at a time
 - **Non-cartesian**: robust to motion, more flexible sampling, prone to gradient errors
 - The time matters: the same sampling schemes can be played in different ways (e.g. radial in-out vs radial-out)
- **Single shot vs multi-shot**
 - Multiple shot = longer acquisition times, better image quality
 - Single shot = faster acquisition, e.g. for dynamic MR imaging mainly
- **MRI acquisition constraints**
 - T_1 -w: Short TE, TR → short readout and thus straight lines (Cartesian or radial) are unavoidable
 - T_2 , T_2^* -w : long TE, TR → enable long readout → more fancy trajectories are feasible

Outline

Part II: Compressed Sensing in MRI

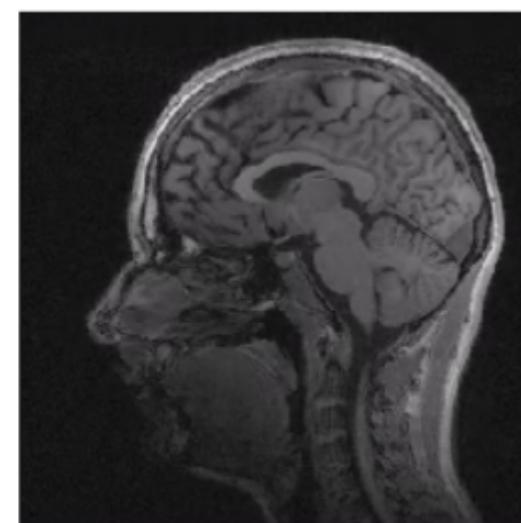
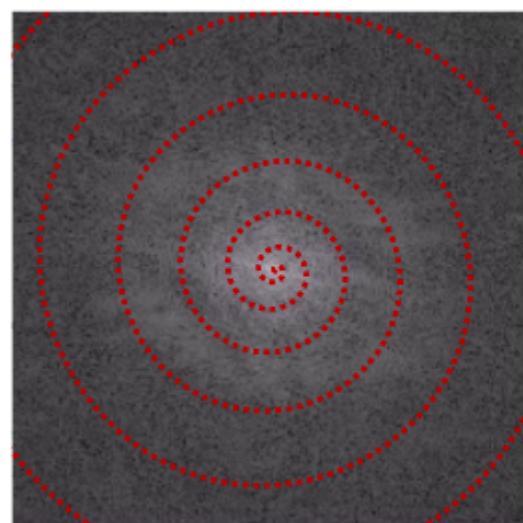
- Evolution of Compressed Sensing
- CS with blocks of measurements

Modeling of k-space Trajectories

Sampling the curve $\mathbf{k}(t)$ generates a set of measurements:

$$E(\mathbf{k}) = \{\hat{x}(\mathbf{k}(p\Delta t))\}_{p \in \{0, \dots, T/(\Delta t)\}}$$

with \hat{x} the Fourier transform of x



Challenges

- **Questions in sampling theory**

- How to choose the set of measurements E ?
 - How to generate a set of segmented trajectories k ?

- How to reconstruct x given E ?



Jeff Fessler's part!

- **Practical consequences**

- Shorten acquisition times
 - Increase image resolution
 - Improve medical diagnosis and reduce exam costs

A fundational result

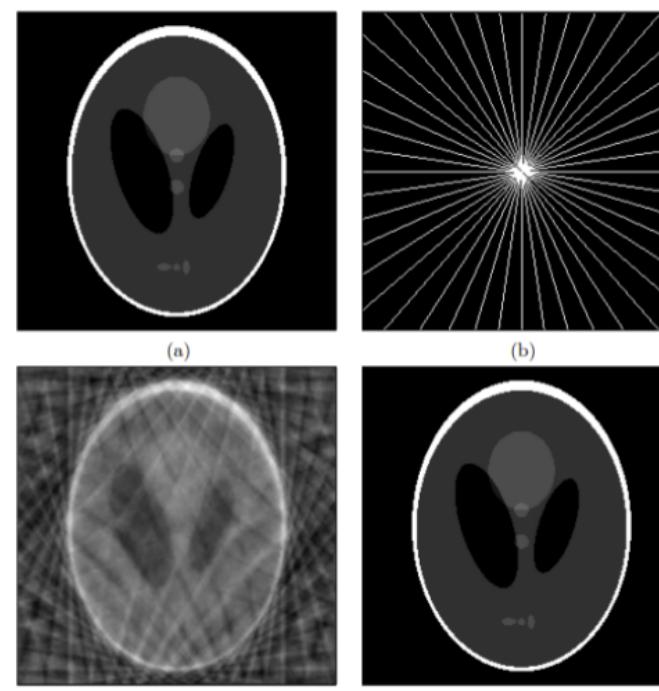
Supporting Thorem [Candes, Romberg and Tao, IEEE IT 2006]

Let $\boldsymbol{x} \in \mathbb{C}^n$ denote an s -sparse signal. Samples m values $y_k = \hat{\boldsymbol{x}}(J_k)$ where indexes $(J_k)_{1 \leq k \leq m}$ are i.i.d. and uniformly distributed.
If

$$m \geq C \cdot s \cdot \log(n)$$

\boldsymbol{x} can be recovered exactly, with high probability by solving:

$$\min_{\boldsymbol{x} \in \mathbb{C}^n, \hat{\boldsymbol{x}}(J_k) = y_k} \|\boldsymbol{x}\|_1$$



As the measurements are not *independent*, this theorem does not explain this result!

Preliminaries

Wavelet expansions

- Let $\Psi = (\Psi_1, \dots, \Psi_n) \in \mathbb{C}^{n \times n}$ denote an **orthogonal transform**. The image to reconstruct writes:

$$\mathbf{x} = \Psi \mathbf{z} = \sum_{i=1}^n z_i \Psi_i \quad \text{where} \quad z_i = \langle \mathbf{x}, \Psi_i \rangle$$

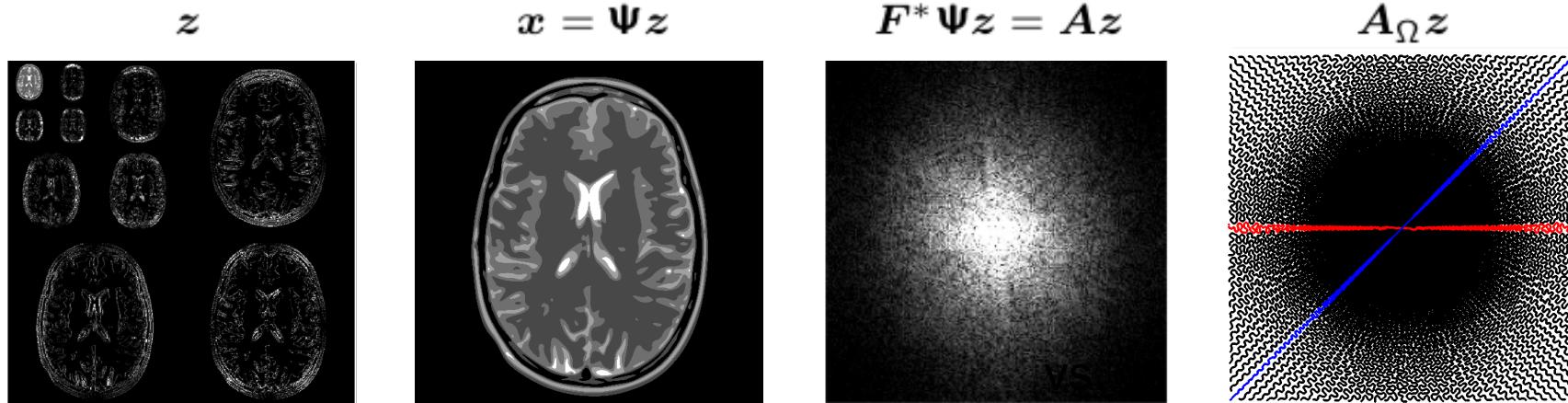
- Objective: retrieve \mathbf{z} (not \mathbf{x})

Fourier sampling

- Sensing matrix:** $A_0 = F^* \Psi = \begin{pmatrix} a_1^* \\ a_2^* \\ \vdots \\ a_n^* \end{pmatrix}$. One measurement reads $y_k = \langle \mathbf{a}_k, \mathbf{z} \rangle$
- Let $\Omega \subseteq \{1, \dots, n\}$ and $A_\Omega = (\mathbf{a}_i^*)_{i \in \Omega}$ the sampling matrix. We get:

$$\mathbf{y} = A_\Omega \mathbf{z} (+ \mathbf{b})$$

Preliminaries



Sparsity: Let $S = \{i, z_i \neq 0\}$ denote the support of \mathbf{z} .

We assume that: $|S| = s \ll n$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noiseless case):

$$\widehat{\mathbf{z}} = \arg \min_{\mathbf{z} \in \mathbb{C}^n, \mathbf{A}_\Omega \mathbf{z} = \mathbf{y}} \|\mathbf{z}\|_1$$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noisy case):

$$\widehat{\mathbf{z}} = \arg \min_{\mathbf{z} \in \mathbb{C}^n} \frac{1}{2} \|\mathbf{y} - \mathbf{A}_\Omega \mathbf{z}\|_2^2 + \lambda \|\mathbf{z}\|_1$$

$$\widehat{\mathbf{x}} = \Psi \widehat{\mathbf{z}}$$

Evolution of Compressed Sensing

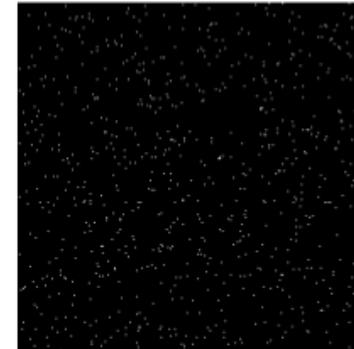
Uniform drawings: the early results

Coherence: $\kappa(\mathbf{A}_0) = n \max_{1 \leq k \leq n} \|\mathbf{a}_k\|_\infty^2$

Assume that \mathbf{x} is s -sparse.

Draw m samples uniformly at random in $\{1, \dots, n\}$.

$m \geq C \cdot \kappa(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right)$ \Rightarrow exact recovery with probability $1 - \epsilon$



Uniform sampling

- [1] **Emmanuel Candes and Terence Tao.**
Near-optimal signal recovery from random projections: Universal encoding strategies?
Information Theory, IEEE Transactions on, 52(12):5406–5425, 2006.
- [2] **Emmanuel Candes and Yaniv Plan.**
A probabilistic and ripless theory of compressed sensing.
Information Theory, IEEE Transactions on, 57(11):7235–7254, 2011.
- [3] **Simon Foucart and Holger Rauhut.**
A mathematical introduction to compressive sensing.
Springer, 2013.

Evolution of Compressed Sensing

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Information Theory, IEEE Transactions on, 57(11):7235–7254, 2011.

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Springer, 2013.

Fourier-Wavelet system satisfies
 $\kappa(\mathbf{A}_0) \propto n!$

$m \gg n$

Coherence barrier!

Evolution of Compressed Sensing

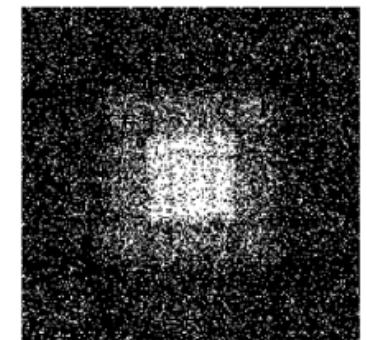
Variable density sampling

Local coherence: $\mu(\mathbf{A}_0) = \sum_{k=1}^n \|\mathbf{a}_k\|_\infty^2$.

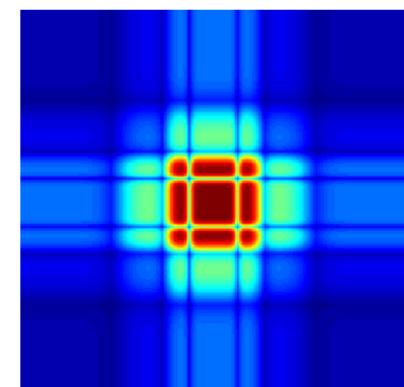
Assume that \mathbf{x} is s -sparse.

Set $\mathbf{A}_\Omega = \begin{pmatrix} \mathbf{a}_{J_1} \\ \vdots \\ \mathbf{a}_{J_m} \end{pmatrix}$ with $\mathbb{P}(J_k = \ell) = \pi_\ell \propto \|\pi_\ell\|_\infty^2$

$$m \geq C \cdot \mu(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Variable density sampling



π in 2D

- [1] **Gilles Puy, Pierre Vandergheynst, and Yves Wiaux.**
On variable density compressive sampling.
Signal Processing Letters, IEEE, 18(10):595–598, 2011.

- [2] **Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.**
Variable density sampling with continuous sampling trajectories.
SIAM Journal on Imaging Science, 2014.

- [3] **Felix Krahmer and Rachel Ward.**
Stable and robust sampling strategies for compressive imaging.
Image Processing, IEEE Transactions on, 23(2):612–622, 2014.



Go to 2nd notebook

Evolution of Compressed Sensing

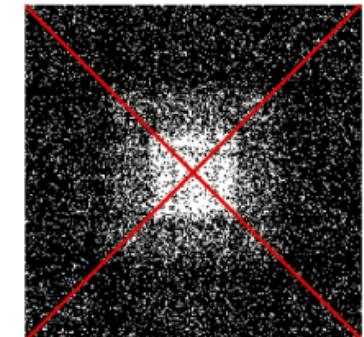
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$m \geq C \cdot \mu(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow$ exact recovery with probability $1 - \epsilon$



Variable density sampling

Fourier-Wavelet system satisfies $\mu(\mathbf{A}_0) \propto \log(n)$!

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Signal Processing Letters, IEEE, 18(10):595–598, 2011.

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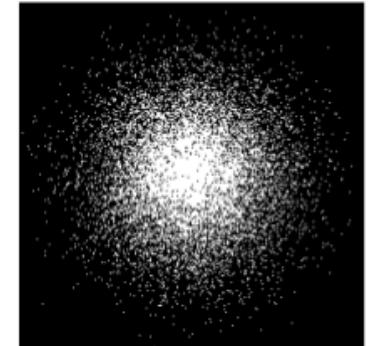
Evolution of Compressed Sensing

Variable density sampling with structure

Assume that x has a certain “sparsity structure”.

$$\text{Set } A_\Omega = \begin{pmatrix} a_{J_1} \\ \vdots \\ a_{J_m} \end{pmatrix} \text{ with } \mathbb{P}(J_k = \ell) = \tilde{\pi}_\ell$$

$$m \geq C \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Variable density
sampling +
structure

- [1] **Ben Adcock and Anders C Hansen.**
Generalized sampling and infinite-dimensional compressed sensing.
Foundations of Computational Mathematics, to appear, 2015.
- [2] **Ben Adcock, Anders C. Hansen, Clarice Poon, and Bogdan Roman.**
Breaking the coherence barrier: A new theory for compressed sensing.
arXiv preprint arXiv:1302.0561, 2013.
- [3] **Jérémie Bigot, Claire Boyer, and Pierre Weiss.**
Compressed sensing with structured sparsity and structured acquisition.
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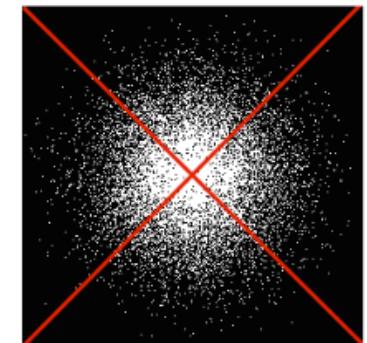
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Compressed sensing with structured sparsity and structured acquisition.
Applied and Computational Harmonic Analysis, 2017.

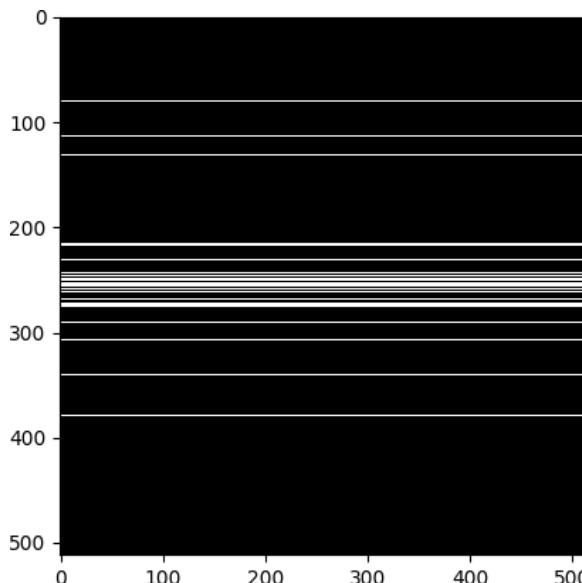
Only works
for isolated
measurements!



Current compressed sensing theories are partially disconnected from applications!

Possible 1D VDS implementation

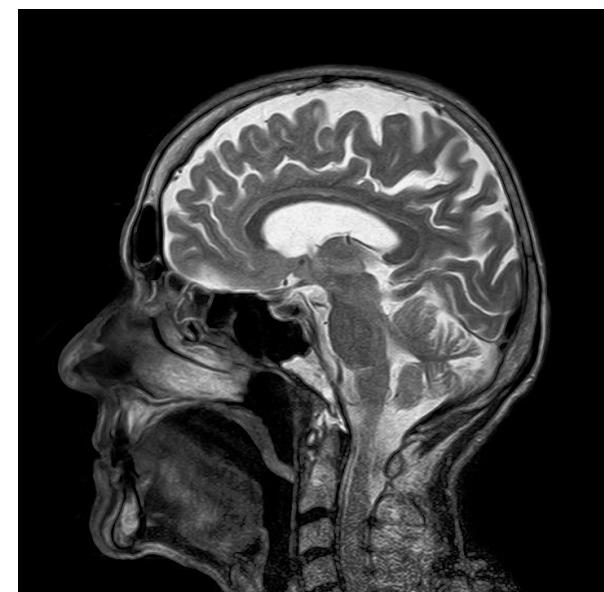
k-space



Nonlinear
reconstructions

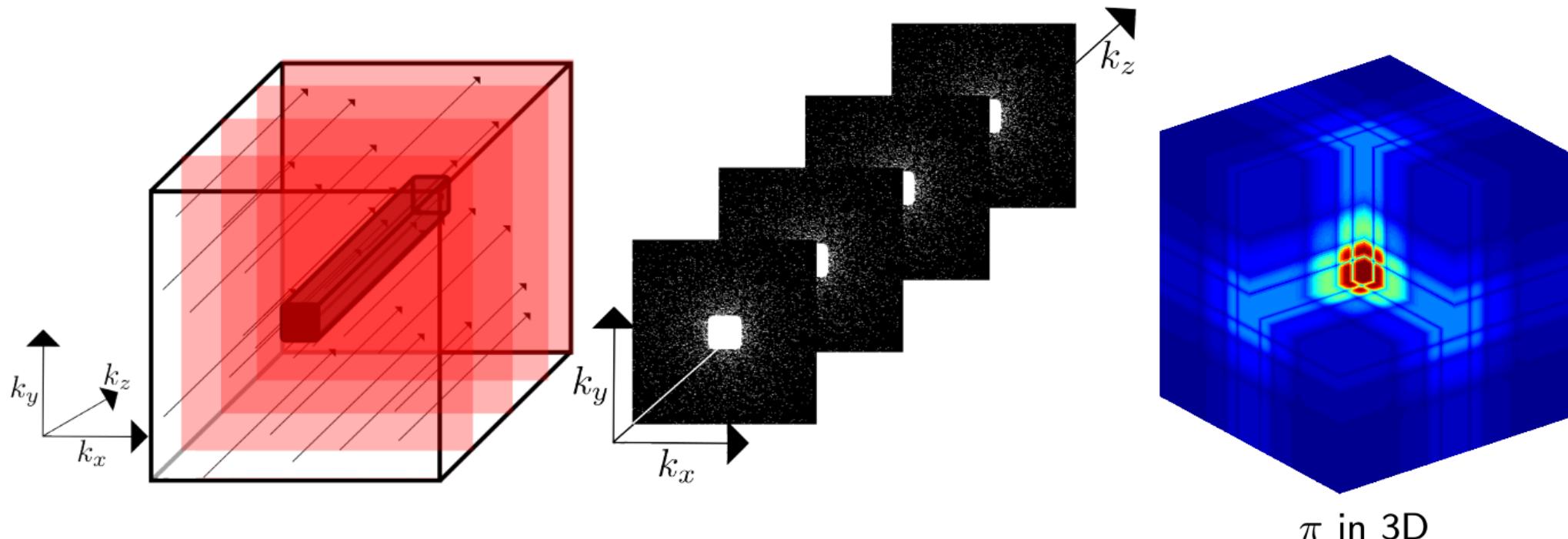


Image



Go the 4th notebook

Possible 2D VDS implementation



→ Perform the readout along the **third dimension!**
Suboptimal as compared to π in 3D.

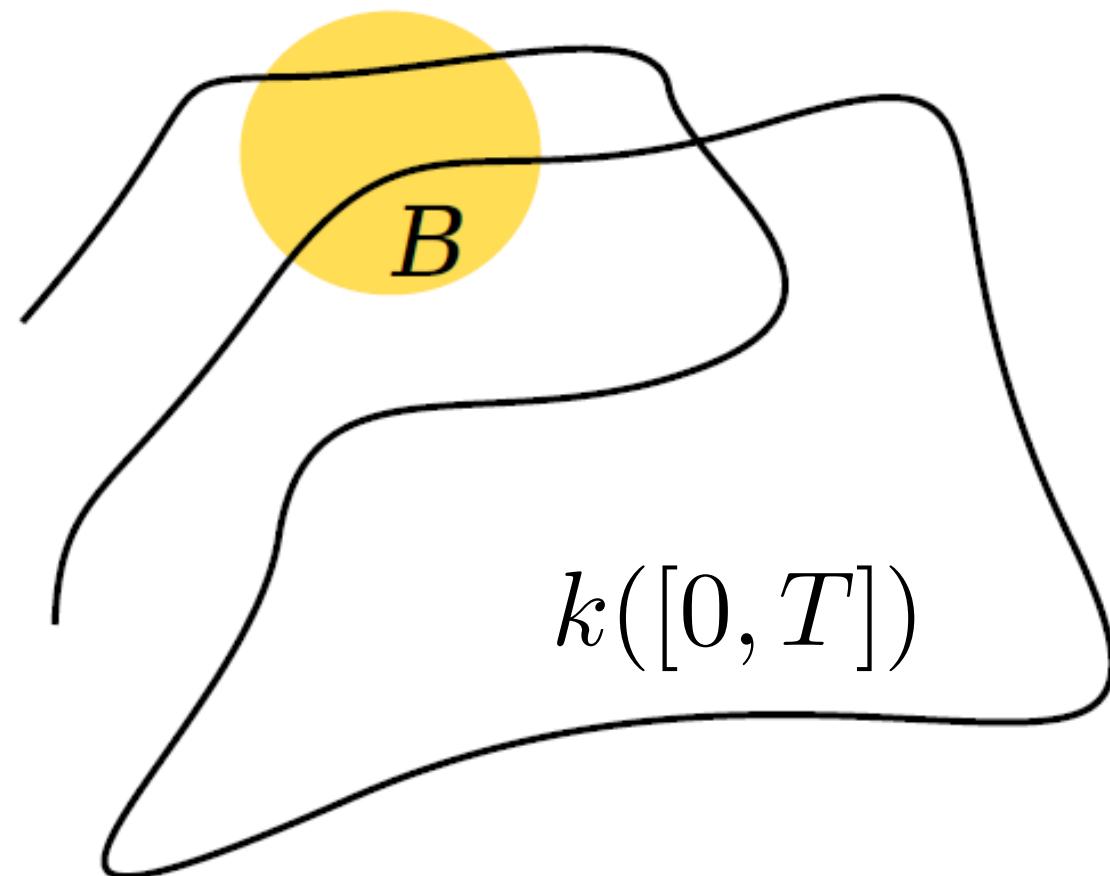
Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

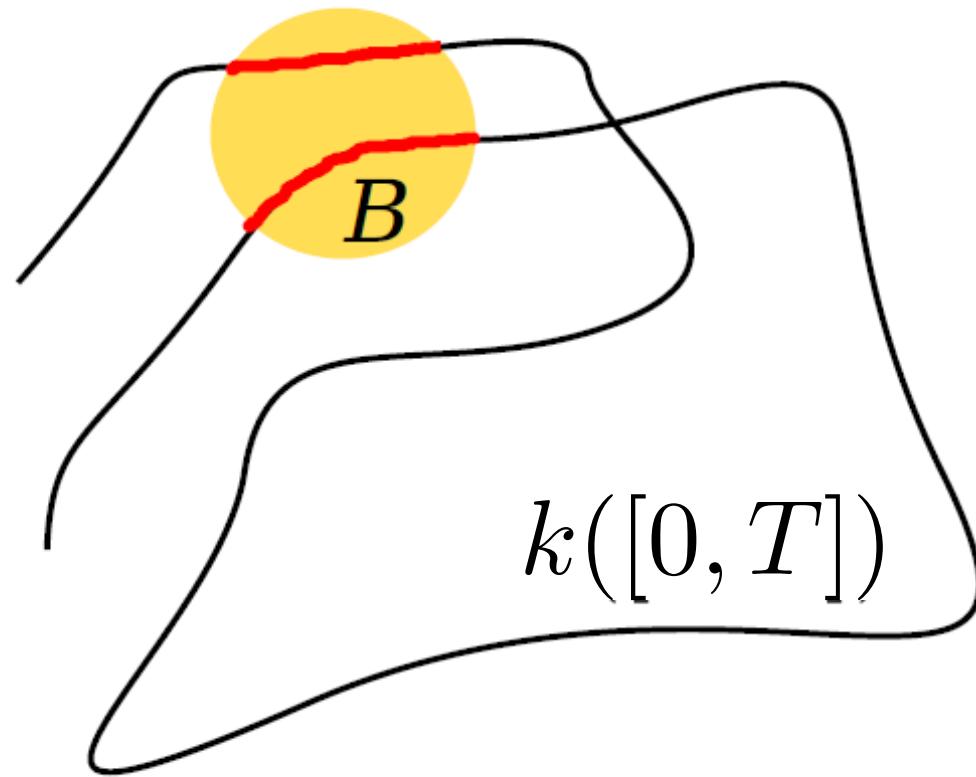
Continuous Variable Density Sampler

Pushforward measure - illustration



Continuous Variable Density Sampler

Pushforward measure - illustration



$$\nu(B) = k_* \lambda_T(B) = \lambda_T(k^{-1}(B))$$

λ_T is the (normalized) Lebesgue measure.

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

Push Forward Measure - Definition

Pushforward measure

Let $\Omega = [0, 1]^d$, where $d = 2$ or 3 denote the space dimension. We equip Ω with the Borel algebra \mathcal{B} . Let (X, Σ) be a measurable space and $k: X \mapsto \Omega$ be a measurable mapping. $\mu: X \mapsto [0; +\infty)$ denote a measure. The *pushforward measure* ν of μ is defined by:

$$\nu(B) = k_*\mu(B) = \mu(k^{-1}(B)), \quad \forall B \in \mathcal{B}$$

Ex. 1: Measures supported by curves

Ex. 2: Atomic measures

$k: \{1, \dots, m\} \mapsto \Omega$ where $k[i] = p_i$ denotes the i -th point. Set μ as the *counting measure* defined for any set $I \subseteq \{1, \dots, m\}$ by $\mu(I) = \frac{|I|}{m}$. Then ν is defined by

$$\nu = \frac{1}{m} \sum_{i=1}^m \delta_{p_i}$$

Push Forward Measure - Theorem

Weak convergence

A sequence of measures (μ_n) is said to weakly converge to μ , if for any bounded continuous function ϕ ,

$$\int_{\Omega} \phi(x) d\mu_n(x) \xrightarrow{n \rightarrow +\infty} \int_{\Omega} \phi(x) d\mu(x)$$

Shorthand notation: $\mu_n \rightharpoonup \mu$.

Variable density sampler

A sequence of (random) trajectories $k_n : X_n \mapsto \Omega$ is said to be a π -Variable Density Sampler if

$$k_{n*}\mu \rightharpoonup \pi \text{ almost surely}$$

Examples

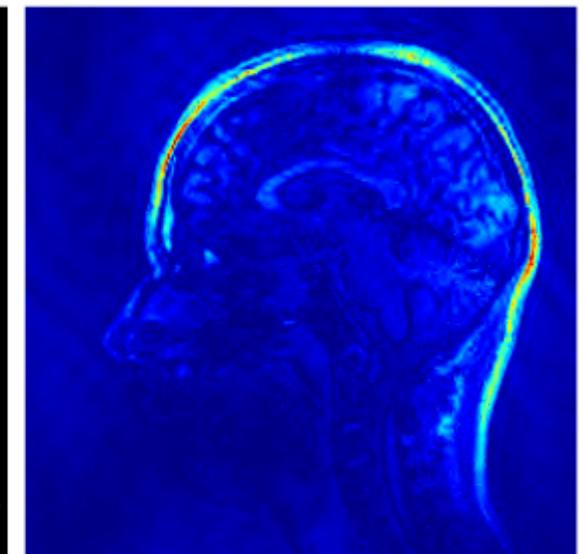
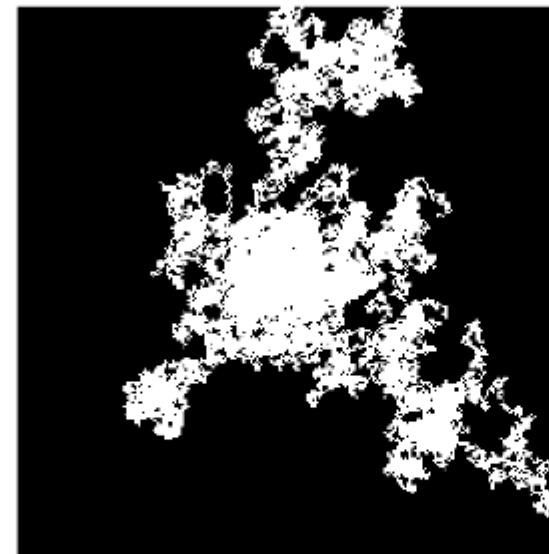
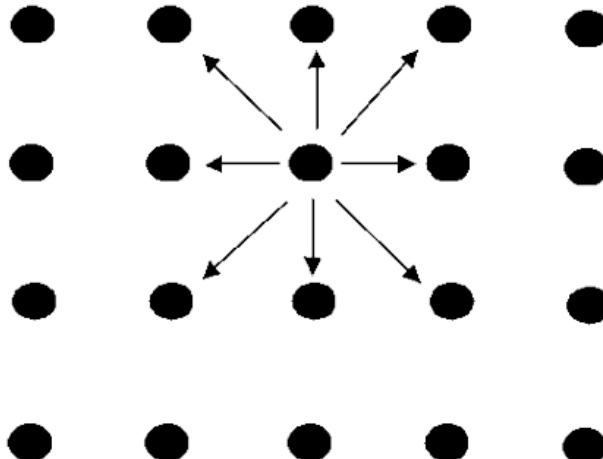
i.i.d. drawing, random walks ...

Sampling along Random Walks

Construction of a discrete Markov chain

Given a target probability distribution $\pi \in \mathbb{R}^n$.

Define a Markov chain $K = (k_i)_{i \in \mathbb{N}}$ on the set $\{1, \dots, n\}$. Use the Metropolis algorithm to construct a stochastic transition matrix $P \in \mathbb{R}^{n \times n}$ such that π is the stationary distribution of K .



- Time to cover the k -space is slow
- Local approach

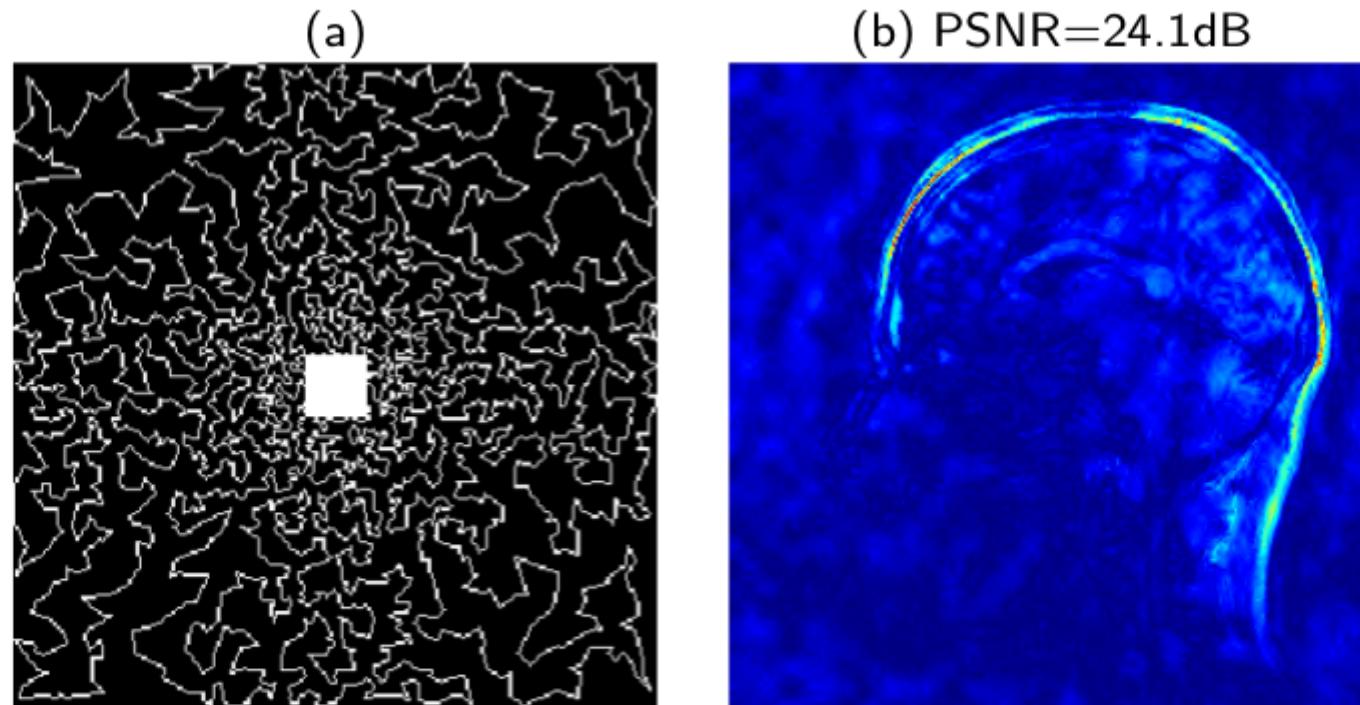
Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

Solving the Travelling Salesman Problem

Idea : cover the k -space more quickly with a global approach.



- Pushforward measure far from π . From which distribution should we sample the initial points to reach a given target distribution?

Go to Matlab code:
TSP-Markov folder

TSP Sampler

- Let

$$q = \frac{\pi^{d/(d-1)}}{\int_{\Omega} \pi^{d/(d-1)}}$$

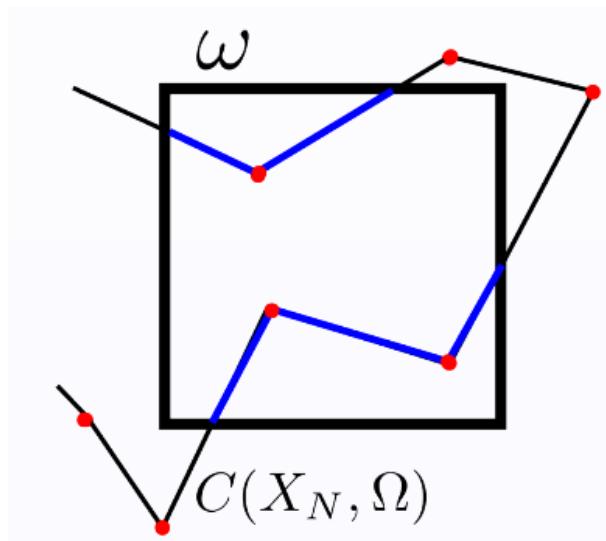
- $(k[i])_{i \in \mathbb{N}^*}$ a sequence of points in Ω , i.i.d. drawn $\sim q$.
- $\mathbf{k}_N = (k[i])_{i \leq N}$
- Denote $T(\mathbf{k}_N)$ the length of the TSP amongst \mathbf{k}_N .
- $\gamma_N : [0, T(\mathbf{k}_N)] \mapsto \Omega$ denotes the parametrization of the curve at speed 1.

Theorem

Almost surely w.r.t. the law $q^{\otimes \mathbb{N}}$ of the sequence $(k[i])_{i \in \mathbb{N}^*}$ of random points in the hypercube, $(\gamma_N)_{N \in \mathbb{N}}$ is a π -variable density sampler, i.e.,

$$\gamma_{N*} \lambda_{T(\mathbf{k}_N)} \rightharpoonup \pi$$

Intuition of the Proof

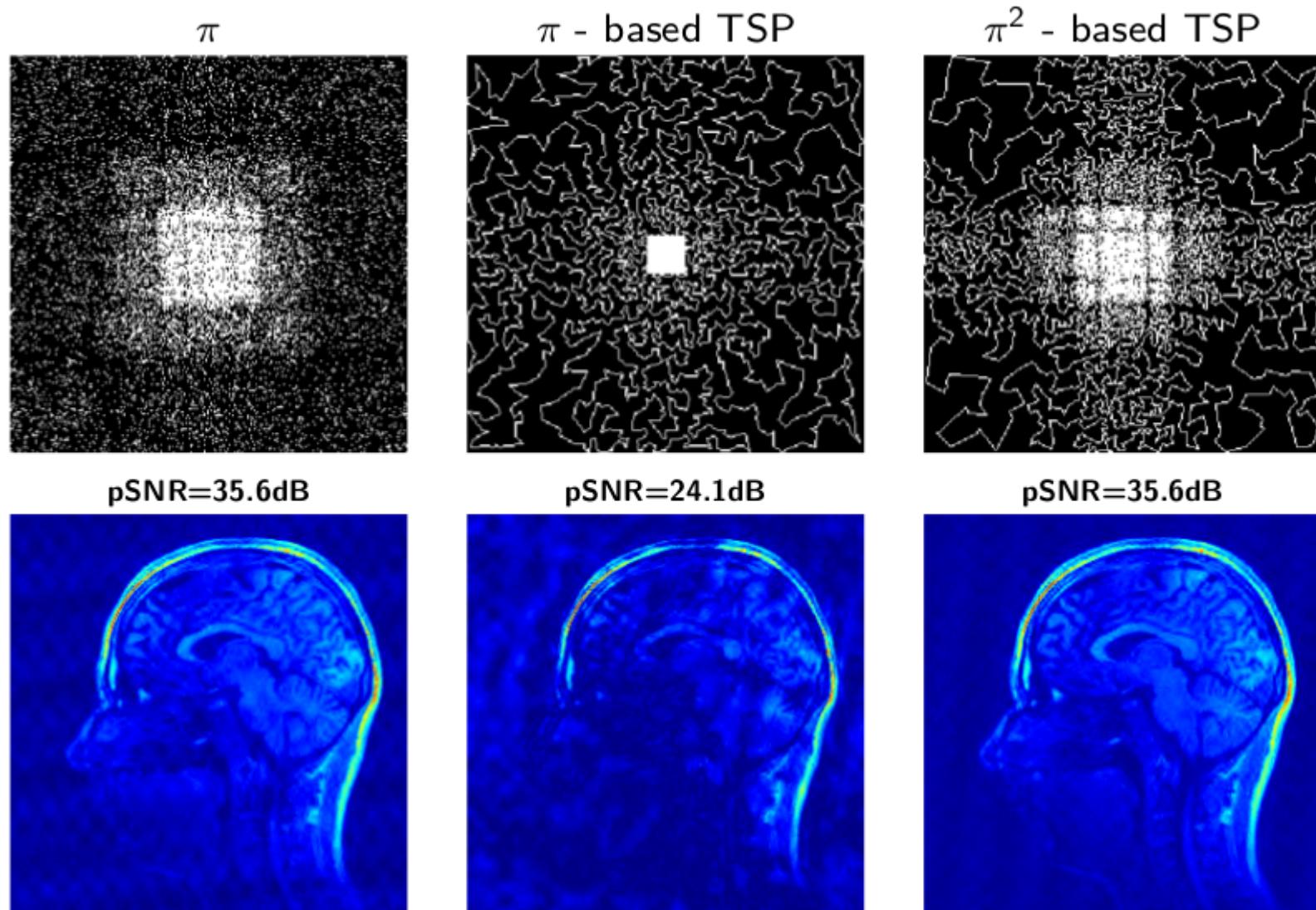


Definition of the TSP distribution

For any Borelian $\omega \in \Omega$:

$$P_N(\omega) = \frac{T_{|\omega}(X_N)}{T(X_N)}$$

The Traveling Salesman Sampler in 2D

Sampling schemes
($r = 5$)

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.
 Variable density sampling with continuous sampling trajectories.
 SIAM Journal on Imaging Science, 2014.

Go to Matlab code:
TSP-Markov folder

PAGE 46

The Traveling Salesman Sampler in 3D

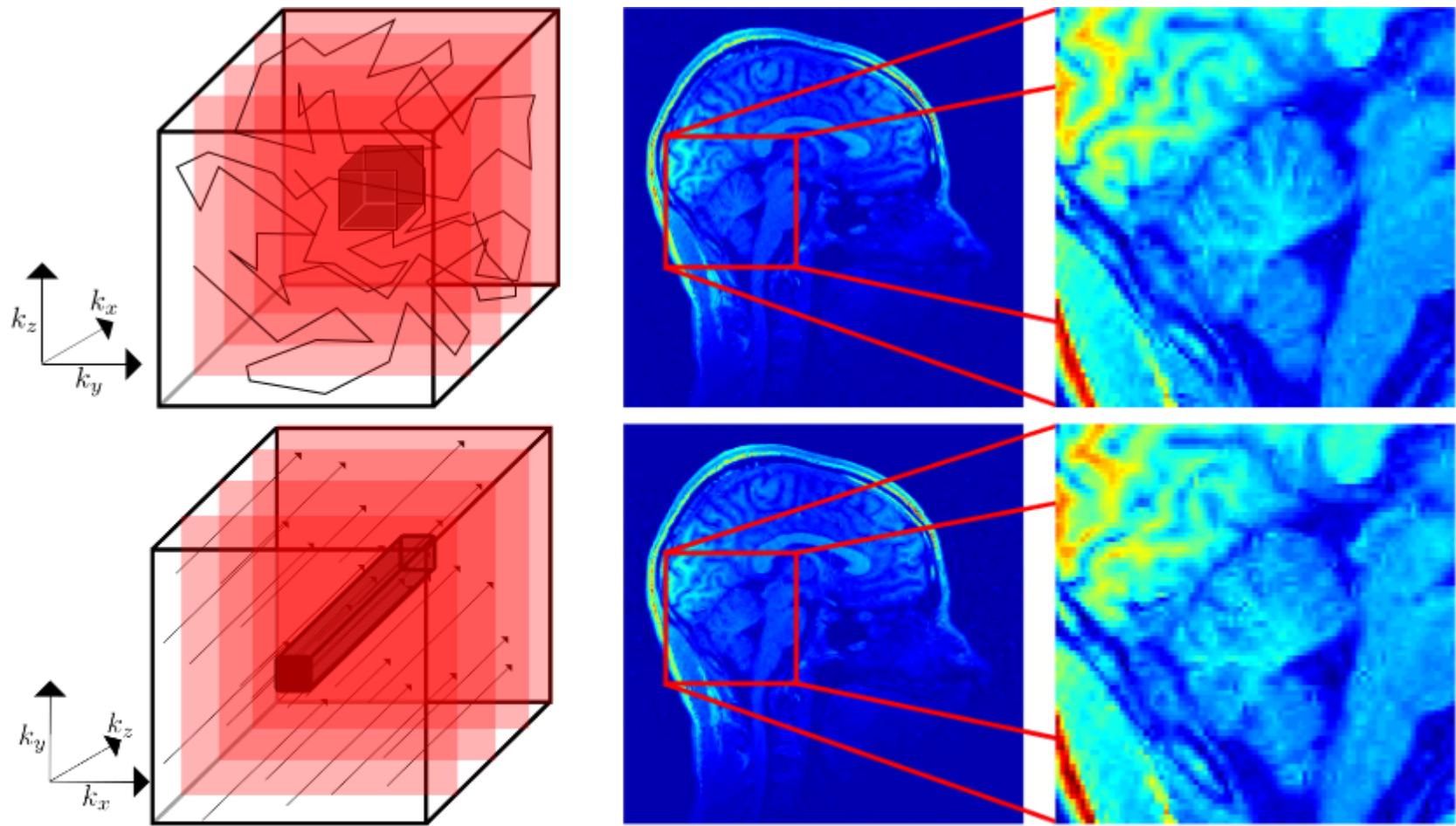


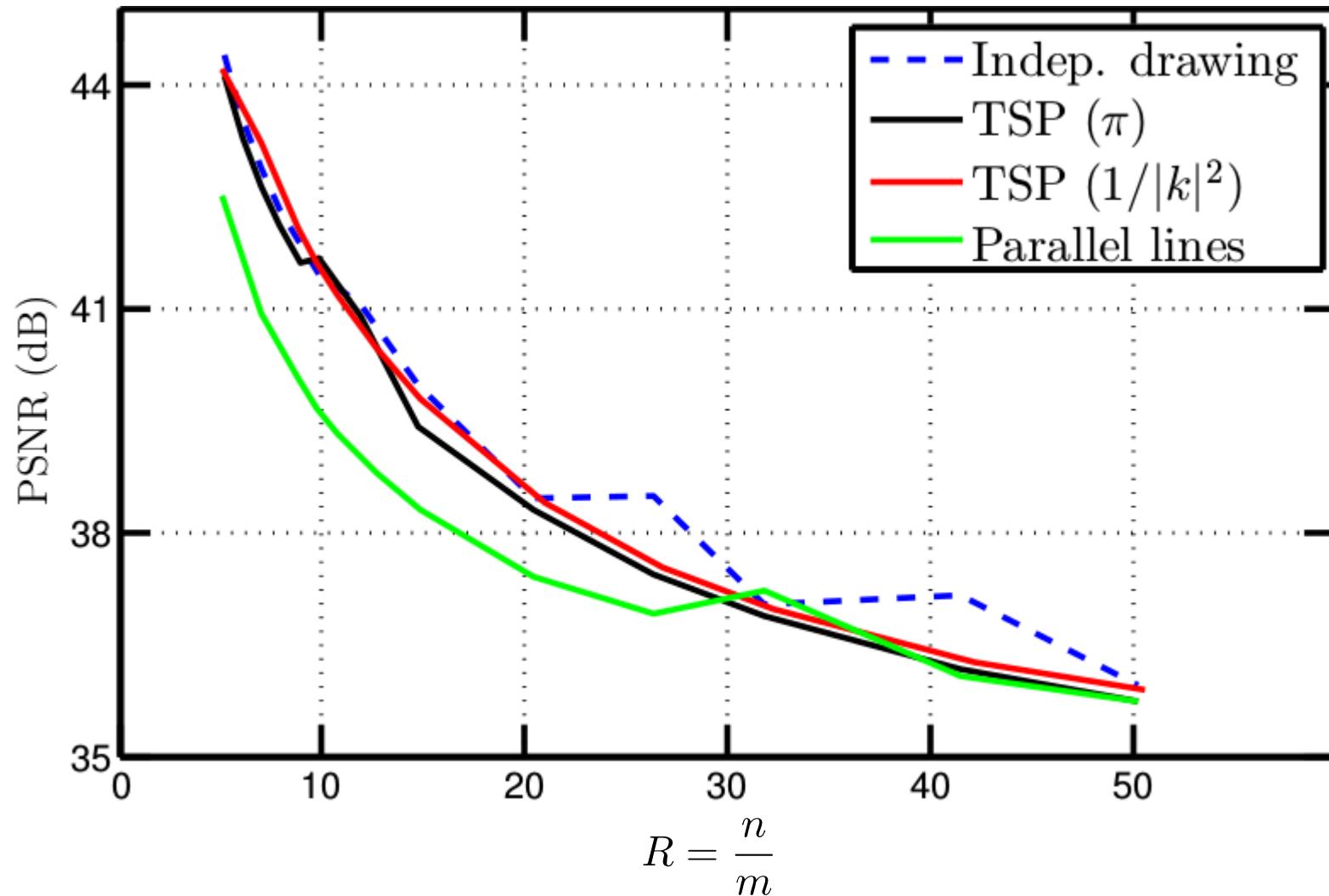
Figure: 3D reconstruction results for $r = 8.8$ for various sampling strategies. **Top row:** TSP-based sampling schemes (PSNR=42.1 dB). **Bottom row:** 2D random drawing and acquisitions along parallel lines [Lustig et al., 2007] (PSNR=40.1 dB).

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

The Traveling Salesman Sampler in 3D

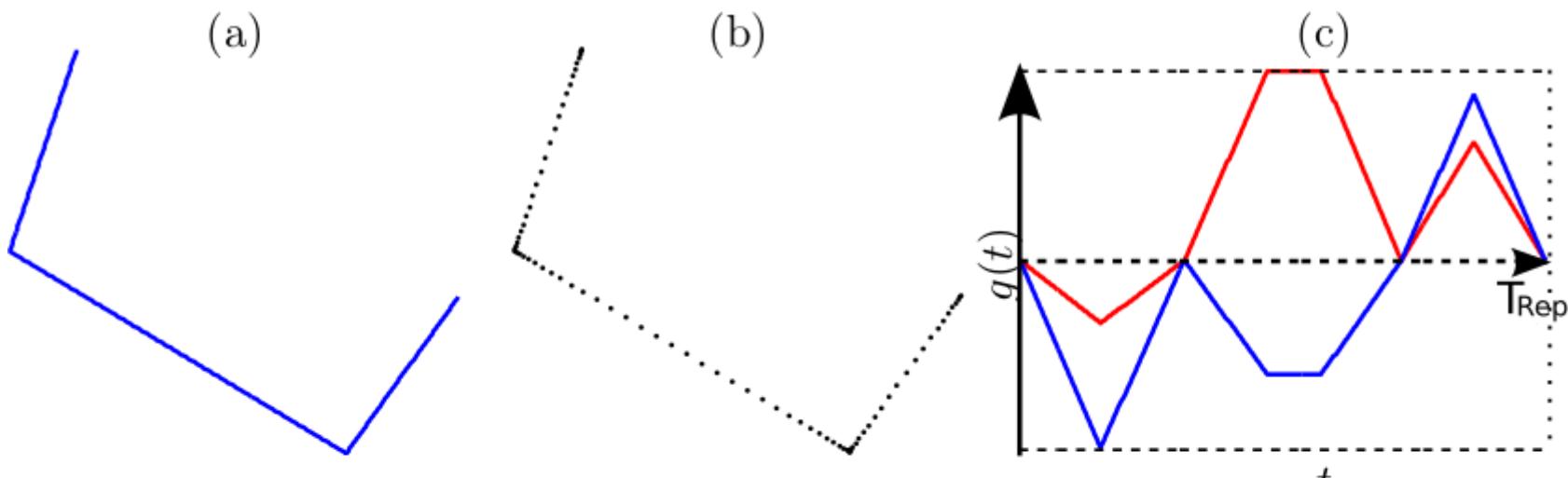


The Parameterization Problem

Finding a parameterization in \mathcal{S} – corresponding to a curve support is not easy !

- Classical approach, find an admissible parameterization
[Hargreaves et al., 2004, Lustig et al., 2008]:

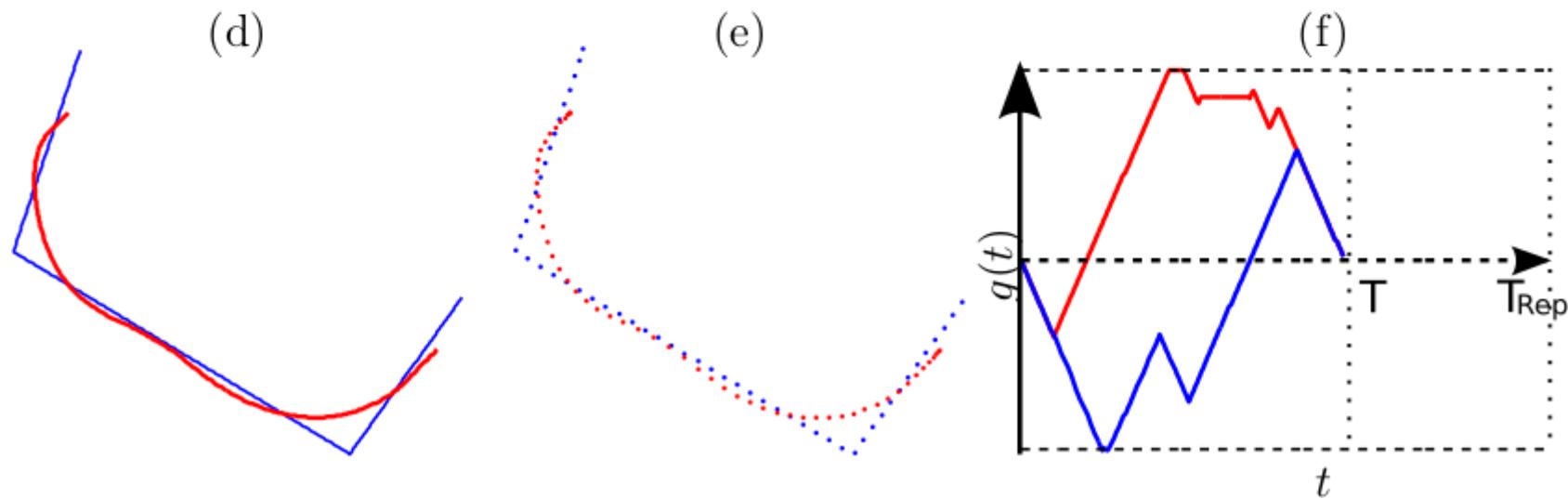
$$T_{\text{rep}} = \min T' \text{ such that } \exists r : [0, T'] \mapsto [0, T], c \circ r \in \mathcal{S}$$



Stick to the support of C

Projection onto the MRI constraints

- Projection onto \mathcal{S} [Chauffert et al, IEEE TMI 2016]



➡ Leave the support of \mathcal{C} but keep control on the sampling density

Go to Matlab code:
[Gradient_waveform_design](#)

The Projection Algorithm

For an input parameterized curve $c : [0, T] \mapsto \Omega$, define:

$$P_{\mathcal{S}}(c) = \arg \min_{k \in \mathcal{S}} \int_0^T (k(t) - c(t))^2 dt$$

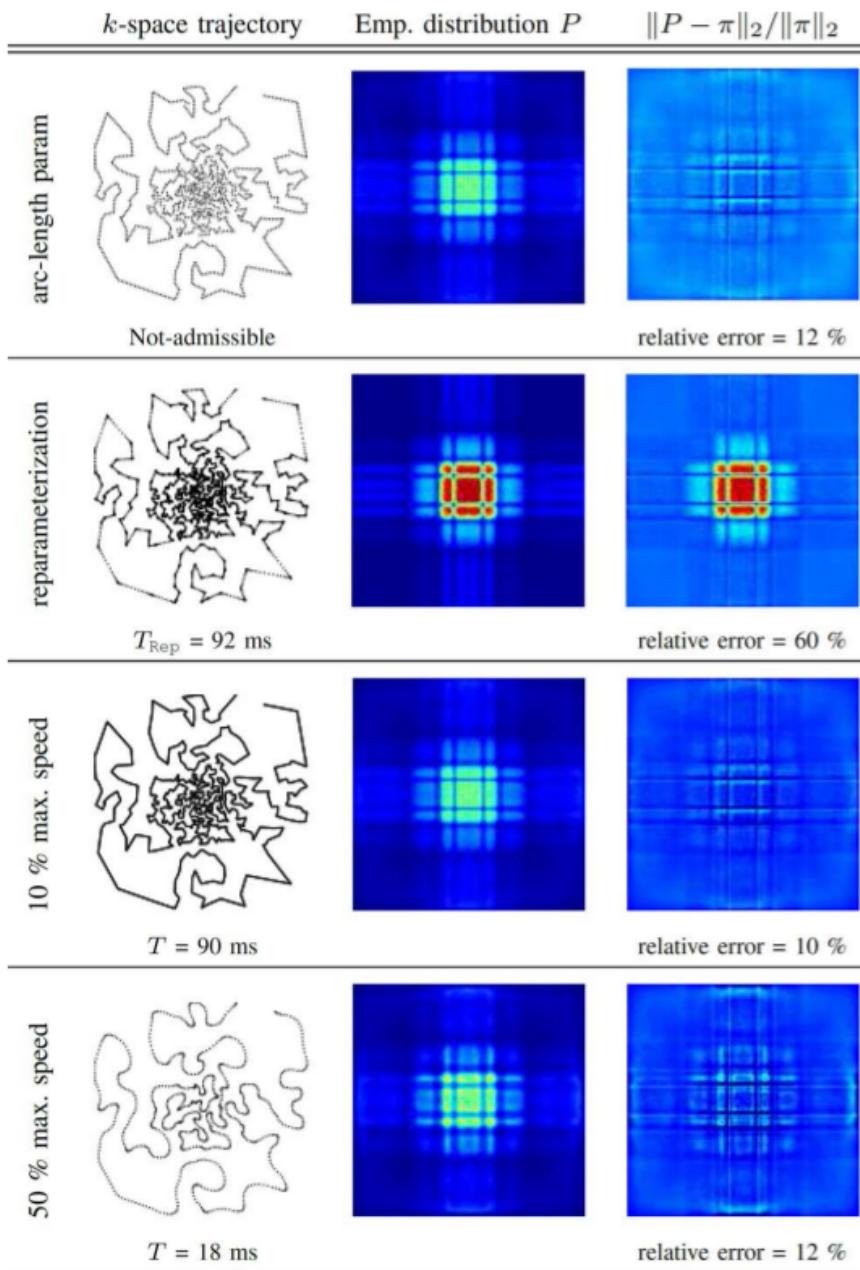
Main properties [Chauffert et al., 2014b]

- Fast resolution using accelerated proximal gradient descent on the dual.
- The sampling time is fixed (equal to T).
- The sampling distribution is well preserved (approximation of Wasserstein distance \mathcal{W}_2).

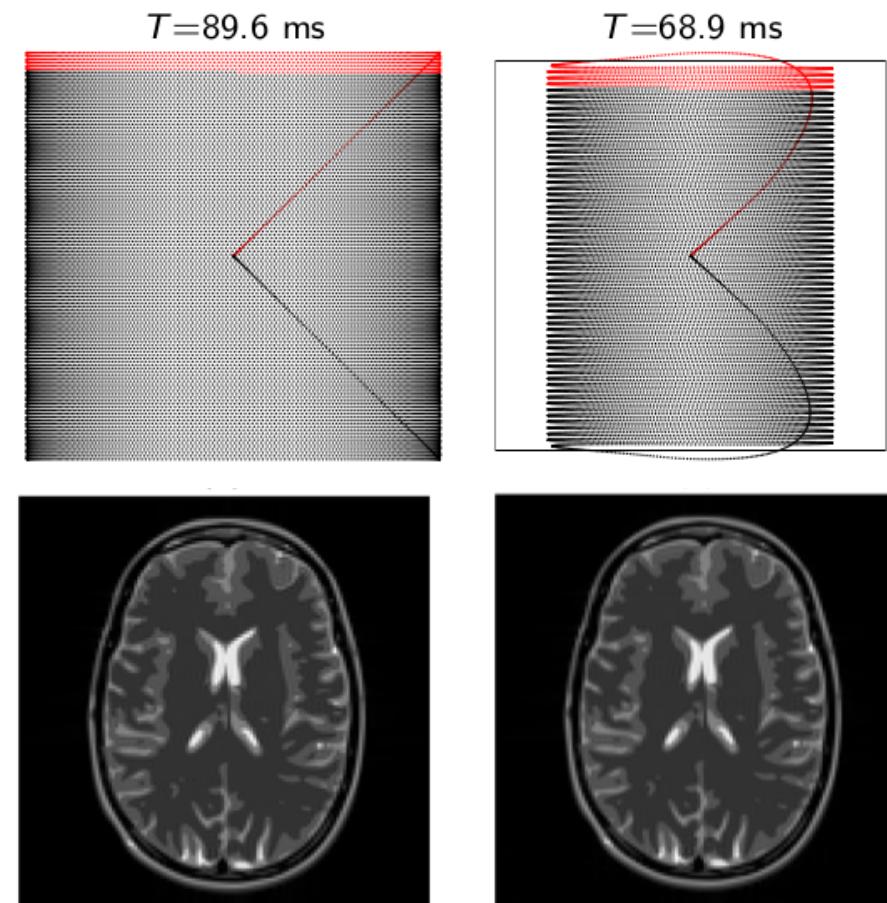


Leave the support of c but keep control on the sampling density

Illustration & Results



EPI trajectories



[Chauffert et al, IEEE TMI 2016]

Interim Summary

- **Heuristic conclusions**

The main features characterizing the efficiency of a given sampler are :

- Its empirical (pushforward) measure/density
- Its mixing time, i.e. its ability to cover k-space rapidly

- **The new quest ...**

Design a variable density sampler with:

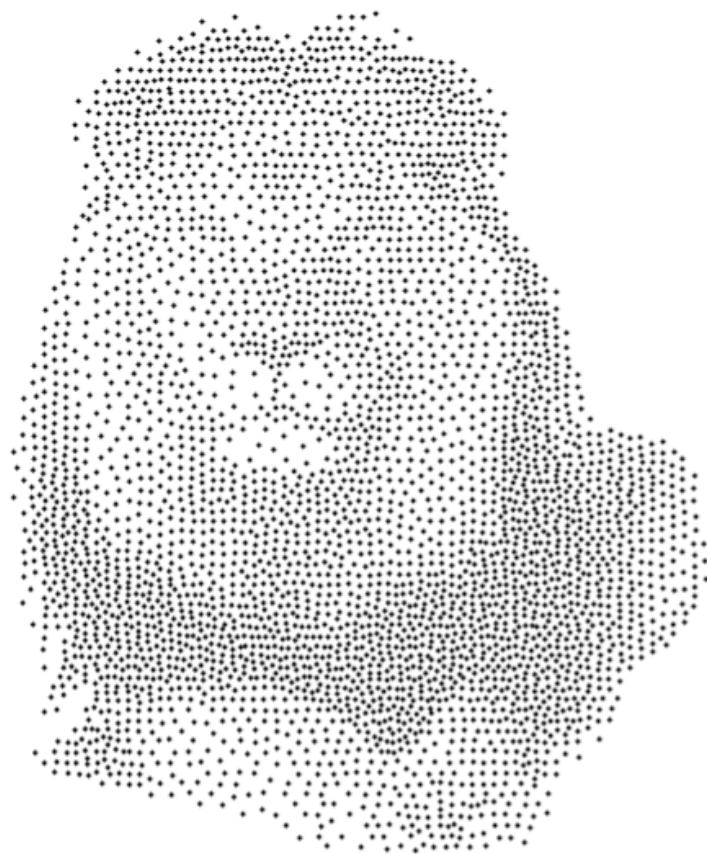
- 1) Prescribed density
 - 2) Good mixing properties
- Belonging to a certain class, i.e. being compliant with MR hardware constraints and a given imaging contrast (e.g. T_1 , T_2 , etc)

Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- **Image stippling techniques**
- SPARKLING

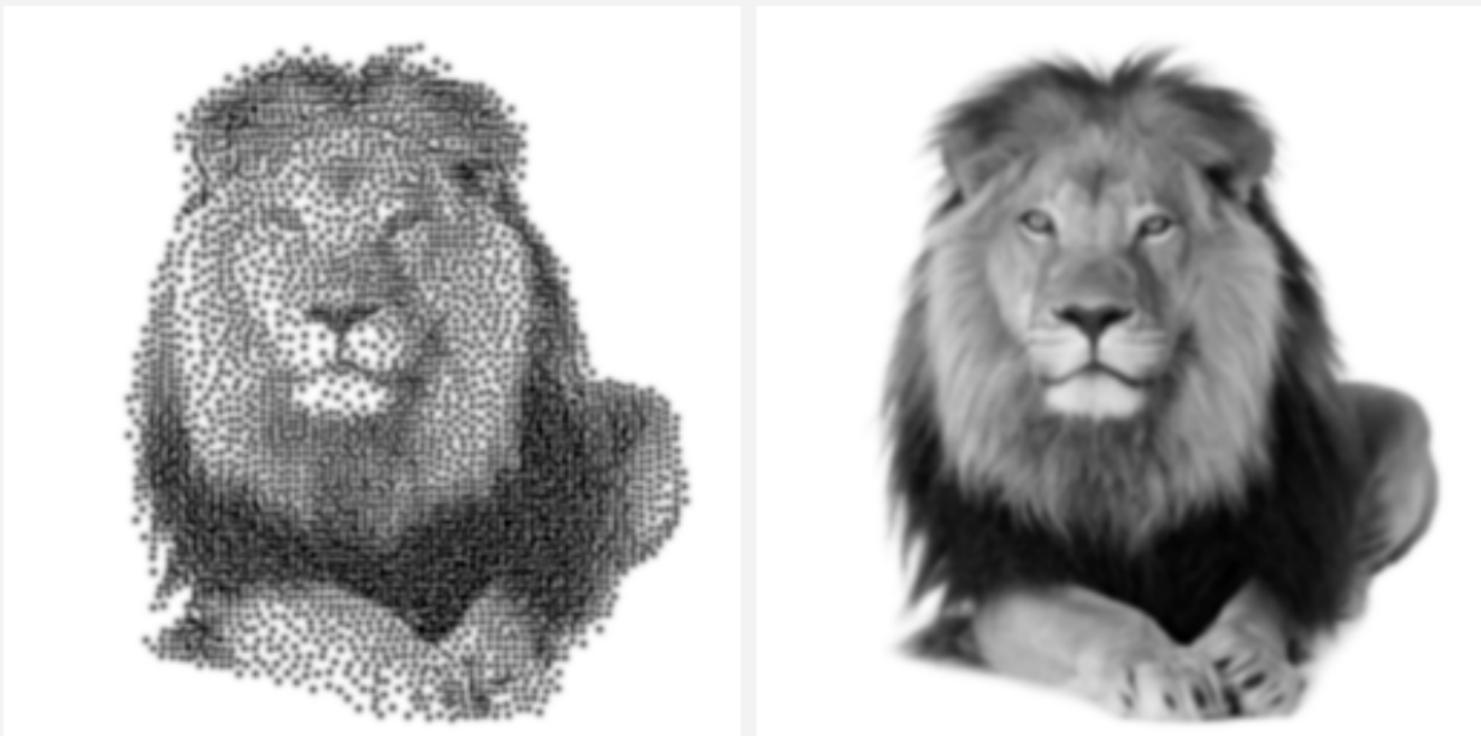
How to measure similarity between images?



Halftoned and reference images

Why are the images alike?

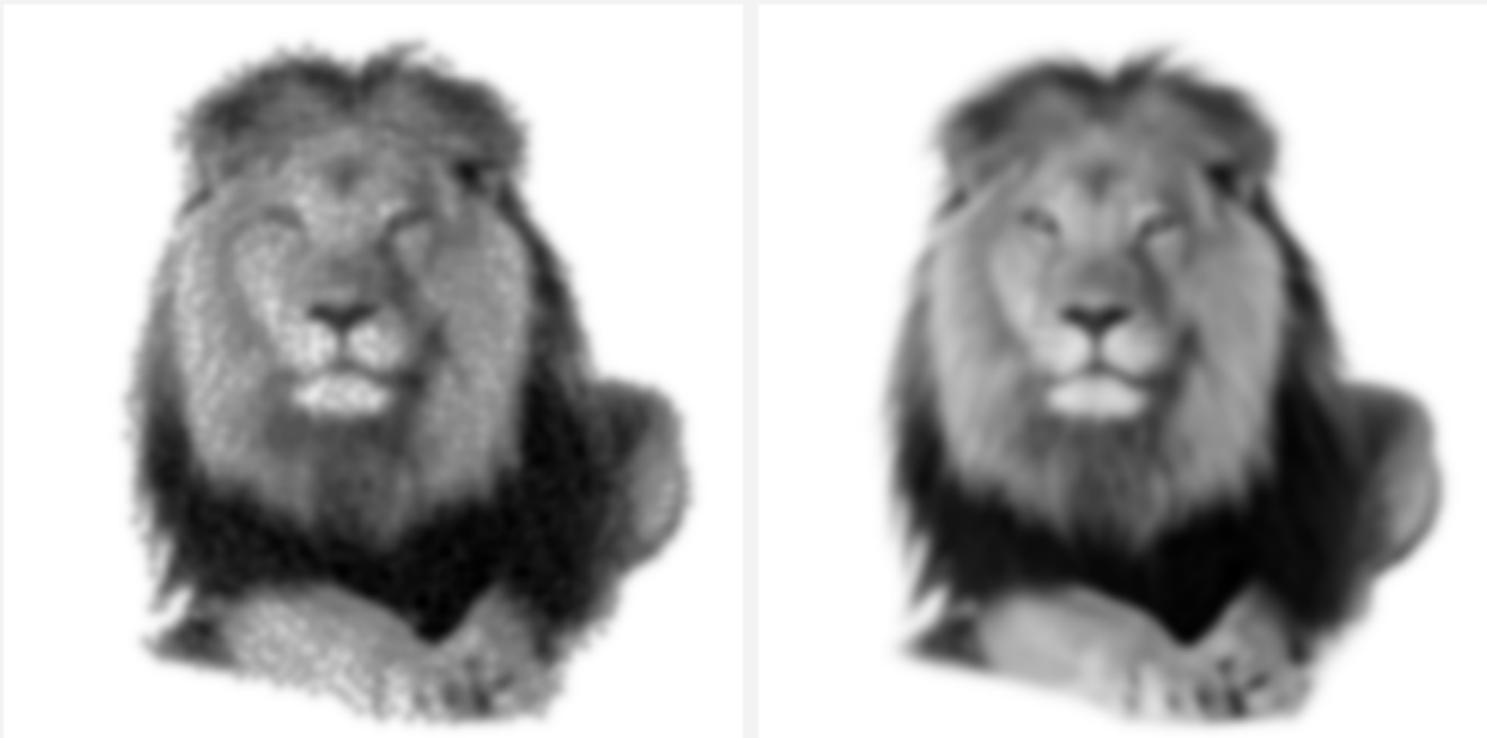
The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 1$.

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 2$.

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 3$.



The images are similar only at **low resolution**

Constructing a metric

Let π denote the **target image** (the lion).

Let ν denote the **atomic measure** (the dots).

A natural distance to quantify image similarity reads:

$$\mathcal{D}_h(\nu - \pi) = \|h \star \nu - h \star \pi\|_2 .$$

The kernel h can be chosen as the impulse response of the visual system.

Halftoning as a projection problem

How to find the point locations?

Let $\Omega \subseteq \mathbb{R}^d$ denote the **image domain**.

Let $\mathcal{M}(\Omega^N)$ denote the **set of atomic measures** with N points.

$$\mathcal{M}(\Omega^N) = \left\{ \nu = \frac{1}{N} \sum_{i=1}^N \delta_{p_i}, p_i \in \Omega \right\}$$

The **halftoning problem** can be formulated as a projection problem:

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \mathcal{D}_h(\nu - \pi).$$

Link with Attraction-Repulsion

Proposition

The projection problem

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \frac{1}{2} \|h \star \nu - h \star \pi\|_2^2$$

can be rewritten

$$\min_{(p_1, \dots, p_N) \in \Omega^N} \underbrace{\frac{1}{N} \sum_{i=1}^N \int_{\Omega} H(p_i - x) \pi(x) dx}_{\text{Attraction potential}} - \underbrace{\frac{1}{2N^2} \sum_{1 \leq i, j \leq N} H(p_i - p_j)}_{\text{Repulsion potential}}$$

where $\hat{H} = |\hat{h}|^2$.

Modeling MRI kinematic constraints

$$\min_{\mathbf{k} \in \mathcal{Q}_N} \frac{1}{2} \|h \star \nu(\mathbf{k}) - h \star \pi\|_2^2 \quad \text{with } \nu(\mathbf{k}) = \frac{1}{N} \sum_{i=1}^N \delta_{k[i]}, \quad \mathbf{k} = (k[i])_{1 \leq i \leq N} \in \mathcal{Q}_N$$

where $\mathcal{Q}_N = \{\|\dot{\mathbf{k}}\|_{2,\infty} \leq \gamma G_{\max}, \|\ddot{\mathbf{k}}\|_{2,\infty} \leq \gamma S_{\max}\}$

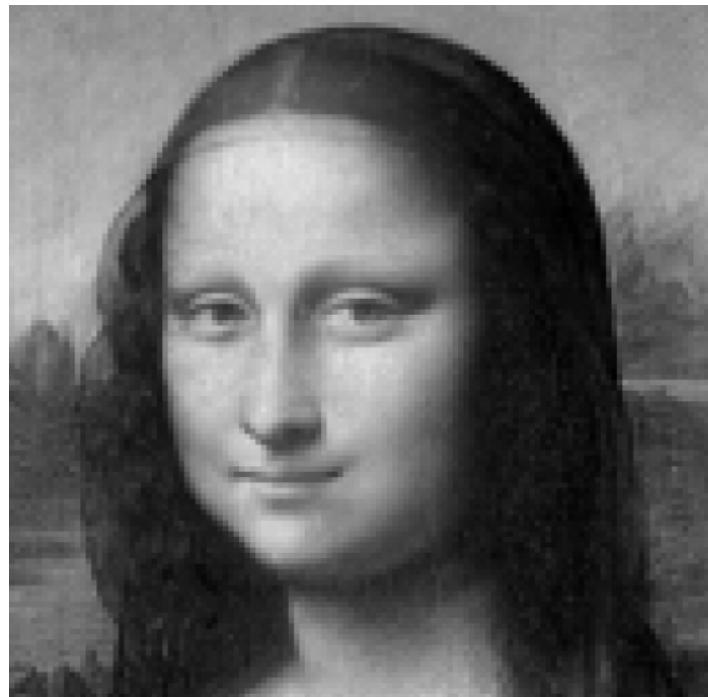
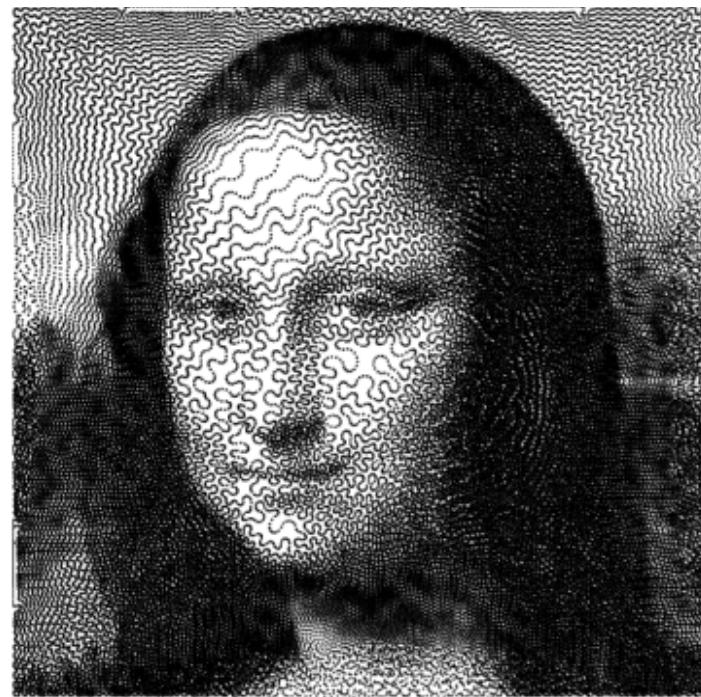
$$\underbrace{\min_{\mathbf{k} \in \mathcal{Q}_N} \frac{1}{N} \sum_{i=1}^N \int_{\Omega} H(x - k[i]) \pi(x) dx}_{F_a(\mathbf{k}) \atop \text{attraction}} - \underbrace{\frac{1}{2N^2} \sum_{1 \leq i, j \leq N} H(k[i] - k[j])}_{F_r(\mathbf{k}) \atop \text{repulsion}}$$

Projected gradient descent: $\mathbf{k}^{(t+1)} = P_{\mathcal{Q}_N}(\mathbf{k}^{(t)} - \tau^{(k)} \nabla(F_a - F_r)(\mathbf{k}^{(t)}))$

[Chauffert et al, IEEE TMI 2016]

Inertial adaptive step-size (BB update)

Image stippling with projection onto MRI kinematic constraints

 $h_\sigma \star \pi$  $h_\sigma \star \nu$

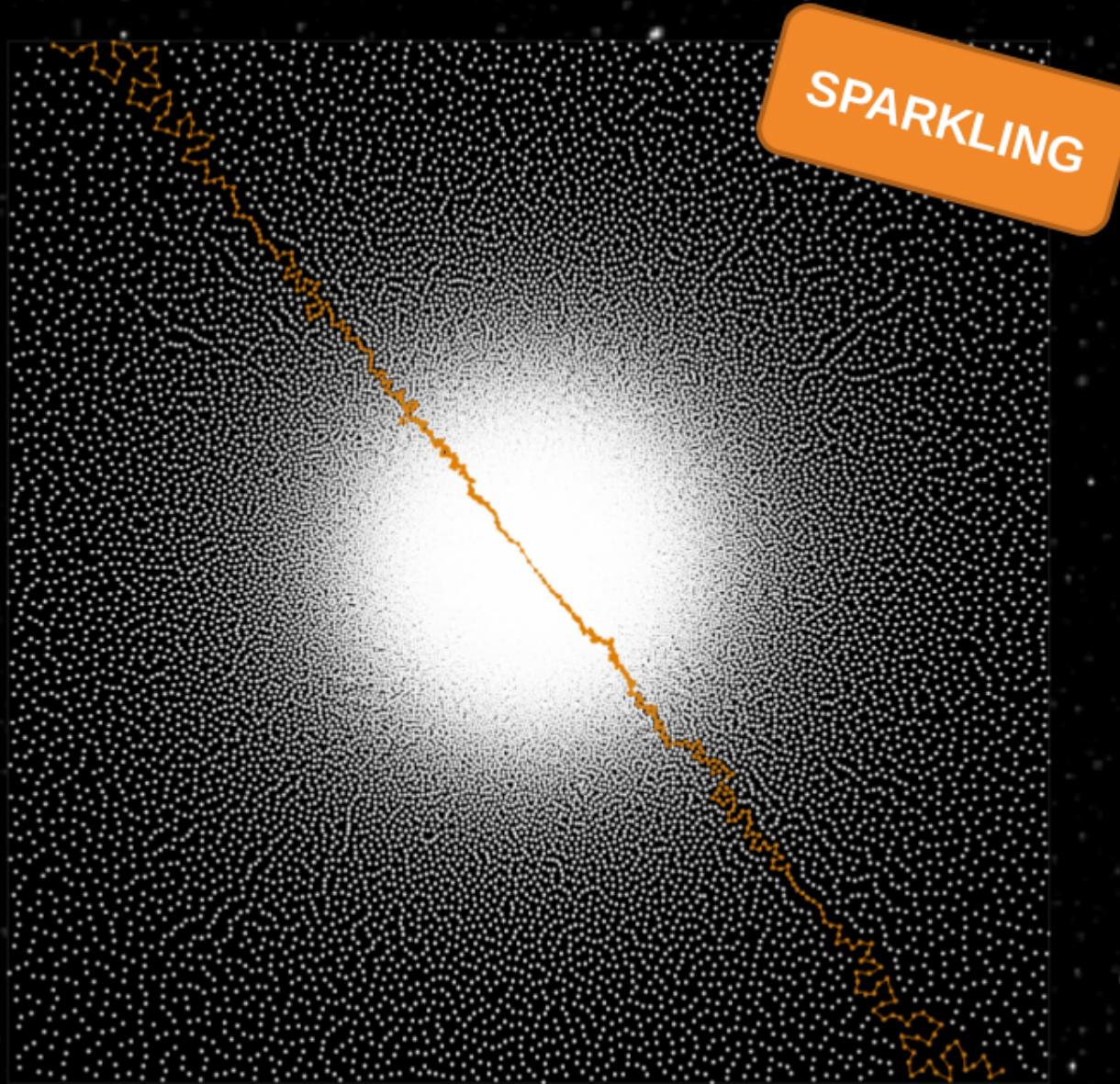
Outline

Part III: CS sampling trajectories

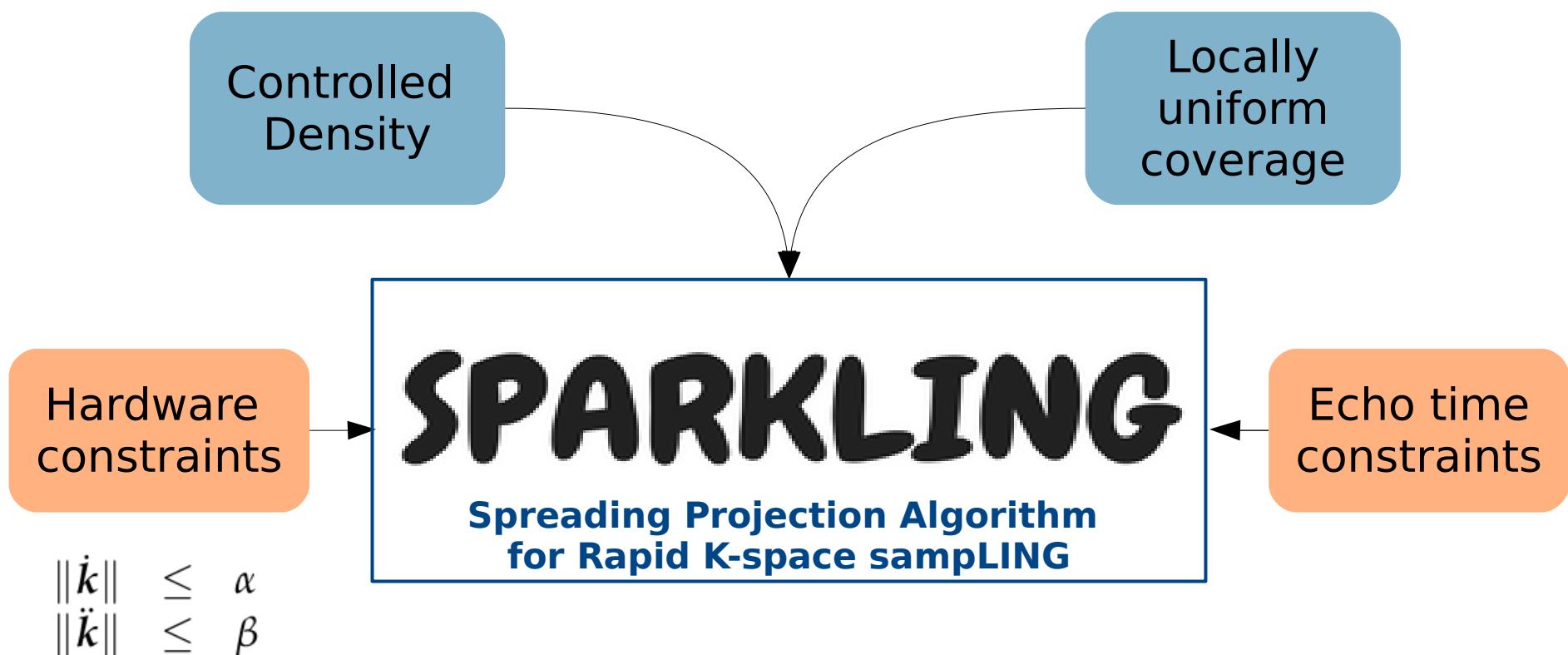
- Continuous Variable Density Samplers
- Image stippling techniques
- **SPARKLING**

SPARKLING

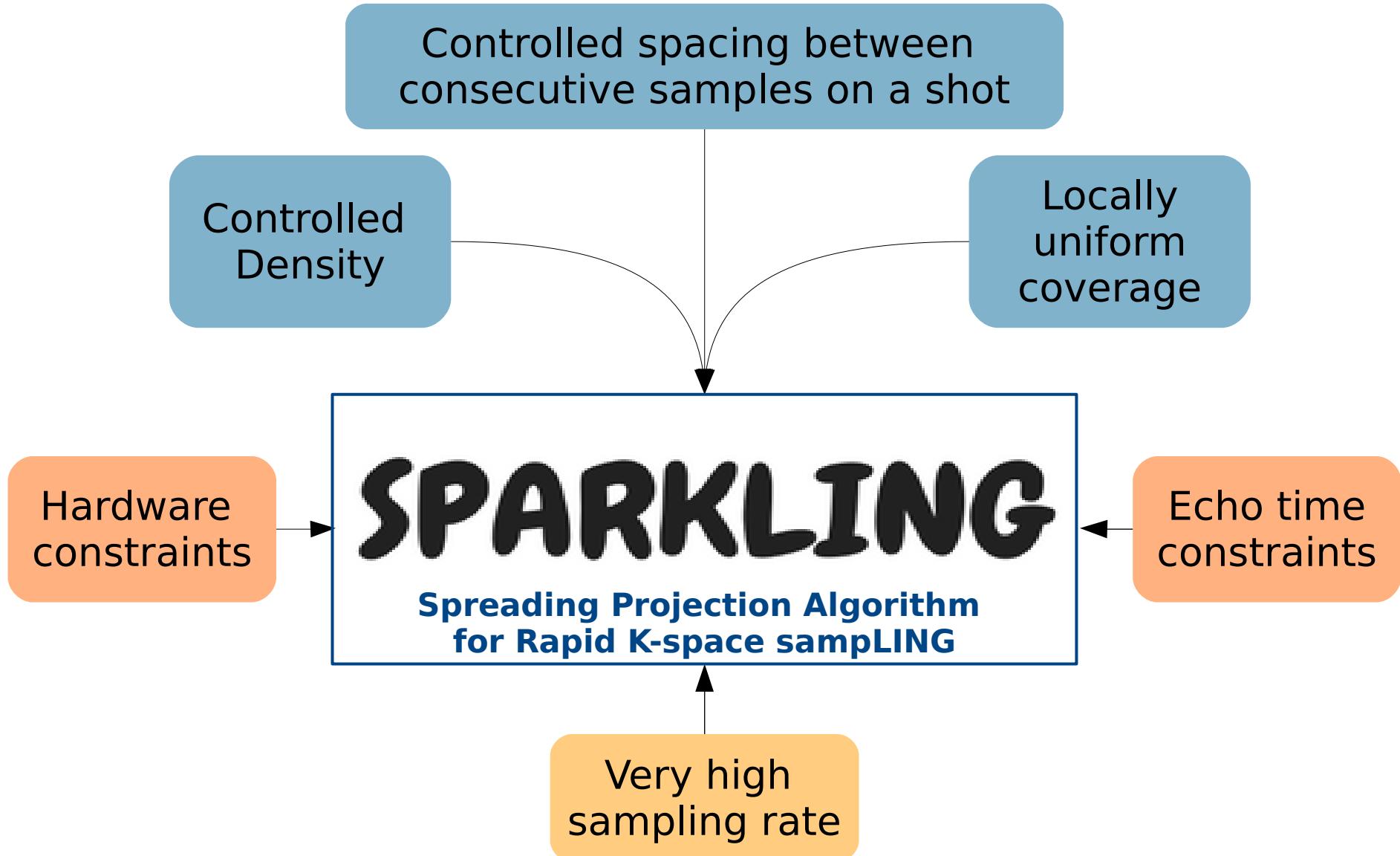
Spreading Projection Algorithm for Rapid K-space sampLING



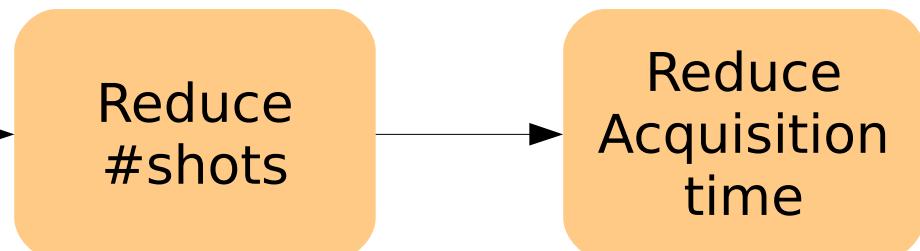
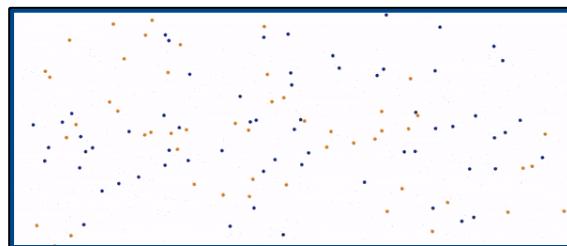
Sparkling recipe



Sparkling recipe



Sparkling acceleration



Acceleration Factor AF = N/ns

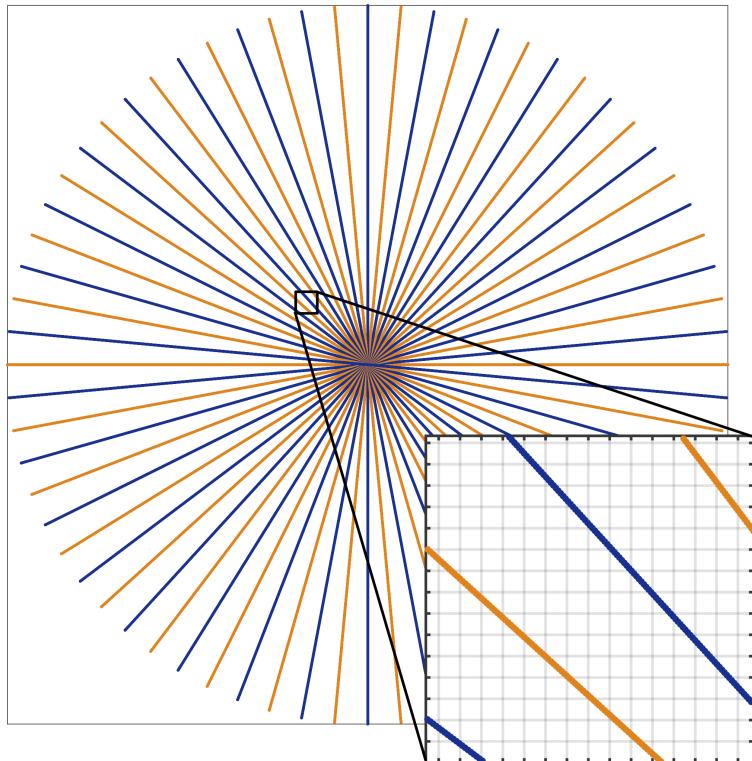
Subsampling Factor R = n/m

(with $n=N \times N = \# \text{pixels}$, $ns = \# \text{shots}$ and $m = \# \text{samples}$)

$$\mathbf{AF} \geq \mathbf{R}$$

Radial VDS Sparkling

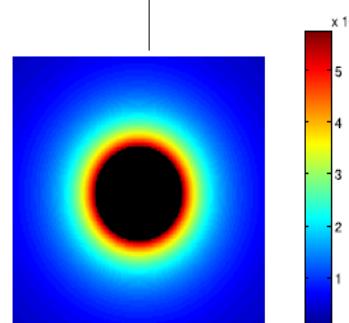
Input



$T_{obs} = 30 \text{ ms}$

$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

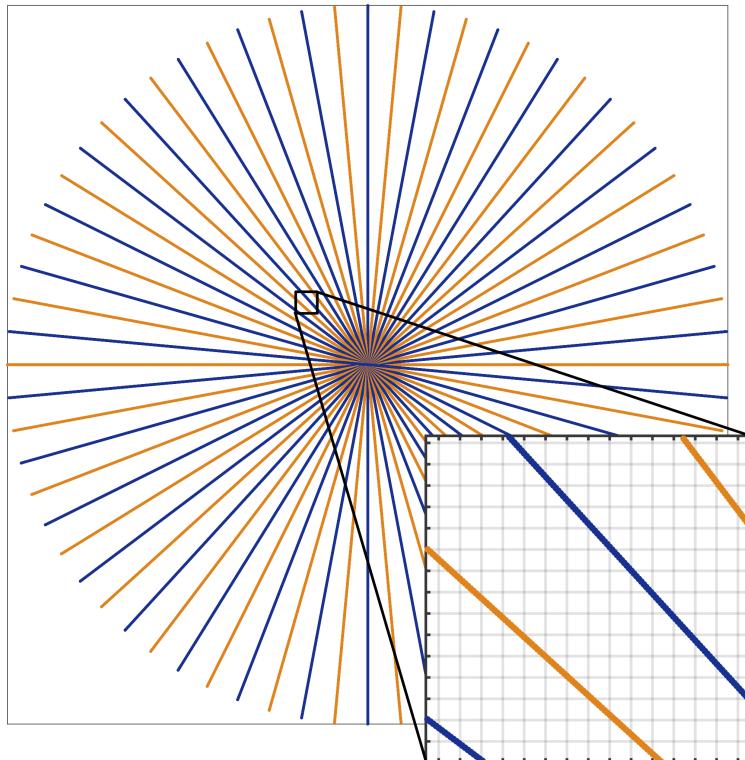
SPARKLING



Acceleration factor in time:
 $AF = N/nc = 512/34 = 15$

Radial VDS Sparkling

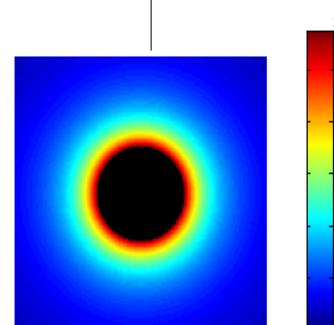
Input



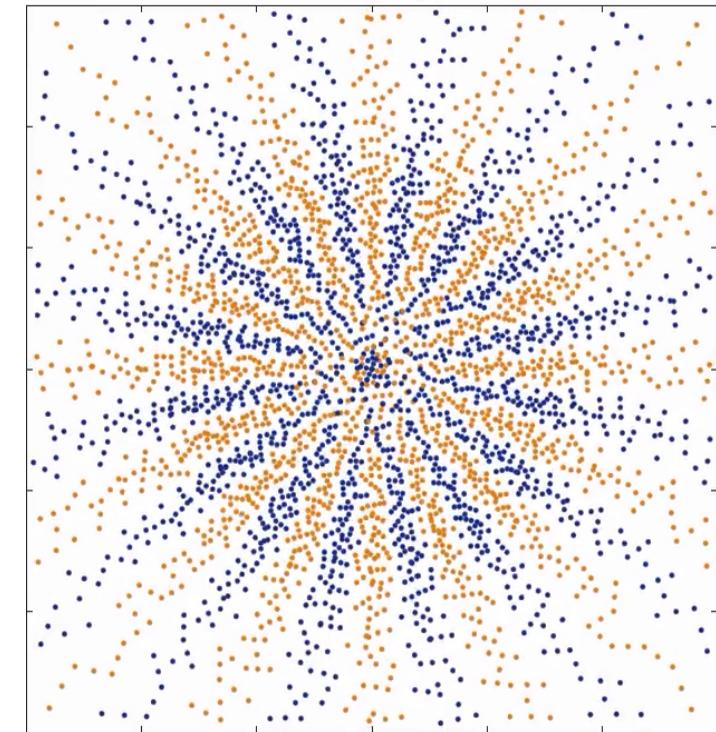
$T_{obs} = 30 \text{ ms}$

$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

SPARKLING

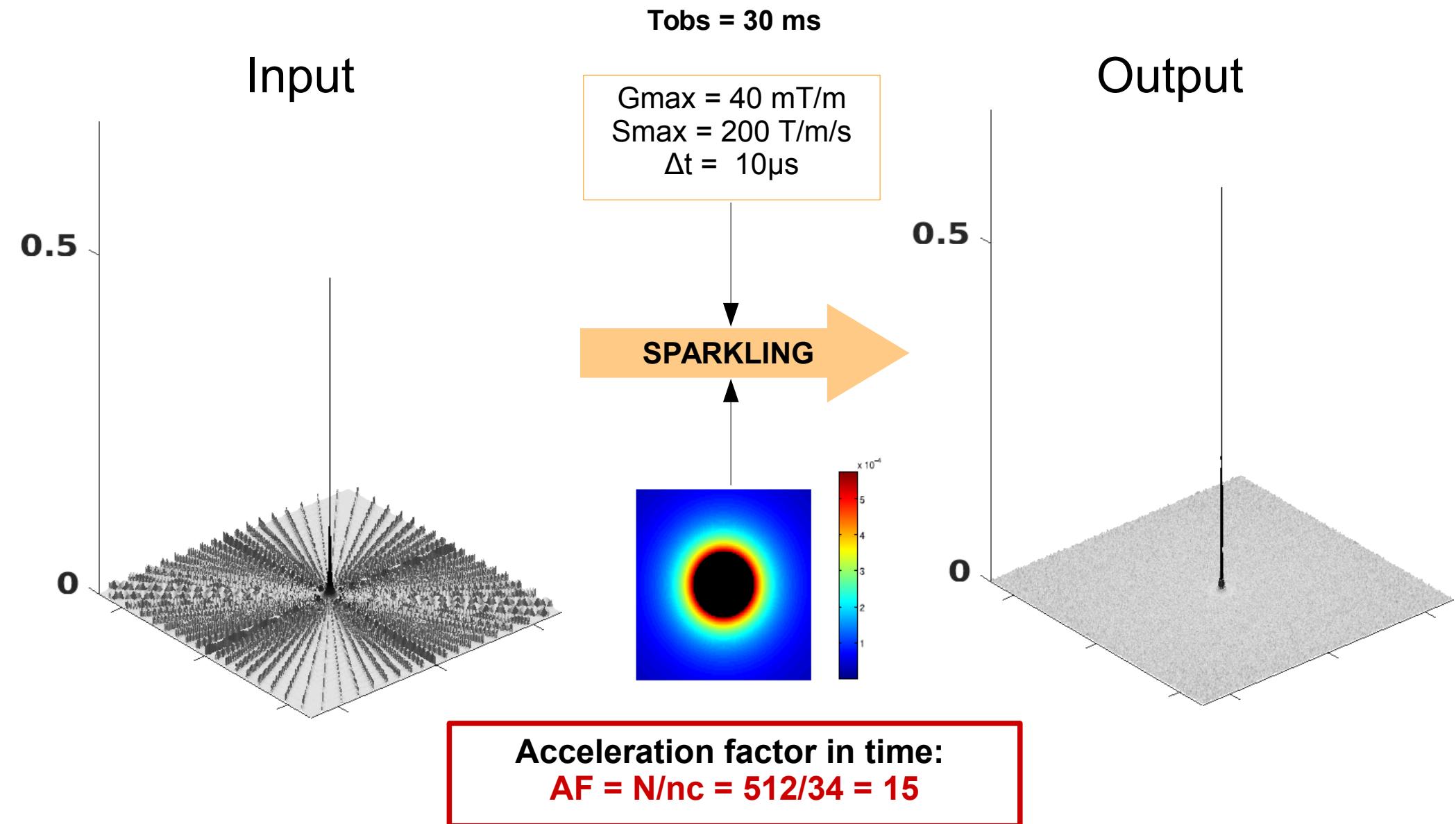


Decimation level 2/6
Iteration n°1



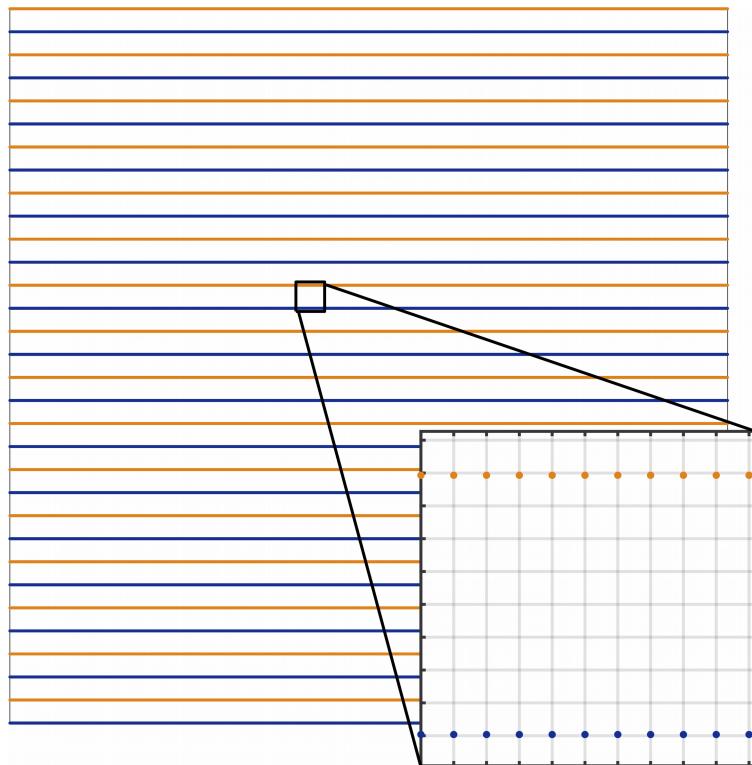
Acceleration factor in time:
 $AF = N/nc = 512/34 = 15$

Radial VDS Sparkling



Uniform Sparkling

Input



$T_{obs} = 20 \text{ ms}$

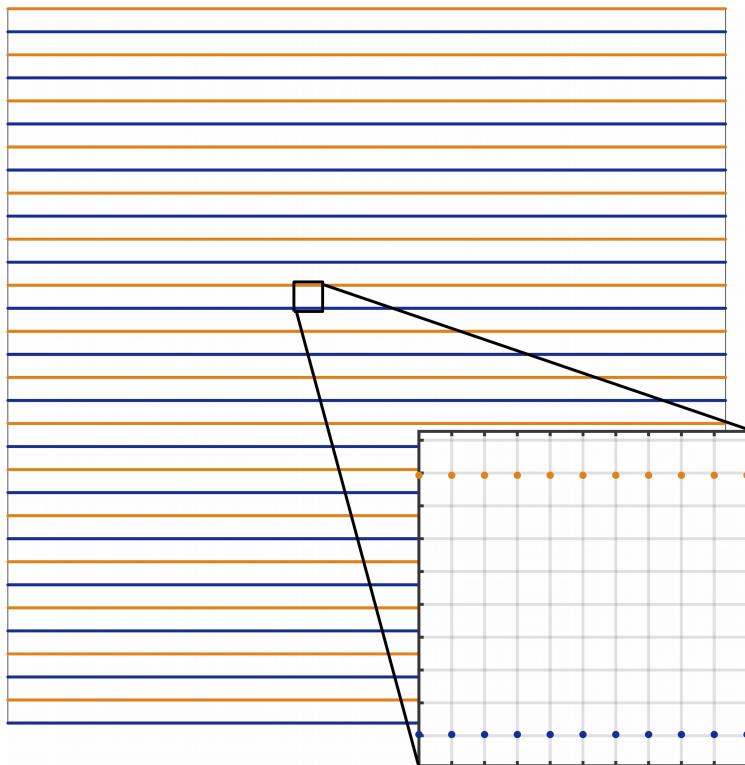
$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

SPARKLING

Acceleration factor in time:
 $AF = N/n_c = 256/32 = 8$

Uniform Sparkling

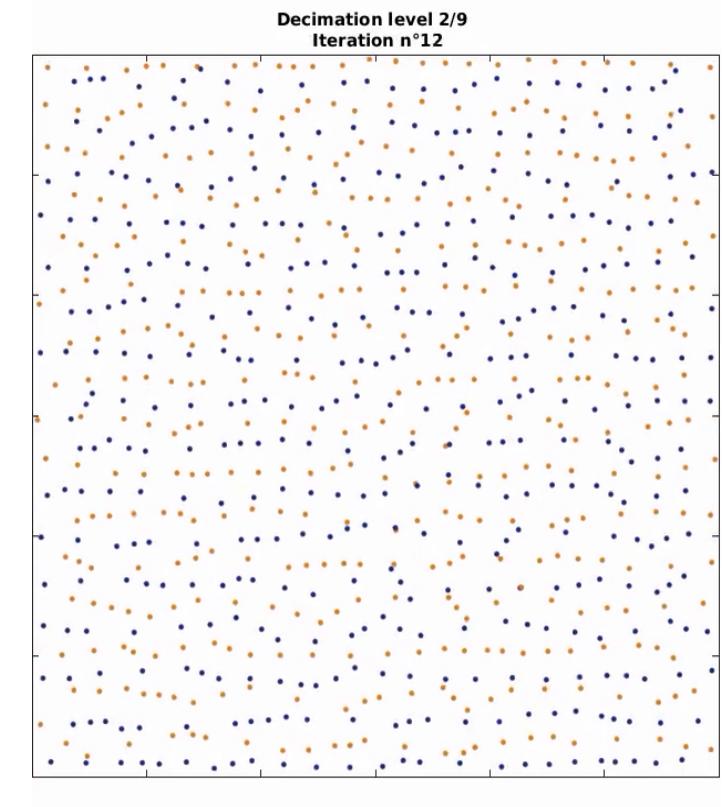
Input



$T_{obs} = 20 \text{ ms}$

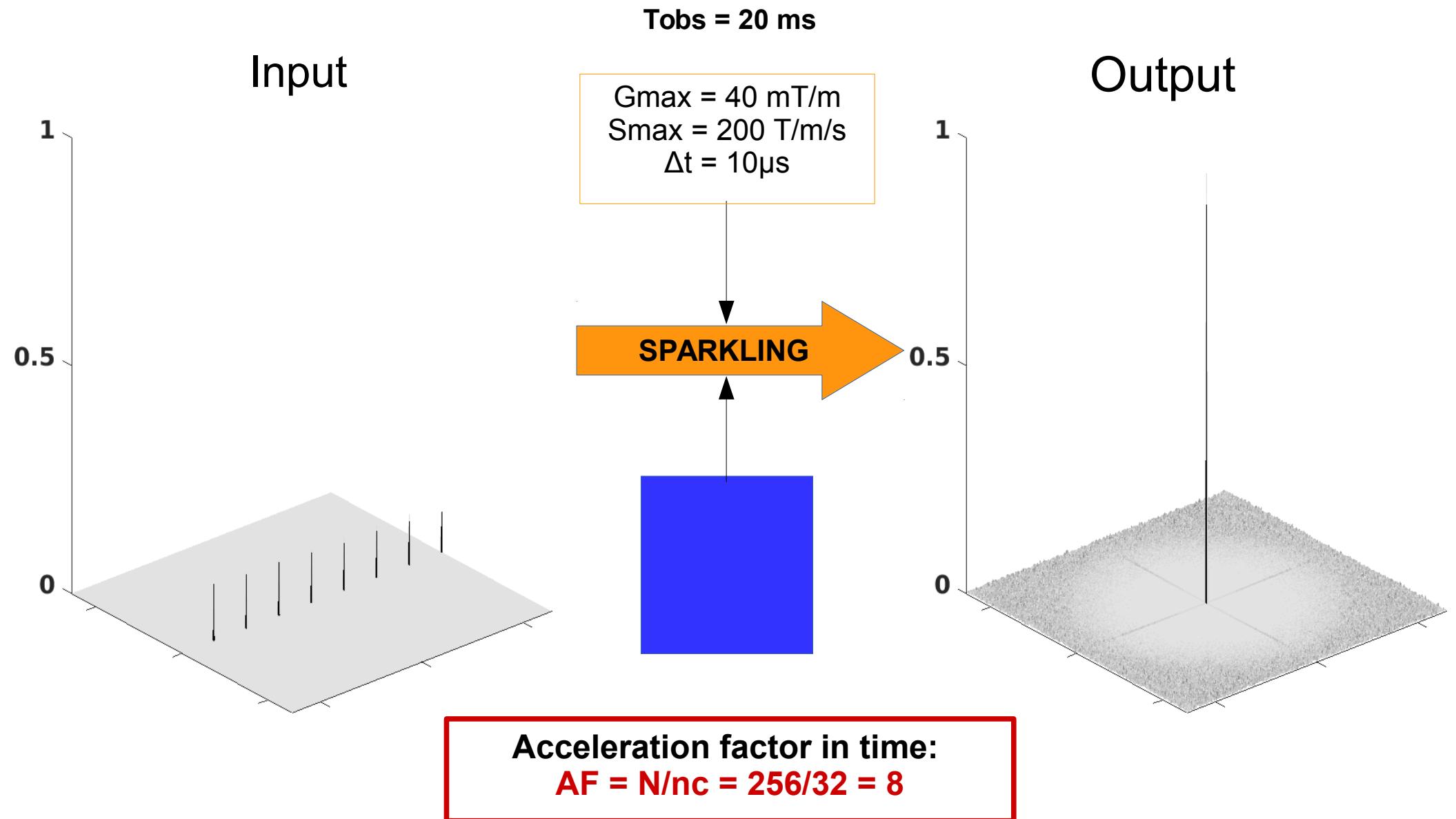
$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

SPARKLING



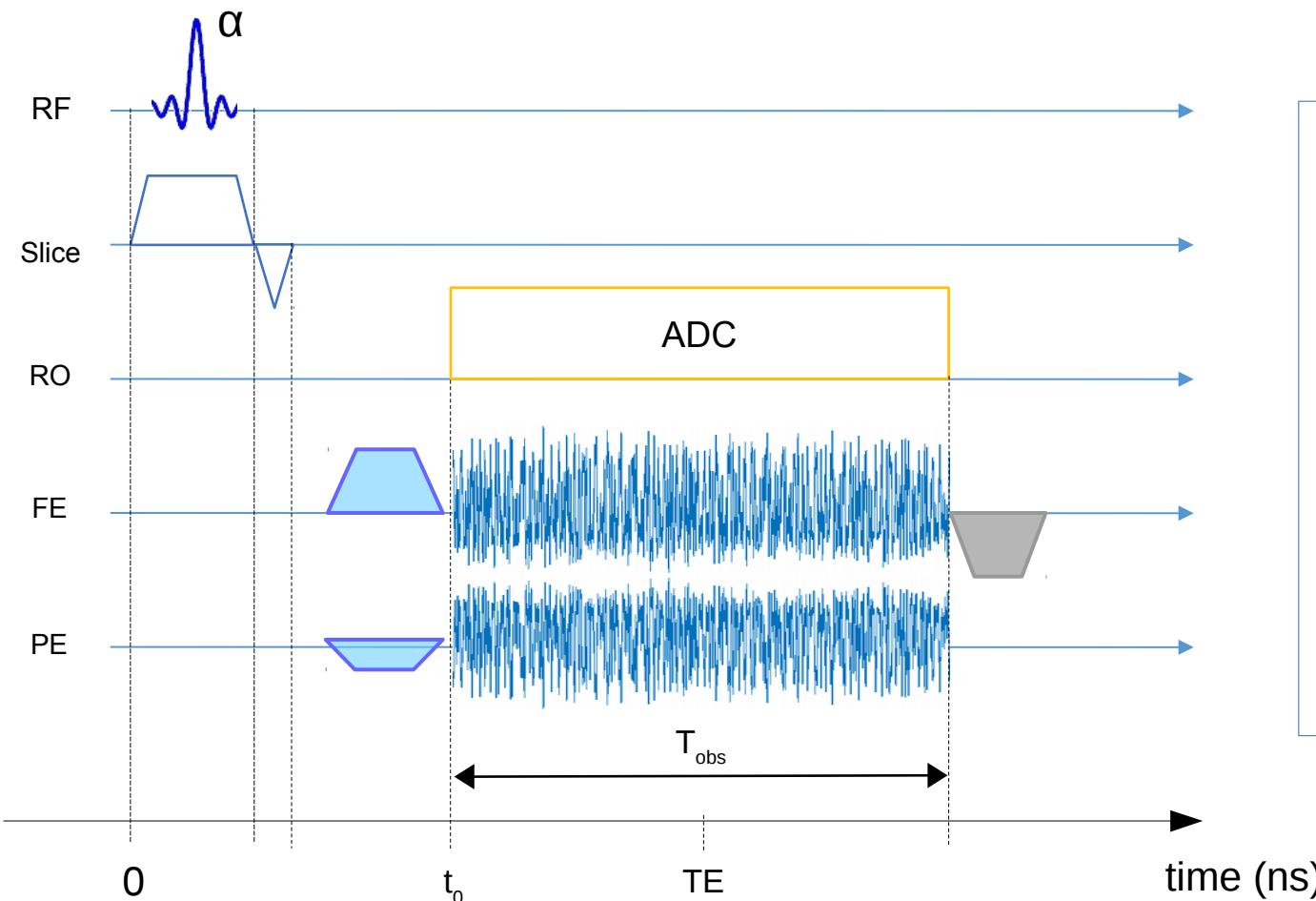
Acceleration factor in time:
 $AF = N/n_c = 256/32 = 8$

Uniform Sparkling



Sequence Implementation

- T_2^* -weighted Gradient-recalled echo sequence (GRE)
- Based on Siemens FLASH sequence

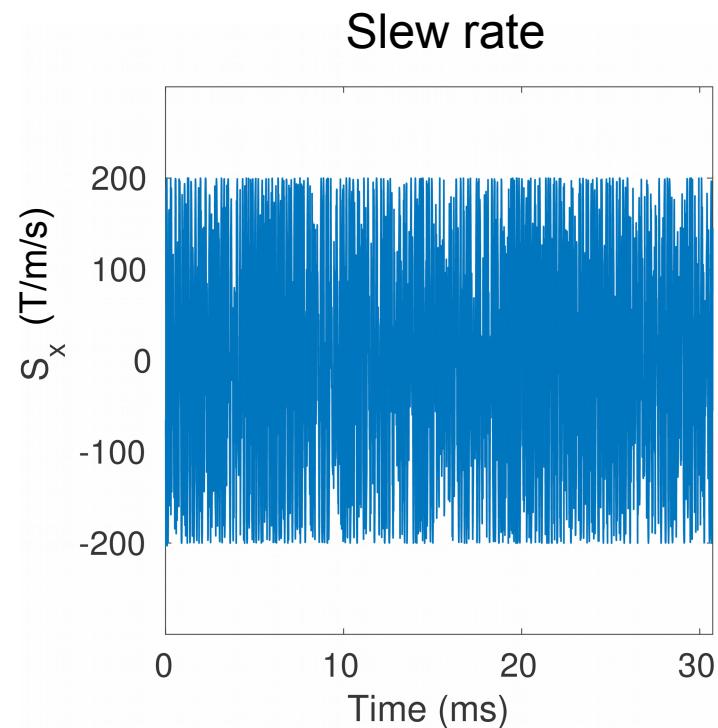
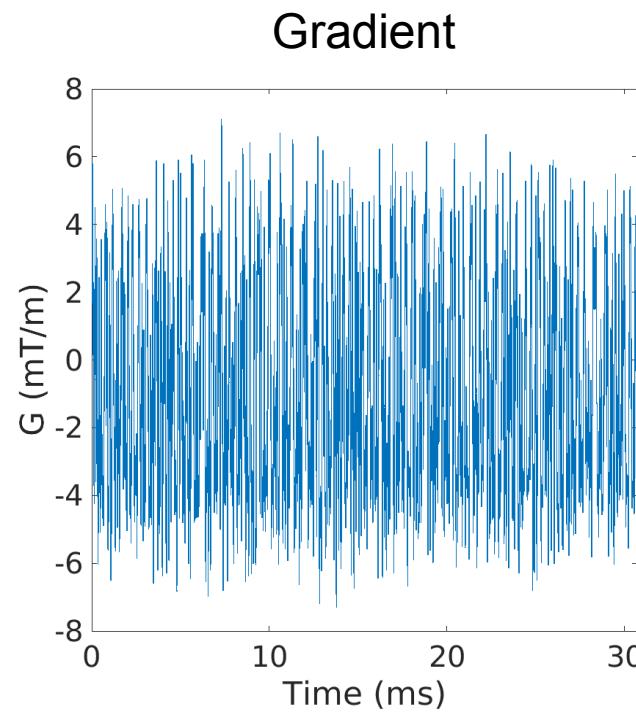


Sequence parameters:

- $N=512$
- $FOV=200 \times 200 \text{ mm}^2$
- $TR=550 \text{ ms} \rightarrow \text{interleaved}$
- $TE=30 \text{ ms}$
- $\alpha=25^\circ$
- $BW=32.55 \text{ Hz/px}$
- $T_{obs}=30.72 \text{ ms}$
- Slice thickness: 3 mm
- 32-channel receiver coil
- Gradient raster time = 10 μs

Sequence Implementation

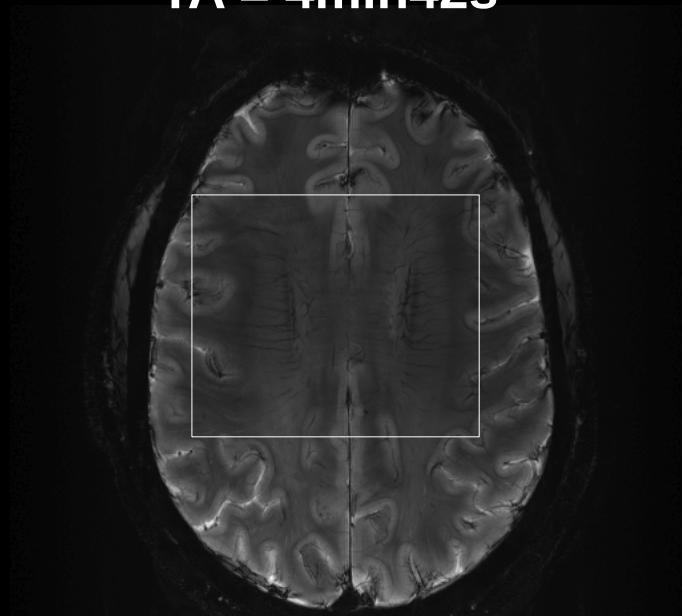
Siemens 7 Tesla scanner



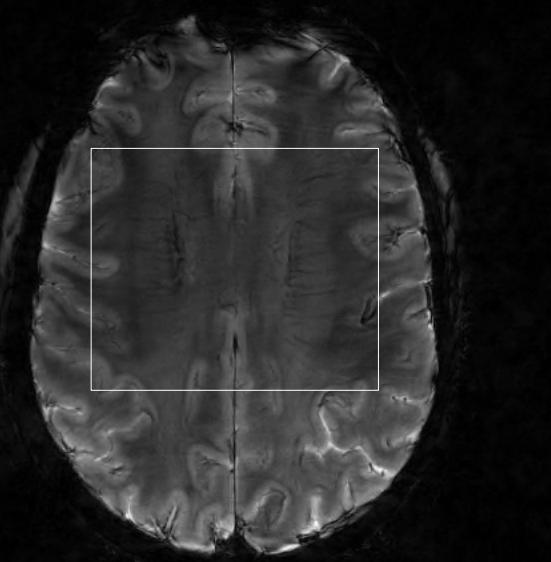
In vivo results: 0.39 mm – 11 slices of 3-mm thickness

Subject 1

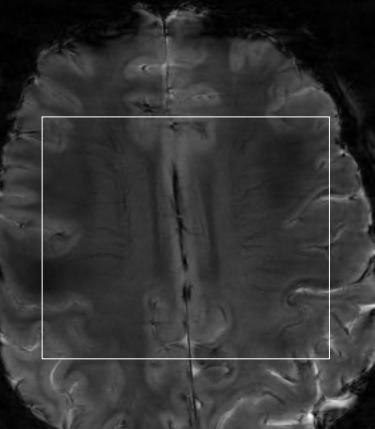
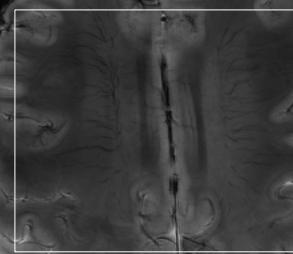
REFERENCE
TA = 4min42s



In-out SPARKLING
TA = 18s (AF=15)



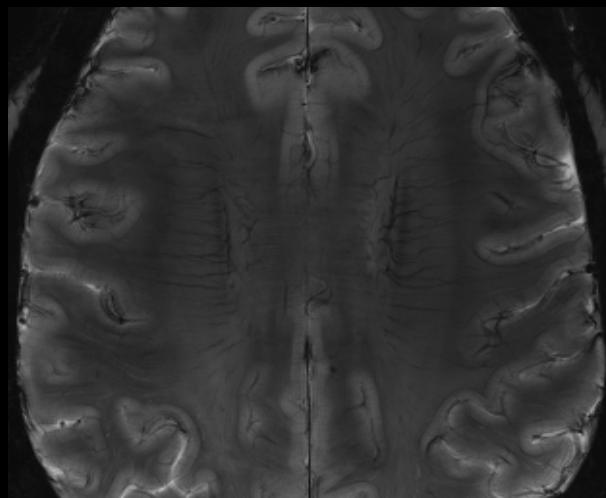
Subject 2



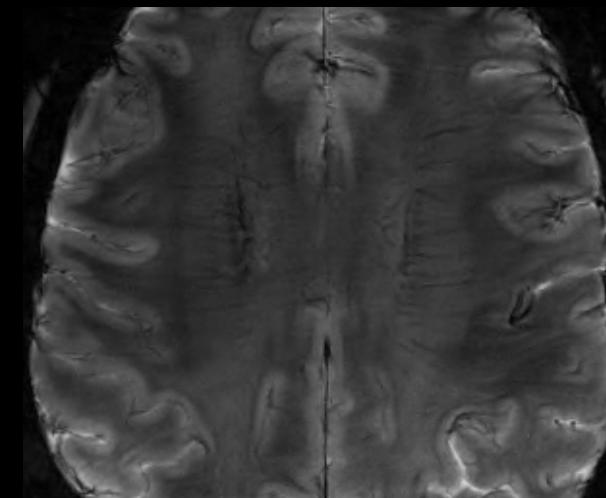
In vivo results: 0.39 mm – 11 slices of 3-mm thickness

Subject 1

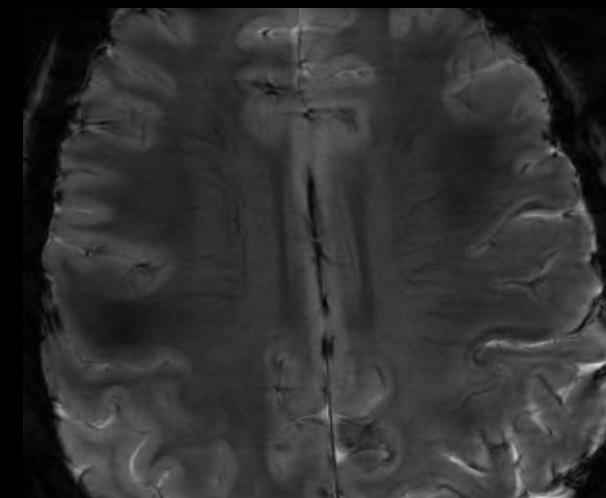
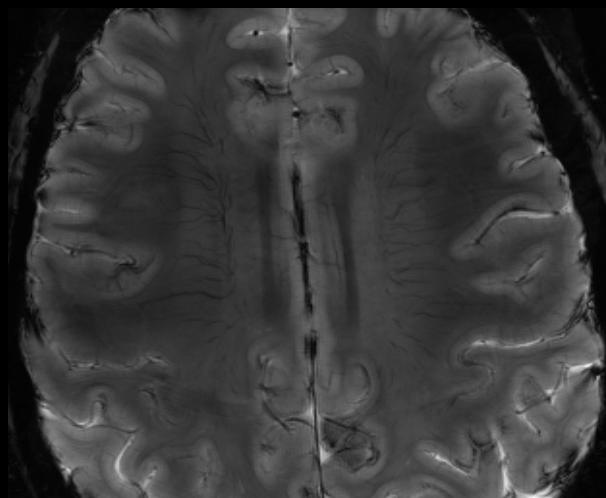
REFERENCE
TA = 4 min42s



In-out SPARKLING
TA = 18s (AF=15)



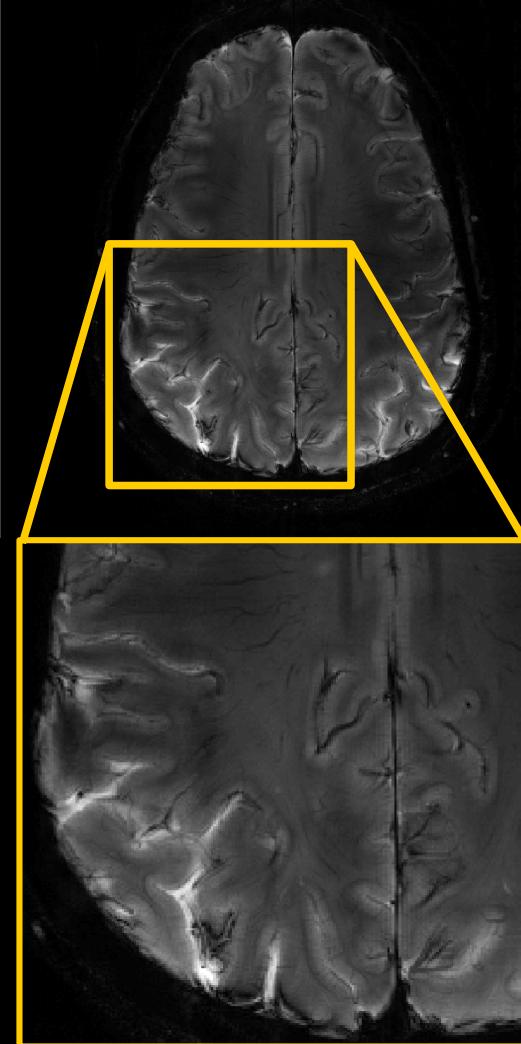
Subject 2



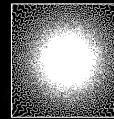
In vivo results at 0.39mm resolution 26 shots – 11 slices

REFERENCE

TA=4min42s

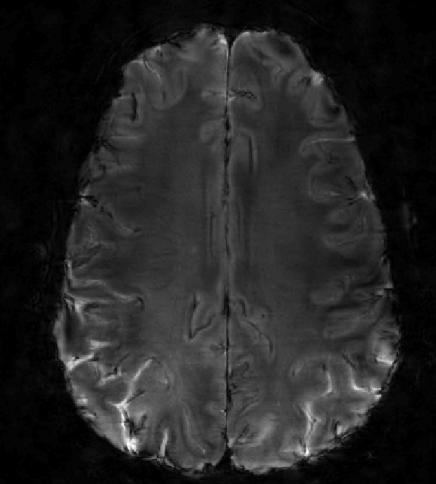


In-out SPARKLING

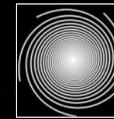


AF=20

TA=14s

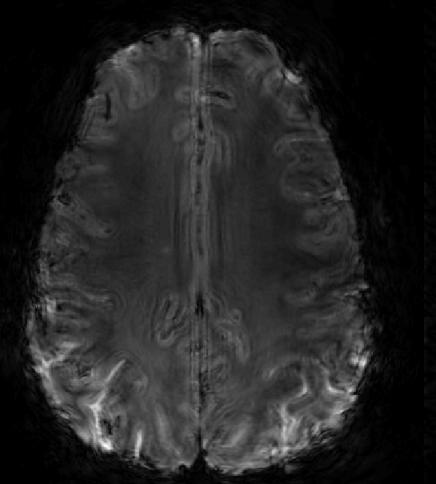


In-out SPIRAL

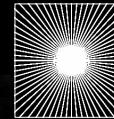


AF=20

TA=14s

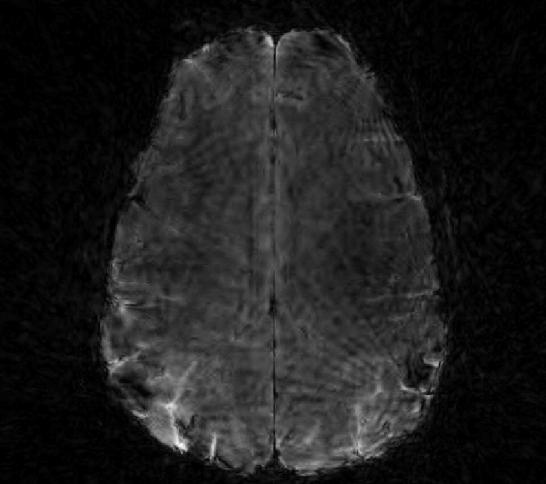


In-out RADIAL



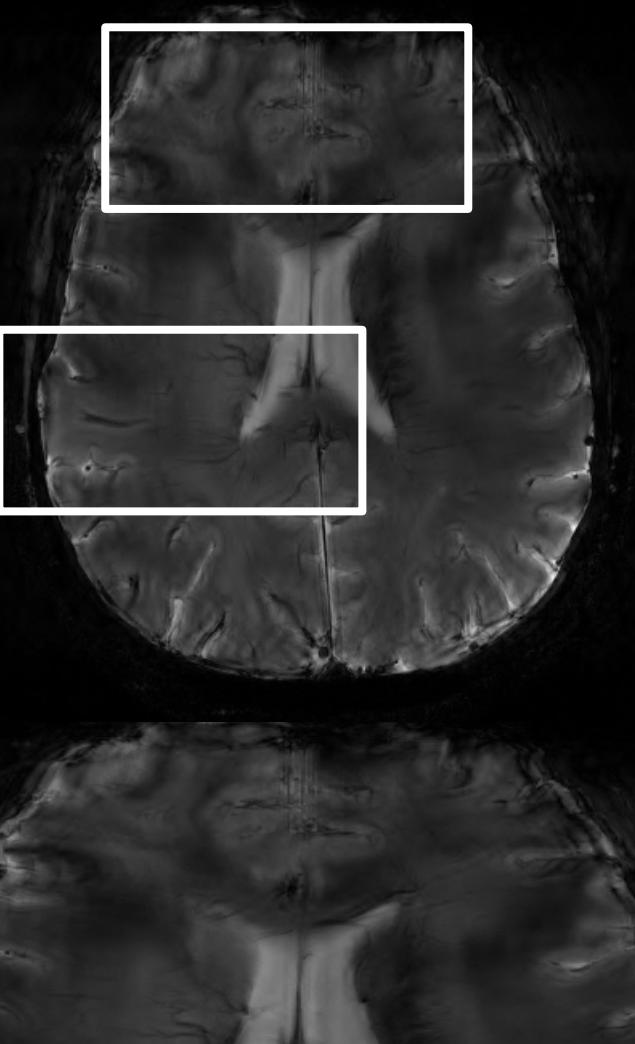
AF=20

TA=14s

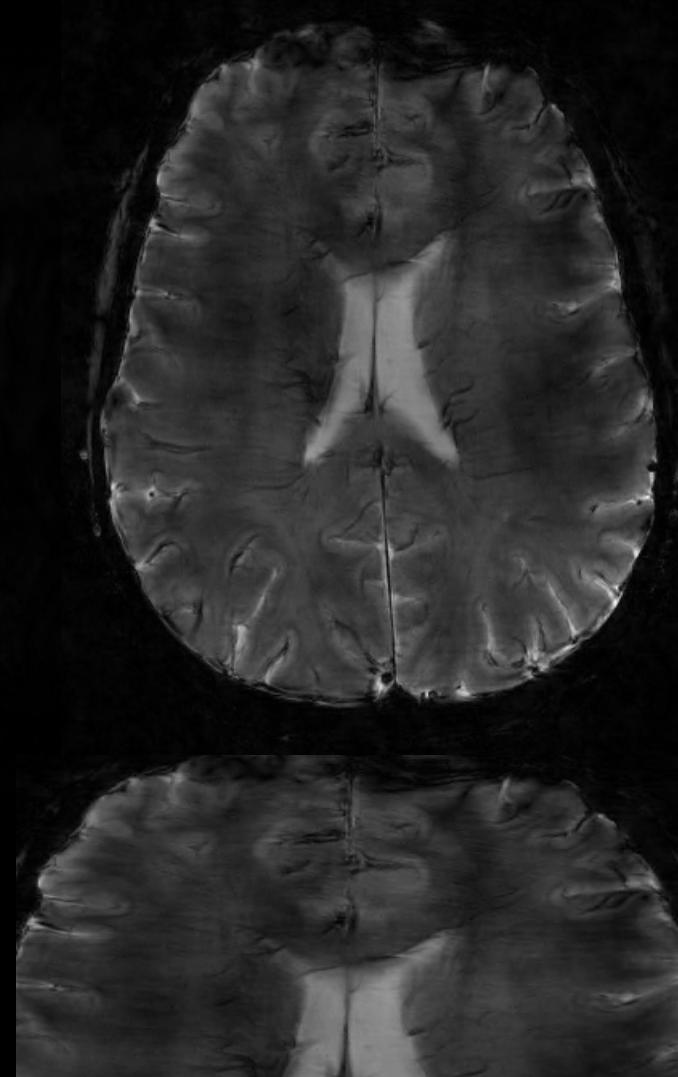


In vivo results at 0.39mm – 43 shots – 11 slices – BW=400kHz – TA=23s

**Segmented EPI
(fully-sampled)**



In-out Sparkling

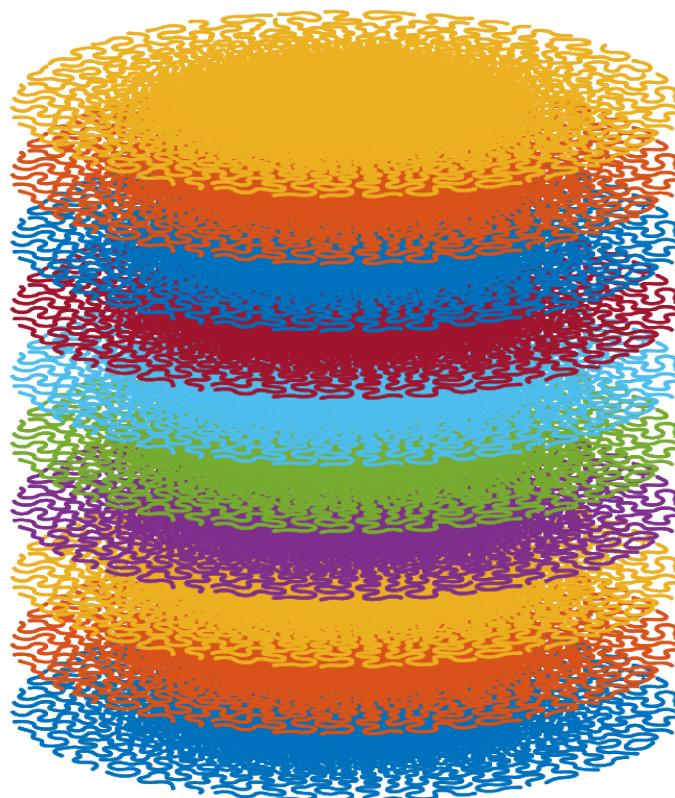


In-out spiral



Stack-of-Sparkling (SOS)

Regular SOS



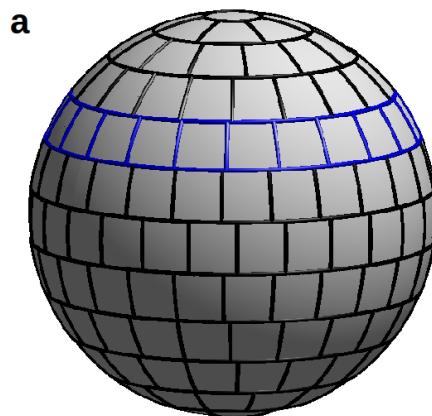
Z-vd SOS



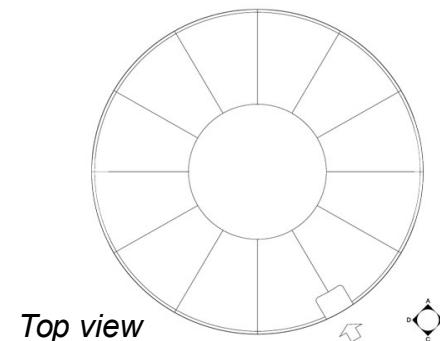
$$n(k_z) = n(0) \frac{\int \pi(:,:,k_z)}{\int \pi(:,:,0)},$$

Fully 3D Sparkling

→ Shot-by-shot generation



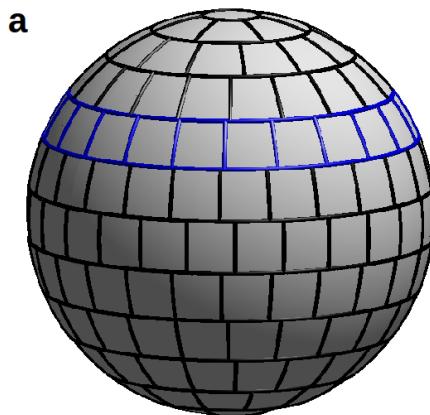
Equal area
tessellation



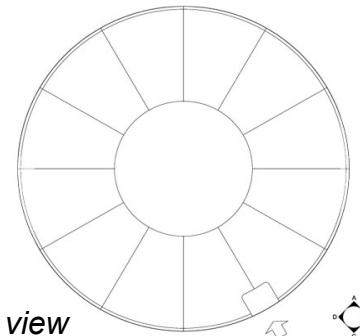
From 1 year to 20 min of
computation time

Fully 3D Sparkling

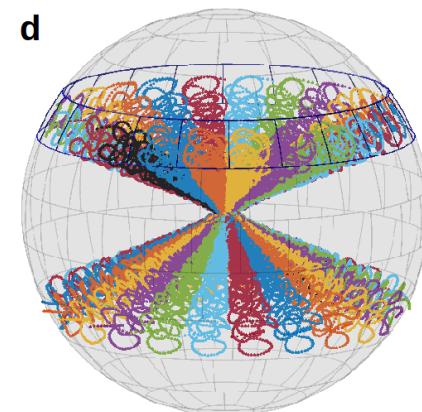
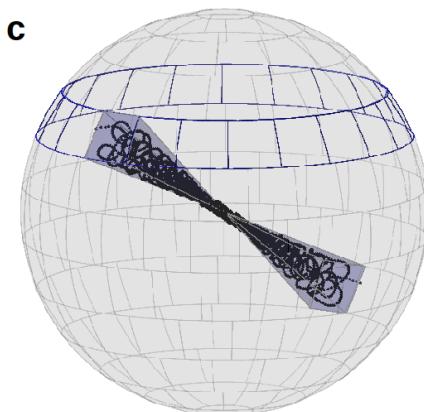
→ Shot-by-shot generation



Equal area
tessellation

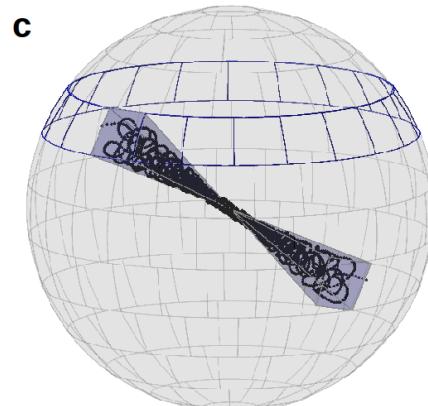
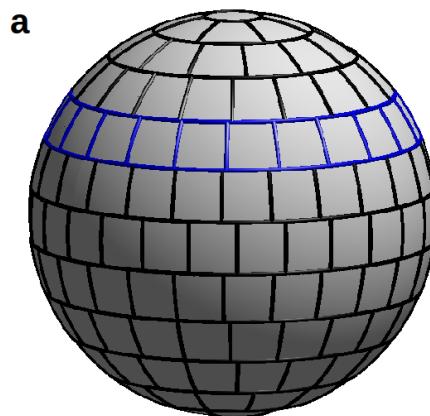


From 1 year to 20 min of
computation time

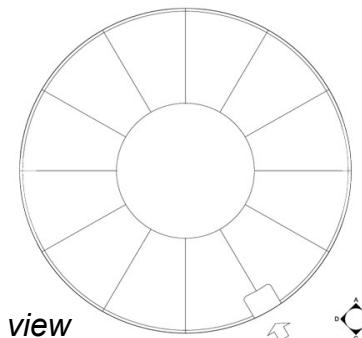


Fully 3D Sparkling

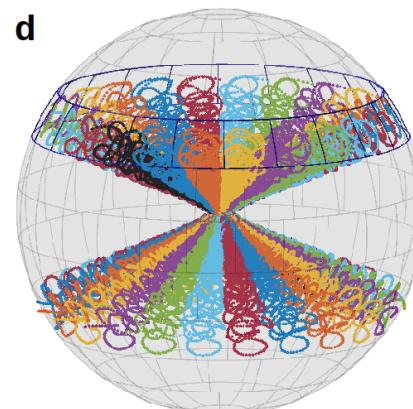
→ Shot-by-shot generation



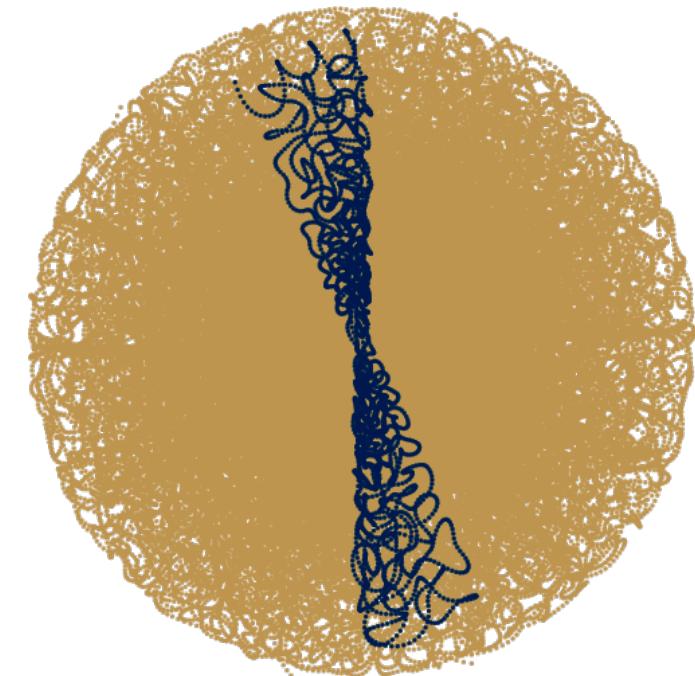
Equal area
tessellation



Top view



From 1 year to 20 min of computation time

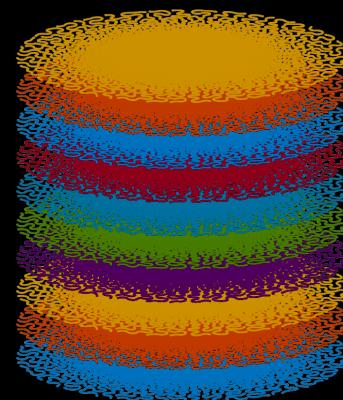


***Ex vivo* comparison of different Sparkling strategies**

T2* contrast

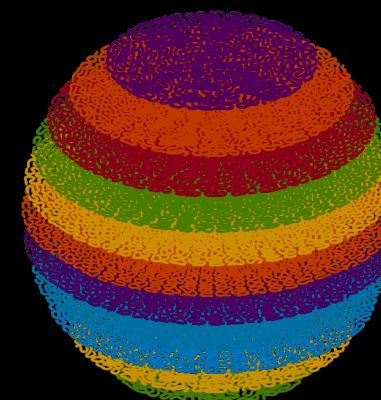
Isotropic resolution of 0.6 mm

Tobs=15 ms - FOV=20x20x14 cm³



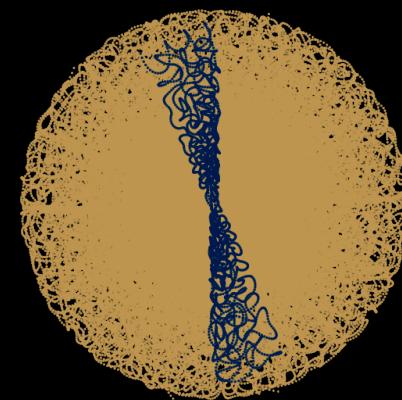
Regular SOS

VS.



z-vd SOS

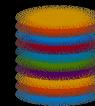
VS.



Fully 3D

Comparison of different Sparkling methods 4000 shots – AF=20

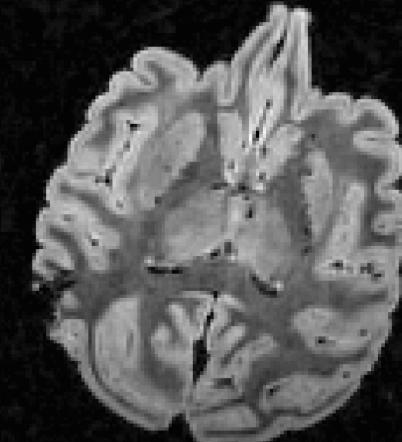
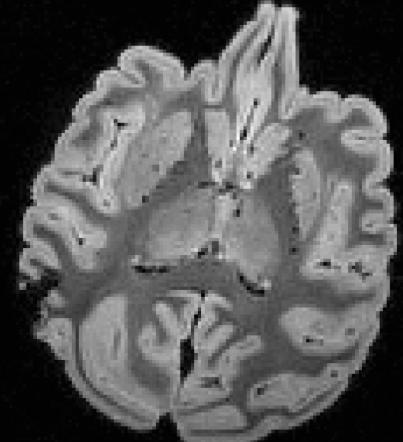
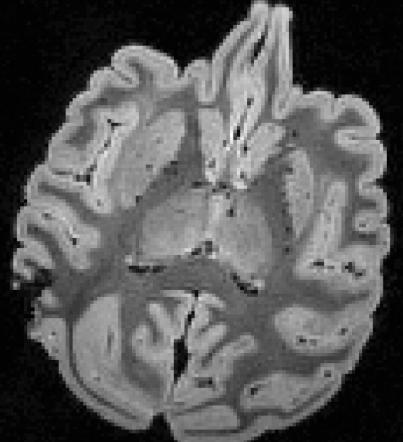
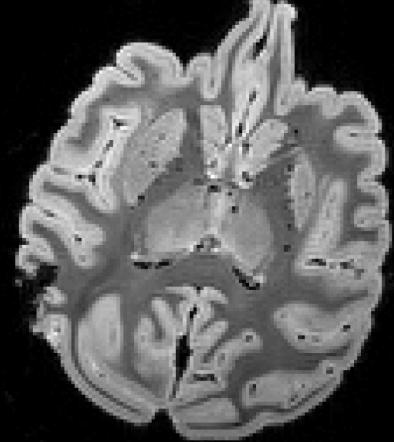
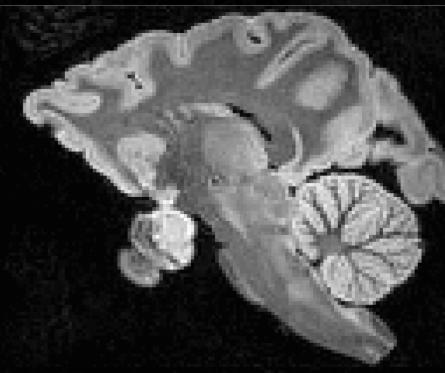
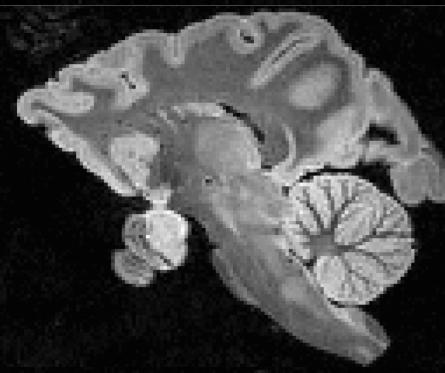
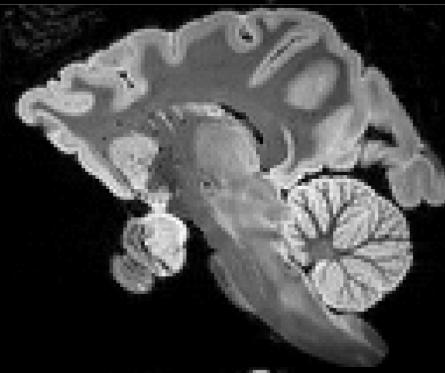
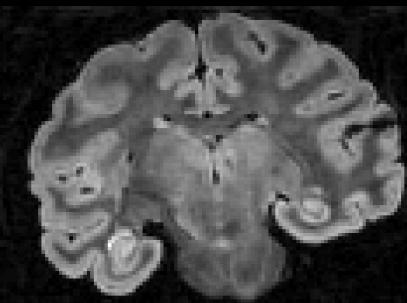
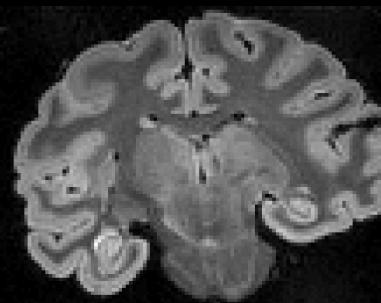
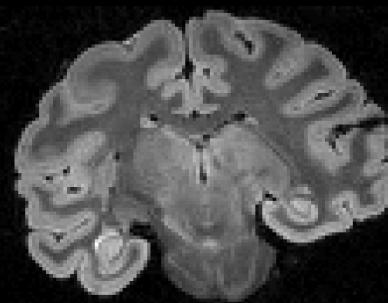
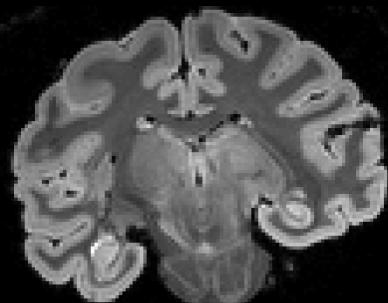
IPAT 4
TA=14min31s



Regular SOS
TA=2min40s
SSIM=0.87

z-vd SOS
TA=2min40s
SSIM=0.88

Fully 3D
TA=2min40s
SSIM=0.79

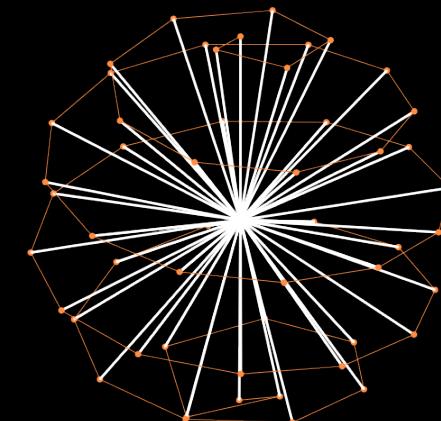


Ex vivo comparison with other strategies

T2* contrast
Isotropic resolution of 0.6 mm
TA=45s

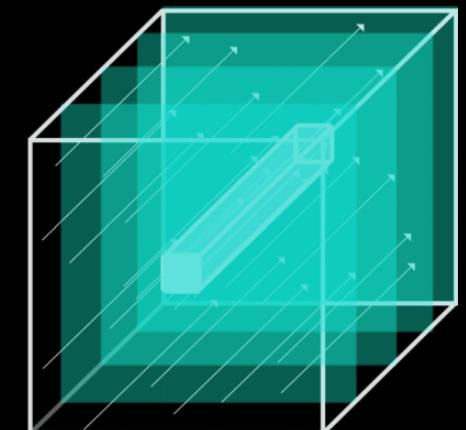


vs.



Larson et al. 2007

vs.

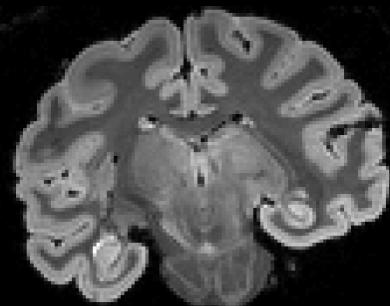


Lustig et al. 2008

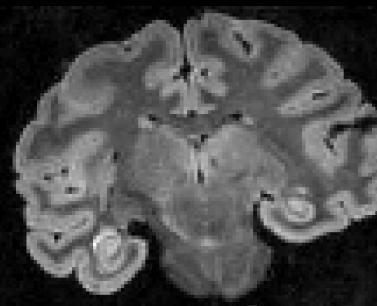
SPARKLING vs. other strategies

1140 shots - AF=69

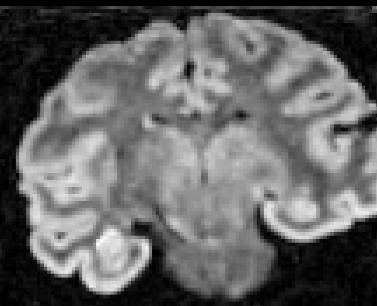
IPAT 4
TA=14min31s



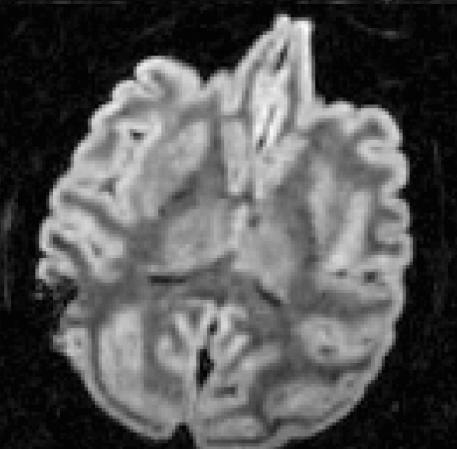
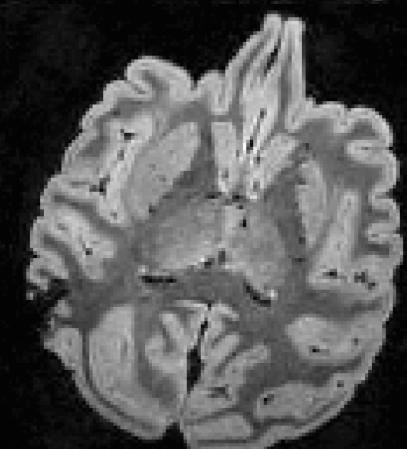
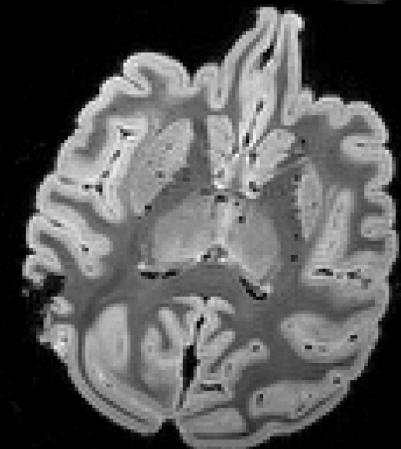
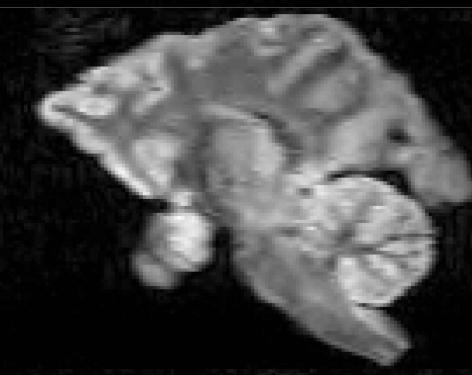
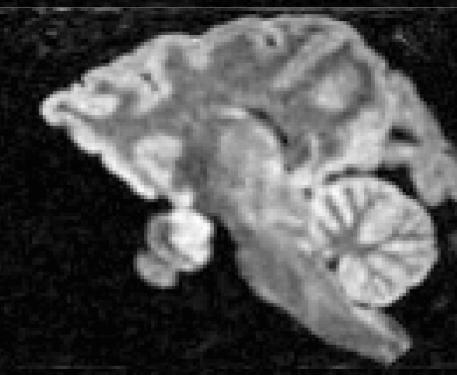
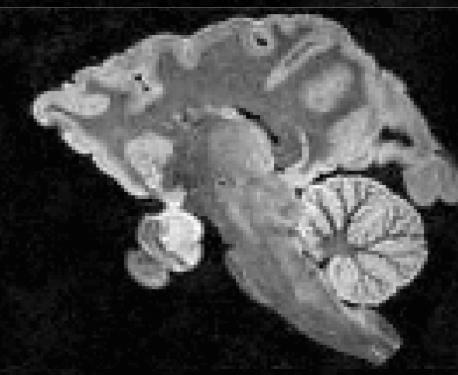
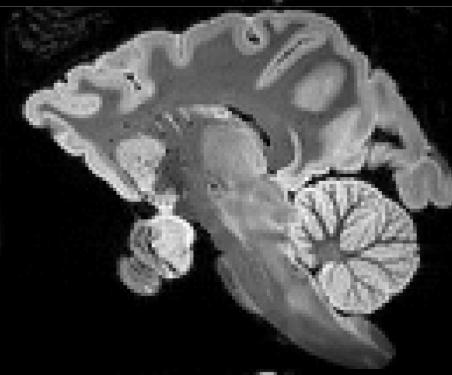
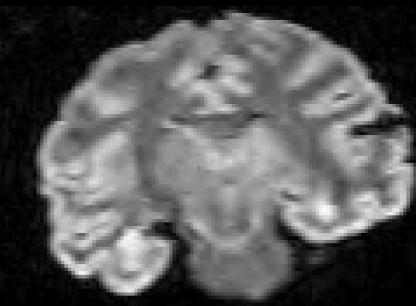
SPARKLING
TA=45s
SSIM=0.86



RADIAL
TA=45s
SSIM=0.72



Poisson disk lines
TA=45s
SSIM=0.58



Conclusion

- **SPARKLING 2D**
 - *Ex vivo* and *in vivo* validation of SPARKLING 2D:
Demonstration of its **superiority** over both radial and spiral trajectories
 - Sampling **efficiency**, robustness to subsampling
 - Adaptability to hardware constraints and various densities (e.g. anisotropic)
- **SPARKLING 3D**
 - Higher acceleration factors
 - *Ex vivo* validation of SPARKLING 3D
- **MR image reconstruction**
 - 2D **offline/online** multichannel image recon with sensitivity maps extraction
 - 3D **offline** single/multi-channel image recon
 - **2D/3D calibrationless** image recon with group sparsity promotion
 - **Analysis-based** regularization using redundant decompositions (e.g. curvelets) and primal/dual recon algorithms



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NeuroSpin:

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