

DE LA RECHERCHE À L'INDUSTRIE



www.cea.fr

New Acquisition strategies for Compressed Sensing in Magnetic Resonance Imaging

Philippe Ciuciu

Philippe.ciuciu@cea.fr



April, 8th 2019
Venice, Italy

Acknowledgements

Nicolas Chauffert
CEA/NeuroSpin



Carole Lazarus
CEA/NeuroSpin



Alexandre Viggnaud
CEA/NeuroSpin



Pierre Weiss
CNRS/ITAV



Claire Boyer
IMT, U. Toulouse



Jonas Kahn
CNRS/IMT



Outline

Part I: Background in Magnetic Resonance Imaging

- Sampling k-space & Cartesian reconstruction
- Trajectories and acquisition strategies

Part II: Compressed Sensing in MRI

- Evolution of Compressed Sensing
- (CS with blocks of measurements)

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

Online Material

Code for MRI sampling available for reproducible research

- 7 Jupyter notebooks (Python 3.5) for Parts I & III
- Matlab code for Part II (TSP+ Markov)
- Sparkling code for Part III *not yet* provided

Code for image reconstruction in Fourier imaging

- PySAP python package developed at CEA
- Off-line discussion for PR contributions, installation issues

Test data

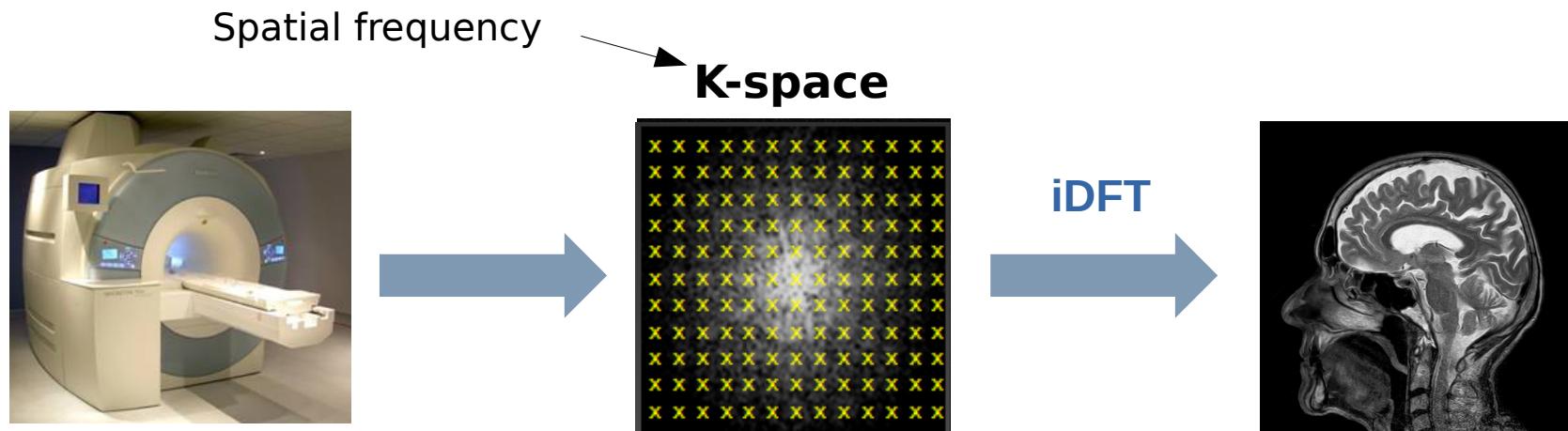
- Numerical phantoms at different image resolution
- Real T2*-w images collected at 7 Tesla at NeuroSpin
- Sparkling trajectories provided for demos

Outline

Part I: Background in Magnetic Resonance Imaging

- Sampling k-space & Cartesian reconstruction
- Trajectories and acquisition strategies

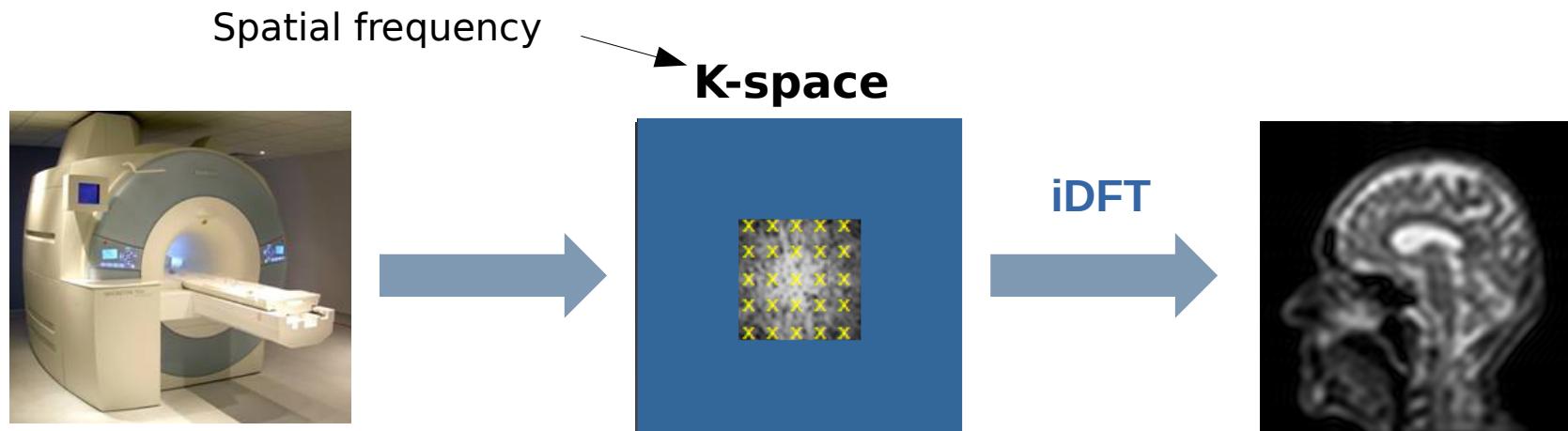
Sampling in MRI



Perfect reconstruction of an object would require measurement of *all* locations in k-space (infinite!)

Data is acquired **point-by-point** in k-space (sampling) along **curves** parametrized by **time**.

Sampling in MRI

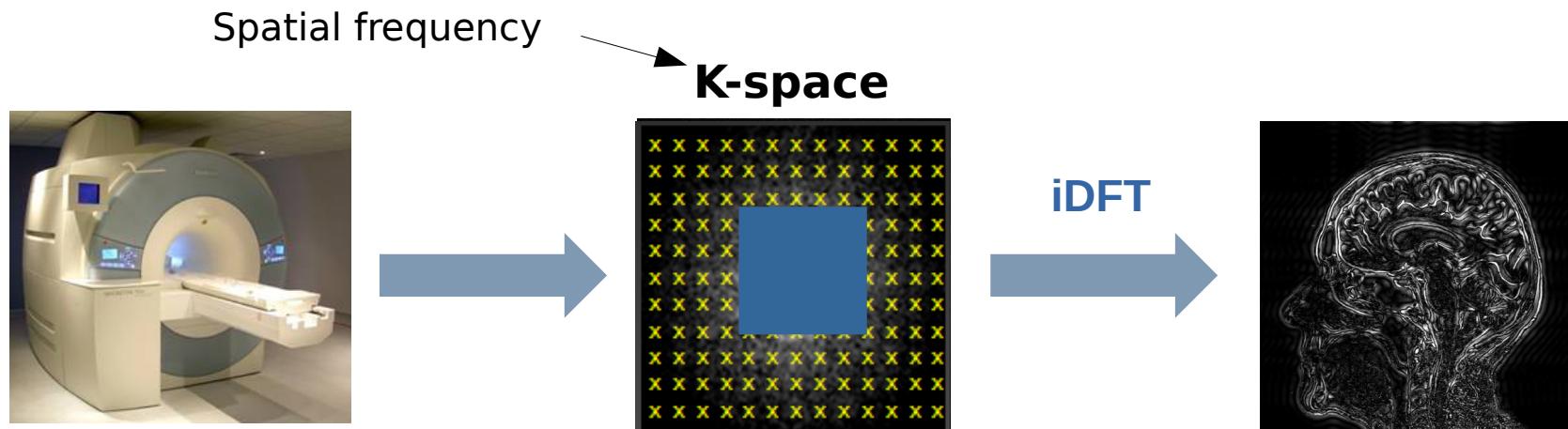


Low frequencies = Contrast



Go to 1st demo/notebook

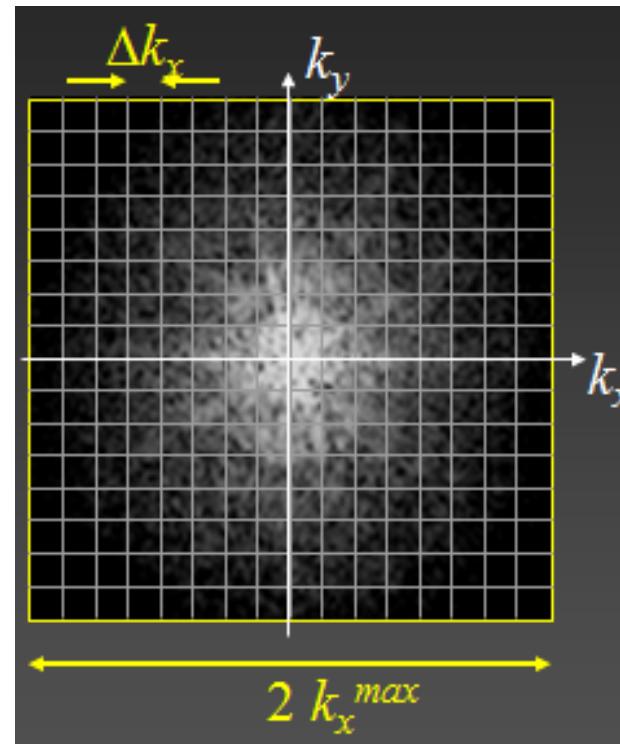
Sampling in MRI



High frequencies = boundaries/edges

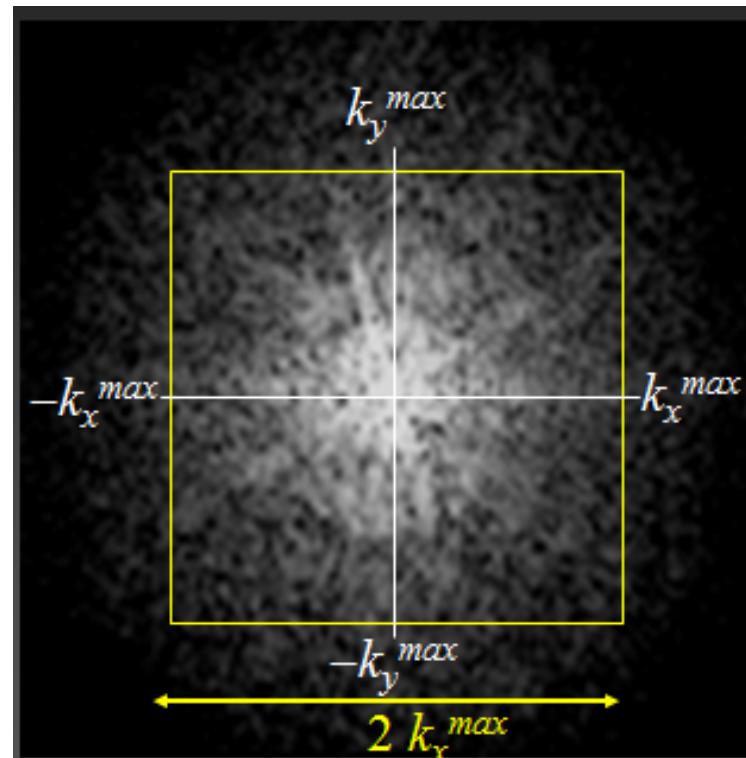
 Go to 1st demo/notebook

Sampling the k-space



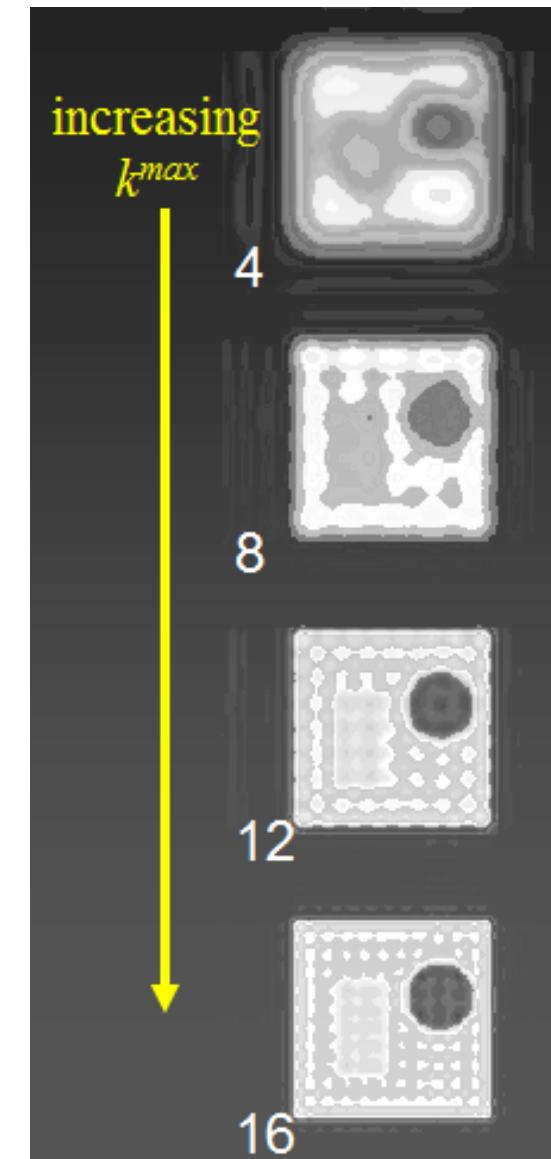
1. What is the highest frequency we need to sample in k -space (k^{\max})?
2. How close should the samples be in k -space (Δk)?

Choosing the maximal frequency



k^{max} determines image resolution

Large k^{max} means high resolution!



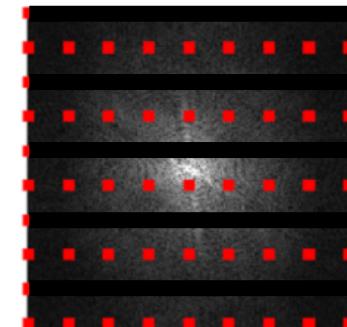
Go to 1st demo/notebook

| PAGE 10

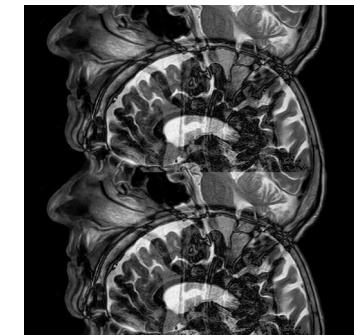
Nyquist Sampling Theorem



K-space



iDFT



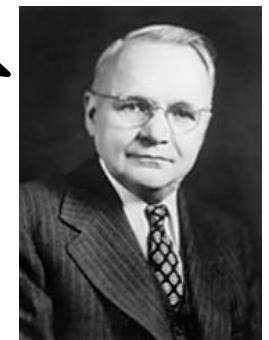
Nyquist-Shannon theory

↑ resolution \Rightarrow ↑ #samples

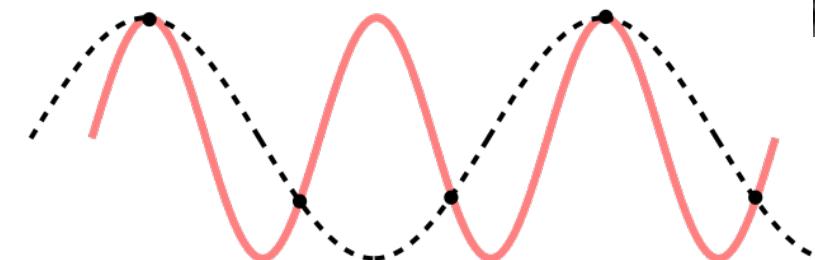


Long acquisition times

The sampling frequency should be at least twice the highest frequency contained in the signal



Harry Nyquist

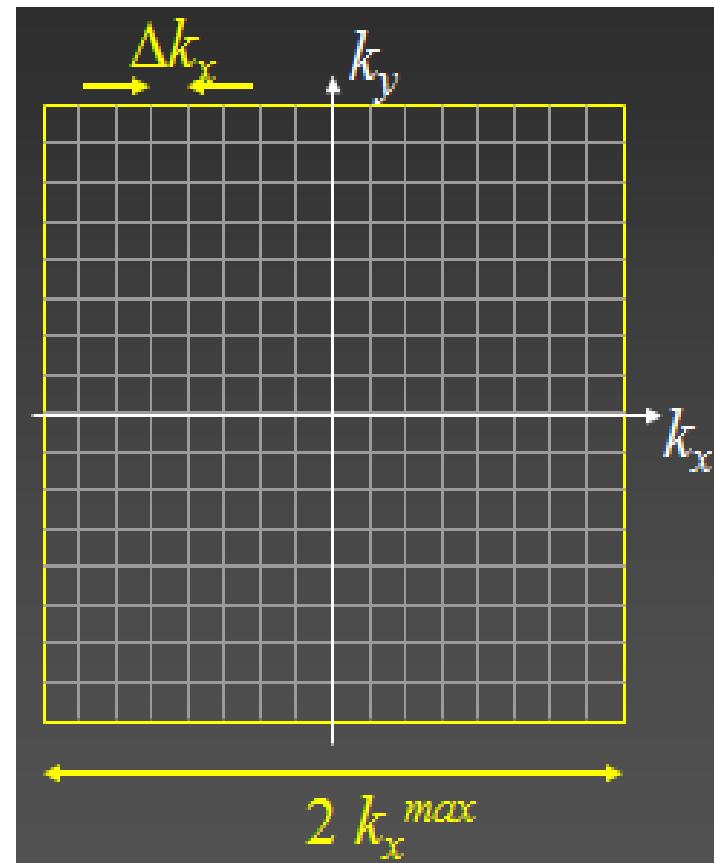


Go to 3rd notebook

Nyquist Sampling Theorem

$$\text{FOV}_x = \frac{1}{\Delta k_x}$$

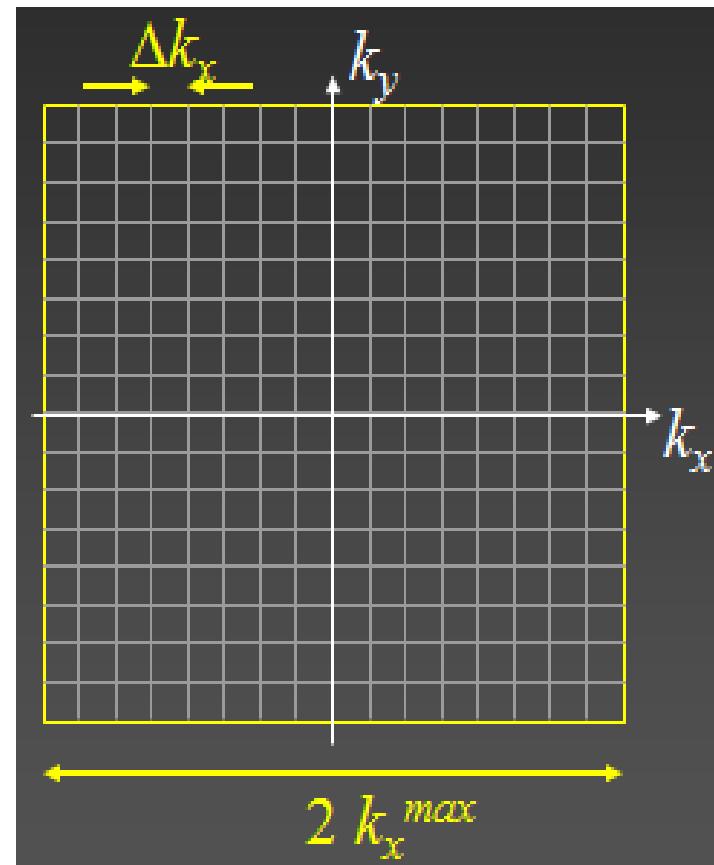
$$\Delta x = \frac{1}{2k_x^{\max}}$$



Nyquist Sampling Theorem

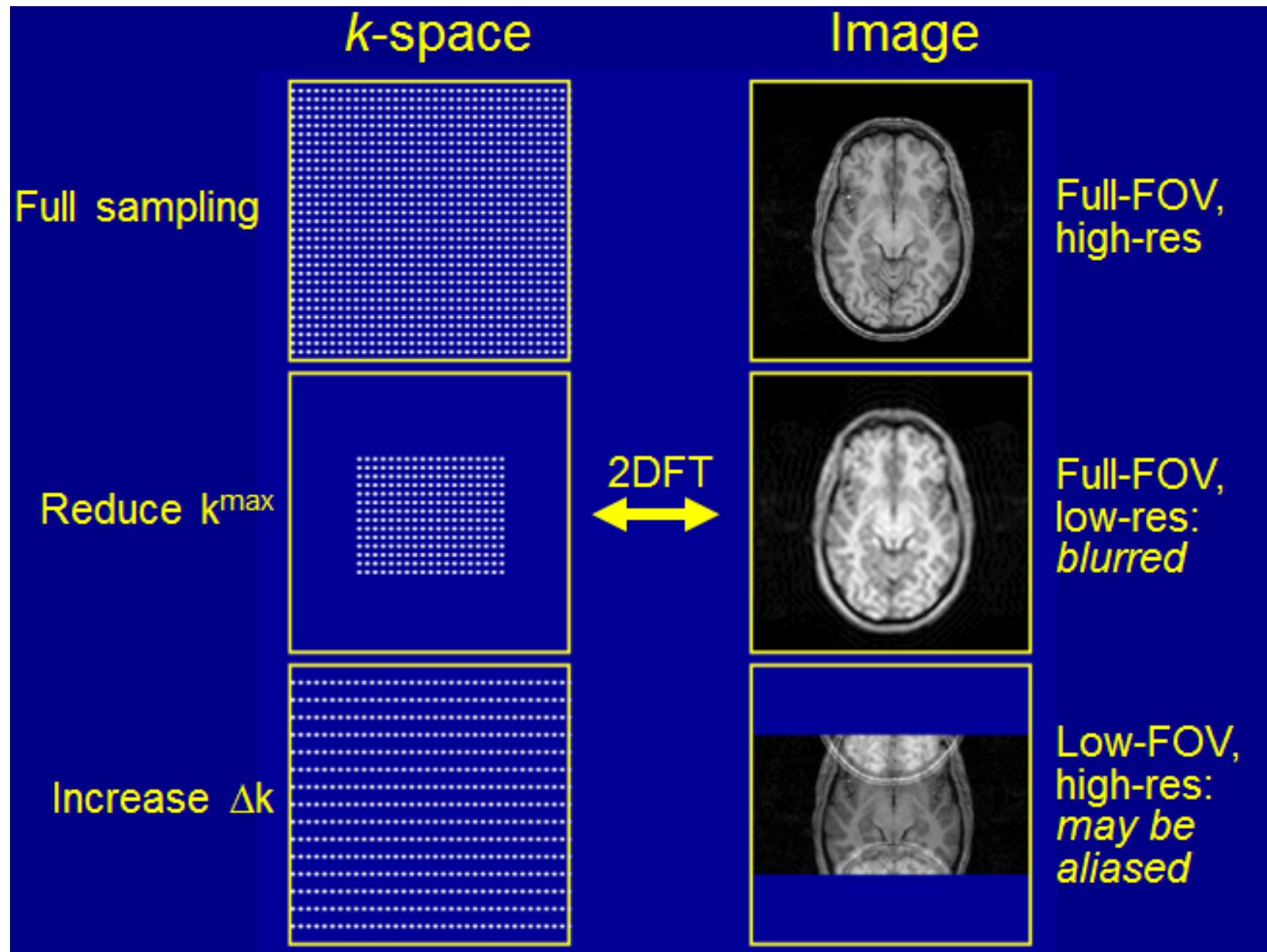
$$x_{\max} = \frac{1}{\Delta k_x}$$

$$2k_x^{\max} = \frac{1}{\Delta x}$$



k-space and image resolution are inversely related:
resolution in one domain determines extent in other.

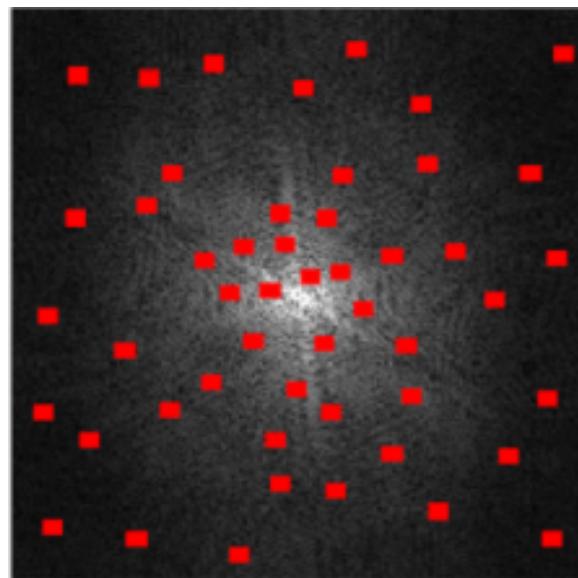
Nyquist Sampling Theorem



Go to 3rd notebook

Beyond Nyquist Sampling Theorem

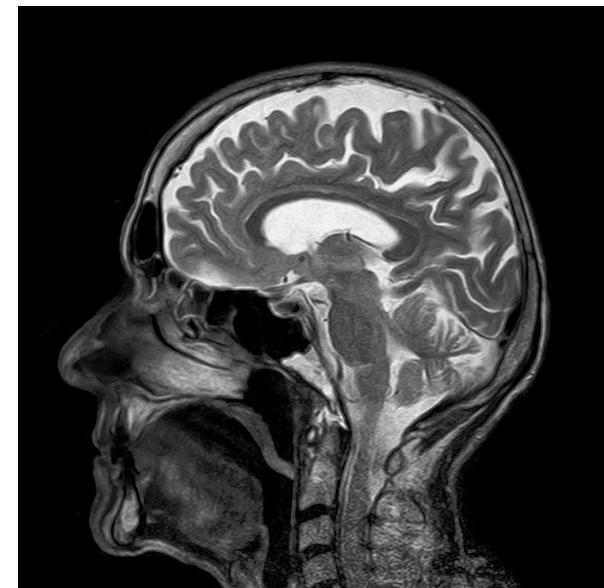
k-space



Nonlinear
reconstructions



Image



Go to 2nd notebook

Lustig, M., Donoho, D. L., and Pauly, J. M. (2007).

Sparse MRI: The application of compressed sensing for rapid MR imaging.

Magn. Reson. Med., 58(6):1182–1195.

Vasanawala, S., Murphy, M., Alley, M. T., Lai, P., Keutzer, K., Pauly, J. M., and Lustig, M. (2011).

Practical parallel imaging compressed sensing MRI: Summary of two years of experience in accelerating body MRI of pediatric patients.

In *Proc. ISBI*, pages 1039–1043.

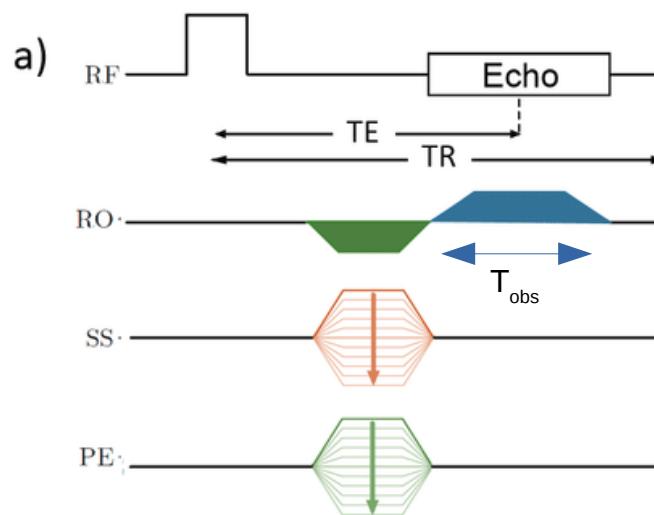
Outline

Part I: Background in Magnetic Resonance Imaging

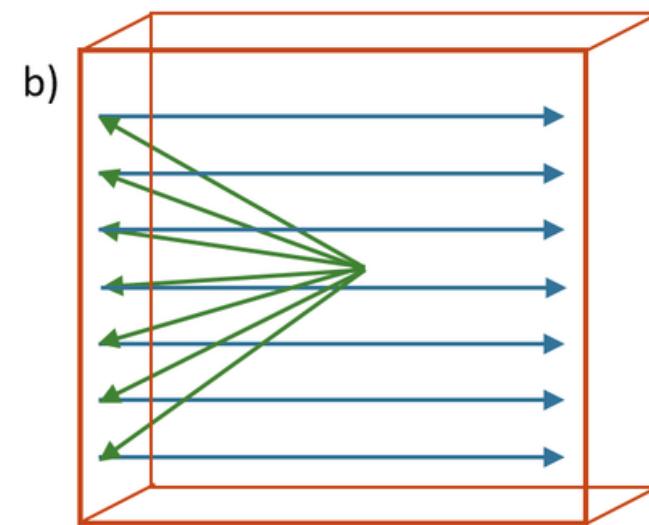
- Sampling k-space & Cartesian reconstruction
- Trajectories and acquisition strategies

Background in MR Acquisition

Chronogram of gradient-echo pulse sequence



K-space sampling



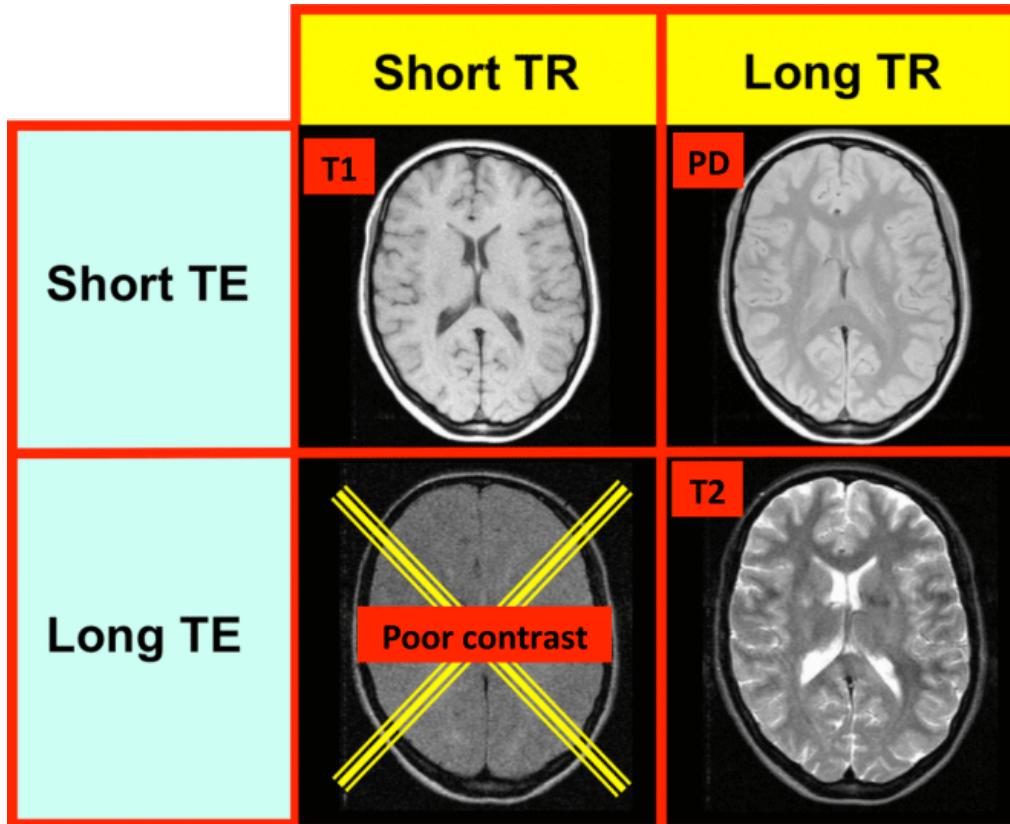
TR: Time between 2 successive RF pulses

TE: time point associated with the center
of echo: $k_x(TE) = 0$

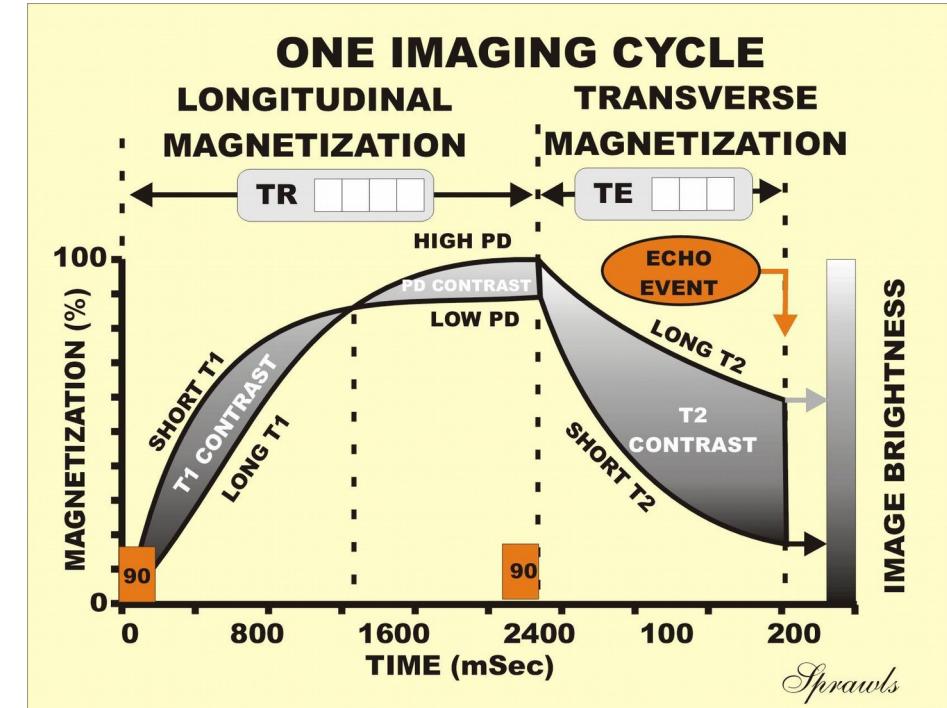
T_{obs} or readout: duration of each segment/shot

Short background in MRI

MR is versatile: multiple imaging contrasts



The MR signal has a short lifespan



Simplified Bloch equations:

- Longitudinal magnetization:

$$M_z(t) \simeq M_0(1 - e^{-t/T_1})$$

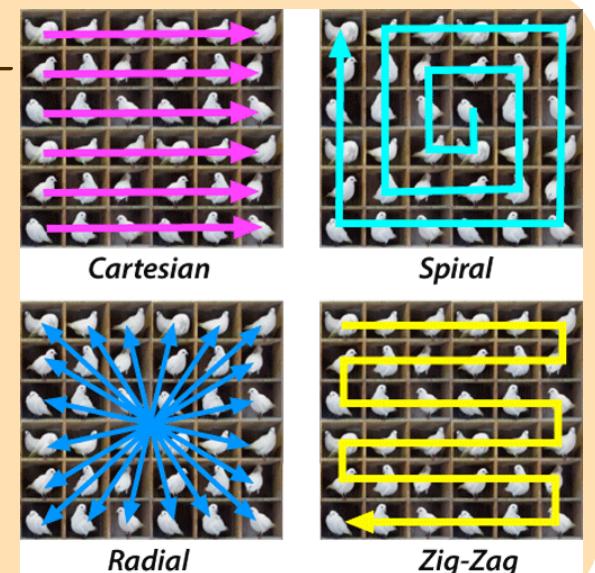
- Transverse magnetization:

$$M_{xy}(t) = M_0 e^{-t/T_2}$$

Constrained Sampling Trajectories

Sampling in MRI: $k(t) = k(0) + \gamma \int_0^t G(\tau) d\tau$

- **Segmented acquisition:**
Scan time proportional to number of shots
- **Hardware constraints on gradients:**
 $G_{\max} < 40 \text{ mT/m}$; $S_{\max} < 200 \text{ T/m/s}$
 → bounded velocity and acceleration



Let $\mathbf{k} : [0, T] \rightarrow \mathbb{R}^d$, ($d = 2, 3$) denote the sampling curve:

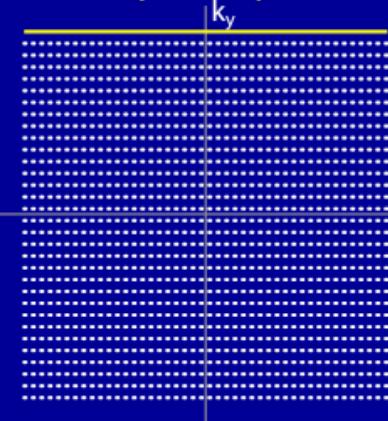
$$\mathbf{k}(t) = \mathbf{k}(0) + \gamma \int_0^t \mathbf{G}(\tau) d\tau \quad d = 2 \rightarrow \mathbf{k} = (k_x, k_y), \mathbf{G} = (G_x, G_y)$$

$$\mathcal{S} = \begin{cases} \|\dot{\mathbf{k}}\|_{2,\infty} &< \gamma G_{\max} & \text{(bounded speed)} \\ \|\ddot{\mathbf{k}}\|_{2,\infty} &< \gamma S_{\max} & \text{(bounded acceleration)} \end{cases} \quad \text{where } \|\mathbf{c}\|_{2,\infty} = \sup_{1 \leq i \leq p} (\lvert c_x[i] \rvert^2 + \lvert c_y[i] \rvert^2)^{1/2}$$

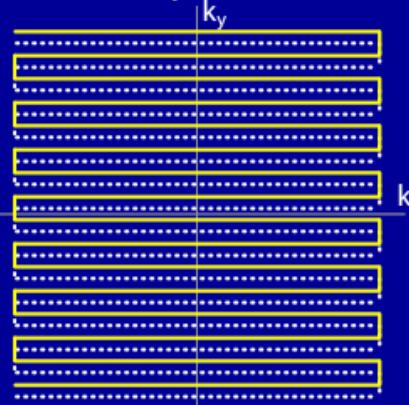
K-space location is proportional to accumulated area under gradient waveforms

Examples of k-space Trajectories

(a) 2DFT (spinwarp)



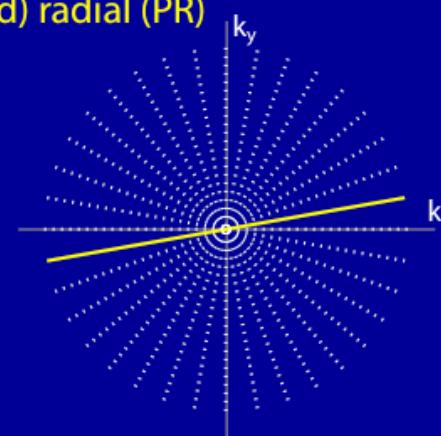
(b) EPI (echo-planar)



(c) spiral



(d) radial (PR)



Line acquisition vs. EPI

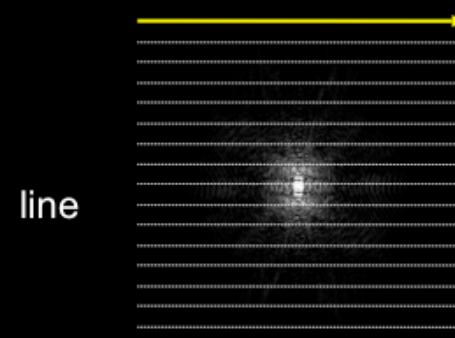


Image accumulated over multiple line acquisitions

Slow: 5-10 minutes

Excellent image quality

k-space

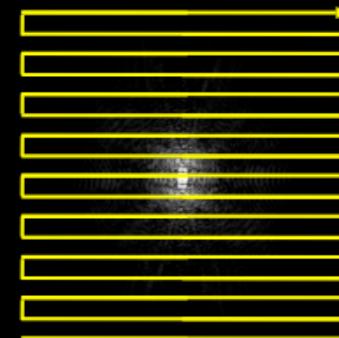


Image acquired in single acquisition

Fast: 3 seconds

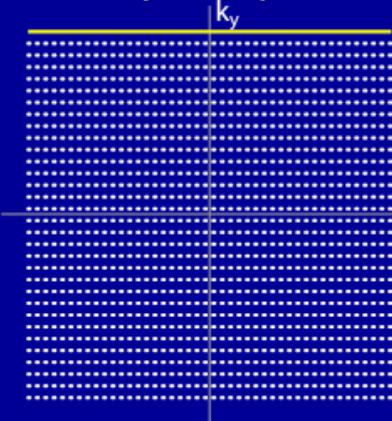
Image artefacts



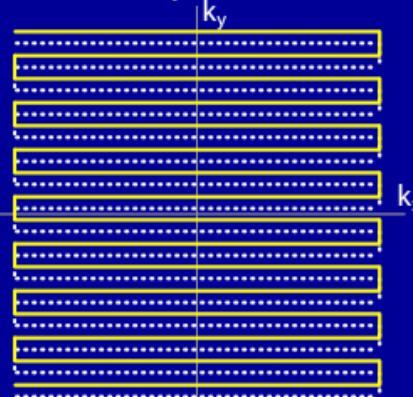
Go to the 5th and the 6th notebooks

Examples of k-space Trajectories

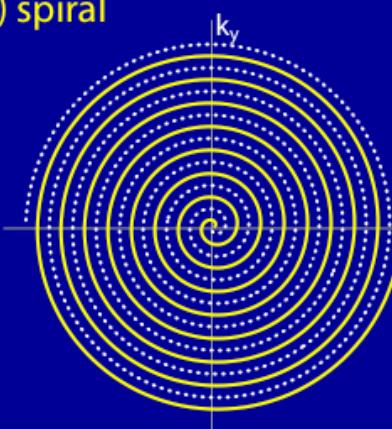
(a) 2DFT (spinwarp)



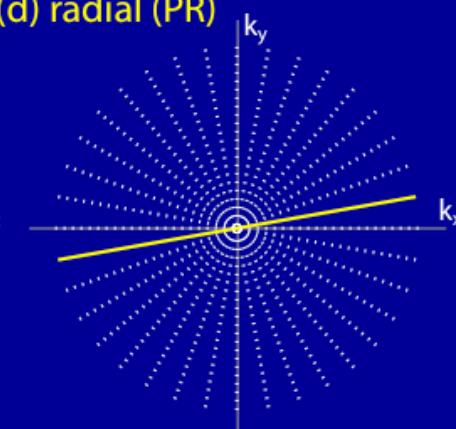
(b) EPI (echo-planar)



(c) spiral



(d) radial (PR)



Line acquisition vs. EPI

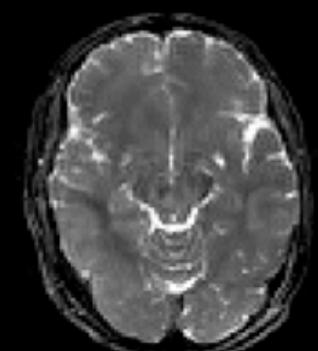
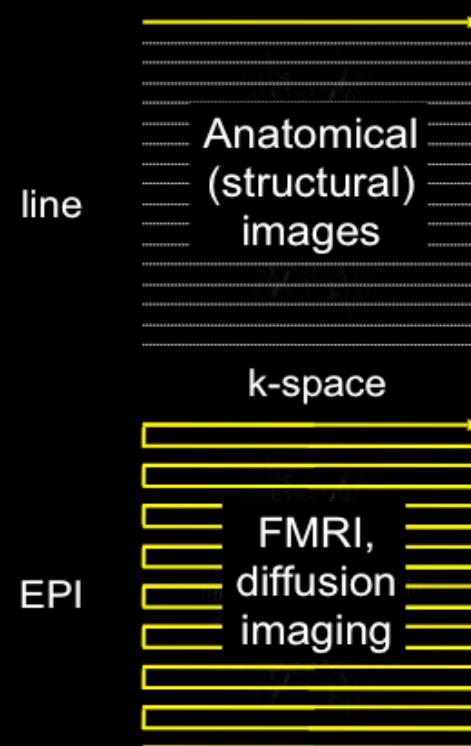
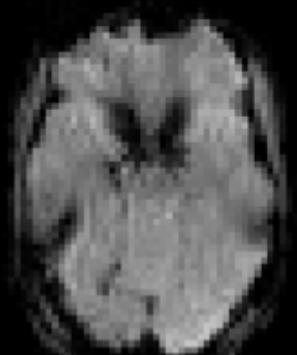


image space



Interim Summary (1)

- **Cartesian vs non-Cartesian trajectories**
 - **Cartesian**: easier to implement less prone to gradient errors, play one gradient at a time
 - **Non-cartesian**: robust to motion, more flexible and efficient sampling but prone to gradient errors
 - The time matters: the same sampling schemes can be played in different ways (e.g. radial in-out vs radial-out)
- **Single shot vs multi-shot**
 - Multiple shot = longer acquisition times, better image quality
 - Single shot = faster acquisition, e.g. for dynamic MR imaging mainly
- **MRI acquisition constraints**
 - T_1 -w: Short TE, TR → short readout and thus straight lines (Cartesian or radial) are unavoidable
 - T_2 , T_2^* -w : long TE, TR → enable long readout → more fancy trajectories are feasible

Outline

Part II: Compressed Sensing in MRI

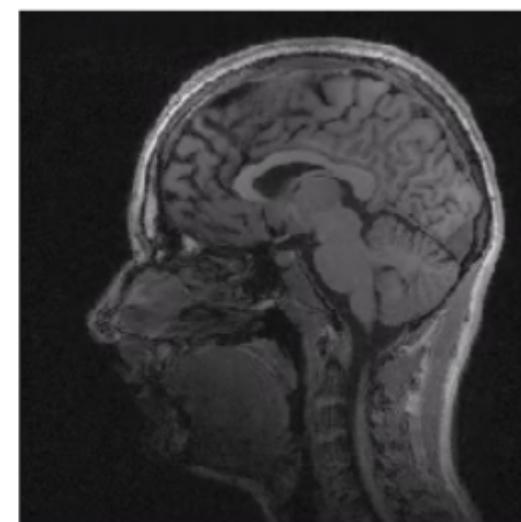
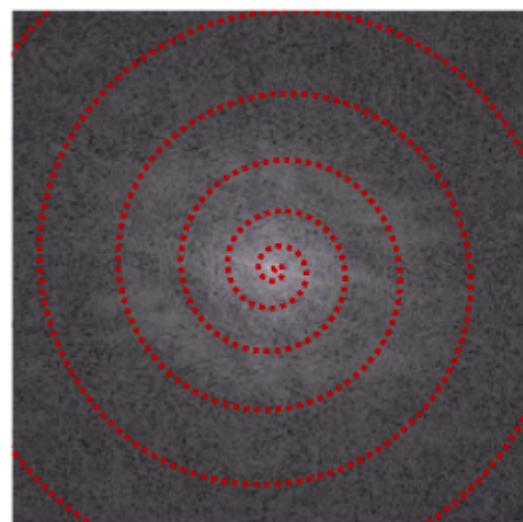
- Evolution of Compressed Sensing
- CS with blocks of measurements

Modeling of k-space Trajectories

Sampling the curve $\mathbf{k}(t)$ generates a set of measurements:

$$E(\mathbf{k}) = \{\hat{x}(\mathbf{k}(p\Delta t))\}_{p \in \{0, \dots, T/(\Delta t)\}}$$

with \hat{x} the Fourier transform of x



Challenges

- **Questions in sampling theory**

- How to choose the set of measurements E ?
 - How to generate a set of segmented trajectories k ?

- How to reconstruct x given E ?



Jeff Fessler's part!

- **Practical consequences**

- Shorten acquisition times
 - Increase image resolution
 - Improve medical diagnosis and reduce exam costs

A fundational result

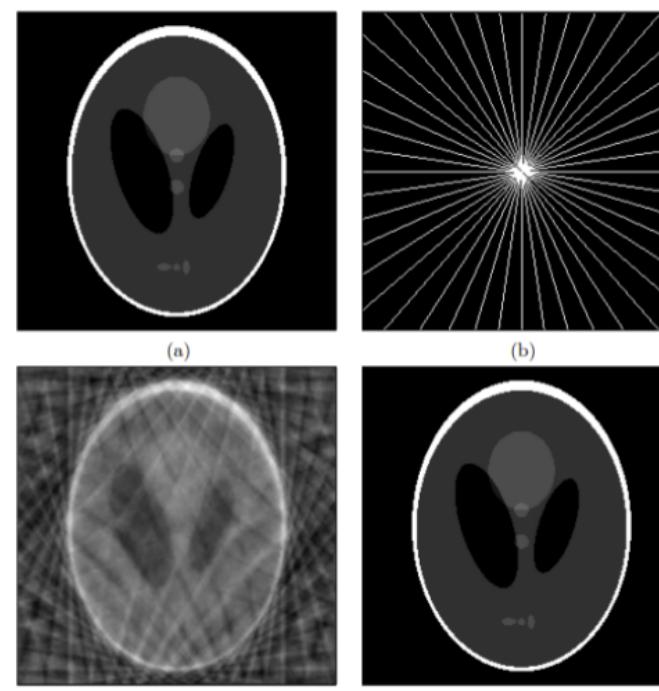
Supporting Thorem [Candes, Romberg and Tao, IEEE IT 2006]

Let $\boldsymbol{x} \in \mathbb{C}^n$ denote an s -sparse signal. Samples m values $y_k = \hat{\boldsymbol{x}}(J_k)$ where indexes $(J_k)_{1 \leq k \leq m}$ are i.i.d. and uniformly distributed.
If

$$m \geq C \cdot s \cdot \log(n)$$

\boldsymbol{x} can be recovered exactly, with high probability by solving:

$$\min_{\boldsymbol{x} \in \mathbb{C}^n, \hat{\boldsymbol{x}}(J_k) = y_k} \|\boldsymbol{x}\|_1$$



As the measurements are not *independent*, this theorem does not explain this result!

Preliminaries

Wavelet expansions

- Let $\Psi = (\Psi_1, \dots, \Psi_n) \in \mathbb{C}^{n \times n}$ denote an **orthogonal transform**. The image to reconstruct writes:

$$\mathbf{x} = \Psi \mathbf{z} = \sum_{i=1}^n z_i \Psi_i \quad \text{where} \quad z_i = \langle \mathbf{x}, \Psi_i \rangle$$

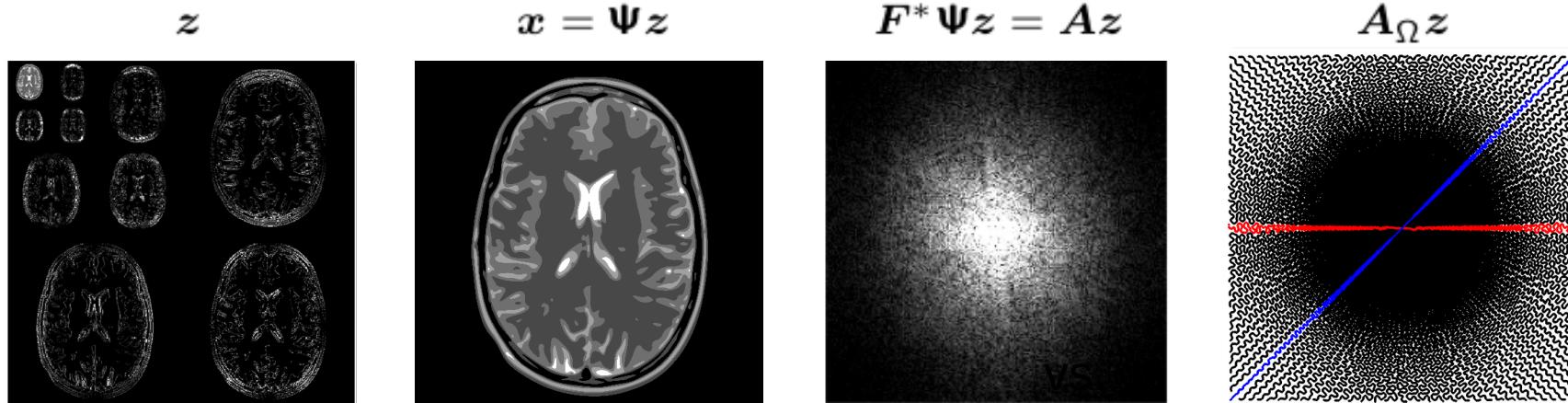
- Objective: retrieve \mathbf{z} (not \mathbf{x})

Fourier sampling

- Sensing matrix:** $A_0 = F^* \Psi = \begin{pmatrix} a_1^* \\ a_2^* \\ \vdots \\ a_n^* \end{pmatrix}$. One measurement reads $y_k = \langle \mathbf{a}_k, \mathbf{z} \rangle$
- Let $\Omega \subseteq \{1, \dots, n\}$ and $A_\Omega = (\mathbf{a}_i^*)_{i \in \Omega}$ the sampling matrix. We get:

$$\mathbf{y} = A_\Omega \mathbf{z} (+ \mathbf{b})$$

Preliminaries



Sparsity: Let $S = \{i, z_i \neq 0\}$ denote the support of \mathbf{z} .

We assume that: $|S| = s \ll n$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noiseless case):

$$\widehat{\mathbf{z}} = \arg \min_{\mathbf{z} \in \mathbb{C}^n, \mathbf{A}_\Omega \mathbf{z} = \mathbf{y}} \|\mathbf{z}\|_1$$

Nonlinear ℓ_1 reconstruction (synthesis formulation, noisy case):

$$\widehat{\mathbf{z}} = \arg \min_{\mathbf{z} \in \mathbb{C}^n} \frac{1}{2} \|\mathbf{y} - \mathbf{A}_\Omega \mathbf{z}\|_2^2 + \lambda \|\mathbf{z}\|_1$$

$$\widehat{\mathbf{x}} = \Psi \widehat{\mathbf{z}}$$

Evolution of Compressed Sensing

Uniform drawings: the early results

Coherence: $\kappa(\mathbf{A}_0) = n \max_{1 \leq k \leq n} \|\mathbf{a}_k\|_\infty^2$

Assume that \mathbf{x} is s -sparse.

Draw m samples uniformly at random in $\{1, \dots, n\}$.

$$m \geq C \cdot \kappa(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Uniform sampling

Emmanuel Candes and Terence Tao.

Near-optimal signal recovery from random projections: Universal encoding strategies?
Information Theory, IEEE Transactions on, 52(12):5406–5425, 2006.

Emmanuel Candes and Yaniv Plan.

A probabilistic and ripless theory of compressed sensing.
Information Theory, IEEE Transactions on, 57(11):7235–7254, 2011.

Simon Foucart and Holger Rauhut.

A mathematical introduction to compressive sensing.
Springer, 2013.

Evolution of Compressed Sensing

Uniform drawings: the early results

Coherence: $\kappa(\mathbf{A}_0) = n \max_{1 \leq k \leq n} \|\mathbf{a}_k\|_\infty^2$

Assume that \mathbf{x} is s -sparse.

Draw m samples uniformly at random in $\{1, \dots, n\}$.

$$m \geq C \cdot \kappa(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Uniform sampling

Emmanuel Candes and Terence Tao.

Near-optimal signal recovery from random projections: Universal encoding strategies?
Information Theory, IEEE Transactions on, 52(12):5406–5425, 2006.

Emmanuel Candes and Yaniv Plan.

A probabilistic and ripless theory of compressed sensing.
Information Theory, IEEE Transactions on, 57(11):7235–7254, 2011.

Simon Foucart and Holger Rauhut.

A mathematical introduction to compressive sensing.
Springer, 2013.

Fourier-Wavelet system satisfies
 $\kappa(\mathbf{A}_0) \propto n!$



$m \gg n$

Coherence barrier!

Evolution of Compressed Sensing

Variable density sampling

Local coherence: $\mu(\mathbf{A}_0) = \sum_{k=1}^n \|\mathbf{a}_k\|_\infty^2$.

Assume that \mathbf{x} is s -sparse.

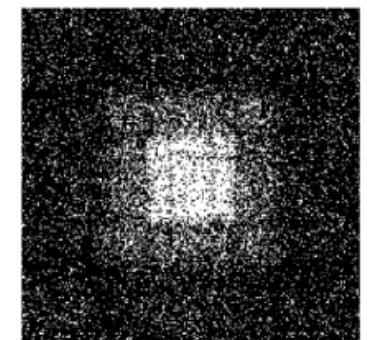
Set $\mathbf{A}_\Omega = \begin{pmatrix} \mathbf{a}_{J_1} \\ \vdots \\ \mathbf{a}_{J_m} \end{pmatrix}$ with $\mathbb{P}(J_k = \ell) = \pi_\ell \propto \|\pi_\ell\|_\infty^2$

$$m \geq C \cdot \mu(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$

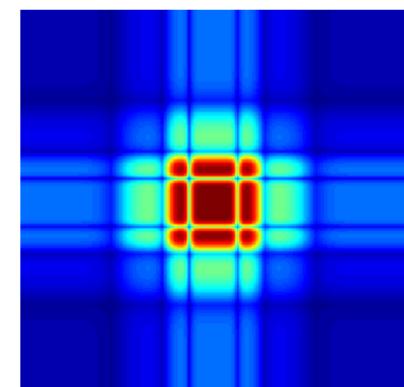
Gilles Puy, Pierre Vandergheynst, and Yves Wiaux.
 On variable density compressive sampling.
[Signal Processing Letters, IEEE, 18\(10\):595–598, 2011.](#)

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.
 Variable density sampling with continuous sampling trajectories.
[SIAM Journal on Imaging Science, 2014.](#)

Felix Krahmer and Rachel Ward.
 Stable and robust sampling strategies for compressive imaging.
[Image Processing, IEEE Transactions on, 23\(2\):612–622, 2014.](#)



Variable density sampling



π in 2D



Go to 2nd notebook

Evolution of Compressed Sensing

Variable density sampling

Local coherence: $\mu(\mathbf{A}_0) = \sum_{k=1}^n \|\mathbf{a}_k\|_\infty^2$.

Assume that \mathbf{x} is s -sparse.

Set $\mathbf{A}_\Omega = \begin{pmatrix} \mathbf{a}_{J_1} \\ \vdots \\ \mathbf{a}_{J_m} \end{pmatrix}$ with $\mathbb{P}(J_k = \ell) = \pi_\ell \propto \|\pi_\ell\|_\infty^2$

$m \geq C \cdot \mu(\mathbf{A}_0) \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow$ exact recovery with probability $1 - \epsilon$

Gilles Puy, Pierre Vandergheynst, and Yves Wiaux.

On variable density compressive sampling.

Signal Processing Letters, IEEE, 18(10):595–598, 2011.

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

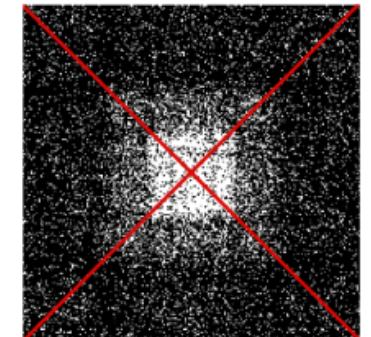
Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

Felix Krahmer and Rachel Ward.

Stable and robust sampling strategies for compressive imaging.

Image Processing, IEEE Transactions on, 23(2):612–622, 2014.



Variable density
sampling

Fourier-Wavelet
system satisfies
 $\mu(\mathbf{A}_0) \propto \log(n)!$

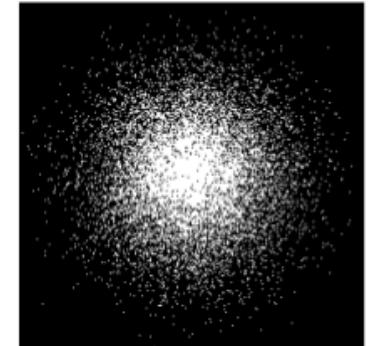
Evolution of Compressed Sensing

Variable density sampling with structure

Assume that x has a certain “sparsity structure”.

$$\text{Set } A_\Omega = \begin{pmatrix} a_{J_1} \\ \vdots \\ a_{J_m} \end{pmatrix} \text{ with } \mathbb{P}(J_k = \ell) = \tilde{\pi}_\ell$$

$$m \geq C \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Variable density
sampling +
structure

Ben Adcock and Anders C Hansen.

Generalized sampling and infinite-dimensional compressed sensing.

Foundations of Computational Mathematics, to appear, 2015.

Ben Adcock, Anders C. Hansen, Clarice Poon, and Bogdan Roman.

Breaking the coherence barrier: A new theory for compressed sensing.

arXiv preprint arXiv:1302.0561, 2013.

Jérémie Bigot, Claire Boyer, and Pierre Weiss.

Compressed sensing with structured sparsity and structured acquisition.

Applied and Computational Harmonic Analysis, 2017.

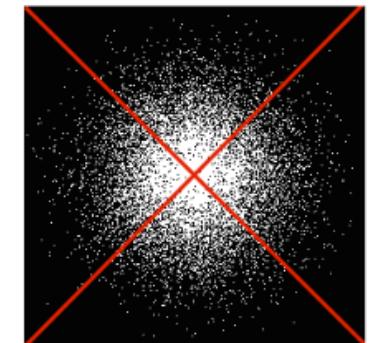
Evolution of Compressed Sensing

Variable density sampling with structure

Assume that x has a certain “sparsity structure”.

$$\text{Set } A_\Omega = \begin{pmatrix} a_{J_1} \\ \vdots \\ a_{J_m} \end{pmatrix} \text{ with } \mathbb{P}(J_k = \ell) = \tilde{\pi}_\ell$$

$$m \geq C \cdot s \cdot \log\left(\frac{n}{\epsilon}\right) \Rightarrow \text{exact recovery with probability } 1 - \epsilon$$



Variable density
sampling +
structure

Ben Adcock and Anders C Hansen.

Generalized sampling and infinite-dimensional compressed sensing.

Foundations of Computational Mathematics, to appear, 2015.

Ben Adcock, Anders C. Hansen, Clarice Poon, and Bogdan Roman.

Breaking the coherence barrier: A new theory for compressed sensing.

arXiv preprint arXiv:1302.0561, 2013.

Jérémie Bigot, Claire Boyer, and Pierre Weiss.

Compressed sensing with structured sparsity and structured acquisition.

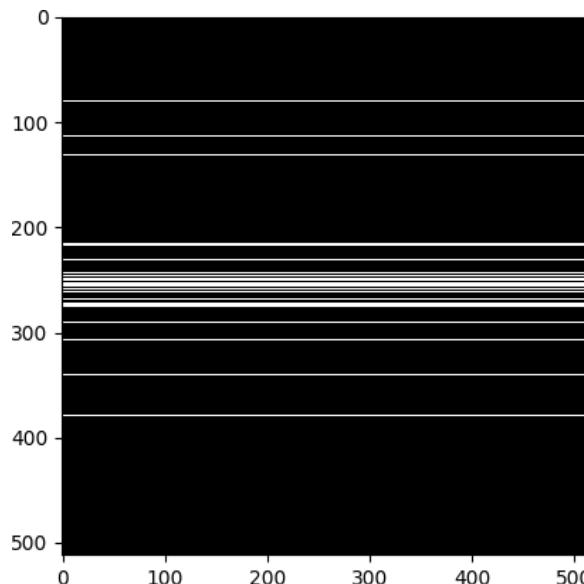
Applied and Computational Harmonic Analysis, 2017.

Only works
for isolated
measurements!

→ Current compressed sensing theories are partially disconnected
from applications!

Possible 1D VDS implementation

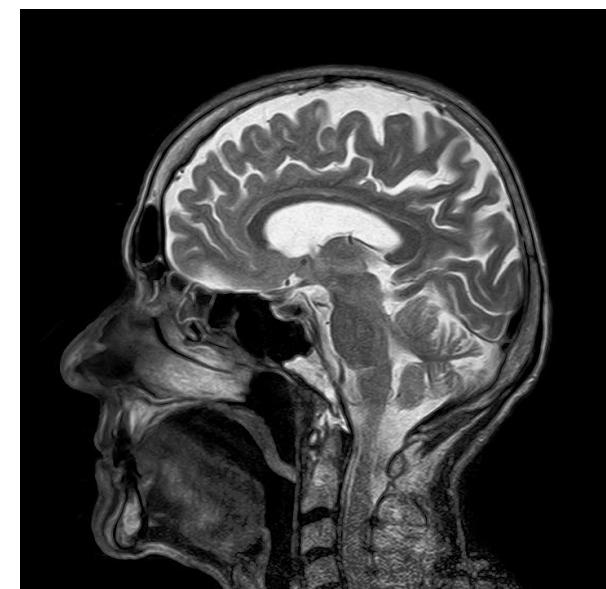
k-space



Nonlinear
reconstructions



Image

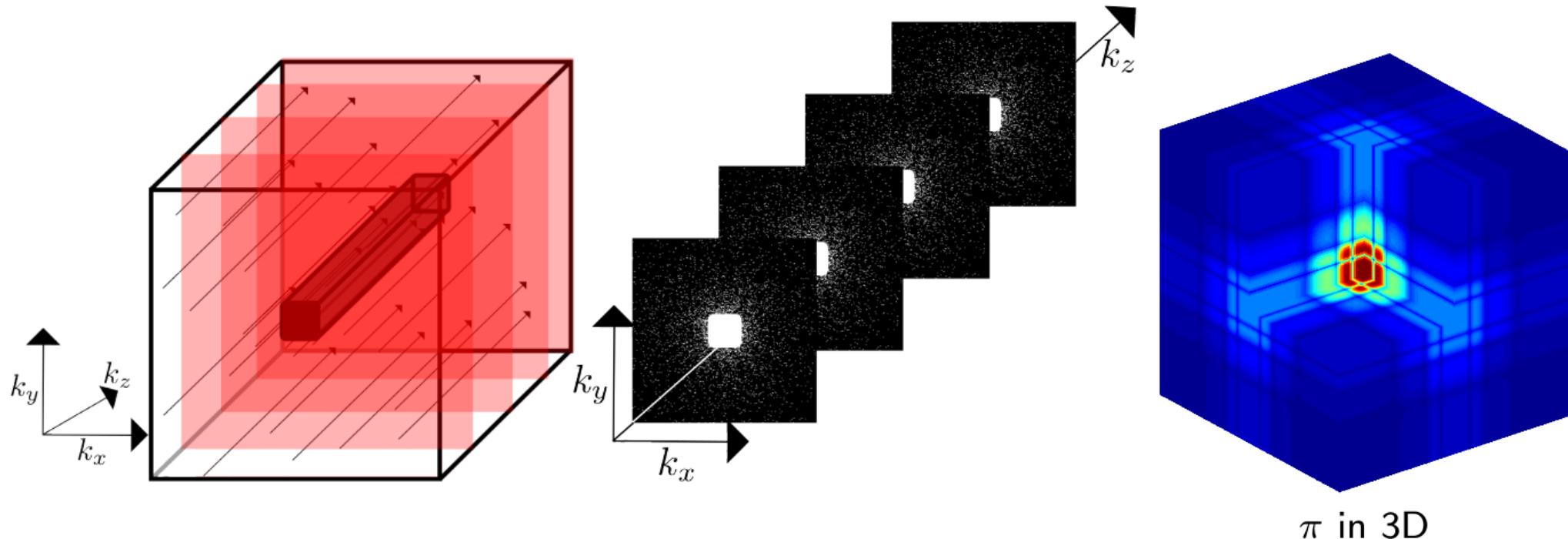


[Haldar et al, IEEE TMI, 2011]



Go to the 4th notebook

Possible 2D VDS implementation



Perform the readout along the **third dimension!**
Suboptimal as compared to π in 3D.

Lustig, M., Donoho, D. L., and Pauly, J. M. (2007).

Sparse MRI: The application of compressed sensing for rapid MR imaging.
Magn. Reson. Med., 58(6):1182–1195.

Chaussert, N., Ciuciu, P., and Weiss, P. (2013b).

Variable density compressed sensing in MRI. Theoretical vs. heuristic sampling strategies.
In Proc. of 10th IEEE ISBI conference, pages 298–301, San Francisco, USA.

Interim Summary (2)

- **Original CS theories**
 - Assume **independent** sampling → not satisfied in MRI
 - Rely either on
 - RIP conditions: **not testable** in practice
 - Coherence tools (global, local, multi-level sampling) → satisfied only **asymptotically** (i.e. at very high resolution)
 - Assume **fixed** sparsifying transforms (e.g. wavelets, frames)
 - **Variable density sampling is required** to minimize the number of measurements
- **Most recent CS theories**
 - Are able to account for blocks of measurements (e.g. Cartesian or radial lines)
 - Stick to **isotropic** sampling densities
- **The way the data is collected has not really been taken into account**

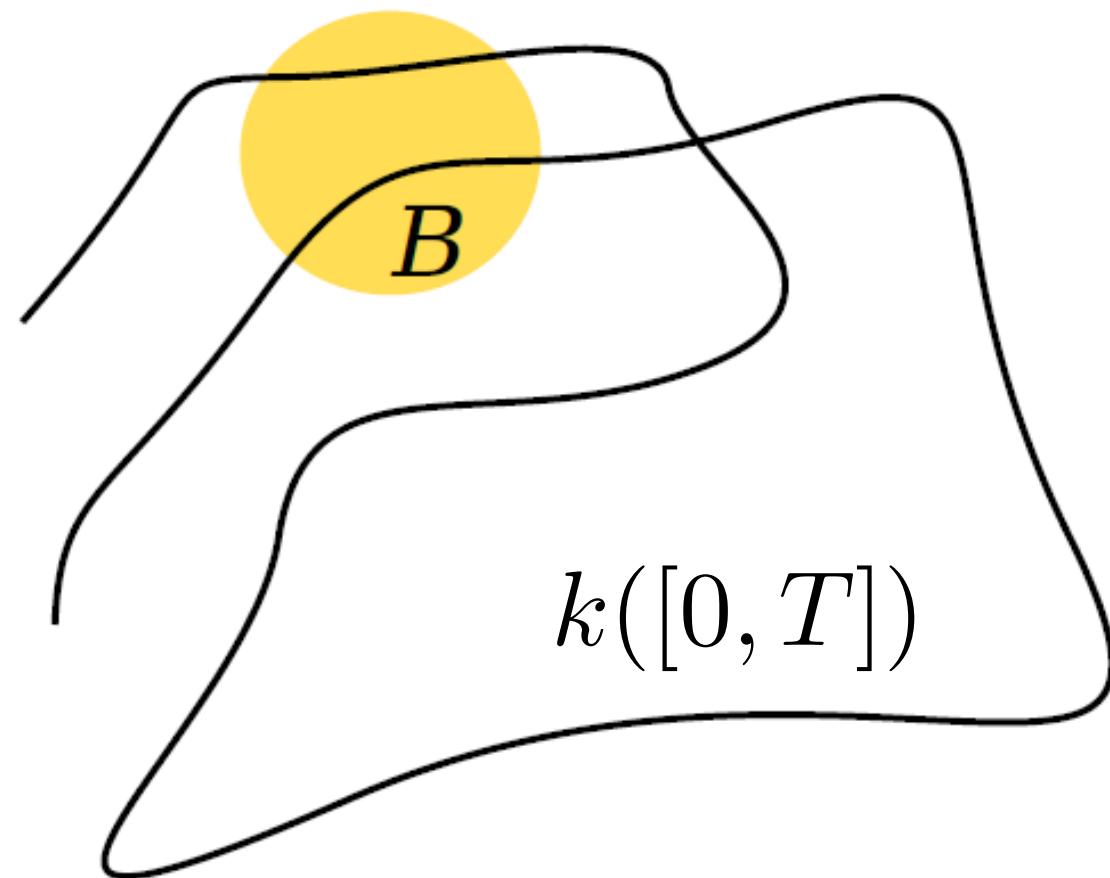
Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

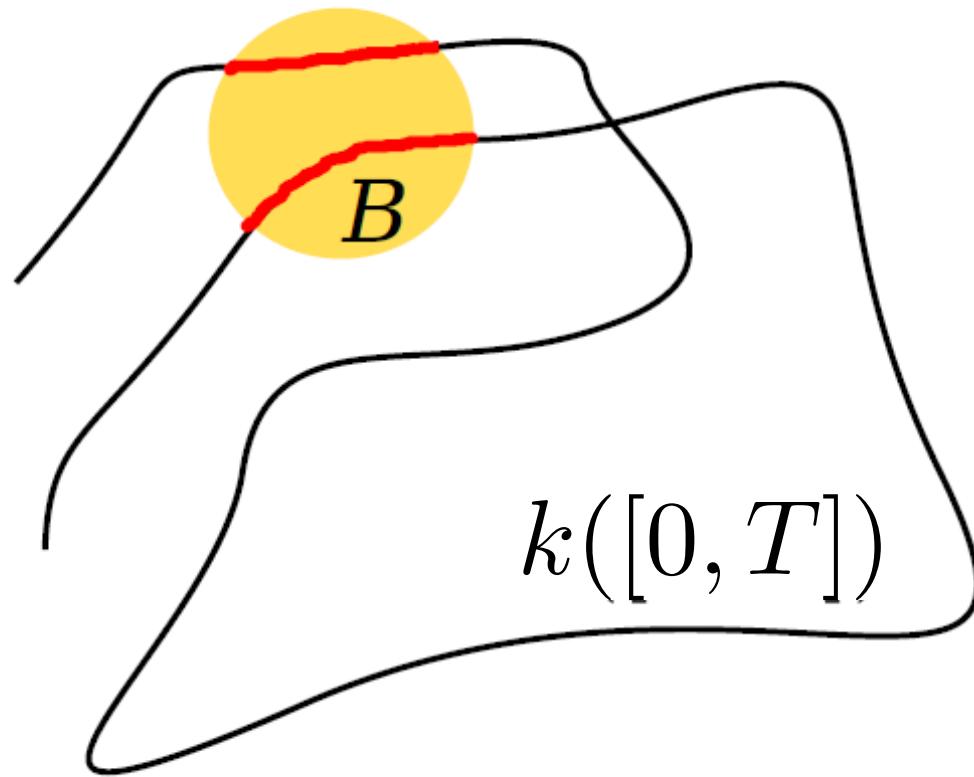
Continuous Variable Density Sampler

Pushforward measure - illustration



Continuous Variable Density Sampler

Pushforward measure - illustration



$$\nu(B) = k_*\lambda_T(B) = \lambda_T(k^{-1}(B))$$

λ_T is the (normalized) Lebesgue measure.

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

Push Forward Measure - Definition

Pushforward measure

Let $\Omega = [0, 1]^d$, where $d = 2$ or 3 denote the space dimension. We equip Ω with the Borel algebra \mathcal{B} . Let (X, Σ) be a measurable space and $k: X \mapsto \Omega$ be a measurable mapping. $\mu: X \mapsto [0; +\infty)$ denote a measure. The *pushforward measure* ν of μ is defined by:

$$\nu(B) = k_*\mu(B) = \mu(k^{-1}(B)), \quad \forall B \in \mathcal{B}$$

Ex. 1: Measures supported by curves

Ex. 2: Atomic measures

$k: \{1, \dots, m\} \mapsto \Omega$ where $k[i] = p_i$ denotes the i -th point. Set μ as the *counting measure* defined for any set $I \subseteq \{1, \dots, m\}$ by $\mu(I) = \frac{|I|}{m}$. Then ν is defined by

$$\nu = \frac{1}{m} \sum_{i=1}^m \delta_{p_i}$$

Push Forward Measure - Theorem

Weak convergence

A sequence of measures (μ_n) is said to weakly converge to μ , if for any bounded continuous function ϕ ,

$$\int_{\Omega} \phi(x) d\mu_n(x) \xrightarrow{n \rightarrow +\infty} \int_{\Omega} \phi(x) d\mu(x)$$

Shorthand notation: $\mu_n \rightharpoonup \mu$.

Variable density sampler

A sequence of (random) trajectories $k_n : X_n \mapsto \Omega$ is said to be a π -Variable Density Sampler if

$$k_{n*}\mu \rightharpoonup \pi \text{ almost surely}$$

Examples

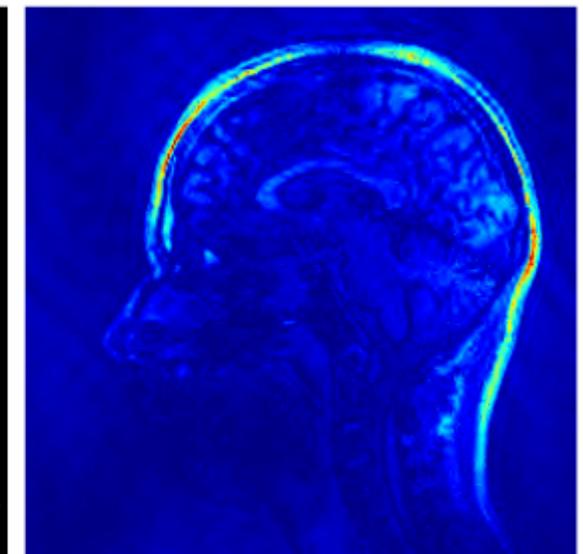
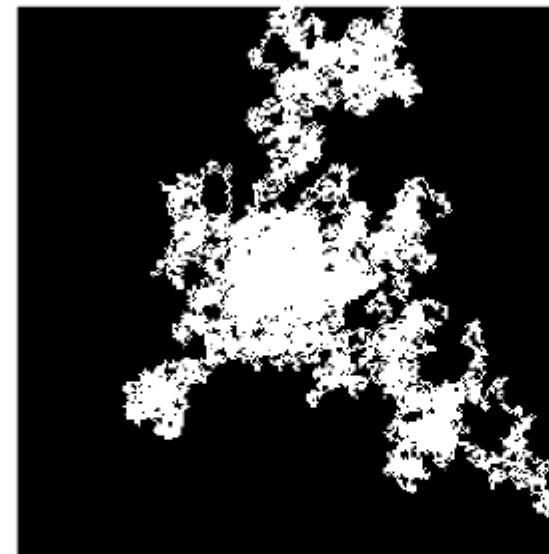
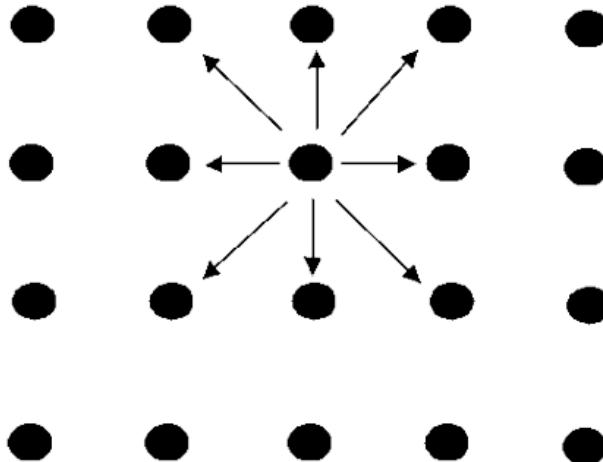
i.i.d. drawing, random walks ...

Sampling along Random Walks

Construction of a discrete Markov chain

Given a target probability distribution $\pi \in \mathbb{R}^n$.

Define a Markov chain $K = (k_i)_{i \in \mathbb{N}}$ on the set $\{1, \dots, n\}$. Use the Metropolis algorithm to construct a stochastic transition matrix $P \in \mathbb{R}^{n \times n}$ such that π is the stationary distribution of K .



- Time to cover the k -space is slow
- Local approach

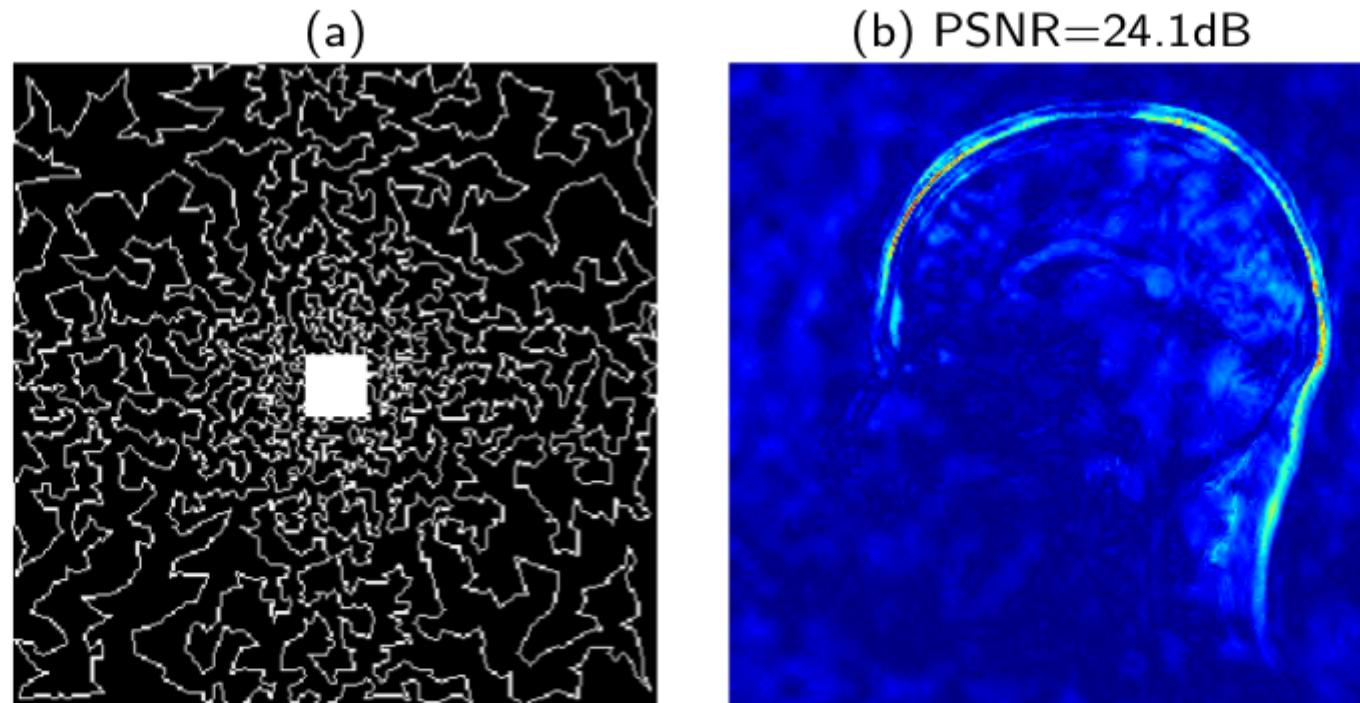
Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

Solving the Travelling Salesman Problem

Idea : cover the k -space more quickly with a global approach.



- Pushforward measure far from π . From which distribution should we sample the initial points to reach a given target distribution?

Go to Matlab code:
TSP-Markov folder

TSP Sampler

- Let

$$q = \frac{\pi^{d/(d-1)}}{\int_{\Omega} \pi^{d/(d-1)}}$$

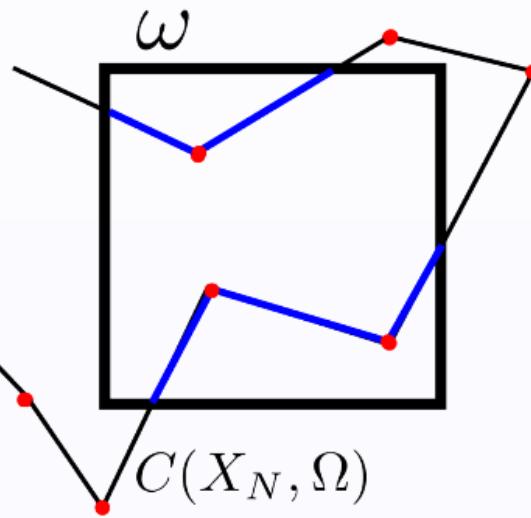
- $(k[i])_{i \in \mathbb{N}^*}$ a sequence of points in Ω , i.i.d. drawn $\sim q$.
- $\mathbf{k}_N = (k[i])_{i \leq N}$
- Denote $T(\mathbf{k}_N)$ the length of the TSP amongst \mathbf{k}_N .
- $\gamma_N : [0, T(\mathbf{k}_N)] \mapsto \Omega$ denotes the parametrization of the curve at speed 1.

Theorem

Almost surely w.r.t. the law $q^{\otimes \mathbb{N}}$ of the sequence $(k[i])_{i \in \mathbb{N}^*}$ of random points in the hypercube, $(\gamma_N)_{N \in \mathbb{N}}$ is a π -variable density sampler, i.e.,

$$\gamma_{N*} \lambda_{T(\mathbf{k}_N)} \rightharpoonup \pi$$

Intuition of the Proof



Definition of the TSP distribution

For any Borelian $\omega \in \Omega$:

$$P_N(\omega) = \frac{T_{|\omega}(\mathbf{k}_N)}{T(\mathbf{k}_N)} = \gamma_{N*}\lambda_{T_{|\omega}(\mathbf{k}_N)}$$

Intuition

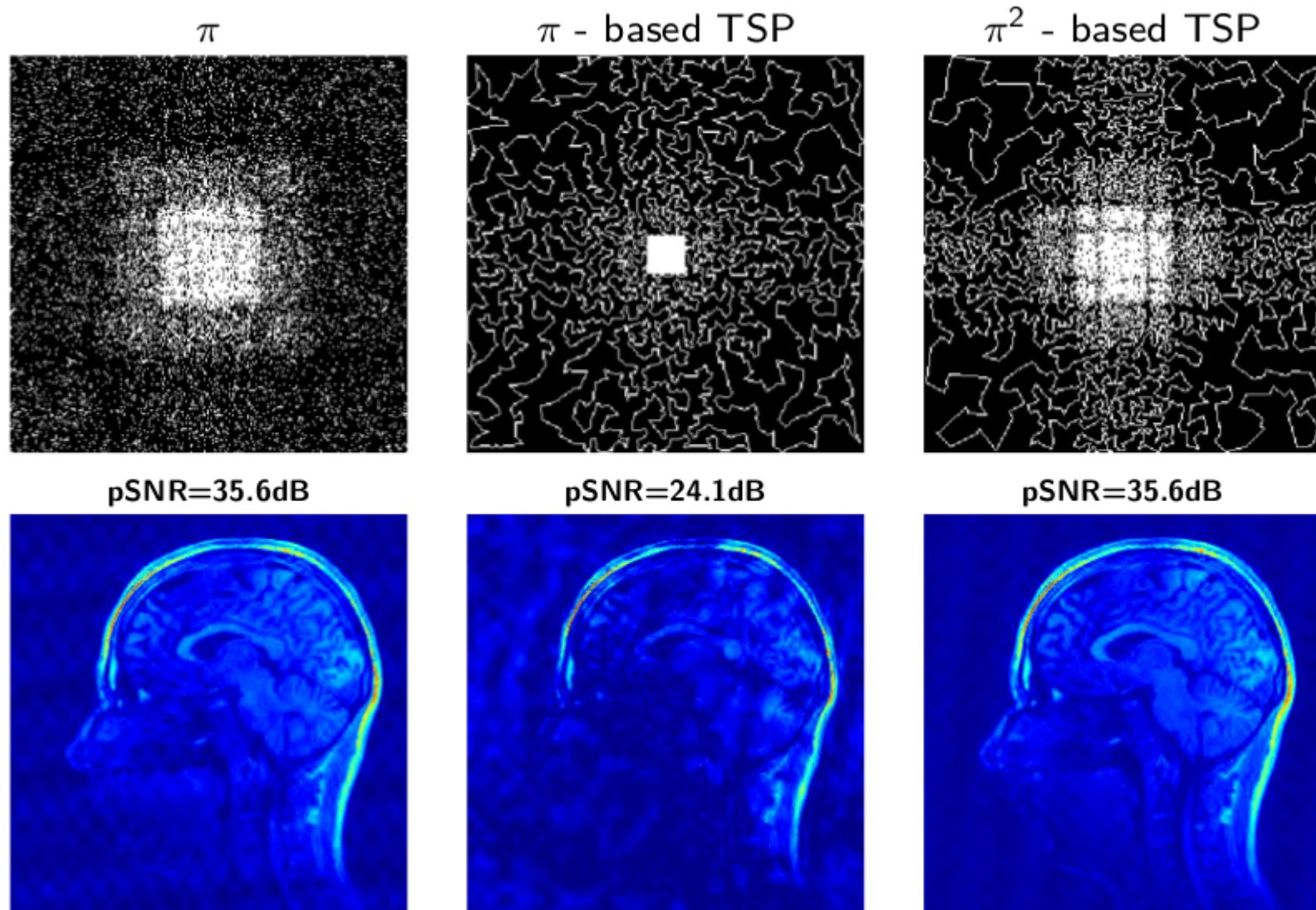
Consider a small hypercube:

- The number of points $n \propto q$;
- The typical distance is proportional to $n^{-1/d}$ (or $q^{-1/d}$);
- \Rightarrow The length of the TSP in the small cube is $\propto q q^{-1/d} = q^{(d-1)/d}$

Conclusion

To reach a target density π one should choose $q \propto \pi^{d/(d-1)}$!

The Traveling Salesman Sampler in 2D

Sampling schemes
($r = 5$)

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.
 Variable density sampling with continuous sampling trajectories.
 SIAM Journal on Imaging Science, 2014.

Go to Matlab code:
TSP-Markov folder

The Traveling Salesman Sampler in 3D

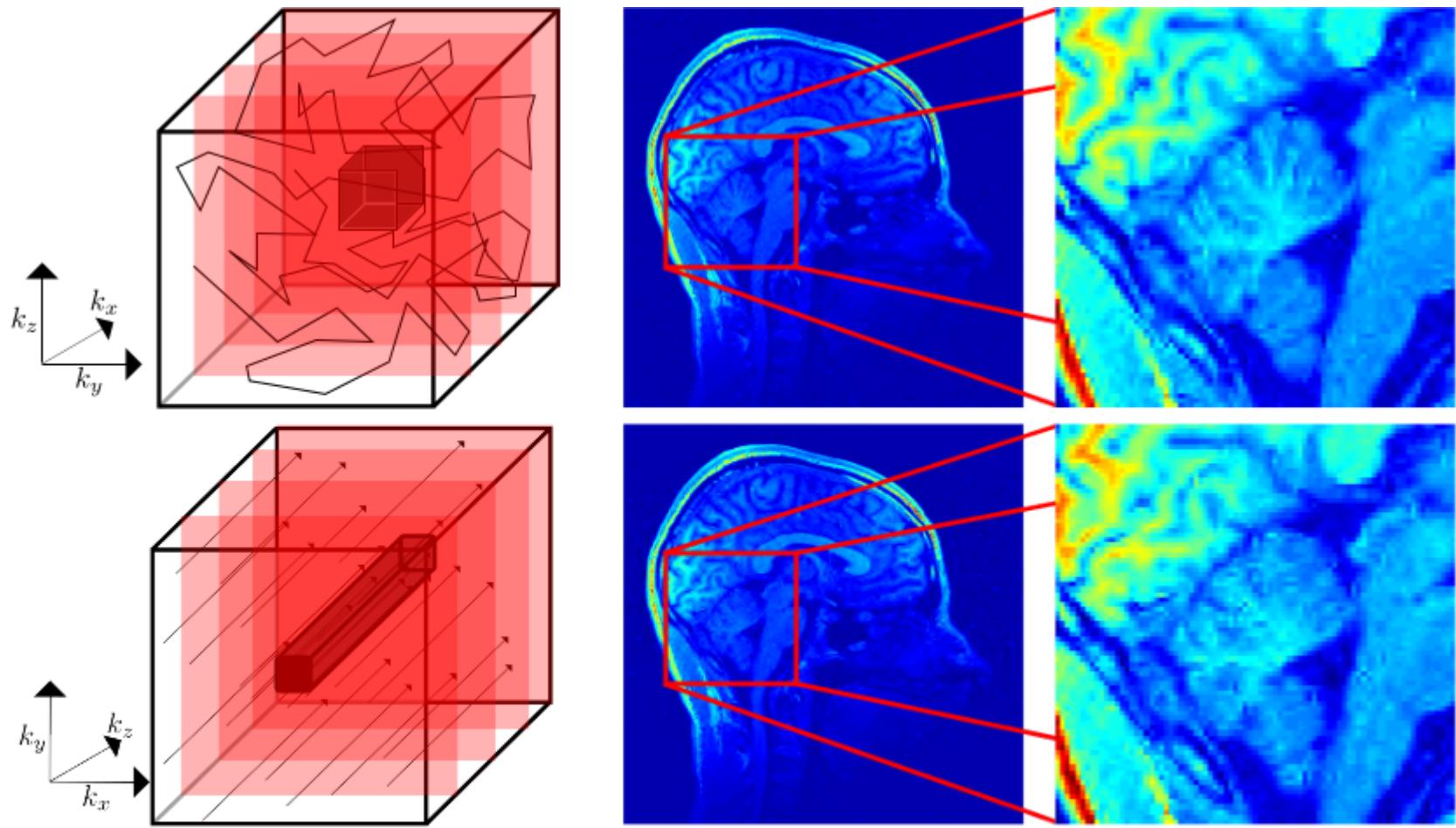


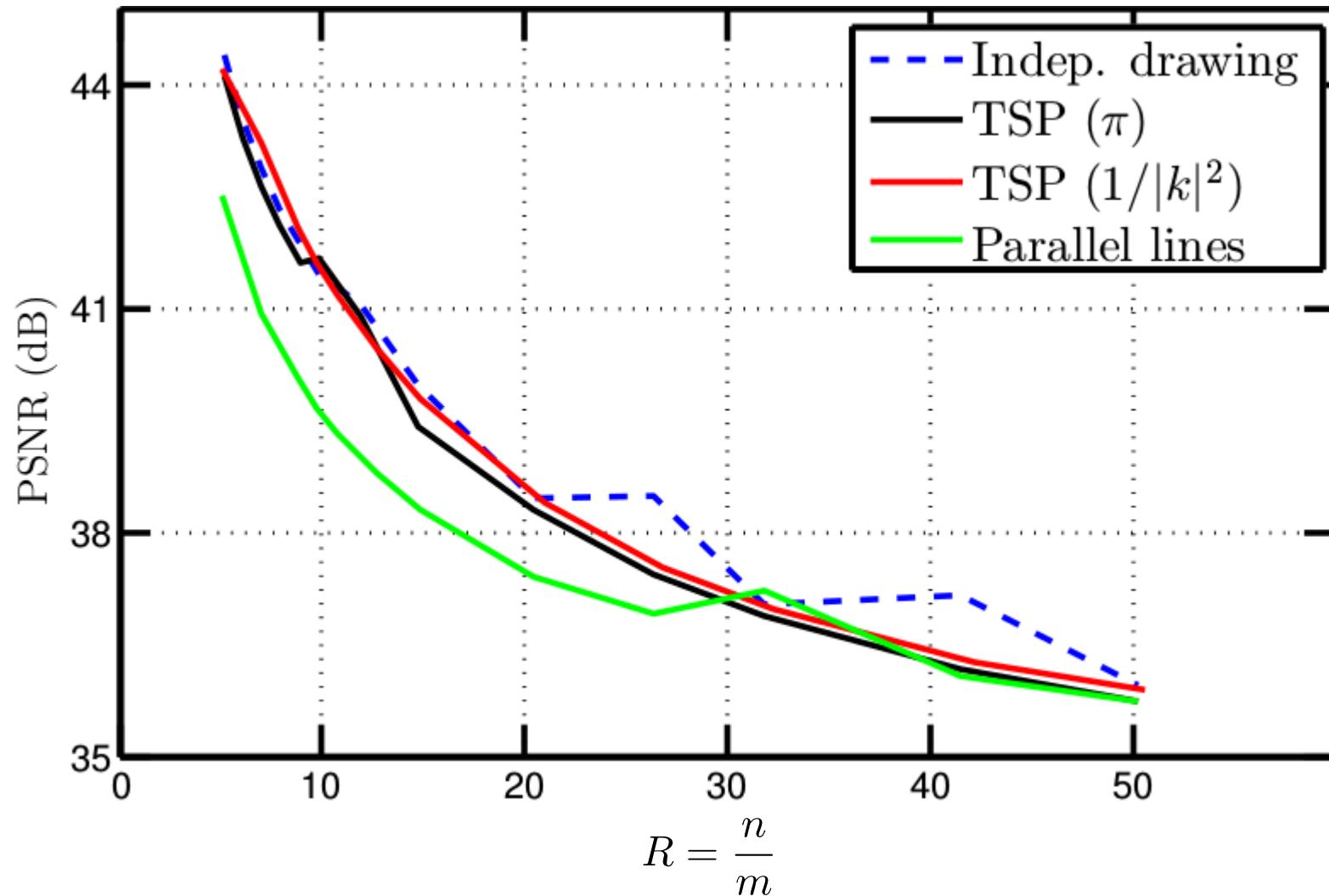
Figure: 3D reconstruction results for $r = 8.8$ for various sampling strategies. **Top row:** TSP-based sampling schemes (PSNR=42.1 dB). **Bottom row:** 2D random drawing and acquisitions along parallel lines [Lustig et al., 2007] (PSNR=40.1 dB).

Nicolas Chauffert, Philippe Ciuciu, Jonas Kahn, and Pierre Weiss.

Variable density sampling with continuous sampling trajectories.

SIAM Journal on Imaging Science, 2014.

The Traveling Salesman Sampler in 3D



Interim Summary (3)

- **Heuristic conclusions**

The main features characterizing the efficiency of a given sampler are:

- Its empirical (pushforward) measure/density
- Its mixing time, i.e. its ability to cover k-space rapidly
- No CS result wrt the number of measurements for exact recovery!

- **The new quest ...**

Design a variable density sampler with:

- 1) Prescribed density
 - 2) Good mixing properties
- Belonging to a certain class, i.e. being compliant with MR hardware constraints and a given imaging contrast (e.g. T_1 , T_2 , etc)

Back to MRI constraints

MRI constraints:

An **admissible sampling curve in MRI** is a curve belonging either to the set:

$$\mathcal{S}_{\text{MRI}} = \left\{ \mathbf{k} : [0, T] \mapsto \mathbb{R}^2, \|\dot{\mathbf{k}}\|_\infty \leq \alpha, \|\ddot{\mathbf{k}}\|_\infty \leq \beta \right\}$$

or

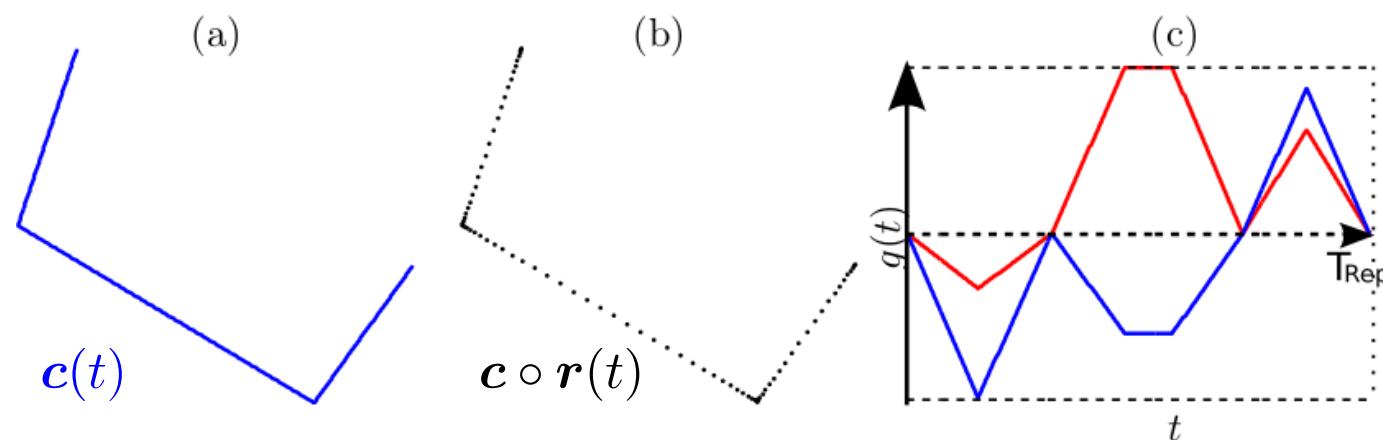
$$\mathcal{S}_{\text{MRI}} = \left\{ \mathbf{k} : [0, T] \mapsto \mathbb{R}^2, \|\dot{\mathbf{k}}\|_{2,\infty} \leq \alpha, \|\ddot{\mathbf{k}}\|_{2,\infty} \leq \beta \right\}$$

depending on the scanner.

Finding a parameterization in \mathcal{S}_{MRI} corresponding to a curve support is not easy !

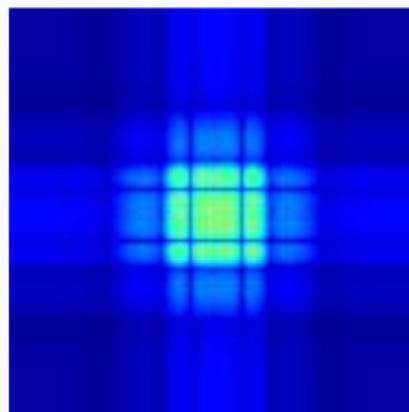
- Classical approach, find an admissible parameterization [Hargreaves et al., 2004, Lustig et al., 2008]:

$$T_{\text{rep}} = \min T' \text{ such that } \exists \mathbf{r} : [0, T'] \mapsto [0, T], \mathbf{c} \circ \mathbf{r}(t) \in \mathcal{S}_{\text{MRI}}$$



Issues with reparametrization

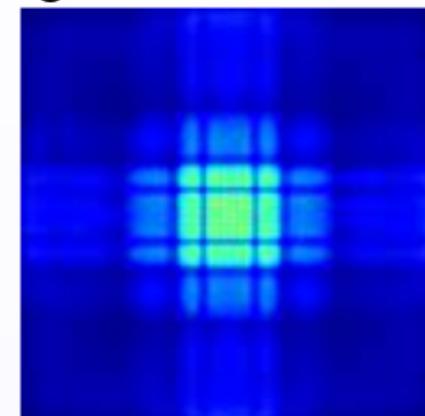
Monte Carlo study over 4000 independent drawings:



target distribution



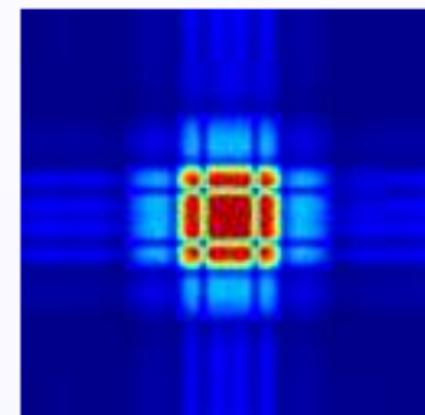
constant speed



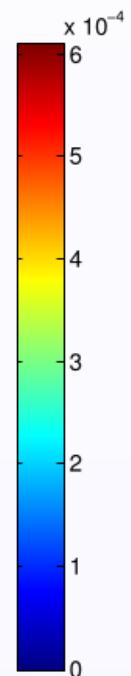
empirical distribution



reparameterization

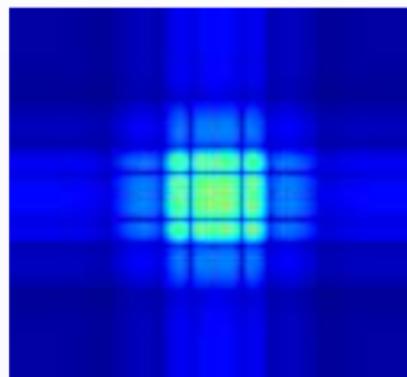


empirical distribution



Issues with reparametrization

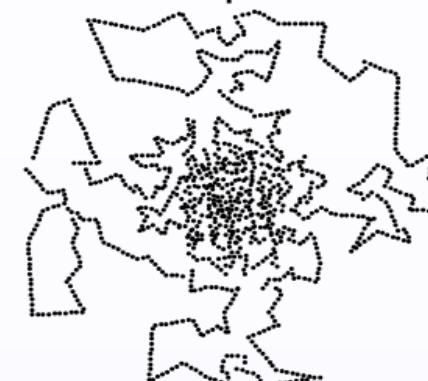
Monte Carlo study over 4000 independent drawings:



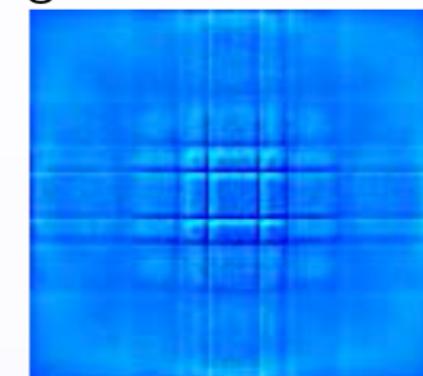
target distribution

Normalized error

$$\frac{\sum_i (\pi(\mathbf{k}(i)) - \hat{\pi}(\mathbf{k}(i)))^2}{\sum_i \pi(\mathbf{k}(i))^2}$$



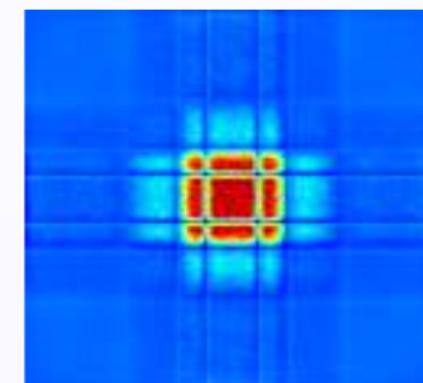
constant speed



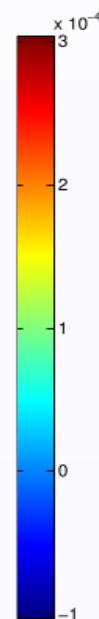
normalized error: 14%



reparameterization
[Lustig et al., 2008]

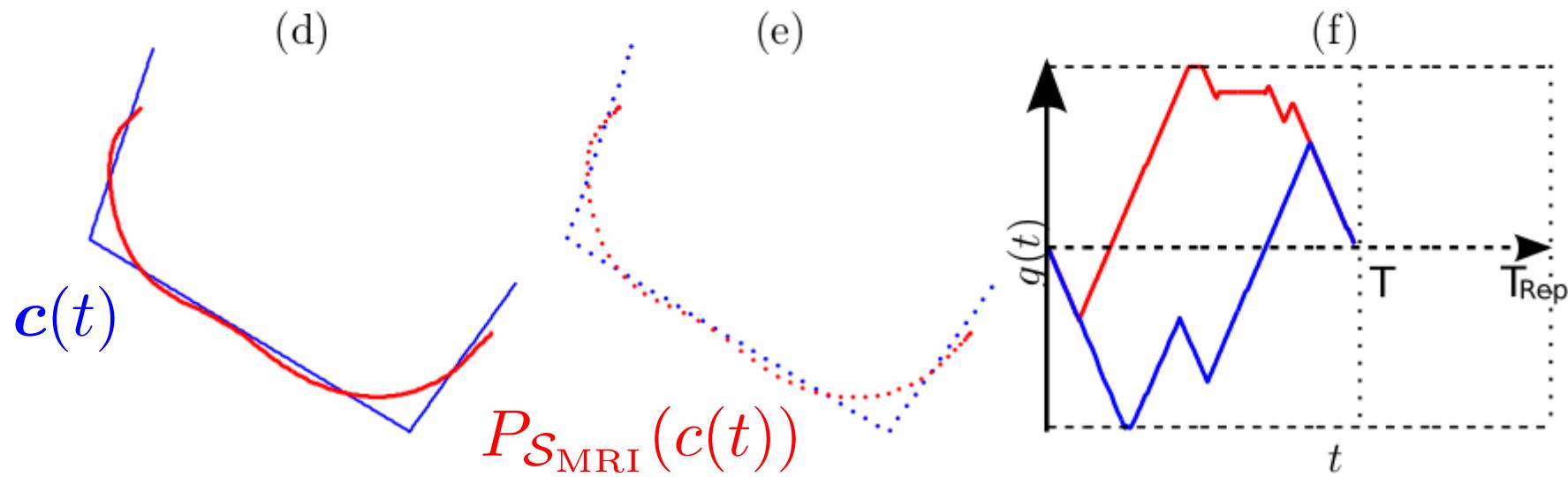


normalized error: 60%



Projection onto the MRI constraints

- Projection onto \mathcal{S}_{MRI} [Chauffert et al, IEEE TMI 2016]



➡ Leave the support of c but keep control on the sampling density

Go to Matlab code:
[Gradient_waveform_design](#)

The Projection Algorithm

For an input parameterized curve $c : [0, T] \mapsto \Omega$, define:

$$P_{\mathcal{S}_{\text{MRI}}}(c) = \arg \min_{k \in \mathcal{S}_{\text{MRI}}} \int_0^T (\mathbf{k}(t) - \mathbf{c}(t))^2$$

Main properties

- Fast resolution using accelerated proximal gradient descent on the dual.
- The sampling time is fixed (equal to T).
- The sampling distribution is well preserved (approximation of Wasserstein distance \mathcal{W}_2).



Leave the support of c but keep control on the sampling density

Mathematical details: Discretization

- A discrete curve $\mathbf{k} = \begin{pmatrix} \mathbf{k}[1] \\ \vdots \\ \mathbf{k}[n] \end{pmatrix}$ defined as a vector in $\mathbb{R}^{n \cdot d}$.
- $\mathbf{k}[i] \in \mathbb{R}^d$ denotes the curve location at time $(i - 1)\delta t$ with $\delta t = \frac{T}{n-1}$ and $i \in \{1, \dots, n\}$.
- The discrete derivative $\dot{\mathbf{k}} \in \mathbb{R}^{nd}$ is defined using first order differences:

$$\dot{\mathbf{k}}[i] = \begin{cases} 0 & \text{if } i = 1 \\ (\mathbf{k}[i] - \mathbf{k}[i - 1])/\delta t & \text{if } i \in \{2, \dots, n\} \end{cases}$$

- Let $\mathbf{A}_1 \in \mathbb{R}^{nd \times nd}$ be such that $\dot{\mathbf{k}} = \mathbf{A}_1 \mathbf{k}$
- We define the discrete second order differential operator by $\mathbf{A}_2 = -\mathbf{A}_1^* \mathbf{A}_1 \in \mathbb{R}^{nd \times nd}$
- More generally, the m -th order differential operator will be denoted \mathbf{A}_m

Objective function

[Chauffert et al, IEEE TMI 2016]

Constraint set

$$\mathcal{S} = \{\mathbf{k} \in \mathbb{R}^{nd}, \|\mathbf{A}_j \mathbf{k}\|_{N_j} \leq \alpha_j, j = 1, \dots, J\}$$

Input: $\mathbf{k}_0 \in \mathbb{R}^{nd}$.

Problem: find

$$\hat{\mathbf{k}} = \underset{\mathbf{k} \in \mathcal{S}}{\operatorname{Arg\,min}} \frac{1}{2} \|\mathbf{k} - \mathbf{k}_0\|_2^2$$

Using **Fenchel duality**:

$$\begin{aligned} \min_{\mathbf{k} \in \mathcal{S}} \frac{1}{2} \|\mathbf{k} - \mathbf{k}_0\|_2^2 &= \min_{\mathbf{k} \in \mathbb{R}^{nd}} \frac{1}{2} \|\mathbf{k} - \mathbf{k}_0\|_2^2 + \sum_{j=1}^J \sup_{\mathbf{k}_j^* \in \mathbb{R}^{nd}} \langle \mathbf{A}_j \mathbf{k}, \mathbf{k}_j^* \rangle - \alpha_j \|\mathbf{k}_j^*\|_{N_j^*} \\ &= \sup_{\mathbf{k}_j^* \in \mathbb{R}^{nd}} -\alpha_j \|\mathbf{k}_j^*\|_{N_j^*} + \underbrace{\min_{\mathbf{k} \in \mathbb{R}^{nd}} \left(\frac{1}{2} \|\mathbf{k} - \mathbf{k}_0\|_2^2 + \sum_{j=1}^J \langle \mathbf{A}_j \mathbf{k}, \mathbf{k}_j^* \rangle \right)}_{\text{red line}} \end{aligned}$$

We have the duality relation:

$$\mathbf{k} = \mathbf{k}_0 - \sum_{j=1}^K \mathbf{A}_j^* \mathbf{k}_j^*$$

Optimization in the dual space

The dual criterion to be minimized reads:

[Chauffert et al, IEEE TMI 2016]

$$\underset{\mathbf{k}^* = (\mathbf{k}_j^*)}{\text{Arg min}} \underbrace{\frac{1}{2} \left\| \sum_{j=1}^J \mathbf{A}_j^* \mathbf{k}_j^* \right\|_2^2 - \sum_{j=1}^J \langle \mathbf{A}_j^* \mathbf{k}_j^*, \mathbf{k}_0 \rangle}_{g(\mathbf{k}^*)} + \underbrace{\sum_{j=1}^J \alpha_j \|\mathbf{k}_j^*\|_{N_j^*}}_{h(\mathbf{k}^*)}$$

- g is a smooth convex function with L -Lipschitz gradient.
- h is a non-smooth convex function.

Algorithm 2 Accelerated proximal gradient method [Nesterov, 1983]

Initialize $\mathbf{k}_j^{(0)}, \mathbf{y}_j^{(0)}$, for $1 \leq j \leq J$.

Set $\eta = 1/L$.

for $n = 1, \dots, n_{\text{iter}}$ **do**

$$\mathbf{k}_j^{(n)} = \text{Prox}_{\mu h_j} \left(\mathbf{y}_j^{(n-1)} - \nu \nabla_j g(\mathbf{y}^{(n-1)}) \right)$$

$$\mathbf{y}_j^{(n)} = \mathbf{k}_j^{(n)} + \frac{n-1}{n+2} \left(\mathbf{k}_j^{(n)} - \mathbf{k}_j^{(n-1)} \right)$$

return $\mathbf{k} = \mathbf{k}_0 - \sum_{j=1}^J \mathbf{A}_j^* \mathbf{k}_j^{(n_{\text{iter}})}$

The convergence rate of this algorithm is $\mathcal{O}(1/n^2)$.

Nesterov, Y. (1983).

A method of solving a convex programming problem with convergence rate $\mathcal{O}(1/k^2)$.

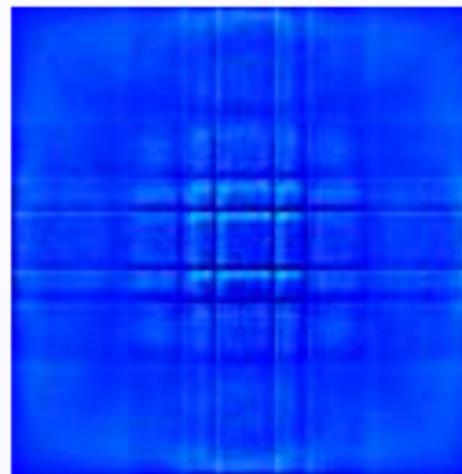
In Soviet Mathematics Doklady, volume 27, pages 372–376.

Sampling density of projected trajectories

[Chauffert et al, IEEE TMI 2016]



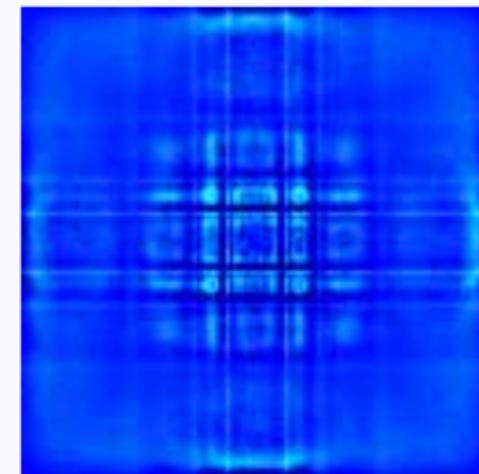
10% of max. speed



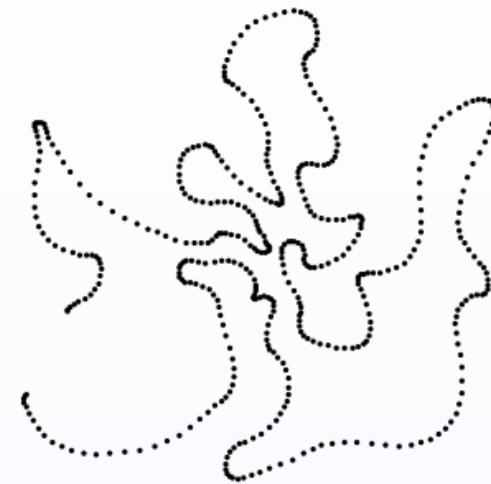
normalized error: 10%



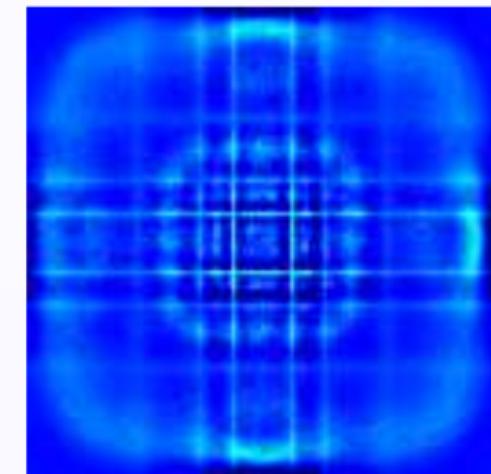
50%



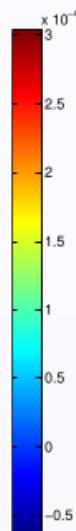
14%



100%



17%



For optimal control, error was 60%!

Illustration & Results

[Chauffert et al, IEEE ISBI 2015]

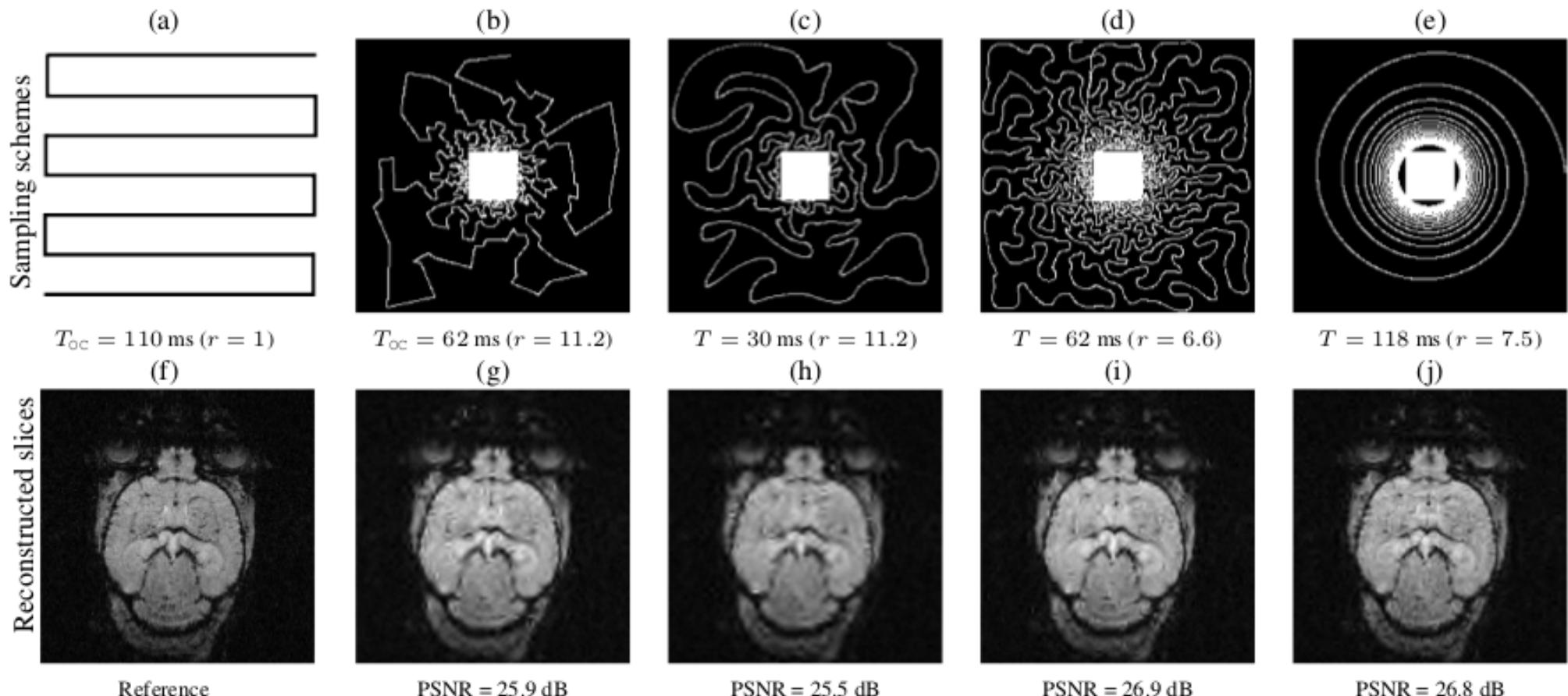
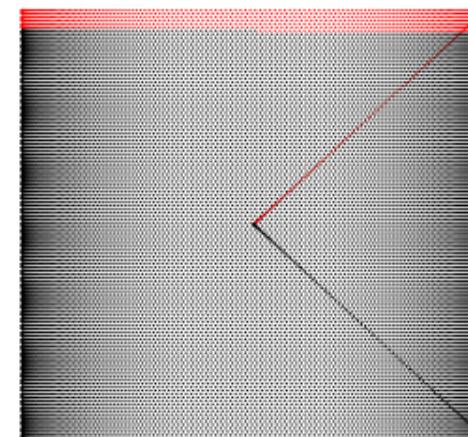


Illustration & Results

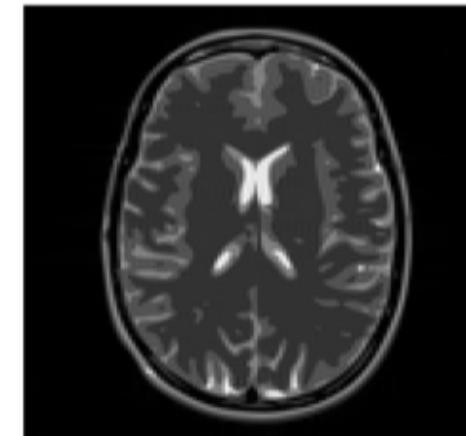
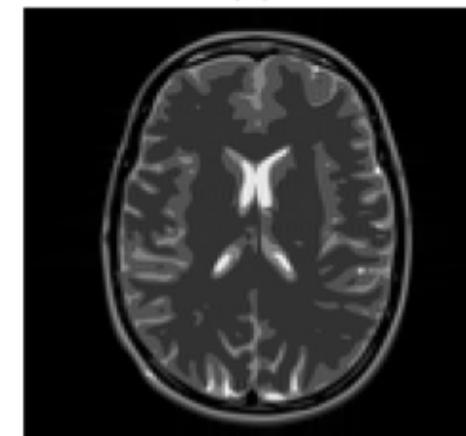
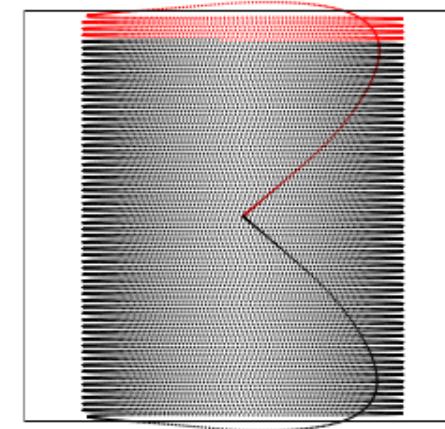
EPI trajectories

$T=89.6 \text{ ms}$



[Chauffert et al, IEEE TMI 2016]

$T=68.9 \text{ ms}$



Interim Summary (4)

- **Physically plausible retrospective CS-MRI**

Key ingredients:

- Variable (possibly optimal in a certain sense) density sampling
- Variable density sampler that meets the gradient constraints
- Fast projection algorithm based on convex optimization

- **But still ...**

- *we observed a discrepancy between the target and empirical densities ...*
- Need for improved mixing properties

- **Conclusion**

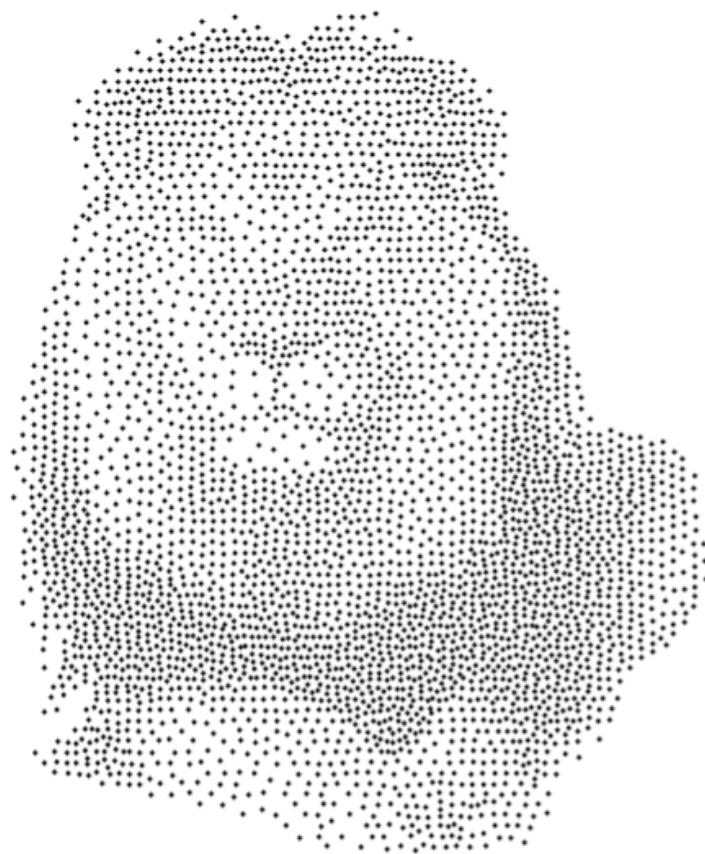
- Instead of a 2-step approach where you draw first iid samples and then seek the physically feasible trajectory that gets through these points, try to solve the two concerns in 1 step!

Outline

Part III: CS sampling trajectories

- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

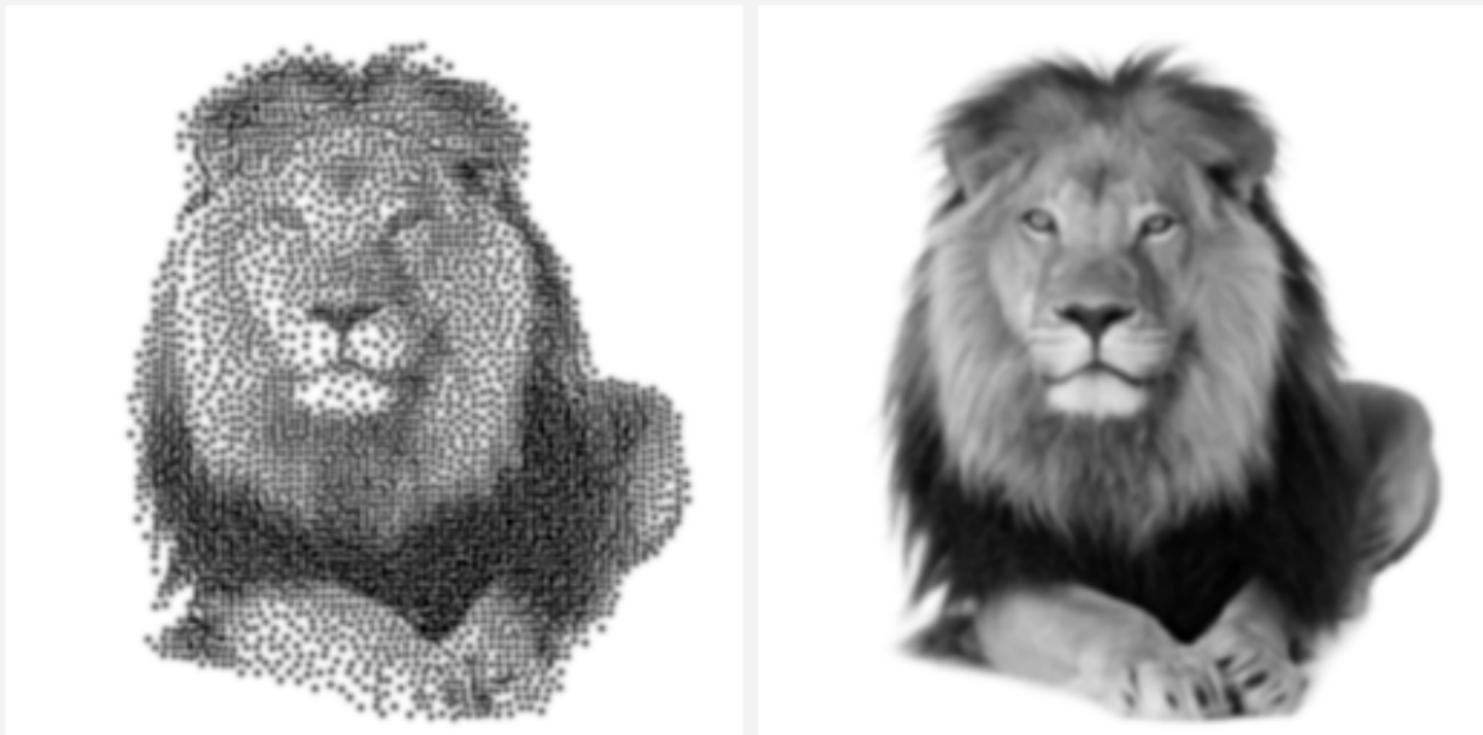
How to measure similarity between images?



Halftoned and reference images

Why are the images alike?

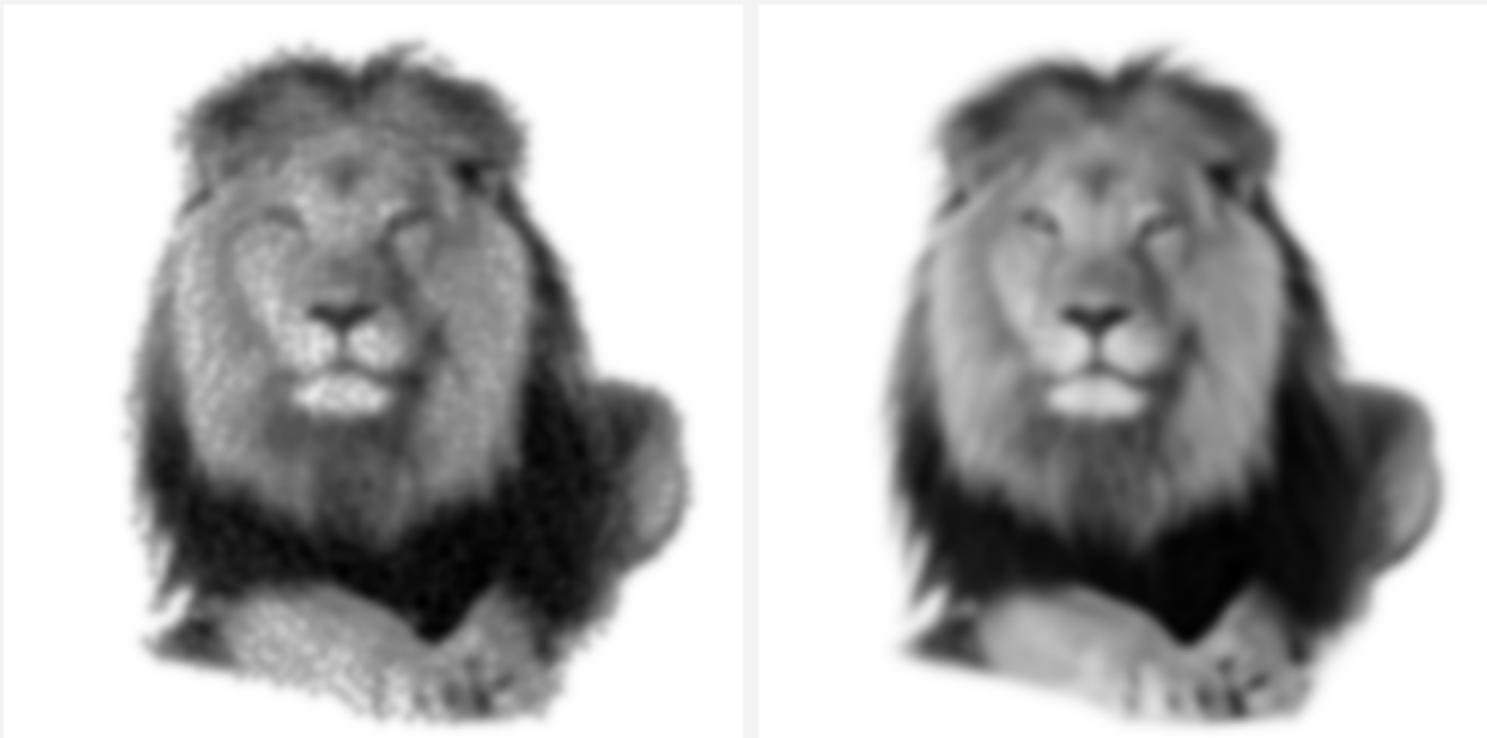
The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 1$.

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 2$.

Why are the images alike?

The multiresolution feature of the human visual system.



Convolution with a Gaussian $\sigma = 3$.



The images are similar only at **low resolution**

Constructing a metric

Let π denote the **target image** (the lion).

Let ν denote the **atomic measure** (the dots).

A natural distance to quantify image similarity reads:

$$\mathcal{D}_h(\nu - \pi) = \|h \star \nu - h \star \pi\|_2 .$$

The kernel h can be chosen as the impulse response of the visual system.

[Teuber et al, SIAM Imagin Sci, 2011]

[Chauffert et al, Construct Approx, 2017]

Halftoning as a projection problem

How to find the point locations?

Let $\Omega \subseteq \mathbb{R}^d$ denote the **image domain**.

Let $\mathcal{M}(\Omega^N)$ denote the **set of atomic measures** with N points.

$$\mathcal{M}(\Omega^N) = \left\{ \nu = \frac{1}{N} \sum_{i=1}^N \delta_{p_i}, p_i \in \Omega \right\}$$

The **halftoning problem** can be formulated as a projection problem:

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \mathcal{D}_h(\nu - \pi).$$

Link with Attraction-Repulsion

Proposition

The projection problem

$$\min_{\nu \in \mathcal{M}(\Omega^N)} \frac{1}{2} \|h \star \nu - h \star \pi\|_2^2$$

can be rewritten

$$\min_{(p_1, \dots, p_N) \in \Omega^N} \underbrace{\frac{1}{N} \sum_{i=1}^N \int_{\Omega} H(p_i - x) \pi(x) dx}_{\text{Attraction potential}} - \underbrace{\frac{1}{2N^2} \sum_{1 \leq i, j \leq N} H(p_i - p_j)}_{\text{Repulsion potential}}$$

where $\hat{H} = |\hat{h}|^2$.

Modeling MRI kinematic constraints

$$\min_{\mathbf{k} \in \mathcal{Q}_N} \frac{1}{2} \|h \star \nu(\mathbf{k}) - h \star \pi\|_2^2 \quad \text{with } \nu(\mathbf{k}) = \frac{1}{N} \sum_{i=1}^N \delta_{k[i]}, \quad \mathbf{k} = (k[i])_{1 \leq i \leq N} \in \mathcal{Q}_N$$

where $\mathcal{Q}_N = \{\|\dot{\mathbf{k}}\|_{2,\infty} \leq \gamma G_{\max}, \|\ddot{\mathbf{k}}\|_{2,\infty} \leq \gamma S_{\max}\}$

$$\underbrace{\min_{\mathbf{k} \in \mathcal{Q}_N} \frac{1}{N} \sum_{i=1}^N \int_{\Omega} H(x - k[i]) \pi(x) dx}_{F_a(\mathbf{k}) \atop \text{attraction}} - \underbrace{\frac{1}{2N^2} \sum_{1 \leq i, j \leq N} H(k[i] - k[j])}_{F_r(\mathbf{k}) \atop \text{repulsion}}$$

Projected gradient descent: $\mathbf{k}^{(t+1)} = P_{\mathcal{Q}_N}(\mathbf{k}^{(t)} - \tau^{(k)} \nabla(F_a - F_r)(\mathbf{k}^{(t)}))$

Attouch, H., Bolte, J., and Svaiter, B. F. (2013).

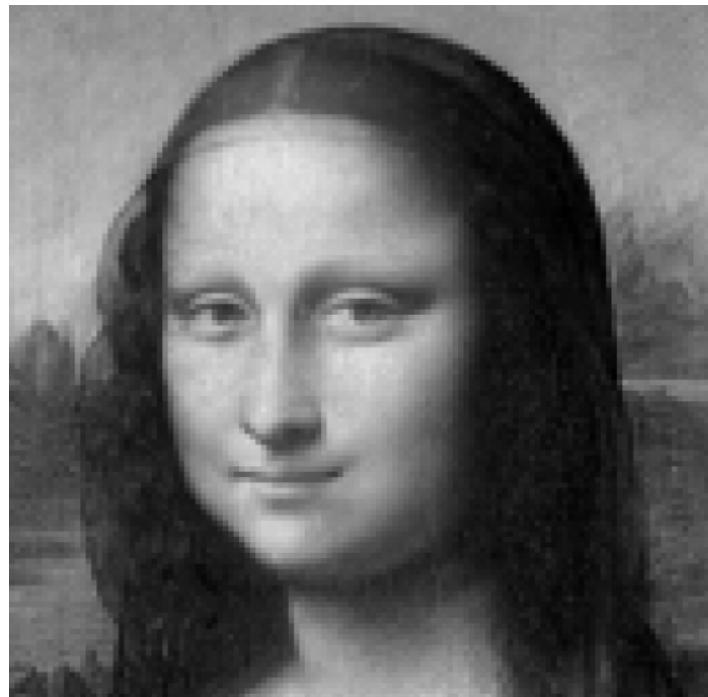
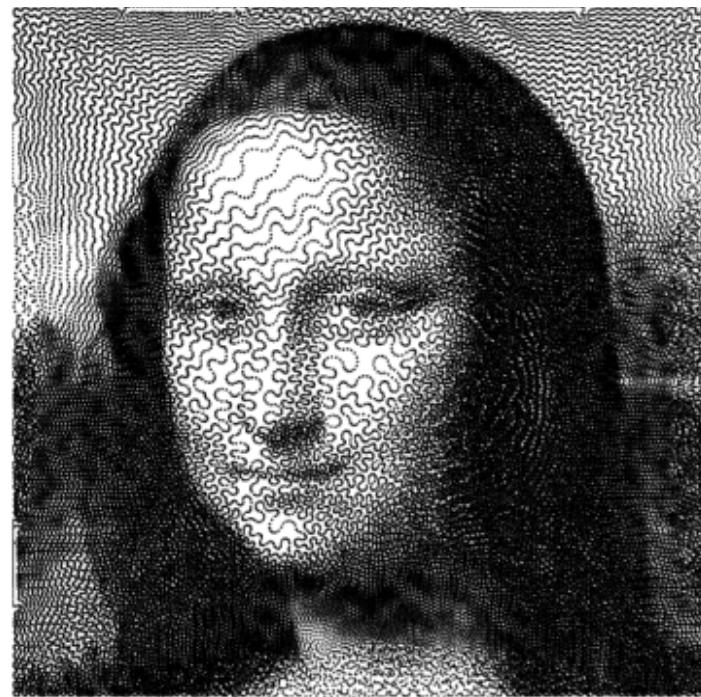
Convergence of descent methods for semi-algebraic and tame problems: proximal algorithms, forward-backward splitting, and regularized gauss-seidel methods.

Mathematical Programming, 137(1-2):91–129.

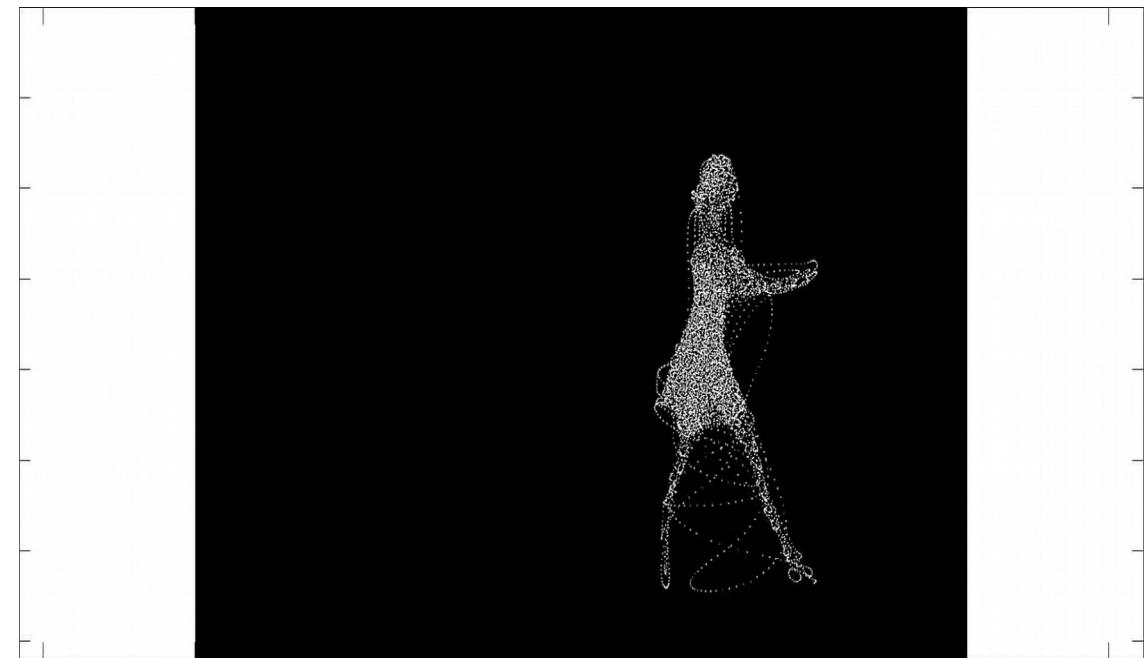


Inertial adaptive step-size (BB update)

Image stippling with projection onto MRI kinematic constraints


$$h_\sigma \star \pi$$

$$h_\sigma \star \nu$$

Video stippling with projection onto MRI kinematic constraints



Interim Summary (5)

- **Computer graphics approximation**
 - The target density is well approximated
 - The constraints are fulfilled
 - However the length of the trajectories is not compatible with MR
- **What is missing for MR sampling trajectories?**
 - Compatibility with realistic readout T_{obs}
 - Clear modeling of imaging contrast: affine constraints for TE
 - Better control of sampling over each shot

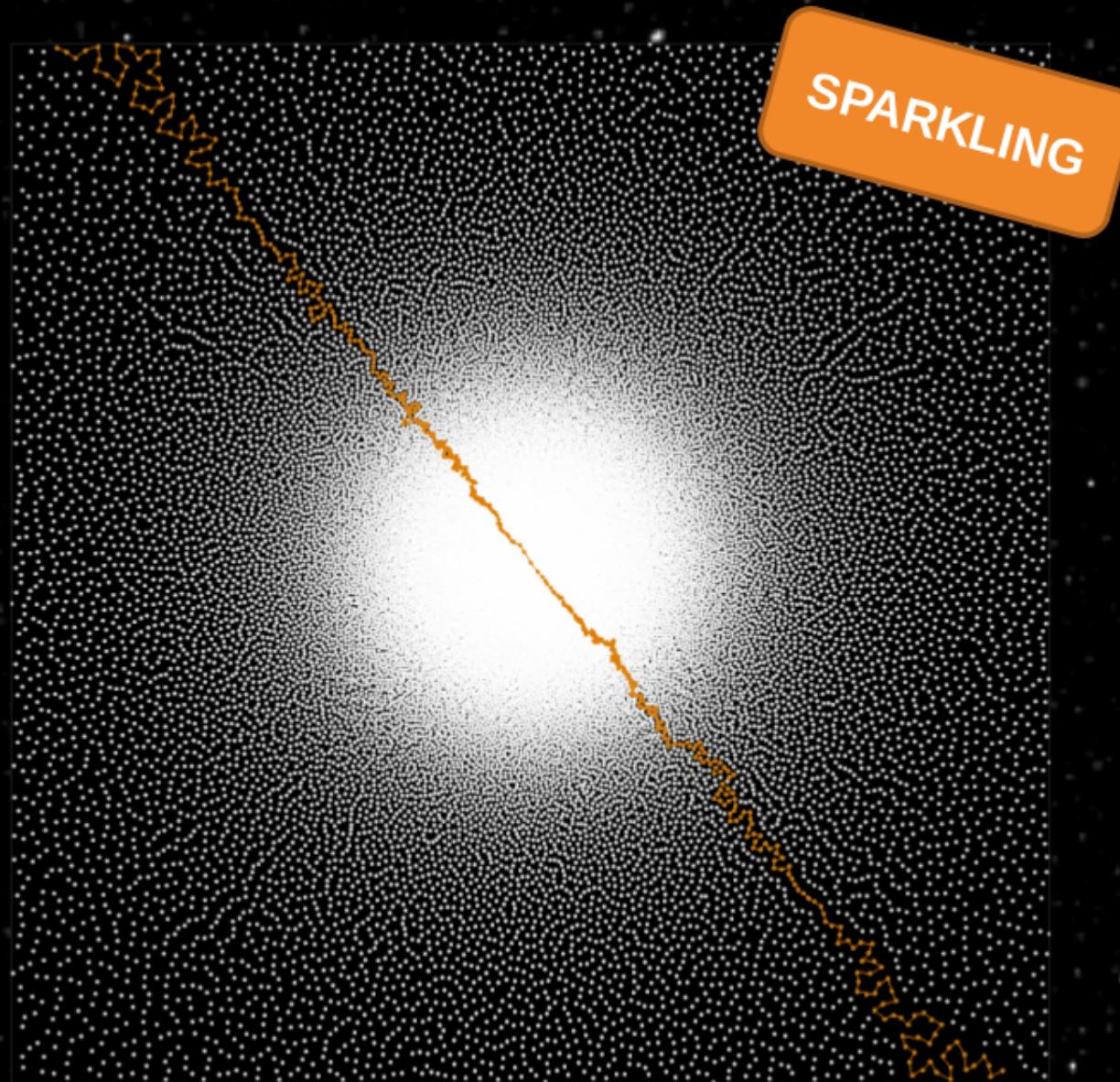
Outline

Part III: CS sampling trajectories

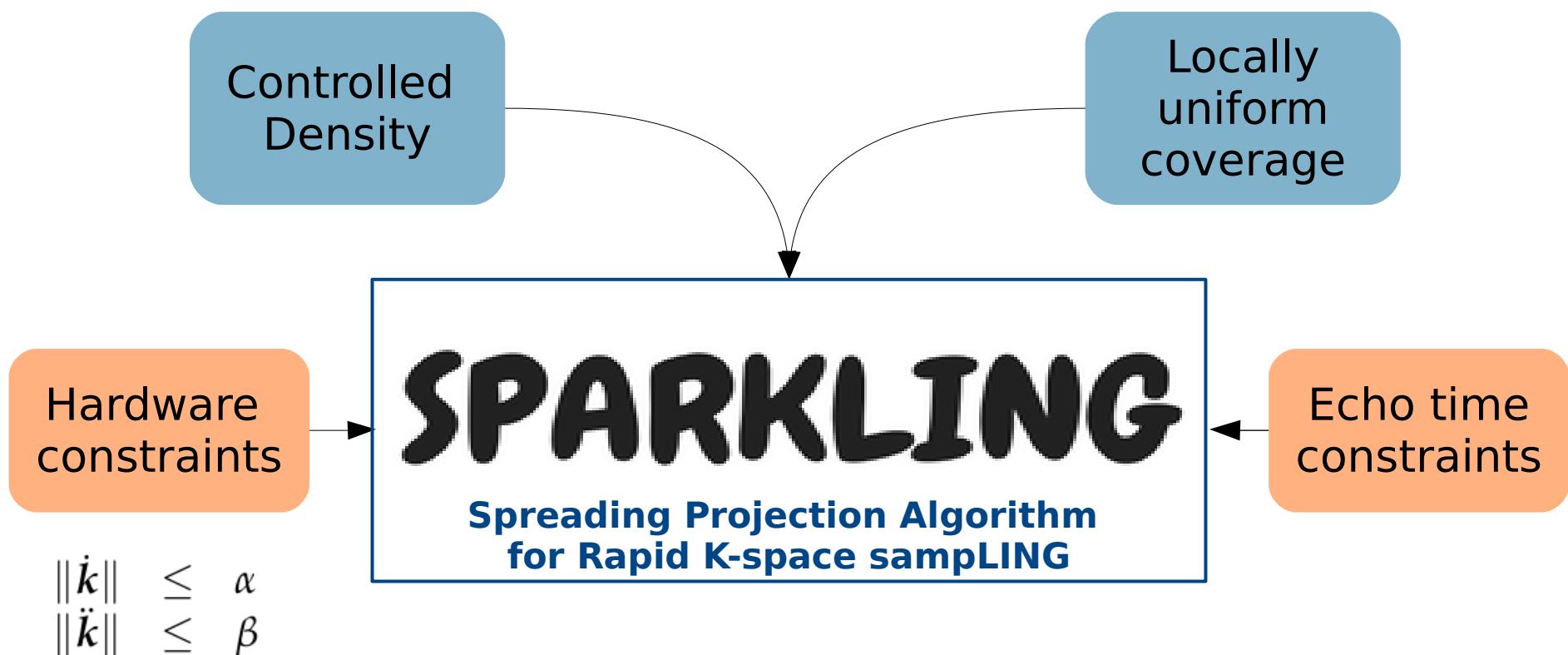
- Continuous Variable Density Samplers
- Image stippling techniques
- SPARKLING

SPARKLING

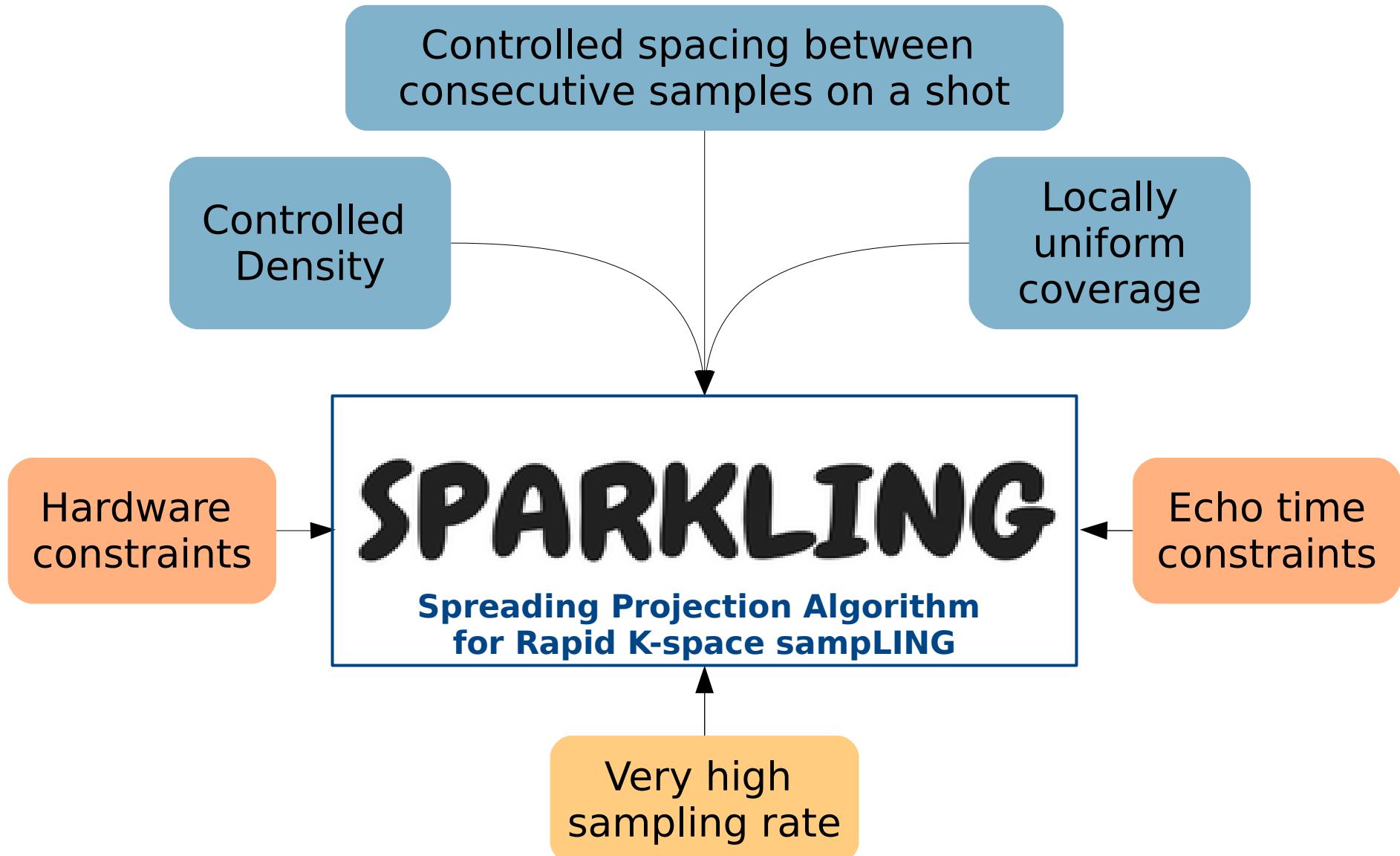
Spreading Projection Algorithm for Rapid K-space sampLING



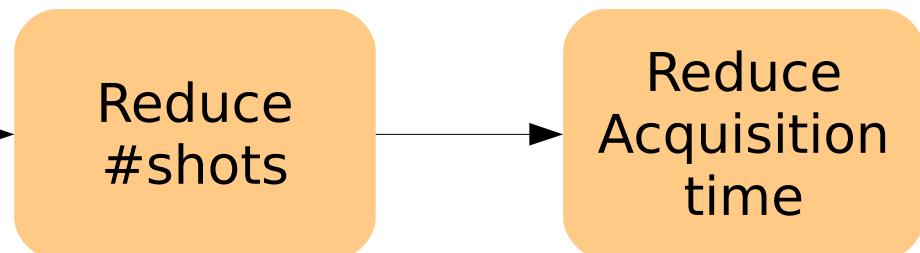
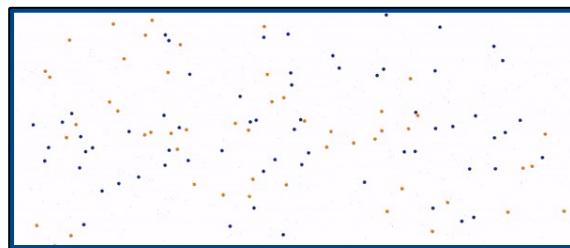
Sparkling recipe



Sparkling recipe



Sparkling acceleration



Acceleration Factor AF = N/ns

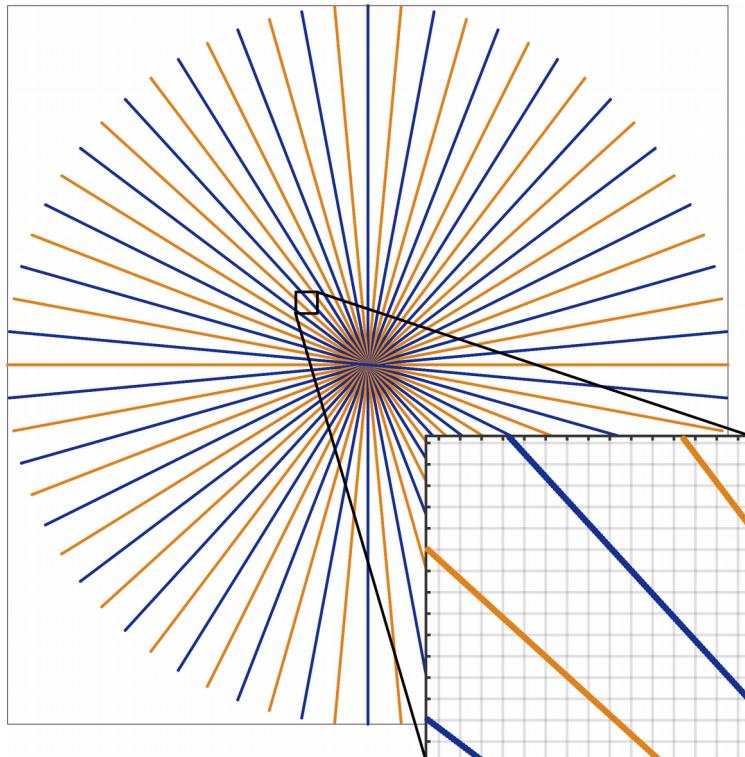
Subsampling Factor R = n/m

(with $n=N \times N = \# \text{pixels}$, $ns = \# \text{shots}$ and $m = \# \text{samples}$)

$$\mathbf{AF} \geq \mathbf{R}$$

Radial VDS Sparkling

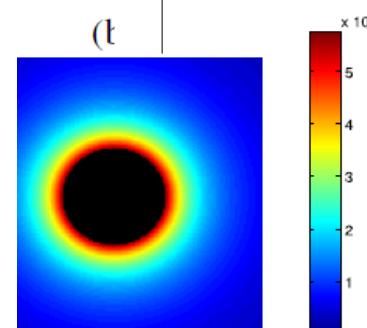
Input



$T_{obs} = 30 \text{ ms}$

$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

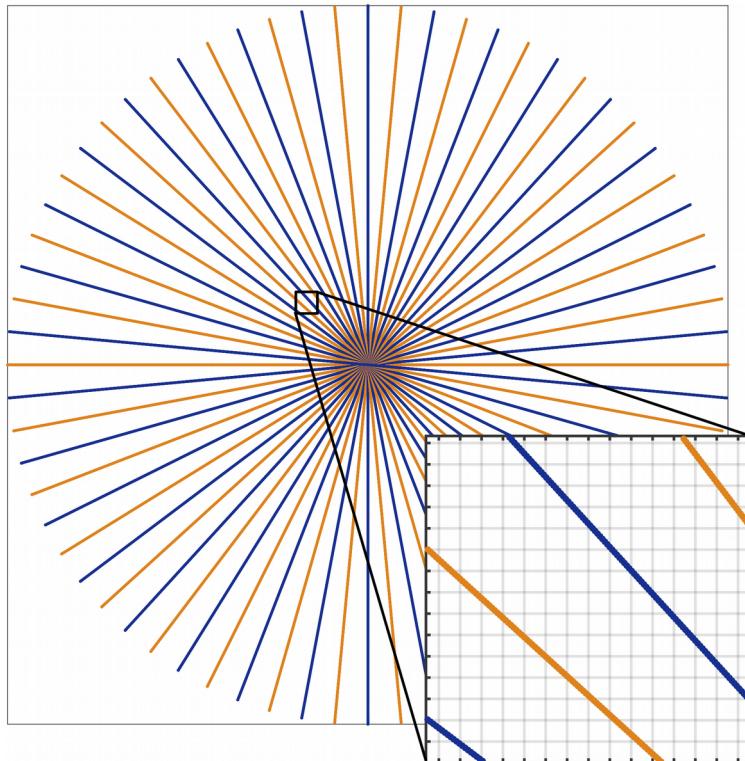
SPARKLING



Acceleration factor in time:
 $AF = N/nc = 512/34 = 15$

Radial VDS Sparkling

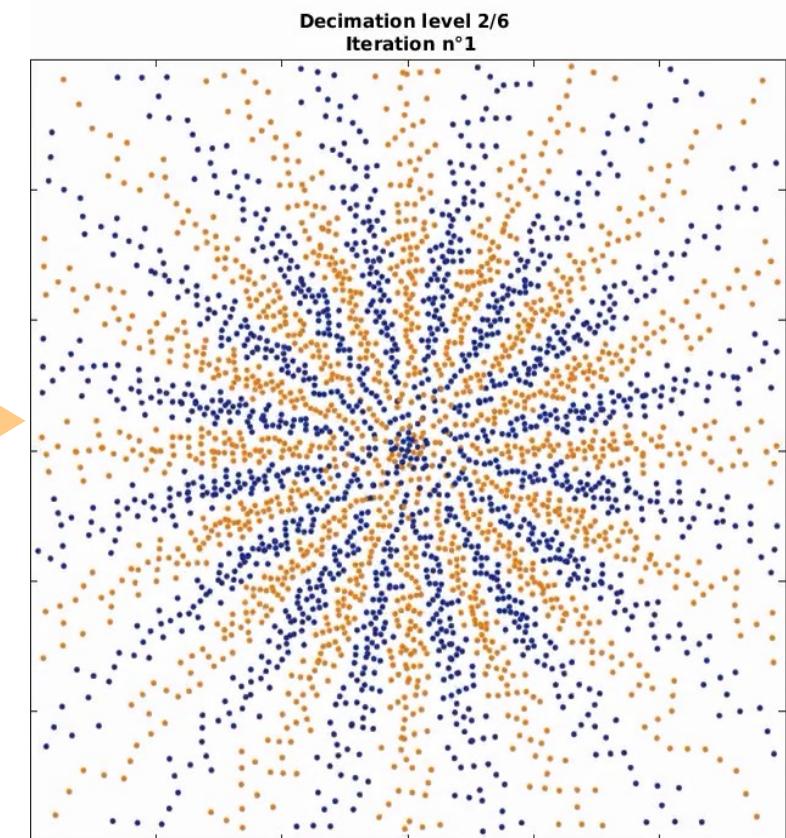
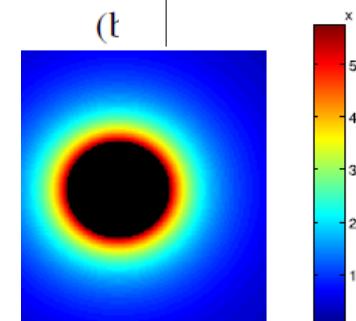
Input



$T_{obs} = 30 \text{ ms}$

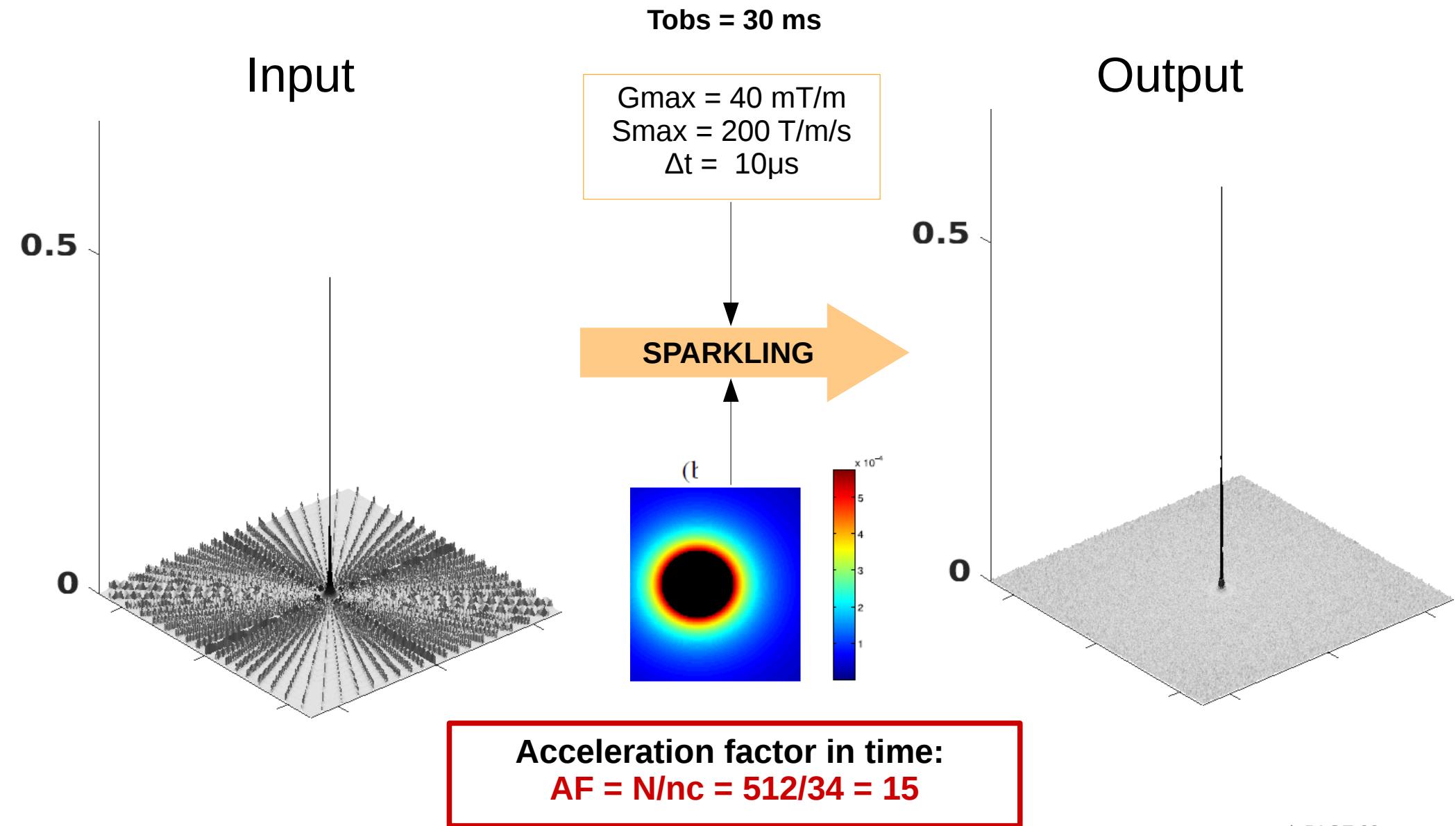
$G_{\max} = 40 \text{ mT/m}$
 $S_{\max} = 200 \text{ T/m/s}$
 $\Delta t = 10 \mu\text{s}$

SPARKLING



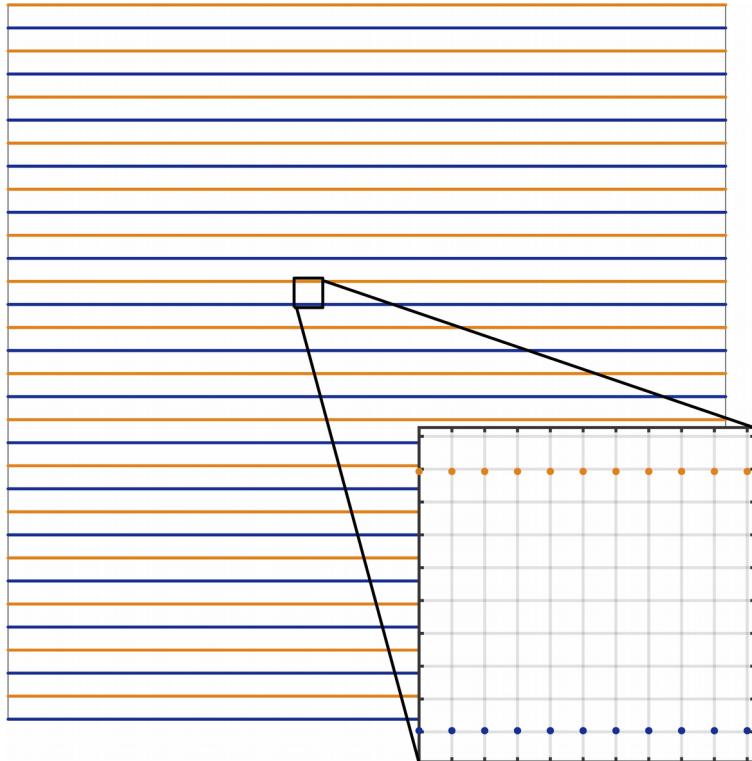
Acceleration factor in time:
 $\text{AF} = N/n_c = 512/34 = 15$

Radial VDS Sparkling



Uniform Sparkling

Input



$T_{obs} = 20 \text{ ms}$

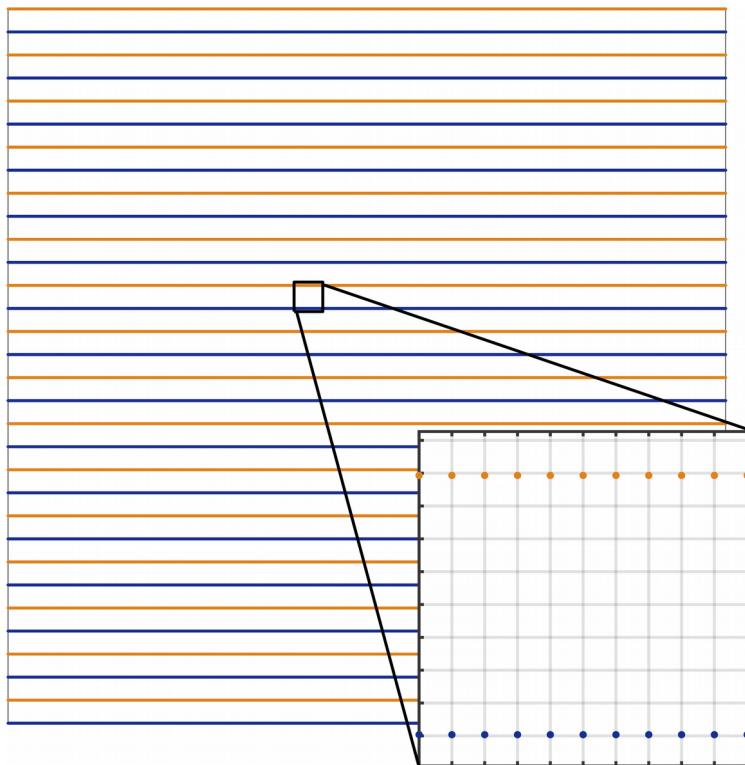
$G_{max} = 40 \text{ mT/m}$
 $S_{max} = 200 \text{ T/m/s}$
 $\Delta t = 10\mu\text{s}$

SPARKLING

Acceleration factor in time:
 $AF = N/n_c = 256/32 = 8$

Uniform Sparkling

Input

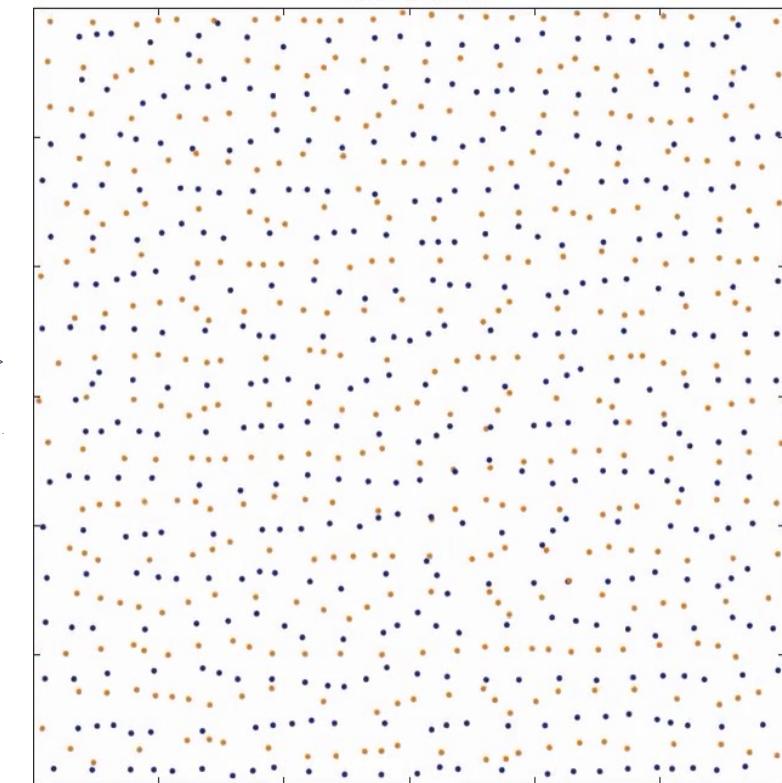


$T_{obs} = 20 \text{ ms}$

$G_{\max} = 40 \text{ mT/m}$
 $S_{\max} = 200 \text{ T/m/s}$
 $\Delta t = 10 \mu\text{s}$

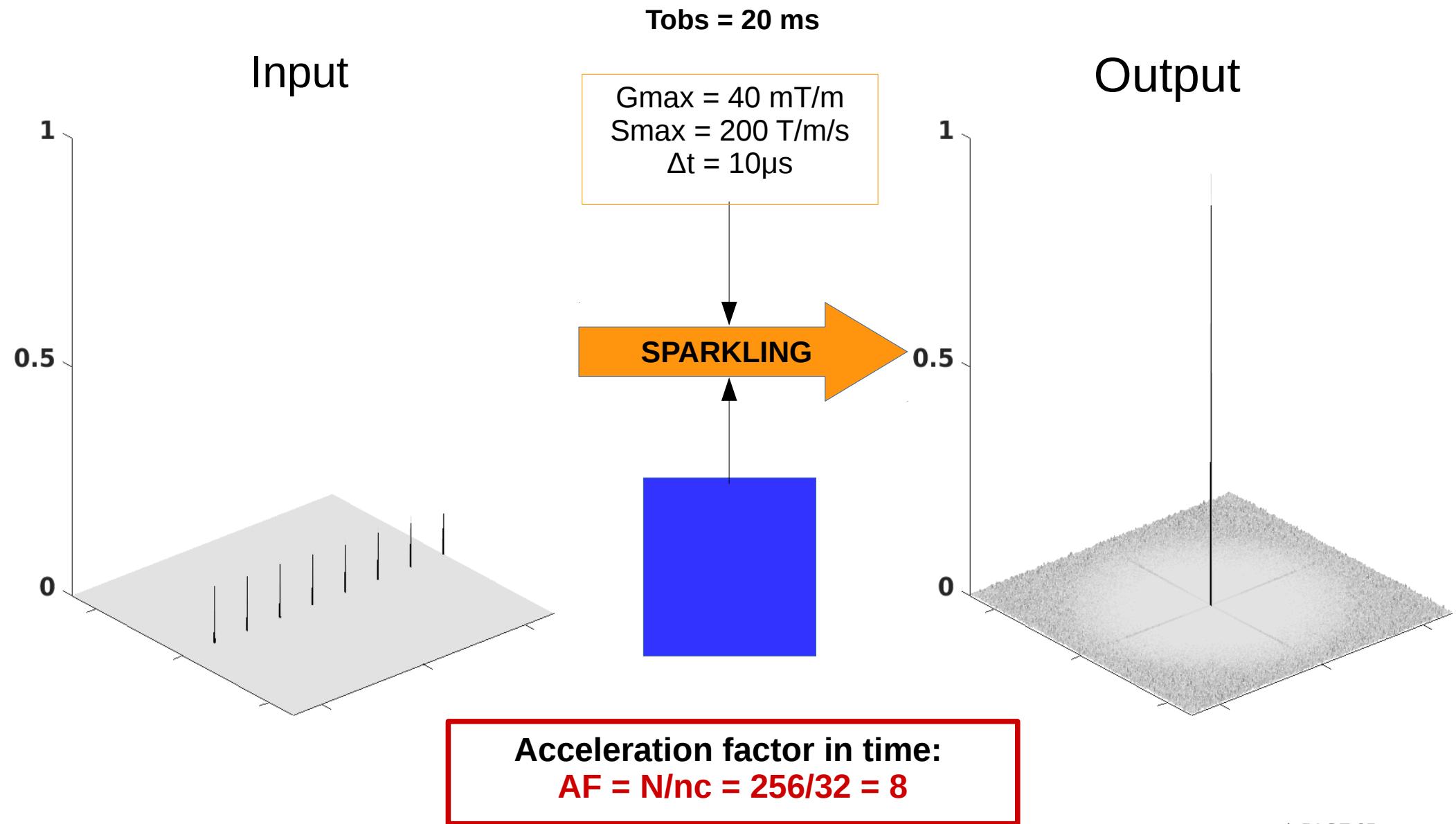
SPARKLING

Decimation level 2/9
Iteration n°12



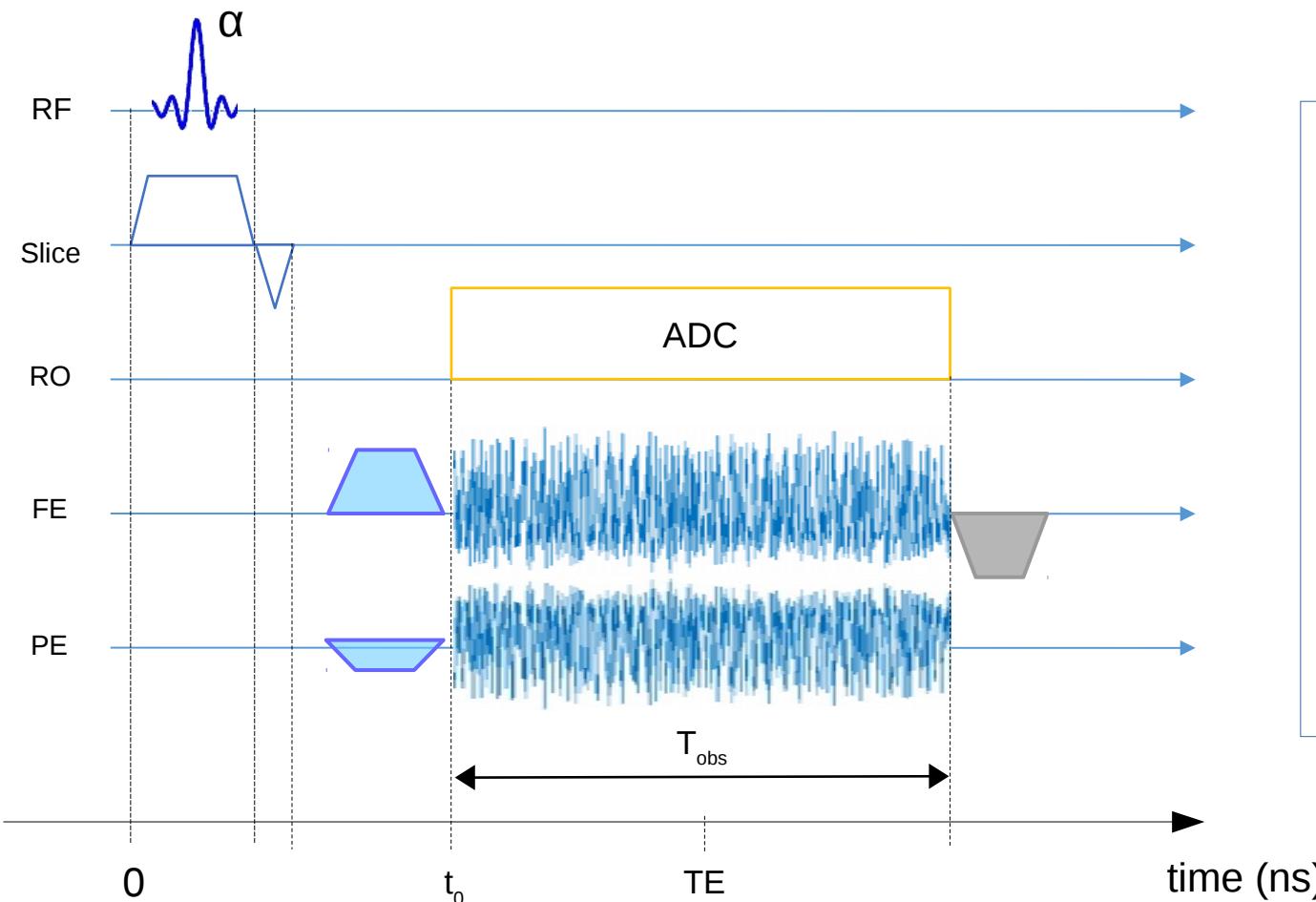
Acceleration factor in time:
 $AF = N/n_c = 256/32 = 8$

Uniform Sparkling



Sequence Implementation

- T_2^* -weighted Gradient-recalled echo sequence (GRE)
- Based on Siemens FLASH sequence

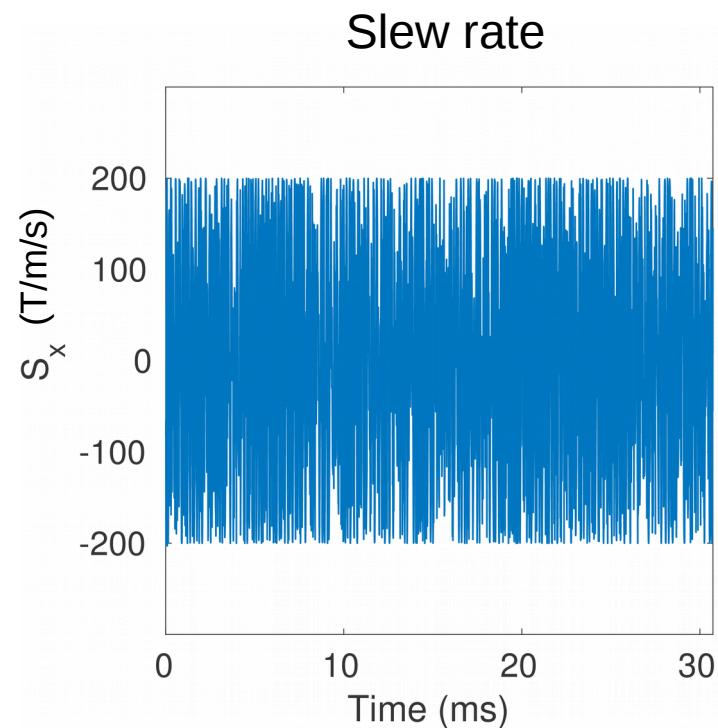
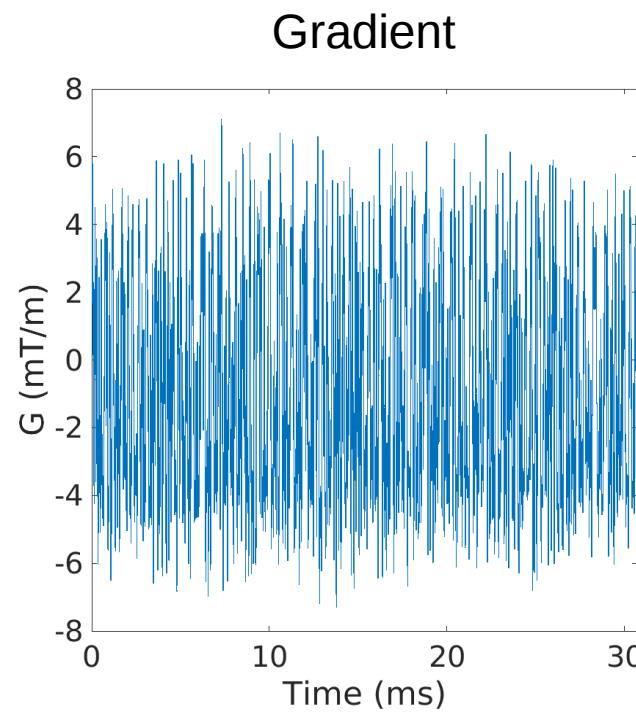


Sequence parameters:

- $N=512$
- $FOV=200 \times 200 \text{ mm}^2$
- $TR=550 \text{ ms} \rightarrow \text{interleaved}$
- $TE=30 \text{ ms}$
- $\alpha=25^\circ$
- $BW=32.55 \text{ Hz/px}$
- $T_{obs}=30.72 \text{ ms}$
- Slice thickness: 3 mm
- 32-channel receiver coil
- Gradient raster time = 10 μs

Sequence Implementation

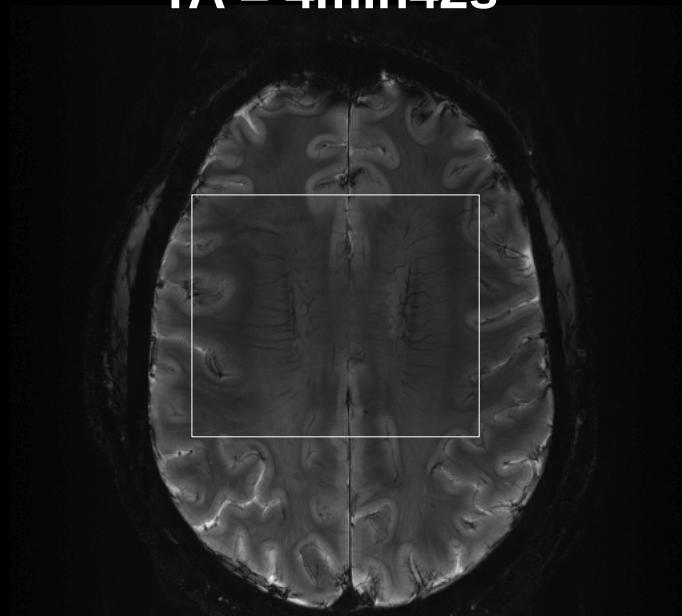
Siemens 7 Tesla scanner



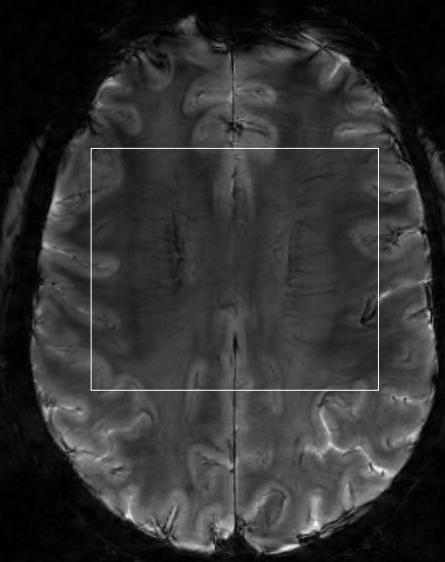
In vivo results: 0.39 mm – 11 slices of 3-mm thickness

Subject 1

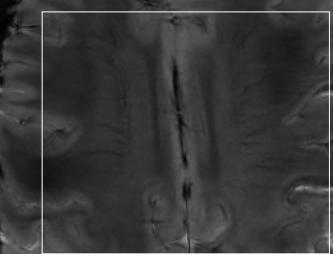
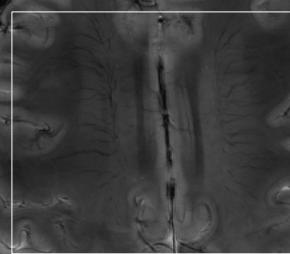
REFERENCE
TA = 4min42s



In-out SPARKLING
TA = 18s (AF=15)



Subject 2

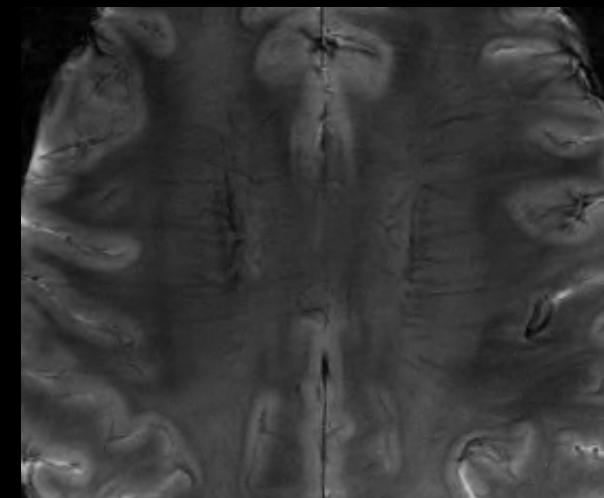
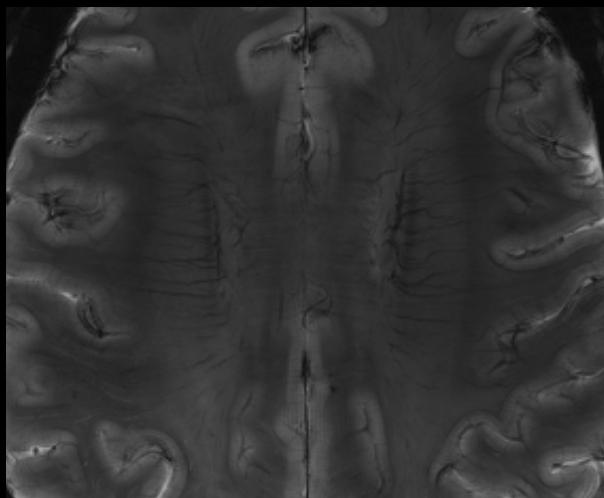


In vivo results: 0.39 mm – 11 slices of 3-mm thickness

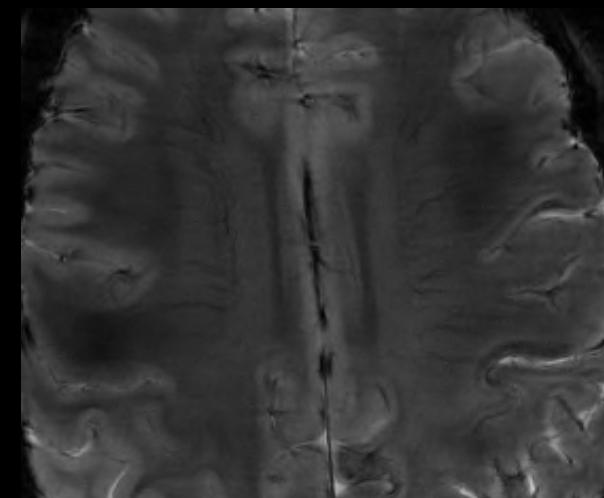
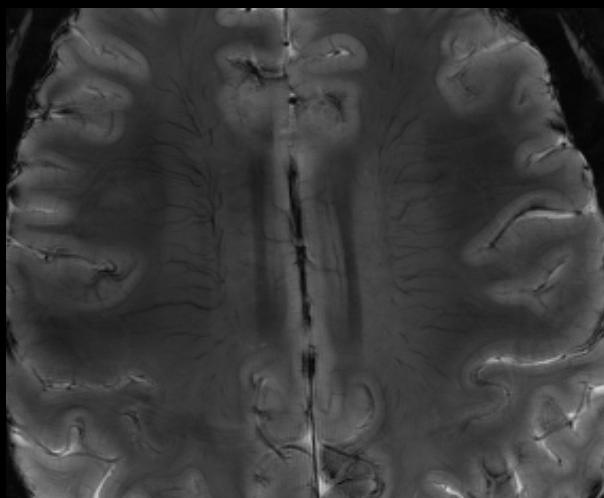
REFERENCE
 $TA = 4 \text{ min}42\text{s}$

In-out SPARKLING
 $TA = 18\text{s (AF=15)}$

Subject 1



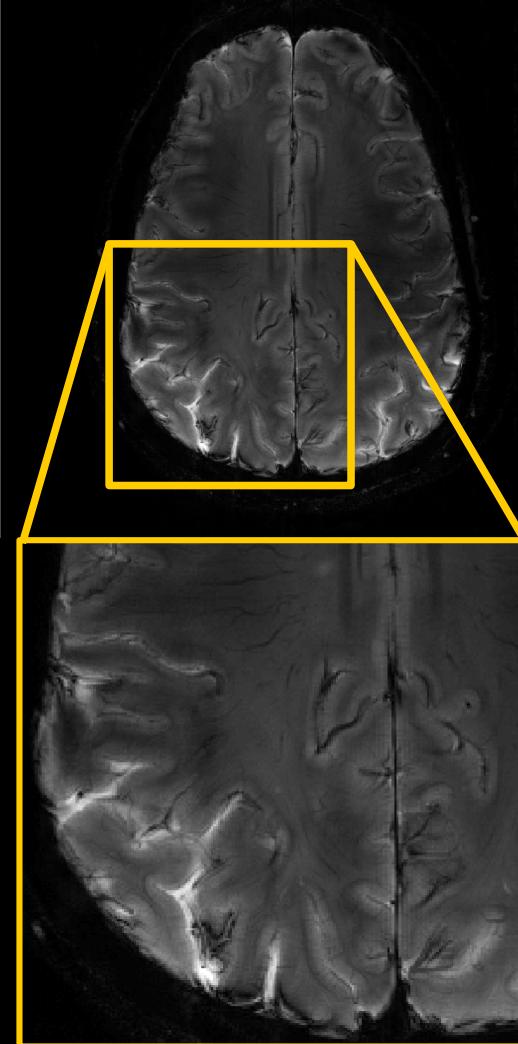
Subject 2



In vivo results at 0.39mm resolution 26 shots – 11 slices

REFERENCE

TA=4min42s



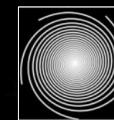
In-out SPARKLING



AF=20

TA=14s

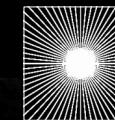
In-out SPIRAL



AF=20

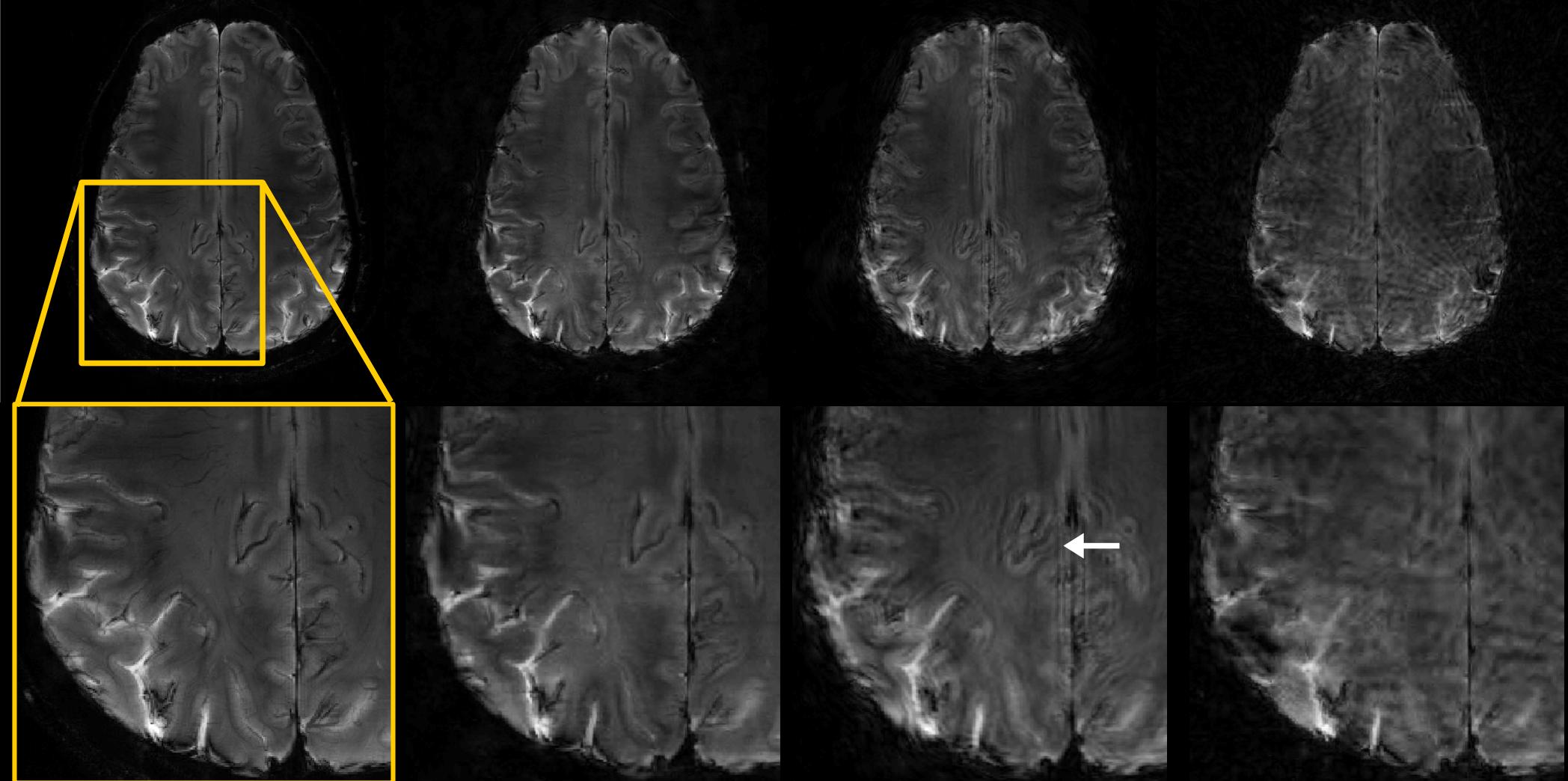
TA=14s

In-out RADIAL



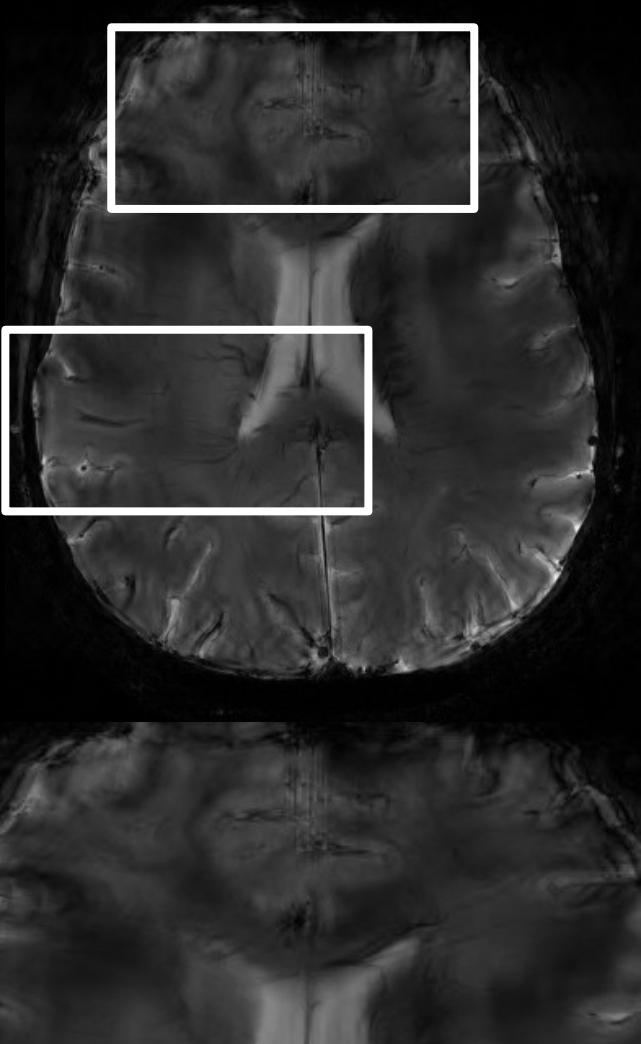
AF=20

TA=14s

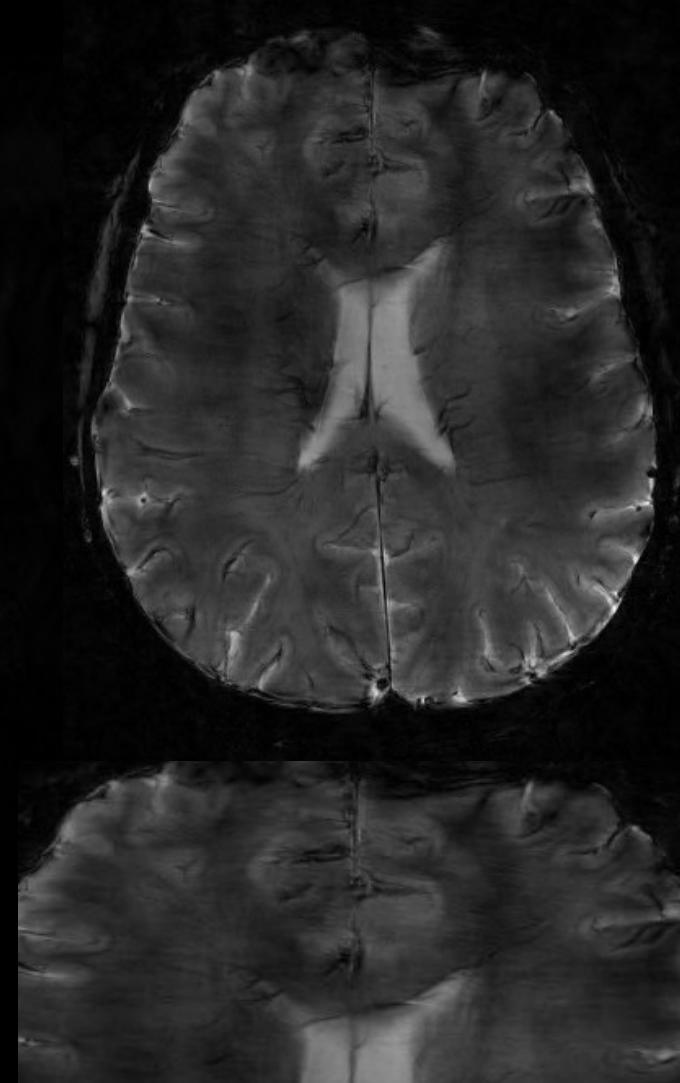


In vivo results at 0.39mm – 43 shots – 11 slices – BW=400kHz – TA=23s

**Segmented EPI
(fully-sampled)**



In-out Sparkling

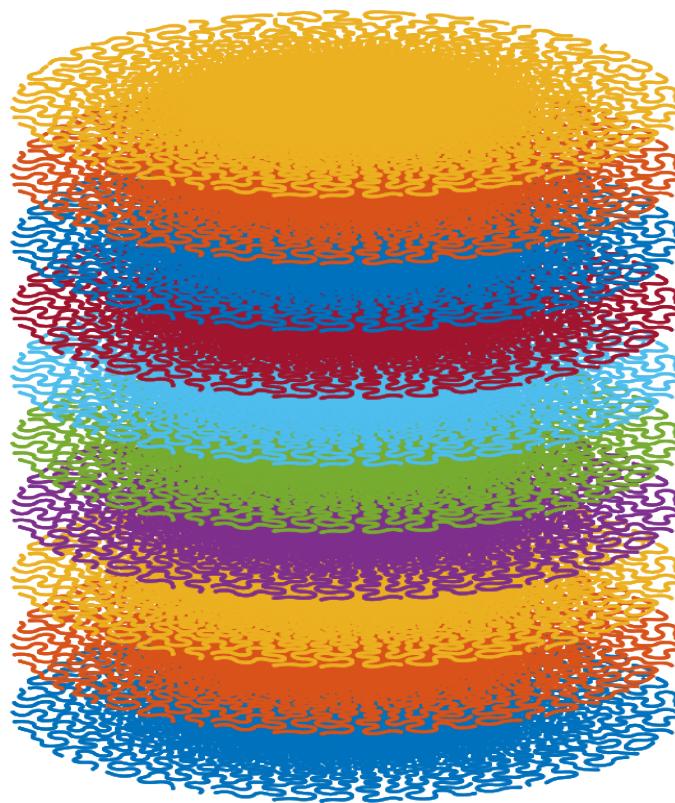


In-out spiral

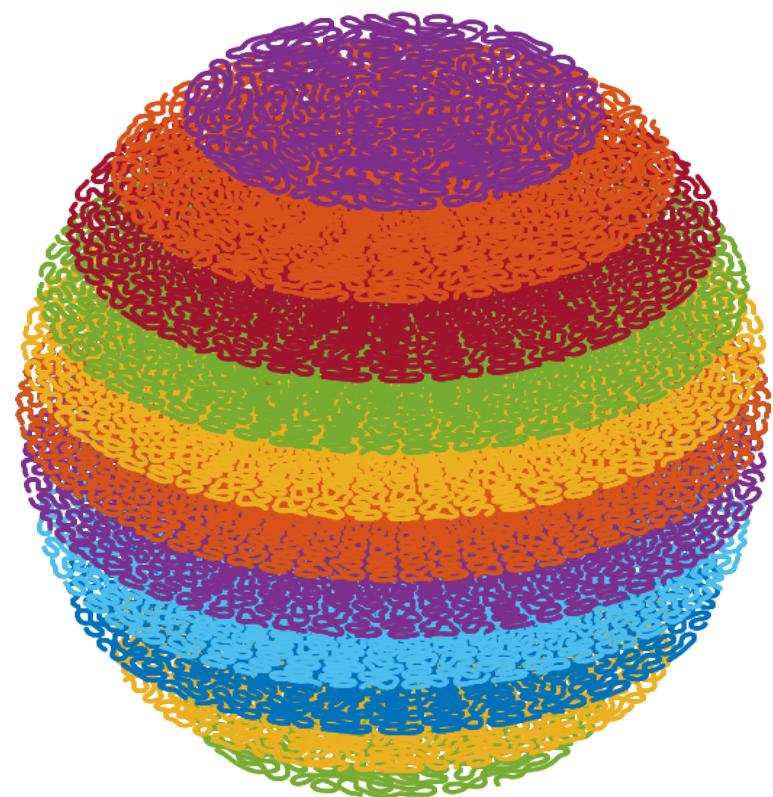


Stack-of-Sparkling (SOS)

Regular SOS



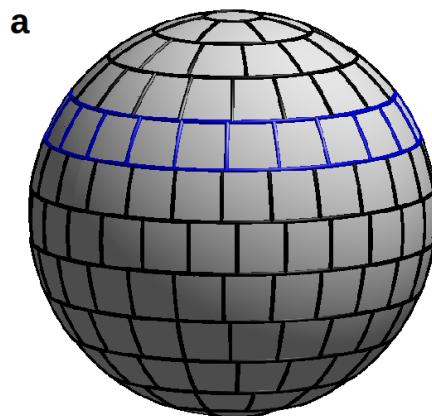
Z-vd SOS



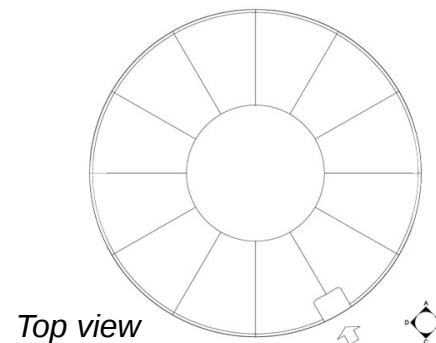
$$n(k_z) = n(0) \frac{\int \pi(:,:,k_z)}{\int \pi(:,:,0)},$$

Fully 3D Sparkling

→ Shot-by-shot generation



Equal area
tessellation



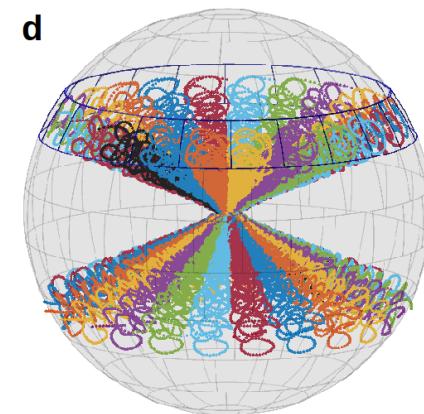
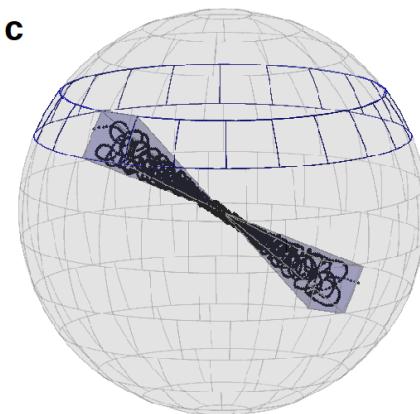
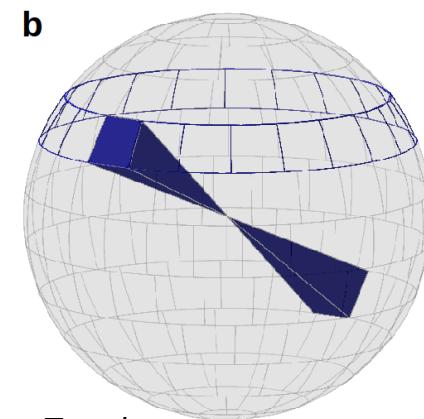
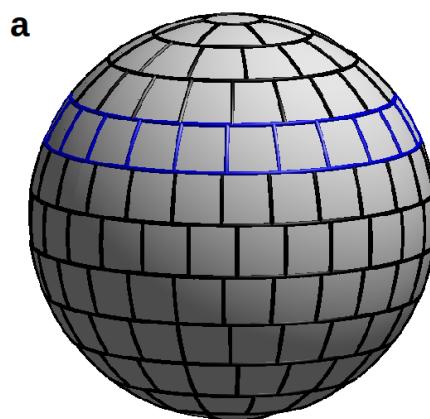
From 1 year to 20 min of
computation time

[Leopardi et al, Elect Trans on Num Anal, 2006]

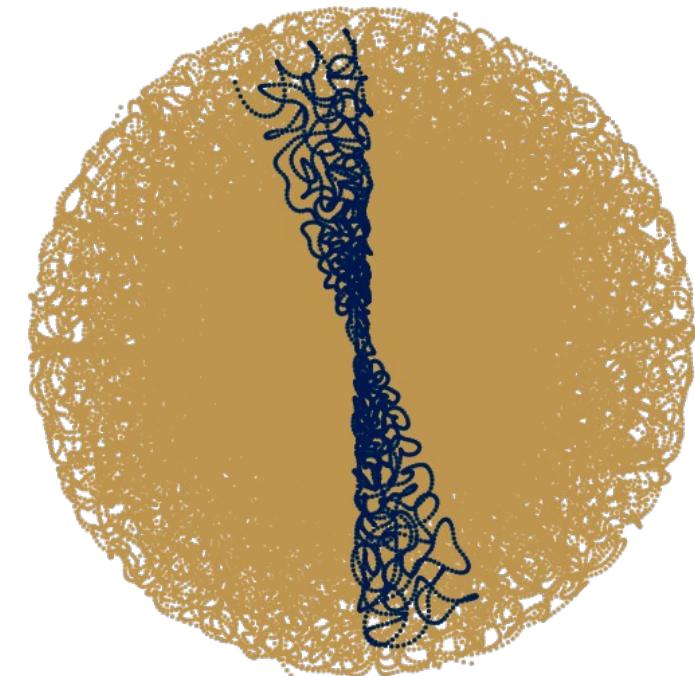
[Lazarus et al , sub. to NMR Biomed 2019]

Fully 3D Sparkling

→ Shot-by-shot generation



From 1 year to 20 min of computation time



[Leopardi et al, Elect Trans on Num Anal, 2006]

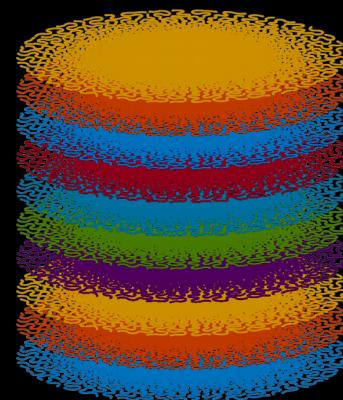
[Lazarus et al , sub. to NMR Biomed 2019]

Ex vivo comparison of different Sparkling strategies

T2* contrast

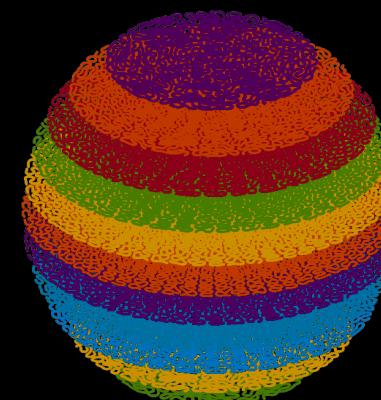
Isotropic resolution of 0.6 mm

Tobs=15 ms - FOV=20x20x14 cm³



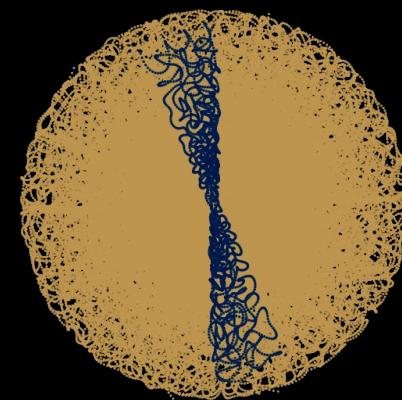
Regular SOS

VS.



z-vd SOS

VS.



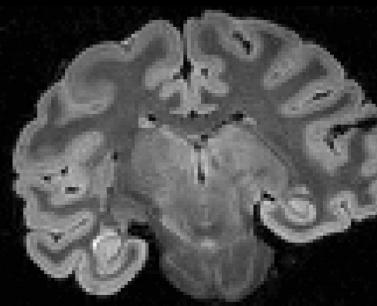
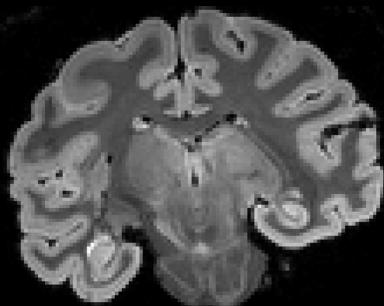
Fully 3D

Comparison of different Sparkling methods 4000 shots – AF=20

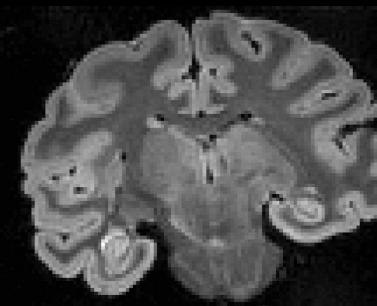
IPAT 4
TA=14min31s



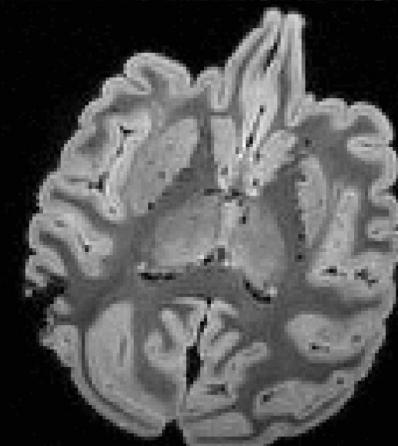
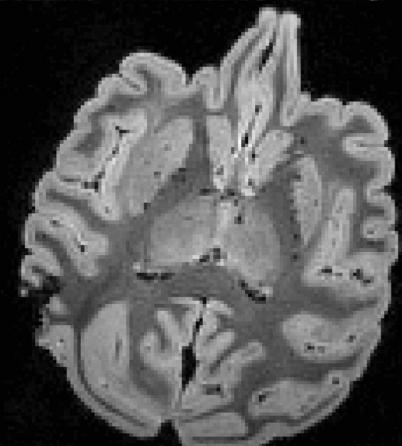
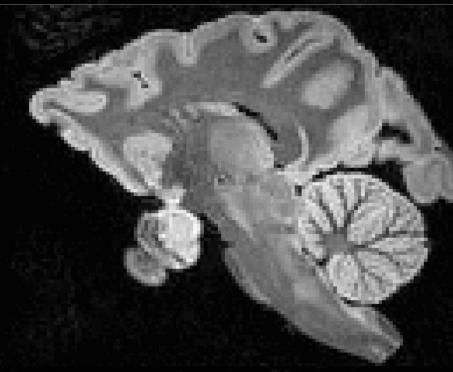
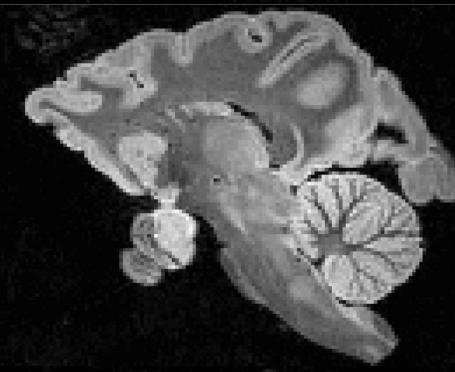
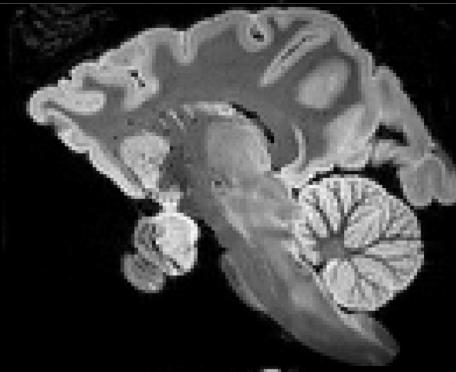
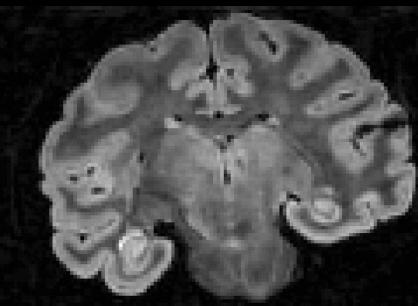
Regular SOS
TA=2min40s
SSIM=0.87



z-vd SOS
TA=2min40s
SSIM=0.88

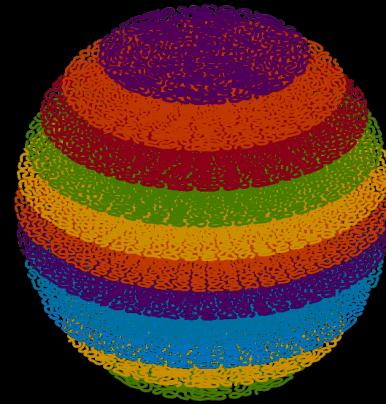


Fully 3D
TA=2min40s
SSIM=0.79

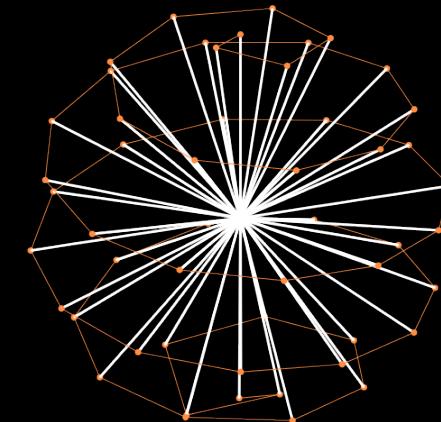


Ex vivo comparison with other strategies

T2* contrast
Isotropic resolution of 0.6 mm
TA=45s

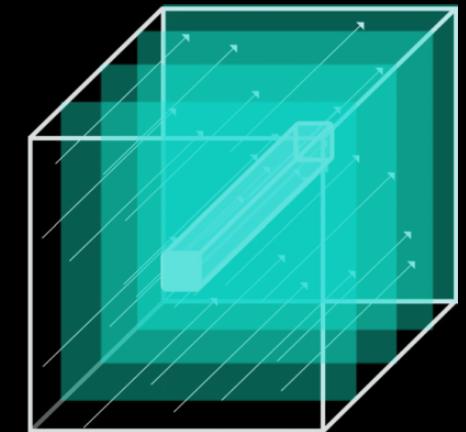


vs.



Larson et al. 2007

vs.

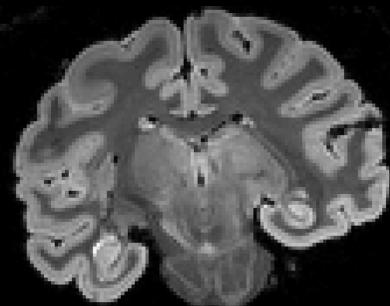


Lustig et al. 2008

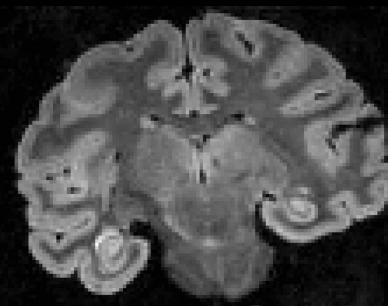
SPARKLING vs. other strategies

1140 shots - AF=69

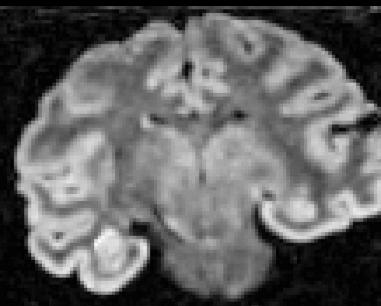
IPAT 4
TA=14min31s



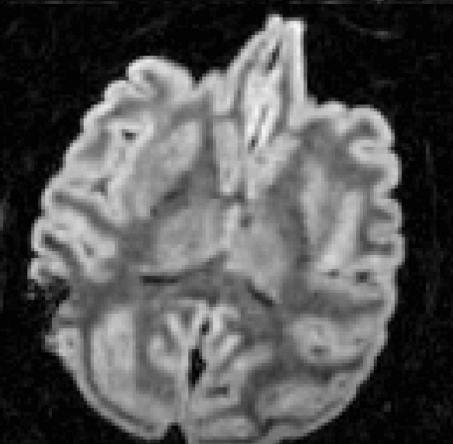
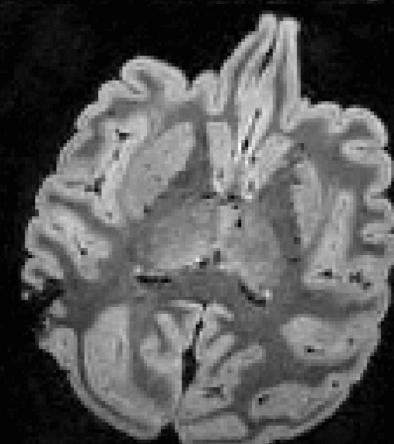
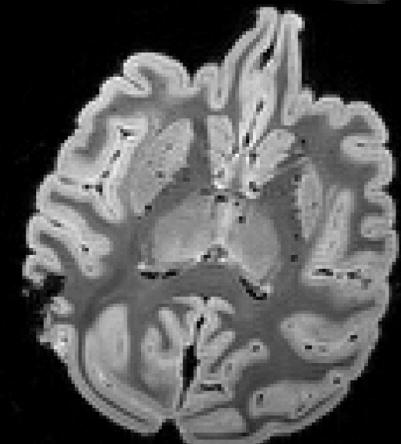
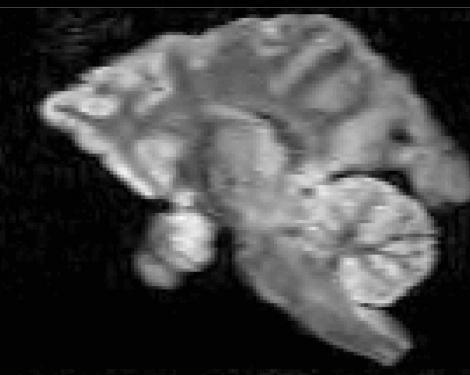
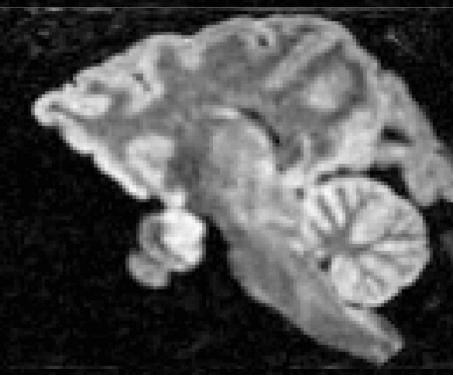
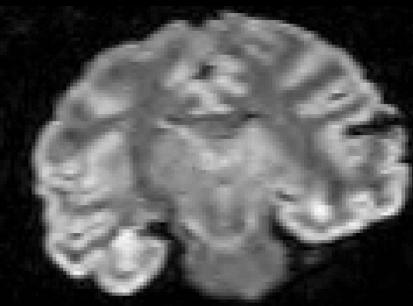
SPARKLING
TA=45s
SSIM=0.86



RADIAL
TA=45s
SSIM=0.72



Poisson disk lines
TA=45s
SSIM=0.58



Interim Summary (6)

- **SPARKLING 2D**
 - **Prospective** *Ex vivo* and *in vivo* validation of SPARKLING 2D:
Demonstration of its **superiority** over both radial and spiral trajectories
 - Improved sampling **efficiency, and** robustness to subsampling
 - Adaptability to hardware constraints and various densities
- **SPARKLING 3D**
 - Higher acceleration factors
 - **Prospective** *Ex vivo* validation of SPARKLING 3D
- **No CS theory for justifying SPARKLING so far**

Conclusion & Outlook

- **New algorithms for accelerated MR acquisition**
 - No existing CS theorem currently supports SPARKLING
 - Still there are degrees of freedom to choose the best target density
 - Existing CS theories stick to Cartesian sampling!
- **Outlook**
 - Deep learning for optimizing the sampling density
 - Optimize the acquisition/reconstruction pair in various imaging scenarios
 - Think about sampling in higher dimensions to perform smarter compression and acceleration



Acknowledgements:

NeuroSpin:

Nicolas Chauffert
Carole Lazarus
Loubna El Gueddari
Alexandre Vignaud
Antoine Grigis
Benoît Sarthou
Nicolas Chartier

CNRS Toulouse:

Pierre Weiss
Jonas Kahn
Claire Boyer

Siemens Healthineers:

Franck Mauconduit