MicrophysicsNotes

May 2, 2016

consider the equation

$$\frac{\partial a}{\partial t} = \frac{\bar{a} - a}{\tau}$$

move a/τ to LHS, multiply by integrating factor $e^{(t/\tau)}$ to give

$$\frac{\partial}{\partial t}(ae^{t/\tau}) = \frac{\bar{a}}{\tau}e^{t/\tau}$$

integrate from t_0 to t_1

$$(a(t_1)e^{t_1/\tau}) - (a(t_0)e^{t_0/\tau}) = \bar{a}(e^{t_1/\tau} - e^{t_0/\tau})$$

multiply by $e^{-t_0/\tau}$

$$a(t_1)e^{(t_1-t_0)/\tau} - a(t_0) = \bar{a}(e^{(t_1-t_0)/\tau} - 1)$$

Let $t_1 - t_0 = \Delta t$, then

$$a(t_1) - a(t_0)e^{-\Delta t/\tau} = \bar{a}(1 - e^{-\Delta t/\tau})$$

or

$$a(t_1) - a(t_0) = (\bar{a} - a(t_0))(1 - e^{-\Delta t/\tau})$$

observve that if $\Delta t = 0$ then $a(t_1) = a(t_0)$ and otherwise the increment for changing $a(t_0)$ is just controlled by the e-folding time τ.

Dividing by Δt to form a "tendency" gives

$$\frac{a(t_1) - a(t_0)}{\Delta t} = \frac{\bar{a} - a(t_0)}{\Delta t}(1 - e^{-\Delta t/\tau})$$

```
In [2]: %matplotlib inline
        import matplotlib.pyplot as plt
        import numpy as np
```

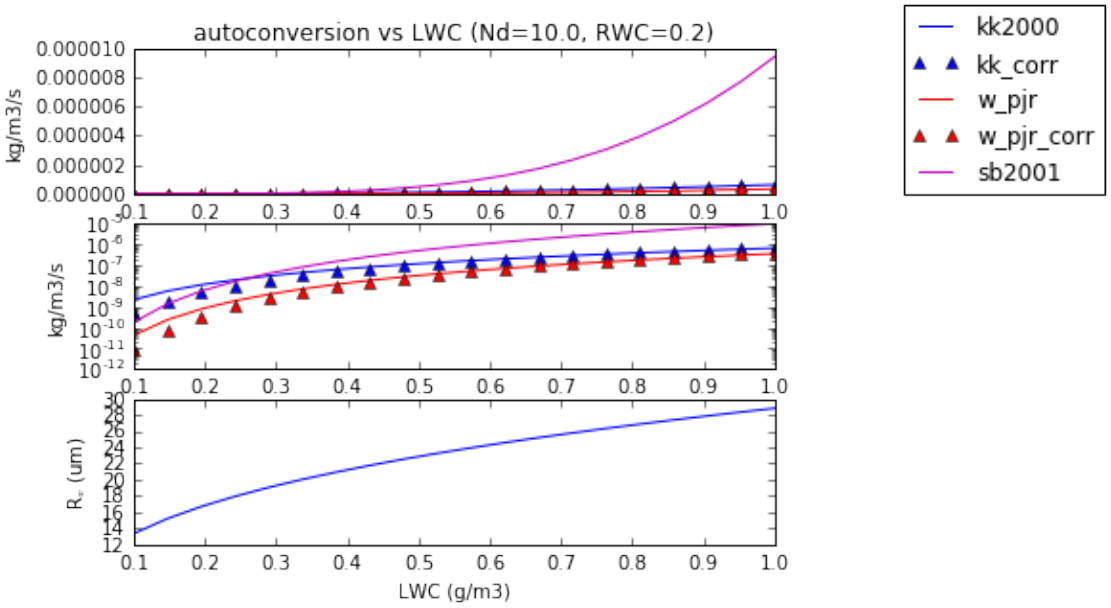
```
In [23]: # the conversion factor from KK(2000) to Wood 2005
         # wood 2005 uses equations in density
         # kk 2000 uses equations in mixing ratio
         print 1350*1.e-6**(-1.79)
         print 1.22**(-1.47)
```

```
7.41880179708e+13
0.746536504479
```

```
In [82]: rhoair = 1.22 # kg/m3
         # using the units found in wood 2005
         LWC_gm3 = np.linspace(0.1, 1., 20) # g/m3
         LWC_kgm3 = LWC_gm3/1000. # convert to kg/m3
         LWC_mr = LWC_kgm3/rhoair
         Nd_cm3 = 10. # per cm3
```

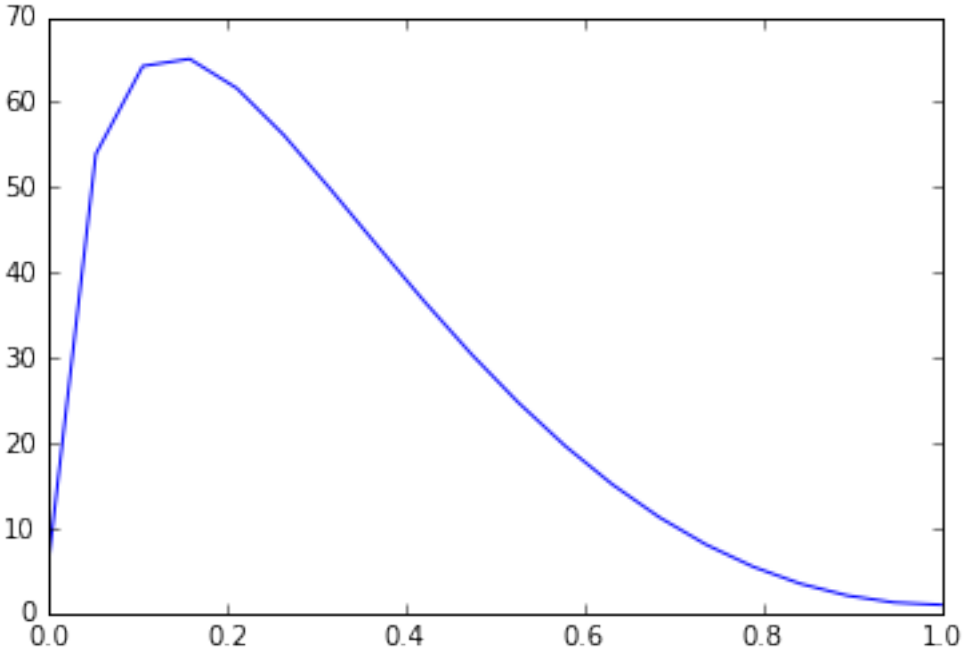
```
Nd_m3 = Nd_cm3 * 1.e6 # per m3
rv = (LWC_gm3*1.e-3/(Nd_m3*4./3.*np.pi*1000.))**(1./3.)*1.e6
#print "rv", rv
a = 2.8
b = -1.42
K = 1.6e13
autowbe   = K*(LWC_kgm3**a)*(Nd_m3**b)*rhoair**(1-a)
#print "w best estimate", autowbe
a = 2.67
b = -0.92
K = 9.06e-7
autow_uk = K*(LWC_gm3**a)*(Nd_cm3**b)
#print "w_uk", autow_uk
a = 3.19
b = -1.42
K = 2.66e-5
autow_vocals = K*(LWC_gm3**a)*(Nd_cm3**b)
#print "w_vocals", autow_vocals
a = 2.47
b = -1.79
K = 7.42e13
autokk_w = K*(LWC_kgm3**a)*(Nd_m3**b)*rhoair**(1-a)
#print "kk_w", autokk_w
autokk = 1350*(LWC_mr**a)*(Nd_cm3**b)*rhoair
#print "kk", autokk
# Seifert and Beheng
kc = 9.44e9 # cm3/g2
xstar = 2.6e-7 # g
nu = 0.
RWC_gcm3 = 0.
RWC_gcm3 = 0.2e-6
LWC_gcm3 = LWC_gm3*1.e-6
tau = 1.-(LWC_gcm3/(LWC_gcm3+RWC_gcm3))
xbar = LWC_gcm3/Nd_cm3
phi_au =600.*tau**0.68*(1-tau**0.68)**3
#print "tau,, phi_au", tau, phi_au
coef = kc/20./xstar*(nu+2)*(nu+4)/((nu+1)**2)
corr = (1.+phi_au/(1.-tau**2))
#print "coef, corr, tau", coef, corr, tau
sb_au = coef*LWC_gcm3**2*xbar**2*corr # probably in g/cm3/s
sb_au = sb_au/1000.*1.e6 # convert to kg/m3/s
#print "sf_au", sf_au
a = 3.19 # from wood vocals email
b = -1.42 # from wood vocals email
K = 1.6e13 # from wood best estimate
autopjr   = K*(LWC_kgm3**a)*(Nd_m3**b)*rhoair**(1-a)
autopjr = autopjr*corr/60.
#print "w pjr", autowbe
plt.figure(1)
plt.subplot(311)
plt.plot(LWC_gm3, autokk,label="kk2000")
plt.plot(LWC_gm3, autokk*corr/60.,'b^',label="kk_corr")
#plt.plot(LWC_gm3, autowbe,'r-')
#plt.plot(LWC_gm3, autow_uk,'g-',label='w_UK')
```

```
#plt.plot(LWC_gm3, autow_vocals,'g^',label="w_Vocals")
plt.plot(LWC_gm3, autopjr,'r',label="w_pjr")
plt.plot(LWC_gm3, autopjr*corr/60,'r^',label="w_pjr_corr")
plt.plot(LWC_gm3, sb_au,'m-',label="sb2001")
plt.title('autoconversion vs LWC (Nd='+str(Nd_cm3)+', RWC='+str(RWC_gcm3*1.e6)+')');
plt.ylabel('kg/m3/s')
plt.legend(loc=(1.2,4.e-8))
plt.subplot(312)
plt.semilogy(LWC_gm3, autokk)
plt.semilogy(LWC_gm3, autokk*corr/60.,'b^',label="kk_corr")
#plt.semilogy(LWC_gm3, autowbe,'r-')
#plt.semilogy(LWC_gm3, autow_uk,'g-')
#plt.semilogy(LWC_gm3, autow_vocals,'g^')
plt.semilogy(LWC_gm3, autopjr,'r')
plt.semilogy(LWC_gm3, autopjr*corr/60,'r^',label="w_pjr_corr")
plt.semilogy(LWC_gm3, sb_au,'m-')
plt.ylabel('kg/m3/s')
##print "w1", autow1
#print "w2", autow2
plt.subplot(313)
plt.plot(LWC_gm3, rv)
plt.ylabel('R$_v$ (um)')
plt.xlabel('LWC (g/m3)')
plt.subplots_adjust(bottom=0.)
plt.show()
```



In [22]:
```
tau = np.linspace(0.001, 0.999, 20) # g/m3
phi_au =600.*tau**0.68*(1-tau**0.68)**3
corr = (1.+phi_au/(1.-tau**2))
print min(corr)
plt.plot(tau, corr)
plt.show()
```

```
1.0000943579
```



In [56]:
```
rhoair = 1.22 # kg/m3
a = 3.19 # from wood vocals email
b = -1.42 # from wood vocals email
K = 1.6e13 # from wood best estimate
#convert from LWC1 in kg/m3 to LWC2 in g/m3
#              N1 in #/m3 to N2 in #/cm3
#              auto1 in kg/m3/s to kg/kg/s
#auto1 = K*(LWC1**a)*(N1**b)  # auto1 in units of kg/m3/s
#auto1 = K*((LWC2*1.e-3)**a)*((N2*1.e6)**b
#      = K*(1.e-3**a)*(LWC2**a)*(N2**b)*(1.e6**b)
#auto1 = auto2/rhoair
print rhoair**(1.-a)
Kfix   = K*(1.e-3**a)*(1.e+6**b)#/rhoair**(1-a)
print Kfix
```

```
0.646951867468
1.30052882586e-05
```