Bargaining problem

The two person **bargaining problem** is a problem of understanding how two agents should cooperate when non-cooperation leads to Pareto-inefficient results. It is in essence an equilibrium selection problem; Many games have multiple equilibria with varying payoffs for each player, forcing the players to negotiate on which equilibrium to target. The quintessential example of such a game is the Ultimatum game. The underlying assumption of bargaining theory is that the resulting solution should be the same solution an impartial arbitrator would recommend. Solutions to bargaining come in two flavors: an axiomatic approach where desired properties of a solution are satisfied and a strategic approach where the bargaining procedure is modeled in detail as a sequential game.

The bargaining game

The **bargaining game** or **Nash bargaining game** is a simple two-player game used to model bargaining interactions. In the Nash Bargaining Game two players demand a portion of some good (usually some amount of money). If the total amount requested by the players is less than that available, both players get their request. If their total request is greater than that available, neither player gets their request. A **Nash bargaining solution** is a (Pareto efficient) solution to a Nash bargaining game. According to Walker (2005), Nash's bargaining solution was shown by John Harsanyi to be the same as Zeuthen's solution of the bargaining problem (*Problems of Monopoly and Economic Warfare*, 1930).

An example

	Opera	Football
Opera	3,2	0,0
Football	0,0	2,3

Battle of the Sexes 1

The Battle of the sexes, as shown, is a two player coordination game. Both Opera/Opera and Football/Football are Nash equilibria. Any probability distribution over these two Nash equilibria is a correlated equilibrium. The question then becomes which of the infinitely many possible equilibria should be chosen by the two players. If they disagree and choose different distributions, they are likely to receive 0 payoffs. In this symmetric case the natural choice is to play Opera/Opera and Football/Football with equal probability. Indeed all bargaining solutions described below prescribe this solution. However, if the game is asymmetric --- for example, Football/Football instead yields payoffs of 2,5 --- the appropriate distribution is less clear. The problem of finding such a distribution is addressed by the bargaining theory.

Formal description

A two person bargain problem consists of a disagreement, or threat, point $v = (v_1, v_2)$, where v_1 and v_2 are the respective payoffs to player 1 and player 2, and a feasibility set F, a closed convex subset of $\mathbf{R}^2$, the elements of which are interpreted as agreements. Set F is convex because an agreement could take the form of a correlated combination of other agreements. The problem is nontrivial if agreements in F are better for both parties than the disagreement. The goal of bargaining is to choose the feasible agreement ϕ in F that could result from negotiations.

Feasibility set

Which agreements are feasible depends on whether bargaining is mediated by an additional party. When binding contracts are allowed, any joint action is playable, and the feasibility set consists of all attainable payoffs better than the disagreement point. When binding contracts are unavailable, the players can defect (moral hazard), and the feasibility set is composed of correlated equilibria, since these outcomes require no exogenous enforcement.

Disagreement point

The disagreement point v is the value the players can expect to receive if negotiations break down. This could be some focal equilibrium that both players could expect to play. This point directly affects the bargaining solution, however, so it stands to reason that each player should attempt to choose his disagreement point in order to maximize his bargaining position. Towards this objective, it is often advantageous to increase one's own disagreement payoff while harming the opponent's disagreement payoff (hence the intrepretation of the disagreement as a threat). If threats are viewed as actions, then one can construct a separate game wherein each player chooses a threat and receives a payoff according to the outcome of bargaining. It is known as Nash's variable threat game. Alternatively, each player could play a minimax strategy in case of disagreement, choosing to disregard personal reward in order to hurt the opponent as much as possible shoud the opponent leave the bargaining table.

Equilibrium analysis

Strategies are represented in the Nash bargaining game by a pair (x, y). x and y are selected from the interval [d, z], where z is the total good. If $x + y$ is equal to or less than z, the first player receives x and the second y. Otherwise both get d. d here represents the disagreement point or the threat of the game; often $d = 0$.

There are many Nash equilibria in the Nash bargaining game. Any x and y such that $x + y = z$ is a Nash equilibrium. If either player increases their demand, both players receive nothing. If either reduces their demand they will receive less than if they had demanded x or y. There is also a Nash equilibrium where both players demand the entire good. Here both players receive nothing, but neither player can increase their return by unilaterally changing their strategy.

Bargaining solutions

Various solutions have been proposed based on slightly different assumptions about what properties are desired for the final agreement point.

Nash bargaining solution

John Nash proposed that a solution should satisfy certain axioms:

1. Invariant to affine transformations or Invariant to equivalent utility representations
2. Pareto optimality
3. Independence of irrelevant alternatives
4. Symmetry

Let u and v be the utility functions of Player 1 and Player 2, respectively. In the *Nash bargaining solution*, the players will seek to maximize $(u(x) - u(d)) * (v(y) - v(d))$, where $u(d)$ and $v(d)$, are the status quo utilities (i.e. the utility obtained if one decides not to bargain with the other player). The product of the two excess utilities is generally referred to as the *Nash product*.

Kalai-Smorodinsky bargaining solution

Independence of Irrelevant Alternatives can be substituted with a monotonicity condition, as demonstrated by Ehud Kalai and Meir Smorodinsky. It is the point which maintains the ratios of maximal gains. In other words, if player 1 could receive a maximum of g_1 with player 2's help (and vice-versa for g_2), then the Kalai-Smorodinsky bargaining solution would yield the point ϕ on the Pareto frontier such that $\phi_1/\phi_2 = g_1/g_2$.

Egalitarian bargaining solution

The egalitarian bargaining solution, introduced by Ehud Kalai, is a third solution which drops the condition of scale invariance while including both the axiom of Independence of irrelevant alternatives, and the axiom of monotonicity. It is the solution which attempts to grant equal gain to both parties. In other words, it is the point which maximizes the minimum payoff among players.

Applications

Some philosophers and economists have recently used the Nash bargaining game to explain the emergence of human attitudes toward distributive justice (Alexander 2000; Alexander and Skyrms 1999; Binmore 1998, 2005). These authors primarily use evolutionary game theory to explain how individuals come to believe that proposing a 50-50 split is the only just solution to the Nash Bargaining Game.

References

- Alexander, Jason McKenzie (2000). "Evolutionary Explanations of Distributive Justice". *Philosophy of Science* **67** (3): 490–516. JSTOR 188629 [1].
- Alexander, Jason; Skyrms, Brian (1999). "Bargaining with Neighbors: Is Justice Contagious". *Journal of Philosophy* **96** (11): 588–598. JSTOR 2564625 [2].
- Binmore, K.; Rubinstein, A.; Wolinsky, A. (1986). "The Nash Bargaining Solution in Economic Modelling". *RAND Journal of Economics* **17**: 176–188. JSTOR 2555382 [3].
- Binmore, Kenneth (1998). *Game Theory and The Social Contract Volume 2: Just Playing*. Cambridge: MIT Press. ISBN 0-262-02444-6.
- Binmore, Kenneth (2005). *Natural Justice*. New York: Oxford University Press. ISBN 0-19-517811-4.
- Kalai, Ehud (1977). "Proportional solutions to bargaining situations: Intertemporal utility comparisons". *Econometrica* **45** (7): 1623–1630. JSTOR 1913954 [4].
- Kalai, Ehud & Smorodinsky, Meir (1975). "Other solutions to Nash's bargaining problem". *Econometrica* **43** (3): 513–518. JSTOR 1914280 [5].
- Nash, John (1950). "The Bargaining Problem". *Econometrica* **18** (2): 155–162. JSTOR 1907266 [6].
- Walker, Paul (2005). "History of Game Theory" [7].

External links

- Nash Bargaining Solutions [8]

References

[1] http://www.jstor.org/stable/188629
[2] http://www.jstor.org/stable/2564625
[3] http://www.jstor.org/stable/2555382
[4] http://www.jstor.org/stable/1913954
[5] http://www.jstor.org/stable/1914280
[6] http://www.jstor.org/stable/1907266
[7] http://www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm#ref94
[8] http://www.cse.iitd.ernet.in/~rahul/cs905/lecture15/

Article Sources and Contributors

Bargaining problem *Source*: http://en.wikipedia.org/w/index.php?oldid=551918388 *Contributors*: Akalai, Allliam, Bender235, Cfp, Cretog8, E.qrqy, Ego White Tray, Ever wonder, Floquenbeam, Giraffedata, Henning Makholm, Jessieliaosha, Josiahstevenson, MGM08314, PigFlu Oink, Toyokuni3, 24 anonymous edits

License