Estimating MCMC convergence rates using CRN

Let $X_1, X_2, \ldots$ be a Markov chain. That is, $X_n = f_{\theta_n}(X_{n-1})$ where $\{\theta_n\}_{n\geq 1}$ are i.i.d. random variables.

Suppose that $\mathcal{L}(X_n) \to \pi$, where $\mathcal{L}(X_n)$ indicates the law of X_n and π represents the stationary distribution.

At what iteration is the distribution of the chain approximately π?

That is, what is the n such that $\mathcal{L}(X_n) \approx \pi$?

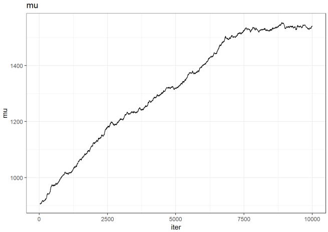

Figure: Traceplot of $\mu_n \in \vec{X}_n$

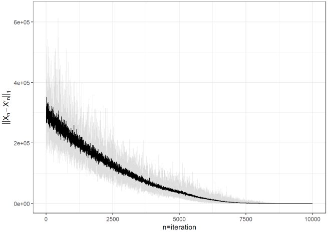

Figure: CRN simulation of $\vec{X}_n$

Estimating MCMC convergence rates using CRN

Algorithm 1 An estimate of $E[d(X_N, Y_N)^p] \approx \frac{1}{M}\sum_{i=1}^{M} d(x_{N,i}, y_{N,i})^p$ using CRN

for $i = 1, \ldots, M$ **do**
  $x_{0,i} \sim \mu$, $y_{0,i} \sim \nu$ where $x_{0,i} \perp\!\!\!\perp y_{0,i}$
  for $n = 1, \ldots, N$ **do**
    $\theta_n \sim \Theta$
    $x_{n,i} \leftarrow f_{\theta_n}(x_{n-1,i})$
    $y_{n,i} \leftarrow f_{\theta_n}(y_{n-1,i})$
  end for
end for
return $\frac{1}{M}\sum_{i=1}^{M} d(x_{N,i} - y_{N,i})^p$

Theorem 3.3. *Let $\{X_n\}_{n\geq 1}$ and $\{Y_n\}_{n\geq 1}$ be two copies of a Markov chain simulated according to Algorithm 1 with $\mathcal{L}(Y_0) = \nu$, $Y_0 \perp\!\!\!\perp X_0$, and $\mathcal{L}(X_\infty) = \mathcal{L}(Y_\infty) = \pi$. Suppose that for each $n \geq 0$, $E[||X_n||_q^p], E[||Y_n||_q^p] < \infty, q, p \in [1, \infty)$ and that ν is defined on the same support as π. Then*

$$W_{||\cdot||_q,p}(\mathcal{L}(X_N), \pi) \leq \left(K \lim_{M\to\infty} \frac{1}{M} \sum_{i=1}^{M} ||x_{N,i} - y_{N,i}||_q^p \right)^{1/p} \tag{4}$$

holds almost surely where $K \geq K' = \frac{1}{1-s(\pi,\nu)} = \frac{1}{1-r(\pi,\nu)}$.