§1 BACK-20Q INTRO 1

1. Intro. This program finds all solutions to Don Woods's Twenty Questions, having a given pattern of true/false grades, assuming that at least 18 answers will be correct. The command line lists two answers that must be incorrect, if nonzero.

I've tried to write this in a straightforward way that will avoid errors in my logic, yet leave it flexible enough to experiment with slight tweaks to the problem.

```
#define A = 0
#define B 1
#define C 2
#define D 3
#define E 4
#define AA (1 \ll A)
#define BB
               (1 \ll B)
#define CC
               (1 \ll C)
#define DD
               (1 \ll D)
#define EE (1 \ll E)
\#define o mems ++
#define oo mems += 2
#define stacksize 1000000
                                   /* I think 20 \times 20 \times 6 is actually an upper bound */
#define delta 10000000000
                                   /* print status every this many mems */
#include <stdio.h>
#include <stdlib.h>
  typedef unsigned long long ull;
  ull mems;
                 /* memory references */
                 /* size of search tree */
  ull nodes;
  ull count:
                 /* solutions found */
  ull thresh = delta;
                          /* time to print next report */
  ull profile [22];
                         /* command-line parameters */
  int false1, false2;
                 /* the score of each solution that is found */
  int score;
  char falsity[21];
                       /* is this answer to be false or not? */
  char believe3;
                     /* is falsity [3] zero? */
  char mem[41];
                      /* status subject to backtracking */
                   /* print lots of debugging info? */
  char vbose;
  (Other global variables 6);
  \langle \text{Subroutines } 7 \rangle;
  main(\mathbf{int} \ argc, \mathbf{char} * argv[])
    register int i, j, k, l, p, q, t, u, x, y, really_bad;
    \langle \text{Process the command line } 3 \rangle;
     \langle Set the initial constraints 5\rangle;
     ⟨ Backtrack through all possibilities 12⟩;
    fprintf(stderr, "Altogether_\'\%llu_\solutions_\(\%llu_\mems, \\\), \\\ n", count, mems, nodes);
    if (vbose) \langle Print the profile 2\;
```

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```
2.
      \langle \text{ Print the profile 2} \rangle \equiv
     fprintf(stderr, "Profile: "\lumber 1\n");
     for (k = 2; k \le 21; k++) fprintf (stderr, "\%1911d\n", profile[k]);
This code is used in section 1.
3. \langle \text{Process the command line } 3 \rangle \equiv
  if (argc < 3 \lor sscanf(argv[1], "%d", \&false1) \neq 1 \lor sscanf(argv[2], "%d", \&false2) \neq 1) {
     fprintf(stderr, "Usage: \_\%s \_false1 \_false2 \_[verbose] \n", argv[0]);
     exit(-1);
  score = 20;
  if (false1 > 0 \land false1 \le 20) falsity[false1] = 1, score ---;
  if (false2 > 0 \land false2 \le 20 \land false2 \ne false1) falsity[false2] = 1, score --;
  believe3 = \neg falsity[3];
  vbose = argc - 3;
                           /* extra arguments are ignored but they increase verbosity */
  if (falsity [6]) {
     if (vbose) fprintf(stderr, "Question_6_can't_be_wrong.\n");
     goto done;
This code is used in section 1.
```

§4 BACK-20Q THE STRATEGY 5

4. The strategy. All the critical data about partial information is kept in a small array called mem, consisting of 8-bit quantities. Locations mem[1] thru mem[20] are bitmaps that show the nonexcluded possibilities for each of the 20 answers. (For example, the bitmap AA + DD means that A and D are still possible.) The next 20 locations hold tag information, which is nonzero if the answer for that question should be doublechecked at level 21. (We catch easy inconsistencies early if it's convenient, but defer the complicated tests.)

When the entry in mem[p] changes from a to b, we put $(p \ll 8) + a$ on the stack so that the old value can be restored later. The number of items on the stack upon entry to level l is stored in frame[l].

I thought about maintaining upper and lower bounds for the number of A's, B's, C's, D's, E's. But that seemed prone to error, and experiments by hand indicated that reasonable cutoffs were possible without such fancy footwork.

```
#define tag 20 /* offset for tag data */
```

5. Initially all of the answers are unrestricted.

```
\langle Set the initial constraints 5\rangle \equiv for (q=1;\ q\leq 20;\ q++)\ o, mem[q]=AA+BB+CC+DD+EE; This code is used in section 1.
```

6. Some of the questions are easy to deal with near the root of the search tree, so I've chosen a heuristic order in which to build the partial solutions. This ordering remains fixed; thus, for example, I'll know that when I'm working on question 1, I've already given answers to several others including questions 2 and 3. It's nice to put question 3 first, because it usually rules out many possibilities immediately: If that answer is supposed to be correct, no two consecutive answers can match, with the exception of *loguy* and *higuy*, which are defined at root level.

This code is used in section 1.

4 THE STRATEGY BACK-20Q §7

7. Here's a subroutine that illustrates the conventions. If option x is presently available for question q, it is removed and the residual bitmap is returned.

Furthermore, if question 3 is not falsified, the question number is placed on pstack whenever the bitmap has been reduced to exactly one bit.

```
\langle \text{Subroutines } 7 \rangle \equiv
   int remov(int q, int x)
   {
      register int t, b, bb;
      t=1\ll x;
      o, b = mem[q];
      if (b \& t) {
          \textbf{if} \ (vbose > 2) \ \textit{fprintf} \ (\textit{stderr}, \texttt{"(%d:\_not\_\%d\%c)} \\ \texttt{`n"}, \textit{stackptr}, q, x + (\textit{falsity}[q] ? \texttt{`a'} : \texttt{`A'})); 
         o, mem[q] = bb = b - t;
         o, stack[stackptr++] = (q \ll 8) + b;
         if (believe 3 \land bb \land \neg (bb \& (bb-1))) o, pstack[pstackptr ++] = q;
         return bb;
      return b;
See also sections 8, 9, 10, and 11.
This code is used in section 1.
       The innerforce routine throws out all options but x.
\langle \text{Subroutines } 7 \rangle + \equiv
   int innerforce(int q, int x)
      register int t, b;
      t=1\ll x;
      o, b = mem[q];
      if ((b \& t) \equiv 0) return 0;
      if (b \neq t) {
          \textbf{if} \ (\textit{vbose} > 2) \ \textit{fprintf} \ (\textit{stderr}, \texttt{"(%d:} \texttt{\_\%d\%c)} \texttt{\n"}, \textit{stackptr}, q, x + (\textit{falsity}[q] ? \texttt{\a'} : \texttt{\A'})); \\ 
         o, mem[q] = t;
         o, stack[stackptr++] = (q \ll 8) + b;
         if (believe3) o, pstack[pstackptr++] = q;
      return t;
```

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```
I'll need some help when debugging.
\langle Subroutines 7\rangle + \equiv
  void print_state(void)
    register int b, c, q, z;
     for (q = 1; q \le 20; q++)
       if (q < 10) fprintf (stderr, "%d_{\sqcup\sqcup\sqcup\sqcup}", q);
       else fprintf(stderr, "%d_{\sqcup\sqcup\sqcup}", q);
     printf("\n");
     for (q = 1; q \le 20; q++) {
       for (b = mem[q], c = z = 0; c < 5; c++) {
          if (b \& (1 \ll c)) fprintf (stderr, "%c", c + (falsity[q]? `a': `A'));
          else z++;
       if (mem[q + tag]) z ---, fprintf(stderr, "*");
       for (c = 0; c < z; c \leftrightarrow) fprintf (stderr, " ");
     printf("\n");
     \langle \text{Subroutines } 7 \rangle + \equiv
  void print_cols(void)
     register int c, q;
     for (c = 0; c < 5; c++) {
       fprintf(stderr, "%c:", 'A' + c);
       for (q = 1; q \le 20; q++)
          if (mem[q] \& (1 \ll c)) fprintf (stderr, " \sqcup \%d", q);
       fprintf(stderr, "\n");
     }
  }
       \langle \text{Subroutines } 7 \rangle + \equiv
  void print_stack(int p)
     register int i, l, b, ii;
     for (i = 0, l = 1; i < p; i ++) {
       ii = stack[i] \gg 8;
       fprintf(stderr, "mem[%d%s]:", ii > 20 ? ii - tag: ii, ii > 20 ? "*":"");
       for (b = 0; b < 5; b++)
          if (stack[i] \& (1 \ll b)) fprintf (stderr, "%c", b + `A');
       while (i \equiv frame[l]) {
          fprintf(stderr, "ll%dg%d%s", l, order[l], falsity[l]? "-":"");
          l++;
       fprintf(stderr, "\n");
    }
  }
```

6 BACKTRACKING BACK-20Q $\S12$

12. Backtracking. Here's the environment/context in which I'll embed the logic for individual questions.

```
#define pack(u, q, x) (((u) \ll 8) + ((q) \ll 3) + (x))
\langle Backtrack through all possibilities 12 \rangle \equiv
b1: l = 1, stackptr = 0;
b2: nodes ++, profile[l]++;
  if (mems \ge thresh) (Print a status report and reset thresh 14);
  if (l > 20) (Check for solution and goto b5 16);
  oo, q = order[l], u = falsity[q];
  o, y = mem[q];
b3: if (y \equiv 0) {
    fprintf(stderr, "I'm_{\square}confused!\n"); exit(-69);
  p = stackptr;
  o, x = rho[y];
  pstackptr = 0;
  if (vbose > 1) fprintf(stderr, "Level_\%d(%d), _trying_\%d%c\n", l, p, q, x + (u? 'a': 'A'));
  switch (pack(u, q, x)) {
     (Cases for the big switch 19)
  default: fprintf(stderr, "Impossible_case_%d%c!\n", q, x + (u? 'a': 'A'));
    exit(-30);
defer: oo, mem[tag + q] = 1, stack[stackptr ++] = (q + tag) \ll 8;
                                                                      /* must check condition later */
  if (vbose > 1) fprintf(stderr, "(%d:_deferring_,%d%c)\n", stackptr - 1, q, x + (u? 'a' : 'A'));
okay: force(q, x);
  \langle Propagate forced consequences, possibly going to bad 15\rangle;
  o, frame[l] = p;
  l ++; goto b2;
bad: if (really_bad) {
                         /* we can't choose x nor can we not choose it */
    really\_bad = 0;
    goto b5;
  while (stackptr > p) \langle Restore saved state 13\rangle;
                        /* we've found that x isn't a good answer for question q *
b4: really\_bad = 1;
  pstackptr = 0;
                  /* this will backtrack if x was our last choice for q */
  \langle Propagate forced consequences, possibly going to bad 15\rangle;
  really\_bad = 0;
                    /* now (it says here) y should equal mem[q] */
  y -= 1 \ll x;
  if (y \equiv mem[q]) goto b3;
  fprintf(stderr, "I_{\square}screwed_{\square}up! \n"); exit(-666);
b5: if (--l) {
    oo, q = order[l], u = falsity[q];
    o, p = frame[l];
    while (stackptr > p) \langle Restore saved state 13\rangle;
    oo, y = mem[q], x = rho[y];
    goto b4;
This code is used in section 1.
```

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```
13.  ⟨Restore saved state 13⟩ ≡
{
    o, t = stack[--stackptr];
    o, mem[t ≫ 8] = t & #1f;
}
This code is used in section 12.

14.  ⟨Print a status report and reset thresh 14⟩ ≡
{
    fprintf(stderr, "After_\%llu\mems,\ull=\%d,\ulletstackptr=\%d\n", mems, l, stackptr);
    print_state();
    thresh += delta;
}
This code is used in section 12.
```

15. All changes to mem[1] thru mem[20] are made by the remov and innerforce routines. Macros deny and force are used to ensure that those routines don't remove an answer's final option.

If question 3 hasn't been falsified on the command line, we can't have consecutive equal answers except in certain cases. This means that one change to *mem* can propagate to its neighbors. For example, suppose the remaining choices for answers 5, 6, 7, 8, 9 are respectively CD, AC, BD, ABE, BCD. Then if answer 8 is forced to be B, answer 7 can only be D; hence answer 6 is also forced to be A. Also answer 9 can no longer be B. Such propagations are handled by the simple *pstack* mechanism implemented here.

This code is used in section 12.

8 BACKTRACKING BACK-20Q §16

```
16.
        Finally all the deferred tests must be made.
\langle\,\text{Check for solution and goto}\,\,\,b5\,\, 16 \rangle\equiv
      \langle Compute the distribution of answers 30\rangle;
      for (q = 1; q \le 20; q ++)
        if (o, mem[tag + q]) {
            oo, x = rho[mem[q]];
            o, u = falsity[q];
            if (vbose > 1) fprintf(stderr, "Checking_{\sqcup}%d%c\n", q, x + (u? 'a' : 'A'));
            switch (pack(u,q,x)) {
               \langle Cases for the deferred switch 20\rangle
            \mathbf{default}: fprintf(stderr, "Impossible\_deferred\_case\_%d%c! \n", q, x + (u ? 'a' : 'A'));
               exit(-31);
      \langle \text{Print a solution } 17 \rangle;
      goto b5;
This code is used in section 12.
         \langle \text{ Print a solution } 17 \rangle \equiv
17.
   count ++;
   printf("\%lld:_{\sqcup}", count);
   \mathbf{for}\ (q=1;\ q\leq 20;\ q++)\ \mathit{printf}\ (\texttt{"%c"},(\mathit{falsity}[q]\ ?\ \texttt{`a'}\ :\texttt{`A'}) + \mathit{rho}[\mathit{mem}[q]]);
   printf("\n");
This code is used in section 16.
```

18. The twenty questions. I'll quote each of the questions here, verbatim, because they're a bit of a moving target. (Don Woods learned in 2001 that his original questions, proposed in 2000, could be "cooked" by dozens of unintended answer lists. So he changed them at that time—only to discover, alas, that the new set could also be "cooked"! Evidently problems such as this are by no means easy to solve correctly. Further tweaking by the author in 2015, with the help of this CWEB program and in consultation with Don himself, has led to the present version of the questionnaire, which retains the original flavor and—it says here—is uncookable.)

Logical reasoning can be tricky. Hopefully the code below is transparent enough to reveal bugs? In each case I'll try to state what ordering assumptions I'm making, when they are relevant.

Each case in the big switch exits either to okay or bad or defer.

"1. The first question whose answer is A is: (A) 1 (B) 2 (C) 3 (D) 4 (E) 5"

We can use the fact that questions 3 and 2 have preceded question 1 in the ordering, hence mem[2] and mem [3] already contain definite values.

```
\langle \text{ Cases for the big switch } 19 \rangle \equiv
case pack(0,1,A): goto okay;
case pack(0,1,B): deny(1,A); force(2,A); goto okay;
case pack(0,1,C): deny(1,A); deny(2,A); force(3,A); goto\ okay;
case pack(0,1,D): deny(1,A); deny(2,A); deny(3,A); force(4,A); goto okay;
case pack(0,1,E): deny(1,A); deny(2,A); deny(3,A); deny(4,A); force(5,A);
  goto okay;
case pack(1, 1, A): goto bad;
case pack(1, 1, B): deny(2, A); goto okay;
                                                  /* 2 < 1 in the ordering */
case pack(1, 1, C): if (o, mem[2] \neq AA) deny(3, A); goto okay;
                                                                       /* 3 < 1 */
case pack(1, 1, D): if ((o, mem[2] \neq AA) \land (o, mem[3] \neq AA)) deny(4, A); goto okay;
case pack(1, 1, E): if ((o, mem[2] \equiv AA) \lor (o, mem[3] \equiv AA)) goto okay;
  goto defer:
See also sections 21, 23, 25, 27, 28, 29, 33, 35, 37, 38, 40, 42, 44, 46, 48, 50, 51, 53, and 55.
This code is used in section 12.
```

20. Deferred cases exit to b5 if the postponed test fails.

```
\langle \text{ Cases for the deferred switch } 20 \rangle \equiv
case pack(1, 1, E): if ((o, mem[4] \neq AA) \land (o, mem[5] \equiv AA)) goto b5; break;
See also sections 22, 24, 26, 32, 34, 36, 39, 41, 43, 45, 47, 49, and 54.
This code is used in section 16.
```

case pack(0,3,E): loguy = 19, higuy = 20; goto okay;

goto defer:

10

21. "2. The next question with the same answer as this one is: (A) 4 (B) 6 (C) 8 (D) 10 (E) 12" \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0, 2, A): deny(3, A); force(4, A); **goto** okay; case pack(0,2,B): deny(3,B); deny(4,B); deny(5,B); force(6,B); goto okay; case pack(0, 2, C): deny(3, C); deny(4, C); deny(5, C); deny(6, C); deny(7, C); force(8, C); **goto** okay; **case** pack(0, 2, D): deny(3, D); deny(4, D); deny(5, D); deny(6, D); deny(7, D); deny(8,D); deny(9,D); force(10,D);**goto** okay; case pack(0,2,E): deny(3,E); deny(4,E); deny(5,E); deny(6,E); deny(7,E); deny(8, E); deny(9, E); deny(10, E); deny(11, E); force(12, E);**goto** okay; /* 3 < 2 */case pack(1,2,A): if $(o, mem[3] \neq AA)$ deny(4,A); goto okay; case pack(1, 2, B): if $(o, mem[3] \equiv BB)$ goto okay; goto defer; case pack(1, 2, C): if $(o, mem[3] \equiv CC)$ goto okay; goto defer; case pack(1, 2, D): if $(o, mem[3] \equiv DD)$ goto okay; goto defer; case pack(1, 2, E): if $(o, mem[3] \equiv EE)$ goto okay; goto defer; 22. $\langle \text{ Cases for the deferred switch } 20 \rangle + \equiv$ case pack(1,2,B): if $((o,mem[4] \neq BB) \land (o,mem[5] \neq BB) \land (o,mem[6] \equiv BB))$ goto b5; break; case pack(1, 2, C): if $((o, mem[4] \neq CC) \land (o, mem[5] \neq CC) \land (o, mem[6] \neq CC) \land$ $(o, mem[7] \neq CC) \land (o, mem[8] \equiv CC))$ goto b5; case pack(1, 2, D): if $((o, mem[4] \neq DD) \land (o, mem[5] \neq DD) \land (o, mem[6] \neq DD) \land$ $(o, mem[7] \neq DD) \land (o, mem[8] \neq DD) \land (o, mem[9] \neq DD) \land (o, mem[10] \equiv DD))$ goto b5; case pack(1, 2, E): if $((o, mem[4] \neq EE) \land (o, mem[5] \neq EE) \land (o, mem[6] \neq EE) \land$ $(o, mem[7] \neq EE) \land (o, mem[8] \neq EE) \land (o, mem[9] \neq EE) \land (o, mem[10] \neq EE) \land$ $(o, mem[11] \neq EE) \land (o, mem[12] \equiv EE))$ goto b5; break; "3. The only two consecutive questions with identical answers are questions: (A) 15 and 16 (B) 16 and 17 (C) 17 and 18 (D) 18 and 19 (E) 19 and 20" \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0,3,A): loguy = 15, higuy = 16; goto okay; case pack(0,3,B): loguy = 16, higuy = 17; **goto** okay; case pack(0,3,C): loguy = 17, higuy = 18; goto okay; case pack(0,3,D): loguy = 18, higuy = 19; goto okay;

case pack(1,3,A): case pack(1,3,B): case pack(1,3,C): case pack(1,3,D): case pack(1,3,E):

```
24.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(1, 3, A): if ((oo, mem[15] \equiv mem[16]) \land (o, mem[16] \neq mem[17]) \land
          (o, mem[17] \neq mem[18]) \land (o, mem[18] \neq mem[19]) \land (o, mem[19] \neq mem[20])) goto test3;
  break;
case pack(1, 3, B): if ((oo, mem[15] \neq mem[16]) \land (o, mem[16] \equiv mem[17]) \land
          (o, mem[17] \neq mem[18]) \land (o, mem[18] \neq mem[19]) \land (o, mem[19] \neq mem[20])) goto test3;
  break:
case pack(1, 3, C): if ((oo, mem[15] \neq mem[16]) \land (o, mem[16] \neq mem[17]) \land
          (o, mem[17] \equiv mem[18]) \land (o, mem[18] \neq mem[19]) \land (o, mem[19] \neq mem[20])) goto test3;
case pack(1, 3, D): if ((oo, mem[15] \neq mem[16]) \land (o, mem[16] \neq mem[17]) \land
          (o, mem[17] \neq mem[18]) \land (o, mem[18] \equiv mem[19]) \land (o, mem[19] \neq mem[20])) goto test3;
  break:
case pack(1, 3, E): if ((oo, mem[15] \neq mem[16]) \land (o, mem[16] \neq mem[17]) \land
          (o, mem[17] \neq mem[18]) \land (o, mem[18] \neq mem[19]) \land (o, mem[19] \equiv mem[20])) goto test3;
  break;
test3: if ((oo, mem[1] \neq mem[2]) \land (o, mem[2] \neq mem[3]) \land (o, mem[3] \neq mem[4]) \land
          (o, mem[4] \neq mem[5]) \land (o, mem[5] \neq mem[6]) \land (o, mem[6] \neq mem[7]) \land
          (o, mem[7] \neq mem[8]) \land (o, mem[8] \neq mem[9]) \land (o, mem[9] \neq mem[10]) \land
          (o, mem[10] \neq mem[11]) \land (o, mem[11] \neq mem[12]) \land (o, mem[12] \neq mem[13]) \land
          (o, mem[13] \neq mem[14]) \land (o, mem[14] \neq mem[15])) goto b5; break;
       "4. The answer to this question is the same as the answers to questions: (A) 10 and 13 (B) 14 and
16 (C) 7 and 20 (D) 1 and 15 (E) 8 and 12"
  Questions 1, 10, 15, and 20 precede this one in the ordering.
\langle Cases for the big switch 19\rangle + \equiv
case pack(0,4,A): force(10,A); force(13,A); goto okay;
case pack(0,4,B): force(14,B); force(16,B); goto okay;
case pack(0, 4, C): force(7, C); force(20, C); goto okay;
case pack(0, 4, D): force(1, D); force(15, D); goto okay;
case pack(0, 4, E): force(8, E); force(12, E); goto okay;
case pack(1, 4, A): if (o, mem[10] \neq AA) goto okay; goto defer;
case pack(1, 4, B): case pack(1, 4, E): goto defer;
case pack(1, 4, C): if (o, mem[20] \neq CC) goto okay; goto defer;
case pack(1, 4, D): if ((o, mem[1] \neq DD) \lor (o, mem[15] \neq DD)) goto okay; goto bad;
       \langle Cases for the deferred switch 20 \rangle + \equiv
26.
case pack(1, 4, A): if ((o, mem[13] \equiv AA)) goto b5; break;
case pack(1,4,B): if ((o,mem[14] \equiv BB) \land (o,mem[16] \equiv BB)) goto b5; break;
case pack(1, 4, C): if (o, mem[7] \equiv CC) goto b5; break;
case pack(1, 4, E): if ((o, mem[8] \equiv EE) \land (o, mem[12] \equiv EE)) goto b5; break;
```

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27. "5. The answer to question 14 is: (A) B (B) E (C) C (D) A (E) D" \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0, 5, A): force(14, B); goto okay; case pack(0,5,B): force(14,E); goto okay; case pack(0, 5, C): force(14, C); goto okay; case pack(0, 5, D): force(14, A); goto okay; case pack(0, 5, E): force(14, D); goto okay; case pack(1, 5, A): deny(14, B); goto okay; case pack(1, 5, B): deny(14, E); goto okay; case pack(1, 5, C): deny(14, C); goto okay; case pack(1, 5, D): deny(14, A); goto okay; case pack(1, 5, E): deny(14, D); goto okay; 28. "6. The answer to this question is: (A) A (B) B (C) C (D) D (E) none of those" \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0,6,A): case pack(0,6,B): case pack(0,6,C): case pack(0,6,D): case pack(0,6,E): goto okay; case pack(1,6,A): case pack(1,6,B): case pack(1,6,C): case pack(1,6,D): case pack(1,6,E): goto bad; 29. "7. An answer that appears most often is: (A) A (B) B (C) C (D) D (E) E" $\langle \text{ Cases for the big switch } 19 \rangle + \equiv$ case pack(0,7,A): case pack(0,7,B): case pack(0,7,C): case pack(0,7,D): case pack(0,7,E): case pack(1,7,A): case pack(1,7,B): case pack(1,7,C): case pack(1,7,D): case pack(1,7,E): **goto** defer; 30. \langle Compute the distribution of answers 30 $\rangle \equiv$ register int dA = 0, dB = 0, dC = 0, dD = 0, dE = 0; for $(q = 1; q \le 20; q++)$ { o, y = mem[q];if $(y \equiv AA) dA ++$; else if $(y \equiv BB)$ dB +++;else if $(y \equiv CC)$ dC ++; else if $(y \equiv DD) dD ++$; else dE ++; o, dist[A] = dA;o, dist[B] = dB;o, dist[C] = dC;o, dist[D] = dD;o, dist[E] = dE; $trick = (1 \ll dA) + (1 \ll dB) + (1 \ll dC) + (1 \ll dD) + (1 \ll dE);$ /* see 14 below */ This code is used in section 16. 31. $\langle \text{Other global variables } 6 \rangle + \equiv$ /* the number of A's, B's, C's, D's, E's */ int dist[5]; int tie[5]; /* cases that are tied with at least one other */ int trick;

```
32.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(0,7,A): case pack(0,7,B): case pack(0,7,C): case pack(0,7,D): case pack(0,7,E):
    case pack(1,7,A): case pack(1,7,B): case pack(1,7,C): case pack(1,7,D): case pack(1,7,E):
    for (o, i = 0, j = dist[x]; i < 5; i++)
    if (o, dist[i] > j) break;
  if (\neg u \land i < 5) goto b5;
  if (u \wedge i \equiv 5) goto b5; break;
33.
       "8. Ignoring answers that appear equally often, the least common answer is: (A) A (B) B (C) C
(D) D (E) E"
\langle \text{ Cases for the big switch } 19 \rangle + \equiv
case pack(0,8,A): case pack(0,8,B): case pack(0,8,C): case pack(0,8,D): case pack(0,8,E):
  case pack(1,8,A): case pack(1,8,B): case pack(1,8,C): case pack(1,8,D): case pack(1,8,E):
  goto defer;
34.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(0,8,A): case pack(0,8,B): case pack(0,8,C): case pack(0,8,D): case pack(0,8,E):
    case pack(1,8,A): case pack(1,8,B): case pack(1,8,C): case pack(1,8,D): case pack(1,8,E):
    for (i = 0; i < 5; i++) o, tie[i] = 0;
  for (i = 0; i < 5; i++)
    for (j = i + 1; j < 5; j ++)
       if (oo, dist[i] \equiv dist[j]) oo, tie[i] = tie[j] = 1;
  for (i = 0, j = 100; i < 5; i++)
    if ((o, tie[i] \equiv 0) \land (o, dist[i] < j)) j = dist[i];
                                                         /* possibly j \equiv 100 */
  if (\neg u \land j \neq dist[x]) goto b5;
  if (u \wedge j \equiv dist[x]) goto b5; break;
35.
        "9. The sum of all question numbers whose answers are correct and the same as this one is:
(A) \in [59..62] (B) \in [52..55] (C) \in [44..49] (D) \in [59..67] (E) \in [44..53]"
\langle Cases for the big switch 19\rangle + \equiv
case pack(0,9,A): case pack(0,9,B): case pack(0,9,C): case pack(0,9,D): case pack(0,9,E):
  case pack(1,9,A): case pack(1,9,B): case pack(1,9,C): case pack(1,9,D): case pack(1,9,E):
  goto defer;
36.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(0,9,A): case pack(0,9,B): case pack(0,9,C): case pack(0,9,D): case pack(0,9,E):
  case pack(1,9,A): case pack(1,9,B): case pack(1,9,C): case pack(1,9,D): case pack(1,9,E):
  for (i = 0, i = 1; i < 20; i ++)
    if ((o, falsity[i] \equiv 0) \land (o, mem[i] \equiv (1 \ll x))) \ j += i;
  switch (x) {
  case A: i = (j \ge 59 \land j \le 62); break;
  case B: i = (j \ge 52 \land j \le 55); break;
  case C: i = (j \ge 44 \land j \le 49); break;
  case D: i = (j \ge 59 \land j \le 67); break;
  case E: i = (j \ge 44 \land j \le 53); break;
  if (\neg u \wedge \neg i) goto b5;
  if (u \wedge i) goto b5; break;
```

14

```
37.
       "10. The answer to question 17 is: (A) D (B) B (C) A (D) E (E) wrong"
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 10, A): force(17, D); goto okay;
case pack(0, 10, B): force(17, B); goto okay;
case pack(0, 10, C): force(17, A); goto okay;
case pack(0, 10, D): force(17, E); goto okay;
case pack(0, 10, E): if (o, falsity[17] \equiv 1) goto okay; goto bad;
case pack(1, 10, A): deny(17, D); goto okay;
case pack(1, 10, B): deny(17, B); goto okay;
case pack(1, 10, C): deny(17, A); goto okay;
case pack(1, 10, D): deny(17, E); goto okay;
case pack(1, 10, E): if (o, falsity[17] \equiv 1) goto bad; goto okay;
38.
       "11. The number of questions whose answer is D is: (A) 2 (B) 3 (C) 4 (D) 5 (E) 6"
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 11, A): case pack(0, 11, B): case pack(0, 11, C): case pack(0, 11, D): case pack(0, 11, E):
  case pack(1, 11, A): case pack(1, 11, B): case pack(1, 11, C): case pack(1, 11, D): case pack(1, 11, E):
  goto defer;
39.
       \langle \text{ Cases for the deferred switch } 20 \rangle + \equiv
case pack(0, 11, A): case pack(0, 11, B): case pack(0, 11, C): case pack(0, 11, D): case pack(0, 11, E):
  case pack(1, 11, A): case pack(1, 11, B): case pack(1, 11, C): case pack(1, 11, D): case pack(1, 11, E):
  if (\neg u \land (o, dist[D] \neq x + 2)) goto b5;
  if (u \land (o, dist[D] \equiv x + 2)) goto b5; break;
       "12. The number of other questions with the same answer as this one is the same as the number of
questions with answer: (A) B (B) C (C) D (D) E (E) none of those"
  Here we see that (E) is quite different from 'A'. "None of those" means that options A, B, C, and D are
all false.
  And there's more subtlety too, because the dist table would be different if we had really chosen A instead
of E. Suppose 12E has been selected. Then 12A does not mean "the number of other questions with answer A
is the same as the number questions with answer B"; it means "the number of other questions with answer E
is the same as the number questions with answer B." Thus 12E is true if and only if dist[B], dist[C], dist[D],
and dist[E] all differ from dist[E] - 1. But 12A is true if and only if dist[A] - 1 = dist[B]. Got that?
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 12, A): case pack(0, 12, B): case pack(0, 12, C): case pack(0, 12, D): case pack(0, 12, E):
  case pack(1, 12, A): case pack(1, 12, B): case pack(1, 12, C): case pack(1, 12, D): case pack(1, 12, E):
  goto defer;
41.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(0, 12, A): case pack(0, 12, B): case pack(0, 12, C): case pack(0, 12, D):
  if (oo, dist[x] - 1 \neq dist[x+1]) goto b5; break;
case pack(0, 12, E): if ((oo, dist[E] - 1 \equiv dist[B]) \lor (o, dist[E] - 1 \equiv dist[C]) \lor
         (o, dist[E] - 1 \equiv dist[D]) goto b5; break;
case pack(1, 12, A): case pack(1, 12, B): case pack(1, 12, C): case pack(1, 12, D):
  if (oo, dist[x] - 1 \equiv dist[x+1]) goto b5; break;
case pack(1, 12, E): if ((oo, dist[E] - 1 \neq dist[B]) \land (o, dist[E] - 1 \neq dist[C]) \land
         (o, dist[E] - 1 \neq dist[D])) goto b5; break;
```

```
42.
       "13. The number of questions whose answer is E is: (A) 5 (B) 4 (C) 3 (D) 2 (E) 1"
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 13, A): case pack(0, 13, B): case pack(0, 13, C): case pack(0, 13, D): case pack(0, 13, E):
  case pack(1, 13, A): case pack(1, 13, B): case pack(1, 13, C): case pack(1, 13, D): case pack(1, 13, E):
  goto defer;
43.
       \langle Cases for the deferred switch 20 \rangle + \equiv
case pack(0, 13, A): case pack(0, 13, B): case pack(0, 13, C): case pack(0, 13, D): case pack(0, 13, E):
  case pack(1, 13, A): case pack(1, 13, B): case pack(1, 13, C): case pack(1, 13, D): case pack(1, 13, E):
  if (\neg u \land (o, dist[E] \neq 5 - x)) goto b5:
  if (u \land (o, dist[E] \equiv 5 - x)) goto b5; break;
       "14. No answer appears exactly this many times: (A) 2 (B) 3 (C) 4 (D) 5 (E) none of those"
  Here I can't help using a dirty trick. Case (E) means that the digits 2, 3, 4, 5 all appear in the dist table.
But the dist table sums to 20, so it must also contain 6! This condition happens if and only if the trick
computed above is (1 \ll 2) + (1 \ll 3) + (1 \ll 4) + (1 \ll 5) + (1 \ll 6), which is #7c.
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 14, A): case pack(0, 14, B): case pack(0, 14, C): case pack(0, 14, D): case pack(0, 14, E):
  case pack(1, 14, A): case pack(1, 14, B): case pack(1, 14, C): case pack(1, 14, D): case pack(1, 14, E):
  goto defer;
       \langle \text{ Cases for the deferred switch } 20 \rangle + \equiv
case pack(0, 14, A): case pack(0, 14, B): case pack(0, 14, C): case pack(0, 14, D):
  for (i = 0; i < 5; i++) if (o, dist[i] \equiv x + 2) goto b5;
case pack(0, 14, E): if (trick \neq {}^{\#}7c) goto b5; break;
case pack(1, 14, A): case pack(1, 14, B): case pack(1, 14, C): case pack(1, 14, D):
  for (i = 0; i < 5; i++) if (o, dist[i] \equiv x + 2) break;
  if (i \equiv 5) goto b5;
  break:
case pack(1, 14, E): if (trick \equiv {}^{\#}7c) goto b5; break;
       "15. The set of odd-numbered questions with answer A is: (A) {7} (B) {9} (C) not {11} (D) {13}
46.
(E) {15}"
  In the falsifying case, I note that question 3 has been treated earlier in the ordering.
\langle Cases for the big switch 19\rangle + \equiv
case pack(0, 15, A): case pack(0, 15, E): goto bad;
case pack(0, 15, B): force(9, A); deny(11, A); deny(13, A); goto odd\_denials;
case pack(0, 15, D): deny(9, A); deny(11, A); force(13, A); goto odd\_denials;
odd\_denials: deny(1, A); deny(3, A); deny(5, A); deny(7, A);
  deny(17, A); deny(19, A); goto okay;
case pack(1, 15, A): case pack(1, 15, E): goto okay;
case pack(0, 15, C): case pack(1, 15, B): case pack(1, 15, D):
  if (o, mem[3] \equiv AA) goto okay; goto defer;
case pack(1, 15, C): goto defer;
```

case pack(0, 17, D): force(10, A); goto okay;

case pack(1,17,A): deny(10,C); goto okay; case pack(1,17,B): deny(10,D); goto okay; case pack(1,17,C): deny(10,B); goto okay; case pack(1,17,D): deny(10,A); goto okay;

case pack(0, 17, E): deny(10, E); if $(o, falsity[10] \equiv 0)$ goto okay; goto bad;

case pack(1, 17, E): deny(10, E); if $(o, falsity[10] \equiv 0)$ goto bad; goto okay;

16

51. "18. The number of prime-numbered questions whose answers are vowels is: (A) prime (B) square (C) odd (D) even (E) zero" All of the prime-numbered questions appear before this one in the ordering. #define isvowel(q) (o, (mem[q] & #11) ? 1 : 0) \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0, 18, A): case pack(0, 18, B): case pack(0, 18, C): case pack(0, 18, D): case pack(0, 18, E): case pack(1, 18, A): case pack(1, 18, B): case pack(1, 18, C): case pack(1, 18, D): case pack(1, 18, E): i = isvowel(2) + isvowel(3) + isvowel(5) + isvowel(7) + isvowel(7) + isvowel(8) +isvowel(11) + isvowel(13) + isvowel(17) + isvowel(19); $j = (1 \ll j) \& magic[x];$ if $(\neg u \land j)$ goto okay; goto bad; if $(u \wedge \neg j)$ goto okay; goto bad; **52.** $\langle \text{ Other global variables } 6 \rangle + \equiv$ int $magic[] = \{(1 \ll 2) + (1 \ll 3) + (1 \ll 5) + (1 \ll 7), \}$ $(1 \ll 0) + (1 \ll 1) + (1 \ll 4),$ $(1 \ll 1) + (1 \ll 3) + (1 \ll 5) + (1 \ll 7),$ $(1 \ll 0) + (1 \ll 2) + (1 \ll 4) + (1 \ll 6) + (1 \ll 8),$ $(1 \ll 0)$ 53. "19. The last question whose answer is B is: (A) 14 (B) 15 (C) 16 (D) 17 (E) 18" Question 20 has been considered before this one in the ordering. \langle Cases for the big switch 19 $\rangle + \equiv$ case pack(0, 19, A): force(14, B); deny(15, B); deny(16, B); deny(17, B); deny(18, B); deny(20, B); **goto** okay; case pack(0, 19, B): goto bad; case pack(0,19,C): force(16,B); deny(17,B); deny(18,B); deny(20,B); **goto** okay; case pack(0, 19, D): force(17, B); deny(18, B); deny(20, B); **goto** okay; case pack(0, 19, E): force(18, B); deny(20, B); **goto** okay; case pack(1, 19, B): goto okay; case pack(1, 19, A): case pack(1, 19, C): case pack(1, 19, D): case pack(1, 19, E): if $(o, mem[20] \equiv BB)$ goto okay; goto defer; $\langle \text{ Cases for the deferred switch } 20 \rangle + \equiv$ case pack(1, 19, A): if $((o, mem[14] \equiv BB) \land (o, mem[15] \neq BB) \land (o, mem[16] \neq BB) \land$ $(o, mem[17] \neq BB) \land (o, mem[18] \neq BB))$ goto b5; break; case pack(1, 19, C): if $((o, mem[16] \equiv BB) \land (o, mem[17] \neq BB) \land (o, mem[18] \neq BB))$ goto b5; break; case pack(1, 19, D): if $((o, mem[17] \equiv BB) \& (o, mem[18] \neq BB))$ goto b5; break;

case pack(1, 19, E): if $(o, mem[18] \equiv BB)$ goto b5; break;

18

55. "20. The maximum score that can be achieved on this test is: (A) 18 (B) 19 (C) 20 (D) indeterminate (E) achievable only by getting this question wrong"

This climactic final question, which Don Woods admits was expressly "designed to create difficulties," obviously needs some special consideration. Discussion in my book shows that option (D) is always false. I assume here that (E) is also false, although that hypothesis must be confirmed by examining outputs of this program. As to (A), (B), (C), I essentially reword the problem to say not "the maximum score" but "the score achieved by all outputs of this run."

```
\langle \text{ Cases for the big switch } 19 \rangle + \equiv
case pack(0, 20, A): case pack(0, 20, B): case pack(0, 20, C):
  if (score \equiv 18 + x) goto okay; goto bad;
case pack(0, 20, D): case pack(0, 20, E): goto bad;
case pack(1, 20, A): case pack(1, 20, B): case pack(1, 20, C):
  if (score \neq 18 + x) goto okay; goto bad;
case pack(1, 20, D): case pack(1, 20, E): goto okay;
```

§56 BACK-20Q INDEX 19

56. Index.

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```
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B: \underline{1}.
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bb: \underline{7}.
BB: \underline{1}, 5, 21, 22, 26, 30, 53, 54.
                                                                             odd\_denials: \underline{46}.
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                                                                                   48, 50, 51, 53, 55.
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b2: \underline{12}.
                                                                             oo: <u>1</u>, 12, 15, 16, 24, 34, 41, 48.
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                                                                             ull: \underline{1}.
higuy: \underline{6}, 15, 23.
                                                                             vbose: 1, 3, 7, 8, 12, 15, 16.
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                                                                             x: \quad \underline{1}, \quad \underline{7}, \quad \underline{8}.
ii: \underline{11}.
                                                                             y: <u>1</u>.
innerforce: 8, 15.
                                                                             z: 9.
isvowel: \underline{51}.
j: \underline{1}.
k: <u>1</u>.
l: \underline{1}, \underline{11}.
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20 NAMES OF THE SECTIONS BACK-20Q

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 \left\langle \text{Backtrack through all possibilities } 12 \right\rangle \quad \text{Used in section 1.}   \left\langle \text{Cases for the big switch } 19, 21, 23, 25, 27, 28, 29, 33, 35, 37, 38, 40, 42, 44, 46, 48, 50, 51, 53, 55} \right\rangle \quad \text{Used in section } 12.   \left\langle \text{Cases for the deferred switch } 20, 22, 24, 26, 32, 34, 36, 39, 41, 43, 45, 47, 49, 54} \right\rangle \quad \text{Used in section } 16.   \left\langle \text{Check for solution and } \mathbf{goto} \quad b5 \quad 16 \right\rangle \quad \text{Used in section } 12.   \left\langle \text{Compute the distribution of answers } 30 \right\rangle \quad \text{Used in section } 16.   \left\langle \text{Other global variables } 6, 31, 52 \right\rangle \quad \text{Used in section } 1.   \left\langle \text{Print a solution } 17 \right\rangle \quad \text{Used in section } 16.   \left\langle \text{Print a status report and reset } thresh \quad 14 \right\rangle \quad \text{Used in section } 12.   \left\langle \text{Print the profile } 2 \right\rangle \quad \text{Used in section } 1.   \left\langle \text{Propagate forced consequences, possibly going to } bad \quad 15 \right\rangle \quad \text{Used in section } 12.   \left\langle \text{Restore saved state } 13 \right\rangle \quad \text{Used in section } 12.   \left\langle \text{Set the initial constraints } 5 \right\rangle \quad \text{Used in section } 1.   \left\langle \text{Subroutines } 7, 8, 9, 10, 11 \right\rangle \quad \text{Used in section } 1.   \left\langle \text{Subroutines } 7, 8, 9, 10, 11 \right\rangle \quad \text{Used in section } 1.
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