

Really good 2D systems of high purity can now be made. The Quantum Hall effect blew minds.

The hall resistance is quantized, while the magnetoresistance vanishes (magnetoresistance is where the regular resistance changes in response to applied field.)

electrons

Statistics change from boson to fermion and back.

The electrons absorb flux quanta?

Epitaxial growth of semiconductors with different band gaps make thin potential wells. GaAs/AlGaAs 2d carriers have mean free path λ of 0.1mm. HUGE

Magnetic fields cause splitting in Landau levels. Energy now comes in quanta of cyclotron frequency. Spin degeneracy is lifted. degeneracy is eB/h states per area.

Cyclotron $\omega = qB/m$.

$\nu = n/d$ The filling factor is actual density divided by Landau level state density.

$h/e = \phi$ the flux quantum. ν is electron to flux density (Duality between vortices and particles? Circuit duality? Circuit duality with respect to some reference impedance. Riemann Sphere. Inversion. Poles to zeros). Electrons per flux quantum.

h/e^2 is the resistance quantum

Experimental condition. Measure Current. Measure voltage orthogonal and along. Divide to get magnetoresistance and hall resistance

Classical hall resistance is $R = B/ne$. Convenient for measuring n the density of conducting electrons. The quantum hall effect has plateaus that follow around this line. So it does increase roughly linearly.

At vanishing magnetoresistance, the system is insulating.

The width of a Landau band is visible in the diagram?

FQHE the filling factor is an exact fraction.

Coulomb force drives correlation between electrons,

Presence of magnetic field requires wavefunction to have as many zeros and flux quantum penetrate per unit area. What is a zero in a many body wavefunction? Fixing all variables except one?

Magnetic Length $l = \sqrt{\hbar/eB}$. Each zero of FQHE state "heals" on this length scale?

Vortices are charge deficits. That's bad? So they attract electrons. Wait. What? The vortex is a zero of the electron wavefunction. How can it attract electrons? The other electrons?

Is the introduction of magnetic field moving the electrons such that they cancel the incoming flux?

Vortex generating flux quanta

Composite Fermions. Some shaky way you combine vortices and electrons and then you couple those to more vortices?

0.1 Conductance viewed as transmission

Traditional view is response to applied electric field. Different view is conduction as due to injection of electrons at one end.

Semiclassical thinking dominates traditional theory. Use quantum band structure and quantum scattering cross sections, but then use classical movement otherwise using distributions in these propagating states. Boltzmann like

Kubo gave first totally quantum version. Extended theory of polarization.

Circuit theory does not give voltage as cause and current as effect. New approach is current causes self consistent buildup of charge, hence electric field

Field in tunnelling barrier is localized around barrier. Not mean free path inside contacts.

0.2 Nakahara Homotopy

Homotopy is the study of maps from well understood spaces to complicated ones. S^1 is the circle, which maps to loops in the space. Continuous deformations between images

Arcwise connectedness: If there exist a path between any two points

Means fundamental group is independent of basepoint

Conjugation by the connection path

Topologically distinct states of fields -

Consider the following geometry. You have a function defined in space $S(x)$. You can divide the space into distinct classes by specifying surfaces $S < a$. This is the analog of topologically inequivalent field configurations $F[\phi] < k_B T$, divides the function space into different classes. For actions, you can only borrow something something \hbar of action. Of you can only borrow such and such energy for a time that is under the Heisenberg uncertainty. Or momentum for a space less than \hbar .

The mixing of the topology of the domain (Balls of various dimensionality - loops spheres etc) going into

Two field configurations are physically (homeomorphic, homotopic) if they can be deformed into one another while never passing through a state of energy greater than kT .

The functional integral (volume integral) may be subdivided into an integral of loops (sum of tubes) or an integral of spheres (sum of shells).

Berry phase is kind of like an order parameter. The degeneracy points are the vortex cores of parametrized hamiltonians

Model manifolds as order parameters

r dimensional contours external to a core a of d' dimension + 1 radial direction = d of space.

On the contour (Which is S_r) we map to the order parameter (Which is often $S_{d'}$ for n vector models, but could be a projective space for nematic crystals or whatever). Therefore, each core gets a member of the homotopy group assigned to it.

Magnetic field. Vector potential due to magnetic field. Some sort of potential for A? In gauge $\nabla \cdot A = 0$, then $A = \nabla \times C$. Hence C = coulomb of B, A = biot savart of B, just like sliding down one using A,B,J.

If we have two wires move around each other, the ensuing loop integral of B is analogous to phase

Phase dynamics

$$\nabla \times A + \dot{F} = B$$

$$\nabla \times F + \dot{A} = E$$

Abelianized homotopy = homology

Edge groups and ability to contract