# Contents

# Chapter 1

# Triangle

Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}, \mathbf{c} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \tag{1.1}$$

## 1.1. Vectors

## 1.2. Median

## 1.3. Altitude

1.3.1. $\mathbf{D}_1$ is a point on $BC$ such that

$$AD_1 \perp BC \tag{1.3.1.1}$$

and $AD_1$ is defined to be the altitude. Find the normal vector of $AD_1$.

**Solution:** Given

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.3.1.2}$$

$$\mathbf{B} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1.3.1.3}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{1.3.1.4}$$

The normal vector of $AD_1$ is orthogonal to $AD_1$ and hence parallel to $BC$

Direction vector $\mathbf{m_{BC}}$

$$= \mathbf{C} - \mathbf{B} \tag{1.3.1.5}$$

$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1.3.1.6}$$

$$m_{BC} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{1.3.1.7}$$

Normal vector of $AD_1$ is

$$\mathbf{n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{1.3.1.8}$$

1.3.2. Find the equation of $AD_1$.

**Solution:** from (**??**)

$$\mathbf{n} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{1.3.2.1}$$

The equation of $AD_1$ is

$$\mathbf{n}^\top (\mathbf{x} - \mathbf{A}) = 0 \tag{1.3.2.2}$$

$$\implies \begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.3.2.3}$$

$$\begin{pmatrix} 3 & 4 \end{pmatrix} \mathbf{x} = 25 \tag{1.3.2.4}$$

see Fig. 1.1

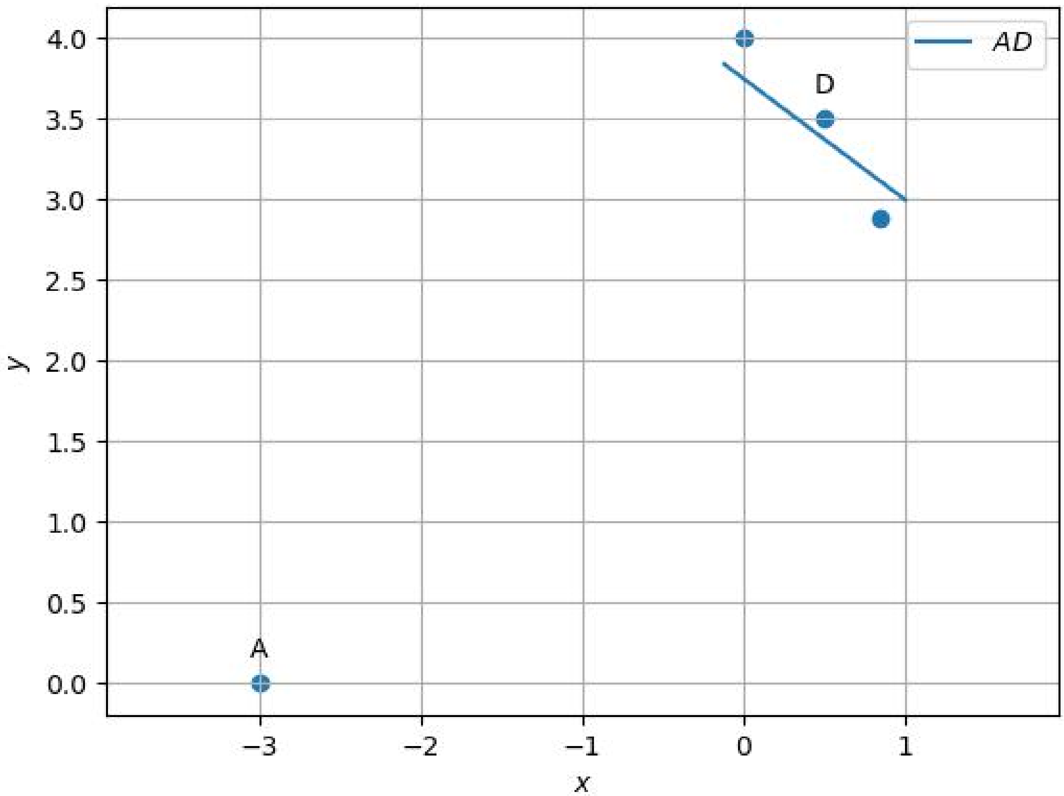


Figure 1.1: Line AD

1.3.3. Find the equations of the altitudes $BE_1$ and $CF_1$ to the sides $AC$ and $AB$ respectively.

**Solution:** Given

$$\mathbf{A} = \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.3.3.1}$$

$$\mathbf{B} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{1.3.3.2}$$

$$\mathbf{C} = \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{1.3.3.3}$$

Direction vector

$$\mathbf{m_{AB}} = \mathbf{B} - \mathbf{A} \tag{1.3.3.4}$$

$$= \begin{pmatrix} -3 \\ 0 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.3.3.5}$$

$$= \begin{pmatrix} -4 \\ -3 \end{pmatrix} \tag{1.3.3.6}$$

$$\mathbf{m_{AC}} = \mathbf{C} - \mathbf{A} \tag{1.3.3.7}$$

$$= \begin{pmatrix} 0 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 3 \end{pmatrix} \tag{1.3.3.8}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{1.3.3.9}$$

$$\tag{1.3.3.10}$$

Normal vector of $BE_1$ is orthogonal to $BE_1$ and hence parallel to $AC$ and normal vector of $CF_!$ is orthogonal to $CF_1$ and hence parallel to $AB$

$$\mathbf{n_{BE_1}} = \mathbf{m_{AC}} \tag{1.3.3.11}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{1.3.3.12}$$

$$\mathbf{n_{CF_1}} = \mathbf{m_{AB}} \tag{1.3.3.13}$$

$$= \begin{pmatrix} -4 \\ -3 \end{pmatrix} \tag{1.3.3.14}$$

$$\tag{1.3.3.15}$$

Equation of line is represented by :

$$\mathbf{n}^\top \left(\mathbf{x} - \mathbf{p}\right) = 0 \tag{1.3.3.16}$$

(a) The equation of line $CF_1$

$$\mathbf{n}_{CF_1}^\top \left(\mathbf{x} - \mathbf{C}\right) = 0 \tag{1.3.3.17}$$

$$\mathbf{n}_{CF_1}^\top \mathbf{x} = \mathbf{n}_{CF_1}^\top \mathbf{C} \tag{1.3.3.18}$$

$$\begin{pmatrix} -4 \\ -3 \end{pmatrix}^\top \mathbf{x} = \begin{pmatrix} -4 \\ -3 \end{pmatrix}^\top \begin{pmatrix} 0 \\ 4 \end{pmatrix} \tag{1.3.3.19}$$

$$\begin{pmatrix} -4 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{1.3.3.20}$$

(b) The equation of line $BE_1$

$$\mathbf{n}_{BE_1}^{\top}\left(\mathbf{x}-\mathbf{B}\right)=0 \tag{1.3.3.21}$$

$$\mathbf{n}_{CF_1}^{\top}\mathbf{x}=\mathbf{n}_{BE_1}^{\top}\mathbf{B} \tag{1.3.3.22}$$

$$\begin{pmatrix}-1\\1\end{pmatrix}^{\top}\mathbf{x}=\begin{pmatrix}-1\\1\end{pmatrix}^{\top}\begin{pmatrix}-3\\0\end{pmatrix} \tag{1.3.3.23}$$

$$\begin{pmatrix}-1 & 2\end{pmatrix}\mathbf{x}=0 \tag{1.3.3.24}$$

see Fig. 1.2

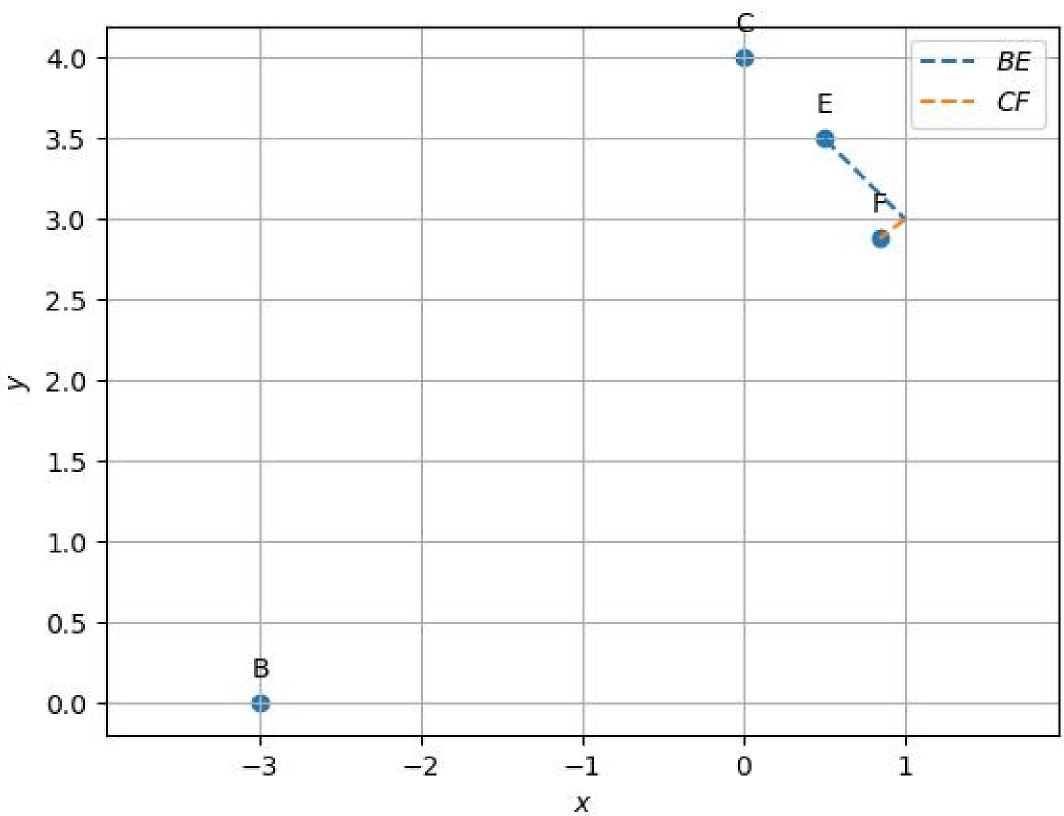


Figure 1.2: Lines $\mathbf{BE_1} and \mathbf{CF_1}$

1.3.4. Find the intersection $\mathbf{H}$ of $BE_1$ and $CF_1$.

**Solution:** Equation of $\mathbf{BE_1}$

$$\begin{pmatrix} -1 & 1 \end{pmatrix} \mathbf{x} = 0 \tag{1.3.4.1}$$

Equation of $\mathbf{CF_1}$ //

$$\begin{pmatrix} -4 & -3 \end{pmatrix} \mathbf{x} = 0 \tag{1.3.4.2}$$

Therefore ,we need to solve the following equation to get $\mathbf{H}$ :

$$\begin{pmatrix} -1 & 1 \\ -4 & -3 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{1.3.4.3}$$

which can be solved as

$$\begin{pmatrix} -1 & 1 & 0 \\ -4 & -3 & 0 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow -R_1} \begin{pmatrix} 1 & -1 & 0 \\ -4 & -3 & 0 \end{pmatrix} \tag{1.3.4.4}$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 6R_1} \begin{pmatrix} 1 & -1 & 0 \\ -2 & 9 & 0 \end{pmatrix} \tag{1.3.4.5}$$

$$\xleftrightarrow{R_2 \leftarrow \frac{R_2}{12}} \begin{pmatrix} 1 & -1 & 0 \\ \frac{-1}{6} & \frac{3}{4} & 0 \end{pmatrix} \tag{1.3.4.6}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 + 2R_2} \begin{pmatrix} \frac{2}{3} & \frac{1}{2} & 0 \\ \frac{-1}{6} & \frac{3}{4} & 0 \end{pmatrix} \tag{1.3.4.7}$$

yielding

$$\mathbf{H} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{1.3.4.8}$$

See Fig. 1.3

Figure 1.3: Intersection point $\mathbf{H}$ of altitudes B$E_1$ and C$F_1$ plotted using python

1.3.5. Verify that

$$\left(\mathbf{A}-\mathbf{H}\right)^{\top}\left(\mathbf{B}-\mathbf{C}\right)=0 \tag{1.3.5.1}$$

**Solution:**

$$\mathbf{A}=\begin{pmatrix}1\\3\end{pmatrix} \tag{1.3.5.2}$$

$$\mathbf{B}=\begin{pmatrix}-3\\0\end{pmatrix} \tag{1.3.5.3}$$

$$\mathbf{C}=\begin{pmatrix}0\\4\end{pmatrix} \tag{1.3.5.4}$$

$$\mathbf{H}=\begin{pmatrix}0\\0\end{pmatrix} \tag{1.3.5.5}$$

$$\mathbf{A}-\mathbf{H}=\begin{pmatrix}1\\3\end{pmatrix},\ \mathbf{B}-\mathbf{C}=\begin{pmatrix}-3\\-4\end{pmatrix} \tag{1.3.5.6}$$

$$\implies \left(\mathbf{A}-\mathbf{H}\right)^{\top}\left(\mathbf{B}-\mathbf{C}\right)=\begin{pmatrix}1 & 3\end{pmatrix}\begin{pmatrix}-3\\-4\end{pmatrix}=-15 \tag{1.3.5.7}$$

see Fig. 1.4

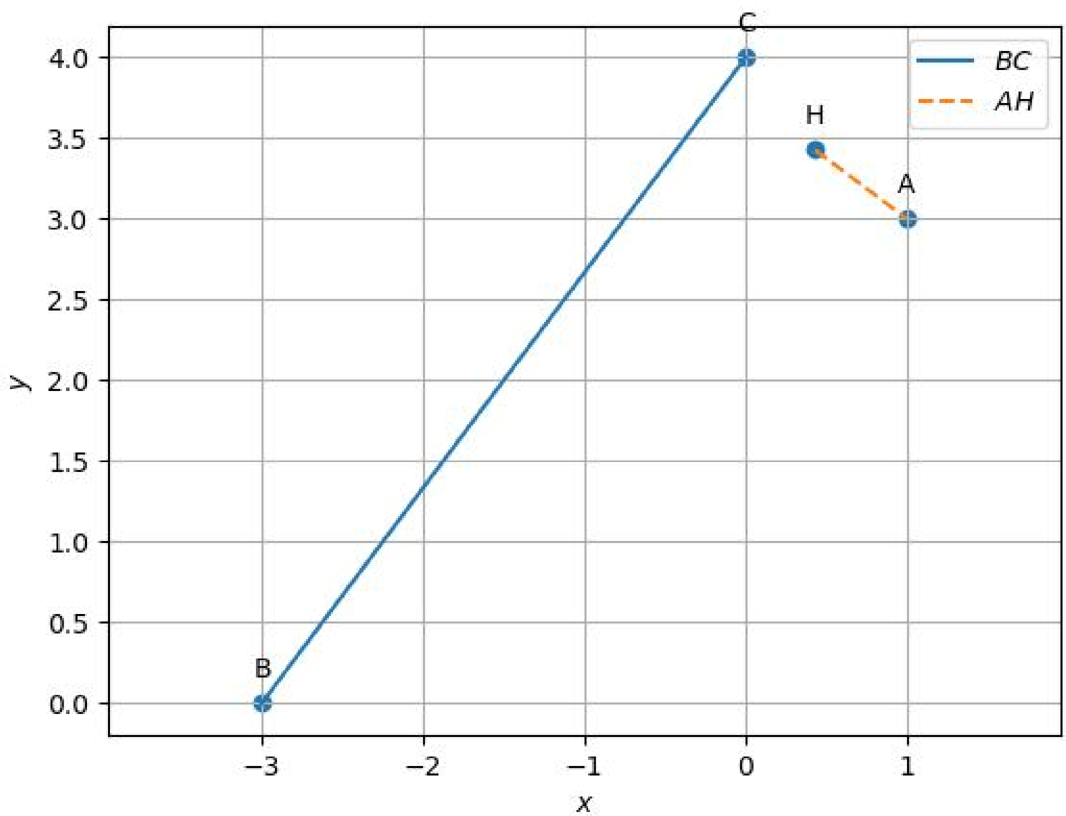


Figure 1.4: Plot of points A,B,C and H