

MATH-COMPUTING

January 20, 2024

1. **Question(MATH-12.10.5.17):** Let \mathbf{a} and \mathbf{b} be two unit vectors and θ is the angle between them. Then $\mathbf{a} + \mathbf{b}$ is a unit vector.

(A) $\theta = \frac{\pi}{4}$

(B) $\theta = \frac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$

(D) $\theta = \frac{2\pi}{3}$

solution:

Given,

$$\|\mathbf{a}\| = \|\mathbf{b}\| = 1, \quad (1)$$

$$\|\mathbf{a} + \mathbf{b}\| = 1 \quad (2)$$

Squaring on both sides of (2), we get

$$\|\mathbf{a} + \mathbf{b}\|^2 = 1^2 \quad (3)$$

$$\implies \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}^T \mathbf{b} = 1 \quad (4)$$

Substituting (1) in (4), we get

$$\implies 1 + 1 + 2(\|\mathbf{a}\| \|\mathbf{b}\| \cos \theta) = 1 \quad (5)$$

$$\implies 2 + 2(\|\mathbf{a}\| \|\mathbf{b}\| \cos \theta) = 1 \quad (6)$$

$$\implies 2(\|\mathbf{a}\| \|\mathbf{b}\| \cos \theta) = -1 \quad (7)$$

$$\implies \|\mathbf{a}\| \|\mathbf{b}\| \cos \theta = \frac{-1}{2} \quad (8)$$

Substituting (1) in (8), we get

$$\implies \cos \theta = \frac{-1}{2} \quad (9)$$

$$\implies \theta = \frac{2\pi}{3} \quad (10)$$

Let,

$$\mathbf{a} = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} \quad (11)$$

$$\mathbf{b} = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} \quad (12)$$

Matrix multiplication of $\mathbf{a} \cdot \mathbf{b}$ is:

$$\mathbf{a}^\top \mathbf{b} = \cos (\theta_1 - \theta_2) = \frac{-1}{2} \quad (13)$$

$$\theta_1 - \theta_2 = \cos^{-1} \left(\frac{-1}{2} \right) \quad (14)$$

$$\theta_1 - \theta_2 = \frac{2\pi}{3} \quad (15)$$

$$\theta_1 = \theta_2 + \frac{2\pi}{3} \quad (16)$$

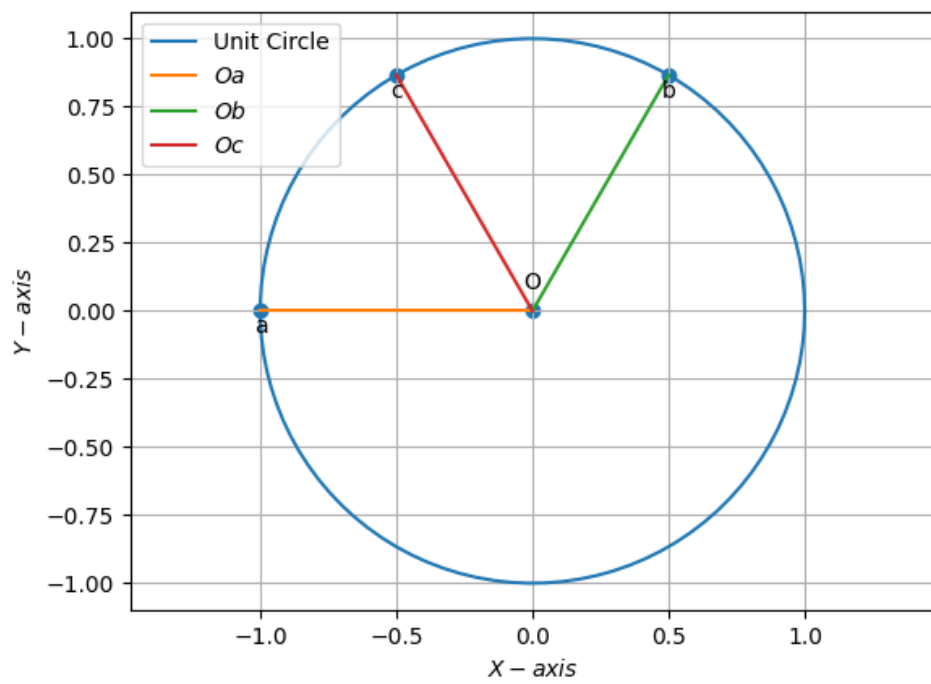


Figure 1