MATH-COMPUTING

December 20, 2023

1. **Question(MATH-12.10.5.17):** Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a}$+$\vec{b}$ is a unit vector.

   (A) $\theta = \frac{\pi}{4}$
   (B) $\theta = \frac{\pi}{3}$
   (C) $\theta = \frac{\pi}{2}$
   (D) $\theta = \frac{2\pi}{3}$

   solution:

   Assuming the co-ordinates:
   A=$\begin{pmatrix} -2.31 \\ 3.98 \end{pmatrix}$, B=$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, c=$\begin{pmatrix} 1.69 \\ 0 \end{pmatrix}$

   To find angle B in a triangle ABC:
   $\cos B \triangleq \frac{(A-B)^\top (C-B)}{\|A-B\|\|C-B\|}$

   A - B = $\begin{pmatrix} -2.31 \\ 3.98 \end{pmatrix}$-$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ = $\begin{pmatrix} -2.31 \\ 3.98 \end{pmatrix}$

   C - B = $\begin{pmatrix} 1.69 \\ 0 \end{pmatrix}$-$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ = $\begin{pmatrix} 1.69 \\ 0 \end{pmatrix}$

   $$\|A-B\| \triangleq \sqrt{(A-B)^\top (A-B)} = \sqrt{(-2.31-3.98)\begin{pmatrix} -2.31 \\ 3.98 \end{pmatrix}} = 4.60$$

   $$\|C-B\| \triangleq \sqrt{(C-B)^\top (C-B)} = \sqrt{(1.69-0)\begin{pmatrix} 1.69 \\ 0 \end{pmatrix}} = 1.69$$

Therefore:

$$\cos B = \frac{(-2.31-3.98)\begin{pmatrix}1.69\\0\end{pmatrix}}{(4.60)(1.69)} = \frac{-3.903}{7.774} = 0.501$$

$$B = \cos^{-1}(0.5) = 120^{\circ}$$

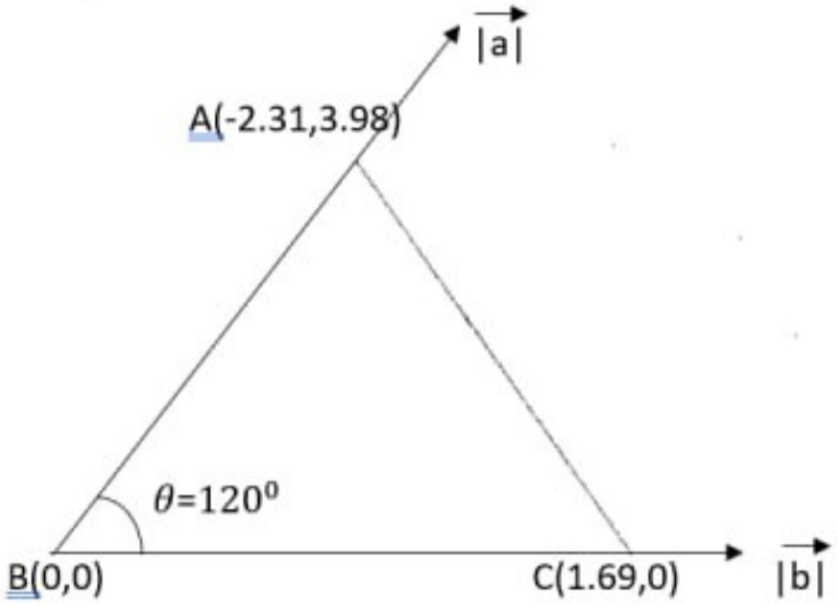

Figure 1: vectors