Investigation of some Stochastic Scheduling Problems

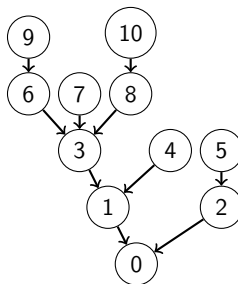## Master's Thesis in Computer Science

Philipp Müller

Technische Universität München

November 20, 2013

Problem statement

- Set of tasks
- Task times exponentially distributed with parameter 1
- Dependencies form intree structure
- Non-preemptive scheduling
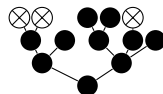- Goal: Minimize total expected make span
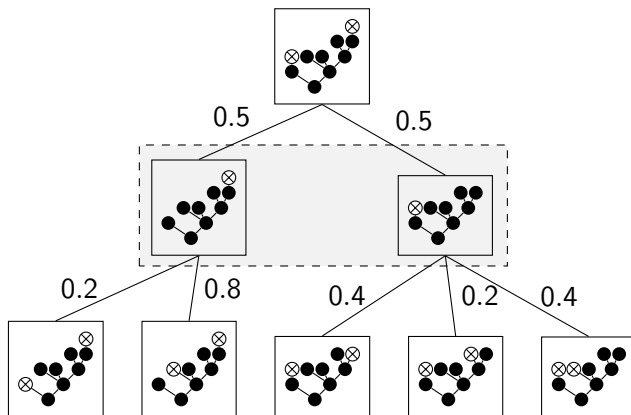
Schedules and snapshots

Schedules

- Describes the order of tasks
- Our scenario: non-deterministic and non-preemptive
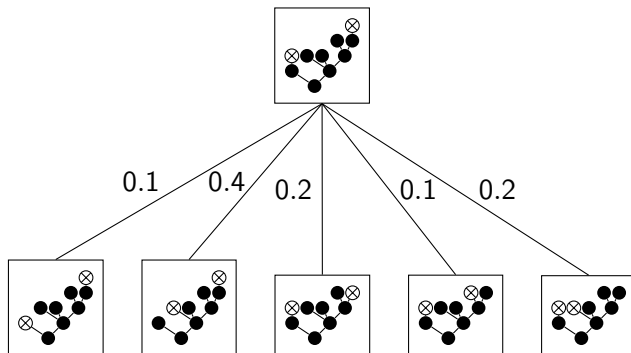- Scheduling *strategy* influences expected make span

Snapshots

- States of a schedule
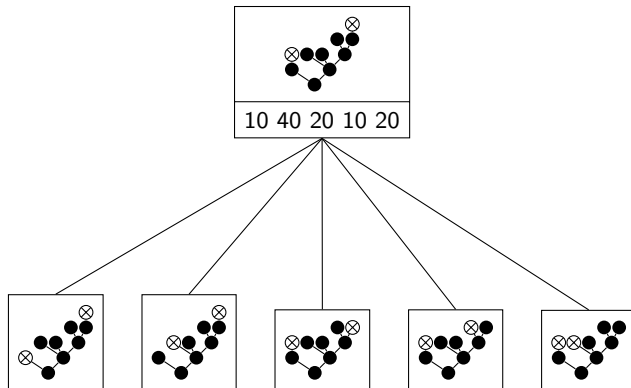- Consist of current intree and the set of scheduled tasks
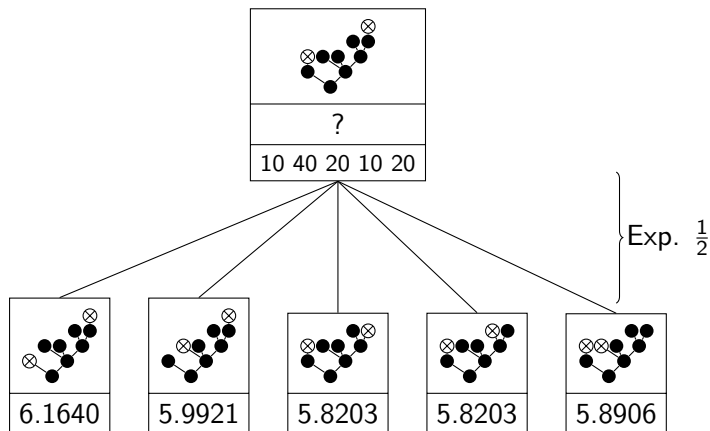
Schedule visualization

Schedule visualization

Schedule visualization

Schedule visualization

$$\frac{1}{2} + 0.1 \cdot 6.1640 + 0.4 \cdot 5.9921 + 0.2 \cdot 5.8203 + \cdots = 6.43745$$

Schedule visualization
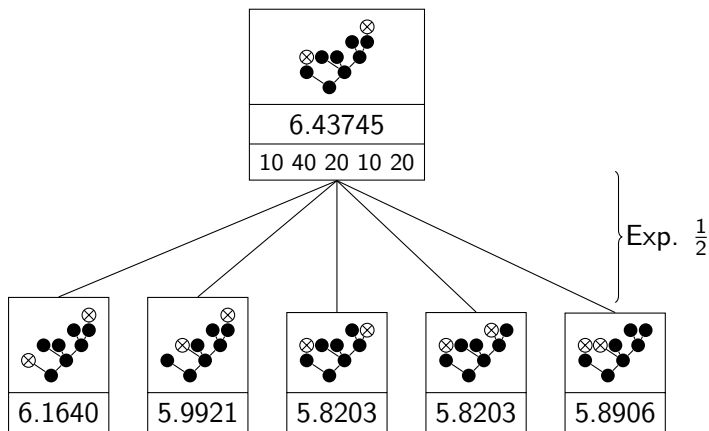
$$\frac{1}{2} + 0.1 \cdot 6.1640 + 0.4 \cdot 5.9921 + 0.2 \cdot 5.8203 + \cdots = 6.43745$$

Equivalent snapshots

These two snapshots describe the same:

Equivalent snapshots

- Isomorphic intrees
- Scheduled tasks are mapped onto each other
- Equivalent snapshots excluded/re-used if they result from the same finishing task
- *Canonical* snapshots constructable in $O(n)$

Equivalent snapshots — example

Each transition shown has a probability of $\frac{1}{6}$.

Task 4 finished first · Task 6 finished first · Task 2 finished first

4 equivalent snapshots · 2 equivalent snapshots
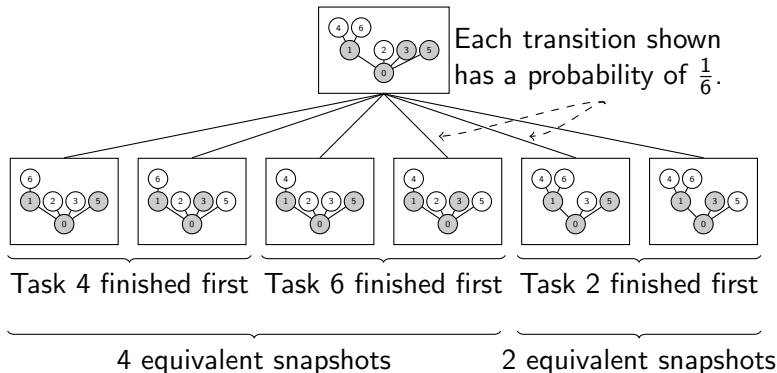
Equivalent snapshots — example
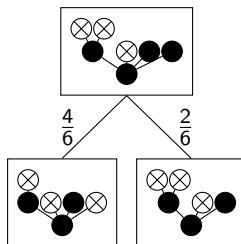
⇒ Reduces number of snapshots

Two processors — optimal solution

Known results (Chandy, Reynolds 1975/1979)

- No deliberate idleness (also for more processors)
- Highest-level-first (HLF) optimal for two processors
- Profile (number of tasks per level) completely determines run time
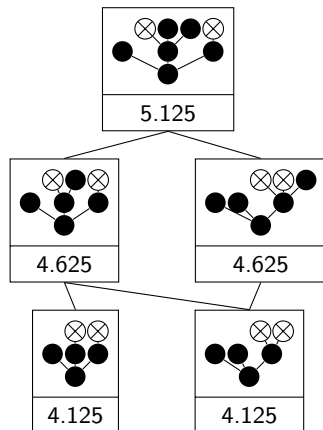- Reduce computation of expected run time to profiles
  $\Rightarrow$ Formula computing expected run time for intrees with 2 leaves (Maaß 2001)

Expected run time — special intrees

Theorem
*Intrees with profile $\left[\!\left[n_1, (1)^{j-2}, n_j, (1)^{r-j} \right]\!\right]$
have expected run time*

$$r + \frac{A_0(n_1 - 2)}{2^{n_1 - 1}} + \frac{A_{j-1}(n_j - 2)}{2^{n_j + j - 2}},$$

where A_i is inductively defined as follows:

$$A_0(n) = (n+1) \cdot 2^n \qquad A_{i+1}(n) = \sum_{k=0}^{n} A_i(k)$$

n_1 tasks

$j - 2$ tasks

n_j tasks

$r - j$ tasks
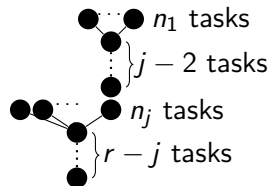
Profile DAG

- Use profiles instead of whole snapshots
- Scheduled tasks given implicitly (HLF)
- Worst case profile $\left\|\left[(1)^{\lfloor \frac{n}{2} \rfloor - 1}, \lceil \frac{n}{2} \rceil, 1\right]\right\|$
- Worst case profile DAG size has $\lfloor \frac{n}{2} \rfloor \cdot \lceil \frac{n}{2} \rceil + 1$ "profile snapshots"
  $\Rightarrow$ Optimal expected run time computable in polynomial time

Computing optimal schedules

- Exhaustive search (*LEAF* scheduler)
- "Optimize" result from exhaustive search by recursively discarding bad choices
- Exponential run time

Size of the snapshot DAG

- Number of subtrees grows exponentially
  ⇒ Number of overall snapshots as well

- Number of snapshots in optimal DAG remarkably smaller

Maximum

Average

11
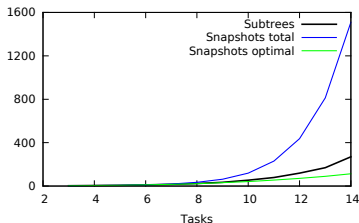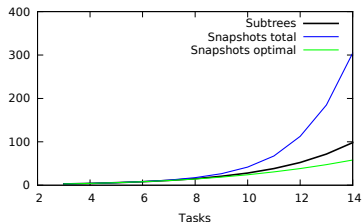
Compressing the snapshot DAG

- Optimal snapshot DAG allows (recursive) merging of some snapshots
- LEAF snapshot DAG prevents us from merging because the run times are not the same for some snapshots

P3: Optimal solution
000

P3: Strategies
●○○○○○○○○

Conjectures
○○○○

Results

HLF may be ambiguous . . .

HLF may result in different run times:

P3: Optimal solution
000

P3: Strategies
0●0000000

Conjectures
0000

Results
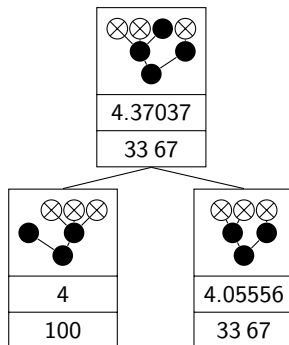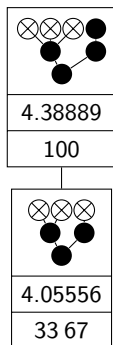
. . . or even strictly suboptimal

P3: Optimal solution
000

P3: Strategies
00●000000

Conjectures
0000

Results
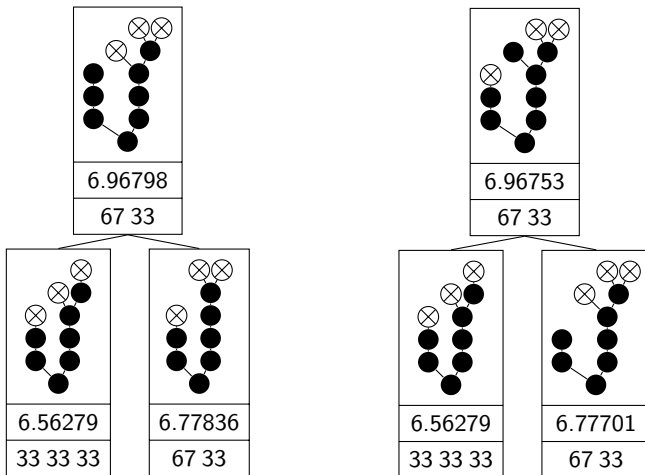
HLF and (dynamic) list scheduling

- HLF is (like every dynamic list scheduling strategy) suboptimal . . .
- . . . but asymptotically good (Papadimitriou, Tsitsiklis 1987)
- Optimal schedule has to consider previous choices
- Optimal schedule may be one particular run of HLF $\Rightarrow$ HLF is "can-optimal" for these intrees

P3: Optimal solution
000

P3: Strategies
000●00000

Conjectures
0000

Results

"2-HLF plus 1"

Motivation

Non-HLF intrees up to 13 tasks have at most one non-HLF task scheduled.

Strategy

Discard snapshots with more than one non-HLF task scheduled.

Counterexample

16

P3: Optimal solution
000

P3: Strategies
00000●0000

Conjectures
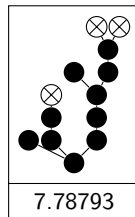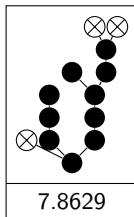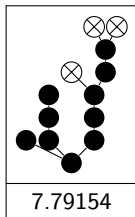0000

Results

Only highest and lowest leaves

Motivation
Optimal schedules seen so far scheduled only topmost and lowest leaves.

Strategy
Restrict to snapshots where only topmost and lowest leaves are scheduled.

Counterexample

| 7.79154 | | 7.8629 | | 7.78793 |

P3: Optimal solution
000

P3: Strategies
000000●000

Conjectures
0000

Results
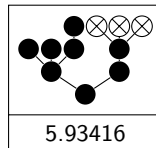
Subtree with fewer topmost tasks

Motivation
In many cases, a topmost task being the single requirement for its direct successor has to be scheduled

Strategy
Prefer tasks whose corresponding subtree has fewer topmost tasks.

Counterexample

5.93467

5.93416

P3: Optimal solution
000

P3: Strategies
000000●00

Conjectures
0000

Results
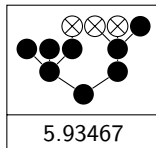
Recursive approach

Strategy

Prefer root's predecessors with maximal processing time.

Counterexample

5.20028

2.83 resp. 3   2.5   2
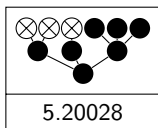
Filling up subtrees

Motivation

Subtrees seem to be filled up one after another.

Counterexample

P3: Optimal solution
000

P3: Strategies
00000000●

Conjectures
0000

Results
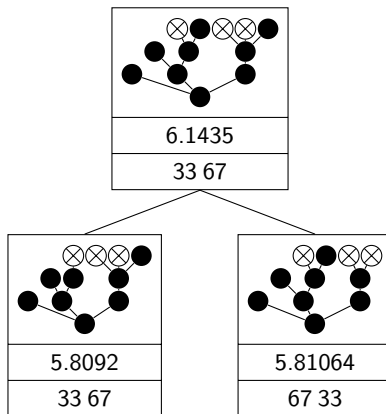
Maximizing 3-processor time, minimizing 1-processor time

Optimal schedules do *not* necessarily . . .

- . . . maximize the expected time span T_3 where 3 processors are busy
- . . . minimize the expected time span T_1 where only 1 processor can work

5.39455

$T_1 = 2.98$

5.36111

$T_3 = 2.36$

5.3287

$T_3 = 2.33, T_1 = 2.99$

21
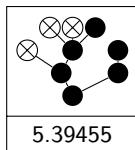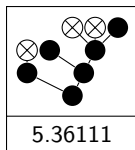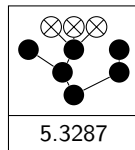
Optimal strategies — conjectures

- *In the beginning*, as many topmost task as possible shall be scheduled
- If the scheduler has a choice, it shall pick a topmost task, if available

P3: Optimal solution
000

P3: Strategies
000000000

Conjectures
0●00

Results

Special classes of intrees

Degenerate intrees

- On each level, at most one task has predecessors
- Degenerate intrees are optimally scheduled by HLF (proven)
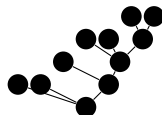
Parallel chains

- Each task, except the root, has at most one predecessor
- Parallel chains with up to 27 tasks are optimally scheduled by HLF

23
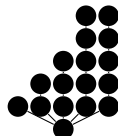
Degenerate intrees and parallel chains — outcome

- HLF is deterministic for these classes ...
- ... and needs remarkably less snapshots than LEAF
- Example: Number of snapshots for degenerate binary trees
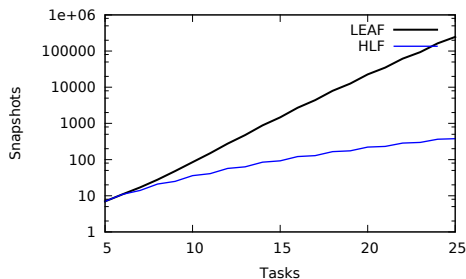
- Analogous results for parallel chains

Improving LEAF with conjectures

SCLEAF

- Simple LEAF scheduler, but . . .
- . . . using HLF when encountering degenerate intrees or parallel chains . . .
- . . . restricts to snapshots where as many topmost task as possible are scheduled

P3: Optimal solution
000

P3: Strategies
000000000

Conjectures
0000
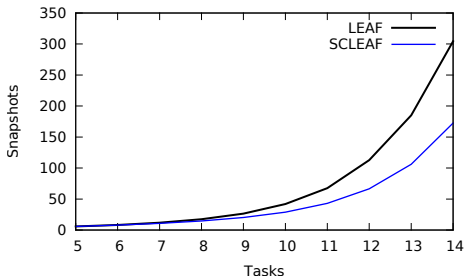
Results

Results

Practical results

- Optimal schedules for all intrees with up to 15 tasks: 11 minutes, less than 2Gb
- More tasks $\Rightarrow$ too much memory $\Rightarrow$ Subdivide into smaller sets of trees
- Optimal schedules for non-trivial intrees with 19 tasks: 1 day

Theoretical results

- New formula for run time for 2 processors
- Conjectures about optimal schedules for 3 processors . . .
- . . . that could help improving exhaustive search

Features

```
./build/tasks_cs1 -p3 -s leaf --optimize --random 6
--tikz

Normalized: [1, 0] [4, 2, 0] [5, 2, 0] [6, 3, 0]
  !   PTM! [3, 4, 5]  4.27161  (1.90123|1.92593|2.44444)  (
  !    TM! [3, 4, 6]  4.26235  (1.93827|1.86111|2.46296)  (
*H        [4, 5, 6]  4.2037   (2|1.7963|2.40741)          (
Total expected run time: 4.25
Total number of snaps:    16
Writing tikz to default.tex
Compiling to PDF.
Pool sizes: 17, 17,
```

27

Extensibility

- Different number representations
- Other distributions
- New checks
- Preemptiveness
- Other dependency structures
- Exporters

Further reading: Original problem I

Bruno, John (1985). "On scheduling tasks with exponential service times and in-tree precedence constraints". English. In: *Acta Informatica* 22.2, pp. 139–148. ISSN: 0001-5903.

Chandy, K. M. and P. F. Reynolds (1975a). "Scheduling partially ordered tasks with probabilistic execution times". In: *Proceedings of the fifth ACM symposium on Operating systems principles*. SOSP '75. New York, NY, USA: ACM, pp. 169–177.

— (Nov. 1975b). "Scheduling partially ordered tasks with probabilistic execution times". In: *SIGOPS Oper. Syst. Rev.* 9.5, pp. 169–177. ISSN: 0163-5980.

— (1979). *Scheduling Partially Ordered Tasks with Exponentially Distributed Times*.

Further reading: Original problem II

Maaß, Moritz G. (Sept. 2001). "Scheduling Independent and Identically Distributed Tasks with In-Tree Constraints on Three Machines in Parallel". MA thesis. Lehrstuhl für Effiziente Algorithmen, Institut für Informatik, TU München.

Pinedo, M. and G. Weiss (1985). "Scheduling Jobs with Exponentially Distributed Processing Times and Intree Precedence Constraints on Two Parallel Machines". In: *Operations Research* 33 (6), pp. 1381–1388.

Further reading: Equivalent snapshots

Aho, A.V., J.E. Hopcroft, and J.D. Ullman (1974). *The design and analysis of computer algorithms.* Addison-Wesley series in computer science and information processing. Addison-Wesley Pub. Co.

Matula, D.W. (1968). "A natural rooted tree enumeration by prime factorization". In: *SIAM Review* 10, p. 273.

Zemlyachenko, V.N., N.M. Korneenko, and R.I. Tyshkevich (1985). "Graph isomorphism problem". English. In: *Journal of Soviet Mathematics* 29.4, pp. 1426–1481. ISSN: 0090-4104.

Further reading: Scheduling I

Coffman E.G., Jr. and R.L. Graham (1972). "Optimal scheduling for two-processor systems". English. In: *Acta Informatica* 1.3, pp. 200–213. ISSN: 0001-5903.

Garey, M. R., D. S. Johnson, and Ravi Sethi (1976). "The Complexity of Flowshop and Jobshop Scheduling". In: *Mathematics of Operations Research* 1.2, pp. 117–129.

Graham, R.L. et al. (1979). "Optimization and Approximation in Deterministic Sequencing and Scheduling: a Survey". In: *Discrete Optimization II Proceedings of the Advanced Research Institute on Discrete Optimization and Systems Applications of the Systems Science Panel of NATO and of the Discrete Optimization Symposium co-sponsored by IBM Canada and SIAM Banff, Aha. and Vancouver*. Ed. by E.L. Johnson P.L. Hammer and B.H. Korte. Vol. 5. Annals of Discrete Mathematics. Elsevier, pp. 287 –326.

Further reading: Scheduling II

Pinedo, M. (2008). *Scheduling: Theory, Algorithms, and Systems*. Springer. ISBN: 9780387789347.