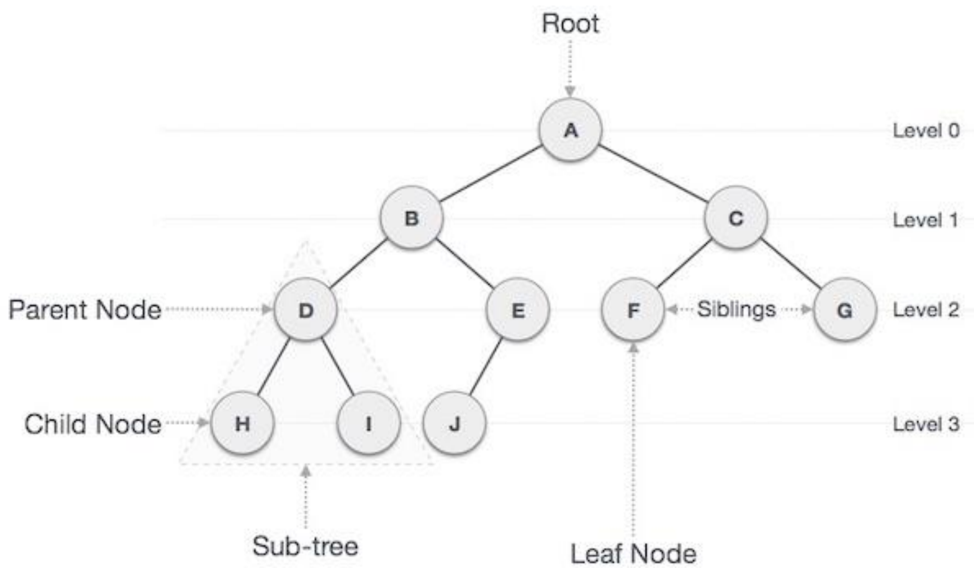
TREE DATA STRUCTURE

Tree represents the nodes connected by edges. We will discuss binary tree or binary search tree specifically.

Binary Tree is a special data structure used for data storage purposes. A binary tree has a special condition that each node can have a maximum of two children. A binary tree has the benefits of both an ordered array and a linked list as search is as quick as in a sorted array and insertion or deletion operation are as fast as in linked list.



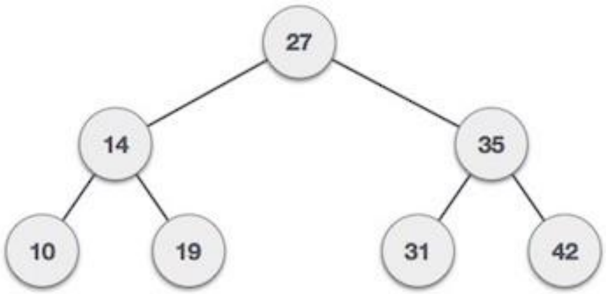
Important Terms

Following are the important terms with respect to tree.

- **Path** − Path refers to the sequence of nodes along the edges of a tree.
- **Root** − The node at the top of the tree is called root. There is only one root per tree and one path from the root node to any node.
- **Parent** − Any node except the root node has one edge upward to a node called parent.
- **Child** − The node below a given node connected by its edge downward is called its child node.
- **Leaf** − The node which does not have any child node is called the leaf node.
- **Subtree** − Subtree represents the descendants of a node.
- **Visiting** − Visiting refers to checking the value of a node when control is on the node.
- **Traversing** − Traversing means passing through nodes in a specific order.
- **Levels** − Level of a node represents the generation of a node. If the root node is at level 0, then its next child node is at level 1, its grandchild is at level 2, and so on.
- **keys** − Key represents a value of a node based on which a search operation is to be carried out for a node.

Binary Search Tree Representation

Binary Search tree exhibits a special behavior. A node's left child must have a value less than its parent's value and the node's right child must have a value greater than its parent value.



We're going to implement tree using node object and connecting them through references.

Tree Node

The code to write a tree node would be similar to what is given below. It has a data part and references to its left and right child nodes.

```
struct node {
   int data;
   struct node *leftChild;
   struct node *rightChild;
};
```

In a tree, all nodes share common construct.

BST Basic Operations

The basic operations that can be performed on a binary search tree data structure, are the following −

- **Insert** − Inserts an element in a tree/create a tree.
- **Search** − Searches an element in a tree.
- **Preorder Traversal** − Traverses a tree in a pre-order manner.
- **Inorder Traversal** − Traverses a tree in an in-order manner.
- **Postorder Traversal** − Traverses a tree in a post-order manner.

Insert Operation

The very first insertion creates the tree. Afterwards, whenever an element is to be inserted, first locate its proper location. Start searching from the root node, then if the data is less than the key value, search for the empty location in the left subtree and insert the data. Otherwise, search for the empty location in the right subtree and insert the data.

Algorithm

```
If root is NULL
   then create root node
return

If root exists then
   compare the data with node.data

   while until insertion position is located

      If data is greater than node.data
         goto right subtree
      else
         goto left subtree

   endwhile

   insert data

end If
```

Implementation

The implementation of insert function should look like this −

```
void insert(int data) {
   struct node *tempNode = (struct node*) malloc(sizeof(struct node));
   struct node *current;
   struct node *parent;

   tempNode->data = data;
   tempNode->leftChild = NULL;
   tempNode->rightChild = NULL;

   //if tree is empty, create root node
   if(root == NULL) {
      root = tempNode;
   } else {
      current = root;
      parent  = NULL;

      while(1) {
         parent = current;

         //go to left of the tree
         if(data < parent->data) {
            current = current->leftChild;

            //insert to the left
            if(current == NULL) {
               parent->leftChild = tempNode;
               return;
            }
         }
```

```
         //go to right of the tree
         else {
            current = current->rightChild;

            //insert to the right
            if(current == NULL) {
               parent->rightChild = tempNode;
               return;
            }
         }
      }
   }
}
```

Search Operation

Whenever an element is to be searched, start searching from the root node, then if the data is less than the key value, search for the element in the left subtree. Otherwise, search for the element in the right subtree. Follow the same algorithm for each node.

Algorithm

```
If root.data is equal to search.data
   return root
else
   while data not found

      If data is greater than node.data
         goto right subtree
      else
         goto left subtree

      If data found
         return node
   endwhile

   return data not found

end if
```

The implementation of this algorithm should look like this.

```
struct node* search(int data) {
   struct node *current = root;
   printf("Visiting elements: ");

   while(current->data != data) {
      if(current != NULL)
      printf("%d ",current->data);

      //go to left tree

      if(current->data > data) {
```

```
            current = current->leftChild;
         }
         //else go to right tree
         else {
            current = current->rightChild;
         }

         //not found
         if(current == NULL) {
            return NULL;
         }

         return current;
      }
   }
```

Traversal is a process to visit all the nodes of a tree and may print their values too. Because, all nodes are connected via edges (links) we always start from the root (head) node. That is, we cannot randomly access a node in a tree. There are three ways which we use to traverse a tree −

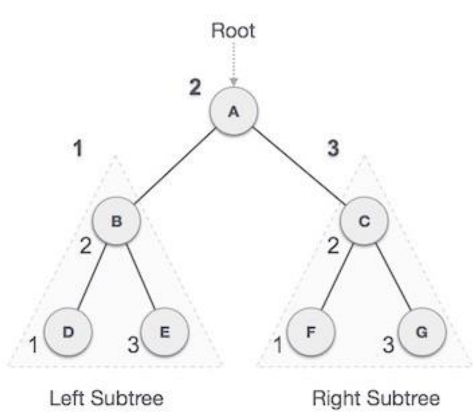
- In-order Traversal
- Pre-order Traversal
- Post-order Traversal

Generally, we traverse a tree to search or locate a given item or key in the tree or to print all the values it contains.

In-order Traversal

In this traversal method, the left subtree is visited first, then the root and later the right sub-tree. We should always remember that every node may represent a subtree itself.

If a binary tree is traversed **in-order**, the output will produce sorted key values in an ascending order.



We start from **A**, and following in-order traversal, we move to its left subtree **B**. **B** is also traversed in-order. The process goes on until all the nodes are visited. The output of inorder traversal of this tree will be −

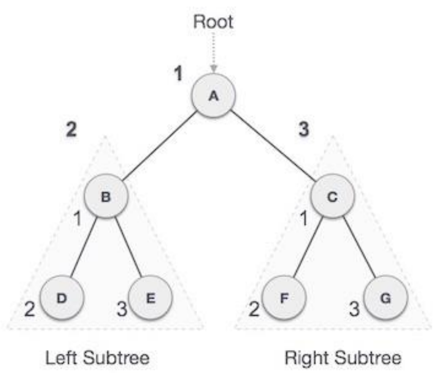
$$D \rightarrow B \rightarrow E \rightarrow A \rightarrow F \rightarrow C \rightarrow G$$

Algorithm

```
Until all nodes are traversed −
Step 1 − Recursively traverse left subtree.
Step 2 − Visit root node.
Step 3 − Recursively traverse right subtree.
```

Pre-order Traversal

In this traversal method, the root node is visited first, then the left subtree and finally the right subtree.



We start from **A**, and following pre-order traversal, we first visit **A** itself and then move to its left subtree **B**. **B** is also traversed pre-order. The process goes on until all the nodes are visited. The output of pre-order traversal of this tree will be −

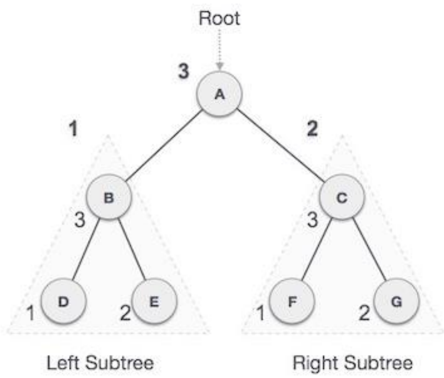
$$A \rightarrow B \rightarrow D \rightarrow E \rightarrow C \rightarrow F \rightarrow G$$

Algorithm

```
Until all nodes are traversed −
Step 1 − Visit root node.
Step 2 − Recursively traverse left subtree.
Step 3 − Recursively traverse right subtree.
```

Post-order Traversal

In this traversal method, the root node is visited last, hence the name. First we traverse the left subtree, then the right subtree and finally the root node.



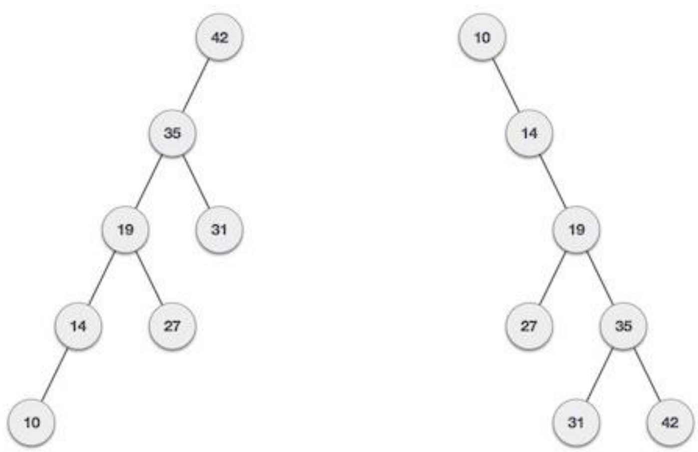
We start from **A**, and following Post-order traversal, we first visit the left subtree **B**. **B** is also traversed post-order. The process goes on until all the nodes are visited. The output of post-order traversal of this tree will be −

$$D \to E \to B \to F \to G \to C \to A$$

Algorithm

```
Until all nodes are traversed −
Step 1 − Recursively traverse left subtree.
Step 2 − Recursively traverse right subtree.
Step 3 − Visit root node.
```

What if the input to binary search tree comes in a sorted (ascending or descending) manner? It will then look like this −



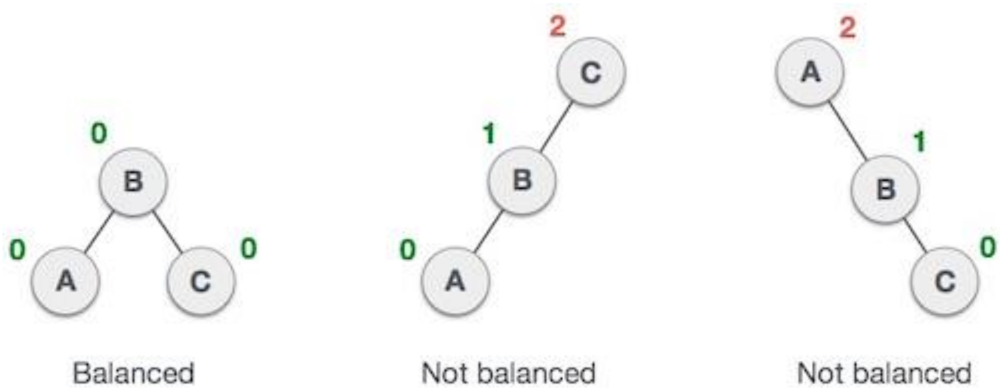
If input 'appears' non-increasing manner   If input 'appears' in non-decreasing manner

It is observed that BST's worst-case performance is closest to linear search algorithms, that is O(n). In real-time data, we cannot predict data pattern and their frequencies. So, a need arises to balance out the existing BST.

AVL TREES

Named after their inventor **Adelson**, **Velski** & **Landis**, **AVL trees** are height balancing binary search tree. AVL tree checks the height of the left and the right sub-trees and assures that the difference is not more than 1. This difference is called the **Balance Factor**.

Here we see that the first tree is balanced and the next two trees are not balanced −



In the second tree, the left subtree of **C** has height 2 and the right subtree has height 0, so the difference is 2. In the third tree, the right subtree of **A** has height 2 and the left is missing, so it is 0, and the difference is 2 again. AVL tree permits difference (balance factor) to be only 1.

```
BalanceFactor = height(left-sutree) − height(right-sutree)
```

If the difference in the height of left and right sub-trees is more than 1, the tree is balanced using some rotation techniques.

AVL Rotations

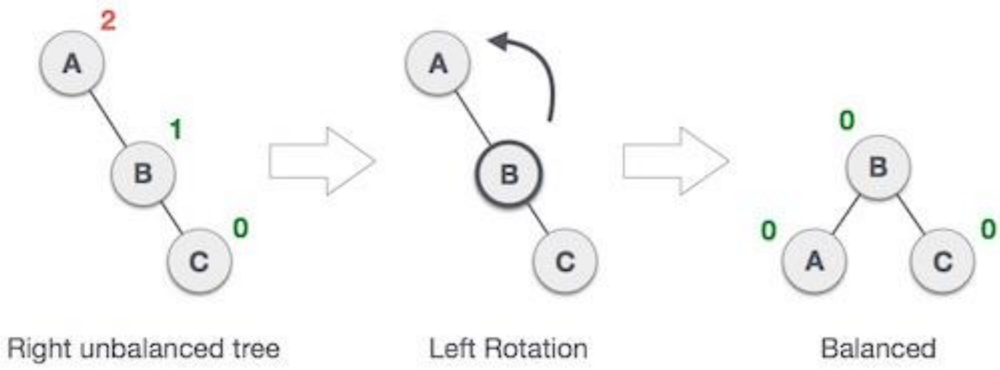
To balance itself, an AVL tree may perform the following four kinds of rotations −

- Left rotation
- Right rotation
- Left-Right rotation
- Right-Left rotation

The first two rotations are single rotations and the next two rotations are double rotations. To have an unbalanced tree, we at least need a tree of height 2. With this simple tree, let's understand them one by one.

Left Rotation

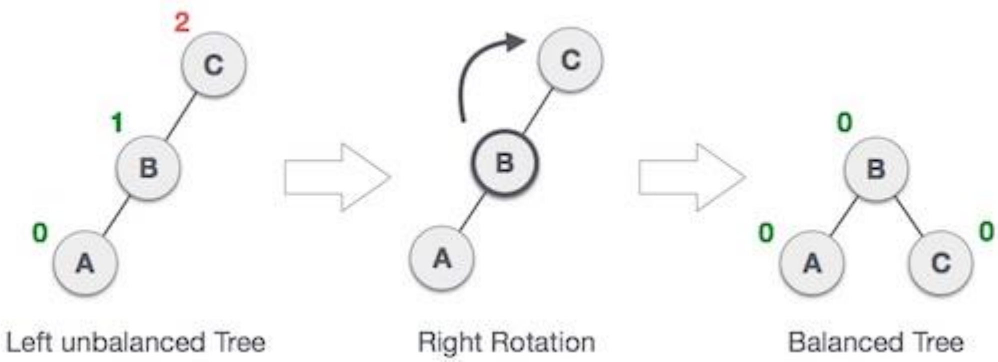
If a tree becomes unbalanced, when a node is inserted into the right of the right subtree, then we perform a single left rotation −



In our example, node **A** has become unbalanced as a node is inserted in the right subtree of A's right subtree. We perform the left rotation by making **The** left-subtree of B.

Right Rotation

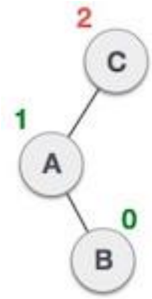
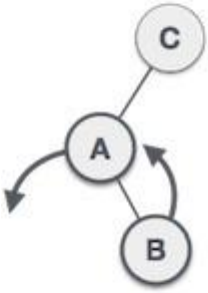
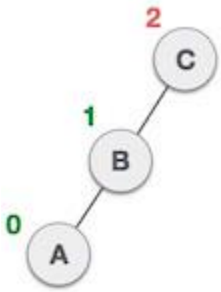
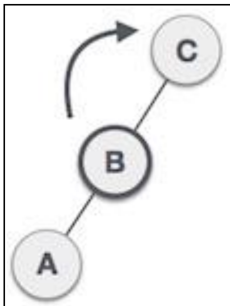
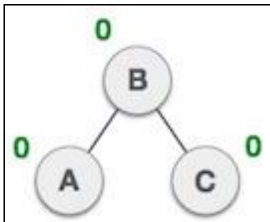
AVL tree may become unbalanced, if a node is inserted in the left subtree of the left subtree. The tree then needs a right rotation.



As depicted, the unbalanced node becomes the right child of its left child by performing a right rotation.

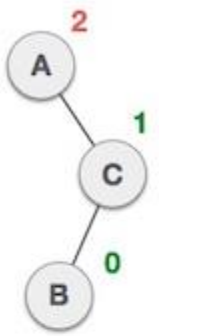
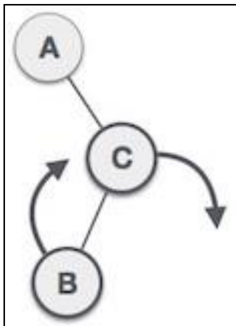
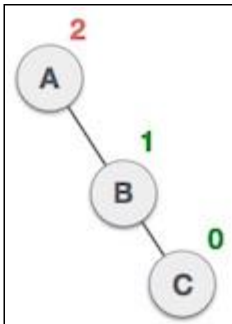
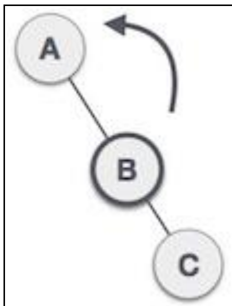
Left-Right Rotation

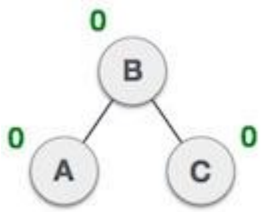
Double rotations are slightly complex version of already explained versions of rotations. To understand them better, we should take note of each action performed while rotation. Let's first check how to perform Left-Right rotation. A left-right rotation is a combination of left rotation followed by right rotation.

State	Action
	A node has been inserted into the right subtree of the left subtree. This makes **C** an unbalanced node. These scenarios cause AVL tree to perform left-right rotation.
	We first perform the left rotation on the left subtree of **C**. This makes **A**, the left subtree of **B**.
	Node **C** is still unbalanced, however now, it is because of the left-subtree of the left-subtree.
	We shall now right-rotate the tree, making **B** the new root node of this subtree. **C** now becomes the right subtree of its own left subtree.
	The tree is now balanced.

Right-Left Rotation

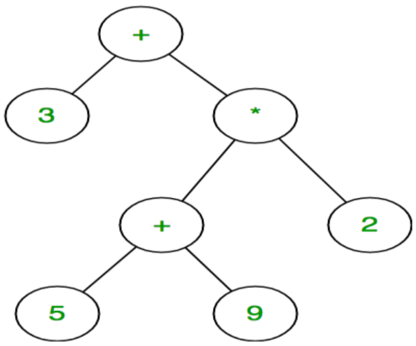
The second type of double rotation is Right-Left Rotation. It is a combination of right rotation followed by left rotation.

State	Action
	A node has been inserted into the left subtree of the right subtree. This makes **A**, an unbalanced node with balance factor 2.
	First, we perform the right rotation along **C** node, making **C** the right subtree of its own left subtree **B**. Now, **B** becomes the right subtree of **A**.
	Node **A** is still unbalanced because of the right subtree of its right subtree and requires a left rotation.
	A left rotation is performed by making **B** the new root node of the subtree. **A** becomes the left subtree of its right subtree **B**.

	The tree is now balanced.

EXPRESSION TREE

The expression tree is a binary tree in which each internal node corresponds to the operator and each leaf node corresponds to the operand so for example expression tree for 3 + ((5+9)*2) would be:



Inorder traversal of expression tree produces infix version of given postfix expression (same with postorder traversal it gives postfix expression)

Evaluating the expression represented by an **expression tree:**

```
Let t be the expression tree
If  t is not null then
      If t.value is operand then
                Return  t.value
      A = solve(t.left)
      B = solve(t.right)

      // calculate applies operator 't.value'
      // on A and B, and returns value
      Return calculate(A, B, t.value)
```

Construction of Expression Tree:

Now for constructing an expression tree we use a stack. We loop through input expression and do the following for every character.

1. If a character is an operand push that into the stack

2. If a character is an operator pop two values from the stack, make them its child and push the current node again.

In the end, the only element of the stack will be the root of an expression tree.

HEAP DATA STRUCTURE

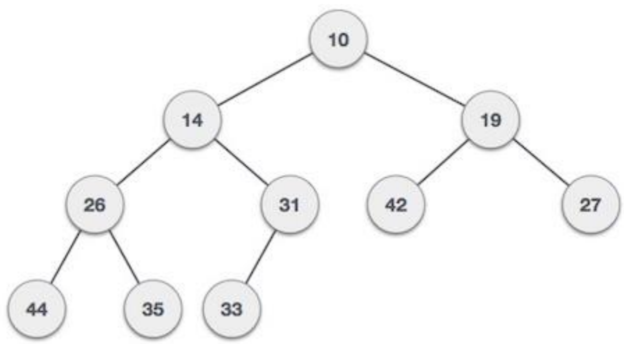
Heap is a special case of balanced binary tree data structure where the root-node key is compared with its children and arranged accordingly. If **α** has child node **β** then −

$$\mathbf{key(\alpha) \geq key(\beta)}$$

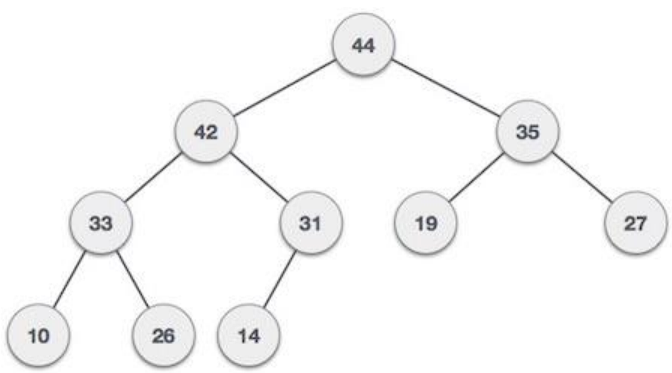
As the value of parent is greater than that of child, this property generates **Max Heap**. Based on this criteria, a heap can be of two types −

```
For Input → 35 33 42 10 14 19 27 44 26 31
```

Min-Heap − Where the value of the root node is less than or equal to either of its children.



Max-Heap − Where the value of the root node is greater than or equal to either of its children.



Both trees are constructed using the same input and order of arrival.

Max Heap Construction Algorithm

We shall use the same example to demonstrate how a Max Heap is created. The procedure to create Min Heap is similar but we go for min values instead of max values.

We are going to derive an algorithm for max heap by inserting one element at a time. At any point of time, heap must maintain its property. While insertion, we also assume that we are inserting a node in an already heapified tree.

```
Step 1 − Create a new node at the end of heap.
Step 2 − Assign new value to the node.
Step 3 − Compare the value of this child node with its parent.
Step 4 − If value of parent is less than child, then swap them.
Step 5 − Repeat step 3 & 4 until Heap property holds.
```

Note − In Min Heap construction algorithm, we expect the value of the parent node to be less than that of the child node.

Max Heap Deletion Algorithm

Let us derive an algorithm to delete from max heap. Deletion in Max (or Min) Heap always happens at the root to remove the Maximum (or minimum) value.

```
Step 1 − Remove root node.
Step 2 − Move the last element of last level to root.
Step 3 − Compare the value of this child node with its parent.
Step 4 − If value of parent is less than child, then swap them.
Step 5 − Repeat step 3 & 4 until Heap property holds.
```