



MACHAKOS UNIVERSITY

University Examinations 2016/2017

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR DEGREE IN

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION(ARTS)

SMA 104: CALCULUS 1

Date: 6/6/2017

Time: 2:00 – 4:00 PM

INSTRUCTIONS

Attempt question **one (compulsory)** and any other **two questions**.

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Consider the function $f(x) = \log_e(3x^2 - 4x + 5)$. Determine range of the function (2 marks)
- b) Consider the function
- $$f(x) = \begin{cases} 7x - 2 & x \geq 2 \\ 3x + 5 & x < 2 \end{cases}$$
- Determine the one-sided limits
- i) $\lim_{x \rightarrow 2^+} f(x)$ (2 marks)
- ii) $\lim_{x \rightarrow 2^-} f(x)$ (2 marks)
- c) Prove that the function $f(x) = \sin^2 x$ is continuous for every value of x on R (4 marks)
- d) Given $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x}{1-x}$ Determine $(f \cdot g)^{-1}$ (4 marks)
- e) A ladder 20m long leans against a vertical building. If the bottom of the Ladder slides away from the building horizontally at a rate of $2m/s$, how fast is the ladder sliding down the building when the top of the ladder is 12m above the ground (4 marks)
- f) Calculate $\frac{dy}{dx}$ for $y = \sin^{-1} \frac{2x}{1+x^2}$ (4 marks)
- g) Prove that $\frac{d}{dx}(\sin x) = \cos x$ from the first principles the derivative of (4 marks)
- h) Determine $\frac{dy}{dx}$ given that $y = (x^2 + 3x)^7$ (4 marks)

QUESTION TWO (20 MARKS)

Consider the function $f(x) = x^4 - 2x^2$

- a) Determine using the second derivative
- i.) Local maximum point (3 marks)
- ii.) Local minimum point (3 marks)
- b) Discuss the concavity of $f(x)$ (7 marks)
- c) Find the points of inflection of $f(x)$ (3 marks)
- d) Sketch the graph of $f(x)$ (4 marks)

QUESTION THREE (20 MARKS)

Determine $\frac{dy}{dx}$

- i.) $5y^2 + \sin y = x^2$ (4 marks)
ii.) $xe^x \sin x$ (4 marks)
iii.) $ye^x = \sinh(xy)$ (4 marks)
iv.) $y = \tan^{-1}(2t+1)$, $x = e^{-t^2}$ (4 marks)
v.) $y = 3^{\cos x}(x^x)$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) A projectile is fired straight upwards with a velocity of $40m/s$; its distance above the ground t sec after being fired is given by $S(t) = -16t^2 + 400t$. $S(t)$ is the distance of the particle from the ground after it has been fired?
i. Calculate the time and the velocity at which the projectile hits the ground (5 marks)
ii. Determine the maximum height achieved by the projectile (4 marks)
iii. Work out the acceleration at any time (t) (2 marks)
- b) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through $(1, 0)$ and has a stationary point at $(1, 0)$. Find the value of α , β , the other turning points and hence sketch the curve (9 marks)

QUESTION FIVE (20 MARKS)

- a) Determine;
i. $\lim_{x \rightarrow \infty} \frac{-3n + (-1)^n}{4n - (-1)^n}$ (5 marks)
ii. $\lim_{x \rightarrow y} \frac{\sin^2 x - \sin^2 y}{x^2 - y^2}$ (4 marks)
iii. $\lim_{x \rightarrow 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (3 marks)
- b) If $y = 3e^{2x} \cos(2x - 3)$, Verify that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (4 marks)
c) Determine the gradient of the curve $x^2 + 2xy - 2y^2 + x = 2$ at the point $(-4, 1)$ (4 marks)