



Machakos University College

(A Constituent College of Kenyatta University)

UNIVERSITY EXAMINATIONS 2013/2014

SCHOOL OF COMPUTING AND APPLIED SCIENCES

FIRST YEAR SECOND SEMESTER EXAMINATION FOR THE DEGREE OF

BACHELOR OF EDUCATION

SMA 104: CALCULUS 1

DATE: Monday, 7th April, 2014

TIME: 8.30 a.m. – 10.30 a.m.

INSTRUCTIONS:

Answer Question ONE which is compulsory and any other TWO

Question 1.

(a) Given $f(x) = \frac{x}{x+1}$ and $g(x) = \frac{x}{1-x}$. Determine $(f \cdot g)^{-1}$ (6 marks)

(b) (i) At what points is the function continuous

$$f(x) = \frac{x^2+x-6}{x^2-4} \quad (4 \text{ marks})$$

(c) Obtain and classify the stationary points of $y = x^2 e^x$ (6 marks)

(i) Evaluate the limits

$$\lim_{x \rightarrow 1} \frac{x^2+x-2}{x^2-x} \quad (4 \text{ marks})$$

(d) Determine the dimensions for which the total surface area of the cube having a volume of 32m^3 is minimum. (4 marks)

(e) A point is moving on the graph of $y^3 = x^2$. When the point is at $(-8, 4)$ its y coordinate is decreasing at 2 units per sec. How fast is the x coordinate changing?

(6 marks)

Question 2

(a) (i) Given that $x^2 \sin\theta - 3x^2 = \sec\theta$ Determine the value of $\frac{dx}{d\theta}$ when $\theta = \pi$

(ii) If $y = 3e^{2x} \cos(2x - 3)$, verify that $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 8y = 0$ (8 marks)

(b) (i) Given $y = x^2 \cos 3x \ln 2x$

Find $\frac{dy}{dx}$

(ii) Find the inflection point of $f(x) = x^3 - 6x^2 + 9x + 1$ (12 marks)

Question 3

(a) Given $z = f(x, y)$ and $z = x \cos(x + y)$ show that $\frac{d^2z}{dxdy} = \frac{d^2z}{dydx}$

(b) Determine the gradient function of the curve $x^2 + 2xy - 2y^2 + x = 2$ Hence determine the gradient of the curve at $(-4, 1)$ (6 marks)

(c) Differentiate $y = 3e^{\tan x}$ (4 marks)

Question 4

(a) Determine the stationary points on the surface $z = (x^2 + y^2)^2 - 2(x^2 - y^2)$ and state their nature. (10 marks)

(b) Given the Hyperbola $x = 2.3 \sec \theta$ and $y = 3.4 \tan \theta$ find:

(i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$ at $\theta = 1 \text{ radian}$. (10 marks)

Question 5

(a) Investigate the critical points on the curve

(i) $y = x^2 e^{-x}$

(ii) Given that $y = x^2 e^x$

Prove that $y_n = e^x [x^2 + 2nx + n(n - 1)] \forall n > 0$ (10 marks)

(b) The pressure (P) Volume (V) and temperature (T) of unit mass of gas are related by the formula $PV=RT$ where R is a constant. Show that;

(i) $dp = \frac{P}{T}dT - \frac{P}{V}dv$

(ii) $dT = \frac{T}{V}dV + \frac{T}{p}dp$ (6 marks)

(c) The pressure P and Volume V of a gas are related by the equation $PV^{1.4}=C$

Estimate the percentage change In C when the pressure is increased by 2.3% and volume decreased by 0.84%. (4 marks)