Exercise 1: Optical Flow

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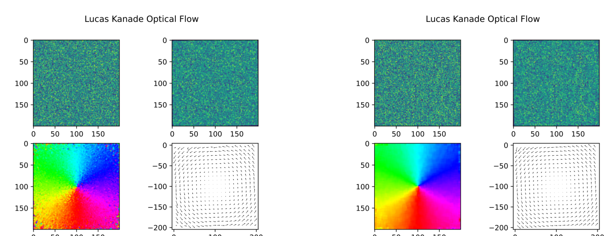
I. Introduction

This work addresses the classic problem of optical flow estimation by implementing two widely used methods: Lucas–Kanade and Horn–Schunck. The Lucas–Kanade method estimates local motion by assuming that the flow is constant in a small neighborhood, while Horn–Schunck imposes a global smoothness constraint. Enhancements such as an optional Harris reliability check and a pyramidal approach are incorporated to improve performance, particularly in low-texture regions and for larger displacements. The code processes grayscale images and visualizes the flow fields, offering insights into both accuracy and computational performance.

II. Experiments

A. Lucas-Kanade

For benchmarking purposes, the basic implementation was evaluated on a synthetic image consisting of random noise that was rotated by 1°. The optical flow field depicted in Figure 1 closely matches the reference results, indicating that the algorithm computes the motion accurately.

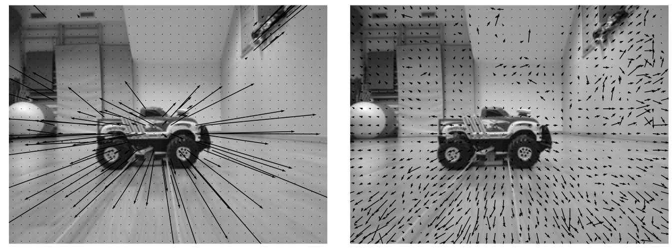


(a) $\sigma = 1, N = 3$ (b) $\sigma = 1, N = 9$

Figure 1: Optical flow response of Lucas-Kanade on a random noise image rotated by 1°.

The two main parameters controlling the algorithm are the neighborhood size N and the standard deviation of the Gaussian kernel σ. Small values of N and σ can lead to a noisy flow field (see Figure 1), whereas larger σ values tend to oversmooth the images and gradients, potentially discarding subtle motion details. In the experiments presented here, N was tuned to suit the specific use case, while σ was maintained at a value of 1.

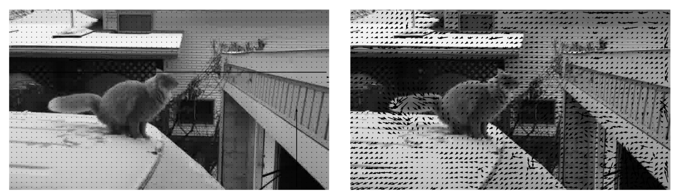
1) **Harris response for flow reliability:** Lucas-Kanade employs the covariance matrix of local gradients to solve the optical flow equation. However, performance may deteriorate in regions where the eigenvalues of the matrix are too small or where their ratio is excessively high—conditions typically encountered in areas of low texture or near edges. As illustrated in Figure 2, incorporating the Harris response filters out unreliable flow estimates in such regions, retaining only those corresponding to the primary object of interest (the toy truck).



(a) With Harris response (b) Without Harris response

Figure 2: Comparison of the Lucas-Kanade algorithm with and without Harris response thresholding ($N = 11$, $\sigma = 1$).

2) **Pyramidial Lucas-Kanade:** In many practical settings, the assumption of small displacements is often invalid. To address this limitation, a Gaussian pyramid is employed: optical flow is initially computed at a coarse resolution, and the estimates are progressively refined as the resolution increases.

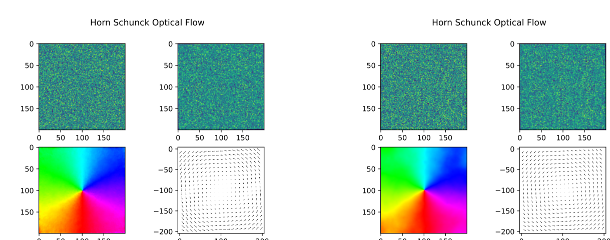


(a) With Harris response (b) Without Harris response

Figure 3: Comparison of the Lucas-Kanade algorithm with and without Gaussian pyramids ($N = 11$, $\sigma = 1$).

As depicted in Figure 3, the basic Lucas–Kanade algorithm struggles to capture the rapid movement of the cat in the video[1]. In contrast, the pyramidal implementation, which processes downscaled versions of the images, effectively detects the "jumping" motion through optical flow.

B. Horn-Schunck



(a) $\lambda = 5$, iterations $= 500$ (b) $\lambda = 5$, iterations $= 1000$

Figure 4: Optical flow response of Horn-Schunck on a random noise image rotated by 1°.

In contrast to the Lucas–Kanade method, the Horn–Schunck approach was evaluated on a random noise image rotated by 1°. As illustrated in Figure 1, the optical flow produced by Horn–Schunck is considerably smoother than the Lucas–Kanade result shown in Figure 4. This improvement is largely attributed to the extra smoothness constraint in Horn–Schunck, which optimizes the flow globally across the image. Furthermore, the number of iterations has a notable impact on the output: higher iteration counts yield a smoother and more accurate flow (see Figure 4b), whereas lower iteration counts result in noticeable noise in the HSV visualization (see Figure 4a). It is important to note, however, that increasing the number of iterations also leads to longer execution times.

C. Algorithms comparison

To assess the strengths and limitations of both algorithms, the flow estimates were evaluated on three additional images.

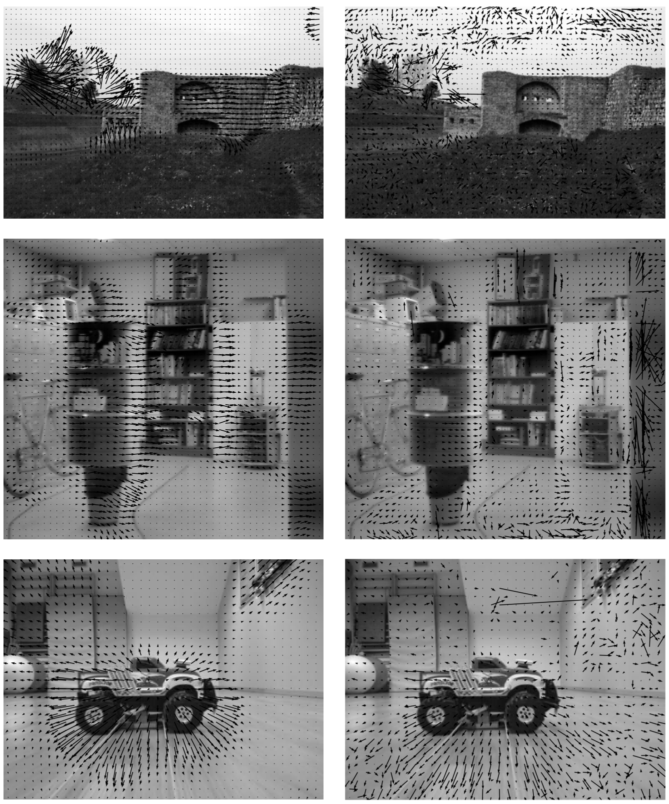


Figure 5: Overall caption for the six images. (Horn-Schunck on the left and Lucas-Kanade on the right)

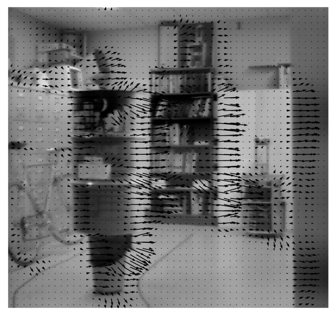
Figure 5 illustrates that, overall, the Horn–Schunck method achieves greater field accuracy and smoother flow estimates than the Lucas–Kanade method. Nonetheless, both techniques tend to encounter difficulties in regions with little or no texture as well as near the object boundaries. Table I summarizes the execution times, measured on images of size 288×384, under various parameter settings. The following abbreviations denote the different algorithms:

- HSLK - Horn-Schunck (initalized with Lucas-Kanade);
- HS - Horn-Schunck;
- LKP - Lucas-Kanade with pyramids;
- LK- Lucas-Kanade.

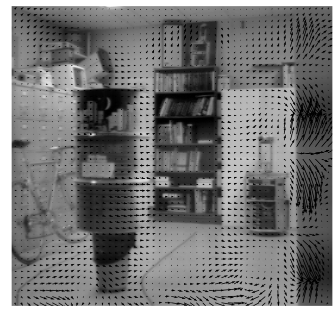
Table I: Comparison of Execution Times for Different Optical Flow Methods

Method	**Iterations**	**N**	**Time (s)**
HSLK	1000	/	1.923
HS	1000	/	0.639
HS	500	/	0.234
LKP	/	9	0.046
LK	/	9	0.029
LK	/	3	0.014

Initializing Horn–Schunck with a Lucas–Kanade–based estimate yielded poorer results. The initial guess did not accelerate convergence, and the resulting flow field became noisier and less accurate.



(a) Normal Horn-Schunck



(b) Horn-Schunck with Lucas-Kanade

Figure 6: Comparison of Horn-Schunck initialization techniques.

Although Lucas–Kanade produced less accurate flow estimates in these examples, its faster execution time could be advantageous in scenarios with stringent time constraints.

III. Conclusion

In this assignment, two widely used optical flow methods—Lucas–Kanade and Horn–Schunck—were implemented. Experimental results indicate that Horn–Schunck produces more accurate flow estimates, albeit with a longer execution time. Both algorithms face challenges when the small motion assumption is violated; the Lucas–Kanade method can address this limitation by employing Gaussian pyramids. Additionally, the instability of Lucas–Kanade can be mitigated by incorporating the Harris detector response. However, initializing Horn–Schunck with Lucas–Kanade estimates did not reduce the number of iterations required for convergence.

References

[1] YouTube, “Waffles The Terrible - Funny Cat Fails Epic Jump,” https://www.youtube.com/watch?v=5d7aruKYkKs, youTube video, accessed March 8, 2025.