# Block Diagram Derivation

May 4, 2014

Our dynamics are modeled by

$$P_g = P_m - D\Delta\omega - M\Delta\dot{\omega} \quad (1)\,,$$

$$\dot{\delta} = \omega_0\Delta\omega \quad (2)\,,$$

$$T_g\dot{P}_m = P_{ref} + \Delta P_c - P_m - \frac{\Delta\omega}{R} \quad (3)\,,$$

and

$$\dot{P}_c = -k \cdot ACE \quad (4)\,.$$

The first equation, known as the Swing Equation, serves as the physical model of the generator, where $P_g$ is the electrical power desired by the system, $P_m$ is the mechanical power output of the turbine, $\triangle\omega$ is the deviation in generator frequency (a.k.a. generator angular speed) from the nominal frequency of 60 Hz, $\triangle\dot{\omega}$ is the first derivative with respect to time of the change in generator frequency (a.k.a. generator angular acceleration), and $M$ and $D$ are damping constants. This equation shows that, with the mechanical power held constant, a decrease (increase) in desired generator power $P_g$ will result in an increase (decrease) in generator frequency. This is analogous to what we experience in a car when we begin going down a hill. If you were to keep pressing on the gas with the same pressure as you begin going down the hill (i.e. hold the mechanical power of your engine constant while decreasing the resistance against the car), the engine would speed up; this is why most of us decrease our gas pedal pressure as we descend.

The second equation simply states that the generator frequency (angular speed) $\omega$ is equal to the derivative of the generator angle $\delta$.

The third equation governs droop control, where $\dot{P}_m$ is the change with respect to time of the mechanical power, $P_{ref}$ is the set point of the system derived by economic dispatch, $\triangle P_c$ is the change in the SOMETHINGGGG, and $T_g$ and $R$ are scaling constants. Droop, also known as "primary frequency control," is a control on the time scale of milliseconds to seconds which allows the mechanical output to change in order to help to correct deviation from nominal frequency. Without droop control, the mechanical power output cannot change and instead frequency must provide all of the buffer for when $P_g$ changes, causing frequency swings and significant grid performance issues. We see that the equation relates $\dot{P}_m$ to the error between the set point of power and the actual power output $(P_{ref} + \Delta P_c - P_m)$ and to the error between the current frequency and the nominal frequency $\left(\frac{\triangle\omega}{R}\right)$. This control, then, alters the mechanical output in response to these errors; if the desired power output is wrong, change the output, and if the frequency is too high or low, change the output (the frequency is directly related to the output via the swing equation). Comparing this with our above downhill car example, this is analogous to decreasing the pressure on the pedal; the difference between the actual power output and the necessary power output is non-zero, and thus we changing the mechanical output by moving our foot, instead of allowing the engine to increase speed.

The fourth equation is a secondary frequency control, known as automatic generation control (AGC), which operates on the time scale of seconds to minutes, where $\dot{P}_c$ is SOMETHINGGG, and, as the North American Electricity Reliability Council explains, ACE "is used to determine a control area's control performance with respects to its' impact on system frequency."

To obtain our control diagram, we take the Laplace transform of each of the above equations. Starting with equation 4, we have

$$\triangle\dot{P}_c(t) = -k \cdot ACE(t) \rightarrow \mathcal{L} \rightarrow s\triangle P_c(s) = -k \cdot ACE(s) \rightarrow \triangle P_c(s) = \frac{-k}{s} \cdot ACE(s).$$

Thus, we see on our block diagram that ACE passes through a gain block of $\frac{-k}{s}$ to produce $\triangle P_c$. Additionally, we note that the limits to $\triangle P_c$ are applied within this block, producing $\triangle P_{c_{lim}}$. Within the 3rd equation, we define

$$P_c(t) = P_{ref}(t) + \triangle P_{c_{lim}}(t) - \frac{\triangle\omega(t)}{R},$$

and taking the Laplace transform, we simply have

$$P_c(s) = P_{ref}(s) + \triangle P_{c_{lim}}(s) - \frac{\triangle\omega(s)}{R},$$

shown in the summation on the next step of the block diagram. Limiting this gives us $P_{c_{lim}}$. Then, equation 3 can be written as

$$T_g\dot{P}_m(t) = P_{c_{lim}}(t) - P_m(t).$$

Taking the Laplace transform, the result is

$$sT_gP_m(s) = P_{c_{lim}}(s) - P_m(s).$$

Adding $P_m$ to the other side, we have

$$(sT_g + 1)\, P_m(s) = P_{c_{lim}}(s) \rightarrow P_m = \left(\frac{1}{T_gs + 1}\right) P_{c_{lim}}.$$

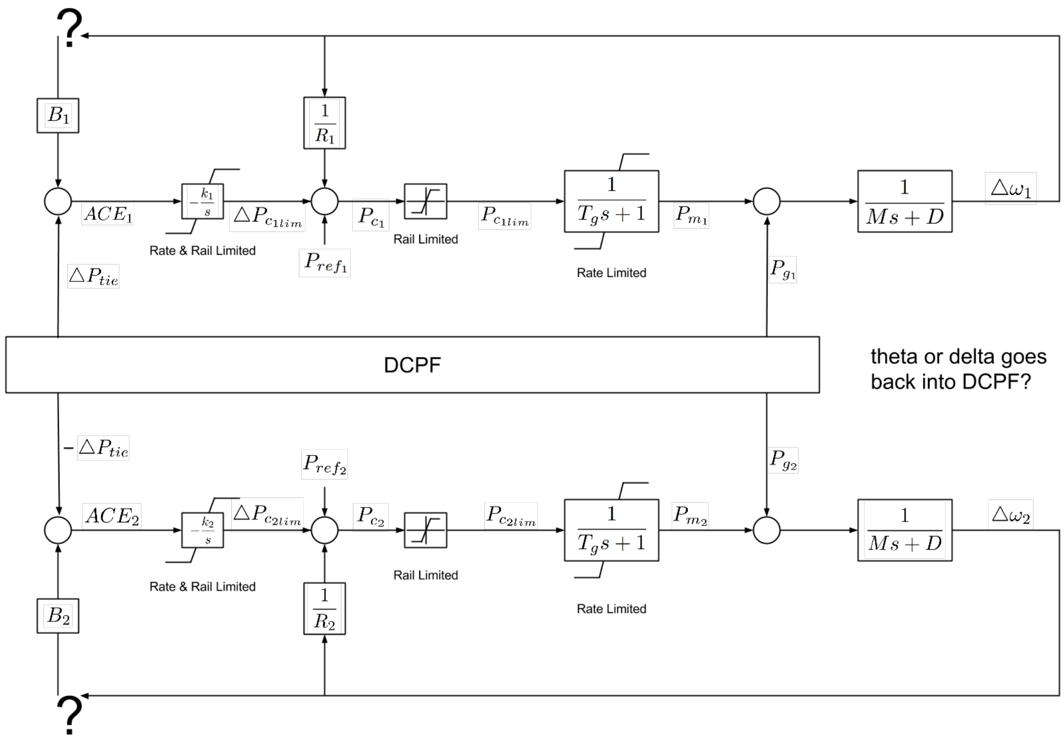
Lastly, we take the Laplace transform of equation one, resulting in

$$sM\triangle\omega(s) = P_m(s) - P_g(s) - D\triangle\omega(s).$$

Solving for $\triangle\omega$, we have

$$(Ms + D)\,\triangle\omega(s) = P_m(s) - P_g(s) \rightarrow \triangle\omega(s) = \frac{1}{Ms + D}\left(P_m(s) - P_g(s)\right).$$

As noted above, a number of limits must be applied to the outputs, which is done to more accurately reflect the operation of a realistic system. The limit on $\triangle P_c$ is applied to ensure that the regulation provided by a given generator stays within the defined percentage. The limit on $P_c$ ensures that the generator stays within its absolute maximum and minimum power output. We place a limit on $\dot{P}_m$ to ensure that the generator ramping does not occur faster than is physically possible; a given generator can only change its output so quickly. Lastly, we apply a limit to $\triangle\dot{P}_c$ to ensure that the system is not attempting to change $\triangle\dot{P}_c$ when $\triangle P_c$ is already at its limit.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$B_1$
$\frac{1}{R_1}$
$\triangle\omega_1$
$ACE_1$
$-\frac{k_1}{s}$
$\triangle P_{c_{1lim}}$
$P_{c_1}$
$P_{c_{1lim}}$
$\frac{1}{T_g s+1}$
$P_{m_1}$
$\frac{1}{Ms+D}$
Rate & Rail Limited
Rail Limited
Rate Limited
$P_{ref_1}$
$\triangle P_{tie}$
$P_{g_1}$
DCPF
theta or delta goes
back into DCPF?
$-\triangle P_{tie}$
$P_{g_2}$
$P_{ref_2}$
$ACE_2$
$-\frac{k_2}{s}$
$\triangle P_{c_{2lim}}$
$P_{c_2}$
$P_{c_{2lim}}$
$\frac{1}{T_g s+1}$
$P_{m_2}$
$\frac{1}{Ms+D}$
$\triangle\omega_2$
Rate & Rail Limited
Rail Limited
Rate Limited
$B_2$
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