Analysis and Graphical Representation of Health and Labor Force Participation among the Elderly in Europe

Humboldt-Universität zu Berlin School of Business and Economics Ladislaus von Bortkiewicz Chair of Statistics



Statistical Programming Languages

Winter 2017/18

Seminar Paper by

Claudia Günther, Phi Nguyen, Julian Winkel 576419, 526624, 562959

Berlin, 2018-03-16

List of Tables

| 1 | Summary Statistics over all groups | G |
|---|--|----|
| 2 | Men: Labor Force Participation Rate by age | 10 |
| 3 | Women: Labor Force Participation Rate by age | 10 |
| 4 | Marginal effects of health and age related variables on the probability of labor market partici- | |
| | pation of Austrian women | 14 |
| 5 | Wald test on no joint impact of health on participation | 17 |
| 6 | Women's current and counterfactual labor share | 21 |
| 7 | Men's current and counterfactual (CF) labor share for different age groups | 22 |

List of Figures

| 1 | Full-time labor employment tile grid maps by age group | 26 |
|---|---|----|
| 2 | Distribution of self-perceived health by country and gender | 27 |

Contents

| 1 Introduction | | | | | |
|----------------|---|----|--|--|--|
| 2 | Data Cleaning and Manipulation | 3 | | | |
| | 2.1 Theory and Design | 3 | | | |
| | 2.2 Implementation | 3 | | | |
| | 2.3 Empirical Results | 6 | | | |
| 3 | Multidisciplinary and Cross-National Summary Statistics | 7 | | | |
| | 3.1 Theory and Design | 7 | | | |
| | 3.2 Implementation | 7 | | | |
| | 3.3 Empirical Results | 9 | | | |
| 4 | Crosssectional Probit Regression | 11 | | | |
| | 4.1 Theory and Design | 11 | | | |
| | 4.2 Implementation | 12 | | | |
| | 4.3 Empirical Results | 13 | | | |
| 5 | Wald Test | 15 | | | |
| | 5.1 Theory and Design | 15 | | | |
| | 5.2 Implementation | 15 | | | |
| | 5.3 Empirical Results | 17 | | | |
| 6 | Counterfactual Exercise | 19 | | | |
| | 6.1 Theory and Design | 19 | | | |
| | 6.2 Implementation | 19 | | | |
| | 6.3 Empirical Results | 21 | | | |
| 7 | Graphical Representation | 22 | | | |
| | 7.1 Theory and Design | 23 | | | |
| | 7.2 Implementation | 23 | | | |
| | 7.3 Empirical Results | 25 | | | |
| 8 | Conclusion | 28 | | | |
| 9 | References | 29 | | | |
| | | | | | |
| 10 | O Appendix | 31 | | | |
| | 10.1 Quantlet Code | 31 | | | |
| | 10.1.1 Quantlet 1 | 31 | | | |
| | 10.1.2 Quantlet 2 | 35 | | | |
| | 10.1.3 Quantlet 3 | 35 | | | |
| | 10.1.4 Quantlet 4 | 35 | | | |
| | 10.1.5 Quantlet 5 | 39 | | | |
| | 10.1.6 Quantlet 6 | 43 | | | |

1 Introduction

The proportion of elderly people in Europe continues to rise and is expected to impose major social, fiscal and economic challenges in the future. According to official projections, half of the European population is expected to be 50 years or older by 2050 (Commission and others 2012). Analyzing the relationship between aging populations, health indicators and labor force participation is essential to come up with suitable policy responses to those demographic changes. Empirical results on the relation between health, well-being and employment are ambiguous. While some studies show that poor health contributes to early retirement (Cai and Kalb 2006; Karpansalo et al. 2004; Monden 2005), another branch of research suggests the opposite effect: that unemployment has negative effects on well-being and health (Shields and Price 2005; Kassenboehmer and Haisken-DeNew 2008).

The Survey of Health, Ageing and Retirement in Europe (SHARE) launched in 2004 is an excellent research base for aging related research designs. This multidisciplinary and cross-national panel database of data on health, socio-economic status and social networks contains data of more than 120,000 individuals aged 50 or older (Borsch-Supan 2017). Based on the first SHARE data release, Kalwij and Vermeulen (2005) analyze the impact of different health indicator on individuals' participation decisions in 11 European countries. They find that health effects on labor force participation behavior of individuals aged 50-64 differ considerable between countries, age groups and gender.

In this paper, we present the developed code that can be utilized to replicate all the results from Kalwij and Vermeulen (2005) and furthermore provides a basis for further data exploration and visualization. For this purpose, we use the recently released easySHARE data set. We design the code in a way that allows others to explore the easySHARE data set, generate comprehensible and instructive summary statistic tables or graphics, and conduct regression-based analysis.

Our code is divided into six quantlets, which are meant to be executed chronologically. In Quantlet 1Q, we transform, clean and aggregate the variables in the raw data set and generate a list of data frames for subsequent data analysis. Quantlet 2Q generates a series of summary statistics tables that present health indicators, labor force participation and labor supply choice by country, age group and gender. In Quantlet 3Q, we conduct probit regression for each country and both genders, calculate marginal effects and average employment probability. Quantlet 4Q lays the foundation for two Wald tests that can be used to test the joint significant of a subset of variables as well as any linear hypothesis. Quantlet 5Q presents counterfactual exercises that are based on the previously carried out probit estimations. By manipulating the population health indicators, we calculate current and counterfactual labor participation rates for different age groups. In Quantlet 6Q, we introduce new methods to graphically represent the summary statistics fo exploratory data analysis. Specifically, we provide tools to generate carefully selected graphics for a subset of desired variables and sample subsets.

2 Data Cleaning and Manipulation

The original SHARE data set used by Kalwij and Vermeulen (2005), released in spring 2005, contains data on the life circumstances of eligible members from approximately 18,000 households. The SHARE survey contains a variety of different health, social, and socioeconomic indicators. SHARE contains data from eleven countries: Austria, Belgium, Denmark, France, Germany, Greece, Italy, the Netherlands, Spain, Sweden and Switzerland.

2.1 Theory and Design

The easySHARE data set, released in spring 2017, is the data set for which we will be focusing on in this paper. It is a panel data set of more than 100,000 individuals covering data from six survey waves carried out between 2004 and 2015. As can be derived from its name, it is designed to be a simplified version of the broader SHARE data set. As a result, the data set includes slightly different observations and does not provide the same level of granularity in variables broader SHARE data set, which we shall show below.

The rest of this section describes our process of data transformation, aggregation, and cleaning , not only to match as closely as possible the initial SHARE data set, but also to convert our data sets into a more accessible state for future analyses.

2.2 Implementation

The read.and.clean() function is designed to take the raw easySHARE data set file, which is provided in an .rda file format, and convert that into a list of data frames. The function additionally accepts a wave number as an argument. Although we are only concerned with the initial wave for our study, this additional parameter may allows us to conduct supplementary panel studies. The function only runs specifically with the raw easySHARE data set (easySHARE_rel6_0_0.rda), which is the default dataset argument. The dataset argument may alternatively be specified to point to the directory where that data set is located. Otherwise it is assumed that it is located in the root directory.

We begin by initializing and downloading the packages (if necessary) required for this function. Much of the function utilizes the functions in the tidyverse family of functions developed by Hadley Wickham, namely dplyr and magrittr. The dplyr package is designed for uncomplicated data manipulation, and it works specifically with data frames. The package provides a consistent set of verbs that do a small thing very well, such as aggregation, selection, filtering, or creation of new variables. The magrittr package expands upon this further with the pipe operator %>%, which enables the chaining of smaller functions to create something more complex.

```
[16]    neededPackages = c("dplyr", "magrittr", "infuser", "countrycode")
[17]    allPackages = c(neededPackages %in% installed.packages()[,"Package"])
[18]    if (!all(allPackages)) {
[19]        missingIDX = which(allPackages == FALSE)
[20]        needed = neededPackages[missingIDX]
[21]    lapply(needed, install.packages)
```

```
[22] }
[23] invisible(lapply(neededPackages, function(x) suppressPackageStartupMessages(
[24] library(x, character.only = TRUE))))
```

We continue with basic filtering to select only variables that are pertinent to our study. Then we encode missing values. The specific reason for the missing values are documented well in the "Guide to easySHARE release 6.0.0" and specifically coded. For example, the numbers -13 and -14 refer to "not asked in this wave" and "not asked in this country". Since this coding scheme is not useful for the purpose of our analysis, we recode all of the missing values as "NA".

```
[53]
        # Encode missing values according to SHARE data set quidelines
        a = c(-2, -3, -4, -7, -9, -12, -13, -14, -15, -16)
[54]
[55]
        b = c("tocheck", "implausible", "tocheck", "uncoded", "notApplicable",
              "dontKnow", "notAskedWave", "notAskedCountry", "noInformation",
[56]
[57]
              "noDropOff")
[58]
        missing.value.codes = data.frame(a,b)
        # Find NA locations and declare them as such
[62]
[63]
        df.decl = apply(dat, 2, function(z) {
[64]
            na.loc
                      = which(z %in% a)
[65]
            z[na.loc] = NA
[66]
           return(z)
[67]
        })
```

We use the countrycode package to change the country codes into human-friendly country names. Then we make minor transformations to the variables. There are minor differences between the data set that Kalwij and Vermeulen (2005) use and the easySHARE data set, which means we must make certain assumptions in our data cleaning process. For example, certain questions in the survey are an aggregated total of a series of questions, such as the types of chronic diseases the subjects suffer from. Whereas the original authors have access to the specific diseases, we are only provided a count of the total number. Therefore we replace certain variables with the closest relative we have available. In other cases, multiple variables are provided in our data set but it is ambiguous which one the original authors use. An interesting example is employment status: our data set contains one variable for type of employment (full-time, part-time, or not working) and one for labor hours (the number of hours worked by the survey respondent). However, some respondents gave answers that ran counter (for example, saying they worked full-time but only working 20 hours a week). The differences in answers can be attributed to varying interpretations of survey responses.

A simplified example of the final script is shown below.

```
[91]
        df.out
                     = df %>%
                dplyr::left join(country data, by = c("country mod" = "iso3n")) %%
[92]
                dplyr::filter(iso3c %in% country_list) %>%
[93]
[94]
                dplyr::mutate(country
                                             = factor(country.name.en),
                                             = factor(ifelse(female, "FEMALE", "MALE")),
[95]
                               gender
[99]
                                             = factor(floor(age)),
                               age
[100]
                                             = isced1997_r \%in\% c(0, 1),
                               edu_low
```

```
[102]
                                edu high
                                               = isced1997_r \frac{1}{n} c(5, 6),
[105]
                                h chronic
                                               = chronic_mod,
[108]
                                h_overweight
                                              = bmi2 == 3,
[115]
                                labor_ft
                                               = ep013 mod >= 32,
[118]
                                labor_hrs
                                               = floor(ep013_mod)) %>%
[119]
                 dplyr::select(country, gender,
                                starts_with("h_"),
[121]
[124]
                                starts_with("labor_")) %>%
[125]
                 na.omit() %>%
                                                                # remove missing values
[126]
                 set rownames(NULL)
                                                                # reset row numbering
```

Our last step involves splitting the data sets, standardizing any numeric variables, converting the data frames into a model matrix, then storing the resultant data frames into a list that is the final object returned by the function.

The data frames are split into separate country and gender splits, since we will create separate regressions for each of these splits. Then we apply a custom-built standardization function, which finds and standardizes all numeric variables and ignores all other variables. Finally, we use a custom-built function that takes advantage of the base model.matrix() function to convert all split data frames into a model matrix. This useful function converts all logical vectors into dummies to replicate the process used in the paper. A corollary benefit is that the resultant data frames can be directly used in R's in-built regression functions.

```
[146]
         standardize.df = function(df) {
[148]
             idx = sapply(df, is.numeric)
[149]
             idx = seq(1:length(idx))[idx]
[154]
             df.reg = df %>%
[155]
                 mutate_at(.vars = vars(idx),
[156]
                            .funs = standardize) %>%
[157]
                 mutate(labor participation = !labor np) %>%
[158]
                 dplyr::select(country, gender, age,
Γ1597
                                h_chronic, h_adla, h_obese, h_maxgrip,
Γ160]
                                edu_second, edu_high, children, couple,
[161]
                                labor participation)
Γ1627
[163]
             return(df.reg)
         }
[164]
[167]
         splits
                   = split(df.out, f = list(df.out$country, df.out$gender),
[168]
                            drop = TRUE)
[169]
         df.reg
                   = standardize.df(df.out)
[170]
         df.splits = lapply(splits, standardize.df)
Γ1737
         dummify = function(df) {
```

```
[174]
             df = df \%
[175]
                  dplyr::select(-country, -gender)
[176]
             model
                         = \sim 0 + .
[177]
             new.df
                         = model.matrix(model, df)
                         = data.frame(new.df)
[178]
             new.df
             return(new.df)
[179]
[180]
         }
[181]
[182]
         df.splits = lapply(df.splits, dummify)
```

2.3 Empirical Results

The validity of our read.and.clean() can be verified by comparing the overall results when running the read.and.clean() function with the ones presented in the original paper. As the function is called, messages are printed to the console to show the status of the data reading process and the data frames contained within the outputted list.

In the original paper, the authors retain a sample of 12,237 observations, which is slightly smaller than our retained observations of 12,689. However, at the individual country level, some peculiarities in number of observations emerge. Namely, we have almost twice as many observations in France, while having relative sparsity in Austria. This again is likely due to differences in data set and in the data gathering process. We will discuss more details about the summary statistics in future sections.

```
source('Quantlets/SPL_ELP_ReadandClean/SPL_ELP_ReadandClean.R')
datasets = read.and.clean(dataset = "easySHARE_rel6_0_0.rda", wav = 1)

## Loading data set...
## Selecting values only from Wave 1 and between ages 50 and 64.
## Removing missing values.
## Rows removed: 276047

## Rows remaining: 12689

##

## Final output is a list containing 3 data.frames:
## 'df.out' is a data.frame containing variables for calculating summary statistics.
## 'df.reg' is a data.frame containing standardized variables for regression.
## 'df.splits' splits the regression data.frame into individual data.frames for each country/
## gender split. 22 data.frames are contained in this list.
```

3 Multidisciplinary and Cross-National Summary Statistics

3.1 Theory and Design

Since we are working with a cross-national survey, it is useful to to present an overview on a country-by-country basis. For each country, survey participants will be classified according to their age group. Within each class, shares of gender-specific as well as overall labor participation rates and health indicators are calculated and discussed.

3.2 Implementation

An efficient way of separating the easySHARE data set into country, gender, and age is explained in the following. In order to minimize hard-coding and possibly related errors, we use the expand.grid function to expand a grid between named variables in the data set. This allows us to calculate the necessary summary statistics for all combinations of country, gender, and age.

We create the function labor(), which calculates labor participation rates for the corresponding country, gender, and age cell within the grid. The group.share() function calculates the average or percentage for an input data frame and a column name by country. The particular share is calculated by finding the sum of occurences and dividing by their total, represented by the length of the vector.

```
# Function for calculation group shares per country
     group.share = function(df, y, agg.country = TRUE) {
[53]
[54]
          if (agg.country) {
[55]
              final = tapply(df[[y]], df$country, function(x) {
                  val = sum(x)/length(x)
[56]
[57]
                  return(val)
              })
[58]
[59]
          } else { # return total if country-level aggregation not specified
[60]
              final = sum(df[[y]])/length(df[[y]])
[61]
          }
[62]
          return(final)
[63]
     }
[64]
[65]
      # Function for calculating labor participation rate per country / gender / age
[66]
     labor = function(df, .country, .gender, .age) {
[67]
          dataset = df %>%
[68]
              filter(country == .country & gender == .gender & get(.age) == TRUE)
          denom = nrow(dataset)
[69]
          numer = with(dataset, sum(labor ft | labor pt))
[70]
[71]
          perc = round(numer/denom, 4)
[72]
[73]
          return(perc)
[74] }
```

Within the code below, args.list contains three sub-lists divided by country (c), gender (g) and age (a). The list is passed as an argument to labor, which are simulatenously iterated over the sub-lists using pmap. This function has an advantage over a similar functions, since it allows multiple arguments and uses parallelization by default.

```
[76]
     # use expand.grid to get all combinations of country / gender / age
      agegroups = names(df.out[, 4:6])
[78]
     args
                = with(df.out,
[79]
                     expand.grid(levels(country), levels(gender), agegroups,
                                   stringsAsFactors = FALSE))
[80]
     args.list = list(c = as.vector(t(args[1])),
[81]
[82]
                       g = as.vector(t(args[2])),
[83]
                       a = as.vector(t(args[3])))
[84]
[85]
      # Map over multiple arguments
[86]
      output = pmap(args.list, function(c, g, a) labor(df.out, c, g, a))
      output.v = do.call("c", output)
[87]
[88]
[89]
                          = cbind(args, output.v)
     labor.part.share
[90]
     labor.part.share.df = spread(labor.part.share, Var3, output.v) %>%
[91]
          arrange(desc(Var2)) %>%
[92]
          set_colnames(c("Country", "Gender",
                         paste0("Age ", rep(c("50-54", "55-59", "60-64")))))
[93]
```

The resulting output list can be easily bound into a data frame and arranged using dplyr. We then use tidyr::spread in order to spread the variable Var3, being the three age groups, into separate columns. Using the pipe operator %>% keeps our code compact and avoids unnecessary variable assignment and inefficient memory requirements.

In the next code chunk, vars is assigned to be a name vector that specifies whether labor force participation is full time, part time or not prevalent. Fetching it using grep from the cleaned data set makes it robust against name changes in the underlying data frame and ensures consistency. Conveniently, grep does not necessarily require regular expressions as an input. Instead, it is able to perform matches on simple strings and, in this case, returns the column names that contain 'labor'. This is forwarded as a name call to the second argument in group.share. It is used to subset the target data frame df by the extracted variable names containing the string "labor".

Since this task involves invoking a function upon multiple list elements, lapply is the best candidate in order to construct labor participation rates grouped by gender, (optionally) by country and classified into non participation, part time and full time work.

```
[105] # Labor supply choice tables
[106] df.female = df.out %>% filter(gender == "FEMALE")
[107] vars = names(df.out)[grep("labor", names(df.out))][3:1] # invert
[108]
[109] labor.supply.f = lapply(vars, function(x) group.share(df.female, x, 1))
```

```
[110] labor.supply.f.df = do.call(cbind.data.frame, labor.supply.f) %>%
[111] set_colnames(c("Nonparticipation", "Half time", "Full time"))
```

3.3 Empirical Results

We can verify the validity of our results by comparing them directly with the findings in the original paper. Before interpreting the data, we advise to note that some country's participants are generally older than other ones. As the evaluation of the probit model shows, we figure age to have a negative impact on health. Hence certain samples may appear less healthy than others due to the fact that they are older on average.

As depicted in the table below, the mean value of chronic diseases is 75%. Conversely, 79% out of the 12689 participants have a good self-perceived health. At first sight, one may deduce that the majority considers themselves to be healthy despite the prevalence of a chronic disease. But the use of the arithmetic mean is sensitive towards outliers and the amount of chronic diseases is in fact counted numerically. In combination, these values indicate an uneven distribution. Surprisingly, the share of overweight people is only 42% compared to an EU-average which Eurostat (2014) reports reports to be 59.6% among 45 to 64 year old citizens. However, information on obesity grouped by age is not available to us.

Table 1: Summary Statistics over all groups

| | TOTAL |
|--------------------------------------|----------|
| Observations | 12689.00 |
| Age 50-54 | 0.36 |
| Age 55-59 | 0.37 |
| Age 60-64 | 0.27 |
| Num chronic diseases (mean) | 0.75 |
| ADLs (in %) | 0.04 |
| Max grip strength (mean) | 37.87 |
| Overweight (in %) | 0.42 |
| Obese (in %) | 0.18 |
| Bad mental health (in $\%$) | 0.21 |
| Good self-perceived health (in $\%)$ | 0.79 |

Having obtained the share of labor force participation grouped by gender, country and age, striking values can be presented. Among the selected countries, France, the Netherlands and Belgium have the fewest laborers among the oldest age group, which is ranging from 60 to 64 years. In the middle age group, ranging from 54-59, the Belgian and Italian population have sparse labor participation rates among men and women. Within the youngest age group including people from 50 to 54 years old, variance among men is generally low. This is not the case female workers, among which Greek, Italian and Spanish participation rates do not exceed 45.5%.

Concerning health statistics, Switzerland and the Scandinavian countries stand out positively. Together, they from a group that has an overall low percentage of obese people while achieving high shares of self-reported good health. Among all other countries, the Swiss have the least number of chronic diseases and

fewest overweight. Only the Greek have lower share of activities of daily living (ADL). Switzerland and the Nordic countries additionally present the least non-participation rates and the highest full-time working rates. Summary statistics show that those countries, which strive in health topics, are related to high labor force participation rates in all categories.

The overall age distribution is almost even in the younger age brackets (36% and 37%). The participants beween 54 and 59 years old make up 27%. Generally, labor force participation is higher for males than for females. This applies to part-time work, too.

The overall results of the descriptive statistics align well with Kalwij and Vermeulen (2005).

Table 2: Men: Labor Force Participation Rate by age

| | Age 50-54 | Age 55-59 | Age 60-64 |
|-------------|-----------|-----------|-----------|
| Austria | 0.8200 | 0.5700 | 0.1250 |
| Belgium | 0.7879 | 0.5398 | 0.2139 |
| Denmark | 0.8333 | 0.7537 | 0.5625 |
| France | 0.8549 | 0.5746 | 0.1729 |
| Germany | 0.8025 | 0.7538 | 0.3704 |
| Greece | 0.8359 | 0.7040 | 0.4545 |
| Italy | 0.8443 | 0.5220 | 0.2738 |
| Netherlands | 0.8688 | 0.7585 | 0.2969 |
| Spain | 0.8031 | 0.7410 | 0.3966 |
| Sweden | 0.9254 | 0.8147 | 0.7005 |
| Switzerland | 0.9114 | 0.8947 | 0.6349 |

Table 3: Women: Labor Force Participation Rate by age

| | Age 50-54 | Age 55-59 | Age 60-64 |
|-------------|-----------|-----------|-----------|
| Austria | 0.6395 | 0.3235 | 0.1129 |
| Belgium | 0.6200 | 0.3105 | 0.1261 |
| Denmark | 0.8467 | 0.7099 | 0.2768 |
| France | 0.7007 | 0.5815 | 0.1938 |
| Germany | 0.7738 | 0.5767 | 0.2350 |
| Greece | 0.3849 | 0.2929 | 0.1507 |
| Italy | 0.4337 | 0.2410 | 0.0762 |
| Netherlands | 0.5939 | 0.4836 | 0.1616 |
| Spain | 0.4550 | 0.3698 | 0.2063 |
| Sweden | 0.8378 | 0.8109 | 0.6000 |
| Switzerland | 0.7234 | 0.6234 | 0.4643 |

4 Crosssectional Probit Regression

4.1 Theory and Design

Probit models allow for the estimation of binary, random response variables (Schild 2017). Probit and Logit models mitigate problems that would occur in case a (linear) regression would be used for a similar task. In this case, the range of the response variable would not be restricted to the interval (0,1). Consequently, as probabilities are commonly defined on this space, non-interpretable probability estimations would be delivered. While literature often refers to use maximum likelihood to estimate coefficients, the glm function in R uses iterative re-weighted least squares (R Core Team 2013). The estimated coefficients are subject to a Wald Test on joint significance.

Schild (2017) defines the model formula as

$$P(Y = 1|X = x) = \Phi(x'\beta) \tag{1}$$

The error term is assumed to be independent and identically distributed (Wooldridge (2012))

Using this model, it is investigated how the probability of labor force participation is influenced by health and age variables.

Whereas a simple linear regression allows for direct interpretation of the estimated coefficients, this is not the case for probit models. As we are particularly interested in the effects of certain regressors on the probability of labor force participation, we are especially focusing on the calculation and evaluation of marginal effects. The marginal effect of x_j on P(Y = 1|X = x) is defined as the effect of a ceteris paribus change of x_j on the conditional probability that the response variable Y is realized as one. In case of a probit model, the marginal effect of x_j on P(Y = 1|X = x) is dependent on x.

Wooldridge (2012) specifies two possible interpretations of the mariginal effect. The "Partial Effect at the Average" (PEA) denotes the marginal effect of a regressor on an average person $(\frac{1}{n}\Sigma_{x_i})$. Problematic are discrete x_i because the resulting averages do not represent anybody in particular. Furthermore, in case of a continuous explanatory variable wrapped inside a non-linear function, it remains unclear whether to apply the average over the function or insert the average into named function.

According to Bauer (2014), the Partial Effect at the Average is defined as

$$PEA = \Phi(\hat{\beta}_0 + \hat{\beta}_1 \sum_{i=1}^n x_i) \beta_1 \tag{2}$$

The "Average Partial Effect" (or "Average Marginal Effect", APE) describes the effect of a regressor on the probability that Y = 1. Bauer (2014) defines it as

$$APE = \frac{1}{n} \sum_{i=1}^{n} \Phi(\hat{\beta}_0 + x_i \hat{\beta}_1) \beta_1$$
 (3)

where β is the vector of coefficients and x_i is the vector of observations at index i. Φ represents the cumulative standard normal distribution function.

4.2 Implementation

As stated in the 'Read and Clean' section, df.splits is obtained using the split function. A list is returned, that seperates the cleaned data frame by country and gender. This structure is helpful throughout the following sections, as it enables us to build a series of concatenated lapply functions. This function invokes a c-loop that iterates over each list element. As lapply takes a list as an input and returns a list, too; it preserves the initial structure. First, it is used to create a probit model for each subset within the list. Since we are estimating with an intercept, the age 50 dummy variable is perfectly correlated with the other age dummies. Hence it is excluded due to multicollinearity.

```
# Probit for each country and gender
[54]
      allModels = lapply(df.splits, function(z){
[55]
[56]
                = z[-z$age50] # Multicollinearity
[57]
          model = glm(z$labor_participationTRUE ~., family = binomial(link = "probit"), data = z)
[58]
[59]
          return(model)
[60]
[61]
     })
[62]
[63]
      # Return summaries
Γ641
      allSummaries = lapply(allModels, summary)
```

The loop below is a robust way to apply the Wald Test for the dataset subsetted by country and gender and utilises only health-related variables. A list intended to log is created, which tracks failure of applying the Wald Test on joint significance of the selected variables. Try ensures continuation of the loop despite possible errors. In case an error occurs, the class of testOutput will be try-error and consequently logged into named list at the index i. Since it preserves the structure as specified before, results can be easily transformed and named. This setup has proven to be useful for debugging our test as well as finding out that some health variables are empty for certain subgroups, leading to selective failure of the test.

```
# Wald Test for all models
[70]
[71]
     wald.log = list() # Save Wald Test Output
[72]
     for(i in 1:length(allSummaries)){
[73]
[74]
[75]
        # Get Element
[76]
        SummaryElement = allSummaries[[i]]
[77]
[78]
        # Specify the of coefficients to be tested: only health variables
[79]
        health = c(16:19)
[80]
        # Test only the joint significance of health variables
[81]
[82]
        testOutput = try(joint.wald.test(allSummaries[[i]], 0.95, health))
[83]
```

```
if(class(testOutput) == "try-error"){
[84]
[85]
          msg = pasteO("Wald Test failed for Model Element ", i)
[86]
          warning(msg)
          wald.log[[i]] = "Error"
[87]
[88]
        } else{
[89]
          wald.log[[i]] = testOutput
[90]
[91]
        rm(SummaryElement) # clean up
[92]
[93]
[94]
      wald.bound = as.data.frame(wald.log)
      colnames(wald.bound) = names(allModels)
```

Furthermore, the application of the Wald Test can easily be substitued. We use a different Wald Test to validate our results.

```
[82] testOutput = try(wald.test(b = coef(ModelElement),

[83] Sigma = vcov(ModelElement), Terms = health)$result)
```

Marginal effects are calculated using probitmfx, which firstly recalculates a probit model. As the boolean 'atmean' evaluates to true, the marginal effects are defined as the partial effects for the average observation (Fernihough 2014).

```
[117] # Calculate marginal effects and standard errors
[118]
[119] # Same structure as before, but must calculate model again.
[120] mfx.Models = lapply(df.splits, function(z){
[121]
[122] z = z[-z$age50] # Multicollinearity
[123] res = probitmfx(z$labor_participationTRUE ~.,atmean = TRUE, data = z)
[124]
[125] return(res)
[126] })
```

4.3 Empirical Results

By means of the presented model, we evaluate how labor force participation behavior of individuals aged 50-64 is influenced by age and demographic as well as health indicators across 11 European countries. Kalwij and Vermeulen (2005) summarize their own results as follows:

- The influence of health (as captured by the indicators) on labor force participation differs among individuals and across countries
- Health improvements are associated with higher labor force participation rates
- Decreasing health explains decreasing labor force participation of the elderly in Europe

These main results are validated by the analysis. As shown representatively on the example below, which is a

summary of the probability of Austrian women participating in the labor market as explained by the probit model, age and the prevalence of chronic diseases are negatively related to employment. The overall negative influence and the statistical significance of age on the dependent variable is growing with increasing age. Conversely, high education and having second education positively predict labor force participation for the specified group, but statistical significance usually restricted to one of them. Additionally, we find maximum grip strength to be statistically significant in multiple models. Having children or a partner does not appear to make a difference. Especially the negative influence of age is extraordinarily stable across different models.

Table 4: Marginal effects of health and age related variables on the probability of labor market participation of Austrian women

| | Estimate | Std. Error | z value | $\Pr(> \mathbf{z})$ |
|-------------------|----------|------------|---------|----------------------|
| (Intercept) | 0.6844 | 0.4351 | 1.57 | 0.1157 |
| age 51 | -0.2492 | 0.4880 | -0.51 | 0.6096 |
| age 52 | -0.3909 | 0.4369 | -0.89 | 0.3709 |
| age53 | -0.7001 | 0.4576 | -1.53 | 0.1260 |
| age 54 | -0.1578 | 0.5613 | -0.28 | 0.7786 |
| age55 | -0.9350 | 0.4169 | -2.24 | 0.0249 |
| age 56 | -0.7092 | 0.4398 | -1.61 | 0.1069 |
| age 57 | -1.4925 | 0.4367 | -3.42 | 0.0006 |
| age 58 | -1.1910 | 0.4340 | -2.74 | 0.0061 |
| age 59 | -1.6635 | 0.5006 | -3.32 | 0.0009 |
| age 60 | -1.8610 | 0.4525 | -4.11 | 0.0000 |
| age 61 | -1.6772 | 0.4641 | -3.61 | 0.0003 |
| age 62 | -1.7351 | 0.4652 | -3.73 | 0.0002 |
| age63 | -2.2090 | 0.4923 | -4.49 | 0.0000 |
| age64 | -6.2803 | 203.4545 | -0.03 | 0.9754 |
| h_chronic | -0.1722 | 0.0908 | -1.90 | 0.0578 |
| $h_adlaTRUE$ | 0.2082 | 0.4569 | 0.46 | 0.6486 |
| $h_obeseTRUE$ | -0.1314 | 0.1912 | -0.69 | 0.4920 |
| $h_{maxgrip}$ | 0.0201 | 0.0814 | 0.25 | 0.8049 |
| $edu_secondTRUE$ | -0.0076 | 0.2424 | -0.03 | 0.9751 |
| $edu_highTRUE$ | 0.6778 | 0.2885 | 2.35 | 0.0188 |
| children | -0.0162 | 0.0837 | -0.19 | 0.8468 |
| coupleTRUE | -0.1606 | 0.1683 | -0.95 | 0.3397 |

5 Wald Test

In the context of probit regression, the Wald test can be used to test multiple hypotheses regarding the model specifications. For example, we test whether the fit of the model is improved if a subset of regression coefficients are all set equal to zero, an exclusion restriction. Kalwij and Vermeulen (2005) conduct a Wald test to check the null hypothesis that none of the included health variables has an impact on labor participation in order to investigate the joint impact of health on participation.

5.1 Theory and Design

Depending on the estimation method and distributional assumptions, the Wald statistic can be formulated in different ways. The general form of the Wald statistic after MLE for testing hypothesis regarding our $k \times 1$ parameter vector θ is given by

$$W = c(\hat{\theta})' [\nabla_{\theta} c(\hat{\theta}) \hat{V} \nabla_{\theta} c(\hat{\theta})]^{-1} c(\hat{\theta})$$
(4)

where $c(\hat{\theta})$ is a $m \times 1$ vector of linear or nonlinear restrictions, $\nabla_{\theta} c(\hat{\theta})$ is the $m \times k$ Jacobian of $c(\hat{\theta})$ evaluated at $\hat{\theta}$ and \hat{V} is the estimated asymptotic covariance matrix (Wooldridge 2010, 463). Under H0, the Wald test statistic is asymptotically $\chi 2_m$ distributed, with m being the number of specified restrictions. In order to assure the test statistic W has the assumed limiting distribution, we need to impose some restrictions: Under H0, θ must lie within parameter space and R must be of rank m (Wooldridge 2010, 362). We limit our attention to testing a set of general linear restrictions since the Wald test is not invariant to the re-formulation of non-linear hypothesis. We thus formulate our null hypothesis in accordance with the common linear restriction structure of H0: $R\hat{\beta} = r$ against the alternative H1: $R\hat{\beta} \neq r$ to facilitate the derivation of our test statistics, where R is a $m \times k$ matrix of rank m (equivalent to the Jacobian), whereas the restriction function r is a $m \times 1$ vector. Given the conditions mentioned above, the Wald statistic can then be rewritten as

$$W = (R\hat{\beta} - r)'[R\hat{V}R']^{-1}(R\hat{\beta} - r)$$
(5)

which facilitates our calculations (Greene 2012, 527–29; Wooldridge 2010, 362). When we are interested in the joint significance of a subset of s coefficients, the Wald test statistic $W \stackrel{a}{\sim} \chi 2_s$ can be simplified further (Wooldridge 2010, 362):

$$W = \hat{\beta}_s' [R\hat{V}R']^{-1} \hat{\beta}_s = \hat{\beta}_s' [\hat{V}_s]^{-1} \hat{\beta}_s$$
 (6)

5.2 Implementation

In order to test the linear hypothesis regarding the model specifications, we design two Wald test versions. The function joint.wald.test can be used to test the joint significance of a subset of model coefficients, whereas general.wald.test allows for us to check any linear hypothesis. The two tests are designed in a way that they align well with the glm estimation output. The only required input is the model summary from glm estimation, whereas the significance level and test specifications are optional.

```
[47] names(joint.wald.test) = c("W","p-value", "df", "Decision")

[48] beta = model.summary$coefficients[,1]

[49] Var_beta_est = vcov(model.summary)
```

In order to set up default values for the hypothesis specification, we make use of a local if-else statement inside the joint.wald.test function. In line 26 we re-assign a sequence vector to spec that includes the number of all model coefficients in increasing order, unless spec is specified differently.

```
[25]
        if (is.null(spec)) {
[26]
            spec = 1:length(beta) # default joint is significance test
[27]
        } else if (is.logical(spec)) {
[28]
            stop("spec cannot be a logical vector, tranform to integer vector!")
[29]
        } else if (any(spec == 0)) {
            stop("spec cannot contain zero values!")
[30]
[31]
        } else if (!is.integer(spec)) {
[32]
            spec.len = length(spec)
                     = as.integer(spec, length = spec.len)
[33]
[34]
            warning("Converting spec to integer and proceeding")
[35]
        }
[51]
        # Wald test statistic
[52]
        W = t(beta[spec]) %*% solve(Var_beta_est[spec,spec]) %*% beta[spec]
```

This means joint.wald.test will conduct a joint significance test on all model coefficients by default. The Wald test statistic is calculated via simple matrix algebra based on the provided input by the model summary. This formula is equivalent to equation 6.

The proper format of the specification vector spec is crucial as it is used to extract the needed estimates from the model coefficient vector and covariance matrix. To select the right covariance elements and degrees of freedom, spec must be a vector of integers of length $0 < m \le k$. Specifically, we make sure spec is neither logical nor contains any zero in line 27-30. Furthermore, we only allow for integers since we are testing the joint significance of coefficients.

The general Wald test is designed in a similar manner, except it allows for the general formulation of linear hypothesis of the form $R\hat{\beta} = r$. The test statistic is given by equation 5. As in the joint.wald.test function, we use local local if-else statements to setup a default settings. If only the model summary is given as an input, the general.wald.test conducts a joint significance test at a 95% confidence level.

The crucial elements for usage of this function are the correctly specified Jacobian matrix R and restriction vector r, which must be of size $m \times k$ and $m \times 1$ respectively. We additionally need to assure that R is of rank m. These conditions are tested in line 123-131.

```
[100] if (class(model.summary) != "summary.glm") {
[101]     stop("model.summary must be a glm summary!")
[102] }
[103] if (confidence.level > 1 | confidence.level < 0) {
[104]     stop("confidence.level out of bounds!")</pre>
```

```
[105] }

[123] dim_R = dim(R) # Testing R

[124] k = length(beta) # Columns
[125] m = length(r) # Rows; general hypothesis test, therefore m = length(r)

[128] if (dim_R[1] != m | dim_R[2] != k) stop("R has wrong dimension!")
[131] if (rankMatrix(R)[1] != m) stop("R has wrong rank!")
```

We furthermore check whether the model.summary has the right class and whether the significance level lies within the expected interval. Otherwise, the execution of the function will be stopped. In the second step, we make sure the matrix R and the restriction vector r have matching dimensions.

5.3 Empirical Results

We use our designed Wald test functions to check the null hypothesis that none of the four included health variables has an impact on labor participation for each country and for both men and women. The results for men can be seen in table 5.

| Table 5: | Wald | test | on 1 | no | ioint | impact | of | health | on | participation | n |
|----------|------|------|------|----|-------|--------|----|--------|----|---------------|---|
| | | | | | | | | | | | |

| | W statistic | p-value | Test decision |
|-------------|-------------|--------------|------------------|
| Austria | 13.63 | 0.00859 | Reject H0 |
| Belgium | 12.59 | 0.01346 | Reject H0 |
| Denmark | 24.32 | 6.886 e - 05 | Reject H0 |
| France | 5.318 | 0.2562 | Cannot reject H0 |
| Germany | 39.42 | 5.701 e-08 | Reject H0 |
| Greece | 0.9592 | 0.9159 | Cannot reject H0 |
| Italy | 8.334 | 0.08009 | Cannot reject H0 |
| Netherlands | 19.95 | 0.0005113 | Reject H0 |
| Spain | 34.96 | 4.738e-07 | Reject H0 |
| Sweden | 25.42 | 4.141e-05 | Reject H0 |
| Switzerland | 9.906 | 0.04205 | Reject H0 |

Our results are in line with the findings of Kalwij and Vermeulen (2005). As expected, the null hypothesis of no impact of health on labor participation is rejected at a 95% confidence level for most countries. One difference is the inability to reject the null for France. We explain this with our differing sample size and overall lower employment. Regarding the Wald tests for women, our results are very similar to those Kalwij and Vermeulen (2005). The inability to reject the null for some countries is due to relatively low health-related probit coefficients (in absolute terms), which implies that these health indicators are not strongly related with the labor participation decision.

The results of our joint.wald.test function are comparable those of our packages, like the wald.test from

the aod package. For example, for German women, the aod package and our joint.wald.test return similar output:

```
wald.test(Sigma = vcov(allModels$Germany.FEMALE),
              b = allModels$Germany.FEMALE$coefficients, Terms = 16:19)
joint.wald.test(allSummaries$Germany.FEMALE, spec = 16:19)
## Wald test:
## -----
##
## Chi-squared test:
## X2 = 9.2, df = 4, P(> X2) = 0.057
                                 p-value
                                                          df
##
                                "0.05688"
                                                         "4"
##
              "9.175"
##
             Decision
## "Cannot reject HO"
```

A weakness of the aod package is that its Wald test cannot deal with empty classes. For the case of Switzerland, where our male sample does not include any 64-year-olds, the wald.test from the aod package leads to an error message.

```
## Error in L %*% V: non-conformable arguments
```

Our Wald test function is robust against the presence of empty classes since we work with model summaries, meaning that the coefficients of empty classes have been dropped instead of being a missing value.

```
joint.wald.test(allSummaries$Switzerland.MALE, spec = 16:19)

## W p-value df Decision
## "9.906" "0.04205" "4" "Reject HO"
```

Both our Wald test functions are sensitive to the improper formulation of tested model restrictions. For example, the general.wald.test function stop from executing if the Jacobian matrix R and the restriction vector r have non-matching dimensions.

```
matrix.R = diag(1, 23)
vector.wrong = rep(1,22)
vector.right = rep(1,23)
general.wald.test(allSummaries$Switzerland.MALE, R = matrix.R, r = vector.wrong)
general.wald.test(allSummaries$Switzerland.MALE, R = matrix.R, r = vector.right)
```

Error in general.wald.test(allSummaries\$Switzerland.MALE, R = matrix.R, : R has wrong dimension!

```
W p-value df Decision "1050" "0" "22" "Reject HO"
```

In this example, Ris an identity matrix of size 22×22 , whereas vector wrong is 21×1 . As soon as we add

another element to the r vector, the function works properly due to dimension matching.

6 Counterfactual Exercise

Counterfactual exercises are a common tool in economic policy analysis that allow assessing and quantifying possible effects of policies. In order to evaluate the quantitative importance of a healthy population on labor participation, Kalwij and Vermeulen (2005) conduct a simple counterfactual exercise. They compare the current national employment rate to the predicted counterfactual rate. This counterfactual employment rate is obtained by assuming that all individuals are in perfect health. According to their definition, a perfectly healthy individual has never had a severe or mild medical condition, suffers from no restrictions in activities of daily living, is not obese, and has a grip strength of an average 50-51 year-old (fe)male individual.

6.1 Theory and Design

Counterfactual exercises are based on regression methods that can be carried out in numerous ways. The simplest method to is to replace the existing covariates values X with counterfactual values X_{cf} (Chernozhukov, Fernández-Val, and Melly 2013). For the case of probit regression with a binary response variable y for employment, we can interpret the fitted values \hat{y} and \hat{y}_{cf} as current and counterfactual employment probabilities for each individual. Based on these calculated individual employment probabilities, we can calculate the current and counterfactual population employment rates (Π_{ac}, Π_{cf}) by taking the mean, making use of the Law of Large Numbers.

Obtaining the counterfactual probabilities \hat{y}_{cf} requires a two-step procedure. First, the regular probit regression of y on X is tun to obtain the estimated coefficients $\hat{\beta}$. Second, the counterfactual probabilities for each individuals Π_i are calculated by predicting y based on X_{cf} and the estimated coefficients.

$$\Pi_{icf} = Pr(Yi = 1|Xi = xi_{cf}) = \Phi(xi'_{cf}\beta)$$
(7)

We can assess counterfactual health-related effects by comparing the actual and counterfactual employment rates Π_{ac} and Π_{cf} . We gain closer insights into the non-linear relation between health effects and participation by calculating Π_{ac} and Π_{cf} for different age groups. This also allows assessing the proportion of decline in participation that is due to decline in health condition according to our model estimates.

6.2 Implementation

Our counterfactual effects are based on the manipulation of explanatory variables according to some specified criteria. We design this counterfactual exercise in a way that allows us to directly use the output from our probit estimation. As a first step, we create a simple function empl.rate that takes an estimation model object as input and predicts the employment rate, i.e. the mean of fitted values, as an output. In order to calculate the both current and counterfactual employment rates Π_{ac} and Π_{cf} conveniently, we manipulate the original covariates contained in our model objects. For this purpose we design the function X.cf, whose only

input the model object, allowing us to treat each country separately later on. As can be seen in line 54, the function creates a duplicate version of original data X, whose values will be replaced later on.

In line 60, we calculate the minimum value for each variable in X, which corresponds to the perfect health conditions of the variables h_chronic, h_adlaTRUE, and h_obeseTRUE since we are dealing with standardized variables. We obtain these values with making use of the apply function and store them in the vector X_min. We also calculate the mean average grip strength of 50- to 51-years old in the sample X. As can be seen in line 64, we derive this mean with the tapply command because it allows us to specify an age group index and returns a vector whose elements we can access.

```
[50]
        X.cf = function(model) {
[52]
            X
                                     = model$data
                                     = X
[54]
            X_cf
[57]
            X_{\min}
                                     = data.frame(t(apply(X, 2, min)))
[59]
            names.vec
                                     = c("h chronic", "h adlaTRUE", "h obeseTRUE")
[60]
            X cf[, names.vec]
                                     = X min[names.vec]
                                     = ifelse(X$age50 == 1 | X$age51 == 1, 1,0)
[62]
            age_50_51
[64]
            X_{cf}[, c("h_maxgrip")] = tapply(X$h_maxgrip, age_50_51 == 1, mean)[[2]]
[65]
            model$data
                                     = X cf
```

To obtain the counterfactual covariates, the values in X_cf are replaced with perfect health values. As a final step, we replace the original model data in X_cf. This allows us to access both the model estimates and counterfactual covariates in one object. By running our empl.rate function on the returned model, we directly obtain the counterfactual employment rate for a given country.

Since we are also interested in the current and counterfactual employment rates of different age groups, we create a more advanced function empl.rate.age. This function requires the estimation model as mandatory argument and has two optional arguments (group.size and group.low) that allow to change the age group and group sizes. Given the specified arguments, the function then calculates the current and counterfactual employment rate for each age group.

The proper value assignment of the numerical vector group.low is essential since we base the dynamic creation of age groups on it. As can be seen in line 104, we use a small for loop to determine the age group upper bound and create a label of each age group. Furthermore, we assign all of the corresponding age values (e.g. age50, age51) to the vector z, which correspond to the age variable names in X.

We need to assign the age values to z in order to locate the right individuals in our data frame X. We find the individuals belonging to a given age group by summing over rows of X and storing this vector inside the list

IND_row_vec displayed in line 118 We can take the sum because the age values assigned to z correspond to the column names of the data frame X. Thus, for a given individual, the row sum of X[, z] is either zero or one, depending on wether the individual belongs to the age group or not. Following this step, we filter out only the relevant age groups via indexing.

```
[118] IND_row_vec[[i]] = apply(X[, z], 1, sum)
[123] IND_row_vec = IND_row_vec[group.low]
[128] empl.rate[k] = empl.probability %*% IND_row_vec[[k]] / sum(IND_row_vec[[k]])
```

Finally, we calculate the employment rate for each age group in line 128 For this, we first sum the predicted employment probability for all individuals in age group k via vector multiplication. Next, we divide this sum by the overall number of individuals in that age group, corresponding to the vector length IND_row_vec[[k]].

Running the empl.rate.age function on a given model object will return a numerical vector containing the employment rate of the specified age groups.

6.3 Empirical Results

Running our created counterfactual functions on all the created subsamples allows us to assess the labor participation potential of a healthy population for different countries and both genders. Table 6 display these results on an aggregate level. On average, the predicted participation rate is 4.5 percentage points given the perfectly healthy individuals (ceteris paribus), although there are quite some outliers. For Dutch women, the estimated counterfactual participation rate is 8 percentage points higher, corresponding to a relative increase of 18%. This is due to the high coefficients of chronic disease and obesity in absolute terms, combined with the above average prevalence of chronic disease and obesity among Dutch women. For men, the results are even more pronounced. In Spain, for example, the counterfactual participation rate is almost 12 percentage points higher, driven by high coefficients of chronic disease, medical conditions and obesity in absolute terms. As expected, our results are in line with Kalwij and Vermeulen (2005) general, though the estimated participation rates differ for some countries, as a result of differing samples.

Table 6: Women's current and counterfactual labor share

| | Employment Current | Employment Counterfactual |
|-------------|--------------------|---------------------------|
| Austria | 0.33 | 0.37 |
| Belgium | 0.38 | 0.43 |
| Denmark | 0.64 | 0.69 |
| France | 0.54 | 0.57 |
| Germany | 0.56 | 0.60 |
| Greece | 0.30 | 0.31 |
| Italy | 0.25 | 0.27 |
| Netherlands | 0.45 | 0.52 |
| Spain | 0.36 | 0.38 |
| Sweden | 0.76 | 0.83 |
| | | |

| | Employment Current | Employment Counterfactual |
|-------------|--------------------|---------------------------|
| Switzerland | 0.63 | 0.63 |

It must be stressed that the counterfactual estimates presented by Kalwij and Vermeulen (2005) should be interpreted with caution. First of all, the transformation of the status quo populations with perfectly health individuals is artificial and represents a state that could not realistically be reached with any policy. Second, some of the high counterfactual rates are driven by relatively high model coefficients that are not statistically significant, indicating non-robust results. Third, the aggregate country estimates do not properly display the variation of health effects among different age groups. In order to address this, we also analyze the results of the counterfactual exercise with help of our empl.rate.age function. The output of this for men is displayed in table XXX. As expected, both the current and counterfactual labor shares decrease at higher age. The difference between current and counterfactual shares is also increasing in age, though some exceptions (Austria, Italy and Greece) exist. Again, our results are in line with those obtained by Kalwij and Vermeulen (2005).

Table 7: Men's current and counterfactual (CF) labor share for different age groups

| | Current 50-54 | CF 50-54 | Current 55-59 | CF 55-59 | Current 60-64 | CF 60-64 |
|-------------|---------------|----------|---------------|----------|---------------|----------|
| Austria | 0.82 | 0.85 | 0.57 | 0.64 | 0.12 | 0.16 |
| Belgium | 0.79 | 0.81 | 0.54 | 0.58 | 0.22 | 0.26 |
| Denmark | 0.83 | 0.87 | 0.75 | 0.83 | 0.56 | 0.65 |
| France | 0.86 | 0.87 | 0.57 | 0.62 | 0.17 | 0.20 |
| Germany | 0.81 | 0.86 | 0.75 | 0.85 | 0.37 | 0.51 |
| Greece | 0.84 | 0.84 | 0.70 | 0.70 | 0.45 | 0.46 |
| Italy | 0.85 | 0.87 | 0.52 | 0.56 | 0.28 | 0.30 |
| Netherlands | 0.87 | 0.90 | 0.76 | 0.81 | 0.30 | 0.36 |
| Spain | 0.81 | 0.90 | 0.74 | 0.85 | 0.40 | 0.55 |
| Sweden | 0.93 | 0.95 | 0.82 | 0.86 | 0.70 | 0.77 |
| Switzerland | 0.92 | 0.94 | 0.89 | 0.93 | 0.63 | 0.72 |

7 Graphical Representation

In this section, we want to introduce novel graphical elements not present in the paper to augment the existing summary statistic tables. Tables, although useful for organizing and displaying many discrete data points, lack both detail and readability. Furthermore, tables offer no visibility into distributions inherent to the data. We believe that a necessary and important enhancement to the paper would be to include graphs that allow readers to view distributions, analyze trends, and compare data points across groups or geographies.

7.1 Theory and Design

An increasingly commonly used method for representing country-level data graphically is with choropleth maps. Choropleth maps shade or pattern areas within a defined geographic region (such as a country, state, or city) in relation to a data variable. They are useful for identifying values that are associated with a given geographic location and quickly comparing those values across different regions.

However, we opt not to use choropleths, instead focusing on an approach called tile grid maps. Rather than using actual geographic borders, countries are reduced to a uniform shape and size (in our case, a simple square) in order to ensure equal visual weight. This is done to avoid visual imbalances inherent to choropleth maps: larger geographic regions appear to have more value, influencing the perception for a reader on the relative importance of such a region.

Additionally, we choose to look at distributions of certain variables, rather than just looking at the mean or median values. By looking at the overall spread of each numeric data variable, we are able to gather a more holistic picture of the nature of the data variable.

7.2 Implementation

We develop two new graphics-oriented functions to generate both graphs listed above. The first is the tile grid maps, defined in the function health.gridmap(). Instead of just generating a single tile grid map, we create facets of tile grid maps, where the facetting is determined by the user. Either they can choose to split by gender or by age group

In order to draw the grid map, we must first draw the coordinates. This is done in a simple Excel spreadsheet to align each country with a specific point on a two-dimensional grid. Rather than sourcing this Excel file each time, we load the file once, then use the dput function to replicate the structure of the table (this is not shown in the script). This structure is then defined explicitly within the function.

We define a "base" plotting function that draws the initial outline of the tile grid, using the ggplot2 package. Additional plotting parameters are added later based on the function parameters passed. This is an advantage of using ggplot2: we define the initial plot, then 'layer' on additional graphical elements as needed.

```
[100]
         plot_function = function(df) {
Γ1017
             p = ggplot(data = df, aes(x = X, y = Y)) +
[102]
                 geom_tile(aes(fill = get(xvar), color = ""),
[103]
                            color = "white", size = 0.6) +
[104]
                 geom_text(aes(label = iso3c), color = "white") +
[105]
                 geom_text(aes(label = round(get(xvar), 2)), vjust = 2,
[106]
                            color = "white", size = 3, na.rm = TRUE) +
[107]
[108]
                 coord_fixed(ratio = 1) +
[109]
                 theme_minimal() +
[110]
                 theme(axis.line
                                           = element_blank(),
[111]
                                            = element blank(),
                        axis.text
[112]
                                            = element blank(),
                        axis.title
```

```
[113]
                       panel.background
                                            = element blank(),
[114]
                       panel.grid
                                            = element_blank(),
Γ115]
                       legend.position
                                            = "bottom") +
                 theme(plot.title
[116]
                                        = element_text(size = 16),
                          plot.subtitle = element_text(size = 10,
[117]
[118]
                                                       color = "#7F7F7F"))
[119]
             return(p)
[120]
```

The range of the color scales is always set to range from the minimum to maximum values between all facets of data. This is done to ensure that there are no values that fall out of bounds, as well as to apply the same color scale for all graphs in view.

```
[174] minval = min(df.f[[xvar]], df.m[[xvar]], na.rm = TRUE)
[175] maxval = max(df.f[[xvar]], df.m[[xvar]], na.rm = TRUE)
```

If a logical variable is selected, we display the percentage of survey respondents for which that value is true. If a numeric variable is selected, we display the average value within survey respondents. The final output is stored as a grob, an element from the gridExtra package, which allows us to organize mulliple graphs side-by-side in a single window. We can either draw the final output using the grid.draw() function or we can save to disk using the ggsave() function.

The second function creates a series of individual bar charts, split by a collection of selected genders and countries, for a selected numeric variable. A vector of countries and/or genders can be passed as arguments to the function, in the event a user only cares about a subset of the overall data set. In the event no vector of countries and/or genders are supplied, the entire data set is used. An optional parameter allows for the removal of outliers.

```
# Remove outliers (outside 1.5 IQR of 25% and 75% percentiles)
[370]
         rm.outliers = function(x, na.rm = TRUE) {
[371]
             qnt = quantile(df.out[[x]], probs = c(0.25, 0.75), na.rm = na.rm)
[372]
             out = 1.5*IQR(df.out[[x]], na.rm = na.rm)
[373]
[374]
             new.out = df.out
[375]
             new.out[(df.out[[x]] < qnt[1] - out),] = NA
[376]
             new.out[(df.out[[x]] > qnt[2] + out),] = NA
[377]
[378]
             new.out = new.out[complete.cases(new.out), ]
[379]
             return(new.out)
[380]
         }
[381]
```

```
[382] if (remove.outliers) df.out = rm.outliers(x = xvar)
```

In addition, bars indicating the average over the entire data set are overlaid on top of each bar chart. This is to enable readers to see how similar or different one distribution is compared to the average.

```
[385] total.dist = table(df.out[[xvar]])/nrow(df.out)
[386]
[387] total = expand.grid(countries, gen, sort(unique(df.out[[xvar]])))
[388] names(total) = c("country", "gender", xvar)
[389] total = arrange(total, country, gender, get(xvar))
[390] total$value = rep(total.dist, times = length(countries)*length(gen))
```

Finally, to prevent overcrowding in the view, we decide on the optimal way to arrange the graphs. If only a single gender is selected, we organize the graphs so that only four columns are shown. If all genders are selected, the graphs are organized so that gender is along the y-axis and countries are along the x-axis.

```
[418]
         if (length(gen) == 1) {
[419]
             plot.bar = plot.bar +
[420]
                 labs(subtitle = paste0("Faceted by country. Gender selected is ", gen,
                 ". Distribution of the entire data set shown is overlaid on each graph.")) +
[421]
                 facet_wrap(~ country, ncol = 4)
[422]
         } else {
[423]
             plot.bar = plot.bar +
[424]
                 facet_grid(gender ~ country)
[425]
```

7.3 Empirical Results

We can test the validity of each of the graphics by generating a few sample outputs. For the tile grid maps, we include some stopping conditions to ensure correct facetting and/or to ensure that only a numeric or logical variable is supplied. A similar sequence of error messages is shown for the health distribution maps when the variable is not numeric, the countries supplied are not contained within the data set, or the correct genders are not supplied.

```
source('Quantlets/SPL_ELP_Graphics/SPL_ELP_Graphics.R')
load('easySHARE_clean.RData')

#Only keep relevant data sets
df.out = datasets$df.out

health.gridmap(xvar = 'country', facetting = 'gender')

## Error in health.gridmap(xvar = "country", facetting = "gender"): 'xvar' must be numeric
## or logical
```

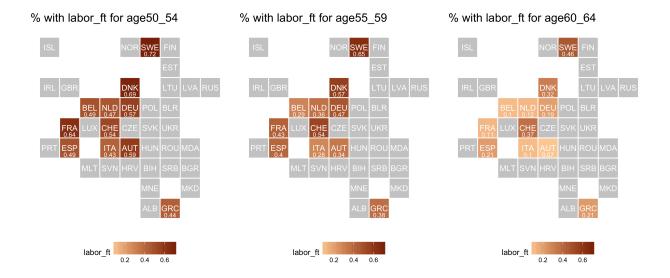


Figure 1: Full-time labor employment tile grid maps by age group

```
health.distribution(xvar = 'age', gen = 'MALEEEEEEE')
## Error in health.distribution(xvar = "age", gen = "MALEEEEEEE"): 'gender' must be one of
```

'all', 'MALE', or 'FEMALE'

##

We show an examples of correct output below. Here, we want to full-time employment percentage, facetted by age group. As we can see in Figure 1, the percentage of the population that works full time generally decreases for all countries as age increases. However, in countries like Sweden and Switzerland, a higher percentage continue to work even in old age.

```
health.gridmap(xvar = 'h_obese', facetting = 'country')
```

Finally, we display an example of the health distribution map. As can be seen in Figure 2, we seek to evaluate the spread of self-perceived health among survey respondents after removing outliers. The black line indicates the overall distribution for the entire data set. As we can see, there are not too many differences between male and female distributions within countries. However, countries like Spain tend to believe they are healthier than average, whereas the opposite effect is true in Switzerland.

```
health.distribution(xvar = 'h_perceived', remove.outliers = FALSE)
```

Distribution of h_perceived by Country

Faceted by country & gender. Distribution of the entire data set shown is overlaid on each graph.

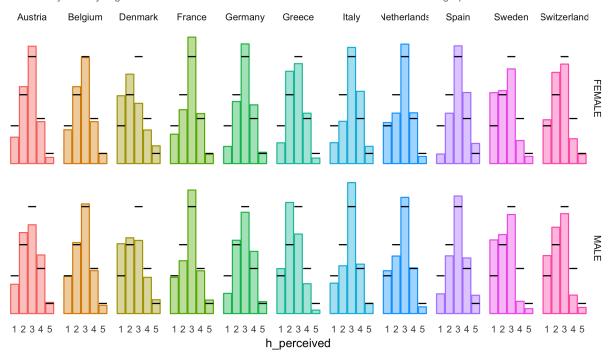


Figure 2: Distribution of self-perceived health by country and gender

8 Conclusion

The purpose of this paper was to present the developed six quantlets that can be used to carry out statistical and graphical analysis based on the easySHARE data set. Our code is designed to simplify the exploration of the easySHARE data set, generate comprehensible and instructive summary statistic tables and graphics, and conduct regression-based analyses. The quantlets have been applied and tested on the easySHARE data set to replicate the results of Kalwij and Vermeulen (2005). The code as well as the empirical results have been saved in a workspace accessible on Github .

In general, our empirical results are in line with those obtained by Kalwij and Vermeulen (2005). However, some minor differences emerge as a result of differing sample sizes by country and differing availability in preciseness of survey responses. Based on using a probit regression, we also find that higher age and worse health conditions are associated with lower labor participation, although there is great variation in the relationship across countries. Our results on the joint impact of health variables on participation (Wald test) are also similar except that we are unable to reject the null in a higher number of cases. The same pattern emerges in the counterfactual exercise, where our results are diverging for some countries. This indicates that the results obtained by Kalwij and Vermeulen (2005) are not entirely robust and should be validated.

Overall, our quantlets perform well on the tested data set. The output generated by our functions is as expected and can be validated with other packages. It works fine for the replication of the chosen paper and should perform well for other tasks performed on the easySHARE data set. Since the quantlet or constructed with a modular design, they can be quickly adjusted for other research tasks. While some of our functions go beyond the functionality of other packages (e.g. wald.test from aod package), our main focus lies on the development of tools to comprehensively analyze the easySHARE data set. One notable source of weakness of our code is its specificity and limitation of scope. Since it has been developed for the special usage of the easySHARE data set, it might not perform well on other data. As a future task, our quantlets could therefore be modified to also be applicable to different panel data sets.

9 References

Arel-Bundock, Vincent. 2017. Countrycode: Convert Country Names and Country Codes. https://CRAN. R-project.org/package=countrycode.

Auguie, Baptiste. 2016. GridExtra: Miscellaneous Functions for "Grid" Graphics. https://CRAN.R-project.org/package=gridExtra.

Bache, Stefan Milton, and Hadley Wickham. 2014. Magrittr: A Forward-Pipe Operator for R. https://CRAN.R-project.org/package=magrittr.

Bates, Douglas, and Martin Maechler. 2017. Matrix: Sparse and Dense Matrix Classes and Methods. https://CRAN.R-project.org/package=Matrix.

Bauer, Dietmar. 2014. Einfuehrung in Die Mikrooekonometrie. Universitaet Bielefeld.

Borsch-Supan, A. 2017. "Survey of Health, Ageing and Retirement in Europe (Share) Wave 2. Release Version: 6.0. 0. Share-Eric [Data Set]."

Cai, Lixin, and Guyonne Kalb. 2006. "Health Status and Labour Force Participation: Evidence from Australia." *Health Economics* 15 (3). Wiley Online Library: 241–61.

Chang, Winston. 2017. Webshot: Take Screenshots of Web Pages. https://CRAN.R-project.org/package=webshot.

Chernozhukov, Victor, Iván Fernández-Val, and Blaise Melly. 2013. "Inference on Counterfactual Distributions." *Econometrica* 81 (6). Wiley Online Library: 2205–68.

Commission, EuroStat European, and others. 2012. "Active Ageing and Solidarity Between Generations." A Statistical Portrait of the European Union 2012.

Eurostat. 2014. "Share of overweight population by sex and age." http://ec.europa.eu/eurostat/statistics-explained/index.php/File:Share_of_overweight_population_by_sex_and_age,_2014.png/.

Fernihough, Alan. 2014. *Mfx: Marginal Effects, Odds Ratios and Incidence Rate Ratios for Glms.* https://CRAN.R-project.org/package=mfx.

Greene, William H. 2012. Econometric Analysis. Seventh Edition. Pearson Education.

Henry, Lionel, and Hadley Wickham. 2017. Purrr: Functional Programming Tools. https://CRAN.R-project.org/package=purrr.

Kalwij, Adriaan, and Frederic Vermeulen. 2005. "Labour Force Participation of the Elderly in Europe: The Importance of Being Healthy."

Karpansalo, Minna, Pirjo Manninen, Jussi Kauhanen, Timo A Lakka, and Jukka T Salonen. 2004. "Perceived Health as a Predictor of Early Retirement." *Scandinavian Journal of Work, Environment & Health*. JSTOR, 287–92.

Kassenboehmer, Sonja C, and John P Haisken-DeNew. 2008. "You're Fired! The Causal Negative Effect of

Unemployment on Life Satisfaction."

Leeper, Thomas J. 2017. Margins: Marginal Effects for Model Objects.

Lesnoff, M., Lancelot, and R. 2012. Aod: Analysis of Overdispersed Data. http://cran.r-project.org/package=aod.

Monden, Christiaan WS. 2005. "Changing Social Variations in Self-Assessed Health in Times of Transition? The Baltic States 1994–1999." *The European Journal of Public Health* 15 (5). Oxford University Press: 498–503.

R Core Team. 2013. R: A Language and Environment for Statistical Computing. Vienna, Austria: R Foundation for Statistical Computing. http://www.R-project.org/.

Ren, Kun, and Kenton Russell. 2016. Formattable: Create 'Formattable' Data Structures. https://CRAN. R-project.org/package=formattable.

RStudio, and Inc. 2017. Htmltools: Tools for Html. https://CRAN.R-project.org/package=htmltools.

Schild, K.-H. 2017. "Binaere Auswahlmodelle." Abt. Statistik, Fb. Wirtschaftswissenschaften, November. Philipps-Universitaet Marburg.

Shields, Michael A, and Stephen Wheatley Price. 2005. "Exploring the Economic and Social Determinants of Psychological Well-Being and Perceived Social Support in England." *Journal of the Royal Statistical Society:* Series A (Statistics in Society) 168 (3). Wiley Online Library: 513–37.

Smeets, Bart. 2017. Infuser: A Very Basic Templating Engine. https://CRAN.R-project.org/package=infuser.

Wickham, Hadley. 2009. *Ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York. http://ggplot2.org.

Wickham, Hadley, and Winston Chang. 2017. Devtools: Tools to Make Developing R Packages Easier. https://CRAN.R-project.org/package=devtools.

Wickham, Hadley, and Lionel Henry. 2017. Tidyr: Easily Tidy Data with 'Spread()' and 'Gather()' Functions. https://CRAN.R-project.org/package=tidyr.

Wickham, Hadley, Romain Francois, Lionel Henry, and Kirill Müller. 2017. Dplyr: A Grammar of Data Manipulation. https://CRAN.R-project.org/package=dplyr.

Wooldridge, Jeffrey M. 2010. Econometric Analysis of Cross Section and Panel Data. MIT press.

——. 2012. Introductory Econometrics. Vol. 5. South Western College Pub - A Modern Approach.

10 Appendix

10.1 Quantlet Code

10.1.1 Quantlet 1



```
# SPL_ELP_ReadandClean.R
# Description:
# Function loads easySHARE data set and outputs relevant data frames for
# analysis and regression. Additionally, print error messages if data set is
# not the correct easySHARE data set and relevant statistics printed (e.g. num
# rows dropped, variables retained, data type)
read.and.clean <- function(dataset = "easySHARE_rel6_0_0.rda", wav = 1) {</pre>
   # LOAD NECESSARY PACKAGES & DATA
   neededPackages = c("dplyr", "magrittr", "infuser", "countrycode")
               = c(neededPackages %in% installed.packages()[,"Package"])
   allPackages
   if (!all(allPackages)) {
      missingIDX = which(allPackages == FALSE)
               = neededPackages[missingIDX]
      lapply(needed, install.packages)
   }
   invisible(lapply(neededPackages, function(x) suppressPackageStartupMessages(
      library(x, character.only = TRUE))))
   # Load dataset
   cat("Loading data set...", sep = "\n")
   load(dataset)
   dat.input = easySHARE rel6 0 0
   rm(easySHARE_rel6_0_0)
   rows.dat.input = nrow(dat.input)
   if (!(wav %in% 1:6)) stop('Out of bounds.
                        Select a value between 1 and 6.')
```

```
# ENCODE MISSING VALUES
# Organize data.frame by selecting relevant variables
cat(infuse("Selecting values only from Wave {{wav}} and between ages 50 and 64.", wav = wav), sep =
dat = dat.input %>%
   dplyr::filter(wave == wav & (age <= 64 & age >= 50)) %>%
   dplyr::select(wave, country_mod,
                                                # dataset details
          female, age, isced1997_r, ch001_, mar_stat, # demo variables
          chronic_mod, maxgrip, adla, bmi, bmi2, eurod, sphus, # health
          ep013 mod, ep005 )
                                                 # labor (outcome var)
rm(dat.input)
# Encode missing values according to SHARE dataset guidelines
a = c(-2, -3, -4, -7, -9, -12, -13, -14, -15, -16)
b = c("tocheck", "implausible", "tocheck", "uncoded", "notApplicable",
     "dontKnow", "notAskedWave", "notAskedCountry", "noInformation",
     "noDropOff")
missing.value.codes = data.frame(a,b)
# This data frame can be used to verify NA codes easily.
# But for the encoding only the numeric vector a is necessary.
# Find NA locations and declare them as such
df.decl = apply(dat, 2, function(z) {
   na.loc
            = which(z %in% a)
   z[na.loc] = NA
   return(z)
})
df = data.frame(df.decl)
# If working hours is NA and job status is not employed, this means
# individuals don't work
# If working hours is NA and job status is employed, we assume part time
df$ep013_mod[is.na(df$ep013_mod) & df$ep005_ != 2] = 0
df$ep013_mod[is.na(df$ep013_mod) & df$ep005_ == 2] = 1
# CREATE DATA FRAMES FOR ANALYSIS AND ESTIMATION
```

```
# Get country information from "countrycode" package
country_list = c("BEL", "NLD", "FRA", "SWE", "DEU", "GRC", "ITA", "ESP",
                 "DNK", "AUT", "CHE")
country_data = with(countrycode_data,
                   data.frame(iso3c, iso3n, country.name.en))
# Variables are cleaned, converted into human-readable naming conventions,
# and converted to dummies as described in the paper
cat("Removing missing values.", sep = "\n")
df.out
            = df %>%
   dplyr::left_join(country_data, by = c("country_mod" = "iso3n")) %>%
   dplyr::filter(iso3c %in% country_list) %>%
   dplyr::mutate(country
                               = factor(country.name.en),
                               = factor(ifelse(female, "FEMALE", "MALE")),
                 gender
                 age50_54
                               = age < 55,
                               = age >= 55 \& age < 60,
                 age55_59
                               = age >= 60,
                 age60_64
                               = factor(floor(age)),
                 age
                               = isced1997_r \%in% c(0, 1),
                 edu_low
                 edu_second = isced1997_r %in% 2:4,
                 edu high
                               = isced1997_r \frac{1}{n} c(5, 6),
                 children
                               = ch001_{,}
                               = mar_stat %in% 1:3,
                 couple
                              = chronic_mod,
                 h_chronic
                 h_maxgrip
                              = maxgrip,
                 h_adla
                             = adla > 0,
                 h_overweight = bmi2 == 3,
                               = bmi2 == 4,
                 h_{obese}
                 h_badmental = eurod > 3,
                 h_goodsp
                               = sphus < 4,
                               = floor(bmi),
                 h_bmi
                 h_depression = eurod,
                 h_perceived = sphus,
                 labor_ft
                             = ep013 mod >= 32,
                 labor_pt
                               = ep013 \mod < 32 \& ep013 \mod > 0,
                               = ep013 \mod == 0,
                 labor_np
                 labor_hrs
                               = floor(ep013_mod)) %>%
   dplyr::select(country, gender,
                                               # country and gender
                 starts with ("age"),
                                              # age dummy
                 starts_with("h_"),
                                               # health indicators
                                               # eduction dummies
                 starts_with("edu_"),
```

```
children, couple,
                                                # demographic details
                  starts_with("labor_")) %>%
                                                # labor supply outcomes
    na.omit() %>%
                                                # remove missing values
    set_rownames(NULL)
                                                # reset row numbering
              = nrow(df.out)
rows.df.out
              = rows.dat.input - rows.df.out
rows.remove
cat(infuse("Rows removed: {{rows.remove}}", rows.remove = rows.remove),
    sep = "\n")
cat(infuse("Rows remaining: {{rows.dat}}}", rows.dat = rows.df.out),
    sep = "\n")
cat("", sep = "\n")
# Create standardized variables for numeric data
standardize = function(x) {
    mean = sum(x)/length(x)
    std = sd(x)
    val = (x - mean) / std
    return(val)
}
# Standardize and select correct variables
standardize.df = function(df) {
    # Gives a vector of integer column positions of numeric variables
    idx = sapply(df, is.numeric)
    idx = seq(1:length(idx))[idx]
    # Creating separate data set with standardized numeric variables for
    # regression, then reselect variables as described in paper (e.g. self-
    # reported health is removed)
    df.reg = df %>%
        mutate_at(.vars = vars(idx),
                  .funs = standardize) %>%
        mutate(labor_participation = !labor_np) %>% # invert to get labor_pt
        dplyr::select(country, gender, age,
                      h_chronic, h_adla, h_obese, h_maxgrip,
                      edu_second, edu_high, children, couple,
                      labor_participation)
    return(df.reg)
}
```

```
# Split data frames into country/gender splits, then standardize numeric
              = split(df.out, f = list(df.out$country, df.out$gender),
    splits
                      drop = TRUE)
              = standardize.df(df.out)
    df.reg
    df.splits = lapply(splits, standardize.df)
    # Create necessary dummary variables for regression
    dummify = function(df) {
        df = df \%
            dplyr::select(-country, -gender)
                                                    # remove country/gender
                   = ~0 + .
        model
                                                    # needed to remove intercept
        new.df
                   = model.matrix(model, df)# create dummies
                   = data.frame(new.df)
        return(new.df)
    }
    df.splits = lapply(df.splits, dummify)
    # df.out is for analysis, df.reg and df.splits are for estimation
    all.output = list(df.out = df.out, df.reg = df.reg, df.splits = df.splits)
    cat("Final output is a list containing 3 data.frames:", sep = "\n")
    cat("`df.out` is a data.frame containing variables for calculating summary statistics.", sep = "\n".
    cat("`df.reg` is a data.frame containing standardized variables for regression.", sep = "\n")
    cat("`df.splits` splits the regression data.frame into individual data.frames for each country/
        gender split. 22 data.frames are contained in this list.", sep = "\n")
    return(all.output)
}
10.1.2 Quantlet 2
Q
10.1.3 Quantlet 3
Q
10.1.4 Quantlet 4
Q
```

```
# SPL_ELP_LoadWald.R
# Description:
# Contains a series of Wald tests
# Wald Test for Joint Significance
# Note: This a Wald test for the joint signifance of a subset of model
# coefficients
# m = number of restrictions
# Beta is vector of coefficients of size k x 1
# Confidence.level is the desired confidence level (between 0 and 1).
# Default: 0.95
# Spec is vector of integers of length 0 < m <= k specifying the subset of
# coefficients to be jointly tested
joint.wald.test = function(model.summary, confidence.level=0.95, spec=NULL) {
   # List all packages needed for session
   neededPackages = c("Matrix")
   allPackages
              = c(neededPackages %in% installed.packages()[, "Package"])
   # Install packages (if not already installed)
   if (!all(allPackages)) {
      missingIDX = which(allPackages == FALSE)
               = neededPackages[missingIDX]
      needed
      lapply(needed, install.packages)
   }
   # Load all defined packages
   invisible(lapply(neededPackages, function(x) suppressPackageStartupMessages(
      library(x, character.only = TRUE))))
   # Set up test restrictions
   if (is.null(spec)) {
      spec = 1:length(beta) # default joint is significance test
   } else if (is.logical(spec)) {
      stop("spec cannot be a logical vector, tranform to integer vector!")
```

```
} else if (any(spec == 0)) {
       stop("spec cannot contain zero values!")
    } else if (!is.integer(spec)) {
       spec.len = length(spec)
                = as.integer(spec, length = spec.len)
       spec
       warning("Converting spec to integer and proceeding")
    }
    # Check Input
    if (class(model.summary) != "summary.glm") {
       stop("model.summary must be a glm summary!")
    }
    if (confidence.level > 1 | confidence.level < 0) {</pre>
       stop("confidence.level out of bounds!")
    }
    # Define test elements
    joint.wald.test
                        = numeric(4)
    names(joint.wald.test) = c("W","p-value", "df", "Decision")
    beta
                         = model.summary$coefficients[,1]
    Var_beta_est
                        = vcov(model.summary)
    # Wald test statistic
    W = t(beta[spec]) %*% solve(Var_beta_est[spec,spec]) %*% beta[spec]
    # Set up test output
                      = 1-pchisq(W,length(spec))
    pval
    joint.wald.test[1] = format(W, digits = 4)
    joint.wald.test[2] = format(pval, digits = 4)
    joint.wald.test[3] = length(spec)
    joint.wald.test[4] = ifelse(pval <= 1- confidence.level, "Reject HO",</pre>
                               "Cannot reject HO")
    return(joint.wald.test)
}
## Wald Test for Linear Hypothesis
# Note: This a general Wald test for linear hypothesis of model beta
# coefficients
# m = number of restrictions
# Beta is vector of coefficients of size k x 1
# Confidence level is the desired confidence level (between 0 and 1).
```

```
# Default: 0.95
\# R is Jacobian matrix of size m x k
\# r is restriction function size m x 1
# Example: H0: b1 + b2 = 1 and 2*b1 = 0 (2 restrictions, m = 2)
\# R = 1 \ 1 \ 0 \ 0 \ \dots 0
                      and r = 1
     2000...0
general.wald.test = function(model.summary,
                             confidence.level = 0.95,
                             R
                                              = NULL,
                                              = NULL) {
    # Check input
   if (class(model.summary) != "summary.glm") {
        stop("model.summary must be a glm summary!")
   }
    if (confidence.level > 1 | confidence.level < 0) {</pre>
        stop("confidence.level out of bounds!")
   }
    # Define test elements
    general.wald.test
                           = numeric(4)
   names(general.wald.test) = c("W","p-value", "df", "Decision")
                             = model.summary$coefficients[, 1]
   beta
   Var_beta_est
                             = vcov(model.summary)
    # Set up restriction matrix/vector for linear hypothesis
    # default option is joint significants of all coefficients
    if (is.null(R)) {
        R = diag(1, length(beta)) # default of R is identity matrix
   }
    if (is.null(r)) {
        r = rep(0, length(beta)) # default for r is null vector
   }
    # Now R and r definitely exist, check dimensions
   dim_R = dim(R) # Testing R
         = length(beta) # Columns
         = length(r) # Rows; general hypothesis test, therefore m = length(r)
   ## Apply test
    if (dim_R[1] != m | dim_R[2] != k) stop("R has wrong dimension!")
```

```
# Check rank of R
if (rankMatrix(R)[1] != m) stop("R has wrong rank!")

# Wald test statistic
W = t(R%*%beta - r) %*% solve(R%*% Var_beta_est %*% t(R)) %*% (R%*%beta - r)

# Set up test output
pval = 1-pchisq(W,length(r))
general.wald.test[1] = format( W, digits = 4)
general.wald.test[2] = format(pval, digits = 4)
general.wald.test[3] = length(r)
general.wald.test[4] = ifelse(pval <= 1- confidence.level, "Reject HO", "Cannot reject HO")
return(general.wald.test)
}</pre>
```

10.1.5 Quantlet 5

Q

```
# Counterfactual
# Description:
# Calculates current and counterfactual labor participation rates for different
# age groups, countries, and genders
# SOURCE DATA
load('easySHARE_clean.RData')
#Only keep relevant data sets
df.splits = datasets$df.splits
rm(datasets)
# Probit for each country and gender
allModels = lapply(df.splits, function(z) {
```

```
model = glm(z$labor_participationTRUE ~ . -age50,
               family = binomial(link = "probit"),
               data = z
   return(model)
})
# Return summaries
allSummaries = lapply(allModels, summary)
# Counterfactual Exercise
# Calculate employment rate based on whole population with mean probability
# Estimate current employment rate
empl.rate = function(model) {
    # Select data from model
                    = model$data
    # Predict probability of being employed of all individuals
    empl.probability = predict(object = model, newdata = X, type = "response")
    # Calculate predicted current employment rate
    empl.rate
                    = sum(empl.probability) / length(empl.probability)
   return(empl.rate)
}
# Function for defining models with modified dataset (perfect health)
X.cf = function(model) {
    # Select data from model
                         = model$data
    #Create counterfactual data set with ideal health condition
                          = X
   X cf
    # Find ideal health conditions by finding the minimum value for each
    # variable
   X_{\min}
                         = data.frame(t(apply(X, 2, min)))
    # Replace mean values by values representing no health issues
   names.vec
                         = c("h_chronic", "h_adlaTRUE", "h_obeseTRUE")
   X_cf[, names.vec]
                         = X_min[names.vec]
    # Create index vector for 50 and 51 year olds
                         = ifelse(X$age50 == 1 | X$age51 == 1, 1,0)
    age_50_51
    # Calculate max grip mean of individuals age 50-51 and replace
   X_{cf}[, c("h_maxgrip")] = tapply(X_h_maxgrip, age_50_51 == 1, mean)[[2]]
   model$data
                         = X cf
   return(model)
```

```
# Returns a list with all models with modified datasets
allModels.cf = lapply(allModels, X.cf)
# Calculate current and counterfactual employment rates
                = lapply(allModels, empl.rate)
empl.counterfact = lapply(allModels.cf, empl.rate)
# Store results and display them
                    = data.frame(cbind(unlist(empl.current),
employment
                                       unlist(empl.counterfact)))
colnames(employment) = c("Employment Current" , "Employment Counterf.")
# Counterfactual exercise for each age group
### Function for current and counterfactual employment rate among age groups
# Default arguments: group size and lower bound of age groups (group.low)
# Estimate current employment rate for age groups
empl.rate.age = function(model, group.size = 5, group.low = c(50,55,60)) {
   #Initialize storage of results
   empl.rate
                   = numeric(3)
   # Select data from model
   X
                   = model$data
   # Predict probability of being employed of all individuals
   empl.probability = predict(object = model, newdata = X, type = "response")
   # Create age group Index vectors
   # Initialize result list containing age group index vectors
   IND_row_vec = list()
   # Define vector of relevant age groups by youngest age
   for (i in group.low) {
       # Define upper bound k of age group
                = i + group.size -1
       # Create labels and contents of each age group
       vec.label = paste("names.vec.age", i, sep=".")
       # Assign ages to the age groups
```

```
= assign(vec.label, paste0("age",i:k))
       # For age groups of size 1, use simple indexing (cannot use apply)
       if (length(z) == 1) {
           IND_row_vec[[i]] = X[, z]
       # For all age groups greater than size 1
       } else {
           \# Sum over rows to find individuals belonging to age group i
           IND_row_vec[[i]] = apply(X[, z], 1, sum)
       }
   }
    # Extract list of relevant age groups, which are each stored in a list
   IND_row_vec = IND_row_vec[group.low]
# Calculate predicted current employment rate for age groups
# loop over age group list
    for (k in 1: length(IND_row_vec)) {
       empl.rate[k] = empl.probability %*% IND_row_vec[[k]] /
           sum(IND_row_vec[[k]])
   return(empl.rate)
}
# Calculate current and counterfactual employment rates for age groups
empl.current.age
                  = t(data.frame(lapply(allModels, empl.rate.age)))
empl.counterfact.age = t(data.frame(lapply(allModels.cf, empl.rate.age)))
employment.age
                        = data.frame(cbind(empl.current.age,
                                          empl.counterfact.age))
colnames(employment.age) = paste0(c(rep("empl.current", 3),
                                  rep( "empl.counterfact", 3)),
                                c("50.54","55.59", "60.64"))
# Test for other age groups: Example of employment for each age group
empl.rate.age(allModels$Germany.MALE, group.size = 1, group.low = c(50:63))
empl.rate.age(allModels.cf$Germany.MALE, group.size = 1, group.low = c(50:63))
# Decline in participation due to decline in health condition
health.decline = function(X) {
    # Differences between counterf. and current participation rates
```

```
dif.60.64 = X[,"empl.counterfact60.64"] - X[, "empl.current60.64"]
   dif.50.54 = X[,"empl.counterfact50.54"] - X[, "empl.current50.54"]
   # Absolute difference in current participation rate
   dif.abs = X[,"empl.current50.54"] - X[,"empl.current60.64"]
   measure = (dif.60.64 - dif.50.54) / dif.abs
   return(measure)
}
participation.health = data.frame(health.decline(employment.age))
# Some Remarks
# Define outout formate of counterfactual estimates: list
empl.each.age = list()
# Calculate employment for each age for all countries
for (i in 1:length(allModels)) {
   k
                    = allModels[[i]]
   empl.each.age[[i]] = empl.rate.age(k, group.size = 1, group.low = c(50:64))
}
names(empl.each.age) = names(allModels)
```

10.1.6 Quantlet 6

Q

```
# health.distribution(xvar, gen, countries, remove.outliers): show distribution
# of numeric variable in dataset, facetted by gender and country.
# Select a numeric variable in xvar (as a string), a vector of genders, and a
# vector of countries. By default, outliers are removed.
###### GEOGRAPHIC VISUALIZATION OF NUMERIC DATA
# Tile grid map of numeric variables by geographic location
# accept metric and facetting as input. xvar is a character vector
# facetting can take either 'gender' or 'age' as inputs
# return error if neither 'gender' or 'age' is inputted
# output side by side grid maps
health.gridmap = function(xvar, facetting) {
   # List all packages needed for session
   neededPackages = c("dplyr", "ggplot2", "countrycode",
                      "gridExtra", "grid", "infuser")
                 = c(neededPackages %in% installed.packages()[ , "Package"])
   allPackages
   # Install packages (if not already installed)
   if(!all(allPackages)) {
       missingIDX = which(allPackages == FALSE)
                 = neededPackages[missingIDX]
       lapply(needed, install.packages)
   }
   # Load all defined packages (silently)
   invisible(lapply(neededPackages, library, character.only = TRUE))
   # first convert age to numeric, not factor variable
   if (is.factor(df.out$age)) {
       df.out$age = as.numeric(levels(df.out$age)[df.out$age])
   }
   # STOPPING CONDITIONS
   if (!is.numeric(df.out[[xvar]]) & !is.logical(df.out[[xvar]])) {
       stop("'xvar' must be numeric
       or logical")
   if (!(facetting %in% c('gender', 'age'))) {
```

```
stop("'facetting' variable
    must be either 'gender' or 'age'")
}
# ADD TILE GRID LAYOUT
coordinates = structure(list(
    Country = c("Albania", "Austria", "Belarus", "Belgium",
                "Bosnia and Herzegovina", "Bulgaria", "Croatia",
                "Czech Republic", "Denmark", "Estonia", "Finland", "France",
                "Germany", "Greece", "Hungary", "Iceland", "Ireland",
                "Italy", "Latvia", "Lithuania", "Luxembourg", "Malta",
                "Montenegro", "Netherlands", "Norway", "Poland", "Portugal",
                "Republic of Moldova", "Romania", "Russian Federation",
                "Serbia", "Slovakia", "Slovenia", "Spain", "Sweden",
                "Switzerland", "The former Yugoslav Republic of Macedonia",
                "Ukraine",
                "United Kingdom of Great Britain and Northern Ireland"
                ),
    X = c(6, 5, 7, 3, 6, 8, 5, 5, 5, 7, 7, 2, 5, 7, 6, 1, 1, 4, 8, 7, 3, 3,
          6, 4, 5, 6, 1, 8, 7, 9, 7, 6, 4, 2, 6, 4, 8, 7, 2
          ),
    Y = c(1, 4, 6, 6, 3, 3, 3, 5, 7, 8, 9, 5, 6, 1, 4, 9, 7, 4, 7, 7, 5, 3,
          2, 6, 9, 6, 4, 4, 4, 7, 3, 5, 3, 4, 9, 5, 2, 5, 7
          ),
    InSet = c(FALSE, TRUE, FALSE, TRUE, FALSE, FALSE, FALSE, TRUE,
              FALSE, FALSE, TRUE, TRUE, TRUE, FALSE, FALSE, TRUE,
             FALSE, FALSE, FALSE, FALSE, TRUE, FALSE, FALSE, FALSE,
              FALSE, FALSE, FALSE, FALSE, FALSE, TRUE, TRUE, TRUE,
             FALSE, FALSE, FALSE
    ),
    .Names = c("Country", "X", "Y", "InSet"),
    row.names = c(NA, -39L),
    class = c("tbl_df", "tbl", "data.frame")
)
# Get correct country information
codes
           = countrycode_data
df.codes
          = left_join(coordinates, codes,
                        by = c("Country" = "country.name.en")) %>%
    dplyr::select(Country, iso3c, X, Y)
# plot function to be used later
plot_function = function(df) {
```

```
p = ggplot(data = df, aes(x = X, y = Y)) +
        geom_tile(aes(fill = get(xvar), color = ""),
                  color = "white", size = 0.6) +
        geom_text(aes(label = iso3c), color = "white") +
        geom_text(aes(label = round(get(xvar), 2)), vjust = 2,
                  color = "white", size = 3, na.rm = TRUE) +
        coord_fixed(ratio = 1) +
        theme minimal() +
        theme(axis.line
                              = element_blank(),
              axis.text
                             = element_blank(),
                              = element_blank(),
              axis.title
              panel.background = element_blank(),
              panel.grid
                               = element blank(),
              legend.position = "bottom") +
        theme(plot.title
                          = element_text(size = 16),
              plot.subtitle = element_text(size = 10,
                                           color = "#7F7F7F"))
    return(p)
}
# IF GENDER SELECTED:
if (facetting == 'gender') {
    # if logical than calculate percentage
    if (is.logical(df.out[[xvar]])) {
        var.type = '% with'
        df.graph = df.out %>%
            dplyr::select(country, gender, `xvar`) %>%
            group_by(country, gender) %>%
            summarize(value = sum(get(xvar))/length(get(xvar)))
        names(df.graph)[3] = xvar
        # split by gender
        df.m = df.graph %>%
            filter(gender == "MALE")
        df.f = df.graph %>%
            filter(gender == "FEMALE")
        df.m = suppressWarnings(
            left_join(df.codes, df.m, c('Country' = 'country'))) %>%
            dplyr::select(iso3c, X, Y, `xvar`)
```

```
df.f = suppressWarnings(
        left_join(df.codes, df.f, c('Country' = 'country'))) %>%
        dplyr::select(iso3c, X, Y, `xvar`)
}
# if numeric than calculate average
if (is.numeric(df.out[[xvar]])) {
    var.type = 'Avg.'
    df.graph = df.out %>%
        dplyr::select(country, gender, `xvar`) %>%
        group_by(country, gender) %>%
        summarize(value = mean(get(xvar)))
    names(df.graph)[3] = xvar
    # split by gender
    df.m = df.graph %>%
        filter(gender == "MALE")
    df.f = df.graph %>%
        filter(gender == "FEMALE")
    df.m = suppressWarnings(
        left_join(df.codes, df.m, c('Country' = 'country'))) %>%
        dplyr::select(iso3c, X, Y, `xvar`)
    df.f = suppressWarnings(
        left_join(df.codes, df.f, c('Country' = 'country'))) %>%
        dplyr::select(iso3c, X, Y, `xvar`)
}
# set equal range for color scales
minval = min(df.f[[xvar]], df.m[[xvar]], na.rm = TRUE)
maxval = max(df.f[[xvar]], df.m[[xvar]], na.rm = TRUE)
# PLOT
output = lapply(list(df.m, df.f), plot_function)
output[[1]] = output[[1]] +
    labs(title
                  = infuse("{{intro}} {{metric}} for {{split}}",
                           intro = var.type, metric = xvar,
                           split = 'male'),
         fill
                  = xvar) +
    scale_fill_continuous(low = "#fdd0a2", high = "#8c2d04",
                          na.value = "#CCCCCC",
                          limits = c(minval, maxval))
```

```
output[[2]] = output[[2]] +
                    = infuse("{{intro}} {{metric}} for {{split}}",
        labs(title
                               intro = var.type, metric = xvar,
                               split = 'female'),
             fill
                     = xvar) +
        scale_fill_continuous(low = "#fdd0a2", high = "#8c2d04",
                              na.value = "#CCCCCC",
                              limits = c(minval, maxval))
   return(arrangeGrob(output[[1]], output[[2]], ncol = 2))
}
# IF AGE SELECTED:
if (facetting == 'age') {
    # if logical than calculate percentage
    if (is.logical(df.out[[xvar]])) {
        var.type = '% with'
        # split by age group
        df50 = df.out %>%
            filter(age50_54 == TRUE) %>%
            group_by(country) %>%
            summarize(value = sum(get(xvar))/length(get(xvar)))
        df50 = suppressWarnings(
            right_join(df50, df.codes, c('country' = 'Country'))) %>%
            dplyr::select(iso3c, X, Y, value)
        names(df50)[4] = xvar
        df55 = df.out %>%
            filter(age55 59 == TRUE) %>%
            group_by(country) %>%
            summarize(value = sum(get(xvar))/length(get(xvar)))
        df55 = suppressWarnings(
            right_join(df55, df.codes, c('country' = 'Country'))) %>%
            dplyr::select(iso3c, X, Y, value)
        names(df55)[4] = xvar
        df60 = df.out %>%
            filter(age60_64 == TRUE) %>%
            group_by(country) %>%
            summarize(value = sum(get(xvar))/length(get(xvar)))
        df60 = suppressWarnings(
```

```
right_join(df60, df.codes, c('country' = 'Country'))) %>%
        dplyr::select(iso3c, X, Y, value)
    names(df60)[4] = xvar
}
# if numeric than calculate average
if (is.numeric(df.out[[xvar]])) {
    var.type = 'Avg.'
    # split by age group
    df50 = df.out %>%
        filter(age50 54 == TRUE) %>%
        group_by(country) %>%
        summarize(value = mean(get(xvar)))
    df50 = suppressWarnings(
        right_join(df50, df.codes, c('country' = 'Country'))) %>%
        dplyr::select(iso3c, X, Y, value)
    names(df50)[4] = xvar
    df55 = df.out %>%
        filter(age55_59 == TRUE) %>%
        group_by(country) %>%
        summarize(value = mean(get(xvar)))
    df55 = suppressWarnings(
        right_join(df55, df.codes, c('country' = 'Country'))) %>%
        dplyr::select(iso3c, X, Y, value)
    names(df55)[4] = xvar
    df60 = df.out %>%
        filter(age60_64 == TRUE) %>%
        group_by(country) %>%
        summarize(value = mean(get(xvar)))
    df60 = suppressWarnings(
        right_join(df60, df.codes, c('country' = 'Country'))) %>%
        dplyr::select(iso3c, X, Y, value)
    names(df60)[4] = xvar
}
# set equal range for color scales
minval = min(df50[[xvar]], df55[[xvar]], df60[[xvar]], na.rm = TRUE)
maxval = max(df50[[xvar]], df55[[xvar]], df60[[xvar]], na.rm = TRUE)
```

```
# PLOT
        output = lapply(list(df50, df55, df60), plot_function)
        output[[1]] = output[[1]] +
            labs(title
                          = infuse("{{intro}} {{metric}} for {{split}}",
                                   intro = var.type, metric = xvar,
                                   split = 'age50_54'),
                          = xvar) +
                 fill
            scale_fill_continuous(low = "#fdd0a2", high = "#8c2d04",
                                  na.value = "#CCCCCC",
                                  limits = c(minval, maxval))
        output[[2]] = output[[2]] +
            labs(title
                        = infuse("{{intro}} {{metric}} for {{split}}",
                                   intro = var.type, metric = xvar,
                                   split = 'age55_59'),
                 fill
                          = xvar) +
            scale_fill_continuous(low = "#fdd0a2", high = "#8c2d04",
                                  na.value = "#CCCCCC",
                                  limits = c(minval, maxval))
        output[[3]] = output[[3]] +
                         = infuse("{{intro}} {{metric}} for {{split}}",
            labs(title
                                   intro = var.type, metric = xvar,
                                   split = 'age60_64'),
                 fill
                          = xvar) +
            scale_fill_continuous(low = "#fdd0a2", high = "#8c2d04",
                                  na.value = "#CCCCCC",
                                  limits = c(minval, maxval))
        return(arrangeGrob(output[[1]], output[[2]], output[[3]], ncol = 3))
    }
}
##### VIEW DISTRIBUTION OF NUMERIC VARIABLES
# Numeric variable: shows distribution for each country, then overlays average of entire data set
# slicing by dummy variables is not included. that can be done before the function is called and then to
# xvar is a character vector
# by default, gender is "all". can either be "all", "MALE" or "FEMALE"
# by default, countries is "all". can also pass a character vector of countries
health.distribution = function(xvar,
                                                = "all",
                               gen
```

```
countries = "all",
                           remove.outliers = TRUE) {
# List all packages needed for session
neededPackages = c("dplyr", "ggplot2", "scales")
              = c(neededPackages %in% installed.packages()[ , "Package"])
allPackages
# Install packages (if not already installed)
if(!all(allPackages)) {
    missingIDX = which(allPackages == FALSE)
               = neededPackages[missingIDX]
    lapply(needed, install.packages)
}
# Load all defined packages (silently)
suppressMessages(lapply(neededPackages, library, character.only = TRUE))
# first convert age to numeric, not factor variable
if (is.factor(df.out$age)) {
    df.out$age = as.numeric(levels(df.out$age)[df.out$age])
}
# STOPPING CONDITIONS
if (!is.numeric(df.out[[xvar]])) stop("'xvar'
                                      must be numeric")
if (length(gen) > 1 | !all(gen %in% c("all", "MALE", "FEMALE"))) {
    stop("'gender' must be one of
         'all', 'MALE', or 'FEMALE'")
}
if (length(countries) > 1 & !all(countries %in% levels(df.out$country))) {
    stop("'countries'
         must be 'all' or a character vector containing countries
         in the `df.out` data frame")
}
if (length(countries) == 1) {
    if (!(countries %in% c('all', levels(df.out$country)))) {
        stop("'countries'
             must be 'all' or a character vector containing countries
             in the `df.out` data frame")
    }
}
# Set faceting variables
```

```
if (gen == "all") gen = c("MALE", "FEMALE")
if (length(countries) == 1) {
    if (countries == "all") countries = levels(df.out$country)
}
# Filter countries/genders as selected
df.out = df.out %>%
    filter(country %in% countries, gender %in% gen)
# Remove outliers (outside 1.5 IQR of 25% and 75% percentiles)
rm.outliers = function(x, na.rm = TRUE) {
    qnt = quantile(df.out[[x]], probs = c(0.25, 0.75), na.rm = na.rm)
    out = 1.5*IQR(df.out[[x]], na.rm = na.rm)
    new.out = df.out
    new.out[(df.out[[x]] < qnt[1] - out),] = NA
    new.out[(df.out[[x]] > qnt[2] + out), ] = NA
    new.out = new.out[complete.cases(new.out), ]
    return(new.out)
}
if (remove.outliers) df.out = rm.outliers(x = xvar)
# Overlay lines for average over entire data set
total.dist = table(df.out[[xvar]])/nrow(df.out)
             = expand.grid(countries, gen, sort(unique(df.out[[xvar]])))
total
names(total) = c("country", "gender", xvar)
             = arrange(total, country, gender, get(xvar))
total$value = rep(total.dist, times = length(countries)*length(gen))
plot.bar = ggplot(data = df.out, aes(x = get(xvar))) +
    geom_bar(aes(fill = country,
                 color = country,
                 y = ..prop..),
             alpha = 0.4) +
    geom_errorbar(data = total, aes(x = get(xvar),
                                    ymin = value,
                                    ymax = value)) +
```

```
theme_minimal() +
    theme(legend.position = "none") +
    theme(panel.grid.minor = element_blank()) +
    theme(panel.grid.major = element_blank()) +
    theme(axis.text.y = element_blank()) +
    labs(title = paste0("Distribution of ", xvar, " by Country")) +
    labs(subtitle = "Faceted by country & gender. Distribution of the entire data set shown is over
    labs(x = xvar) +
    labs(y = "") +
    theme(plot.title = element_text(size=16)) +
    theme(plot.subtitle = element_text(size=10, color = "#7F7F7F"))
# show correct faceting and titles
if (length(gen) == 1) {
    plot.bar = plot.bar +
        labs(subtitle = paste0("Faceted by country. Gender selected is ", gen, ". Distribution of tl
        facet_wrap(~ country, ncol = 4)
} else {
    plot.bar = plot.bar +
        facet_grid(gender ~ country)
}
return(plot.bar)
```

Declaration of Authorship

We hereby confirm that we have authored this Seminar paper independently and without use of others than the indicated sources. All passages which are literally or in general matter taken out of publications or other sources are marked as such.

Berlin, 2018-03-16

Claudia Günther, Phi Nguyen, Julian Winkel