Lecture 2

PNC
Probability

Lecture 2

- Experiment
- Sample Space
- Events
- Set Theory
- Disjoint Events
- Permutations & Combinations
- Questions

Experiment

An experiment is any activity or process whose outcome is subject to uncertainty.

Examples:

tossing a coin once or several times

selecting a card or cards from a deck

Sample Space

The sample space of an experiment, denoted by S, is the set of all possible outcomes of that experiment.

Toss a coin → Sample Space H, T

Tossing two coins → Sample Space HH, TT, HT, TH

Tossing three coins → HHH, HHT, HTT, HTH,....

Toss n coins: 2^n possibilities

Simple and Compound Event

An event is any collection (subset) of outcomes contained in the sample space S. An event is simple if it consists of exactly one outcome and compound if it consists of more than one outcome.

Coin tossed twice

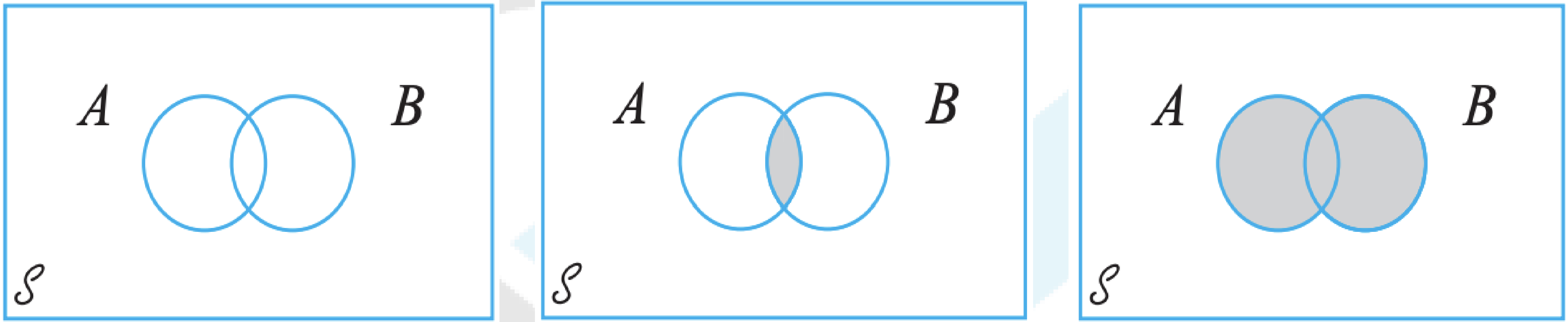
Simple Event → HH

Compound Event → {HT,TH}

Set Theory

An event is just a set, so relationships and results from elementary set theory can be used to study events.

1. The **complement** of an event A, denoted by A', is the set of all outcomes in $\mathcal{S}$ that are not contained in A.
2. The **union** of two events A and B, denoted by $A \cup B$ and read "*A or B*," is the event consisting of all outcomes that are *either in A or in B or in both events* (so that the union includes outcomes for which both A and B occur as well as outcomes for which exactly one occurs)—that is, all outcomes in at least one of the events.
3. The **intersection** of two events A and B, denoted by $A \cap B$ and read "*A and B*," is the event consisting of all outcomes that are in *both A and B*.



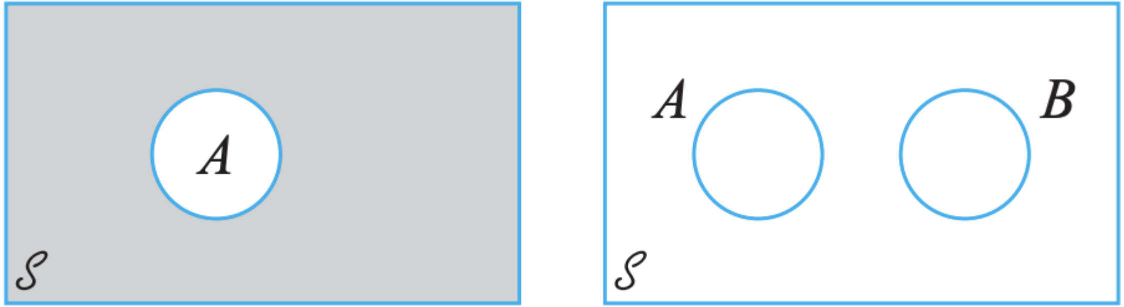
(a) Venn diagram of events A and B

(b) Shaded region is $A \cap B$

(c) Shaded region is $A \cup B$

Mutually Exclusive or Disjoint Events

A and B have no outcomes in common, so that the intersection of A and B contains no outcomes.



(d) Shaded region is A'

(e) Mutually exclusive events

Exercise: De Morgan's Law

Proof the following

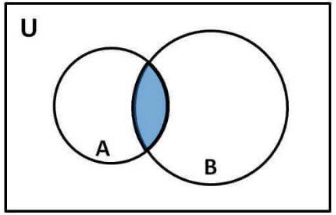
a. $(A \cup B)' = A' \cap B'$

b. $(A \cap B)' = A' \cup B'$

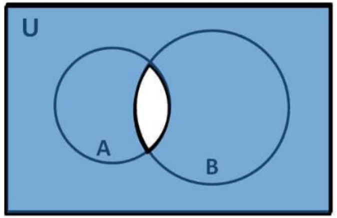
De Morgan’s Law

Proving $(A \cap B)' = A' \cup B'$

$(A \cap B)'$

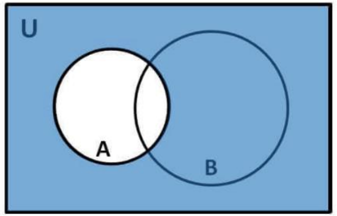


$A \cap B$

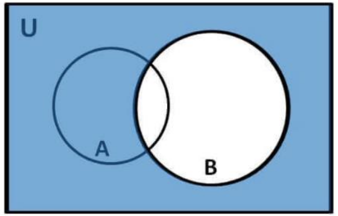


$(A \cap B)'$

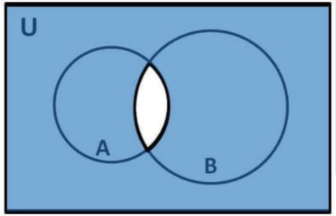
$A' \cup B'$



A'



B'



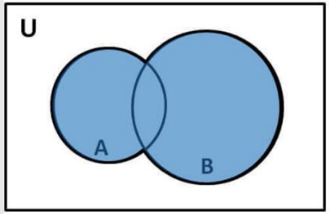
$A' \cup B'$

$\therefore (A \cap B)' = A' \cup B'$

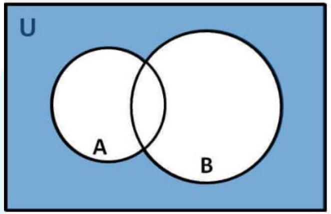
De Morgan’s Law

Proving $(A \cup B)' = A' \cap B'$

$(A \cup B)'$

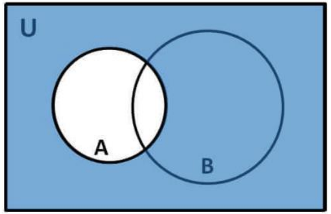


$A \cup B$

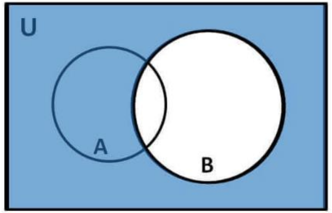


$(A \cup B)'$

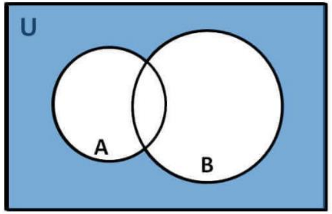
$A' \cap B'$



A'



B'



$A' \cap B'$

$\therefore (A \cup B)' = A' \cap B'$

Properties

Given an experiment and a sample space , the objective of probability is to assign to each event A a number P(A), called the probability of the event A, which will give a precise measure of the chance that A will occur.

For any event A, $P(A) \geq 0$.

$P(\mathcal{S}) = 1$.

If $A_1, A_2, A_3, \ldots$ is an infinite collection of disjoint events, then

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = \sum_{i=1}^{\infty} P(A_i)$$

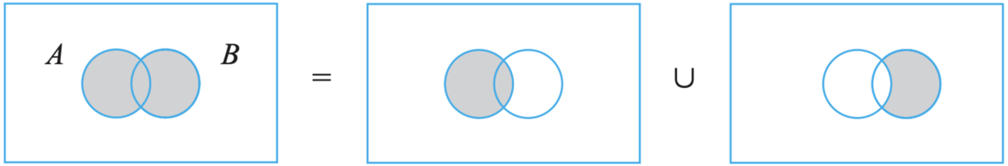
For any event A, $P(A) + P(A') = 1$, from which $P(A) = 1 - P(A')$.

Properties

For any two events A and B,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = P(A) + P(B \cap A') = P(A) + [P(B) - P(A \cap B)]$$
$$= P(A) + P(B) - P(A \cap B)$$



For any three events A, B, and C,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C)$$
$$-P(B \cap C) + P(A \cap B \cap C)$$

Permutation and Combination(^nC_k)

$$P_{k,n} = \frac{n!}{(n-k)!}$$

$$\binom{n}{k} = \frac{P_{k,n}}{k!} = \frac{n!}{k!(n-k)!}$$

A permutation is used for the list of data (where the order of the data matters) and the combination is used for a group of data (where the order of data doesn't matter).

Question

The computers of six faculty members in a certain department are to be replaced. Two of the faculty members have selected laptop machines and the other four have chosen desktop machines.

Suppose that only two of the setups can be done on a particular day, and the two computers to be set up are randomly selected from the six (implying 15 equally likely outcomes; if the computers are numbered 1, 2, . . . , 6, then one outcome consists of computers 1 and 2, another consists of computers 1 and 3, and so on).

a.What is the probability that both selected setups are for laptop computers?

b. What is the probability that both selected setups are desktop machines?

c. What is the probability that at least one selected setup is for a desktop computer?

d. What is the probability that at least one computer of each type is chosen for setup?

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a. What is the probability that both selected setups are for laptop computers? ${}^{2}C_{2}/15$

b. What is the probability that both selected setups are desktop machines? ${}^{4}c_{2}/15$

c. What is the probability that at least one selected setup is for a desktop computer? (15-1)/15 =14/15

d. What is the probability that at least one computer of each type is chosen for setup? (2*4)/15

Propositions

If the first element or object of an ordered pair can be selected in n_1 ways, and for each of these n_1 ways the second element of the pair can be selected in n_2 ways, then the number of pairs is $n_1 n_2$.

A homeowner doing some remodeling requires the services of both a plumbing contractor and an electrical contractor. If there are 12 plumbing contractors and 9 electrical contractors available in the area, in how many ways can the contractors be chosen? 108

Product Rule for k-Tuples

Suppose a set consists of ordered collections of k elements (k-tuples) and that there are n_1 possible choices for the first element; for each choice of the first element, there are n_2 possible choices of the second element; . . . ; for each possible choice of the first $k - 1$ elements, there are n_k choices of the kth element. Then there are $n_1 n_2 \cdot \cdots \cdot n_k$ possible k-tuples.

Question

A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews.

Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

a. How many selections result in all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?

Question

A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews.

Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

a. How many selections result would lead to all 6 workers coming from the day shift? What is the probability that all 6 selected workers will be from the day shift?

${}^{20}C_{6}$, ${}^{20}C_6/{}^{45}C_6$

Question

A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews.

Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

b. What is the probability that all 6 selected workers will be from the same shift?

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b. What is the probability that all 6 selected workers will be from the same shift?

$({}^{20}C_6 + {}^{15}C_6 + {}^{10}C_6)/{}^{45}C_6$

Question

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c. What is the probability that at least two different shifts will be represented among the selected workers?

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Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

c. What is the probability that at least two different shifts will be represented among the selected workers?

$(1-({}^{20}C_6 + {}^{15}C_6 + {}^{10}C_6))/{}^{45}C_6$

Question

A production facility employs 20 workers on the day shift, 15 workers on the swing shift, and 10 workers on the graveyard shift. A quality control consultant is to select 6 of these workers for in-depth interviews. Suppose the selection is made in such a way that any particular group of 6 workers has the same chance of being selected as does any other group (drawing 6 slips without replacement from among 45).

d. What is the probability that at least one of the shifts will be unrepresented in the sample of workers?

Question

$$P_4 = P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_3 \cap A_2) + P(A_1 \cap A_2 \cap A_3)$$

A1 = Day shift workers unrepresented

P(A1) = $^{25}C_6$ / $^{45}C_6$

A2 = Swing shift workers unrepresented

P(A2) = $^{30}C_6$ / $^{45}C_6$

A3 = Graveyard shift workers unrepresented

P(A3) = $^{35}C_6$ / $^{45}C_6$

P(A1 ∩ A2) = $^{10}C_6$ / $^{45}C_6$

P(A1 ∩ A2 ∩ A3) = 0

Question

An academic department with five faculty members— Anderson, Box, Cox, Cramer, and Fisher—must select two of its members to serve on a personnel review committee. Because the work will be time-consuming, no one is anxious to serve, so it is decided that the representative will be selected by putting the names on identical pieces of paper and then randomly selecting two.

a. What is the probability that both Anderson and Box will be selected?

b. What is the probability that at least one of the two members whose name begins with C is selected?

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

Question

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a. What is the probability that both Anderson and Box will be selected? 0.1

b. What is the probability that at least one of the two members whose name begins with C is selected?

0.7

c. If the five faculty members have taught for 3, 6, 7, 10, and 14 years, respectively, at the university, what is the probability that the two chosen representatives have a total of at least 15 years' teaching experience there?

0.6

Question

The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events.

a. The next purchaser will request at least one of the three options.

b. The next purchaser will select none of the three options.

c. The next purchaser will request only a built in GPS and not either of the other two options.

d. The next purchaser will select exactly one of these three options.

Question

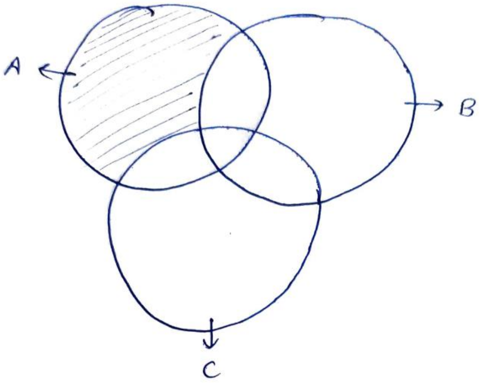
The three most popular options on a certain type of new car are a built-in GPS (A), a sunroof (B), and an automatic transmission (C). If 40% of all purchasers request A, 55% request B, 70% request C, 63% request A or B, 77% request A or C, 80% request B or C, and 85% request A or B or C, determine the probabilities of the following events.

a. The next purchaser will request at least one of the three options. 0.85

b. The next purchaser will select none of the three options. 0.15

c. The next purchaser will request only a built in GPS and not either of the other two options.

d. The next purchaser will select exactly one of these three options.



$P(A) = 0.4 \quad P(B) = 0.55 \quad P(C) = 0.7$

$P(A \cup B) = 0.63 \quad P(A \cup C) = 0.77 \quad P(B \cup C) = 0.8$

$P(A \cup B \cup C) = 0.85$

a) $P(A \cup B \cup C) = 0.85$

b) $1 - P(A \cup B \cup C) = 0.15$

c) shaded region

$P(A \cap B) = 0.4 + 0.55 - 0.63 = 0.32$

$P(A \cap C) = 0.4 + 0.7 - 0.77 = 0.33$

$P(B \cap C) = 0.55 + 0.7 - 0.8 = 0.45$

shaded region $= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$P(A \cap B \cap C) = P(A \cup B \cup C) - P(A) - P(B) - P(C) + P(A \cap B) + P(A \cap C) + P(B \cap C)$

$= 0.85 - 0.4 - 0.55 - 0.7 + 0.32 + 0.33 + 0.45$

$= 0.3$

shaded region $= 0.4 + 0.3 - 0.32 - 0.33$

$= 0.05$

d) $P(A \cup B \cup C) - P(A \cap C) - P(A \cap B) - P(B \cap C) + 2P(A \cap B \cap C)$

$= 0.85 - 0.33 - 0.32 - 0.45 + 2(0.3)$

$= 0.35$

References

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