Lecture 3

Conditional Probability
Bayes Theorem
Independent Events
Questions

Index

- Conditional Probability
- Bayes Theorem
- Independent Events
- Questions

Conditional Probability

For any two events A and B with $P(B) > 0$, the **conditional probability of A given that B has occurred** is defined by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A \mid B) \cdot P(B)$$

$P(A).P(B|A) = P(B).P(A|B)$

Question

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

1) What is the probability that a randomly selected purchaser has bought a brand 1 DVD player that will need repair while under warranty?

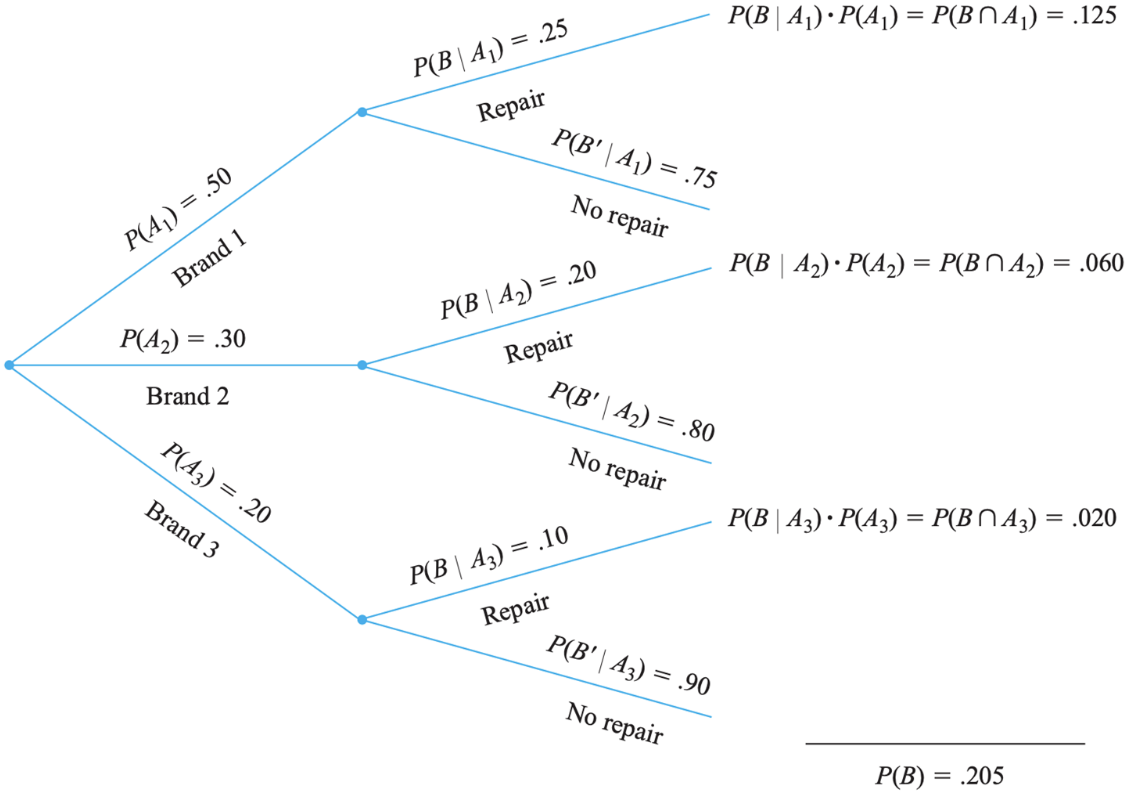
Question

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1’s DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

1) What is the probability that a randomly selected purchaser has bought a brand 1 DVD player that will need repair while under warranty?

0.125

Question



$P(A_1) = .50$
Brand 1
$P(A_2) = .30$
Brand 2
$P(A_3) = .20$
Brand 3
$P(B \mid A_1) = .25$
Repair
$P(B' \mid A_1) = .75$
No repair
$P(B \mid A_2) = .20$
Repair
$P(B' \mid A_2) = .80$
No repair
$P(B \mid A_3) = .10$
Repair
$P(B' \mid A_3) = .90$
No repair
$P(B \mid A_1) \cdot P(A_1) = P(B \cap A_1) = .125$
$P(B \mid A_2) \cdot P(A_2) = P(B \cap A_2) = .060$
$P(B \mid A_3) \cdot P(A_3) = P(B \cap A_3) = .020$
$P(B) = .205$

Question

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

2. What is the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty?

Question

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

2. What is the probability that a randomly selected purchaser has a DVD player that will need repair while under warranty?

0.205

Question

A chain of video stores sells three different brands of DVD players. Of its DVD player sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's DVD players require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

3. If a customer returns to the store with a DVD player that needs warranty repair work, what is the probability that it is a brand 1 DVD player? A brand 2 DVD player? A brand 3 DVD player?

Question

3. If a customer returns to the store with a DVD player that needs warranty repair work, what is the probability that it is a brand 1 DVD player? A brand 2 DVD player? A brand 3 DVD player?

$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.125}{.205} = .61$$

$$P(A_2|B) = \frac{P(A_2 \cap B)}{P(B)} = \frac{.060}{.205} = .29$$

$$P(A_3|B) = 1 - P(A_1|B) - P(A_2|B) = .10$$

Bayes Theorem

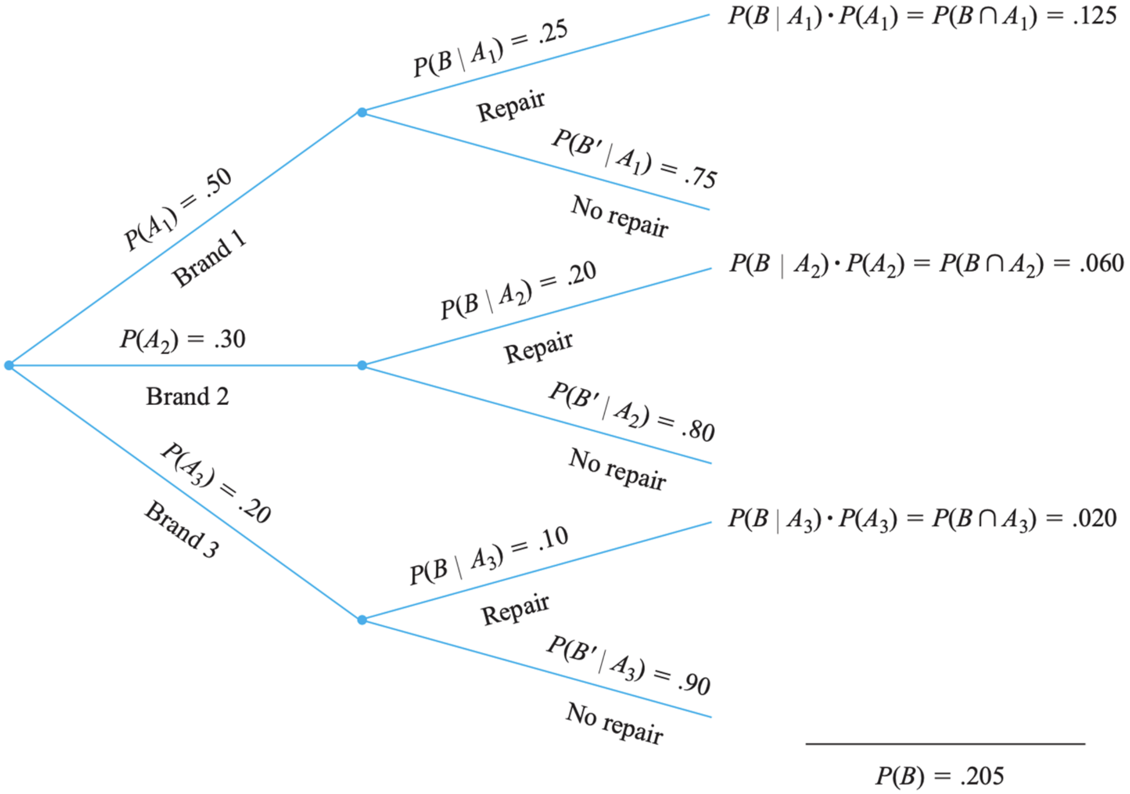
Let $A_1, \ldots, A_k$ be mutually exclusive and exhaustive events. Then for any other event B,

$$P(B) = P(B|A_1)P(A_1) + \cdots + P(B|A_k)P(A_k)$$
$$= \sum_{i=1}^{k} P(B|A_i)P(A_i)$$

Let $A_1, A_2, \ldots, A_k$ be a collection of k mutually exclusive and exhaustive events with *prior* probabilities $P(A_i)$ ($i = 1, \ldots, k$). Then for any other event B for which $P(B) > 0$, the *posterior* probability of A_j given that B has occurred is

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)} = \frac{P(B|A_j)P(A_j)}{\sum_{i=1}^{k} P(B|A_i) \cdot P(A_i)} \quad j = 1, \ldots, k$$

Question



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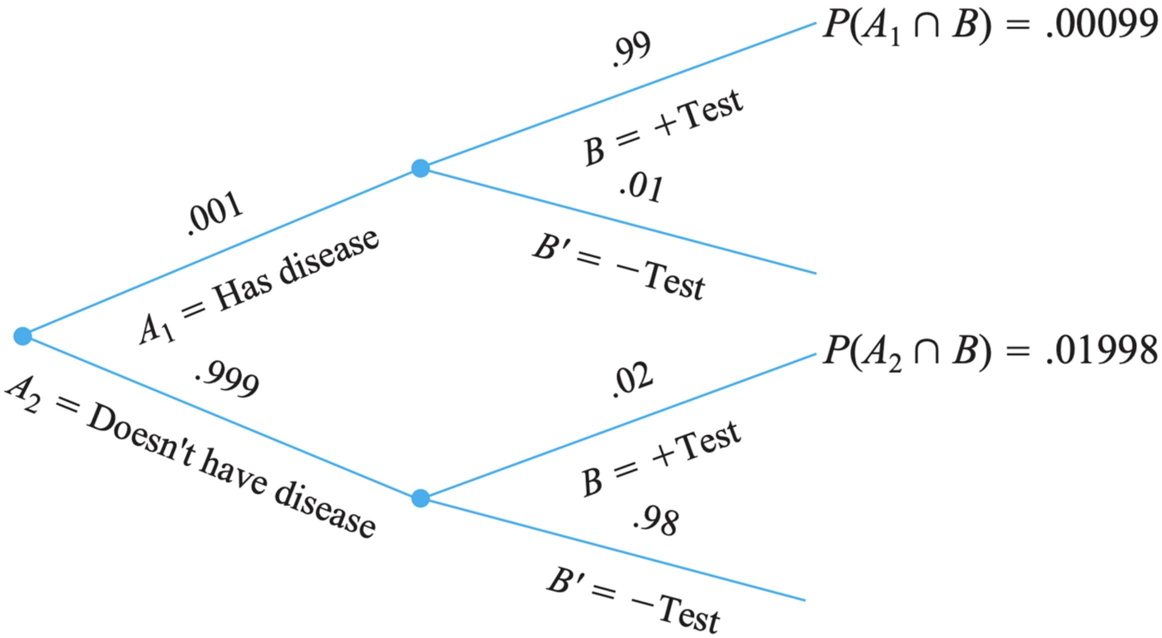
$P(B \mid A_3) \cdot P(A_3) = P(B \cap A_3) = .020$
$P(B) = .205$

Question

Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur 99% of the time, whereas an individual without the disease will show a positive test result only 2% of the time. If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?



$$P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{.00099}{.02097} = .047$$

Independent Events

Two events A and B are **independent** if $P(A|B) = P(A)$ and are **dependent** otherwise.

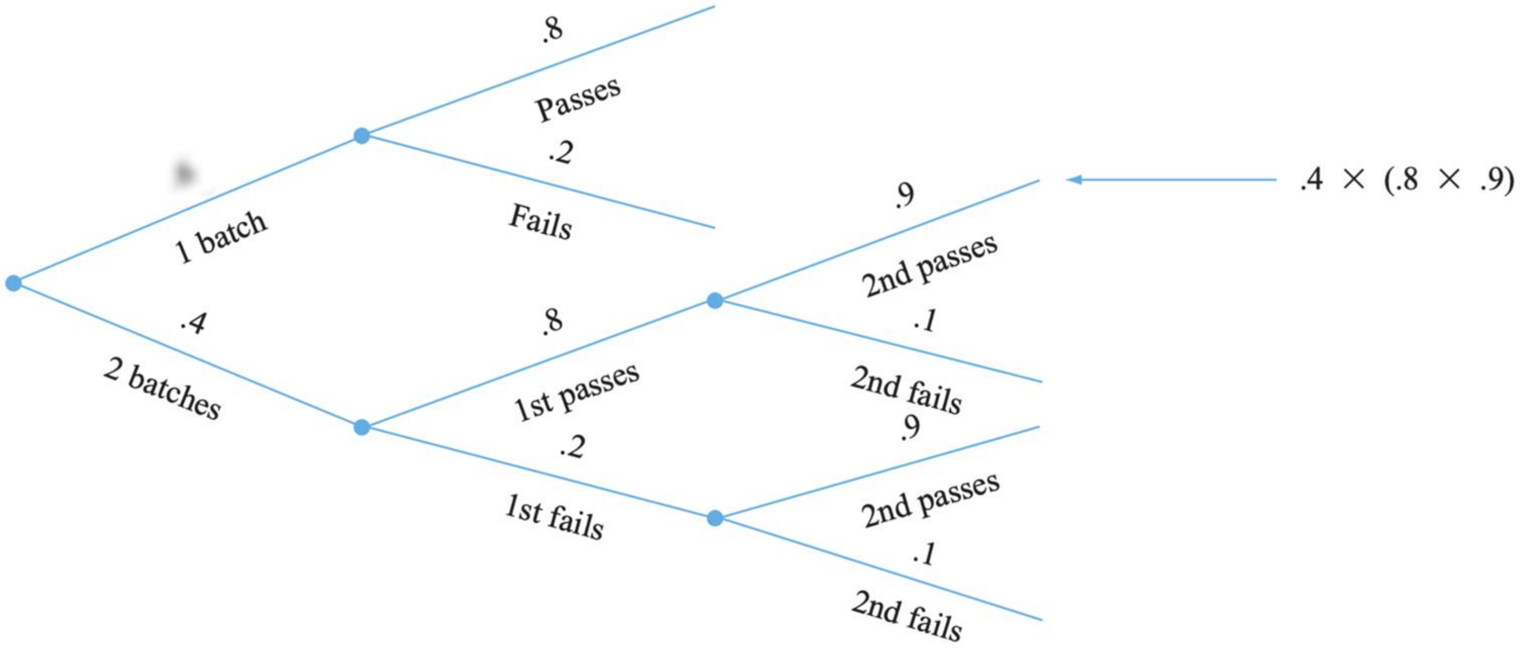
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A and B are independent if and only if (iff)

$$P(A \cap B) = P(A) \cdot P(B)$$

Question

Each day, Monday through Friday, a batch of components sent by a first supplier arrives at a certain inspection facility. Two days a week, a batch also arrives from a second supplier. Eighty percent of all supplier 1’s batches pass inspection, and 90% of supplier 2’s do likewise. What is the probability that, on a randomly selected day, two batches pass inspection?


.8
Passes
.2
Fails
1 batch
.4
2 batches
.8
1st passes
.2
1st fails
.9
2nd passes
.1
2nd fails
.9
2nd passes
.1
2nd fails
.4 × (.8 × .9)

$$\begin{aligned} P(\text{two pass}) &= P(\text{two received} \cap \text{both pass}) \\ &= P(\text{both pass} \mid \text{two received}) \cdot P(\text{two received}) \\ &= [(.8)(.9)](.4) = .288 \end{aligned}$$

Question

Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

Question

Two pumps connected in parallel fail independently of one another on any given day. The probability that only the older pump will fail is .10, and the probability that only the newer pump will fail is .05. What is the probability that the pumping system will fail on any given day (which happens if both pumps fail)?

P(Pump 1 fails) = P(only Pump 1 failing) + P(both pumps failing) = 0.1+0.005 = 0.105
P(Pump 2 fails) = P(only Pump 2 failing) + P(both pumps failing) = 0.05+0.05 = 0.055
Since independent events then answer is P(Pump 1 fails)*P(pump 2 fails) = 0.105*0.055 = 0.005775

Question

Individual A has a circle of five close friends (B, C, D, E, and F). A has heard a certain rumor from outside the circle and has invited the five friends to a party to circulate the rumor. To begin, A selects one of the five at random and tells the rumor to the chosen individual. That individual then selects at random one of the four remaining individuals and repeats the rumor. Continuing, a new individual is selected from those not already having heard the rumor by the individual who has just heard it, until everyone has been told.

Question

1. What is the probability that the rumor is repeated in the order B, C, D, E, and F?

   1/(5.4.3.2.1) = 1/120 = 0.0083

Question

2. What is the probability that F is the third person at the party to be told the rumor?

? ? F ? ?
4.3.1.2.1
=24
Total possibilities = 5.4.3.2.1 =120
So answer is 24/120 = 0.2

Question

3. What is the probability that F is the last person to hear the rumor?

? ? ? ? F
4.3.2.1.1
=24
Total possibilities = 5.4.3.2.1 =120
So answer is 24/120 = 0.2

Question

4. If at each stage the person who currently “has” the rumor does not know who has already heard it and selects the next recipient at random from all five possible individuals, what is the probability that F has still not heard the rumor after it has been told ten times at the party?

4.4.4.4…../ 5.5.5.5…..
$4^{10}/5^{10}$
=0.1074

References

Probability and statistics for engineers RA Johnson, I Miller, JE Freund - 2000 - 117.239.47.98

Statistics for business & economics DR Anderson, DJ Sweeney, TA Williams, JD Camm

Probability and statistics for engineering and science J Deovre