1. Consider a joint probability distribution of the random variables X, Y given as

$$f_{X,Y}(x, y) = \begin{cases} 4xy, & \text{for} \quad 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal densities of X and Y and the conditional density of X given $Y = y$.

2. Determine k so that

$$f_{X,Y}(x, y) = \begin{cases} kxy(x - y) \quad , & 0 < x < 1, \quad -x < y < x \\ 0, & \text{otherwise} \end{cases}$$

can be a joint p.d.f. of the random variables X and Y.

3. Let us consider the joint p.d.f. of X and Y given as

$$f_{X,Y}(x, y) = \begin{cases} x + y, & ,\text{if} \quad 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find $f_{Y|X}(y|x)$ and the conditional distribution function.

4. If X and Y are two random variables, then show that

   a) $E[g_1(Y) + g_2(Y) \mid X = x] = E[g_1(Y) \mid X = x] + E[g_2(Y) \mid X = x]$

   a) $E[g_1(Y)g_2(Y) \mid X = x] = g_2(x)E[g_1(Y) \mid X = x]$

5. If X and Y have joint distribution given by

$$f_{X,Y}(x, y) = \begin{cases} 2, & \text{for} \quad 0 < x < y, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find a) $Cov(X, Y)$ and b) $f_{Y|X}(y \mid x)$

6. Let us consider an urn which contains three balls numbered 1,2 and 3. A "sample" of size 2 is drawn without replacement from the urn. Let X denote the number on the first ball while Y denotes the larger of the two.

   a) Find the joint p.m.f. of the functions X and Y

   b) Find $P[X = 1 \mid Y = 3]$

   c) Compute $Cov[X, Y]$

7. Let X and Y are jointly distributed as a bivariate normal distribution with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ and ρ. Show that the conditional density function of Y given $X = x$ follows a normal distribution with mean

$$\mu_{Y|X} = \mu_2 + \rho\frac{\sigma_2}{\sigma_1}(x - \mu)$$

and variance

$$\sigma^2_{Y|X} = \sigma^2_2(1 - \rho^2)$$