ESO 208A: Computational Methods in Engineering
Programming Assignment 4: *Ordinary Differential Equations*

Due date: Saturday, 9th November 24:00 mid-night

1) Write a computer program for solving IVPs.

Input: (i) Ordinary differential equation to be solved $\frac{dy}{dx} = f(x, y)$; (ii) initial values x_0 and y_0; (iii) final value x_f (iv) interval size h; (v) maximum tolerance tol.

Options: The user should have the option of selecting one or more of the following –
   (a) Euler Forward
   (b) Euler Backward
   (c) Trapezoidal
   (d) 4th -order Adams-Bashforth
   (e) 4th -order Adams-Moulton
   (f) 4th -order Backward Difference Formulation (BDF)
   (g) 4th Order Runge-Kutta

Obtaining analytical solution is optional.

Output: The output from the program should be:
   (a) A text file containing the values of x_i and corresponding y_i;
   (b) A figure showing y vs x.

Test Problem: Provided at the end as well other problems in Tutorial 12.

2) Write a computer program to solve the following BVP:

The diagram shows a body of conical section fabricated from stainless steel immersed in air at zero temperature. It is of circular cross section that varies with x. The large end is located at $x = 0$ and is held at temperature $T_A = 5$. The small end is located at $x = L = 2$ and is insulated (i.e, the temperature gradient is zero).
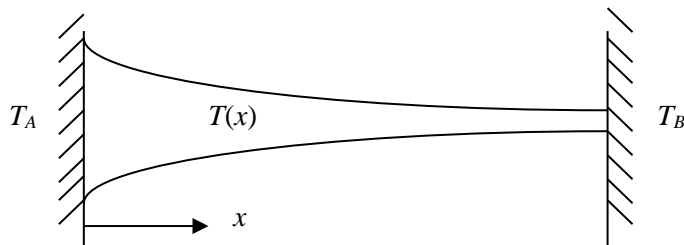
Conservation of energy can be used to develop a heat balance equation at any cross section of the body. When the body is not insulated along its length and the system is at steady state, its temperature satisfies the following ODE:

$$\frac{d^2T}{dx^2} + a(x)\frac{dT}{dx} + b(x)T = f(x)$$

where, a(x), b(x), and f(x) are functions of the cross-sectional area, heat transfer coefficients, and the heat sinks inside the body. In the present case, they are given by

$$a(x) = -\frac{x+3}{x+1}, \qquad b(x) = \frac{x+3}{(x+1)^2}, \qquad \text{and} \quad f(x) = 2(x+1) + 3b(x).$$

a) Discretize the above equation using 2nd order central difference approximation and formulate the set of linear simultaneous equations. Incorporate the boundary conditions such that the accuracy of the scheme is preserved. Use $\Delta x = 0.5$.

b) Solve the system of equations using Thomas Algorithm and draw the temperature profile indicating the values at the nodes.

In the program, keep the following user options:

- Grid size (h)
- Implementation options for the small end boundary: 2nd order Backward Difference or 2nd order Central Difference with Ghost Node

You may use the Thomas Algorithm program module that you wrote in Programming Assignment 2.

Test Problem: Problems in Tutorial 13.

Due date: Saturday, November 9th, 2019, 24:00 midnight

Make a single zip folder with all your program file(s) name it roll_number.zip (e.g., If your roll no. is 123456, the folder name should be 'P3_123456.zip'). The folder should include -

(i) All the computer program file(s), input file(s) and output file(s)

(ii) A PDF file of the plots and the solution of the test cases given in this assignment.

Send the zip file by e-mail to: eso208.sec*@gmail.com, where * is section number 1-10. Example: for section O5, it is eso208.sec5@gmail.com; for section O10, it is eso208.sec10@gmail.com

Additional Test Data: Part 1:

Sample input file

$$\frac{dy}{dx} = 5e^{-100(x-2)^2} - 0.5y$$

$x_0 = 0.0$

$y_0 = 0.5$

$x_f = 4.0$

$h = 0.2$

$h_{max} = 2.0$

$\alpha = 0.25$

$tol = 10^{-5}$

Sample output files

x	y_analytical	y_eulerF	y_trapezoidal	y_RK4
0.00000	0.50000	0.50000	0.50000	0.50000
0.20000	0.45242	0.45000	0.45250	0.45242
0.40000	0.40937	0.40500	0.40951	0.40937
0.60000	0.37041	0.36450	0.37061	0.37041
0.80000	0.33516	0.32805	0.33540	0.33516
1.00000	0.30327	0.29525	0.30354	0.30327
1.20000	0.27441	0.26572	0.27470	0.27441
1.40000	0.24829	0.23915	0.24861	0.24829
1.60000	0.22466	0.21523	0.22499	0.22466
1.80000	0.20534	0.19371	0.20374	0.20642
2.00000	0.61483	0.19265	0.55135	0.58950
2.20000	0.96673	1.17339	0.81685	0.92054
2.40000	0.87663	1.07437	0.73845	0.83578
2.60000	0.79321	0.96693	0.66830	0.75625
2.80000	0.71773	0.87024	0.60481	0.68428
3.00000	0.64942	0.78321	0.54736	0.61916
3.20000	0.58762	0.70489	0.49536	0.56024
3.40000	0.53170	0.63440	0.44830	0.50693
3.60000	0.48111	0.57096	0.40571	0.45869
3.80000	0.43532	0.51387	0.36717	0.41504
4.00000	0.39390	0.46248	0.33229	0.37554