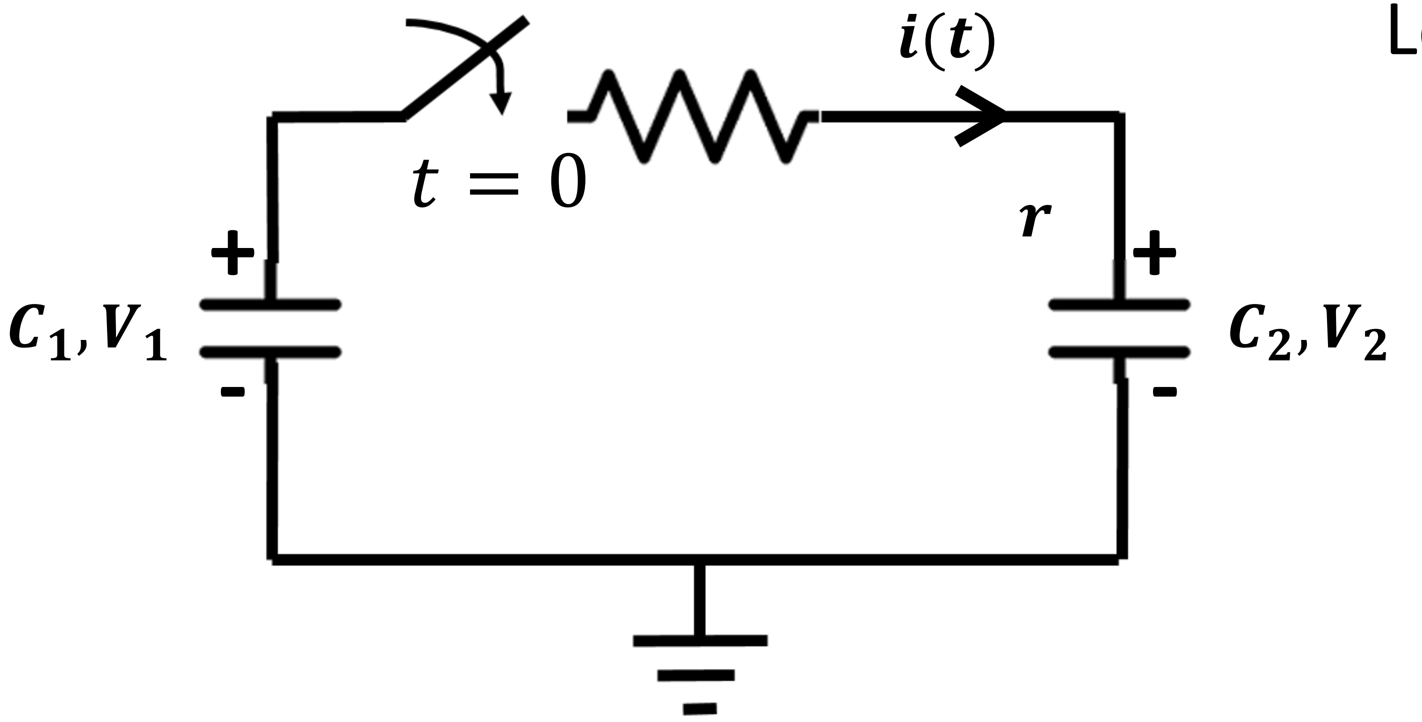
Introduction to Electronics

- Man's interaction with nature and systems designed by him is an interplay of energy and information.
- Contrast between the notion of a *source* in *circuits* v/*s* *information theory.*
- In 80's consumption of energy/person in USA was 75 times that of India and served as a benchmark of advancement.
- In current times, usage of information plays a dominant role .
- In 1956, white collar workers in USA acceded blue collar workers.
- Information age started around 1990, although information theory as a discipline began in 1948.
- Professor Gallager of MIT while visiting IIT Bombay in 2008, shared with me that theoretical developments in information theory had little impact for first 3 decades whereas in subsequent years the theoretical results have had a major impact on practice.
- Take compression and error correcting codes as examples.

Introduction to Electronics

- In this course, our focus will be on basic **circuit analysis**
- In particular, it will deal with basic concepts related to circuits made up of components / elements such as resistors, inductors, capacitors, diodes, transistors, operational amplifiers, logic gates and flip-flops as memory devices
- These circuits give rise to systems achieving objectives in terms of control, communication, storage, retrieval of information, computing and so on.
- Telegraph and *Morse code* (a compact representation of text) came into existence long before light *bulb* was invented by Edison
- **Transistor** in **1948** was developed at AT &T labs in response to the need of communication industry for an effective **switch.**
- 1948 also marks the birth of ***information theory*** which guides our advancement into information age.

Conservation of energy v/s Conservation of charge



Let $V_1 > V_2$

$$\mathrm{i(t)} = \frac{v_1(t) - v_2(t)}{r}$$

$\mathrm{i}(\infty) = 0$ and

$v_1(\infty) = v_2(\infty) = \mathrm{V}$ (say)

Conservation of energy v/s Conservation of charge

By conservation of charge,
$C_1V_1 + C_2V_2 = (C_1 + C_2)V$

$$V = \frac{C_1}{C_1 + C_2}V_1 + \frac{C_2}{C_1 + C_2}V_2$$

(a convex combination of V_1 and V_2)

Initial energy= $\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = E_1$, Final energy= $\frac{1}{2}(C_1 + C_2)V^2 = E_2$

Final energy = $\frac{1}{2}(C_1 + C_2)\left[\left(\frac{C_1}{C_1+C_2}\right)^2 V_1^2 + \left(\frac{C_2}{C_1+C_2}\right)^2 V_2^2 + \frac{2C_1C_2}{C_1+C_2}V_1V_2\right]$

$$\longrightarrow E_1 = \frac{1}{2}(C_1 + C_2)\left[\left(\frac{C_1}{C_1 + C_2}\right) V_1^2 + \left(\frac{C_2}{C_1 + C_2}\right) V_2^2\right] > E_2$$

As $V^2 < \frac{C_1}{C_1+C_2}V_1^2 + \frac{C_2}{C_1+C_2}V_2^2$ by Jensen's inequality

Jensen’s Inequality

$$\phi\left(\sum_{i=1}^{i=n} a_i x_i\right) \leq \sum_{i=1}^{i=n} a_i \phi(x_i)$$

where $0 \leq a_i \leq 1, \sum_{i=1}^{i=n} a_i = 1,$
and $\phi(.)$ is a concave upward function.
eg. $\phi(x) = x^2$.

Conservation of energy v/s Conservation of charge

Optional problem: difference is **dissipated** energy through r

Verify that $\int_0^\infty i(t)^2 r dt = E_1 - E_2$ is independent of r.

Note that $i(t) = C_2 \frac{dv_2(t)}{dt} = -C_1 \frac{dv_1(t)}{dt} = \frac{v_1(t)-v_2(t)}{r}$

$$-\frac{dv_2(t)}{dt} = -\frac{i(t)}{C_2}, \frac{dv(t)}{dt} = -\frac{i(t)}{C_1}$$

$$\frac{d(v_1(t) - v_2(t))}{dt} = -i\left[\frac{1}{C_1} + \frac{1}{C_2}\right] = \frac{-(v_1(t) - v_2(t))}{r(C_1||C_2)}$$

Let $\mathrm{v}_1(t) - \mathrm{v}_2(t)$ =v(t)

Then $\frac{\mathrm{dv}(t)}{\mathrm{dt}} = -\frac{v(t)}{rC}$ where C $= \frac{C_1 C_2}{C_1+C_2}$

vOr $\int_{V1-V2}^{v(t)} \frac{dv(t)}{v(t)} = -\frac{1}{rE}\int_0^t dt$

Or $-\frac{t}{rC} = log v(t) - \log(V_1 - V_2)$

$$= \log\left(\frac{v(t)}{V_1 - V_2}\right)$$

Or $\mathrm{v}(t) = (V_1 - V_2)\exp^{-\frac{t}{rC}} \rightarrow 0$ as t $\rightarrow \infty$.

Conservation of energy v/s Conservation of charge

Optional problem: difference is dissipated energy through r

$$\int_0^\infty p(t)\,dt = \int_0^\infty \frac{v(t)^2}{r} = \frac{(V_1 - V_2)^2}{r}\int_0^\infty \exp^{-\frac{2t}{rC}}\,dt$$

$$= -\frac{(V_1 - V_2)^2}{r}\frac{\mathrm{rC}}{2}\mathrm{e}^{-\frac{2t}{rC}}\Big|_0^\infty$$

$$= \frac{1}{2}C(V_1 - V_2)^2 = \quad \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

Since $E_1 - E_2$ is independent of r,

we have $\lim_{\{r\to 0\}}(E_1 - E_2) = E_1 - E_2$

If $r = 0$ before switch closes, the energy will radiate.

If $V_2 < 0$, then both capacitors have potential in same direction.

Then $i(t) = C_2\frac{dv_2(t)}{dt} = C_1\frac{dv_1(t)}{dt} = \frac{v_1(t)+v_2(t)}{r}$

Or$\frac{i(t)}{C_1} + \frac{i(t)}{C_2} = \frac{d(v_1(t)+v_2(t))}{dt} = \frac{(v_1(t)+v_2(t))(C_1+C_2)}{rC_1C_2}$

And $v(t) = v_1(t) + v_2(t) = (V_1 + V_2)e^{-\frac{t}{\tau}}$

Where $\tau = \frac{C_1C_2}{C_1+C_2}r$.

Conservation of energy v/s Conservation of charge

Optional problem: difference is dissipated energy through r

Again $\int_0^\infty p(t) = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1+V_2)^2$

Since $v_1(t)+v_2(t) \to 0$ as $t \to \infty$.

Let V_1' & V_2' be the final voltages on C_1 & C_2.

Then $V_1'+V_2' = 0 \longrightarrow V_1' = -V_2'$

Or $V_1'^2 = V_2'^2$

Final energy $= \frac{1}{2}C_1V_1'^2 + \frac{1}{2}C_2V_2'^2 = \frac{1}{2}(C_1+C_2)V'^2$

Where $|V_1'| = |V_2'| = |V'|$

By direct computation,

$$V_1' = V_1 - \frac{Q_1}{C_1} = V_1 - \frac{1}{C_1}\int_0^\infty i(t)dt$$

$$= V_1 - \frac{(V_1+V_2)}{rC_1}\int_0^\infty e^{-\frac{t}{\tau}}dt = V_1 - \frac{(V_1+V_2)}{C_1}\, C_{eq}$$

$$= V_1 - \frac{(V_1+V_2)C_2}{C_1+C_2} = \frac{V_1C_1}{C_1+C_2} - \frac{V_2C_2}{C_1+C_2}$$

Conservation of energy v/s Conservation of charge

Optional problem: difference of initial and final energy is dissipated energy through r.

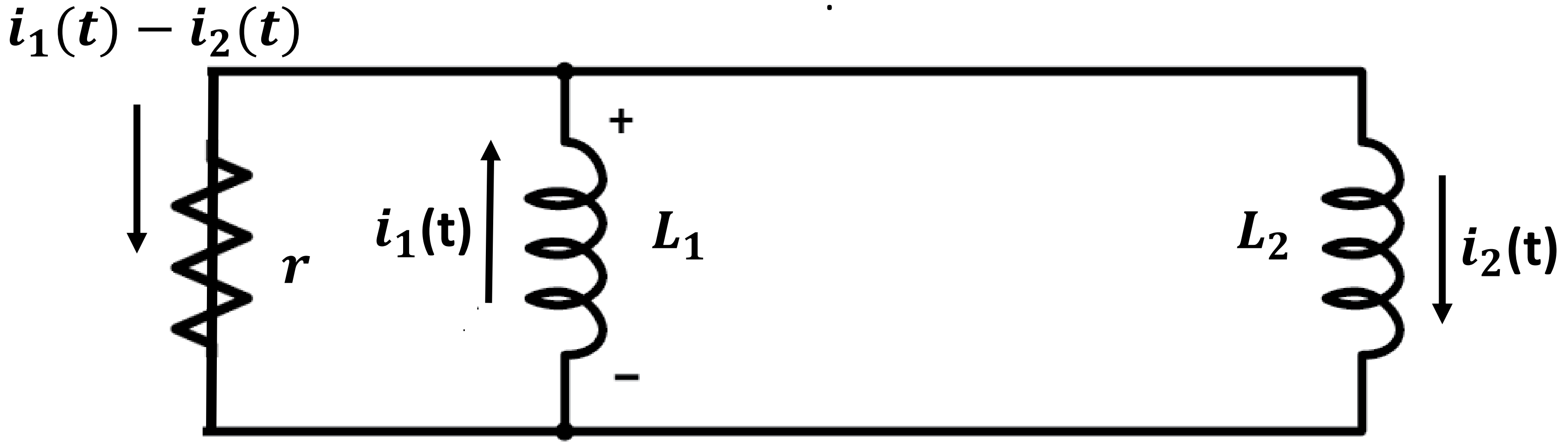
Then $\mathrm{V}_2' = V_2 - \frac{(V_1+V_2)C_{eq}}{C_2} = \frac{V_2 C_2}{C_1+C_2} - \frac{V_1 C_1}{C_1+C_2} = -\mathrm{V}_1'$

If $C_1V_1 = C_2V_2$ then $\mathrm{V}_1' = V_2' = 0$ and net charge on individual capacitors is zero or the two capacitors discharge completely.

Conservation of energy,

the dual problem

Let following configuration come into place at t=0, with initial currents I_1 and I_2.



$$\boxed{p(t) = (i_1(t) - i_2(t))^2 r}$$

$$-L_1\frac{di_1(t)}{dt} = L_2\frac{di_2(t)}{dt} = (i_1(t) - i_2(t))r$$

$$-\frac{di_1(t)}{dt} = (i_1(t) - i_2(t)\frac{r}{L_1},$$

$$\frac{di_2(t)}{dt} = (i_1(t) - i_2(t))\frac{r}{L_2}$$

Conservation of energy

The Dual problem

$$\frac{d(i(t)-i_2(t))}{dt} = -r\left[\frac{1}{L_1}+\frac{1}{L_2}\right](i_1(t)-i_2(t))$$

Or $\frac{di(t)}{dt} = -\frac{r}{L_1||L_2}i(t) = -\frac{i(t)}{\tau}$ where $\tau = \frac{(L_1||L_2)}{r}$

$$\Rightarrow \int \frac{di(t)}{i(t)} = -\int \frac{dt}{\tau}$$

Or $lni(t) - lni(0) = -\frac{t}{\tau} = \ln\left(\frac{i(t)}{i(0)}\right)$

$$i(t) = Ie^{-\frac{t}{\tau}} \rightarrow 0 \text{ as } t \rightarrow 0,$$

$where\ I = I_1 - I_2.$

Conservation of energy

The Dual problem

$$\int p(t)dt \ = \int i(0)^2 r e^{-\frac{2t}{\tau}} \ = \frac{i(0)^2 r e^{-\frac{2t}{\tau}}}{-\frac{2}{\tau}}\Big|_0^{\infty} = \frac{\tau . r}{2} i(0)^2$$

$$= \frac{1}{2}(L_1 || L_2)(I_1 - I_2)^2$$

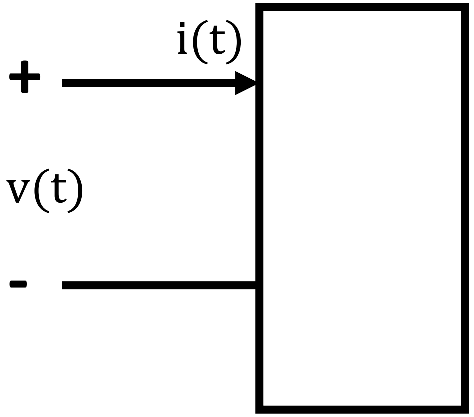
So in steady state,

$$E = \frac{1}{2}(L_1 + L_2)I^2 = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 - \frac{1}{2}(L_1||L_2)(I_1 - I_2)^2.$$

Exercise :Think about the situation when in steady state ,
zero current flows through each inductor.

Power sources and elements

Charge,	Current,	Voltage,	Power
q	$\frac{\mathrm{d}q}{dt}$	$\mathrm{v(t)}$	$\mathrm{p}(t)$



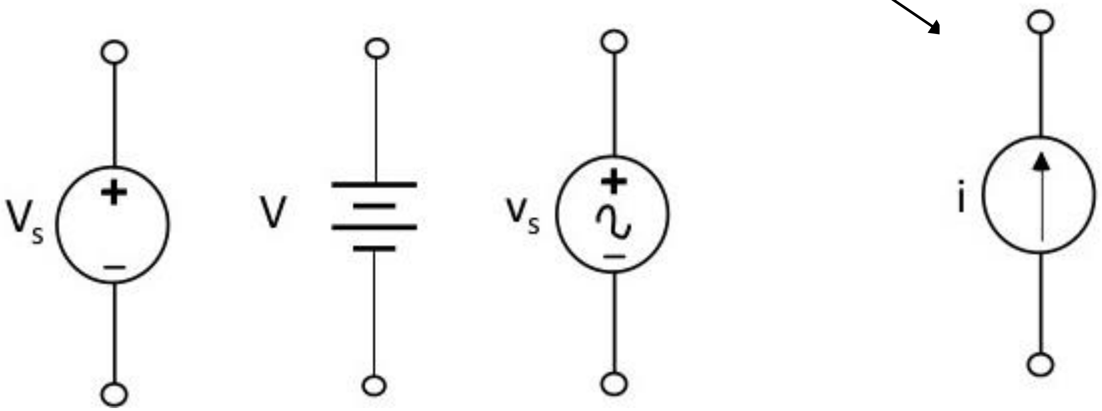
Power absorbed

$p(t) = v(t)i(t) > 0$ if either $\mathrm{v}(t), i(t) > 0$ or $\mathrm{v}(t), i(t) < 0$

Independent Sources

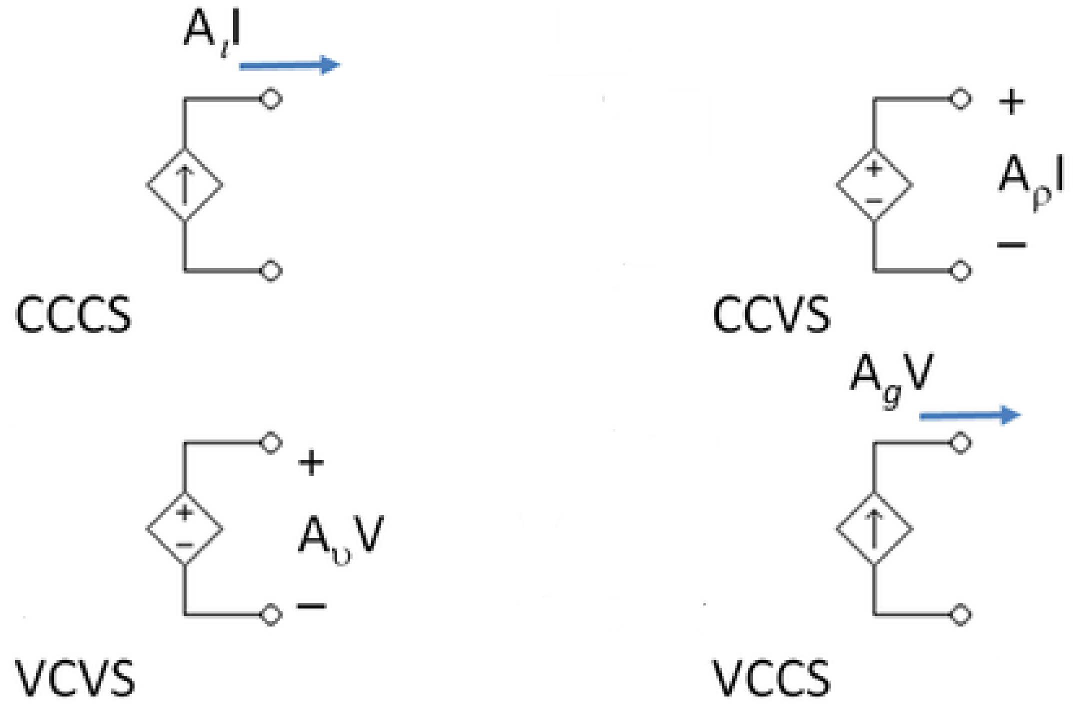
Voltage

Current



Power sources and elements

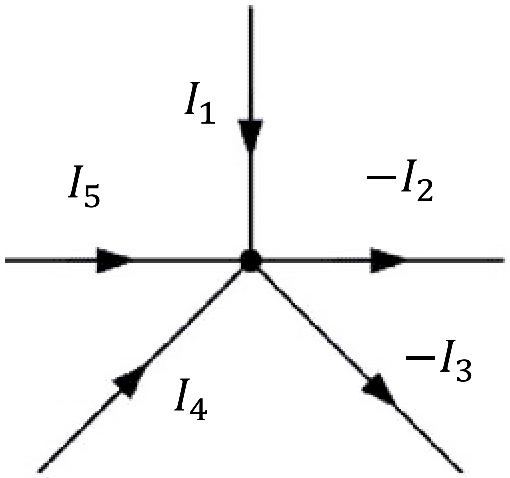
Dependent Sources:
current / voltage controlled current / voltage sources



Kirchhoff’s Laws

Kirchhoff’s current law (KCL)

- A consequence of *charge* conservation
- Node: a point where two or more elements meet

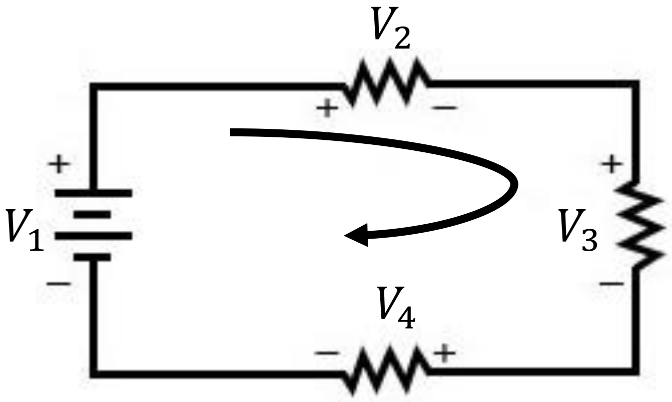


$$\sum_{j=1}^{n} I_j = 0$$

(KCL)

Kirchhoff’s voltage law (KVL)

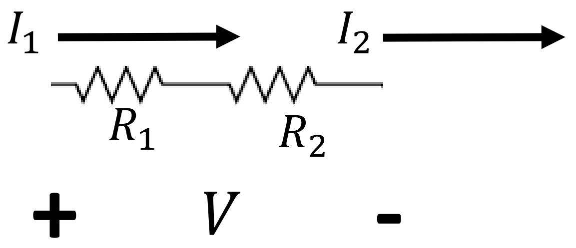
- A consequence of *energy* conservation
- Loop: a closed path starting from a particular node and ending there *without* revisiting that particular node



$$\sum_{i=2}^{m} V_i = V_1$$

(KVL)

Resistors in series and in parallel



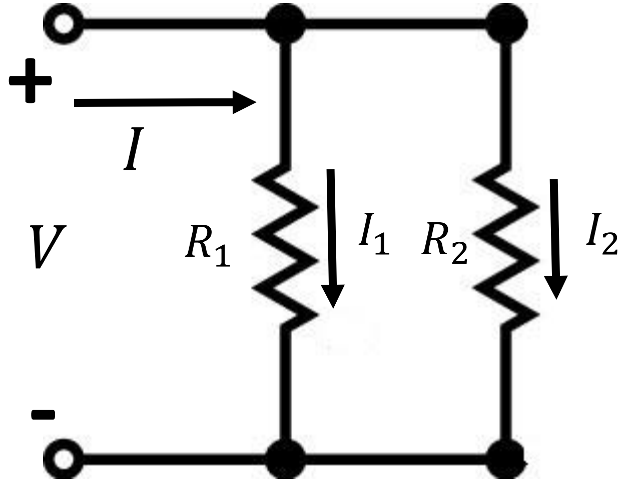
$I_1 = I_2 = I(suppose)$ by KCL

$$V = I_1 R_1 + I_2 R_2 = I(R_1 + R_2)$$

$$\frac{V}{I} = R_1 + R_2$$

The equivalent resistance for n resistors by induction will be given by

$$R_{eq} = \sum_{i=1}^{n} R_i$$



$$V = I_1 R_1 = I_2 R_2$$

$$I_1 + I_2 = I$$

$$V\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = I$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For n resistors,

$$R_{eq} = \left(\sum_{i=1}^{n} R_i^{-1}\right)^{-1}$$

A similar approach can be followed for finding equivalent Inductance in series and parallel