1. Let X be a continuous random variable with p.d.f.
$$f_X(x) = \frac{1}{\pi(x^2+1)}, \quad ; -\infty < x < \infty \tag{1}$$
Show that f_X is a p.d.f. and compute its Expectation.

2. Let X be a continuous random variable with m.g.f. given as
$$m_X(t) = e^{t^2/2} \tag{2}$$
Compute the mean and variance. Can you recognize the distribution?

3. Suppose that the proportion of erroneous tax returns is given by a random variable. Suppose this has been observed to fit well with a Beta random variable with parameters a & b, which are related to taxation issues. What is the probability that in a randomly chosen year the proportion of erroneous tax returns is at most γ?

4. Suppose the amount of time a customer spends in a restaurant has an Exponential distribution, with mean of 6 minutes. If we select a customer randomly, what is the probability that he/she will spend more than 12 minutes in the restaurant?

5. Suppose a particular make of a bathroom water heater has a life modelled as an exponential random variable with mean 12 years. Mr. J. buys this house along with a water heater of the same brand. Mr. J. will sell the house in 4 years. What is the probability that Mr. J. won't have to buy a new heater for replacing the old one?

6. Let X be a continuous random variable following Gamma distribution with parameters γ and λ. Write down the p.d.f. if we set $\lambda = \frac{1}{\beta}$. In this new form set $\gamma = 2$. Then that particular form is called the Weibull distribution. Let X be a Weibull distribution. Show that
$$P(X > t) = e^{-t^2/\beta} \tag{3}$$

Further if s and $t > 0$ are given, and say

$$P(X > s + t | x > t) = \gamma > 0 \tag{4}$$

then show that

$$\beta = \frac{s(s+2t)}{\ln\left(\frac{1}{\gamma}\right)} \tag{5}$$

7. Suppose we roll two fair dice 600 times. Let X denote the random variable that the total of the two die is 7. Then compute $P[90 \leq X \geq 100]$.