1. Given a number N, consider the following function

$$f_X(x) = \begin{cases} \frac{1}{N}, & x = 1, 2, \ldots, N \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

where N is positive. For a given natural number N, is f_X a probability mass function? Draw its graph. Can you suggest a name for the distribution? Find $E(X)$ and $Var(X)$.

2. We call a discrete random variable a Bernoulli variable if it has a distribution as follows

$$f_X(x) = \begin{cases} p^x(1-p)^{1-x}, & for \quad x = 0 \quad or \quad x = 1 \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

Find $E(X)$ and $Var(X)$. Let X_N is a random variable giving the number of successes in n trials. Show that

$$X_N = X_1 + X_2 + \cdots + X_n \tag{3}$$

where each X_i is a Bernoulli random variable.

3. <u>Probability Generating Functions:</u> Let $X : \Omega \longmapsto \mathbb{R}$, be a discrete random variable possibly taking countably infinite values.

The probability generating function $G_X : [-1, +1] \longmapsto \mathbb{R}$ is defined as

$$G_X(s) = E[s^X], \quad -1 \leq s \leq +1$$

a) Consider a random variable X, with probability generating function

$$G_X(s) = e^{\lambda(s-1)}, \quad -1 \leq s \leq +1$$

What is the distribution of X?

b) Show that $G_X(1) = 1$ and under the convention $0^0 = 1$, we have $G_X(0) = P(X = 0)$

c) Show that $G'_X(1) = E[X]$.

d) Show that $G_{X+Y}(s) = G_X(x)G_Y(s)$ if X and Y are independent random variables (to be done later).

Why is $s \in [-1, +1]$? Can you guess the reason?

4. "Sampling with Replacement": Consider sampling with replacement from a box having M light-bulbs of which K are defective. Let X be the random variable denoting the number of defective bulbs in n successive draws, i.e a sample of size n. Find the distribution of X.

5. "Sampling without Replacement": Again let there be a box of light-bulbs with M bulbs of which N are alright and the rest defective. What is the probability that if we choose a sample of $n < M$, bulbs x will be found to be alright? Can you write it up as a probability distribution? If you can let me tell you that such a distribution is called a Hypergeometric distribution. Compute its mean and variance.

6. If $X \sim Poisson(\lambda)$, such that $P[X = 1] = P[X = 2]$. Then compute $P[X = 1 \quad \text{or} \quad X = 2]$.