1. Let A and B are events with $P(A) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$. Prove that
$$\frac{1}{12} \leq P(A \cap B) \leq \frac{1}{3}$$

2. A fair coin is tossed repeatedly. Show that with probability 1, head turns up sooner or later.

3. Show that the probability that exactly one of the events A and B occurs is given as
$$P(A) + P(B) - 2P(A \cap B)$$

4. Let A be an event that accused in a crime is actually guilty and B is the event that some testimony is true. The lawyers have argued based on the assumption that
$$P(A|B) = P(B|A)$$
Show that this is only possible if $P(A) = P(B)$.

5. Let A and B be independent events. What can you say about A^c and B & A^c and B^c ?

6. <u>Galton's Paradox:</u> Let us flip three fair coins. At least two are alike and thus it is an even chance that the third toss will be either head or tail. Thus $P(all\, alike) = \frac{1}{2}$. Can we agree on this?

7. Let $\Omega = \{\omega_1, \ldots, \omega_n\}$; and let P be a function which satisfies,
$$P(\omega_j) = p_j,\ j = 1, \ldots, n, \text{ with } p_j \geq 0 \ \&\ \sum_{i=1}^{n} p_j = 1.$$
Describe an event A in this setting and how will you define $P(A)$ such that P satisfies the Kolmogorov axioms. What happens if Ω has countably infinite members?

8. If $A \cap B = \phi$, then A and B cannot be independent unless $P(A) = 0$ or $P(B) = 0$.

9. Polya's Urn: An urn contains r red balls and b blue balls. A ball is chosen at random from the urn; its colour is noted and returned to the urn by adding d more balls of the same colour. This is repeated indefinitely. What is the probability that the second ball drawn is blue? Further find the probability that the first ball drawn is blue, given that the second ball drawn is blue.

10. An insurance company insures an equal number of male and female drivers. In any given year a male driver makes a claim with probability α, independent of other years. For the female driver it is β. The company selects a driver at random.

    (a) What is the probability that the selected driver will make a claim this year?

    (b) What is the probability that the selected driver makes a claim in two consecutive years?