1. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let $X : \Omega \longmapsto \mathbb{R}$, be a random variable given as

$$X(\omega) = c, \quad \forall\, \omega \in \Omega$$

Find the distribution function F_X and draw its graph. Is the variable continuous or discrete?

2. A distribution function F_X is continuous at the point y if and only if $P(\{y\}) = 0$ or $P(X = y) = 0$.

3. Let X be a random variable with a finite variance and let $a \in \mathbb{R}$, then $Var(aX) = a^2 Var(X)$. Is this true? Give reasons for your answer.

4. If $a < b$, then find the following probabilities in terms of their distribution function:

   i) $P(a < x \leq b)$

   ii) $P(a \leq x \leq b)$

   iii) $P(a < x < b)$

   iv) $P(a \leq x < b)$

5. Let f_x and h_x be two probability density functions & let $\lambda \in [0, 1]$. Then $\lambda f_x + (1 - \lambda) h_x$ is a probability density. Prove this. In general $f_x h_x$ is not a probability density. Construct an example where it is.

6. Find the parameters c & d for which the following will be a density function,

$$f_X(x) = \begin{cases} cx^{-d}, & x > 1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

Compute the distribution function.

7. Check for yourself the following properties of the moment generating function

   a) $m_{X+c}(t) = e^{ct} m_X(t);\ c \in \mathbb{R}$, a constant

b) $m_{bX}(t) = m_X(bt);\ b \in \mathbb{R}$, a constant

c) $m_{\frac{X+a}{b}}(t) = e^{\frac{a}{b}t} m_X(\frac{t}{b});\ a, b \in \mathbb{R}$, constants & $b > 0$

8. Consider the following density function

$$f_X(x) = \begin{cases} \dfrac{1}{x^2}, & x \geq 1 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Show that it is a density function. Try to compute its expectation and variance and see what happens.

9. Consider $f_X(x) = e^{-x}$, if $x > 0$ & zero otherwise. Show it is a density function. Note that f_X is not continuous. But show that its distribution function is continuous.

10. Let f_X be a density which is symmetric. Show that $F_X(0) = \dfrac{1}{2}$, where F_X is the distribution function. Also show that

$$P(x \geq a) = \frac{1}{2} - \int_0^a f_X(x)dx$$

and

$$F(-a) + F(a) = 1$$

11. Let us consider the experiment of repeatedly tossing a coin. Let S_n be the number of heads that appear in n tosses. Let us assume the coin is fair. Then show that for each $\epsilon > 0$

$$\lim_{n\to\infty} P\left(\left|\frac{S_n}{n} - \frac{1}{2}\right| > \epsilon\right) = 0$$

(This is a version of the weak law of large numbers). Can you interpret what it means? If you can you will start enjoying probability theory!