Homework Assignment 1

Practice Solved Problems

1. Smith, J. M., Van Ness, H. C., & Abbott, M. M. (2005). Introduction to Chemical Engineering Thermodynamics, 7th ed.
    - Chapter 1: 1.3, 1.4
    - Chapter 2: 2.3, 2.4, 2.7, 2.8, 2.9, 2.10
    - Chapter 3: 3.1 to 3.6
2. Callen, H. B. (1998). Thermodynamics and an Introduction to Thermostatistics.
    - Chapter 1: Example 1

Homework Assignment (Due 17th Jan, 2020)

1. [10 points] You are given that following are two possible equation of state for a rubber band

    (a) $S = L_0\gamma(\theta E/L_0)^{1/2} - L_0\gamma\left[\frac{1}{2}\left(\frac{L}{L_0}\right)^2 + \frac{L_0}{L} - \frac{3}{2}\right], \qquad L_0 = nl_0$

    (b) $S = L_0\gamma e^{(\theta n E/L_0)} - L_0\gamma\left[\frac{1}{2}\left(\frac{L}{L_0}\right)^2 + \frac{L_0}{L} - \frac{3}{2}\right], \qquad L_0 = nl_0$

    where γ, l_0 and θ are constants, L is the length of the rubber band, and the other symbols have their usual meaning. Which of the two possibilities is acceptable? Why?

2. [10 points] Construct Legendre transforms of the entropy that are natural functions of $(1/T, V, n)$ and of $(1/T, V, \mu/T)$

3. [10 points] Use Maxwell's relation to prove the following:

    (a) $\left(\frac{\partial C_V}{\partial V}\right)_{T,N} = T\left(\frac{\partial^2 P}{\partial T^2}\right)$

    (b) $C_P - C_V = -T\left(\frac{\partial P}{\partial V}\right)_{T,N}\left[\left(\frac{\partial V}{\partial T}\right)_{P,N}\right]^2$

4. [30 points] Consider the fluid whose behaviour is described by the equation:

    $$p\left(\frac{V}{n} - b\right) = RT \tag{1}$$

    Find expressions for $\left(\frac{\partial S}{\partial V}\right)_T$, $\left(\frac{\partial S}{\partial P}\right)_T$, $\left(\frac{\partial U}{\partial V}\right)_T$, $\left(\frac{\partial U}{\partial P}\right)_T$, and $\left(\frac{\partial H}{\partial P}\right)_T$.
    Also find expressions for ΔU, ΔH, ΔA, ΔG, and ΔS for an isothermal change in pressure from P_1 to P_2.

5. [30 points] Prove the following using both the derivatives method and the Jacobian method:

    (a) $C_P - C_V = \frac{TV\beta^2}{\kappa_T}$

    (b) $\left(\frac{\partial H}{\partial A}\right)_P = -\frac{C_P}{S + P\left(\frac{\partial V}{\partial T}\right)_P}$

    (c) $\left(\frac{\partial G}{\partial U}\right)_S = \frac{-S\left(\frac{\partial V}{\partial T}\right)_P + V\left(\frac{\partial S}{\partial T}\right)_P}{-P\left[\left(\frac{\partial V}{\partial T}\right)_P^2 + \left(\frac{\partial V}{\partial P}\right)_T \cdot \left(\frac{\partial S}{\partial T}\right)_P\right]}$

    [HINT: You may use the Shaw's table done in the class.]

6. [20 points] You are given a rigid insulated tank of volume 0.2 m^3. It is divided into two equal compartments. One of the compartments initially contains 1 kmol of CO_2 at 500 K while the second is evacuated. Now, you remove the middle partition and let the system attain equilibrium. Determine the final temperature of CO_2 in the tank. Assume CO_2 to be a van der Waal's gas with $C_V = 37$ J/mol K. Mention the source of van der Waal's parameters.

7. [20 points] You are given that an ideal gas changes its initial state (V_1, P_1, T_1) to a final state (V_2, P_2, T_2) by following any of the three quasi-static processes as shown in the following figure: (i) 1A2; (ii) 1B2; and (iii) 1DC2. Calculate the increase in internal energy for $1 \rightarrow 2$. Also obtain the work that must be done to the system and the heat that must be added for each of the three processes. Assume specific heat is constant.