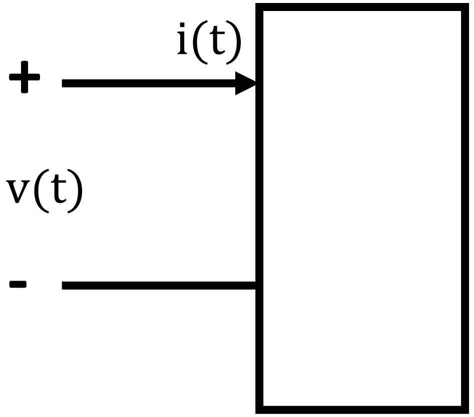
Power sources and elements

Charge, Current, Voltage, Power

q $\frac{\mathrm{d}q}{dt}$ $\mathrm{v(t)}$ $\mathrm{p}(t)$



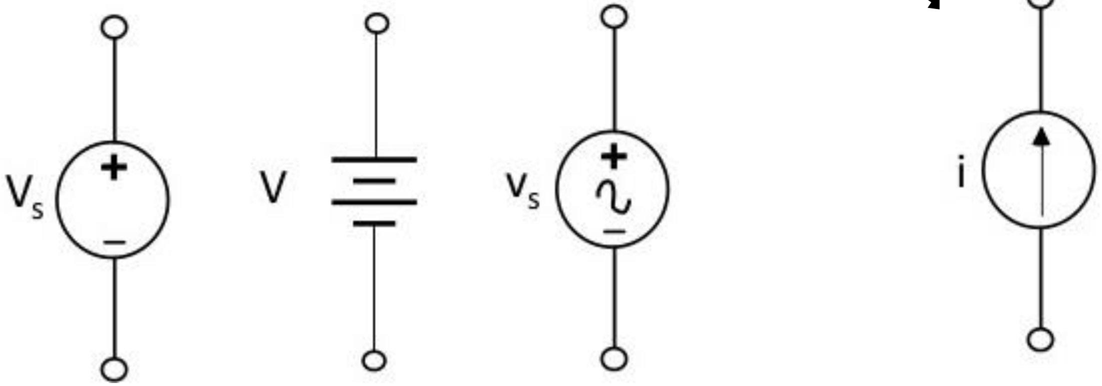
Power absorbed

$P(t) = V(t)I(t) > 0$ if either $\mathrm{V}(t), I(t) > 0$ or $\mathrm{V}(t), I(t) < 0$

Independent Sources

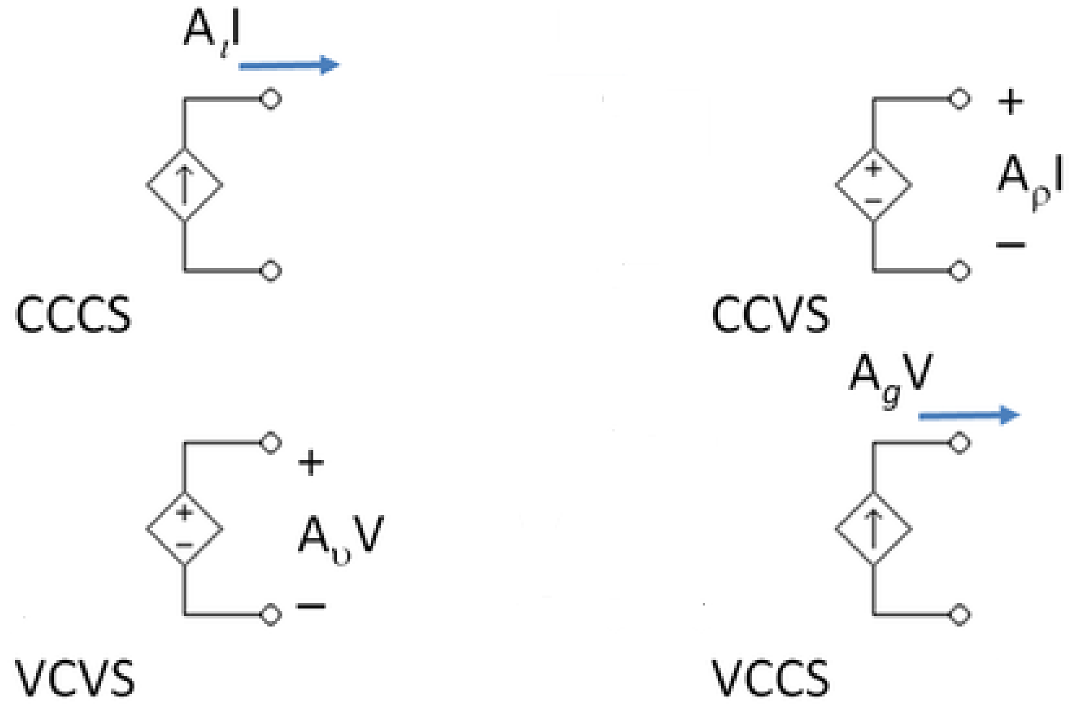
Voltage

Current



Power sources and elements

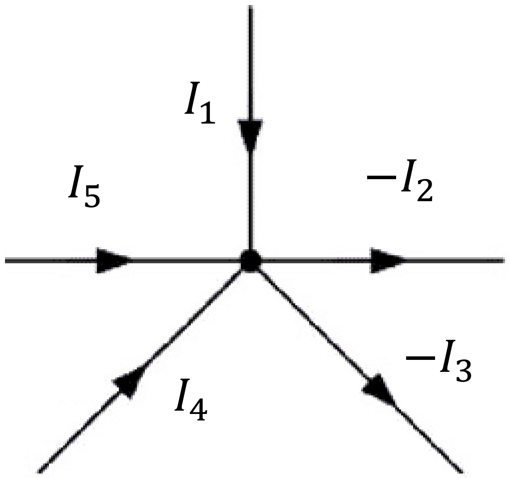
Dependent Sources:
current / voltage controlled current / voltage sources



Kirchhoff’s Laws

Kirchhoff’s current law (KCL)

- A consequence of *charge* conservation
- Node: a point where two or more elements meet

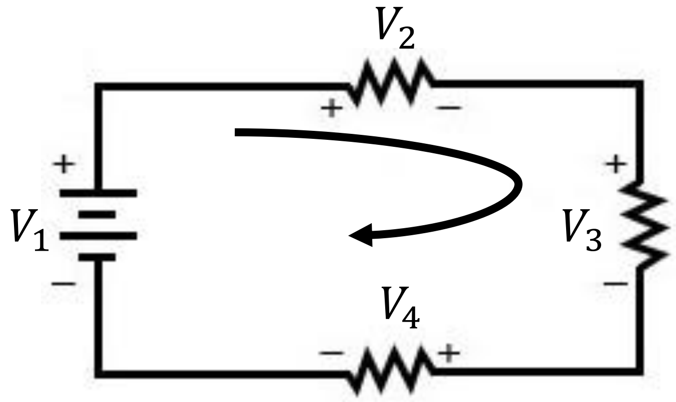


$$\sum_{j=1}^{n} I_j = 0$$

(KCL)

Kirchhoff’s voltage law (KVL)

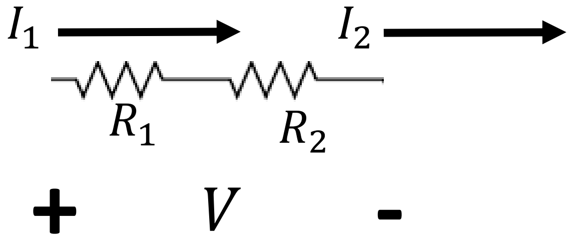
- A consequence of *energy* conservation
- Loop: a closed path starting from a particular node and ending there *without* revisiting that particular node

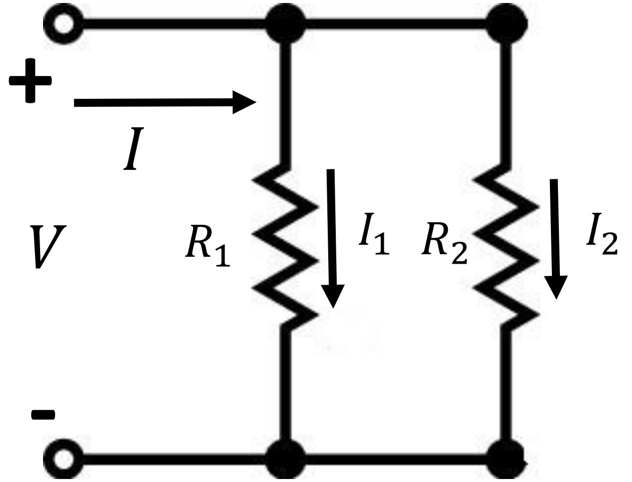


$$\sum_{i=2}^{m} V_i = V_1$$

(KVL)

Resistors in series and in parallel





$I_1 = I_2 = I(suppose)$ by KCL

$$V = I_1R_1 + I_2R_2 = I(R_1 + R_2)$$

$$\frac{V}{I} = R_1 + R_2$$

The equivalent resistance for n resistors by induction will be given by

$$R_{eq} = \sum_{i=1}^{n} R_i$$

$$V = I_1R_1 = I_2R_2$$

$$I_1 + I_2 = I$$

$$V\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = I$$

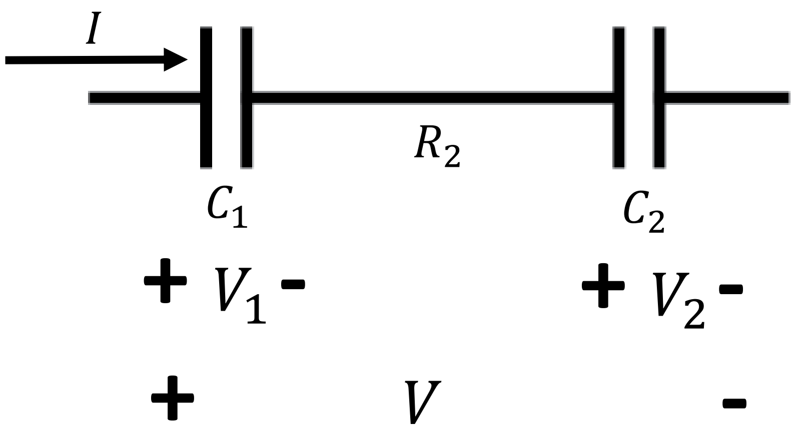
$$R_{eq} = \frac{R_1R_2}{R_1 + R_2}$$

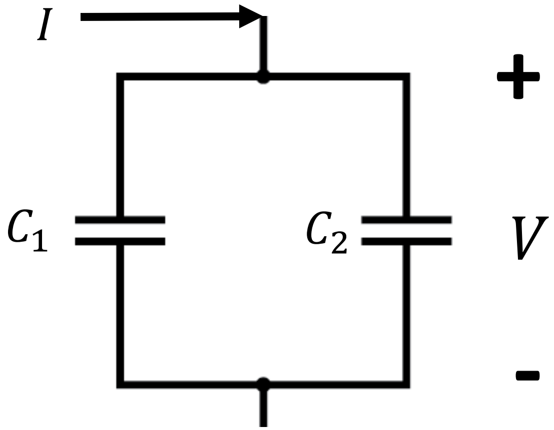
For n resistors,

$$R_{eq} = \left(\sum_{i=1}^{n} R_i^{-1}\right)^{-1}$$

A similar approach can be followed for finding equivalent Inductance in series and parallel

Capacitors in series and in parallel





$$I = C_1 \frac{dV_1}{dt} = C_2 \frac{dV_2}{dt} \quad \text{(KCL)}$$

$$I\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = \frac{d(V_1 + V_2)}{dt} = \frac{dV}{dt}$$

$$I = \frac{C_1 C_2}{C_1 + C_2} \frac{dV}{dt}$$

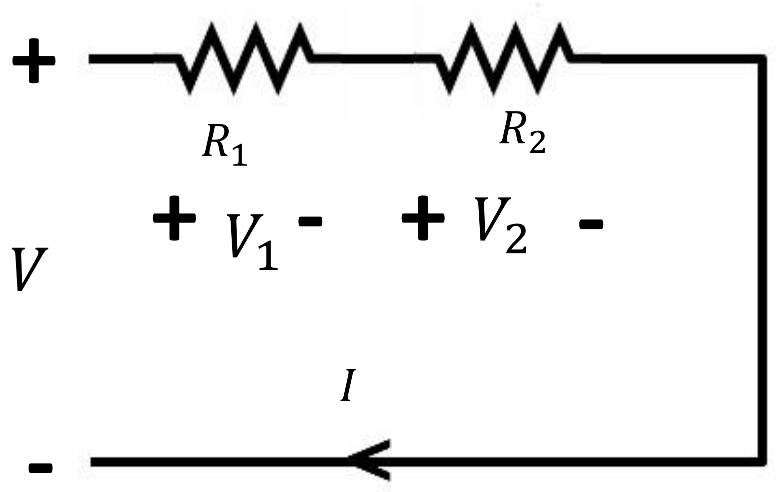
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$I = I_1 + I_2 = C_1 \frac{dV_1}{dt} + C_2 \frac{dV_2}{dt}$$

$$I = (C_1 + C_2) \frac{dV}{dt}$$

$$C_{eq} = C_1 + C_2$$

Voltage and Current division

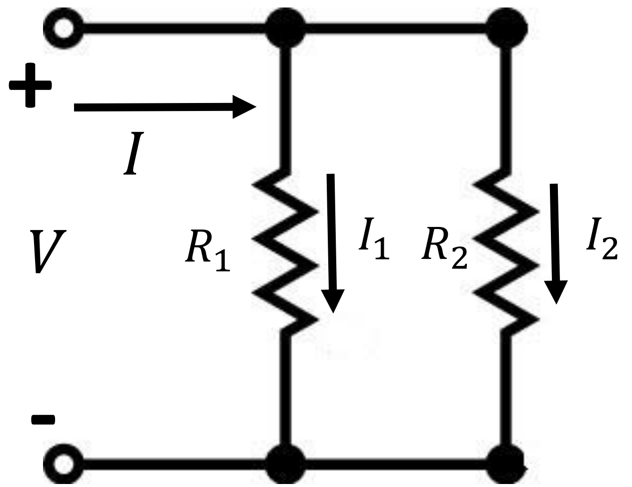


Voltage Division

$$I = \frac{V}{R_1 + R_2}$$

$$V_1 = IR_1 = \frac{R_1}{R_1 + R_2}V$$

$$V_2 = IR_2 = \frac{R_2}{R_1 + R_2}V$$



Current Division

$$V = I_1R_1 = I_2R_2$$

$$I_1 = \frac{V}{R_1}, \mathrm{I}_2 = \frac{V}{R_2}$$

$$I = V\left[\frac{1}{R_1} + \frac{1}{R_2}\right]$$

$$V = I(R_1||R_2)$$

$$I_1 = \frac{I(R_1||R_2)}{R_1} = \mathrm{I}\frac{\mathrm{R}_2}{\mathrm{R}_1 + \mathrm{R}_2}$$

$$I_2 = \frac{I(R_1||R_2)}{R_2} = \mathrm{I}\frac{\mathrm{R}_1}{\mathrm{R}_1 + \mathrm{R}_2}$$

Node and Mesh analysis

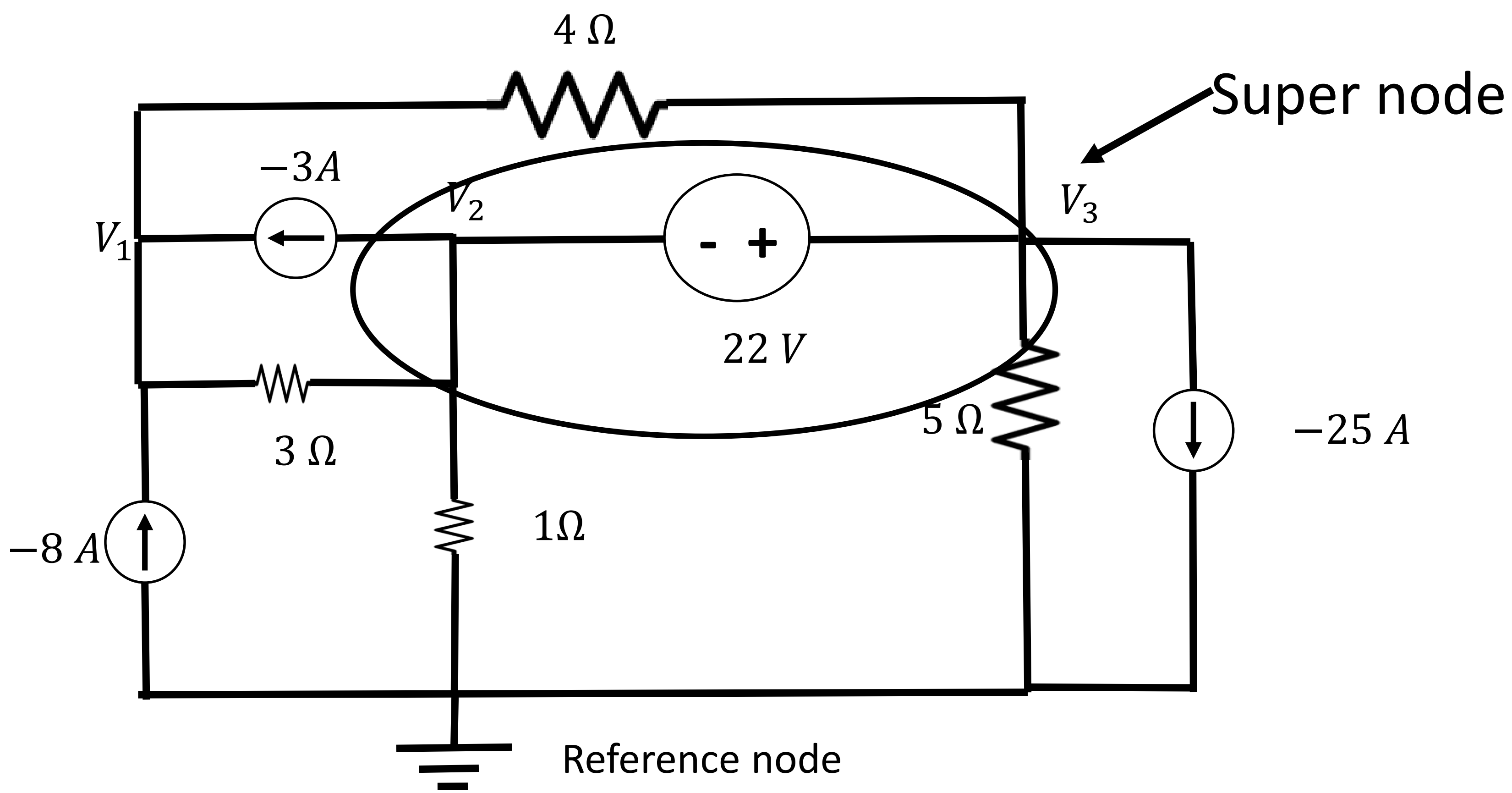
Node Analysis

- Applicable to all networks
- Choose the node with maximum number of branches as a reference node or the ground node as reference node.
- If there are N nodes, then we use one as ground and for remaining ones provide voltage as $V_1, V_2, V_3, \cdots V_{N-1}$.
- We get $N - 1$ equations using KCL
- Presence of a voltage source may force us to merge two nodes into a super node to apply KCL .

Mesh Analysis

- Applicable only to planar networks
- If current source exists then super mesh may be used so that KVL can be applied successfully or define an unknown voltage across the current sources.
- The number of equations formed are equal to number of all the closed loops present in the circuit

Node Analysis Example



Node Analysis Example

KCL at node 1: $-8-3+\frac{V_2-V_1}{3}+\frac{V_3-V_1}{4}=0$ **(1)**

KCL at 'super node'(currents entering add up to zero)

$$3+\frac{V_1-V_2}{3}-\frac{V_2}{1}-\frac{V_3}{5}+25+\frac{V_1-V_3}{4}=0 \quad \textbf{(2)}$$

$$V_2-V_3=-22 \qquad \textbf{(3)}$$

Now we have three equations.

If 22 V source is replaced by a resistor R then no need of super node and

(2) →(2') and (3) →(3')

$$3+\frac{V_1-V_2}{3}-\frac{V_2}{1}+\frac{V_3-V_2}{R}=0 \quad \textbf{(2')}$$

$$\frac{V_1-V_3}{4}+\frac{V_2-V_3}{R}-\frac{V_3}{5}+25=0 \quad \textbf{(3')}$$

Node Analysis Example

If we do not want to define a super node in case 1, then we can define current I from node 2 to node 3, then we will have

$$3+\frac{V_1-V_2}{3}-\frac{V_2}{1}-I=0 \quad \textbf{(Node 2)}$$

$$I+\frac{V_1-V_3}{4}-\frac{V_3}{5}+25=0 \qquad\qquad \textbf{(Node 3)}$$

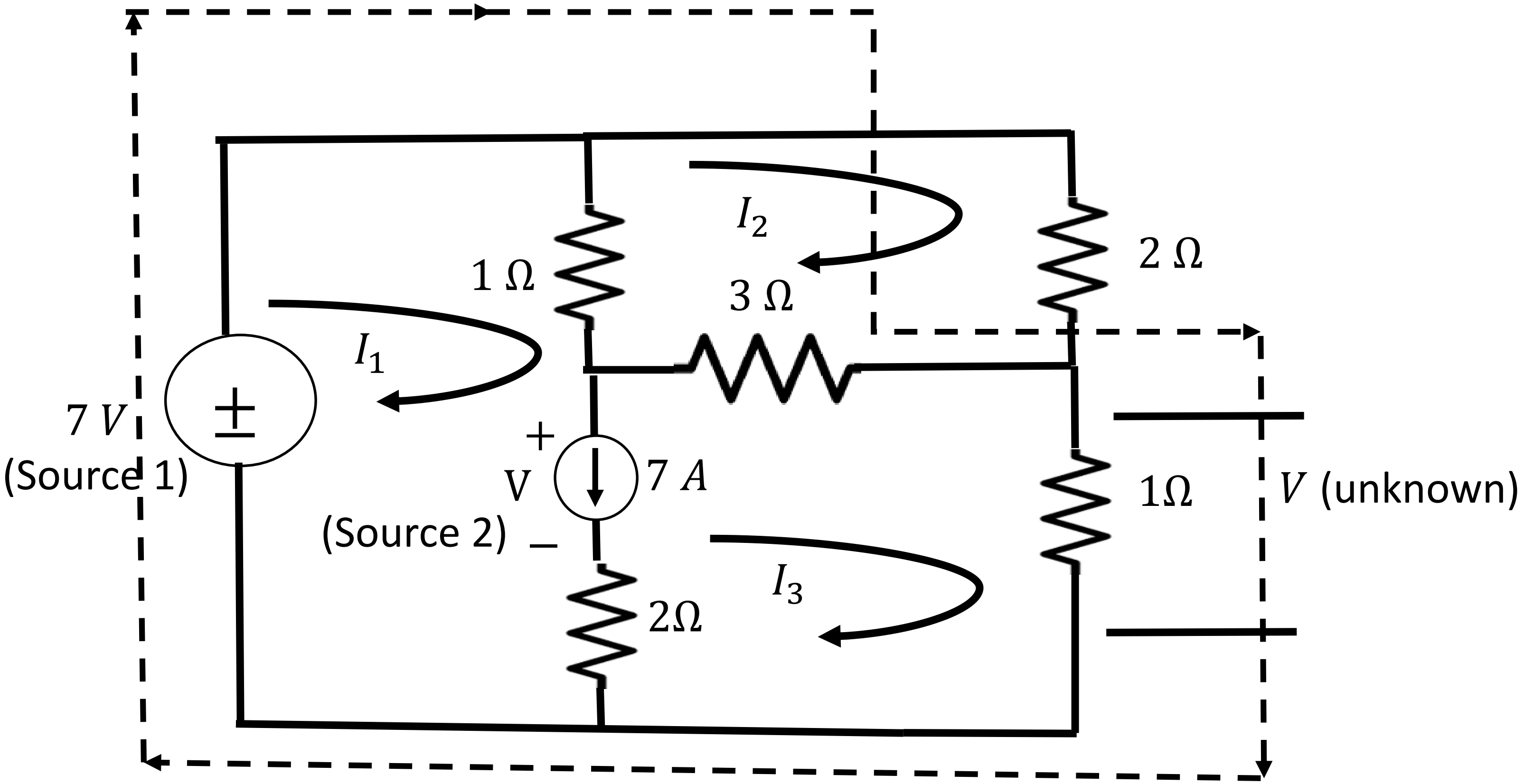
And $V_2 - V_3 = 22$

Now we have 4 variables V_1, V_2, V_3 & I with 4 equations. So there is a trade off. Supernode leads to less number of equations. Similar situation arises for a need of super mesh if ∃ a current source.

Add Node 2 and Node 3 equations

$$3+\frac{V_1-V_2}{3}+-\frac{V_1-V_3}{4}-\frac{V_2}{1}-\frac{V_3}{5}+25=0 \quad \textbf{(same as (2))}$$

Supermesh Analysis Example



Supermesh Analysis Example

Since voltage across $7\ A$ source is not clear so we need to avoid this branch.

So we take the 'union' of node 1 & 3 and then proceed.

Now, moving clockwise for left bottom corner, we get

$-7 + (I_1 - I_2).1 + (I_3 - I_2).3 + I_3.1 = 0$ **(1)** (all voltage drops adding to zero)

In mesh 2, we get

$(I_2 - I_1).1 + 2.I_2 + (I_2 - I_3).3 = 0$ **(2)**

and finally to get one more equation, we use the source which forced us to take the detour.

We have $I_1 - I_3 = 7$ **(3)**

Alternately, we can define an extra variable (voltage across the current source)

Mesh 1: $-7 + (I_1 - I_2).1 + V + 7 \times 2 = 0$ **(1')**

Mesh 2: **(2)** $\longleftarrow\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!-\!\!\longrightarrow$ **(2')**

Mesh 3: $(I_3 - I_2).3 + I_3.1 - 2 \times 7 - V = 0$ **(3')**

4th equation: $I_1 - I_3 = 7$

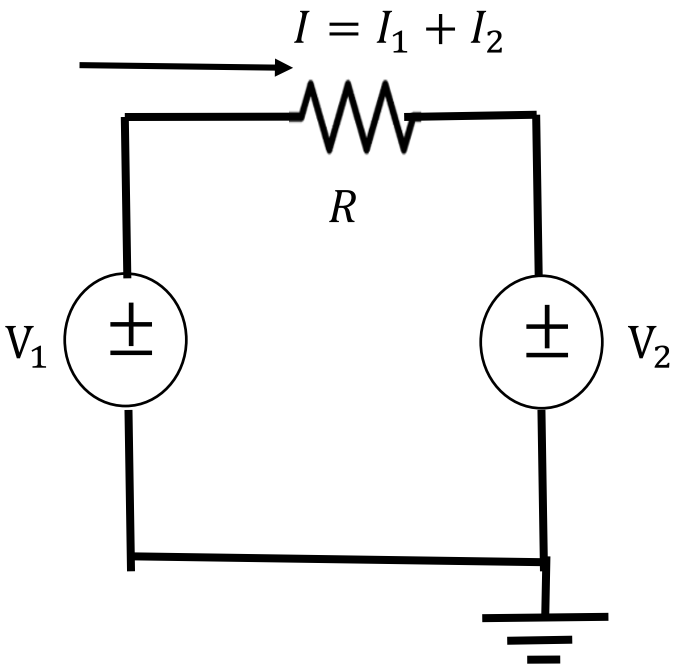
Solution:

$I_1 = 9\ A$

$I_2 = 2.5\ A$

$I_3 = 2\ A$

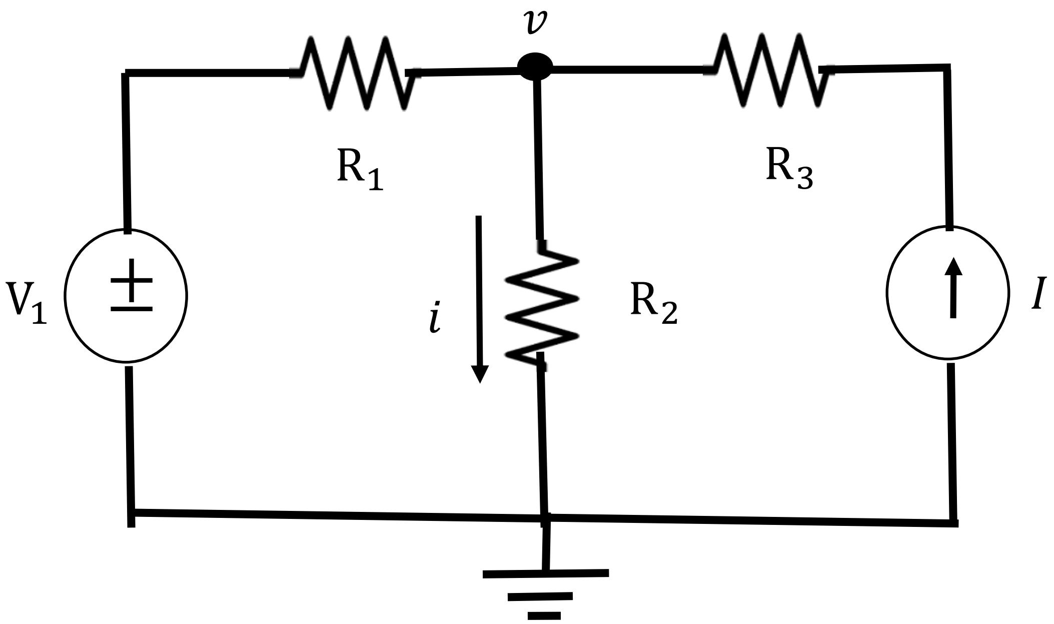
Superposition Theorem



$$I = \frac{V_1 - V_2}{R}$$

I_1(current due to V_1 alone) $= \frac{V_1}{R}$

I_2(current due to V_2 alone) $= -\frac{V_2}{R}$



To compute v and i

Superposition Theorem

$$\frac{V-v}{R_1} + I = \frac{v}{R_2} \text{ or } \frac{V}{\mathrm{R}_1} + \mathrm{I} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$v = \left(\frac{V}{R_1} + I\right)(R_1||R_2) = V.\frac{R_2}{R_1 + R_2} + I.\frac{R_1 R_2}{R_1 + R_2}$$

Now let $v_1 =$ contribution of V alone $= V.\frac{R_2}{R_1+R_2}$ (with current source opened)

$v_2 =$ contribution of I alone $= I(R_1||R_2)$ (V replaced by a short, i.e. 0 volts)

So $v = v_1 + v_2$

This property holds for all linear circuits.

For linear circuits, superposition principle states that the net current through a given branch or the voltage across a given branch is the algebraic sum of the contribution made by each individual source that acts independently.

Superposition Theorem

Examples of linear systems

$$x \xrightarrow{\text{Input}} \boxed{\text{System}} \xrightarrow{\text{Output}} y$$

$$a_1x_1 + a_2x_2 \rightarrow \mathrm{b}_1\mathrm{y}_1 + \mathrm{b}_2\mathrm{y}_2$$

The inputs and outputs could be scalar or vectors.
And vectors they could be finite dimensional or infinite dimensional.

Example 1

Consider all Riemann-integrable functions on the interval $[a, b]$, $f\colon\ [a, b] \rightarrow \Re$ (real line)

Then, $f_1 \rightarrow \int_a^b f_1(x)dx$ and $f_2 \rightarrow \int_a^b f_2(x)dx$

$$a_1f_1 + a_2f_2 \rightarrow \int_a^b (a_1f_1 + a_2f_2)dx = a_1\int_a^b f_1dx + a_2\int_a^b f_2dx$$

Example 2

$\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \Re^n$, $A\colon \Re^n \rightarrow \Re^m$

$Ax = y$ is a linear transformation

As

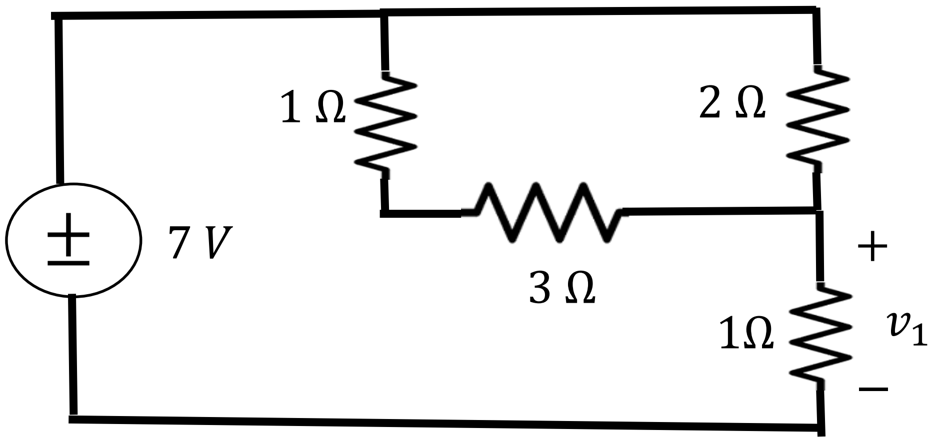
$$A(a_1x_1 + a_2x_2) = a_1Ax_1 + a_2Ax_2$$
$$= a_1y_1 + a_2y_2$$

$y(t) \triangleq \int h(t-\tau)x(\tau)d\tau$ is a mapping from $x(t) \rightarrow y(t)$

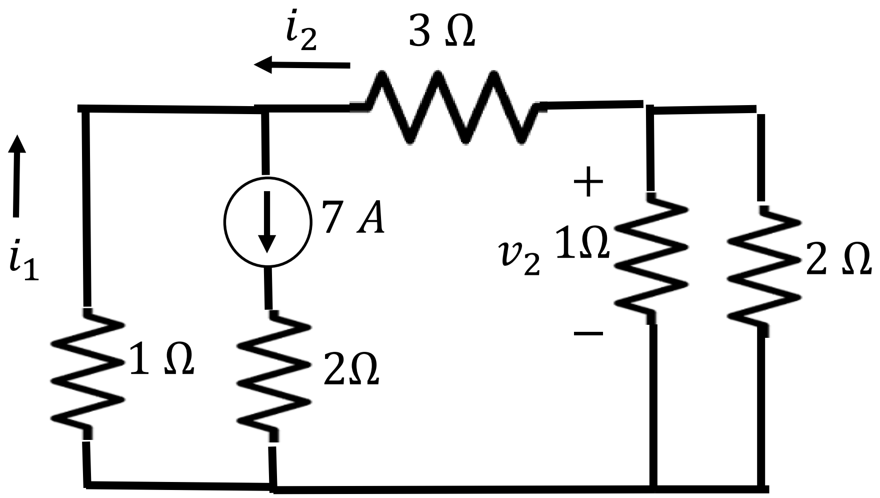
Where $h(.)$ is the so called impulse response of an LTI system.

Superposition Theorem

Let us solve the previous example of super mesh analysis through superposition theorem.



Retaining source 1



Retaining source 2

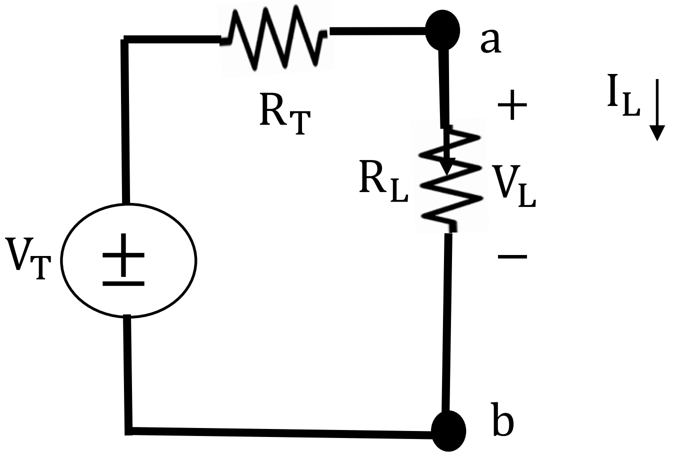
$$v_1 = 7.\left[\frac{1}{1 + (4||2)}\right] = \frac{7}{1 + 8/6} = \frac{7 \times 6}{14}$$
$$v_1 = 3 \; volts$$

$$i_1 + i_2 = 7\,A$$
$$v_2 = -(2||1)i_2 = -\frac{2}{3}i_2$$
$$i_2 = \frac{7.(1)}{1 + 3 + (1||2)} = \frac{7}{4 + 2/3} = \frac{3}{2}\,A$$
$$v_2 = -\frac{2}{3} \times \frac{3}{2} = -1\,V$$

$$v = v_1 + v_2 = 2\,V$$
Verify that $i_3 = 2\,A$

Thevenin equivalent circuit

For linear circuits where principle of superposition applies, Thevenin equivalent of a circuit between a pair of terminals is a two parameter characterization .

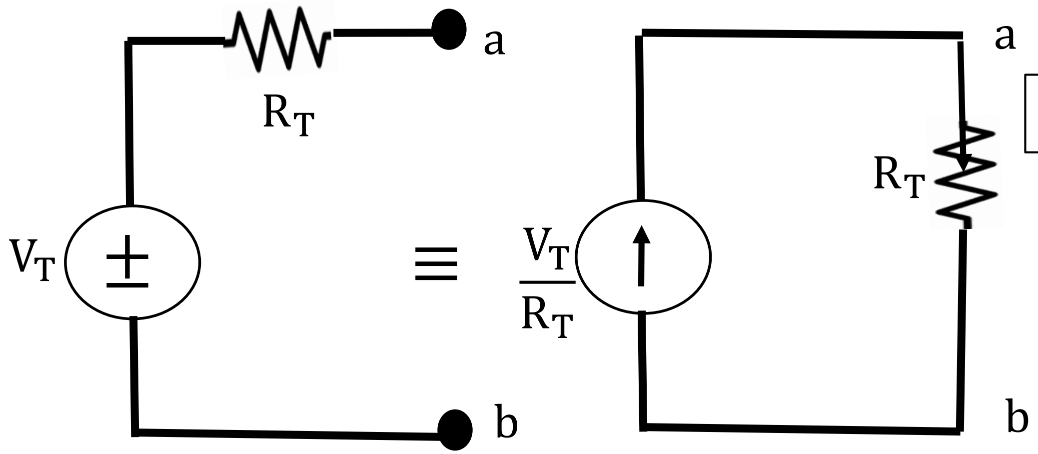


$$V_L = I_L R_L$$

$$R_L \to +\infty \Longrightarrow V_{ab} = V_{oc} = V_T \text{ (max voltage, min current)}$$

$$R_L = 0 \Longrightarrow I_{ab} = I_{sc} = I_T = \frac{V_T}{R_T} \text{ (max current, min voltage)}$$

$$R_T = \frac{V_{oc}}{I_{sc}}$$



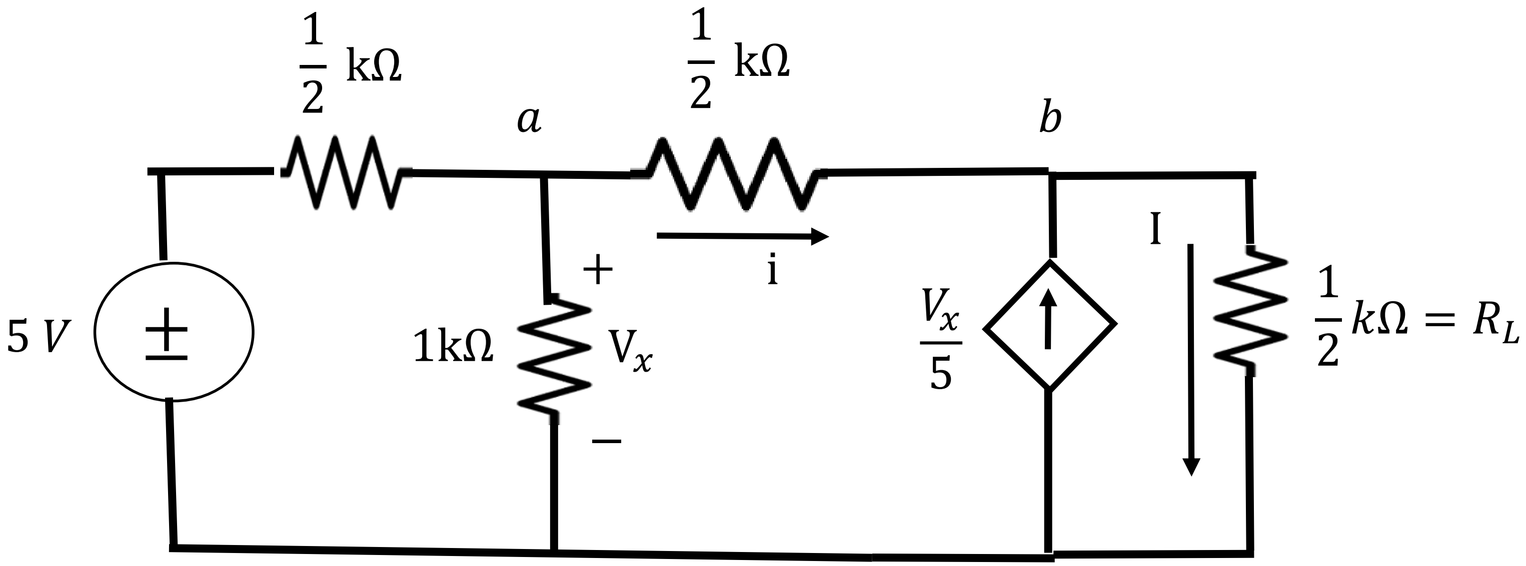
Norton’s Equivalent

Sometimes looking backwards, after suppressing all independent sources, one can easily compute R_T, generally with the help of series/parallel connections

Thevenin equivalent circuit

- Through superposition, we simplify the problem or decompose it into a set of simpler problems.
- While computing Thevenin equivalent we are using linearity to reduce the problem into two subproblems.
- These sub problems are defined by two extreme or limiting cases,
  - Computing the open circuit voltage $(\mathrm{R_L} \to +\infty)$
  - Computing the short circuit current $(\mathrm{R_L} = 0)$
- By doing so, we can focus our attention on the load itself.
- Readings
  - *Dimitri P. Betsekas* (1996), “Thevenin decomposition & large-scale optimization”, *J.O.T.A.*, Vol. 89, pp 1-15.
  - *Gorazza, G.C., Saveda C.G., & Lengo G.,* “Generalized Thevenin’s theorem for linear n-port networks”, *IEEE transactions on circuit theory,* Vol. 16, pp 564-566, 1969.

Thevenin equivalent circuit examples



To compute I, we have to compute V_T, R_T with respect to R_L

Thevenin equivalent circuit examples

For $\mathrm{V_{oc}}$, we open the resistance across $\mathrm{R_L}$ and then apply KCL at point 'a'

We have, $\frac{5-\mathrm{V_x}}{1/2} = \frac{\mathrm{V_x}}{1} - \frac{\mathrm{V_x}}{5} = \frac{4}{5}\mathrm{V_x}$

On solving, we get $5 - \mathrm{V}_x = \frac{2}{5}V_x \Longrightarrow V_x = \frac{5\times5}{7} = \frac{25}{7}$

$$\Longrightarrow V_b = V_x + \frac{V_x}{5} \times \frac{1}{2} = \frac{11}{10}V_x = \frac{11 \times 25}{10 \times 7} = \frac{55}{14} = V_{oc}$$

For $\mathrm{I_{sc}}$, we short the resistance across $\mathrm{R_L}$

We can observe that, $\mathrm{I_{sc}} = \frac{\mathrm{V_x}}{5} + \mathrm{i}$

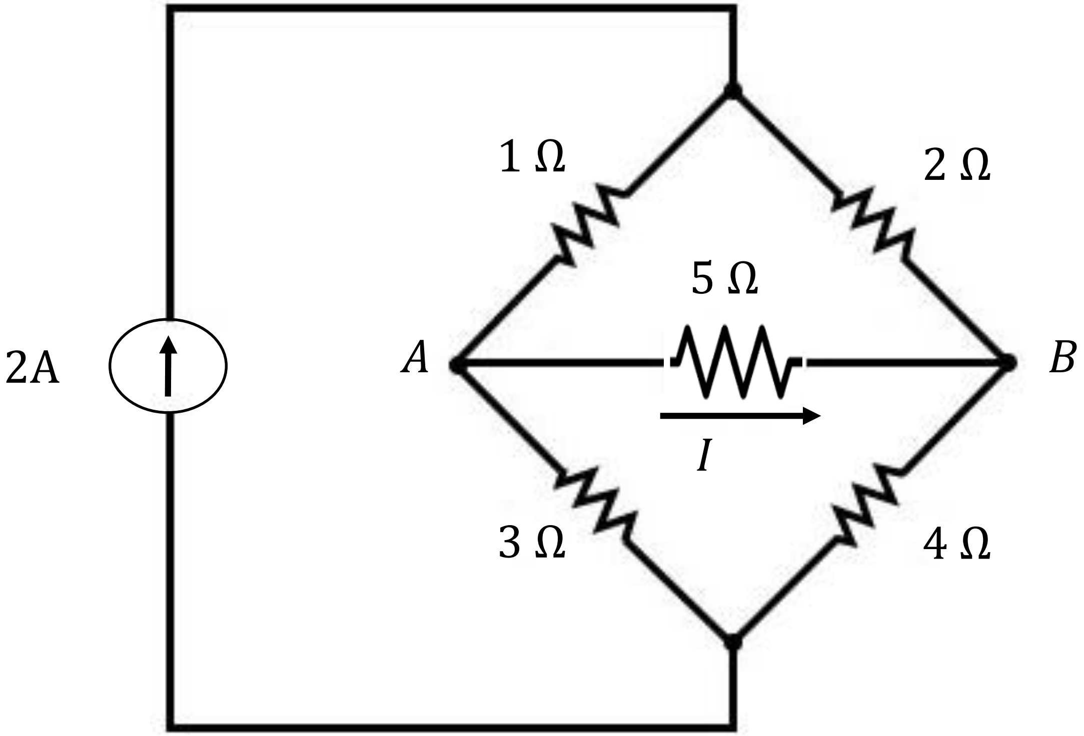
On solving, we get $\mathrm{V}_x = \frac{5\times(1||\frac{1}{2})}{\frac{1}{2}+(1||\frac{1}{2})} = \frac{5\times\frac{1}{2}}{\frac{3}{2}\times(\frac{1}{2}+\frac{1}{3})} = \frac{5}{5/2} = 2\ V$

$$\mathrm{I_{sc}} = \frac{2}{5} + \frac{2}{1/2} = \frac{22}{5}$$

The Thevenin resistance is given by, $R_T = \frac{V_{oc}}{I_{sc}} = \frac{55\times5}{14\times22} = \frac{25}{28}\ k\Omega$

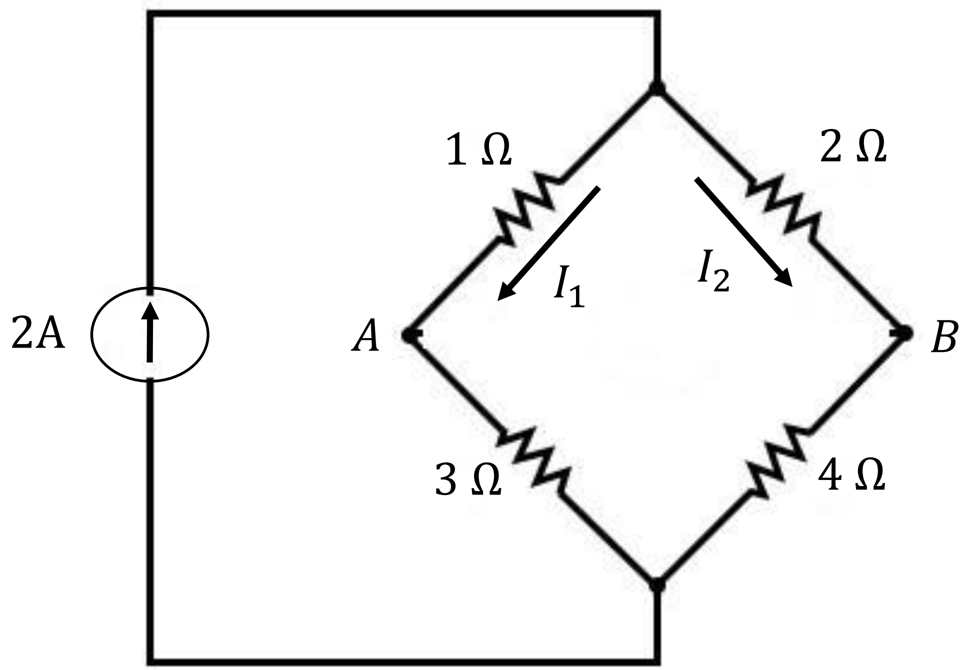
The current through $\mathrm{R_L}$, $\mathrm{I} = \frac{\mathrm{V_{oc}}}{\mathrm{R_T}+1/2} = \frac{55/14}{\frac{25}{28}+\frac{1}{2}} = \frac{55\times2}{25+14} = \frac{110}{39}\ \mathrm{mA}$

Thevenin equivalent circuit examples



Compute I using Thevenin equivalent circuit

Thevenin equivalent circuit examples



For calculating open circuit voltage $\mathrm{V_{oc}}$ or $\mathrm{V_T}$
We have to find $\mathrm{V}_{AB} = \mathrm{V_A} - \mathrm{V_B}$

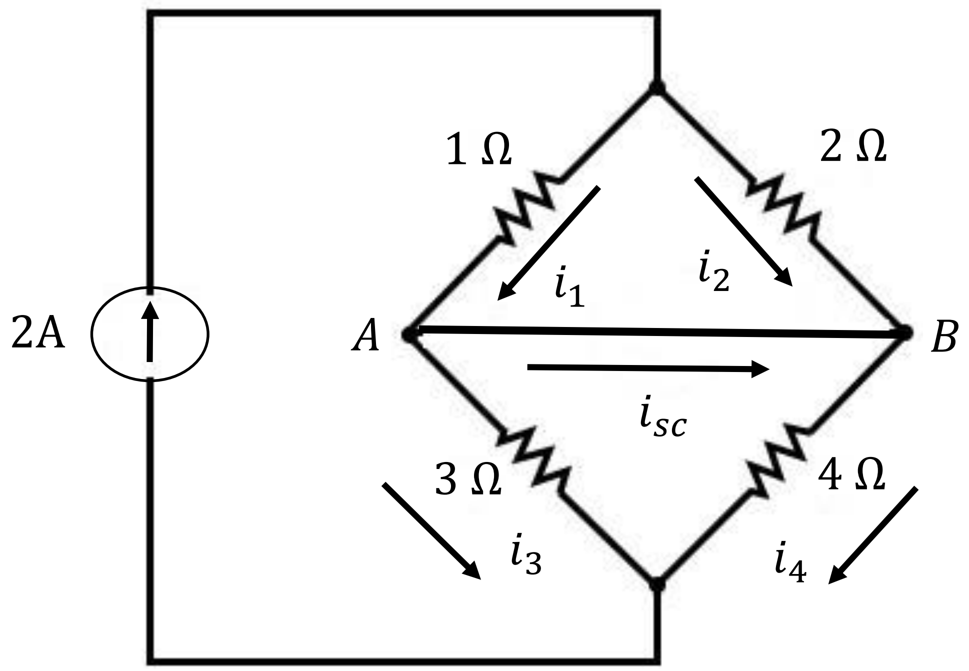
$$I_1 = \frac{2+4}{2+4+1+3} \times 2 = \frac{6}{5}\ A$$

$$I_2 = 2 - I_1 = \frac{4}{5}\ A$$

$$V_{AB} = V_A - V_B = 3I_1 - 4I_2$$

$$V_{AB} = 3.6 - 3.2 = 0.4\ V$$

Thevenin equivalent circuit examples



For calculating short circuit current I_{sc} or V_T

$i_1 = \frac{2\times 2}{2+1} = \frac{4}{3}\ A,\ i_3 = \frac{4\times 2}{3+4} = \frac{8}{7}\ A$

$$\Longrightarrow I_{sc} = \frac{4}{3} - \frac{8}{7} = \frac{4}{21}\ A$$

$\therefore$Thevenin resistance, $R_T = \frac{V_{oc}}{I_{sc}} = \frac{2}{5} \times \frac{21}{4} = \frac{42}{20} = 2.1$

We can also verify $R_T = (3+4)||(1+2) = \frac{7\times 3}{10} = 2.1\ \Omega$

$$I = \frac{0.4}{2.1+5} = 56.3\ mA$$

Thevenin equivalent circuit