

1. Let us consider a random variable  $X$  with the following p.d.f.

$$f_X(x) = \begin{cases} \frac{\theta}{x^{(\theta+1)}}, & x \geq 1 \\ 0, & \text{elsewhere} \end{cases}$$

Let  $Y = \ln X$ , i.e.  $Y = \log_e X$ . Find the p.d.f. of  $Y$ .

2. Let  $X$  be a random variable which follows the standard normal distribution i.e.  $X \sim N(0,1)$ . If  $Y = X^2$ , find the p.d.f. of  $Y$ . Use the moment generating function technique.
3. Let  $X_1$  and  $X_2$  be two independent random variables with  $X_1 \sim \text{Poisson}(\lambda)$  and  $X_2 \sim \text{Poisson}(\lambda)$ . What do you think is the distribution of  $Y = u(X_1, X_2) = X_1 + X_2$ ?
4. Let  $X$  and  $Y$  be two random variables with joint p.d.f.; which is given as

$$f_{(x,y)} = \begin{cases} 12xy(1-y), & \text{for } 0 < x < 1, \quad 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find the joint probability of the random variables given as  $U = XY^2$  &  $V = Y$ .

5. Let  $X$  be a uniformly distributed random variable, on the circumference of a circle. Then find the density function of  $Y = \sin X$ .
6. Let  $X$  be a random variable with probability density function

$$f_X(x) = \begin{cases} 6x(1-x), & \text{for } 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Use the distribution function technique to compute the p.d.f. of  $Y = X^3$ .