Example: Heat loss from ducts

Hot air at atmospheric pressure and 80°C enters an 8 m long uninsulated square duct of cross-section 0.2m x 0.2m that passes through the attic of a house at a rate of $0.15\ m^3/s$. The duct is observed to be nearly isothermal at 60°C. Determine exit temperature of the air and the rate of heat loss from the duct.

Assumptions:

- Steady state
- Surface of the duct is smooth
- Air is an ideal gas
- Air properties at 80°C are approximately the same as those at the unknown bulk average temperature

From table A-15 (Cengel) (or A.4 Incropera), at 80°C

Properties of air:

$\rho = 0.9994\ kg/m^3$ $\qquad C_p = 1008\ J/kg.K$

$k = 0.02953\ W/m.K$ $\qquad Pr = 0.7154$

$\nu = 2.097 \times 10^{-5}\ m^2/s$

$$D_h = \frac{4A_c}{P} = \frac{4a^2}{4a} = 0.2\ m$$

$$U_{avg} = \frac{\text{volumetric flow rate}}{\text{cross-sectional area}} = \frac{0.15}{(0.2)^2} = 3.75\ m/s$$

$$Re_{D_h} = \frac{U_{avg} D_h}{\nu} = 35,767 > 10,000$$

$\Rightarrow$ Flow is turbulent.

$$L_{h,turbulent} \approx L_{t,turbulent} = 10 D_h = 0.2 \times 10 = 2m$$

Length of duct is 8m. Thus, we will assume that the entire duct is under fully developed regime

Also, $T_s = 60°C$ $T_i = 80°C$

Thus, we can assume $T_m > T_s$ and hence, a cooling process. Using Dittus-Boelter equation,

$$Nu = \frac{h D_h}{k} = 0.023 Re^{0.8} Pr^{0.3} = 91.4$$

hence,

$$h = 13.5 \ W/m^2 \cdot K$$

The exit temperature for a constant surface temperature boundary condition is

$$T_e = T_s - (T_s - T_i) \exp\left(-\frac{h A_s}{\dot{m} C_p}\right)$$

with

$$A_s = A a L = 6.4 m^2$$

and

$$\dot{m} = \rho \times \text{volumetric flow rate} \approx 0.15 \ kg/s$$

thus

$$T_e = 71.3°C$$

$$\Delta T_{lm} = \frac{T_i - T_e}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = -15.2°C$$

Hence,

$$Q = h A_s \Delta T_{lm} = -1313 \ W$$

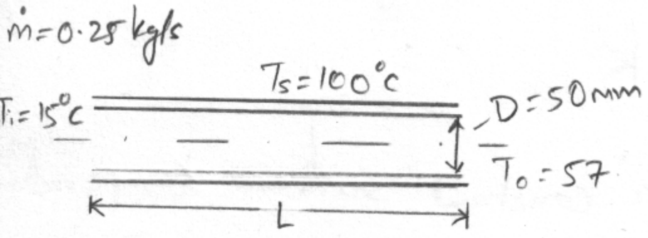
Notes: • Heat is lost by the fluid (air)

• The bulk average temperature is $\frac{71.3 + 80}{2} = 75.7°C$.

Hence, the approximate use of air properties at 80°C is justified.

Example: Heat transfer during turbulent flow of a fluid in a tube

Steam condensing on the outer surface of a thin-walled circular tube of diameter $D = 50$ mm and length $L = 6$ m maintains a uniform outer surface temperature of 100°C. Water flows through the tube at a rate of $\dot{m} = 0.25$ kg/s, and its inlet and outlet temperatures are $T_{m,i} = 15$°C and $T_{m,o} = 57$°C. What is the average convection coefficient associated with the water flow?



Assumptions:

- Negligible resistance due to the tube wall
- Negligible viscous dissipation
- Constant properties

Energy balance on the tube:

$$\dot{m} C_p (T_{m,o} - T_{m,i}) = h_{avg} A_s \Delta T_{lm} = h_{avg} \pi D L \Delta T_{lm} \cdots$$

$$\Delta T_{lm} = \frac{(T_s - T_{m,o}) - (T_s - T_{m,i})}{\ln\left[\dfrac{T_s - T_{m,o}}{T_s - T_{m,i}}\right]} = 61.6\,°C$$

$$h_{avg} = \frac{\dot{m} C_p (T_{m,o} - T_{m,i})}{\pi D L \Delta T_{lm}} = 755 \ \text{W/m}^2\cdot\text{K}$$

If the flow is assumed to be fully developed in most part of the tube, then using properties of water at 35°C (bulk avg. temp = 36°C)

$\rho = 994$ kg/m³ $\quad k = 0.623$ W/m.K $\quad \mu = 0.720 \times 10^{-3}$ kg/m.s

$Pr = 4.83$

Then,

$$Re_D = \frac{D u_{avg} \rho}{\mu} = D \frac{\dot{m}}{\rho\left(\frac{\pi D^2}{4}\right)} \frac{\rho}{\mu} = \frac{4\dot{m}}{\pi D \mu} = 8842$$

Using Dittus-Boelter equation,

$$h = \frac{k}{D} Nu = \frac{k}{D}\, 0.023\, Re^{4/5}\, Pr^{0.4} = 773 \ \text{W/m}^2\cdot\text{K}$$