Thermal Radiation

- Type of electromagnetic radiation pertinent to heat transfer
- IR + visible + UV (partly) spectrum
  (0.1 to 100 μm)
- Result of energy transitions of molecules, atoms and electrons
  at $T > 0\,K$, electrons, atoms and molecules of solids, liquids and gases are constantly in motion
  $\Rightarrow$ radiation is constantly absorbed, emitted and transmitted throughout the volume of matter.

For opaque solids, however, radiation is treated as a surface phenomenon, since any radiation emitted in the interior never reaches surface and any incident radiation is absorbed within a few microns from the surface.

Considering the radiation wavelength spectrum. Planck's law gives the spectral blackbody emissive power.

$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]} \quad W/m^2.\mu m$$

where

$$C_1 = 3.74177 \times 10^8\ W.\mu m^4/m^2$$
$$C_2 = 1.43878 \times 10^4\ \mu m\,K$$

and $T \equiv$ temperature in kelvin, $\lambda \equiv$ wavelength in μm

The total blackbody emissive power is given by

$$E_b(T) = \int_0^\infty E_{b\lambda}(\lambda, T)\, d\lambda = \sigma T^4 \quad (W/m^2)$$

— Stefan-Boltzmann law

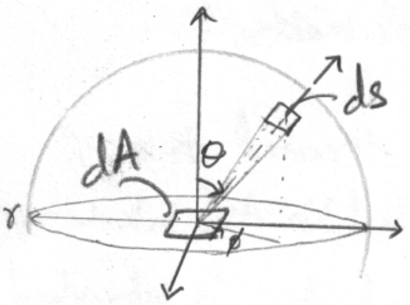
Radiation Intensity

Unlike blackbodies, real surfaces emit radiation non-uniformly. Thus, emissive power alone is not sufficient.

Solid angle

. Area of a surface on a sphere of unit radius is equivalent in magnitude to the solid angle it subtends.

· Unit: steradian



differential solid angle,

$$d\omega = \frac{dS}{r^2} = \sin\theta\, d\theta\, d\phi$$

Here, dS is normal to the direction of viewing, from the center of the sphere.

If the surface is not normal to the direction of viewing, but is at an angle α (angle between normal of the surface and viewing direction), then

$$dA_{normal} = dA \cos\alpha$$ is the projected area normal to the viewing direction

For a hemisphere, $\omega = 2\pi$ steradian (sr)

If the area dA is emitting radiation (which it does in all directions in a hemisphere), the radiation passing through the surface area dS is proportional to the solid angle $d\omega$, and also proportional to the projection of area dA in the direction of dS. The projected area varies from a maximum of dA when dS is at the top of dA, to zero when $\theta = 90°$

For an emitted radiation, thus, radiation intensity

$$I_e(\theta,\phi) = \frac{dQ_e}{dA\cos\theta\, d\omega} = \frac{dQ_e}{dA\cos\theta\sin\theta\, d\theta\, d\phi} \quad W/m^2 \cdot sr$$

Emissive power, dE is the "radiation flux", given by

$$dE = \frac{dQ_e}{dA} = I_e \cos\theta \sin\theta\, d\theta\, d\phi$$

Thus, emission power from the surface to the hemisphere surrounding it is

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi/2} d\theta\, I_e(\theta,\phi).\cos\theta\sin\theta$$

For a diffusely emitting surface, intensity of emitted radiation is independent of the direction

Thus, I_e = constant, and $E = \pi I_e$ for diffusely emitting surfaces

For a blackbody,

$$E_b = \pi I_b$$

Also,

$$E_b = \sigma T^4 \quad \text{Stefan-Boltzmann law}$$

$$\Rightarrow \quad I_b = \sigma T^4/\pi$$

Irradiation

For radiation incident on a surface dA from the direction θ, ϕ, its intensity is denoted by $I_i(\theta,\phi)$. This is the rate at which radiation energy is incident from the θ, ϕ direction per unit area of the receiving surface normal to the incident direction per unit solid angle in the incident direction.

Irradiation is the radiation flux incident from all directions onto the surface

$$G = \int_{\text{hemisphere}} dG = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \; I_i(\theta,\phi) \cos\theta \sin\theta \quad \mathrm{W/m^2}$$

For diffusely incident radiation, $G = \pi I_i$

Radiosity

Surfaces emit radiation as well as reflect radiation. Thus, total radiation leaving a surface is a sum of these, which is characterized using radiosity.

$$J = \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \; I_{e+r}(\theta,\phi) \cos\theta \sin\theta \qquad \mathrm{W/m^2}$$

For a diffuse emitter and reflector, $J = \pi I_{e+r}$

Radiation intensity, emissive power, irradiation and radiosity can also be considered to vary with wavelength. Thus, the latter three can be defined in their spectral form.

Radiative Properties

- For real surfaces, using blackbody as a reference

Emissivity : Measure of how closely a real surface is to blackbody.

spectral directional emissivity

$$\varepsilon_{\lambda,\theta}(\lambda,\theta,\phi,T) = \frac{I_{\lambda,e}(\lambda,\theta,\phi,T)}{I_{b\lambda}(\lambda,T)}$$

Blackbody is a diffusely emitting body

total directional emissivity

$$\varepsilon_{\theta}(\theta,\phi,T) = \frac{I_e(\theta,\phi,T)}{I_b(T)}$$

total hemispherical emissivity (or average emissivity)

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^\infty d\lambda\ \varepsilon_\lambda(\lambda,T)\,E_{b\lambda}(\lambda,T)}{\sigma T^4}$$

Absorptivity, Reflectivity, Transmissivity

Noting that irradiation is denoted by G,

Absorptivity $\quad \alpha = \dfrac{\text{Absorbed radiation}}{\text{Incident radiation}} = \dfrac{G_{abs}}{G}$

Reflectivity $\quad \rho = \dfrac{\text{Reflected radiation}}{\text{Incident radiation}} = \dfrac{G_{ref}}{G}$

Transmissivity $\quad \tau = \dfrac{\text{Transmitted radiation}}{\text{Incident radiation}} = \dfrac{G_{tr}}{G}$

First law of thermodynamics $\Rightarrow \quad \alpha + \rho + \tau = 1$

For blackbody $\rho = 0 \quad \tau = 0 \quad \Rightarrow \alpha = 1$

For most gases $\rho = 0 \quad \Rightarrow \alpha + \tau = 1$

For opaque surfaces $\tau = 0 \quad \Rightarrow \alpha + \rho = 1$

Note: actual reflection on a real surface is dependent on the incident radiation and direction of reflection.

Specular (mirrorlike) reflection $\Rightarrow$ angle of reflection = angle of incidence

Diffuse reflection $\Rightarrow$ radiation is equally reflected in all directions.

View Factor

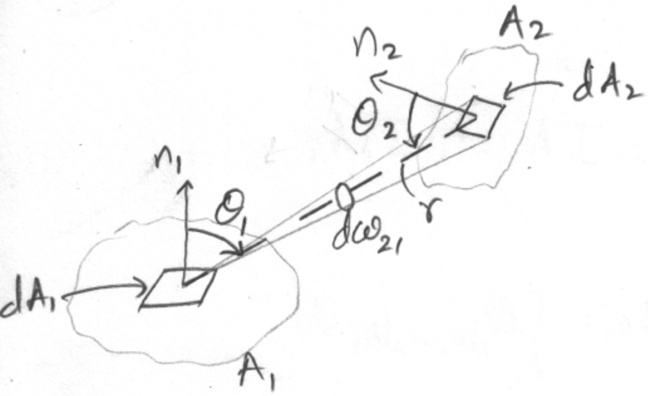
Radiation heat transfer between surfaces depends on their relative orientation.

Assuming that surfaces are diffuse emitters and diffuse reflectors, $\Rightarrow$ diffuse view factor

- also known as shape factor, configuration factor or angle factor
- purely geometric quantity
- defined as

$F_{ij} \equiv$ fraction of the radiation leaving surface i that strikes surface j directly

Note: radiation leaving surface i that is reflected from a surface k and then strikes surface j is not considered in evaluating viewfactors F_{ij}



Consider two arbitrarily oriented surfaces A_1 and A_2

Let dA_1 and $dA_2 \equiv$ differential areas on A_1 and A_2 respectively

and

$\theta_1, \theta_2 \equiv$ angle between normals of the surfaces dA_1, dA_2 and the line connecting dA_1 and dA_2

Assuming dA_1 emits and reflects radiation diffusely. with intensity I_1 and with $d\omega_{21}$ as the solid angle subtended by dA_2 when viewed from dA_1,

rate at which radiation leaves dA_1 in the direction θ_1 $= I_1 \cos\theta_1 dA_1$

$$d\omega_{21} = \frac{dA_2 \cos\theta_2}{r^2}$$

part of this radiation that strikes dA_2,

$$Q_{dA_1 \to dA_2} = I_1 \cos\theta_1 dA_1 \, d\omega_{21}$$

Now, total rate at which radiation leaves dA_1 (e+r) $Q_{dA_1} = J_1 dA_1 = \pi I_1 dA_1$

Thus, diferential view factor,

$$dF_{dA_1 \to dA_2} = \frac{Q_{dA_1 \to dA_2}}{Q_{dA_1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$

$dF_{dA_2 \to dA_1}$ can be determined similarly

Similarly,

Total rate at which radiation leaves the entire area A_1: $Q_{A_1} = J_1 A_1 = \pi I_1 A_1$

Portion of this radiation that strikes dA_2: $Q_{A_1 \to dA_2} = \int_{A_1} Q_{dA_1 \to dA_2}$

Portion that strikes A_2: $Q_{A_1 \to A_2} = \int_{A_2} \int_{A_1} Q_{dA_1 \to dA_2}$

Thus,

$$Q_{A_1 \to A_2} = \int_{A_2} \int_{A_1} \frac{I_1 \cos\theta_1 \cos\theta_2}{r^2} \, dA_1 dA_2$$

Hence,

$$F_{12} = \frac{Q_{A_1 \to A_2}}{Q_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} \, dA_1 dA_2$$

Similarly,

$$F_{21} = \frac{1}{A_2} \int_{A_1} \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} \, dA_1 dA_2$$

Since integration limits are constants, the order of integration can be readily changed. Thus,

$$F_{12} A_1 = F_{21} A_2 \qquad \text{(reciprocity relation)}$$

Table 13-1 (Incropera's) (and 13.2) or Table 13-1 (Cengel's) (and 13.2) ← 30 geometries

View Factor Relations

[1] $F_{ij} = F_{ji}$ iff $A_i = A_j$ **Reciprocity**

$F_{ij} \neq F_{ji}$ when $A_i \neq A_j$

In general, $A_i F_{ij} = A_j F_{ji}$

[2] For an enclosure with N surfaces, for any surface i such that $i \in (1, N)$, **Summation**

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad i \in (1, N)$$

For an enclosure, [1] gives $N(N-1)/2$ relations
[2] gives N relations

There are a total of N^2 view factors defined

Thus, the number of viewfactors to be evaluated reduces from N^2 to $N^2 - (N + N(N-1)/2) = N(N-1)/2$

[3] For radiation originating from one surface that strikes a composite surface (i.e. from 1 to (2,3), when 2 and 3 together make a composite surface), **Superposition**

$$F_{1,(2,3)} = F_{12} + F_{13}$$

to obtain the viewfactor from (2,3) to 1, we multiply A_1 to both sides of the above equation

$$A_1 F_{1,(2,3)} = A_1 F_{1,2} + A_1 F_{1,3}$$

Using [1],

$$(A_2 + A_3) F_{(2,3),1} = A_2 F_{2,1} + A_3 F_{3,1}$$

$$F_{(2,3),1} = \frac{A_2 F_{21} + A_3 F_{31}}{A_2 + A_3}$$

[A] If surfaces j and k are symmetric about the surface i, then

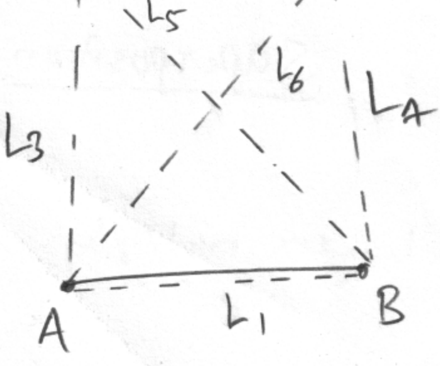
Symmetry

$$F_{ij} = F_{ik}$$

j, k are symmetric about i

Cross-strings method

- For geometries that are very long in one direction relative to other directions, and hence can be assumed infinitely long.
- Lines connecting endpoints are used to determine the view factors.



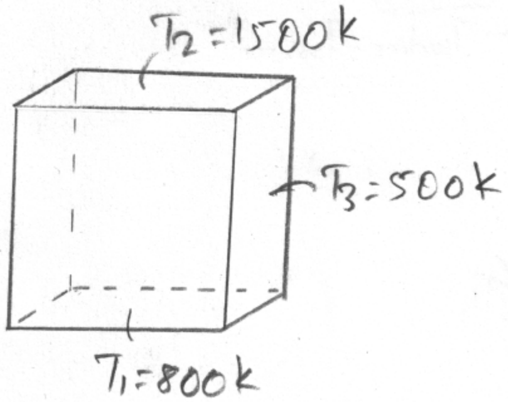
$$F_{12} = \frac{(L_5 + L_6) - (L_3 + L_A)}{2L_1}$$

i.e.

$$F_{ij} = \frac{\Sigma \text{ crossed strings} - \Sigma \text{ Uncrossed strings}}{2 \times \text{strings on surface}}$$

Example: Heat transfer by radiation

For a 5m× 5m× 5m cubical furnace has surfaces that can be approximated as blackbody. The base, top and side surfaces of the furnace are maintained at 800 k, 1500k and 500k, respectively. Determine the net rate of heat transfer between the following: base and the side surfaces, base and the top surface, and the net rate of heat transfer from the base surface, all for radiation heat transfer.



Since all sides are maintained at the same temperature, these can be considered as a single surface, to simplify the analysis.

Goal: to find

$Q_{1\to3}$, $Q_{1\to2}$ and Q_1

Since surfaces 1, 2 and 3 are blackbodies,

$$Q_{1\to3} = A_1 F_{13} \sigma (T_1^4 - T_3^4)$$

$$Q_{1\to2} = A_1 F_{12} \sigma (T_1^4 - T_2^4)$$

From figures in the book, for aligned parallel rectangles with

$\bar{X} = 1$ and $\bar{Y} = 1$, $F_{12} = 0.2$

Also, $F_{11} = 0$ (since radiation from 1 cannot strike 1)

Since 1, 2 & 3 form an enclosure,

$F_{11} + F_{12} + F_{13} = 0$ using summation rule

$\Rightarrow$ $F_{13} = 0.8$, $Q_{1\to3} = 394$ kW

$Q_{1\to2} = -1319$ kW and $Q_1 = \sum_{j=1}^{3} Q_{ij} = -925$ kW.

Net radiation heat transfer

- From a surface i

$$Q_i = \text{Radiation leaving the surface} - \text{Radiation incident on the surface}$$

$$= \underbrace{A_i J_i}_{\text{Emitted + Reflected}} - A_i G_i$$

For a gray and opaque surface, $\underbrace{\varepsilon_i = \alpha_i}_{\text{when } T_{surface} = T_{source}}$ and $\alpha_i + \rho_i = 1$

radiation properties independent of λ

$$J_i = \varepsilon_i E_{bi} + \rho_i G_i$$

$$= \varepsilon_i E_{bi} + (1-\varepsilon_i) G_i$$

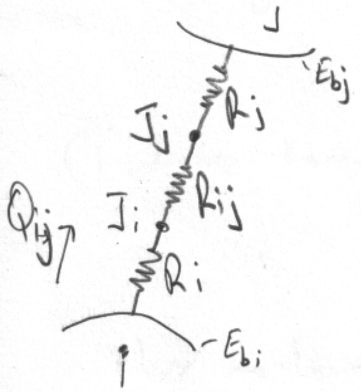
Thus;

$$Q_i = (E_{bi} - J_i)\frac{A_i \varepsilon_i}{1-\varepsilon_i} = \frac{E_{bi} - J_i}{R_i}$$

where

$$R_i \equiv \text{surface resistance} = \frac{1-\varepsilon_i}{A_i \varepsilon_i}$$

- Between any two surfaces



$$Q_{i\to j} = \left(\begin{array}{c}\text{Radiation leaving i}\\ \text{that strikes j}\end{array}\right) - \left(\begin{array}{c}\text{Radiation leaving j}\\ \text{that strikes i}\end{array}\right)$$

$$= A_i J_i F_{ij} - A_j J_j F_{ji}$$

$$= A_i F_{ij}(J_i - J_j) \quad \text{--- Using reciprocity relation}$$

$$= \frac{J_i - J_j}{R_{ij}}$$

where

$$R_{ij} \equiv \text{space resistance} = \frac{1}{A_i F_{ij}}$$

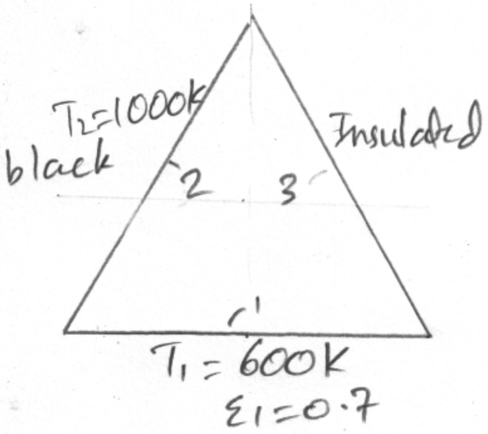
For an N-surface enclosure, the resistance analogy can be extended

$$Q_i = \sum_{j=1}^{N} \frac{J_i - J_j}{R_{ij}} \quad , \text{ noting that } Q_{ii} = 0$$

Table 13.3 (Incropera) or 13-3 (Cengel) gives heat transfer relations for two-surface enclosures (eg: parallel plates, concentric cylinders, concentric spheres).

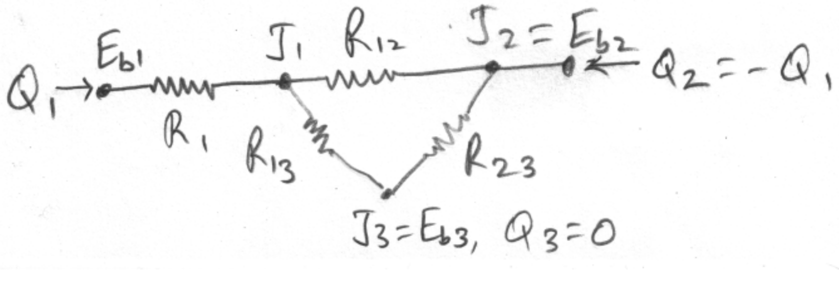
Example:

A furnace is shaped like a long equilateral triangular duct with a side of 1 m. The base surface has emissivity of 0.7 and is maintained at 600 K. The heated left side surface closely approximates a blackbody at 1000 K, while the right surface is well insulated. Determine the rate at which heat must be supplied to the heated surface externally per unit length of the duct to maintain these operating conditions.



$$Q_1 = \frac{E_{b1} - E_{b2}}{R_1 + \left(\frac{1}{R_{12}} + \frac{1}{R_{13} + R_{23}}\right)^{-1}}$$

$$= \frac{E_{b1} - E_{b2}}{\frac{1-\varepsilon_1}{A_1 \varepsilon_1} + \left(A_1 F_{12} + \frac{1}{\left(\frac{1}{A_1 F_{13}} + \frac{1}{A_2 F_{23}}\right)}\right)^{-1}}$$



$A_1 = A_2 = A_3 = w \times L = 1 m^2$ per unit length of the duct

$F_{12} = F_{13} = F_{23} = 0.5$ from table for infinitely long triangle with $w_1 = w_2 = w_3$

$E_{b1} = \sigma T_1^4 = 7348 \ W/m^2$

$E_{b2} = \sigma T_2^4 = 56{,}700 \ W/m^2$

$Q_1 = 28 \ kW$ ← rate at which heat must be supplied