SUMMARY

In this chapter, the basics of heat transfer are introduced and discussed. The science of *thermodynamics* deals with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, whereas the science of *heat transfer* deals with the rate of heat transfer, which is the main focus of interest in the design and evaluation of heat transfer equipment. The sum of all forms of energy of a system is called *total energy,* and it includes the internal, kinetic, and potential energies. The *internal energy* represents the molecular energy of a system, and it consists of sensible, latent, chemical, and nuclear forms. The sensible and latent forms of internal energy can be transferred from one medium to another as a result of a temperature difference, and are referred to as *heat* or *thermal energy.* Thus, *heat transfer* is the exchange of the sensible and latent forms of internal energy between two mediums as a result of a temperature difference. The amount of heat transferred per unit time is called *heat transfer rate* and is denoted by $\dot{Q}$. The rate of heat transfer per unit area is called *heat flux*, $\dot{q}$.

A system of fixed mass is called a *closed system,* and a system that involves mass transfer across its boundaries is called an *open system* or *control volume.* The *first law of thermodynamics* or the *energy balance* for any system undergoing any process can be expressed as

$$E_{\text{in}} - E_{\text{out}} = \Delta E_{\text{system}}$$

When a stationary closed system involves heat transfer only and no work interactions across its boundary, the energy balance relation reduces to

$$Q = mc_v\Delta T$$

where Q is the amount of net heat transfer to or from the system. When heat is transferred at a constant rate of $\dot{Q}$, the amount of heat transfer during a time interval Δt can be determined from $Q = \dot{Q}\Delta t$.

Under steady conditions and in the absence of any work interactions, the conservation of energy relation for a control volume with one inlet and one exit with negligible changes in kinetic and potential energies can be expressed as

$$\dot{Q} = \dot{m}c_p\Delta T$$

where $\dot{m} = \rho VA_c$ is the mass flow rate and $\dot{Q}$ is the rate of net heat transfer into or out of the control volume.

Heat can be transferred in three different modes: conduction, convection, and radiation. *Conduction* is the transfer of heat from the more energetic particles of a substance to the adjacent less energetic ones as a result of interactions between the particles, and is expressed by *Fourier's law of heat conduction* as

$$\dot{Q}_{\text{cond}} = -kA\frac{dT}{dx}$$

where k is the *thermal conductivity* of the material in W/m·K or Btu/h·ft·R, A is the *area* normal to the direction of heat transfer, and dT/dx is the *temperature gradient.* The magnitude of the rate of heat conduction across a plane layer of thickness L is given by

$$\dot{Q}_{\text{cond}} = kA\frac{\Delta T}{L}$$

where ΔT is the temperature difference across the layer.

Convection is the mode of heat transfer between a solid surface and the adjacent liquid or gas that is in motion, and involves the combined effects of conduction and fluid motion. The rate of convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{\text{convection}} = hA_s\,(T_s - T_\infty)$$

where h is the *convection heat transfer coefficient* in W/m²·K or Btu/h·ft²·R, A_s is the *surface area* through which convection heat transfer takes place, T_s is the *surface temperature,* and T_∞ is the *temperature of the fluid* sufficiently far from the surface.

Radiation is the energy emitted by matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules. The maximum rate of radiation that can be emitted from a surface at a thermodynamic temperature T_s is given by the *Stefan–Boltzmann law* as $\dot{Q}_{\text{emit, max}} = \sigma A_s T_s^4$, where $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴ or 0.1714×10^{-8} Btu/h·ft²·R⁴ is the *Stefan–Boltzmann constant.*

When a surface of emissivity ε and surface area A_s at a temperature T_s is completely enclosed by a much larger (or black) surface at a temperature T_{surr} separated by a gas (such as air) that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s\,(T_s^4 - T_{\text{surr}}^4)$$

In this case, the emissivity (ε) and the surface area of the surrounding surface do not have any effect on the net radiation heat transfer.

The rate at which a surface absorbs radiation is determined from $\dot{Q}_{\text{absorbed}} = \alpha\dot{Q}_{\text{incident}}$ where $\dot{Q}_{\text{incident}}$ is the rate at which radiation is incident on the surface and α is the absorptivity of the surface.

SUMMARY

In this chapter we have studied the heat conduction equation and its solutions. Heat conduction in a medium is said to be *steady* when the temperature does not vary with time and *unsteady* or *transient* when it does. Heat conduction in a medium is said to be *one-dimensional* when conduction is significant in one dimension only and negligible in the other two dimensions. It is said to be *two-dimensional* when conduction in the third dimension is negligible and *three-dimensional* when conduction in all dimensions is significant. In heat transfer analysis, the conversion of electrical, chemical, or nuclear energy into heat (or thermal) energy is characterized as *heat generation.*

The heat conduction equation can be derived by performing an energy balance on a differential volume element. The one-dimensional heat conduction equation in rectangular, cylindrical, and spherical coordinate systems for the case of constant thermal conductivities are expressed as

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\dot{e}_{\text{gen}}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

where the property $\alpha = k/\rho c$ is the *thermal diffusivity* of the material.

The solution of a heat conduction problem depends on the conditions at the surfaces, and the mathematical expressions for the thermal conditions at the boundaries are called the *boundary conditions.* The solution of transient heat conduction problems also depends on the condition of the medium at the beginning of the heat conduction process. Such a condition, which is usually specified at time $t = 0$, is called the *initial condition,* which is a mathematical expression for the temperature distribution of the medium initially. Complete mathematical description of a heat conduction problem requires the specification of two boundary conditions for each dimension along which heat conduction is significant, and an initial condition when the problem is transient. The most common boundary conditions are the *specified temperature, specified heat flux, convection,* and *radiation* boundary conditions. A boundary surface, in general, may involve specified heat flux, convection, and radiation at the same time.

For steady one-dimensional heat transfer through a plate of thickness L, the various types of boundary conditions at the surfaces at $x = 0$ and $x = L$ can be expressed as

Specified temperature:

$$T(0) = T_1 \quad \text{and} \quad T(L) = T_2$$

where T_1 and T_2 are the specified temperatures at surfaces at $x = 0$ and $x = L$.

Specified heat flux:

$$-k\frac{dT(0)}{dx} = \dot{q}_0 \quad \text{and} \quad -k\frac{dT(L)}{dx} = \dot{q}_L$$

where $\dot{q}_0$ and $\dot{q}_L$ are the specified heat fluxes at surfaces at $x = 0$ and $x = L$.

Insulation or thermal symmetry:

$$\frac{dT(0)}{dx} = 0 \quad \text{and} \quad \frac{dT(L)}{dx} = 0$$

Convection:

$$-k\frac{dT(0)}{dx} = h_1[T_{\infty 1} - T(0)] \quad \text{and} \quad -k\frac{dT(L)}{dx} = h_2[T(L) - T_{\infty 2}]$$

where h_1 and h_2 are the convection heat transfer coefficients and $T_{\infty 1}$ and $T_{\infty 2}$ are the temperatures of the surrounding mediums on the two sides of the plate.

Radiation:

$$-k\frac{dT(0)}{dx} = \varepsilon_1\sigma[T^4_{\text{surr, 1}} - T(0)^4] \quad \text{and}$$

$$-k\frac{dT(L)}{dx} = \varepsilon_2\sigma[T(L)^4 - T^4_{\text{surr, 2}}]$$

where ε_1 and ε_2 are the emissivities of the boundary surfaces, $\sigma = 5.67 \times 10^{-8}$ W/m^2·K^4 is the Stefan–Boltzmann constant, and $T_{\text{surr, 1}}$ and $T_{\text{surr, 2}}$ are the average temperatures of the surfaces surrounding the two sides of the plate. In radiation calculations, the temperatures must be in K or R.

Interface of two bodies A and B in perfect contact at $x = x_0$:

$$T_A(x_0) = T_B(x_0) \quad \text{and} \quad -k_A\frac{dT_A(x_0)}{dx} = -k_B\frac{dT_B(x_0)}{dx}$$

where k_A and k_B are the thermal conductivities of the layers A and B.

Heat generation is usually expressed *per unit volume* of the medium and is denoted by $\dot{e}_{\text{gen}}$, whose unit is W/m^3. Under steady conditions, the surface temperature T_s of a plane wall of thickness $2L$, a cylinder of outer radius r_o, and a sphere of radius r_o in which heat is generated at a constant rate of

$\dot{e}_{\text{gen}}$ per unit volume in a surrounding medium at T_∞ can be expressed as

$$T_{s,\text{ plane wall}} = T_\infty + \frac{\dot{e}_{\text{gen}} L}{h}$$

$$T_{s,\text{ cylinder}} = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{2h}$$

$$T_{s,\text{ sphere}} = T_\infty + \frac{\dot{e}_{\text{gen}} r_o}{3h}$$

where h is the convection heat transfer coefficient. The maximum temperature rise between the surface and the midsection of a medium is given by

$$\Delta T_{\text{max, plane wall}} = \frac{\dot{e}_{gen} L^2}{2k}$$

$$\Delta T_{\text{max, cylinder}} = \frac{\dot{e}_{gen} r_o^2}{4k}$$

$$\Delta T_{\text{max, sphere}} = \frac{\dot{e}_{gen} r_o^2}{6k}$$

When the variation of thermal conductivity with temperature $k(T)$ is known, the average value of the thermal conductivity in the temperature range between T_1 and T_2 can be determined from

$$k_{\text{avg}} = \frac{\int_{T_1}^{T_2} k(T)dT}{T_2 - T_1}$$

Then the rate of steady heat transfer through a plane wall, cylindrical layer, or spherical layer can be expressed as

$$\dot{Q}_{\text{plane wall}} = k_{\text{avg}} A \frac{T_1 - T_2}{L} = \frac{A}{L}\int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{cylinder}} = 2\pi k_{\text{avg}} L \frac{T_1 - T_2}{\ln(r_2/r_1)} = \frac{2\pi L}{\ln(r_2/r_1)}\int_{T_2}^{T_1} k(T)dT$$

$$\dot{Q}_{\text{sphere}} = 4\pi k_{\text{avg}} r_1 r_2 \frac{T_1 - T_2}{r_2 - r_1} = \frac{4\pi r_1 r_2}{r_2 - r_1}\int_{T_2}^{T_1} k(T)dT$$

The variation of thermal conductivity of a material with temperature can often be approximated as a linear function and expressed as

$$k(T) = k_0(1 + \beta T)$$

where β is called the *temperature coefficient of thermal conductivity.*

REFERENCES AND SUGGESTED READING

1. W. E. Boyce and R. C. Diprima. *Elementary Differential Equations and Boundary Value Problems.* 4th ed. New York: John Wiley & Sons, 1986.

2. S. S. Kutateladze. *Fundamentals of Heat Transfer.* New York: Academic Press, 1963.

PROBLEMS*

Introduction

2–1C How does transient heat transfer differ from steady heat transfer? How does one-dimensional heat transfer differ from two-dimensional heat transfer?

2–2C Is heat transfer a scalar or vector quantity? Explain. Answer the same question for temperature.

2–3C Does a heat flux vector at a point P on an isothermal surface of a medium have to be perpendicular to the surface at that point? Explain.

*Problems designated by a "C" are concept questions, and students are encouraged to answer them all. Problems designated by an "E" are in English units, and the SI users can ignore them. Problems with the icon are solved using EES, and the complete solutions together with parametric studies are included on the text website. Problems with the icon are comprehensive in nature, and are intended to be solved with an equation solver such as EES. Problems with the icon are Prevention through Design problems.

2–4C From a heat transfer point of view, what is the difference between isotropic and anisotropic materials?

2–5C What is heat generation in a solid? Give examples.

2–6C Heat generation is also referred to as energy generation or thermal energy generation. What do you think of these phrases?

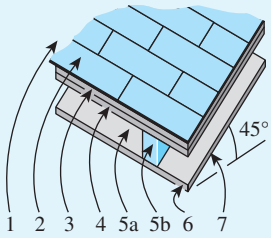
2–7C In order to size the compressor of a new refrigerator, it is desired to determine the rate of heat transfer from the kitchen air into the refrigerated space through the walls, door, and the top and bottom section of the refrigerator. In your analysis, would you treat this as a transient or steady-state heat transfer problem? Also, would you consider the heat transfer to be one-dimensional or multidimensional? Explain.

2–8C In order to determine the size of the heating element of a new oven, it is desired to determine the rate of heat loss through the walls, door, and the top and bottom section of the oven. In your analysis, would you consider this to be a steady

Properties The *R*-values of different materials are given in Table 3–8.

Analysis The schematic of the pitched roof as well as the different elements used in its construction are shown below. Following the approach described above and using the available *R*-values from Table 3–8, the overall *R*-value of the roof can be determined in the table here.

Schematic



	Construction	*R*-value, h·ft²·°F/Btu: Between Studs	At Studs
1.	Outside surface, 15 mph wind	0.17	0.17
2.	Asphalt shingle roofing	0.44	0.44
3.	Building paper	0.06	0.06
4.	Plywood deck, $\frac{5}{8}$ in	0.78	0.78
5a.	Nonreflective air space, 3.5 in	0.86	—
5b.	Wood stud, 2 in × 4 in	—	3.58
6.	Gypsum wallboard, 0.5 in	0.45	0.45
7.	Inside surface, 45° slope, still air	0.63	0.63
	Total unit thermal resistance of each section, *R*	3.39	6.11
	The *U*-factor of each section, $U = 1/R$, in Btu/h·ft²·°F	0.292	0.163
	Area fraction of each section, f_{area}	0.75	0.25

Overall *U*-factor: $U = \Sigma f_{area,\,i}\, U_i = 0.75 \times 0.292 + 0.25 \times 0.163$
$= \mathbf{0.260\ Btu/h\cdot ft^2\cdot °F}$

Overall unit thermal resistance: $R = 1/U = \mathbf{3.85\ h\cdot ft^2\cdot °F/Btu}$

Therefore, the overall unit thermal resistance of this pitched roof is 3.85 h·ft²·°F/Btu and the overall *U*-factor is 0.260 Btu/h·ft²·°F. Note that the wood studs offer much larger thermal resistance to heat flow than the air space between the studs.

SUMMARY

One-dimensional heat transfer through a simple or composite body exposed to convection from both sides to mediums at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ can be expressed as

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

where R_{total} is the total thermal resistance between the two mediums. For a plane wall exposed to convection on both sides, the total resistance is expressed as

$$R_{total} = R_{conv,\,1} + R_{wall} + R_{conv,\,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

This relation can be extended to plane walls that consist of two or more layers by adding an additional resistance for each additional layer. The elementary thermal resistance relations can be expressed as follows:

Conduction resistance (plane wall): $R_{wall} = \dfrac{L}{kA}$

Conduction resistance (cylinder): $R_{cyl} = \dfrac{\ln(r_2/r_1)}{2\pi Lk}$

Conduction resistance (sphere): $R_{sph} = \dfrac{r_2 - r_1}{4\pi r_1 r_2 k}$

Convection resistance: $R_{conv} = \dfrac{1}{hA}$

Interface resistance: $R_{interface} = \dfrac{1}{h_c A} = \dfrac{R_c}{A}$

Radiation resistance: $R_{rad} = \dfrac{1}{h_{rad} A}$

where h_c is the thermal contact conductance, R_c is the thermal contact resistance, and the radiation heat transfer coefficient is defined as

$$h_{\text{rad}} = \varepsilon\sigma(T_s^2 + T_{\text{surr}}^2)(T_s + T_{\text{surr}})$$

Once the rate of heat transfer is available, the *temperature drop* across any layer can be determined from

$$\Delta T = \dot{Q}R$$

The thermal resistance concept can also be used to solve steady heat transfer problems involving parallel layers or combined series-parallel arrangements.

Adding insulation to a cylindrical pipe or a spherical shell increases the rate of heat transfer if the outer radius of the insulation is less than the *critical radius of insulation,* defined as

$$r_{\text{cr, cylinder}} = \frac{k_{\text{ins}}}{h}$$

$$r_{\text{cr, sphere}} = \frac{2k_{\text{ins}}}{h}$$

The effectiveness of an insulation is often given in terms of its *R-value,* the thermal resistance of the material for a unit surface area, expressed as

$$R\text{-value} = \frac{L}{k} \qquad \text{(flat insulation)}$$

where L is the thickness and k is the thermal conductivity of the material.

Finned surfaces are commonly used in practice to enhance heat transfer. Fins enhance heat transfer from a surface by exposing a larger surface area to convection. The temperature distribution along the fin are given by

Very long fin:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-x\sqrt{hp/kA_c}}$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x)}{\cosh mL}$$

Specified temperature at fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{[(T_L - T_\infty)/(T_b - T_\infty)]\sinh mx + \sinh m(L - x)}{\sinh mL}$$

Convection from fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L - x) + (h/mk)\sinh m(L - x)}{\cosh mL + (h/mk)\sinh mL}$$

where $m = \sqrt{hp/kA_c}$, p is the perimeter, and A_c is the cross-sectional area of the fin. The rates of heat transfer for these cases are given to be

Very long fin:

$$\dot{Q}_{\text{long fin}} = -kA_c\left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)$$

Adiabatic fin tip:

$$\dot{Q}_{\text{adiabatic tip}} = -kA_c\left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh mL$$

Specified temperature at fin tip:

$$\dot{Q}_{\text{specified temp.}} = \sqrt{hpkA_c}(T_b - T_\infty)\frac{\cosh mL - [(T_L - T_\infty)/(T_b - T_\infty)]}{\sinh mL}$$

Convection from the fin tip:

$$\dot{Q}_{\text{convection}} = \sqrt{hpkA_c}(T_b - T_\infty)\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

Fins exposed to convection at their tips can be treated as fins with adiabatic tips by using the corrected length $L_c = L + A_c/p$ instead of the actual fin length.

The temperature of a fin drops along the fin, and thus the heat transfer from the fin is less because of the decreasing temperature difference toward the fin tip. To account for the effect of this decrease in temperature on heat transfer, we define *fin efficiency* as

$$\eta_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{fin, max}}} = \frac{\text{Actual heat transfer rate from the fin}}{\text{Ideal heat transfer rate from the fin if the entire fin were at base temperature}}$$

When the fin efficiency is available, the rate of heat transfer from a fin can be determined from

$$\dot{Q}_{\text{fin}} = \eta_{\text{fin}}\dot{Q}_{\text{fin, max}} = \eta_{\text{fin}}hA_{\text{fin}}\,(T_b - T_\infty)$$

The performance of the fins is judged on the basis of the enhancement in heat transfer relative to the no-fin case and is expressed in terms of the *fin effectiveness* ε_{fin}, defined as

$$\varepsilon_{\text{fin}} = \frac{\dot{Q}_{\text{fin}}}{\dot{Q}_{\text{no fin}}} = \frac{\dot{Q}_{\text{fin}}}{hA_b\,(T_b - T_\infty)} = \frac{\text{Heat transfer rate from the fin of } \textit{base area } A_b}{\text{Heat transfer rate from the surface of } \textit{area } A_b}$$

Here, A_b is the cross-sectional area of the fin at the base and $\dot{Q}_{\text{no fin}}$ represents the rate of heat transfer from this area if no fins are attached to the surface. The *overall effectiveness* for a finned surface is defined as the ratio of the total heat transfer from the finned surface to the heat transfer from the same surface if there were no fins,

$$\varepsilon_{\text{fin, overall}} = \frac{\dot{Q}_{\text{total, fin}}}{\dot{Q}_{\text{total, no fin}}} = \frac{h(A_{\text{unfin}} + \eta_{\text{fin}}A_{\text{fin}})(T_b - T_\infty)}{hA_{\text{no fin}}\,(T_b - T_\infty)}$$

Fin efficiency and fin effectiveness are related to each other by

$$\varepsilon_{\text{fin}} = \frac{A_{\text{fin}}}{A_b}\,\eta_{\text{fin}}$$

Certain multidimensional heat transfer problems involve two surfaces maintained at constant temperatures T_1 and T_2. The steady rate of heat transfer between these two surfaces is expressed as

$$\dot{Q} = Sk(T_1 - T_2)$$

where S is the *conduction shape factor* that has the dimension of *length* and k is the thermal conductivity of the medium between the surfaces.

(*b*) Heat is transferred to air at the rate determined above, and the temperature of the air rises from −2°C to 0.7°C as a result. Therefore, the mass flow rate of air is

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{air}}}{(c_p \Delta T_{\text{air}})} = \frac{277 \text{ kW}}{(1.006 \text{ kJ/kg·K})[0.7 - (-2)^\circ\text{C}]} = 102.0 \text{ kg/s}$$

Then the volume flow rate of air becomes

$$\dot{V}_{\text{air}} = \frac{\dot{m}_{\text{air}}}{\rho_{\text{air}}} = \frac{102 \text{ kg/s}}{1.292 \text{ kg/m}^3} = \mathbf{78.9 \text{ m}^3\text{/s}}$$

(*c*) Normally the heat transfer load of the evaporator is the same as the refrigeration load. But in this case the water that enters the evaporator as a liquid is frozen as the temperature drops to −2°C, and the evaporator must also remove the latent heat of freezing, which is determined from

$$\dot{Q}_{\text{freezing}} = (\dot{m} h_{\text{latent}})_{\text{water}} = (0.080 \text{ kg/s})(333.7 \text{ kJ/kg}) = 27 \text{ kW}$$

Therefore, the total rate of heat removal at the evaporator is

$$\dot{Q}_{\text{evaporator}} = \dot{Q}_{\text{total, chill room}} + \dot{Q}_{\text{freezing}} = 277 + 27 = 304 \text{ kW}$$

Then the heat transfer surface area of the evaporator on the air side is determined from $\dot{Q}_{\text{evaporator}} = (UA)_{\text{air side}} \Delta T$,

$$A = \frac{\dot{Q}_{\text{evaporator}}}{U \Delta T} = \frac{304{,}000 \text{ W}}{(20 \text{ W/m}^2\text{·K})(5.5^\circ\text{C})} = \mathbf{2764 \text{ m}^2}$$

Obviously, a finned surface must be used to provide such a large surface area on the air side.

SUMMARY

In this chapter, we considered the variation of temperature with time as well as position in one- or multidimensional systems. We first considered the *lumped systems* in which the temperature varies with time but remains uniform throughout the system at any time. The temperature of a lumped body of arbitrary shape of mass m, volume V, surface area A_s, density ρ, and specific heat c_p initially at a uniform temperature T_i that is exposed to convection at time $t = 0$ in a medium at temperature T_∞ with a heat transfer coefficient h is expressed as

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

where

$$b = \frac{hA_s}{\rho c_p V} = \frac{h}{\rho c_p L_c}$$

is a positive quantity whose dimension is (time)$^{-1}$. This relation can be used to determine the temperature $T(t)$ of a body at time t or, alternatively, the time t required for the temperature to reach a specified value $T(t)$. Once the temperature $T(t)$ at time t is available, the *rate* of convection heat transfer between the body and its environment at that time can be determined from Newton's law of cooling as

$$\dot{Q}(t) = hA_s[T(t) - T_\infty]$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval $t = 0$ to t is simply the change in the energy content of the body,

$$Q = mc_p[T(t) - T_i]$$

The *maximum* heat transfer between the body and its surroundings is

$$Q_{\max} = mc_p(T_\infty - T_i)$$

The error involved in lumped system analysis is negligible when

$$\text{Bi} = \frac{hL_c}{k} < 0.1$$

where Bi is the *Biot number* and $L_c = V/A_s$ is the *characteristic length.*

When the lumped system analysis is not applicable, the variation of temperature with position as well as time can be determined using the *transient temperature charts* given in Figs. 4–17, 4–18, 4–19, and 4–31 for a large plane wall, a long cylinder, a sphere, and a semi-infinite medium, respectively. These charts are applicable for one-dimensional heat transfer in those geometries. Therefore, their use is limited to situations in which the body is initially at a uniform temperature, all surfaces are subjected to the same thermal conditions, and the body does not involve any heat generation. These charts can also be used to determine the total heat transfer from the body up to a specified time t.

Using the *one-term approximation,* the solutions of one-dimensional transient heat conduction problems are expressed analytically as

Plane wall: $$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L)$$

Cylinder: $$\theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o)$$

Sphere: $$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$

where the constants A_1 and λ_1 are functions of the Bi number only, and their values are listed in Table 4–2 against the Bi number for all three geometries. The error involved in one-term solutions is less than 2 percent when $\tau > 0.2$.

Using the one-term solutions, the fractional heat transfers in different geometries are expressed as

Plane wall: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{wall}} = 1 - \theta_{0,\text{ wall}} \frac{\sin\lambda_1}{\lambda_1}$$

Cylinder: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{cyl}} = 1 - 2\theta_{0,\text{ cyl}} \frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere: $$\left(\frac{Q}{Q_{\max}}\right)_{\text{sph}} = 1 - 3\theta_{0,\text{ sph}} \frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3}$$

The solutions of transient heat conduction in a semi-infinite solid with constant properties under various boundary conditions at the surface are given as follows:

Specified Surface Temperature, T_s = constant:

$$\frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \quad \text{and} \quad \dot{q}_s(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}}$$

Specified Surface Heat Flux, $\dot{q}_s$ = constant:

$$T(x,t) - T_i = \frac{\dot{q}_s}{k}\left[\sqrt{\frac{4\alpha t}{\pi}}\exp\left(-\frac{x^2}{4\alpha t}\right) - x\,\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)\right]$$

Convection on the Surface, $\dot{q}_s(t) = h[T_\infty - T(0,t)]$:

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right) \times \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)$$

Energy Pulse at Surface, e_s = constant:

$$T(x,t) - T_i = \frac{e_s}{k\sqrt{\pi t/\alpha}}\exp\left(-\frac{x^2}{4\alpha t}\right)$$

where $\text{erfc}(\eta)$ is the *complementary error function* of argument η.

Using a superposition principle called the *product solution* these charts can also be used to construct solutions for the *two-dimensional* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, or a semi-infinite cylinder or plate, and even *three-dimensional* problems associated with geometries such as a rectangular prism or a semi-infinite rectangular bar, provided that all surfaces of the solid are subjected to convection to the same fluid at temperature T_∞, with the same convection heat transfer coefficient h, and the body involves no heat generation. The solution in such multidimensional geometries can be expressed as the product of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.

The total heat transfer to or from a multidimensional geometry can also be determined by using the one-dimensional values. The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is given by

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3\left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]$$

Knowing that the global discretization error is proportional to the step size is not much help either since there is no easy way of determining the value of the proportionality constant. Besides, the global discretization error alone is meaningless without a true estimate of the round-off error. Therefore, we recommend the following practical procedures to assess the accuracy of the results obtained by a numerical method.

- Start the calculations with a reasonable mesh size Δx (and time step size Δt for transient problems) based on experience. Then repeat the calculations using a mesh size of $\Delta x/2$. If the results obtained by halving the mesh size do not differ significantly from the results obtained with the full mesh size, we conclude that the discretization error is at an acceptable level. But if the difference is larger than we can accept, then we have to repeat the calculations using a mesh size $\Delta x/4$ or even a smaller one at regions of high temperature gradients. We continue in this manner until halving the mesh size does not cause any significant change in the results, which indicates that the discretization error is reduced to an acceptable level.
- Repeat the calculations using double precision holding the mesh size (and the size of the time step in transient problems) constant. If the changes are not significant, we conclude that the round-off error is not a problem. But if the changes are too large to accept, then we may try reducing the total number of calculations by increasing the mesh size or changing the order of computations. But if the increased mesh size gives unacceptable discretization errors, then we may have to find a reasonable compromise.

It should always be kept in mind that the results obtained by any numerical method may not reflect any trouble spots in certain problems that require special consideration such as hot spots or areas of high temperature gradients. The results that seem quite reasonable overall may be in considerable error at certain locations. This is another reason for always repeating the calculations at least twice with different mesh sizes before accepting them as the solution of the problem. Most commercial software packages have built-in routines that vary the mesh size as necessary to obtain highly accurate solutions. But it is a good engineering practice to be aware of any potential pitfalls of numerical methods and to examine the results obtained with a critical eye.

SUMMARY

Analytical solution methods are limited to highly simplified problems in simple geometries, and it is often necessary to use a numerical method to solve real world problems with complicated geometries or nonuniform thermal conditions. The numerical *finite difference method* is based on replacing derivatives by differences, and the finite difference formulation of a heat transfer problem is obtained by selecting a sufficient number of points in the region, called the *nodal points* or node*s,* and writing *energy balances* on the volume elements centered about the nodes.

For *steady* heat transfer, the *energy balance* on a volume element can be expressed in general as

$$\sum_{\text{All sides}} \dot{Q} + \dot{e}\dot{V}_{\text{element}} = 0$$

whether the problem is one-, two-, or three-dimensional. For convenience in formulation, we always assume all heat transfer to be *into* the volume element from all surfaces toward the node under consideration, except for specified heat flux whose direction is already specified. The finite difference

formulations for a general interior node under *steady* conditions are expressed for some geometries as follows:

One-dimensional steady conduction in a plane wall:

$$\frac{T_{m-1} - 2T_m + T_{m+1}}{(\Delta x)^2} + \frac{\dot{e}_m}{k} = 0$$

Two-dimensional steady conduction in rectangular coordinates:

$$T_{\text{left}} + T_{\text{top}} + T_{\text{right}} + T_{\text{bottom}} - 4T_{\text{node}} + \frac{\dot{e}_{\text{node}} l^2}{k} = 0$$

where Δx is the nodal spacing for the plane wall and $\Delta x = \Delta y = l$ is the nodal spacing for the two-dimensional case. Insulated boundaries can be viewed as mirrors in formulation, and thus the nodes on insulated boundaries can be treated as interior nodes by using mirror images.

The finite difference formulation at node 0 at the left boundary of a plane wall for steady one-dimensional heat conduction can be expressed as

$$\dot{Q}_{\text{left surface}} + kA\frac{T_1 - T_0}{\Delta x} + \dot{e}_0(A\Delta x/2) = 0$$

where $A\Delta x/2$ is the volume of the volume, $\dot{e}_0$ is the rate of heat generation per unit volume at $x = 0$, and A is the heat transfer area. The form of the first term depends on the boundary condition at $x = 0$ (convection, radiation, specified heat flux, etc.).

The finite difference formulation of heat conduction problems usually results in a system of N algebraic equations in N unknown nodal temperatures that need to be solved simultaneously.

The finite difference formulation of *transient* heat conduction problems is based on an energy balance that also accounts for the variation of the energy content of the volume element during a time interval Δt. The heat transfer and heat generation terms are expressed at the previous time step i in the *explicit method,* and at the new time step $i + 1$ in the *implicit method.* For a general node m, the finite difference formulations are expressed as

Explicit method:

$$\sum_{\text{All sides}} \dot{Q}^i + \dot{e}_m^i V_{\text{element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

Implicit method:

$$\sum_{\text{All sides}} \dot{Q}^{i+1} + \dot{e}_m^{i+1} V_{\text{element}} = \rho V_{\text{element}} c_p \frac{T_m^{i+1} - T_m^i}{\Delta t}$$

where T_m^i and T_m^{i+1} are the temperatures of node m at times $t_i = i\Delta t$ and $t_{i+1} = (i + 1)\Delta t$, respectively, and $T_m^{i+1} - T_m^i$ represents the temperature change of the node during the time interval Δt between the time steps i and $i + 1$. The explicit and implicit formulations given here are quite general and can be used in any coordinate system regardless of heat transfer being one-, two-, or three-dimensional.

The explicit formulation of a general interior node for one- and two-dimensional heat transfer in rectangular coordinates can be expressed as

One-dimensional case:

$$T_m^{i+1} = \tau(T_{m-1}^i + T_{m+1}^i) + (1 - 2\tau)\, T_m^i + \tau \frac{\dot{e}_m^i \Delta x^2}{k}$$

Two-dimensional case:

$$T_{\text{node}}^{i+1} = \tau(T_{\text{left}}^i + T_{\text{top}}^i + T_{\text{right}}^i + T_{\text{bottom}}^i) + (1 - 4\tau)\, T_{\text{node}}^i + \tau \frac{\dot{e}_{\text{node}}^i l^2}{k}$$

where $\tau = \alpha\Delta t/\Delta x^2$ is the dimensionless *mesh Fourier number* and $\alpha = k/\rho c_p$ is the *thermal diffusivity* of the medium.

The implicit method is inherently stable, and any value of Δt can be used with that method as the time step. The largest value of the time step Δt in the explicit method is limited by the *stability criterion,* expressed as: *the coefficients of all T_m^i in the T_m^{i+1} expressions (called the primary coefficients) must be greater than or equal to zero for all nodes m.* The maximum value of Δt is determined by applying the stability criterion to the equation with the smallest primary coefficient since it is the most restrictive. For problems with specified temperatures or heat fluxes at all the boundaries, the stability criterion can be expressed as $\tau \leq \frac{1}{2}$ for one-dimensional problems and $\tau \leq \frac{1}{4}$ for the two-dimensional problems in rectangular coordinates.

REFERENCES AND SUGGESTED READING

1. D. A. Anderson, J. C. Tannehill, and R. H. Pletcher. *Computational Fluid Mechanics and Heat Transfer.* New York: Hemisphere, 1984.
2. C. A. Brebbia. *The Boundary Element Method for Engineers.* New York: Halsted Press, 1978.
3. G. E. Forsythe and W. R. Wasow. *Finite Difference Methods for Partial Differential Equations.* New York: John Wiley & Sons, 1960.
4. B. Gebhart. *Heat Conduction and Mass Diffusion.* New York: McGraw-Hill, 1993.
5. K. H. Huebner and E. A. Thornton. *The Finite Element Method for Engineers.* 2nd ed. New York: John Wiley & Sons, 1982.
6. Y. Jaluria and K. E. Torrance. *Computational Heat Transfer.* New York: Hemisphere, 1986.
7. W. J. Minkowycz, E. M. Sparrow, G. E. Schneider, and R. H. Pletcher. *Handbook of Numerical Heat Transfer.* New York: John Wiley & Sons, 1988.
8. G. E. Myers. *Analytical Methods in Conduction Heat Transfer.* New York: McGraw-Hill, 1971.

3. S. Roy, R. Raju, H. Chuang, B. Kruden and M. Meyyappan, "Modeling Gas Flow Through Microchannels and Nanopores." *Journal of Applied Physics*, 93, 8 (2003), pp. 4870–79.
4. M. von Smoluchowski, "Ueber Wärmeleitung in Verdünnten Gasen," *Annalen der Physik und Chemi.* 64 (1898), pp. 101–130.
5. C. L. Tien, A. Majumdar, and F. Gerner. *Microscale Energy Transport*. New York: Taylor & Francis Publishing, 1998.

SUMMARY

Convection heat transfer is expressed by *Newton's law of cooling* as

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty)$$

where h is the convection heat transfer coefficient, T_s is the surface temperature, and T_∞ is the free-stream temperature. The convection coefficient is also expressed as

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty}$$

The *Nusselt number,* which is the dimensionless heat transfer coefficient, is defined as

$$\text{Nu} = \frac{hL_c}{k}$$

where k is the thermal conductivity of the fluid and L_c is the characteristic length.

The highly ordered fluid motion characterized by smooth streamlines is called *laminar.* The highly disordered fluid motion that typically occurs at high velocities characterized by velocity fluctuations is called *turbulent.* The random and rapid fluctuations of groups of fluid particles, called *eddies,* provide an additional mechanism for momentum and heat transfer.

The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt is called the *velocity boundary layer.* The *boundary layer thickness,* δ, is defined as the distance from the surface at which $u = 0.99V$. The hypothetical line of $u = 0.99V$ divides the flow over a plate into the *boundary layer region* in which the viscous effects and the velocity changes are significant, and the *irrotational flow region,* in which the frictional effects are negligible.

The friction force per unit area is called *shear stress,* and the shear stress at the wall surface is expressed as

$$\tau_w = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \quad \text{or} \quad \tau_w = C_f \frac{\rho V^2}{2}$$

where μ is the dynamic viscosity, V is the upstream velocity, and C_f is the dimensionless *friction coefficient.* The property $\nu = \mu/\rho$ is the *kinematic viscosity.* The friction force over the entire surface is determined from

$$F_f = C_f A_s \frac{\rho V^2}{2}$$

The flow region over the surface in which the temperature variation in the direction normal to the surface is significant is the *thermal boundary layer.* The *thickness* of the thermal boundary layer δ_t at any location along the surface is the distance from the surface at which the temperature difference $T - T_s$ equals $0.99(T_\infty - T_s)$. The relative thickness of the velocity and the thermal boundary layers is best described by the dimensionless *Prandtl number,* defined as

$$\text{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

For external flow, the dimensionless *Reynolds number* is expressed as

$$\text{Re} = \frac{\text{Inertia forces}}{\text{Viscous forces}} = \frac{VL_c}{\nu} = \frac{\rho V L_c}{\mu}$$

For a flat plate, the characteristic length is the distance x from the leading edge. The Reynolds number at which the flow becomes turbulent is called the *critical Reynolds number.* For flow over a flat plate, its value is taken to be $\text{Re}_{\text{cr}} = Vx_{\text{cr}}/\nu = 5 \times 10^5$.

The continuity, momentum, and energy equations for steady two-dimensional incompressible flow with constant properties are determined from mass, momentum, and energy balances to be

$$\textit{Continuity:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\textit{x-momentum:} \quad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}$$

$$\textit{Energy:} \quad \rho c_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

where the *viscous dissipation function* Φ is

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2$$

Using the boundary layer approximations and a similarity variable, these equations can be solved for parallel steady incompressible flow over a flat plate, with the following results:

Velocity boundary layer thickness: $\delta = \dfrac{4.91}{\sqrt{V/\nu x}} = \dfrac{4.91x}{\sqrt{\mathrm{Re}_x}}$

Local friction coefficient: $C_{f,x} = \dfrac{\tau_w}{\rho V^2/2} = 0.664\,\mathrm{Re}_x^{-1/2}$

Local Nusselt number: $\mathrm{Nu}_x = \dfrac{h_x x}{k} = 0.332\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}$

Thermal boundary layer thickness: $\delta_t = \dfrac{\delta}{\mathrm{Pr}^{1/3}} = \dfrac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$

The average friction coefficient and Nusselt number are expressed in functional form as

$$C_f = f(\mathrm{Re}_L) \qquad \text{and} \qquad \mathrm{Nu} = g(\mathrm{Re}_L, \mathrm{Pr})$$

The Nusselt number can be expressed by a simple power-law relation of the form

$$\mathrm{Nu} = C\,\mathrm{Re}_L^m\,\mathrm{Pr}^n$$

where m and n are constant exponents, and the value of the constant C depends on geometry. The *Reynolds analogy* relates the convection coefficient to the friction coefficient for fluids with $\mathrm{Pr} \approx 1$, and is expressed as

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \mathrm{St}_x$$

where

$$\mathrm{St} = \frac{h}{\rho c_p V} = \frac{\mathrm{Nu}}{\mathrm{Re}_L\,\mathrm{Pr}}$$

is the *Stanton number*. The analogy is extended to other Prandtl numbers by the *modified Reynolds analogy* or *Chilton–Colburn analogy,* expressed as

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\mathrm{Pr}^{-1/3}$$

or

$$\frac{C_{f,x}}{2} = \mathrm{St}_x\,\mathrm{Pr}^{2/3} = j_H \quad (0.6 < \mathrm{Pr} < 60)$$

These analogies are also applicable approximately for turbulent flow over a surface, even in the presence of pressure gradients.

REFERENCES AND SUGGESTED READING

1. H. Blasius. "The Boundary Layers in Fluids with Little Friction (in German)." *Z. Math. Phys.,* 56, 1 (1908); pp. 1–37; English translation in National Advisory Committee for Aeronautics Technical Memo No. 1256, February 1950.
2. Y. A. Çengel and J. M. Cimbala. *Fluid Mechanics: Fundamentals and Applications*. New York: McGraw-Hill, 2006.
3. W. M. Kays, M. E. Crawford and B. Weigand. *Convective Heat and Mass Transfer.* 4th ed. New York: McGraw-Hill, 2005.
4. O. Reynolds. "On the Experimental Investigation of the Circumstances Which Determine Whether the Motion of Water Shall Be Direct or Sinuous, and the Law of Resistance in Parallel Channels." *Philosophical Transactions of the Royal Society of London* 174 (1883), pp. 935–82.
5. H. Schlichting. *Boundary Layer Theory.* 7th ed. New York: McGraw-Hill, 1979.
6. G. G. Stokes. "On the Effect of the Internal Friction of Fluids on the Motion of Pendulums." *Cambridge Philosophical Transactions,* IX, 8, 1851.

SUMMARY

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{Vx}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\mathrm{Re}_{x,\,\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $C_{f,x} = \dfrac{0.664}{\mathrm{Re}_x^{1/2}} \quad \mathrm{Re}_x < 5 \times 10^5$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \mathrm{Pr} > 0.6$$

Turbulent:

$$C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}} \quad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{matrix}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}} \quad \mathrm{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} \quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

Combined: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}} - \dfrac{1742}{\mathrm{Re}_L} \quad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log\dfrac{\varepsilon}{L}\right)^{-2.5}$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $\mathrm{Nu} = \dfrac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3} \quad \begin{matrix} \mathrm{Pr} > 0.6 \\ \mathrm{Re}_L < 5 \times 10^5 \end{matrix}$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3} \quad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{matrix}$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3} \quad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{matrix}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $\mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\,\xi=0)}}{[1 - (\xi/x)^{3/4}]^{1/3}} = \dfrac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1 - (\xi/x)^{3/4}]^{1/3}}$

Turbulent: $\mathrm{Nu}_x = \dfrac{\mathrm{Nu}_{x\,(\mathrm{for}\,\xi=0)}}{[1 - (\xi/x)^{9/10}]^{1/9}} = \dfrac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1 - (\xi/x)^{9/10}]^{1/9}}$

Laminar: $h = \dfrac{2[1 - (\xi/x)^{3/4}]}{1 - \xi/L} h_{x=L}$

Turbulent: $h = \dfrac{5[1 - (\xi/x)]^{9/10}}{(1 - \xi/L)} h_{x=L}$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux*, the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \quad \begin{matrix} \mathrm{Pr} > 0.6 \\ \mathrm{Re}_x < 5 \times 10^5 \end{matrix}$

Turbulent: $\mathrm{Nu}x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \quad \begin{matrix} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{matrix}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for $\mathrm{Re\,Pr} > 0.2$, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 8 \times 10^4$, $0.7 \le \mathrm{Pr} \le 380$ and $1.0 \le (\mu_\infty/\mu_s) \le 3.2$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

In tube banks, the Reynolds number is based on the maximum velocity $V_{\max}$ that is related to the approach velocity V as

In-line and *Staggered* with $S_D < (S_T + D)/2$:

$$V_{\max} = \frac{S_T}{S_T - D}V$$

Staggered with $S_D < (S_T + D)/2$:

$$V_{\max} = \frac{S_T}{2(S_D - D)}V$$

where S_T the transverse pitch and S_D is the diagonal pitch. The average Nusselt number for cross flow over tube banks is expressed as

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

where the values of the constants C, m, and n depend on Reynolds number. Such correlations are given in Table 7–2. All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and exit temperatures of the fluid defined as $T_m = (T_i + T_e)/2$.

The average Nusselt number for tube banks with less than 16 rows is expressed as

$$\mathrm{Nu}_{D,\,N_{L<16}} = F\mathrm{Nu}_D$$

where F is the *correction factor* whose values are given in Table 7–3. The heat transfer rate to or from a tube bank is determined from

$$\dot{Q} = hA_s\Delta T_{\mathrm{lm}} = \dot{m}c_p(T_e - T_i)$$

where ΔT_{lm} is the logarithmic mean temperature difference defined as

$$\Delta T_{\mathrm{lm}} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

and the exit temperature of the fluid T_e is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right)$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho V(N_T S_T L)$ is the mass flow rate of the fluid. The pressure drop ΔP for a tube bank is expressed as

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2}$$

where f is the friction factor and χ is the correction factor, both given in Fig. 7–27.

REFERENCES AND SUGGESTED READING

1. R. D. Blevin. *Applied Fluid Dynamics Handbook*. New York: Van Nostrand Reinhold, 1984.
2. S. W. Churchill and M. Bernstein. "A Correlating Equation for Forced Convection from Gases and Liquids to a Circular Cylinder in Cross Flow." *Journal of Heat Transfer* 99 (1977), pp. 300–306.
3. S. W. Churchill and H. Ozoe. "Correlations for Laminar Forced Convection in Flow over an Isothermal Flat Plate and in Developing and Fully Developed Flow in an Isothermal Tube." *Journal of Heat Transfer* 95 (Feb. 1973), pp. 78–84.
4. W. M. Edmunds. "Residential Insulation." *ASTM Standardization News* (Jan. 1989), pp. 36–39.
5. W. H. Giedt. "Investigation of Variation of Point Unit-Heat Transfer Coefficient around a Cylinder Normal to an Air Stream." *Transactions of the ASME* 71 (1949), pp. 375–381.

3. A. J. Ghajar and L. M. Tam. "Flow Regime Map for a Horizontal Pipe with Uniform Wall Heat Flux and Three Inlet Configurations." *Experimental Thermal and Fluid Science*, Vol. 10 (1995), pp. 287–297.

4. A. J. Ghajar, L. M. Tam, and S. C. Tam. "Improved Heat Transfer Correlation in the Transition Region for a Circular Tube with Three Inlet Configurations Using Artificial Neural Networks." *Heat Transfer Engineering,* Vol. 25, No. 2 (2004), pp. 30–40.

5. A. J. Ghajar, C. C. Tang, and W. L. Cook. "Experimental Investigation of Friction Factor in the Transition Region for Water Flow in Minitubes and Microtubes." *Heat Transfer Engineering,* Vol. 31, No. 8 (2010), pp. 646–657.

6. L. M. Tam and A. J. Ghajar. "Effect of Inlet Geometry and Heating on the Fully Developed Friction Factor in the Transition Region of a Horizontal Tube." *Experimental Thermal and Fluid Science,* Vol. 15 (1997), pp. 52–64.

7. L. M. Tam and A. J. Ghajar. "The Unusual Behavior of Local Heat Transfer Coefficient in a Circular Tube with a Bell-Mouth Inlet." *Experimental Thermal and Fluid Science,* Vol. 16 (1998), pp. 187–194.

8. L. M. Tam and A. J. Ghajar. "Transitional Heat Transfer in Plain Horizontal Tubes." *Heat Transfer Engineering,* Vol. 27, No. 5 (2006), pp. 23–38.

9. L. M. Tam, A. J. Ghajar, and H. K. Tam. "Contribution Analysis of Dimensionless Variables for Laminar and Turbulent Flow Convection Heat Transfer in a Horizontal Tube Using Artificial Neural Network." *Heat Transfer Engineering,* Vol. 29, No. 9 (2008), pp. 793–804.

10. H. K. Tam, L. M. Tam, and A. J. Ghajar. "Effect of Inlet Geometries and Heating on the Entrance and Fully-Developed Friction Factors in the Laminar and Transition Regions of a Horizontal Tube." *Experimental Thermal and Fluid Science*, Vol. 44 (2013), pp. 680–696.

11. H. K. Tam, L. M. Tam, A. J. Ghajar, S. C. Tam, and T. Zhang. "Experimental Investigation of Heat Transfer, Friction Factor, and Optimal Fin Geometries for the Internally Microfin Tubes in the Transition and Turbulent Regions." *Journal of Enhanced Heat Transfer*, Vol. 19, No. 5 (2012), pp. 457–476. Reprinted with permission from Begell House, Inc.

SUMMARY

Internal flow is characterized by the fluid being completely confined by the inner surfaces of the tube. The mean or average velocity and temperature for a circular tube of radius R are expressed as

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r)rdr \quad \text{and} \quad T_m = \frac{2}{V_{\text{avg}}R^2}\int_0^R u(r)T(r)rdr$$

The Reynolds number for internal flow and the hydraulic diameter are defined as

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{V_{\text{avg}} D}{\nu} \quad \text{and} \quad D_h = \frac{4A_c}{p}$$

The flow in a tube is laminar for Re $<$ 2300, turbulent for about Re $>$ 10,000, and transitional in between.

The length of the region from the tube inlet to the point at which the flow becomes fully developed is the *hydrodynamic entry length* L_h. The region beyond the entrance region in which the velocity profile is fully developed is the *hydrodynamically fully developed region.* The length of the region of flow over which the thermal boundary layer develops and reaches the tube center is the *thermal entry length* L_t. The region in which the flow is both hydrodynamically and thermally developed is the *fully developed flow region.* The entry lengths are given by

$$L_{h,\ \text{laminar}} \approx 0.05\ \text{Re}\ D$$

$$L_{t,\ \text{laminar}} \approx 0.05\ \text{Re Pr}\ D = \text{Pr}\ L_{h,\ \text{laminar}}$$

$$L_{h,\ \text{turbulent}} \approx L_{t,\ \text{turbulent}} = 10D$$

For $\dot{q}_s$ = constant, the rate of heat transfer is expressed as

$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_e - T_i)$$

For T_s = constant, we have

$$\dot{Q} = hA_s \Delta T_{\text{lm}} = \dot{m} c_p (T_e - T_i)$$

$$T_e = T_s - (T_s - T_i)\exp(-hA_s/\dot{m}c_p)$$

$$\Delta T_{\text{lm}} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

The irreversible pressure loss due to frictional effects and the required pumping power to overcome this loss for a volume flow rate of $\dot{V}$ are

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{\text{avg}}^2}{2} \quad \text{and} \quad \dot{W}_{\text{pump}} = \dot{V}\,\Delta P_L$$

For *fully developed laminar flow* in a circular pipe, we have:

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

$$f = \frac{64\mu}{\rho D V_{\text{avg}}} = \frac{64}{\text{Re}}$$

$$\dot{V} = V_{\text{avg}}\,A_c = \frac{\Delta P R^2}{8\mu L}\pi R^2 = \frac{\pi R^4\,\Delta P}{8\mu L} = \frac{\pi R^4\,\Delta P}{128\mu L}$$

Circular tube, laminar ($\dot{q}_s$ = constant): $\text{Nu} = \dfrac{hD}{k} = 4.36$

Circular tube, laminar (T_s = constant): $\text{Nu} = \dfrac{hD}{k} = 3.66$

For *developing laminar flow* in the entrance region with constant surface temperature, we have

Circular tube: $$\text{Nu} = 3.66 + \frac{0.065(D/L)\,\text{Re}\,\text{Pr}}{1 + 0.04[(D/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

Circular tube: $$\text{Nu} = 1.86\left(\frac{\text{Re}\,\text{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

Parallel plates: $$\text{Nu} = 7.54 + \frac{0.03(D_h/L)\,\text{Re}\,\text{Pr}}{1 + 0.016[(D_h/L)\,\text{Re}\,\text{Pr}]^{2/3}}$$

For *fully developed turbulent flow with smooth surfaces,* we have

$$f = (0.790\ln\text{Re} - 1.64)^{-2} \quad 10^4 < \text{Re} < 10^6$$

$$\text{Nu} = 0.125 f\,\text{Re}\,\text{Pr}^{1/3}$$

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\begin{pmatrix}0.7 \le \text{Pr} \le 160\\ \text{Re} > 10{,}000\end{pmatrix}$$

$$\text{Nu} = 0.023\,\text{Re}^{0.8}\,\text{Pr}^{n} \text{ with } n = 0.4 \text{ for } \textit{heating} \text{ and } 0.3 \text{ for } \textit{cooling} \text{ of fluid}$$

$$\text{Nu} = \frac{(f/8)(\text{Re} - 1000)\,\text{Pr}}{1 + 12.7(f/8)^{0.5}(\text{Pr}^{2/3} - 1)}\begin{pmatrix}0.5 \le \text{Pr} \le 2000\\ 3\times10^3 < \text{Re} < 5\times10^6\end{pmatrix}$$

The fluid properties are evaluated at the *bulk mean fluid temperature* $T_b = (T_i + T_e)/2$. For liquid metal flow in the range of $10^4 < \text{Re} < 10^6$ we have:

$$T_s = \text{constant:} \quad \text{Nu} = 4.8 + 0.0156\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$$

$$\dot{q}_s = \text{constant:} \quad \text{Nu} = 6.3 + 0.0167\,\text{Re}^{0.85}\,\text{Pr}_s^{0.93}$$

For *fully developed turbulent flow with rough surfaces,* the friction factor f is determined from the Moody chart or

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \approx -1.8\log\left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

For a *concentric annulus,* the hydraulic diameter is $D_h = D_o - D_i$, and the Nusselt numbers are expressed as

$$\text{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \text{Nu}_o = \frac{h_o D_h}{k}$$

where the values for the Nusselt numbers are given in Table 8–4.

REFERENCES AND SUGGESTED READING

1. J. P. Abraham, E. M. Sparrow, and W. J. Minkowycz. "Internal-Flow Nusselt Numbers for the Low-Reynolds-Number End of the Laminar-to-Turbulent Transition Regime." *International Journal of Heat and Mass Transfer* 54 (2011)." pp. 584–588.
2. J. P. Abraham, E. M. Sparrow, and J. C. K. Tong. "Heat Transfer in All Pipe Flow Regimes—Laminar, Transitional/Intermittent, and Turbulent." *International Journal of Heat and Mass Transfer* 52 (2009), pp. 557–563.
3. M. S. Bhatti and R. K. Shah. "Turbulent and Transition Flow Convective Heat Transfer in Ducts." In *Handbook of Single-Phase Convective Heat Transfer,* S. Kakaç, R. K. Shah, and W. Aung (Eds.). New York: Wiley Interscience, 1987.
4. Y. A. Çengel and J. M. Cimbala. *Fluid Mechanics: Fundamentals and Applications,* Third Edition. New York: McGraw-Hill, 2014.
5. A. P. Colburn. *Transactions of the AIChE* 26 (1933), p. 174.
6. C. F. Colebrook. "Turbulent Flow in Pipes, with Particular Reference to the Transition between the Smooth and Rough Pipe Laws." *Journal of the Institute of Civil Engineers London.* 11 (1939), pp. 133–156.
7. R. G. Deissler. "Analysis of Turbulent Heat Transfer and Flow in the Entrance Regions of Smooth Passages." 1953.
8. D. F. Dipprey and D. H. Sabersky. "Heat and Momentum Transfer in Smooth and Rough Tubes at Various Prandtl Numbers." *International Journal of Heat Mass Transfer* 6 (1963), pp. 329–353.
9. F. W. Dittus and L. M. K. Boelter. *University of California Publications on Engineering* 2 (1930), p. 433.
10. D. K. Edwards, V. E. Denny, and A. F. Mills. *Transfer Processes.* 2nd ed. Washington, DC: Hemisphere, 1979.
11. V. Gnielinski. "New Equations for Heat and Mass Transfer in Turbulent Pipe and Channel Flow." *International Chemical Engineering* 16 (1976), pp. 359–368.

For aspect ratios greater than 10, Eqs. 9–54 and 9–55 should be used. For inclined enclosures, Eqs. 9–48 through 9–51 should be used.

For *concentric horizontal cylinders,* the rate of heat transfer through the annular space between the cylinders by natural convection per unit length is

$$\dot{Q} = \frac{2\pi k_{\text{eff}}}{\ln(D_o/D_i)}(T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.386\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{cyl}}\text{Ra}_L)^{1/4}$$

and

$$F_{\text{cyl}} = \frac{[ln(D_o/D_i)]^4}{L_c^3(D_i^{-3/5} + D_o^{-3/5})^5}$$

For a *spherical enclosure,* the rate of heat transfer through the space between the spheres by natural convection is expressed as

$$\dot{Q} = k_{\text{eff}}\frac{\pi D_i D_o}{L_c}(T_i - T_o)$$

where

$$\frac{k_{\text{eff}}}{k} = 0.74\left(\frac{\text{Pr}}{0.861 + \text{Pr}}\right)^{1/4}(F_{\text{sph}}\text{Ra}_L)^{1/4}$$

$$L_c = (D_o - D_i)/2$$

$$F_{\text{sph}} = \frac{L_c}{(D_iD_o)^4(D_i^{-7/5} + D_o^{-7/5})^5}$$

The quantity kNu is called the *effective thermal conductivity* of the enclosure, since a fluid in an enclosure behaves like a quiescent fluid whose thermal conductivity is kNu as a result of convection currents. The fluid properties are evaluated at the average temperature of $(T_i + T_o)/2$.

For a given fluid, the parameter Gr/Re^2 represents the importance of natural convection relative to forced convection. Natural convection is negligible when $\text{Gr}/\text{Re}^2 < 0.1$, forced convection is negligible when $\text{Gr}/\text{Re}^2 > 10$, and neither is negligible when $0.1 < \text{Gr}/\text{Re}^2 < 10$.

REFERENCES AND SUGGESTED READING

1. American Society of Heating, Refrigeration, and Air Conditioning Engineers. *Handbook of Fundamentals.* Atlanta: ASHRAE, 1993.
2. J. N Arnold, I. Catton, and D. K. Edwards. "Experimental Investigation of Natural Convection in Inclined Rectangular Region of Differing Aspects Ratios." ASME Paper No. 75-HT-62, 1975.
3. L. S. Asgeirsson and A. J. Ghajar. "Prediction of Thermal Conductivity and Viscosity for Some Fluids in the Near-Critical Region." *Chemical Engineering Communications*, Vol. 43, Nos. 1–3 (1986), pp. 165–184.
4. P. S. Ayyaswamy and I. Catton. "The Boundary–Layer Regime for Natural Convection in a Differently Heated Tilted Rectangular Cavity." *Journal of Heat Transfer* 95 (1973), p. 543.
5. A. Bar-Cohen. "Fin Thickness for an Optimized Natural Convection Array of Rectangular Fins." *Journal of Heat Transfer* 101 (1979), pp. 564–566.
6. A. Bar-Cohen and W. M. Rohsenow. "Thermally Optimum Spacing of Vertical Natural Convection Cooled Parallel Plates." *Journal of Heat Transfer* 106 (1984), p. 116.
7. B. M. Berkovsky and V. K. Polevikov. "Numerical Study of Problems on High-Intensive Free Convection." In *Heat Transfer and Turbulent Buoyant Convection,* eds. D. B. Spalding and N. Afgan, pp. 443–445. Washington, DC: Hemisphere, 1977.
8. I. Catton. "Natural Convection in Enclosures." *Proceedings of Sixth International Heat Transfer Conference.* Toronto: Canada, 1978, Vol. 6, pp. 13–31.
9. T. Cebeci. "Laminar Free Convection Heat Transfer from the Outer Surface of a Vertical Slender Circular Cylinder." *Proceedings of Fifth International Heat Transfer Conference* paper NCI.4, 1974 pp. 15–19.
10. Y. A. Çengel and P. T. L. Zing. "Enhancement of Natural Convection Heat Transfer from Heat Sinks by Shrouding." *Proceedings of ASME/JSME Thermal Engineering Conference.* Honolulu: HA, 1987, Vol. 3, pp. 451–475.
11. S. W. Churchill. "A Comprehensive Correlating Equation for Laminar Assisting Forced and Free Convection." *AIChE Journal* 23 (1977), pp. 10–16.
12. S. W. Churchill. "Free Convection around Immersed Bodies." In *Heat Exchanger Design Handbook,* ed. E. U. Schlünder, Section 2.5.7. New York: Hemisphere, 1983.
13. S. W. Churchill. "Combined Free and Forced Convection around Immersed Bodies." In *Heat Exchanger Design Handbook,* Section 2.5.9. New York: Hemisphere Publishing, 1986.
14. S. W. Churchill and H. H. S. Chu. "Correlating Equations for Laminar and Turbulent Free Convection from a Horizontal Cylinder." *International Journal of Heat Mass Transfer* 18 (1975), p. 1049.
15. S. W. Churchill and H. H. S. Chu. "Correlating Equations for Laminar and Turbulent Free Convection from a Vertical Plate." *International Journal of Heat Mass Transfer* 18 (1975), p. 1323.
16. E. R. G. Eckert and E. Soehngen. "Studies on Heat Transfer in Laminar Free Convection with Zehnder–Mach Interferometer." USAF Technical Report 5747, December 1948.

SUMMARY

Boiling occurs when a liquid is in contact with a surface maintained at a temperature T_s sufficiently above the saturation temperature T_{sat} of the liquid. Boiling is classified as *pool boiling* or *flow boiling* depending on the presence of bulk fluid motion. Boiling is called *pool boiling* in the absence of bulk fluid flow and *flow boiling* (or *forced convection boiling*) in its presence. Pool and flow boiling are further classified as *subcooled boiling* and *saturated boiling* depending on the bulk liquid temperature. Boiling is said to be *subcooled* (or *local*) when the temperature of the main body of the liquid is below the saturation temperature T_{sat} and *saturated* (or *bulk*) when the temperature of the liquid is equal to T_{sat}. Boiling exhibits different regimes depending on the value of the excess temperature ΔT_{excess}. Four different boiling regimes are observed: natural convection boiling, nucleate boiling, transition boiling, and film boiling. These regimes are illustrated on the *boiling curve*. The rate of evaporation and the rate of heat transfer in nucleate boiling increase with increasing ΔT_{excess} and reach a maximum at some point. The heat flux at this point is called the *critical* (or *maximum*) *heat flux*, $\dot{q}_{\max}$. The rate of heat transfer in nucleate pool boiling is determined from

$$\dot{q}_{\text{nucleate}} = \mu_l h_{fg}\left[\frac{g(\rho_l - \rho_v)}{\sigma}\right]^{1/2}\left[\frac{c_{pl}(T_s - T_{\text{sat}})}{C_{sf} h_{fg}\,\text{Pr}_l^n}\right]^3$$

The *maximum* (or *critical*) *heat flux* in nucleate pool boiling is determined from

$$\dot{q}_{\max} = C_{cr} h_{fg}[\sigma g \rho_v^2 (\rho_l - \rho_v)]^{1/4}$$

where the value of the constant C_{cr} is about 0.15. The minimum heat flux is given by

$$\dot{q}_{\min} = 0.09 \rho_v h_{fg}\left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l + \rho_v)^2}\right]^{1/4}$$

The heat flux for stable *film boiling* on the outside of a *horizontal cylinder* or *sphere* of diameter D is given by

$$\dot{q}_{\text{film}} = C_{\text{film}}\left[\frac{g k_v^3 \rho_v(\rho_l - \rho_v)h_{fg} + 0.4 c_{pv}(T_s - T_{\text{sat}})}{\mu_v D(T_s - T_{\text{sat}})}\right]^{1/4} \times (T_s - T_{\text{sat}})$$

where the constant $C_{\text{film}} = 0.62$ for horizontal cylinders and 0.67 for spheres. The *vapor* properties are to be evaluated at the *film temperature* $T_f = (T_{\text{sat}} + T_s)/2$, which is the *average temperature* of the vapor film. The liquid properties and h_{fg} are to be evaluated at the saturation temperature at the specified pressure.

Two distinct forms of condensation are observed in nature: film condensation and dropwise condensation. In *film condensation,* the condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity. In *dropwise condensation,* the condensed vapor forms countless droplets of varying diameters on the surface instead of a continuous film.

The Reynolds number for the condensate flow is defined as

$$\text{Re} = \frac{D_h \rho_l V_l}{\mu_l} = \frac{4 A_c \rho_l V_l}{p \mu_l} = \frac{4\dot{m}}{p\mu_l}$$

and

$$\text{Re} = \frac{4\dot{Q}_{\text{conden}}}{p\mu_l h_{fg}^*} = \frac{4 A_s h(T_{\text{sat}} - T_s)}{p\mu_l h_{fg}^*}$$

where h_{fg}^* is the *modified latent heat of vaporization,* defined as

$$h_{fg}^* = h_{fg} + 0.68 c_{pl}(T_{\text{sat}} - T_s)$$

and represents heat transfer during condensation per unit mass of condensate.

Using some simplifying assumptions, the *average heat transfer coefficient* for film condensation on a vertical plate of height L is determined to be

$$h_{\text{vert}} = 0.943\left[\frac{g\rho_l(\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{\text{sat}} - T_s) L}\right]^{1/4}$$

All properties of the *liquid* are to be evaluated at the film temperature $T_f = (T_{\text{sat}} + T_s)/2$. The h_{fg} and ρ_v are to be evaluated at T_{sat}. Condensate flow is *smooth* and *wave-free laminar* for about $\text{Re} \le 30$, *wavy-laminar* in the range of $30 < \text{Re} < 1800$, and *turbulent* for $\text{Re} > 1800$. Heat transfer coefficients in the wavy-laminar and turbulent flow regions are determined from

$$h_{\text{vert, wavy}} = \frac{\text{Re}\, k_l}{1.08\,\text{Re}^{1.22} - 5.2}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \quad \begin{matrix} 30 < \text{Re} < 1800 \\ \rho_v \ll \rho_l \end{matrix}$$

$$h_{\text{vert, turbulent}} = \frac{\text{Re}\, k_l}{8750 + 58\,\text{Pr}^{-0.5}(\text{Re}^{0.75} - 253)}\left(\frac{g}{\nu_l^2}\right)^{1/3}, \quad \begin{matrix} \text{Re} > 1800 \\ \rho_v \ll \rho_l \end{matrix}$$

Equations for vertical plates can also be used for laminar film condensation on the upper surfaces of the plates that are inclined by an angle θ from the vertical, by replacing g in that equation by $g \cos\theta$. Vertical plate equations can also be used to calculate the average heat transfer coefficient for laminar film condensation on the outer surfaces of vertical tubes provided that the tube diameter is large relative to the thickness of the liquid film.

The average heat transfer coefficient for film condensation on the outer surfaces of a *horizontal tube* is determined to be

$$h_{\text{horiz}} = 0.729\left[\frac{g\rho_l(\rho_l - \rho_v) h_{fg}^* k_l^3}{\mu_l (T_{\text{sat}} - T_s) D}\right]^{1/4}$$

where D is the diameter of the horizontal tube. This relation can easily be modified for a *sphere* by replacing the constant 0.729 by 0.815. It can also be used for *N horizontal tubes* stacked on top of each other by replacing D in the denominator by ND.

For low vapor velocities, film condensation heat transfer *inside horizontal tubes* can be determined from

$$h_{\text{internal}} = 0.555\left[\frac{g\rho_l(\rho_l - \rho_v)k_l^3}{\mu_l(T_{\text{sat}} - T_s)D}\left(h_{fg} + \frac{3}{8}c_{pl}(T_{\text{sat}} - T_s)\right)\right]^{1/4}$$

$$\text{Re}_{\text{vapor}} = \left(\frac{\rho_v V_v D}{\mu_v}\right)_{\text{inlet}} < 35{,}000$$

where the Reynolds number of the vapor is to be evaluated at the tube inlet conditions using the internal tube diameter as the characteristic length. Finally, the heat transfer coefficient for *dropwise condensation* of steam on copper surfaces is given by

$$h_{\text{dropwise}} = \begin{cases} 51{,}104 + 2044T_{\text{sat}}, & 22°\text{C} < T_{\text{sat}} < 100°\text{C} \\ 255{,}510, & T_{\text{sat}} > 100°\text{C} \end{cases}$$

where T_{sat} is in °C and the heat transfer coefficient h_{dropwise} is in W/m^2·°C or its equivalent W/m^2·K.

REFERENCES AND SUGGESTED READING

1. N. Arai, T. Fukushima, A. Arai, T. Nakajima, K. Fujie, and Y. Nakayama. "Heat Transfer Tubes Enhancing Boiling and Condensation in Heat Exchangers of a Refrigeration Machine." *ASHRAE Journal 83* (1977), p. 58.
2. P. J. Berensen. "Film Boiling Heat Transfer for a Horizontal Surface." *Journal of Heat Transfer* 83 (1961), pp. 351–358.
3. P. J. Berensen. "Experiments in Pool Boiling Heat Transfer." *International Journal of Heat Mass Transfer* 5 (1962), pp. 985–999.
4. L. A. Bromley. "Heat Transfer in Stable Film Boiling." *Chemical Engineering Prog.* 46 (1950), pp. 221–227.
5. J. C. Chato. "Laminar Condensation inside Horizontal and Inclined Tubes." *ASHRAE Journal* 4 (1962), p. 52.
6. S. W. Chi. *Heat Theory and Practice.* Washington, D.C.: Hemisphere, 1976.
7. M. T. Cichelli and C. F. Bonilla. "Heat Transfer to Liquids Boiling under Pressure." *Transactions of AIChE* 41 (1945), pp. 755–787.
8. R. A. Colclaser, D. A. Neaman, and C. F. Hawkins. *Electronic Circuit Analysis.* New York: John Wiley & Sons, 1984.
9. P. Griffith. "Dropwise Condensation." In *Heat Exchanger Design Handbook,* ed. E. U. Schlunder, Vol 2, Ch. 2.6.5. New York: Hemisphere, 1983.
10. H. Hampson and N. Özişik. "An Investigation into the Condensation of Steam." *Proceedings of the Institute of Mechanical Engineers,* London 1B (1952), pp. 282–294.
11. J. J. Jasper. "The Surface Tension of Pure Liquid Compounds." *Journal of Physical and Chemical Reference Data* 1, No. 4 (1972), pp. 841–1009.
12. S. S. Kutateladze. *Fundamentals of Heat Transfer.* New York: Academic Press, 1963.
13. S. S. Kutateladze. "On the Transition to Film Boiling under Natural Convection." *Kotloturbostroenie* 3 (1948), p. 48.
14. D. A. Labuntsov. "Heat Transfer in Film Condensation of Pure Steam on Vertical Surfaces and Horizontal Tubes." *Teploenergetika* 4 (l957), pp. 72–80.
15. J. H. Lienhard and V. K. Dhir. "Extended Hydrodynamic Theory of the Peak and Minimum Pool Boiling Heat Fluxes." NASA Report, NASA-CR-2270, July 1973.
16. J. H. Lienhard and V. K. Dhir. "Hydrodynamic Prediction of Peak Pool Boiling Heat Fluxes from Finite Bodies." *Journal of Heat Transfer* 95 (1973), pp. 152–158.
17. W. H. McAdams. *Heat Transmission.* 3rd ed. New York: McGraw-Hill, 1954.
18. W. M. Rohsenow. "A Method of Correlating Heat Transfer Data for Surface Boiling of Liquids." *ASME Transactions* 74 (1952), pp. 969–975.
19. W. M. Rohsenow. "Film Condensation." In *Handbook of Heat Transfer,* ed. W. M. Rohsenow and J. P. Hartnett, Ch. 12A. New York: McGraw-Hill, 1973.
20. I. G. Shekriladze, I. G. Gomelauri, and V. I. Gomelauri. "Theoretical Study of Laminar Film Condensation of Flowing Vapor." *International Journal of Heat Mass Transfer* 9 (1966), pp. 591–592.
21. N. V. Suryanarayana. *Engineering Heat Transfer.* St. Paul, MN: West Publishing, 1995.
22. J. W. Westwater and J. G. Santangelo. *Industrial Engineering Chemistry* 47 (1955), p. 1605.
23. N. Zuber. "On the Stability of Boiling Heat Transfer." *ASME Transactions* 80 (1958), pp. 711–720.

REFERENCES

1. S. Weinbaum and L. M. Jiji. "A New Simplified Bioheat Equation for the Effect of Blood Flow on Local Average Tissue Temperature." *Journal of Biomechanical Engineering 107* (1985), pp. 131–139.

SUMMARY

Heat exchangers are devices that allow the exchange of heat between two fluids without allowing them to mix with each other. Heat exchangers are manufactured in a variety of types, the simplest being the *double-pipe* heat exchanger. In a *parallel-flow* type, both the hot and cold fluids enter the heat exchanger at the same end and move in the same direction, whereas in a *counter-flow* type, the hot and cold fluids enter the heat exchanger at opposite ends and flow in opposite directions. In *compact* heat exchangers, the two fluids move perpendicular to each other, and such a flow configuration is called *cross-flow*. Other common types of heat exchangers in industrial applications are the *plate* and the *shell-and-tube* heat exchangers.

Heat transfer in a heat exchanger usually involves convection in each fluid and conduction through the wall separating the two fluids. In the analysis of heat exchangers, it is convenient to work with an *overall heat transfer coefficient U* or a *total thermal resistance R*, expressed as

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{\text{wall}} + \frac{1}{h_oA_o}$$

where the subscripts i and o stand for the inner and outer surfaces of the wall that separates the two fluids, respectively. When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the relation simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The effects of fouling on both the inner and the outer surfaces of the tubes of a heat exchanger can be accounted for by

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R$$

$$= \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln (D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

where $A_i = \pi D_iL$ and $A_o = \pi D_oL$ are the areas of the inner and outer surfaces and $R_{f,i}$ and $R_{f,o}$ are the fouling factors at those surfaces.

In a well-insulated heat exchanger, the rate of heat transfer from the hot fluid is equal to the rate of heat transfer to the cold one. That is,

$$\dot{Q} = \dot{m}_c c_{pc}(T_{c,\text{out}} - T_{c,\text{in}}) = C_c(T_{c,\text{out}} - T_{c,\text{in}})$$

and

$$\dot{Q} = \dot{m}_h c_{ph}(T_{h,\text{in}} - T_{h,\text{out}}) = C_h(T_{h,\text{in}} - T_{h,\text{out}})$$

where the subscripts c and h stand for the cold and hot fluids, respectively, and the product of the mass flow rate and the specific heat of a fluid $\dot{m}c_p$ is called the *heat capacity rate*.

Of the two methods used in the analysis of heat exchangers, the *log mean temperature difference* (or LMTD) method is best suited for determining the size of a heat exchanger when all the inlet and the outlet temperatures are known. The *effectiveness–NTU* method is best suited to predict the outlet temperatures of the hot and cold fluid streams in a specified heat exchanger. In the LMTD method, the rate of heat transfer is determined from

$$\dot{Q} = UA_s\,\Delta T_{\text{lm}}$$

where

$$\Delta T_{\text{lm}} = \frac{\Delta T_1 - \Delta T_2}{\ln (\Delta T_1/\Delta T_2)}$$

is the *log mean temperature difference,* which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature differences between the two fluids at the two ends (inlet and outlet) of the heat exchanger. For cross-flow and multipass shell-and-tube heat exchangers, the logarithmic mean temperature difference is related to the counter-flow one $\Delta T_{\text{lm, CF}}$ as

$$\Delta T_{\text{lm}} = F\,\Delta T_{\text{lm, CF}}$$

where F is the *correction factor,* which depends on the geometry of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams.

The *effectiveness* of a heat exchanger is defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{\max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

where

$$\dot{Q}_{\max} = C_{\min}(T_{h,\text{in}} - T_{c,\text{in}})$$

and $C_{\min}$ is the smaller of $C_h = \dot{m}_h c_{ph}$ and $C_c = \dot{m}_c c_{pc}$. The effectiveness of heat exchangers can be determined from effectiveness relations or charts.

The selection or design of a heat exchanger depends on several factors such as the heat transfer rate, cost, pressure drop, size, weight, construction type, materials, and operating environment.

The implementation cost of installing films is

$$\text{Implementation cost} = (\$20/\text{m}^2)(40\ \text{m}^2) = \$800$$

This gives a simple payback period of

$$\text{Simple payback period} = \frac{\text{Implementation cost}}{\text{Annual cost savings}} = \frac{\$800}{\$92/\text{year}} = \mathbf{8.7\ years}$$

Discussion The reflective film will pay for itself in this case in about nine years. This may be unacceptable to most manufacturers since they are not usually interested in any energy conservation measure that does not pay for itself within three years. But the enhancement in thermal comfort and thus the resulting increase in productivity often makes it worthwhile to install reflective film.

SUMMARY

Radiation propagates in the form of electromagnetic waves. The *frequency* ν and *wavelength* λ of electromagnetic waves in a medium are related by $\lambda = c/\nu$, where c is the speed of propagation in that medium. All matter continuously emits *thermal radiation* as a result of vibrational and rotational motions of molecules, atoms, and electrons of a substance.

A *blackbody* is defined as a perfect emitter and absorber of radiation. At a specified temperature and wavelength, no surface can emit more energy than a blackbody. A blackbody absorbs *all* incident radiation, regardless of wavelength and direction. The radiation energy emitted by a blackbody per unit time and per unit surface area is called the *blackbody emissive power* E_b and is expressed by the *Stefan–Boltzmann law* as

$$E_b(T) = \sigma T^4$$

where $\sigma = 5.670 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ is the *Stefan–Boltzmann constant* and T is the absolute temperature of the surface in K. At any specified temperature, the spectral blackbody emissive power $E_{b\lambda}$ increases with wavelength, reaches a peak, and then decreases with increasing wavelength. The wavelength at which the peak occurs for a specified temperature is given by *Wien's displacement law* as

$$(\lambda T)_{\text{max power}} = 2897.8\ \mu\text{m}\cdot\text{K}$$

The *blackbody radiation function* f_λ represents the fraction of radiation emitted by a blackbody at temperature T in the wavelength band from $\lambda = 0$ to λ. The fraction of radiation energy emitted by a blackbody at temperature T over a finite wavelength band from $\lambda = \lambda_1$ to $\lambda = \lambda_2$ is determined from

$$f_{\lambda_1 - \lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$

where $f_{\lambda_1}(T)$ and $f_{\lambda_2}(T)$ are the blackbody radiation functions corresponding to $\lambda_1 T$ and $\lambda_2 T$, respectively.

The magnitude of a viewing angle in space is described by solid angle expressed as $d\omega = dA_n/r^2$. The *radiation intensity* for emitted radiation $I_e(\theta, \phi)$ is defined as the rate at which radiation energy is emitted in the (θ, ϕ) direction per unit area normal to this direction and per unit solid angle about this direction. The *radiation flux* for emitted radiation is the *emissive power* E, and is expressed as

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_e(\theta, \phi) \cos\theta \sin\theta\, d\theta\, d\phi$$

For a *diffusely emitting* surface, intensity is independent of direction and thus

$$E = \pi I_e$$

For a blackbody, we have

$$E_b = \pi I_b \qquad \text{and} \qquad I_b(T) = \frac{E_b(T)}{\pi} = \frac{\sigma T^4}{\pi}$$

The radiation flux incident on a surface from all directions is *irradiation* G, and for diffusely incident radiation of intensity I_i it is expressed as

$$G = \pi I_i$$

The rate at which radiation energy leaves a unit area of a surface in all directions is *radiosity* J, and for a surface that is both a diffuse emitter and a diffuse reflector it is expressed as

$$J = \pi I_{e+r}$$

where I_{e+r} is the sum of the emitted and reflected intensities. The spectral emitted quantities are related to total quantities as

$$I_e = \int_0^\infty I_{\lambda,e}\, d\lambda \qquad \text{and} \qquad E = \int_0^\infty E_\lambda\, d\lambda$$

They reduce for a diffusely emitting surface and for a blackbody to

$$E_\lambda = \pi I_{\lambda, e} \qquad \text{and} \qquad E_{b\lambda}(\lambda, T) = \pi I_{b\lambda}(\lambda, T)$$

The *emissivity* of a surface represents the ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature. Different emissivities are defined as

Spectral directional emissivity:

$$\varepsilon_{\lambda,\theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda, e}(\lambda, \theta, \phi, T)}{I_{b\lambda}(\lambda, T)}$$

Total directional emissivity:

$$\varepsilon_\theta(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

Spectral hemispherical emissivity:

$$\varepsilon_\lambda(\lambda, T) = \frac{E_\lambda(\lambda, T)}{E_{b\lambda}(\lambda, T)}$$

Total hemispherical emissivity:

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\displaystyle\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{b\lambda}(\lambda, T)\, d\lambda}{\sigma T^4}$$

Emissivity can also be expressed as a step function by dividing the spectrum into a sufficient number of *wavelength bands* of constant emissivity as, for example,

$$\varepsilon(T) = \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T)$$

The *total hemispherical emissivity* ε of a surface is the average emissivity over all directions and wavelengths.

When radiation strikes a surface, part of it is absorbed, part of it is reflected, and the remaining part, if any, is transmitted. The fraction of incident radiation (intensity I_i or irradiation G) absorbed by the surface is called the *absorptivity,* the fraction reflected by the surface is called the *reflectivity,* and the fraction transmitted is called the *transmissivity.* Various absorptivities, reflectivities, and transmissivities for a medium are expressed as

$$\alpha_{\lambda,\theta}(\lambda, \theta, \phi) = \frac{I_{\lambda,\text{abs}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)} \text{ and } \rho_{\lambda,\theta}(\lambda, \theta, \phi) = \frac{I_{\lambda,\text{ref}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)}$$

$$\alpha_\lambda(\lambda) = \frac{G_{\lambda,\text{abs}}(\lambda)}{G_\lambda(\lambda)}, \rho_\lambda(\lambda) = \frac{G_{\lambda,\text{ref}}(\lambda)}{G_\lambda(\lambda)}, \text{ and } \tau_\lambda(\lambda) = \frac{G_{\lambda,\text{tr}}(\lambda)}{G_\lambda(\lambda)}$$

$$\alpha = \frac{G_{\text{abs}}}{G}, \rho = \frac{G_{\text{ref}}}{G}, \text{ and } \tau = \frac{G_{\text{tr}}}{G}$$

The consideration of wavelength and direction dependence of properties makes radiation calculations very complicated. Therefore, the *gray* and *diffuse* approximations are commonly utilized in radiation calculations. A surface is said to be *diffuse* if its properties are independent of direction and *gray* if its properties are independent of wavelength.

The sum of the absorbed, reflected, and transmitted fractions of radiation energy must be equal to unity,

$$\alpha + \rho + \tau = 1$$

For *opaque* surfaces ($\tau = 0$): $\alpha + \rho = 1$

For surfaces with *no reflectance* ($\rho = 0$): $\alpha + \tau = 1$

Surfaces are usually assumed to reflect in a perfectly *specular* or *diffuse* manner for simplicity. In *specular* (or *mirrorlike*) *reflection,* the angle of reflection equals the angle of incidence of the radiation beam. In *diffuse reflection,* radiation is reflected equally in all directions. Reflection from smooth and polished surfaces approximates specular reflection, whereas reflection from rough surfaces approximates diffuse reflection. *Kirchhoff's law* of radiation is expressed as

$$\varepsilon_{\lambda,\theta}(T) = \alpha_{\lambda,\theta}(T), \quad \varepsilon_\lambda(T) = \alpha_\lambda(T), \qquad \text{and} \qquad \varepsilon(T) = \alpha(T)$$

Gas molecules and the suspended particles in the atmosphere emit radiation as well as absorbing it. The atmosphere can be treated as a blackbody at some lower fictitious temperature, called the *effective sky temperature* T_{sky} that emits an equivalent amount of radiation energy,

$$G_{\text{sky}} = \sigma T_{\text{sky}}^4$$

The net rate of radiation heat transfer to a surface exposed to solar and atmospheric radiation is determined from an energy balance expressed as

$$\dot{q}_{\text{net, rad}} = \alpha_s G_{\text{solar}} + \varepsilon\sigma(T_{\text{sky}}^4 - T_s^4)$$

where T_s is the surface temperature in K, and ε is the surface emissivity at room temperature.

REFERENCES AND SUGGESTED READING

1. American Society of Heating, Refrigeration, and Air Conditioning Engineers, *Handbook of Fundamentals*, Atlanta, ASHRAE, 1993.
2. A. G. H. Dietz. "Diathermanous Materials and Properties of Surfaces." In *Space Heating with Solar Energy,* R. W. Hamilton, (Ed.) Cambridge, MA: MIT Press, 1954.
3. J. A. Duffy and W. A. Beckman. *Solar Energy Thermal Process.* New York: John Wiley & Sons, 1974.
4. H. C. Hottel. "Radiant Heat Transmission." In *Heat Transmission.* 3rd ed., W. H. McAdams, (Ed.) New York: McGraw-Hill, 1954.
5. M. F. Modest. *Radiative Heat Transfer.* New York: McGraw-Hill, 1993.

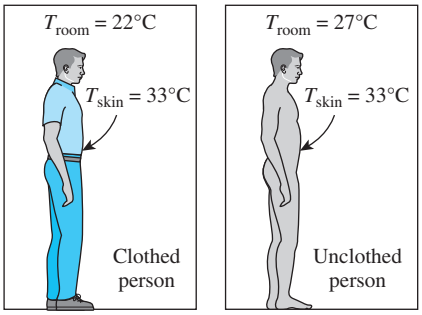


FIGURE 13–48
Clothing serves as insulation, and the room temperature needs to be raised when a person is unclothed to maintain the same comfort level.

To maintain thermal comfort after taking the clothes off, the skin temperature of the person and the rate of heat transfer from him must remain the same. Then setting the equation above equal to 95.2 W gives

$$T_{ambient} = \mathbf{26.9°C}$$

Therefore, the air temperature needs to be raised from 22 to 26.9°C to ensure that the person feels comfortable in the room after he takes his clothes off (Fig. 13–48).

Discussion Note that the effect of clothing on latent heat is assumed to be negligible in the solution above. We also assumed the surface area of the clothed and unclothed person to be the same for simplicity, and these two effects should counteract each other.

SUMMARY

Radiaton heat transfer between surfaces depends on the orientation of the surfaces relative to each other. In a radiation analysis, this effect is accounted for by the geometric parameter *view factor*. The *view factor* from a surface i to a surface j is denoted by $F_{i \to j}$ or F_{ij}, and is defined as the fraction of the radiation leaving surface i that strikes surface j directly. The view factors between differential and finite surfaces are expressed as

$$dF_{dA_1 \to dA_2} = \frac{\dot{Q}_{dA_1 \to dA_2}}{\dot{Q}_{dA_1}} = \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$

$$F_{dA_1 \to A_2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_2$$

$$F_{12} = F_{A_1 \to A_2} = \frac{\dot{Q}_{A_1 \to A_2}}{\dot{Q}_{A_1}} = \frac{1}{A_1} \int_{A_2} \int_{A_1} \frac{\cos\theta_1 \cos\theta_2}{\pi r^2} dA_1\, dA_2$$

where r is the distance between dA_1 and dA_2, and θ_1 and θ_2 are the angles between the normals of the surfaces and the line that connects dA_1 and dA_2.

The view factor $F_{i \to i}$ represents the fraction of the radiation leaving surface i that strikes itself directly; $F_{i \to i} = 0$ for *plane* or *convex* surfaces and $F_{i \to i} \neq 0$ for *concave* surfaces. For view factors, the *reciprocity rule* is expressed as

$$A_i F_{i \to j} = A_j F_{j \to i}$$

The sum of the view factors from surface i of an enclosure to all surfaces of the enclosure, including to itself, must equal unity. This is known as the *summation rule* for an enclosure. The *superposition rule* is expressed as the view factor from a surface i to a surface j is equal to the sum of the view factors from surface i to the parts of surface j. The symmetry rule is expressed as if the surfaces j and k are symmetric about the surface i then $F_{i \to j} = F_{i \to k}$.

The rate of net radiation heat transfer between two *black* surfaces is determined from

$$\dot{Q}_{1 \to 2} = A_1 F_{1 \to 2} \sigma (T_1^4 - T_2^4)$$

The *net* radiation heat transfer from any surface i of a *black* enclosure is determined by adding up the net radiation heat transfers from surface i to each of the surfaces of the enclosure:

$$\dot{Q}_i = \sum_{j=1}^{N} \dot{Q}_{i \to j} = \sum_{j=1}^{N} A_i F_{i \to j} \sigma (T_i^4 - T_j^4)$$

The total radiation energy leaving a surface per unit time and per unit area is called the *radiosity* and is denoted by J. The *net* rate of radiation heat transfer from a surface i of surface area A_i is expressed as

$$\dot{Q}_i = \frac{E_{bi} - J_i}{R_i}$$

where

$$R_i = \frac{1 - \varepsilon_i}{A_i \varepsilon_i}$$

is the *surface resistance* to radiation. The *net* rate of radiation heat transfer from surface i to surface j can be expressed as

$$\dot{Q}_{i \to j} = \frac{J_i - J_j}{R_{i \to j}}$$

where

$$R_{i \to j} = \frac{1}{A_i F_{i \to j}}$$

is the *space resistance* to radiation. The *network method* is applied to radiation enclosure problems by drawing a surface resistance associated with each surface of an enclosure and

connecting them with space resistances. Then the problem is solved by treating it as an electrical network problem where the radiation heat transfer replaces the current and the radiosity replaces the potential. The *direct method* is based on the following two equations:

Surfaces with specified net heat transfer rate $\dot{Q}_i$

$$\dot{Q}_i = A_i \sum_{j=1}^{N} F_{i\to j}(J_i - J_j)$$

Surfaces with specified temperature T_i

$$\sigma T_i^4 = J_i + \frac{1-\varepsilon_i}{\varepsilon_i} \sum_{j=1}^{N} F_{i\to j}(J_i - J_j)$$

The first and the second groups of equations give N linear algebraic equations for the determination of the N unknown radiosities for an N-surface enclosure. Once the radiosities J_1, J_2, . . . , J_N are available, the unknown surface temperatures and heat transfer rates can be determined from the equations just shown.

The net rate of radiation transfer between any two gray, diffuse, opaque surfaces that form an enclosure is given by

$$\dot{Q}_{12} = \frac{\sigma(T_1^4 - T_2^4)}{\dfrac{1-\varepsilon_1}{A_1\varepsilon_1} + \dfrac{1}{A_1 F_{12}} + \dfrac{1-\varepsilon_2}{A_2\varepsilon_2}}$$

Radiation heat transfer between two surfaces can be reduced greatly by inserting between the two surfaces thin, high-reflectivity (low-emissivity) sheets of material called *radiation shields.* Radiation heat transfer between two large parallel plates separated by N radiation shields is

$$\dot{Q}_{12,\,N\text{ shields}} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\dfrac{1}{\varepsilon_1} + \dfrac{1}{\varepsilon_2} - 1\right) + \ldots + \left(\dfrac{1}{\varepsilon_{N,1}} + \dfrac{1}{\varepsilon_{N,2}} - 1\right)}$$

The radiation effect in temperature measurements can be properly accounted for by

$$T_f = T_{\text{th}} + \frac{\varepsilon\sigma(T_{\text{th}}^4 - T_w^4)}{h}$$

where T_f is the actual fluid temperature, T_{th} is the temperature value measured by the thermometer, and T_w is the temperature of the surrounding walls, all in K.

Gases with asymmetric molecules such as H_2O, CO_2 CO, SO_2, and hydrocarbons H_nC_m participate in the radiation process by absorption and emission. The *spectral transmissivity, absorptivity,* and *emissivity* of a medium are expressed as

$$\tau_\lambda = e^{-\kappa_\lambda L},\quad \alpha_\lambda = 1 - \tau_\lambda = 1 - e^{-\kappa_\lambda L},\qquad \text{and}$$
$$\varepsilon_\lambda = \alpha_\lambda = 1 - e^{-\kappa\lambda L}$$

where κ_λ is the *spectral absorption coefficient* of the medium.

The emissivities of H_2O and CO_2 gases are given in Figure 13–36 for a total pressure of $P = 1$ atm. Emissivities at other pressures are determined from

$$\varepsilon_w = C_w\varepsilon_{w,\,1\text{ atm}} \qquad \text{and} \qquad \varepsilon_c = C_c\varepsilon_{c,\,1\text{ atm}}$$

where C_w and C_c are the *pressure correction factors.* For gas mixtures that contain both of H_2O and CO_2, the emissivity is determined from

$$\varepsilon_g = \varepsilon_c + \varepsilon_w - \Delta\varepsilon = C_c\varepsilon_{c,\,1\text{ atm}} + C_w\varepsilon_{w,\,1\text{ atm}} - \Delta\varepsilon$$

where $\Delta\varepsilon$ is the *emissivity correction factor,* which accounts for the overlap of emission bands. The gas absorptivities for radiation emitted by a source at temperature T_s are determined similarly from

$$\alpha_g = \alpha_c + \alpha_w - \Delta\alpha$$

where $\Delta\alpha = \Delta\varepsilon$ at the source temperature T_s and

$$CO_2: \qquad \alpha_c = C_c \times (T_g/T_s)^{0.65} \times \varepsilon_c(T_s,\, P_c LT_s/T_g)$$
$$H_2O: \qquad \alpha_w = C_w \times (T_g/T_s)^{0.45} \times \varepsilon_w(T_s,\, P_w\, LT_s/T_g)$$

The rate of radiation heat transfer between a gas and a surrounding surface is

Black enclosure:

$$\dot{Q}_{\text{net}} = A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)$$

Gray enclosure, with $\varepsilon_s > 0.7$:

$$\dot{Q}_{\text{net, gray}} = \frac{\varepsilon_s + 1}{2} A_s\sigma(\varepsilon_g T_g^4 - \alpha_g T_s^4)$$

REFERENCES AND SUGGESTED READING

1. D. K. Edwards. *Radiation Heat Transfer Notes.* Washington, D.C.: Hemisphere, 1981.

2. D. K. Edwards and R. Matavosian. "Scaling Rules for Total Absorptivity and Emissivity of Gases." *Journal of Heat Transfer* 106 (1984), pp. 684–689.

3. D. K. Edwards and R. Matavosian. "Emissivity Data for Gases." Section 5.5.5, in *Hemisphere Handbook of Heat Exchanger Design,* G. F. Hewitt, (Ed.) New York: Hemisphere, 1990.

4. D. C. Hamilton and W. R. Morgan. "Radiation Interchange Configuration Factors." National Advisory Committee for Aeronautics, Technical Note 2836, 1952.