ChE641 Mathematical Methods in Chemical Engineering

Assignment 6 **Due Date: 20 November 2020**
Linear Algebra – Solution of Linear System of Equations

1. Use the Fredholm alternative theorem to determine under what conditions does the following linear system has a solution.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

If $[b_1, b_2, b_3] = [1, 1, 7/3]$, is there a solution? If so, use Gauss elimination to determine the solution. What is the rank of the coefficient matrix? Is the solution unique?

2. Show the solvability conditions that the Fredholm alternative theorem requires for $\mathbf{b}$ when the coefficient matrix is

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Find a $\mathbf{b}$ satisfying the solvabilty conditions and find the most general solution to $\mathbf{Ax} = \mathbf{b}$ for this case.

3. Find the *general* solution to $\mathbf{Ax} = \mathbf{b}$ for the following:

(a)

$$\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 6 & -10 & 10 & 4 \\ -2 & 4 & -3 & -1 \\ 2 & -2 & 4 & 2 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 1 \\ -3 \\ 2 \\ 3 \end{bmatrix}$$

(b)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 & 0 \\ 2 & 1 & 2 & 1 & 1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

(c)

$$\mathbf{A} = \begin{bmatrix} 3 & -1 & 1 \\ 0 & 2 & 2 \\ 6 & -4 & 0 \\ 3 & 1 & 3 \\ 3 & -3 & -1 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 6 \\ 4 \\ 8 \\ 10 \\ 2 \end{bmatrix}$$

4. Consider the set of equations

$$\begin{aligned} x - 3y &= -2 \\ 2x + y &= 3 \\ 3x - 2y &= \alpha \end{aligned}$$

(a) Are there values of α for which this set has no solution? If so, what are they?

(b) Are there values of α for which this set has a solution? If so, given an example.

5. For what values of k will the following system have a nontrivial solution?

$$\begin{aligned} 2x + ky + z &= 0 \\ (k-1)x - y - 2z &= 0 \\ 4x + y + 4z &= 0 \end{aligned}$$

6. Consider the augmented matrix

$$\mathbf{A}|\mathbf{b} = \begin{bmatrix} 1 & 3 & -8 & 2 \\ 1 & -9 & -10 & -3 \\ -1 & 3 & 9 & 0 \end{bmatrix}.$$

(a) Use simple Gauss elimination to find the rank of $[\mathbf{A}|\mathbf{b}]$ and $\mathbf{A}$.

(b) How many of the column vectors of $[\mathbf{A}|\mathbf{b}]$ are linearly independent? How many of the rows are linearly independent?

(c) If the problem $\mathbf{Ax} = \mathbf{b}$ has a solution, find the most general one. If there is no solution, why not?

(d) Is there a solution to $\mathbf{A}^{\dagger}\mathbf{z} = \mathbf{0}$? What are the implications of the answer to this question to the solvability of the linear system considered above?