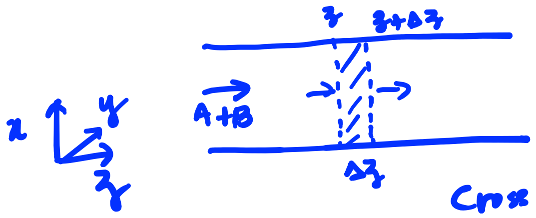
Lecture 4: Mass Transfer & Its Applications

Mass Balance:

Mixture of components A & B flowing in 1-D.



Rxn: $A \rightarrow B$

Cross section area $= \Delta y \Delta x$ (uniform)

From Mass Balance:

$$\begin{pmatrix}\text{Rate of accumulation}\\ \text{of } A\end{pmatrix} = \begin{pmatrix}\text{Rate of mass}\\ \text{of } A \text{ in}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{mass of } A\\ \text{out}\end{pmatrix} + \underbrace{\begin{pmatrix}\text{Rate of generation}\\ \text{of } A\end{pmatrix}}_{\text{Rxn term}}$$

$$\frac{\partial}{\partial t}\left(\rho_A \Delta x \Delta y \Delta z\right) = \overbrace{n_A \Delta x \Delta y\Big|_{z} - n_A \Delta x \Delta y\Big|_{z+\Delta z}}^{\text{Flux}} + M_A \dot{R}_A \Delta x \Delta y \Delta z$$

ρ_A: mass conc.

$\frac{1}{s} \cdot (kg/m^3) \cdot m^3 = kg/s$

$n_A = n_{A_z}$

$\frac{kg}{m^2 \cdot s} \times m^2 = kg/s$

M_A: molar mass of A

$\dot{R}_A$ = Rate of consumption of A

$\frac{kg}{mol} \times \frac{mol}{m^3 \cdot s} \times m^3 = kg/s$

$$\boxed{\frac{\partial \rho_A}{\partial t} = -\frac{\partial n_A}{\partial z} + \dot{R}_A M_A} \quad -(i)$$

(Divide by previous eq^n by $\Delta x \Delta y \Delta z$ & assume $\Delta z \rightarrow 0$)

Similarly, for component B

$$\frac{\partial \rho_B}{\partial t} = -\frac{\partial n_B}{\partial z} + \dot{R}_B M_B \quad -(ii)$$

M_B: Molar mass of component B

(i)+(ii): $$\frac{\partial}{\partial t}\left(\rho_A + \rho_B\right) = -\frac{\partial}{\partial z}\left(n_A + n_B\right)$$

$\rho = \rho_A + \rho_B$

$$\begin{bmatrix} A \rightarrow B \\ \dot{R}_A M_A + \dot{R}_B M_B = 0 \end{bmatrix}$$

$\rho_A + \rho_B = \underline{\rho}$; ρ : Solution/mixture density

$n_A + n_B = \underline{n}$ (Total mass flux)

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial z}(\rho u) \Rightarrow \boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(\rho u) = 0}$$

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla.(\rho u) = 0} \Leftarrow \underline{\text{In 3-D}}$$

$$\frac{\partial \rho}{\partial t} + \underline{\frac{\partial}{\partial z}(\rho u)} = 0 \Rightarrow \frac{\partial \rho}{\partial t} + \rho\frac{\partial u}{\partial z} + u\frac{\partial \rho}{\partial z} = 0$$

If you assume that density is constant :

$$\underline{\frac{\partial \rho}{\partial z}} = 0 \quad ; \quad \underline{\frac{\partial \rho}{\partial t}} = 0$$

$$\frac{\partial u}{\partial z} = 0 \quad (\text{in 1-D})$$

In 3-D: $\underline{\nabla.u = 0}$

Lets use equation (i) :

$$\frac{\partial \rho_A}{\partial t} = -\frac{\partial}{\partial z}(n_A) + \dot{R}_A M_A \qquad \left[\frac{kg}{s}\right]$$

$$\underline{\rho_A} : \frac{kg}{m^3} \times \frac{mol}{mol} = \frac{kg}{mol} \times \frac{mol}{m^3} = \underline{M_A C_A}$$

$$\underline{n_A} : \frac{kg}{m^2.s} \times \frac{mol}{mol} = \frac{kg}{mol} \times \frac{mol}{m^2.s} = M_A N_A$$

$$\cancel{\partial \rho_A} \quad \frac{\partial}{\partial t}(M_A C_A) = -\frac{\partial}{\partial z}(M_A N_A) + \dot{R}_A M_A$$

$$\frac{\partial C_A}{\partial t} = -\frac{\partial (N_A)}{\partial z} + \dot{R}_A$$ ← Molar terms $\left[\frac{mol}{s}\right]$

$$N_A = \frac{C_A}{c}\underbrace{(N_A + N_B)}_{\substack{N \\ \text{(Total flux)}}} - D_{AB}\frac{dC_A}{dz}$$

$$N \equiv N_A + N_B = cU$$ ← molar average velocity

↑ total concentration

$$= (C_A + C_B)\left(\frac{C_A u_A + C_B u_B}{C_A + C_B}\right)$$

$$N = \underbrace{C_A u_A + C_B u_B}_{\text{Total flux}}$$

If you assume that the mixture is stationary,
No bulk motion ⇒ $U = 0$

$$N_A = -D_{AB}\frac{dC_A}{dz}$$

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2} + \dot{R}_A$$

Assume that there is no reaction/ generation/ consumption of A ; then
$\dot{R}_A = 0$

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$ ← Unsteady state sol^n

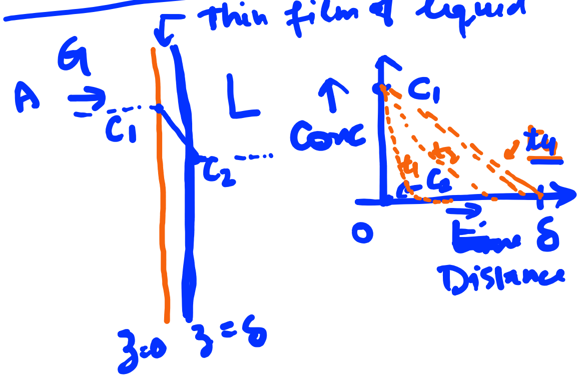
S1 Assume, steady state assumption is valid

$$\frac{\partial C_A}{\partial t} = 0$$

The above equation reduces to:

$$D_{AB} \frac{\partial^2 C_A}{\partial z^2} = 0$$

Steady vs Unsteady state



$$\frac{\delta^2}{D_{AB}\, t_0} = \text{Dimensionless number}$$

t_0: observation time

$$\frac{m^2}{\frac{m^2}{s} \cdot s}$$

Unsteady: $\delta^2 / D_{AB}\, t_0 > 1$

Steady: $\delta^2 / D_{AB}\, t_0 < 1$