Symmetry

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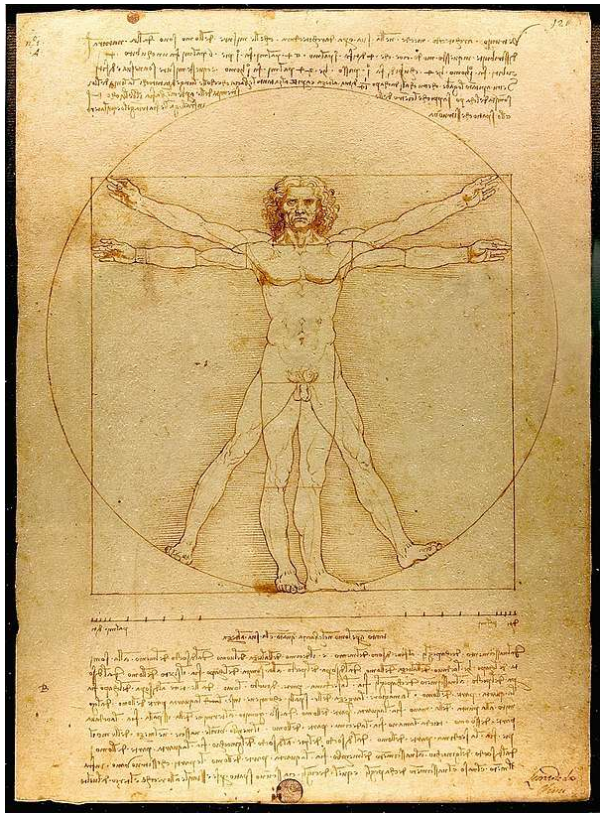
Introduction

- Symmetry comes from “symmetria” in Greek
- Harmony, proportion and balance
- Perception of beauty
- In mathematical terms, it is invariant with respect to certain operations



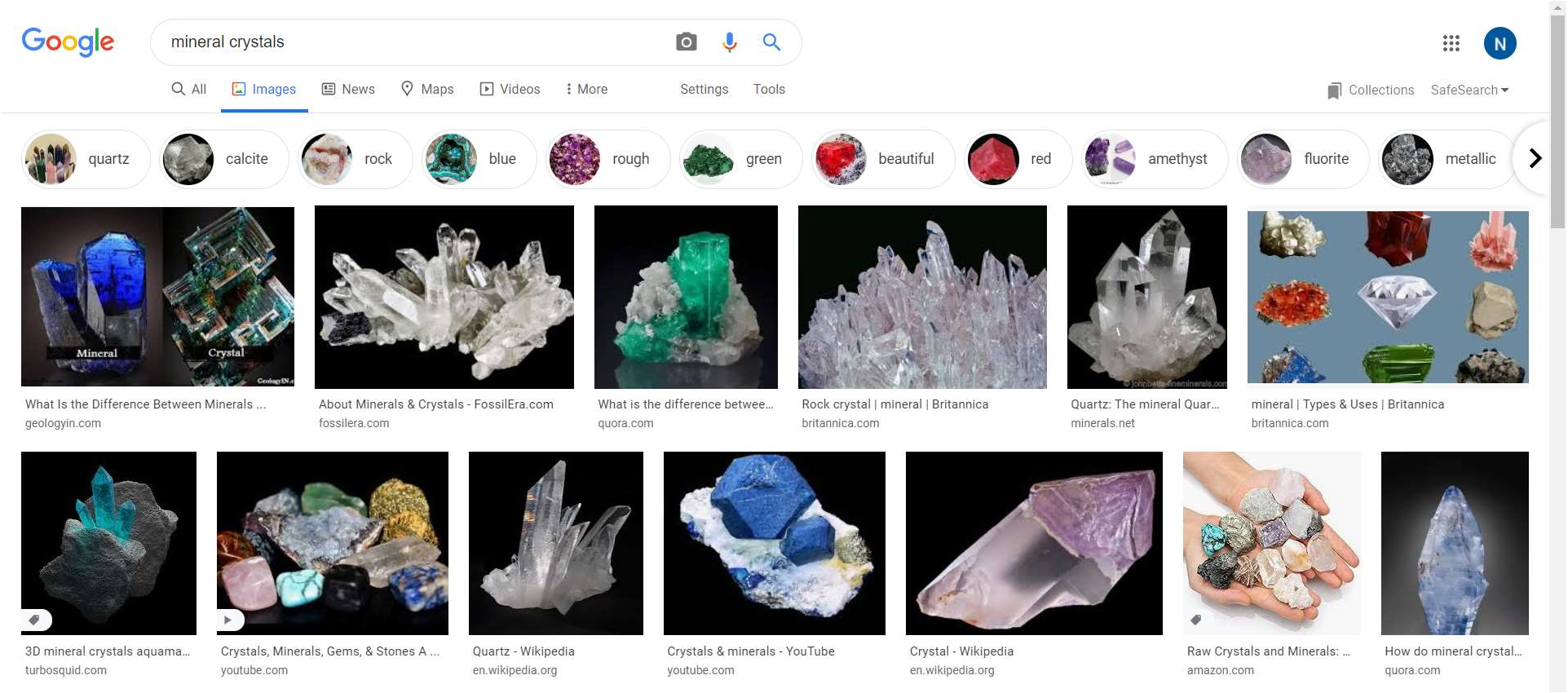
https://en.wikipedia.org/wiki/Vitruvian_Man

- Symmetry is the preservation of form and configuration about a point, line, or plane
- Nature full of symmetry
- Art mimicked nature

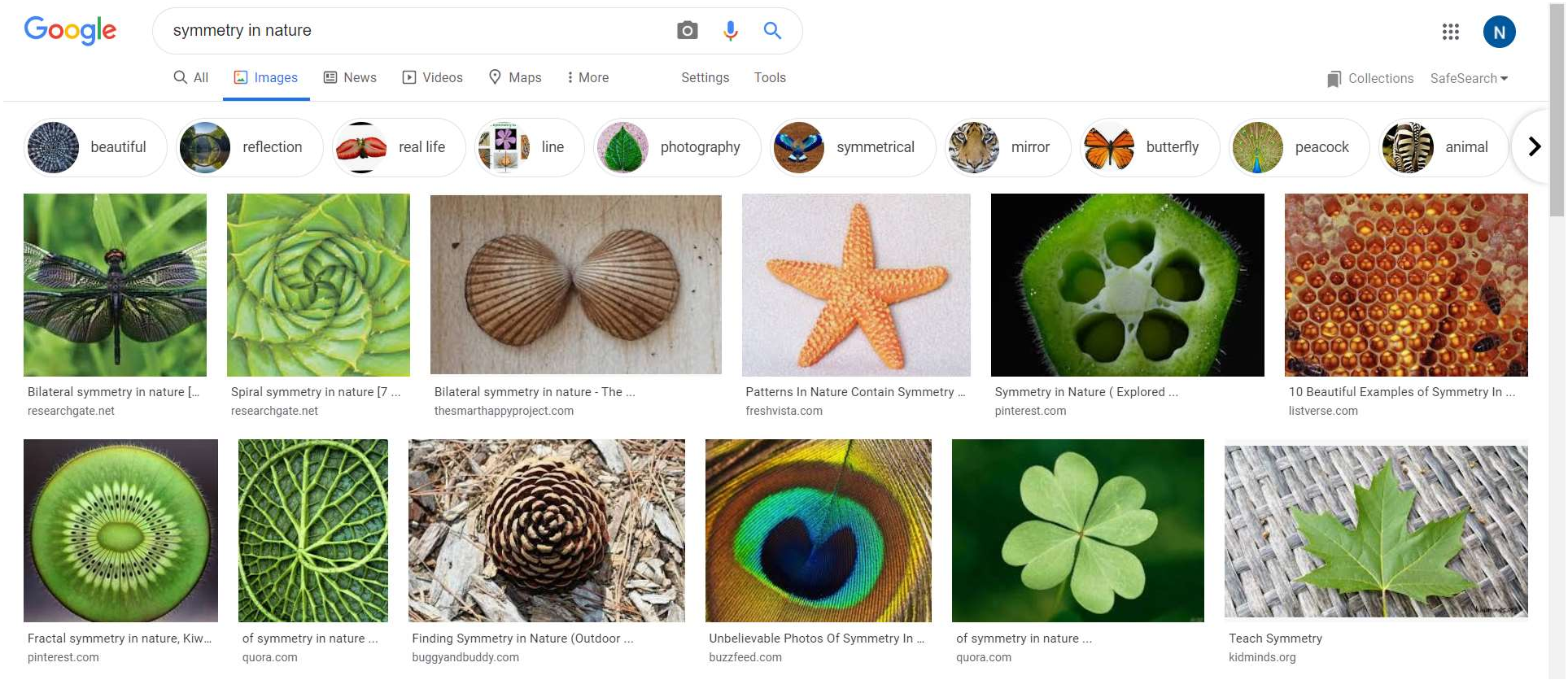
- Plants, flowers, animals, humans
- Architecture, Dance, aerobics
- Symmetry creates perception of beauty
- Symmetry + Proportion = Beauty ?

http://www.redbubble.com/people/marjoleink/works/4727248-masjed-e-shah-symmetry

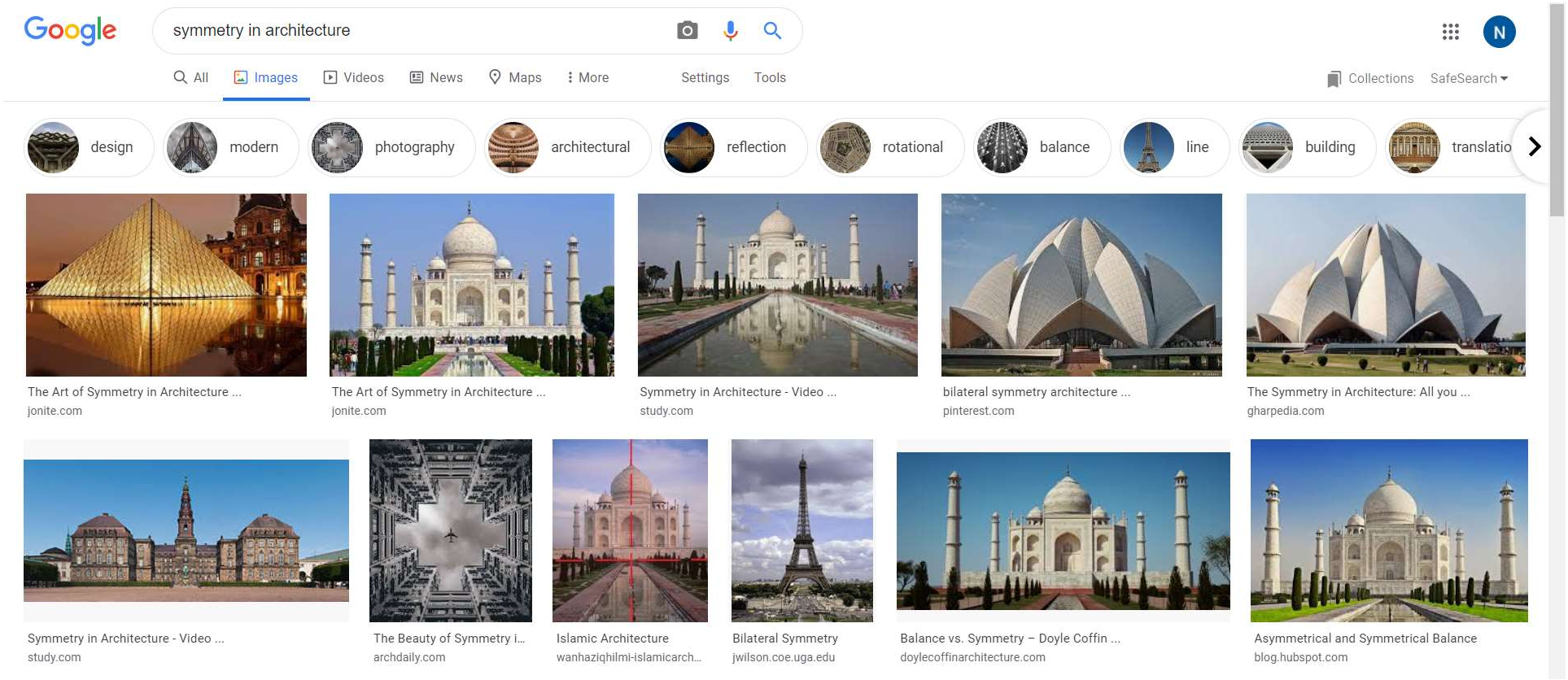
Mineral crystals



Symmetry in nature

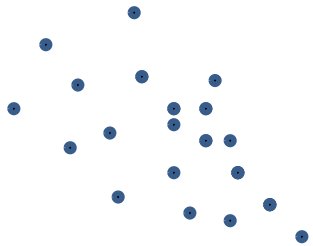


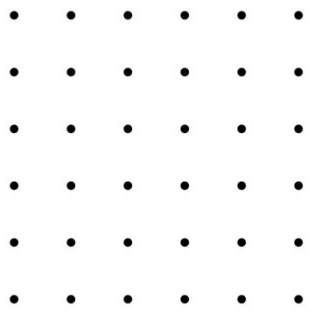
Symmetry in architecture



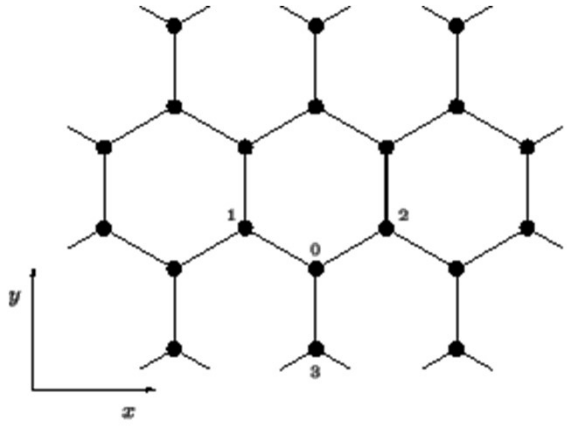
- Inorganic crystals like minerals have shapes that reflect the intrinsic symmetry
- Symmetry is self-similarity after transformation
- Symmetry element is an imaginary point, line, plane about which symmetry operation takes place
- Symmetry operation is a permutation that brings atoms/molecules/pattern into self coincidence

- Repetition in space (and time)
- Space can be divided into lattice points
- 1D lattice, 2D lattice and 3D lattice
- Periodic with identical neighbourhood

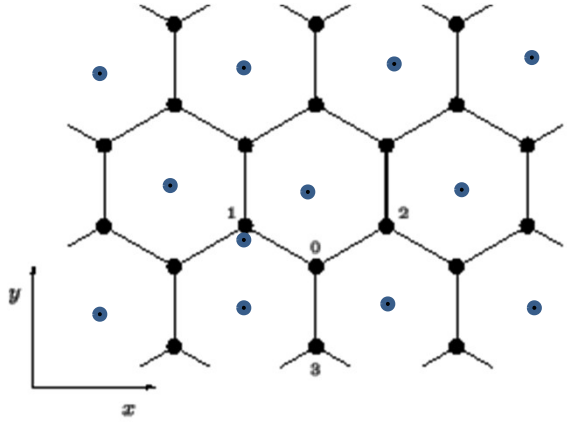




- Periodic set of points
- Identical neighbourhood

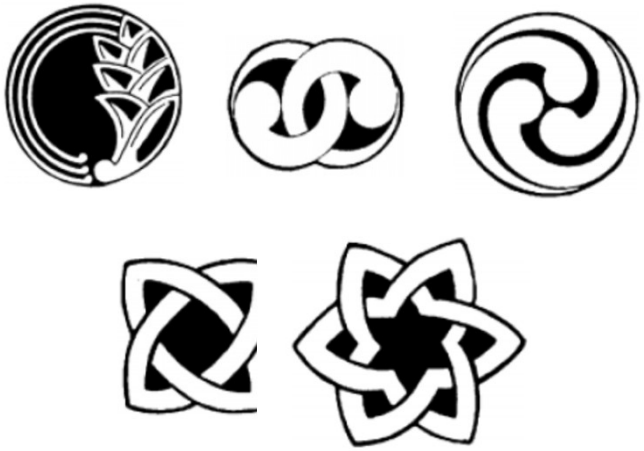


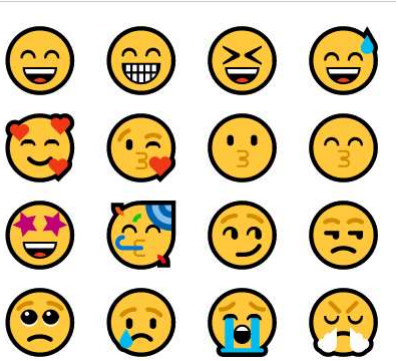
Not a lattice



Lattice

- Motif or Basis is the repeating unit of a pattern
- Atom, molecule,, group of atom or even an emoji
- Can be an asymmetric unit or have some symmetry





https://getemoji.com/

Lattice

\+ = Crystal structure

Motif

From patterns in tiling to minerals, metals and alloys and semiconductors

- Crystallography is the study of pattern and their symmetry
- Symmetry is ubiquitous
- Let intuition take over formal training
- Find symmetry in English alphabets

A B H N F

- Symmetry in 2D
  - Translations
  - Rotations
  - Reflections
  - Glide reflections

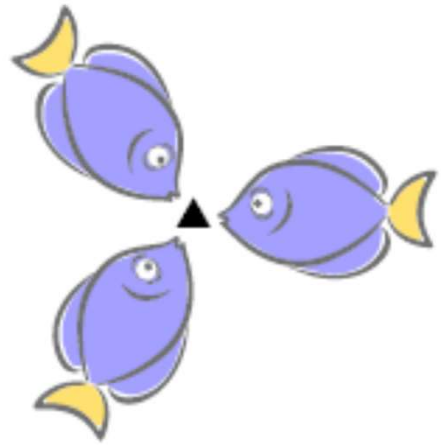
Identity or no symmetry is also a symmetry but it is redundant

Translation

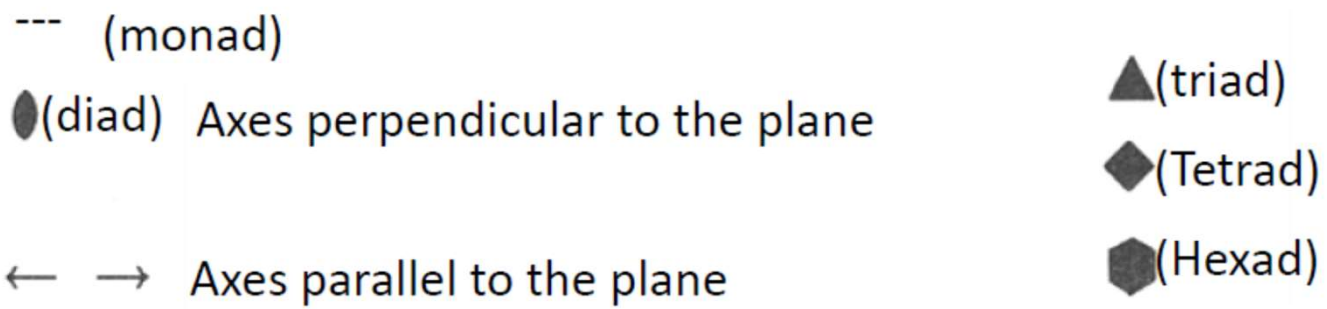
- Move the asymmetric unit in a specific direction by a specific magnitude
- 1 step in 1D, 2 steps in 2D and 3 steps in 3D
- Results in 1D, 2D and 3D crystal

Rotation

- Rotate the asymmetric unit (all the points in the asymmetric unit) about an axis
- Axis and angle of rotation
- Rotation axis only invariant point (does not move)

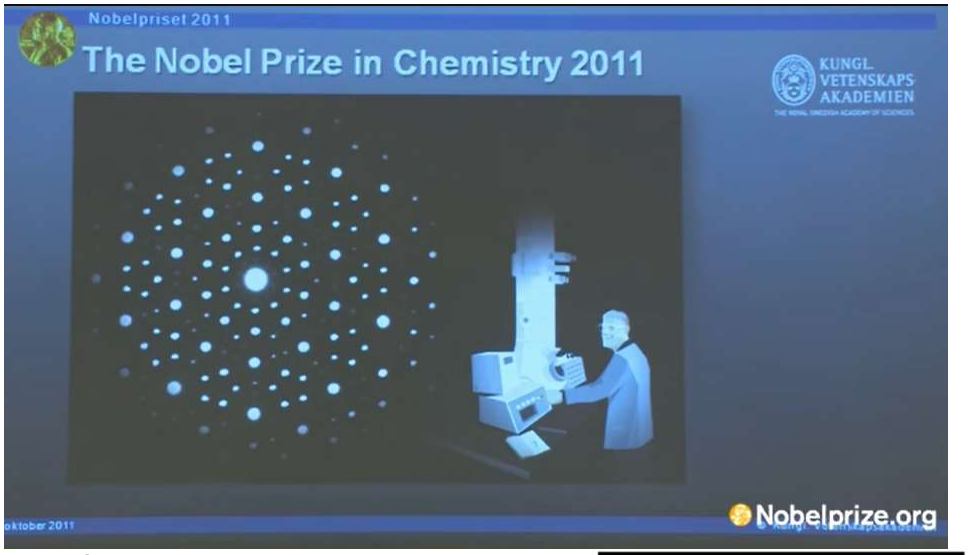


	Crystals	Molecules
	Hermaan-Mauguin symbol	Schoenflies symbol
1 fold rotation axis	1	C_1
2 fold rotation axis	2	C_2
3 fold rotation axis	3	C_3
4 fold rotation axis	4	C_4
6 fold rotation axis	6	C_6



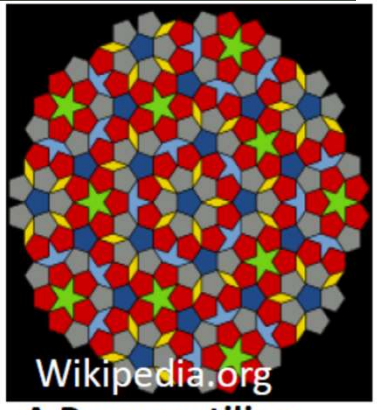
Eyn chaya kazo: There can be no such creature

- 10 fold diffraction pattern in Al-Mn alloy
- April 8, 1982
- Prof. David Shechtman
- 5 fold symmetry



- Quasi scientist or Quasicrystals

- A quasiperiodic crystal, or **quasicrystal**, is a structure that is ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but it lacks translational symmetry.



A Penrose tiling

https://www.theguardian.com/science/2013/jan/06/dan-shechtman-nobel-prize-chemistry-interview

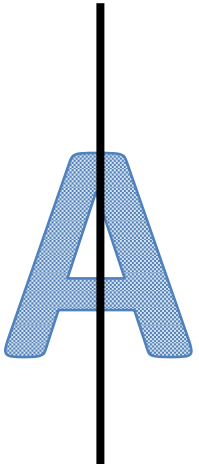
- Took 2 years to publish
- Aperiodic patterns in 3D
- Penrose tiling
- Medieval Arabic shrines

- Quasicrystals have opened up new avenues of research

https://www.nobelprize.org/prizes/chemistry/2011/prize-announcement/

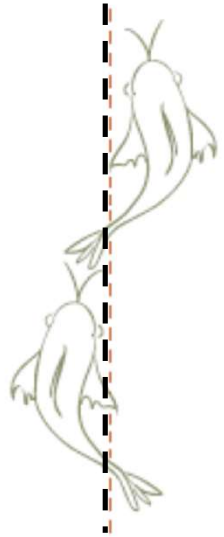
Reflection

- Flips all points in an asymmetric unit over a line called mirror
- Reflection changes chirality
- All points on the mirror are invariant
- Symbol “m” and solid line representation



Glide reflection

- Reflects asymmetric unit across a mirror and translates it parallel to the mirror
- Change in chirality of the unit
- No invariant point or line
- Symbol “g” and representation is dashed line



Point group

- Point group is collection of symmetry element of an isolated shape
- It does not consider translation
- Every point in the lattice should be identical after symmetry operation/transformation
- Lattice point symmetry
- Plane symmetry/crystallographic group: mathematical classification of 2D repetitive pattern that captures symmetry of the pattern

Why worry about symmetry ?

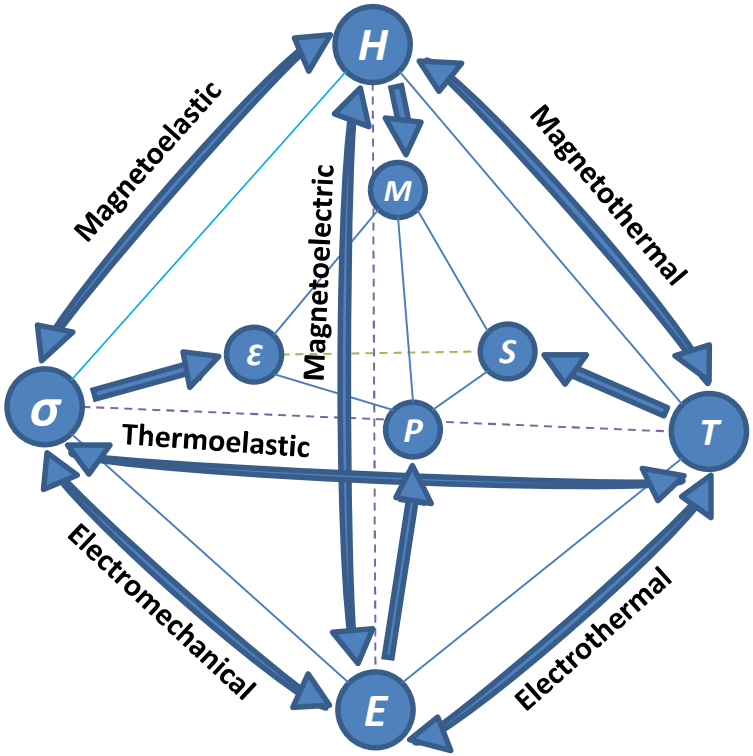
Neumann's Principle

Original 1885:

A fundamental natural law: *Neumann's Principle*: *the symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of the crystal*. The property may have additional symmetry elements to those of the crystal (point group) symmetry.

Modified version:

The symmetry elements of any physical property of a crystal must include the symmetry elements that are common to the point group of the crystal and the defect structure contained within the crystal.



Tensors

Perturbation	Response	Susceptibility
σ	ε	Elasticity
	P	Piezoelectric
	S	Piezocaloric
	M	Converse Magnetostriction
E	ε	Converse Piezoelectric
	P	Dielectric
	S	Electrocaloric
	M	Electromagnetic
T	ε	Thermoelastic
	P	Pyroelectric
	S	Heat Capacity
	M	Pyromagnetic
H	ε	Magnetostriction
	P	Magnetoelectric
	S	Magnetocaloric
	M	Magnetic

Vector like entity that require more than two directions for complete description

Property	Symbol	Field	Response	Type #
Tensors of Rank 0 (Scalars)				
Specific Heat	C	ΔT	$T\Delta S$	*E1*
Tensors of Rank 1 (Vectors)				
Electrocaloric	p_i	E_i	ΔS	*E3*
Magnetocaloric	q_i	H_i	ΔS	*E3*
Pyroelectric	p'_i	ΔT	D_i	*E3*
Pyromagnetic	q'_i	ΔT	B_i	*E3*
Tensors of Rank 2				
Thermal expansion	α_{ij}	ΔT	ϵ_{ij}	*E6*
Piezocaloric effect	α'_{ij}	σ_{ij}	ΔS	*E6*
Dielectric permeability	κ_{ij}	E_j	D_i	*E6*
Magnetic permeability	μ_{ij}	H_j	B_i	*E6*
Optical activity	g_{ij}	$l_i l_j$	G	*E6*
Magnetoelectric polarization	λ_{ij}	H_j	D_i	*E9*
Converse magnetoelectric polarization	λ'_{ij}	E_j	B_i	*E9*
Electric conductivity (resistivity)	σ_{ij} (ρ_{ij})	E_j (j_j)	j_j (E_j)	*T6*
Thermal conductivity	K_{ij}	$\Delta_j T$	h_i	*T6*
Diffusivity	D_{ij}	$\Delta_j c$	m_i	*T6*
Thermoelectric power	Σ_{ij}	$\Delta_j T$	E_i	*T9*
Hall effect	R_{ij}	B_j	ρ_i^a	*T9*

Property	Symbol	Field	Response	Type #
Tensors of Rank 3				
Piezoelectricity	d_{ijk}	σ_{jk}	D_i	*E18*
Converse piezoelectricity	d'_{ijk}	E_k	ϵ_{ij}	*E18*
Piezomagnetism	Q_{ijk}	σ_{jk}	B_i	*E18*
Converse piezomagnetism	Q'_{ijk}	H_k	ϵ_{ij}	*E18*
Electro-optic effect	r_{ijk}	E_k	$\Delta\beta_{ij}$	*E18*
Nernst tensor	Σ_{ijk}	$\Delta_j T B_k$	E_i	*T27*
Tensors of Rank 4				
Elasticity	$S_{ijkl}\,(c_{ijkl})$	$\sigma_{kl}\,(\epsilon_{kl})$	$\epsilon_{ij}\,(\sigma_{ij})$	*E21*
Electrostriction	γ_{ijkl}	$E_k E_l$	ϵ_{ij}	*E36*
Photoelasticity	q_{ijkl}	σ_{kl}	$\Delta\beta_{ij}$	*E36*
Karl effect	p_{ijkl}	$E_k E_l$	$\Delta\beta_{ij}$	*E36*
Magnetoresistance	ξ_{ijkl}	$B_k B_l$	ρ^{δ}_{ij}	*T36*
Piezoresistance	Π_{ijkl}	σ_{kl}	$\Delta\rho_{ij}$	*T36*
Magnetothermoelectric power	Σ_{ijkl}	$\Delta_j T B_k B_l$	E_i	*T54*
Second order Hall effect	ρ_{ijkl}	$B_j B_k B_l$	$\rho^2{}_i$	*T30*
Tensors of Rank 6				
Third order elasticity	c_{ijklmn}	$\epsilon_{kl}\,(\epsilon_{kl})$	σ_{ij}	*E56*