

# Lecture # 11.1 CHE331A

- Basics of reactions and ideal reactors
- Design/Analysis of CSTRs, PFRs, PBRs, Batch and Semi-batch reactors
- Collection and Analysis of Rate Data
- Non-linear regression and other methods of rate data analysis

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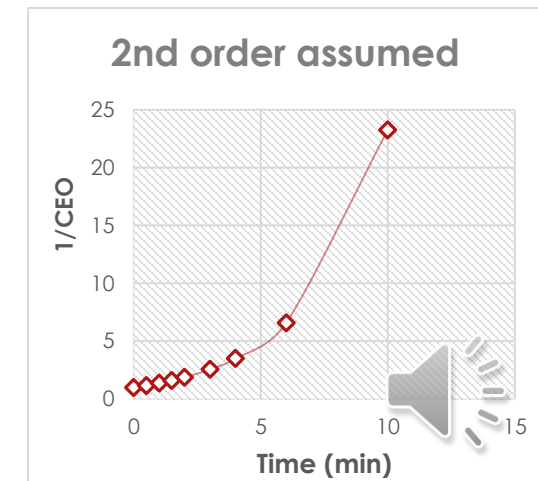
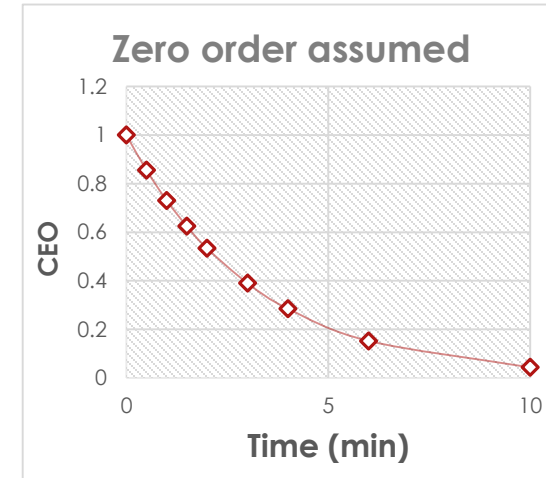
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## Differential method

Given  $C_A$  vs  $t$ , methods to determine  $\frac{dC_A}{dt}$  are:

- Graphical differentiation
- Numerical differentiation
- Differentiation of a polynomial fit

## Integral Method



# Nonlinear regression involves search for parameters that minimize “errors”

- ▶ Minimize the sum of the square of difference between measured values and estimated values
  - Determines the best parameter values by “searching” different values – this usually required an initial guess obtained by linear regression
  - Also used to discriminate rate law models, → heterogeneous catalysis
- ▶ For example, we want to find  $k$  and  $\alpha$  from set of rate data
- ▶ Minimize the difference between measured reaction rates,  $r_m$ , and the reaction rates calculated,  $r_c$ , based on “search” values →  $(r_m - r_c)^2$
- ▶ Based on the initial value the best value  $k$  and  $\alpha$  is obtained by following some search technique



# The function to be minimized also depends on the number of experiments and parameters

- (Objective) Function to be minimized given by:

$$\sigma^2 = \frac{s^2}{N - K} = \sum_{i=1}^N \frac{(r_{im} - r_{ic})^2}{N - K}$$

- N = number of runs                      K = number of parameters to be determined
- $r_{im}$  = measured reaction rate       $r_{ic}$  = calculated reaction rate for run  $i$
- $EO + H_2O \rightarrow EG$     measured 10  $C_{EG}$  values (N=10) → 10 values of  $r_{EG,m}$
- For a specific  $k$  and  $\alpha$  (K = 2), we have 10 values of  $r_{EG,c}(= k'_{EO} C_{EO}^\alpha)$

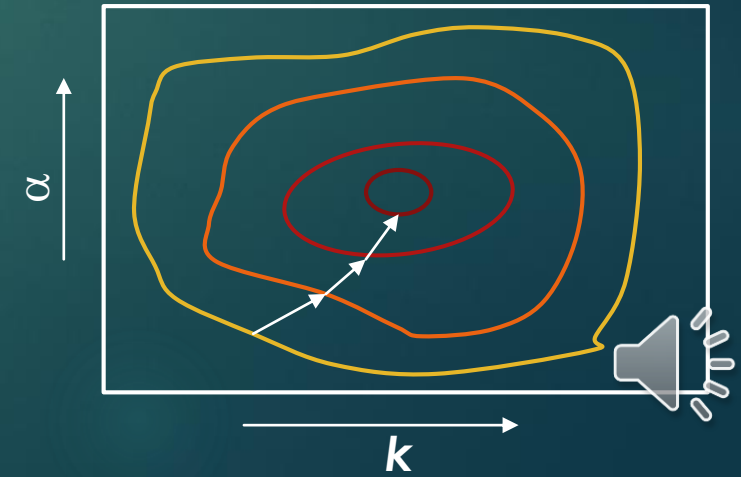
$$\sigma^2 = \sum_{i=1}^{10} \frac{(r_{im} - r_{ic})^2}{10 - 2}$$



# Calculations for the objective function

time (min)	Rate, meas.	Rate, calcu ( $k=0.5, \alpha = 1$ )	$\sigma^2$	Rate, calcu ( $k=0.5, \alpha = 2$ )	$\sigma^2$	Rate, calcu ( $k=0.31, \alpha = 2$ )	$\sigma^2$	Rate, calcu ( $k=0.31, \alpha = 1$ )	$\sigma^2$
0	0.31	0.50000	0.00451	0.50000	0.00451	0.31000	0.00000	0.31000	0.00000
0.5	0.27	0.42750	0.00310	0.36551	0.00114	0.22662	0.00024	0.26505	0.00000
1	0.231	0.36500	0.00224	0.26645	0.00016	0.16520	0.00054	0.22630	0.00000
1.5	0.197	0.31200	0.00165	0.19469	0.00000	0.12071	0.00073	0.19344	0.00000
2	0.234	0.26650	0.00013	0.14204	0.00106	0.08807	0.00266	0.16523	0.00059
3	0.248	0.19500	0.00035	0.07605	0.00370	0.04715	0.00504	0.12090	0.00202
4	0.238	0.14250	0.00114	0.04061	0.00487	0.02518	0.00566	0.08835	0.00280
6	0.242	0.07600	0.00344	0.01155	0.00664	0.00716	0.00689	0.04712	0.00475
10	0.194	0.02150	0.00372	0.00092	0.00466	0.00057	0.00468	0.01333	0.00408
Date Measured		total =	0.02030		0.02673		0.02644		0.01424

- ▶ Parameter values are found by minimizing  $\sigma^2$  using some optimization technique (MatLab/PolyMath/Mathematica)



# The objective function can be also in terms of concentration vs. time data

- ▶ Combined mol balance-stoichiometry for constant volume Batch Reactor:  $\frac{dC_A}{dt} = -k \cdot C_A^\alpha$  and for  $\alpha \neq 1$  by integrating

$$C_{A0}^{1-\alpha} - C_A^{1-\alpha} = (1 - \alpha)k \cdot t \rightarrow C_A = [C_{A0}^{1-\alpha} - (1 - \alpha)k \cdot t]^{1/1-\alpha}$$

- ▶ For 10 runs, where  $C_A$  vs. *time* is measured and 2 parameters

$$\sigma^2 = \sum_{i=1}^{10} \frac{(C_{A,im} - C_{A,ic})^2}{10 - 2}$$

- ▶ Time, instead of conc is used then

$$\sigma^2 = \sum_{i=1}^{10} \frac{(t_{im} - t_{ic})^2}{10 - 2} \text{ where } t_c = \frac{C_{A0}^{1-\alpha} - C_A^{1-\alpha}}{(1 - \alpha)k}$$

**Example 5-3**  
as a reading  
assignment