- Non-ideal Reactors
- Residence time distribution
- Tank-in-Series model
- Applications to design

# Lecture # 28 CHE331A

## Residence time distribution

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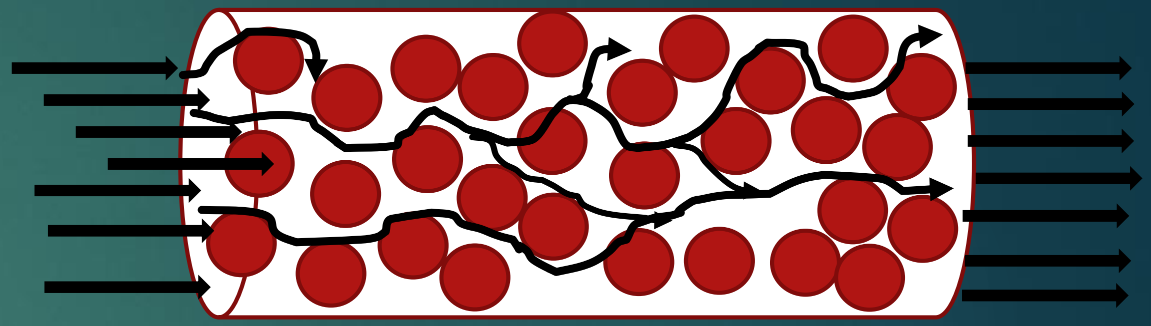
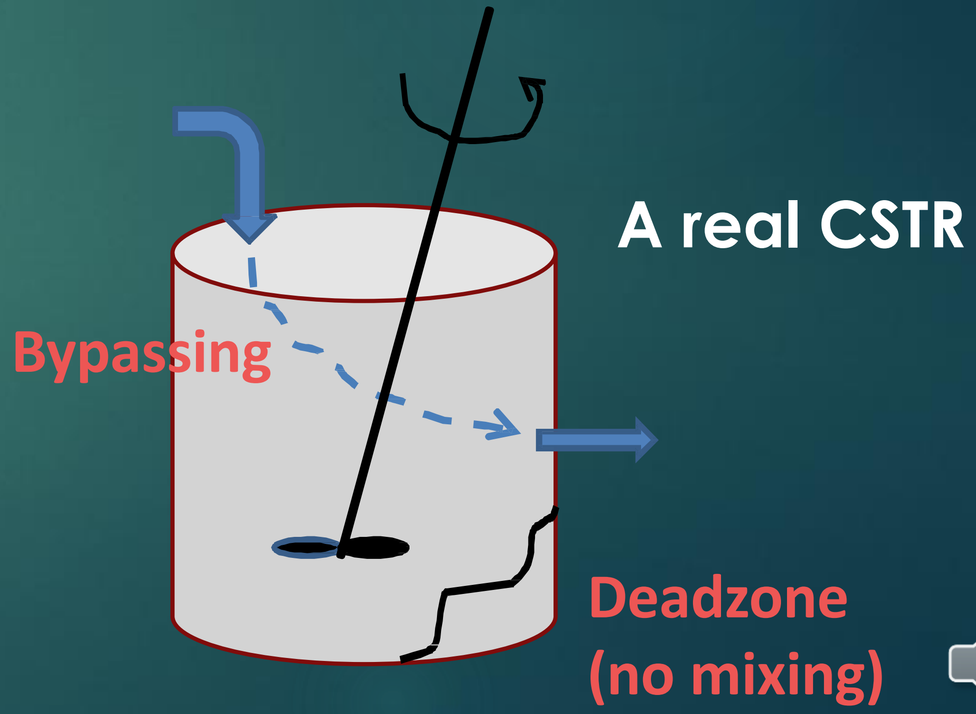
# Previously …

- Causes for non-ideality
  - Flows insider ideal reactors
  - CSTR and PFR/PBR
- Time spent by fluid elements are different
  - Have an effect on conversion
  - Residence time distribution
- Methods of determining RTD
  - Detection of response to tracers



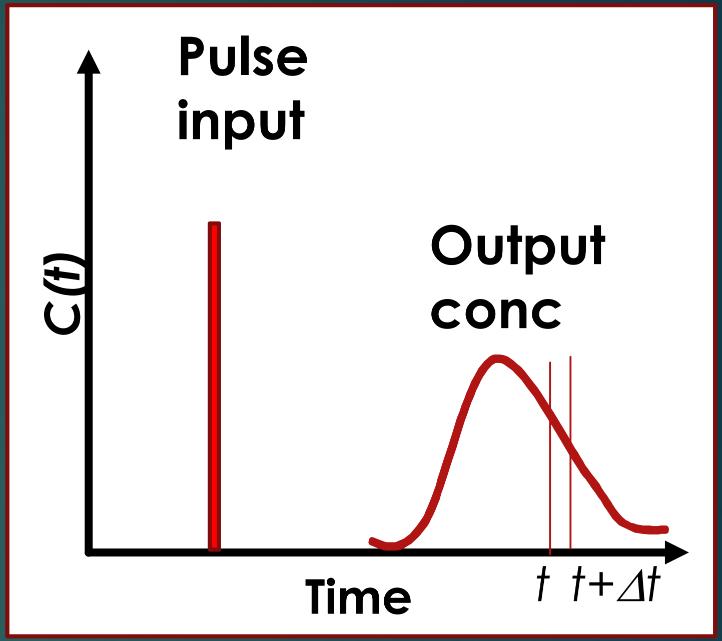
A real Packed Bed Reactor



# Experimental determination of RTD – a diagnostic tool for determining non-idealities in flow

- Residence time distribution function:

$$E(t) = \frac{C(t)}{\int_0^\infty C(t)dt}$$




- $E(t)dt$ is the fraction of the fluid spending time between

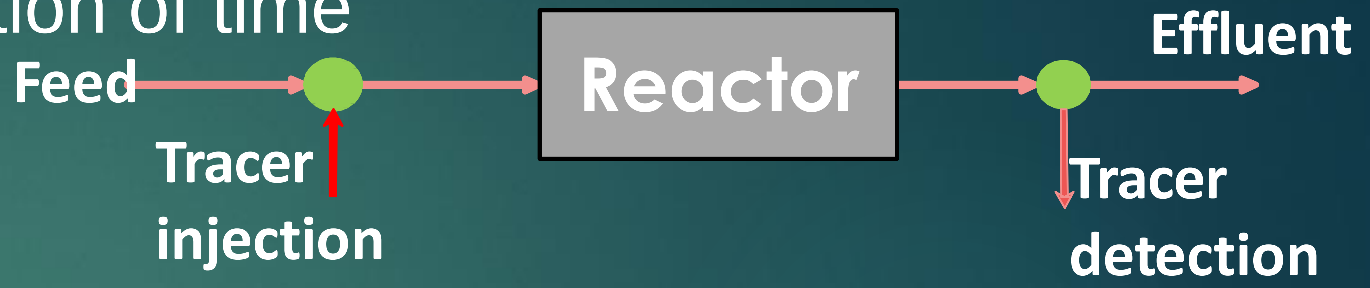
  time $t$ *and* $t + dt$ in the reactor

- Also, $\int_{t_1}^{t_1} E(t)dt$ is the fraction of fluid exiting the reactor between time $t_1$ *and* $t_2$

- Further, $\int_0^\infty E(t)dt = 1$

# Constructing the C(t) and E(t) curves

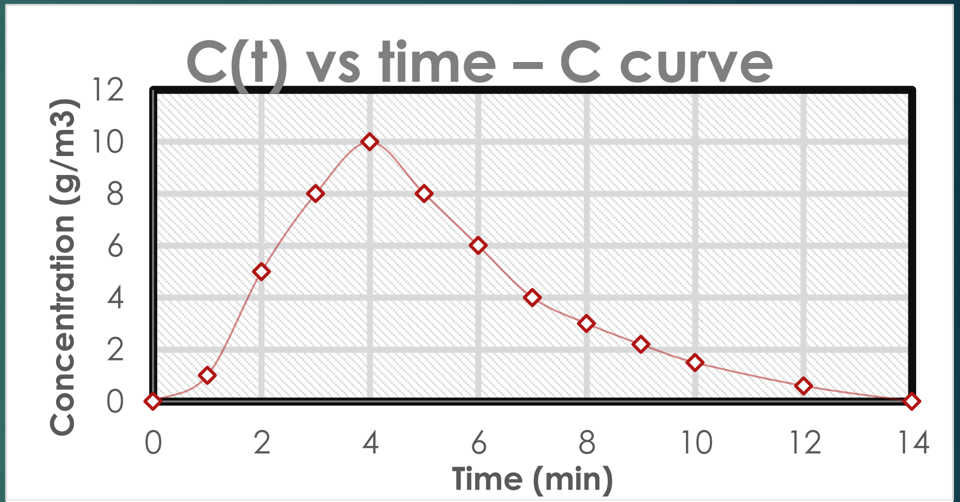
- Concentration measured as a function of time

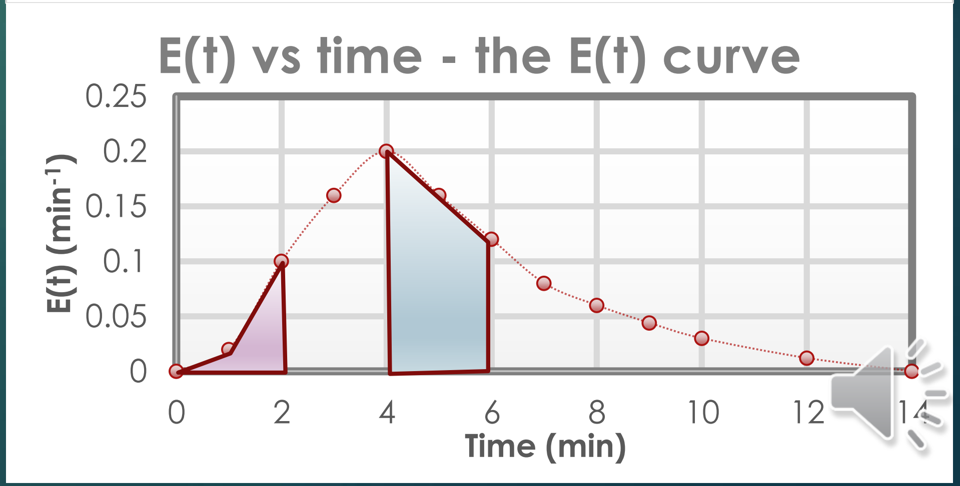



- To obtain the E(t) curve from the C(t) curve, we divide C(t) by the $N_0$ or the integral of the C(t) curve → $E(t) = \frac{C(t)}{\int_0^\infty C(t)dt}$; $\int_0^\infty C(t)dt$ = 50.0 g.min/m$^3$

- Fraction of fluid exiting the reactor that has resided in the reactor between $t_1 (4\ min)\ and\ t_2\ (6\ min) = \sim 0.32$ (32%)

- Fraction spending less than 2 mins is the corresponding area



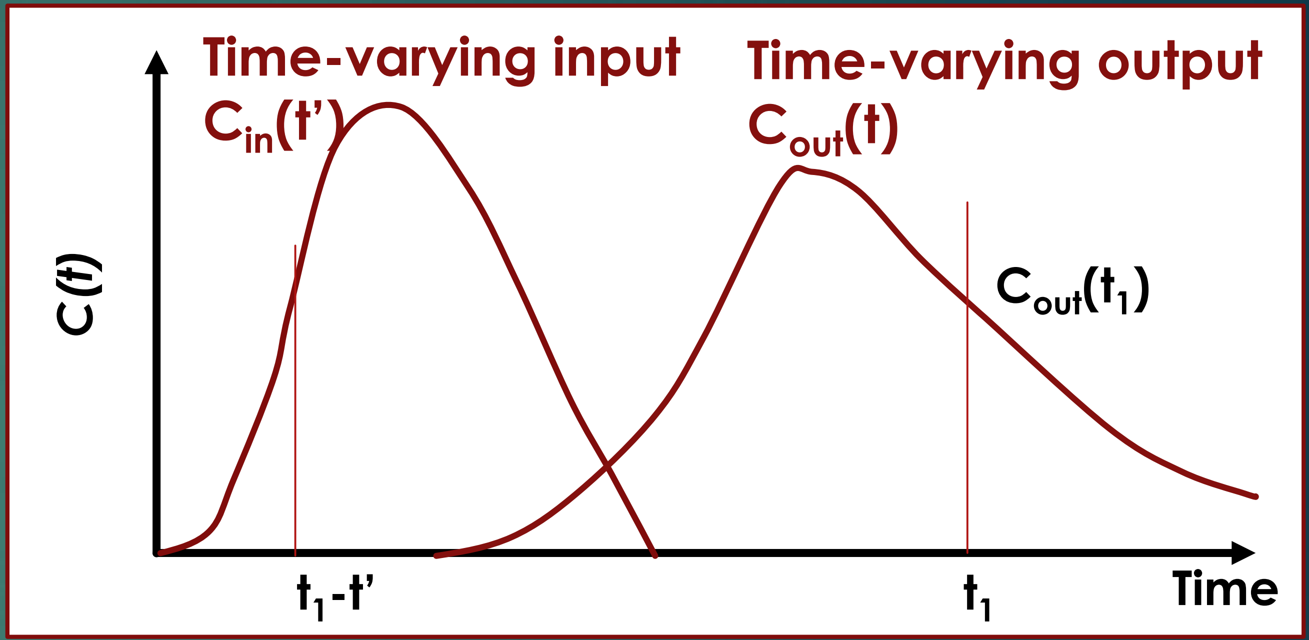
# C(t) versus t curve for a arbitrary input tracer signal – the convolution integral

- The concentration at the output coming out at $t_1$ depends on the concentration of the material injected at $t_1$ - t’ (as shown) and having a residence time of t’
- For example, if $t_1$ = 20 minutes and

  $t_1$ – t’ = 12 minutes and t’ = 8
- Similarly, $t_1$ – t’ = 8 minutes and

  t’ = 12
- In general

$$C_{out}(t) = \int_0^t C_{in}(t - t')\,E(t')\,dt'$$



# Step input also used to determine E(t) to overcome difficulties in the pulse input method

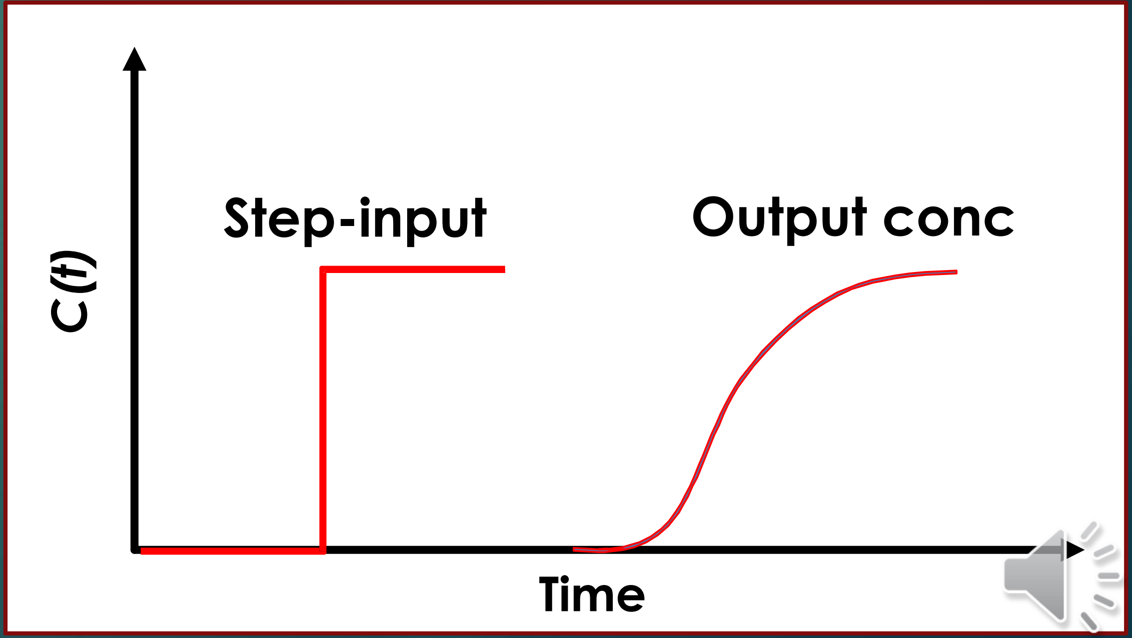
- The inlet concentration can take the form of a perfect pulse input, imperfect pulse injection, or step input
  - Perfect pulse input → Dirac delta function
- For a step-input: $C_0(t) = 0 \; for \; t < 0 \; and \; C_0(t) = C_0 \; for \; t \geq 0$
- Conc. of feed is kept at a level until the conc in the effluent is the same as the feed
- Applying the convolution integral:

$$C_{out}(t) = \int_0^t C_{in}(t - t')E(t')dt$$

- To the step input given by:

$$C_0(t) = 0 \; for \; t < 0 and \; C_0(t) = C_0 \; for \; t \geq 0$$

- We have: $C_{out}(t) = C_0 \int_0^t E(t')dt'$



# Positive step input is easier to carry out

- Applying the convolution integral $C_{out}(t) = \int_0^t C_{in}(t-t')E(t')dt$ to the step input given by $C_0(t) = 0 \; for \; t < 0 \; and \; C_0(t) = C_0 \; for \; t \geq 0$
- We have: $C_{out}(t) = C_0 \int_0^t E(t')dt'$
- Dividing by $C_0$: $\left[\frac{C_{out}(t)}{C_0}\right]_{step} = \int_0^t E(t')dt' = F(t)$
- By differentiating we obtain the RTD function E(t): $E(t) = \frac{d}{dt}\left[\frac{C_{out}(t)}{C_0}\right]_{step}$
- Drawbacks of step input include:
  - maintain a constant tracer concentration in the feed,
  - Differentiation of the data, and
  - large amount of tracer required for this test.

# The exit-age distribution function, E(t), and the cumulative-age distribution function, F(t)

- E(t) is sometimes called the exit-age distribution function and it signifies the age of the atom residing in the reaction environment
- The fraction of the atom that has resided in the reactor for a period less than t is given by: $\int_0^t E(t)dt = F(t)$
- F(t) is the cumulative RTD function (Dankwerts)
- We also have: $\int_t^\infty E(t)dt = 1 - F(t)$
  - These two relationships are called integral relationships
  - E(t) and F(t) are normalized
  - F(t) is calculated from E(t) vs t or vice-versa