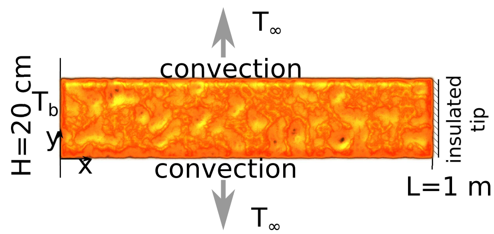
CHE312A: Homework 3

Instructions: Upload your original, legible and handwritten solution in pdf format, with the exact filename ⟨yourrollnumber⟩.pdf (e.g. 06302016.pdf). Include your name and roll number on the top of the first page. Include all necessary steps in detail, define all symbols and state all assumptions made in your solution. The solution to each problem must be accompanied with your working MATLAB/Octave code, uploaded as a separate file ⟨yourrollnumber⟩_p1.txt for problem 1 and ⟨yourrollnumber⟩_p2.txt for problem 2. Each code must run error-free and produce necessary figures upon simply changing the file extension from .txt to .m. Submit only your original work. Submitted codes may also be checked for plagiarism.

A 1 m long (along x-direction) 20 cm thick (along y-direction) copper fin ($k = 401$ W/m.K)maintained at the base temperature $T_b = 30°$C has an insulated tip. Heat is transferred by convection between the upper, lower surfaces, and the surrounding air at $T_\infty = 20°$C, with a convection heat transfer coefficient $h = 10$ W/m^{2}K.



1. With the goal of finding the steady state 2D temperature distribution T(x,y) in the fin,

   (a) Write the simplified steady state conduction equation appropriate for the problem, assuming no temperature gradient in the z direction, along with boundary conditions specified in the problem. (1 points)

   (b) Write a finite difference based numerical scheme with the following specifications and re-arrange in the form of A·T = C, where A is the coefficient matrix, T is the 1D array (i.e. a vector) of grid point temperatures and C is a vector of rest of the terms not involving T. Clearly indicate all unique terms in the matrix A, and the vectors T, C that satisfy all boundary conditions and the governing equation at all interior grid points. (2 points)

   - Draw a uniform grid spanning the fin using Nx and Ny discrete points in x and y directions, respectively.
   - Use central finite difference approximation with second order accuracy to represent all derivatives at each internal grid point.
   - Use forward (left, bottom) or backward (right, top) finite difference approximation with first order accuracy at the boundary points, as necessary.
   - Implement respective x-directional boundary conditions at corner gridpoints (for example, at the corner grid point x=0,y=0 apply the condition for x=0 instead of that at y=0).

   (c) Write your own Matlab/Octave function `steadytemperaturedistribution()` to solve for the temperature distribution T using the numerical scheme, with Nx = 10, Ny = 5. Map the 1D array T back to a 2D distribution T2D. Use MATLAB/Octave function `surf(y,x,T2D` to plot the 2D temperature distribution T2D using the grid point temperature. (2 points)

   (d) Assume the fin to be 1 m deep (along z-direction) for finding the analytical temperature profile $T_{\text{analytical}}(x)$ as discussed in the lecture material. Calculate the y-averaged numerical temperature profile, T_i, y-avg.(x_i). Compare the two profiles by calculating the root mean square deviation $\sqrt{\frac{\sum_{i=1}^{N_x}\left(T_{i,\text{y-avg}}(x_i)-T_{\text{analytical}}(x_i)\right)^2}{N_x}}$. (1 points)

2. With the goal of finding the transient state 2D temperature distribution T(x,y,t) in the fin,

   (a) Write the simplified unsteady conduction equation appropriate for the problem, with the same boundary conditions as above, and with the initial condition that the temperature in the fin is uniformly $T_i = T_b$. Assume thermal diffusivity to be 1 m^2/s (although unrealistically high). (1 points)

   (b) Use the matrix A and the vectors T,C in the above finite difference scheme to adapt for the unsteady problem, by writing a new MATLAB/Octave function `F = myode(t,T)` that returns F = A·T-C. Integrate this function for a 15 seconds time span, using the MATLAB/Octave function `ode45`. (2 points)

   (c) Plot the 2D temperature distribution at 15 seconds and the steady state 2D temperature distribution in problem 1 on the same figure (using the MATLAB/Octave feature `hold on`). Does the temperature distribution at 15 seconds resemble the steady state temperature distribution? (1 points)

Information:

The following MATLAB/Octave functions for mapping the 2D temperature field to a 1D array/vector and vice-a-versa may be borrowed, if useful:

```
function f = mypack(i,j)
global Nx
f = (j-1)*Nx+i;
end

function [ii,jj] = myunpack(idx)
global Nx
jj = ceil(idx/Nx);
ii = idx-(jj-1)*Nx;
end
```

Hint: `T(mypack(i,j))` is the temperature at the grid point `(i,j)` mapped to a 1D array. `A(mypack(i,j),mypack(p,q))` corresponds to the coefficient of `T(p,q)` that appears in the 2D conduction equation applied at the grid point `(i,j)`.