Complex Numbers and Complex Analysis
Part 1

ChE641, IIT Kanpur

ChE : complex numbers → process control, transport phenomena, fluid mechanics

→ Fourier series
Fourier transform
Laplace transform

Need for complex numbers?

$z_{1,2} = 2 \pm i$ $\qquad z^2 - 4z + 5 = 0$

$(z - z_1)(z - z_2) = 0$

$$z_{1,2} = \frac{4 \pm \sqrt{16-20}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 2 \pm i$$

$$= 2 \pm \frac{\cancel{\sqrt{4}}\sqrt{-1}}{\cancel{2}}$$

$\sqrt{-4} = ?$

$x - 4 = 0 \rightarrow x = 4$

$x + 4 = 0 \rightarrow$ negative integers $\quad x = -4$

Complex numbers : $\sqrt{-4}$ –

$$\sqrt{-4} = \sqrt{4 \times -1} = \sqrt{4} \times \sqrt{-1}$$

$$\sqrt{-a} = \sqrt{a}\,\sqrt{-1}$$

$$\boxed{i = \sqrt{-1}}$$ "imaginary unity"

$2+i, \quad 2-i$ → "complex number"

real part, imaginary part

Complex number $= z = x + iy$

real part, imag. part → real qty

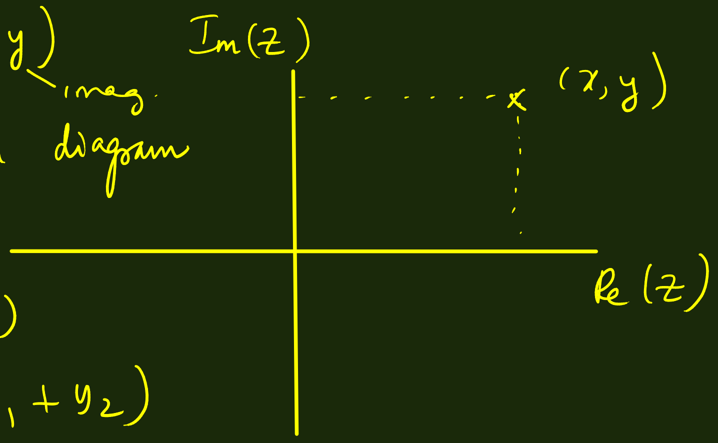
Notation: $i = \sqrt{-1}$ ← This course

$j = \sqrt{-1}$ (electrical engg)
$i \to$ current.

$z = x + iy \qquad z = (x, y)$

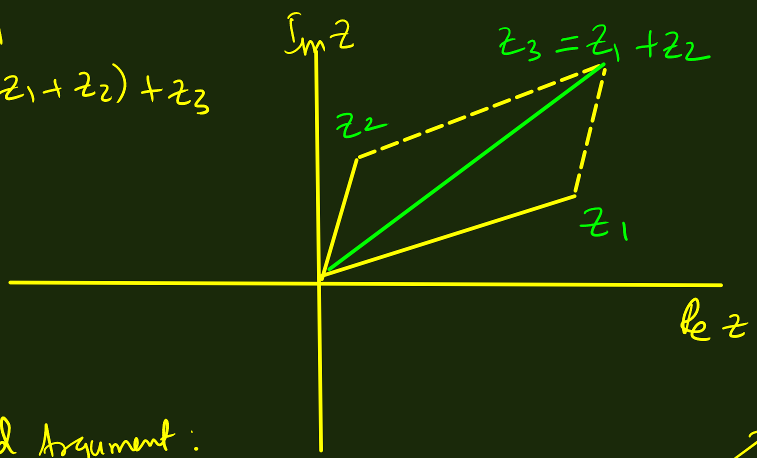
real, imag.

"complex plane" ← Argand diagram

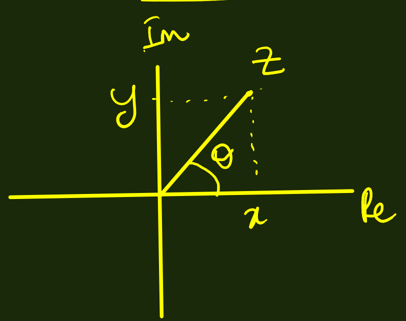


$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$
$$= (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 + z_2 = z_2 + z_1$$
$$z_1 + (z_2 + z_3) = (z_1 + z_2) + z_3$$



Modulus and Argument:

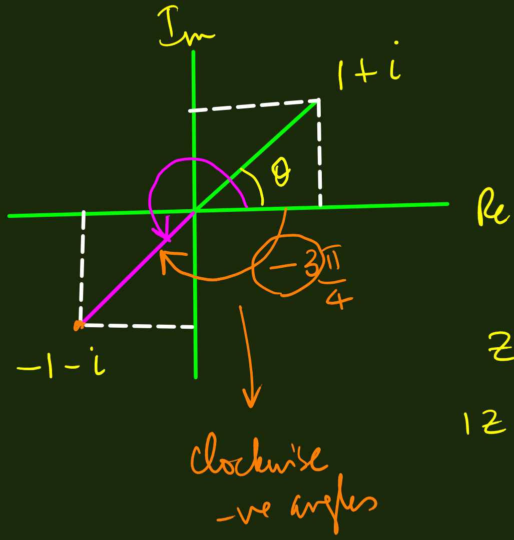


distance from the origin: $r = |z| = \sqrt{x^2 + y^2}$ → Modulus / absolute value.

$$\tan\theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

almost.



$z = 1+i$

$|z| = \sqrt{1^2+1^2}$

$= \sqrt{2}$

$\tan\theta = \frac{1}{1} = 1$

$\theta = 45^\circ = \frac{\pi}{4}$ radians

$z = -1-i$

$|z| = \sqrt{2}$

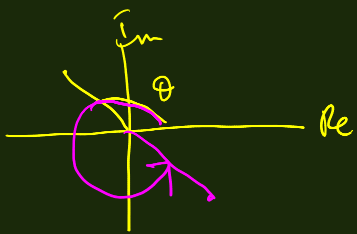
$\tan\theta = \frac{-1}{-1} = 1$

$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{\pi}{2} + \frac{\pi}{4}$

$= \frac{3\pi}{4}$

$z = -1+i \qquad |z| = \sqrt{2} \qquad \tan\theta = \frac{1}{-1} = -1$



$\theta = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

$z = 1-i \longrightarrow \theta = \frac{3\pi}{2} + \frac{\pi}{4} = \frac{7\pi}{4}$

$\theta : (0 - 2\pi)$

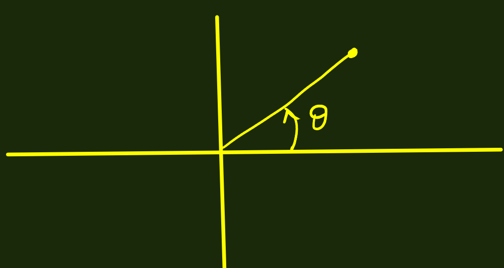
$\theta : (-\pi \text{ to } \pi)$

$z = x + iy \qquad |z| = (x^2+y^2)^{1/2}$

$\theta \underset{[\text{almost}]}{=} \tan^{-1}\left(\frac{y}{x}\right)$ {Quadrant in which the point lies}

<u>Argument</u>: θ (up to additive factor of 2π)

$\theta + 2\pi \rightarrow$ same pt in complex plane.



$\theta + 2n\pi$ (n integer).

Radians (degrees). $\longrightarrow$ $\frac{d}{dx}\sin x = \cos x$ — x is measured in radians

2π radians $= 360^\circ$

π radians $= 180^\circ$

Can use degrees: adding and subtracting an angle.

$$\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}(1) = \frac{\pi}{4} \quad (\cancel{\text{NOT } 45^\circ!})$$

$\theta + (2n\pi)$ $\quad$ $(n$