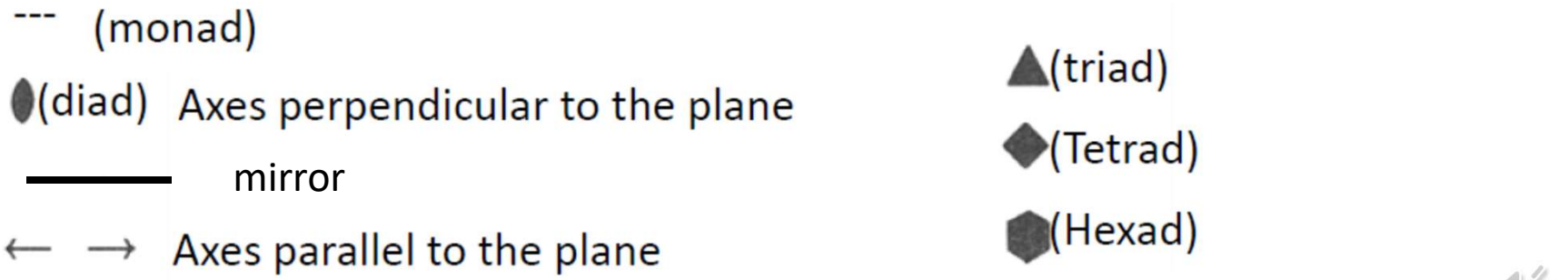
Point group reminder

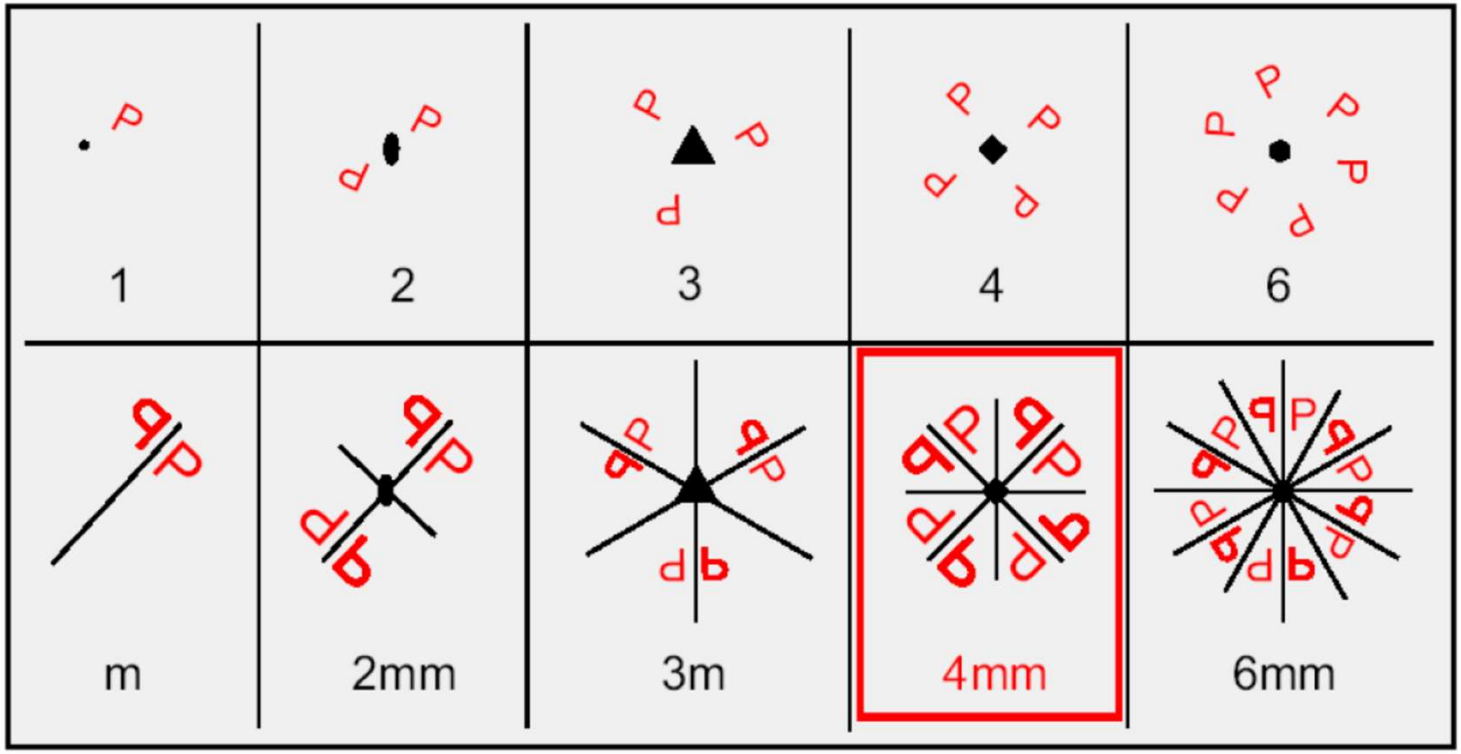
- Point group is collection of symmetry element of an isolated shape
- Combination of rotation and reflection in 2D
- Translation is not considered and hence glide is also ignored
- 5 rotation elements: 1, 2, 3, 4, 6 fold
- Mirror or reflection changes handedness
- Odd rotation have one mirror (m), even have two (mm)

	Crystals	Molecules
	Hermaan-Mauguin symbol	Schoenflies symbol
mirror	m	C_v
2 fold with mirrors	2mm	C_{2v}
3 fold with mirror	3m	C_{3v}
4 fold with mirrors	4mm	C_{4v}
6 fold with mirrors	6mm	C_{6v}

https://en.wikipedia.org/wiki/Wallpaper_group

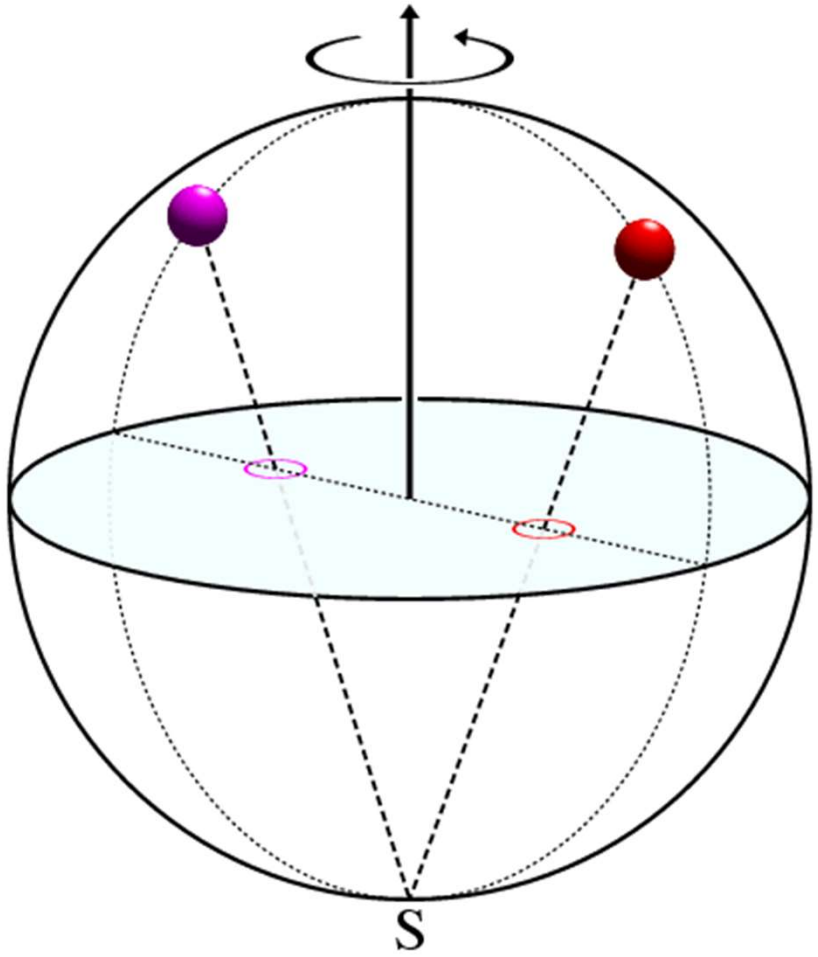


Point group symmetry



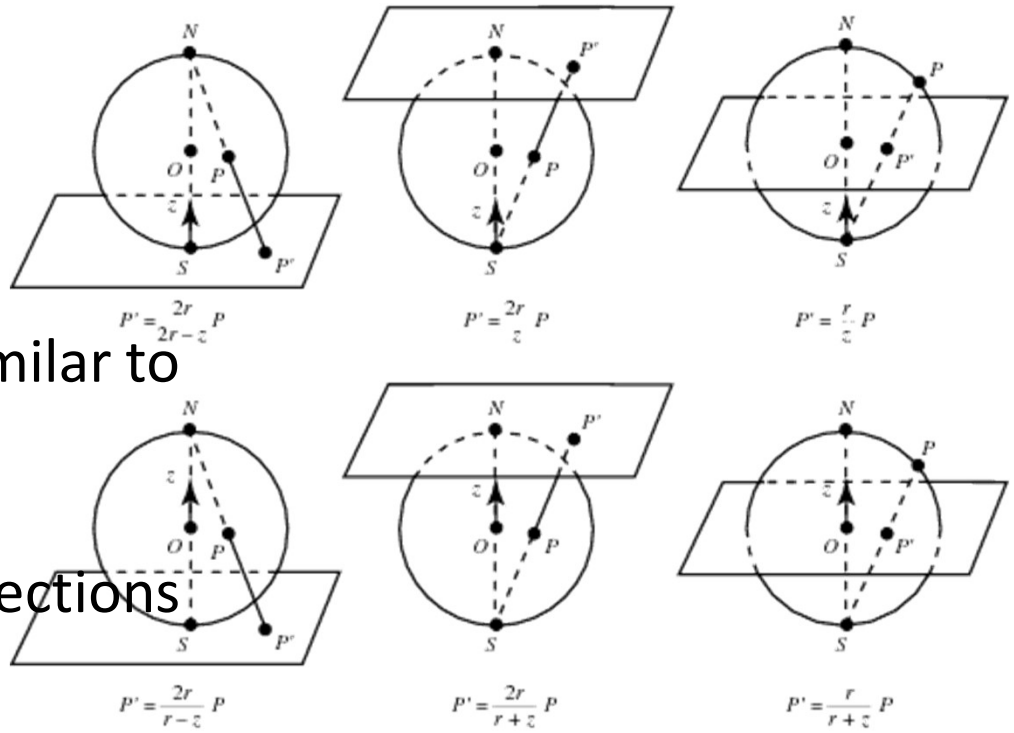
- m on major and minor axis of ellipse
- m on altitude
- m on sides and diagonal
- m on line joining corner to corner and edge center to edge center

- Symmetry in 1D, 2D and 3D
- Cartesian space is 3D
- We can view 2D
- Stereographic projection is angle true
- We can appreciate symmetry
- Spherical trignomentry



http://pd.chem.ucl.ac.uk/pdnn/symm2/project2.htm

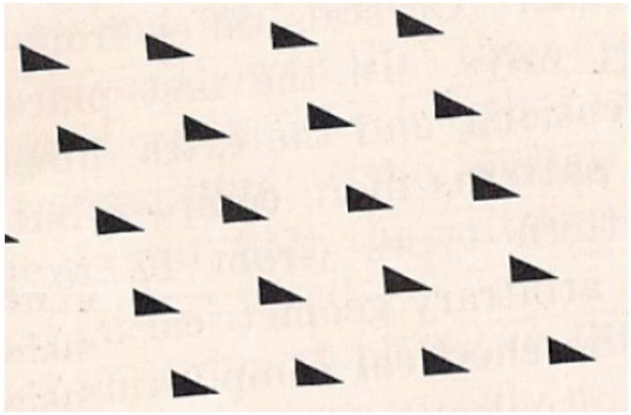
- center of inversion
- 3D not in 2D
- Northern hemisphere similar to southern hemisphere
- Positive and negative directions



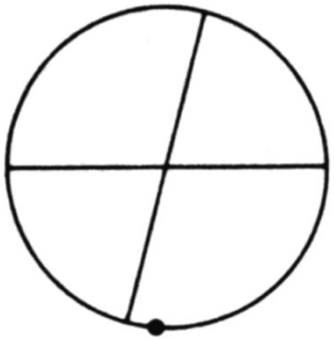
https://mathworld.wolfram.com/StereographicProjection.html

Pattern, stereographic projection and representation

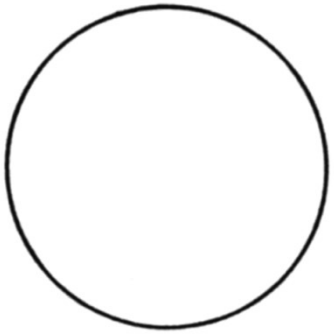
1



Stereographic projection



Representation of symmetry elements



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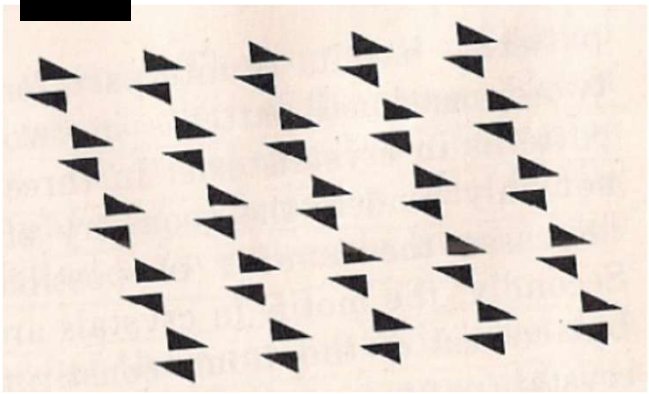

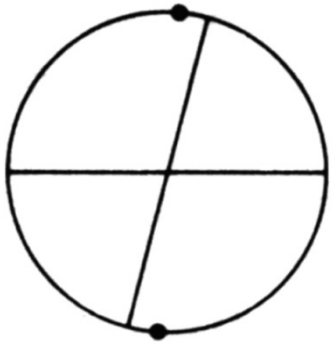

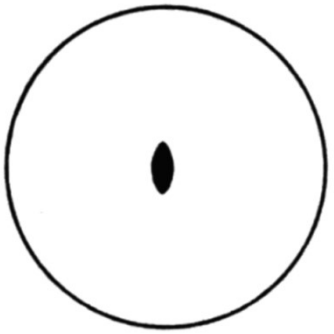

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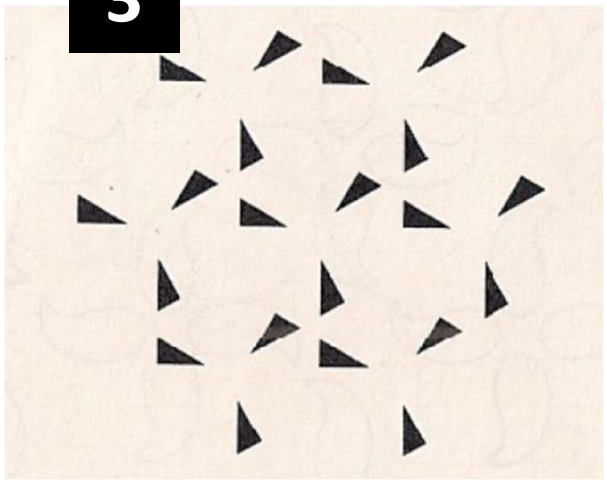

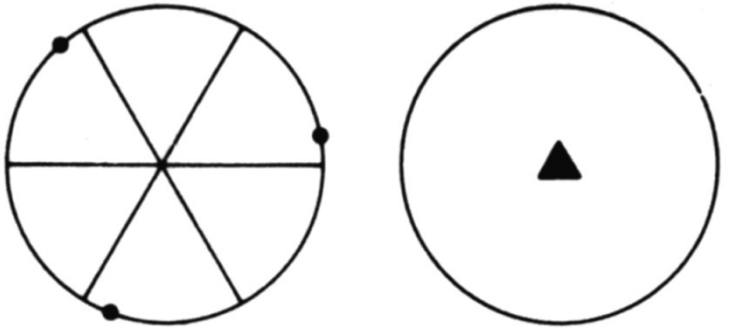

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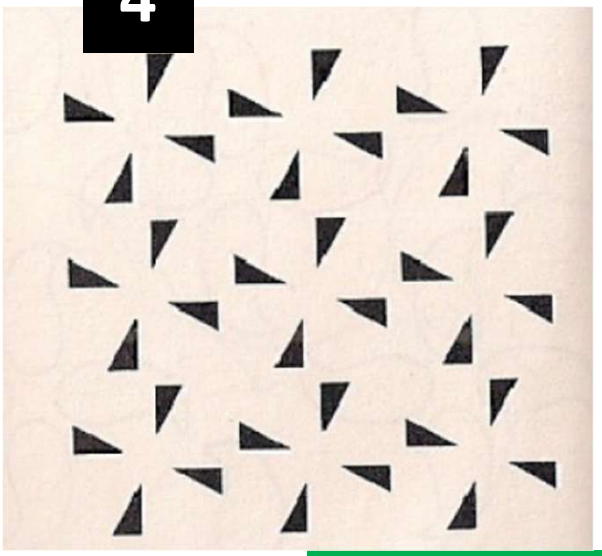

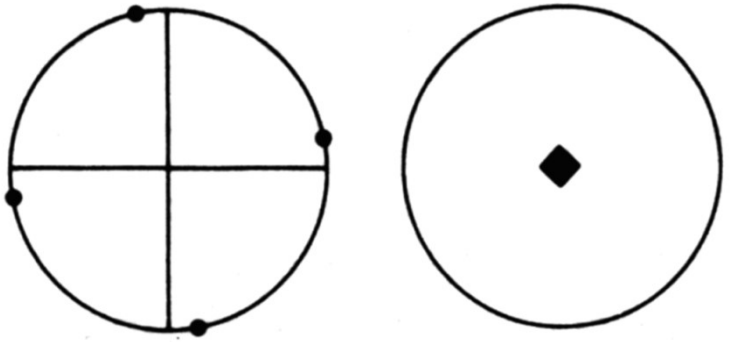

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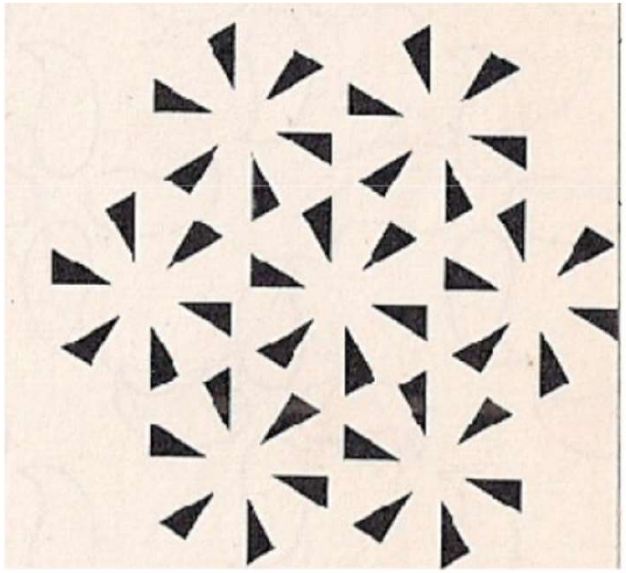
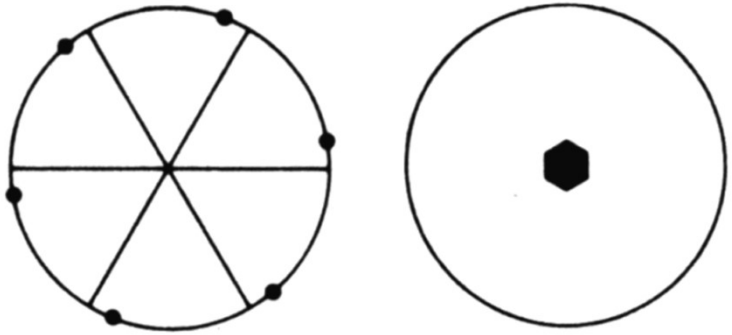
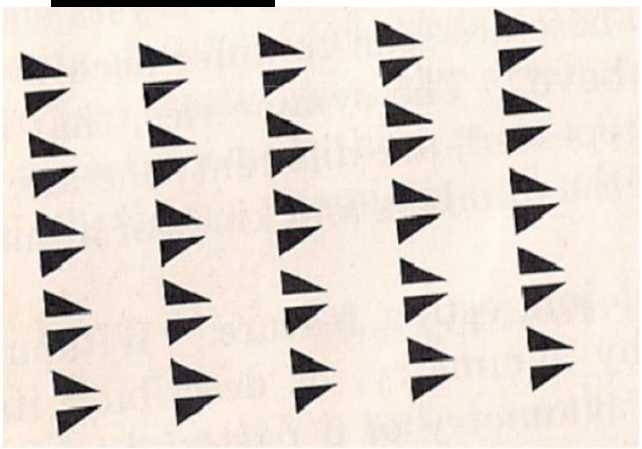
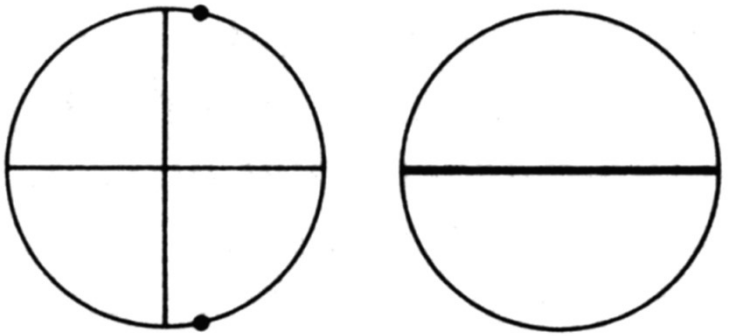


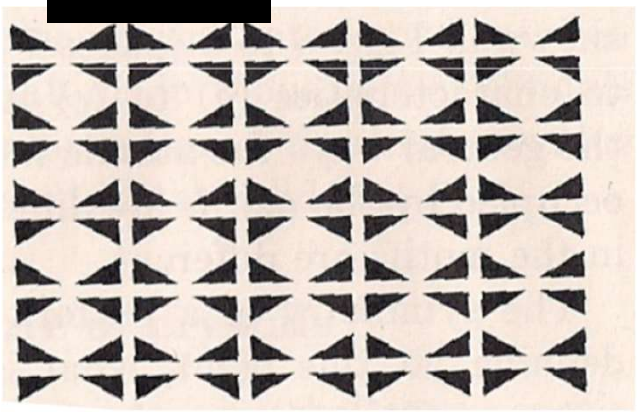


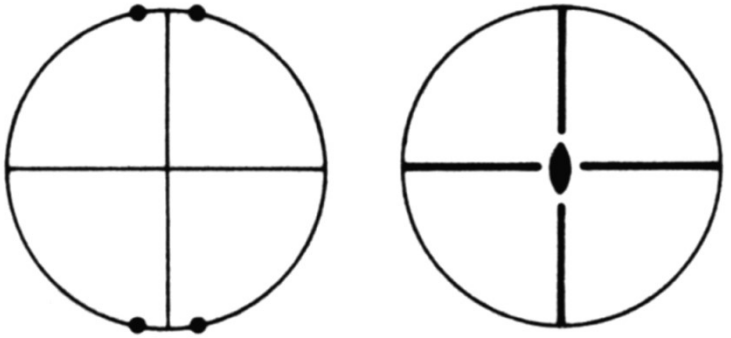
m



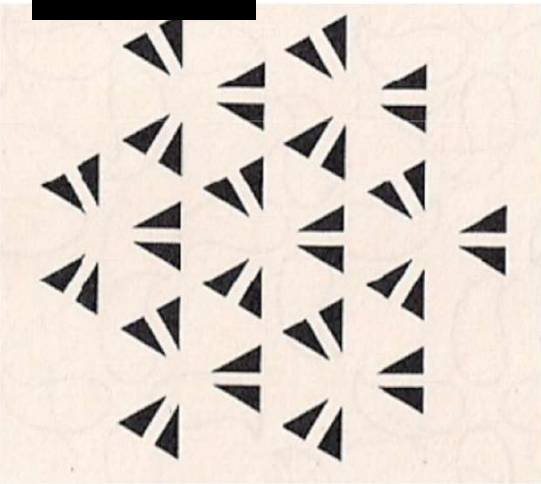


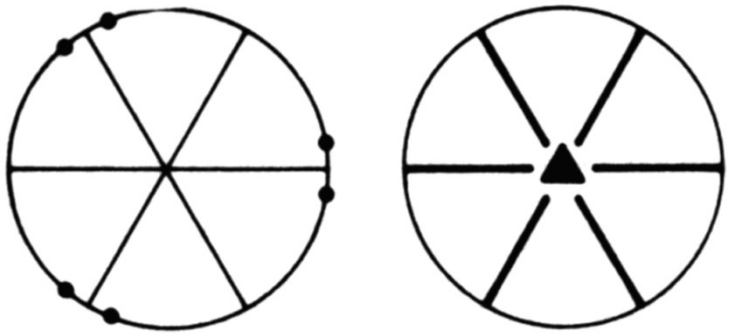
2mm



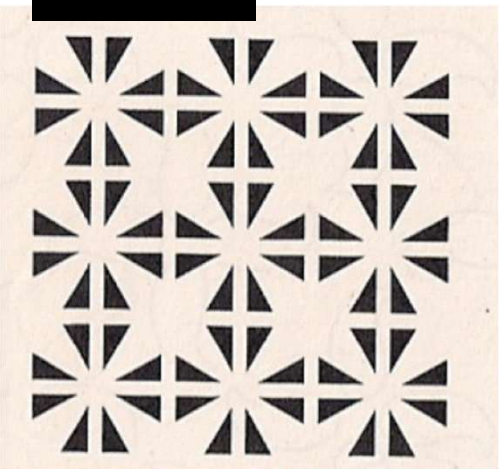


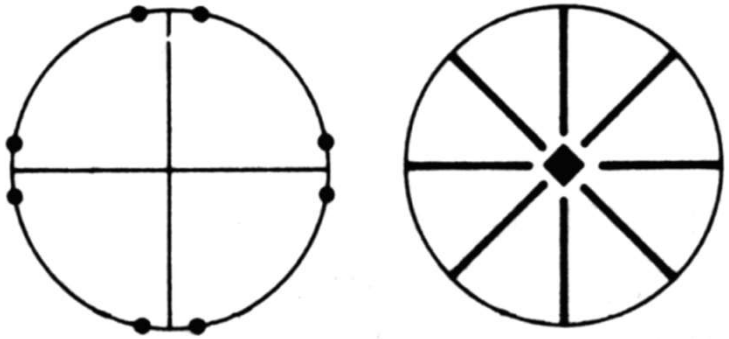
3m



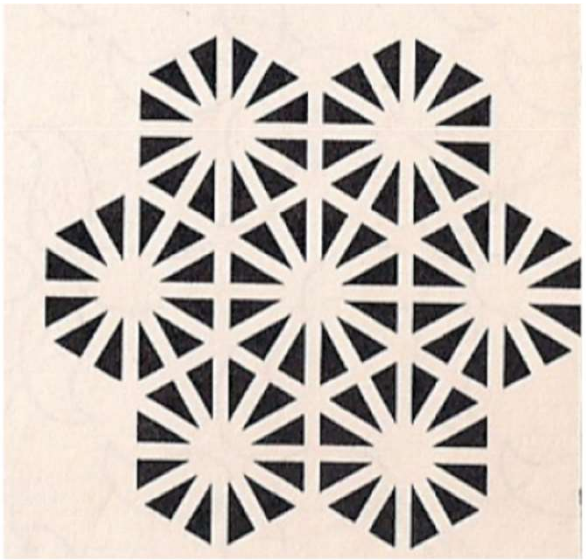


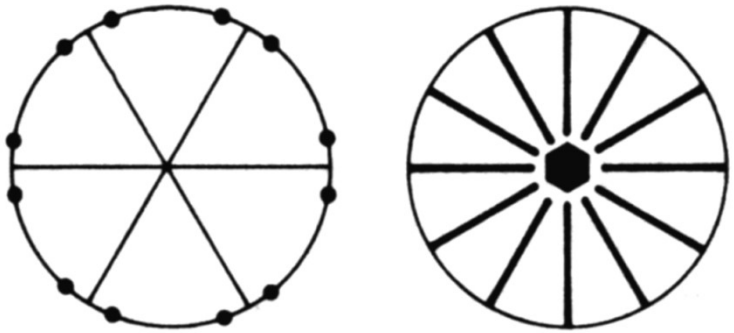
4mm





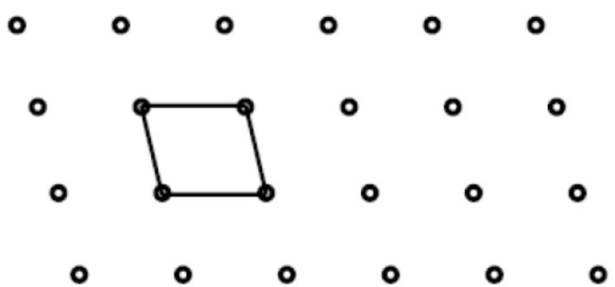
6mm



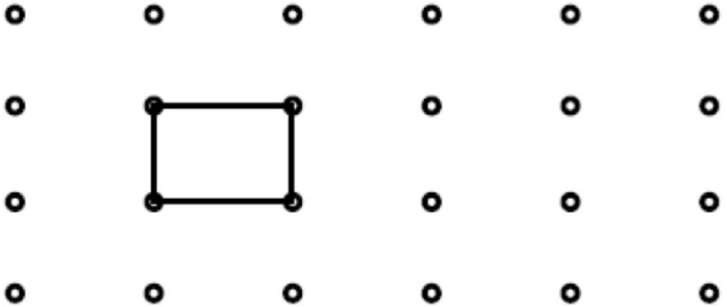


- Five different cell types
  - Oblique or parallelogram ($a \neq b$, $\angle \neq 90^\circ$)
  - Rectangular ($a \neq b$, $\angle = 90^\circ$)
  - Centered rectangular ($a \neq b$, $\angle = 90^\circ$)
  - Square ($a = b$, $\angle = 90^\circ$)
  - Rhombic or hexagonal ($a = b$, $\angle = 120^\circ$)

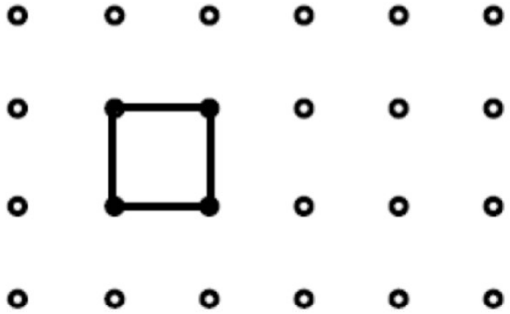
Oblique or parallelogram



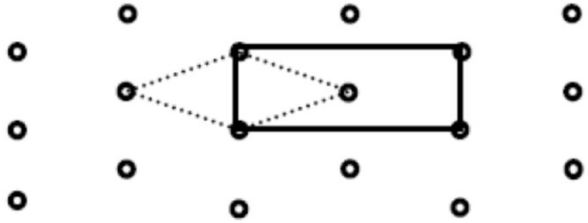
Rectangular



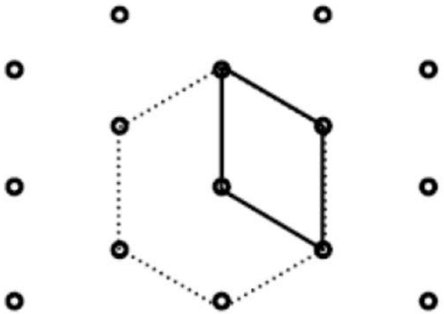
Square



Centered rectangular



Rhombic or hexagonal



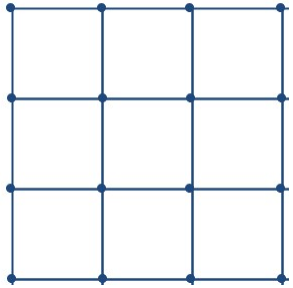
Plane groups

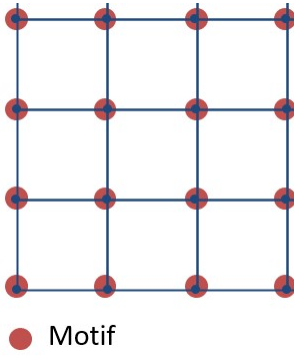
- Combining rotation and mirror symmetry with translation and glide reflection gives plane groups

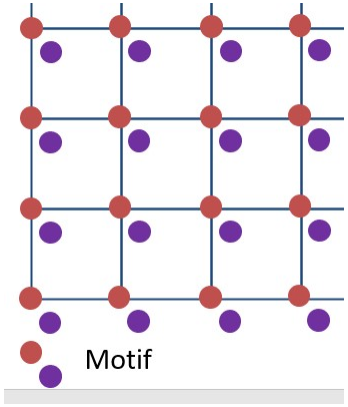
- Notation for plane groups
  - p for primitive and c for centered
  - 1 lattice point per unit cell in p
  - 2 lattice points per unit cell in c
  - Symmetry elements arranged along x, y and z axis
  - Highest symmetry element first even if along z axis p4mm
  - Lower symmetry axes omitted p4mm 2 fold along x and y are omitted
  - For 2 fold, the sequence is always x, y, z

Plane groups

- 10 2D point groups
- 5 2D lattices
- 17 plane groups







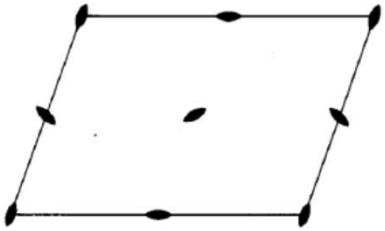
https://en.wikipedia.org/wiki/Wallpaper_group

https://mcescher.com/

https://www2.clarku.edu/faculty/djoyce/wallpaper/seventeen.html

Plane groups from oblique cell

p1



p2

- p stands for primitive that is one point per unit cell
- Motif with 1 fold and 2 fold symmetry respectively for p1 and p2

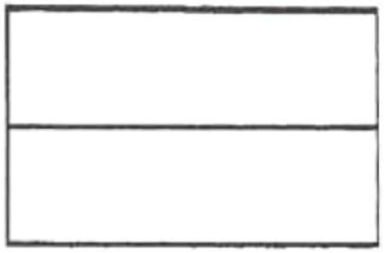
p1

Remember p1 and p2 can have all lattices !

p2

Visions of Symmetry: Notebooks, Periodic Drawings a
Related Work of M. C. Escher by Doris Schattschneide

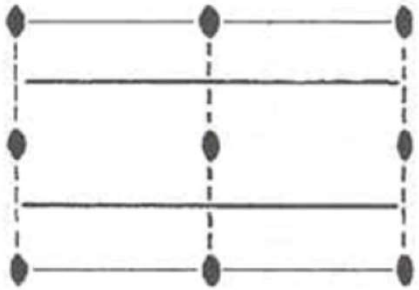
Plane groups from rectangular cell



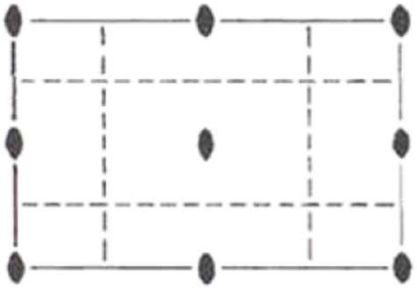
pm

pg

pmm2



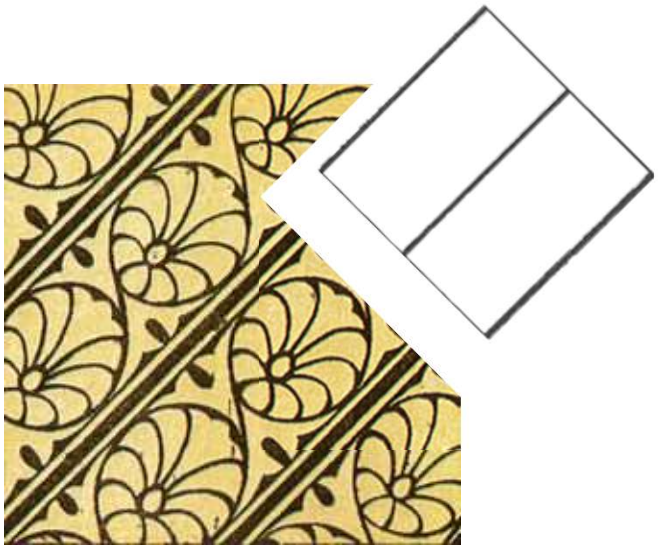
pmg2



pgg2

m

2mm



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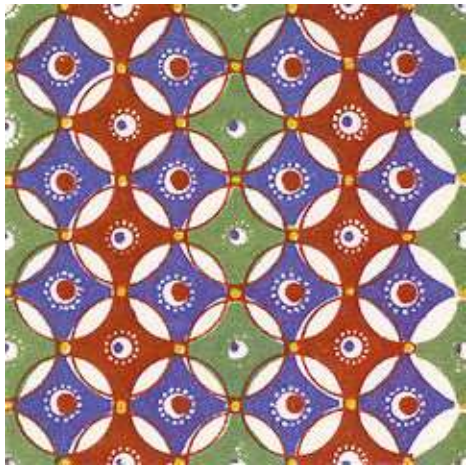
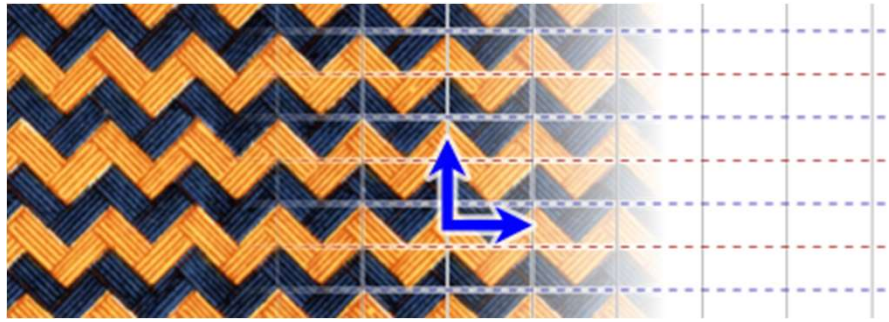
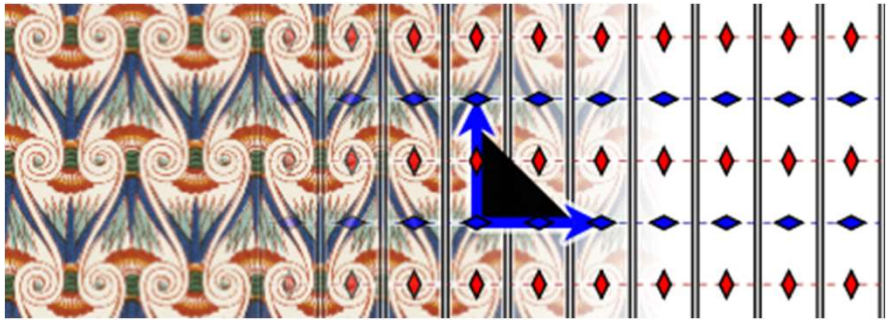


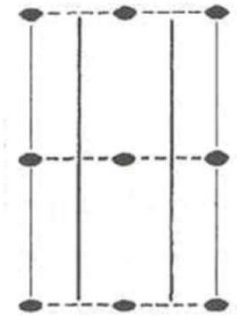
pmm2

pg



pmg2





pgg2