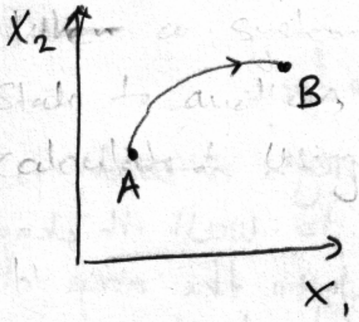
Heat Transfer

Introduction:

Heat : form of energy that can be transferred from one System to another as a result of temperature difference



If

$(X_1, X_2) \equiv$ state of a system

$A, B \equiv$ two thermodynamic states

$A \to B \equiv$ process

then

Thermodynamics: amount of heat transferred in the process

Heat Transfer : rate of heat transferred during the process.

Common Examples Involving Heat Transfer

- Household appliances: Heaters, Air Conditioners, Refrigerators, Ovens, Fans
- Cooking, Electronic equipments, Human Body

Industrial Examples

- Heat Exchangers, Evaporators, Condensers, Radiators, Furnaces, Reactors, Polymer Processing Equipment

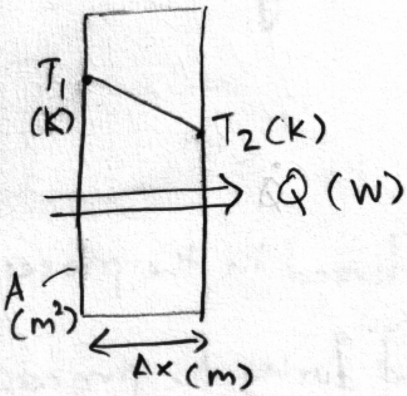
Why worry about heat transfer?

- How to design an efficient equipment (furnace)?
- Does adding insulating material always help retain heat?
- Is a given process or design safe?

Heat Transfer Mechanisms

Conduction:

- can take place in solids, liquids and gases
- Molecular diffusion, vibrations, flow of electrons



Heat conduction through a wall

Observation / Empirical knowledge

- rate of heat conduction through a wall is
  - inversely proportional to wall thickness
  - directly proportional to the area of heat transfer
  - directly proportional to the temperature difference across the wall

i.e.

$$Q = kA\frac{(T_1 - T_2)}{\Delta x} = -kA\frac{dT}{dx} \quad \text{.... In the limit that } \Delta x \to 0$$

$$(W) \quad \left(\frac{W}{m.K}\right)(m^2)\ \frac{(K)}{(m)}$$

$$Q = -kA\frac{dT}{dx} \quad \ldots\ldots \quad \text{Fourier's law of heat conduction}$$

Thus, heat flux

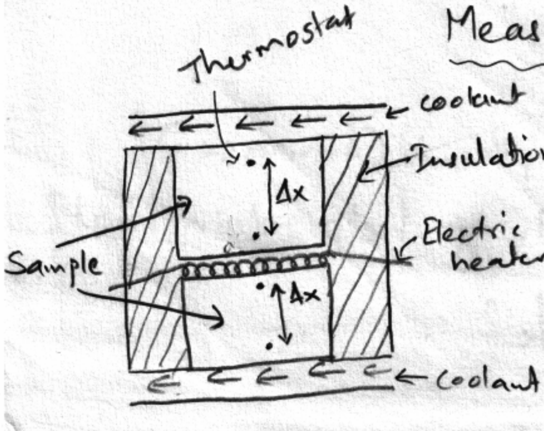
$$q = \frac{Q}{A} = -k\frac{dT}{dx}$$

$$\left(\frac{W}{m^2}\right)$$

where

$k \equiv$ thermal conductivity

Measuring thermal conductivity



- Direct measurement of ΔT across Δx
- Known current passing through heater

Electric power $= VI$, where $V \equiv$ Voltage applied

Typical values of thermal conductivity at 0°C

Metals: $O(10^1) - O(10^2)$ W/m·K

e.g. Silver 410 W/m·K, Copper 385 W/m·K

Non-metal solids: $O(10^{-3}) - O(10^0)$ W/m·K

eg. Styrofoam 3×10^{-3} W/m·K
Ice 2.2 W/m·K

Diamond 2300 W/m·K

Liquids $O(10^{-1}) - O(10^0)$

eg. Mercury 8.21 W/m·K
Water 0.556 W/m·K

Gases $O(10^{-2}) - O(10^{-1})$ W/m·K

eg. Hydrogen 0.175 W/m·K
sat. Water vapor 0.0206 W/m·K

Typically, solids > liquids > gases in thermal conductivity

Example: Flow of heat through a wall

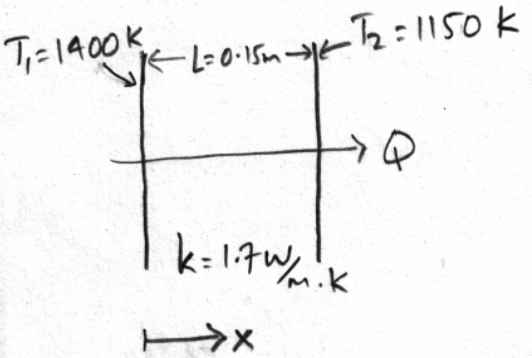
(Example 1.1, Incropera) The wall of an industrial furnace is constructed from 0.15 m thick fireclay brick having a thermal conductivity of 1.7 W/m·K. Measurements made during steady-state operation reveal temperatures of 1400 K and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m × 1.2 m on a side?

Solution:

Find: rate of heat loss through wall

Known: Steady state condition
Wall geometry (thickness, area)
Thermal conductivity
Surface temperatures

Schematic:



Assumptions:

Steady state
One-dimensional heat conduction
Constant thermal conductivity
Constant temperature gradient *

Heat transfer by conduction ⇒ Fourier's law of Conduction

$$q = -k\frac{dT}{dx} \quad ; \quad Q = -kA\frac{dT}{dx} = -kA\frac{T_2 - T_1}{L} = kA\frac{T_1 - T_2}{L}$$

$A \equiv$ Area of heat transfer $= H \times W$ (height, width)

Substituting values,

$$Q = 1.7 \times 0.5 \times 1.2 \times \frac{250}{0.15} \quad W$$

$$(W) \quad (W/m.K) \; (m) \; (m) \; \left(\frac{K}{m}\right)$$

Note:

On the "Constant thermal conductivity" assumption
- Reasonable for most practical problems
- Solids: conductivity may increase or decrease with temperature

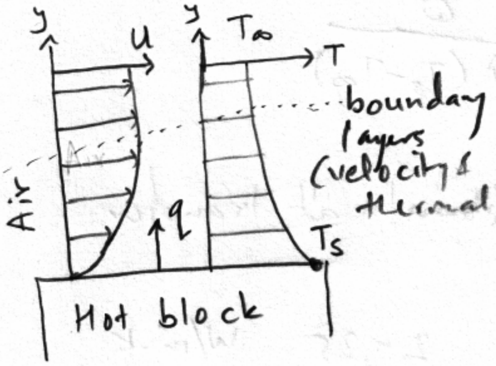
Liquids: mostly decreases with increasing temperature

Gases: Increases with increasing temperature.

* Can also be derived using energy balance equation at steady state

Convection

- between a solid surface and adjacent fluid in motion
- combination of conduction and fluid motion
- Types:
    - Forced convection: fluid flow is externally imposed (using a pump/fan)
    - Natural (free) convection: fluid flow is caused by buoyancy forces induced by density differences due to temperature gradients
    - Additional latent heat exchange associated with phase change



Observation/Empirical knowledge:

- rate of heat transfer is
    - proportional to the temperature difference
    - proportional to the area of the surface

Heat transfer by convection $\Rightarrow$ Newton's Law of Cooling

$$Q = hA(T_s - T_\infty)$$
$$(W) \quad \left(\frac{W}{m^2 \cdot K}\right)(m^2) \quad (K)$$

and heat flux

$$q = h(T_s - T_\infty)$$
$$\left(\frac{W}{m^2}\right) \quad \left(\frac{W}{m^2 \cdot K}\right) \quad (K)$$

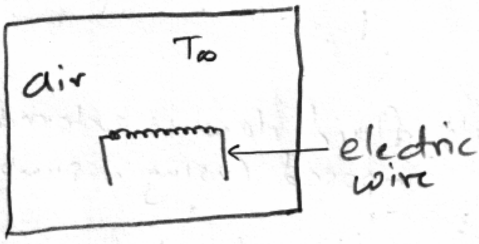
with

$h \equiv$ convection heat transfer coefficient

Unlike thermal conductivity, convection heat transfer coefficient is not a material property, but also depends on the surface geometry, nature of fluid flow, etc.
$\Rightarrow$ Must be determined for every setup.

Measuring convection heat transfer coefficient

Example: Electrical wire in a room



- Known rate of heat generation (Q
- Known area of surface (A)
- Known surface and ambient (T_s, T_∞ temperatures

$$h = \frac{Q}{A(T_s - T_\infty)}$$

Typical Values of convection heat transfer coefficient

Free convection	gases	2 - 25 $W/m^2 \cdot K$
	liquids	10 - 1000 $W/m^2 \cdot K$
Forced convection	gases	25 - 250 $W/m^2 \cdot K$
	liquids	50 - 20 000 $W/m^2 \cdot K$
Boiling and condensation		2500 - 100 000 $W/m^2 K$

Example: Measuring convection heat transfer coefficient

A 2 m long, 0.3 cm diameter electrical wire extends across a room at 15°C. Heat is generated in the wire as a result of resistance heating, and the surface temperature of the wire is measured to be 152°C in steady operation. Also, the voltage drop and electric current through the wire are measured to be 60 V and 1.5 A, respectively. Disregarding any heat transfer by radiation, determine the convection heat transfer coefficient for heat transfer between the outer surface of the wire and the air in the room.

Solution:

Find: convection heat transfer coefficient

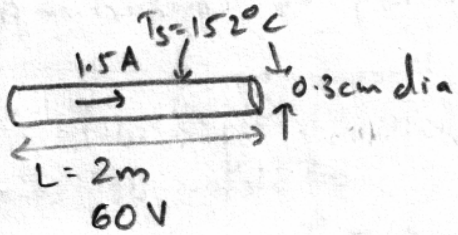
Known: Steady state conditions

Wire geometry

Surface and ambient temperatures

Schematic:

$T_\infty = 15°C$



Assumptions:

Steady state

Negligible heat transfer by radiation

Heat transfer by convection ⇒ Newton's law of cooling

$$Q = hA(T_s - T_\infty)$$

Resistance heating

$$Q = VI$$

$$A = \pi D L$$

Substituting values

$$h = \frac{Q}{A(T_s - T_\infty)} = \frac{VI}{A(T_s - T_\infty)} = \frac{60 \times 1.5}{\pi \times 0.003 \times 2 \times 137}$$

$\left(\frac{W}{m^2 K}\right)$

Note: Convection - typically appears as a boundary condition in conduction problems

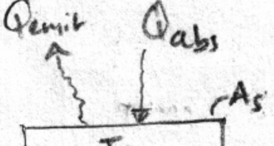
- typically involves determining convection heat transfer coefficient (h).

Radiation

- Emitted by matter at any non-zero absolute temperature
- Emitted by solids, liquids and gases
- Heat transfer through electromagnetic waves
  ∴ a medium is not necessary
  (contrary to conduction & convection)

Observation/Empirical Knowledge

- Maximum rate of radiation emitted by a surface
  ⇒ Stefan-Boltzmann Law



$$Q_{emit,max} = \sigma A_s T_s^4$$

An idealized surface that emits radiation at the maximum rate is called a blackbody.

For real surfaces,

$$Q_{emit} = \varepsilon \sigma A_s T_s^4$$

where

$$\varepsilon = \text{emissivity of the surface}$$

$$0 \leq \varepsilon \leq 1$$

- The portion of any incident radiation (from surroundings onto the surface) absorbed by the surface is given by its absorptivity

$$\alpha = \frac{Q_{absorbed}}{Q_{incident}}$$

$$0 \leq \alpha \leq 1$$

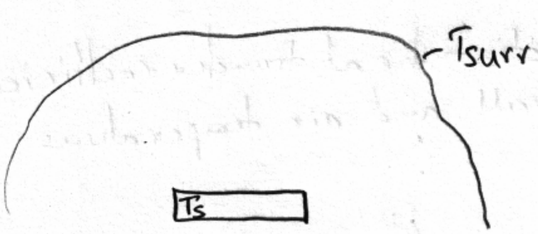
For a blackbody, $\alpha = 1$.

<u>Note:</u>

- Both ε and α of a surface depend on the temperature and wavelength of the radiation
- Kirchoff's law of radiation $\Rightarrow$ $\varepsilon = \alpha$ for a surface at a given temperature and wavelength are equal
- Reflected or transmitted radiation do not affect thermal energy of matter. Typically,

  gases: transparent
  liquids: opaque
  Solids: opaque (metals) or semitransparent (polymers)
- The difference between rates of radiation emission and absorption gives the net radiation heat transfer

<u>Special case:</u>



- small surface fully enclosed in a surrounding surface (large).
- Surrounding surface ~ blackbody

Net rate of radiation heat transfer from/to the surface

$$q = \frac{Q}{A} = \varepsilon\sigma\left(T_s^4 - T_{surr}^4\right)$$

which can be re-written as

$$q = h_r\left(T_s - T_{surr}\right)$$

where

$$h_r = \varepsilon\sigma\left(T_s + T_{surr}\right)\left(T_s^2 + T_{surr}^2\right)$$

$\equiv$ radiation heat transfer coefficient

Thus, if a medium such as air is present between the surface and surrounding surface (and does not intervene with radiation), the convection + radiation heat transfer rate

$$Q = h_{combined} A\left(T_s - T_{surr}\right), \text{ with } h_{combined} = h_{conv} + h_r$$

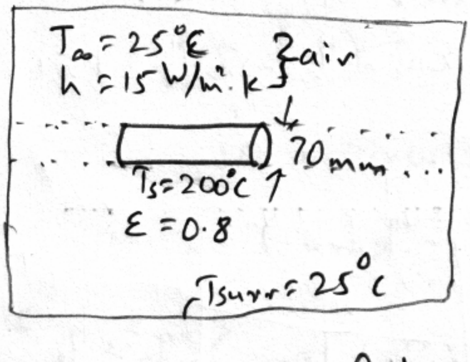
Example: An insulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, resp. What is the radiation heat loss from the surface per unit length of the pipe. If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/m²·K, what is the combined rate of heat loss from the surface per unit length of the pipe?

Solution:

Find: rate of heat loss due to (a) radiation alone and (b) convection + radiation

Known: Pipe diameter
Emissivity and convection heat transfer coefficient
Surface, surrounding wall and air temperatures

Schematic:



Assumptions:
- Steady state
- Small surface enclosed in a large room
- Kirchoff's law of radiation is valid here

Rate of heat loss due to radiation alone

$$Q = \varepsilon\,(\pi D L)\,\sigma\,(T_s^4 - T_{surr}^4)$$

Thus

$$\frac{Q}{L} = \varepsilon \pi D \sigma\,(T_s^4 - T_{surr}^4) = 421\ W/m$$

Note that T_s, T_{surr} must be converted to kelvin scale

Rate of combined heat loss per unit length

$$\frac{Q}{L} = h \pi D (T_s - T_\infty) + \varepsilon \pi D \sigma (T_s^4 - T_{surr}^4)$$

$$= (577 + 421)\ W/m = 998\ W/m$$

In this example, the rates of heat transfer by convection and radiation are comparable, because of (a) large temperature difference between surface and surroundings and (b) free convection heat transfer coefficient is small. But in case of forced convection, radiation heat transfer is often relatively small.

Thermal Resistance

In all three modes of heat transfer, the heat transfer rate can be written as

$$Q = \frac{\Delta T}{R_t}$$

where ΔT is the relevant temperature difference
and A is the surface area normal to the direction of heat transfer.

Mechanism:	conduction	convection	radiation
R_t:	$\frac{L}{kA}$	$\frac{1}{hA}$	$\frac{1}{h_r A}$

This equation is analogous to electric current flow rate

$$I = \frac{\Delta V}{R_e}$$

With this analogy, one can borrow concepts of series and parallel resistances.

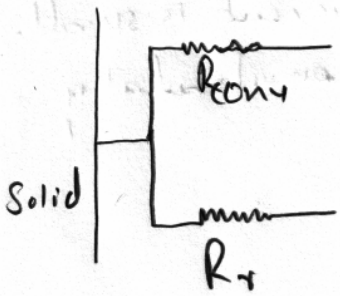
For example, a solid wall surrounded by a gas involves radiation effects in addition to convection.

We had used

$$h_{combined} = h_{conv.} + h_r$$

This is equivalently described by two thermal resistances in parallel (convection and radiation). The net thermal resistance

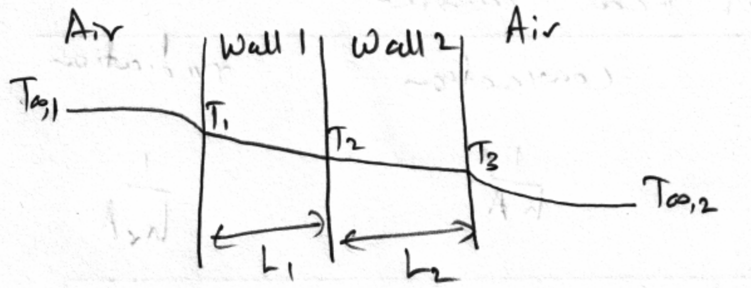
$$\frac{1}{R_{combined}} = \frac{1}{R_{conv.}} + \frac{1}{R_r} \qquad \text{(parallel)}$$



$$\Rightarrow \frac{1}{\left(\frac{1}{h_{combined}A}\right)} = \frac{1}{\left(\frac{1}{h_{conv}A}\right)} + \frac{1}{\left(\frac{1}{h_r A}\right)}$$

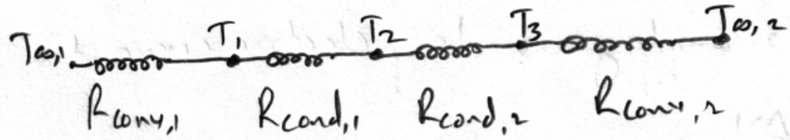
$$\Rightarrow h_{combined} = h_{conv} + h_r$$

A composite (multi-layer) plane wall surrounded by a gas on both sides. Even if the radiation heat transfer is ignored, there are multiple resistances (thermal) in series (convection, conduction), resulting into a net thermal resistance



$R_{combined}$ (series)

$= R_{conv,1} + R_{cond,1} + R_{cond,2} + R_{conv,2}$



$$\frac{1}{h_1 A} \qquad \frac{L_1}{k_1 A} \qquad \frac{L_2}{k_2 A} \qquad \frac{1}{h_2 A}$$