Fin Performance

Fins :
- increase surface area
- pose a conduction resistance

$\Rightarrow$ Performance must be assessed

Fin Effectiveness

Enhancement in heat transfer rate relative to the no-fin case.

$$\varepsilon_f = \frac{Q_f}{Q_{no\text{-}fin}} = \frac{Q_f}{h A_{c,b} \theta_b}$$

where

$A_{c,b} \equiv$ fin cross-sectional area at the base.

Fins add to the cost and complexity of equipment. Thus, their use is justified only if the effecti-eness compensates for the added cost and complexity.

For an infinitely long fin of uniform cross-section, (case A in the table),

$$Q_f = M = \sqrt{hPkA_c}\ \theta_b$$

Thus

$$\varepsilon_f = \frac{\sqrt{hPkA_c}\,\theta_b}{h A_{c,b} \theta_b}$$

Also, $A_c = A_{c,b}$ $\because$ uniform cross-section

$$\Rightarrow \quad \varepsilon_f = \left(\frac{kP}{hA_c}\right)^{1/2}$$

Note:

- Fin effectiveness is enhanced by choosing material of high thermal conductivity
  eg. Copper or Aluminium alloys

- Fin effectiveness is also enhanced by increasing the perimeter to cross-sectional area ratio.
  thus, thin fins are preferred, as long as the flow is not affected significantly and hence the heat transfer coefficient is not reduced significantly

- Fins are better justified when convection heat transfer coefficient is small.
  thus, the same fin can enhance heat transfer to a greater extent if placed on the air side vs on the liquid side, when heat is exchanged between a liquid and a gas seperated by a solid

- Fin resistance $R_{th,f} = \frac{\theta_b}{Q_f}$ and convection thermal resistance at the base in absence of the fin $R_{th,b} = \frac{1}{hA_{c,b}}$ gives an alternate expression for fin effectiveness

$$\varepsilon_f = \frac{Q_f}{Q_{nofin}} = \frac{R_{th,b}}{R_{th,f}}$$

Thus, a fin should not have more resistance than that of the exposed base.

Fin Efficiency

- With respect to ideal fin material

$$\eta_f = \frac{Q_f}{Q_{f,max}} = \frac{Q_f}{h A_f \theta_b} = \frac{1}{h A_f R_{th,f}}$$

- The denominator is the rate of heat transfer if the entire fin is at the temperature at its base.

- For a straight fin of uniform cross-section with an adiabatic tip, $Q_f = M \tanh mL$

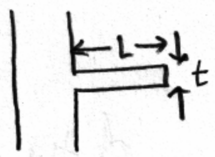
Thus,

$$\eta_f = \frac{M \tanh mL}{h(PL)\theta_b} = \frac{\tanh mL}{mL} \qquad \because \frac{M}{hP\theta_b} = \frac{1}{m}$$

- Corrected fin lengths: For a straight fin of uniform cross-section with convection from tip, instead of dealing with the complicated expression for heat transfer, approximation can be made to enable using expression for an adiabatic tip, if a "corrected length" is used instead of the actual length.

For a rectangular cross-section, the corrected length

$$L_c = L + t/2$$



Then,

$$Q_f = M \tanh mL_c$$

and

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

This assumption gives a reasonable estimate if $ht/k \leq 0.0625$

Here,

$$mL_c = \left(\frac{hP}{kA_c}\right)^{1/2} L_c = \left(\frac{2h}{kt}\right)^{1/2} L_c \, ,$$

by approximating

$$P \approx 2w \qquad \text{(i.e. assuming } w \gg t\text{)}$$

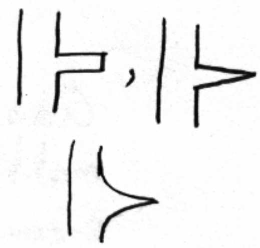
Note that $A_c = wt$ is the cross-sectional area

If $A_p = L_c t$ is a corrected fin profile area
(i.e. corrected side-area)

$$mL_c = \left(\frac{2h}{kA_p}\right)^{1/2} L_c^{3/2}$$

Plots of η_f vs $\left(\frac{h}{kA_p}\right)^{1/2} L_c^{3/2}$ are used in selection and design of straight fins (Uniform or varying cross-section) — Efficiency curves for fins.

eg: rectangular, triangular, parabolic



Non-uniform Cross-sectional Area

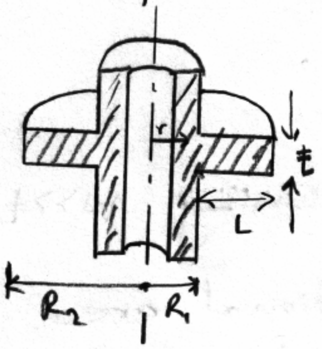
Energy balance was simplified to give

$$0 = \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{hP}{k}(T - T_\infty)$$

$$0 = \frac{d^2T}{dx^2} + \underbrace{\left(\frac{1}{A_c}\frac{dA_c}{dx}\right)}_{\text{non-zero}}\frac{dT}{dx} - \frac{hP}{kA_c}(T - T_\infty)$$

Knowledge about fin geometry is required beyond this point.

Special case: Annular fin



- Cross sectional area

$$A_c = 2\pi r t \Rightarrow \text{function of } r$$

- Surface area

$$A_s = 2\pi(r^2 - R_1^2)$$
$$\Rightarrow \frac{dA_s}{dr} = 4\pi r$$

- Thickness t = uniform

Energy balance simplifies to

$$0 = \frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{2h}{kt}(T - T_\infty)$$

In terms of excess temperature

$$0 = \frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - m^2\theta$$

– modified Bessel equation of order zero

i.e. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - \alpha^2)y = 0$ with $\alpha = 0$

General solution is in terms of modified Bessel functions of first and second kinds.

$$\theta(r) = \underbrace{C_1 I_0(mr)}_{} + \underbrace{C_2 K_0(mr)}_{}$$

I_0: zero-order modified Bessel function of first kind

K_0: zero-order modified Bessel function of second kind

With $\theta = \theta_b$ at $r = R_1$ and an adiabatic tip, i.e $\left.\frac{dT}{dr}\right|_{r=R_2} = 0$

$$\frac{\theta(r)}{\theta_b} = \frac{I_0(mr)K_1(mR_2) + K_0(mr)I_1(mR_2)}{I_0(mR_1)K_1(mR_2) + K_0(mR_1)I_1(mR_2)}$$

with

$$Q_f = -k A_{c,b}\left.\frac{dT}{dr}\right|_{r=R_1} = k(2\pi R_1 t)\frac{d\theta}{dr}$$

$$Q_f = 2\pi k R_1 t \theta_b m \frac{K_1(mR_1)I_1(mR_2) - I_1(mR_1)K_1(mR_2)}{K_0(mR_1)I_1(mR_2) + I_0(mR_1)K_1(mR_2)}$$

And fin efficiency is given by

$$\eta_f = \frac{Q_f}{h(2\pi(R_2^2 - R_1^2))\theta_b}$$

For convection at the fin tip, an approximation in terms of a "corrected radius", $R_c = R_2 + t/2$ can be made to use the above set of equations.

Optimum Fin:

- highest heat transfer for minimum amount of metal
- manufacturing costs, material costs, etc need to be considered
- chosen fin may depend on costs and availability and structural adaptability for industrial applications

Overall Effectiveness and Efficiency

- For an array of fins
- Heat transfer for a surface with N fins

$$Q_{total,fin} = Q_{unfinned} + Q_f$$
$$= hA_{unfinned}(T_b - T_\infty)$$
$$+ \eta_f h N A_f (T_b - T_\infty)$$

where A_f is the surface area of each fin and η_f is the efficiency of each fin

Overall effectiveness is given by

$$\varepsilon_{f,overall} = \frac{Q_{total,f}}{Q_{total,no\,fin}} = \frac{h(A_{unfinned} + \eta_f N A_f)(T_b - T_\infty)}{h(A_{no\,fin})(T_b - T_\infty)}$$

Overall surface efficiency is given by

$$\eta_{f,overall} = \frac{Q_{total,f}}{Q_{max,f}} = \frac{Q_{total,f}}{h A_{total} \theta_b}$$

where

$$A_{total} = A_{unfinned} + N A_f$$

Rearranging the total rate of heat transfer,

$$Q_{total,f} = h A_{total}\left[1 - \frac{N A_f}{A_{total}}(1-\eta_f)\right]\theta_b$$

Thus,

$$\eta_{f,overall} = 1 - \frac{N A_f}{A_{total}}(1-\eta_f)$$

In terms of the thermal resistance of the fin array

$$R_{th,overall} = \frac{\theta_b}{Q_{total,f}} = \frac{1}{\eta_{f,overall}\, h A_{total}}$$

Contact resistance may arise depending on how fins are attached.

Equivalent thermal circuits can be used to help with the analysis in these cases.