ChE641 Mathematical Methods in Chemical Engineering

Assignment 8 **Submission not compulsory**
Differential Equations

1. Solve $\frac{\mathrm{d}y}{\mathrm{d}x} + 2xy = 4x$ subject to the initial condition $y(x = 0) = y_0$.
2. Determine the solution of $y'' + y' - 2y = 0$ with $y(0) = 0$ and $y'(0) = 6$, where primes denote derivatives with x.
3. Find the three linearly independent solutions of $y''' - 3y'' + 3y' - y = 0$. Show that they are linearly independent.
4. Solve the differential equation $y'' + 3y' + 2y = 6 + x^2$.
5. Using the method of eigenfunction expansions, solve the following linear partial differential equation: [10 points]

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions $u(0, t) = u(L, t) = 0$ and the initial conditions $u(x, 0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$, with $f(x = 0) = f(x = L) = 0$.

Useful information

Orthnormality of sines and cosines:

$$\int_0^L \sin(n\pi x/L) \sin(m\pi x/L) = \delta_{mn} \frac{L}{2}$$

$$\int_0^L \cos(n\pi x/L) \cos(m\pi x/L) = \delta_{mn} \frac{L}{2}$$