Natural Convection

- Characterized by convection currents
- Buoyancy effect causes natural convection currents

Buoyancy force

$$F_{buoyancy} = \underbrace{\rho_{fluid}}_{\text{fluid density}} \underbrace{g}_{\text{gravitational acceleration}} \underbrace{V_{body}}_{\text{immersed volume of body}}$$

The net force is a balance between weight of an immersed body and buoyancy force acting on it

$$F_{net} = (\rho_{body} - \rho_{fluid})\, g\, V_{body}$$

Examples: · Flow of hot gases upward through a chimney
· Floating of heavy metal ships on water

Density difference results into buoyancy effect

Volume Expansion Coefficient

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P$$

specifies the extent of variation of density of the fluid with temperature, at constant pressure.

In natural convection, β is approximately defined also as

$$\beta \approx -\frac{1}{\rho}\,\frac{\rho_\infty - \rho}{T_\infty - T} \quad \text{at constant } P.$$

where ρ_∞ is the density of fluid far away from the surface
and T_∞ is the temperature "

Larger temperature difference between fluid and an adjacent surface, the larger the density difference

$\Rightarrow$ a larger buoyancy force

$\Rightarrow$ a stronger natural convection current

& a higher heat transfer rate

Thus, rate of natural convection heat transfer is directly dependent on the fluid's flow rate.

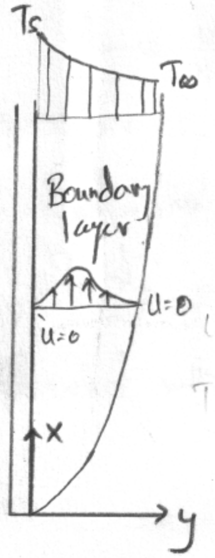
Buoyancy force due to density difference in the same fluid at different temperatures (at different spatial locations) is proportional to

- difference in density at the two temperatures
- Volume occupied by the warmer fluid

Free convection can be unbounded (by a solid surface, such as plumes) or bounded by a surface. We will consider the latter, which is more relevant for heat exchange in chemical processes.

Laminar Boundary Layers

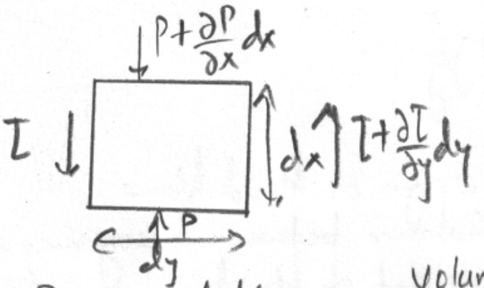
- Consider a vertical flat plate immersed in a quiescent fluid at a different temperature.
- Assume the resulting natural convection flow to be steady, laminar and two-dimensional.
- Properties of the fluid are constant, except its density, which is different at different locations (due to temperature dependance of density) and which results into flow.



Stationary fluid
u_∞, T_∞

- Density difference $\rho - \rho_\infty$ drives flow
- 2D flow, with $u(x,y)$ and $v(x,y)$ as the velocity components

- A boundary layer develops (similar to forced convection), with thickness increasing in the flow direction
- Unlike forced convection, $u=0$ both at the surface of the plate and outside the boundary layer.



For a differential volume element of sides dx, dy and $dz=1$, Newton's second law of motion $\Rightarrow$ $F_x = \underset{\text{mass of fluid}}{\underset{\uparrow}{\delta m}} \cdot a_x$

$$\delta m = \rho(dx \cdot dy \cdot 1) \quad ; \quad a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial t} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

Forces:
- Pressure acting on top and bottom surfaces
- Shear stresses acting on the side surfaces
- Gravity

$$F_x = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx\, dy\, 1)$$

$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx\, dy\, 1) \quad \text{... Newtonian fluid}$$

$$\Rightarrow \quad \rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2} - \rho g$$

Outside the boundary layer, $u = v = 0$

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g \quad \text{... hydrostatic pressure}$$

Assuming that $u >> v$ in the boundary layer, $\frac{\partial v}{\partial x}$ and $\frac{\partial v}{\partial y}$ are negligible. In the y-direction, there is no component of gravitational acceleration. Thus, y-directional momentum balance gives $\frac{\partial P}{\partial y} = 0 \Rightarrow P = P(x)$ only. If P is not varying in y-direction, using $\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$ in the boundary layer equation for x-momentum balance gives

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + \underbrace{(\rho_\infty - \rho)g}$$

Origin of fluid flow
Balance between buoyancy and weight of fluid.

Using the approximate definition of β (the volume expansion coefficient),

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

Buoyancy effects appear in momentum balance. Thus, the continuity and energy balance equations are the same as in forced convection.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

Boundary Conditions

At $y = 0$ $\quad u(x,0) = v(x,0) = 0 \quad ; \quad T(x,0) = T_s$

At $y \to \infty$ $\quad u(x,\infty) \to 0$ and $v(x,\infty) \to 0$; $T(x,\infty) \to T_\infty$

Nondimensionalized Form

Using L as the characteristic lengthscale
u_0 as the characteristic velocity scale

$$x^* = \frac{x}{L}; \quad y^* = \frac{y}{L}; \quad u^* = \frac{u}{u_0}; \quad v^* = \frac{v}{u_0}; \quad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{g\beta (T_s - T_\infty) L}{u_0^2} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Note: Unlike in forced convection, no simple velocity scale is available in this problem

For convenience, let

$$u_0^2 = g\beta (T_s - T_\infty) L.$$

Then,

$$\frac{g\beta (T_s - T_0) L}{u_0^2} = \frac{g\beta (T_s - T_0) L^3}{\nu^2} \times \frac{1}{Re_L^2} = 1$$

Grashof Number

$$Gr_L = \frac{g\beta (T_s - T_\infty) L^3}{\nu^2} \quad (= Re_L^2, \text{ with chosen } u_0)$$

- Represents the ratio of buoyancy force to the viscous force
- Governs the flow regime in Natural Convection (as Reynolds number does for forced convection)

For $Gr_L < 10^9$, flow around a veritical plate due to natural convection is laminar

- If a problem involves free and forced convection, the relative importance is decided by Gr_L/Re_L^2.

$$\frac{Gr_L}{Re_L^2} \gg 1,$$ inertial forces are negligible
$\Rightarrow$ natural convection effects dominate

Rayleigh Number

$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu\alpha}$$

Going back to nondimensional form of governing equations.

The complexities of fluid motion make analytical solution nearly impossible except for a few simple geometries with restrictive assumptions. Instead, correlations are widely available and often used. The guessed solution form, , for the average Nusselt number, is

$$Nu = \frac{hL}{k} = f(Gr_L, Pr)$$ for purely free/natural convection

$$Nu = f(Re_L, Gr_L, Pr)$$ for free and forced convection (combined)

Recall

$$Nu = f(Re_L, Pr)$$ for purely forced convection

The simplest correlations for free convection are of the form

$$Nu = \frac{hL}{k} = C(Gr_L Pr)^n = C\,Ra_L^n$$

Typically, $n = 1/4$ for laminar flow, and $n = 1/3$ for turbulent flow. Properties are evaluated at film temperature $T_f = \frac{1}{2}(T_s + T_\infty)$

Vertical Plate Correlations (plate length ≡ lengthscale)

Isothermal:

Laminar flow $\quad (10^4 \le Ra_L \le 10^9)$

$$Nu = 0.59\, Ra_L^{1/4}$$

Turbulent flow $\quad (10^9 \le Ra_L \le 10^{13})$

$$Nu = 0.1\, Ra_L^{1/3}$$

A more involved correlation applicable over the entire range is

$$Nu = \left[0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2$$

— Churchill and Chu

Constant Surface Flux:

Same as above correlations, using the plate mid-point temperature $T(x = L/2, y = 0)$ as the surface temperature (representative, T_s) in evaluating the film temperature, Ra_L. Using

$$h = \frac{q_s}{(T_{L/2} - T_\infty)},$$ the average Nusselt number is

$$Nu = \frac{hL}{k} = \frac{q_s L}{k\,(T_{L/2} - T_\infty)}$$

Iterations are used to determine $T_{L/2}$ to get matching Nu from the above equation and from the correlation.

These correlations can also be used for vertical cylinders, given that boundary layer thickness is much smaller than the diameter of the cylinder, D, as verified by

$$\frac{D}{L} \geq \frac{35}{Gr_L^{1/4}}$$

For inclined plates, the same correlations can be used for the upper surface of a cold plate and lower surface of a hot plate, with g replaced by $g\cos\theta$, θ being the angle with respect to vertical. For the other cases (lower surface of cold plate or top surface of a hot plate), equivalence cannot be established with these correlations. Specific solutions are presented in literature (e.g. Fujii and Imura, 1972).

Horizontal Plates

Length scale $L = \frac{A_c}{p}$

Upper surface of a hot plate or lower surface of a cold plate:

$$Nu = 0.59\, Ra_L^{1/4} \qquad 10^4 \leq Ra_L \leq 10^7,\ Pr \geq 0.7$$

$$Nu = 0.1\, Ra_L^{1/3} \qquad 10^7 \leq Ra_L \leq 10^{11}$$

Lower surface of a hot plate or upper surface of a cold plate:

$$Nu = 0.27\, Ra_L^{1/4} \qquad 10^5 \leq Ra_L \leq 10^{11}$$