ChE641 Mathematical Methods in Chemical Engineering

Assignment 3 **Due Date: 1 October, 2020**

Complex variables and Complex analysis

1. Find the analogue of the Cauchy-Riemann equations for an "*anti-analytic*" function $f(x,y) = u(x,y) + iv(x,y)$ which is a function only of $z^* = x - iy$. To do this, you need to set $\partial f/\partial z = 0$ and set the real and imaginary parts of this equation to zero.

2. Consider a real-valued function of two variables $f(x,y)$. You can change variables from (x,y) to $x_+ = (x+y)/2$ and $x_- = (x-y)/2$. Derive the conditions such that the function is a function *only* of (a) x_+ , and (b) x_- .

3. Locate and name the type of singularities of $f(z) = 1/(1+z^4)$.

4. Verify if $f(z) = (x-iy)/(x^2+y^2)$ is analytic.

5. Write down the two square roots of (a) i and (b) $(3+4i)$ in Cartesian form.

6. Express the following in Cartesian form: (a) 3^i, (b) $\ln[(1+i)/(1-i)]$.

7. Write $(1+i)$ in polar form and find its cube roots in Cartesian form. Check explicitly that their cubes do give the original complex number.

8. Find all the roots of $\sqrt[5]{i}$.

9. Find the value in Cartesian form: (a) $\ln i$, (b) $\ln(1-i)$.

10. Find the Cauchy-Riemann equations in polar coordinates. *Hint:* Write $z = re^{i\theta}$, and $f(z) = u(r,\theta) + iv(r,\theta)$.