Introduction to Convection

Convection
- Natural or free convection
  - buoyancy causes fluid flow
- Forced convection
  - externally induced flow

In a fluid,
- Presence of bulk fluid flow ⇒ heat transfer by convection
- Absence of bulk fluid flow ⇒ heat transfer by conduction

⇒ Conduction is a limiting case of convection for a quiescent fluid.

⇒ higher the fluid velocity, higher the rate of heat transfer

Empirical knowledge
- Convection heat transfer depends on
  - fluid properties: μ, k, ρ, c_p
  - fluid velocity : v
  - surface properties : geometry and roughness
  - type of flow : laminar / turbulent

Model for convection heat transfer flux (Newton's Law of Cooling)

$$q_{conv} = h\,(T_s - T_\infty) \qquad W/m^2$$

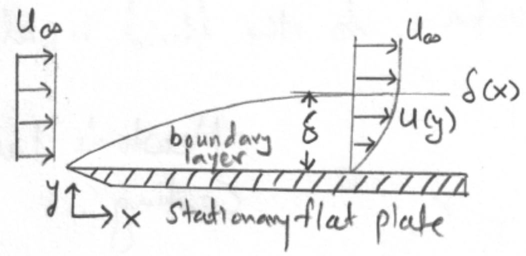
h ≡ convection heat transfer coefficient
depends on several of the above factors, making the flux equation complicated.

Understanding how fluid flow and heat transfer by convection are affected by presence of solid surfaces is crucial - Hence, boundary layers.

Boundary Layers

- presence of solid surfaces affects fluid flow
  - condition on the fluid layer adjacent to a solid surface is that of matching velocity (no slip boundary condition)

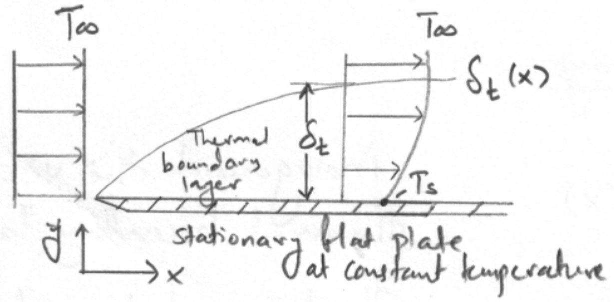
Velocity Boundary Layer



- Consider flow over a flat plate
- Far upstream, velocity is uniformly $(u_\infty, 0, 0)$
- Fluid layer adjacent to the plate has zero velocity in x-direction
- Subsequent layers farther and farther away have increasing velocity, until the layer for which the velocity in x-direction is $\sim 0.99\, u_\infty \equiv$ boundary layer

- Distance of this layer from the plate is called the boundary layer thickness, δ
- As viscous effects reach farther with increasing x-coordinate, the boundary layer becomes thicker. $\Rightarrow \delta = \delta(x)$

Thermal Boundary Layer



- Fluid at uniform upstream temperature T_∞
- Plate at constant surface temperature T_s (say $< T_\infty$)
- Heat is exchanged between subsequent layers farther and farther away from the plate

- Temperature profile develops along y-direction
- The distance of the layer of fluid with temperature $= 0.99(T_\infty - T_s)$ relative to T_s i.e. $T - T_s = 0.99(T_\infty - T_s)$ is $\delta_t \equiv$ boundary layer thickness
- $\delta_t = \delta_t(x)$.

At a distance x from the edge of the plate, the local heat flux at the surface,

$$q\,(x=x, y=0) = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Fourier's Law of Conduction

This is because of the no slip boundary condition, implying no bulk fluid motion at $y=0$. Thus, conduction is the mechanism of heat transfer. Here, k_f = thermal conductivity of the fluid.

The flux of heat from the solid surface to the fluid is also given as

$$q = h\,(T_s - T_\infty)$$

Newton's Law of Cooling

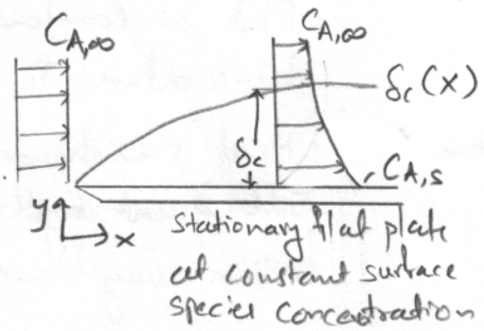
The two equations above represent the same heat flux.

Thus,

$$h = \frac{-k_f \left.\frac{\partial T}{\partial y}\right|_{y=0}}{T_s - T_\infty}$$

As δ_t increases with x (distance from the leading edge), $\left.\frac{\partial T}{\partial y}\right|_{y=0}$ decreases with increasing x. Thus, h and hence q decrease with increasing x.

Concentration Boundary Layer



- Analogous to the viscous and thermal boundary layers
- Thickness of boundary layer δ_c = value of y for which $C_{A,s} - C_A = 0.99\,(C_{A,s} - C_{A,\infty})$
- $\delta_c = \delta_c(x)$

Convection mass transfer coefficient

$$h_m = \frac{-D_{AB} \left.\frac{\partial C_A}{\partial y}\right|_{y=0}}{C_{A,s} - C_{A,\infty}}$$

Relevance of Boundary Layers

- Flow over any surface ⇒ Velocity boundary layer
  - depends on type of flow
  - thickness δ
  - friction coefficient
    $$C_f = \frac{\tau_w}{\frac{\rho u_\infty^2}{2}}$$ — typically determined from experiments

- Temperature difference between free stream and surface ⇒ Thermal boundary layer
  - thickness δ_t
  - Convection heat transfer coefficient
    $$h = \frac{q_s}{(T_s - T_\infty)} = \frac{-k\frac{\partial T}{\partial y}\big|_{y=0}}{T_s - T_\infty}$$

- Concentration difference between free stream and surface ⇒ Concentration boundary layer
  - thickness δ_c
  - convection mass transfer coefficient
    $$h_m = \frac{-D_{AB}\frac{\partial C_A}{\partial y}\big|_{y=0}}{(C_{A_s} - C_{A,\infty})}$$

In many situations, two or three boundary layers might develop. Especially in flow over a heated surface, both velocity and thermal boundary layers develop simultaneously. Fluid velocity is known to have a strong influence on the temperature profile.

Boundary layers' thicknesses vary with location (distance from the leading edge). Further, velocity and thermal boundary layers may not develop to be identical. Thus, their relative thicknesses will have a strong effect on the convection heat transfer coefficient.

Prandtl Number

- Relative thickness of velocity and thermal boundary layers

$$Pr = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$$

$$= \frac{\nu}{\alpha} = \frac{C_p \mu}{k}$$

Gases
$Pr \sim O(1)$

Water
$Pr \sim O(10)$

Liquid metals
$Pr \sim O(0.01)$ or less

Oils
$Pr \sim O(10 - 100\,000)$

Thus, thermal boundary is relatively thicker for liquid metals, and heat diffuses very quickly through them.

Reynolds Number

- Fluid flow is streamlined at low velocities, but turns chaotic beyond a critical velocity
  $\Rightarrow$ laminar $\rightarrow$ turbulent transition
- Intense mixing of fluid in turbulent flow
  $\Rightarrow$ enhanced momentum and heat transfer
- Transition depends on surface geometry, roughness, flow velocity, type of fluid, etc.

· Transition depends primarily on relative strength of inertia forces to viscous forces in the fluid.

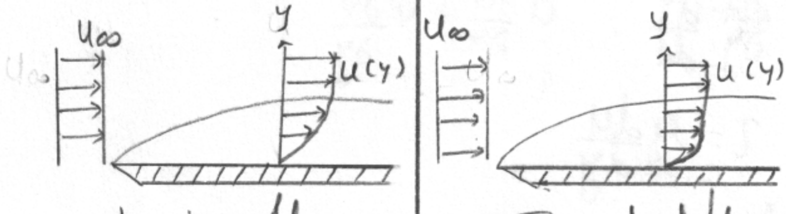
$$Re = \frac{\text{Inertia forces}}{\text{Viscous forces}}$$

$$= \frac{u_\infty L_c}{\nu} = \frac{\rho u_\infty L_c}{\mu}$$

· Flow becomes turbulent beyond a critical Reynolds number.

For flat plate $Re_{cr} = \dfrac{u_\infty x_{cr}}{\nu}$

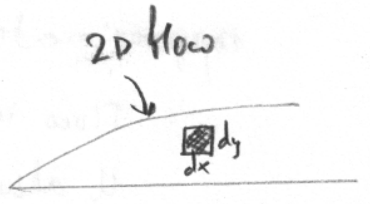
where $x_{cr} \equiv$ distance from leading edge of the plate where flow transitions from laminar to turbulent



Laminar flow | Turbulent flow

Differential Convection Equations

Continuity Equation (mass conservation)

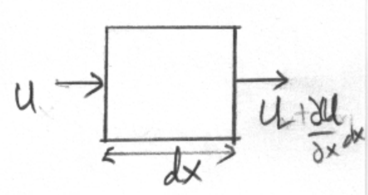


In steady flow of incompressible fluid

Rate of mass flow into the control volume = Rate of mass flow out of the control volume.

$u \equiv$ x-component of velocity
$v = y$-component of velocity

$$\rho u(dy \times 1) + \rho v(dx \times 1) = \rho\left(u + \frac{\partial u}{\partial x}dx\right)dy\,1 + \rho\left(v + \frac{\partial v}{\partial y}dy\right)dx \times 1$$



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

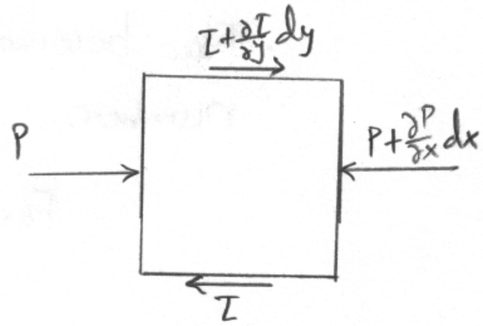
Momentum Equations

- Newton's second law of motion
- 2D steady flow

mass × Acceleration in a given direction = Net force (surface + body) acting in that direction

In x-direction,

$$\rho(dx\,dy\,w) \times \frac{du}{dt} = \left(\frac{\partial \tau}{\partial y} dy\right)(dx\,w)$$
$$\left(-\frac{\partial P}{\partial x} dx\right)(dy\,w)$$



ignoring normal stress and body forces.

$$u = u(x,y) \Rightarrow \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y}$$

For Newtonian fluids, $\tau = \mu \frac{du}{dy}$

Thus, at steady state,

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$

Similar equation is obtained for the y-direction. The y-directional momentum balance can be simplified using boundary layer approximations.

Flow is predominantly in x-direction $\Rightarrow u \gg v$

u also varies strongly in y-direction compared to x-direction $\Rightarrow \frac{\partial u}{\partial y} \gg \frac{\partial u}{\partial x}$

$$0 = -\frac{\partial P}{\partial y}$$