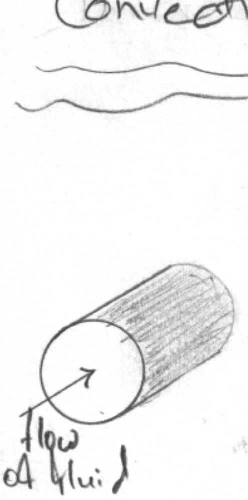
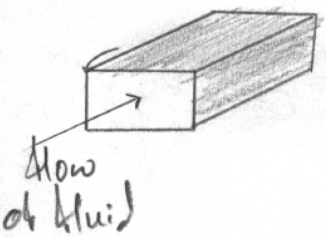
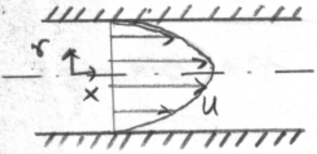
Convective Heat Transfer in Tubes



- Flow in pipes, ducts, conduits
- Common in heating and cooling operations
- Fundamentally different from external flows (over pipes, plates, etc) in the following ways
  - Unlike an unconstrained boundary layer in external flows, confinement limits the boundary layer thickness ⇒ Entry length, Fully developed region
  - Free stream velocity is no longer available as a velocity scale ⇒ Average velocity, mean temperature



Average velocity



Mass conservation ⇒

$$\dot{m} = \rho U_{avg} A_c = \int_{A_c} dA_c\, \rho U(r)$$

$$U_{avg} = \frac{\int_{A_c} dA_c\, \rho U(r)}{\rho A_c}$$

Mean Temperature

Energy Conservation

$$\dot{E}_{fluid} = \dot{m} c_p T_m = \int_{\dot{m}} \delta \dot{m}\, c_p T(r) = \int_{A_c} dA_c\, U(r) T(r) c_p \rho$$

$$T_m = \frac{\int_{A_c} dA_c\, \rho c_p\, U(r) T(r)}{\rho c_p U_{avg} A_c}$$

Hydraulic Diameter

- For flow through non-circular cross-section tubes
- Useful in defining a length-scale for Re, Nu, etc.

$$D_h = \frac{4A_c}{P} \quad ; \quad \begin{aligned} A_c &\equiv \text{cross-sectional area} \\ P &\equiv \text{wetted perimeter} \end{aligned}$$

Entrance and Fully Developed Regions

- Fluid entering a circular pipe at uniform velocity
- No-slip at walls results into increased velocity closer to the centerline $\Rightarrow$ velocity profile
- Boundary layer develops from the entrance of the pipe
- Thickness of boundary layer increases with distance from entrance, until it approaches the pipe radius.
- Due to confinement, boundary layer thickness cannot exceed the pipe radius $\Rightarrow$ fully developed velocity profile.
- Entrance region (hydrodynamic): region between entrance and the point where velocity becomes fully developed. Length of this region is entry length.

  $L_H \equiv$ length where wall shear stress is within 2% of its fully developed value

- Beyond entrance region is fully developed region.
- Similarly, a thermal entrance region, thermal entry length and thermally fully developed region can be defined using the profile of dimensionless temperature, $\frac{T_s - T}{T_s - T_m}$

Fully developed regions in a circular pipe

Hydrodynamically $\qquad \frac{\partial u(x,r)}{\partial x} = 0 \qquad \Rightarrow u = u(r)$ only

Thermally $\qquad \frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right] = 0 \Rightarrow T = T(r)$ only iff $T_s \equiv$ const

$\Rightarrow$ local friction coefficient and local heat transfer coefficient are independent of x.

<u>Entry Length</u>:

Hydrodynamic: $L_{h,\text{laminar}} \approx 0.05\, Re\, D$

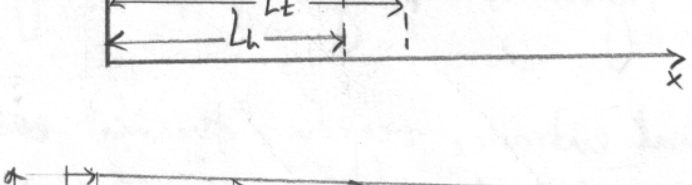
Thermal: $L_{t,\text{laminar}} \approx 0.05\, Re\, Pr\, D = Pr\, L_{h,\text{laminar}}$

Flow transitions to turbulent for $Re > 2300$

$$L_{h,\text{turbulent}} \approx L_{t,\text{turbulent}} \approx 10D$$

Note: • Just below the critical Reynolds number, the hydrodynamic entry length is 115 D, but transitions to a much shorter value as the flow becomes turbulent

• No significant dependence on Re, Pr.

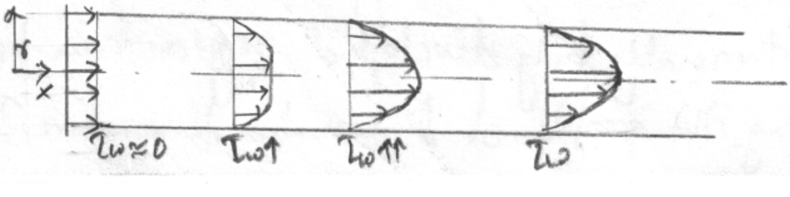


$\because$ At $x \approx 0$

$\frac{\partial u}{\partial r} \approx 0$ uniform fluid velocity

and

$\frac{\partial}{\partial r}\left[\frac{T_s - T}{T_s - T_m}\right] \approx 0$



Energy Balance and Boundary Conditions

- Steady flow of a fluid in a tube

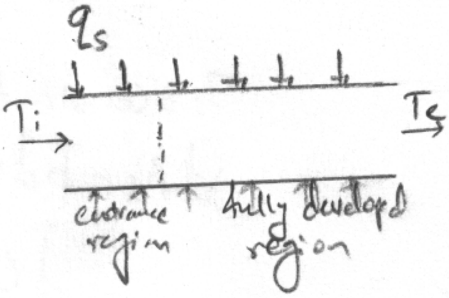
$$Q = \dot{m} C_p (T_e - T_i) \; ; \qquad T_e \equiv \text{exit fluid temperature}$$
$$T_i \equiv \text{inlet fluid temperature}$$

- Boundaries can be at a
  - Constant temperature $\qquad T_s \equiv \text{constant}$

  OR
  - Constant heat flux $\qquad q_s = h_x (T_s - T_m)$

Constant heat flux:

$$Q = q_s A_s = \dot{m} C_p (T_e - T_i)$$



and

$$T_e = T_i + \frac{q_s A_s}{\dot{m} C_p}$$

Area upto location $x \equiv P \times x$

$\Rightarrow T_m \propto x \qquad$ assuming constant fluid properties

$$T_s = T_m + \frac{q}{h} \qquad \because q = h(T_s - T_m)$$

In the fully developed region

$T_s \propto x \qquad \because h \equiv \text{constant}$

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \Rightarrow \frac{1}{T_s - T_m}\left(\frac{dT_s}{dx} - \frac{\partial T}{\partial x}\right) = 0 \Rightarrow \frac{dT_s}{dx} = \frac{\partial T}{\partial x}$$

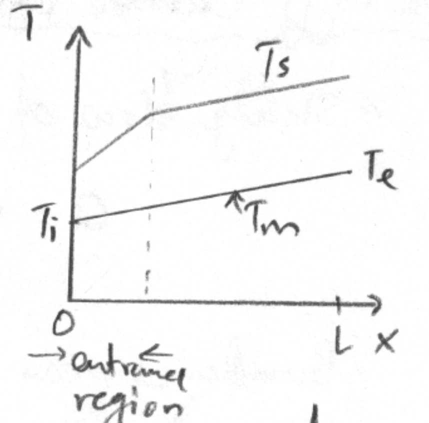
because $\dfrac{dT_s}{dx} = \dfrac{dT_m}{dx}$ from the above expression, at constant q & h in the fully developed region

Also, applying steady state energy balance over a differential length of the tube, dx over which the temperature changes by dT_m.

$$\dot{m} C_p \, dT_m = q_s (p\,dx) \Rightarrow \frac{dT_m}{dx} = \frac{q_s p}{\dot{m} C_p}$$

Thus, $\frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\partial T}{\partial x} = \frac{\dot{q}p}{\dot{m}C_p}$

$$\Rightarrow \quad T_m = T_i + \frac{\dot{q}p}{\dot{m}C_p}x$$



Constant Surface Temperature :

Energy balance over a differential length dx of the tube over which the temperature changes by dT_m

$$\dot{m}c_p dT_m = h(T_s - T_m)\,dA_s$$

$T_s \equiv$ constant $\Rightarrow -dT_m = d(T_s - T_m)$

$dA_s = p\,dx$

$$\frac{d(T_s - T_m)}{(T_s - T_m)} = -\frac{hp}{\dot{m}C_p}dx$$

At $x = 0$, $T_m = T_i$

$x = L$ $\quad T_m = T_e$

$$\Rightarrow \quad \ln\left(\frac{T_s - T_e}{T_s - T_i}\right) = -\frac{hpL}{\dot{m}C_p} \qquad \text{with } pL = A_s$$

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{hA_s}{\dot{m}C_p}\right)$$

-- used on p115

Note :

- Temperature difference between fluid and the surface decays exponentially along the flow direction.
- Decay rate depends on $\frac{hA_s}{\dot{m}C_p} \equiv$ Number of Transfer Units (NTU)

thus,

$$\dot{m} c_p = - \frac{h A_s}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)}$$

Overall energy balance over the entire tube gives

$$Q = \dot{m} c_p (T_e - T_i)$$

$$Q = h A_s \Delta T_{lm}$$

where
$$\Delta T_{lm} = \frac{T_i - T_e}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = \frac{\Delta T_e - \Delta T_i}{\ln\left(\frac{\Delta T_e}{\Delta T_i}\right)}$$

with
$$\Delta T_i = T_s - T_i \quad \text{and} \quad \Delta T_e = T_s - T_e$$

Note:

- ΔT_{lm} is the exact representation of the average temperature difference between the fluid and the surface, consistent with the exponential decay.

- NTU represents the effectiveness of heat transfer

- For NTU>5, exit fluid temperature is approximately equal to surface temperature i.e. $T_e \approx T_s$.

  $\Rightarrow$ heat transfer is not enhanced by further extending the length of the tube

- Instead of T_s, if temperature of external fluid is fixed (T_∞), then

$$\frac{T_\infty - T_e}{T_\infty - T_i} = \exp\left(-\frac{\bar{U} A_s}{\dot{m} c_p}\right) \text{ and } Q = \bar{U} A_s \Delta T_{lm}; \; \bar{U} \equiv \text{average overall heat transfer coefficient}$$