

# Symmetry



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Associate Professor

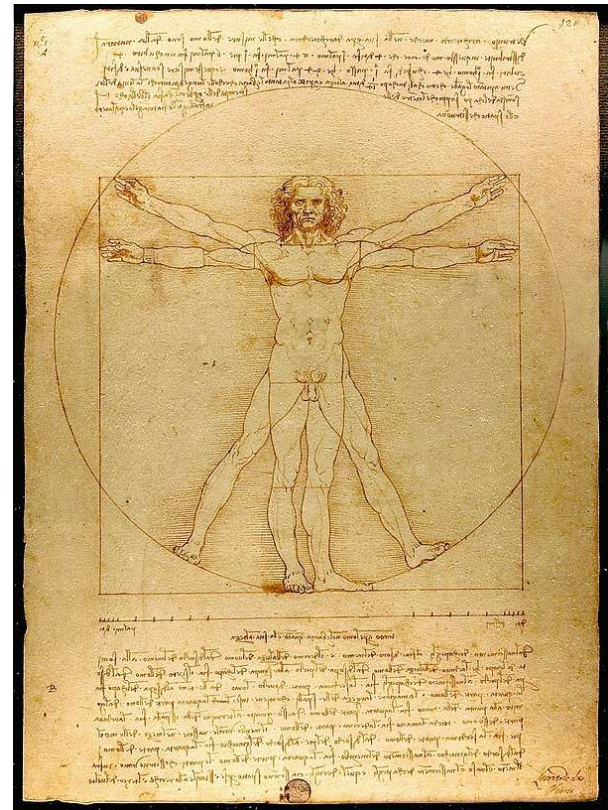
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# Introduction

- Symmetry comes from “symmetria” in Greek
- Harmony, proportion and balance
- Perception of beauty
- In mathematical terms, it is invariant with respect to certain operations



[https://en.wikipedia.org/wiki/Vitruvian\\_Man](https://en.wikipedia.org/wiki/Vitruvian_Man)

- 
- Symmetry is the preservation of form and configuration about a point, line, or plane
  - Nature full of symmetry
  - Art mimicked nature

- Plants, flowers, animals, humans
- Architecture, Dance, aerobics
- Symmetry creates perception of beauty
- Symmetry + Proportion = Beauty ?



<http://www.redbubble.com/people/marjoleink/works/4727248-masjed-e-shah-symmetry>

# Mineral crystals


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What Is the Difference Between Minerals ...  
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# Symmetry in nature

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Bilateral symmetry in nature [7 ... researchgate.net

Spiral symmetry in nature [7 ... researchgate.net

Bilateral symmetry in nature - The ... thesmarthappyproject.com

Patterns In Nature Contain Symmetry ... freshvista.com

Symmetry in Nature ( Explored ... pinterest.com

10 Beautiful Examples of Symmetry In ... listverse.com

Fractal symmetry in nature, Kiw... pinterest.com

of symmetry in nature ... quora.com

Finding Symmetry in Nature (Outdoor ... buggandbuddy.com

Unbelievable Photos Of Symmetry In ... buzzfeed.com

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Teach Symmetry kidminds.org

# Symmetry in architecture



symmetry in architecture



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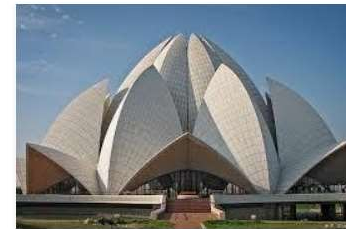
The Art of Symmetry in Architecture ...  
jonite.com



The Art of Symmetry in Architecture ...  
jonite.com



Symmetry in Architecture - Video ...  
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bilateral symmetry architecture ...  
pinterest.com



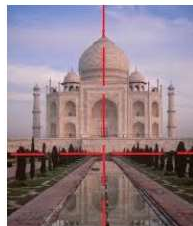
The Symmetry in Architecture: All you ...  
gharpedia.com



Symmetry in Architecture - Video ...  
study.com



The Beauty of Symmetry i...  
archdaily.com



Islamic Architecture  
wanhaziqhilm-i-islamicarch...




Bilateral Symmetry  
jwilson.coe.uga.edu



Balance vs. Symmetry - Doyle Coffin ...  
doylecoffinarchitecture.com



Asymmetrical and Symmetrical Balance  
blog.hubspot.com

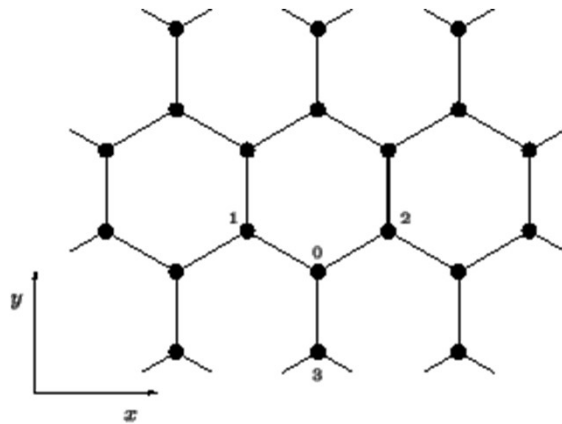
- 
- Inorganic crystals like minerals have shapes that reflect the intrinsic symmetry
  - Symmetry is self-similarity after transformation
  - Symmetry element is an imaginary point, line, plane about which symmetry operation takes place
  - Symmetry operation is a permutation that brings atoms/molecules/pattern into self coincidence



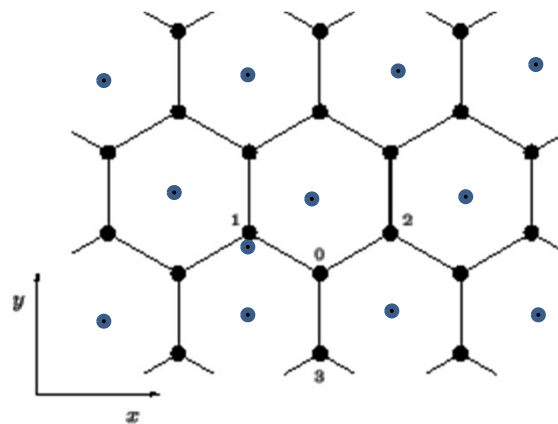
- Repetition in space (and time)
- Space can be divided into lattice points
- 1D lattice, 2D lattice and 3D lattice
- Periodic with identical neighbourhood



- Periodic set of points
- Identical neighbourhood

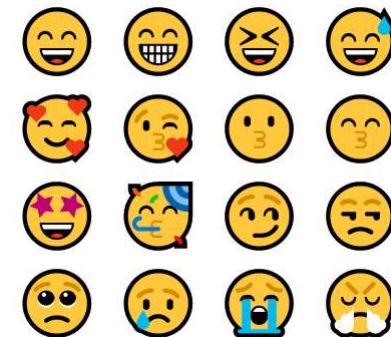
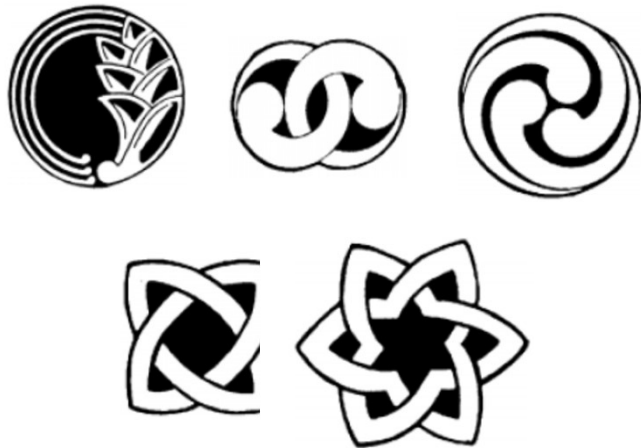


Not a lattice



Lattice

- Motif or Basis is the repeating unit of a pattern
- Atom, molecule,, group of atom or even an emoji
- Can be an asymmetric unit or have some symmetry




<https://getemoji.com/>



Lattice  
+       =       Crystal structure  
Motif

From patterns in tiling to minerals,  
metals and alloys and semiconductors



- 
- Crystallography is the study of pattern and their symmetry
  - Symmetry is ubiquitous
  - Let intuition take over formal training
  - Find symmetry in English alphabets

A B H N F



## ➤ Symmetry in 2D

- Translations
- Rotations
- Reflections
- Glide reflections

Identity or no symmetry is also a symmetry but it is redundant

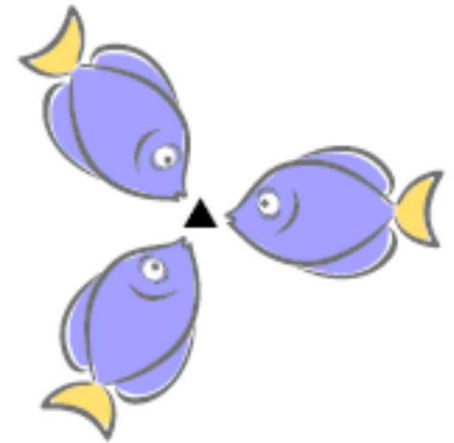


## ➤ Translation

- Move the asymmetric unit in a specific direction by a specific magnitude
- 1 step in 1D, 2 steps in 2D and 3 steps in 3D
- Results in 1D, 2D and 3D crystal

## ➤ Rotation

- Rotate the asymmetric unit (all the points in the asymmetric unit) about an axis
- Axis and angle of rotation
- Rotation axis only invariant point (does not move)







	Crystals	Molecules
	Hermann-Mauguin symbol	Schoenflies symbol
1 fold rotation axis	1	$C_1$
2 fold rotation axis	2	$C_2$
3 fold rotation axis	3	$C_3$
4 fold rotation axis	4	$C_4$
6 fold rotation axis	6	$C_6$

--- (monad)

● (diad) Axes perpendicular to the plane

← → Axes parallel to the plane

▲ (triad)

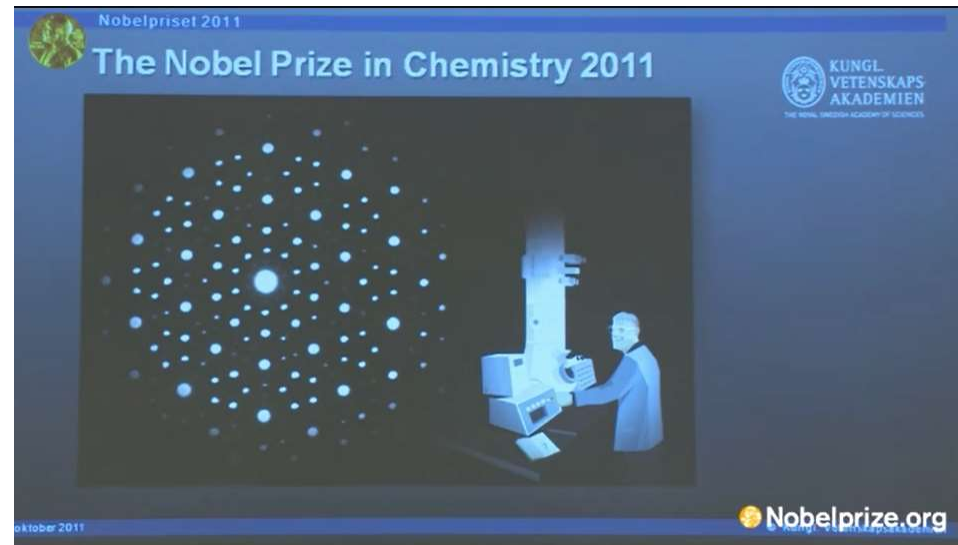
◆ (Tetrad)

⬠ (Hexad)

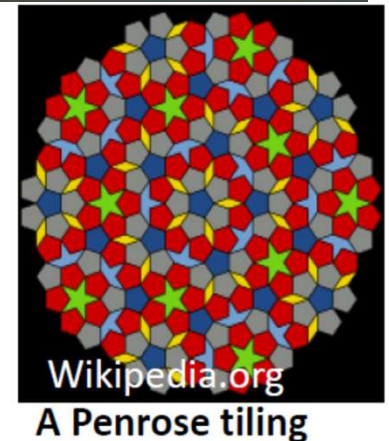


# Eyn chaya kazo: There can be no such creature

- 10 fold diffraction pattern in Al-Mn alloy
- April 8, 1982
- Prof. David Shechtman
- 5 fold symmetry

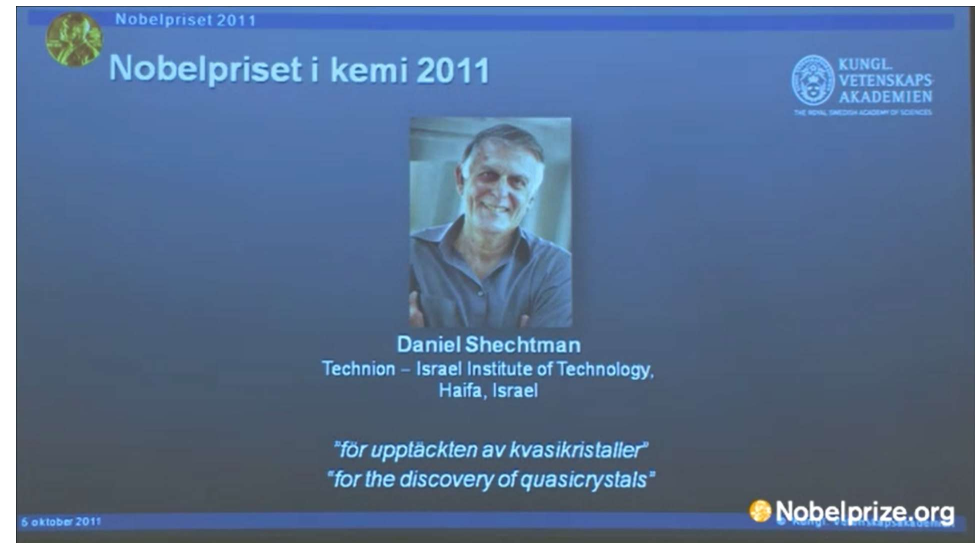


- Quasi scientist or Quasicrystals
- A quasiperiodic crystal, or **quasicrystal**, is a structure that is ordered but not periodic. A quasicrystalline pattern can continuously fill all available space, but it lacks translational symmetry.



<https://www.theguardian.com/science/2013/jan/06/dan-shechtman-nobel-prize-chemistry-interview>

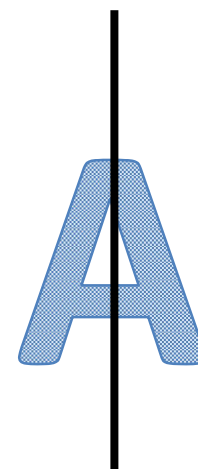
- Took 2 years to publish
- Aperiodic patterns in 3D
- Penrose tiling
- Medieval Arabic shrines
- Quasicrystals have opened up new avenues of research



<https://www.nobelprize.org/prizes/chemistry/2011/prize-announcement/>

## ➤ Reflection

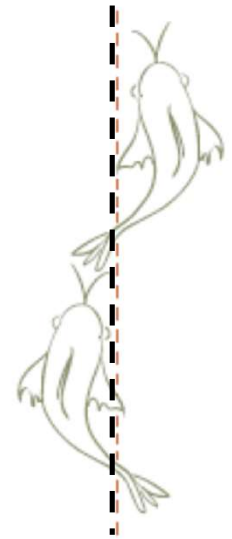
- Flips all points in an asymmetric unit over a line called mirror
- Reflection changes chirality
- All points on the mirror are invariant
- Symbol “m” and solid line representation





## ➤ Glide reflection

- Reflects asymmetric unit across a mirror and translates it parallel to the mirror
- Change in chirality of the unit
- No invariant point or line
- Symbol “g” and representation is dashed line



# Point group

- Point group is collection of symmetry element of an isolated shape
- It does not consider translation
- Every point in the lattice should be identical after symmetry operation/transformation
- Lattice point symmetry
- Plane symmetry/crystallographic group: mathematical classification of 2D repetitive pattern that captures symmetry of the pattern

# Why worry about symmetry ?

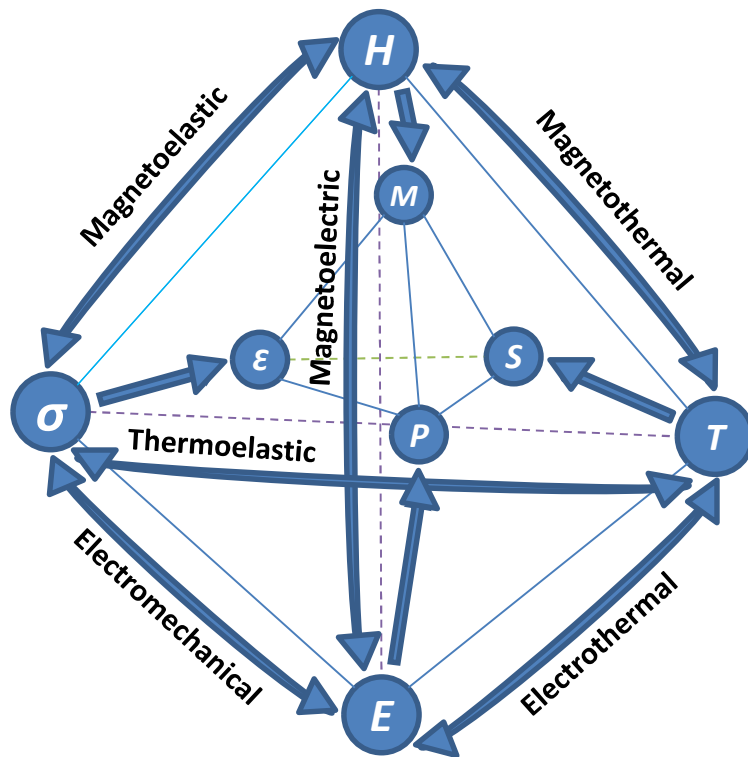
## Neumann's Principle

Original 1885:

A fundamental natural law: *Neumann's Principle: the symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of the crystal.* The property may have additional symmetry elements to those of the crystal (point group) symmetry.

Modified version:

The symmetry elements of any physical property of a crystal must include the symmetry elements that are common to the point group of the crystal and the defect structure contained within the crystal.



## Tensors

Vector like entity that require more than two directions for complete description

Perturbation	Response	Susceptibility
$\sigma$	$\epsilon$	Elasticity
	P	Piezoelectric
	S	Piezocaloric
	M	Converse Magnetostriction
E	$\epsilon$	Converse Piezoelectric
	P	Dielectric
	S	Electrocaloric
	M	Electromagnetic
T	$\epsilon$	Thermoelastic
	P	Pyroelectric
	S	Heat Capacity
	M	Pyromagnetic
H	$\epsilon$	Magnetostriction
	P	Magnetoelectric
	S	Magnetocaloric
	M	Magnetic



Property	Symbol	Field	Response	Type #
Tensors of Rank 0 (Scalars)				
Specific Heat	$C$	$\Delta T$	$T\Delta S$	$E1$
Tensors of Rank 1 (Vectors)				
Electrocaloric	$p_i$	$E_i$	$\Delta S$	$E3$
Magnetocaloric	$q_i$	$H_i$	$\Delta S$	$E3$
Pyroelectric	$p'_i$	$\Delta T$	$D_i$	$E3$
Pyromagnetic	$q'_i$	$\Delta T$	$B_i$	$E3$
Tensors of Rank 2				
Thermal expansion	$\alpha_{ij}$	$\Delta T$	$\epsilon_{ij}$	$E6$
Piezocaloric effect	$\alpha'_{ij}$	$\sigma_{ij}$	$\Delta S$	$E6$
Dielectric permeability	$\kappa_{ij}$	$E_j$	$D_i$	$E6$
Magnetic permeability	$\mu_{ij}$	$H_j$	$B_i$	$E6$
Optical activity	$g_{ij}$	$I_j$	$G$	$E6$
Magnetoelectric polarization	$\lambda_{ij}$	$H_j$	$D_i$	$E9$
Converse magnetoelectric polarization	$\lambda'_{ij}$	$E_j$	$B_i$	$E9$
Electric conductivity (resistivity)	$\sigma_{ij} (\rho_{ij})$	$E_j (j_j)$	$j_j (E_j)$	$T6$
Thermal conductivity	$K_{ij}$	$\Delta_j T$	$h_i$	$T6$
Diffusivity	$D_{ij}$	$\Delta_j c$	$m_i$	$T6$
Thermoelectric power	$\Sigma_{ij}$	$\Delta_j T$	$E_i$	$T9$
Hall effect	$R_{ij}$	$B_j$	$\rho_i^a$	$T9$



Property	Symbol	Field	Response	Type #
<b>Tensors of Rank 3</b>				
Piezoelectricity	$d_{ijk}$	$\sigma_{jk}$	$D_i$	<b>E18</b>
Converse piezoelectricity	$d'_{ijk}$	$E_k$	$\epsilon_{ij}$	<b>E18</b>
Piezomagnetism	$Q_{ijk}$	$\sigma_{jk}$	$B_i$	<b>E18</b>
Converse piezomagnetism	$Q'_{ijk}$	$H_k$	$\epsilon_{ij}$	<b>E18</b>
Electro-optic effect	$r_{ijk}$	$E_k$	$\Delta\beta_{ij}$	<b>E18</b>
Nernst tensor	$\Sigma_{ijk}$	$\Delta_j T B_k$	$E_i$	<b>T27</b>
<b>Tensors of Rank 4</b>				
Elasticity	$S_{ijkl} (c_{ijkl})$	$\sigma_{kl} (\epsilon_{kl})$	$\epsilon_{ij} (\sigma_{ij})$	<b>E21</b>
Electrostriction	$\gamma_{ijkl}$	$E_k E_l$	$\epsilon_{ij}$	<b>E36</b>
Photoelasticity	$q_{ijkl}$	$\sigma_{kl}$	$\Delta\beta_{ij}$	<b>E36</b>
Karl effect	$p_{ijkl}$	$E_k E_l$	$\Delta\beta_{ij}$	<b>E36</b>
Magnetoresistance	$\xi_{ijkl}$	$B_k B_l$	$\rho^{\delta}_{ij}$	<b>T36</b>
Piezoresistance	$\Pi_{ijkl}$	$\sigma_{kl}$	$\Delta\rho_{ij}$	<b>T36</b>
Magnetothermoelectric power	$\Sigma_{ijkl}$	$\Delta_j T B_k B_l$	$E_i$	<b>T54</b>
Second order Hall effect	$\rho_{ijkl}$	$B_j B_k B_l$	$\rho^2_i$	<b>T30</b>
<b>Tensors of Rank 6</b>				
Third order elasticity	$c_{ijklmn}$	$\epsilon_{kl} (\epsilon_{kl})$	$\sigma_{ij}$	<b>E56</b>

