Heat Exchanger Design

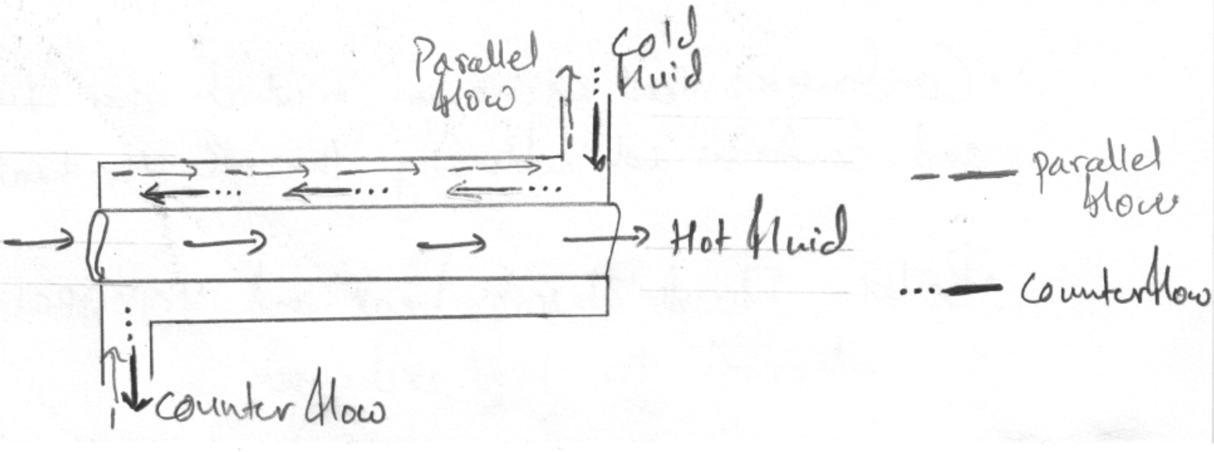
- Devices that allow/facilitate heat exchange between two fluids, while keeping them from mixing.
- Used in many chemical processing industries
  - important in recovering waste heat, thereby intensifying the process, (specifically making it energy efficient)

- Typically involve convection in each of the two fluids, and conduction across the solid wall separating the two fluids.

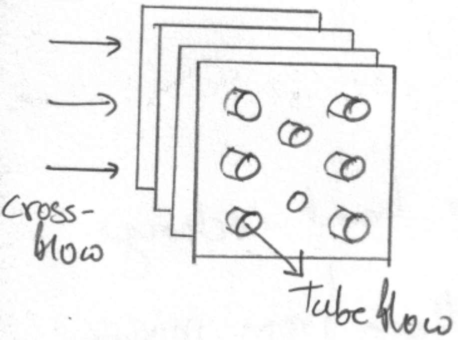
- To be discussed:
  - Overall heat transfer coefficient
  - Log mean temperature difference
  - NTU method (effectiveness)
  - Selection

Types:

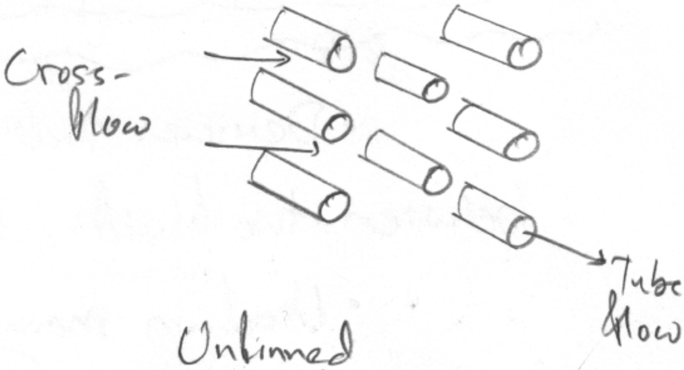
- Double pipe / concentric tube
  - Co-current and Counter current



- Cross-flow
  - finned and unfinned

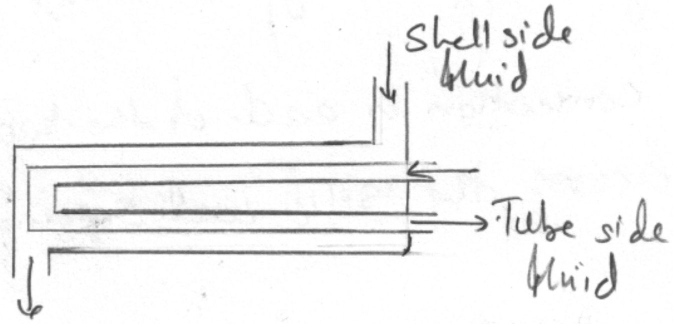


Finned
(both fluids unmixed)

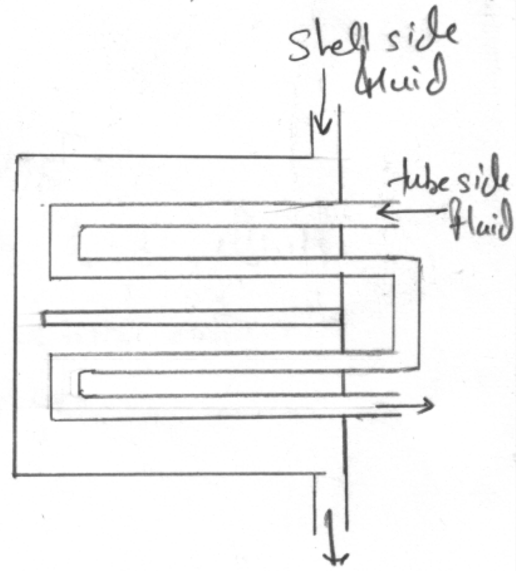


Unfinned
(one fluid mixed)

- Shell-and-tube



One shell pass, two tube passes



two shell passes, four tube passes

- Other types
  - plate and frame
  - regenerative (static or dynamic)

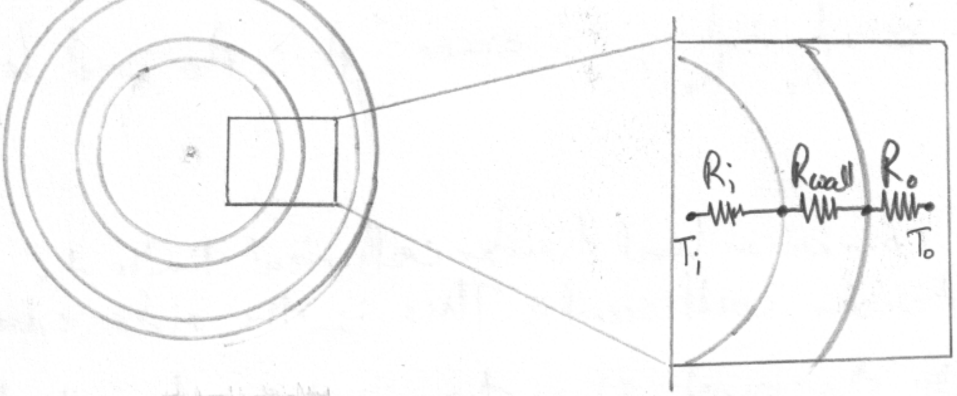
- Condenser: Heat exchanger in which one fluid cools down <u>and</u> condenses while flowing through the heat exchanger

- Boiler: Fluid absorbs heat and vaporizes as it flows through the heat exchanger.

Overall Heat Transfer Coefficient

- Typical heat exchanger: two fluids separated by a solid wall

- Heat transfer by
  - convection between fluid and surface of the wall
  - conduction across the solid wall
  - radiation, if important, is included via a radiation heat transfer coefficient added



$R_i = \frac{1}{h_i A_i}$

$R_o = \frac{1}{h_o A_o}$

For a double pipe heat exchanger

$$R_{wall} = \frac{\ln(D_o/D_i)}{2\pi k L}$$

Total thermal resistance

$$R = R_i + R_{wall} + R_o = \frac{1}{h_i A_i} + \frac{\ln(D_o/D_i)}{2\pi k L} + \frac{1}{h_o A_o}$$

Then,

$$Q = \frac{\Delta T}{R} = U A_s \Delta T$$

U: overall heat transfer coefficient

A_s: area for heat transfer

- Depending on which area is used to calculate the overall heat transfer coefficient

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o}$$

- Thus, $U_i \neq U_o$ unless $A_i = A_o$

- For typical double pipe heat exchangers with thin walls with high thermal conductivity,

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}, \quad \text{since } A_i \approx A_o \text{ and } k \text{ is high}$$

- The smaller convection heat transfer coefficient limits the overall heat transfer coefficient. This is the side where fins will help the most, since this is the side with largest heat transfer resistance.

  → typically on gas side (vs liquid side).

Reminder, for the finned side, (Notes p47)

$$A_s = A_{unfinned} + \eta_{fin} A_{fin}$$

- Fouling Factor
  - Deposits accumulate on heat exchanger surface, with time ⇒ fouling
  - Additional resistance to heat transfer
  - Fouling Factor R_f ≡ thermal resistance due to fouling

- Chemical fouling (corrosion, etc), biological fouling (growth of algae in hot water)

- Fouling must be taken into account, along with the downtime associated with periodic cleaning.

Table 11-2, Cengel's book gives values of fouling factor for a few fluids

- Two resistances: one on each surface of the wall

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$

Heat Exchanger Analysis

Typical goals

- Select a heat exchanger to achieve a specific temperature change in a fluid stream $\equiv$ LMTD

- predict outlet temperature of fluid streams for a given heat exchanger $\equiv$ NTU

Consider a heat exchanger

- Operating at steady state
- Insulated from outside
- With properties of the fluid assumed to be constant (at some average temperature)

Let $Q \equiv$ rate of heat transfer from the hot to the cold fluid

Energy equation (or first law of thermodynamics with no change in k.e., p.e or loss to surroundings)

$\Longrightarrow$

$$Q = \dot{m} C_{pc} (T_{c,out} - T_{c,in})$$

$$Q = \dot{m} C_{ph} (T_{h,in} - T_{h,out})$$

Assuming no phase change in either fluid

If a fluid undergoes phase change on one of the sides

$$Q = \dot{m} h_{fg}$$

Eg: Boilers, Condensers

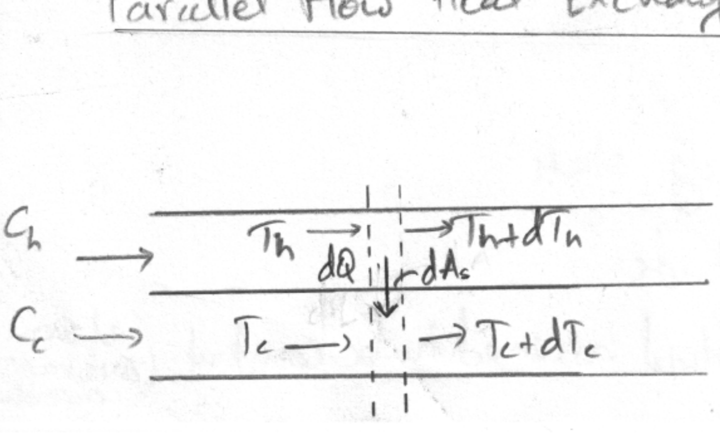
The rate of heat transfer can also be written based on the temperature difference between the two fluids, using Newton's law of cooling.

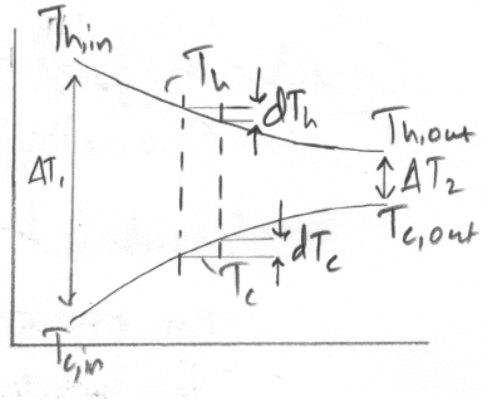
$$Q = U A_s \Delta T_{mean}$$

$\Delta T_{mean} \equiv$ an "appropriate" average temperature difference

- depends on heat exchanger flow arrangement

<u>Parallel Flow Heat Exchanger</u>





To determine ΔT_{lm} for this flow arrangement, consider a small element of thickness dx corresponding to heat transfer area dA_s. Energy balance on each fluid in the differential element, assuming no losses to the surroundings and C_p, U being nearly constant over the range of temperature variation in each fluid, gives

$$dQ = -\dot{m}_h C_{ph}\, dT_h = -C_h\, dT_h$$

$$dQ = \dot{m}_c C_{pc}\, dT_c = C_c\, dT_c$$

C_c, C_h = heat capacity rates

$$\Rightarrow dT_h - dT_c = d(T_h - T_c) = -dQ\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

In terms of heat capacity rates $C_h = \dot{m}_h C_{ph}$ and $C_c = \dot{m}_c C_{pc}$

$$d(T_h - T_c) = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Using Newtons law of cooling,

$$dQ = U\, dA_s\, (T_h - T_c)$$

$$\frac{d(T_h - T_c)}{(T_h - T_c)} = -U\, dA_s\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

Integrating over the entire length of the heat exchanger

$$\ln\left(\frac{T_{h,out} - T_{c,out}}{T_{h,in} - T_{c,in}}\right) = -UA_s\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$= -UA_s\left(\frac{T_{h,in} - T_{h,out}}{Q} + \frac{T_{c,out} - T_{c,in}}{Q}\right)$$

Let $\Delta T_{lm} = \dfrac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \dfrac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$

where
$$\Delta T_1 = T_{h,in} - T_{c,in} \ ; \qquad \Delta T_2 = T_{h,out} - T_{c,out}$$

Substituting,
$$Q = U A_s \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)} = U A_s \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

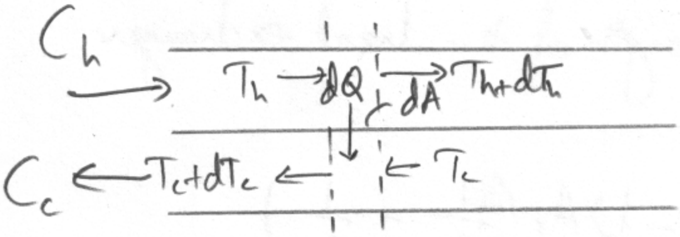
$$Q = U A \Delta T_{lm}$$

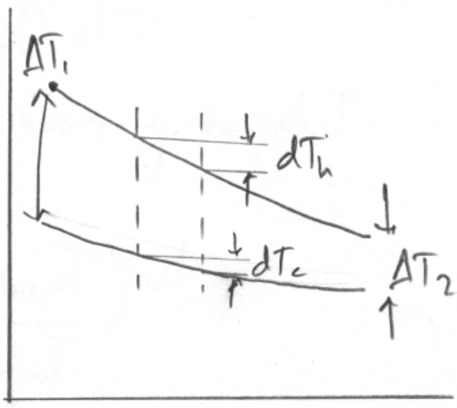
Thus, log mean temperature difference is the appropriate average temperature difference.

Note

- ΔT_{lm} and $Q = UA\Delta T_{lm}$ are appropriate for any heat exchanger as long as ΔT_1 and ΔT_2 are defined correctly.

Counter-Flow Heat Exchanger





- Maximum ΔT in counterflow setup is much smaller than that in parallel flow setup.
- Outlet cold fluid temperature can exceed that of hot fluid

Using a similar approach as that used in the previous section (parallel flow),

$$Q = \dot{m} c_{ph} (T_{h,in} - T_{h,out})$$

$$Q = \dot{m} c_{ph} (T_{c,out} - T_{c,in})$$

Defining a small element and writing energy balance, using Newton's law of cooling (similar to the previous section),

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}, \quad \text{with} \quad \Delta T_1 = T_{h,in} - T_{c,out}$$

$$\Delta T_2 = T_{h,out} - T_{c,in}$$

this is the only difference w.r.t. the previous section

and

$$Q = U A_s \Delta T_{lm}$$

Note

- ΔT_{lm} in counterflow $>$ ΔT_{lm} in parallel flow

  $\Rightarrow$ smaller surface area is required to achieve a given rate of heat transfer in counterflow setup.

  $\Rightarrow$ Counterflow is preferred over parallel flow

- When $C_h = C_c$ in counterflow setup, ΔT is constant throughout the heat exchanger.

  $\Rightarrow \Delta T_2 = \Delta T_1 \Rightarrow \Delta T_{lm}$ giving $0/0$.

  Use L'Hopital's rule, to confirm $\Delta T_{lm} = \Delta T_1 = \Delta T_2$

Multipass and Cross-flow Heat Exchangers

Instead of the approach used in earlier sections that leads to complicated expressions, an equivalent to ΔT_{lm} is used.

- Example: shell-and-tube heat exchanger

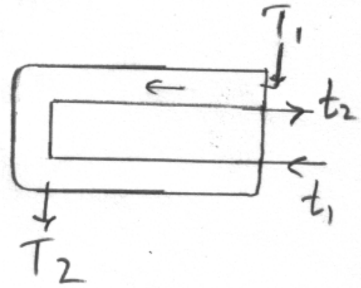
$$\Delta T_{lm} = F \, \Delta T_{lm,\,counter\text{-}flow}$$

$F \equiv$ correction factor $\qquad 0 \leq F \leq 1$

Charts of F vs P for different R are available
(Cengel's book Figure 11-19)

where
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{\dot{m} C_p}{\dot{m} C_p}$$



One shell pass
two tube passes

Example: Heating of Glycerin in a multipass heat exchanger

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin waled 2-cm diameter tubes at 80°C and leaves at 40°C. The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m²·K on the glycerin side (shell) and 160 W/m²·K on the water (tube) side. Determine the rate of heat transfer in the heat exchanger before any fouling and after, with a fouling factor of 0.0006 m²·K/W on tubes.

Assumptions

Inner and outter surface area of tubes is identical (thin-walled)

$$A_s = \pi D L = \pi \times 0.02 \times 60 = 3.77\ m^2$$

The rate of heat transfer

$$Q = U A_s F \Delta T_{lm,counterflow}$$

Based on given inlet and outlet temperatures,

$$\left.\begin{aligned} \Delta T_1 &= 80-50 = 30^\circ C \\ \Delta T_2 &= 40-20 = 20^\circ C \end{aligned}\right\} \text{Counter flow arrangement}$$

$$\Rightarrow \Delta T_{lm,counterflow} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = 24.7^\circ C$$

$$P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{40-80}{20-80} = 0.67$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{20-50}{40-80} = 0.75$$

Using Figure 11-19 (Cengel's book),

$$F = 0.91$$

In absence of fouling

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o}} = 21.6\ W/m^2 \cdot K$$

and

$$Q = U A_s F \Delta T_{lm,counterflow} = 1830\ W$$

When fouling takes place on the surface,

$$U = \frac{1}{\frac{1}{h_i} + \frac{1}{h_o} + R_{fouling}} = \frac{1}{\frac{1}{160} + \frac{1}{25} + 0.0006} = 21.3 \ W/m^2\text{-}K$$

and $\quad Q = U A_s F \Delta T_{lm,counterflow} = 1805 \ W$

Notes: • A commonly encountered task is selecting type+size of a heat exchanger for a given heat exchange problem. Steps involved are.

- Select heat exchanger type
- Determine any missing inlet or outlet temperatures or heat transfer rate using energy balance
- Calculate ΔT_{lm} and F
- Calculate U
- Calculate the area requirement A_s

Then, select a heat exchanger with larger or equal area.

• Another commonly encountered task is determining heat transfer rate and outlet temperatures for a given heat exchanger type+size. Although LMTD method (above steps) can still be used, they require iterations.

Instead, a simpler approach is effectiveness–NTU method

Effectiveness - NTU Method

- Alternative to LMTD method, to determine heat transfer rate and outlet temperatures of the fluids.

Heat transfer effectiveness

$$\varepsilon = \frac{Q}{Q_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

Actual heat transfer rate
(from energy balance)

$$Q = C_c(T_{c,out} - T_{c,in}) = C_h(T_{h,in} - T_{h,out})$$

Maximum heat transfer rate

The heat transfer in an heat exchanger is maximum when either the cold fluid is heated to the inlet temperature of the hot fluid or the hot fluid is cooled to the inlet temperature of the cold fluid. Both cannot be true, unless ΔT is a constant (i.e. $C_h = C_c$). Depending on C_h and C_c, the fluid with lowest heat capacity rate will experience the maximum temperature change.

If $C_c < C_h$, then $|dT_c| > |dT_h|$, since

$$dQ = -C_h dT_h = C_c dT_c$$

$$\Rightarrow Q_{max} = C_c(T_{h,in} - T_{c,in}) \qquad \because T_{c,out} \to T_{h,in}$$

If $C_c > C_h$, then $|dT_c| < |dT_h|$

and $Q_{max} = C_h (T_{h,in} - T_{c,in})$ since $T_{h,out} = T_{c,in}$

Thus, for any C_c and C_h,

$$Q_{max} = C_{min} (T_{h,in} - T_{c,in})$$

Substituting in the definition of heat transfer effectiveness

$$Q = \varepsilon Q_{max} = \varepsilon C_{min} (T_{h,in} - T_{c,in})$$

and

if $C_c \equiv C_{min}$
$$\varepsilon = \frac{Q}{Q_{max}} = \frac{C_c (T_{c,out} - T_{c,in})}{C_c (T_{h,in} - T_{c,in})} = \frac{T_{c,out} - T_{c,in}}{T_{h,in} - T_{c,in}}$$

if $C_h = C_{min}$
$$\varepsilon = \frac{Q}{Q_{max}} = \frac{C_h (T_{h,in} - T_{h,out})}{C_h (T_{h,in} - T_{c,in})} = \frac{T_{h,in} - T_{h,out}}{T_{h,in} - T_{c,in}}$$

If ε is known, the outlet temperature of the fluid can be calculated.

For any heat exchanger, $\varepsilon = f\left(NTU, \frac{C_{min}}{C_{max}}\right)$

For parallel-flow heat exchanger, we had derived

$$\ln\left(\frac{T_{h,out}-T_{c,out}}{T_{h,in}-T_{c,in}}\right) = -UA_s\left(\frac{1}{\dot{m}_h c_{p_h}}+\frac{1}{\dot{m}_c c_{p_c}}\right)$$

$$= -UA_s\left(\frac{1}{C_h}+\frac{1}{C_c}\right)$$

$$= -\frac{UA_s}{C_c}\left(1+\frac{C_h}{C_c}\right)$$

From energy balance,

$$Q = C_c(T_{c,out}-T_{c,in}) = -C_h(T_{h,out}-T_{h,in})$$

$$\Rightarrow \quad T_{h,out} = T_{h,in} - \frac{C_c}{C_h}(T_{c,out}-T_{c,in})$$

Substituting in the above expression and

$$\ln\left(\frac{T_{h,in}-T_{c,out}-\frac{C_c}{C_h}(T_{c,out}-T_{c,in})}{T_{h,in}-T_{c,in}}\right)$$

$$= \ln\left(1-\left(\frac{T_{c,out}-T_{c,in}}{T_{h,in}-T_{c,in}}\right)\left(1+\frac{C_c}{C_h}\right)\right) = -\frac{UA_s}{C_c}\left(1+\frac{C_h}{C_c}\right)$$

From definition of ε, (and energy balance above),

$$\varepsilon = \frac{C_c(T_{c,out}-T_{c,in})}{C_{min}(T_{h,in}-T_{c,min})} \Rightarrow \frac{T_{c,out}-T_{c,in}}{T_{h,in}-T_{c,in}} = \varepsilon\frac{C_{min}}{C_c}$$

$$\Rightarrow \quad \varepsilon_{\substack{parallel\\flow}} = \frac{1-\exp\left(-\frac{UA_s}{C_c}\left(1+\frac{C_c}{C_h}\right)\right)}{\left(1+\frac{C_c}{C_h}\right)\frac{C_{min}}{C_c}} = \frac{1-\exp\left(-\frac{UA_s}{C_{min}}\left(1+\frac{C_{min}}{C_{max}}\right)\right)}{1+(C_{min}/C_{max})}$$

Number of transfer units (NTU)

$$NTU = \frac{UA_s}{C_{min}} = \frac{UA_s}{(\dot{m} C_p)_{min}}$$

Note: NTU, for a given U and C_{min}, is a measure of the area of heat exchanger

⇒ for parallel-flow,

$$\varepsilon_{\text{parallel flow}} = \frac{1-\exp\left(-NTU\left(1+\frac{C_{min}}{C_{max}}\right)\right)}{1+\frac{C_{min}}{C_{max}}}$$

Capacity ratio (c) ((C_r in Incropera's book)

$$c = \frac{C_{min}}{C_{max}}$$

$$\Rightarrow \varepsilon_{\text{parallel flow}} = \frac{1-\exp(-NTU(1+c))}{1+c}$$

Similarly,

$$\varepsilon_{\text{counterflow}} = \frac{1-\exp[-NTU(1-c)]}{1-c\exp[-NTU(1-c)]} \quad \text{for } c<1$$

$$= \frac{NTU}{1+NTU} \quad \text{for } c=1$$

Effectiveness-NTU relations,
i.e. $\varepsilon = f(NTU, c)$ for other heat exchanger types are available in
Table 11.3 (with 11.4) in Incropera's book
OR Table 11-4 (with 11-5) in Cengel's book

Note:

- $0 < \varepsilon < 1$
- ε increases rapidly with NTU for up to NTU $= 1.5$ and slowly beyond.

  $\Rightarrow$ Heat exchangers with NTU > 3 are rarely justified

- Counterflow heat exchanger has the highest effectiveness for the same NTU and c.

- $0 < c < 1$. For a given NTU, ε is maximum for $c = 0$ and minimum for $c = 1$.

  For phase change process, $c = 0 \Rightarrow C_{max} \to \infty$

  then $\varepsilon = \varepsilon_{max} = 1 - \exp(-NTU)$ independent of the type of heat exchanger.

Example: Multipass heat exchanger

Hot oil is cooled by water in a 1-shell pass and 8-tube passes heat exchanger. Tubes are thin walled, made of copper, and with internal diameter of 1.4 cm. Length of each tube pass in the exchanger is 5 m, and overall heat transfer coefficient is $310\ W/m^2 K$. Water flows through the tubes at 0.2 kg/s and the oil at 0.3 kg/s through the shell. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of oil and water.

Given: $C_{p,water} = 4.18\ kJ/kg\cdot{}^\circ C$ $\quad C_{p,oil} = 2.13\ kJ/kg\cdot{}^\circ C$

Will use effectiveness-NTU method, assuming steady operation, insulated heat exchanger, and thin-walled tube.

$$\left.\begin{aligned} C_h &= \dot{m}_h C_{ph} = 0.639\ kW/K \\ C_c &= \dot{m}_c C_{pc} = 0.836\ kW/K \end{aligned}\right\} \Rightarrow C_{min} = C_h$$

and $c = \frac{C_{min}}{C_{max}} = 0.764$

$$Q_{max} = C_{min}(T_{h,in} - T_{c,in}) = 0.639(150-20) = 83.1 \text{ kW}$$

$$A_s = \underset{\substack{\uparrow \\ \text{no. of} \\ \text{passes}}}{n}(\pi D \underset{\substack{\nearrow \\ \text{per pass}}}{L}) = 8\pi \times (0.014 \times 5) = 1.76 \text{ m}^2$$

$$\Rightarrow NTU = \frac{UA_s}{C_{min}} = 0.854$$

(pay attention to units)

$$\varepsilon_1 = 2\left[1 + c + \sqrt{1+c^2}\ \frac{1+\exp[-NTU_1\sqrt{1+c^2}]}{1-\exp[-NTU_1\sqrt{1+c^2}]}\right]^{-1}$$

$$\varepsilon_1 = 0.4622$$

(here subscript 1 denotes 1-shell pass)

$$Q = \varepsilon Q_{max} = 38.4 \text{ kW}$$

The outlet temperatures can be calculated as

$$Q = C_c(T_{c,out} - T_{c,in}) \Rightarrow T_{c,out} = T_{c,in} + \frac{Q}{C_c}$$

and

$$Q = C_h(T_{h,in} - T_{h,out}) \Rightarrow T_{h,out} = T_{h,in} - \frac{Q}{C_h}$$

$$\Rightarrow \quad T_{c,out} = 65.9°C$$

and $T_{h,out} = 89.9°C$

Selection of Heat Exchangers

- Heat transfer rate
- Cost
- Pumping power
- Size and weight
- Type
- Materials
- Servicing, maintenance
- Safety
- Reliability