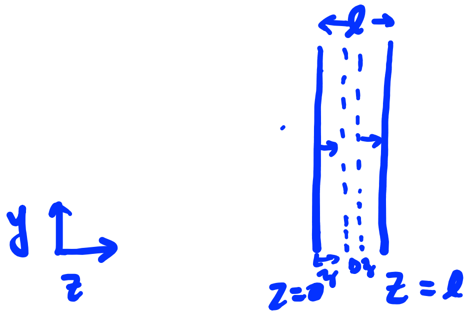
Lecture 5: Example Problems

1) Steady state diffusion across a thin film



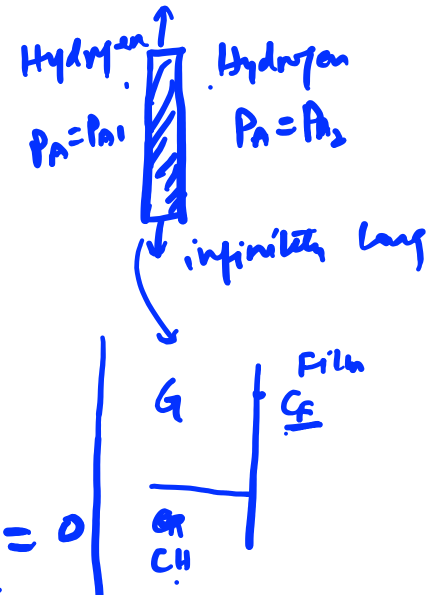
$z=0,\ C_A = C_{A0}$ | $W = \Delta x$
$z=l,\ C_A = C_{Al}$ | $L = \Delta y$

Questions:

1) Conc. profile
2) Flux across the film

At SS: $\dfrac{\partial(\ )}{\partial t} = 0$

Example:



$$\left(\cancel{\begin{matrix}\text{Rate of accumulation}\\ \text{of A}\end{matrix}}\right)_0 = \left(\begin{matrix}\text{Rate of}\\ \text{A in}\end{matrix}\right) - \left(\begin{matrix}\text{Rate of}\\ \text{A out}\end{matrix}\right) + \left(\cancel{\text{Rate of generation of A}}\right)_0$$

At SS: $\left(\begin{matrix}\text{Rate of A}\\ \text{in}\end{matrix}\right) - \left(\begin{matrix}\text{Rate of}\\ \text{A out}\end{matrix}\right) = 0$

$$N_A\, \Delta x\, \Delta y\big|_z - N_A\, \Delta x\, \Delta y\big|_{z+\Delta z} = 0$$

$\Delta x \Delta y$ = constant

$$\Rightarrow \underbrace{N_A\big|_z = N_A\big|_{z+\Delta z}}\ = \text{Constant} \qquad \left[\ \frac{dN_A}{dz} = 0\ \right] \quad (i)$$

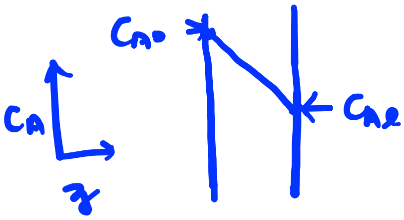
$$N_A = -D_{AB}\frac{dC_A}{dz} \quad -(ii)$$

(i) & (ii)

$$\boxed{D_{AB}\frac{d^2C_A}{dz^2} = 0}$$

[SS; No rxn; purely diffusive]

Concentration profile: $\underline{C_A} = C_{A0} + (C_{Al} - C_{A0})(z/l)$



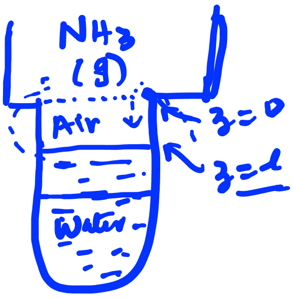
Flux: $N_A = -D_{AB}\dfrac{dC_A}{dz} = \dfrac{D_{AB}}{l}(C_{A0} - C_{Al})$

$$\boxed{N_A = \frac{D_{AB}}{l}(\underline{C_{A0}} - \underline{C_{Al}})}$$

(ii) Diffusion of A through non-diffusing B :

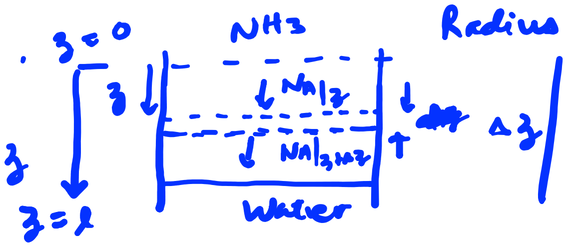
Ideal Gas mixture having components $\underline{A}$ & $\underline{B}$.

1) Air is stagnant
2) NH_3 is diffusing.



1) Ammonia is highly soluble in water
2) Air has very low solubility in water.

A : Ammonia
B : Air



Radius of Tube = 'r'.

Area of cross section = $\underline{\pi r^2}$

From Mass Balance:

$$\left(\text{Rate of acc. of } A\right) = \left(\text{Rate of mass of } A \text{ in}\right) - \left(\text{Rate of mass of } A \text{ out}\right) + \cancel{\left(\text{Rate of generation of } A\right)}^{\,0}$$

$$\frac{\partial}{\partial t}\left(M_A C_A \pi r^2 \Delta z\right) = M_A N_A \pi r^2\big|_z - M_A N_A \pi r^2\big|_{z+\Delta z}$$

$$\Rightarrow \quad \frac{\partial C_A}{\partial t} = -\frac{\partial N_A}{\partial z}$$

Assuming steady state : $\frac{\partial C_A}{\partial t} = 0$

$$\frac{\partial N_A}{\partial z} = 0 \Rightarrow N_A = \text{Constant}$$

From Fick's law: $N_A = \frac{P_A}{P}(N_A + N_B) - \frac{D_{AB}}{RT}\frac{dP_A}{dz}$

P = Total pressure

P_A = Partial pressure of A at same z

1) N_A = constant

2) $N_B = 0$

$$N_A = \frac{P_A}{P}(N_A) - \frac{D_{AB}}{RT}\frac{dP_A}{dz}$$

$$\Rightarrow \left(1 - \frac{P_A}{P}\right) N_A = -\frac{D_{AB}}{RT}\frac{dP_A}{dz}$$

$$\Rightarrow \int_0^l N_A\, dz = \int -\frac{D_{AB}P}{(P-P_A)RT}\, dP_A$$

$z = 0,\ P_A = P_{A0}$

$z = l,\ P_A = P_{Al}$

$$\Rightarrow N_A\, l = \frac{D_{AB}P}{RT}\ln\left(\frac{P-P_{Al}}{P-P_{A0}}\right)$$

$$\Rightarrow \boxed{N_A = \frac{D_{AB}P}{RTl}\ln\left(\frac{P-P_{Al}}{P-P_{A0}}\right)}$$

Assumptions:

1) Total pressure is constant

$P_{A0} + P_{B0} = P_{Al} + P_{Bl} = P$ (total pressure)

(ii) Ideal gas mixture

(iii) Temp. is uniform

(iv) Steady state

(v) Dilute solutions

$P - P_{Al} = P_{Bl}$

$P - P_{A0} = P_{B0}$

$$\Rightarrow N_A = \frac{D_{AB}P}{RTl} \cdot \frac{P_{A0}-P_{Al}}{P_{A0}-P_{Al}} \cdot \ln\left(\frac{P_{Bl}}{P_{B0}}\right)$$

$$p_{A0} + p_{B0} = p_{Al} + p_{Bl}$$

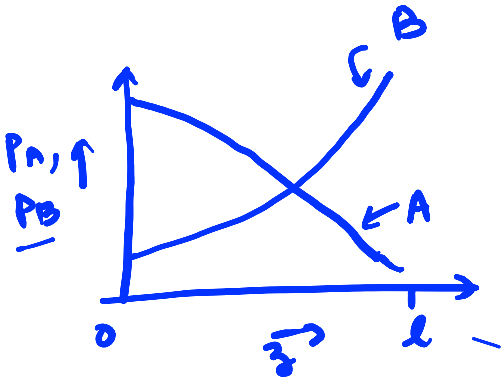
$$\Rightarrow p_{A0} - p_{Al} = \underline{p_{Bl} - p_{B0}}$$

$$N_A = \frac{D_{AB}P}{RTl} \cdot \frac{p_{A0} - p_{Al}}{(p_{Bl} - p_{B0}) / \ln\left(\frac{p_{Bl}}{p_{B0}}\right)}$$

$$\boxed{N_A = \frac{D_{AB}P}{RTl} \cdot \frac{p_{A0} - p_{Al}}{p_{BM}}}$$

where p_{BM} = log mean partial pressure of B

$$= (p_{Bl} - p_{B0}) / \ln(p_{Bl}/p_{B0})$$



$$p_{A0} + p_{B0} = P$$

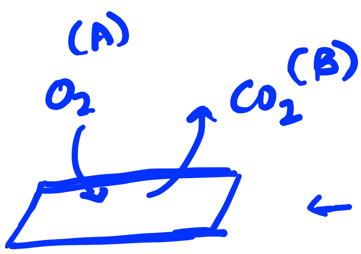
$$\underline{p_{Al}} + p_{Bl} = P$$

$\underline{N_B = 0}$ ← ?

$$J_B = \frac{-D_{BA}}{RT} \frac{dp_B}{dz}$$

$$\underline{N_B} = \underbrace{\frac{p_B}{P}(\cancel{N_A} + N_B)}_{\text{Bulk}} - \underbrace{\cancel{\frac{D_{AB}}{RT} \frac{dp_B}{dz}}}_{\text{Diffusion}}$$

(ii) <u>Equimolar counter diffusion of A & B:</u>



← Flat plate covered with Carbon

$$\underline{N_A} + \underline{N_B} = 0$$

$$N_A = -N_B$$

For diffusion taking place across a cross section of uniform/constant area, then at steady state

$$\frac{dN_A}{dz} = 0 \Rightarrow \underline{N_A = \text{constant}}$$

$$\underline{N_A} = \frac{p_A}{P}\underbrace{(\cancel{N_A + N_B})}_{0} - \frac{D_{AB}}{RT}\frac{dp_A}{dz}$$

$$N_A = \frac{-D_{AB}}{RT}\frac{dp_A}{dz}$$

Assumption
(i) S.S
(ii) Cross section area is constant
(iii) Temp is uniform
(iv) Ideal gas mixture
(v) Total pressure is constant

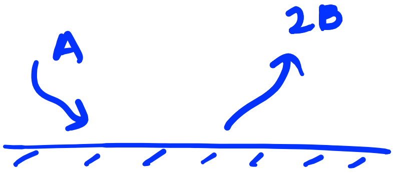
$$\Rightarrow N_A\int_0^{l} dz = -\frac{D_{AB}}{RT}\int_{p_{A0}}^{p_{Al}} dp_A$$

$$\Rightarrow \boxed{N_A = \frac{D_{AB}(p_{A0} - p_{Al})}{RTl}}$$

(iii) Non-equimolar counter-diffusion of A & B

$$N_A = -f N_B$$

$$A \rightarrow 2B$$



$$N_A = -\frac{1}{2}N_B \quad ; \quad (\underline{N_A + N_B}) \rightarrow \text{flux term}$$