Assignment 4

ChE641 Mathematical Methods in Chemical Engineering

Due Date: 10 October, 2020 **Maximum Marks: 100**

1. Find an analytic function of $z = x + iy$ whose imaginary part is:

$$(y \cos y + x \sin y) \exp x$$

2. Determine the types of singularities (if any) possessed by the following functions at $z = 0$:

   (a) $(z - 2)^{-1}$,

   (b) $(1 + z^3)/z^2$,

   (c) $\sinh(1/z)$,

   (d) e^z/z^3,

   (e) $z^{1/2}/(1 + z^2)^{1/2}$.

3. Prove that if $f(0)$ has a simple zero at z_0, then $1/f(z)$ has residue $1/f'(z_0)$ there. Hence evaluate

$$\int_{-\pi}^{\pi} \frac{\sin\theta}{a - \sin\theta} d\theta,$$

   where a is real and > 1.

4. Prove that, for $\alpha > 0$, the integral

$$\int_0^{\infty} \frac{t \sin \alpha t}{1 + t^2} dt$$

   has the value $(\pi/2)\exp(-\alpha)$.

5. Find out whether the given vectors are linearly dependent or independent; if they are linearly dependent, find a linearly independent subset. Write each of the given vectors as a linear combination of the independent vectors.

   (a) (1,-2, 3), (1, 1, 1), (-2, 1,-4), (3, 0, 5)

   (b) (0, 1, 1), (-1, 5, 3), (1, 0, 2), (2,-15, 1)

6. Show that any vector $\mathbf{V}$ in a plane can be written as a linear combination of two non-parallel vectors $\mathbf{A}$ and $\mathbf{B}$ in the plane; that is, find a and b so that $\mathbf{V} = a\mathbf{A} + b\mathbf{B}$. **Hint**: Find the cross products $\mathbf{A} \times \mathbf{V}$ and $\mathbf{B} \times \mathbf{V}$; what are $\mathbf{A} \times \mathbf{A}$ and $\mathbf{B} \times \mathbf{B}$? Take components perpendicular to the plane to show that

$$a = \frac{(\mathbf{B} \times \mathbf{V}) \cdot \mathbf{n}}{(\mathbf{B} \times \mathbf{A}) \cdot \mathbf{n}}$$

   where $\mathbf{n}$ is normal to the plane, and find a similar formula for b.

7. Express $\mathbf{V} = 3\mathbf{i} + 5\mathbf{j}$ as a linear combination of $\mathbf{A} = 2\mathbf{i} + \mathbf{j}$ and $\mathbf{B} = 3\mathbf{i} - 2\mathbf{j}$.

8. For the given sets of vectors, find the dimension of the space spanned by them and a basis for this space.

   (a) $(1, -1, 0, 0), (0, -2, 5, 1), (1, -3, 5, 1), (2, -4, 5, 1)$;

   (b) $(0, 1, 2, 0, 0, 4), (1, 1, 3, 5, -3, 5), (1, 0, 0, 5, 0, 1), (-1, 1, 3, -5, -3, 3),$
   $(0, 0, 1, 0, -3, 0)$.

9. Find the norms of $\mathbf{A}$ and $\mathbf{B}$ and the inner product of $\mathbf{A}$ and $\mathbf{B}$:

   (a) $\mathbf{A} = (3 + i, 1, 2 - i, -5i, i + 1), \mathbf{B} = (2i, 4 - 3i, 1 + i, 3i, 1)$;

   (b) $\mathbf{A} = (2, 2i - 3, 1 + i, 5i, i - 2), \mathbf{B} = (5i - 2, 1, 3 + i, 2i, 4)$.

10. (a) Show that, in n-dimensional space, any $n + 1$ vectors are linearly dependent.

    (b) Show that two different sets of basis vectors for the same vector space must contain the same number of vectors.
    Hint: Suppose a basis for a given vector space contains n vectors. Use Part (a) to show that there cannot be more than n vectors in a basis for this space. Conversely, if there were a correct basis with less than n vectors, what can you say about the claimed n-vector basis?