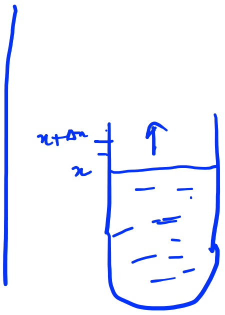
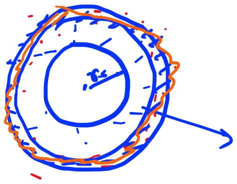
Diffusion Through Variable Area



Ex : Evaporation of a drop of water in stagnant air.

Let us assume:

Water vapor = Component A
Air = Component B

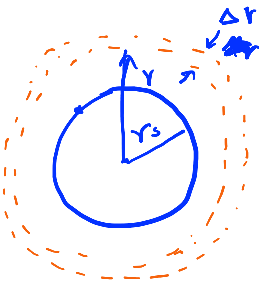


@ $t = t_1$, $r = r_s$

Since air is stagnant, $N_B = 0$

From mass balance:

$$\underset{0(SS)}{\left(\text{Rate of accumulation of } A\right)} = \left(\text{Rate of } A \text{ in}\right) - \left(\text{Rate of } A \text{ out}\right) + \underset{0}{\left(\text{Rate of generation of } A\right)}$$



steady state

At SS:

$$\left(\text{Rate of } A \text{ in}\right) = \left(\text{Rate of } A \text{ out}\right)$$

Q: (i) Rate of evaporation
(ii) Radius changes from $r_s \to r_s'$, what is time required ?

$$\text{Rate of moles of } A \text{ in} = N_A(4\pi r^2)\big|_r$$

$$\text{Rate of moles of } A \text{ out} = N_A(4\pi r^2)\big|_{r+\Delta r}$$

$$N_A(4\pi r^2)\big|_r - N_A(4\pi r^2)_{r+\Delta r} = 0$$

$$\lim_{\Delta r \to 0}\left[\frac{N_A(4\pi r^2)\big|_r - N_A(4\pi r^2)\big|_{r+\Delta r}}{\Delta r}\right] = 0$$

$$-\frac{d}{dr}(4\pi r^2 N_A) = 0 \Rightarrow 4\pi r^2 N_A = \text{constant}$$

Area × Flux = Constant

$$4\pi r^2 \times N_A \equiv m^2 \times \frac{mol}{m^2.s} = \frac{mol}{s} = \text{Rate of evaporation of } A\ (W)$$

W = Rate of evaporation of A

$$\underline{W = 4\pi r^2 N_A}$$

Boundary conditions: $r = r_s$, $p_A = p_{As}$

$r \to \infty$, $p_A \to \underline{p_{A\infty}}$

$$N_A = \frac{p_A}{p}(N_A + N_B) - \frac{D_{AB}}{RT}\frac{dp_A}{dr}$$

Air is stagnant $\Rightarrow$ $\underline{N_B = 0}$

$$\Rightarrow N_A = \frac{p_A}{p}(N_A) - \frac{D_{AB}}{RT}\frac{dp_A}{dr}$$

$$\Rightarrow \underline{N_A} = -\frac{D_{AB}\,p}{RT(p - p_A)}\frac{dp_A}{dr}$$

; p_A = Partial pressure of A at 'r'

p = Total pressure (constant)

$$\Rightarrow W = N_A(4\pi r^2)$$

$$= \left[-\frac{D_{AB}\,p}{RT(p - p_A)}\frac{dp_A}{dr}\right][4\pi r^2]$$

$$\Rightarrow -\int_{p_{As}}^{p_{A\infty}} \frac{dp_A}{p - p_A} = \frac{WRT}{4\pi p D_{AB}}\int_{r_s}^{\infty}\frac{dr}{r^2}$$

Solving the above equation $\Rightarrow$

$$\underline{W} = \frac{4\pi D_{AB}\,p\,r_s}{RT}\ln\left(\frac{p - p_{A\infty}}{p - p_{As}}\right) \leftarrow$$

Rate of evaporation of A [mol/time]

Q2: Time required for radius of drop to change from $r_s \to r_s'$?

$$-\frac{d}{dt}\left[\frac{4}{3}\pi r_s^3 \frac{\rho_A}{M_A}\right] = \frac{4\pi D_{AB}\,p\,r_s}{RT}\ln\left(\frac{p - p_{A\infty}}{p - p_{As}}\right)$$

$$-\int_{r_s}^{r_s'} r_s\,dr = \frac{D_{AB}\, p M_A}{RT \rho_A} \ln\left(\frac{p-p_{A\infty}}{p-p_{As}}\right)\int_0^{t'} dt$$

$$\Rightarrow \left(r_s^2 - r_s'^2\right) = \frac{2 D_{AB} P M_A t'}{RT\rho_A} \ln\left(\frac{p-p_{A\infty}}{p-p_{AS}}\right)$$

$$t' = \frac{RT\rho_A\left(r_s^2 - r_s'^2\right)}{2 D_{AB} P M_A \ln\left(\frac{p-p_{A\infty}}{p-p_{AS}}\right)}$$

Ex2: