Bank of Tubes

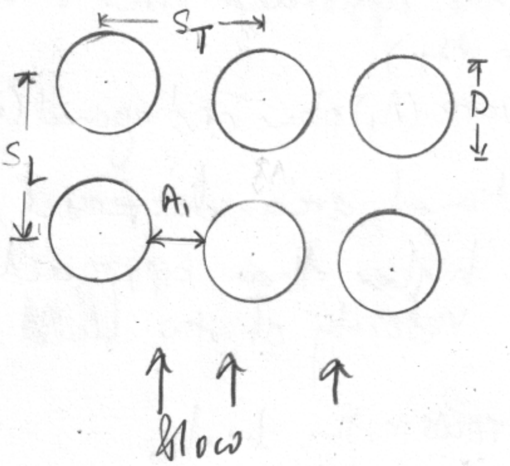
- Example: shell and tube heat exchanger
  tubes placed in a shell

Internal flows:
Flow through N_L tubes $=$ $N_L \times$ flow through each tube

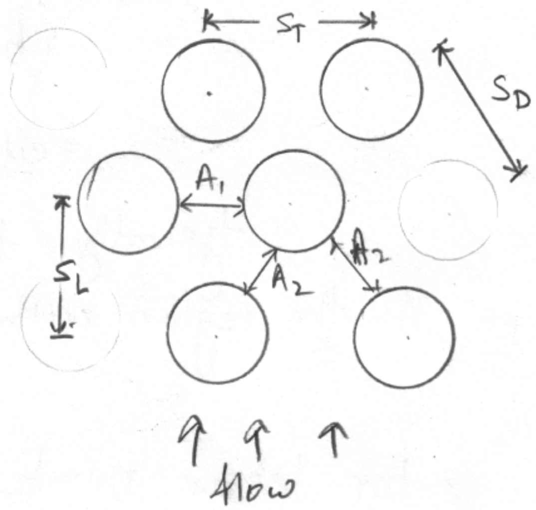
External flows:
Flow across N_L tubes $\neq$ $N_L \times$ flow across each tube

- Flow affected by neighboring tubes
  $\Rightarrow$ heat transfer affected by neighboring tubes
- Tube arrangements affect flow differently

Typical arrangements with N_L rows:



In-line tube bank
(Aligned)



Staggered tube bank

In-line tube bank:

- Beyond first row, tubes are in the 'wake' of upstream tubes.
- Upto the fifth row, h of a row increases. No significant increase beyond that row.
- As S_L increases, enhancement in h decreases.
  $\Rightarrow$ $S_T/S_L < 0.7$ is not desirable

Staggered tube bank:

- Fluid flow is complicated, and results into enhanced mixing
- Relative to In-line, h is enhanced to a greater extent, especially at low Re_D

Average Heat Transfer Coefficient of the bank

$$Nu_D = C_1 Re_{D,max}^{m} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Table 7.5 (Incropera) for $N_L > 16$,
Table 7-2 (Cengel)

$0.7 \leq Pr \leq 500$

$10 \leq Re_{D,max} \leq 2 \times 10^6$

where

$$Re_{D,max} = \frac{\rho U_{max} D}{\mu}$$

with

$U_{max} \equiv$ maximum velocity inside the tube bank (not the approach/free stream velocity)

$\equiv$ either at transverse (A_1) plane or diagonal (A_2)

typically, the cross-sectional area decreases in the region with banks $\Rightarrow$ higher than approach velocity of the fluid

- For lesser number of tube rows in the bank,

$$Nu_{D,correct} = \text{Correction factor} \times Nu_D$$

- Once Nusselt number for the bank of tubes is known, Newton's law of cooling gives the rate of heat transfer in terms of the "correct" temperature gradient. However, $T_s - T_\infty$ or $T_s - T_{avg}$ overpredict $\dot{Q}$, because as the fluid moves along the bank of tubes, it gets heated, and the gradient for heat transfer decreases.

Assuming the correct form of temperature difference (will be derived for internal flows later)

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[\frac{T_s - T_i}{T_s - T_o}\right]}$$

where
$T_s \equiv$ surface temperature
$T_i \equiv$ inlet fluid temperature
$T_o \equiv$ outlet fluid temperature
$= T_s - (T_s - T_i)\exp\left[-\frac{A_s h}{\dot{m} C_p}\right]$; $\dot{m} \equiv$ mass flow rate

The net rate of heat transfer

$$Q = h A_s \Delta T_{lm}$$

where $A_s = N\pi D L$ is the total surface area.

and $N \equiv$ total number of tubes $= N_L \times N_T$

N_L: no. of rows; N_T: tubes per row

Example : Staggered tube bank

Pressurized water is often available at elevated temperatures and may be used for space heating. In such cases, it is customary to use a tube bundle in which the water is passed through the tubes, while air is passed in cross flow over the tubes. Consider a staggered arrangement for which the tube diameter (outside) is 16.4 mm, and $S_L = 34.3$ mm, $S_T = 31.3$ mm. $N_L = 7$, $N_T = 8$. The cylinder surface temperature is at $70°C$, while the air (upstream) temperature, velocity are $15°C$ and 6 m/s. Determine the airside convection heat transfer coefficient and the rate of heat transfer from the bundle.

Assumptions:

- Steady, incompressible flow
- Negligible radiation effects
- Air properties nearly constant
- Per unit width of the tubes.

Table A.4 (Incropera)

At 15°C

$\rho = 1.217\ kg/m^3$

$C_p = 1007\ J/kg\cdot K$

$\nu = 14.82 \times 10^{-6}\ m^2/s$

$k = 0.0253\ W/m\cdot K$

$Pr = 0.710$

At 70°C

$Pr_s = 0.701$

At 43°C

$\nu = 17.4 \times 10^{-6}\ m^2/s$

$k = 0.0274\ W/m\cdot K$

$Pr = 0.705$

For $N_L < 16$,

$$Nu_{D,correct} = \text{correction factor}\ Nu_D$$

$$= \text{correction factor} \times C_1 Re_{D,max}^{m} Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Comparison of A_1 and A_2 and mass balance

$$A_1 = S_T - D = 31.3 - 16.4 = 14.9\,mm$$

$$A_2 = S_D - D = \left(S_L^2 + \left(\frac{S_T}{2}\right)^2\right)^{1/2} - D = 37.7 - 16.4 = 21.3\,mm$$

$\Rightarrow$ $A_1 < 2A_2$

Thus, A_1 is where velocity will be maximum

$$u_{max} = \frac{S_T}{S_T - D} u_\infty = 12.6\ m/s$$

$$Re_{D,max} = \frac{u_{max} D}{\nu} = \frac{12.6 \times 0.0164}{14.82 \times 10^{-6}} = 13943$$

$$\frac{S_T}{S_L} = 0.91 < 2$$

From tables, $C_1 = 0.35\left(\frac{S_T}{S_L}\right)^{1/5} (= 0.34)$; $m = 0.6$

For 7 rows ($N_L = 7$), correction factor $= 0.95$

$$Nu_{D,correct} = 0.95 \times 0.34 (13943)^{0.6} (0.71)^{0.36} \left(\frac{0.710}{0.701}\right)$$
$$= 87.9$$

$$\Rightarrow \quad h_{corrected} = Nu_{D,corrected} \frac{k}{D} = 135.6 \ W/m^2 \cdot K$$

$$T_s - T_i = 55^\circ C$$

$$T_s - T_o = (T_s - T_i) \exp\left(- \frac{\pi D N h_{corrected}}{\underbrace{\rho U_\infty N_T S_T}_{\dot{m}} C_p} \right)$$

$$= 44.5^\circ C$$

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left[\frac{T_s - T_i}{T_s - T_o}\right]} = 49.6^\circ C$$

$$\frac{Q}{W} = N (h_{corrected} \pi D \Delta T_{lm})$$
$$= 19.4 \ kW/m$$

Note: • Fluid properties could be calculated at the average of inlet-outlet temperature

• $\Delta T_i = T_s - T_i = 55^\circ C$ vs $\Delta T_{lm} = 49.6^\circ C$

Thus, over 10% overprediction in the rate of heat transfer would result due to the use of ΔT_i.

• With increasing N_L, air temperature (outlet temperature as an indicator) would approach the surface temperature, $\Rightarrow$ diminishing advantage of adding tubes.