

SOLUTIONS MANUAL

Principles of Mass Transfer and Separation Processes

Binay K. Dutta

Universiti Teknologi Petronas

Malaysia

Prentice-Hall of India Private Limited

Regd. Office: M-97, CONNAUGHT CIRCUS, NEW DELHI-110001, INDIA
Phones: 011-23411779, 23415326, 23418078 • Fax: 011-23417179

Patparganj Office: 'Rimjhim House', 111, Patparganj Industrial Estate, Delhi-110092
Phones: 011-43031100, 22143344 • Fax: 011-43031144, 22141144
E-Mail: phl@phindia.com

Contents

- 2. MOLECULAR DIFFUSION
- 3. CONVECTIVE MASS TRANSFER AND MASS TRANSFER COEFFICIENT
- 4. INTERPHASE MASS TRANSFER
- 5. GAS-LIQUID CONTACTING EQUIPMENT
- 6. GAS ABSORPTION AND STRIPPING 
- 7. DISTILLATION
- 8. LIQUID-LIQUID EXTRACTION
- 9. SOLID-LIQUID EXTRACTION
- 10. HUMIDIFICATION AND WATER COOLING 
- 11. DRYING OF WET SOLIDS 
- 12. ADSORPTION
- 13. CRYSTALLIZATION 
- 14. MEMBRANE SEPARATION

CHAPTER 2: MOLECULAR DIFFUSION

Short and MCQ's

(2.1)

1. Diffusion of the reactants and products within the pores of a solid catalyst; diffusion of moisture in a wet solid during drying; diffusion through a membrane; diffusion of a solute within a 'rigid' drop in liquid extraction; diffusion of an ionic species in an ion-exchange resin.
2. If the molecular weights of all the species in a mixture are equal.
3. (i) Concentration, (ii) length of the diffusion path, (vi) partial pressure are scalar quantities; others are vectors.
4. (i) ft^2/s ; (ii) $\text{Mmol}/\text{ft}^2 \cdot \text{s}$; (iii) $\text{Mmol}/(\text{ft}^3) \cdot (\text{ft})$
5. (i) If the bulk $\frac{\text{flow}}{\text{term}}$ is small, or (ii) for transport in a dilute solution.
6. Ideal gas mixture.
7. (a) Adsorption of a solute from an inert (insoluble) carrier; (b) distillation, if the components have equal molar heats of vaporization.
8. Since $N_B = 0$, $N_A = N_A \cdot \frac{P_A}{P} - \frac{D_{AB}}{RT} \frac{dp_A}{dz} = \text{constant}$ ($A \rightarrow \text{ethanol}$)
 P_A is maximum at the liquid surface \Rightarrow contribution of the bulk flow term is the maximum at the liquid surface.
9. From Eq.(2.19), $-\frac{dp_A}{dz} = N_A \cdot \frac{P - p_A}{P} \cdot \frac{RT}{D_{AB}}$. At $z = z_2$, $p_A = p_{A2}$ is the minimum and the magnitude of $\frac{dp_A}{dz}$ is maximum.
10. If the other two components are non-diffusing and these are present in the same volume ratio along the diffusion path.
11. Yes, by virtue of the Cauchy's Mean Value Theorem.
12. Compare the fluxes given by Eqs(2.27) and (2.30).

2.2

12 contd... $\frac{(N_A)_{\text{nondiffusing } B}}{(N_A)_{\text{equimolar count}}} = \frac{P}{P_{B^M}} > 1$. Reason: molecules of B in countercurrent diffusion offer more 'resistance' to diffusion of A compared to the case when these are non-diffusing.

13. D_{AB} depends inversely with total pressure, P, since at a higher pressure there are more molecular collisions (less mean free path) offering more 'resistance' to diffusion.

D_{AB} increases as $T^{3/2}$ to $T^{1.75}$, since higher temperature means higher velocity or mobility of the molecules.

14. The flux will be very high and pseudosteady state assumption will not be applicable.

15. The diffusivity increases with T and decreases with μ .

16. (a) \rightarrow (ii) zero; (b) \rightarrow (i) $U > V$; (c) \rightarrow (iii) multicomponent...;
 (d) \rightarrow (i) -2; (e) \rightarrow (i) $-78/94 = -0.83$; (f) \rightarrow Diffusion vel., $v_{A,d}$
 $= U_A - V = J_A / C_A = - \frac{D_{AB}}{C_A} \cdot \frac{dC_A}{dz} = - \frac{D_{AB}}{P_A} \cdot \frac{dp_A}{dz}$.

Equimolar countercurrent diffusion for $p_{A_1} = 0.3 \text{ atm}$, $p_{A_2} = 0.1 \text{ atm}$ midway in the film, $p_A = 0.2 \text{ atm}$; $\frac{dp_A}{dz} = \frac{0.1 - 0.3}{1} \frac{\text{atm}}{\text{cm}} = -0.2 \frac{\text{atm}}{\text{cm}}$

$$v_{A,d} = - \frac{0.2 \text{ cm}^2/\text{s}}{0.2 \text{ atm}} \cdot (-0.2 \frac{\text{atm}}{\text{cm}}) = 0.2 \text{ cm/s} = 2 \text{ mm/s} \quad (\text{i})$$

$$(g) N_A \left(\frac{P - p_A}{P} \right) = - \frac{D_{AB}}{RT} \cdot \frac{dp_A}{dz} \Rightarrow \frac{(dp_A/dz)_1}{(dp_A/dz)_2} = \frac{(P - p_A)_1}{(P - p_A)_2}$$

$$= \frac{2 - (2)(0.1)}{2 - (2)(0)} = \frac{1.8}{2} = 0.9, \quad (\text{iii})$$

$$(h) (dp_B/dz) = - (dp_A/dz) = \text{constant everywhere (since } p_A + p_B = P)$$

$$= - (-7 \text{ atm/m}) = 7 \text{ atm/m} \quad (\text{iii})$$

(i) After 10 hr, partial pressures of A and B in bulb 1 are:

$$p_{A_1} = 0.5 \text{ atm}; p_{B_1} = 0.5 \text{ atm} \quad (\text{since half of A diffuses out}).$$

(2.3)

$$16(i) \text{ contd... Material balance of } A : V_1 \cdot \frac{1 \text{ atm}}{RT} = V_1 \cdot \frac{0.5 \text{ atm}}{RT} + V_2 \cdot \frac{P_{A2}}{RT}$$

$$\Rightarrow P_{A2} = \frac{(V_1)(0.5)}{V_2} = \left(\frac{5}{10}\right)(0.5) = 0.25 \text{ atm.} \quad \begin{array}{l} I \rightarrow \text{initial} \\ II \rightarrow \text{after 10-hour} \end{array}$$

$$\frac{(N_A)_I}{(N_A)_{II}} = \frac{(D_{AB}/RTl)(P_{A1} - P_{A2})_I}{(D_{AB}/RTl)(P_{A1} - P_{A2})_{II}} = \frac{1 - 0.0}{0.5 - 0.25} = \frac{1}{0.25} = \underline{\underline{4}}$$

(j) Equimolar countercurrent diffusion of A and B occurs. Since the area of diffusion is variable,

$$\pi r_i^2 \cdot (N_A)_I = \pi r_2^2 (N_B)_{II} \Rightarrow (N_B)_{II} = -\left(\frac{r_1^2}{r_2^2}\right) \cdot (N_A)_I = -\left(\frac{1}{2}\right)^2 (10^{-6} \text{ kmol/m}^2 \text{s}) \\ = \underline{\underline{2.5 \times 10^{-7} \text{ kmol/m}^2 \text{s towards vessel 1}}}$$

(k) \rightarrow (ii). It is the geometric mean dia = $(1 \times 4)^{1/2} = \underline{\underline{2 \text{ cm}}}$

(l) \rightarrow (ii). Since $\pi r^2 N_A = \text{constant}$, N_A is the highest where the radius is the smallest.

(m) For equimolar countercurrent diffusion through a spherical film,

$$4\pi r^2 N_A = \text{constant}, N_A = -\frac{D_{AB}}{RT} \cdot \frac{dp_A}{dr} \Rightarrow 4\pi r^2 \left(-\frac{D_{AB}}{RT} \frac{dp_A}{dr}\right) = W$$

$$\Rightarrow -\int_{P_{A1}}^{P_{A2}} \frac{dp_A}{D_{AB} \cdot \frac{W \cdot RT}{4\pi}} \int_{r_i}^{r_i+\delta} \frac{dr}{r^2} \Rightarrow P_{A1} - P_{A2} = \Delta p = \frac{W \cdot RT}{D_{AB} \cdot 4\pi} \left(\frac{1}{r_i} - \frac{1}{r_i + \delta}\right)$$

$$\Rightarrow \text{Flux at } r = r_i \text{ is } (N_A)_I = \frac{W}{4\pi r_i^2} = \frac{D_{AB}}{RT} \cdot \frac{r_i + \delta}{r_i \cdot \delta} \cdot \Delta p$$

For diffusion through a flat film of thickness, $(N_A)_{II} = \frac{D_{AB}}{RT \cdot \delta} \cdot \Delta p$

$$\frac{(N_A)_I}{(N_A)_{II}} = 1.05 \text{ (given)} \Rightarrow \frac{r_i + \delta}{r_i} = 1.05 \Rightarrow \frac{\delta}{r_i} = 0.05 \Rightarrow \underline{\underline{\delta = 0.05 r_i}}$$

(n) \rightarrow (i), because of spherical symmetry; (o) \rightarrow (i);

(p) \rightarrow (iii), other values are out of range; (q) \rightarrow (iii), other values are out of range; (r) \rightarrow (iii) methanol, since it has the smallest molecular size among the three; (s) \rightarrow (ii) largest molecular size/weight; (t) \rightarrow (iii), since D_{AB}/P is constant for gas-phase diffusion; (u) \rightarrow (i); use the relation $\frac{D_1 \mu_1}{T_1} = \frac{D_2 \mu_2}{T_2}$.

- 16(v) \rightarrow (ii), $0.09 \text{ cm}^2/\text{s}$, use the relation $D_{AB} \propto T^{1.5}/P$;
 (w) \rightarrow (ii), velocity at the interface, $\frac{dz}{dt} \propto \frac{1}{z}$, see Eq (2.51a);
 (x) \rightarrow (i), surface diffusion; (y) \rightarrow (ii); (z) \rightarrow (ii).

Problems

- 2.1 Water vapor (A) diffuses through a stagnant film of air (B). Use Eq (2.26) to calculate the flux.

Water temp. = $23^\circ\text{C} = 296\text{ K}$, vapor pressure $P_{A0} = 13.8573 - \frac{5160.2}{296}$

Ambient air temp. = 27°C ; $P_{Ae} = 0.0353 \text{ bar} = 0.0280 \text{ bar}$

R. Humidity of air = 65% \Rightarrow partial pressure of moisture in bulk air, $P_{Ax} = (0.0353)(0.65) = 0.023 \text{ bar}$.

Mean value of diffusivity of water vapor in the film = $(0.257 + 0.262)/2 = 0.2595 \text{ cm}^2/\text{s}$; mean film temperature = $(296 + 300)/2 = 298 \text{ K}$.

Film thickness, $l = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$; $P = 1 \text{ atm} = 1.013 \text{ bar}$

Flux of water vapor at steady state, $N_A = \frac{D_{AB} P}{RT l} \cdot \ln \frac{P - P_{Ax}}{P - P_{A0}}$

$$\Rightarrow N_A = \frac{(2.595 \times 10^{-5})(1.013)}{(0.08317)(298)(3 \times 10^{-3})} \ln \frac{1.013 - 0.023}{1.013 - 0.028} = 1.79 \times 10^{-6} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$= \frac{0.116 \text{ kg/m}^2 \cdot \text{h}}{* * *} = \text{rate of evaporation per m}^2$$

- 2.2 Use Eq (2.26). The flux of moisture is given, calculate film thickness.

Amount water evaporated per m^2 of surface in 10 days

$$= (20 \text{ mm})(1 \text{ m}^2) = 0.02 \text{ m}^3 = 20 \text{ kg} = 20/18, \text{ or } 1.111 \text{ kmol}$$

$$\text{Flux} = \frac{1.111 \text{ kmol}}{(1 \text{ m}^2)(10 \times 24 \times 3600 \text{ s})} = 1.286 \times 10^{-6} \text{ kmol/m}^2 \cdot \text{s} = N_A$$

$$\text{Given: } D_{AB} = 1 \text{ ft}^2/\text{h} = (30.48)^2/3600 \rightarrow 0.258 \text{ cm}^2/\text{s} = 2.58 \times 10^{-5} \text{ m}^2/\text{s}$$

$$P = 1.013 \text{ bar}; R = 0.08317 \text{ m}^3 \cdot \text{bar/kmol} \cdot \text{K}; T = 23^\circ\text{C} = 296 \text{ K}$$

Humidity of bulk air = $0.0135 \text{ kg moisture/kg dry air}$.

(2.5)

$$2.2) \text{ Cond... } Y_{Al} = \frac{0.0135/18}{(0.0135/18) + (1/28.9)} = \underline{0.0212} \quad [\text{mol wt of air} = 28.9]$$

$$P_{Al} = P \cdot Y_{Al} = (1.013)(0.0212) = \underline{0.0215} \text{ bar.}$$

Vapor pressure of water at 296 K = 0.028 bar = P_{AO} ; $\underline{l = ?}$
(from Antoine eq.)

$$\text{Substitute in Eq (2.26)} \rightarrow 1.286 \times 10^{-6} = \frac{(2.58 \times 10^5)(1.013)}{(0.08317)(296)(l)} \cdot \ln \frac{1.013 - 0.0215}{1.013 - 0.028}$$

$$\Rightarrow l = 5.4 \times 10^{-3} \text{ m} = \underline{5.4 \text{ mm}} = \text{thickness of the air film}$$

* * * *

$$2.3) \text{ Temperature, } T = 35^\circ C = 308^\circ C; \text{ vapor press. of water, } \underline{P_A = 0.0552 \text{ bar}}$$

$$\text{Relative humidity of bulk air} = 30\% \Rightarrow P_{AO} = (0.0552)(0.3) = \underline{0.01656 \text{ bar}}$$

$$\text{Partial pressure of moisture at the acid surface, } P_{Al} \approx 0$$

$$\text{Thickness of air film, } l = 1.2 \times 10^{-3} \text{ m} \quad (\text{since the acid has a high affinity for water})$$

$$\text{Moisture flux} \quad P = \underline{1.013 \text{ bar}} \quad D_{AB} = 0.257 \text{ cm}^2/\text{s at } 23^\circ C;$$

$$\text{from Eq (2.26),} \quad N_A = \frac{(2.728 \times 10^5)(1.013)}{(0.08317)(308)(1.2 \times 10^{-3})} \cdot \ln \frac{1.013 - 0.01656}{1.013} \quad \text{at } 308K, D_{AB} = (0.257)(308/296)^{1.5} \\ = 0.2728 \text{ cm}^2/\text{s}$$

$$= 1.482 \times 10^{-6} \text{ kmol/m}^2 \cdot \text{s} = 9.602 \times 10^{-2} \text{ kg/m}^2 \cdot \text{h}$$

* * * *

$$2.4) \text{ Temperature, } T = 300K; \text{ total pressure} = 400 \text{ mm Hg; partial pressure of ammonia in the bulk (40% by volume)} = (400)(0.4) = \underline{160 \text{ mm Hg}}$$

$$\text{Diffusivity of NH}_3 \text{ at 1 atm and } 333K = 2.53 \times 10^{-5} \text{ m}^2/\text{s} \quad [\text{Table 2.1}]$$

$$\text{Diffusivity at 400 mm pressure and } 300K = (2.53 \times 10^{-5}) \left(\frac{760}{400} \right) \left(\frac{300}{333} \right)^{1.5}$$

$$P_{Al} = 0 \text{ at the acid surface; } l = 0.01 \text{ m} \quad = 4.11 \times 10^{-5} \text{ m}^2/\text{s} \quad P = \underline{0.533 \text{ bar}}$$

$$P_{AO} = 160 \text{ mm Hg} = \underline{0.2133 \text{ bar}} \quad \text{Flux of ammonia, } N_A = \frac{(4.11 \times 10^{-5})(0.533)}{(0.08317)(300)(0.01)} \cdot \ln \frac{0.533 - 0}{0.533 - 0.2133}$$

$$= \underline{4.488 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{s}}$$

$$(i) \text{ Rate of absorption of ammonia through an area, } a = 0.1 \text{ m}^2 \\ = (4.488 \times 10^{-5})(0.1) = \underline{4.488 \times 10^{-6} \text{ kmol/s}}$$

(ii) In order to calculate the partial pressure gradient at

(2.6)

2.4 contd... at a point in the film, calculate the partial pressure of the species at the particular point using Eq.(2.28), and then calculate the gradient ($\frac{dp_A}{dz}$) using the Fick's law, Eq.(2.24).

The given point is 4 mm from the acid surface $\Rightarrow z = 10 - 4 = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$.

$$\text{Put the values in Eq.(2.28)} \rightarrow 4.488 \times 10^{-5} = \frac{(4.11 \times 10^{-5})(0.533)}{(0.08317)(300)(0.006)} \ln \frac{0.533 - p_A}{0.533 - 0.2133}$$

From Eq.(2.24),

$$4.488 \times 10^{-5} \left(1 - \frac{0.0985}{0.533}\right) = \frac{(4.11 \times 10^{-5})}{(0.08317)(300)} \cdot \frac{dp_A}{dz} \quad \Rightarrow p_A = 0.0985 \text{ bar}$$

$$\Rightarrow \frac{dp_A}{dz} = -22.2 \text{ bar/m} \quad * \quad * \quad *$$

(iii) Acid-side end of the film $\rightarrow z = 0.01 \text{ m}; p_A = 0$

Contribution of 'bulk flow' to the flux [see Eq.(2.15)] $\boxed{N_B = 0}$

$$= (N_A + N_B) C / C = N_A \cdot p_A / P = (4.488 \times 10^{-5}) \left(\frac{0}{0.533}\right) = 0$$

$$\text{Contribution of the diffusion term} = N_A - N_A \cdot p_A / P = \frac{4.488 \times 10^{-5} \text{ kmol}}{\text{m}^2 \cdot \text{s}}$$

[i.e. there is no contribution of the bulk flow term at $z = 0.01 \text{ m}$]

Gas-side end of the film $\rightarrow z = 0; p_A = 0.2133 \text{ bar}$

$$\text{Contribution of the bulk flow term} = N_A \cdot p_A / P = (4.488 \times 10^{-5}) \left(\frac{0.2133}{0.533}\right) = 1.796 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{s}$$

$$\text{Contribution of the diffusion term} = 4.488 \times 10^{-5} - 1.796 \times 10^{-5} \text{ to the flux} = 2.692 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{s.}$$

2.5 Estimation of diffusivity of Camphor, $C_{10}H_{16}O$ (A)

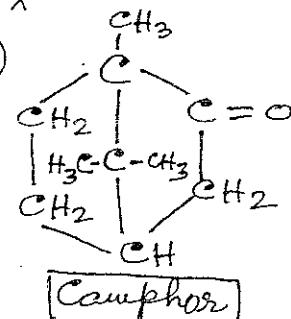
'Diffusion volume', see Table 7.4

$$10 \text{ carbons} \rightarrow (10)(16.5) = 165$$

$$16 \text{ hydrogen atoms} \rightarrow (16)(1.98) = 31.68$$

$$1 \text{ oxygen atom} \rightarrow 5.48$$

$$\text{Total, } (\sum v)_A = 165 + 31.68 + 5.48 = 202.16$$



$$\text{For air, } (\sum v)_B = 20.1; M_A = 152.13; M_B = 28.9; T = 293 \text{ K; } P = 1.013 \text{ bar}$$

$$\text{Eq.(2.57)} \rightarrow D_{AB} = \frac{(1.013 \times 10^{-7})(293)^{1.75}}{(1.013) \left[(202.16)^{1/3} + (20.1)^{1/3} \right]} \cdot \left[\frac{1}{152.13} + \frac{1}{28.9} \right]^{1/2} = 5.644 \times 10^{-6} \text{ m}^2/\text{s}$$

2.7

2.5) contd... Eq (2.26), $\text{flux} = \frac{(5.644 \times 10^{-6})(1.013)}{(0.08317)(293)(0.12)} \ln \frac{1.013 - 0}{1.013 - 0.13} = 2.685 \times 10^{-7} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$

$l = 0.12 \text{ m}; P_{A0} = 97.5 \text{ mmHg}$
 $= 0.13 \text{ bar}, P_{Al} = 0$

of camphor in the tube = 0.4 gm

Area of the tube = $\frac{\pi}{4}(0.015)^2 = 1.767 \times 10^{-4} \text{ m}^2 = a$

$N_A \cdot a \cdot t = m_A = 2.63 \times 10^{-6} \Rightarrow t = \frac{2.63 \times 10^{-6}}{(2.685 \times 10^{-7})(1.767 \times 10^{-4})} s = 15.4 \text{ hr}$

* * * *

2.6) For diffusion of A through non-diffusing B at steady state we write in the spherical geometry, the following equation [see Eq(2.73)].

$-\frac{dp_A}{P - p_A} = \frac{WRT}{4\pi D_{AB}P} \cdot \frac{dr}{r^2} ; W = 4\pi r^2 N_A = \text{constant rate of transport at steady state.}$

Integrate from $r = r_i$, $p_A = p_{Ai}$ to $r = r_o = r_i + \delta$, $p_A = p_{Ao}$ and simplify.

$$W = \frac{4\pi D_{AB}P}{RT\delta} \cdot r_i(r_i + \delta) \cdot \ln \frac{P - p_{Ao}}{P - p_{Ai}} \dots (i)$$

$$= \frac{(4)(\pi)(1.744 \times 10^{-5})(1.013)(0.0035)(0.0035 + 0.0035)}{(0.0821)(313)(0.0035)} \times \ln \frac{1 - 0.3}{1 - 0} = -2.13 \times 10^{-8} \frac{\text{kmol}}{\text{s}}$$

$T = 40^\circ C = 313K; p_{Ao} = 0.3 \text{ atm}$
 $D_{AB} \text{ at } 313K, 1 \text{ atm}$
 $= (1.62 \times 10^{-5}) \left(\frac{313}{298}\right)^{1.5} = 1.744 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$
 $p_{Ai} = 0; \delta = 0.0035 \text{ m};$
 $r_i = 7/2 \text{ mm} = 0.0035 \text{ m}$

Flux of CO_2 midway in the film :

Midway in the film, $r_m = r_i + \delta/2 = 0.0035 + 0.0035/2 = 0.00525 \text{ m}$

$4\pi r^2 N_A = W = 2.13 \times 10^{-8} \text{ kmol/s}$ (constant). Put $r = 0.00525 \text{ m}$ to get

$$N_A \text{ (midway in the film)} = \frac{-2.13 \times 10^{-8}}{4\pi (0.00525)^2} = -6.15 \times 10^{-5} \text{ kmol/m}^2 \text{ s}$$

This means that diffusion occurs towards decreasing r , i.e. towards the pellet.

'Diffusion velocity' at $r_m = 0.00525 \text{ m}$:

Eq.(2.18) is to be used. (i) calculate p_A from Eq(i) above written for $r = r_m$, (ii) calculate dp_A/dr from the expression for Fick's law, Eq(2.19).

Eq(i) above written for $r = r_m$ and $P = p_A$ is

$$W = \frac{4\pi D_{AB}P}{RT(\delta/2)} \cdot (r_i)(r_m) \cdot \ln \frac{P - p_A}{P - p_{Ai}} \quad (i) \quad \text{Equate (i) and (ii) to get}$$

$$2\pi r_m \ln \frac{P - p_A}{P - p_{Ai}} = (r_i + \delta) \ln \frac{P - p_{Ao}}{P - p_{Ai}}$$

$$\Rightarrow (2)(0.00525) \ln \frac{1 - p_A}{1 - 0} = (0.007) \ln \frac{1 - 0.3}{1 - 0} \Rightarrow p_A = 0.2116 \text{ atm, midway in the film.}$$

(2.8)

(2.6) contd... Eq(2.19) $\rightarrow N_A = (N_A + N_B) \cdot \frac{P_A}{P} - \frac{D_{AB}}{RT} \frac{dP_A}{dr}$
 $\Rightarrow (6.15 \times 10^{-5}) \left(1 - \frac{0.2}{1.0}\right) = - \frac{D_{AB}}{RT} \cdot \frac{dP_A}{dr} = -4.848 \times 10^{-5} \text{ kmol/m}^2\text{s}$

Diffusion velocity [Eq(2.18)] $\rightarrow v_{A,d} = - \frac{D_{AB}}{C_A} \cdot \frac{dC_A}{dr}$

$\Rightarrow v_{A,d} = - \frac{D_{AB}}{P_A} \cdot \frac{dP_A}{dr} = (-4.848 \times 10^{-5}) \left(\frac{(0.0821)(313)}{0.2116} \right) = -5.9 \times 10^{-3} \text{ m/s}$

\Rightarrow The diffusion velocity (5.9 mm/s) is directed towards the pellet, i.e. in the direction of decreasing r .

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(2.7) $N_2 \rightarrow A; CO_2 \rightarrow B; P = 1.013 \text{ bar}$

$P_{A1} = 10^4 \text{ Pa} = 0.1 \text{ bar}; P_{A2} = 0.8 \text{ bars};$

$P_{B1} = 0.913 \text{ bars}; P_{B2} = 0.213 \text{ bars}; l = 0.06 \text{ m}$

Equimolar countercurrent diffusion occurs at steady state.

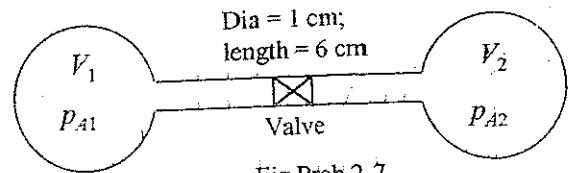


Fig Prob 2.7

(i) Use Eq(2.30) to calculate the flux. Since the vessels are 'large', the partial pressures remain virtually unchanged.

$\text{Flux of } N_2 (A) = \frac{D_{AB}}{RTl} (P_{A1} - P_{A2}); D_{AB} = 1.69 \times 10^{-5} \text{ m}^2/\text{s}, \text{ calculated from the value at } 298 \text{ K, Table 2.1.}$

$\Rightarrow N_A = \frac{(1.69 \times 10^{-5})}{(0.08317)(303)(0.06)} \cdot (0.1 - 0.8) = -7.824 \times 10^{-6} \text{ kmol/m}^2\text{s}$

(The flux is negative, diffusion occurs from vessel 2 to 1)

Velocity of the molecules of species A w.r.t. a stationary observer is

$u_A = N_A / C_A. \text{ At vessel 1 end of the tube, } C_{A1} = P_{A1} / RT = \frac{0.1}{(0.08317)(303)}$

$u_{A1} = \frac{-7.824 \times 10^{-6}}{(0.1)/(0.08317)(303)} = -0.00197 \text{ m/s.}$

$\text{Similarly, } u_{A2} = \frac{-7.824 \times 10^{-6}}{(0.8)/(0.08317)(303)} = -2.46 \times 10^{-4} \text{ m/s}$

$\text{Average velocity of the nitrogen molecules} = -\frac{1.97 \times 10^{-3} + 2.46 \times 10^{-4}}{2} = 1.108 \times 10^{-3} \text{ m/s}$

$\Rightarrow \text{velocity of the observer, } u_o = -1.108 \times 10^{-3} \text{ m/s}$

Molar flux of N_2 w.r.t. the observer moving with the above velocity,

$= (u_A - u_o) C_A = u_A C_A - u_o C_A = N_A - (-1.108 \times 10^{-3})(P_A / RT)$

Since the partial pressure is linear in distance for equimolar countercurrent diffusion,

(2.7) contd... $P_A = (P_{A1} + P_{A2})/2 = (0.1 + 0.8)/2 = 0.45 \text{ bar}$, midway in the tube.

$$\text{Required flux} = -7.824 \times 10^{-6} + (1.108 \times 10^{-3}) \left[\frac{0.45}{(0.08317)(303)} \right] = 1.19 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

This flux occurs in the direction of vessel 1 \rightarrow vessel 2.

(ii) To calculate the rate of transport through a tapered tube, use Eq. (2.79), $(\pi r^2 N_A) = W = \frac{\pi D_{AB}}{RT} \cdot \frac{r_1 r_2}{l} (P_{A1} - P_{A2})$ (compare Fig 2.9)

$$\text{Given: } r_1 = 0.005 \text{ m}, r_2 = 0.01 \text{ m. } W = \frac{(\pi)(1.69 \times 10^{-5})}{(0.08317)(303)} \cdot \frac{(0.005)(0.01)}{0.06} \cdot (0.1 - 0.8) \\ = -1.229 \times 10^{-9} \frac{\text{kmol}}{\text{s}}$$

(from vessel 2 to 1, in reality)

(iii) From the above equation it appears that the rate of transport does not change if the tube is reversely fitted.

(iv) The theoretical approach of Section 2.5.1 applies. Start with Eq. (2.49) and integrate from $t=0, P_{A10} - P_{A20}$ to $t=t, P'_{A1} - P'_{A2} \Rightarrow \ln \frac{P_{A10} - P_{A20}}{P'_{A1} - P'_{A2}} = \frac{a \cdot D_{AB}}{l} \left(\frac{1}{V_1} + \frac{1}{V_2} \right) \cdot t \dots (i)$

$$P_{A10} = 0.1 \text{ bar; } P_{A20} = 0.8 \text{ bar; } a = \frac{\pi}{4} (0.01)^2 = 7.853 \times 10^{-5} \text{ m}^2;$$

$$P'_{A1} = 2P_{A0} \text{ (given)} \Rightarrow P'_{A1} = 0.2 \text{ bar; } V_1 = 0.005 \text{ m}^3; V_2 = 0.003 \text{ m}^3.$$

Obtain P'_{A2} by material balance.

$$V_1 C_{A10} + V_2 C_{A20} = V_1 C'_{A1} + V_2 C'_{A2} \Rightarrow V_1 P_{A10} + V_2 P_{A20} = V_1 P'_{A1} + V_2 P'_{A2}$$

$$\Rightarrow (0.005)(0.1) + (0.003)(0.8) = (0.005)(0.2) + (0.003)(P'_{A2}) \Rightarrow P'_{A2} = 0.633 \text{ bar}$$

$$\Rightarrow \ln \frac{0.1 - 0.8}{0.2 - 0.633} = \frac{(7.853 \times 10^{-5})(1.69 \times 10^{-5})}{0.06} \cdot \left(\frac{1}{0.005} + \frac{1}{0.003} \right) \cdot t$$

$$\Rightarrow t = 11.3 \text{ hr}$$

Partial pressure of N_2 midway in the connecting tube after 20 hr

Put the values in the above equation (i) to get

$$\ln \frac{0.1 - 0.8}{P_{A1f} - P_{A2f}} = \frac{(7.853 \times 10^{-5})(1.69 \times 10^{-5})}{0.06} \cdot \left(\frac{1}{0.005} + \frac{1}{0.003} \right) \cdot (20 \times 3600)$$

$$\Rightarrow \ln \frac{-0.7}{P_{A1f} - P_{A2f}} = 0.85 \Rightarrow (P_{A1f} - P_{A2f}) = -0.3 \text{ bar} \dots (ii)$$

P_{A1f} and P_{A2f} are also related by material balance.

(2.10)

2.7) constd... $(0.005)(0.1) + (0.003)(0.8) = (0.005)(P_{A1f}) + (0.003)(P_{A2f})$

$$\Rightarrow 5P_{A1f} + 3P_{A2f} = 2.9 \quad \dots \text{(ii)}$$

Solve eqs. (ii) and (iii) to get $P_{A1f} = 0.2502 \text{ bar}$, $P_{A2f} = 0.5496 \text{ bar}$

Partial pressure of N_2 midway in the tube at $t=20 \text{ hr}$,

$$(P_A)_{av} = (0.2502 + 0.5496)/2 = \underline{\underline{0.4 \text{ bar}}}$$

* * * *

2.8) Partial pressures of A and B in the bulk $\rightarrow P_{A0}$, P_{B0} ; those at the catalyst surface ($z=\delta$) $\rightarrow P_{AS}$, P_{BS} ; $P_{AS}=0$. Also $P=P_{A0}+P_{B0}$.

(a) Reaction: $A \rightarrow B$. $N_A = -N_B$ at steady state. Use Eq(2.19).

$$N_A = (N_A + N_B) \frac{P_A}{P} - \frac{D_{AB}}{RT} \frac{dp_A}{dz} \Rightarrow N_A \int_0^{\delta} dz = - \frac{D_{AB}}{RT} \int_0^{P_{AS}=0} dp_A$$

$$\Rightarrow N_A = \frac{D_{AB}}{RT\delta} \cdot P_{A0} \Rightarrow \text{Reaction rate of A per unit area} = N_A = \frac{D_{AB}}{RT\delta} \cdot (P - P_{B0})$$

(b) Reaction: $2A \rightarrow B \Rightarrow N_A = -2N_B$ at steady state. From Eq(2.19)

$$N_A = (N_A - N_A/2) \frac{P_A}{P} - \frac{D_{AB}}{RT} \frac{dp_A}{dz} \Rightarrow N_A \int_0^{\delta} dz = - \frac{2D_{AB}P}{RT} \int_0^{P_{A0}} \frac{dp_A}{2P - p_A}$$

$$\Rightarrow N_A = \frac{2D_{AB}P}{RT\delta} \ln \frac{2P}{2P - P_{A0}} \Rightarrow \text{Rate of reaction, } N_A = \frac{2D_{AB}P}{RT\delta} \ln \frac{2P}{P + P_{B0}}$$

(c) Reaction: $2A \rightarrow 3B \Rightarrow 3N_A = 2N_B$ (at steady state) $\Rightarrow N_B = -\frac{3}{2}N_A$.

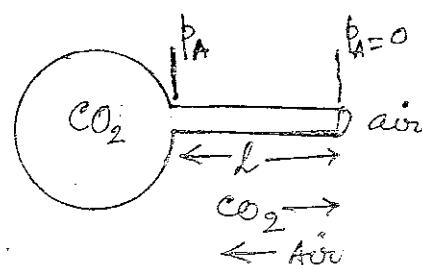
$$N_A = (N_A - \frac{3}{2}N_A) \frac{P_A}{P} - \frac{D_{AB}}{RT} \frac{dp_A}{dz} \Rightarrow N_A \int_0^{\delta} dz = - \frac{2D_{AB}P}{RT} \int_0^{P_{A0}} \frac{dp_A}{2P + p_A}$$

$$\Rightarrow N_A = \frac{2D_{AB}P}{RT\delta} \ln \frac{2P + P_{A0}}{2P} \Rightarrow \text{Rate of reaction, } N_A = \frac{2D_{AB}P}{RT\delta} \ln \frac{3P - P_{B0}}{2P}$$

* * *

2.9) Equimolar countercurrent diffusion of CO_2 and air occurs through the tube. Assume Pseudo-steady state.

Refer to the theoretical analysis in Section 2.5.1.



(2.11)

2.9) Contd... The time rate of change of the partial pressure of CO_2 in the bulb is given by:

$$-\frac{V}{RT} \frac{dp_A}{dt} = \text{Rate of diffusion of } \text{CO}_2 \text{ through the tube} = a \cdot N_A$$

$$\Rightarrow -\frac{V}{RT} \frac{dp_A}{dt} = a \cdot \frac{D_{AB}}{R \cdot l} (p_A - p_f); p_A = \text{CO}_2 \text{ partial pressure at any time, } t; a = \text{area of cross-section of the tube.}$$

The partial pressure of CO_2 at the air-side end of the tube

$$\text{Integrate to get } -\int \frac{\frac{dp_A}{dt}}{p_A} = \frac{a \cdot D_{AB}}{V \cdot l} \int dt \Rightarrow \ln \frac{p_{Ai}}{p_{Af}} = \frac{a D_{AB}}{V \cdot l} \cdot t_f \dots (i)$$

Given: p_{Ai} = initial partial pressure of CO_2 in the bulb = 1 atm.

p_{Af} = final partial pressure = 0.5 atm; $V = 2 \text{ liters} = 2 \times 10^{-3} \text{ m}^3$

$$l = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}; a = \frac{\pi}{4} (0.005)^2 = 1.96 \times 10^{-5} \text{ m}^2; D_{AB} = 1.62 \times 10^{-5} \text{ m}^2$$

Substituting in Eq.(i) above,

$$\text{Time of diffusion, } t_f = \frac{(0.002)(0.03)}{(1.96 \times 10^{-5})(1.62 \times 10^{-5})} \ln \frac{1.0}{0.5} = \frac{1.31 \times 10^8}{8} = 36.38 \text{ hr.}$$

2.10) Reaction: $2 \text{CO} + \text{O}_2 \rightarrow 2 \text{CO}_2$; $\text{CO} \rightarrow \text{component 1}, \text{O}_2 \rightarrow 2, \text{CO}_2 \rightarrow 3$

Flux relation: $N_1 = 2N_2 = -N_3$; $y \rightarrow \text{mole fraction}$, $C = \text{total conc.}$

Use Eq.(2.37) for concentration distribution of component 1 (CO) in the film.

$$-C \frac{dy_1}{dz} = N_1 \left(\frac{y_2}{D_{12}} + \frac{y_3}{D_{13}} \right) - y_1 \left(\frac{N_2}{D_{12}} + \frac{N_3}{D_{13}} \right)$$

$$\Rightarrow -C \frac{dy_1}{dz} = N_1 \left(\frac{y_2}{D_{12}} + \frac{y_3}{D_{13}} \right) - y_1 \cdot N_1 \left(\frac{1}{2D_{12}} - \frac{1}{D_{13}} \right) \quad (i)$$

Similarly, putting the flux relations in Eq.(2.38),

$$-C \frac{dy_2}{dz} = \frac{N_1}{2} \left(\frac{y_1}{D_{21}} + \frac{y_3}{D_{23}} \right) - y_2 \cdot N_1 \left(\frac{1}{D_{21}} - \frac{1}{D_{23}} \right) \quad (ii)$$

Put the diffusivity values in Eq.(i) and (ii) above,

take $D_{ij} = D_{ji}$ for an ideal mixture

and use the relation $y_1 + y_2 + y_3 = 1$ to get

$$-\frac{C}{N_1} \cdot \frac{dy_1}{dz} = 2.755 - 1.02 y_1 - 0.714 y_2 \quad (iii)$$

$$\text{And } -\frac{C}{N_1} \frac{dy_2}{dz} = 1.359 - 0.338 y_1 - 0.682 y_2 \quad (iv)$$

For the CO-O₂ pair
 $D_{12} = 0.185 \text{ cm}^2/\text{s}$, at 1 atm
 and at 273 K

$$\text{At } 250 = 523 \text{ K,}$$

$$D_{12} = (0.185) \left(\frac{523}{273} \right)^{1.5}$$

$$= 0.363 \text{ cm}^2/\text{s}$$

Similarly,
 $D_{23} = 0.368$; $D_{12} = 0.49 \text{ cm}^2/\text{s}$

Since the given mixture has three components, two simultaneous ordinary differential equations are to be solved

(2.10) contd... The rate of reaction of CO or the CO_2 flux (N_1) can be obtained by solving the above two linear, simultaneous, first order ordinary differential equations in y_1 and y_2 . The following conditions apply.

Bulk of the gas : $z=0$; $y_1 = 0.2$ (for CO), $y_2 = 0.2$ (for O_2).

Surface of the catalyst; $z=\delta = 0.8\text{ mm}$; $y_1 = 0$, $y_2 = ?$, $y_3 = ?$

There are different techniques available. We use the Laplace Transform technique. Put $\xi = z \cdot N_1/c$ and take the L-transform of Eqs. (iii) and (iv) above.

$$\begin{aligned} \text{Eq. (iii)} \rightarrow - \int_{0}^{\infty} \frac{dy_1}{d\xi} e^{-\delta\xi} d\xi &= 2.755 \int_{0}^{\infty} e^{-\delta\xi} d\xi - 1.02 \int_{0}^{\infty} y_1 e^{-\delta\xi} d\xi - 0.714 \int_{0}^{\infty} y_2 e^{-\delta\xi} d\xi \\ \Rightarrow - \left[(y_1 e^{-\delta\xi}) \right]_{\xi=0}^{\infty} + \delta \int_{0}^{\infty} y_1 e^{-\delta\xi} d\xi &= \frac{2.755}{-\delta} \left[e^{-\delta\xi} \right]_{0}^{\infty} - 1.02 \bar{y}_1 - 0.714 \bar{y}_2 \quad (\text{v}) \\ \Rightarrow 0.2 - \delta \bar{y}_1 &= \frac{2.755}{\delta} - 1.02 \bar{y}_1 - 0.714 \bar{y}_2 ; \text{ where } \bar{y}_1 = \bar{y}_1(\delta) = \int_{0}^{\infty} y_1 e^{-\delta\xi} d\xi \end{aligned}$$

\bar{y}_1 and \bar{y}_2 are the L-transforms of y_1 and y_2

$$\text{Similarly, from Eq.(iv), } 0.2 - \delta \bar{y}_2 = \frac{1.359}{\delta} - 0.338 \bar{y}_1 - 0.682 \bar{y}_2 \quad \dots \text{(vi)}$$

The algebraic equations (v) and (vi) can be solved for the transforms \bar{y}_1 and \bar{y}_2 .

$$\bar{y}_1 = \frac{0.2\delta^2 - 2.7486\delta + 0.9086}{\delta(\delta^2 - 1.7028 + 0.4543)} = \frac{0.2\delta^2 - 2.7468\delta + 0.9086}{\delta(\delta - 1.3705)(\delta - 0.3315)}$$

To obtain the inverse transform, the r.h.s. is split in terms of partial fractions.

$$\bar{y}_1 = \frac{0.2\delta^2 - 2.7468\delta + 0.9086}{\delta(\delta - 1.3705)(\delta - 0.3315)} = \frac{A}{\delta} + \frac{B}{\delta - 1.3705} + \frac{C}{\delta - 0.3315}$$

By following the usual procedure, $A = 2$, $B = -1.7436$, $C = -0.0564$

$$\Rightarrow \bar{y}_1 = \frac{2}{\delta} - \frac{1.7436}{\delta - 1.3705} - \frac{0.0564}{\delta - 0.3315}. \text{ Now take the "inverse".}$$

$$\Rightarrow y_1(\xi) = \underline{2 - 1.7436 \exp(-1.3705 \xi) - 0.0564 \exp(0.3315 \xi)}$$

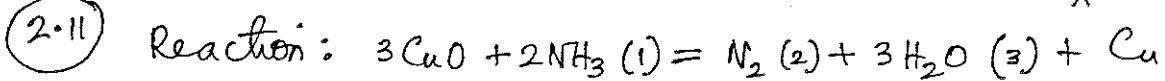
2.10 Contd... At $z=0$ or $\xi=z \cdot N_1/C = 0$, the mole fraction of CO is $y_1=0.2$ (in the bulk gas) \rightarrow satisfied.

Put $y_1=0$ at the surface of the catalyst (instantaneous reaction) and obtain the value of ξ .

$$0 = 2 - 1.7436 \exp(1.3705 \xi) - 0.0564 \exp(0.3315 \xi) \Rightarrow \xi = 0.0787$$

$$\text{At the Catalyst Surface, } z = \delta = 0.8 \text{ mm} = 0.08 \text{ cm}; C = \frac{P}{RT} = \frac{1 \text{ atm}}{(0.0821)(523)}$$

$$\begin{aligned} \xi &= \frac{z \cdot N_1}{C} \Rightarrow 0.0787 = \frac{(0.08) \cdot N_1}{2.329 \times 10^5} \\ &\Rightarrow N_1 = 2.29 \times 10^5 \frac{\text{gmol}}{\text{cm}^2 \text{s}} = \text{flux of CO} \\ &\quad = \text{Rate of formation of CO}_2 / \text{cm}^2 \text{area} \end{aligned}$$



$$\text{Flux relation: } N_1 = -2N_2 \quad ; \quad N_1 = -\frac{2}{3}N_3$$

At steady state

$$\text{Eq.(2.37)} \rightarrow -C \frac{dy_1}{dz} = N_1 \left(\frac{y_2}{D_{12}} + \frac{y_3}{D_{13}} \right) - y_1 \left(\frac{N_2}{D_{12}} + \frac{N_3}{D_{13}} \right)$$

$$\Rightarrow -C \frac{dy_1}{dz} = N_1 \left(\frac{y_2}{D_{12}} + \frac{y_3}{D_{13}} \right) - y_1 \left(\frac{-N_1/2}{D_{12}} + \frac{-3N_1/2}{D_{13}} \right)$$

Put the values of diffusivities, use the relation $D_{ij} = D_{ji}$, and put $\xi = z \cdot N_1/C$. Note that $z=0$ in the bulk-gas side of the film; $z=\delta$ at the catalyst surface.

$$\Rightarrow -\frac{dy_1}{d\xi} = 0.6738 + 0.8616 y_1 + 0.3755 y_2 \dots \text{(i)}$$

Similarly, using Eq.(2.38), we get

$$-\frac{dy_2}{d\xi} = -0.4517 - 0.073 y_1 + 0.7574 y_2 \dots \text{(ii)}$$

Given conditions: At $\xi=0$ (bulk gas), $y_1=0.1$, $y_2=0.9$.

Take Laplace transforms of (i) and (ii) to get

$$0.1 - 8\bar{y}_1 = \frac{0.6738}{8} + 0.8616 \bar{y}_1 + 0.3755 \bar{y}_2 \quad \text{(iii)} \quad \left| \begin{array}{l} \bar{y}_1(\xi) = \int_0^\infty y_1(\xi) e^{-8\xi} d\xi \\ \bar{y}_2(\xi) = \int_0^\infty y_2(\xi) e^{-8\xi} d\xi \end{array} \right.$$

$$0.9 - 8\bar{y}_2 = -\frac{0.4517}{8} - 0.073 \bar{y}_1 + 0.7574 \bar{y}_2 \quad \text{(iv)} \quad \left| \begin{array}{l} \bar{y}_1(\xi) = \int_0^\infty y_1(\xi) e^{-8\xi} d\xi \\ \bar{y}_2(\xi) = \int_0^\infty y_2(\xi) e^{-8\xi} d\xi \end{array} \right.$$

(2-14)

(2-11) contd... Solve the algebraic equations (iii) and (iv) to get

$$\bar{y}_1(\delta) = \frac{0.1\delta^2 - 0.936\delta - 0.68}{\delta(\delta^2 + 1.619\delta + 0.68)} = \frac{0.1\delta^2 - 0.936\delta - 0.68}{\delta(\delta + \omega_1)(\delta + \omega_2)} \dots (v)$$

$$c_0 = 0.8095 - 0.1572i; \omega_2 = 0.8095 + 0.1572i; i = \sqrt{-1}$$

The transform given by (v) above can be obtained by evaluating the 'residues at the poles'.

Residue at $\delta = -1$; at $\delta = -\omega_1 \rightarrow (0.55 + 0.66i)[\cos(0.1572\xi) + i\sin(0.1572\xi)]e^{-0.8095\xi}$
 Residue at $\delta = -\omega_2 = (0.55 - 0.66i)[\cos(0.1572\xi) - i\sin(0.1572\xi)]\exp(-0.8095\xi)$

Solution for $y_1(\xi) = \text{sum of the residues at the poles}$

$$\underline{y_1(\xi)} = -1 + 2[0.55 \cos(0.1572\xi) - 0.668 \sin(0.1572\xi)]\exp(-0.8095\xi)$$

$y_2(\xi)$ can be directly obtained from Eq (i) above using the expression for $y_1(\xi)$.

$$\underline{y_2(\xi)} = 0.5 + [0.4 \cos(0.1572\xi) + 0.64378 \sin(0.1572\xi)]\exp(-0.8095\xi)$$

It is obvious that the above solutions for $y_1(\xi)$ and $y_2(\xi)$ satisfy the given conditions at $\xi = 0$ where $y_1 = 0.1$, $y_2 = 0.9$. To calculate the value of ξ at the catalyst surface, put $y_1 = 0$, since the reaction is diffusion-controlled (very fast).

$$0 = -1 + 2[0.55 \cos(0.1572\xi) - 0.668 \sin(0.1572\xi)]\exp(-0.8095\xi)$$

Solving, $\xi = 0.095 = z \cdot N_i/C$. At the catalyst surface,

$$z = \delta = 2 \text{ mm} = 0.2 \text{ cm}$$

C = total concentration of the gas mixture at 1 atm and 450°C (723K).

$$C = \frac{P}{RT} = \frac{1}{(0.0821)(723)} = 1.685 \times 10^2 \frac{\text{gmol}}{\text{litres}} = 1.685 \times 10^{-5} \frac{\text{gmol}}{\text{cm}^3}$$

N_i = flux of ammonia to the catalyst surface

$$= \xi \cdot C/8 = (0.095)(1.685 \times 10^{-5})/0.2 = \underline{8 \times 10^{-6} \frac{\text{gmol}}{\text{cm}^2 \cdot \text{s}}}$$

This is the rate of reaction of NH_3 per unit area of the catalyst.

Note: at the catalyst surface ($\xi = 0.095$), the value of y_2 is

$$y_2 = 0.5 + [0.4 \cos(0.1572 \times 0.095) + 0.64378 \sin(0.1572 \times 0.095)]\exp(-0.8095 \times 0.095)$$

$$= 0.5 + [(0.4)(0.9999) + (0.6437)(0.01493)](0.926) = \underline{0.8792}$$

(2.15)

(2.11) contd... This is an interesting observation. Nitrogen, which is a product of the reaction and is called component 2 here, is formed at the catalyst surface and diffuses to the bulk. The given value of its mole fraction in the bulk is 0.9; the calculated value of its mole fraction at the catalyst surface is 0.8792 which is less than the bulk mole fraction. Still it diffuses from the catalyst surface to the bulk - i.e. it diffuses against its concentration gradient. This is an example to show that diffusional flux of a species in a multicomponent mixture depends upon the fluxes and driving forces of other species present.

* * * *

(2.12) Here two of the components - CO_2 (1) and H_2S (2) - diffuse through a stagnant gas film, reach the gas-liquid interface and get absorbed. The other component, CH_4 (3), is not soluble in the liquid (alkaline solution) and does not diffuse. Fluxes of two components diffusing through a non-diffusing third component can be calculated from Eqs. (2.44) and (2.45). Assume $D_{12} = D_{21}$, $D_{23} = D_{32}$, and $D_{31} = D_{13}$ [$D_{ij} = D_{ji}$]

$$\text{First calculate } \xi = \frac{D_{23}(D_{13} - D_{12})}{D_{13}(D_{23} - D_{12})} = \frac{0.0417}{0.0367} \cdot \frac{(0.0367 - 0.0303)}{(0.0417 - 0.0303)} = 0.638$$

Total molar concentration of the gas mixture ($P = 5 \text{ atm}$, $T = 30^\circ\text{C} = 303\text{K}$)

$$C = \frac{P}{RT} = \frac{5}{(0.0821)(303)} = 0.201 \frac{\text{kmol}}{\text{m}^3} \quad [\text{assuming ideal behavior}]$$

Given: film thickness, $l = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$; mole fractions:

Bulk gas ($z=0$) $\rightarrow y_{10} = 0.2$, $y_{20} = 0.1$, $y_{30} = 0.7$; gas-liquid interface ($z=l$) $\rightarrow y_{1l} = 0$, $y_{2l} = 0$, $y_{3l} = 1.0$ [no CO_2 or H_2S at the interface]

Put the values in Eq. (2.44) $D_{12} = 0.0303 \frac{\text{cm}^2/\text{s}}{\text{interface}} = 3.03 \times 10^{-6} \frac{\text{m}^2/\text{s}}{\text{interface}}$

$$(N_1 + N_2) = \frac{(3.03 \times 10^{-6})(0.201)}{2 \times 10^{-3}} \cdot \ln \frac{\left(\frac{N_1 + N_2}{N_1}\right)(0) - \left(\frac{N_1 + N_2}{N_2}\right) \cdot \xi \cdot (0) + (0.638 - 1)}{\left(\frac{N_1 + N_2}{N_1}\right)(0.2) - \left(\frac{N_1 + N_2}{N_2}\right)(0.638)(0.1) + (0.638 - 1)}$$

(2.12) Contd... Simplify to get

2.16

$$N_1 \left(1 + \frac{N_2}{N_1}\right) = -3.045 \times 10^{-4} \cdot \ln \left[0.6237 + 0.1762 \left(\frac{N_1}{N_2}\right) - 0.5525 \left(\frac{N_2}{N_1}\right)\right]$$

From Eq (2.45), $\frac{N_1}{3.67 \times 10^6} + \frac{N_2}{4.17 \times 10^6} = \frac{0.201}{2 \times 10^{-3}} \cdot \ln \frac{1.0}{0.7}$... (i)

$$\Rightarrow N_1 [1 + (0.8807)(N_2/N_1)] = 1.3155 \times 10^{-4} \quad \dots \text{(ii)}$$

Solving Eqs. (i) and (ii), $N_1 = 8.8465 \times 10^{-5}$, $N_2 = 4.892 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$

N_1 and N_2 are the rates of absorption of CO_2 and H_2S .

(2.13) Components in the mixture: $\text{CO}_2 \rightarrow 1$, $\text{N}_2 \rightarrow 2$, and $\text{H}_2 \rightarrow 3$.

Binary diffusivities (Table 7.1): $D_{12} = 1.65 \times 10^{-5} \text{ m}^2/\text{s}$ at 1 atm, 298 K;

$D_{23} = 7.79 \times 10^{-5} \text{ m}^2/\text{s}$ at 297 K, 1 atm; $D_{31} = 6.46 \times 10^{-5} \text{ m}^2/\text{s}$ at 1 atm, 298 K.

At the given condition ($50^\circ\text{C} = 323 \text{ K}$, 5 bar),

$$D_{12} = (1.65 \times 10^{-5}) \left(\frac{323}{298}\right)^{1.5} \left(\frac{1.013}{5}\right) = 0.377 \times 10^{-5} \text{ m}^2/\text{s}$$

Similarly, $D_{23} = 7.79 \times 10^{-5}$ and $D_{31} = 1.477 \times 10^{-5} \text{ m}^2/\text{s}$ at 323 K, 5 bar.

CO_2 (1) diffuses through nondiffusing N_2 (2) and H_2 (3). Also the ratio y_2/y_3 remains constant along the diffusion path.

Use Eq. (2.35) to calculate the 'effective diffusivity' of CO_2

$$D_{1m} = \frac{1}{(y'_2/D_{12}) + (y'_3/D_{13})} = \frac{1}{\frac{0.25}{0.377 \times 10^{-5}} + \frac{0.75}{1.477 \times 10^{-5}}} = 8.54 \times 10^{-6} \frac{\text{m}^2}{\text{s}}$$

y'_2 and y'_3 are the mole fractions of N_2 and H_2 on ' CO_2 -free basis'.

Use Eq. (2.36) to calculate the flux of CO_2

$$N_1 = \frac{D_{1m} C}{l} \cdot \ln \frac{1-y_{1e}}{1-y_{10}} \quad \text{Given: } l = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$\Rightarrow N_1 = \frac{(8.54 \times 10^{-6})(0.1861)}{1.5 \times 10^{-3}} \cdot \ln \frac{1-0}{1-0.12} = \frac{P}{RT} = \frac{5}{(0.08317)(323)} = 0.1861 \frac{\text{kmol}}{\text{m}^3}$$

$$= 1.355 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}} \quad y_{10} = \text{mole fraction of } \text{CO}_2 \text{ in the bulk gas} = 0.12$$

$$= \text{flux of } \text{CO}_2 \quad y_{12} = \text{mole fraction of } \text{CO}_2 \text{ at the gas-liquid interface} = 0.$$

(2.14) Initial diffusion flux of the ester can be obtained by using Eq.(2.26) for diffusion of A through non-diffusing B at steady state.

$$N_A = \frac{D_{AB} P}{RTl} \ln \frac{P - P_{Ae}}{P - P_{AO}}$$

$$= \frac{(8.66 \times 10^{-6})(1.013)}{(0.08317)(298)(0.075)} \ln \frac{1.013 - 0}{1.013 - 0.1264}$$

$$= 6.29 \times 10^{-7} \frac{\text{Rmol}}{\text{m}^2 \cdot \text{s}}$$

Given: $P = 1.013 \text{ bar}$; $T = 298 \text{ K}$;
 $D_{AB} = 8.66 \times 10^{-6} \text{ m}^2/\text{s}$; $P_{AO} = \text{partial pressure of ethyl acetate at the liquid surface} = 0.1264 \text{ bar}$
 $P_{Ae} = \text{that at the open end of the tube} = 0$
 $l = \frac{15}{2} = 7.5 \text{ cm} = 0.075 \text{ m at the beginning}$

Time for vaporization of half of the liquid can be calculated from Eq.(2.52).

$$t' = \frac{RT \bar{P}_{BM} \rho (z'^2 - z_0^2)}{2PM_A (\bar{P}_{A1} - \bar{P}_{A2}) \cdot D_{AB}}$$

$\rho_A = \text{liquid density} = 900 \text{ kg/m}^3$

$M_A = \text{mol. wt of the liquid} = 88$

$z_0 = \text{initial length of the diffusion path} = 15/2 \text{ cm} = 0.075 \text{ m}$

$z' = \text{final length of the diffusion path} = 15/7 \text{ cm} = 0.08 \text{ m}$

$$t' = \frac{(0.08317)(298)(0.9484)(900)[(0.08)^2 - (0.075)^2]}{(2)(1.013)(88)(0.1264 - 0) (8.66 \times 10^{-6})} = \frac{8.4 \times 10^4}{= 23.3 \text{ hr.}}$$

$$\bar{P}_{BM} = \frac{\bar{P}_{B2} - \bar{P}_{B1}}{\ln(\bar{P}_{B2}/\bar{P}_{B1})} ; \bar{P}_{B2} = 1.013 \text{ bar}$$

$$\bar{P}_{B1} = 1.013 - 0.1264 = 0.8866$$

position 2 → open end of the tube
position 1 → liquid surface
 $\Rightarrow \bar{P}_{BM} = 0.9484 \text{ bar}$

(2.15) Start with Eq.(2.49). Integrate it over the experimental time:

$$-\int \frac{d(P_{A1f} - P_{A2f})}{(P_{A1i} - P_{A2i})} = \frac{a D_{AB}}{l} \cdot \left(\frac{1}{V_1} + \frac{1}{V_2} \right) \int dt$$

$i \rightarrow \text{initial}; f \rightarrow \text{final}$

$$\Rightarrow \ln \frac{P_{A1i} - P_{A2i}}{P_{A1f} - P_{A2f}} = \frac{a D_{AB}}{l} \cdot \left(\frac{1}{V_1} + \frac{1}{V_2} \right) \cdot t_f$$

$$\Rightarrow \ln \left(\frac{0.725 - 0.275}{0.543 - 0.457} \right) = \frac{(7.04 \times 10^{-4}) \cdot D_{AB}}{0.08} \left(\frac{2}{5 \times 10^{-3}} \right) (5.7) \times (3600)$$

$$\Rightarrow D_{AB} = \underline{2.29 \times 10^{-5} \frac{\text{m}^2/\text{s}}{}}$$

Given: $P_{A1i} = 0.725 \text{ atm}$;
 $P_{A2i} = 0.275 \text{ atm}$; $P = 1 \text{ atm}$;
 $P_{A1f} = 0.543$, $P_{A2f} = 0.457 \text{ atm}$;
 $a = \text{cross-section of the connecting tube}$

$$= \frac{\pi}{4} (0.03)^2 = 7.07 \times 10^{-4} \text{ m}^2$$

$$l = 0.08 \text{ m}; t_f = 5.7 \text{ hr.}$$

$$V_1 = V_2 = 5 \text{ liters} = 5 \times 10^{-3} \text{ m}^3$$

2-16 a(i) Diffusivity of NH_3 (A) in H_2O vapor (B) at $T = 373 \text{ K}$, $1 \text{ atm} = 1.013 \text{ bar}$

Use Fuller's equation: $M_A = 17$, $M_B = 18$; $(\sum v)_A = 14.9$, $(\sum v)_B = 12.7$
Eq. (2.57)

$$D_{AB} = \frac{1.0133 \times 10^{-7} (373)^{1.75}}{(1.013) [(14.9)^{1/3} + (12.7)^{1/3}]^2} \cdot \left[\frac{1}{17} + \frac{1}{18} \right]^{1/2} = 4.66 \times 10^{-5} \text{ m}^2/\text{s}$$
Table 2.4

(ii) H_2S (A) and N_2 (B); $T = 298 \text{ K}$, $1 \text{ atm} = 1.013 \text{ bar}$

Table 2.4 $\text{H}_2\text{S} \rightarrow (\sum v)_A = 17.0 + (2)(1.98) = 20.96$; $(\sum v)_B = 17.9$; $M_A = 34$, $M_B = 28$

$$D_{AB} = \frac{1.0133 \times 10^{-7} (298)^{1.75}}{(1.013) [(20.96)^{1/3} + (17.9)^{1/3}]^2} \left[\frac{1}{34} + \frac{1}{28} \right]^{1/2} = 1.9 \times 10^{-5} \text{ m}^2/\text{s}$$

(iii) Ethylene oxide (A) - $\text{Ar}(B)$; $T = 298 \text{ K}$; 1.013 bar

Table 2.4 Ethylene oxide, $\begin{array}{c} \text{CH}_2 \\ | \\ \text{CH}_2 \end{array} > \text{O} \rightarrow (\sum v)_A = (16.5)(2) + (1.98)(4) + 5.48 - 20.2$
 $(\sum v)_B = 20.1$; $M_A = 44$, $M_B = 28.9$ $\begin{array}{l} 2 \text{ carbon} \\ 4 \text{ hydrogen} \\ \text{Oxygen} \uparrow \\ \text{for heterocyclic} \\ \text{ring} \end{array}$

$$D_{AB} = \frac{1.0133 \times 10^{-7} (298)^{1.75}}{(1.013) [(26.2)^{1/3} + (20.1)^{1/3}]^2} \left[\frac{1}{44} + \frac{1}{28.9} \right]^{1/2} = 5.8 \times 10^{-5} \text{ m}^2/\text{s}$$

(b) n -Hexane (A) - Oxygen (B). Use Chapman-Enskog Eq. 2.54

Table 2.2 $T = 323 \text{ K}; 1 \text{ atm}$

n -Hexane: $\alpha_A = 5.949 \text{ \AA}^3$, $\epsilon_A/k = 399.3$

Oxygen: $\alpha_B = 3.467 \text{ \AA}^3$, $\epsilon_B/k = 106.7$

Table 2.3 for $\frac{kT}{\epsilon_{AB}} = \frac{323}{206.4} = 1.565$

By interpolation, collision integral

$$\sigma_D = 1.198 + \frac{1.167 - 1.198}{1.6 - 1.5} \times (1.565 - 1.5) = 1.178$$

Eq. 2.54 $D_{AB} = \frac{1.858 \times 10^{-7} \times (323)^{3/2} \left[\frac{1}{78} + \frac{1}{32} \right]^{1/2}}{(1)(4.709)^2 (1.178)} = 8.67 \times 10^{-6} \text{ m}^2/\text{s}$

Experimental value = $0.0753 \text{ cm}^2/\text{s}$ at 273 K , 1 atm .

At 323 K and $1 \text{ atm} \rightarrow D_{AB} = (0.0753) (323/273)^{3/2} = 0.097 \text{ cm}^2/\text{s}$

[The estimated value matches well with $= 9.7 \times 10^{-6} \text{ m}^2/\text{s}$
the experimental value.]

* * *

2.17 $D_{AB} = 2.1 \times 10^{-9} \text{ m}^2/\text{s}$ (Table 2.5). Integrated form of Eq (2.14) for liquid-phase diffusion is given by Eq (2.59).

Take the density of the solution, $\rho' = \frac{1000 \text{ kg/m}^3}{\text{assumed}}$. Since the solution (O_2 dissolved in water) is very dilute, take its molecular weight $M \approx 18$ — i.e. the same as that of water.

$$\text{Total molar concentration} = \rho/M = 1000/18 = 55.55 \text{ kmol/m}^3$$

Mole fraction of oxygen on two sides of the film:

$$x_{A0} = \frac{0.0144/32}{55.55} = 8.1 \times 10^{-6}; \quad x_{Al} = \frac{0.008/32}{55.55} = 4.5 \times 10^{-6}$$

$$\text{Film thickness, } l = 0.5 \text{ mm} = 5 \times 10^{-4} \text{ m}; \quad x_{Bm} \approx 1$$

$$\text{Flux} = \frac{(2.1 \times 10^{-9})(55.55)}{(5 \times 10^{-4})(1)} (8.1 \times 10^{-6} - 4.5 \times 10^{-6}) = 8.4 \times 10^{-10} \frac{\text{kmol}}{\text{m}^2 \text{s}}$$

Since the solution is very dilute, it is easy to verify that the result remains the same if the bulk flow term in Eq (2.14) is neglected to derive the equation for flux.

2.18 For diffusion of A through non-diffusing B ($N_B=0$),

$$N_A = N_A \cdot \frac{C_A}{C} - D_{AB} \cdot \frac{dC_A}{dz} \Rightarrow N_A \left(\frac{C - C_A}{C} \right) = - (K_1 + K_2 C_A) \cdot \frac{dC_A}{dz}$$

$$\Rightarrow \frac{N_A}{C} \cdot dz = - \frac{(K_1 + K_2 C_A)}{C - C_A} \cdot dC_A = \left(K_2 - \frac{K_1 + K_2 C}{C - C_A} \right) dC_A$$

Integrate from $z=0$, $C_A = C_{A0}$ to $z=l$, $C_A = C_{Al}$

$$\frac{N_A}{C} \int_0^l dz = \int_{C_{A0}}^{C_{Al}} \left[K_2 - \frac{K_1 + K_2 C}{C - C_A} \right] dC_A$$

$$\Rightarrow N_A = \frac{l}{C} \left[(K_1 + K_2 C) \ln \left(\frac{C - C_{Al}}{C - C_{A0}} \right) - K_2 (C_{A0} - C_{Al}) \right]$$

For equimolar countercurrent diffusion ($N_A + N_B = 0$),

$$N_A = - D_{AB} \cdot \frac{dC_A}{dz} = - (K_1 + K_2 C_A) \cdot \frac{dC_A}{dz}. \quad \text{Integrating,}$$

$$\Rightarrow N_A = \frac{1}{l} \left[K_1 (C_{A0} - C_{Al}) + \frac{K_2}{2} (C_{A0}^2 - C_{Al}^2) \right]$$

2.20

2.19 (a) Diffusivity of methanol(A) in water(B); $T=298K$

Association factor for the solvent (water), $\phi=2.26$; $M_B=18$;

μ (for the solution) = $0.9 \text{ CP} = 9 \times 10^{-4} \text{ kg/m.s}$; $\nu_A = 0.0425 \text{ m}^3/\text{kmol}$
 \equiv that for water (see Poling et al.)

Put in Wilke-Chang Eq (2.66).

$$D_{AB} = \frac{(1.173 \times 10^{-16})(2.26 \times 18)^{1/2}(298)}{(9 \times 10^{-4})(0.0425)^{0.6}} = 1.65 \times 10^{-9} \text{ m}^2/\text{s} \quad \begin{array}{l} \text{Experimental value:} \\ = 1.6 \times 10^{-9} \text{ m}^2/\text{s} \\ \text{see Perry, 1997} \end{array}$$

The match with the expt. value is excellent.

(b) Benzene (A) - Ethanol (B) $M_B=46$; $\phi=1.5$ (for ethanol); calculate ν_A for benzene from the Tyn and Clous equation; take $\nu_c = 259 \text{ cm}^3/\text{gmol}$ from Poling (2001). $\nu_A = 0.285(259)^{1.048} = 96.38 \text{ cm}^3/\text{gmol} = 0.09638 \frac{\text{m}^3}{\text{kmol}}$

(Compare with the experimental value of molar volume at normal b.p., $96.5 \frac{\text{cm}^3}{\text{gmol}}$)
 Viscosity of the solution \approx viscosity of the solvent, ethanol, at 25°C

$$D_{AB} = \frac{(1.173 \times 10^{-16})(1.5 \times 46)^{1/2}(298)}{(1.04 \times 10^{-3})(0.09638)^{0.6}} = 1.136 \times 10^{-9} \text{ m}^2/\text{s}$$

(c) Ethylene oxide (A) - Water(B). $M_B=18$, $\phi=2.26$;

$\mu = 9 \times 10^{-4} \text{ kg/m.s}$. Use Tyn and Clous equation to calculate ν_A .
 Critical molar volume of ethylene oxide = $140 \frac{\text{cm}^3}{\text{gmol}}$ (from Poling, 2001)

$$\nu_A = (0.285)(140)^{1.048} = 50.6 \text{ cm}^3/\text{gmol} = 0.0506 \frac{\text{m}^3}{\text{kmol}}$$

$$D_{AB} = \frac{(1.173 \times 10^{-16})(2.26 \times 18)^{1/2}(298)}{(9 \times 10^{-4})(0.0506)^{0.6}} = 1.484 \times 10^{-9} \text{ m}^2/\text{s}$$

2.20 Calculate the cell constant using Eq (2.65b).

$$a = \frac{\pi}{4}(0.028)^2 = 6.157 \times 10^{-4} \text{ m}^2; l = 1.8 \text{ mm} = 0.0018 \text{ m}; V_1 = 4.52 \times 10^{-5} \text{ m}^3;$$

$$V_2 = 4.6 \times 10^{-5} \text{ m}^3; \epsilon = ?; \tau = ?$$

$$\beta = \frac{6.157 \times 10^{-4}}{1.8 \times 10^{-3}} (\epsilon/l) \left(\frac{1}{4.52 \times 10^{-5}} + \frac{1}{4.6 \times 10^{-5}} \right) = 1.5 \times 10^4 (\epsilon/l)$$

Put the data on KCl

diffusion in Eq (2.65a).

$$1.86 \times 10^{-9} = (\epsilon/l) \left(\frac{1}{4.52 \times 10^{-5}} \right) \ln \frac{1.0 - 0}{0.8 - 0.1965} \cdot \frac{1}{t_f}$$

$$\Rightarrow (\epsilon/l) \frac{1}{t_f} = 5.525 \times 10^{-5}$$

(2.21)

2.20 contd...

Note that $C_{2f} = 0.1965$ can be obtained by material balance.

$$V_1 C_{1,0} + V_2 C_{2,0} = V_1 C_{1,f} + V_2 C_{2,f} \Rightarrow (4.52)(1) + (4.6)(0) = (4.52)(0.8) + (4.6)C_{2f}$$

$$\Rightarrow C_{2f} = 0.1965$$

Obtain the methanol concentration at the end of the run to obtain C_{2f} for methanol.

$$(4.52)(1) + (4.6)(0) = (4.52)(0.815) + (4.6) \cdot C_{2f} \Rightarrow 0.1818$$

Diffusivity of methanol, $D_{AB} = (\tau/\epsilon) \left(\frac{1}{1.5 \times 10^4} \right) \frac{\ln \frac{1.0 - 0}{0.815 - 0.1818}}{t_f} \cdot \frac{1}{t_f}$

$$\Rightarrow D_{AB} = \frac{5.525 \times 10^{-5}}{1.5 \times 10^4} \ln \frac{1}{0.6332} = \frac{1.683 \times 10^{-9}}{m^2/s}$$

* * * *

(2.21) Given: $V_1 = 5.02 \times 10^{-5} m^3$; $V_2 = 5.1 \times 10^{-5} m^3$; $a = \frac{\pi}{4}(0.02)^2 = 3.464 \times 10^{-4} m^2$
 $l = 1.8 \times 10^{-3} m$; $\epsilon = 0.35$; D_{AB} of acetic acid = $1.2 \times 10^{-9} m^2/s$; $t_f = 50$ hr.

Calculate the end concentration of compartment 2 (bigger compartment)
 $(50.2)(0) + (51)(0.9) = (50.2)(0.0816) + (51)(C_{Af}) \Rightarrow C_{Af} = 0.8197$ molar.

Put the values in Eq(2.65a).

$$1.2 \times 10^{-9} = \frac{1.8 \times 10^{-3} \cdot \tau}{(3.464 \times 10^{-4})(0.35)(50 \times 3600)} \cdot \ln \frac{0.9 - 0}{0.8197 - 0.0816} \left(\frac{1}{5.02 \times 10^{-5}} + \frac{1}{5.1 \times 10^{-5}} \right)^{-1}$$

$$\Rightarrow \frac{\tau = 2.9}{*} = \text{tortuosity of the pores}$$

(2.22) The reactant A diffuses through a spherical film of thickness δ from the bulk gas towards the surface
 $\Rightarrow p_A$ increases with r , $\frac{dp_A}{dr}$ is +ve.

The Ficks law takes the form:

$$N_A = (N_A + N_B) \frac{p_A}{P} + \frac{D_{AB}}{RT} \cdot \frac{dp_A}{dr} \dots (i)$$

For diffusion through a spherical film at steady state,
 $4\pi r^2 N_A = \text{constant} = W$, say [Eq(2.70)] ... (ii)

(a) Reaction: $A \rightarrow B$. $N_A = -N_B \Rightarrow N_A + N_B = 0$

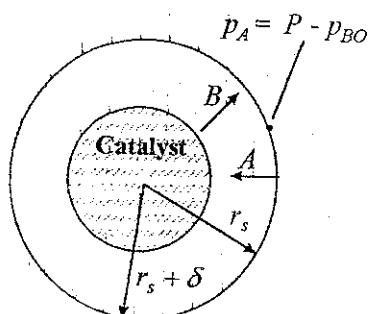


Fig Prob 2.22

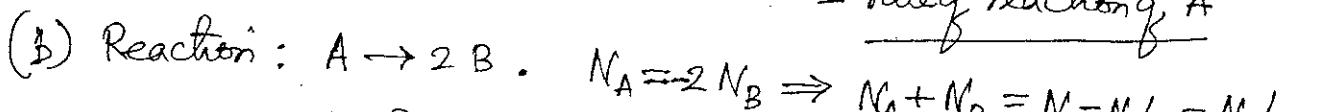
(2.22) contd.. From (i) and (ii), $4\pi r^2 \cdot \left(\frac{D_{AB}}{RT} \cdot \frac{dp_A}{dz} \right) = W$ (2.22)

Boundary conditions : $r = r_s, p_A = 0$; $r = r_s + \delta, p_A = P - p_{B_0}$

Integrate to get $4\pi \cdot \frac{D_{AB}}{RT} \int_{r_s}^{P-p_{B_0}} dp_A = W \cdot \int_{r_s}^{r_s+\delta} \frac{dr}{r^2}$

$$\Rightarrow \frac{4\pi D_{AB}}{RT} (P - p_{B_0}) = W \cdot \left(\frac{1}{r_s} - \frac{1}{r_s + \delta} \right) \Rightarrow W = \frac{4\pi D_{AB}}{RT} \cdot (P - p_{B_0}) \cdot \frac{r_s(r_s + \delta)}{\delta}$$

= rate of reaction A



$$\Rightarrow 4\pi r^2 \left[\frac{2 D_{AB} P}{(2P - p_A) RT} \cdot \frac{dp_A}{dr} \right] = W \Rightarrow \frac{8\pi D_{AB} P}{RT} \int_{r_s}^{P-p_{B_0}} \frac{dp_A}{2P - p_A} = W \cdot \int_{r_s}^{r_s+\delta} \frac{dr}{r^2}$$

$$\Rightarrow \frac{8\pi D_{AB} P}{RT} \ln \frac{2P}{P + p_{B_0}} = W \left(\frac{1}{r_s} - \frac{1}{r_s + \delta} \right) \Rightarrow W = \frac{8\pi D_{AB} P}{RT} \cdot \ln \frac{2P}{P + p_{B_0}} \cdot \frac{r_s(r_s + \delta)}{\delta}$$

* * *

= rate of reaction A

(2.23) The radius of the carbon sphere = r_s at any time t . The film thickness = δ ; outer radius of the film = $r_s + \delta$.

Oxygen ^(A) diffuses towards the sphere and CO_2 ^(B) diffuses away from it; i.e., equimolar countercurrent diffusion occurs through the film.

(a) The initial rate of combustion can be calculated directly from the result of part (a) of the previous problem.

$$\text{Rate of diffusion of oxygen} = W = \frac{4\pi D_{AB}}{RT} (P - p_{B_0}) \cdot \frac{r_s(r_s + \delta)}{\delta}$$

Given : $D_{AB} = 3.4 \text{ cm}^2/\text{s}$; $R = 82.1 \text{ cm}^3 \text{ atm}/\text{K} \cdot \text{mol}$; $T = 1573 \text{ K}$;

$P = 1 \text{ atm}$; $p_{B_0} = 0.2 \text{ atm}$; $r_s = 12/2 = 6 \text{ mm} = 0.6 \text{ cm}$; $\delta = 0.3 \text{ cm}$.

$$W = \frac{(4\pi)(3.4)}{(82.1)(1573)} \cdot (1 - 0.2) \cdot \frac{(0.6)(0.6 + 0.3)}{0.3} = 4.764 \times 10^{-4} \frac{\text{g mol O}_2}{\text{s}}$$

(b) Time for combustion of the sphere to 8 mm size

Assume (i) pseudo-steady state condition; (ii) no residue after combustion (pure carbon).

(2.23)

2.23) contd... Moles carbon in the sphere at any time $= \frac{4}{3}\pi r_s^3 \rho_s \cdot \frac{M}{M}$
The rate of change of size is given by $\rho_s = \text{density}, M = \text{mol. wt.}$

$$-\frac{d}{dt} \left(\frac{4}{3}\pi r_s^3 \rho_s \cdot \frac{M}{M} \right) = \frac{4\pi D_{AB} P_{AO}}{RT} \cdot \frac{r_s(r_s+\delta)}{\delta} \cdot (\text{rate of diffusing } O_2)$$

$$\Rightarrow -4\pi r_s^2 (\rho_s/M) \cdot \frac{dr_s}{dt} = \frac{4\pi D_{AB} P_{AO}}{RT} \cdot \frac{r_s(r_s+\delta)}{\delta} \quad P_{AO} = P - P_{BO}$$

Integrate from $t=0, r_s=r_{s_i}$ to $t=t, r_s=r_{s_f}$.

$$-\int_{r_{s_i}}^{r_{s_f}} \frac{r_s dr_s}{r_s + \delta} = \frac{D_{AB} P_{AO}}{RT \delta} \left(\frac{M}{\rho_s} \right) \cdot \int_0^t dt \quad \begin{aligned} & \text{l.h.s.} = - \int_{r_{s_i}}^{r_{s_f}} \frac{(r_s + \delta) - \delta}{r_s + \delta} dr_s \\ & = - \left[r_s - \delta \ln(r_s + \delta) \right]_{r_{s_i}}^{r_{s_f}} \end{aligned}$$

$$\Rightarrow t = \frac{RT \delta}{D_{AB} P_{AO}} \left(\frac{\rho_s}{M} \right) \cdot \left[(r_{s_i} - r_{s_f}) - \delta \ln \left(\frac{r_{s_i} + \delta}{r_{s_f} + \delta} \right) \right]$$

Put $\rho_s = 0.9 \text{ gm/cm}^3; M = 12; P_{AO} = 1 - 0.2 = 0.8 \text{ atm}; r_{s_i} = 0.6 \text{ cm}; r_{s_f} = 0.4 \text{ cm}$

$$t = \frac{(82.1)(1573)(0.3)}{(3.4)(0.8)} \cdot \left(\frac{0.9}{12} \right) \cdot \left[(0.6 - 0.4) - (0.3) \ln \frac{0.6 + 0.3}{0.4 + 0.3} \right] = \underline{\underline{133 \text{ second}}}$$

(2.24) Naphthalene (A) diffuses through non-diffusing air (B).

$$\Rightarrow N_A = \frac{D_{AB} P}{RT(P-P_A)} \cdot \frac{dp_A}{dr} \quad [\text{see Eq}(2.72)] \cdot \text{Now use } 4\pi r^2 N_A = \text{constant} = W$$

Put the expression for N_A and integrate from $r=r_s, p_A=p_{AS}$

$$-\frac{4\pi D_{AB} P}{RT} \int_{P_{AS}}^0 \frac{dp_A}{P-p_A} = W \int_{r_s}^{r_s+\delta} \frac{dr}{r^2} = W \left(\frac{1}{r_s} - \frac{1}{r_s + \delta} \right) \quad r = r_s + \delta, p_A = 0$$

$$\Rightarrow W = \frac{4\pi D_{AB} P}{RT \delta} \cdot r_s(r_s + \delta) \cdot \ln \frac{P}{P - p_{AS}}$$

Following the procedure adopted in the previous problem, the time of reducing the radius from r_{s_i} to r_{s_f} is given by

$$(r_{s_i} - r_{s_f}) - \delta \ln \frac{r_{s_i} + \delta}{r_{s_f} + \delta} = \frac{D_{AB} \cdot P}{RT \delta} \cdot \left(\frac{M_A}{\rho_s} \right) \cdot \ln \frac{P}{P - p_{AS}} \quad \begin{aligned} & r_{s_i} = 0.75 \text{ cm} \\ & \rho_s = 1.14 \frac{\text{gm}}{\text{cm}^3} \end{aligned}$$

Initial mass of the ball $= \frac{4}{3}\pi r_{s_i}^3 \rho_s = \frac{4\pi}{3} (1.5/2)^3 (1.14) = 2.014 \text{ gm}$

80% sublimes \Rightarrow remaining mass $= (2.014)(0.2) = 0.403 \text{ gm}$

$$\Rightarrow \frac{4}{3}\pi r_{s_f}^3 \cdot (1.14) = 0.403 \Rightarrow r_{s_f} = 0.4386 \text{ cm}; \delta = 0.4 \text{ cm}$$

$$D_{AB} = 0.0892 \text{ cm}^2/\text{s}; P_{AS} = 0.4303 \text{ mm Hg.} = 5.662 \times 10^{-4} \text{ atm.}; P = 1 \text{ atm.}$$

$$T = 308 \text{ K}, M_A = 128 \text{ (C}_1\text{O}_2\text{H}_8\text{)}$$

2.24

(2.24) Contd... Put the values in the above equation.

$$(0.75 - 0.4386) - (0.4) \ln \frac{0.75 + 0.4}{0.4386 + 0.4} = \frac{(0.0892)(1)}{(82.1)(308)(0.4)} \cdot \frac{(128)}{1.14} \left(\ln \frac{1}{1 - 5.662 \times 10^{-4}} \right) t$$

Time of sublimation, $t = 3.3 \times 10^5 s = 91.7 \text{ hr}$

(2.25) Temperature of the drop = 18°C , ambient temperature = 30°C .

Mean air film temperature = $(18+30)/2 = 24^\circ\text{C} = 297\text{K}$ is used in

Diffusivity at $T=297$ is $D_{AB} = (0.102) \left(\frac{297}{273} \right)^{1.5} = 0.116 \text{ cm}^2/\text{s}$ [$D_{AB} = 0.102 \frac{\text{cm}^2}{\text{s}}$ calculation at 273K]

Use the equation derived in the previous problem to calculate the time of evaporation from 2 mm to 1 mm diameter.

$$r_{Si} = 1 \text{ mm} = 0.1 \text{ cm}; r_{Sf} = 0.5 \text{ mm} = 0.05 \text{ cm}; \delta = 0.15 \text{ cm}; P = 1 \text{ atm};$$

P_{AB} = vapor pressure of ethanol at $18^\circ\text{C} = 0.0516 \text{ bar} = 0.051 \text{ atm}$;

$M_A = 46$; $P_e = 0.789 \text{ gm/cm}^3$. Put the values in the working equation.

$$(0.1 - 0.05) - 0.15 \ln \frac{0.1 + 0.15}{0.05 + 0.15} = \frac{(0.116)(1)}{(82.1)(297)(0.15)} \cdot \frac{46}{0.789} \cdot \ln \frac{1}{1 - 0.051} \cdot t$$

$t = 171 \text{ second}$

To calculate the time of disappearance of the drop in stagnant air, use Eq.(2.77) directly.

$$t = \frac{(82.1)(297)(0.789)(0.1)^2}{(2)(0.116)(1)(46) \cdot \ln \frac{1}{1 - 0.051}} = \underline{\underline{345 \text{ second}}} \\ * * *$$

(2.26) The situation corresponds to diffusion of toluene (A) through nondiffusing nitrogen (B), and the gas film is cylindrical.

Outer diameter of the gas film (= inner dia of the tube), $2r_o = 15.8 \text{ mm}$;

inner diameter of the film, $2r_i = 15.8 - (2 \times 3 \text{ mm}) = 9.8 \text{ mm}$

For steady-state diffusion of A through a cylindrical film, the following flux relation is applicable [see Example 2.15].

$$(2\pi r L) N_A = (2\pi r L) \left[- \frac{D_{AB} P}{RT(P - P_A)} \right] \cdot \frac{dp_A}{dr} = W, \text{ constant.}$$

L = length of a section of the tube.

(2.25)

Boundary conditions : at $r=r_0$, $p=p_{AS}$ = vapor pressure of toluene at the temperature of the condensate liquid film ; at $r=r_i$, $p=p_{Ab}$ = partial press. of toluene in the bulk gas.

Integrate the above equation.

$$-\frac{2\pi L D_{AB} P}{RT} \int_{p_{Ab}}^{p_{AS}} \frac{dp_A}{P-p_A} = W \cdot \int_{r_i}^{r_0} \frac{dr}{r}$$

$$\Rightarrow W \cdot \ln\left(\frac{r_0}{r_i}\right) = \frac{2\pi L D_{AB} P}{RT} \cdot \ln \frac{P-p_{AS}}{P-p_{Ab}} \dots (i)$$

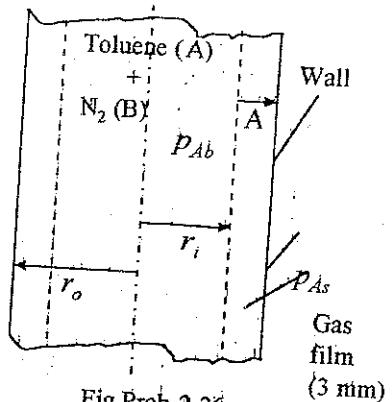


Fig Prob 2.26

The rate W of diffusion of toluene (= the rate of condensation) to the condensate surface over a length L of the tube can be calculated from the above equation.

Calculate D_{AB} using the Fuller's equation. Take the mean gas-film temp., $T = (15+6.5)/2 = 10.75^\circ C = 284 K$

$$(\sum v)_A = 111.14; (\sum v)_B = 17.9; P = 1.2 \text{ bar}; M_A = 92; M_B = 28.$$

$$\text{Put the values in Eq. (2.57). } D_{AB} = \frac{1.0133 \times 10^7 (284)^{1.75}}{(1.2)[(111.14)^{1/3} + (17.9)^{1/3}]^2} \cdot \left[\frac{1}{92} + \frac{1}{28} \right]^{1/2} = 6.4 \times 10^{-6} \frac{m^2}{s}$$

$$\text{Other quantities: } p_{AS} = 10 \text{ mm Hg} = 10/760 = 0.01316 \text{ atm}; p_{Ab} = 25 \text{ mm Hg} \\ = 0.0329 \text{ atm}; r_i = 9.8/2 \text{ mm} = 4.9 \times 10^{-3} \text{ m}; r_0 = 15.8/2 \text{ mm} = 7.9 \times 10^{-3} \text{ m.}$$

$$\text{Take } L = 1 \text{ m} \Rightarrow W \cdot \ln\left(\frac{r_0}{r_i}\right) = \frac{(2\pi)(1)(6.4 \times 10^{-6})(1.2)}{(0.08317)(284)} \ln \frac{1.2 - 0.01316}{1.2 - 0.0329}$$

$$\Rightarrow \text{Rate of condensation, } W = 7.17 \times 10^{-8} \text{ kmol/s}$$

(2.27) Benzene (A) diffuses through non-diffusing air (B) in the empty space above the liquid in the tank. The area available for diffusion is a function of z (the vertical distance above the liquid level).

Area of diffusion (see Figure)

$$= \pi r^2 = \pi(r_0^2 - z^2) \Rightarrow r = \sqrt{r_0^2 - z^2}$$

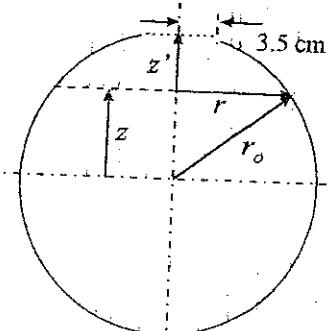


Fig Prob 2.27

(2.28) Contd...

(2.28)

For diffusion through the variable area, $\pi r^2 N_A = \text{constant} = W$

$$\Rightarrow \pi r^2 \left(-\frac{D_{AB} P}{RT} \right) \frac{1}{P - P_A} \frac{dp_A}{dz} = \pi r^2 N_A = W$$

Integrate from the surface of the liquid, $z=0, p_A = p_{AS}$, to the opening at the top, $z=z'$, $p_A = 0$ (bulk air outside the tank is benzene-free).

$$W \cdot \int_{z=0}^{z'} \frac{dz}{r^2} = - \frac{\pi D_{AB} P}{RT} \int_{p_{AS}}^0 \frac{dp_A}{P - p_A} \Rightarrow W \cdot \int_{z=0}^{z'} \frac{dz}{r_0^2 - z^2} = - \frac{\pi D_{AB} P}{RT} \ln \frac{P}{P - p_{AS}}$$

$$\text{The integral on the left} = \frac{1}{2r_0} \left[\ln \frac{r_0 + z}{r_0 - z} \right]_{z=0}^{z'} = \frac{1}{2r_0} \ln \frac{r_0 + z'}{r_0 - z'}$$

$$\Rightarrow \text{Rate of loss of benzene vapor}, W = - \frac{\pi D_{AB} P}{RT} \ln \frac{P}{P - p_{AS}} \cdot \left[\ln \frac{r_0 + z'}{r_0 - z'} \right]^{-1} \cdot 2r_0$$

$$\text{Given: } D_{AB} = 9 \times 10^{-6} \text{ m}^2/\text{s}; P = 1 \text{ atm}; T = 298 \text{ K}$$

$$p_{AS} = \frac{0.127}{1.013} = 0.1254 \text{ atm. Radius of the opening} = \frac{70}{2} \text{ mm} = 3.5 \times 10^{-2} \text{ m}$$

$$W = - \frac{\pi \cdot (9 \times 10^{-6})(1)}{(0.0821)(298)} \cdot \ln \frac{1}{1 - 0.1254} \cdot \left[\ln \frac{0.4 + 0.3985}{0.4 - 0.3985} \right]^{-1} \cdot (2)(0.4)$$

$$= 1.974 \times 10^{-8} \text{ kmol/s}$$

* * * *

(2.29) Let V = volume of the gas space in vessel 1; Q_V = flowrate of nitrogen; P_A = partial pressure of ethanol vapor in the gas space at any time t . Assume pseudo-steady state condition. The time rate of change of P_A in the vessel is given by

$$-\frac{V}{RT} \frac{dp_A}{dt} = \frac{Q_V p_A}{RT} - a N_A \quad \begin{cases} a = \text{area of cross-section of vessel 1.} \\ N_A = \text{ethanol flux through the gas film} \end{cases}$$

$$p_{AS} = \text{partial pressure of ethanol at the liquid surface} = \text{vap. pressure} \quad \begin{cases} = \frac{D_{AB} P}{RT \delta} \cdot \frac{p_{AS} - p_A}{p_{BM}} \quad \text{of thickness } \delta. \end{cases}$$

Since p_{AS} is rather small (0.058 bar compared to the total pressure), we assume $p_{BM} \approx P$. This will greatly simplify the theoretical analysis.

$$\Rightarrow -\frac{V}{RT} \frac{dp_A}{dt} = \frac{Q_V p_A}{RT} - \frac{a \cdot D_{AB}}{RT \delta} \cdot (p_{AS} - p_A)$$

$$\Rightarrow -V \frac{dp_A}{dt} = (Q_V + \frac{a D_{AB}}{\delta}) p_A - \frac{a D_{AB}}{\delta} \cdot p_{AS}$$

| Integrate from
 $t=0, p_A = p_{AS}$ to
 $t=t_f, p_A = p_{Af}$

$$2.29 \text{ contd...} \quad \frac{V}{(Q_v + \alpha D_{AB}/\delta)} \ln \left[\left(Q_v + \frac{\alpha D_{AB}}{\delta} \right) p_A - \frac{\alpha D_{AB}}{\delta} \cdot p_{AS} \right] = t_f \quad 2.27$$

$$\Rightarrow t_f = \frac{V}{Q_v + (\alpha D_{AB}/\delta)} \ln \frac{Q_v p_{AS}}{(Q_v + \alpha D_{AB}/\delta) p_{AS} - \alpha D_{AB} p_{AS}/\delta}$$

Given : $V = 3 \times 10^{-3} \text{ m}^3$; $Q_v = 4 \times 10^{-3}/60 = 6.667 \times 10^{-5} \text{ m}^3/\text{s}$; $D_{AB} = 1.02 \times 10^{-5} \text{ m}^2/\text{s}$
 $p_{AS} = 0.058 \text{ bar}$; $p_{Af} = 0.045 \text{ bar}$; $\alpha = \frac{\pi}{4} (0.16)^2 = 0.0201 \text{ m}^2$; $\delta = 2.5 \times 10^{-3} \text{ m}$.

$$\Rightarrow t_f = \frac{3 \times 10^{-3}}{6.667 \times 10^{-5} + (0.0201)(1.02 \times 10^{-5}) / 2.5 \times 10^{-3}} \cdot \ln \frac{(6.667 \times 10^{-5})(0.058)}{\left[6.667 \times 10^{-5} + \frac{(0.0201)(1.02 \times 10^{-5})}{2.5 \times 10^{-3}} \right] (0.045)} - \frac{(0.0201)(1.02 \times 10^{-5})(0.058)}{2.5 \times 10^{-3}}$$

$$\Rightarrow t_f = 14 \text{ second}$$

(ii) Calculation of the steady state concentration in the vessel, p_A

The rate of removal of ethanol with the nitrogen stream
= rate of vaporization of ethanol

$$\Rightarrow Q_v \cdot p_A / RT = \alpha N_A = \frac{\alpha \cdot D_{AB} P}{RT \delta} \cdot \ln \frac{P - p_A}{P - p_{AS}}$$

$$\Rightarrow (6.667 \times 10^{-5}) \cdot p_A = \frac{(0.0201)(1.02 \times 10^{-5})(1.013)}{2.5 \times 10^{-3}} \cdot \ln \frac{1.013 - p_A}{1.013 - 0.058}$$

Solving the above equation, $p_A = 0.0325 \text{ bar}$

30 Det volume of a bulb = $V \text{ m}^3$; flow rate of gas through bulb 1 = $Q_v \text{ m}^3/\text{s}$; partial pressure of NH_3 in this gas = p'_A ; partial pressures of NH_3 in the bulbs at any time $t \rightarrow p_{A1}$ and p_{A2} . Assume pseudo-steady state. Rate of transport of NH_3 from bulb 2 to bulb 1 through the connecting tube (cross-section = $a \text{ m}^2$), $N_A = \frac{\alpha D_{AB}}{RTl} \cdot (p_{A2} - p_{A1})$; l = tube length.

'Unsteady state' material balance:

$$\text{Bulb 1 : } \frac{V}{RT} \cdot \frac{dp_{A1}}{dt} = \frac{Q_v}{RT} (p'_A - p_{A1}) + \frac{\alpha D_{AB}}{RTl} \cdot (p_{A2} - p_{A1}) \quad \dots \dots (i)$$

$$\text{Bulb 2 : } -\frac{V}{RT} \cdot \frac{dp_{A2}}{dt} = \frac{\alpha D_{AB}}{RTl} \cdot (p_{A2} - p_{A1}) \quad \dots \dots (ii)$$

$$\text{Eq (i)} \rightarrow \frac{dp_{A1}}{dt} = \beta' p'_A + \beta p_{A2} - (\beta + \beta') p_{A1}; \text{ Eq (ii)} \rightarrow \frac{dp_{A2}}{dt} = -\beta (p_{A2} - p_{A1})$$

$$2.30) \text{ contd... } \beta' = Q_r/V; \beta = a_{AB}/V \cdot l.$$

(2.28)

Given: $V = 1 \text{ liter} = 10^{-3} \text{ m}^3$; $Q_r = 2 \text{ liters/l} = 5.555 \times 10^{-7} \text{ m}^3/\text{s}$; $l = 0.03 \text{ m}$
 $a = \pi(0.01)^2 = 3.1416 \times 10^{-4} \text{ m}^2$; $D_{AB} = 2.3 \times 10^{-5} \text{ m}^2/\text{s}$; $T = 298 \text{ K}$; $p_A' = 0.1013 \text{ bar}$
 $\Rightarrow \beta' = 5.555 \times 10^{-4}$; $\beta = \frac{(3.1416 \times 10^{-4})(2.3 \times 10^{-5})}{(10^{-3})(3 \times 10^{-2})} = 2.41 \times 10^{-4}$

Putting the values in the above two equations for p_{A_1} and p_{A_2} ,

$$\frac{dp_{A_1}}{dt} = 5.627 \times 10^{-5} + 2.41 \times 10^{-4} p_{A_2} - 7.965 \times 10^{-4} p_{A_1} \quad \dots \quad (\text{iii})$$

$$\frac{dp_{A_2}}{dt} = -2.41 \times 10^{-4} (p_{A_2} - p_{A_1}) \quad \dots \quad (\text{iv})$$

The above two coupled first order linear ordinary differential equations have to be solved for $t=0$, $p_{A_1}=0$ and $p_{A_2}=0.05$ (0.013)

Take the L-transform of eq (iii)

$$\int_0^\infty \frac{dp_{A_1}}{dt} e^{-st} dt = 5.627 \times 10^{-5} \int_0^\infty e^{-st} dt + 2.41 \times 10^{-4} \int_0^\infty p_{A_2} e^{-st} dt - 7.965 \times 10^{-4} \int_0^\infty p_{A_1} e^{-st} dt$$

$$\Rightarrow s\bar{p}_{A_1} = \frac{5.627 \times 10^{-5}}{s} + 2.41 \times 10^{-4} \bar{p}_{A_2} - 7.965 \times 10^{-4} \bar{p}_{A_1} \quad \text{and } \bar{p}_{A_2}$$

Similarly, taking L-transform of eq (iv), | are the L-transforms

$$s\bar{p}_{A_2} = -2.41 \times 10^{-4} (\bar{p}_{A_2} - \bar{p}_{A_1}) \quad \dots \quad (\text{v}) \quad \text{of } p_{A_1} \text{ and } p_{A_2}$$

Solving the above two algebraic equations (v) and (vi),

$$\bar{p}_{A_2} = \frac{0.1013}{s} + \frac{0.027}{(s+8.864 \times 10^{-4})} - \frac{0.07768}{(s+1.51 \times 10^{-4})} \quad \text{Expressed as partial fractions}$$

Taking the inverse transform,

$$p_{A_2}(t) = 0.1013 + 0.027 \exp(-8.864 \times 10^{-4} t) - 0.07768 \exp(-1.51 \times 10^{-4} t)$$

$$\text{From Eq (ii), } p_{A_1}(t) = 0.1013 - 0.0723 \exp(-8.864 \times 10^{-4} t) - 0.02901 \exp(-1.51 \times 10^{-4} t)$$

These will be no flux of NH_3 when $p_{A_1} = p_{A_2}$.

Equating the above two solutions, $t = 969 \text{ second}$ when $p_{A_1} = p_{A_2} = 0.0456$

The partial pressure of $\text{NH}_3(A)$ increases in bulb 1 because of

(i) Supply of NH_3 with the gas stream passing through it,

(2.30) const... and (ii) diffusion of NH_3 from bulb 2 to 1. When p_{A1} and p_{A2} become equal, the flux through the connecting tube is zero.

Since p_{A1} continues to increase even after that (because the incoming gas stream has a higher concentration of NH_3 in it), transport of NH_3 (A) now occurs from bulb 1 to bulb 2 (i.e. in the reverse direction). Eventually, the partial pressures of NH_3 in both the bulbs become equal again. (0.1013 atm) which is the partial pressure of NH_3 in the flowing gas stream. This can be checked by putting $t = \infty$ in $p_{A1}(t)$ and $p_{A2}(t)$.

* * * *

(2.31) This is a case of non-equimolar countercurrent diffusion through a gas-film. Reaction: $\text{Ni} + 4\text{CO} \rightarrow \text{Ni}(\text{CO})_4 \Rightarrow N_A = -4N_B$.

$$\text{Put in Eq. (2.19) to get } N_A = (N_A - \frac{1}{4}N_B) \frac{p_A}{P} - \frac{D_{AB}}{RT} \cdot \frac{dp_A}{dz}$$

$$\Rightarrow N_A \int_0^{\delta} dz = - \frac{4D_{AB}P}{RT} \int_{p_{AO}}^{p_{AS}} \frac{dp_A}{4P - 3p_A} \Rightarrow N_A = \frac{4D_{AB}P}{3RT\delta} \ln \frac{4P - 3p_{AS}}{4P - 3p_{AO}}$$

$$\text{Given: } \delta = 2 \text{ mm} = 2 \times 10^{-3} \text{ m; } P = 1 \text{ atm; } D_{AB} = 1.91 \times 10^{-5} \text{ m}^2/\text{s; } p_{AO} = \text{bulk}$$

concentration of CO = 0.6 atm; $p_{AS} = 0$ (since CO reacts instantaneously at the nickel surface).

$$\begin{aligned} \text{Put the values to get, } N_A &= \frac{(4)(1.91 \times 10^{-5})(1)}{(3)(0.0821)(313)(2 \times 10^{-3})} \cdot \ln \frac{4-0}{4-(3)(0.6)} \\ &= 2.962 \times 10^{-4} \text{ kmol/m}^2\text{s} \end{aligned}$$

$$\text{Rate of forming the carbonyl} = N_A/4 = \frac{7.406 \times 10^{-5}}{* * *} \text{ kmol/m}^2\text{s}$$

(2.32) Example 2.6 deals with a situation in which equimolar countercurrent diffusion occurs.

(i) Total mass flux, $N_A + N_B = 0$ if the observer is stationary.

(ii) Flux of oxygen (B) molecules = $N_B = -N_A = -3.36 \times 10^{-6} \text{ kmol/m}^2\text{s}$.

\Rightarrow Flux of oxygen = $3.36 \times 10^{-6} \text{ kmol/m}^2\text{s}$ from vessel 2 to vessel 1.

Midway in the tube, $p_B = (0.8 + 0.2)(2 \text{ atm}) = 1 \text{ atm}; T = 293 \text{ K}$

$$\text{and } C_B = \frac{p_B}{RT} = \frac{1}{(0.0821)(293)} = 0.04157 \text{ kmol/m}^3$$

(2.30)

$$(2.32) \text{ contd. Flux of } B, N_B = k_B C_B \Rightarrow N_B = \frac{N_B}{C_B} = \frac{3.36 \times 10^{-6}}{0.04157}$$

So an observer midway between the vessels moves at a velocity 0.291 m/s towards vessel 1, he finds oxygen molecules non-diffusing.

(2.33) Following Eq (2.52), the time for fall in level of the liquid in the pores from a depth z_1 to a depth z_2 is given by

$$t' = \frac{RT p_{BM} P_A (z_1^2 - z_2^2)}{2D_{AB} P M_A (P_{A1} - P_{A2}) \cdot \epsilon}$$

Given: $p_{BM} \approx P$, since the vapor pressure of diesel is small.

The quantity ϵ is incorporated since a fraction ϵ (porosity of the soil) of the area is available for diffusion. All other quantities have usual significances.

$$T = 298 \text{ K}; P_A = \text{mass of diesel per unit volume of soil} = 143 \text{ kg/m}^3;$$

$$D_{AB} = 6.28 \times 10^{-6} \text{ m}^2/\text{s}; M_A = 200; P_{A1} = \text{vapor pressure of diesel} = 0.268 \text{ mm Hg} = 3.526 \times 10^{-4} \text{ atm}; P_{A2} = \text{partial pressure of diesel in the air at the open surface of soil} = 0; z_1 = 100 \text{ mm} = 0.1 \text{ m}; z_2 = 0; \epsilon (\text{porosity of diesel}) = 0.4.$$

$$\Rightarrow t' = \frac{(0.082)(298)(P)(143)(0.1)^2}{(2)(6.28 \times 10^{-6})(P)(200)(3.526 \times 10^{-4})(0.4)} = 9.875 \times 10^7 \text{ second}$$

$$= 2.743 \times 10^4 \text{ hr.} = 1143 \text{ days}$$

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CHAPTER 3: MASS TRANSFER COEFFICIENTS

Short and MCQ's

3.1

② Relation between k_Y and k'_Y :

$N_A = k_Y (Y_{A1} - Y_{A2})$, for diffusion of A through non-diffusing B

$= k'_Y (Y_{A1} - Y_{A2})$, when equimolar countercdiffusion of A and B occurs.

Refer to Example 3.1. $k_Y = \frac{D_{AB} P_{B1} P_{B2}}{RT \delta P_{BM}}$. Note that $Y_{A1} - Y_{A2} = \frac{P(P_{A1} - P_{A2})}{P_{B1} P_{B2}}$

For equimolar countercdiffusion, $N_A = \frac{D_{AB}}{RT \delta} (P_{A1} - P_{A2})$

$$\Rightarrow N_A = \frac{D_{AB}}{RT \delta} \cdot \frac{P_{B1} P_{B2}}{P} \cdot (Y_{A1} - Y_{A2}) = k'_Y (Y_{A1} - Y_{A2})$$

$$\Rightarrow \frac{k_Y}{k'_Y} = \frac{D_{AB} P_{B1} P_{B2} / RT \delta P_{BM}}{D_{AB} P_{B1} P_{B2} / RT \delta P} = \frac{P_{BM}}{P}. \text{ This is the relation.}$$

④ From "shell balance" for diffusion in a spherical geometry at steady state [see Eq. (2.70)] we may write

$$4\pi r^2 N_A = \text{constant} = W. \text{ At 'low solute concentration', } N_A = -D_{AB} \frac{dC_A}{dr}$$

$$\Rightarrow 4\pi r^2 \left(-D_{AB} \frac{dC_A}{dr} \right) = W \quad [\text{see Eq. (2.17)}]$$

Integrate from the surface of the sphere where $r = r_s$ and $C_A = C_{AS}$ to $r = \infty$, $C_A = 0$ to get

$$-\int_{r=r_s}^{\infty} \frac{W \cdot dr}{r^2} = \int_{C_{AS}}^0 4\pi D_{AB} dC_A \Rightarrow \frac{W}{r_s} = 4\pi D_{AB} \cdot C_{AS} \Rightarrow W = 4\pi D_{AB} C_{AS} \cdot r_s \quad \dots (i)$$

Since $W = \text{rate of mass transfer from the sphere}$, at $r = r_s$ we can write $W = 4\pi r_s^2 N_A|_{r_s} = 4\pi r_s^2 \cdot k_L (C_{AS} - 0)$ $\boxed{N_A|_{r_s} = k_L (C_{AS} - 0)}$

Equating (i) and (ii), $4\pi r_s^2 k_L (C_{AS}) = 4\pi D_{AB} C_{AS} r_s \quad \dots (ii)$

$$\Rightarrow \frac{k_L r_s}{D_{AB}} = 1 \Rightarrow \frac{k_L d_p}{D_{AB}} = 2, \text{ i.e. } Sh = 2 \quad \text{where the characteristic length is the diameter, } d_p.$$

⑥ See chapter 16

Q. ① The expression for k_Y in the case of diffusion of A through non-diffusing B has a term $\frac{P_{BM}}{P}$ in the denominator. P_{BM} is obviously a concentration term. Thus k_Y depends upon concentration according to the film theory.

- ⑦ If $C_A = C_A(z)$ is the true concentration profile then molar flux at the interface (at the interface, $z=0$) is $N_A = -D_A \left[\frac{dC_A}{dz} \right]_{z=0}$.
 But, according to the film theory, $N_A = \frac{D_A}{s} (C_{As} - C_{Ab})$ where
 C_{As} = Solute concentration at the interface, and C_{Ab} = that in the bulk.
 $\Rightarrow -D_A \left[\frac{dC_A}{dz} \right]_{z=0} = D_A \cdot \frac{C_{As} - C_{Ab}}{s} \Rightarrow \left[\frac{dC_A}{dz} \right]_{z=0} = \frac{C_{As} - C_{Ab}}{s}$
 \Rightarrow the true concentration profile at the interface = the concentration profile in the film.

⑨ Colburn-Drew mass transfer coefficient, $k'_y = \frac{N_{A1}}{\Delta y}$
 $\Rightarrow k'_y = \frac{5 \times 10^{-3}}{0.3 - 0.05} = 2 \times 10^{-2} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)} ; k'_y = \frac{D_{AB}}{s} \cdot \frac{P}{RT}$

Flux for diffusion of A through non-diffusing B

$$N_{A2} = \frac{D_{AB} P}{RT s Y_{BM}} \cdot (\Delta y) = \frac{k'_y \cdot \Delta y}{Y_{BM}} \quad \begin{cases} \Delta y = 0.2 - 0.03 = 0.17 \\ Y_{BM} = \frac{0.97 - 0.8}{\ln(0.97/0.8)} = 0.8823 \end{cases}$$

$$\Rightarrow N_{A2} = \frac{2 \times 10^{-2}}{0.8823} \cdot (0.17) = \underline{3.853 \times 10^{-3} \text{ kmol/m}^2 \cdot \text{s}}$$

- ⑩ (a) \rightarrow (i) diffusivity, (ii) hydrodynamics, (iii) viscosity; (10b) \rightarrow (ii);
 (10c) \rightarrow (i) increase; (10d) \rightarrow (i) increase; (10e) \rightarrow (i) $St = \frac{Sh}{Re \cdot Sc}$;
 (10f) \rightarrow (i) $Pe = Re \cdot Sc$; (10g) \rightarrow (ii) $jd = \frac{Sh}{Re \cdot Sc} \sqrt{3}$; (10h) \rightarrow (ii) increases;
 (10i) \rightarrow (i) $0.45 \text{ kmol}/\text{ft}^2 \cdot \text{atm} \cdot \text{h}$ (Other two values are either too small or too large); (10j) \rightarrow (ii) 0.665 and (iii) 650; (10k) \rightarrow (iii);
 (10l) \rightarrow (iii); (10m) \rightarrow (ii) penetration theory; (10n) \rightarrow (ii) less;
 (10o) \rightarrow (i) greater than 1; (10p) \rightarrow (iii) hr^{-1} ; (10q) \rightarrow (ii) decreases;
 (10r) $\rightarrow k_L = D/8 = 1.1 \times 10^{-9} / 2 \times 10^{-5} = 0.55 \times 10^{-4} \text{ m/s} = \sqrt{Ds} \Rightarrow s = 2.75 \bar{s}^2$ (iii);
 (10s) \rightarrow Life of individual liquid elements = $d/u = 0.5 \text{ cm} / 2 \text{ cm/s} = 0.025 \text{ s}$
 \rightarrow Average mass transfer coefficient, $k_L = 2 \sqrt{\frac{D}{\pi t_c}} = 2 \left[\frac{3.14 \times 10^{-9}}{\pi \cdot (0.025)} \right]^{1/2} = 4 \times 10^{-4} \text{ m/s}$
 (10t) $\rightarrow t_c = d/u \Rightarrow t_{c1} = \frac{1.0 \text{ cm}}{25 \text{ cm/s}} = 0.04 \text{ s}; t_{c2} = \frac{0.5 \text{ cm}}{10 \text{ cm/s}} = 0.05 \text{ s} \quad \boxed{= 0.04 \text{ cm/s}}$
 $\Rightarrow k_{L2} = k_{L1} (t_{c1}/t_{c2})^{1/2} = (0.013)(0.04/0.05)^{1/2} = 0.0116 \text{ cm/s}$, (ii);
 (10u) \rightarrow (i) 0.005, since H_2S has a slightly smaller diffusivity;

- (10v) \rightarrow Since $k_L \propto (L)^{-\frac{1}{2}}$, for a plate having twice the length, $k_L = \frac{0.00705}{\sqrt{2}}$ (3.3)
- (10w) $\rightarrow j_D = j_H \Rightarrow (St)_M = (St)_H (Le)^{-\frac{1}{2}} = 0.00425$, (iii); $\boxed{= 0.005 \text{ cm/s}}$, (iii);
- (10x) \rightarrow (ii) $S_c = 1$; (10y) $\rightarrow k_L = (0.012)(0.4/0.2)^{0.5} = 0.017 \text{ cm/s}$, (iii).

PROBLEMS

- (3.1) Here NH_3 (A) diffuses through non-diffusing N_2 (B). Use Eq. (2.26) to calculate the flux of NH_3 .

$$N_A = \frac{D_{AB}P}{RTl} \cdot \ln \frac{P - P_{AO}}{P - P_{AL}} \quad \left| \begin{array}{l} \text{Given: } D_{AB} = 2.3 \times 10^{-5} \text{ m}^2/\text{s}; P = 1 \text{ atm}; T = 298 \text{ K}; \\ l = 1 \text{ mm} = 10^{-3} \text{ m}; P_{AO} = 40 \text{ mm Hg} = 40/760 \\ = 0.05263 \text{ atm}; P_{AL} = 0 \text{ at the interface.} \end{array} \right.$$

$$\frac{(2.3 \times 10^{-5})(1)}{(0.0821)(298)(10^{-3})} \ln \frac{1 - 0}{1 - 0.05263} = 5.083 \times 10^{-5} \text{ kmol/m}^2 \cdot \text{s} = \text{Absorption flux of NH}_3.$$

Calculation of mass transfer coefficients

$$\text{Use Eq. (3.3). } k_G = \frac{N_A}{P_{AO} - P_{AL}} = \frac{5.083 \times 10^{-5}}{0.05263 - 1} = 9.658 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{atm} \cdot \text{s}}$$

$$k_y = \frac{N_A}{y_{AO} - y_{AL}} = \frac{5.083 \times 10^{-5}}{0.05263 - 1} = 9.658 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{atm} \cdot \text{s}} \quad \left[\begin{array}{l} \text{Since } P = 1 \text{ atm} \\ y_A \text{ and } P_A \text{ are numerically equal} \end{array} \right]$$

$$k_C = \frac{N_A}{C_{AO} - C_{AL}} = \frac{5.083 \times 10^{-5}}{(P_{AO}/RT) - 0} = \frac{(5.083 \times 10^{-5})(0.0821)(298)}{0.05263} = 0.02363 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\frac{\text{kmol}}{\text{m}^3})}$$

Calculation of F_G

$$\text{Use Eq. (3.14). } F_G = k_G P_{BM} = (9.658 \times 10^{-4})(0.9734) \quad \left| \begin{array}{l} P_{BM} = \frac{1 - (1 - 0.05263)}{\ln \frac{1}{1 - 0.05263}} \\ = 0.9734 \text{ atm} \end{array} \right.$$

The mass transfer coefficients, k_G , k_y and k_C can be calculated directly also using Eq. (3.5).

- (3.2) Use the conversion relation given at the bottom of Table 3.1.

$$P = 1.2 \text{ bar}; T = 35^\circ\text{C} = 308 \text{ K}; y_b = 0.075 \text{ (7.5% benzene in the bulk);}$$

$$y_i = 0.01 \text{ (mole fraction of benzene at the gas-liquid interface);}$$

$$Y_b = \frac{y_b}{1 - y_b} = \frac{0.075}{1 - 0.075} = 0.0811 \text{ (molar ratio); } Y_i = \frac{y_i}{1 - y_i} = \frac{0.01}{1 - 0.01} = 0.0101.$$

$$\text{Given: } k_G = 9.8 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}} \Rightarrow k_y = P \cdot k_G = (1.2)(9.8 \times 10^{-4}) \\ = 1.176 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$$

(3-4)

$$3.2) \text{ contd... } k_c = k_{\infty} RT = (9.8 \times 10^{-4})(0.08317)(308) = 0.0251 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\text{kmol/m}^3)}$$

To calculate k_y , use the relation $k_y (Y_b - Y_i) = k_T (Y_b - Y_i)$

$$\Rightarrow (1.176 \times 10^{-3})(0.075 - 0.01) = k_T (0.0811 - 0.010) \Rightarrow k_T = 1.077 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta Y)}$$

Calculation of k_y in FPS unit

$$k_y = 1.176 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta Y)} = 1.176 \times 10^{-3} \frac{1}{(1/0.3048) \text{ft}^2 \cdot (1/3600) \text{hr} \cdot \Delta Y} = 0.867 \frac{\text{kmol}}{\text{ft}^2 \cdot \text{hr} \cdot (\Delta Y)}$$

$$k_c = 0.0251 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\text{kmol/m}^3)} = 0.0251 \frac{\text{m}}{\text{s}} = (0.0251)(3.281) = 0.823 \frac{\text{kmol}}{\text{ft}^2 \cdot \text{s} \cdot (\text{lbmol/ft}^3)}$$

$$k_{\infty} = 9.8 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}} = 9.8 \times 10^{-4} \frac{2.0046 \text{ kmol}}{(3.281 \text{ ft})^2 \cdot \text{s} \cdot (14.51 \text{ psi})} = 1.383 \times 10^{-5} \frac{\text{kmol}}{\text{ft}^2 \cdot \text{s} \cdot \text{psi}}$$

3.3) Let the radius of the bubble be r at any time t .

Molar mass of CO_2 in the bubble $= \left(\frac{4}{3}\pi r^3 \frac{P}{RT}\right)$; P = pressure of pure CO_2 in the bubble.

If k_L is the mass transfer coefficient of CO_2 ,

$$-\frac{d}{dt} \left[\frac{4}{3}\pi r^3 \frac{P}{RT} \right] = k_L \cdot 4\pi r^2 (C_i - C_b) \quad \boxed{C_i = \text{conc. of } \text{CO}_2 \text{ at the gas-liquid interface}, \quad C_b \rightarrow \text{in the bulk}}$$

$$\Rightarrow -\frac{P}{RT} \cdot 4\pi r^2 \frac{dr}{dt} = k_L \cdot 4\pi r^2 (C_i - C_b) \Rightarrow -\frac{dr}{dt} = \frac{RT}{P} \cdot k_L (C_i - C_b) \quad \boxed{\text{liquid}}$$

$$\text{Integrating (note that at } t=0, r=r_0), \quad r = r_0 - \frac{RT}{P} \cdot k_L (C_i - C_b) \cdot t \dots (i)$$

So the bubble diameter is linear in time.

Solubility of CO_2 in water $= 1.45 \times 10^{-3}$ mass fraction $= x_i$

$$x_i = \frac{C_i}{C_i + C_w} \approx \frac{C_i}{C_w} \quad (\text{since } C_w \gg C_i), \quad \boxed{\text{Molar concentration of solvent, water} = \frac{1000}{18} = 55.5 \text{ kmol/m}^3}$$

$$C_i = x_i \cdot C_w = (1.45 \times 10^{-3})(55.5 \frac{\text{kmol}}{\text{m}^3}) = 0.0805 \text{ kmol/m}^3; \quad C_b = 0, \text{ since the volume of water is large.}$$

Given: $t=0, r=r_0 = 0.004 \text{ m}; t=808, r=0.001 \text{ m}$

$$\text{Put in Eq (i) above. } 10^{-3} = 4 \times 10^{-3} - \frac{(0.0821)(298)}{(1)} \cdot k_L \cdot (0.0805 - 0)(80)$$

$$\text{If the bubble dissolves completely, } r=0 \quad \Rightarrow k_L = \frac{1.904 \times 10^{-5}}{(0.0821)(298)(1.904 \times 10^{-5})(0.0805)} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\text{kmol/m}^3)}$$

$$t = \frac{(4 \times 10^{-3})(1)}{(0.0821)(298)(1.904 \times 10^{-5})(0.0805)} = 107 \text{ second}$$

(3.5)

- 3.3) contd... Mass transfer coefficient, $k_x = C \cdot k_L = (55.5)(1.904 \times 10^{-5})$
 Under a constant hydrodynamic condition, the mass transfer coefficient will be smaller when the bubble dia. is less.
 \Rightarrow The assumption of a constant value of k_L is only approximate.

- 3.4) This is a case of equimolar countercurrent diffusion of the reactant A and the product B through a 'stagnant' film.

(i) The reaction rate = flux of A = $N_A = k'_c (C_b - C_s)$. Given: $N_A = 0.08 \text{ kmol/m}^2 \text{ h}$ $= k'_c (0.002 - 0)$

 $\Rightarrow k'_c = \frac{0.08 \text{ kmol/m}^2 \text{ h}}{0.002 \text{ kmol/m}^3} = 40 \frac{\text{m}}{\text{h}} = 0.0111 \text{ m/s}$

Eq (3.9a) $\rightarrow k'_c = \frac{D_{AB}}{8}$; $D_{AB} = (0.16 \text{ cm}^2/\text{s}) \left(\frac{1}{1.5}\right) \left(\frac{523}{373}\right)^{1.5}$

 $\Rightarrow \text{Film thickness, } \delta = \frac{1.77 \times 10^{-5} \text{ m}^2/\text{s}}{0.0111 \text{ m/s}} = 1.6 \text{ mm}$

Bulk conc. of A
 $C_b = 0.002 \text{ kmol/m}^3$
 Concentration at the surface of the catalyst,
 $C_s = 0$, since the reaction is instantaneous.
 $T = 273 + 250 = 523 \text{ K}$
 $P = 1.5 \text{ bar}$
 (after correction for pressure and temperature)

(ii) Calculation of k'_G and k'_Y

$$\text{Eq (3.9a)} \rightarrow k'_G = \frac{D_{AB}}{RT\delta} = \frac{0.0111}{(0.08317)(523)} = 2.552 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}}$$

$$k'_Y = \frac{D_{AB}P}{RT\delta} = (2.552 \times 10^{-4})(1.5) = 3.828 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

- (iii) For equimolar countercurrent diffusion, the concentration profile in the gas film is linear. The concentration of A halfway in the tube, $C_A = \frac{C_b + C_s}{2} = \frac{0.002 + 0}{2} = 0.001 \text{ kmol/m}^3$

- 3.5) Radius of the drop at time t is r . Density of water = ρ_w , molecular wt = M_w ; vapor pressure at the drop temperature = P_g^∞ , partial pressure of moisture in the bulk gas = P_b ; mass transfer coefficient at the drop surface = k_G ; D_{AB} = diffusivity of moisture in air at the 'mean film temperature'; μ_a and ρ_a = viscosity and density of the surrounding air.
- Molar mass of the drop at time t = $\left(\frac{4}{3}\pi r^3 \rho_w / M_w\right)$

(3.5) Contd... Rate of evaporation of the drop = $-\frac{d}{dt} \left(\frac{4}{3} \pi r^3 p_w / M_w \right) = k_G (P - P_b) \cdot 4\pi r^2$

 $\Rightarrow -\frac{dr}{dt} = k_G (P - P_b) \cdot (M_w / P_w) \dots \text{(i)}$

Sherwood number, $Sh = \frac{k_G P_{BM} R T}{D_{AB} \cdot P}$
(Eq 3.16)

Since the partial pressure of moisture is small, take $P_{BM}/P \approx 1$

$$\Rightarrow Sh = \frac{(k_G)(0.0821)(298) \cdot (2r)}{2.57 \times 10^{-5}}$$

$$= (k_G \cdot r) (1.904 \times 10^6)$$

Using the correlation for Sh ,

$$(k_G \cdot r) (1.904 \times 10^6) = 2 + (0.6) \cdot 5777 \left(\frac{r^{3/2}}{2.935 \times 10^{-6}} \right)^{1/2} (0.842)$$

$$\Rightarrow k_G = \frac{1.532 \times 10^{-3}}{r} \left[6.854 \times 10^{-4} + \left(\frac{r^{3/2}}{2.935 \times 10^{-6}} \right)^{1/2} \right]$$

Put this expression for k_G in Eq (i) above. Note that k_G depends upon the drop size, r . Also put $P = 0.02605$ atm (at the drop temp.)

$$P_b = (0.7) (\text{vapor pressure of water at } 25^\circ\text{C}) = (0.7) (0.03147) \text{ bar} = 0.02203 \text{ bar}$$

Substitute in Eq (i) above; $P_w = 995 \frac{\text{kg}}{\text{m}^3}$, $M_w = 18$ [see vapor pressure relation in Example 2.2]

$$-\frac{dr}{dt} = \frac{1.532 \times 10^{-3}}{r} \left[6.854 \times 10^{-4} + \left(\frac{r^{3/2}}{2.935 \times 10^{-6}} \right)^{1/2} \right] (0.02605 - 0.02203) \left(\frac{18}{995} \right)$$

$$\Rightarrow -\frac{dr}{dt} = \frac{1.114 \times 10^{-7}}{r} \left[6.854 \times 10^{-4} + \left(\frac{r^{3/2}}{2.935 \times 10^{-6}} \right)^{1/2} \right]$$

The radius of the drop changes from $r = 4 \times 10^{-4} \text{ m}$ to $3 \times 10^{-4} \text{ m}$.

$$\text{Time required, } t = - \int_{r=4 \times 10^{-4}}^{3 \times 10^{-4}} \frac{4.617 \times 10^{-6} \cdot r \, dr}{\left[6.854 \times 10^{-4} + \left(\frac{r^{3/2}}{2.935 \times 10^{-6}} \right)^{1/2} \right]}$$

Evaluate the integral numerically to get, time required, $t = 638$

(3.6) Flux of toxin through the dialysis tubing, $N_A = k_L (c_i - c_o)$,
 $k_L = k_L (u, P, \mu, D, d)$ [D = diffusivity; d = tube diameter]

Assume a power law dependence : $k_L = \beta \cdot u^{n_1} P^{n_2} \mu^{n_3} D^{n_4} d^{n_5} \dots \text{(i)}$

Equate the units on both sides

$$\frac{L}{t} = [] \left(\frac{L}{t} \right)^{n_1} \left(\frac{M}{L^3} \right)^{n_2} \left(\frac{M}{L \cdot t} \right)^{n_3} \left(\frac{L^2}{t} \right)^{n_4} (L)^{n_5}$$

Dimensions : Mass $\rightarrow M$; Length $\rightarrow L$
 Time $\rightarrow t$

3.7

(3.6) contd... Equate the powers of L : $1 = n_1 - 3n_2 - n_3 + 2n_4 + n_5 \dots (ii)$
 Equate the powers of M : $0 = n_2 + n_3$. Equate the power of t : $-1 = -n_1 - n_3 - n_4 \dots (iv)$
 We have three equations (ii), (iii) and (iv) in five unknowns.
 Solve for n_2 , n_3 and n_5 in terms of n_1 and n_4 .

$$\underline{n_1 = n_1}; \underline{n_2 = n_1 + n_4 - 1}; \underline{n_3 = 1 - n_1 - n_4}; \underline{n_4 = n_4}; \underline{n_5 = n_1 - 1}$$

$$\text{From Eq. (i), } k_L = \beta \cdot \mu^{n_1} \cdot \rho^{n_1 + n_4 - 1} \cdot \mu^{1 - n_1 - n_4} \cdot D^{n_4 - 1} \cdot d^{n_1}$$

$$\text{Rearrange to get } \frac{k_L d}{D} = \beta \cdot \left[\frac{\mu \rho}{\mu} \right]^{n_1} \cdot \left[\frac{\mu}{\rho D} \right]^{1 - n_4}$$

$$\Rightarrow Sh = \beta \cdot Re^{n_1} \cdot Sc^{1 - n_4}$$

This is the type of correlation that can be used.

(3.7) Local flux of H_2S , $N_A = k_c(C_b - C_i)$; k_c = gas-phase mass transfer coefficient; C_b , C_i = gas phase concentration of H_2S (kmol/m^3) in the bulk and at the gas-liquid interface respectively.

$$\text{Given: } C_b = \frac{P_b}{RT} = \frac{P \cdot Y_b}{RT} = \frac{(1.6)(0.15)}{(0.08317)(298)} = 9.683 \times 10^{-3} \text{ kmol/m}^3$$

Liquid-side concentration at the interface, $x_i = H \cdot p_i$

H = solubility coefficient or Henry's law constant [see Section 4.2.2]

$$H = 0.00337 \text{ mass fraction/atm} = \frac{0.00337}{1.013} = 0.003327 \text{ /bar}$$

$$H \rightarrow \frac{0.003327/34 \text{ mol H}_2S}{\frac{0.003327}{34} \text{ mol H}_2S + \frac{1 - 0.003327}{18} \text{ mol H}_2O} = 0.00176 \text{ mol fraction} \quad \text{Res bar}$$

Given: mole fraction of H_2S on the liquid-side of the interface

$$= x_i = 1.8 \times 10^{-5} \Rightarrow \text{partial pressure, } p_i = \frac{x_i}{H} = \frac{1.8 \times 10^{-5}}{0.00176} = 0.01023 \text{ bar}$$

Corresponding concentration at the gas-side of the interface,

$$C_i = \frac{p_i}{RT} = \frac{0.01023}{(0.08317)(298)} = 4.13 \times 10^{-4} \frac{\text{kmol}}{\text{m}^3}$$

$$\text{Local flux of } H_2S, N_A = k_c(C_b - C_i) = (3.42)(9.683 \times 10^{-3} - 4.13 \times 10^{-4})$$

$$\Rightarrow \underline{N_A = 0.0317 \text{ kmol/m}^2 \cdot \text{h}}$$

3.8 Area of cross-section of the column = $\frac{\pi}{4}(0.03)^2 = 7.068 \times 10^{-4} \text{ m}^2$

$$\text{Gas (air) flow rate } G = \frac{1 \text{ kg}/60 \text{ s}}{7.068 \times 10^{-4} \text{ m}^2} = 23.58 \text{ kg/m}^2 \text{ s; } T=38^\circ\text{C} = 311 \text{ K}$$

$$\text{Air density (1 atm, } 38^\circ\text{C}), \rho_a = \frac{P}{RT} \cdot M_a = \frac{(1)(28.9)}{(0.0821)(273+38)} = 1.132 \text{ kg/m}^3$$

$$\text{Viscosity, } \nu_a = 1.85 \times 10^{-5} \frac{\text{lb}}{\text{ft} \cdot \text{s}} / 1.132 \text{ kg/m}^3 = 2.432 \times 10^{-5} \text{ m}^2/\text{s; } d = 0.03 \text{ m}$$

$$\text{Reynolds number, } Re = \frac{dG}{\mu_a} = \frac{0.03)(23.58)}{2.753 \times 10^{-5}} = 36,340 \quad \mu_a = 1.85 \times 10^{-5} \frac{\text{lb}}{\text{ft} \cdot \text{s}} = 2.753 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

Diffusivity of moisture in air

$$\text{Take the mean air temperature, } T_{av} = \frac{305+311}{2} = 308 \text{ K}$$

$$\text{Diffusivity at } 308 \text{ K, } D_{AB} = 2.6 \times 10^{-5} \left(\frac{308}{298} \right)^{1.5} = 2.732 \times 10^{-5} \text{ m}^2/\text{s}$$

$$\text{Schmidt number, } Sc = \frac{\nu_a}{D_{AB}} = \frac{2.432 \times 10^{-5}}{2.732 \times 10^{-5}} = 0.89$$

$$D = 2.6 \times 10^{-5} \text{ m}^2 \text{ at } 298 \text{ K}$$

See Table 2.2

$$\text{Sherwood number, } Sh = (0.023)(36,340)^{0.8} (0.89)^{0.33} = 98.5$$

$$\text{Eq. (3.16)} \rightarrow Sh = \frac{k_G P_{BM} RT \cdot d}{D_{AB} \cdot P} = 98.5 \quad \boxed{\text{Take } P_{BM}/P \approx 1 \text{ since the vapor pressure water is small}}$$

$$\Rightarrow k_G = (98.5) \cdot \frac{(2.732 \times 10^{-5})}{(0.0821)(308)(0.03)} = 3.547 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{atm}}$$

Driving force for moisture transport, Δp

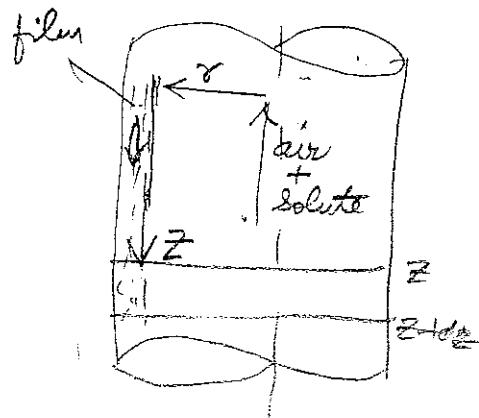
$$\text{Bulk gas } \rightarrow Y_b = 0.0098 \Rightarrow p_b = (0.0098)(1 \text{ atm}) = 0.0098 \text{ atm}$$

$$\text{Water surface, } p_i = \text{vapor pressure of water at } 305 \text{ K} = 0.0462 \text{ atm}$$

$$\text{Evaporation flux of water, } N_A = k_G (p_i - p_b)$$

$$= (3.547 \times 10^{-3})(0.0462 - 0.0098) = 1.291 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$= 2.324 \times 10^{-3} \frac{\text{kg H}_2\text{O}}{\text{m}^2 \cdot \text{s}}$$



3.9 Mass transfer coefficient is given by: $(k_L z/D) = 0.69 (Z^2/D)^{0.5}$

$$\text{Given: } Z = 2.1 \text{ m/s, and } D = 1.7 \times 10^{-9} \frac{\text{m}^2}{\text{s}}$$

$$\Rightarrow k_L = (0.69)(1.7 \times 10^{-9})^{0.5} (2.1)^{0.5} \frac{1}{z^{1/2}} = 4.183 \times 10^{-5} \cdot z^{-1/2} \text{ m/s}$$

Let r = radius of the column. The film is thin.

(3.9)

3.9 contd... 'Local' concentration of the solute in the film = \underline{c} and the equilibrium concentration (solubility) of the solute gas in the liquid (water) = \underline{c}^* , kmol/m³, film thickness = $\underline{\delta}$.

If the increase in liquid concentration over a distance is \underline{dc} , the following mass balance can be written:

$$(2\pi r \cdot \delta)(v)dc = k_L(2\pi r) \cdot dz \cdot (c^* - c) \quad [c^* - c = \text{local driving force for mass transfer}]$$

$$\Rightarrow \delta v \cdot dc = k_L \cdot dz \cdot (c^* - c). \text{ Put the expression for } k_L \text{ and integrate}$$

$$\int_{C=0}^C \frac{dc}{c^* - c} = \frac{1}{\delta \cdot v} \int_{z=0}^Z 4 \cdot 1.83 \times 10^{-5} z^{-1/2} dz \Rightarrow \ln \frac{c^*}{c^* - c} = \frac{2 \times 4 \cdot 1.83 \times 10^{-5}}{\delta \cdot v} \cdot z^{1/2}$$

Solute concentration in the exit liquid is $c = 0.1c^*$; $\delta = 10^{-3} \text{ m}$.

$$\Rightarrow \ln \frac{c^*}{c^* - 0.1c^*} = \frac{2 \times 4 \cdot 1.83 \times 10^{-5}}{(10^{-3})(2 \cdot 1)} \cdot z^{1/2} \Rightarrow z = 7 \text{ m} = \text{tower height}$$

3.10 Values of the parameters: diameter of the spherical particle, $d = 2.5 \times 10^{-3} \text{ m}$; liquid velocity, $u = 0.12 \text{ m/s}$; viscosity, $\mu = 10^{-3} \frac{\text{kg}}{\text{m s}}$; diffusivity of NaCl, $D = 1.4 \times 10^{-9} \text{ m}^2/\text{s}$; liquid density, $\rho_L = 1300 \text{ kg/m}^3$; saturation concentration of NaCl (39 kg salt in 139 kg solution), $c^* = 36.3 / (136.3 / 1300) = 346.2 \text{ kg/m}^3$; bulk concentration of NaCl, $c_b = 39 / (139 / 1300) = 364.7 \text{ kg/m}^3$.

Calculation of mass transfer coefficient at the crystal surface:

Correlation: $Sh = 2 + 0.79 \cdot Re^{1/2} Sc^{1/3}$

$$\Rightarrow Sh = 2 + (0.79)(390)^{1/2} (549)^{1/3} = 129.7$$

$$Sh = \frac{k_L \cdot d}{D} \Rightarrow k_L = \frac{(129.7)(1.4 \times 10^{-9})}{2.5 \times 10^{-3}} = 7.26 \times 10^{-5} \frac{\text{m}}{\text{s}}$$

$$Re = \frac{du\rho}{\mu} = \frac{(2.5 \times 10^{-3})(0.12)(1300)}{10^{-3}} = 390$$

$$Sc = \frac{\mu}{\rho D} = \frac{10^{-3}}{(1300)(1.4 \times 10^{-9})} = 549$$

Flux of NaCl to the crystal surface, $N_A = k_L(c_b - c^*)$

$$\Rightarrow N_A = (7.26 \times 10^{-5})(364.7 - 346.2) = 1.343 \times 10^{-3} \frac{\text{kg}}{\text{m}^2 \text{s}}$$

Now we calculate the experimental flux and compare with the theoretically calculated flux to determine if the process is 'diffusion controlled'.

Mass of the crystal = $\frac{4}{3}\pi r^3 \rho_S$; ρ_S = density of the solid = 2160 kg/m^3 .

$$\text{Rate of change of mass} = \frac{d}{dt} \left(\frac{4}{3}\pi r^3 \rho_S \right) = 4\pi r^2 \rho_S \cdot \frac{dr}{dt}$$

(3.10) contd... Flux of salt to the crystal surface = $\frac{1}{4\pi r^2} \rho_s \frac{dr}{dt}$
 $= \rho_s \cdot \frac{dr}{dt} = (2160 \text{ kg/m}^3)(9.5 \times 10^{-8} \text{ m/s}) = 2.052 \times 10^{-4} \text{ kg/m}^2 \text{s}$

The true flux is about 9% of the flux theoretically calculated if the crystallization process is 'diffusion controlled'.

⇒ The crystal growth process cannot be considered to be diffusion controlled.
 * * * *

(3.11) Bubble diameter, $d = 5 \times 10^{-3} \text{ m}$; rise velocity, $v = 0.1 \text{ m/s}$.

Contact time, $t_c = d/v = 5 \times 10^{-3}/0.1 = 0.05 \text{ second}$.

Calculate the liquid-side mass transfer coefficient using the penetration theory, Eq. (3.38). Temperature = $30^\circ\text{C} = 303\text{K}$

$$k_{L,av} = 2 \left[\frac{D_{AB}}{\pi t_c} \right]^{1/2}$$

$$= 2 \left[\frac{2.381 \times 10^{-9}}{\pi \cdot (0.05)} \right]^{1/2}$$

$$= 2.462 \times 10^{-4} \text{ m/s}$$

The diffusivity value is given at $25^\circ\text{C} = 298\text{K}$. Make temperature and viscosity correction using Eq. (2.67), Stokes-Einstein equation.

$$\frac{D_1 \mu_1}{T_1} = \frac{D_2 \mu_2}{T_2} \Rightarrow D = (2.1 \times 10^{-9}) \left(\frac{303}{298} \right) \left(\frac{0.911}{0.817} \right)$$

$$= 2.381 \times 10^{-9} \text{ m}^2/\text{s at } 303\text{K}$$

Rate of mass transfer from a single bubble = $k_L (C^* - C_b) \cdot a$

To calculate C^* , use the Henry's law, $P_{O_2} = H \cdot x^*$

$H = 4.75 \times 10^4 \text{ atm/mole fraction}$; $P_{O_2} = 0.21 \text{ atm}$ (in air).

$$x^* = \frac{0.21}{4.75 \times 10^4} = 4.42 \times 10^{-6} \text{ mole fraction} \approx \frac{4.42 \times 10^{-6} \text{ mol oxygen}}{1 \text{ mole solution}}$$

$$\Rightarrow \text{Solubility of oxygen} \approx \frac{4.42 \times 10^{-6} \text{ mol O}_2}{1 \text{ mol water}} = \frac{4.42 \times 10^{-6}}{(18/1000) \text{ m}^3 \text{ water}} = 2.455 \times 10^{-4} \frac{\text{kmol}}{\text{m}^3}$$

Bulk water is oxygen-free $\Rightarrow C_b = 0$

$$\text{Area of a bubble, } a = \pi \cdot (d^2) = \pi \cdot (0.005)^2 = 7.854 \times 10^{-5} \text{ m}^2$$

$$\text{Rate of mass transfer} = k_L (C^* - C_b) \cdot a = (2.462 \times 10^{-4})(2.455 \times 10^{-4} - 0)(7.854 \times 10^{-5})$$

$$= 4.747 \times 10^{-12} \frac{\text{kmol O}_2}{\text{second}}$$

(3.12) We calculate the rate of absorption of CO_2 using the Penetration Theory and compare with the experimental value.

$$\text{Contact time, } t_c = \frac{\text{bubble length}}{\text{velocity}} = \frac{0.05 \text{ m}}{0.2 \text{ m/s}} = 0.25 \text{ s}$$

$$\text{Diffusivity of CO}_2 \text{ in water at } 25^\circ\text{C} = 1.92 \times 10^{-9} \text{ m}^2/\text{s} [\text{Table 2.5}]$$

(3.12) Contd..

(3.11)

Average mass transfer coefficient, $k_L = 2 \left[\frac{1.92 \times 10^{-9}}{\pi \cdot (0.25)} \right]^{1/2} = 9.89 \times 10^{-5} \text{ m/s}$

Assume cylindrical bubble shape; dia, $d = 3 \times 10^{-3} \text{ m}$; length = 0.05 m

Area, $a \approx \pi \cdot (3 \times 10^{-3})^2 (0.05) + (2)(\frac{\pi}{4})(3 \times 10^{-3})^2 = 4.85 \times 10^{-4} \text{ m}^2$

Driving force = $C^* - C_b$. $C^* = 1.5 \text{ kg/m}^3$, $C_b = 0$ (negligible CO_2 conc. in the bulk H_2)

Rate of CO_2 absorption from a single bubble,

$$Q = k_L \cdot a \cdot (C^* - C_b) = (9.89 \times 10^{-5})(1.5 - 0)(4.85 \times 10^{-4}) = 7.195 \times 10^{-8} \text{ kg/s}$$

Residence time of a bubble in the tube = $\frac{\text{tube length}}{\text{bubble vel.}} = \frac{0.6 \text{ m}}{0.2 \text{ m/s}} = 3 \text{ sec.}$

Amount of CO_2 absorbed from a single bubble during its passage through the tube = $Q \times \text{Residence time} = (7.195 \times 10^{-8})(3) = 2.158 \times 10^{-7} \text{ kg}$

Number of bubbles passing through the tube per sec. = 3 (per sec.)

Total rate of absorption = $(2.158 \times 10^{-7} \text{ kg})(3/8) = 6.474 \times 10^{-7} \text{ kg CO}_2/\text{s}$
 $= (6.474 \times 10^{-7})(60)(10^6) \text{ mg/min} = 38.8 \text{ mg/s}$

Experimental rate of absorption = 40 mg/s

⇒ The Penetration Theory is applicable in the present case.

(3.13) Volume of a single bubble = $\frac{\pi}{6} d^3 = \frac{\pi}{6} (6 \times 10^{-3})^3 = 1.131 \times 10^{-7} \text{ m}^3$

Fractional gas hold-up = 0.02 ; dispersion volume = $2.5 \text{ lit.} = 2.5 \times 10^{-3} \text{ m}^3$

Total volume of gas in the dispersion = $(0.02)(2.5 \times 10^{-3}) = 5 \times 10^{-5} \text{ m}^3$

Number of bubbles in the dispersion at any moment = $\frac{5 \times 10^{-5}}{1.131 \times 10^{-7}} = 442$

Total area of the bubbles in the dispersion = $(442) \frac{\pi}{4} (6 \times 10^{-3})^2 = 1.25 \times 10^{-2} \text{ m}^2$

Average mass transfer coefficient, $k_L = 2 \left[\frac{D}{\pi a} \right]^{1/2} = 2 \left[\frac{1.8 \times 10^{-9}}{(\pi)(0.006/0.18)} \right]^{1/2}$
 $= 2.623 \times 10^{-4} \text{ m/s}$ [Contact time
 $= d/U = \frac{0.006}{0.18} = 0.033$]

Let C_b be the concentration of the solute in the solution at any time t , C^* = solubility, V = liquid volume.

⇒ $\frac{d}{dt}(VC_b) = \text{rate of absorption} = k_L \cdot a \cdot (C^* - C_b)$; $t=0$, $C_b=0$.

Integrate → $V \cdot \int_{C_b=0}^{C_b} \frac{dc_b}{C^* - C_b} = k_L a \cdot \int_0^t dt \Rightarrow \ln \frac{C^*}{C^* - C_b} = \frac{k_L a}{V} \cdot t$

To calculate the time when C_b reaches 50% of the saturation concentration,

put $C_b = C^*/2$ in the above equation

$$\ln \frac{C^*}{C^* - C^*/2} = \frac{(2.623 \times 10^{-4})(1.25 \times 10^{-2})}{(2.5)(0.98) \times 10^{-3}} \cdot t$$

⇒ Required time, $t = 518 \text{ sec.}$

Volume of liquid;
 $V = 98\% \text{ of the dispersion}$
 $= (2.5)(0.98) \text{ liters}$

(3.12)

- (3.14) Use Eq. (3.41) to calculate the average mass transfer coefficient.

Parameters: length of the plate, $l = 0.5 \text{ m}$; ^{free stream} liquid velocity, $v_\infty = 0.25 \text{ m/s}$; viscosity = $8.9 \times 10^{-4} \text{ kg/m.s}$; density of liquid, $\rho = 1000 \text{ kg/m}^3$; diffusivity of benzoic acid, $D = 10^{-9} \text{ m}^2/\text{s}$; solubility, $C^* = 3.01 \text{ kg/m}^3$.

$$\text{Reynolds number, } Re = \frac{l v_\infty \rho}{\mu} = \frac{(0.5)(0.25)(1000)}{8.9 \times 10^{-4}} = 140450$$

$$\text{Schmidt number, } Sc = \mu/D = \frac{8.9 \times 10^{-4}}{(1000)(10^{-9})} = 890$$

$$\text{Eq. (3.41)} \rightarrow Sh = \frac{(k_L)_{av} \cdot l}{D} = (0.664) Re^{1/2} Sc^{1/3} = (0.664)(140450)^{1/2} (890)^{1/3}$$

$$(k_L)_{av} = \frac{(2393)(10^{-9})}{0.5} = 4.786 \times 10^{-6} \text{ m/s} = 2393$$

Average flux of dissolution: $N_A = (k_L)_{av} (C^* - C_b)$; $C_b = 0$, pure water

$$\Rightarrow N_A = (4.786 \times 10^{-6})(3.01 - 0) = 1.441 \times 10^{-5} \text{ kg/m}^2 \text{s}$$

If δ is the equivalent thickness of a stagnant film,

$$k_L = \frac{D}{\delta} \Rightarrow \delta = \frac{D}{k_L} = \frac{10^{-9}}{4.786 \times 10^{-6}} = 2.39 \times 10^{-4} \text{ m} = 0.239 \text{ mm}$$

(3.15) $Re = \frac{l v_\infty \rho}{\mu} = \frac{(0.4 \text{ m})(5 \text{ m/s})}{1.59 \times 10^{-5} \text{ m}^2/\text{s}} = 1.258 \times 10^5$; $Sc = \frac{\nu}{D_{AB}} = \frac{1.59 \times 10^{-5}}{2.62 \times 10^{-5}} = 0.607$

Take the water temperature to be 300 K. vapor pressure, $P^v = 0.0353 \text{ bar}$
Relative humidity of bulk air = 60%; $P_b = (0.6)(0.0353) = 0.0212 \text{ bar}$

Sherwood number (use Eq. 3.41) $\Rightarrow Sh = \frac{k_G P_{BM} R T L}{D_{AB} P} = (0.664)(1.258 \times 10^5)^{1/2} (0.607)^{1/3} = 199.4$

Vapor pressure of water is low, take $P_{BM}/P \approx 1$; $L = 40 \text{ cm} = 0.4 \text{ m}$;
 $\Rightarrow k_G = (199.4) \cdot \frac{(2.62 \times 10^{-5})(1.0)}{(0.0821)(300)(0.4)} = 5.303 \times 10^{-4} \text{ kmol/m}^2 \cdot \text{s} \cdot \text{atm}$

Evaporation flux, $N_A = k_G (P^v - P_b) = (5.303 \times 10^{-4})(0.0353 - 0.0212) = 0.0574 \text{ kg water/h} \cdot \text{m}^2$

Area of pan = $(0.4)(0.3) = 0.12 \text{ m}^2$ $\Rightarrow 7.38 \times 10^{-6} \text{ kmol/m}^2 \cdot \text{s} = 0.478 \text{ kg water/h} \cdot \text{m}^2$

Rate of evaporation = $(0.478)(0.12) = 0.0574 \text{ kg water/h}$

- (3.16) From the analogy of heat and mass transfer, we use the similar correlation to estimate Sherwood numbers.

$$Sh = 0.664 Re^{1/2} Sc^{1/3}$$

$$Re = \frac{(0.25)(15)}{1.6 \times 10^{-5}} = 2.344 \times 10^5; Sc = \frac{1.6 \times 10^{-5}}{6 \times 10^{-6}} = 2.667$$

(3.16) Contd... $S_h = \frac{k_G P_{BM} RT}{D_{AB} \cdot P} = (0.664)(2.344 \times 10^5)^{1/2} (2.667)^{1/3} = 446$

Take $P_{BM}/P \approx 1$

 $k_G = \frac{(446)(6 \times 10^{-6})(1)}{(0.0821)(300)(0.25)} = 4.346 \times 10^{-4} \text{ kmol/m}^2 \cdot \text{s. atm}$

(i) Molar concentration of naphthalene at the surface
Vapor pressure = $0.235 \text{ mm Hg} = \frac{0.235}{760} = 3.092 \times 10^{-4} \text{ atm} = \text{partial pressure of naphthalene at the surface} = P^*$
Corresponding molar concentration = $\frac{P^*}{RT} = \frac{3.092 \times 10^{-4}}{(0.0821)(300)} = 1.255 \times 10^{-5} \frac{\text{kmol}}{\text{m}^3}$

(ii) Average mass transfer coefficient, k_G , is calculated above.

(iii) Area of the plate per unit width, $a = (0.25 \text{ m})(1 \text{ m}) = 0.25 \text{ m}^2$
Rate of loss of naphthalene = $k_G \cdot a \cdot (P^* - P_b)$; $P_b = 0$ since there is no naphthalene in bulk air.
Mol wt = 128
 $= (4.346 \times 10^{-4})(0.25)(3.092 \times 10^{-4})$
 $= 3.36 \times 10^{-8} \text{ kmol/s} = (3.36 \times 10^{-8})(128) = 4.3 \times 10^{-6} \text{ kg/s}$

* * * *

(3.17) If the liquid (water) is in laminar flow, a small region near the free surface may be considered to have a 'flat' velocity profile. So we may assume that the penetration theory is applicable, provided the 'penetration depth' is small.

Free surface velocity of water, $u = 40 \text{ cm/s} = 0.4 \text{ m/s}$ (See illustration below)

Length of the column = $30 \text{ cm} = 0.3 \text{ m} \Rightarrow \text{Contact time, } t_c = \frac{0.3}{0.4} = 0.75 \text{ sec}$

Diffusivity of chlorine in water = $1.26 \times 10^{-9} \text{ m}^2/\text{s}$

Mass transfer coefficient, $(k_L)_{av} = 2 \left[\frac{D}{\pi \cdot t_c} \right]^{1/2} = 2 \left[\frac{1.26 \times 10^{-9}}{\pi \cdot (0.75)} \right]^{1/2} = 4.625 \times 10^{-5} \frac{\text{m}}{\text{s}}$

Concentration of chlorine at the liquid surface = solubility of chlorine, $C^* = 0.823 \text{ kg/100 kg water} \approx \frac{0.823/71}{0.1 \text{ m}^3} = 0.116 \text{ kmol/m}^3$

Concentration of chlorine in bulk water, $C_b \approx 0$

Area of gas-liquid contact (column dia = 2 cm , height = 0.3 m)
 $a = \pi(0.02)(0.03) = 1.885 \times 10^{-2} \text{ m}^2$

Mol wt. of $\text{Cl}_2 = 71$

Rate of absorption = $k_L \cdot a \cdot (C^* - C_b) = (4.625 \times 10^{-5})(1.885 \times 10^{-2})(0.116 - 0)$
 $= 1.0113 \times 10^{-7} \text{ kmol/s} = 7.18 \times 10^{-6} \text{ kg/s}$

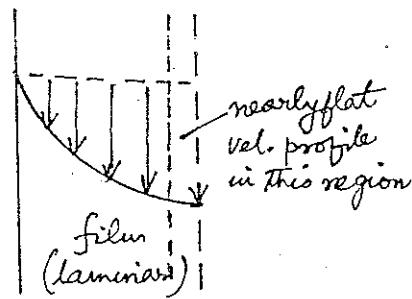


Figure Prob 3.17

(3.18) This problem is similar to Problem 3.13. Only an outline of the solution is given.

$$\text{Contact time, } t_C = \frac{0.005 \text{ m}}{0.26 \text{ m/s}} = 0.01928; D = 2.1 \times 10^{-9} \text{ m}^2/\text{s}.$$

$$(i) (k_L)_{av} = 2 \left[\frac{2.1 \times 10^{-9}}{\pi \cdot (0.0192)} \right]^{1/2} = 3.732 \times 10^{-4} \text{ m/s}$$

(ii) Time to reach 60% of the saturation conc.

Use the integral
(See Prob 3.13)

$$\int_{C_0}^{C_b} \frac{dC_b}{C^* - C_b} = \frac{k_L a}{V} \cdot t$$

$$\begin{aligned} &\text{Number of bubbles in the tank} \\ &= \frac{10^3 \text{ m}^3}{(\pi/6)(0.005)^3} = 15280 \\ &\text{Area, } a = (15280)(\pi)(0.005)^2 \\ &= 1.2 \text{ m}^2 \end{aligned}$$

$$\text{Put } C^* = 0.0085 \text{ kg/m}^3; C_b = (0.6)(0.0085) = 0.0051; C_0 = 0.001; V = 0.099 \text{ m}^3$$

$$\ln \frac{0.0085 - 0.001}{0.0085 - 0.0051} = \frac{(3.732 \times 10^{-4})(1.2)}{0.099} \cdot t \Rightarrow t = 175 \text{ second}$$

(iii) Maximum oxygen flux occurs at the beginning, $t=0$.

$$\text{Flux, } N_A = k_L (C^* - C_0) = (3.732 \times 10^{-4})(0.0085 - 0.001) = 2.8 \times 10^{-6} \frac{\text{kg O}_2}{\text{m}^2 \cdot \text{s}}$$

Over the given time,

Minimum oxygen flux occurs at $t=175 \text{ s}$ when $C_b = 0.0051$

$$N_A = k_L (C^* - C_b) = (3.732 \times 10^{-4})(0.0085 - 0.0051) = 1.27 \times 10^{-6} \frac{\text{kg O}_2}{\text{m}^2 \cdot \text{s}}$$

* * * *

(3.19) Let ϵ be the fractional gas holdup in 100 liters dispersion

$$\text{Volume of air in the tank} = (\epsilon)(100/1000) = 0.1\epsilon \text{ m}^3$$

$$\text{Number of bubbles} = \frac{0.1\epsilon}{(\pi/6)(0.005)^3} = 1.528 \times 10^6 \epsilon$$

$$\text{Air-water contact area} = (1.528 \times 10^6 \epsilon)(\pi)(0.005)^2 = 120\epsilon$$

$$\begin{aligned} &\text{Rate of oxygen consumption by 50 fish} \\ &\approx (50)(1.5 \times 10^{-8}) = 7.5 \times 10^{-7} \text{ kg/s} \end{aligned}$$

$$\begin{aligned} &\text{Volume of a bubble} \\ &= \frac{\pi}{6} (0.005)^3 \text{ m}^3 \end{aligned}$$

(3.19) contd...

This must be equal to the rate of O_2 absorption at steady state.

$$\Rightarrow 7.5 \times 10^{-7} = k_L a \cdot (C^* - C_0) \Rightarrow 7.5 \times 10^{-7} = (3.732 \times 10^{-4}) (120 \epsilon) (0.0085 - 0.0051)$$

$$\Rightarrow \text{Air holdup fraction, } \epsilon = 0.005, \text{ or } 0.5\% \quad \boxed{\begin{array}{l} \text{Steady state } O_2 \\ \text{concentration} = 0.0051 \\ \text{kg/m}^3 \end{array}}$$

* * * *

(3.20) Since CO_2 reacts instantaneously with a strong NaOH solution, the conc. of CO_2 at the gas-liquid interface is virtually zero (i.e. $y_i = 0$).

Consider a thin section of the column of thickness dz at a location z from the bottom. Radius of the tube = r ; local molar gas flow rate = G ; local mole fraction of CO_2 in the gas = y_b . By mass balance of CO_2 over the differential section,

$$-\frac{d}{dz}(G \cdot y_b) = 2\pi r dz \cdot N_A ; \text{ also } -dG = 2\pi r dz \cdot N_A,$$

$$\Rightarrow -G dy_b - y_b dG = 2\pi r dz \cdot N_A$$

$$\Rightarrow -G dy_b + y_b \cdot 2\pi r dz \cdot N_A = 2\pi r dz \cdot N_A$$

Since the change in the flowrate of the gas occurs only because of absorption of the solute, CO_2 .

$$\Rightarrow -G dy_b = 2\pi r dz \cdot N_A (1 - y_b) ; N_A = k_y (y_b - y_i) = k_y y_b$$

$$\Rightarrow -G dy_b = 2\pi r dz \cdot k_y y_b (1 - y_b)$$

$$\Rightarrow - \int_{y_1}^{y_2} \frac{G \cdot dy_b}{y_b \cdot (1 - y_b)} = 2\pi r k_y \int_0^h dz ;$$

Take the average value of $G = (G_1 + G_2)/2$ for simplicity

$$\Rightarrow G_{av} \int_{y_1}^{y_2} \frac{dy_b}{y_b (1 - y_b)} = 2\pi r k_y h \Rightarrow G_{av} \left[\ln \frac{y_1}{y_2} + \ln \frac{1 - y_2}{1 - y_1} \right] = 2\pi r k_y h \quad \dots \dots \dots (i)$$

Given: $y_1 = 0.05$ (5% CO_2 in the gas); molar rate of input.

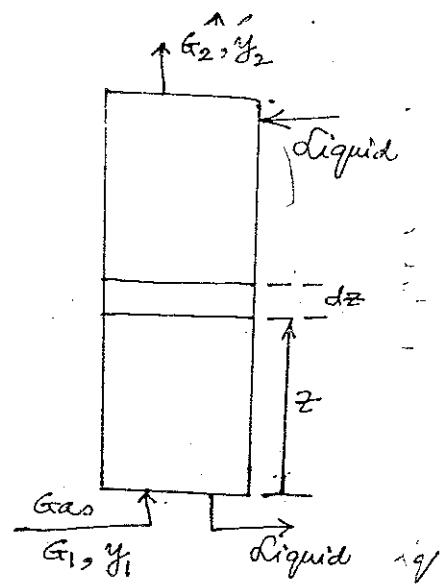


Fig. Prob. 3.20

(3.20) Contd...

(3-16)

$$\text{of the gas} = (\pi r^2 u) \frac{P}{RT} = \pi (0.02)^2 (4) \left(\frac{1.0}{0.0821 \times 303} \right) = 2.0206 \times 10^{-4} \frac{\text{kmol}}{\text{s}}$$

Moles CO_2 entering = $(2.0206 \times 10^{-4}) (0.05) = 1.0103 \times 10^{-5} \text{ kmol/s}$

75% of the CO_2 is absorbed

$$= (1.0103 \times 10^{-5}) (0.75) = 7.577 \times 10^{-6} \frac{\text{kmol}}{\text{s}}$$

$$\begin{aligned} \text{Inlet gas - } & \text{CO}_2 \text{ conc.} = y_1 = 0.05 \\ & u = 4 \text{ m/s} \\ \text{Tubed dia} & = 4 \text{ cm; } r = 0.02 \text{ m} \end{aligned}$$

$$\text{Moles of gas leaving, } G_2 = 2.0206 \times 10^{-4} - 7.577 \times 10^{-6} = 1.9448 \times 10^{-4} \frac{\text{kmol}}{\text{s}}$$

$$\text{Average gas flow rate, } G_{av} = \frac{(G_1 + G_2)}{2} = \frac{2.0206 \times 10^{-4} + 1.9448 \times 10^{-4}}{2}$$

$$\begin{aligned} \text{Moles of } & \text{CO}_2 \text{ leaving the column} \\ = (1.0103 \times 10^{-5}) (0.25) & = 2.5257 \times 10^{-6} \frac{\text{kmol}}{\text{s}} \quad = 1.9827 \times 10^{-4} \frac{\text{kmol}}{\text{s}} \end{aligned}$$

$$\text{Mole fraction of } \text{CO}_2 \text{ in the exit gas, } y_2 = \frac{2.5257 \times 10^{-6}}{1.9448 \times 10^{-4}} = 0.013$$

Mole fraction of CO_2 at the inlet, $y_1 = 0.05$ (given).

Substitute the values in Eq(i) above

$$(1.9827 \times 10^{-4}) \left[\ln \frac{0.05}{0.013} + \ln \frac{1-0.013}{1-0.05} \right] = (2\pi)(0.02) \cdot k_f \cdot (3) \quad \begin{array}{l} [\text{Column height,}] \\ h = 3 \text{ m} \end{array}$$

$$\Rightarrow \text{Mass transfer coefficient for } \text{CO}_2 \text{ absorption, } k_f = 7.286 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$$

Estimation of k_f using the given correlation

To use the correlation, calculate Re and Sc first.

$$\begin{aligned} Re : \text{ Molecular wt of the feed gas, } M_1 &= y_1 M_{\text{CO}_2} + (1-y_1) M_{\text{air}} \\ &= (0.05)(44) + (0.95)(28.8) = 29.6 \end{aligned}$$

$$\text{Mol. wt. of the exit gas} = y_2 M_{\text{CO}_2} + (1-y_2) M_{\text{air}} = (0.013)(44) + (0.987)(28.8) = 29 = M_2$$

$$\text{Gas velocity at the inlet} = 4 \text{ m/s (given)}$$

$$\text{That at the exit} = 3.85 \text{ m/s (calculation not shown)}$$

$$\text{Mean gas velocity} = (4+3.85)/2 = 3.925 \text{ m/s}$$

$$\text{Gas velocity relative to the liquid surface (liquid surface velocity)} \\ u = 3.925 + 0.15 = 4.075 \text{ m/s} \quad = 0.15 \text{ m/s, given}$$

$$\text{Density of the feed gas, } \rho_1 = \frac{P \cdot M_1}{RT} = \frac{(1)(29.6)}{(0.0821)(303)} = 1.19 \frac{\text{kg}}{\text{m}^3} ; \rho_2 = 1.166 \frac{\text{kg}}{\text{m}^3}$$

$$\text{Mean density, } \bar{\rho} = \frac{1.19 + 1.166}{2} = 1.178 \text{ kg/m}^3$$

$$\text{Viscosity of the gas} = \nu \cdot \bar{\rho} = (1.6 \times 10^{-5})(1.178) = 1.885 \times 10^{-5} \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

[Kinematic viscosity of the gas is taken as that of air]

$$Re = \frac{d \cdot u}{\nu} = \frac{(0.04)(4.075)}{1.6 \times 10^{-5}} = 10190 ; Sc = \frac{\nu}{D} = \frac{1.6 \times 10^{-5}}{1.65 \times 10^{-5}} = 0.97$$

3.17

$$3.20) \text{ Contd... } Sh = \frac{k_c d}{D} = (0.026)(10190)^{0.8}(0.97)^{0.33} = 41.4$$

$$k_c = \frac{(41.4)(1.65 \times 10^{-5})}{0.04} = \frac{0.0171}{m^2 \cdot s \cdot (\Delta C)} \frac{\text{kmol}}{m^2 \cdot s \cdot (\Delta y)}$$

$$\text{From Table 2.1, } k_y = \frac{P}{RT} \cdot k_c = \frac{(1)(0.0171)}{(0.0821)(303)} = \frac{6.87 \times 10^{-4}}{m^2 \cdot s \cdot (\Delta y)} \frac{\text{kmol}}{m^2 \cdot s \cdot (\Delta y)}$$

This is close to the value of k_y calculated from the experimental data.

* * * *

3.21 Let V be the volume of liquid in the cell and C_b its concentration at any time, t . The time rate of change of C_b is given by

$$V \frac{dc}{dt} = k_L \cdot a \cdot (C^* - C_b) ; \text{ initial condition; } t=0, C_b=0 \text{ (pure water)}$$

$$\text{Integrating, } \ln \frac{C^* - 0}{C^* - C_b} = \frac{k_L \cdot a}{V} \cdot t \dots \dots \dots \text{ (i)}$$

$$\text{Given: } V = 100 \text{ ml} = 10^{-4} \text{ m}^3; a = 31.5 \text{ cm}^2 = 3.15 \times 10^{-3} \text{ m}^2; t = 12 \text{ min} = 720 \text{ s}$$

$$C_b = 0.112 \text{ mass\%} = \frac{1.12 \times 10^{-3} \text{ kg}}{\text{kg liquid}} \approx \frac{1.12 \times 10^{-3} \text{ kg}}{10^{-3} \text{ m}^3} = 1.12 \text{ kg/m}^3$$

Calculation of C^* : partial pressure of $\text{CO}_2 = 1 \text{ atm}$; from Henry's law, the solubility (in mole fraction), $x^* = P/H = 1/1.64 \times 10^3 = 6.097 \times 10^{-4}$ $\equiv 6.097 \times 10^{-4} \text{ kmol CO}_2 \text{ per kmol solution} \approx (6.097 \times 10^{-4})(44) \text{ kg CO}_2 \text{ per 18 kg solution}$ (the average molecular wt. of solution is virtually the same as of water since the CO_2 concentration is very small). $\Rightarrow C^* = 0.02683 \text{ kg CO}_2 / (18/1000) \text{ m}^3 = 1.49 \text{ kg CO}_2 / \text{m}^3 \text{ solution.}$

Substitute in Eq (i) above to get $k_L = 6.17 \times 10^{-5} \text{ m/s}$

Fractional rate of surface renewal:

$$k_L = \sqrt{DS}, \text{ according to this model.}$$

$$\text{Put } D = 1.92 \times 10^{-9} \text{ m}^2/\text{s} \text{ and } k_L = 6.17 \times 10^{-5} \text{ m/s} \Rightarrow S = 1.98 \text{ per second}$$

* * * *

3.22 The amount of NH_3 absorbed over a period of $t = 20 \text{ min}$ $= (0.82 \text{ gmol/liter})(100 \text{ ml acid solution}) = 0.082 \text{ gmol} = 8.2 \times 10^{-5} \text{ kmol}$. Since the absorbed ammonia reacts instantaneously with

(3.18)

- (3.22) contd... the acid, the partial pressure of NH_3 at the gas-liquid interface is zero $\Rightarrow y_A^* = 0$. Mole fraction of NH_3 in the bulk gas = mole fraction of NH_3 in the gas leaving, $y_{AB} = 0.03$. Rate of absorption = $k_y \cdot a \cdot (y_{AB} - y_A^*) = 8.2 \times 10^{-5} \text{ kmol}/20 \text{ min}$
 Put the values to get $k_y \cdot (3.15 \times 10^3)(0.03 - 0) = 6.833 \times 10^{-8} \text{ kmol/s}$
 \Rightarrow Gas phase mass transfer coefficient, $k_y = 7.23 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$
- * * * *

(3.23) Area of the catalyst particles:

Volume of the catalyst = $\frac{50 \text{ gm}}{3800 \text{ kg/m}^3} = 1.316 \times 10^{-5} \text{ m}^3$; volume of a single particle (dia = 1 mm) = $\frac{\pi}{6} (10^{-3})^3 = 5.236 \times 10^{-10} \text{ m}^3$.
 Number of particles = $\frac{1.316 \times 10^{-5}}{5.236 \times 10^{-10}} = 25130$.

Area of 50 gm catalyst particles, $a = (25130)(\pi)(10^{-3})^2 = 0.079 \text{ m}^2$

The rate of reaction = the rate of transport of A from the bulk solution to the catalyst surface. The rate of change of concentration of A in the solution is

$$-V \frac{dC_b}{dt} = k_L \cdot a \cdot (C_b - C_s)$$

$$\Rightarrow - \int_{C_{b0}}^{C_b} \frac{dC_b}{C_b} = \frac{k_L a}{V} \int_0^t dt$$

$$\Rightarrow \ln \frac{C_{b0}}{C_b} = \frac{k_L a}{V} \cdot t$$

$$\Rightarrow \frac{C_{b0}}{C_b} = e^{\frac{k_L a}{V} \cdot t}$$

$$\Rightarrow \frac{C_{b0}}{C_b} = 12.06 \Rightarrow C_b = (0.083) \cdot C_{b0}$$

$$\Rightarrow \text{Conversion of A} = (1 - 0.083) \times 100\% = \underline{\underline{91.7\%}}$$

Given: $V = 2 \text{ liters} = 2 \times 10^{-3} \text{ m}^3$; $t = 1800 \text{ s}$
 $k_L = 3.5 \times 10^{-5} \text{ m/s}$
 $C_s = 0$ since the reaction is diffusion controlled (i.e. very fast)
 $V = \text{volume of solution in the reactor}$

(3.24) Diameter of a salt particle = $0.7 \text{ mm} = 7 \times 10^{-4} \text{ m}$; mass of a particle = $\frac{\pi}{6} (7 \times 10^{-4})^3 (2160 \text{ kg/m}^3) = 3.88 \times 10^{-7} \text{ kg}$

Number of particles in 500 gm (= 0.5 kg) salt = $\frac{0.5}{3.88 \times 10^{-7}} = 1.289 \times 10^6$

Calculation of mass transfer coefficient, k_L :

Area of the particles at the beginning = $\pi (7 \times 10^{-4})^2 \cdot (1.289 \times 10^6) = 1.984 \text{ m}^2 = a$

(3.19)

(3.24) Contd... Initial rate of dissolution = Initial rate of mass transfer

$$\Rightarrow 20.8 \text{ gm/s} = 0.0208 \text{ kg/s} = k_L \cdot a_t \cdot (C_s - C_b)$$

$$\Rightarrow k_L = \frac{0.0208}{(1.984)(350-0)} = 2.995 \times 10^{-5} \text{ m/s}$$

$C_s = \text{Surface concentration}$
 $\text{or solubility} = 350 \text{ kg/m}^3$
 $C_b = 0, \text{ initial conc.}$

Calculation of the time for complete dissolution

Note that the area of the particles and bulk concentration, both change with time.

Let m = mass of the particles at any time, t .

Diameter of a particle at time t $\Rightarrow \frac{\pi}{6} \cdot d^3 \rho_s = m / 1.289 \times 10^6 ; \rho_s = 2160 \text{ kg/m}^3$

$$\Rightarrow d = \sqrt[3]{\frac{m}{1.289 \times 10^6}} \text{ meter}$$

Area of the particles, $a_t = \pi [8.82 \times 10^{-4} \text{ m}^{1/3}]^2 (1.289 \times 10^6) = (3.15) m^{2/3} \text{ square meters}$

Initial mass of salt = 0.5 kg, salt dissolved = $(0.5 - m)$ kg

Bulk solution concentration at time t , $C_b = \frac{(0.5 - m) \text{ kg}}{10 \text{ liters}} = 100(0.5 - m) \frac{\text{kg}}{\text{m}^3}$

(Assume that the volume of solution \approx volume of water taken)

Rate of dissolution, $-\frac{dm}{dt} = k_L \cdot a_t \cdot (C_s - C_b)$

$$\Rightarrow -\frac{dm}{dt} = (2.995 \times 10^{-5}) \cdot [(3.15) m^{2/3}] \cdot [350 - 100(0.5 - m)] ; t=0, m=0.5$$

Integrate to get the time for complete dissolution, t_f , when $m=0$.

$$-\int_{m=0.5}^0 \frac{dm}{m^{2/3} (3+m)} = 9.434 \times 10^{-3} \int_{t=0}^{t_f} dt$$

The integral on the L.H.S.
is evaluated numerically,
value = 0.763
Note: make a change of variable
 $m = w^3$ before integration.

$$\Rightarrow 0.763 = 9.434 \times 10^{-3} \cdot t_f \Rightarrow t_f = 81 \text{ second.}$$

*

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*

(3.25) Volume of solution to be prepared in a batch = $1.2 \text{ m}^3 = 1200 \text{ kg}$

Final solution concentration = 5% \Rightarrow mass of solute = $(1200)(0.05) = 60 \text{ kg}$

\Rightarrow Initial charge = 1140 kg water + 60 kg solid reagent (0.75 mm particle size)

Let m = mass of the solid particles in the tank at any time, t .

Number of particles = $\frac{60 \text{ kg}}{(\pi/6) d_p^3 \cdot \rho_s} = \frac{60}{(\pi/6)(7.5 \times 10^{-4})^3 (1580)} = 1.719 \times 10^8$

Mass of a particle at time t , $\frac{m}{1.719 \times 10^8} = \frac{\pi}{6} (d_p^3) \cdot (1580)$

$$\Rightarrow d_p = 1.916 \times 10^{-4}, \text{ m.}^{1/3}, \text{ meters} \dots \dots (i)$$

(3.20)

5.25 Contd... Area of the particles at time $t \rightarrow a_t = \pi \cdot (1.916 \times 10^{-4} \text{ m}^{1/3})^2 / (1.719 \times 10^{-8})$

Solution concentration at time t ,

$$C_b = \frac{(60-m) \text{ kg}}{(1200-m) \times 10^{-3} \text{ meter}^3} = \frac{(60-m) \times 10^3}{(1200-m) \text{ meter}^3} \quad \dots \text{(ii)}$$

The dissolution mass transfer coefficient:

Correlation to be used [see Example 3.4]: $\frac{k_L d_p}{D} = 2 + 0.44 \left(\frac{N d_p \rho_L}{\mu} \right)^{0.504} \left(\frac{\mu}{\rho_L D} \right)^{0.355} \dots \text{(iii)}$
Given:

$$N = \text{agitator speed} = \frac{120}{60} = 2 \text{ r.p.s} ; \quad d_p \text{ given in Eq. (i)} ; \quad \rho_L = 1000 \text{ kg/m}^3$$

$$\mu = 0.98 \text{ cP} = 9.8 \times 10^{-4} \text{ kg/m.s} ; \quad D = 1.5 \times 10^{-9} \text{ m}^2/\text{s}$$

$$\text{Agitator Reynolds number} = \frac{(2)(1.916 \times 10^{-4} \text{ m}^{1/3})^2 \cdot (1000)}{9.8 \times 10^{-4}} = 0.0749 \text{ m}^{2/3}$$

$$Sc = \frac{9.8 \times 10^{-4}}{(1000)(1.5 \times 10^{-9})} = 653 . \quad \text{From Eq. (iv) above,}$$

$$k_L = \frac{1.5 \times 10^{-9}}{1.916 \times 10^{-4} \text{ m}^{1/3}} \cdot \left[2 + 0.44 \left(0.0749 \text{ m}^{2/3} \right)^{0.504} \left(653 \right)^{0.355} \right]$$

$$\Rightarrow k_L = 7.83 \times 10^{-6} \text{ m}^{-1/3} \left[2 + 1.19 \text{ m}^{0.336} \right] ; \quad \begin{aligned} &\text{Solubility of the solid,} \\ &\text{The instantaneous rate of dissolution is} \end{aligned}$$

$$-\frac{dm}{dt} = k_L \cdot a_t \cdot (C_s - C_b) ; \quad a_t \text{ is given in Eq. (ii), } C_b \text{ in Eq. (iii) above}$$

$$\Rightarrow -\frac{dm}{dt} = 7.83 \times 10^{-6} \text{ m}^{-1/3} \left[2 + 1.19 \text{ m}^{0.336} \right] \cdot (19.82) \text{ m}^{2/3} \cdot \left[70.6 - \frac{(60-m) \times 10^3}{1200-m} \right]$$

$$\Rightarrow -\frac{dm}{dt} = 1.552 \times 10^{-4} \text{ m}^{1/3} \left(2 + 1.19 \text{ m}^{0.336} \right) \left[70.6 - \frac{60-m \times 10^3}{1200-m} \right]$$

Integrate the above from $m=60 \text{ kg}$ (initial) to $m=0$ and obtain the dissolution time.

(3.21)

(3.26) Consider unit cross-section of the bed and a bed thickness dz at a height z from the end where air enters into it. Let C_b be the local bulk concentration of the naphthalene vapor in the air and dC_b be the increase in the concentration over this thickness dz . Superficial air velocity = u , m/s, assumed constant (since the naphthalene concentration is low). The differential mass balance equation over the thickness dz is

$$\frac{u \cdot dC_b}{C_{b1}} = k_c \bar{a} dz \cdot (C^* - C_b) \quad \boxed{\begin{array}{l} C^* = \frac{P^v}{RT}, \frac{\text{kmol}}{\text{m}^3}; P^v = \text{vap. pressure of} \\ \text{naphthalene} \\ \bar{a} = \text{specific surface area of the} \\ \text{bed, } \frac{\text{m}^2}{\text{m}^3 \text{ bed}} \\ k_c = \text{mass transfer coeff. for sublimation} \\ \text{volume} \end{array}}$$

$$\Rightarrow \int_{C_{b1}}^{C_{b2}} \frac{dC_b}{C^* - C_b} = \frac{k_c \bar{a}}{u} \int_{z=0}^z dz$$

$$\Rightarrow \ln \frac{C^* - C_{b1}}{C^* - C_{b2}} = \frac{k_c \bar{a}}{u} \cdot z \quad \dots \text{(i)}$$

$$\text{Given: } C^* = \frac{P^v}{RT} = \frac{1.7 \times 10^{-4}}{(0.08317)(293)} = 6.98 \times 10^{-6} \frac{\text{kmol}}{\text{m}^3}$$

$$C_{b1} = 0 \text{ (inlet air does not have any naphthalene);}$$

$$\frac{0.0138 \text{ mole \% naphthalene}}{100 \text{ kmol gas (20°C, 1 atm)}} = \frac{0.0138 \text{ kmol}}{(100)(22.4) \left(\frac{293}{273} \right) \text{ m}^3 \text{ gas}} = 5.74 \times 10^{-6} \frac{\text{kmol}}{\text{m}^3}$$

$$k_c = 70 \text{ m/h} = 0.01944 \text{ m/s; } \bar{a} = 140 \text{ m}^2/\text{m}^3; u = 0.2 \text{ m/s}$$

Put the values in Eq.(i) above.

$$\ln \frac{6.98 \times 10^{-6} - 0}{6.98 \times 10^{-6} - 5.74 \times 10^{-6}} = \frac{(0.01944)(140)}{0.2} \cdot z \Rightarrow \text{bed height, } z = 12.7 \text{ cm}$$

Estimation of k_c from the given correlation for the Colburn factor?

$$\text{Correlation: } j_D = 1.17 (Re)^{-0.415}; Re = \frac{dp \cdot u \cdot \rho}{\mu}; 10 \leq Re \leq 2500$$

$$\text{Given: } dp = 5 \times 10^{-3} \text{ m; } u = 0.2 \text{ m/s; } \mu/\rho = \nu = 1.5 \times 10^{-5} \text{ m}^2/\text{s.}$$

$$Re = \frac{(5 \times 10^{-3})(0.2)}{1.5 \times 10^{-5}} = 66.7 \Rightarrow j_D = 1.17 (66.7)^{-0.415} = 0.205$$

$$j_D = \frac{Sh}{Re \cdot Sc^{1/3}} \Rightarrow Sh = (0.205)(66.7)(2.586)^{1/3} \quad \boxed{Sc = \frac{\nu}{D} = \frac{1.5 \times 10^{-5}}{5.8 \times 10^{-6}} = 2.586}$$

$$\Rightarrow Sh = 18.76 = \frac{k_c \cdot dp}{D} \Rightarrow k_c = \frac{(18.76)(5.8 \times 10^{-6})}{5 \times 10^{-3}} = 0.0218 \text{ m/s}$$

This value compares well with the given value of 70 m/h
or 0.01944 m/s

3.27 Two correlations for Sh for two ranges of Re are given; a change from one correlation to the other is to be done at $Re = 2000$. First find out the sphere diameter d' corresponding to $Re = 2000$

$$Re = 2000 = \frac{d' u \rho_s}{\mu} = \frac{(d')(1m/s)(1000 kg/m^3)}{9.5 \times 10^{-4} kg/m \cdot s} \Rightarrow d' = 1.9 mm$$

$u = 1 m/s$ given
 $\rho_s = 1000 kg/m^3$
density of water

Let at any time t , the diameter of the sphere = d_s

Correlation 1: $Sh = 0.3 Re^{0.593}$ $Sc = 0.3 \left[\frac{(d_s)(1)(1000)}{9.5 \times 10^{-4}} \right]^{0.593} \cdot (633)^{1/3}$

$$Sc = \frac{\mu}{\rho_s D} = \frac{9.5 \times 10^{-4}}{(1000)(1.5 \times 10^{-9})} = 633$$

$$\Rightarrow \frac{k_L d_s}{1.5 \times 10^{-9}} = (0.3)(1.053 \times 10^6 d_s)^{0.593} (8.584) \Rightarrow k_L = 1.44 \times 10^{-5} \frac{d_s^{-407}}{(d_s)^{-0.526}} \dots \dots \text{(i)}$$

Correlation 2: $Sh = \frac{k_L d_s}{1.5 \times 10^{-9}} = (0.803) \left[\frac{(d_s)(1)(1000)}{9.5 \times 10^{-4}} \right]^{0.474} (633)^{1/3}$

$$\Rightarrow k_L = 7.4 \times 10^{-6} \frac{d_s^{-0.526}}{d_s} \dots \dots \text{(ii)}$$

Rate of change of radius of the sphere because of dissolution:

$$-\frac{d}{dt} \left[\frac{\pi}{6} d_s^3 \rho_s \right] = k_L \cdot \pi d_s^2 (C_s - C_b) \Rightarrow -\frac{d}{dt} (d_s) = \frac{2 k_L}{\rho_s} (C_s - C_b)$$

There is no solute in the bulk water $\Rightarrow C_b = 0$. The time for change of diameter from d_1 to d_2 can be calculated by integrating the above equation. The expression for k_L is to be substituted from Eq (i) and (ii) over the two ranges of diameter.

$$-\int_{d_1}^{d_2} \frac{d(d_s)}{k_L} = \int_{t=0}^t \frac{2 \cdot C_s}{\rho_s} dt \Rightarrow \int_{d_s=d_2}^{d'} \frac{d(d_s)}{7.4 \times 10^{-6} \frac{d_s^{-0.526}}{d_s}} + \int_{d'=d_1}^{d_1} \frac{d(d_s)}{1.44 \times 10^{-5} \frac{d_s^{-407}}{d_s}} = \frac{2 \cdot C_s}{\rho_s} t$$

Given: $d_2 = 10^{-3} m$; $d_1 = 5 \times 10^{-3} m$; $d' = 1.9 \times 10^{-3} m$ (see above); $C_s = 30 \frac{kg}{m^3}$,

$\rho_s = 1800 kg/m^3$ [C_s = solubility of the solid, ρ_s = solid density]

$$\Rightarrow \frac{(1.9 \times 10^{-3})^{1.526} - (10^{-3})^{1.526}}{(7.4 \times 10^{-6})(1.526)} + \frac{(5 \times 10^{-3})^{1.407} - (1.9 \times 10^{-3})^{1.407}}{(1.44 \times 10^{-5})(1.407)} = \frac{(2)(30)}{1800} \cdot t$$

\Rightarrow Required time, $t = 754 \text{ second}$

(3.28) Radius of the jet = r ; length = l ; velocity = u ; since the jet has a 'rod-like flow', flowrate = $\pi r^2 \cdot u = 15 \text{ cm}^3/\text{s} \Rightarrow u = \frac{15}{\pi r^2} \text{ cm/s}$. Contact time of the liquid elements (on the jet surface) with the gas, $t_c = l/u = l(\pi r^2/15)$.

The 'Penetration theory' applies. Mass transfer coeff, $k_L = 2 \left[\frac{D}{\pi t_c} \right]^{1/2} = 2 \left[\frac{D}{\pi} \cdot \frac{15}{\pi r^2} \right]^{1/2}$

Area of the jet, $a = 2\pi rl$. Rate of gas absorption, $Q = k_L \cdot a \cdot (C^* - C_b) \dots (i)$

Calculation of solubility of CO_2 in water:

Use Henry's law. $x^* = P/H = \frac{1.05 \text{ atm}}{1540 \text{ atm}} = 6.82 \times 10^{-4}$ mole fraction.

This concentration is pretty small. So

$$x^* = \frac{6.82 \times 10^{-4} \text{ mol CO}_2}{1 \text{ mol solution}} = \frac{6.82 \times 10^{-4} \text{ mol CO}_2}{18 \text{ gm water}} = \frac{6.82 \times 10^{-4} \text{ mol CO}_2}{18 \text{ cm}^3 \text{ water}}$$

$$\Rightarrow C^* = 3.789 \times 10^{-5} \text{ g mol CO}_2/\text{cm}^3 \text{ water. } [\text{The mol wt. of solution} \approx \text{mol wt. of water}]$$

$$\text{From Eq (i) above, } Q = 2 \left[\frac{D}{\pi} \cdot \frac{15}{\pi r^2 l} \right]^{1/2} \cdot 2\pi r l \cdot (3.789 \times 10^{-5} - 0) \quad [\text{from CO}_2]$$

$$= 4 \left[D \cdot 15 l \right]^{1/2} (3.789 \times 10^{-5}) = 1.174 \times 10^{-3} \sqrt{D} \text{ g mol/s}$$

$$\text{Given absorption rate} = (30.5 - 23.5) \text{ cm}^3/\text{min} = \left(\frac{7 \times 1.05}{22414} \right) \left(\frac{273}{298} \right) = 3.004 \times 10^{-4} \text{ g mol/min}$$

$$\Rightarrow 1.174 \times 10^{-3} \sqrt{D} = 3.004 \times 10^{-4} / 60 \Rightarrow D = 1.82 \times 10^{-5} \text{ cm}^2/\text{s}$$

* * * *

(3.29) Given: wind speed, $u = 12 \text{ km/h} = 12000/3600 = 3.33 \text{ m/s}$; mol. wt. of toluene, $M_w = 92.1$; $M_w = 18$.

$$k_e = (0.67)(3.33)^{0.78} (18/92.1)^{1/3} = 0.994 \text{ cm/s} = 9.94 \times 10^{-3} \text{ m/s}$$

Evaporation flux, $N_A = k_e(C_s - C_b)$; $C_b = \text{toluene conc. in bulk air} = 0$; $C_s = \text{toluene conc. at the liquid surface (it corresponds to the vapor pressure of toluene)} = \frac{(28.44/760) \text{ atm}}{(0.0821)(298)} = 1.53 \times 10^{-3} \text{ kmol/m}^3$

$$\text{Evaporation flux} = (9.94 \times 10^{-3})(1.53 \times 10^{-3} - 0) = 1.52 \times 10^{-5} \text{ kmol/m}^2 \text{s}$$

Time for complete disappearance:

$$\text{Amount of toluene spilled} = 100 \text{ gal} = [(100/7.48)/35.3 \text{ m}^3] \times 860 \text{ kg/m}^3$$

$$= 325.7 \text{ kg. Area of spill} = 75 \text{ m}^2. \text{ Moles toluene/m}^2 = \frac{325.7}{(92.1)(75)} = 0.04715$$

$$\text{Evaporation time} = \frac{0.04715 \text{ kmol/m}^2}{1.52 \times 10^{-5} \text{ kmol/m}^2 \text{s}} = \frac{3100 \text{ s}}{0.86 \text{ hr}} = \frac{\text{kmol/m}^2}{\text{mole/m}^2}$$

(3.30) Follow the procedure of solution of Prob. 3.6

(3.24)

(3.31) First we check if the boundary layer over water is partly turbulent. Transition Reynolds number = 3×10^5 .

$$\Rightarrow Re = \frac{U_z z}{\nu} = 3 \times 10^5 \Rightarrow \frac{(5.364)(z)}{1.58 \times 10^{-5}} = 3 \times 10^5 \quad \begin{cases} U_z = 5.364 \text{ m/s} \\ \nu = 1.7 \times 10^{-4} \text{ ft}^2/\text{s} \\ = 1.58 \times 10^{-5} \text{ m}^2/\text{s} \end{cases}$$

$$\Rightarrow z = 0.884 \text{ m}$$

So, the boundary layer becomes turbulent after $z = 0.884$. The mass transfer correlation for the laminar boundary layer should be used upto this distance; beyond this the turbulent boundary layer correlation applies.

Laminar region: $Sh_z = 0.332 \frac{U_z}{\nu} z^{1/2} Sc^{1/3}$

$$\Rightarrow \frac{k_c z}{2.6 \times 10^{-5}} = (0.332) \left[\frac{z \cdot (5.364)}{1.58 \times 10^{-5}} \right]^{1/2} (0.607)^{1/3}$$

$$\Rightarrow k_c = \frac{0.00426 \cdot z^{-1/2} \text{ m/s}}{0.884}$$

Schmidt number,
 $Sc = \frac{\nu}{D} = \frac{1.7 \times 10^{-4}}{2.8 \times 10^{-5}} = 0.607$
 $D = 2.8 \times 10^{-5} \text{ ft}^2/\text{s} = 2.6 \times 10^{-5} \frac{\text{m}^2}{\text{s}}$

Average mass transfer coefficient, $(k_c)_{l,av} = \frac{1}{0.884} \int_{z=0}^{0.884} (0.00426) z^{-1/2} dz = 0.00906 \text{ m/s}$

Turbulent region: $Sh_z = 0.0292 \frac{U_z}{\nu} z^{0.8} Sc$

$$\Rightarrow \frac{k_c z}{2.6 \times 10^{-5}} = (0.0292) \left[\frac{z \cdot (5.364)}{1.58 \times 10^{-5}} \right]^{0.8} (0.607) \Rightarrow k_c = 0.01225 z^{0.2} \text{ m/s}$$

Average value of k_c in the turbulent region ($0.884 \leq z \leq 3 \text{ m}$)

$$(k_c)_{t,av} = \frac{1}{(3 - 0.884)} \int_{z=0.884}^3 (0.01225) z^{-0.2} dz = \frac{(0.01225)}{(0.8)(2.116)} \left[(3)^{0.8} - (0.884)^{0.8} \right]$$

$$= 0.0109 \text{ m/s}$$

Consider 1 m breadth of the pan. Area of the laminar region,

$$a_l = (0.884)(1) = 0.884 \text{ m}^2; \text{ area of the turbulent region} = 2.116 \text{ m}^2 = a_t$$

Vapor pressure of water = $26 \text{ mmHg} = \frac{26}{760} = 0.0342 \text{ atm}; T = 80^\circ\text{F} = 26.6^\circ\text{C}$

Molar concentration of water vapor at the surface = $\frac{0.0342}{(0.0821)(299.6)}$

Concentration of water vapor in bulk air, $C_b = 0$

Rate of vaporization of water

$$Q = (k_c)_{l,av} a_l (C_s - C_b) + (k_c)_{t,av} a_t (C_s - C_b)$$

$$= 1.39 \times 10^{-3} \frac{\text{kmol}}{\text{m}^3} = C_s$$

(3.31) Contd...

$$\Rightarrow Q = (0.00906)(0.884)(1.39 \times 10^{-3}) + (0.0109)(2.116)(1.39 \times 10^{-3}) \\ = 1.113 \times 10^{-5} + 3.206 \times 10^{-5} = 4.319 \times 10^{-5} \text{ kmol/s}$$

Amount of water per meter breadth of the pan (depth = 10 mm = 0.01 m)

$$= (3 \text{ m} \times 1 \text{ m} \times 0.01 \text{ m})(1000/18 \text{ kmol/m}^3) = \underline{1.666 \text{ kmol}}$$

$$\text{Time of evaporation, } t = \frac{1.666 \text{ kmol}}{4.319 \times 10^{-5} \text{ kmol/s}} = 385908 = \underline{10.72 \text{ hr}}$$

(3.32)

Oxygen concentration in water $\frac{C_b}{C_s} = 5 \text{ ppm} = \frac{5/32 \text{ kmol}}{10^6 \text{ kg}/(1000 \text{ kg/m}^3)} = 1.562 \times 10^{-4}$

Partial pressure of oxygen in air, $p = 0.21 \text{ atm.}$ Equilibrium mole fraction in water, $x = p/H = 0.21/4.2 \times 10^4 = \underline{5 \times 10^{-6}}$

Concentration at saturation (Very dilute solution \Rightarrow Av. mol. wt of solution \approx mol wt of the solvent.) $= \frac{5 \times 10^{-6} \text{ kmol}}{(18/1000) \text{ m}^3} = \frac{2.78 \times 10^{-4} \text{ kmol/m}^3}{C_s}$

Calculation of the mass transfer coefficient for oxygen absorption.

Contact time of a liquid element on the rotating disk with air, (by Penetration Theory)

$$t_c = \frac{2\pi - \theta}{2\pi N_r} = \frac{360 - 120}{(360)(0.5)} = \underline{1.333 \text{ s.}} \quad [N_r = \text{r.p.s. of the disk} = 30/60 = 0.5]$$

Mass transfer coefficient [Eq.(3.38)], $k_L = 2 \left[\frac{D_{AB}}{\pi t_c} \right]^{1/2} = 2 \left[\frac{2.05 \times 10^{-9}}{(\pi)(1.333)} \right]^{1/2}$

Oxygen flux, $N_A = k_L (C_s - C_b)$

$$= 4.425 \times 10^{-5} (2.78 \times 10^{-4} - 1.562 \times 10^{-4}) = \frac{4.425 \times 10^{-4} \text{ m/s}}{5.39 \times 10^{-9} \text{ kmol/m}^2 \text{ s}}$$

Calculation of the exposed area of disk

Area of the section APB (Submerged in water)

= (Area of the sector OAPB) - (Area of $\triangle OAB$)

$$= \pi R^2 \cdot \frac{2\theta_1}{2\pi} - \frac{1}{2} \cdot R \cos \theta_1 \cdot R \sin \theta_1; \theta_1 = 60^\circ, R = 1.5 \text{ m}$$

$$= \frac{\pi}{3} R^2 - \left(\frac{1}{2} \cdot \frac{R}{2} \cdot \frac{\sqrt{3}R}{2} \right) \times 2 = \frac{\pi R^2}{3} - \frac{R^2 \sqrt{3}}{4} = 1.382 \text{ m}^2$$

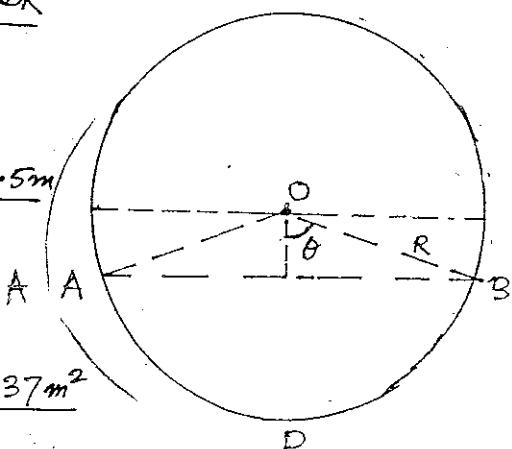
Considering both sides of a disk,

$$\text{Exposed area (both sides), } A = 2\pi R^2 = 2.764 \text{ m}^2$$

Rate of oxygen absorption = $a N_A$

$$= (11.37 \text{ m}^2)(5.39 \times 10^{-9} \text{ kmol/m}^2 \text{ s}) = 6.13 \times 10^{-8} \text{ kmol/s} \quad \underline{\text{Fig. Prob 3.32}} \\ = 7.06 \times 10^{-3} \text{ kg/hr}$$

(3.25)



3.26

3.33 Since the velocity of water is u (uniform and constant), a liquid element at position $z=z$ has travelled for a time t (measured from time $t=0$ when it was $z=0$) given by $t = z/u$. or dissolved oxygen (DO) sag,

Let $D = C_s - C$ be the 'BOD deficit', where C_s is the solubility (or saturation concentration) of oxygen in water and C is the actual concentration. The rate of change of oxygen concentration is

$$\frac{dC}{dt} = \frac{-dD}{dt}$$

Consider unit volume of water at position z and make an oxygen balance.

Rate of change of oxygen concentration

$$= \text{Rate of absorption of oxygen from air} - \text{Rate of consumption of oxygen for BOD destruction}$$

$$\Rightarrow -\frac{dC}{dt} = \frac{k_L \cdot b dz (C_s - C)}{a' \cdot dz} - k_1 L t = \frac{k_L b}{a'} (C_s - C) - k_1 L o e^{-k_1 t}$$

$$\Rightarrow -\frac{dD}{dt} = k_m D - k_1 L o e^{-k_1 t}; k_1 = \text{mass transfer coefficient of oxygen, and } k_m = \frac{k_L b}{a'} = \text{volumetric mass transfer coefficient.}$$

$$\Rightarrow \frac{dD}{dt} + k_m D = k_1 L o e^{-k_1 t} \dots (i); \text{at } t=0 \text{ (or, } z=0\text{)}, D_0 = C_s - C_0 \text{ and } L = L_o. \text{ Here } C_0 \text{ is the oxygen concentration in water at } z=0 \text{ (or } t=0\text{).}$$

Multiply by the 'integration factor' $I = \exp[\int k_m dt] = e^{k_m t}$

$$e^{k_m t} \cdot \frac{dD}{dt} + k_m D e^{k_m t} = k_1 L o e^{-k_1 t} \cdot e^{k_m t} \Rightarrow \frac{d}{dt}[D e^{k_m t}] = k_1 L o e^{(k_m - k_1)t}$$

$$\Rightarrow D e^{k_m t} = \frac{k_1 L o}{(k_m - k_1)} e^{(k_m - k_1)t} + K'; K' = \text{integration constant.}$$

$$\text{At } t=0, D=D_0 \Rightarrow D_0 = \frac{k_1 L o}{(k_m - k_1)} + K' \Rightarrow K' = D_0 - \frac{k_1 L o}{(k_m - k_1)}$$

Solution for D is

$$D = \frac{k_1 L o}{(k_m - k_1)} e^{-k_1 t} + \left[D_0 - \frac{k_1 L o}{(k_m - k_1)} \right] e^{-k_m t} \dots . . . (ii)$$

Maximum DO sag (or minimum dissolved oxygen) occurs when $\frac{dD}{dt} = 0$.

Put $\frac{dD}{dt} = 0$ in Eq. (i) to get the corresponding time (the 'critical time'), i.e.

3.27

$$3.33 \text{ Contd...} \Rightarrow k_m D_c = k_1 L_o e^{-k_1 t_c} \Rightarrow D_c = (k_1 L_o / k_m) e^{-k_1 t_c} \dots (iii)$$

From Eqs(ii) and (iii), $\frac{k_1 L_o}{k_m} e^{-k_1 t_c} = \frac{k_1 L_o}{k_m - k_1} e^{-k_1 t_c} + \left[D_o - \frac{k_1 L_o}{(k_m - k_1)} \right] e^{-k_m t_c}$

$$\Rightarrow k_1 L_o \left(\frac{1}{k_m} - \frac{1}{k_m - k_1} \right) e^{-k_1 t_c} = \left[D_o - \frac{k_1 L_o}{(k_m - k_1)} \right] e^{-k_m t_c}$$

$$\Rightarrow -\frac{k_1^2 L_o}{k_m (k_m - k_1)} \exp(k_m - k_1) t_c = D_o - \frac{k_1 L_o}{k_m - k_1}$$

$$\Rightarrow \exp(k_m - k_1) t_c = -\frac{k_m (k_m - k_1)}{k_1^2 L_o} \left[\frac{k_1 L_o}{k_m - k_1} - D_o \right] = \frac{k_m}{k_1} - \frac{D_o \cdot k_m (k_m - k_1)}{k_1^2 L_o}$$

$$\Rightarrow t_c = \frac{1}{k_m - k_1} \ln \left[\frac{k_m}{k_1} \left\{ 1 - \frac{D_o}{L_o} \cdot \left(\frac{k_m - k_1}{k_1} \right) \right\} \right] \therefore t_c = Z_c / u$$

(or the minimum O₂ conc.)

Thus the maximum DO lag occurs at a time t_c when the water has moved through a distance Z_c from the reference point.

* * * *

3.34 Refer to the figure under the problem. The outer radius of the pellet (consisting of a porous ash layer on the unreacted solid 'core') is r_o (assumed constant). At any time, t, the radius of the core is r_c. There are effectively two resistances in series so far as diffusion of the gaseous reactant, A, is concerned — (i) the external gas-film resistance (the corresponding mass transfer coefficient is k_c), and (ii) diffusional resistance of the porous ash layer. An effective diffusivity, D_e is often used to describe diffusion through a porous layer; often we take D_e = ε · D, ε = porosity of the layer.

Steady state diffusion of the gaseous reactant, B, through the ash layer from r = r_o to r = r_c (surface of the 'unreacted' core) can be described by Eq(2.70).

$4\pi r^2 N_B = W$; $N_B = D_e \frac{dc_B}{dr}$ (at a low concentration; since C_B increases with r, the -ve sign is not used)
The equation is integrated from r = r_o, C_B = C_{B0} to r = r_c, C_B = 0.

$$3.34) \text{ contd...} \quad \int_{C_{B0}}^0 dC_B = \frac{W}{4\pi D_e} \cdot \int_{r_0}^{r_c} \frac{dr}{r^2} \Rightarrow C_{B0} = \frac{W}{4\pi D_e} \left(\frac{1}{r_c} - \frac{1}{r_0} \right)$$

$$\Rightarrow W = \frac{4\pi D_e C_{B0} r_c}{(1 - r_c/r_0)} \quad \dots \text{(i)}$$

(3.28)

The rate of diffusion of B through the gas film = $4\pi r_0^2 k_c (C_{Bb} - C_{B0}) \dots \text{(ii)}$
Here C_{Bb} is the concentration of B in the bulk gas. Eliminate C_{B0} by equating (i) and (ii). Note that C_{B0} is unknown.

$$C_{Bb} - C_{B0} = \frac{W}{4\pi r_0^2 k_c}, \text{ and } C_{B0} = \frac{W(1 - r_c/r_0)}{4\pi D_e r_c}$$

$$\Rightarrow C_{Bb} = W \left[\frac{1}{4\pi r_0^2 k_c} + \frac{1 - r_c/r_0}{4\pi D_e r_c} \right] \Rightarrow W = \frac{4\pi r_0^2 C_{Bb}}{(1/k_c) + (1 - r_c/r_0) \cdot r_0^2 / D_e r_c} \dots \text{(iii)}$$

If ρ_A is the density of the solid, and M_A is its molecular weight, moles of A in the core at any time = $\frac{(4/3)\pi r_c^3 \rho_A / M_A}{\text{assumed}}$. The rate of consumption of the core is now equated to the rate of supply of B through the ash layer (pseudo-steady state assumed).

$$-\frac{d}{dt} \left(\frac{4}{3}\pi r_c^3 \rho_A / M_A \right) = W = \frac{4\pi r_0^2 C_{Bb}}{(1/k_c) + (1 - r_c/r_0) \cdot r_0^2 / D_e r_c}$$

$$\Rightarrow -\frac{dr_c}{dt} = \frac{\left(\frac{r_0}{r_c}\right)^2 C_{Bb} \cdot (M_A / \rho_A)}{(1/k_c) + (1 - r_c/r_0) \cdot r_0^2 / D_e r_c} \quad \dots \text{(iv)}$$

Integration of this equation gives the time required for reduction of the radius of the core from the initial value of r_0 to the value r_c .

A Special case : In many situations the ash layer is found to offer the 'controlling resistance'. This means that the film mass transfer coefficient, k_c , is large and $1/k_c \approx 0$.

Putting $1/k_c = 0$ and rearranging, eq. (iv) reduces to :

$$-\left[\xi(1-\xi)\right] d\xi = \frac{C_{Bb} \cdot D_e \cdot M_A}{r_0^2 \rho_A}, \text{ where } \xi = \text{dimensionless core radius} = r_c/r_0$$

Integrate from $\xi=1$ at $t=0$, to $\xi=\xi$ at $t=t$

$$-\int_1^\xi (\xi - 1) d\xi = \frac{C_{Bb} \cdot D_e \cdot M_A}{r_0^2 \rho_A} \int_0^t dt \Rightarrow (1 - 3\xi^2 + 2\xi^3) = \frac{6 C_{Bb} D_e \cdot M_A \cdot t}{r_0^2 \cdot \rho_A} \quad \text{if } \xi = T$$

3.29

3.34) Contd... The above equation gives the relation between the radius of the shrinking core and time. Here $\tau = \frac{6 C_{Bb} D_e M_A}{r_0^2 P_A} \cdot t$ is the dimensionless time.

The time for complete reaction, t_f , of the sphere is obtained by putting $\xi = 0 \Rightarrow \tau = 1$

$$\Rightarrow t_f = \frac{r_0^2}{6 C_{Bb} D_e M_A} \cdot \frac{P_A}{t}$$

* * *

3.35) Let r_0 = initial radius of a particle ($r_0 = d_p/2$); r = particle radius at any time, t ; C_b = bulk concentration of the solute ≈ 0 . The rate of dissolution can be expressed as

$$-\frac{d}{dt} \left(\frac{4}{3} \pi r^3 P_s \right) = k_L \cdot 4\pi r^2 (C_s - C_b) \Rightarrow -\frac{dr}{dt} = \frac{k_L C_s}{P_s}; \text{ at } t=0, r=r_0$$

Integrating, $r_0 - r = \frac{k_L C_s}{P_s} \cdot t$. Note that $M_0 = \frac{4}{3} \pi r_0^3 P_s \cdot n$ and $M = \frac{4}{3} \pi r^3 P_s \cdot n$; M_0 and M \rightarrow mass of particles at time $t=0$ and $t=t$, $n = \text{total number of particles}$.

$$\Rightarrow \left(\frac{3M_0}{4\pi P_s \cdot n} \right)^{1/3} - \left(\frac{3M}{4\pi P_s \cdot n} \right)^{1/3} = \frac{k_L C_s}{P_s} \cdot t \Rightarrow 1 - \left(\frac{M}{M_0} \right)^{1/3} = \frac{k_L C_s}{r_0 P_s} \cdot t$$

$$\Rightarrow 1 - \left(\frac{M}{M_0} \right)^{1/3} = k' t$$

Data : Initial mass of particles = 75 mg ; $\overset{= M_0}{\text{solution volume}} = 1000 \text{ ml}$, $t, \text{ min}$

$t, \text{ min}$	0	10	20	30	40	50
liqu. conc., mg/ml	0	0.0197	0.0374	0.0510	0.0595	0.0650
Mass of undissolved particles, M , mg	75	75 - 19.7	37.6	24	15.5	10
$(M/M_0)^{1/3}$	0	$(55.3/75)^{1/3}$ = 0.902	0.794	0.684	0.592	0.511

Prepare a plot of $(M/M_0)^{1/3}$ against t .

It is a straight line of slope -0.96

\Rightarrow "Dissolution rate constant",

$$k' = 0.96/\text{min.} = 0.016 \text{ sec}^{-1}$$

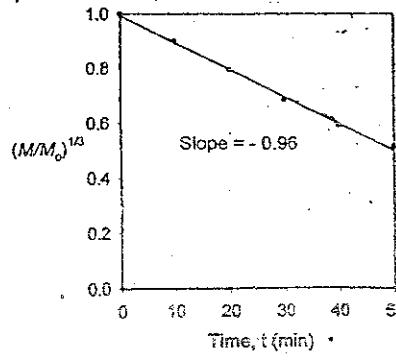


Fig Prob 3.35

CHAPTER 4: INTERPHASE MASS TRANSFER

Short and MC Q's

(4.1)

- ① (a) Absorption of $O_2 \rightarrow$ (ii) liquid-film resistance controlling; CO_2 in water \rightarrow (ii); $H_2S \rightarrow$ (ii); $SO_2 \rightarrow$ more resistance in the liquid phase but not controlling; $Cl_2 \rightarrow$ (iii) both resistances are substantial (moderate solubility in water); $HCl \rightarrow$ gas-film resistance controlling (very high solubility in water); $NH_3 \rightarrow$ gas-film controlling (i).

(b) Absorption in $NaOH$ solution: Absorption of all the three (CO_2 , H_2S and SO_2 have 'high solubility' in aqueous caustic since they irreversibly react with $NaOH$).

(c) All the three cases are gas-film controlled (high solubility in amine oil).

(d) Desorption of $NH_3 \rightarrow$ gas-film controlled; TCA \rightarrow liquid-film controlled; benzene \rightarrow liquid phase controlled (the latter two have low solubility in water).

(e) It is normally controlled by both the resistances. Since the reaction of CO_2 with the amine is 'reversible' the solubility is limited.

- ② The relevant form of Henry's law: $p = H'C$, $H' = 990 \text{ kPa}/(\text{gmol/liter})$
 If p is low, C is small and the total molar concentration of the solution is virtually the same as that of the solvent, here water.
 $\Rightarrow C_t = \frac{1000 \text{ gm/liter}}{18} = 55.55 \text{ gmol/liter} \Rightarrow$ mole fraction of the solute $= \frac{C}{C_t} = x \Rightarrow C = 55.5x$. If p is the partial pressure of H_2S , $p = 990 \cdot C$ (given) $= (990)(55.5x) = 5.5 \times 10^4 x \Rightarrow p = H \cdot x$, $H = 5.5 \times 10^4 \text{ kPa}$.

Given: the total pressure, $P = \text{total pressure} = 1 \text{ atm} = 101.3 \text{ kPa}$,

$$\Rightarrow \text{Mole fraction of } H_2S \text{ in the gas, } y = \frac{p}{P} = \frac{(5.5 \times 10^4 \text{ kPa}) \cdot x}{101.3 \text{ kPa}}$$

$$\Rightarrow y = 542.4x; \text{ i.e. } m = 542.4$$

$$\text{If } p = 50 \text{ mm Hg} = (50/760)(101.3) = 6.66 \text{ kPa}, C = \frac{p}{H} = \frac{6.66 \text{ kPa}}{990 \text{ kPa}/(\text{gmol/liter})}$$

$$\Rightarrow C = 6.73 \times 10^{-3} \text{ gmol/liter} = (6.73 \times 10^{-3})(34) \text{ gm/liter} = \underline{\underline{229 \text{ mg/liter}}}$$

(Mol. wt. of $H_2S = 34$)

(4.2)

- (3) Refer to fig. 4.7. $G_1 = 35 \text{ kmol/h}$, $y_1 = 0.1$; $y_1 = \frac{0.1}{0.9} = 0.111$ molar ratio;
 $G_s = G(1-y_1) = (35)(1-0.1) = 31.5 \text{ kmol/h}$ (solute-free basis);
 $L_s = 40 \text{ kmol/h}$; inlet liquid is solute-free $\Rightarrow x_2 = 0$.

Driving force at the top, $(\Delta Y)_{\text{top}} = y_2 - y_2^* = 0.00555$ (given).
 (Overall, gas phase basis)

$$y_2^* = \alpha x_2 = (1.1)(0) = 0 \Rightarrow y_2 = 0.00555.$$

Overall material balance, $G_s(y_1 - y_2) = L_s(x_1 - x_2)$

$$\Rightarrow (31.5)(0.111 - 0.00555) = 40(x_1 - 0) \Rightarrow x_1 = 0.0831.$$

$$y_1^* = \alpha x_1 = (1.1)(0.0831) = 0.0914$$

Driving force at the bottom, $(\Delta Y)_{\text{bottom}} = y_1 - y_1^* = 0.111 - 0.0914 = 0.0196$.

- (4) (a) Countercurrent absorption; (b) Countercurrent stripping;
 (c) Cocurrent absorption; (d) Cocurrent stripping.

- (5) Mole ratio, units have to be used in general. At low concentrations (say $x, y \leq 0.05$), mole fraction (or mass fraction) can be used without much error.

All other quantities remaining constant, the number of theoretical trays required decreases with \bar{A} for absorption. Similarly,

N decreases with decreasing \bar{A} (or increasing y_A = stripping factor) in case of stripping.

- (6) Given $y = 2.5x \Rightarrow \frac{Y}{1+Y} = 2.5 \frac{x}{1+x} \Rightarrow Y = \frac{2.5x}{1-1.5x} = \phi(x)$

Slope of this curve, $\frac{dy}{dx} = \frac{2.5}{1-1.5x} + \frac{3.75x}{(1-1.5x)^2}$

At $x = 0$, $\frac{dy}{dx} = 2.5$; at $x = 0.1$, $\frac{dy}{dx} = \frac{2.5}{1-(1.5)(0.1)} + \frac{(3.75)(0.1)}{(1-0.15)^2} = 3.46$

- (7) (ii) $\rightarrow S_A < S_B$; (8) (i) increases with temperature; (9) (ii) \rightarrow molar flux; (10) (ii) $\rightarrow 0.02 \text{ mm/s}$; (11) (ii) $\rightarrow 5 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}}$;

[For gas-phase transport, $k_c \sim 10^2 \text{ m/s} \Rightarrow k_g = \frac{P}{RT} \cdot k_c \sim \frac{10^2}{25} \rightarrow 4 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}}$

$$k_g = \frac{1}{P} k_y \sim 4 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}} \quad \text{kmol/m}^2 \cdot \text{s}^2 (\Delta y)$$

- (12) (ii) \rightarrow gas phase; (13) (ii) 1.5 [$k_y \sim 2 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot \text{bar}} \rightarrow 0.72 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot \text{bar}}$]
 O_2 undergoes instantaneous reaction
 $\Rightarrow k_y \sim 1 \frac{\text{m}^2 \cdot \text{h}}{\text{kmol}}$

(7.3)

- (14) Henry's law $\rightarrow p = H \cdot x$, $H = 600 \text{ bar/mole fraction}$;
 Mole fraction of chlorine in the bulk gas $= \frac{P_b}{P} = \frac{0.1 \text{ bar}}{1.2 \text{ bar}} = 0.0833$.
 (partial pressure of chlorine = 0.1 bar; total press = 1.2 bar)
 Since absorption of chlorine in water occurs, the interfacial conc. (y_i) of chlorine in nitrogen must be less than y_b .

$y_i = \frac{P_i}{P} = \frac{600}{1.2} x_i$. Consider the set of values of x_i and y_i , given under (i). If we take $x_i = 0.01$, $y_i = \frac{(600)}{1.2}(0.01) = 5$. Since this calculated value is more than the value in the bulk ($y_b = 0.0833$), $x_i = 0.01$ cannot be the interfacial conc. Thus the set of values of $x_i, y_i \rightarrow (0.01, 0.0833)$ cannot be a probable set of interfacial concentrations. It can be shown that the other two sets are not improbable.

- (15) The problem is similar to No. 3 solved before. $x_2 = 0 \Rightarrow y_2^* = 0$.
 $(\Delta Y)_{\text{top}} = Y_2 - Y_2^* = 0.001$ given. $\Rightarrow Y_2 = 0.001$.

Write down the material balance: $70(0.1 - 0.001) = 50(x_1 - 0)$
 $\Rightarrow x_1 = 0.1385$. Corresponding equil. value, $Y_1^* = 6.6(0.1385) = 0.0831$

$$(\Delta Y)_{\text{bottom}} = Y_1 - Y_1^* = 0.1 - 0.0831 = 0.0169$$

- (16) Feed solution $\rightarrow 30 \text{ kmol/h, } 10^\circ\text{C. Solute-free flowrate} = \frac{27 \text{ kmol/R}}{A_s}$
 Maximum solute recovery in a cross-current contact unit is achieved if equal amounts of the solvent is used in each stage $\Rightarrow 27/3 = 9 \text{ kmol/h} = B_s \Rightarrow -B_s/A_s = -\frac{9}{27} = -0.333$.

- (17) Given: $p = 40.7x$. The gas is 'pure' SO_2 and $p = 0.1 \text{ atm}$.
 $\Rightarrow x = 0.1/40.7 = 0.002457 = x_i$ (since the gas is pure and there is no gas-side resistance.). Also, the bulk liquid concentration is $x_b = 0.0001$ which is less than x_i . Driving force for absorption $= x_i - x_b = 0.002357$ and occurs as a 'step function' at the instant of exposure \Rightarrow instantaneous flux is extremely large.

- (18) Absorption factor, $A = L_s/\alpha G_s = 1 \Rightarrow \alpha = L_s/G_s = \frac{49.5}{38} = 1.3026$ (since the concentration gradient is infinite).

$$\Rightarrow \ln \alpha = \ln(1.3026) = 0.2644 = 0.3512 - (26.462/T)$$

$$\Rightarrow T = 305 \text{ K} = \underline{\underline{32^\circ\text{C}}} \text{ (iii)}$$

(4.4)

- (19) $G_1 = 80 \text{ kmol/h}$, 10% A, i.e. $y_1 = 0.1 \Rightarrow G_S = (80)(1-0.1) = 72 \text{ kmol/h}$,
 and $\gamma_1 = \frac{0.1}{0.9} = 0.111$. Total solute entering = $(80)(0.1) = 8 \text{ kmol/h}$.
 95% of A is removed \Rightarrow Moles solute removed = $(80)(0.1)(0.95) = 7.6 \frac{\text{kmol}}{\text{h}}$;
 moles solute remaining = $8 - 7.6 = 0.4 \text{ kmol/h}$.
 Liquid input, $L_2 = 152 \text{ kmol/h}$, $x_2 = 0 \Rightarrow L_S = 152$, $x_2 = 0$.
 Material balance: Solute removed = $7.6 = L_S(x_1 - x_2) = 152 \cdot (x_1 - 0)$
 $\Rightarrow x_1 = 0.05$ and $x_1 = \frac{x_1}{1+x_1} = \frac{0.05}{1.05} = 0.0476$.
 $y_2 = \frac{0.4 \text{ kmol/h}}{72 \text{ kmol/h}} = 0.00555 \Rightarrow y_2 = \frac{y_2}{1+y_2} = 0.005525$

$$(a) (\Delta y)_2 = y_2 - mx_2 = 0.005525 - 0 = 0.005525$$

$$(\Delta y)_1 = y_1 - mx_1 = 0.1 - (2.0)(0.0476) = 0.0048 \quad \left[\begin{array}{l} \text{since } y^* = mx \\ = 2.0x \\ m = 2.0 \end{array} \right]$$

- (b) If $L_S = 200 \text{ kmol/h}$, $x_1 = 7.6/200 = 0.038$; $x_2 = 0$ (pure solvent).
 50% of the solute is absorbed over the section $\Rightarrow y = \frac{4}{72} = 0.0555$.
 Material balance: $72(0.0555 - 0.00555) = 200(x - 0) \Rightarrow x = 0.018$.
 In mole fraction unit, $y = \frac{0.0555}{1.0555} = 0.0526$; $x = \frac{0.018}{1.018} = 0.0177$
 Driving force at the section = $(\Delta y) = y - y^* = y - mx$
 $= 0.0526 - (2.0)(0.0177) = 0.0172$

- (c) At this section, $L = 200(1+x) = (200)(1.0177) = 203.5 \text{ kmol/h}$;
 $G = G_S(1+y) = (72)(1.0526) = 75.79 \text{ kmol/h}$
 Slope of the operating line = $L/G = 203.5/75.79 = 2.685$

$$(d) \text{Absorption factor, } A = L/mG = \frac{2.685}{2.0} = 1.342$$

PROBLEMS

- (4.1) Total pressure, $P = 10 \text{ bar} = (10 \cdot 1.013)(760) \text{ mmHg} = 7502 \text{ mmHg}$

(i) Given equilibrium relation: $p_A \text{ (mmHg)} = 10^5 x_A$.

$$\Rightarrow y_A = \frac{p_A}{P} = 10^5 x_A / 7502 \Rightarrow y_A = 13.33 x_A$$

$$(ii) x_A = \frac{C_A \text{ mol solute/liter}}{C_A \text{ mol solute/liter} + C_W \text{ mol H}_2\text{O/liter}} \sim \frac{C_A \text{ gmol solute/liter}}{(1000/18) \text{ gmol H}_2\text{O per liter}}$$

$$= \frac{C_A}{55.55} \text{ mole fraction (since the solution is 'dilute').}$$

$$\text{Given: } p_A = 10^5 x_A \Rightarrow p_A = 10^5 \cdot \frac{C_A}{55.55} = 1800 C_A, \text{ i.e. } m' = 1800 \text{ mm/(gmol/liter)}$$

4.5

4.1 Consd... Equilibrium relation with concentrations in the mole ratio unit:

$$Y_A = 13.33 X_A \text{ from part (i). } Y_A = \frac{Y_A}{1+Y_A} \text{ and } X_A = \frac{X_A}{1+X_A}$$

$$\Rightarrow \frac{Y_A}{1+Y_A} = \frac{13.33 X_A}{1+X_A} \Rightarrow Y_A = \frac{13.33 X_A}{1+13.33 X_A}$$

4.2 Bulk gas concentration, $y_b = 0.095$; bulk liquid, $x_b = 0.02$.

(i) Total resistance, gas-phase basis is: $\frac{1}{k_y} = \frac{1}{k_y} + \frac{m}{k_x}$; $y^* = 0.85x$; $m = 0.85$
60% of the total resistance lies in the liquid film.

$$\Rightarrow 0.6 \left[\frac{1}{k_y} + \frac{m}{k_x} \right] = \frac{m}{k_x} \Rightarrow 0.6 \left[\frac{1}{10} + \frac{0.85}{k_x} \right] = \frac{0.85}{k_x}$$

$$\Rightarrow k_x = 5.67 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot \Delta x}$$

Overall mass transfer coefficient, liquid-phase basis, k_x see Eq (4.21)

$$K_x = \frac{1}{k_x} + \frac{1}{m k_y} = \frac{1}{5.67} + \frac{1}{(0.85)(10)} = 3.41 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot (\Delta x)}$$

$$(ii) \text{ Molar flux of A, } N_A = k_x (x_b^* - x_b) = 3.41 \left[\frac{y_b}{0.85} - x_b \right], \text{ (Eq. 4.12)}$$

$$\Rightarrow N_A = 3.41 \left(\frac{0.095}{0.85} - 0.02 \right) = 0.313 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2}$$

(iii) Interfacial concentrations, x_i , y_i :

$$\text{Flux relation: } N_A = k_y (y_b - y_i) \Rightarrow 0.313 = (10)(0.095 - y_i) \Rightarrow y_i = 0.0637$$

* * * *

4.3 Equilibrium relation: $p_A = 25 x_A$, p_A in atm; total pressure, $P = 10 \text{ atm}$

$$\Rightarrow Y_A = \frac{p_A}{P} = \frac{25 x_A}{10} \Rightarrow y_A = 2.5 x_A = m x_A, m = 2.5$$

$$\frac{1}{k_y} = \frac{1}{k_y} + \frac{m}{k_x} = \frac{1}{8} + \frac{2.5}{10} = 0.375 \Rightarrow k_y = 2.667 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 (\Delta y)}$$

$$K_G = K_y / P = \frac{2.667}{10} = 0.2667 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 (\Delta p, \text{atm})} = 3.51 \times 10^{-4} \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 (\Delta p, \text{mm Hg})}$$

Interfacial concentrations, x_{Ai} and p_{Ai} : [mol wt: $SO_2 = 64$; $H_2O = 18$]

$$\text{Bulk concentration of } SO_2 \text{ in the liquid} = 4 \text{ mass\%} = \frac{4/64}{4/64 + 96/18} = 0.0116 = x_{Ab}$$

$$\text{At steady state, } N_A = k_y (y_{Ab} - y_{Ai}) = k_y (x_{Ai} - x_{Ab}) \Rightarrow y_{Ai} = 2.5 x_{Ai}$$

$$\Rightarrow 10(0.1 - y_{Ai}) = 8(x_{Ai} - 0.0116) \Rightarrow 10(0.1 - 2.5 x_{Ai}) = 8(x_{Ai} - 0.0116) \Rightarrow x_{Ai} = 0.0331$$

$$p_{Ai} = 25 x_{Ai} \text{ (from the equil. relation)} \Rightarrow p_{Ai} = (25)(0.0331) = 0.827 \text{ atm.}$$

(4.6)

4.4 Use the two-film model. Note that the solubility of O_2 in water is small.

Use Eq. (3.5) to calculate k_y from the given gas film thickness ($\delta_L = 2 \text{ mm}$);
 $k_y = \frac{D_{AB}^G \cdot P^2}{R \cdot T \cdot \delta_L \cdot P_{BM}}$; $P_{BM} = \frac{(P_{Bb} - P_{Bi})}{\ln(P_{Bb}/P_{Bi})}$; P_{AB} is known ($= 0.21 \text{ atm}$), P_{Ai} is unknown; P_{Ai} is to be calculated to obtain k_y .

Similarly, the individual liquid-phase mass transfer coefficient is:

$$k_x = \frac{D_{AB}^L \cdot (P/M)_{av}}{\delta_L \cdot x_{BM}}; x_{BM} = \frac{x_{Bb} - x_{Bi}}{\ln(x_{Bb}/x_{Bi})}; \frac{x_{Bb}}{b \rightarrow \text{bulk}} = 1 \quad (\text{pure water}); \\ x_{Bi} \text{ is not known}$$

Use the above two expressions for k_y and k_x together with the following relation to calculate P_{Ai} (or P_{Bi}) and x_{Ai} (or x_{Bi}) at steady state.

$$N_A = k_y (y_{Ab} - y_{Ai}) = k_x (x_{Ai} - x_{Ab}) \Rightarrow \frac{D_{AB}^G P^2}{R T \delta_L \cdot P_{BM}} (y_{Ab} - y_{Ai}) = \frac{D_{AB}^L (P/M)_{av}}{\delta_L x_{BM}} \cdot (x_{Ai}) \dots (i)$$

$$x_{Ab} = 0 \quad (\text{pure water}); D_{AB}^G = \frac{1.76 \times 10^{-5}}{\text{m}^2/\text{s}}; D_{AB}^L = \frac{2.1 \times 10^{-9}}{\text{m}^2/\text{s}}; \delta_L = 2 \times 10^{-3} \text{ m}; \\ \delta_L = 0.02 \text{ mm} = 2 \times 10^{-4} \text{ m}; P_{Ab} = 0.21 \text{ atm}; x_{Ai} = \frac{P_{Ai}}{4.36 \times 10^{-4}} = 2.3 \times 10^{-5} P_{Ai}, P = 1 \text{ atm}.$$

Put in Eq. (i) to get the following:

$$\frac{(1.76 \times 10^{-5})(1.0)^2}{(0.0821)(298)(2 \times 10^{-3})} \ln \frac{1 - P_{Ai}}{1 - 0.21} = \frac{(2.1 \times 10^{-9})(55.55)}{2 \times 10^{-4}} \cdot (2.3 \times 10^{-5} P_{Ai})$$

The following relation has been used:
 $\frac{P_{Ab} - P_{Ai}}{P_{BM}} = \frac{P - P_{Ai}}{P - P_{Ab}}$

Solving the above equation, $P_{Ai} \approx 0.21 \text{ atm}$

$P_{BM} = P_B = 0.79 \text{ atm}$. Note that the difference of partial pressure of the solute (O_2) across the gas-film is negligible; the gas-film resistance is very small.

$$k_y = \frac{(1.76 \times 10^{-5})(1.0)^2}{(0.0821)(298)(2 \times 10^{-3})(0.79)} = \frac{4.553 \times 10^{-4} \text{ kmol}}{\text{s} \cdot \text{m}^2 \cdot (\Delta y)}.$$

$$k_x = \frac{(2.1 \times 10^{-9})(55.55)}{(2 \times 10^{-4})(1)} = \frac{5.833 \times 10^{-4} \text{ kmol}}{\text{s} \cdot \text{m}^2 \cdot (\Delta x)}$$

$$\text{Equilibrium relation: } P_A = 4.36 \times 10^{-4} x_A \Rightarrow y_A = \frac{P_A}{P} = 4.36 \times 10^{-4} x_A; m = 4.36 \times 10^{-4}$$

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x} = \frac{1}{4.553 \times 10^{-4}} + \frac{4.36 \times 10^{-4}}{5.833 \times 10^{-4}} \Rightarrow K_y = 1.338 \times 10^{-8} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$$

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m k_y} = 5.833 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)} \quad (= k_x, \text{since the gas-film resistance is negligible})$$

Use the Penetration Theory to calculate k_x

$$(k_x)_{\text{av}} = 2 \left[\frac{D}{\pi k_c} \right]^{1/2} = 2 \left[\frac{2.1 \times 10^{-9}}{\pi \cdot (1.0)} \right]^{1/2} = 5.171 \times 10^{-5} \text{ m/s}$$

$$\Rightarrow k_x = k_x \cdot (P/M)_{av} = (5.171 \times 10^{-5})(55.55) = 2.872 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)}$$

$$\Rightarrow K_y = 6.587 \times 10^{-8} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)} \text{ and } K_x = 2.872 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)} \quad [\text{Calculation not shown}]$$

$(P/M)_{av} \approx \text{average molar conc. of water} = 5.55 \frac{\text{kmol}}{\text{m}^3}$

(4.7)

4.5 $x \rightarrow$ mole fraction of isomone in the aqueous phase; $y \rightarrow$ organic phase.
Conversion of the mass transfer coefficients:

$$k_y = (k_L)_{\text{org}} \cdot (P/M)_{\text{org}} = (3.5 \times 10^{-6})(10) = \frac{3.5 \times 10^{-5}}{\text{Molar conc. of the organic phase}} = \frac{(P/M)_{\text{org}}}{(P/M)_{\text{air}}} = 10 \text{ kmol/m}^3 \text{ given}$$

$$k_x = (k_L)_{\text{aq}} \cdot (P/M)_{\text{aq}} = (2.7 \times 10^{-5})(55.55) = \frac{1.5 \times 10^{-3}}{P/M \text{ for the aq. phase}} \approx 55.55 \text{ kmol/m}^3$$

Equilibrium relation: $y = 125x$

$$\frac{1}{k_y} = \frac{1}{k_x} + \frac{m}{k_x} = \frac{1}{3.5 \times 10^{-5}} + \frac{175}{1.5 \times 10^{-3}} = 28570 + 116700 = 145270 = \text{total resistance}$$

$$k_y = \frac{1}{145270} = 6.88 \times 10^{-6} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$$

Fractional resistance to mass transfer offered by the organic phase $= 28570/145270 = 0.2 \Rightarrow 20\%$; aqueous phase $\rightarrow 80\%$.

None of the resistances control.

* * * *

4.6 Mass transfer resistances (B-phase basis):

$$\text{Resistance offered by B-phase} = \frac{1}{k_y} = \frac{1}{4.35} = 0.23 \frac{\text{h} \cdot \text{ft}^2 \cdot (\Delta y)}{\text{kmol}}$$

$$\text{Resistance offered by A-phase} = \frac{m}{k_x} = \frac{10.5}{10.21} = 1.028 \frac{\text{h} \cdot \text{ft}^2 \cdot (\Delta x)}{\text{kmol}}$$

$$\text{Fractional resistance offered by B-phase} = \frac{0.23}{0.23 + 1.028} = 18.3\%$$

\Rightarrow Neither of the phases can be considered to offer controlling resistance.

* * * *

4.7 Concentrations of the VOC in both liquid and air are small. The total molar concentrations of the phases may be taken as:

$$\text{Water} \rightarrow C_L = \frac{1000}{18} = 55.55 \text{ kmol/m}^3; \text{air} \rightarrow C_A = \frac{P}{RT} = \frac{1}{(0.0821)(298)}$$

(i) Overall coefficients:

$$\text{Eq. (4.24)} \rightarrow \frac{1}{K_G} = \frac{1}{k_G} + \frac{H'}{k_L}; P_A = H' C_A \text{ at equilibrium,} = 0.04087 \text{ kmol/m}^3$$

$$\text{Given: } k_C = 0.028 \text{ m/s} \Rightarrow k_G = \frac{k_C}{RT} = \frac{0.028}{(0.0821)(298)} = 1.145 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta p, \text{atm})}$$

$$\text{Equil. relation given: } P_A = H' x_A \Rightarrow P_A = (5.5) \cdot \frac{C_A}{C_L} = \frac{5.5}{55.55} \cdot C_A = 0.099 C_A$$

$$\Rightarrow P_A = H' C_A, H' = 0.099. \text{ Given: } k_L = 3 \text{ cm/s} = 8.333 \times 10^{-6} \text{ m/s}$$

$$\frac{1}{K_G} = \frac{1}{1.145 \times 10^{-3}} + \frac{0.099}{8.333 \times 10^{-6}} = 87.3 + 1.188 \times 10^4 \Rightarrow K_G = 8.356 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta p, \text{atm})}$$

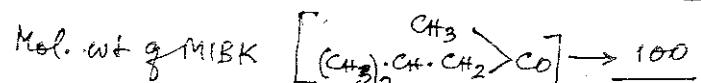
$$\text{Eq. (4.25)} \rightarrow \frac{1}{k_L} = \frac{1}{H' k_G} + \frac{1}{k_L} = \frac{1}{(0.099)(1.145 \times 10^{-3})} + \frac{1}{8.333 \times 10^{-6}} = 8.822 \times 10^3 + 1.2 \times 10^5 \Rightarrow K_L = 8.273 \times 10^{-6} \text{ m/s}$$

It is obvious that the liquid-phase resistance controls. This is quite expected since MIBK has a low solubility in water.

(4.8)

(4.7) contd... (ii) Fractional gas-phase resistance = $\frac{8.222 \times 10^{-8}}{1.21 \times 10^5} \Rightarrow \underline{\text{About } 7\%}$

(iii) Flux of MIBK



Concentration of MIBK in water = 8.7 ppm = $\frac{8.7 \text{ gm}}{\text{m}^3} = \frac{(8.7/100) \times 10^{-3}}{\text{m}^3} = 8.7 \times 10^{-5} \frac{\text{kmol}}{\text{m}^3}$

$$\begin{aligned} \text{Flux} &= K_L \Delta C = K_L (8.7 \times 10^{-5} - 0) = (8.273 \times 10^{-6}) (8.7 \times 10^{-5} \frac{\text{kmol}}{\text{m}^3}) \\ &= 7.2 \times 10^{-10} \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}} = 2.59 \times 10^{-4} \frac{\text{kg}}{\text{m}^2 \cdot \text{h}} \end{aligned}$$

* * * *

(4.8) Overall gas-phase resistance = $1/K_y = 1/5.35 = 0.187 \frac{\text{h} \cdot \text{m}^2 \cdot \Delta y}{\text{kmol}}$

(i) Individual Gas phase resistance = $(0.7)(0.187) = 0.1308 = 1/K_y \Rightarrow K_y = 7.645 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot (\Delta y)}$

Individual liquid-phase resistance = $(0.3)(0.187) = 0.0562 = m/k_x$

Given: $y = 3.2x \Rightarrow m = 3.2 \cdot k_x = 3.2 \cdot 0.0562 = 56.94 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta x)}$

(ii) $\frac{1}{K_x} = \frac{1}{m k_y} + \frac{1}{k_x} = \frac{1}{(3.2)(7.645)} + \frac{1}{56.94} \Rightarrow K_x = 17.11 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta x)}$

(iii) $K_G = \frac{1}{P} \cdot K_y = \frac{5.35}{120} \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta p, \text{ atm})} \quad [\text{Since } P = \text{Total pressure} = 120 \text{ atm}]$

Determination of K_y : If the concentrations (x and y) are small, $K_y \approx K_{y^*}$.
If x and y are not small, the bulk gas and liquid concentrations should be known for calculation of K_y from K_{y^*} .

Calculate Y_b and Y_b^* and the flux. Obtain K_y from $N_A = K_y (Y_b - Y_b^*)$

* * * *

(4.9) Total pressure, $P = 120 \text{ kPa}$. $K_y = P \cdot K_G = (120)(0.05) = 6 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot (\Delta y)}$

$Y_{cb} = 0.15$; $X_{cb} = 0.045$; flux of the solute C, $N_c = 0.08 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$

(i) Absorption flux, $N_c = 0.08 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}} = K_y (Y_{cb} - Y_{ci}) = (6)(0.15 - Y_{ci}) \Rightarrow Y_{ci} = 0.1367$

Equilibrium relation: $y = 2.5x \Rightarrow X_{ci} = 0.1367/2.5 = 0.05468$

Liquid-phase coefficient, $k_x = N_c / (X_{ci} - X_{cb}) = \frac{0.08}{0.05468 - 0.045} = 8.264 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta x)}$

Total resistance to mass transfer, gas-phase basis:

$$\frac{1}{K_y} + \frac{m}{k_x} = \frac{1}{6} + \frac{2.5}{8.264} = 0.1667 + 0.3025 = 0.4692$$

Fraction of resistance lying in the gas phase = $\frac{0.1667}{0.4692} = 0.355$

(ii) New gas-phase mass transfer coefficient = $(K_y)_2 = 1.5 K_y = 9 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta y)}$

Overall coefficient $\rightarrow \frac{1}{(K_y)_2} = \frac{1}{9} + \frac{2.5}{8.264} \Rightarrow (K_y)_2 = 2.35 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta y)}$

Absorption flux, $(N_c)_2 = (K_y)_2 [Y_{cb} - (2.5) \cdot X_{cb}] = (2.35)(0.15 - 0.1125)$
 $= 0.0881 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2}$

4.9

4.10 Calculations of $x_A - y_A$ and $G_A - Y_A$ for a single data point are shown.

Take the data point corresponding to $P_{NH_3} = p_A = 227 \text{ mm Hg at } 20^\circ\text{C}$.

$$\text{NH}_3 \text{ concentration} = \frac{0.25 \text{ kg NH}_3}{1 \text{ kg water}} = \frac{(25/17) \text{ kmol NH}_3}{(1.25 \text{ kg})/(907 \text{ kg/m}^3)} = 10.67 \text{ kmol NH}_3/\text{m}^3 \text{ solution}$$

$$y_A = \frac{227 \text{ mm Hg}}{760 \text{ "}} = 0.2987 ; x_A = \frac{(0.25/17)}{(0.25/17) + (1/18)} = 0.21$$

Calculated equilibrium data : $(x_A, y_A) \rightarrow (0.21, 0.2987)$; $(G_A - Y_A) \rightarrow (10.67, 0.2987)$

The equilibrium diagrams can be drawn after calculating the data

* * * *

4.11 Calculation of moisture concentrations in air and in H_2SO_4 at the top, bottom and at the given intermediate section of the tower by material balance will be done first to obtain the bulk values.

Vapor pressure of water at 25°C (298K)

$$\ln P^v(\text{bar}) = 11.9648 - \frac{3984.923}{298 - 39.72} \Rightarrow P^v = 0.0313 \text{ bar} = 0.0309 \text{ atm.}$$

$$\text{Feed air} \rightarrow 80\% \text{ relative humidity (R.H.)} \Rightarrow \frac{(0.0309)(18)(0.8)}{(1-0.0309)(28.8)} \text{ kg moisture}$$

$$\text{Exit air} \rightarrow 2\% \text{ R.H.} \Rightarrow \frac{(0.0309)(18)(0.02)}{(1-0.0309)(28.8)} \text{ kg dry air}$$

$$= 0.01584 \frac{\text{kg moisture}}{\text{kg dry air}} = Y_1$$

$$\text{Consider } 1 \text{ m}^2 \text{ area} \quad \frac{0.00039 \text{ kg moisture}}{\text{kg dry air}} = Y_2$$

$$\text{Feed acid} \rightarrow 70\% H_2SO_4, 30\% \text{ water} \Rightarrow x_2 = \frac{0.3}{0.7} = 0.4286 \frac{\text{kg water}}{\text{kg } H_2SO_4}$$

$$\text{Feed air rate} \rightarrow G_1 = 3000 \text{ kg/h.m}^2 \Rightarrow G_S = (3000)(1-Y_1) = (3000)\left(\frac{1}{1+0.01584}\right)$$

$$\text{Feed acid rate} \rightarrow L_2 = 1000 \text{ kg/m}^2 \text{ h} \Rightarrow L_S = 700 \text{ kg/h}$$

water-free basis

Overall moisture balance:

$$G_S(Y_1 - Y_2) = L_S(x_1 - x_2)$$

$$\Rightarrow (2953.2)(0.01584 - 0.00039) = 700(x_1 - 0.4286)$$

$$\Rightarrow x_1 = 0.4938 \text{ kg H}_2\text{O/kg dry acid}$$

$$\Rightarrow \text{Exit acid concentration} = \frac{0.4938}{1.4938} \rightarrow 33.1\% \text{ H}_2\text{O} \& 66.9\% \text{ acid}$$

Intermediate section

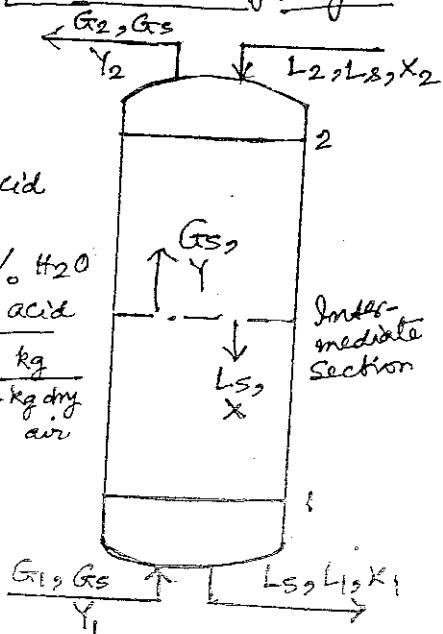
$$\text{R.H. of air} = 40\% \Rightarrow Y = \frac{(0.0309)(18)(0.4)}{(1-0.0309)(0.6)(28.8)} = 0.0078 \frac{\text{kg}}{\text{kg dry air}}$$

Material balance:

$$G_S(Y - Y_2) = L_S(x - x_2)$$

$$\Rightarrow (2953.2)(0.0078 - 0.00039) = 700(x - 0.4286)$$

$$\Rightarrow x = 0.46 \text{ kg H}_2\text{O/kg dry acid.}$$



4.10

4.11 Contd... The overall gas-phase mass transfer coefficient, K_y , and the interfacial concentrations (x_i, y_i) at the given intermediate section can be calculated by graphical construction on the $x-y$ plane. The given equilibrium data are converted to $x-y$ data.

Mass % H ₂ SO ₄	40	45	50	55	60	65	70	75	80
P _{H₂O} , mm Hg	13.5	11.1	8.74	6.36	4.09	2.34	1.23	0.546	0.19
x, mol fraction water in H ₂ SO ₄	0.891	0.87	0.845	0.8167	0.784	0.7456	0.7	0.6447	0.5765
y, mol fraction moisture in air	0.01776	0.0146	0.0115	0.00837	0.00538	0.00308	0.00162	7.2 × 10 ⁻⁴	2.5 × 10 ⁻⁴

Sample calculation: Take the data point, 45 mass % H₂SO₄ and $P_{H_2O} = 11.1$ mm Hg at equilibrium. Mass % water = 55%.
 $x = \frac{55/18}{55/18 + 45/98} = 0.869$ mole fraction water in H₂SO₄;
 $y = \frac{11.1}{760} = 0.0146$ mole fraction moisture in air at equilibrium.

Plot the x-y data to get the equilibrium curve.

Moisture concentration in the bulk air and bulk of the liquid at the given section of the tower;

$$x = \frac{0.46 \text{ kg H}_2\text{O}}{\text{kg dry acid}} \Rightarrow \frac{0.46/18}{(0.46/18) + (0.54/98)}$$

$$y = \frac{0.0078}{\frac{0.0078}{18} + \frac{1}{28.8}} \Rightarrow x = 0.714$$

$$= 0.01236 \text{ mole fraction}$$

Individual mass transfer coefficients:

$$K_x = 0.96 \text{ kmol}/\text{h.m}^2(\Delta x)$$

$$K_y = P \cdot K_G = (260 \text{ mm Hg}) \cdot (0.016 \text{ kmol}/\text{h.m}^2(\text{mm Hg}))$$

$$= 12.16 \text{ kmol}/\text{h.m}^2(\Delta y)$$

Refer to Section 4.4

Locate the point L (0.714, 0.01236) which represents the bulk phase concentrations (at the given section) on the x-y plane.

Draw the line LM of slope $-k_x/k_y = -0.96/12.16 = -0.079$ that meets the equilibrium curve at the point M (x_i, y_i) → (0.796, 0.006). This is the set of interfacial concentrations.

The vertical line through L meets the equil. curve at the point N. The slope of NM = 0.0624. $\Rightarrow \frac{1}{K_y} = \frac{1}{K_x} + \frac{m'}{K_x} = \frac{1}{12.16} + \frac{0.0624}{0.96} = 0.0822 + 0.065$
 $\Rightarrow K_y = 6.8 \text{ kmol}/\text{h.m}^2(\Delta y)$. Note that the liquid phase resistance is appreciable.

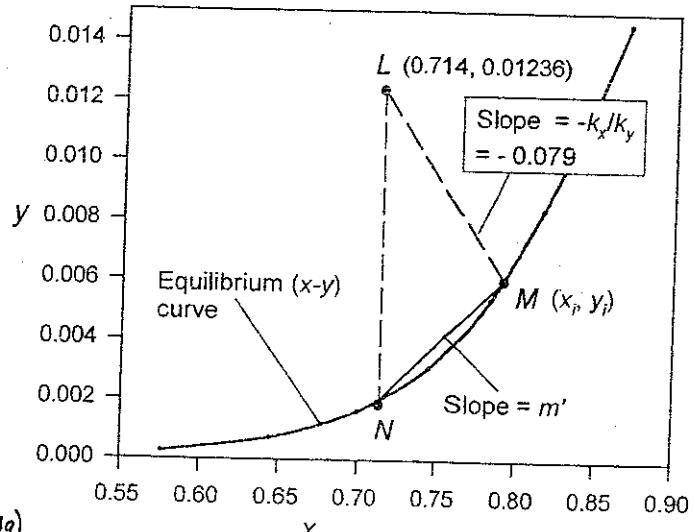


Fig. Prob Solution 4.11

4.11

- (4.12) Refer to Figure 4.12. Calculate the flows on 1-hour basis.
- Feed gas, $G_{N+1} = 30 \text{ kmol}$; $y_{N+1} = 0.015$; moles acetone in = $(30)(0.015) = 0.45 \text{ mol}$. 95% of the solute is absorbed. Mole acetone leaving = $(0.45)(1 - 0.95) = 0.0225$. The given equilibrium relation is linear in mole fraction unit, i.e. $y = 2.53x$ (i.e. $m = 2.53$). The concentrations are low ('dilute' solution). So the Kremser equation can be used to calculate the number of ideal trays. It is advisable to use the average absorption factor.
- Flow rate of the exit gas = $(30 - 0.45) + 0.0225 = 29.57 \text{ kmol} = G_1$
- Liquid flow rates: Inlet, $L_0 = 90 \text{ kmol}$; outlet, $L_N = 90 + (0.45)(0.95) = 90.43$
- Absorption factor: top, $\bar{A}_1 = L_0/m G_1 = \frac{90}{(2.53)(29.57)} = 1.203$;
- bottom, $\bar{A}_N = \frac{L_N}{m G_{N+1}} = \frac{90.43}{(2.53)(30)} = 1.191 \Rightarrow \text{Average, } \bar{A} = 1.197$

(Note: if \bar{A} varies substantially, the geometric mean should be taken)

Calculate the number of ideal stages from Eq. (4.4) taking mole fraction units

$$N = \frac{\log \left[\frac{y_{N+1} - mx_0}{y_1 - mx_0} \left(1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\log \left(\frac{y_{N+1} - 0}{y_1 - 0} \left(1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right)}$$

$$\Rightarrow N = \frac{\log \left[\frac{0.015 - 0}{0.00076 - 0} \left(1 - \frac{1}{1.197} \right) + \frac{1}{1.197} \right]}{\log (1.197)} = \frac{1.407}{0.1798} = 7.8$$

$x_0 = 0$ (inlet water in solute-free)
 $y_1 = \frac{0.0225}{29.57} = 0.00076$

Graphical construction of the number of ideal stages can be done following Example 4.6. It is not shown here.

- (4.13) Take the gas and liquid flow rates on the basis of 1-hr operation.

Feed gas: $G_{N+1} = 40 \text{ kmol}$; $y_{N+1} = 0.02$; $L_0 = 110 \text{ kmol}$; $x_0 = 0$. The solutions are very dilute. Use Kremser equation in mole fraction unit and take the absorption factor calculated on the basis of above flow rates. The results will be reasonably accurate.

Take $\bar{A} = L/m G = 110/(2.5)(40) = 1.1$. Now given, calculate y_1 .

$$N = 4 = \frac{\log \left[\frac{0.02 - 0}{y_1 - 0} \left(1 - \frac{1}{1.1} \right) + \frac{1}{1.1} \right]}{\log (1.1)} \Rightarrow y_1 = 0.00328$$

$$\text{Moles Solute absorbed} = (40)(0.02) - \left(\frac{40}{0.98} \right) \left(\frac{0.00328}{1 - 0.00328} \right) = 0.8 - 0.1343 = 0.6657 \text{ kmol}$$

$$\text{Mole fraction of the solute in the exit liquid} = \frac{0.6657}{110 + 0.6657} = 0.00601 \approx x_1$$

4.14 Feed wastewater: flowrate, $L_0 = 4000 \text{ kg/h}$, $x_0 = 0.1 \text{ kg VOC per kg water}$
 $L_s = (4000)(1 - 0.001) = 3996 \text{ kg/h}$
 Water leaving $\rightarrow x_N = 50 \text{ ppm}$, i.e. $\frac{50}{10^6} = 5 \times 10^{-5} \text{ kg/kg water}$
 Air rate for stripping, $G_s = 1500 \text{ kg/h}$; $y_{N+1} = 0$

Material balance: $G_s (y_1 - y_{N+1}) = L_s (x_0 - x_N)$
 $\Rightarrow 1500 (y_1 - 0) = (3996)(0.001 - 5 \times 10^{-5}) \Rightarrow y_1 = 0.00253$

Equilibrium relation: $y = 4.35 x \Rightarrow \alpha = 4.35$

Stripping factor, $S = \frac{\alpha G_s}{L_s} = \frac{(4.35)(1500)}{3996} = 1.633 = \frac{1}{A} \Rightarrow A = 0.6124$

Use the Kremser equation for stripping, Eq.(4.41). Number of ideal trays available = number of real trays \times tray efficiency

$$N = \frac{\log \left[\frac{x_0 - y_{N+1}/\alpha}{x_N - y_{N+1}/\alpha} \right] (1 - \bar{A}) + \bar{A}}{\log(1/\bar{A})} = \frac{(20)(0.5)}{\log(1/0.6124)} = \frac{10}{\log(1/0.6124)}$$

$\Rightarrow N = 4.5$ = required number of ideal trays for the given separation.

Number of ideal trays available = 10. So the column is definitely suitable.

4.15 Basis: 1-hr operation. Refer to figure 4.7.

Feed gas: $G_1 = 100 \text{ kmol}$, $y_1 = 0.15$, $G_s = 100(1 - 0.15) = 85$, $y_1 = \frac{0.15}{0.85} = 0.1765$

Solute entering = 15 kmol; Solute absorbed = $(0.95)(15) = 14.25 \text{ kmol}$;

Solute leaving with the gas = $15 - 14.25 = 0.75 \text{ kmol}$; $y_2 = \frac{0.75}{85} = 0.00882$.

Solvent rate = 80 kmol/h; $x_2 = 0$.

(a) Equation of the operating line: Use Eq.(4.26).

$$G_s (y - y_2) = L_s (x - x_2) \Rightarrow 85(y - 0.00882) = 80(x - 0) \Rightarrow y = 0.941x + 0.00882. \quad \dots (1)$$

(b) Slope of the operating line in mole fraction unit.

The equation of the op. line $\Rightarrow \frac{y}{1-y} = 0.941 \frac{x}{1-x} + 0.00882 = \frac{0.9322x + 0.00882}{1-x}$

$$\Rightarrow y = \frac{0.9322x + 0.00882}{1.00882 - 0.0678x} \Rightarrow \frac{dy}{dx} = \frac{0.9322}{(1.00882 - 0.0678x)^2} + \frac{(0.0678)(0.9322x + 0.00882)}{(1.00882 - 0.0678x)^2}$$

At a section of the column where the gas concentration is $y = 0.1$ $\Rightarrow y = \frac{0.1}{0.9} = 0.1111$, $X = 0.1087$ or, $x = \frac{0.1087}{1.1087} = 0.09804$ $\dots (ii)$

$$[\text{from eq (1)}]$$

(4.13)

Put $x = 0.09804$ in eq (ii) to get $\frac{dy}{dx} = \underline{\underline{0.937}}$, slope of the op. line for $x = 0.1$.

Maximum slope of the operating line \rightarrow it will occur at one of the terminal points since both x and y varies monotonically.

At the top terminal of the column, $x_2 = 0 \Rightarrow \frac{dy}{dx}\Big|_{x_2} = \underline{\underline{0.9246}}$

$$\text{At the bottom terminal, } x_1 = \frac{G_s(Y_1 - Y_2)}{L_s} = \frac{(85)(0.1765 - 0.00882)}{80}$$

$$\Rightarrow x_1 = 0.1782, \text{ and } x_1 = \frac{0.1782}{1.1782} = \underline{\underline{0.1512}}$$

Put $x_1 = 0.1512$ in eq (ii) to get $\frac{dy}{dx}\Big|_{x_1} = \underline{\underline{0.9437}}$

So the maximum slope of the operating line occurs at the bottom or the gas inlet end of the column.

Note : Even though the feed gas concentration is 15 vol%, the slope of the operating line in mole fraction unit varies by only about 1% along the column. In other words, it is nearly linear.

(4.16) Basis : 1 hr. operation. Feed gas: $G_1 = 70 \text{ kmol}$; $Y_1 = 0.07$;

$$\text{Moles Solute in the feed} = (70)(0.07) = 4.9 \text{ kmol} \quad G_s = 65.1 \text{ kmol}, Y_1 = \frac{0.07}{0.93} = \underline{\underline{0.07527}}$$

$$\text{Moles absorbed in the column} = (4.9)(0.98) = \underline{\underline{4.802}} \quad X_2 = 0, \text{ fresh solvent}$$

$$\text{Moles leaving unabsorbed} = 0.098 \Rightarrow Y_2 = \frac{0.098}{65.1} = \underline{\underline{0.001505}}; Y_2 = \frac{Y_2}{1+Y_2} = \underline{\underline{0.001503}}$$

$$\begin{aligned} &\text{Material balance over the column: } G_s(Y_1 - Y_2) = L_s(X_1 - X_2); L_s = 80 \text{ kmol} \\ &\Rightarrow (65.1)(0.07527 - 0.001505) = 80(X_1 - 0) \Rightarrow X_1 = \underline{\underline{0.06002}}. \end{aligned}$$

(a) Equation of the operating line : Use Eq (4.26)

$$G_s(Y - Y_2) = L_s(X - X_2) \Rightarrow (65.1)(Y - 0.001505) = 80(X - 0) \Rightarrow Y = 1.2289X + 0.001505 \dots (i)$$

(b) Overall gas phase driving force at a section where $x = 0.04$

$$\Rightarrow X = \frac{0.04}{1-0.04} = 0.04167. \text{ From Eq (i), } Y = (1.2289)(0.04167) + 0.001505 = \underline{\underline{0.05271}}$$

$$Y = 0.05271 \Rightarrow Y = \frac{0.05271}{1.05271} = \underline{\underline{0.05007}} = \text{bulk gas conc. at the particular section.}$$

$$\text{Equilibrium gas phase concentration corresponding to the liquid phase concentration } (x = 0.04) \text{ is } Y^* = (1.2)(0.04) - (0.62)(0.04)^2$$

$$\text{Overall gas-phase driving force, } Y - Y^* = \underline{\underline{0.05007 - 0.04701}} = \underline{\underline{0.00306}} = \underline{\underline{0.04701}}$$

(4.17) For convenience of calculation, change the variable X (kg color per 1000 kg water) to x' (kg color per kg water) $\Rightarrow x' = 10^3 X$ or $X = 10^{-3} x'$
Equilibrium relation: $Y = 0.62 X^{0.48} = (0.62)(10^3 x')^{0.48} \Rightarrow Y = 17.08 (x')^{0.48}$
Refer to figure 4.16

(a) Notations used: amount of 'color-free' water = L_s ; conc. = x' kg/kg water
Amount of adsorbent = G_s ; concentration = Y , kg color/kg adsorbent.

(i) Given: $L = 100$ kg water having 1 kg color $\Rightarrow L_s = 99$ kg; $x'_0 = \frac{1}{99} = 0.0101$

The stage is ideal $\Rightarrow G_s$ supplied = 1 kg; $Y_0 = 0$

At equilibrium, $G_s(Y_1 - Y_0) = L_s(x'_0 - x'_1)$; $Y_1 = 17.08(x'_1)^{0.48}$ at equilibrium

$$\Rightarrow (1)[17.08(x'_1)^{0.48} - 0] = 99(0.0101 - x'_1). \text{ Solving, } x'_1 = 0.0018 \text{ kg color/kg water.}$$

$$\text{Total color removed} = 99(0.0101 - 0.0018) = 0.8217; \% \text{ removal} = 82.17\%$$

Graphical solution:

The equilibrium line, Y vs x' is plotted.

The point $(x'_0, Y_0) \rightarrow (0.0101, 0)$ is located

on the graph. This is point P.

Draw a line of slope $L_s/G_s = -99$ from P to meet the equilibrium line at the point Q $\rightarrow (x'_1, Y_1) \rightarrow (0.0018, 0.8217)$.

x'_1, Y_1 are the concentrations of the streams leaving the equilibrium stage.

The % removal is as calculated above.

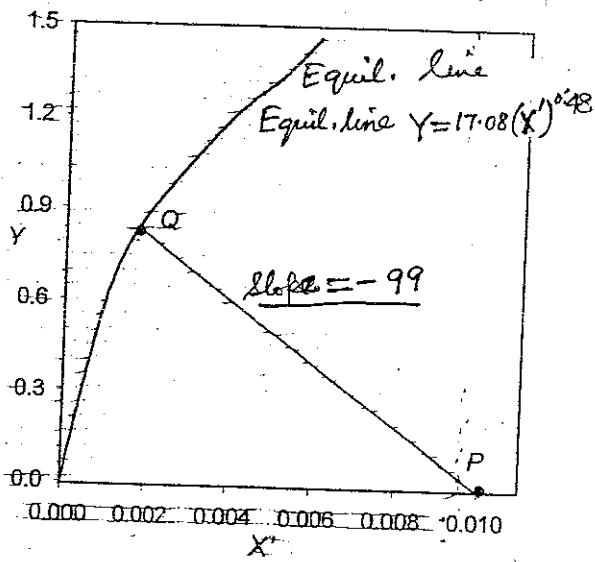


Fig Prob 4.17(a)

(ii) 0.5 kg adsorbent is used in each stage $\Rightarrow G_{s1} = 0.5 \text{ kg} = G_{s2}$

Stage 1: $G_{s1}(Y_1 - Y_0) = L_s(x'_0 - x'_1) \Rightarrow (0.5)[(17.08)(x'_1)^{0.48} - 0] = 99(0.0101 - x'_1)$

Solution of the above non-linear equation: $x'_1 = 0.004; Y_1 = 1.206$

Stage 2: $G_{s2}(Y_2 - Y_1) = L_s(x'_1 - x'_2) \Rightarrow (0.5)[17.08(x'_2)^{0.48} - 0] = 99(0.004 - x'_2)$

Solution of this material balance equation for Stage 2: $x'_2 = 0.00095$

Amount of dye leaving the second stage with water $= (99)(0.00095) = 0.094 \text{ kg}$. Dye adsorbed $= 1 - 0.094 \Rightarrow 90.6\%$

Note: A larger fraction of the dye (90.6%) is removed by using 1 kg total amount of adsorbent in two stages compared to 82.17% removed in a single stage contact (using the same amount of adsorbent).

Graphical solution : A line of slope $-L_s/G_{S_1} = -99/0.5 = -198$ is drawn from the point $P_1(x'_0, Y_0) \rightarrow (0.0101, 0)$ to meet the equilibrium curve at the point $Q_1(0.004, 1.206)$.

A vertical line through Q_1 meets the x' -axis at $P_2(0.004, 0)$.

The operating line for stage 2 is drawn through P_2 with the same slope of -198 to meet the equl. line at $Q_2(x'_2, Y_2) \rightarrow (0.00095, 0.605)$.

Percentage removal of the dye is calculated as before.

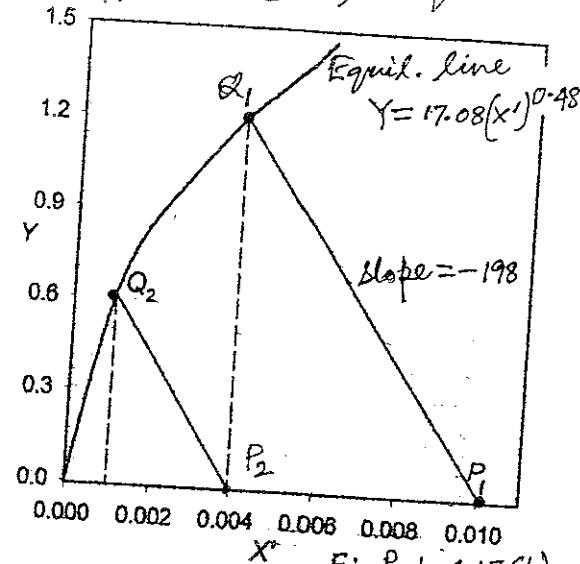


Fig Prob. 4.17(b)

(b) Feed : 100 kg waste water having 0.8% dye $\Rightarrow L_s = 99.2 \text{ kg}$; $x'_0 = \frac{0.8}{99.2} = 0.00806$

99% of the color is removed \Rightarrow color remaining in solution $= (0.8)(0.01) = 0.008 \text{ kg}$

$$x'_1 = \frac{0.008}{99.2} = \frac{8.06 \times 10^{-5}}{99.2} \Rightarrow Y_1 = 17.08(x'_1)^{0.48} = (17.08)(8.06 \times 10^{-5})^{0.48} = 0.1852$$

G_s = amount of adsorbent used. $\Rightarrow G_s(Y_1 - Y_0) = L_s(x'_0 - x'_1)$; $Y_0 = 0$

$$\Rightarrow G_s(0.1852) = (99.2)(0.00806 - 8.06 \times 10^{-5}) \Rightarrow G_s = 4.27 \text{ kg}$$

(ii) Ideal cross-current contact (Refer to Fig. 4.18b)

In a differential contact stage, dG_s amount of adsorbent is added, and the color concentration in the solution changes by dx' .

$$\Rightarrow dG_s(Y - Y_0) = -L_s \cdot dx'; \quad dx' \text{ is negative}; \quad Y_0 = 0 \text{ (fresh adsorbent)}$$

Put $Y = 17.08(x')^{0.48}$ for the equl. stage,

$$\Rightarrow dG_s = -L_s \cdot \frac{dx'}{17.08(x')^{0.48}} \Rightarrow G_s = -L_s \int_{x'_0}^{x'} \frac{dx'}{(17.08)(x')^{0.48}}$$

$L_s = 99.2$; $x'_0 = 0.00806$; x' = final dye concentration in the effluent
 $= 8.06 \times 10^{-5}$

$$\text{Integrating, } G_s = (-99.2) \cdot \frac{1}{(17.08)(1-0.48)} \left[(8.06 \times 10^{-5})^{0.52} - (0.00806)^{0.52} \right]$$

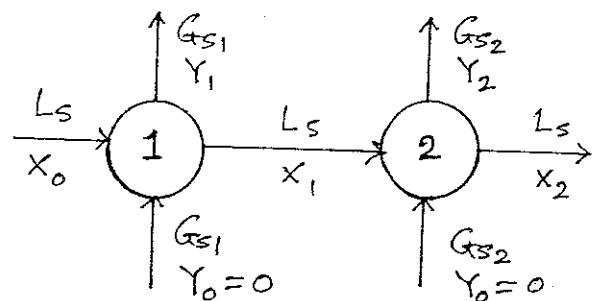
$$\Rightarrow G_s = 0.82 \text{ kg}$$

4.18 Feed : L_s kg A, x_0 kg C per kg A ($A \rightarrow$ carrier).

Solvent used : Stage 1 $\rightarrow G_{S1}$ kg; Stage 2 $\rightarrow G_{S2}$; total $G_S = G_{S1} + G_{S2}$

Material balance, Stage 1

$$L_s(x_0 - x_1) = G_{S1}(Y_1 - Y_0); Y_0 = 0 \\ \Rightarrow x_1 = \frac{L_s x_0}{L_s + \alpha G_{S1}} \quad \& Y_1 = \alpha x_1$$



Material balance, Stage 2

$$L_s(x_1 - x_2) = G_{S2}(Y_2 - Y_0) = (G_S - G_{S1}) \cdot \alpha x_2 \\ \Rightarrow x_2 = \frac{L_s x_1}{L_s + \alpha(G_S - G_{S1})} = \frac{L_s}{L_s + \alpha(G_S - G_{S1})} \cdot \frac{L_s x_0}{L_s + \alpha G_{S1}} \dots (i)$$

For maximum removal of the solute (or minimum x_2), put $\frac{dx_2}{dG_{S1}} = 0$

Differential (i) w.r.t. G_{S1}

$$\frac{dx_2}{dG_{S1}} = \frac{\frac{L_s^2 x_0}{\left[\{L_s + \alpha(G_S - G_{S1})\}\right]^2} \cdot (\alpha) + (L_s + \alpha G_{S1}) \cdot (-\alpha)}{\left[\{L_s + \alpha(G_S - G_{S1})\}\right] \cdot (L_s + \alpha G_{S1})} = 0 \\ \Rightarrow \alpha L_s + \alpha^2 G_S - \alpha^2 G_{S1}^2 - \alpha L_s - \alpha^2 G_{S1} = 0 \Rightarrow 2\alpha^2 G_{S1} = \alpha^2 G_S \\ \Rightarrow G_{S1} = G_S/2$$

Half of the total solvent is to be used in each stage.

Note : The second derivative d^2x_2/dG_{S1}^2 can be found to be positive to ensure a minimum of x_2 .

4.19 $X \rightarrow$ Solute concentration in water, kg solute / 1000 kg water.
If $X' =$ Solute concn. per kg water, Then $X = 1000X'$.

Equilibrium relation : $Y = 0.6X = Y = (1000)(0.6X') \Rightarrow Y = 600X' \dots (i)$

Refer to the figure in the previous problem, Prob. 4.18.

Use equal amounts of the absorbent (1 kg each) in the two stages to achieve maximum recovery of the solute.

Since the equilibrium relation is linear, eq. (i), it will be convenient to solve the problem algebraically.

(4.19) contd...

(4.17)

Feed : 100 kg, Solute concentration = 1 gm per kg.

$$\Rightarrow x_0 = \frac{1}{1000} = 0.001; L_s = (100)(1 - 0.001) = 99.9 \text{ kg. } G_{S1} = G_{S2} = 1 \text{ kg}$$

Material balance, Stage 1: $L_s(x_0 - x'_1) = G_s(Y_1 - Y_0); Y_0 = 0, \text{ fresh}$
use Eq (i), equil. relation

$$\Rightarrow (99.9)(0.001 - x'_1) = (1)[(600x'_1) - 0] \Rightarrow x'_1 = 1.427 \times 10^{-4} \text{ adsorbent}$$

Material balance, Stage 2: $L_s(x'_1 - x'_2) = G_s(Y_2 - Y_0); Y_2 = 600x'_2$

$$\Rightarrow (99.9)(1.427 \times 10^{-4} - x'_2) = (1)(600x'_2 - 0) \Rightarrow x'_2 = 2.037 \times 10^{-5}$$

Solute leaving with the solution from Stage 2

$$= L_s \cdot x'_2 = (99.9)(2.037 \times 10^{-5}) = 0.002035 \text{ kg}$$

$$\text{Fractional removal of the solute} = 1 - \frac{0.002035}{(99.9)(0.001)} \Rightarrow 98\%$$

(4.20) Given: feed = 1000 kg, $W_{w0} = 0.12 \text{ kg solute per kg water.}$

$$L_s = (1000) \left(\frac{1}{1.12} \right) = 892.9. \text{ Solute entering} = 107.1 \text{ kg; Solute leaving}$$

with the solution from Stage 3 (96% removal), $W_{w3} = \frac{(107.1)(0.04)}{892.9} = 0.0048$

Material balance, Stage 1

$$\text{Equil. relation: } W_s = 1.25 W_w \dots \text{ (i)}$$

$$L_s(W_{w0} - W_{w1}) = G_{S1}(W_{s1} - W_{s0})$$

$$\text{Given: } W_{w0} = 0; W_{s1} = 1.25 W_{w1}$$

$$\Rightarrow (892.9)(0.12 - W_{w1}) = (1200)(1.25 W_{w1} - 0)$$

$$\Rightarrow W_{w1} = 0.04478 \frac{\text{kg solute}}{\text{kg water}}$$

Material balance Stage 2 :

$$L_s(W_{w1} - W_{w2}) = G_s(W_{s2} - W_{s0}) \dots \text{ (ii)}$$

Use the equil. relation (i), $W_{s2} = 1.25 W_{w2}$ and $W_{s3} = 1.25 W_{w3} = (1.25)(0.0048)$

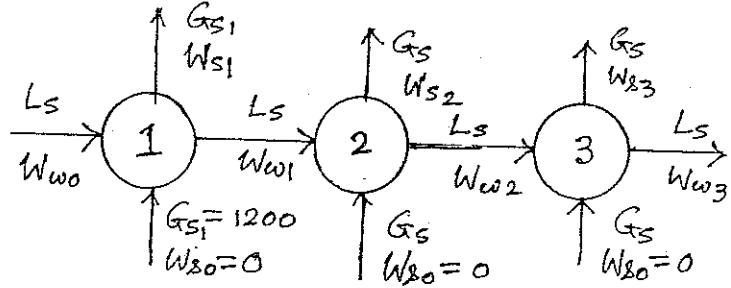
Divide (ii) by (iii)

$$\frac{W_{w1} - W_{w2}}{W_{w2} - W_{w3}} = \frac{W_{s2} - W_{s0}}{W_{s3} - W_{s0}} \Rightarrow \frac{0.04478 - W_{w2}}{W_{w2} - 0.0048} = \frac{1.25 W_{w2}}{0.006} \Rightarrow W_{w2} = 0.01466$$

$$\text{Put the values in eq (ii)} \rightarrow (892.9)[0.04478 - 0.01466] = G_s[(1.25)(0.01466) - 0]$$

$$\Rightarrow G_s = 1467.6 \text{ kg}$$

$$\text{Total Solvent required} = 1200 + 2 \times (1467.6) = 4135 \text{ kg}$$



Material balance, Stage 3 :

$$L_s(W_{w2} - W_{w3}) = G_s(W_{s3} - W_{s0}) \dots \text{ (iii)}$$

$$\Rightarrow W_{w3} = 0.006$$

$$\Rightarrow W_{w2} = 0.01466$$

(4.18)

4.21) The general scheme is similar to that sketched in the previous page. The problem can be solved algebraically or graphically. Basis of calculation: 1-hr operation

Feed: 2000 kg, 10% nicotine $\Rightarrow L_s = 1800 \text{ kg}$, $W_{L0} = \frac{10}{90} = 0.1111$

Nicotine leaving with stream L = 5% of the feed nicotine = $(200)(0.05) = 10 \text{ kg}$

$$W_{L3} = \frac{10}{1800} = 0.00555 \\ W_{L3} = 1.176 W_{L1} = 0.00653$$

Equilibrium relation:

$$W_L = 0.85 W_S \Rightarrow W_S = 1.176 W_L \dots (i)$$

Material balance equations:

$$\underline{\text{Stage 1:}} \quad L_s (W_{L0} - W_{L1}) = G_S (W_{S1} - W_{S0})$$

$$\Rightarrow 1800 (0.1111 - W_{L1}) = G_S (1.176 W_{L1} - 0.001) \quad : \quad W_{S0} = 0.001 \text{ mass\%} \approx \frac{0.001 \text{ kg solute}}{\text{kg solvent}}$$

$$\underline{\text{Stage 2:}} \quad L_s (W_{L1} - W_{L2}) = G_S (W_{S2} - W_{S0}) \Rightarrow 1800 (W_{L1} - W_{L2}) = G_S (1.176 W_{L2} - 0.001) \dots (iii)$$

$$\underline{\text{Stage 3:}} \quad L_s (W_{L2} - W_{L3}) = G_S (W_{S3} - W_{S0}) \Rightarrow 1800 (W_{L2} - 0.00555) = G_S (0.00653 - 0.001) \dots (iv)$$

$$\text{Dividing Eq.(ii) by Eq.(iii)} \rightarrow \frac{0.1111 - W_{L1}}{W_{L1} - W_{L2}} = \frac{1.176 W_{L1} - 0.001}{1.176 W_{L2} - 0.001}$$

$$\text{Dividing (ii) by (iv)} \rightarrow \frac{0.1111 - W_{L1}}{W_{L2} - 0.00555} = \frac{1.176 W_{L1} - 0.001}{0.00653 - 0.001} \quad \dots (v)$$

Substitute for W_{L2} from Eq.(v). After a little algebraic manipulation, we get a cubic equation for W_L .

$$W_L^3 - 0.002556 W_L^2 + 2.2 \times 10^{-6} W_L - 5.73 \times 10^{-5} = 0$$

$$\text{Solution: } W_{L1} = 0.0394 ; \quad W_{L2} = 0.01433$$

$$\text{Putting these values in Eq.(ii), } 1800 (0.1111 - 0.0394) = G_S [(1.176)(0.0394) - 0.001]$$

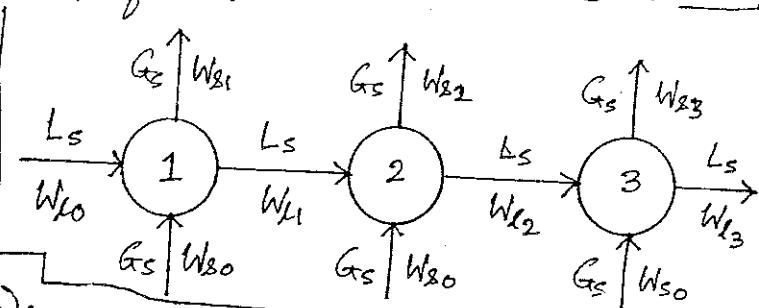
$$\Rightarrow G_S = 2847 \text{ kg}$$

$$\underline{\text{Total solvent required}} = 3 G_S = 3 \times 2847 = 8541 \text{ kg}$$

Graphical solution:

The equilibrium line, Eq.(i), is drawn on the $W_L - W_S$ plane.

The point $P_1 (W_{L0}, W_{S0}) \rightarrow (0.1111, 0.001)$ is located on the graph.



(4.19)

4.21 Contd... Since the same amount of solvent, G_s , is added to each stage, the operating lines will be parallel of slope $-L_s/G_s$. Three operating lines, P_1Q_1 , P_2Q_2 and P_3Q_3 are drawn by trial such that the abscissa of Q_3 is $w_{L3} = 0.00555$ as required by the problem. The slope of the operating lines is found to be -0.63.

$$\Rightarrow -L_s/G_s = -0.63$$

$$\Rightarrow G_s = L_s/0.63 = \frac{1800}{0.63} = 2857$$

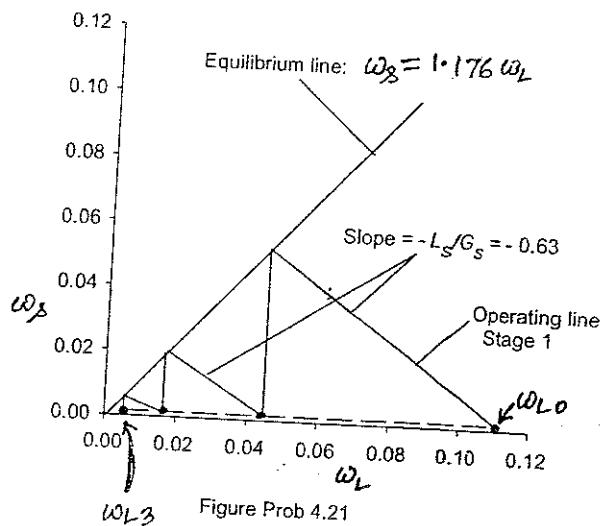


Figure Prob 4.21

Total solvent required = $3G_s = 8570$ kg which is very close to the value calculated algebraically.

4.22 (i) Counter current contact : Refer to the figure below.

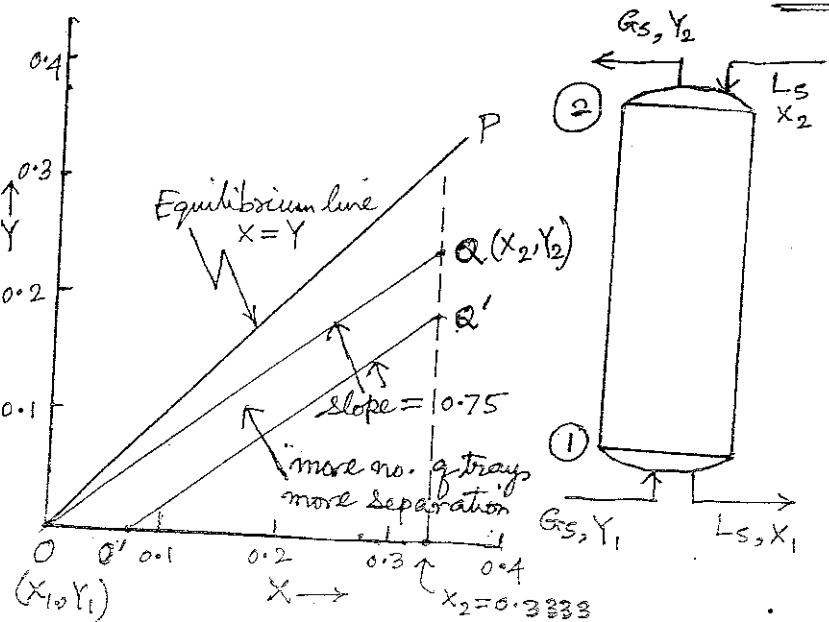
Feed: 10 kmol, 25 mole% Solute $\Rightarrow L_s = 7.5$ kmol, $x_2 = \frac{25}{7.5} = 0.3333$
Basis: 1 hr. operation
 $G_s = 10$ kmol, $y_1 = 0$ (fresh solvent).

$$\text{Slope of the operating line} = L_s/G_s = 7.5/10 = 0.75$$

Mass transfer from L to G \Rightarrow the operating line lies below the equil. line $\downarrow Y = X$

The top terminal of the operating line lies on the vertical line through $x_2 = 0.3333$.

As the operating line $O'Q'$ of constant slope 0.75 slides towards the equilibrium line the number of trays will increase and the extent of separation



(A-20)

4.22) Contd... also will increase. The maximum separation is achieved when it takes the position OQ. The number of plates become infinite since the lower end of the operating line meets the equilibrium line (a "pinch point", see section 6.3).
 ↓(it is $x=Y$)

Terminal concentrations for the operating line OQ:

lower terminal $\rightarrow (x_1, Y_1) \rightarrow (0, 0)$; upper terminal, (x_2, Y_2)

Calculate Y_2 by material balance. $\rightarrow x_2 = 0.3333, Y_2 = ?$

$$G_s(Y_2 - Y_1) = L_s(x_2 - x_1) \Rightarrow 10(Y_2 - 0) = 7.5(0.3333 - 0) \Rightarrow Y_2 = 0.25$$

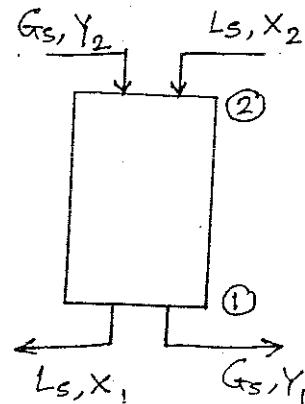
$$\text{Solute recovered} = G_s(Y_2 - Y_1) = 10(0.25 - 0) = 2.5 \text{ kmole}$$

$\Rightarrow \frac{100\% \text{ recovery (theoretical)}}{\text{countercurrent contact.}}$

(ii) Cocurrent Contact

Both the phases enter the cocurrent contact unit at one end (say, the top in the figure) and leave at the bottom. For ideal contact, the phases reach equilibrium as they leave.

$$\text{Equilibrium line, } x = Y \Rightarrow x_1 = Y_1$$



$$\begin{aligned} \text{Material balance: } G_s(Y_1 - Y_2) &= L_s(x_2 - x_1); Y_2 = 0 \quad x_2 = 0.3333 \\ \Rightarrow 10(Y_1 - 0) &= 7.5(0.3333 - Y_1) \Rightarrow Y_1 = 0.1428. \end{aligned}$$

$$\text{Solute removed} = (10)(0.1428) = 14.28$$

$$\text{Percent recovery} = \frac{14.28}{25} \times 100\% = 57.1\%$$

(iii) Cross-current contact — infinite number of stages [See Problem 4.18 Part b(ii)]

A differential amount of solvent dG_s is supplied to each stage of the crosscurrent battery.

$$dG_s(Y - Y_0) = -L_s dx; \text{ put } Y_0 = 0, \text{ and } Y = x \text{ is the equil. relation.}$$

$$\Rightarrow \int_0^{G_s} dG_s = -L_s \int_{x=x_0}^{x=x_f} \frac{dx}{Y} = -L_s \int_{x_0}^{x_f} \frac{dx}{x} \Rightarrow G_s = L_s \ln \frac{x_0}{x_f} \Rightarrow \frac{10}{7.5} = \ln \frac{0.3333}{x_f}$$

$$\Rightarrow x_f = \text{final concentration of stream L} = 0.0878. \quad \% \text{ removal} = \frac{0.3333 - 0.0878}{0.3333} \Rightarrow 73.6\%$$

(4.21)

4.23 Let m_a = mass of dry air, m_s = mass of dry silica gel.

Total mass of gel taken = 0.5 kg, 5% moisture.

$$m_s = (0.5)(1-0.05) = 0.475 \text{ kg}; \text{ moisture, } w_{s_i} = \frac{0.025}{0.475} = 0.0526 \frac{\text{kg H}_2\text{O}}{\text{kg dry gel}}$$

Dry air : 1 m³, 1 atm total pressure, 15 mm Hg partial pressure of moisture, $\frac{25^\circ\text{C}}{273} = 298\text{K}$

$$\text{Mass of water} = (1 \text{ m}^3) \left(\frac{273}{298} \right) \left(\frac{15}{760} \right) \left(\frac{18}{22.414} \right) = 0.01452 \text{ kg}$$

$$\text{Mass of air} = (1 \text{ m}^3) \left(\frac{273}{298} \right) \left(\frac{760-15}{760} \right) \left(\frac{28.8}{22.414} \right) = 1.154 \text{ kg} = m_a,$$

$$\text{Initial humidity of air} = \frac{0.01452}{1.154} = 0.01258 \text{ kg moisture/kg dry air} = w_a$$

$$\text{Material balance : } m_s (w_s - w_{s_i}) = m_a (w_a - w_{a_i}); \quad w_a = 0.0344 w_s$$

$$\Rightarrow (0.475)(w_s - 0.0526) = 1.154(0.01258 - 0.0344 w_s) \quad \boxed{\text{Equil. relation}}$$

$$\Rightarrow w_s = \frac{0.01452 + 0.025}{0.475 + 0.0344} = 0.07678 \text{ kg moisture/kg dry gel.}$$

$$(i) \text{ Amount of moisture picked up by the gel} = m_s (w_s - w_{s_i}) \\ = (0.475)(0.07678 - 0.0526) = 0.0115 \text{ kg}$$

(ii) Final partial pressure of moisture in air (total pressure = 760 mm Hg, constant)

$$= \frac{\text{Final moles of moisture in the air}}{\text{Final total moles, air + moisture}} \times 760 \text{ mm Hg.}$$

$$= \frac{(0.01452 - 0.0115) \text{ kg} / (18 \text{ kg/kmol})}{(1.154 / 28) \text{ kmol dry air} + (0.01452 - 0.0115) / 18 \text{ kmol moisture}} \times 760 \text{ mm Hg} = 3.08 \text{ mm Hg}$$

*

*

*

*

4.24 Molar mass of the adsorbent sphere, $m_s = 0.04 \text{ g/mol} = 4 \times 10^{-5} \text{ kmol}$.

Area of the sphere, $A = \pi (0.01)^2 = 3.1416 \times 10^{-4} \text{ m}^2$.

$$\text{Moisture content of 'bulk air', } Y_b = \frac{15 \text{ mm Hg}}{(760 - 15) \text{ mm Hg}} = 0.02013 \frac{\text{kmol moisture}}{\text{kmol dry air}}$$

Since the volume of air is 'large', Y_b remains constant

If k_y is the gas-phase mass transfer coefficient of moisture, $m_s \frac{dx}{dt} = k_y A \cdot (Y_b - Y) = \text{rate of moisture absorption at any time } t$, $Y = \text{moisture concentration at the surface of the sphere.}$

Considering 'interfacial equilibrium' at the surface of the sphere, we write $Y = 0.1X$, $X = \text{kmol moisture/kmol dry adsorbent}$

4.24 contd... Substituting for Y in the transient mass balance equation,
 $m_s \frac{dx}{dt} = k_y a \cdot (Y_b - 0.1x) ; t=0, x=x_i=0, \text{dry pellet.}$

Integrating, $\ln \left[\frac{Y_b - 0.1x_i}{Y_b - 0.1x} \right] = \frac{k_y a}{m_s} \cdot t$, Given $t=3 \text{ min}, x=0.16 \frac{\text{kmol H}_2\text{O}}{\text{kmol dry adsorbent}}$

$$\Rightarrow \ln \left[\frac{0.02013 - 0}{0.02013 - (0.1)(0.16)} \right] = \frac{k_y \cdot (3 \cdot 1416 \times 10^{-4})}{4 \times 10^5} \times (3 \times 60)$$

$$\Rightarrow k_y = 1.12 \times 10^{-3} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta Y)}$$

* * * *

4.25 Concentration of acetone (the solute) in two phases:

Aqueous (continuous phase): Acetone (mol. wt. = 58) concentration.

$$= 5 \text{ mass\%} \Rightarrow \frac{0.05/58 \text{ mol acetone}}{(0.05/58) + (0.95/18) \text{ total mol}} = 0.01607 \text{ mole fraction} = x_b.$$

(remains constant)

Dispersed (chloroform) phase: concentration at the top, $y_{b_1} = 0$

$$\text{Concentration at the bottom} = 1.02 \text{ mass\%} \Rightarrow \frac{1.02/58}{(1.02/58) + (98.98/119)} = 0.0207 = y_{b_2}$$

(Mol. wt. of chloroform $\rightarrow 119$)

Equilibrium relation: $w_d = 1.72 w_c$, w in mass fraction.

Convert ' w ' to mole fraction (note that the solution is dilute, mol. wt. of the solvent \approx average mol. wt. of solution)

$$\frac{w_d \text{ kg acetone}}{1 \text{ kg solution in chloroform}} = \frac{w_d/58 \text{ mol acetone}}{1/119 \text{ mol soln.}} = 2.051 w_d \text{ mole fraction} = y$$

$$\text{Similarly, } w_c \approx \frac{w_c/58}{1/18} = 0.3103 w_c = x \Rightarrow w_c = \frac{3.222 x}{1.72}$$

$$\text{Equilibrium relation: } w_d = 1.72 w_c \Rightarrow 0.4874 y = 3.222 x \Rightarrow y = 11.37 x$$

Overall mass transfer driving force $\therefore m = 11.37$
 (aqueous or continuous phase basis);

$$\text{At the top, } (\Delta x)_1 = x_b - x_1^* = x_b - y_{1b}/11.37 = x_b - 0 = 0.01607$$

$$\text{At the bottom, } (\Delta x)_{11} = x_b - x_2^* = x_b - y_{2b}/11.37 = 0.01607 - \frac{0.0207}{11.37}$$

Since the change in the driving force along the unit is small,

$$\text{take the average, } (\Delta x)_{av} = \frac{0.01607 + 0.01425}{2} = 0.01516$$

4.25) contd ... Calculation of the amount of acetone transferred to

Mass of the drop = $\frac{\pi}{6} (dp)^3 \cdot \rho_d = \frac{\pi}{6} (6 \times 10^{-3})^3 (1480) = 1.674 \times 10^{-4} \text{ kg}$ a single drop.

If m_a = mass of acetone transferred to it during its fall, mass fraction acetone in it = $\frac{m_a}{m_a + 1.674 \times 10^{-4}} = 0.0102$ (i.e. 1.02%)

$$\Rightarrow m_a = 1.725 \times 10^{-6} \text{ kg} = 1.725 \times 10^{-6} / 58 \rightarrow 2.974 \times 10^{-8} \text{ kmol} = Q_a$$

If K_x is the overall mass transfer coefficient,

$$[\text{area of a drop}] = \pi (6 \times 10^{-3})^2 = 1.131 \times 10^{-4} \text{ m}^2$$

$$Q_a = K_x a \cdot (\Delta x)_{av} \cdot t_f \Rightarrow 2.974 \times 10^{-8} = K_x \cdot (1.131 \times 10^{-4}) (0.01516) (108)$$

$$\Rightarrow K_x = 1.734 \times 10^{-3} \text{ kmol/m}^2 \cdot \text{s} \cdot (\Delta x)$$

$$\boxed{\begin{aligned} \text{Time of contact, } t_f &= h / v_d \\ &= 1 \text{ m} / (0.1 \text{ m/s}) = 10 \text{ s.} \end{aligned}}$$

Calculation of the overall coefficient, K_x , using the given correlations.

Continuous (aqueous) phase mass transfer coefficient, $(k_L)_c$:

$$(S_A)_c = \frac{(k_L)_c \cdot dp}{D_c} = 0.6 (Re)^{1/2} (Sc)_c^{1/2}$$

$$Re = \frac{dp \cdot \rho_c \cdot v_d}{\mu_c} = \frac{(3 \times 10^{-3})(990)(0.1)}{8.8 \times 10^{-4}} = 675$$

$$(Sc)_c = \frac{\mu_c}{\rho_c D_c} = \frac{8.8 \times 10^{-4}}{(990)(2 \times 10^{-9})} = 444.5$$

$$P_c = 990 \text{ kg/m}^3; dp = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$

$$\mu_c = 0.88 \text{ CP} = 8.8 \times 10^{-4} \text{ kg/m.s}$$

$$D_c = 2 \times 10^{-9} \text{ m}^2/\text{s}$$

$$\text{drop velocity, } v_d = 0.1 \text{ m/s}$$

$$(k_L)_c = \frac{D_c}{dp} \cdot (0.6) \cdot Re^{1/2} (Sc)_c^{1/2} = \frac{2 \times 10^{-9}}{6 \times 10^{-3}} \cdot (0.6) \cdot (675)^{1/2} (444.5)^{1/2} = 1.095 \times 10^{-4} \frac{\text{m}}{\text{s}}$$

Dispersed (chloroform) phase mass transfer coefficient, $(k_L)_d$:

$$(k_L)_d = (31.4) \cdot \frac{D_d}{dp} \cdot \left(\frac{4 D_d t_f}{dp^2} \right)^{-0.34} \cdot \left(\frac{\mu_d}{\rho_d D_d} \right)^{-0.125} \cdot \left(\frac{dp v_d^2 \rho_c}{a} \right)^{0.37}$$

$$D_d = 4 \times 10^{-9} \text{ m}^2/\text{s}; t_f = 108 \text{ s} \quad \rho_d = 1480 \text{ kg/m}^3; \mu_d = 0.53 \text{ CP} = 5.3 \times 10^{-4} \text{ kg/m.s}$$

$$\Rightarrow (k_L)_d = (31.4) \cdot \frac{4 \times 10^{-9}}{(6 \times 10^{-3})} \left[\frac{4 \times 4 \times 10^{-9} \times 10}{(6 \times 10^{-3})^2} \right]^{-0.34} \cdot \left(\frac{5.3 \times 10^{-4}}{1480 \times 4 \times 10^{-9}} \right)^{-0.125} \left[a = 30 \frac{\text{dyne}}{\text{cm}} = 0.03 \text{ N/m} \right]$$

$$\times \left[\frac{6 \times 10^{-3} \times (0.1)^2 \times 990}{0.03} \right]^{0.37} = 1.46 \times 10^{-5} \text{ m/s}$$

$$k_x = (k_L)_c \cdot (\rho_c/M)_{av} = (1.095 \times 10^{-4}) (990/18.64) = 5.816 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)}$$

$$k_y = (k_L)_d \cdot (\rho_d/M)_{av} = (1.46 \times 10^{-5}) (1480/119) = 1.82 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)}$$

Average mol. wt. of solution: Aqueous (5% acetone)

$$\Rightarrow (M)_{av} = 1 / \left[\frac{0.05}{58} + \frac{0.95}{18} \right] = 18.64; \text{ chloroform phase, } M_{av} \approx 119$$

4.24

4.25 contd... Overall mass transfer coefficient (aqueous phase),

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m k_y} = \frac{1}{5.816 \times 10^{-3}} + \frac{1}{(11.37)(1.82 \times 10^4)} = 172 + 483 = \underline{\underline{655}}$$

$$K_x = \frac{1}{655} = \underline{\underline{1.527 \times 10^{-3} \text{ kmol/m}^2 \cdot \text{s} \cdot (\Delta x)}}$$

Fractional resistance in the aqueous phase
 $= 172/655 \Rightarrow \underline{\underline{26.3\%}}$

which compares well with the experimental value of $1.734 \times 10^{-3} \text{ kmol/m}^2 \cdot \text{s} \cdot (\Delta x)$, (see above)

Continuous phase mass transfer coefficient, (k_L)_c, from the penetration theory:
Assume the drop to be rigid.

Contact time of a liquid element with the drop surface,

$$t_c = \frac{dp}{\rho g} = \frac{5 \times 10^3 \text{ m}}{0.1 \text{ m/s}} = \underline{\underline{0.06 \text{ s}}} \quad \text{Use Eq.(3.38)}$$

$$(k_L)_c = 2\sqrt{\frac{D_c}{\pi \cdot t_c}} = 2 \left[\frac{2 \times 10^{-9}}{\pi(0.06)} \right]^{1/2} = \underline{\underline{2.06 \times 10^{-4} \text{ m/s}}} \quad \text{which is about double that calculated using the given correlation.}$$

* * * *

4.26 For removal of solute from phase L to phase G (call it 'stripping') use Eq.(4.44), countercurrent contact

$$N = \frac{\log \left[\frac{x_0 - (Y_{N+1}/\alpha)}{x_N} \right]}{\log \left(\frac{Y_A}{x_A} \right)}$$

Given: $Y_{N+1} = 0$, inlet G stream is solute-free.

$$N = 3, \bar{A} = 0.9 = \frac{\alpha G_s}{L_s}$$

$$\Rightarrow \frac{x_0 \cdot (1 - \bar{A}) + \bar{A}}{x_N} = \frac{1}{(\bar{A})^3} \Rightarrow \frac{x_0 (1 - \bar{A})}{x_N} = \frac{1}{(\bar{A})^3} - \bar{A} = \frac{1 - (\bar{A})^4}{(\bar{A})^3}$$

$$\Rightarrow \frac{x_N}{x_0} = \frac{(\bar{A})^3 \cdot (1 - \bar{A})}{1 - (\bar{A})^4} = \frac{(\bar{A})^3}{(1 + \bar{A}^2)(1 + \bar{A})}$$

$$\text{Fraction of the solute separated, } (f) = 1 - \frac{x_N}{x_0} = 1 - \frac{\bar{A}^3}{(1 + \bar{A}^2)(1 + \bar{A})} = \frac{\bar{A}^2 + \bar{A} + 1}{(1 + \bar{A}^2)(1 + \bar{A})} \quad \text{(i)}$$

Crosscurrent contact, $N = 3$; $G_s/3$ amount used in each stage

Use the solution obtained in Example 4.10.

$$N = \frac{\log \left(\frac{x_0}{x_N} \right)}{\log \left(1 + \frac{\alpha G_s}{3 L_s} \right)} \Rightarrow N = \frac{\log \left(\frac{x_0}{x_N} \right)}{\log \left(1 + \frac{1}{3 \bar{A}} \right)} \Rightarrow \frac{x_0}{x_N} = \left(1 + \frac{1}{3 \bar{A}} \right)^3$$

Fractional separation in the three-stage cross-current cascade,

$$(f)_{\text{cross}} = 1 - \frac{x_N}{x_0} = 1 - \left(\frac{3 \bar{A}}{1 + 3 \bar{A}} \right)^3 = \frac{27 \bar{A}^2 + 9 \bar{A} + 1}{(3 \bar{A} + 1)^3} \quad \dots \text{(ii)}$$

4.25

(4.26) contd... Take $\bar{A} = 0.9$. $(f)_{\text{counter}} = \frac{(0.9)^2 + (0.9) + 1}{(1+0.8)(1+0.9)} \Rightarrow 78.8\%$, from Eq(i)
 From Eq.(ii), $(f)_{\text{cross}} = \frac{(2)(0.9)^2 + (9)(0.9) + 1}{(2.7+1)^3} \Rightarrow 61.1\%$

For the same value, $\bar{A} = \frac{\alpha G_s}{L_s} = 0.9$, if an 'ideal' cross-current cascade (infinite number of stages, a differential amount of G is added to each stage) is used [see Example 4.9, part(c)],

$$\ln(\frac{x_0}{x_f}) = \alpha G_s / L_s = 0.9 \Rightarrow \frac{x_0}{x_f} = 3.038$$

$$\text{fractional separation}, f = 1 - \frac{x_f}{x_0} = 1 - \frac{1}{3.038} \Rightarrow 67\%$$

* * * *

(4.27) Equilibrium relation: $y^* = 2.5x + 8x^2$; bulk cone $(x_b, y_b) \rightarrow$
 Mass transfer will occur in the direction of $(0.05, 0.17)$.
 decreasing concentration.

For $x_b = 0.05$, $y_b^* = (2.5)(0.05) + 8(0.05)^2 = 0.145$ which is less than
 the bulk gas conc. $y_b = 0.17$
 $\Rightarrow y_b - y_b^* > 0 \Rightarrow$ mass transfer occurs from the gas to the
 liquid phase

Calculation of the overall coefficients, K_y & K_x

Use Eqs. (4.18) and (4.21). We need the slopes, m' and m'' .

Consider Fig. 4.5. $M \rightarrow (x_b, y_b) \rightarrow (0.05, 0.17)$

Point T \rightarrow ordinate = $y_b = 0.17$

Abscissa (x_b^*) is given by: $y_b = 0.17 = 2.5(x_b^*)^2 + 8(x_b^*)^2 \Rightarrow x_b^* = 0.05744$

T $\rightarrow (x_b^*, y_b) \rightarrow (0.05744, 0.17)$

Point S \rightarrow Abscissa, $x_b = 0.05$; ordinate, $y_b^* = (2.5)(0.05) + 8(0.05)^2 = 0.145$

$\Rightarrow S \rightarrow (x_b, y_b^*) \rightarrow (0.05, 0.145)$

To get the coordinates of the point N (x_i, y_i) , use the flex relations.

$$K_x(x_i - x_b) = k_y(y_b - y_i) \Rightarrow 5 \times 10^{-4}(x_i - 0.05) = 2.9 \times 10^{-4}(0.17 - 2.5x_i - 8x_i^2)$$

Solving this quadratic in x_i , (since x_i, y_i lie on the equil. line)

$$x_i = 0.05494 ; \quad y_i = 2.5x_i + 8x_i^2 = 0.1615$$

Point N $\rightarrow (x_i, y_i) \rightarrow (0.05494, 0.1615)$.

4.26

4.27 contd... Slope of the chord SN, see the figure below

$$= \frac{0.1615 - 0.145}{0.05 \cdot 4 - 0.05} = m' = 3.34$$

(Note that over a small concentration range, the equilibrium line is almost linear, and the chord almost coincides with the given equil. line)

$$\text{Slope of NT} = \frac{0.17 - 0.1615}{0.05744 - 0.05494} = 3.4 = m''$$

Overall gas-phase coefficient, Eq(4.18),

$$\frac{1}{K_y} = \frac{1}{k_y} + \frac{m'}{k_x} = \frac{1}{2.9 \times 10^{-4}} + \frac{3.34}{5 \times 10^{-4}}$$

$$\Rightarrow \frac{1}{K_y} = 3448 + 6668 = 10116 \Rightarrow K_y = \frac{1}{10116} \text{ kmol/m}^2 \cdot \text{s.} (\Delta y)$$

$$\text{Similarly, from Eq (4.21), } \frac{1}{K_x} = \frac{1}{m'' k_y} + \frac{1}{k_x} = \frac{1}{(3.4)(2.9 \times 10^{-4})} + \frac{1}{5 \times 10^{-4}}$$

$$\Rightarrow \frac{1}{K_x} = 1014 + 2000 = 3014 \Rightarrow K_x = \frac{1}{3014} \text{ kmol/m}^2 \cdot \text{s.} (\Delta x)$$

$$\% \text{ resistance offered by the gas phase} = \frac{1014}{3014} = 33.6\%$$

* * * *

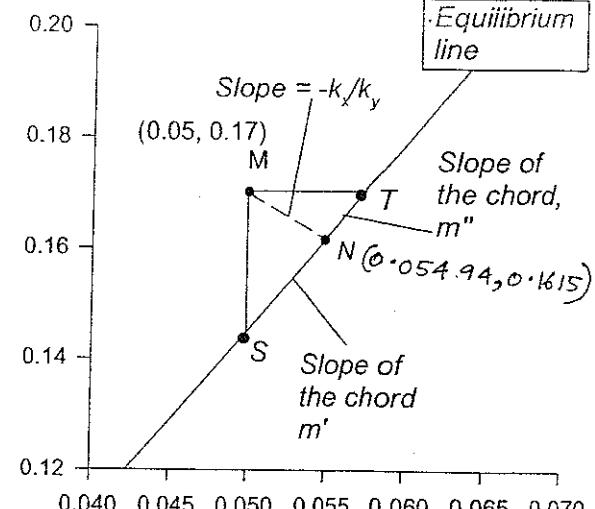


Figure Problem 4.27

4.29 The graphical method of determination of the number of ideal stages in a cross-current cascade has been discussed in Section 4.6.4. If the equilibrium line is straight, and an equal amount of solvent is used in each stage it is easy to determine the number of stages algebraically.

Consider Figure 4.19. Let us take $Y_0 \neq 0$; Equil line: $Y = \alpha X$

Write down the material balance equations for the stages.

$$\text{Stage 1 : } R_s (X_0 - X_1) = E_s (Y_1 - Y_0) \Rightarrow X_0 - X_1 = (\alpha E_s / R_s) \cdot X_1 - (E_s Y_0) / R_s$$

$$\Rightarrow X_0 - (1 + A') X_1 = -(E_s Y_0) / R_s \quad \dots \quad (i) \quad \text{where } A' = (\alpha E_s / R_s)$$

$$\text{Stage 2 : } R_s (X_1 - X_2) = E_s (Y_2 - Y_1) \Rightarrow X_1 - X_2 = (\alpha E_s / R_s) X_2 - (E_s Y_1) / R_s$$

$$\Rightarrow X_1 - (1 + A') X_2 = -(E_s Y_1) / R_s \Rightarrow X_1 (1 + A') - (1 + A')^2 X_2 = -(1 + A') \cdot (E_s Y_1) / R_s \quad \dots \quad (ii)$$

$$\text{Similarly, Stage 3 : } X_2 (1 + A')^2 - (1 + A')^3 X_3 = -(1 + A')^2 \cdot (E_s Y_1) / R_s \rightarrow (iii)$$

(4.27)

4.29 contd... Stage N : $x_{N-1}(1+A')^{N-1} - (1+A')x_N = (1+A')^{N-1}(E_s Y_0)/R_s \dots (i)$

Add equation (i), (ii), (iii) ...

$$x_0 - (1+A')^N \cdot x_N = -\frac{(E_s Y_0)}{R_s} [1 + (1+A') + (1+A')^2 + \dots + (1+A')^{N-1}]$$

$$\Rightarrow x_0 - (1+A')^N \cdot x_N = -\frac{(E_s Y_0)}{R_s} \cdot \frac{1 - (1+A')^N}{1 - (1+A')} = -\frac{(E_s Y_0)}{R_s} \cdot \frac{1 - (1+A')^N}{-(\alpha E_s / R_s)}$$

$$\Rightarrow x_0 - (1+A')^N \cdot x_N = \frac{Y_0 - Y_0/\alpha}{\alpha} (1+A')^N$$

$$\Rightarrow (x_0 - Y_0/\alpha) = (x_N - Y_0/\alpha) \cdot (1+A')^N \Rightarrow N = \frac{\log \left[\frac{x_0 - Y_0/\alpha}{x_N - Y_0/\alpha} \right]}{\log (1+A')}$$

The above equation gives the number of ideal stages for crosscurrent contact.

- * * * * *
- 4.30 The notations used are shown in the figure.

C_s = Solubility of the steroid in the polymer
= Surface concentration in the membrane (inner surface)

C_{mi} = membrane-phase concentration at the liquid-membrane interface.

C_{Li} = liquid-side concentration at the interface

C_{Lb} = steroid concentration in the bulk liquid ($= 0$).

$$\text{At Steady State}, N_A = D_m \cdot \frac{C_s - C_{mi}}{l_m} = D_L \cdot \frac{C_{Li} - C_{Lb}}{\delta_L} = \frac{D_L}{\delta_L} \cdot C_{Li}$$

The given equilibrium relation applies at the interface $\Rightarrow C_{Li} = \beta C_{mi}$

$$\text{Then, } (C_s - C_{mi}) = \frac{N_A}{(D_m/l_m)} \Rightarrow \beta C_s - \beta C_{mi} = \frac{N_A}{(D_m/\beta l_m)}$$

$$\frac{C_{Li}}{C_{mi}} = \frac{N_A}{(D_L/\delta_L)}$$

$$\text{Add to get } \beta C_s = N_A \left[\frac{\beta}{D_m/l_m} + \frac{1}{(D_L/\delta_L)} \right]$$

$$\Rightarrow N_A = \frac{\beta C_s}{\left(\frac{\beta}{D_m/l_m} + \frac{1}{(D_L/\delta_L)} \right)}$$

Given: $\beta = 0.15$; $D_m = 1.083 \times 10^{-7} \text{ m}^2/\text{h}$;
 $D_L = 1.042 \times 10^{-6} \text{ m}^2/\text{h}$;
 $l_m = 10^{-3} \text{ m}$;
 $C_s = 100 \text{ mg/cm}^3$; $N_A = 1 \text{ mg/cm}^2 \text{ h}$

4.30 Contd... $\frac{1 \text{ mg}}{\text{cm}^2 \cdot \text{h}} = \frac{(0.15)(100 \text{ mg/cm}^3)}{\frac{0.15}{(1.083 \times 10^{-3} \text{ cm}^2/\text{h})/0.1 \text{ cm}} + \frac{1}{(1.042 \times 10^{-3} \text{ cm}^2/\text{h})/8_L}}$

$$\Rightarrow \underline{\underline{8_L = 1.2 \times 10^{-3} \text{ cm}}} \quad * \quad * \quad *$$

4.31 Refer to the figure of the problem.

The steady state flux of the diffusant (sodium salicylate) is

$$N_A = D_1 \cdot \frac{C_{Lp} - C_{Lpi}}{l} = D_{eff} \cdot \frac{C_{1i} - C_{2i}}{l_m} = D_2 \cdot \frac{C_{2i} - C_{LW}}{l}$$

$$\Rightarrow C_{Lp} - C_{Lpi} = \frac{N_A}{D_1/l} \Rightarrow \beta C_{Lp} - \beta C_{Lpi} = \frac{\beta N_A}{D_1/l}$$

Use the relation:

$$\underline{\underline{C_{1i} = \beta C_{Lp}}}$$

$$C_{1i} - C_{2i} = \frac{N_A}{D_{eff}/l_m}$$

$$C_{2i} - C_{LW} = \frac{N_A}{D_2/l}$$

Relation at the CMC soln.-membrane interface: $C_{1i} = \beta C_{Lp}$

D_{eff} = effective diffusivity of the solute through the membrane.

$$\text{Add to get } \beta C_{Lp} - C_{LW} = N_A \left[\frac{\beta}{D_1/l} + \frac{1}{D_{eff}/l_m} + \frac{1}{D_2/l} \right]$$

Given: $\beta = 1.16$; $D_1 = 1.33 \times 10^{-5} \text{ cm}^2/\text{s}$; $D_2 = 1.11 \times 10^{-5} \text{ cm}^2/\text{s}$; $l_m = 4.66 \times 10^{-3} \text{ cm}$; $l = 8.2 \times 10^{-3} \text{ cm}$; C_{Lp} = bulk concentration in CMC = 8 mmol/litre = $8 \times 10^{-6} \text{ mol/cm}^3$; C_{LW} = solute conc. in bulk water = 0 (pure water);

$$N_A = 2.18 \times 10^{-9} \text{ gmol/cm}^2 \cdot \text{s}$$

$$\Rightarrow (1.16)(8 \times 10^{-6} \text{ mol/cm}^3) = 2.18 \times 10^{-9} \frac{\text{g mol}}{\text{cm}^2 \cdot \text{s}} \cdot \left[\frac{1.16}{1.33 \times 10^{-5}/8.2 \times 10^{-3}} + \frac{1}{D_{eff}/4.66 \times 10^{-3}} + \frac{1}{1.11 \times 10^{-5}/8.2 \times 10^{-3}} \right]$$

$$\Rightarrow 4.257 \times 10^{-3} = 715.2 + \frac{4.66 \times 10^{-3}}{D_{eff}} + 738.7$$

$$\Rightarrow \underline{\underline{D_{eff} = 1.662 \times 10^{-6} \text{ cm}^2/\text{s}}} = \underline{\underline{\text{effective diffusivity of the solute through the cellophane film}}}$$

5.1

Design of a Sieve tray

(i) Calculation of the tower diameter: Use Fair method

$$L = 6000 \text{ gal/h} = (6000)/(7.48) \text{ ft}^3/\text{h} = 802.14 \text{ ft}^3/\text{h} \quad (Q=100 \text{ gal/min}) \\ = (802.14)(56) = 44920 \text{ lb/h}$$

$$\text{Flow parameter, } F_v = (L/G)(P_g/P_L)^{0.5} = \left(\frac{44920}{30,000}\right) \left(\frac{0.15}{56}\right)^{0.5} = 0.0775$$

Select a tray spacing, $s = 20$ inch

$$\text{From fig 5.12(a), } C_{SB} = 0.27 = U_{S, fl} \cdot (20/\alpha)^{0.2} \left(\frac{P_g}{P_L - P_g}\right)^{0.5}$$

$$U_{S, fl} = (0.27) \left(\frac{21}{20}\right)^{0.2} \left(\frac{56 - 0.15}{0.15}\right)^{0.5} = 5.26 \text{ ft/s}$$

Take the operating gas velocity as 70% of flooding velocity.

$$U_S = (0.7)(5.26) = 3.68 \text{ ft/s}; \text{ vapor rate} = \frac{30,000 \text{ lb/h}}{0.15 \text{ lb/ft}^3} = 55.55 \text{ ft}^3/\text{s}$$

$$\text{Active tray area, } A_a = \frac{55.55 \text{ ft}^3/\text{s}}{3.68 \text{ ft/s}} = 15.1 \text{ ft}^2$$

Take a total downcomer area = 20% of tower cross-section.

$$\text{Tower cross-section} = \frac{15.1}{0.8} = 18.87 \text{ ft}^2$$

$$\text{Tower diameter} = \left[(18.87)(4/\pi) \right]^{0.5} = 4.9 \text{ ft} \\ = 1.49 \text{ m} \sim 1.5 \text{ m (1500 mm)}$$

(ii) Check for the estimated downcomer area

Downcomer area (one side) = 10% of tower cross-section

$$= (0.1)(18.87) = 1.887 \text{ ft}^2$$

$$\text{Clear liquid flowrate} = (802.14/3600) \text{ ft}^3/\text{s} = 0.2228 \text{ ft}^3/\text{s}$$

$$\text{Downcomer volume} = (1.887)(22/12) = 3.46 \text{ ft}^3$$

Liquid residence time in the downcomer

$$= \frac{3.46 \text{ ft}^3}{0.2228 \text{ ft}^3/\text{s}} = 15.58$$

This is acceptable. [A smaller value may be used however - i.e. a smaller downcomer area, say 15% of tower cross-section may be tried.]

(5.2)

(iii) Effective bubbling area and layout of the holes:

Take the effective bubbling area as 90% of active area

$$\Rightarrow (18.87)(0.9) = \underline{17 \text{ ft}^2} \text{ less } 2 \text{ ft}^2 \text{ for support rings etc.}$$

$$\Rightarrow \text{Effective bubbling area} = \underline{15 \text{ ft}^2}$$

Place holes ($\text{dia} = d_{\frac{1}{4}} = 3/8''$) on a triangular pitch, $p' = 3d_{\frac{1}{4}}$

Use Eq.(5.10) to calculate the fractional hole area

$$= (0.905)(\frac{1}{3})^2 = 0.101 \Rightarrow 10.1\%$$

$$\text{Number of holes} = \frac{(15 \text{ ft}^2)(0.101)}{\left(\frac{\pi}{4}\right)[3/(8 \times 12)]^2} = \underline{1975 \text{ number}}$$

(iv) Weir height: Select a weir height, $h_w = 2''$

(v) Pressure drop calculations:

Gas velocity through the holes, $U_h = \frac{55.55 \text{ ft}^3/\text{s}}{(15)(0.101)} = \underline{36.7 \text{ ft}/\text{s}}$

Dry tray pressure drop, $h_d = (0.186/C_0^2)(P_g/P_L) U_h^2$, Eq.(5.15a)

Select tray thickness $1/8''$ for the 5 ft dia column.

Tray thickness / hole dia = $\frac{1/8}{3/8} = 0.33$. From Fig 5.12(d), $C_0 = 0.69$

$$h_d = \frac{0.186}{(0.69)^2} \cdot \frac{0.15}{56} \cdot (36.7)^2 = \underline{1.41'' \text{ liquid head}}$$

Calculation of l_w The weir length, l_w , is calculated

from Eq.(5.11). $\frac{A_d}{A_t} = 0.1 = \frac{\theta - 2\sin\theta}{2\pi} \Rightarrow \theta = 94^\circ$

$$94 = 2\sin^{-1}(l_w/D_c) \Rightarrow l_w/D_c = 0.73 \Rightarrow l_w = (0.73)(4.9 \text{ ft})$$

$$\text{Liquid rate, } Q_L = \frac{100 \text{ gal/min}}{(3.56)(12)} = \underline{2.341 \text{ gal/(min)(inch)}}$$

$$= \underline{3.56 \text{ ft}}$$

$$\alpha / (l_w, \text{ft})^{2.5} = 100 / (3.56)^{2.5} = 4.18$$

$$\text{Weir length / tower dia} = 3.56 / 4.9 = 0.726$$

$$\text{Use Fig 5.12c. } F_w = \underline{1.045}$$

$$h_{ow} = (0.48)(1.045)(2.341)^{2/3} = \underline{0.88'' \text{ liquid head}}$$

Calculation of $h_t = \beta \cdot h_c$ (pressure drop for flow of the gas/vapor through the liquid) :

Neglect hydraulic gradient. Then $\underline{h_c = h_w + h_{ow} = 2.88''}$

Superficial vapor velocity based on the bubbling area,

$$u_s = \frac{55.55 \text{ ft}^{0.3}/s}{15 \text{ ft}^2} = 3.703 \text{ ft/s}$$

$$u_s (P_g)^{1/2} = (3.703)(0.15)^{1/2} = 1.434$$

Use Fig. 5.12(e) to get $\beta = 0.63$

$$\Rightarrow h_t = (0.63)(2.88) = \underline{1.81 \text{ inch}}$$

Calculation of h_{ad} (head loss for liquid flow below the downcomer) :

Select $\frac{1}{2}$ inch clearance below the downcomer.

$$A_{da} = \left(\frac{1}{2} \times 12\right) \cdot l_w = \left(\frac{1}{2}\right)(3.56) \text{ ft}^2 = \underline{0.148 \text{ ft}^2}$$

$$\text{Use Eq(5.17)} \rightarrow h_{ad} = (0.03) \left[\frac{100}{(100)(0.148)} \right]^2 = 1.37''$$

$$\begin{aligned} \text{Total pressure drop} &= h_c + h_t + h_{ad} = (h_w + h_{ow}) + (h_t + h_c) + h_{ad} \\ &= (2 + 0.88) + (1.41 + 1.81) + 1.37 = 7.47'' \text{ liquid head.} \end{aligned}$$

Since the tray spacing is 20", there is no risk of downcomer flooding.

Check for entrainment: Use Eq 5.12(f) (5.4)

$\frac{L}{G} \left(\frac{\rho_G}{\rho_L} \right)^{0.5} = 0.0775$. For 70% approach to flooding,
fractional entrainment = $0.025 < 0.1$
This is acceptable.

Check for weeping and turndown rates:

$$\text{Eq}(5.19) \rightarrow h_a = \frac{(0.04)(21)}{(56)(3/8)} = 0.04 \text{ inch liquid}$$

$$h_w + h_{ow} = 2.88'' \text{ (calculated before)}; h_d + h_a = 1.41 + 0.04 = 1.45''$$

The point $(2.88, 1.45)$ is pretty much above the weep point line in Fig 5.12g. \Rightarrow There is no risk of weeping under normal operating condition.

Calculation of gas velocity at which weeping starts:

For $h_w + h_{ow} = 2.88$, and $A_d/A_a = 0.1$, the ordinate of the weep point curve, Fig 5.12g, is $h_d + h_a = 0.7 \Rightarrow h_d = 0.66''$

$$\text{Put } h_d = 0.66 = \left[\frac{0.186}{(0.69)^2} \right] \left[\frac{0.15}{56} \right] \cdot U_R^2 \Rightarrow U_R = \underline{25.16 \text{ ft/s}}$$

This is $25.16/36.7 = 0.68$, or 68% of the normal velocity.

Turndown = $1/0.68 = 1.46$, which appears to be a bit low. A second round of calculation on the basis of a smaller hole diameter may be done to improve the turndown.

5.4 Design of the column using 1.5" ceramic intalox saddles

$$\text{Flow parameter, } F_{\text{fr}} = \frac{L}{G} \left(\frac{P_G}{P_L} \right)^{0.5} = \frac{L}{G} \left(\frac{P_G}{P_L} \right)^{0.5}$$

$$= \frac{(8500)}{6800} \left(\frac{2.403}{710} \right)^{0.5} = \underline{0.0727} \quad \begin{cases} P_G = 0.15 \text{ lb/ft}^3 \\ = 2.403 \text{ kg/m}^3 \end{cases}$$

Use the Eckert's GPDC chart, fig. 5.33. At flooding, the capacity parameter is :

$$\frac{\left(\frac{G'}{fL}\right)^2 \cdot F_p \cdot \left(P_w/P_L\right) \left(\mu_L\right)^{0.2}}{P_G \cdot P_L \cdot g_c} = 0.16$$

$$\left(\frac{G'}{fL}\right)^2 = \frac{(0.16)(0.15)(44.2)(4.18 \times 10^8)}{(52)(1000/710)(0.73)^{0.2}} \Rightarrow \left(\frac{G'}{fL}\right)^2 = \underline{2540 \text{ lb/ft}^2 \cdot h}$$

$$F_p = 52 \text{ ft}^{-1}; \text{ take } P_w = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$P_L = 710 \text{ kg/m}^3 = \underline{44.2 \text{ lb/ft}^3}$$

$$\mu_L = 0.73 \text{ CP}$$

Take the operating gas rate, $G' = (0.7)\left(\frac{G'}{fL}\right)^2 = 1778 \text{ lb/hr.ft}^2$

$$\text{Tower cross-section} = \frac{G/G'}{1778 \text{ lb/ft}^2 \cdot h} = \frac{6800 \times 2.206 \text{ ft}^2}{1778 \text{ lb/ft}^2 \cdot h} = \underline{8.437 \text{ ft}^2}$$

$$\text{Tower diameter, } D_c = \left[\frac{(8.437)(4/\pi)}{0.73} \right]^{0.5} = \underline{3.28 \text{ ft} = 1 \text{ meter}}$$

Pressure drop calculation

$$\text{Flow parameter} = 0.0727$$

$$\text{Capacity parameter} = \frac{\left(\frac{G'}{fL}\right)^2 (52)(1000/710)(0.73)^2}{(0.15)(44.2)(4.18 \times 10^8)} = 0.0784$$

From the Eckert chart, Fig 5.33,

pressure drop = 1.1 inch water per foot of packed bed.

$$\text{Pressure drop per meter bed} = 1.1/0.3048 = 3.61 \text{ " water}$$

$$= \frac{(3.61)(14.7)}{(12)(34.03)} = \underline{0.13 \text{ psi}}$$

Operating liquid holdup. Use Eq(5.25) in consistent units.

$$U_{Lg} = \left(\frac{8000}{710}\right) / (\pi/4 \cdot 1) = \underline{14.35 \text{ m/h}} = \underline{4 \times 10^{-3} \text{ m/s}}$$

$$\alpha_p = 52 \text{ ft}^{-1} = \frac{52}{0.3048} = \underline{170.6 \text{ m}^{-1}} ; g = \underline{9.81 \text{ m/s}^2}$$

$$\mu_L = \underline{7.3 \times 10^{-4} \text{ kg/m.s}} ; \alpha_L = \underline{23 \text{ dyne/cm}} = \underline{0.023 \text{ kg/s}^2}$$

$$h_{L0} = (0.93) \left[\frac{(4 \times 10^{-3})(170.6)}{9.81} \right]^{1/6} \cdot \left[\frac{(7.3 \times 10^{-4})(170.6)^3}{(710)^2 (9.81)} \right]^{1/10} \cdot \left[\frac{(0.023)(170.6)^2}{(710)(9.81)} \right]^{1/8}$$

$$= (0.93)(0.2554)(0.236)(0.7462) = \underline{\underline{0.042}}$$

Design calculation for # 50 IMTP

Use Strigle's GPDC chart (Figure 5.34). Take the allowable pressure drop = 0.2" water per foot of bed.

For $F_v = 0.0727$, and $\Delta P/L = 0.2$,

$$\text{capacity parameter} = 0.9 = C_s (F_p)^{0.5} \cdot v^{0.05}$$

$$\text{Put } F_p = 18 \text{ ft}^{-1}, v = \mu_L / \rho_L = 1.03 \text{ centistoke}$$

$$C_s = (0.9) / (0.8)^{0.5} \cdot (1.03)^{0.05} = 0.212 = U_{Lg} \left[\frac{\rho_G}{\rho_L - \rho_A} \right]^{0.5}$$

$$\Rightarrow U_{Lg} = (0.212) \left[\frac{710 - 2.403}{2.403} \right]^{1/2} = \underline{\underline{3.638 \text{ ft/s}}} = \underline{\underline{1.11 \text{ m/s}}}$$

$$\text{Volumetric gas flowrate} = \frac{6800}{2.403} \text{ m}^3/\text{h} = \underline{\underline{2830 \text{ m}^3/\text{h}}}$$

$$\text{Cross-section of the tower} = \frac{2830}{(1.11)(3600)} = \underline{\underline{0.708 \text{ m}^2}}$$

$$\text{Diameter} = \underline{\underline{0.95 \text{ m}}}$$

5.5 $L = 378,000 \text{ kg/day} = 15,750 \text{ kg/h} ; G = 20,000 \text{ kg/h}$

$$\rho_L = 470 \text{ kg/m}^3 ; \rho_G = 0.4 \text{ lb/ft}^3 = 6.43 \text{ kg/m}^3 \\ = 29.2 \text{ lb/ft}^3$$

Flow parameter, $F_{LV} = \frac{L}{G} \left(\frac{\rho_G}{\rho_L} \right)^{0.5} = \frac{(15750)}{20,000} \left(\frac{0.4}{29.2} \right)^{0.5} = 0.0922$.

Since the operating pressure is high, a pressure drop on the higher side is acceptable.

Take $\Delta P/L = 0.5$ inch water per foot of packing.

Capacity parameter (fig 5.34), $C_s F_p^{0.5} v^{0.05} = 1.26$

For #25 IMTP, $F_p = 41 \text{ ft}^{-1} ; a_p = 74.7 \text{ ft}^2/\text{ft}^3$,

$$v = \mu_L / \rho_L = 0.1 / 0.47 = 0.213 \text{ centistokes}$$

$$C_s = \frac{1.26}{(41)^{0.5} (0.213)^{0.05}} = 0.2126 = u_{SG} \left(\frac{\rho_G}{\rho_L - \rho_G} \right)^{0.5}$$

$$\Rightarrow u_{SG} = (0.2126) \left(\frac{470 - 6.43}{6.43} \right)^{0.5} = 1.805 \text{ ft/s} = 0.55 \text{ m/s}$$

Volumetric gas flowrate = $\frac{20,000 \text{ kg/h}}{6.43 \text{ kg/m}^3} = 3110 \frac{\text{m}^3}{\text{h}} = 0.864 \frac{\text{m}^3}{\text{s}}$

Tower cross-section = $\frac{0.864 \text{ m}^3/\text{s}}{0.55 \text{ m/s}} = 1.57 \text{ m}^2$

Tower diameter = $\sqrt{[(1.57)(4/\pi)]} = 1.414 \text{ m}$

If 25 mm Pall ring is used, $C_s F_p^{0.5} v^{0.05} = 1.26 ; F_p = 56 \text{ ft}^{-1}$

$$C_s = 0.182 \text{ and } u_{SG} = 0.471 \text{ m/s}$$

Tower cross-section = $\frac{0.864}{0.471} = 1.834 \text{ m}^2 ; \text{ dia} = 1.53 \text{ m}$

- 6.1 Since $N_A = k_G (P_{AB} - P_{A,i}) = k_L (C_{Ai} - C_{ib})$, slope of PM = $-k_L/k_G$ (ii)
- 6.2 (a) and (d) \rightarrow countercurrent; (b) and (c) cocurrent; (ii) \rightarrow 6.16(b)
represents a cocurrent operation with 100% stage efficiency; (c) \rightarrow not.
- 6.3 A horizontal line segment from the point K to the equilibrium line represents the overall liquid-phase driving force; a vertical line segment represents the overall gas-phase driving force.
- 6.4 (iv) \rightarrow none of these, since the operating line cannot cross the equil. line.
- 6.5 (a) \rightarrow (iii) towards E; (b) \rightarrow (iii) somewhere midway.
- 6.6 (iii) $N > N'$, since the available driving force over the latter 40% of the concentration range is more than that used for a linear eq. line.
- 6.7 (iii), (iv), (v) will help in reducing the packed height.
- 6.8 (i) increase. 6.9 (ii). 6.10 Option (iii) is the best, although option (i) may work in some cases. 6.11 (i) . 6.12 (i) . 6.13 (ii).
- 6.14 For absorption of C. 6.15 (i), (ii). 6.16 (iii) 63.2% = 6.17 (ii) 1.3 ft.
- 6.18 (ii) NTU, Since the driving force will be more. 6.19 (i), see Eq(6.25e).
- 6.20 Eq (6.23) $\rightarrow N_{TOG}^{x_1=1} = \frac{y_1 - y_2}{(y - y^*)_{av}} \Rightarrow (y_1 - y_2) = (y - y^*)_{av}$ for $N_{TOG} = 1$
 $\Rightarrow (y_1 - y_2) = \frac{0.014 + 0.0098}{2} = 0.0119 \approx 0.012 \rightarrow$ (iii). 6.21 (iii) $4/k_G = m$.
- 6.22 (ii) . 6.23 $N_{TOL} = \frac{x_1 - x_2}{(\Delta x)_{av}} = \frac{0.06 - 0.002}{(0.00472 - 0.00102) / \ln(\frac{0.00472}{0.00102})} = 21$ (iii).
- 6.24 Use Eq(6.24b). For absorption of a dilute gas, take $y_{IBM}/x_{BM}^* \approx 1$ and $x_{IBM}/x_{BM}^* \approx 1$. $G = 50 \text{ kmol/h}$, $L = 1800 \text{ kg/h} = 100 \text{ kmol/h}$, $m = 1.5$; $G/L = G'/L'$
 $H_{TOL} = (L'/m G') H_{tG} + H_{tL} = \frac{100}{(1.5)(50)} \cdot (1.2) + 1.9 = 1.6 \text{ ft}$
- 6.25 (i) . 6.26 (i) . 6.27 (iii) . 6.28 (i) . 6.29 See eq(6.49), $HETP = H_{TOG} \cdot \frac{\ln(1/\bar{A})}{(\bar{A}-1)}$
 $= (1.2) \cdot \frac{\ln(1/1.6)}{(1/1.6)-1} = 1.5 \text{ ft}$. 6.30 (iii) . 6.31 Use Eq(6.61), evaluate the limit of E_0 as $G'/L' \rightarrow 1$, Ans. $E_0 = E_{MG} = 0.6$. 6.32 $N_{TOG} = 0.8$; $E_{OG} = 1 - e^{-N_{TOG}} = 0.55$; Now use Eq(6.58) $\rightarrow E_{MG} = (1.5) \left[\exp\left(\frac{0.55}{1.5}\right) - 1 \right] = 0.664$ (iii)
Put $L'/m G' = 1.5$

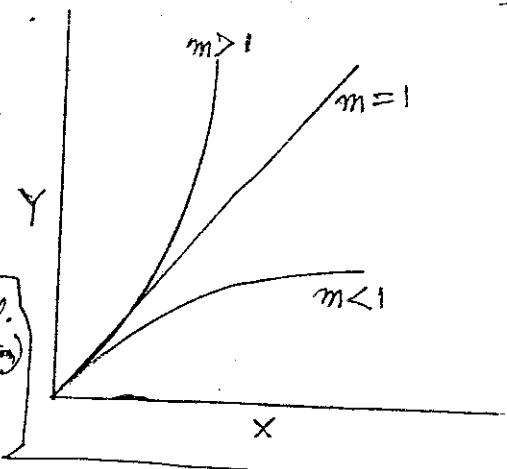
6.2

$$6.33 \quad y = mx \Rightarrow \frac{Y}{1+Y} = \frac{m \cdot X}{1+X} \Rightarrow Y = \frac{mx}{1+(1-m)x} \Rightarrow \frac{dY}{dx} = \frac{m}{[1+(1-m)x]} - \frac{mx(1-m)}{[1+(1-m)x]^2}$$

$$\Rightarrow \frac{dY}{dx} = \frac{m}{[1+(1-m)x]^2} \Rightarrow \frac{d^2Y}{dx^2} = \frac{-2m(1-m)}{[1+(1-m)x]^3}$$

If $m < 1$, $\frac{d^2Y}{dx^2} < 0 \Rightarrow$ X-Y plot is convex upward

$m > 1$, $\frac{d^2Y}{dx^2} > 0 \Rightarrow$ X-Y plot is convex downward



$$6.34 \quad \text{Given: } Y_2 = 0.001; X_2 = 0.0; Y^* = 2.5 \times (\text{equil. reln})$$

Material balance over a section of the column:

$$L_s(X-X_2) = G_s(Y-Y_2); \text{ given: } Y = 0.035$$

Calculate X from the given driving force : $\Delta Y = Y - Y^* = Y - 2.5X$

$$\Rightarrow 0.004 = 0.035 - 2.5 \cdot X \Rightarrow X = 0.0124$$

Put in the material balance eq. $\rightarrow L_s(0.0124 - 0) = G_s(0.035 - 0.001) \Rightarrow \frac{L_s}{G_s} = 2.742$

Now consider the location where $Y = 0.04$.

From material balance $\rightarrow (L_s/G_s)(X-0) = (Y-Y_1)$

$$\Rightarrow (2.742)(X-0) = 0.04 - 0.001 \Rightarrow X = 0.01422$$

Overall liquid phase driving force, $X^* - X = \frac{Y}{2.5} - X = \frac{0.04}{2.5} - 0.01422$

$$\Rightarrow (X^* - X) = 0.00178$$

$$6.35 \quad \text{For 'well-mixed' liquid on a tray, } E_{OG} = \frac{E_{MG}}{0.55} \quad (6.57).$$

$$\text{Eq (6.55)} \rightarrow E_{OG} = 1 - e^{-N_{tOG}} \Rightarrow 0.55 = 1 - e^{-N_{tOG}} \Rightarrow N_{tOG} = 0.8$$

$$6.36 \quad \text{For a 'dilute' gas-liquid system, take } y_{iBM}/x_{BM}^* \approx 1, \frac{x_{iBM}}{x_{BM}^*} \approx 1$$

$$\Rightarrow H_{tOL} = \frac{L'}{mb} \cdot H_{tG} + H_{tL} \Rightarrow H_{tOL} = \bar{A} H_{tG} + H_{tL}$$

6.37 Derivations given at the end.

6.38 Frequent termination of liquid films on a packing surface causes mixing as well as quicker 'surface renewal'. This enhances the mass transfer coefficient and hence the packing efficiency.

6.39 Any of them is the ratio of two slopes - those of the equil. line (G_s/m) and of the operating line (say L_s/G_s).

6.40 (iii) 0.035. (6.41) 200 kmol/k.m.².atm is a reasonable value,

See Table 6.4.

(6.42) (i) $E_{OG} = E_{MG}$. (6.43) (ii) decreases, See Section 6.8.3.

6.44) The gas-phase mass transfer coefficient, k_y , for diffusion of A through non-diffusing B is concentration dependent because of the term y_{BM} (in Eq. 3.5a; also see SQ 2, Ch. 3). Thus $k_y(1-y)_{IM}$ should be concentration-independent (y is the mole fraction of A). Also k_y varies as $G^{0.8}$ for turbulent flow. Effectively $G'/k_y \bar{c}(1-y)_{IM}$ remains almost constant.

6.45) Yes, it is possible, although the exercise does not appear to have any practical use.

Assume that the gas-liquid system is dilute. Consider Eqs. 3.5a & 3.5b.

$$K_y = \frac{D_{AB} P^2}{RT \delta g \cdot P_{BM}} = \frac{D_{AB} P}{RT \delta g} (P \approx P_{BM}); K_x = \frac{D_{AB} (P/M)_{av}}{\delta_L \cdot x_{BM}} = \frac{D_{AB} \bar{c}}{\delta_L} [x_{BM} \approx 1 \text{ &} (P/M)_{av} = \bar{c}]$$

$$\Rightarrow \frac{1}{K_y} = \frac{1}{K_x} + \frac{m}{K_x} = \left(\frac{RT}{D_{AB} P} \right) \cdot \delta g + \left(\frac{1}{D_{AB} \bar{c}} \right) \delta_L$$

$$\text{Also } K_y = \frac{D_{AB} P}{RT (\delta g)_{\text{overall}}} \Rightarrow (\delta g)_{\text{overall}} = \frac{D_{AB} P}{RT K_y}$$

specific surface area, porosity,
shape, surface characteristics including
wettability, etc.

6.46) Packing type and its characteristics; transport properties of the gas and of the liquid (density, viscosity, surface tension, diffusivity) flowrates of the fluids.

6.47) By Schmidt number correction; see Eq. (6.37).

6.48) Yes, the wetted-wall column may simulate a packed bed so far as flow and mass transfer are concerned, see channel model of a packed bed, chapter 5 and Bilek (1995).

6.49) The assumptions are : (i) the liquid is vertically well-mixed, but is in plug flow in the horizontal direction ; (ii) the gas is in plug flow. The assumptions are reasonable if the depth of gas-liquid dispersion is not much and there is not much froth.

$$6.50) N \cdot \text{HETP} = \text{packed height} = H_{toG} \cdot N_{toG} \Rightarrow N_{toG} = N \cdot \frac{\text{HETP}}{H_{toG}} = N \cdot \frac{\ln \bar{s}}{\bar{s} - 1}$$

6.51) (i) concentrated H_2SO_4 ; (ii) water; (iii) di- or tri-ethylene glycol; (iv) concentrated H_2SO_4 ; (v) an alkanolamine; (vi) water; (vii) (non-volatile) absorption oil.

from Eq. (6.49)

$$(6.52) \quad E_{MG} = \frac{L'}{mG'} \left[\exp \left(\frac{mG'}{L'} \cdot E_{OG} \right) - 1 \right] \text{ for plug flow of the liquid; } \bar{A} = L'/mG'$$

If $E_{MG} = E_{OG} = E'$ say, determine \bar{A} .

$$E' = \bar{A} \left[\exp \left(\frac{E'}{\bar{A}} \right) - 1 \right] \Rightarrow \left(E' / \bar{A} \right) = \exp(E' / \bar{A}) - 1.$$

The equation has the solution : $E' / \bar{A} = 0 \Rightarrow \bar{A} = \underline{\text{infinite}}$

Thus E_{MG} for the liquid in plug flow becomes the same as the point efficiency if the absorption factor, \bar{A} , is infinite

(6.5)

PROBLEMS

(6.1) Sample calculation of $x_A, y_A; X_A, Y_A$ (shown in the solution of Prob 4.10 also)

Take the data point, $P_{NH_3} = 51.0 \text{ mm Hg}$; equil. conc. $5 \text{ kg NH}_3 / 100 \text{ kg water}$.

Total pressure, $P = 105.1 \text{ kPa} = (105.1/101.3)(760) = 788.5 \text{ mm Hg}$.

$$Y_A = P_A/P = 51.0/788.5 = 0.06468; Y_A = 51.0/(788.5 - 51.0) = 0.06915$$

$$x_A = \frac{5/17}{5/17 + 100/18} = 0.0502; X_A = \frac{5/17}{100/18} = 0.05294$$

Calculated data for all the data points are given below:

$P_{NH_3} (\text{mm Hg})$	0	19.3	29.6	40.1	51.0	79.5	110
x_A	0	0.0207	0.0308	0.0406	0.0502	0.0736	0.0957
y_A	0	0.0245	0.0375	0.0509	0.0647	0.1008	0.1395
X_A	0	0.02118	0.03178	0.04232	0.05294	0.07941	0.1059
Y_A	0	0.0251	0.0390	0.0536	0.0691	0.112	0.162

The plots x_A, p_A ; $x_A - y_A$; and $X_A - Y_A$ are shown below (Fig. 6.1(a)).

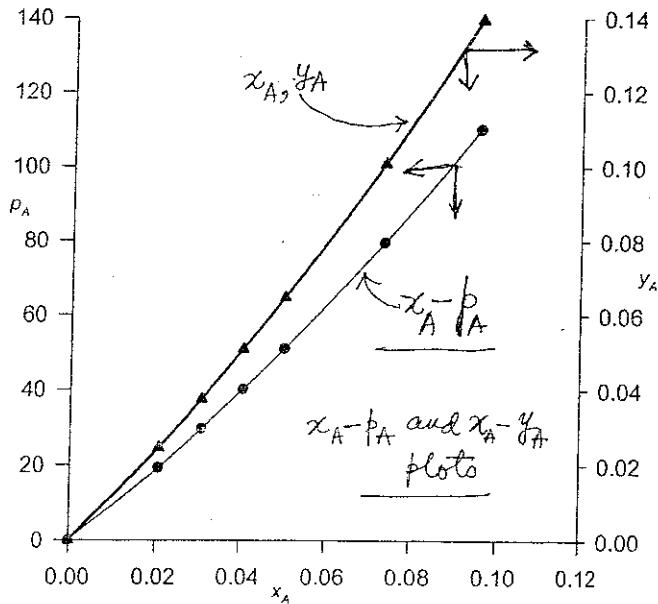


Figure Prob 6.1(a) $x_A - y_A - p_A$ plots

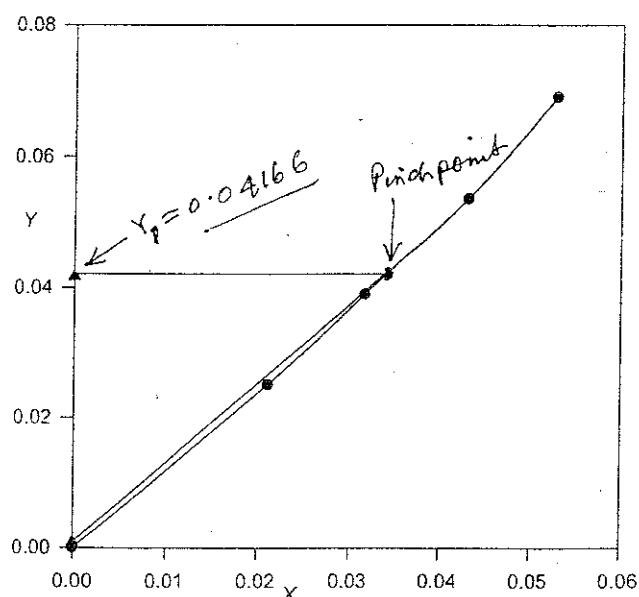


Figure 6.1(b)

(b) Feed gas: $4200 \text{ Nm}^3 = \frac{4200}{22.414} = 1874 \text{ kmol/h}$, 4% $\text{NH}_3 \Rightarrow 7.495 \text{ kmol NH}_3 + 179.89 \text{ kmol air}$. $NH_3 \text{ out} = (7.495)(0.02) = 0.15 \text{ kmol/h}$. $Y_2 = \frac{0.15}{179.89} = 0.000834; Y_1 = \frac{7.495}{179.89} = 0.04166$

or the feed water, $x_2 = 0$. The equilibrium curve is convex downward. The operating line for minimum water rate, $(L_s)_m$, is drawn from $(x_2, y_2) \rightarrow (0, 0.000834)$ and meets the equil. line for $Y_1 = 0.04166$. Slope $= (L_s)_{min}/G_s = 1.136$. Minimum water rate $= (1.136)(179.89) = 3678 \text{ kmol/h}$

(6.2) If the solution is ideal, $y = \frac{P^v}{P} \cdot x = Kx$ (say), $K = \frac{P^v}{P}$.
 $y = Kx \Rightarrow \frac{Y}{1+Y} = \frac{Kx}{1+x} \Rightarrow Y = \frac{Kx}{1+x - Kx} = \frac{Kx}{1+(1-K)x} \dots \dots \text{(i)}$

For a solution obeying Raoult's law, Y and x are related as in eq(i).

Given equil. relation $\rightarrow Y = \frac{0.19x}{1+0.81x} = \frac{0.19x}{1+(1-0.19)x}$ is of the form of eq(i).

\Rightarrow The benzene-oil system is ideal with $K = 0.19 = \frac{P^v}{P}$.

Given $P = 1.07 \text{ bar}$, $P^v = (0.19)(1.07) = 0.2033 \text{ bar} = \underline{\text{vapor pressure of benzene at } 308 \text{ K}} \rightarrow (a)$

(b) Mole fraction of benzene in the rich oil:

$$\text{Feed gas rate} = 1550 \text{ Nm}^3/\text{h} = \frac{1550}{22.414} \text{ kmol/h} = 69.2 \text{ kmol/h}$$

$$\text{Mole fraction benzene in the feed} = 0.095 \Rightarrow \text{Moles benzene} = (69.2)(0.095) = 6.57$$

$$\text{Moles of carrier (nitrogen) in the feed} = 69.2 - 6.57 = 62.63 \text{ kmol/h} = G_s$$

$$\text{Mole fraction of benzene in the outlet gas}, y_2 = 0.005 \Rightarrow Y_2 = \frac{0.005}{0.995} = 0.00502$$

$$\text{Moles benzene leaving} = G_s \cdot Y_2 = (62.63)(0.00502) = 0.31 \text{ kmol/h.}$$

$$\text{Liquid input at the top (solute-free)}, L_s = 4600 \text{ kg/h} = \frac{4600}{230} = 20 \text{ kmol/h.}$$

$$\text{Material balance (overall)}: G_s(Y_1 - Y_2) = L_s(X_1 - X_2) ; Y_1 = \frac{0.095}{0.905} = 0.105.$$

$$\Rightarrow (62.63)(0.105 - 0.00502) = (20)(X_1 - 0) \Rightarrow X_1 = 0.3125 \Rightarrow x_1 = 0.238 \text{ mole fraction}$$

(c) Calculation of tower diameter and flooding velocity: benzene in the rich liquid.

$$\text{Volumetric feed gas rate} = (1550 \text{ Nm}^3/\text{h}) \left(\frac{308}{273} \right) \left(\frac{1.013}{1.07} \right) = 1655.6 \text{ m}^3/\text{h.}$$

$$\text{Superficial gas velocity at the bottom} = 0.32 \text{ m/s}$$

$$\Rightarrow \text{Tower cross-section} = \frac{1655.6}{(3600)(0.32)} = 1.437 \text{ m}^2. \text{ Tower dia} = 1.353 \text{ m}$$

Liquidate at flooding: Use Eq(5.27) to calculate the flooding liquid vel.

$$\text{Feed gas density}, \rho_g = \frac{62.63 \text{ kmol N}_2 + 6.57 \text{ kmol C}_6H_6}{1655.6 \text{ m}^3} = \frac{(62.63)(28) + (6.57)(78)}{1655.6} = 1.385 \frac{\text{kg}}{\text{m}^3}.$$

$$\text{Flow parameter, } F_{LR} = \left(\frac{L'}{G'} \right) \left(\frac{\rho_g}{\rho_L} \right)^{1/2} = \left[\frac{4600}{(1655.6)(1.385)} \right] \left(\frac{1.385}{840} \right)^{1/2} = 0.082.$$

$$\text{Eq}(5.27) \rightarrow \log_{10} (Y_{fl}) = -0.296 [\log(0.082)]^2 - 1.081 \log(0.082) - 1.668 = -0.843$$

$$Y_{fl} = 0.1435 = \frac{(G')^2 \cdot F_p \cdot (\mu_L / \mu_f) (g_c)}{\rho_g \rho_L g_c}$$

$$\Rightarrow G' = 1334 \text{ lb/hr ft}^2 \text{ at flooding}$$

$$F_p = 94.5 \text{ ft}^{-1} \text{ for } 1\frac{1}{2}'' \text{ Raschig ring}$$

$$\mu_L = 4.2 \text{ cP}, \rho_L = 840 \text{ kg/m}^3$$

$$\text{etc. } g_c = 4.18 \times 10^8 \text{ ft/lb}^2$$

$$\rho_g = 0.0851 \text{ lb/ft}^3 \quad = 52.2 \text{ kg/m}^3$$

(6.2)

(6.2) Constd... Actual gas rate = $(62.63)(28) + (6.57)(78) = 2266 \text{ kg/h}$
 $\Rightarrow G' = \frac{2266 \text{ kg/h}}{1.437 \text{ m}^2} = \frac{(2266)(2.046 \text{ kg})}{(1.437)(0.3048)^2 \text{ ft}^2} = 333 \text{ lb/hr ft}^2$

$\frac{\text{Actual gas rate}}{\text{Gas rate at flooding}} = \frac{333}{1334} = 0.242$ (this is a bit too small!)

(d) Number of trays: Plot the equilibrium curve $Y = \frac{0.19X}{1+0.81X}$, and draw the operating line joining the points P $\rightarrow (X_2, Y_2) \rightarrow (0, 0.00502)$, and Q $\rightarrow (X_1, Y_1) \rightarrow (0.3125, 0.105)$. Draw steps as usual. No of ideal trays = 3.7

Tray efficiency (overall)
 $= 35\% \rightarrow 0.35$

Number of real trays
 $= 3.7/0.35$
 $= 10.6 \rightarrow 11$

Eleven real trays are required for the absorption.

Data calculated for equil. line:

X	0	0.05	0.10	0.2	0.3
Y	0	0.0093	0.0176	0.0326	0.0459

X	0.6
Y	0.0767

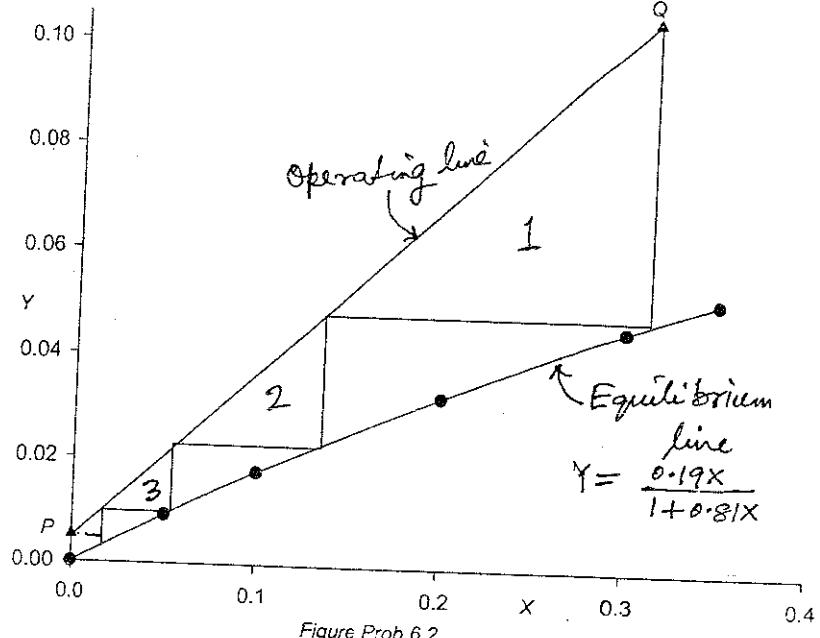


Figure Prob 6.2

(6.3) Concentration of CS₂ in the "rich" oil entering the stripper, $x_0 = 0.0335$.

Equilibrium relation (assume ideal solution): $y = \frac{P^v}{P} \cdot x$; $P^v = 4.7373 \text{ bar}$
 $\Rightarrow y = \frac{4.7373}{1.12} x \Rightarrow y = 4.23x$

Or, in mole ratio units, $\frac{Y}{1+Y} = \frac{4.23X}{1+X} \Rightarrow Y = \frac{4.23X}{1-3.23X}$.

The "lean" oil leaves the tower with 50 ppm CS₂ (mol. wt: CS₂ = 74 oil: 220; Ex 6.6)

$$X_N = \frac{50/74}{10^6/220} = 1.486 \times 10^{-4}; Y_{N+1} = 0 \text{ (no CS}_2 \text{ in the inlet steam);}$$

$$X_0 = \frac{x_0}{1-x_0} = \frac{0.0335}{1-0.0335} = 0.0347.$$

The 'pinch point' for the minimum steam rate occurs at the [since the equil. line is almost linear] top of the column where $x_0 = 0.0347$

(6.3) contd... Corresponding equil. concentration in the gas (2 stream):

$$(Y_1)_{\max} = \frac{4.23X_0}{1-3.23X_0} = \frac{(4.23)(0.0347)}{1-(3.23)(0.0347)} = 0.1653.$$

Use an overall material balance to calculate the minimum steam rate:

$$(G_s)_{\min} [(Y_1)_{\max} - Y_{N+1}] = L_s (X_0 - X_N); \quad L_s = 304.2 \text{ kmol/h, see Ex 6.6.}$$

$$\Rightarrow (G_s)_{\min} [0.1653 - 0] = (304.2) [0.0347 - 1.486 \times 10^{-4}] \Rightarrow (G_s)_{\min} = 63.7 \frac{\text{kmol}}{\text{h}}.$$

Actual steam rate, $G_s = (1.3)(G_s)_{\min} = (1.3)(63.7) = 82.8 \text{ kmol/h.}$

Corresponding C_{S_2} concentration in the exit steam at the top:

$$(82.8)(Y_1 - 0) = (304.2)(0.0347 - 1.486 \times 10^{-4}) \Rightarrow Y_1 = 0.127.$$

Draw the equilibrium line

$$Y = \frac{4.23X}{1-3.23X} \Rightarrow \begin{array}{c|ccccccccccccc} X & 0 & 0.005 & 0.01 & 0.015 & 0.02 & 0.025 & 0.03 \\ \hline Y & 0 & 0.0215 & 0.0437 & 0.0667 & 0.0904 & 0.115 & 0.1405 \end{array} \dots$$

Locate the points $P \rightarrow (X_N, Y_{NH}) \rightarrow (1.486 \times 10^{-4}, 0)$ and $Q \rightarrow (X_0, Y_1) \rightarrow (0.0347, 0.127)$.

Join PQ, which is the operating line and draw steps starting from Q.

We draw seven steps after which the equil. line and the operating line become too close. We end at the

point $(x_7, y_7) \rightarrow (0.007, 0.021)$
on the operating line. The
number of trays below this
point is calculated using
the Kremser equation. In
this concentration range the
equilibrium line is linear
with a slope $\alpha = 4.23$

$$\bar{A} = \frac{L_s}{\alpha G_s} = \frac{304.2}{(4.23)(82.8)} = 0.869$$

Number of trays,

$$N' = \frac{[(0.007 - 0) / (1.486 \times 10^{-4} - 0)](1 - 0.869) + 0.869}{\log(1/0.869)}$$

$$= 14$$

Total number of ideal trays
 $= 7 + 14 = 21$

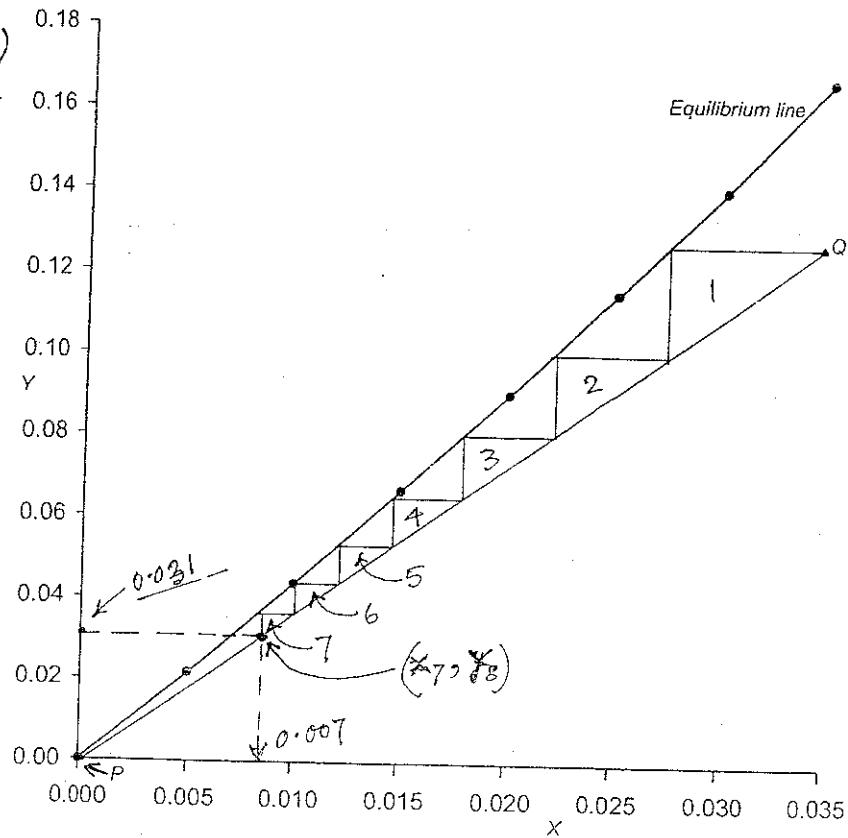


Figure Prob 6.3

(6.9)

(6.4) Feed gas : $150 \text{ m}^3/\text{h}$, 20°C and $1.1 \text{ bar} \Rightarrow G_1 = (150) \left(\frac{273}{293} \right) \left(\frac{1.1}{1.013} \right) \left(\frac{1}{22.4} \right)$

Inlet SO_2 conc, $y_1 = 0.03$. $G_S = 6.775 (1 - 0.03) = 6.572 \frac{\text{kmol}}{\text{h}}$ $= 6.775 \text{ kmol/h}$

Moles SO_2 in $= (6.775)(0.03) = 0.2032 \text{ kmol/h}$; SO_2 absorbed $= (0.2032)(0.95)$
 $= 0.1930 \text{ kmol/h}$; SO_2 leaving $= 0.0102 \text{ kmol/h}$

$\Rightarrow y_2 = \frac{0.0102}{0.0102 + 6.572} = 0.00155$; liquid rate $= 1.4 \text{ kmol/h} = L_S = L_2$.

$x_1 = \frac{0.1930}{0.1930 + 1.4} = 0.1212$; $x_2 = 0$ (feed liquid SO_2 -free).

Since the concentrations are low the problem can be solved using the mole fraction unit of concentration and the average gas rate.

Exit gas rate, $G_2 = 6.572 + 0.0102 = 6.5822$. Take $G = \frac{G_1 + G_2}{2} = 6.679 \frac{\text{kmol}}{\text{h}}$

Tower dia $= 1 \text{ ft} = 0.3048 \text{ m}$; cross-section $= \frac{\pi}{4} (0.3048)^2 = 0.07297 \text{ m}^2$

$(G')_{av} = \frac{6.679}{0.07297} = 91.54 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$; $K_G = 3.2 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta P, \text{bar})}$, given.

$K_y = P \cdot K_G = (1.1)(3.2 \times 10^{-4}) = 3.52 \times 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$; $\bar{a} = 105 \text{ m}^2/\text{m}^3$.

$H_{tOG} = \frac{G'}{K_y \bar{a}} = \frac{91.54}{(3.52 \times 10^{-4})(3600)(105)} = 0.688 \text{ m}$. (given)

Use Eq (6.27b) to calculate N_{tOG} for a dilute system.

$N_{tOG} = \frac{y_1 - y_2}{(y - y^*)_M}$; equil. relation: $y^* = 0.17x$ ($m = 0.17$, given)

$y_1 = 0.03$; $x_1 = 0.1212$; $y_1^* = mx_1 = (0.17)(0.1212) = 0.0206$

$\Rightarrow N_{tOG} = \frac{0.03 - 0.00155}{0.00436}$ $\left| \begin{array}{l} (\Delta y)_1 = 0.03 - 0.0206 = 0.0094 \leftarrow (y_1 - y_1^*) \\ (\Delta y)_2 = 0.00155 - 0 = 0.00155 \leftarrow (y_2 - y_2^*) \\ (y - y^*)_M = \frac{0.0094 - 0.00155}{\ln \frac{0.0094}{0.00155}} = 0.00436 \end{array} \right.$

Packed height $= H_{tOG} \cdot N_{tOG}$
 $= (0.688)(6.52) = 4.5 \text{ m}$

* * * *

(6.5) (a) Henry's law constant for the system TCA-air-water. M.W. of TCA $= 132.5$
 $|P = 1.1 \text{ bar}|$

At $\theta = 25^\circ\text{C}$, $K_i = 0.204 + (0.0182)(25) + (0.000173)(25)^2 = 0.7671 \frac{(\text{mg/L})_{\text{gas}}}{(\text{mg/L})_{\text{water}}}$

$1 \text{ L (liter)} \text{ air at } 1.1 \text{ bar and } 25^\circ\text{C} = \frac{1.1}{(0.08317)(298)} = 0.04438 \text{ kmol}$

$1 \text{ L (liter)} \text{ solution (very dilute)} \equiv 1 \text{ liter water} = 1000/18 = 55.55 \text{ g/mol}$

(6.5) contd... $\Rightarrow K_i \equiv \frac{(0.7671 \times 10^{-3} \text{ gm}/132.5)/0.04438}{(10^{-3} \text{ gm}/132.5)/55.55}$ mole fraction TCA in air
 $\equiv \frac{1.304 \times 10^{-4} \text{ mole fraction TCA in air}}{1.3586 \times 10^{-7} \text{ mole fraction TCA in water}} = \frac{960}{m} \Rightarrow y = 960x$
 = Henry's law const.

(b) TCA conc. in water is small \Rightarrow mol. wt of solution $\approx 18 = M.W.$

Feed: $9000 \text{ kg/h} = 9000/18 = 500 \text{ kmol/h} = L$ (assume constant)

Feed conc. = 600 ppm in water $= \frac{600/132.5}{(10^6/18)} = 8.151 \times 10^{-5} = x_0$

Take $y_1 \rightarrow 2.5 \text{ vol\% TCA} \rightarrow y_1 = 0.025; y_{N+1} = 0$ (Inlet air is TCA-free)

Make total material balance (Take $G \approx G_s$, $L \approx L_s$ for very dilute solute)

$$G(0.025 - 0) = L \cdot (8.151 \times 10^{-5} - x_N) \quad \text{Stripping factor, } S = mG/L = \frac{960}{0.025} (8.151 \times 10^{-5} - x_N) = 3.84 \times 10^4 (8.151 \times 10^{-5} - x_N) = 1/A$$

$$N = \frac{\log \left[\frac{(x_0 - (y_{N+1}/m))}{(x_N - (y_{N+1}/m))} \right] (1 - A) + A}{\log (1/A)}$$

Number of stripping trays, Eq. (4.44) in mole fraction unit

$$N = \frac{\log \left[\frac{(x_0 - (y_{N+1}/m))}{(x_N - (y_{N+1}/m))} \right] (1 - A) + A}{\log (1/A)} \quad \text{Given } N = (8)(0.4) = 3.2 \text{ ideal trays}$$

$$\Rightarrow 3.2 = \frac{\log \left[\frac{(8.151 \times 10^{-5})}{x_N} \right] (1 - \frac{1}{3.84 \times 10^4 (8.151 \times 10^{-5} - x_N)}) - \frac{1}{3.84 \times 10^4 (8.151 \times 10^{-5} - x_N)}}{\log [3.84 \times 10^4 (8.151 \times 10^{-5} - x_N)]}$$

Solution of the above equation $\rightarrow x_N = 1.5 \times 10^{-6}$ mole fraction

$$\text{Removal of TCA} \approx \frac{x_0 - x_N}{x_0} \Rightarrow \frac{8.151 \times 10^{-5} - 1.5 \times 10^{-6}}{8.151 \times 10^{-5}} \times 100 = 98\%$$

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(6.6) Gas flow rates and compositions (total and solute-free basis)

$$G_1 = 10 \text{ kmol/hm}^2, y_1 = 0.08; G'_s = (10)(0.92) = 9.2 \text{ kmol/hm}^2; Y_1 = \frac{0.08}{0.92} = 0.087.$$

$$\text{solute in } = (10)(0.08) = 0.8 \text{ kmol/hm}^2; \text{solute leaving} = (0.8)(0.03) = 0.024 \text{ kmol/hm}^2$$

$$Y_2 = \frac{0.024}{9.2 + 0.024} = 0.026; Y_2 = \frac{0.024}{9.2} = 0.00261.$$

Equilibrium line: $y = 2.5x + 4.5x^2$. It is convex downward.

Hence the operating line for minimum liquid rate meets

(6.11)

6.6 Contd... the equilibrium line at a point corresponding to $y=0.08$.
 \Rightarrow For minimum liquid rate, $y=0.08 = 2.5x + 4.5x^2 \Rightarrow x=0.0303$
 The pinch point is $(0.0303, 0.08)$ on the equilibrium line.

Calculation of the minimum solvent rate:

For the minimum liquid, the material balance eqn. is

$$(G_s') [Y_1 - Y_2] = (L_s')_{\min} [(x_1)_{\max} - x_2]; x_2 = 0 \text{ (fresh liquid)} \\ (x_1)_{\max} = \frac{0.0303}{0.9697} = 0.0312 \\ \Rightarrow (9.2)(0.087 - 0.00261) = (L_s')_{\min} (0.0312 - 0) \Rightarrow (L_s')_{\min} = 24.85 \frac{\text{kmol}}{\text{hm}^2}$$

Actual Solvent rate, $L_s' = 30 \frac{\text{kmol}}{\text{hm}^2}$

\Rightarrow Actual solvent rate is more than the minimum, $\frac{30}{24.85} = 1.2$ times.

Outlet liquid concentration (by overall mass balance),

$$x_1 = \frac{(9.2)(0.087 - 0.00261)}{30} = 0.02588 \Rightarrow x_1 = \frac{0.02588}{1+0.02588} = 0.02523.$$

Now consider the section where $x=0.02 \Rightarrow x=0.0204$.

By mass balance, $G_s'(Y_1 - Y) = L_s'(x_1 - x) \Rightarrow (9.2)(0.087 - Y) = 30(0.02588 - 0.0204)$

$$\Rightarrow Y = 0.0691 \Rightarrow y = 0.0647.$$

$$G' = G_s'(1+Y) = (9.2)(1.0691) = 9.836 \frac{\text{kmol}}{\text{hm}^2}; L' = L_s'(1+x) = 30(1+0.0204) = 30.61 \frac{\text{kmol}}{\text{hm}^2}$$

$$\text{Slope of the equilibrium line at } x=0.02 = \frac{dy}{dx} = [2.5 + 9x]_{x=0.02} = 2.68 = m \cdot \text{Absorption factor, } A = \frac{L'}{mG'} = \frac{30.61}{(2.68)(9.836)} = 1.161$$

Interfacial concentration - calculation
 The bulk liquid concentration at the particular section is given, $x=0.02 \rightarrow x=0.02041$. We first calculate the bulk gas concentration, Y , at the section and then proceed to obtain the interfacial concentrations following the procedure given in Section 4.4. We write the following material balance.

$$G_s(Y - Y_2) = L_s(x - x_2) \Rightarrow (9.2)(Y - 0.00261) = 30(0.02041 - 0)$$

$$\Rightarrow Y = 0.06783 \Rightarrow y = 0.06783 / 1.06783 = 0.0635$$

The interfacial concentrations can be determined without

(6.6) contd... graphical construction since the equation of the equilibrium line is given.

At steady state, $k_y(y - y_i) = k_x(x_i - x)$; $(x, y) \rightarrow (0.02, 0.0635)$.

The Colburn-Drew mass transfer coefficients k'_x and k'_y are given.

From the relations given in Section 3.2, $k_y = k'_y/x_{BM}$; $k_x = k'_x/x_{BM}$

$$\Rightarrow \frac{y - y_i}{x_i - x} = \frac{k_x}{k_y} = \frac{k'_x}{k'_y} \cdot \frac{y_{BM}}{x_{BM}} \quad \text{(i)} \quad \because y_{BM} \text{ & } x_{BM} \text{ are not known.}$$

Put $x = 0.02$, $y = 0.0635$ and $y_i = 2.5x_i + 4.5x_i^2$; k_x & k_y are given.

$$\Rightarrow \frac{0.0635 - (2.5x_i + 4.5x_i^2)}{x_i - 0.02} = \frac{1.2}{0.15} \quad \text{(i)} \Rightarrow x_i = 0.0211 \text{ and } y_i = 0.05475.$$

Once the "first estimate" of x_i and y_i are obtained, we calculate y_{BM} and x_{BM} and recalculate the whole thing from Eq.(i) above.

$$y_{BM} = \frac{(1 - y_i) - (1 - y)}{\ln \frac{1 - y_i}{1 - y}} = \frac{0.0635 - 0.05475}{\ln \frac{1 - 0.05475}{1 - 0.0635}} = 0.0941 \quad \text{once again.}$$

$$x_{BM} = \frac{(1 - x) - (1 - x_i)}{\ln \frac{1 - x}{1 - x_i}} = \frac{(1 - 0.02) - (1 - 0.0211)}{\ln \frac{1 - 0.02}{1 - 0.0211}} = 0.979.$$

$$\text{Put the values in Eq.(i)} \rightarrow \frac{0.0635 - (2.5x_i + 4.5x_i^2)}{x_i - 0.02} = \frac{1.2}{0.15} \cdot \left(\frac{0.941}{0.979} \right)$$

Solution for $x_i \rightarrow x_i = 0.0211$ (upto four places after decimal).

The recalculated x_i is the same as the one obtained in the first cycle of calculation. Thus consideration of y_{BM} and x_{BM} does not substantially affect the calculated values in the given concentration range.

Calculation of the individual gas-phase driving force at the given section.
It is $\Delta y = y - y_i = 0.0635 - 0.05477 = 0.00873$.

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(6.7) Since the solute concentration is pretty low, we assume that the ^{total} _{gas} and liquid flowrates remain essentially constant. We shall first calculate the liquid flowrate, then the absorption factor and therefrom the number of trays using Kremser eq.

(6.7) Contd... Material balance: $G(y_{N+1} - y_1) = L(x_N - x_0) \dots (i)$

For the minimum liquid rate, the pinch point occurs at the bottom of the column.

We use the total flowrate and mole fraction unit of conc.

$$y_{N+1} = 1\% \text{ by volume} \rightarrow 0.01$$

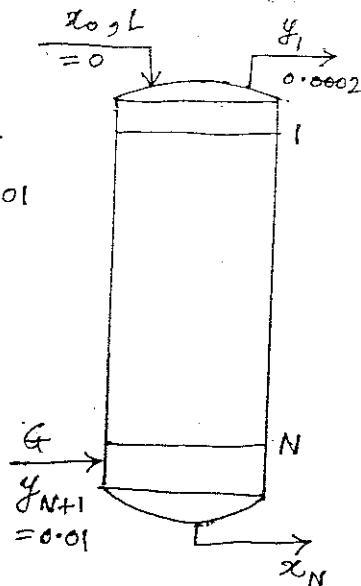
$$y_1 = 0.0002; x_0 = 0$$

Equilibrium relation: $P(\text{bar}) = Hx = 10x$

$$\Rightarrow y = \frac{P}{P} = \frac{H}{P} \cdot x. \text{ Column pressure} = 0.5 \text{ bar(gauge)}$$

$$\Rightarrow P = 1.013 + 0.5 = 1.513 \text{ bar}$$

$$\Rightarrow y = \frac{10}{1.513} x = 6.609 x \Rightarrow m = 6.609$$



At the minimum liquid rate, the exit liquid is in equilibrium with the inlet gas $\Rightarrow x_N^* = y_{N+1}/m = 0.01/6.609 = 0.001513$.

From Eq (i) above, $G(0.01 - 0.0002) = L_{\min}(0.001513 - 0) \Rightarrow L_{\min} = 6.477 G$
Actual liquid rate, $L = 1.3 L_{\min} = (1.3)(6.477 G) = \frac{(8.42)G}{8.42} \Rightarrow \frac{L}{G} = 8.42$

Absorption factor, $\bar{A} = \frac{L}{mG} = \frac{8.42 G}{(6.609)G} = 1.274$.

Actual exit liquid concentration (from Eq (i)), $x_N = \frac{0.01 - 0.0002}{8.42} = 0.001164$.

Calculate the number of ideal trays from Eq. 4.41 (use m for α).
 $N = \frac{\log \left[\frac{y_{N+1} - m x_0}{y_1 - m x_0} \right] (1 - \frac{1}{\bar{A}}) + \frac{1}{\bar{A}}}{\log \bar{A}} = \frac{\log \left[\frac{0.01 - 0}{0.0002 - 0} \right] (1 - \frac{1}{1.274}) + \frac{1}{1.274}}{\log(1.274)} = 10$ ideal trays.

Overall tray efficiency = 0.4 \Rightarrow Number of real tray = $\frac{10}{0.4} = 25 \rightarrow$ Part (a)

(b) Now let us see if the exit gas concentration can be reduced to 0.01% (i.e. $y_1 = 0.0001$) by increasing the column pressure

Number of ideal trays = $10 = \frac{\log \left[\frac{0.01 - 0}{0.0001 - 0} \right] (1 - \frac{1}{\bar{A}}) + \frac{1}{\bar{A}}}{\log(\bar{A})}$ (when 'm' will increase).

Solution of the above equation gives the required value of $\bar{A} = 1.4$.
 $\Rightarrow L/m'G = 1.4 \Rightarrow m' = \frac{8.42}{1.4} = 6.014$.

Since $y = \frac{H}{P} \cdot x$, $m' = H/P \Rightarrow 6.014 = 10/P \Rightarrow P = 1.663 \text{ bar}$

So the desired separation can be achieved in the same column if the operating pressure is raised to 1.663 bar.

6.8

Flowrates and concentrations of the two streams:

Feed gas concentration, $y_1 = 0.0425$, $\alpha_2 = 0$, $\text{position} \rightarrow \text{bottom of the packed tower}, 2 \rightarrow \text{top}$

$$Y_1 = \frac{0.0425}{0.9575} = 0.0444; Y_2 = 0.0008 \Rightarrow Y_2 \approx 0.0008.$$

Liquid rate, $L_s = L_2$ (inlet liquid is solute free). $G_1 = G_s(1+Y_1)$

Given: $(L_s/G_1) = L_s/[G_s(1+Y_1)] = 2.9 \Rightarrow \frac{L_s}{G_s} = (2.9)(1+0.0444) = 3.029$.

(a) Equation of the operating line. Material balance: $G_s(Y - Y_2) = L_s(X - X_2)$

$$\Rightarrow Y = \frac{L_s}{G_s} \cdot X + Y_2 \Rightarrow \frac{Y}{1-Y} = 3.029 \cdot \frac{X}{1-X} + 0.0008 \quad (\text{Given: } X_2 = 0)$$

(b) Calculation of $N_{\text{tot}G}$:

Calculate the exit liquid conc. by total material balance,

$$G_s(Y_1 - Y_2) = L_s(X_1 - X_2) \Rightarrow (0.0444 - 0.0008) = (3.029)(X_1) \Rightarrow X_1 = 0.0144$$

Since the concentrations are low (dilute solution), $\Rightarrow X_1 = 0.0142$

use Eq. (6.27b) to calculate $N_{\text{tot}G}$. (Given, Eq. relation: $Y = 2.5X; m = 2.5$)

$$(\Delta Y)_1 = Y_1 - Y_1^* = Y_1 - mX_1 = 0.0425 - (2.5)(0.0142) = 0.007$$

$$(\Delta Y)_2 = Y_2 - Y_2^* = 0.0008 - (2.5)(0) = 0.0008$$

$$(Y - Y^*)_M = \frac{0.007 - 0.0008}{\ln(\frac{0.007}{0.0008})} = 2.858 \times 10^{-3}$$

$$N_{\text{tot}G} = \frac{Y_1 - Y_2}{(Y - Y^*)_M} = \frac{0.0425 - 0.0008}{2.858 \times 10^{-3}} = 14.6 \rightarrow \text{number of overall gas-phase transfer units}$$

(c) The maximum permissible value of (G_s/L_s) :

This corresponds to the "minimum liquid rate", $(L_s)_{\min}$. we calculate below.

The equilibrium relation in mole ratio units: $\frac{Y}{1+Y} = \frac{2.5X}{1+2.5X} \Rightarrow Y = \frac{2.5X}{1+1.5X}$

This equation plotted on the X-Y plane will be convex downward \Rightarrow The operating line for the minimum liquid rate should meet the equilibrium line where $Y = Y_1$.

For $Y = Y_1 = 0.0444$ we have $0.0444 = \frac{2.5X_1^*}{1+1.5X_1^*} \Rightarrow X_1^* = 0.0173$.

$$\Rightarrow G_s(Y_1 - Y_2) = (L_s)_{\min} (X_1^* - X_2) \Rightarrow G_s(0.0444 - 0.0008) = (L_s)_{\min} (0.0173)$$

$$\Rightarrow G_s/(L_s)_{\min} = 0.397 \rightarrow \text{maximum ratio of gas/lq. rate}$$

Actual value used in part (b) $\rightarrow Y_{3.029} = 0.33$:

6.9 Consider the absorption column of the absorber-stripper combination.

Feed gas : 100 kmol/h, 90% saturated with benzene at 25°C, 1 atm.

Vapor pressure of benzene at 25°C (298K) :

$$\ln P^\circ = 17.5818 - 3867/T = 17.5818 - 3867/298 = 4.605 \Rightarrow P^\circ = 100 \text{ mm Hg}$$

Partial pressure of benzene in the feed gas = $(100)(0.9) = 90 \text{ mm Hg}$.

Mole fraction of benzene, $y_{N+1} = \frac{90}{760} = 0.1184$; $Y_{N+1} = \frac{0.1184}{1-0.1184} = 0.1343$.

$$G_{N+1} = 100 \text{ kmol/h}, G_s = G_{N+1}(1-y_{N+1}) = 88.16 \text{ kmol/h.}$$

The assembly is shown below. The concentration terms in the stripper is marked by a prime (') to differentiate the quantities from those in the absorber.

2.5% of the feed benzene leaves the absorber

$$\Rightarrow Y_1 = (0.025)(0.1343) = 0.00336.$$

Rich oil leaving the absorber $\rightarrow X_N = 0.30$ (given)

Lean oil (from the stripper) entering the absorber at the top, $X_0 = 0.007$ (given)

Overall material balance:

$$G_s(Y_{N+1} - Y_1) = L_s(X_0 - X_N)$$

$$\Rightarrow G_s(0.1343 - 0.00336) = L_s(0.30 - 0.007)$$

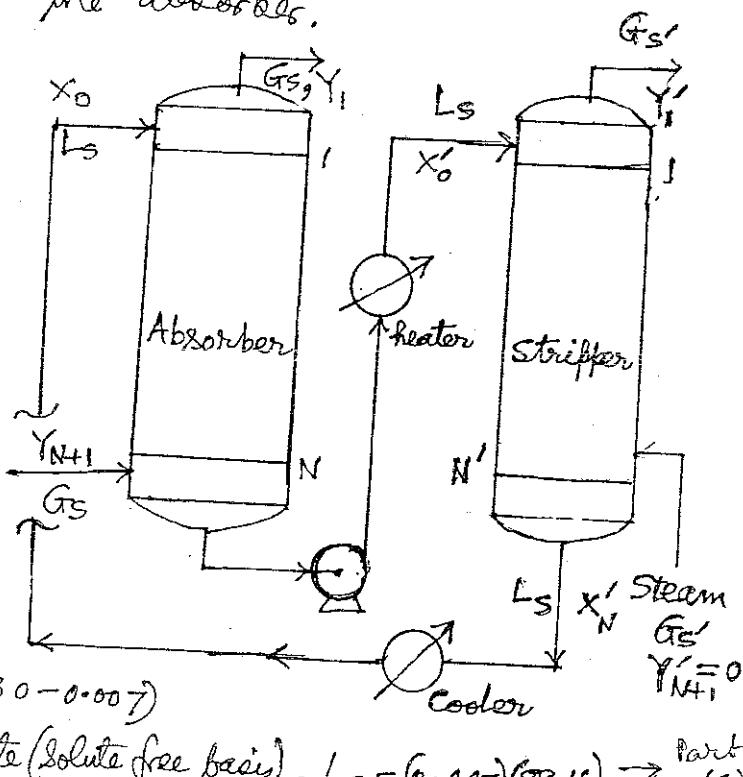
$$\Rightarrow L_s/G_s = 0.447. \text{ Liquid rate (solute free basis), } L_s = (0.447)(88.16) \xrightarrow{\text{Part (a)}} = 39.4 \frac{\text{kmol}}{\text{h}}$$

Equilibrium relation: $X-Y$ data are given.

Incidentally the $X-Y$ data can be nicely fitted by a straight line

(see Figure Prob 6.9a), $Y = 0.25X$ (the correlation coefficient is above 0.996)

So the number of ideal trays in the absorber can be determined using the Kremser equation:



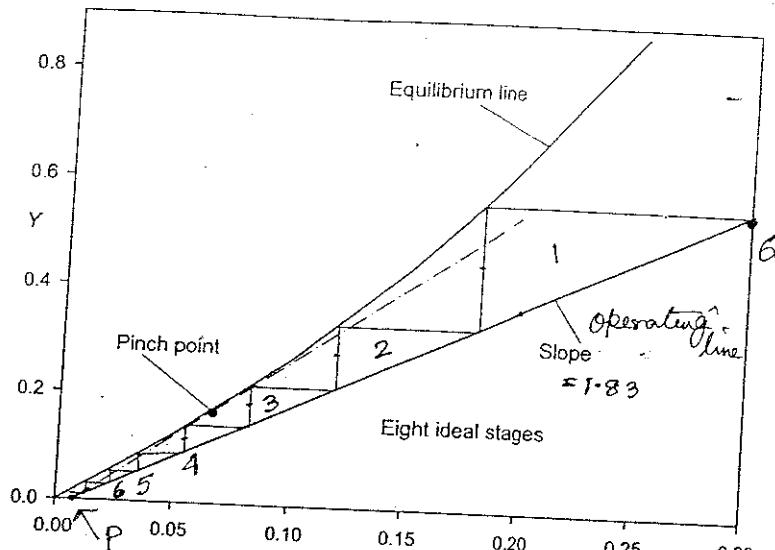


Fig Prob 6.9(b): The operating line for minimum steam rate (broken line), and the pinch point.

The absorption factor, $A = \frac{L_s}{G_s} = \frac{0.447}{0.25} = 1.788$

$$(b) \text{ Number of ideal stages, } N = \log \left[\frac{(Y_{N+1} - X_0 \alpha)}{(Y_1 - X_0 \alpha)} \left(1 - \frac{1}{A} \right) + \frac{1}{A} \right]$$

$$\Rightarrow N = \log \left[\frac{0.1343 - (0.25)(0.007)}{0.00336 - (0.25)(0.007)} \left(1 - \frac{1}{1.788} \right) + \frac{1}{1.788} \right] / \log(1.788)$$

\Rightarrow Number of ideal trays for absorption = 6.2

(c) Number of trays for stripping from minimum steam rate

Minimum steam rate will be determined first.

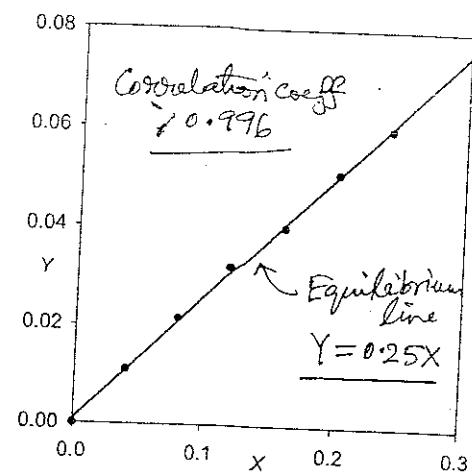
$X'_0 = 0.3$ kmol benzene per kmol oil; $Y'_{N+1} = 0$ (inlet steam does not have any benzene)

$L_s = 39.7$ kmol/h; $X'_1 = 0.007$. [For the stripper, the concentration terms are marked by prime (')]

Locate the point $(X'_N, Y'_{N+1}) \rightarrow (0.007, 0)$ on the X-axis of the equilibrium plot (see above). Draw a tangent to the equilibrium line from the point P describing the terminal concentration, get the Pinch point [see figure Prob (6.9 b)]. Slope of the line gives $(L_s/G_s)_{\max}$ or G_s at minimum.

$$(L_s/G_s)_{\max} = 2.733 \text{ (from the plot.)} \quad (G_s)_{\min} = \frac{L_s}{2.733} = \frac{39.1}{2.733} = 14.5 \text{ kmol/h}$$

Calculated data	X	0	0.05	0.10	0.15	0.20	0.25
	Y	0	0.126	0.271	0.441	0.642	0.883



Fitting of the given equilibrium data at 25°C

Figure 6.9(a)

Equil. plot for stripping at 110°C, 1 atm pressure

P^{∞} for benzene = 1781.5

$$Y = \frac{P^{\infty}}{P} \cdot x = \frac{1781.5}{760} \cdot x$$

$$\Rightarrow Y = 2.345x \quad \text{and } Y = \frac{2.345x}{1 - 1.345x}$$

- (6.9) const... Actual steam rate = $(1.5) G_{S, \text{min}} = (1.5)(14.5) = 21.75 \frac{\text{kmol}}{\text{h}} = G_S$
- Slope of the operating line for stripping = $\frac{39.7}{21.75} = 1.83$
- The operating line, PQ, is drawn through $(X_N, Y_{NH}) \rightarrow (0.007, 0)$ with the above slope (1.83). Steps are drawn between the equilibrium line and the operating line. The number of ideal stages = 8, Fig. Prob 6.9(b)
- (e) Number of real stages, given $EMG = 0.5$

Since $EMG = 0.5$,
the actual enrichment across a tray ($y_n - y_{n+1}$) is 50% of the ideal enrichment ($y^* - y_{n+1}$). We locate a set of points midway between the equil. line and the op. line (black dots on Fig Prob 6.9c).

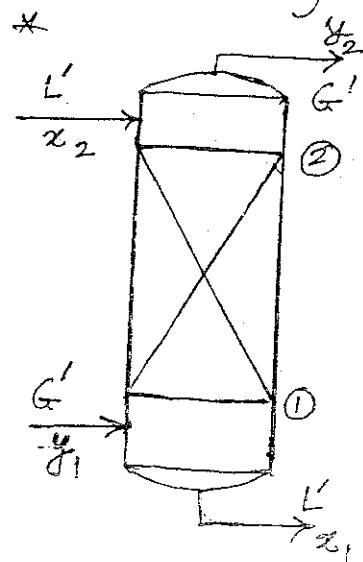
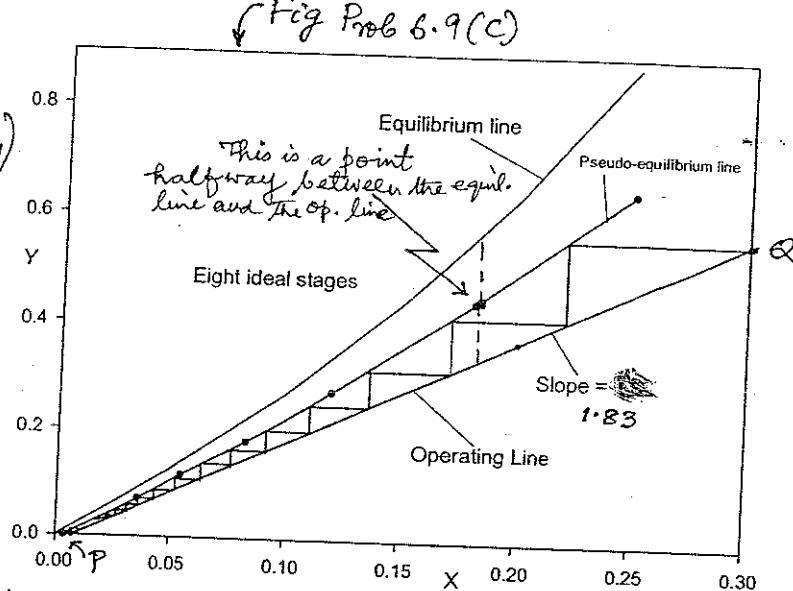
A continuous line joining such points is the "pseudo-equilibrium line". Steps are now drawn between the operating line and the pseudo-equilibrium line to find out the number of real stages.

The number of real stages = 17 (separate construction using enlarged scale is required for drawing step at the lower end)

- (6.10) Since the concentrations are low, we assume that the gas and the liquid flow rate remain essentially constant. Flowrates are given on unit area basis.

$$\text{Gas rate : } G' = 1200 \frac{\text{Nm}^3}{\text{h m}^2} \frac{1200}{22.414}$$

$= 53.54 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$
(Here G' is taken as the 'constant' gas rate. It is better to take the average gas flow rate as G'_1)



6.10 contd... Liquid rate, $L' = 52 \text{ kmol/m}^2 \text{ h}$. The concentration values are: $y_1 = 0.02$, $y_2 = (0.02)(0.01) = 0.0002$ (99% of the feed x_2 = ammonia concentration in feed water = 8×10^{-5} gas is absorbed).

Material balance over the column:

$$G'(Y_1 - Y_2) = L'(x_1 - x_2) \Rightarrow (53.54)(0.02 - 0.0002) = 52(x_1 - 8 \times 10^{-5})$$

$$\Rightarrow x_1 = 0.02047 \text{ mole fraction}$$

(a) Ammonia concentration in kmol/m^3 :

$$x_1 = 0.02047 \text{ mole fraction} \rightarrow \frac{0.02047 \text{ kmole NH}_3}{0.02047 \text{ mole NH}_3 + 0.97953 \text{ mole H}_2\text{O}}$$

$$\rightarrow \frac{0.02047 \text{ kmole NH}_3}{(0.02047)(1) + (0.97953)(8) \text{ kg solution}} = \frac{0.02047 \text{ kmole NH}_3}{(17.98 \text{ kg/1000 kg/m}^3)} = 1.138 \frac{\text{kg}}{\text{m}^3}$$

(b) Henry's law $\rightarrow p = Hx$. Given equil. relation $\rightarrow p = x$, p in bar. By comparing, $H = 1 \text{ bar} = \text{Henry's law constant}$.

(c) At a section of the column where 50% of the ammonia in the feed has been absorbed, ammonia concentration in the bulk gas = $\frac{1 \text{ mole NH}_3}{1 \text{ mole NH}_3 + 99 \text{ mole air}} = 0.0101 \text{ mole fraction}$.

Mole fraction of NH_3 in water at the section is obtained by mass

$$G'(Y_1 - Y) = L'(x_1 - x) \Rightarrow (53.54)(0.02 - 0.0101) = 52(0.02047 - x) \text{ balance.}$$

$$\Rightarrow x = 0.01028 \Rightarrow y^* = \frac{p}{P} = \frac{x}{P} = \frac{0.01028}{1.2 \text{ bar}} = 0.00857.$$

$$\text{Driving force for absorption at the section} = \frac{y - y^*}{y^*} = 0.0101 - 0.00857 = 0.00153.$$

(d) If the operating pressure is doubled, the driving force will increase (because of higher solubility of the gas) \Rightarrow the fractional removal of NH_3 will be more for same values of the other variables except the value of y_2 which will now be less.

If $\frac{1}{2}$ " size packing is used (rather than 1"), the volumetric area of gas-liquid contact will be more \Rightarrow mass transfer efficiency of the column will improve but the pressure drop of the gas across the bed will be more.

6-11 From the given data, H_{TOG} for the bed and, therefore, the N_{TOG} can be calculated. From N_{TOG} , we calculate the driving force at the top, therefrom x_1 and the water flowrate by solute material balance.

To calculate H_{TOG} , we shall use the average gas flowrate.

Gas velocity at the top = $0.8 \text{ m/s} \Rightarrow 0.8 \text{ m}^3/\text{m}^2\cdot\text{s}$; 26°C , 1 atm.

$$\Rightarrow G'_2 = (0.8) \left(\frac{273}{299} \right) \left(\frac{1}{22.4} \right) = 0.03261 \text{ kmol/m}^2\cdot\text{s} = 117.4 \text{ kmol/m}^2\cdot\text{h}$$

Solute conc. in the exit gas, $y_2 = 0.0005$ (0.05 mol%).

$$G'_s = G'_2 (1-y_2) = (117.4)(1-0.0005) = 117.34 \text{ kmol/m}^2\cdot\text{h}$$

(the prime, ' indicates per unit area basis)

Feed gas concentration, $y_1 = 0.035$ (3.5 mol%)

$$G'_1 = \frac{G'_s}{1-y_1} = \frac{117.34}{1-0.035} = 121.6 \text{ kmol/m}^2\cdot\text{h}$$

Average gas rate, $G' = (G'_1 + G'_2)/2 = (117.34 + 121.6)/2 = 119.5 \text{ kmol/m}^2\cdot\text{h}$

$$K_g \bar{a} = 7050 \frac{\text{kg}}{\text{h}\cdot\text{m}^3(\Delta p, \text{bar})} = \frac{7050}{30} = 235 \frac{\text{kmol}}{\text{m}^3\cdot\text{h}\cdot(\Delta p, \text{bar})}$$

$$K_y \bar{a} = K_g \bar{a} \cdot P = K_g \bar{a} \cdot (1 \text{ atm}) = 235 \frac{\text{kmol}}{\text{m}^3\cdot\text{h}\cdot(\Delta y)}$$

$$H_{TOG} = \frac{G'}{K_y \bar{a}} = \frac{119.5}{235} = 0.51 \text{ m} ; \text{ Packed height} = 8.6 \text{ m (given)}$$

$$\Rightarrow N_{TOG} = \frac{8.6}{0.51} = 16.86$$

$$\text{Next, } N_{TOG} = \frac{y_1 - y_2}{(\Delta y)_M} = 16.86$$

$$\Rightarrow (\Delta y)_M = \frac{0.0345}{16.86} = 2.046 \times 10^{-3}$$

$$\Rightarrow 2.046 \times 10^{-3} = \frac{(\Delta y)_1 - (\Delta y)_2}{\ln[(\Delta y)_1 / (\Delta y)_2]} = \frac{(y_1 - mx_1) - y_2}{\ln[(y_1 - mx_1) / y_2]}, \quad y_1 = 0.035 \\ m = 2.53$$

Put the values of y and m to obtain $x_1 = 0.01172$ given

Now make an overall solute balance,

$$G'(y_1 - y_2) = L'(x_1 - e) \Rightarrow (119.5)(0.035) = L'(0.01172)$$

$$\Rightarrow L' = 352 \text{ kmol/m}^2\cdot\text{h} = \text{required liquid flow rate.}$$

6.20

6.11 contd... To calculate HETP of the packing, we need the number of ideal plates required for the given separation. It can be obtained by using the Kremser equation. We shall take the absorption factor at the mean flow rates and the mole fraction unit.

$$\text{Take } \bar{A} = L/G \cdot m = \frac{352}{(119.5)(2.53)} = 1.164. \quad \text{mole fraction unit}$$

$$N = \frac{\log \left[\frac{(y_{N+1} - m x_0)}{(y_1 - m x_0)} \right] (1 - 1/\bar{A}) + 1/\bar{A}}{\ln \bar{A}}$$

In this notation (see Eq. 4.41), $y_{N+1} = 0.035$,

$$x_0 = 0, y_1 = 0.0005.$$

$$15.6 = \text{no. of ideal trays}$$

$$\text{HETP of the bed} = \frac{\text{Packed height}}{\text{number of ideal trays}} = \frac{8.6 \text{ m}}{15.6} = 0.55 \text{ m.}$$

Alternatively, use Eq. (6.49). Put $\bar{s} = 1/\bar{A} = 0.859$. $HETP = H_{\text{tot}} \cdot \frac{\ln \bar{s}}{1 - \bar{s}} = 6.51 \left(\frac{\ln 0.859}{1 - 0.859} \right) = 0.55 \text{ m}$

6.12

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6.13 From the given data on the required fractional removal of the gas and the number of ideal trays available, we can calculate the absorption factor, \bar{A} . The equilibrium relation $Y = \alpha X$ being known, L_s can be calculated.

$$G_{N+1} = 100 \text{ kmol/h, 8% solute.}$$

$$\Rightarrow G_s = (0.08)(1 - 0.08) = 92 \text{ kmol/h; } Y_{N+1} = \frac{0.08}{0.92} = 0.087;$$

$$Y_1 = (0.087)(0.03) = 0.00261 \quad [97\% \text{ removal of the gas}]; \quad X_0 = 0; \quad N = 10$$

Put the values in the Kremser eq: $N = \frac{\log \left[\frac{(0.087 - 0)}{0.00261 - 0} \right] (1 - 1/\bar{A}) + 1/\bar{A}}{\ln \bar{A}} = 10$

Solution of this equation : $\bar{A} = 1.205 = \frac{L_s}{\alpha G_s} \log(\bar{A})$

Solvent 1 : Equil. relation, $Y = 2.5X \Rightarrow \alpha_1 = 2.5$; $G_s = 92 \frac{\text{kmol}}{\text{hr}}$ in both cases.

$$\text{Solvent rate, } L_{s1} = \bar{A} \cdot \alpha_1 \cdot G_s = (1.205)(2.5)(92) = 277.1 \text{ kmol/hr}$$

$$\text{Or, } G_s = (277.1)(200) = 55420 \text{ kg/hr.} \quad [\text{M.W. of Solvent 1 is 200}]$$

Pumping and spillage loss = $(55420)(24)(0.001/100) = 13.3 \text{ kg/day}$

Price = $R. 50/\text{kg}$. Cost due to pumping and spillage $= (13.3)(50) = R. 665/\text{day}$

Solvent 2 : Equil. relation, $Y = 1.5X \Rightarrow \alpha_2 = 1.5$

$$L_{S2} = A \cdot \alpha_2 \cdot G_S = (1.205)(1.5)(92) = \frac{166.3 \text{ kmol/h}}{(166.3)(13.0)(24)} \xrightarrow{\text{M.W. of Solvent 2}} 5.1886 \times 10^5 \frac{\text{kg day}}{\text{day}}$$

Cost of Solvent = Rs. 30 per kg

Pumping and Spillage loss = $(5.1886 \times 10^5)(0.001/100) \Rightarrow 5.2 \text{ kg/day}$

Vaporization loss (0.004%) $\rightarrow (5.1886 \times 10^5)(0.004/100) = 20.75 \text{ kg/day}$

Total loss on the above two accounts

$$= 5.2 + 20.75 = 25.95 \text{ kg/day} \rightarrow (25.95)(Rs. 30/\text{kg}) = \underline{\underline{\text{Rs. 778.5/day}}}$$

Per day cost is more for solvent 2 \Rightarrow Select Solvent 1

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(6.14) The gas flow rates and terminal concentrations & the liquid conc. at the exit are given. So the liquid rate can be obtained by a simple material balance.

Gas rate, $G'_1 = 45 \text{ kmol/h.m}^2$ (a prime '1' indicates flowrate on unit area)
 $y_1 = 0.035$; fractional removal = 99%. Inlet liquid conc., $x_2 = 0$

$$G'_S = 45(1 - 0.035) = 43.42 \text{ kmol/h.m}^2. \text{ Exit liquid conc., } x_1 = 0.015 \Rightarrow$$

(a) Material balance: $G'_1 \cdot y_1 \cdot (0.99) = L'_S(0.015) \Rightarrow L'_S = 104 \text{ kmol/m}^2\text{.h} \quad x_1 = 0.01478$

(b) Exit gas conc. at the top, $y_2 = \frac{(45)(0.035)(0.01)}{43.42} = 0.000363 \frac{\text{kmol H}_2\text{S}}{\text{kmol air}}$
 Equation of the operating line:

$$G'_S(Y - Y_2) = L'_S(X - X_2) \Rightarrow (43.42)(Y - 0.000363) = 104(X - 0)$$

$$\Rightarrow Y = 2.395X + 0.000363$$

At low concentrations we take $Y \approx y$ and $X \approx x$.

\Rightarrow Operating line using mole fraction units $\rightarrow y = 2.395x + 0.000363$

(c) Driving forces

(i) Gas phase basis : At the bottom, $(\Delta y)_1 = y_1 - y_1^* = y_1 - mx_1$
 $\Rightarrow (\Delta y)_1 \approx 0.035 - (1.95)(0.015) = 0.00575 \quad (m = 1.95, \text{ given})$

At the top, $(\Delta y)_2 = y_2 - y_2^* = y_2 - mx_2 = 0.000363 - 0 = 0.000363$

(ii) Liquid phase basis : $(\Delta x)_1 = x_1^* - x_1 = (y_1/m) - x_1$
 $= (0.035/1.95) - 0.015 = 0.00295 ; \quad (\Delta x)_2 = y_2^* - x_2 = (y_2/m) - 0$
 $= (0.000363/1.95) - 0 = 0.000186$

6.14) contd...

6.22

(d) Number of transfer units (low solute concentration; linear equil. line)

$$\text{Use Eq. (6.27b)} \Rightarrow N_{\text{toG}} = \frac{Y_1 - Y_2}{(\Delta y)_M} \cdot (\Delta y)_M = \frac{(Y_1 - Y_2)}{\ln(Y_1/Y_2)}$$

$$\Rightarrow N_{\text{toG}} = \frac{0.035 - 0.000363}{1.95 \times 10^{-3}} = 16$$

$$= \frac{0.00575 - 0.000363}{\ln\left(\frac{0.00575}{0.000363}\right)} = \frac{1.95 \times 10^{-3}}{2}$$

(e) Packed height

$$\text{Calculate } H_{\text{toG}} = \frac{G'}{K_y \bar{a} \cdot y_{\text{BM}}} \cdot \text{Take } y_{\text{BM}} \approx \frac{y_{B1} + y_{B2}}{2} = \frac{(1 - 0.035) + (1 - 0.000363)}{2}$$

Take the average value of G' for better accuracy.

$$= 0.982$$

$$G'_1 = 45 \text{ kmol/h} \cdot \text{m}^2 \text{ (given)}; G'_2 = G'_1(1 + Y_2) \approx (43.42)(1.000363) = 43.43 \text{ kmol/h} \cdot \text{m}^2$$

$$G' = \frac{45 + 43.43}{2} = 44.22 \text{ kmol/h} \cdot \text{m}^2; K_y \bar{a} = 13.0 \frac{\text{kmol}}{\text{m}^3 \cdot \text{h} \cdot (\Delta y)} \text{ given.}$$

$$H_{\text{toG}} = \frac{44.22}{(13.0)(0.982)} = 0.346 \text{ m}$$

$$\text{Packed height} = H_{\text{toG}} \cdot N_{\text{toG}} = (0.346)(16) = 5.54 \text{ m}$$

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6.15) Higher concentration feed gas (enters at the bottom):

$$G_{N+1} = 90 \text{ kmol/h}; Y_{N+1} = 0.1; \text{solute in} = 9 \text{ kmol/h}$$

Feed gas (stream 1) on solute-free basis, $G_{S1} = 81 \text{ kmol/h}$

Lower concentration feed gas (stream 2, enters

$$G_{\text{side}} = 60 \text{ kmol/h}, Y = 0.05; \text{solute} = (60)(0.05) = 3 \text{ kmol/h}$$

$$\text{Side stream feed on solute-free basis, } G_{S2} = 57 \text{ kmol/h}$$

$$\text{Total gas rate (solute-free)} = 81 + 57 = 138 \text{ kmol/h} = G_S$$

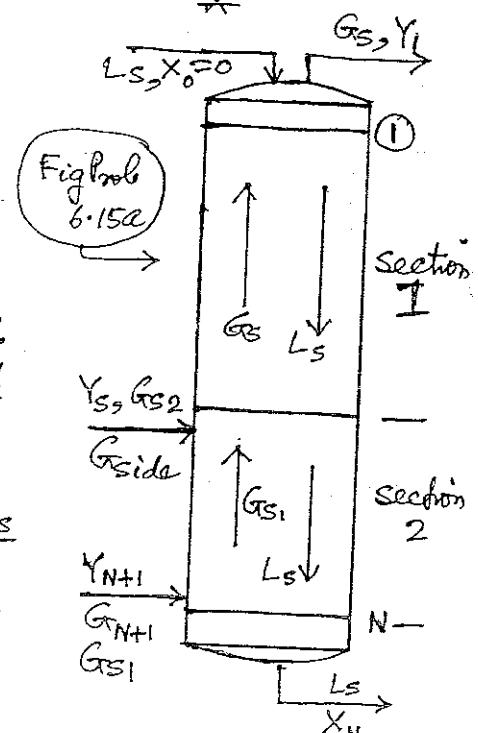
$$\text{Total solute entering} = 9 + 3 = 12 \text{ kmol/h}$$

$$\text{Solute leaving (unabsorbed)} = (12)(0.05) = 0.6 \text{ kmol/h}$$

Gas concentrations on solute-free basis:

$$Y_{N+1} = \frac{Y_{N+1}}{1 - Y_{N+1}} = \frac{0.1}{0.9} = 0.1111; Y_S = \frac{0.05}{0.95} = 0.0526; Y_1 = \frac{0.6}{138} = 0.00435$$

Calculation of the minimum liquid rate: Since the equilibrium



relation is linear, the two operating lines for the two sections of the column will meet on the equilibrium line if the minimum liquid rate is used. Note that the gas flow rates in the two sections of the column (above and below the entry of the side feed gas stream) are different, and hence there will be two operating lines.

For the minimum liquid rate, $L_{S,m}$, we shall write down the equations of the two operating lines and calculate $L_{S,m}$ such that the two operating lines meet on the equilibrium line creating a pinch point. See Fig. Prob 6.15a and Fig. Prob 6.15b

Operating line for section 1 : $G_S(Y - Y_1) = L_{S,m}(x - x_0)$

$$\Rightarrow 138(Y - 0.00435) = L_{S,m}(x - x_0) \Rightarrow Y = \frac{L_{S,m}}{138}x + 0.00435 \dots (i)$$

Exit liquid concentration (in terms of $L_{S,m}$) can be obtained by overall material balance,

$$G_{S1}Y_{N+1} + G_{S2}Y_S + L_{S,m}X_0 = G_S Y_1 + L_{S,m}X_N \Rightarrow X_N = \frac{11.4}{L_{S,m}}$$

Operating line for section 2 : $G_{S1}(Y_{N+1} - Y) = L_{S,m}(X_N - x)$

$$\Rightarrow 81(0.1111 - Y) = L_{S,m}\left(\frac{11.4}{L_{S,m}} - x\right) \Rightarrow Y = \frac{L_{S,m}}{81}x - 0.02963 \dots (ii)$$

The two operating lines (i) and (ii) meet on the equilibrium line ($Y = 1.2x$) for the minimum liquid rate. Put $Y = 1.2x$ in both Eqs. (i) and (ii), and solve for $L_{S,m}$.

$$\text{Eq. (i)} \rightarrow 1.2x = \frac{L_{S,m}}{138}x + 0.00435 \Rightarrow x\left(1.2 - \frac{L_{S,m}}{138}\right) = 0.00435 \quad (iii)$$

$$\text{Eq. (ii)} \rightarrow 1.2x = \frac{L_{S,m}}{81}x - 0.02963 \Rightarrow x\left(1.2 - \frac{L_{S,m}}{81}\right) = -0.02963 \quad (iv)$$

$$\text{Dividing (iv) by (iii), } \frac{1.2 - (L_{S,m}/81)}{1.2 - (L_{S,m}/138)} = -6.8115 \Rightarrow L_{S,m} = 152 \text{ kmol/hr}$$

So the minimum liquid rate is $L_{S,m} = 152 \text{ kmol/hr}$.

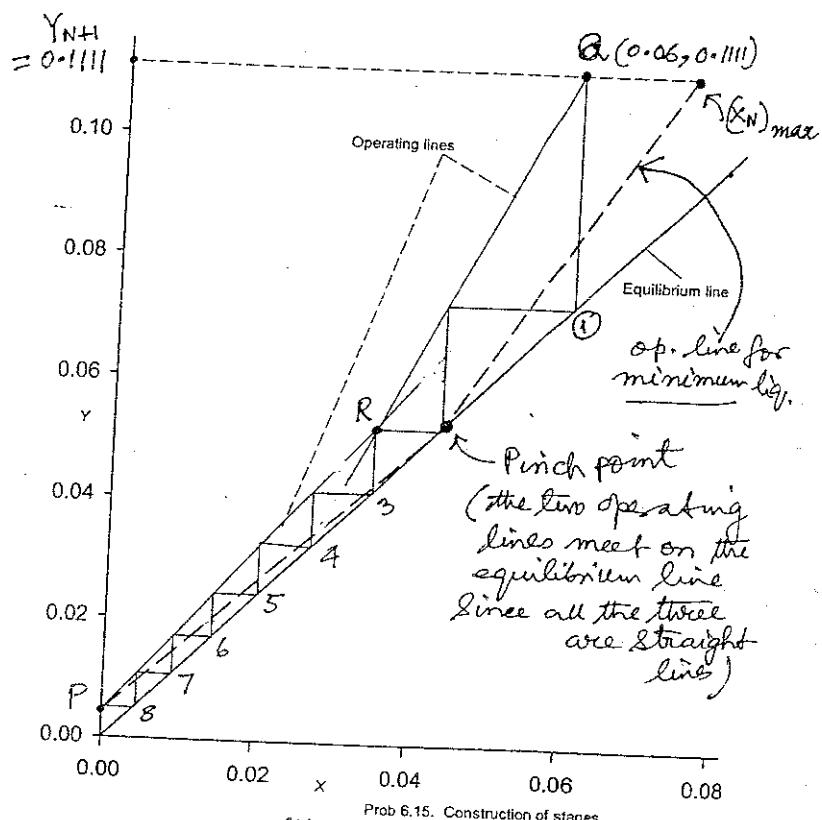
Determination of the number of ideal plates:

Actual liquid rate, $L_S = (1.25)L_{S,m} = (1.25)(152) = 190 \text{ kmol/hr}$

Slope of the operating line for section 1 $\rightarrow (L_S/G_S) = \frac{190}{138} = 1.377$

6-24

- (6-15) contd... Slope of the section 2 operating line = $\frac{L_s}{G_{S_1}} = \frac{19.0}{8.1} = 2.346$
 Concentration of the liquid leaving, $X_N = \frac{11.4}{L_s} = \frac{11.4}{19.0} = 0.06$
 Draw the section 1 operating line; terminal point, $(X_0, Y_1) \rightarrow (0, 0.00435)$
 Draw the same for section 2; terminal $(X_N, Y_{N+1}) \rightarrow (0.06, 0.111)$, slope = $1.377 (QR)$
 The equilibrium line $Y = 1.2X$ is also drawn.
 The operating lines intersect at $R (0.0351, 0.0527)$
 Now draw steps between the two operating lines and the equilibrium line, changing from one to the other operating line after their point of intersection, R, is crossed.



6.16 Feed gas rate, $G_1 = 60 \text{ kmol/h}$; $Y_1 = 0.075$; area of the tower cross-section = $\frac{\pi}{4}(1.2)^2 = 1.131 \text{ m}^2$; $G'_1 = 53.05 \text{ kmol/m}^2 \cdot \text{h}$.

$$G'_1 = (53.05)(1 - 0.075) = \underline{49.07 \text{ kmol/m}^2 \cdot \text{h}}; Y_1 = \frac{0.075}{0.925} = \underline{0.0811}.$$

$$\text{Exit gas} \rightarrow 0.2\% \text{ solute} \Rightarrow Y_2 = \frac{0.02}{99.8} = \underline{0.00204}.$$

$$\text{Feed liquid rate} = 40 \text{ kmol/h} \Rightarrow L'_s = \frac{40}{1.131} = \underline{35.37 \text{ kmol/m}^2 \cdot \text{h}}; X_2 = 0.$$

Make a solute balance to calculate the concentration of exit liquid, x_2 .

$$G'_s(Y_1 - Y_2) = L'_s(X_1 - X_2) \Rightarrow (49.07)(0.0811 - 0.00204) = (35.37)(X_1 - 0)$$

$$\Rightarrow X_1 = \underline{0.1097}.$$

Overall gas-phase mass transfer coefficient (molar ratio unit of concentration):

$$\frac{1}{K_Y} = \frac{1}{k_Y} + \frac{\alpha}{k_X}; \text{ Equil relation: } Y = 0.63X \Rightarrow \alpha = \underline{0.63}$$

$$\Rightarrow \frac{1}{K_Y} = \frac{1}{1.75} + \frac{0.63}{2.05} = \underline{0.8787} \Rightarrow K_Y = \underline{1.138 \text{ kmol/m}^2 \cdot \text{h} \cdot (\Delta Y)}$$

$$H_{t0G} = \frac{G'_s}{K_Y \cdot \bar{a}} = \frac{49.07}{(1.138)(71)} = \underline{0.607 \text{ m}}. \quad Y_1 = \int_{Y_2}^{Y_1} \frac{dY}{Y - Y^*} \dots \dots \text{(i)}$$

Y^* can be expressed in terms of Y through a solute balance - i.e. from the operating line equation.

$$G'_s(Y - Y_2) = L'_s(X - X_2) \Rightarrow (49.07)(Y - 0.00204) = (35.37)(X - 0)$$

$$\Rightarrow X = 1.387(Y - 0.00204) \Rightarrow Y^* = \alpha X = (0.63)(1.387)(Y - 0.00204)$$

$$\Rightarrow Y^* = 0.874Y - 0.00178 \dots \dots \text{(ii)}$$

$$\text{From Eq.(i) and (ii)} \rightarrow N_{t0G} = \int_{0.00204}^{0.0811} \frac{dY}{Y - (0.874Y - 0.00178)}$$

$$\Rightarrow N_{t0G} = \int_{0.00204}^{0.0811} \frac{dY}{0.126Y + 0.00178} = \frac{1}{0.126} \ln \frac{(0.126)(0.0811) + 0.00178}{(0.126)(0.00204) + 0.00178} = \underline{13.9}$$

$$\text{Packed height} = H_{t0G} \cdot N_{t0G} = (0.607 \text{ m})(13.9) = \underline{8.46 \text{ m}}$$

$$\text{If } 99.5\% \text{ of the solute is removed, } Y_2 = (0.0811)(0.005) = \underline{4.05 \times 10^{-4}}.$$

$$\text{Material balance: } (49.07)(Y - 4.05 \times 10^{-4}) = (35.37)X \Rightarrow X = 1.387Y - 5.62 \times 10^{-4}$$

$$Y^* = \alpha X = 0.874Y - 0.000354. \text{ Put in Eq. and integrate} \rightarrow N_{t0G} = \underline{22}.$$

$$\text{Packed height reqd.} = (0.607)(22) = \underline{13.35 \text{ m}}. \% \text{ increase} = \frac{13.35 - 8.46}{8.46} \Rightarrow \underline{58\%}$$

(6.17) Since the column has a packed section below three trays, a little different notations for flow rates and concentrations have been used.

$$\text{Feed gas rate, } G'_i = 70 \text{ kmol/h} \cdot m^2 Y_i = 0.1$$

$$G'_s = (70)(0.9) = 63 \text{ kmol/h} \cdot m^2; Y_i = \frac{0.1}{0.9} = 0.1111$$

$$98\% \text{ of the solute is removed. } Y_1 = (0.1111)(0.02) = 0.00222.$$

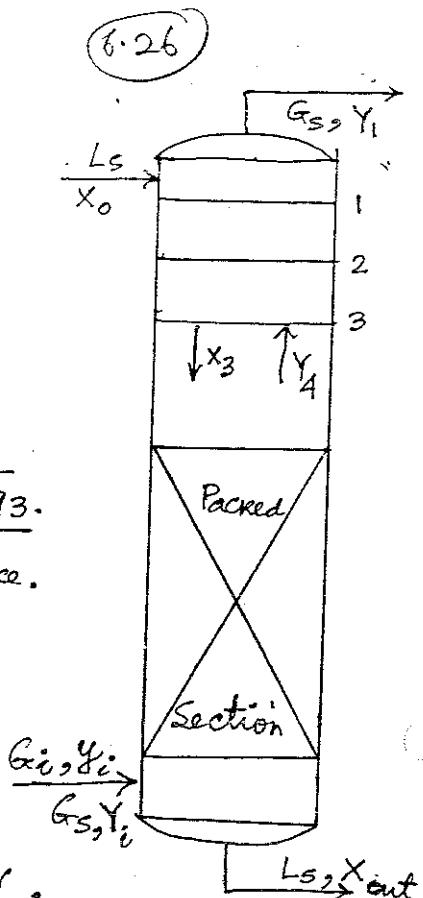
$$\text{Concentration of exit liquid, } x_{\text{out}} = 0.082 \Rightarrow x_{\text{out}} = 0.0893.$$

We first calculate the liquid rate by material balance.

$$G'_s(Y_i - Y_1) = L'_s(x_{\text{out}} - x_0)$$

$$\Rightarrow (63)(0.1111 - 0.00222) = L'_s(0.0893 - 0)$$

$$\Rightarrow L'_s = 76.8 \text{ kmol/m}^2 \cdot \text{h.}$$



For the trayed section of the column, the concentrations are pretty low and $x \approx x$, $y \approx Y$.

We shall use the Kremser equation to calculate x_3 and Y_4 below the trayed section. Number of trays, $N=3$.

$$\text{Equilibrium relation: } Y = 1.05x + 0.32x^2 \Rightarrow Y = 1.05x \text{ for small } x; \alpha = 1.05$$

$$\text{Absorption factor, } \bar{A} = L'_s/\alpha G'_s = (76.8)/(1.05)(63) = 1.161.$$

$$\text{Using Eq (4.41), } x_3 = \frac{\log \left[\frac{Y_4 - \alpha x_0}{Y_1 - \alpha x_0} \right] (1 - \frac{1}{\bar{A}}) + \frac{1}{\bar{A}}}{\log(\bar{A})} = \frac{\log \left[\frac{0.00222}{0.01127} \cdot (1 - \frac{1}{1.161}) + \frac{1}{1.161} \right]}{\log(1.161)}$$

$\Rightarrow Y_4 = 0.01127$ = concentration of the gas leaving the packed section
The liquid concentration, x_3 , can be calculated by material balance,

$$G'_s(Y_4 - Y_1) = L'_s(x_3 - x_0) \Rightarrow (63)(0.01127 - 0.00222) = 76.8(x_3 - 0)$$

$$\Rightarrow x_3 = 0.00743$$

Calculation of the packed height

Since the feed gas concentration is 10%, we shall use Eq(6.28) to calculate N_{tot} . Also we shall use the average gas flow rate to calculate H_{tot} .

$$N_{\text{tot}} = \int_{y_{\text{out}}}^{y_{\text{in}}} \frac{dy}{y - y^*} + \frac{1}{2} \ln \frac{1 - Y_{\text{out}}}{1 - Y_{\text{in}}} \\ y_{\text{out}} \quad (I)$$

$$\left| \begin{array}{l} Y_{\text{in}} = Y_i = 0.10 \\ Y_{\text{out}} = Y_4 = \frac{Y_4}{1+Y_4} = \frac{0.01127}{1.01127} = 0.01114 \\ \frac{1}{2} \ln \frac{1 - Y_{\text{out}}}{1 - Y_{\text{in}}} = \frac{1}{2} \ln \frac{1 - 0.01114}{1 - 0.10} = 0.094 \end{array} \right.$$

6.27

6.17 contd... The integral I has to be evaluated graphically or numerically. For a set of values of y in the given interval, values of y^* are to be obtained from the equations of the operating line and the equilibrium line. An algebraic relation for the equil. line is given here $\rightarrow y = 1.05x + 0.32x^2 \dots (i)$

Operating line equation (for the packed section) :

$$G_s'(Y_i - Y) = L_s(X_{out} - X_3) \Rightarrow (63)(0.1111 - \frac{y}{1-y}) = (76.8)(0.0893 - \frac{x}{1-x})$$

$$\Rightarrow \frac{y}{1-y} = 1.219 \frac{x}{1-x} + 0.0022 \dots (ii)$$

Sample calculations : $y = y_i = 0.1$; $x = 0.082$ (at the inlet) from eq.(i),

$$y^* = (1.05)(0.082) + (0.32)(0.082)^2 = 0.08825; y - y^* = 0.1 - 0.08825 = 0.01175$$

Similarly, take $y = 0.085$; $x = 0.06925$ from eq.(ii);

$$y^* = (1.05)(0.06925) + (0.32)(0.06925)^2 = 0.07425; y - y^* = 0.085 - 0.07425 = 0.01075.$$

Calculated values

y	0.1	0.085	0.07	0.055	0.04	0.025	0.015	0.01114
y^*	0.08825	0.07425	0.0604	0.04673	0.03324	0.01993	0.01113	0.00777
$y - y^*$	0.01175	0.01075	0.0096	0.00827	0.00676	0.00507	0.00387	0.00337
$1/(y - y^*)$	85.1	93	104.2	120.9	148	197.2	258.4	296.7

A plot of $\frac{1}{y - y^*}$ against y is prepared (see the figure below).

The area under the curve is

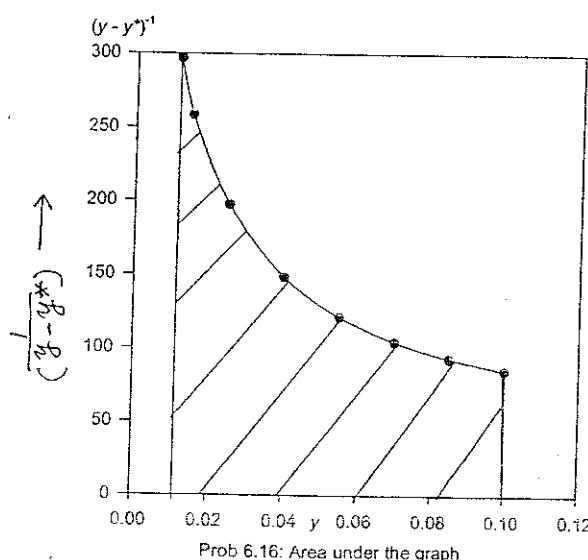
$$\int_{0.01114}^{0.1} \frac{dy}{y - y^*} = 12.72 = I$$

Total area of the integral

$$N_{tOG} = 12.72 + 0.094 = 12.81$$

$$\text{Calculation of } H_{tOG} = \frac{G'}{K_y a \cdot y_{BM}}$$

Since the gas flow rate ranges substantially along the packed bed, an average gas rate will be used.



(6.17) Contd... At the bottom of the bed, gas rate = 70 kmol/m².h

At the exit of the bed, gas rate = $G_s(1+Y_4) = (63)(1+0.01127) = 63.7 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$

Average gas rate, $G' = \frac{70 + 63.7}{2} = 66.85 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$

$y_{BM}^* \approx \left[\frac{(1-y^*) + (1-y)}{2} \right]_{av}$. The value at the bottom (see the calculated values of y^* in the previous page) and top are:

bottom $\rightarrow 1 - \frac{0.1 + 0.08825}{2} = 0.9059$; top $\rightarrow 1 - \frac{0.01114 + 0.00777}{2} = 0.9905$

$y_{BM}^* = \frac{0.9059 + 0.9905}{2} = 0.9482$; $K_y \bar{A} = 95 \frac{\text{kmol}}{\text{m}^3 \cdot \text{h} \cdot (\Delta y)}$, given

$$H_{tOG} = \frac{G'}{K_y \bar{A} \cdot y_{BM}^*} = \frac{66.85}{(95)(0.9482)} = 0.742 \text{ m}$$

$$\text{Packed height} = H_{tOG} \cdot N_{tOG} = (0.742)(12.81) = 9.5 \text{ m}$$

* * * *

(6.18) We shall first calculate the terminal flow rates and concentrations by material balance. These data will be used to calculate the absorption factor at the bottom and at the top of the column.

Neglecting the traces of solute in the feed solvent, solvent rate, $L_s = 8500 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}} = \frac{8500}{240} = 35.4 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}}$.

Concentrations: $y_1 = 0.038 \Rightarrow Y_1 = 0.0395$; $y_2 = 0.0005 \approx Y_2$; $x_1 = 0.152 \Rightarrow X_1 = \frac{0.152}{0.848} = 0.1792$; $x_2 = 0.0008 \approx X_2$.

Material balance: $G'_s (0.0395 - 0.0005) = (35.4)(0.1792 - 0.0008)$
 $\Rightarrow G'_s = 161.9 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2}$; $G'_1 = G'_s (1+Y_1) = 168.3$; $G'_2 = G'_s (1+Y_2) = 162 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2}$

Liquid rate at the bottom, $L'_1 = L_s (1+x_1) = (35.4)(1.1792) = 41.74 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2}$

Equilibrium line: $y = 0.19x \Rightarrow m = 0.19$

Absorption factor at the top, $\bar{A}_2 = \frac{L'_2}{m G'_2} = \frac{35.4}{(0.19)(162)} = 1.15$

At the bottom, $\bar{A}_1 = \frac{L'_1}{m G'_1} = \frac{41.74}{(0.19)(168.3)} = 1.305$

Take the geometric mean, $\bar{A} = (A_1 \cdot A_2)^{1/2} = (1.305 \times 1.15)^{1/2} = 1.225$

(The arithmetic mean = 1.227 which is virtually the same as the above)

6.29

6.18 contd... (a) Use Eq. (6.24c) to calculate H_{tOG}

$$H_{tOG} = H_{tG} + (\bar{A}) H_{tL} = 0.33 + \frac{0.43}{1.225} = 0.68 \text{ m}$$

$$H_{tOL} = \bar{A} H_{tG} + H_{tL} = (1.225)(0.33) + 0.43 = 0.83 \text{ m}$$

(b) Use Eq. (6.49) to calculate the HETP. $\bar{S} = \frac{1}{\bar{A}} = \frac{1}{1.225} = 0.816$

$$\text{HETP} = H_{tOG} \cdot \frac{\ln \bar{S}}{\bar{S} - 1} = (0.68) \cdot \frac{\ln(0.816)}{0.816 - 1} = 0.75 \text{ m}$$

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First we shall calculate $(L'_s)_{\min} \rightarrow L'_s \rightarrow k_x, k_y \rightarrow H_{tL}, H_{tG} \rightarrow H_{tOG}$. Then N_{tOG}

6.19 Feed gas : $60.00 \text{ m}^3/\text{m}^2 \cdot \text{h}$; 3 mol% acetone; 27°C , 1 atm total pressure.

Neglect pressure drop of the gas across the feed \Rightarrow constant pressure,

Average molecular weight of the feed, $M_{av} = (0.03)(58) + 0.97(28.9) = 29.8 \text{ atm}$.

$$\text{Total moles of feed} = (60.00) \left(\frac{273}{300} \right) \left(\frac{1}{22.4} \right) = 243.7 \text{ kmol/m}^2 \cdot \text{h} = G'$$

$$\text{Density of the gas, } \rho_G = \frac{7263.7}{6000} = 1.2106 \text{ kg/m}^3 = (243.7)(29.8) = 7263.7 \text{ kg/m}^2$$

$$\text{Moles of gas on solute-free basis (i.e., av), } G'_s = G' (1 - Y_1) = (243.7)(1 - 0.03) = 236.4 \text{ kmol/h}$$

$$\text{Also, } Y_1 = \frac{0.03}{0.97} = 0.0309$$

$$= 236.4 \text{ kmol/h}$$

$$98\% \text{ of the acetone is absorbed} \Rightarrow Y_2 = (0.0309)(0.02) = 0.000618.$$

Equilibrium relation $\rightarrow Y = 2.31x \rightarrow$ it is convex downward.

The 'pinch point' corresponding to the minimum water rate occurs at $Y = Y_1$ and $(x)_{\max} =$ corresponding equil. concentration.

Equilibrium relation in molar ratio units, $Y = \frac{2.31x}{1 + 1.31x}$.

$$\text{Put } Y = Y_1 = 0.0309 \text{ to get } (x_1)_{\max} = (x_1)^* = 0.0136$$

$$\text{Make a total material balance : } G'_s (Y_1 - Y_2) = (L'_s)_{\min} (x_1, \max x_2)$$

$$\Rightarrow (236.4)(0.0309 - 0.000618) = (L'_s)_{\min} (0.0136 - 0) \quad x_2 = 0 \text{ (solute free)}$$

$$\Rightarrow (L'_s)_{\min} = 586.9 \text{ kmol/m}^2 \cdot \text{h} = 10,557 \text{ kg/m}^2 \cdot \text{h} \quad \rho_L = 997 \text{ kg/m}^3$$

$$\text{Actual water rate, } L'_s = (1.3)(L'_s)_{\min} = (1.3)(10,557) = 13,724 \text{ kg/m}^2 \cdot \text{h}$$

Take the gas and water flow rates reasonably constant (since the solutions are 'dilute').

$$\text{Gas velocity, } u_G = \frac{6000 \text{ m}^3/\text{m}^2 \cdot \text{h}}{3600} = 1.667 \text{ m/s; Liquid vel., } u_L = \frac{(13,724)/997}{3600}$$

$$\Rightarrow u_L = 3.824 \times 10^{-3} \text{ m/s}$$

(6.30)

(6.19) contd... Other relevant property data for use in the correlations:
 For 2-inch (50 mm) metal Pall ring, a_p = specific surface area of the dry packing = $112.6 \text{ m}^2/\text{m}^3$ (Table 6.4); porosity, $\epsilon = 0.951 \text{ m}^3/\text{m}^3$ (Table 6.4); diffusivity of acetone in air, $D_G = 1.08 \times 10^{-5} \text{ m}^2/\text{s}$; in water, $D_L = 1.18 \times 10^{-9} \text{ m}^2/\text{s}$; viscosity - air, $\mu_G = 1.813 \times 10^{-5} \text{ kg/m.s}$; water, $\mu_L = 8.57 \times 10^{-4} \text{ kg/m.s}$; $\alpha_L = 7.2 \times 10^2 \text{ kg/s}^2$. Use these values to calculate different quantities in the correlations.

$$k_T = 4\epsilon/a_p = \frac{(4)(0.951)}{112.6} = 0.0338; Re_L = \frac{U_L \cdot P_L}{a_p \cdot \mu_L} = \frac{(3.824 \times 10^{-3})(997)}{(112.6)(8.57 \times 10^{-4})} = 39.5.$$

$$Re_G = \frac{U_G \cdot P_G}{a_p \cdot \mu_G} = \frac{(1.667)(1.2106)}{(112.6)(1.813 \times 10^{-5})} = 988. \quad \text{Froude number, } Fr_L = \frac{U_L^2 \cdot a_p}{g} = \frac{(3.824 \times 10^{-3})^2 (112.6)}{9.81} = 1.68 \times 10^{-4}$$

$$(Sc)_G = \frac{\mu_G}{P_G \cdot D_G} = \frac{1.813 \times 10^{-5}}{(1.2106)(1.08 \times 10^{-5})} = 1.387.$$

$$\text{Weber no., } We_L = \frac{U_L^2 P_L}{\alpha_L a_p} = \frac{(3.824 \times 10^{-3})^2 (997)}{(7.2 \times 10^2)(112.6)} = 1.8 \times 10^{-3}.$$

$$\text{Liquid holdup in the bed, } h_L = (12^{1/3})(39.5)^{-1/3} (1.68 \times 10^{-4})^{1/3} = 0.0371$$

$$\text{For 2-inch Pall ring, } C_L = 1.92; C_G = 0.41 \text{ (Table 6.4)}$$

$$\text{Eq(6.38)} \rightarrow k_L = 12^{1/6} \cdot C_L \cdot \left(\frac{U_L D_L}{h_L \cdot \alpha} \right)^{1/2} = 12^{1/6} \cdot (1.92) \left[\frac{(3.824 \times 10^{-3})(1.18 \times 10^{-9})}{(0.0371)(0.0338)} \right]^{1/2}$$

$$\text{Eq(6.39)} \rightarrow k_c = (0.4) \left(\frac{112.6}{0.0338} \right)^{1/2} \cdot \frac{1.08 \times 10^{-5}}{(0.951 - 0.0371)^{1/2}} \cdot (988)^{3/4} \cdot (1.387)^{1/3} = 1.082 \times 10^{-4} \text{ m/s}$$

$$= 5.253 \times 10^{-2} \text{ m/s}$$

Effective interfacial area of gas-liquid contact can be calculated from Eq(6.40). per unit packed volume

$$\bar{a} = (112.6) \cdot 3 \cdot (0.951)^{0.5} (39.5)^{-0.2} \cdot (1.68 \times 10^{-4})^{-0.45} \cdot (1.8 \times 10^{-3})^{0.75} = 68.95 \text{ m}^2/\text{m}^3$$

$$\text{Use Eq(3.6)} \rightarrow k_y = \frac{P}{RT} k_c = \frac{1}{(0.082)(300)} \cdot (0.05253) = \frac{2.133 \times 10^{-3}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta y)}$$

$$\text{And, } k_x = (P/M) k_w = (997) \cdot (1.082 \times 10^{-4}) = 0.006 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s} \cdot (\Delta x)}$$

Height of an individual liquid-phase transfer unit: (considering that the liquid, see Table 6.2, take $(-x_i)_M \approx 1$ and the gas both are dilute):

$$H_{tL} = \frac{L'}{k_x \bar{a}} = \frac{L'}{k_x \bar{a}} = \frac{(13.724 \text{ kg/m}^2)/(3600)(18)}{(0.006)(68.95)} = 0.512 \text{ m}$$

$$\text{Similarly, } H_{tG} = \frac{G'}{k_y \bar{a}} = \frac{(240 \text{ kmol/m}^2)/(3600)}{(2.133 \times 10^{-3})(68.95)} = 0.453 \text{ m}$$

[The mean gas flow rate $\approx \frac{(243.7 + 236.4)}{2} \rightarrow 240 \text{ kmol/m}^2 \text{ has been taken as } G'$]

6.31

(6.19) contd... Calculate H_{tOG} from Eq. (6.24c). Equil. relation: $y = 2.31x$

Absorption factor, $\bar{A} = L'/mG' = \frac{(13724)/(18)(3600)}{(2.31)(240/3600)} = 1.375$

$$H_{tOG} = H_{tG} + \frac{1}{\bar{A}} H_{tL} = (0.453) + \frac{0.512}{1.375} = \underline{\underline{0.825 \text{ m}}}$$

The number of overall gas-phase mass transfer units has to be determined.

Top of the tower $\rightarrow y_1 \approx y_2 = \underline{\underline{0.000618}}; x_2 = 0 \rightarrow (\Delta y)_2 = y_2 - y_2^* = \underline{\underline{0.000618}}$

Bottom of the tower $\rightarrow x_1 = \frac{236.4(0.0309 - 0.000618)}{(13724/18)} = \underline{\underline{0.00939}}, \text{ by solute material balance over the tower.}$

$$(\Delta y)_1 = y_1 - y_1^* = y_1 - mx_1 = 0.03 - (2.31)(0.00939) = \underline{\underline{0.00831}}$$

$$(y - y^*)_M = \frac{(\Delta y)_1 - (\Delta y)_2}{\ln[(\Delta y)_1 / (\Delta y)_2]} = \frac{0.00831 - 0.000618}{\ln[0.00831 / 0.000618]} = \underline{\underline{2.96 \times 10^{-3}}}$$

$$\text{Eq. (6.27b)} \rightarrow N_{tOG} = \frac{y_1 - y_2}{(y - y^*)_M} = \frac{0.03 - 0.000618}{2.96 \times 10^{-3}} = \underline{\underline{9.926}}$$

Height of packing required, $-h = H_{tOG} N_{tOG} = (0.825 \text{ m})(9.926)$

* * * * $= \underline{\underline{8.2 \text{ m}}}.$

(6.20) Use Eq.(6.25) to determine the number of overall gas-phase

$$N_{tOG} = \int_{y_2}^{y_1} \frac{dy}{y - y^*} \cdot \text{ Since absorbed ammonia instantaneously reacts with } H_2SO_4, \text{ its vapor pressure over the acid is zero} \Rightarrow y^* = 0$$

$$\Rightarrow N_{tOG} = \int_{y_2}^{y_1} \frac{dy}{dy} = \ln \frac{y_1}{y_2} \Rightarrow \frac{y_1}{y_2} = \exp(-N_{tOG})$$

Fraction absorbed, $f = 1 - \frac{y_2}{y_1} \Rightarrow f = 1 - e^{-N_{tOG}} \dots (i)$

A plot of N_{tOG} against f can be prepared from Eq. (i). Expectedly it is an exponential dependence.

Other data supplied (gas and liquid flowrates, and mass transfer coefficients) are not required to solve the problem.

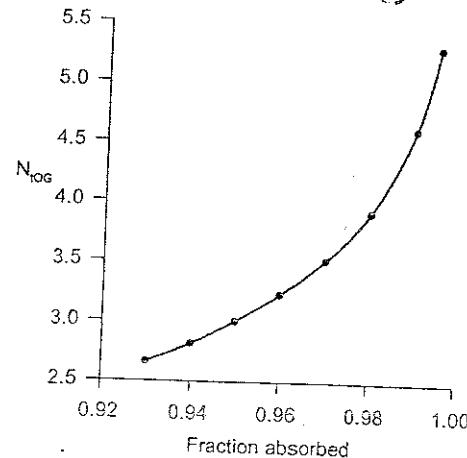


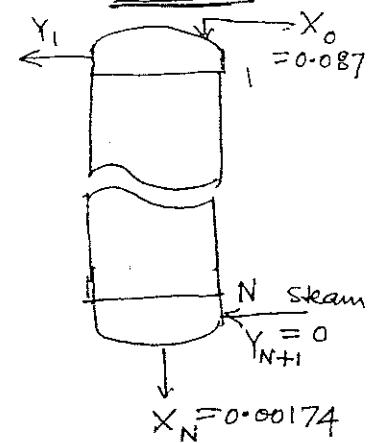
Fig. Prob 6.20

$$6.23) \text{ contd...} \Rightarrow (Y_1)_{\max} = 2.08X_0 = (2.08)(0.087) = \underline{0.181}$$

$L_s/(G_s)_{\min}$ is obtained by material balance over the column.

$$L_s(X_0 - X_N) = (G_s)_{\min} [(Y_1)_{\max} - Y_{N+1}] ; \text{ feed stream is 'pure'} \Rightarrow \underline{Y_{N+1} = 0}$$

$$\Rightarrow \frac{(G_s)}{L_s}_{\min} = \frac{0.087 - 0.00174}{0.181} = 0.471$$



Terminal points of the operating line for minimum steam:

$$(X_N, Y_{N+1}) \rightarrow (0.00174, 0) \text{ and } (X_0, Y_1, \max) \rightarrow (0.087, 0.181).$$

(b) 1.3 times the minimum steam is used.

$$\Rightarrow G_s/L_s = (1.3)(G_s/L_s)_{\min} = (1.3)(0.471) = \underline{0.612}.$$

$$\text{Recalculate } Y_1 \text{ for this } G_s/L_s \Rightarrow (G_s/L_s)(Y_1 - 0) = (X_0 - X_N)$$

$$\Rightarrow (0.612) \cdot Y_1 = 0.181 - 0.00174 \Rightarrow \underline{Y_1 = 0.1393}$$

$$\text{Equation of the operating line: } G_s(Y_n - Y_{N+1}) = L_s(X_n - X_N)$$

$$\Rightarrow (0.612) \cdot (Y_n - 0) = X_n - 0.00174 \Rightarrow \underline{Y_n = 1.634X_n - 0.00248, \dots (i)}$$

Number of ideal trays: Use the Kremser eq. for stripping, Eq.(4.44).

$$\bar{A} = (\gamma_S) = L_s/\alpha G_s = \frac{1}{(2.08)(0.612)} = \underline{0.813}$$

$$N = \frac{\log \left[\frac{(X_0 - Y_{N+1}/\alpha)}{(X_N - Y_{N+1}/\alpha)} \right] (1 - \bar{A}) + \bar{A}}{\log (1/\bar{A})} = \frac{\log \left[\frac{(0.087 - 0)}{(0.00174 - 0)} \right] (1 - 0.813) + 0.813}{\log (1/0.813)}$$

$$= \frac{\log 10.16}{\log 1.23} = \underline{11.2 \text{ ideal trays}}$$

(c) Enrichment of the stripping steam at the 5-th tray from the top:

The required enrichment (in mole ratio unit) = $Y_1 - Y_5$.

Tray-by-tray calculation from one terminal is to be done to determine Y_5 and Y_6 . The operating line eq. (i) and the equil. line $Y = 2.08X$ have to be used. The steps are shown.

$$Y_1 = 0.1393 \Rightarrow X_1 = Y_1/2.08 = \frac{0.1393}{2.08} = \underline{0.067} \quad (\text{Since } X_1 \text{ and } Y_1 \text{ lie on the equil. line if the tray is 'ideal'})$$

(X_1, Y_2) lies on the operating line, eq.(i).

- (6.21) The given concentrations are converted to mole ratios for material balance calculation over the p -th tray.

$$Y_{p+1} = \frac{y_{p+1}}{1-y_{p+1}} = \frac{0.08}{0.92} = 0.08696$$

$$X_{p-1} = \frac{x_{p-1}}{1-x_{p-1}} = \frac{0.044}{0.956} = 0.04602$$

Multiphase efficiency of the p -th tray,

$$\frac{y_p - y_{p+1}}{y_p^* - y_{p+1}} = 0.5 \Rightarrow (y_p - y_{p+1}) = (0.55)(y_p^* - y_{p+1}) = (0.55)(1.8x_p^{1.2} - y_{p+1})$$

$$\Rightarrow (y_p - 0.08) = (0.55)(1.8x_p^{1.2} - 0.08) ; \text{ (i)} \quad y_p^* = 1.8x_p^{1.2}$$

Material balance over the p -th tray: (Given: $L_s/G_s = 1.3$)

$$G_s(Y_{p+1} - Y_p) = L_s(X_p - X_{p-1}) \Rightarrow (0.08696 - \frac{y_p}{1-y_p}) = (1.3)(\frac{x_p}{1-x_p} - 0.04602) \dots \text{(ii)}$$

Solve Eq.(i) and (ii) to get

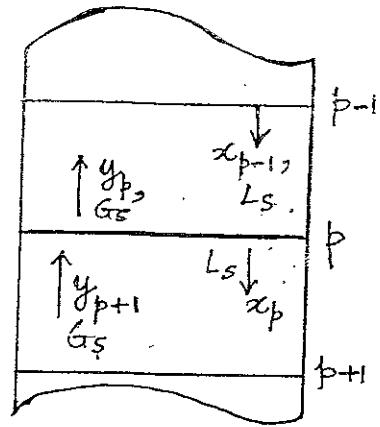
$$x_p = 0.055 ; y_p = 0.06648$$

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- (6.22) Use Eq.(6.55) $\rightarrow E_{OG} = 1 - e^{-N_{TOG}} = 1 - e^{-0.4} = 1 - 0.67 = 0.33$

$$\text{Use Eq.(6.54)} \rightarrow N_{TOG} = \frac{K_y \bar{a} \cdot Z_L}{G'} \quad ; \quad Z_L = 0.28 \text{ m.} \quad = \text{point efficiency}$$

$$\Rightarrow K_y \bar{a} = (G'/Z_L) \cdot N_{TOG} = \frac{120}{0.28} \cdot (0.4) = 171.4 \text{ k/mol/k.m}^3 (\Delta y)$$

= overall volumetric gas-phase mass transfer coefficient

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- (6.23) Given: feed liquid concentration, $x_0 = 0.08 \Rightarrow x_0 = 0.087$.
 98% of the solute is stripped out. Lean (exit) liquid concentration, $x_N = (0.087)(0.02) = 0.00174$.

- (a) The pinch point (corresponding to the minimum steam rate) occurs at the top of the column. At the pinch point the vertical line through x_0 meets the equilibrium line which is the upper terminal of the operating line for minimum steam rate (this happens because the equilibrium line is straight). No graphical construction is necessary to locate the pinch point.

6.34

(6.23) contd... Put $x_1 = 0.067$ in Eq(i) to get $y_2 = 0.107$.

Then find x_2 from the equil. line eqi., $x_2 = \frac{y_2}{2.08} = 0.05144$ and soon, to get $y_5 = 0.0459$ and $y_6 = 0.0336$.

Required enrichment = $y_5 - y_6 = 0.0459 - 0.0336 = \underline{\underline{0.0123}}$

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(6.24) Eq(6.22) $\rightarrow H_{tOG} = \frac{G'}{K_{y\bar{a}} \cdot y^*} = \frac{G'}{K_{y\bar{a}}}$; taking $y_{BM}^* \approx 1$ for dilute gas.

Calculate H_{tOG} and \bar{A} , put in Eq (6.49) to get HETP.

Given, $K_{G\bar{a}} = 0.0113 \frac{\text{kmol}}{\text{s.m}^3 (\Delta P, \text{bar})} \Rightarrow K_{y\bar{a}} = \frac{K_{G\bar{a}}}{P} = \frac{0.0113}{1.05} = \underline{\underline{0.01076}}$.

$\bar{A} = 1.35 = L'/mG'$. Put $m = 1.5$, $L' = 65 \frac{\text{kmol}}{\text{h.m}^2}$ to get $G' = 32.1 \frac{\text{kmol}}{\text{m}^2 \text{h}}$
 $H_{tOG} = \frac{(32.1/3600)}{0.01076} = \underline{\underline{0.83 \text{ m}}} \quad \bar{S} = 1/\bar{A} = \underline{\underline{0.407}}$

Eq.(6.49) $\rightarrow \underline{\underline{\text{HETP}}} = H_{tOG} \cdot \frac{\ln \bar{S}}{\bar{S} - 1} = (0.83 \text{ m}) \cdot \frac{\ln 0.7407}{0.7407 - 1} = \underline{\underline{0.961 \text{ m}}}$

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(6.25) Case I: Feed $\rightarrow 400 \text{ ppm TCE}$ (mol.wt. 119.4), calculate its mole fraction (take the mol.wt. of TCE-containing water as 18) at the top of the column (inlet of contaminated water).

$$x_2 = \frac{(400/119.4)}{(10^6/18)} = \underline{\underline{6.03 \times 10^{-5}}}$$

$$\text{At equilibrium, } y_2^* = m x_2 = (661.1)(6.03 \times 10^{-5}) = \underline{\underline{3.986 \times 10^{-2}}}$$

$$\text{Mole fraction of TCE in the water leaving, } x_1 (= 3 \text{ ppm}) = \frac{3/119.4}{10^6/18} = \underline{\underline{4.523 \times 10^{-7}}}$$

Calculate air rate: The stripping air does not have any solute (TCE) $\Rightarrow y_1 = 0$. For minimum air, the pinch point will occur at the top of column $\Rightarrow (y_2)_{\max} = m x_2 = \underline{\underline{3.986 \times 10^{-2}}}$.

$$\text{Material balance: } (G')_{\min} \cdot [(y_2)_{\max} - y_1] = L'(x_2 - x_1)$$

Take G' and L' to be constant (very dilute solution); put the values of $x_1, x_2, (y_2)_{\max}$ and y_1 to get $(G')_{\min}/L' = \underline{\underline{1.5 \times 10^{-3}}}$
 $[(G')_{\text{actual}}/L'] = (2)(1.5 \times 10^{-3}) = \underline{\underline{3 \times 10^{-3}}}$.

6.25

$$\text{Actual outlet air concentration} = (L'/G') \cdot (x_2 - x_1) = \frac{6.08 \times 10^{-5} - 4.523 \times 10^{-5}}{3 \times 10^{-3}} = \underline{\underline{0.02}} = \underline{\underline{y_2}}$$

Driving force: At the top of the tower, $(\Delta y) = y_2^* - y_2 = 0.03986 - 0.02$

$$\text{At the bottom, } (\Delta y)_1 = y_1^* - y_1 = mx_1 - y_1 = (661 \cdot 1)(4.523 \times 10^{-7}) = \underline{\underline{0.01986}}$$

$$(\Delta y)_M = \frac{(\Delta y)_2 - (\Delta y)_1}{\ln[(\Delta y)_2 / (\Delta y)_1]} = \frac{4.665 \times 10^{-3}}{\ln[0.02 / 0.01986]} = \underline{\underline{3 \times 10^{-4}}} = 0$$

$$N_{TOG} = \frac{y_2 - y_1}{(\Delta y)_M} = \frac{0.02 - 0}{4.665 \times 10^{-3}} = \underline{\underline{4.28}} \text{ for case 1}$$

Case 2: Reduction of TCE concentration from 600 ppm to 0.1 ppm.

$$x_1 = 0.1 \text{ ppm} = \frac{0.1 / 119.4}{10^6 / 18} = \underline{\underline{1.5075 \times 10^{-8}}} ; x_2 = 600 \text{ ppm} = \underline{\underline{9.045 \times 10^{-5}}}$$

$$\text{By material balance, } y_2 = (L'/G')(x_2 - x_1) = \frac{1}{3 \times 10^{-3}} \cdot (9.045 \times 10^{-5} - 1.5075 \times 10^{-8})$$

$$\text{Driving force: } = \underline{\underline{0.03014}} \quad (L' \text{ and } G' \text{ remain as in case 1})$$

$$\text{At the bottom, } (\Delta y)_1 = y_1^* - y_1 = (661 \cdot 1)(1.5075 \times 10^{-8}) - 0 = \underline{\underline{9.97 \times 10^{-6}}}.$$

$$\text{At the top, } (\Delta y)_2 = y_2^* - y_2 = (661 \cdot 1)(9.045 \times 10^{-5}) - 0.03014 = \underline{\underline{0.02966}}.$$

The driving force for stripping is positive at both the terminals of the column \Rightarrow it is possible to strip 600 ppm TCE in water to 0.1 ppm.

Calculation of N_{TOG} :

$$(\Delta y)_M = \frac{(\Delta y)_2 - (\Delta y)_1}{\ln[(\Delta y)_2 / (\Delta y)_1]} = \frac{0.02966 - 9.97 \times 10^{-6}}{\ln(0.02966 / 9.97 \times 10^{-6})} = \underline{\underline{0.003707}}$$

$$N_{TOG} = \frac{y_2 - y_1}{(\Delta y)_M} = \frac{0.03014 - 0}{0.003707} = \underline{\underline{8.13}} \text{ for case 2}$$

Since the gas and liquid flow rates remain unchanged, H_{TOG} also remains the same.
Packed height in case 1 = 3 m

$$\text{Height of packing required} = (3 \text{ m}) \left(\frac{N_{TOG,2}}{N_{TOG,1}} \right) = (3) \left(\frac{8.13}{4.28} \right) = \underline{\underline{5.7 \text{ m}}}$$

* * * *

6.26 We calculate the overall gas-phase mass transfer coefficient first in order to obtain H_{TOG} and then N_{TOG} (for calculating E_{OG}).

Use Eq.(4.24) to calculate $K_G \rightarrow \frac{1}{K_G} = \frac{1}{k_G} + \frac{H'}{k_L} ; H' = 0.3 \frac{\text{atm}}{(\text{kmol/m}^2)} \text{ given}$

$$\Rightarrow \frac{1}{K_G} = \frac{1}{4.07} + \frac{0.3}{5.56} \Rightarrow K_G = 3.75 \frac{\text{kmol}}{\text{h} \cdot \text{m}^2 \cdot (\Delta P, \text{ atm})}$$

(6.26) contd... $K_y = P \cdot K_G = (1.3)(3.75) = 4.875 \frac{\text{kmol}}{\text{m}^2 \text{h. (kg)}} ; P = \text{total pres.}$
 $\stackrel{= 1.3 \text{ atm}, \text{ given}}{\text{Specific interfacial area of the dispersion}}$

Assume dilute gas, $y_{\text{EM}}^* \approx 1$, $\bar{a} = 100 \text{ m}^2/\text{m}^3$, given.

$$H_{\text{t0G}} = \frac{G'}{K_y \bar{a}} = \frac{160 \text{ kmol/m}^2 \text{ h}}{(4.875)(100) \frac{\text{kmol}}{\text{m}^3 \text{ h. kg}}} = 0.328 \text{ m}$$

$$\text{Froth height, } z_L = 0.2 \text{ m} \Rightarrow N_{\text{t0G}} = z_L / H_{\text{t0G}} = 0.2 / 0.328 = 0.61$$

(a) Use Eq.(6.55) to calculate the 'point efficiency'.

$$E_{\text{OG}} = 1 - e^{-N_{\text{t0G}}} = 1 - e^{-0.61} = 1 - 0.543 = 0.457 \Rightarrow 45.7\%$$

(b) Misfree Efficiency

(i) For plug flow with no entrainment, use Eq. 6.58. Given, $A = \frac{L'}{mG'} = 1$.

$$E_{\text{MG}} = \frac{L'}{mG'} \left[\exp \left(\frac{mG'}{L'} \cdot E_{\text{OG}} \right) - 1 \right] = (1.2) \cdot \left[\exp \left(\frac{0.457}{1.2} \right) - 1 \right] = 0.556 \Rightarrow 55.6\%$$

(ii) For plug flow and with an entrainment, $E = 0.06$, use Eq. (6.59).

$$E_{\text{MGE}} = \frac{E_{\text{MG}}}{1 + E_{\text{MG}} [E/(1-E)]} = \frac{0.556}{1 + (0.556) \left(\frac{0.06}{0.94} \right)} = 0.537 \Rightarrow 53.7\%$$

(Note that entrainment causes a fall in the tray efficiency.)

(c) Overall column efficiency : Use Eq.(6.61).

$$E_0 = \frac{\ln \left[1 + E_{\text{MG}} \left(\frac{mG'}{L'} - 1 \right) \right]}{\ln \left(mG'/L' \right)} = \frac{\ln \left[1 + (0.556) \left(\frac{1}{1.2} - 1 \right) \right]}{\ln (1/1.2)} = 0.533 \Rightarrow 53.3\%$$

* * * *

(6.27) The flowrate and inlet and exit concentrations of the gas are given. Also given is the number of "ideal" trays. The required liquid rate should be such that eight ideal trays can be fitted between the operating line and the equilibrium line.

Graphical construction on the X-Y plane will be convenient. Equil. relation : $y = 0.5x \Rightarrow Y = \frac{0.5X}{1+0.5X}$

Feed gas : $G_{\text{NH}_3} = 8000 \text{ m}^3/\text{h}$, 27°C & 1 atm.

$$\Rightarrow G_{\text{NH}_3} = (8000) \left(\frac{27.3}{300} \right) \cdot \left(\frac{1}{22.4} \right) \text{ kmol} = 325 \text{ kmol}; Y_{\text{NH}_3} = 0.12 \Rightarrow Y = 0.1364$$

(6.37)

- 6.27 contd... Flow rate of the carrier gas, $G_s = 300(1-0.12) = 286 \frac{\text{kmol}}{\text{hr}}$.
 Exit gas conc. (96% removal) $\rightarrow Y_1 = (0.1364)(0.04) = 0.00545$; $X_0 = 0$
 The equilibrium line is drawn and the point $(X_0, Y_1) \rightarrow (0, 0.00545)$ is located.

The operating line PQ (see fig Prob 6.27) is drawn by trial such that just eight steps can be drawn between the equilibrium line and PQ while the point Q lies on the horizontal line through $Y_{N+1} = 0.1364$. Corresponding exit liquid concentration, $X_N = 0.22$.

The liquid rate can be obtained from the slope of PQ or from an overall material balance.

$$G_s(Y_{N+1} - Y_1) = L_s(X_N - X_0)$$

$$\Rightarrow (286)(0.1364 - 0.00545)$$

$$= L_s(0.22 - 0) \Rightarrow L_s = 170 \frac{\text{kmol/hr}}{\text{hr}} = \underline{\text{required liquid rate}}$$

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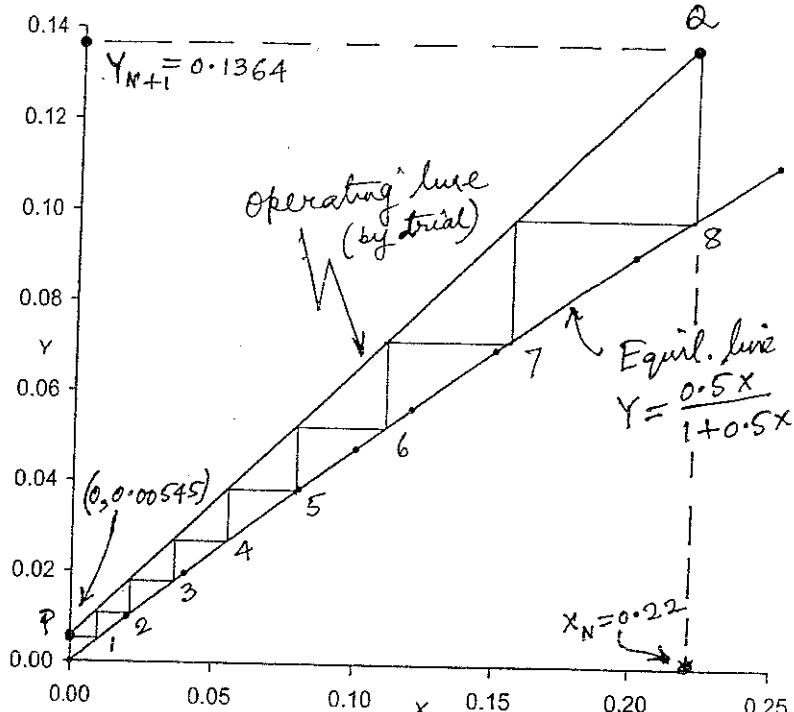


Figure Prob 6.27

- 6.28 This problem also needs trial-and-error solution.

Feed gas, $G_{N+1} = 300 \frac{\text{kmol}}{\text{hr}}$; $Y_{N+1} = 0.1$

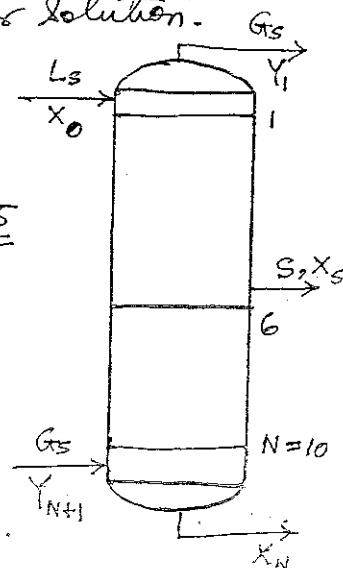
$$\Rightarrow G_s = (300)(1 - 0.1) = 270 \frac{\text{kmol}}{\text{hr}}; Y_{N+1} = \frac{0.1}{0.9} = 0.1111$$

95% of the solute is absorbed $\Rightarrow Y_1 = (0.05)(0.1111) = 0.00555$

Feed liquid concentration, $X_0 = 0$ (solute-free)

Side stream $\rightarrow 0.1$ mole fraction $\Rightarrow X_s = 0.1111$

Since the liquid rate changes below the sixth tray, there will be two operating lines which meet at the sixth tray.



6.28) Contd... Draw the equilibrium line, $y = 0.5x \Rightarrow Y = \frac{0.5X}{1+0.5X}$, on the X-Y plane, and locate the point $P(x_0, Y_1) \rightarrow (0, 0.00555)$, which is one terminal of the operating line for the upper section of the tower having six trays. The other terminal lies on the vertical line through $x = x_s = 0.1111$. This terminal, S, is located by trial such that exactly six steps can be drawn between the line PS and the equil. line.

Now we have to draw the operating line for the lower part of the column having four trays. The point S is one terminal of the operating line. The other terminal, T, is located by trial such that exactly four steps can be drawn between ST and the equil. line while the point T lies on the horizontal line through the point $Y_{NH_3} = 0.1111$.

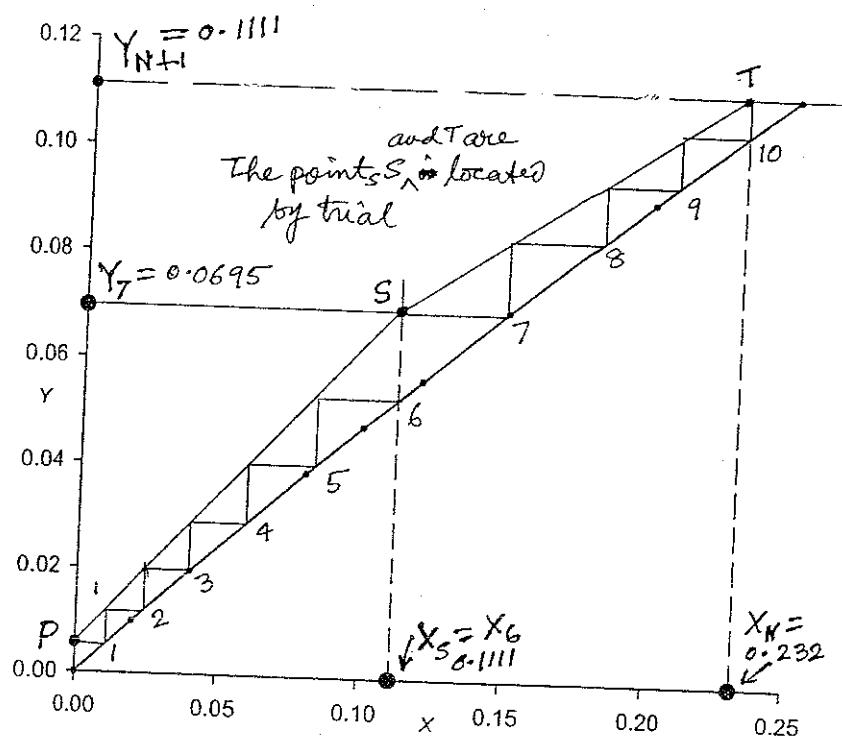


Figure Prob 6.28

The construction is shown in Figure Prob 6.28. The terminal liquid concentration, $X_N = 0.232$ from the graph. To calculate the amount of side stream, S , and the feed liquid rate, L_s , use the following material balance equations. Note that the gas concentration entering the sixth tray is $Y_7 = 0.0695$ (from the graph).

$$G_s(Y_7 - Y_1) = L_s(X_6 - X_0) \Rightarrow (270)(0.0695 - 0.00555) = L_s(0.1111 - 0) \\ \Rightarrow L_s = 155.4 \text{ kmol/hr}$$

Also, $G_s(Y_{NH_3} - Y_1) = S \cdot X_s + (L_s - S)X_N \Rightarrow (270)(0.1111 - 0.00555) = S \cdot (0.1111) \\ \Rightarrow S = \underline{62.5 \text{ kmol/hr (solute free)}} = \underline{62.5 / (1 - 0.1111)} = \underline{70.3 \text{ kmol/hr total}}$

(1a) (ii) 0.5, $\alpha_{BA} = \frac{1}{2}\alpha_{AB}$; (1b) (i) 1.4 since $\alpha_{AB} = K_A/K_B$. (2) Yes.

(3) (i) Heat loss from the still and resulting condensation of vapor. This causes 'rectification' of the vapor. (ii) Superheat of the boiling liquid, indicating an incorrect bubble point. (iii) Entrainment of liquid droplets.

(4) (a) Calculate first the total pressure exerted by a liquid having 0.3 mole fraction benzene at 95°C. Use Table 7.2.

$$\text{At } 95^\circ\text{C} = 368\text{K}, P_A^v = 1.4573 \text{ atm}; P_B^v = 0.6331 \text{ atm}$$

$$\text{Total pressure exerted by the mixture} = (0.3)P_A^v + (0.7)P_B^v = 0.88 \text{ atm}$$

Since this calculated total pressure is less than the ambient pressure of 1 atm, the mixture will exist as a liquid.

(4)(b). The required minimum temperature is the dew point of a vapor having $y_A = 0.3$ at 1 atm total pressure.

$$\text{Ans. } T = 376.8\text{K} \text{ at which } P_A^v = 1.8805; P_B^v = 0.828 \text{ atm}$$

$$\text{and } \frac{y_A}{P_A^v} + \frac{1-y_A}{P_B^v} = 1.005 \Rightarrow \text{the required}$$

$$\text{minimum temp} = 376.8\text{K}.$$

(5) (i), (ii) and (iv); (6) $y_A = \frac{1.5x_A}{1+0.5x_A} \Rightarrow y_A = 0.5, x_A = ? \Rightarrow x_A = 0.4$ (iii).

(7) (i) $x_1 < y_1^*$. (8) Use Eqs. (7.26) and (7.27). F = D + W = 2 + 1 = 3 moles.

$$\frac{x_D/(1-x_D)}{x_W/(1-x_W)} = \frac{1}{0.216}; x_W = 0.15 \Rightarrow x_D = 0.45, F.Z_F = D x_D + W x_W \Rightarrow Z_F = 0.35.$$

(9) (ii) $T_b = T_d$; (10) (a) \rightarrow (i) 0.18; (b) $y_2^* = \frac{x_2}{\sum \alpha_i x_i}$, Eq. (7.13), = 0.277 (ii);

$$(c) \alpha_{32} = K_3/K_2 \Rightarrow 0.43 = K_3/0.688 \Rightarrow K_3 = 0.296 \text{ (iii)};$$

(11) Use Eqs (7.20), (7.21) and (7.22), enthalpy = 1100 kcal/kmol, composition, $x = 0.667$.

(12) $\sum y_i = \sum K_i x_i = (2.25)(0.2) + (1.02)(0.4) + (0.6)(0.2) = 0.978 \rightarrow$ it is below bubble point, (i); (13) (a) Vaporization ratio; $f = \frac{0.62}{0.38} = 1.631$, Eq. 7.36 $\rightarrow (1+f) \sum \frac{K_i z_F}{K_i + f} = 0.803 \neq 1$, the calculation is not correct,

(b) $K_{43} = K_4/K_3 = 0.2/0.3 = 0.33 \rightarrow$ (i); (14) Use Eq. (7.41).

$$m_A = (0.8)(1) = 0.8 \text{ mol}, P_B^v = 1.3 \text{ bar}, P_A^v = 6.55 \text{ mmHg} = 0.00873 \text{ bar}$$

$$m_B = (0.8)(1-3)/(0.00873) = 70.5 \text{ mol steam.}$$

7.2

(15) Capacity and product purity;

(16) dD kmol vapor produced in time dt, h ; benzene mole fraction in the vapor = y^* ; kmol benzene (A) vaporized = $dD \cdot y^*$; kmol p-xylene(B) vaporized = $dD(1-y^*)$; heat absorbed = $[y^* \cdot \Delta H_A + (1-y^*) \cdot \Delta H_B] dD$
 $\Delta H_A = (100)(78) = 7800 \text{ kcal/kmol}$; $\Delta H_B = (85)(106) = 9010 \text{ kcal/kmol}$; -
Heat supplied over a time $\frac{M_A}{dt} = 4000 \cdot dt = [y^* \cdot \Delta H_A + (1-y^*) \cdot \Delta H_B] \cdot dD$
 $\Rightarrow \frac{dD}{dt} = 0.4976 \text{ kmol/h}$; $\alpha = 5.6$, $x_A = 0.4 \Rightarrow y^* = \frac{(5.6)(0.4)}{1+(4.6)(0.4)} = 0.789$

Instantaneous rate of vaporization of p-xylene
 $= (1-y^*) \frac{dD}{dt} = (1-0.789)(0.4976) = 0.105 \frac{\text{kmol}}{\text{h}}$

(17) Feed $\approx 1000 \text{ kg}$; A \rightarrow benzene ($M_A = 78$) ; B \rightarrow nitrobenzene ($M_B = 123$).
 $x_F = 0.5 \Rightarrow \text{mol. wt g feed} = (0.5)(78) + (0.5)(123) = 100.5$.
Moles of feed, $F = 1000/100.5 = 9.95 \text{ kmol}$.
Residue in the still = $W \text{ kmol}$, mole fraction of benzene = x_w ;
mass of residue = $W[78x_w + 123(1-x_w)] = 500 \Rightarrow W = 500/[]$.
Use Eq. (7.51b), put the values of F , x_F and $\alpha (= 7.0)$.
 $\ln \frac{9.95(0.5)[78x_w + 123(1-x_w)]}{500x_w} = 7 \ln \frac{9.95(1-0.5)[78x_w + 123(1-x_w)]}{500(1-x_w)}$.

Solving, $x_w = 0.141$, $W = 4.286$. Use Eq. 7.52 for material balance
 $(9.95)(0.5) = (5.664) y_{D,av} + (4.286)(0.141) \Rightarrow y_{D,av} = 0.77$

The reported concentration of benzene in the accumulate

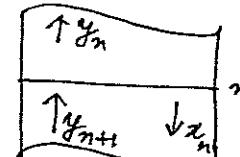
Condensate is not accurate.

(18) Use Eq. (7.57). Fraction distilled = 1 - fraction remaining = 1 - $\frac{L_3, \text{final}}{L_3, \text{initial}}$
 $\frac{L_1, \text{initial}}{L_1, \text{final}} = 1/0.7 = 1.4286 = \left(\frac{L_3, \text{initial}}{L_3, \text{final}} \right)^{1/0.2} \Rightarrow \frac{L_3, \text{final}}{L_3, \text{initial}} = 0.931$

(19) The slope of the rectifying section Op. line is minimum at the minimum reflux. For a saturated feed, the feed line is vertical; for $x_F = 0.5$, $y^* = \frac{\alpha \cdot z_F}{1+(\alpha-1)z_F} = 0.667$; also $x_D = y_D = 0.95$

Slope = $\frac{y_D - y^*}{x_D - z_F} = \frac{0.95 - 0.667}{0.95 - 0.05} = 0.79$. The minimum slope of the stripping section Op. line occurs at total reflux.

- 20) Given: $\alpha = 2.5 \Rightarrow y = \frac{2.5x}{1+1.5x}$; local slope of the equil. line, $\frac{dy}{dx} = m = \frac{2.5}{1+1.5x} - \frac{(2.5)(1.5)x}{(1+1.5x)^2} = 0.595$ at $x=0.7$
 $\Rightarrow R = mV/L = (0.595)(0.9) = 0.661$
 Use Eq(7.123). $H_{OG} = H_G + R H_L = (0.42) + (0.661)(0.15) = 0.52m$

- 21) Assume 'well-mixed' liquid on the tray.
- $x_n = 0.2$, $y_{n+1} = 1.4x_n - 0.02 = (1.4)(0.2) - 0.02 = 0.26$
- $\alpha = 2.6 \Rightarrow y_n^* = \frac{2.6x_n}{1+1.6x_n} = 0.394$.
- $E_{MG} = 0.7 = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{y_n - 0.26}{0.394 - 0.26} \Rightarrow y_n = 0.354$
- 

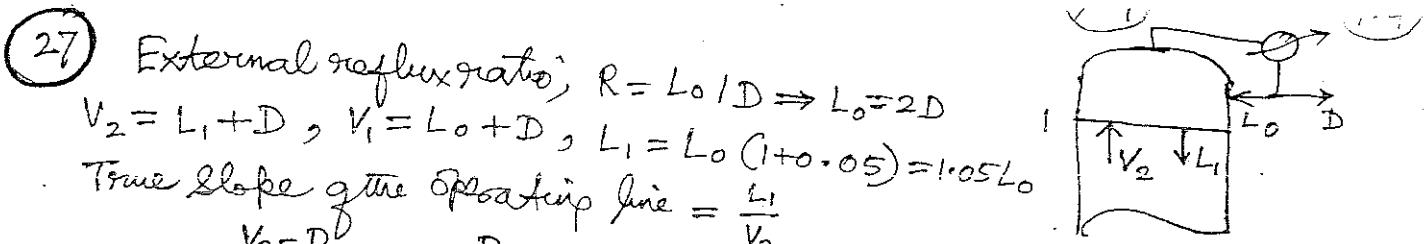
- 22) Since condensed steam accumulates in the column, the total liquid holdup goes on increasing, and the concentration on each tray continues to decrease.
 Also the column will flood if there is no 'product' removal from the bottom.
- 23) Compare with Eq(7.90). For the last tray $y=0 \Rightarrow x=x_w = 0.2$ (ii)
 Note that the value of α is not necessary for solution.

- 24) At the bottom end of the operating line, $(x_w, 0) \rightarrow (0.03, 0)$;
 $q=0.4$; slope of the feed line $= q/q_1 = 0.4/-0.6 = -0.667$, it passes through $(0.5, 0.5) \Rightarrow$ feed line eq. is $y = -0.667x + 0.8333$.
 Find the intersection of the feed line and the equil. line, $y = \frac{2.6x}{1+1.6x}$
 Solving, $x = 0.36$ and $y = 0.594$. At minimum reflux, the bottom operating line passes through $(0.36, 0.594)$ and $(0.03, 0)$
 Eq. $\rightarrow y = 1.8x - 0.054$.

- 25) Use Fenske's equation. Geometric mean of the relative volatility, $\alpha = (1.8 \times 2)^{1/2} = 1.897$. Given; $x_D = 0.97$, $x_w = 0.04$

$$\text{Eq. (7.83)} \rightarrow N_{n+1} = \frac{\log(0.97)(0.96)}{(0.04)(0.03)} = \frac{10.4}{0.012} = 9.4 \text{ ideal trays}$$

- plus reboiler
- 26) $y_4 = 0.925 = 0.6667x_3 + 0.32 \Rightarrow x_3 = 0.907$; $\alpha = 1.85$
 $y_3^* = \frac{(1.85)(0.907)}{1+(0.85)(0.907)} = 0.947$. $E_{MG} = \frac{y_3 - y_4}{y_3^* - y_4} = \frac{0.947 - 0.925}{0.947 - 0.925} = 0.682$



28 $F = 1 \text{ kmol/h} = L + D$; $0.5 = Lx_w + Dx_D$; $D = 0.6$ (given), $L = 0.4$

$$\Rightarrow 0.5 = 0.4x_w + 0.6x_D; x_D \text{ and } x_w \text{ are compositions of the vapor and the liquid at equilibrium} \Rightarrow x_D = \frac{3.33x_w}{1+2.33x_w}; [x = 1/0.33] = 3.33$$

Solve for x_D and x_w . $x_D = 0.616$ and $x_w = 0.325$

29 Assume constant molar overflow. The feed is 40% vapor $\Rightarrow y = 0.6$, $Z_F = 0.8$; the feed line [Eq(7.85)] $\rightarrow y = \frac{y}{Z-1}x - \frac{Z_F}{Z-1} = -1.5x + 2$

(a) Given: $R = 2.2$ and $x_D = 0.98 \Rightarrow$ rectifying section operating line, $y = \frac{R}{R+1}x + \frac{x_D}{R+1} = 0.687x + 0.306$

(b) At total reflux, the rectifying section operating line coincides with the diagonal — i.e. $x = y$; the feed line reduces to a point.

30 Since the reflux is cold, the 'internal reflux ratio' is larger than the external reflux ratio. So the external ratio should be kept smaller than 1.3.

31 (a) The reflux ratio is given by $\frac{R}{R+1} = 0.6 \Rightarrow R = 1.5$.

(b) $\frac{x_D}{R+1} = 0.376 \Rightarrow x_D = (0.376)(1+1.5) = 0.94$.

(c) For a bulk liquid concentration of $x = 0.8$, the bulk vapor concentration is $y = (0.6)(0.8) + 0.376 = 0.856$.

Volumetric rate of transport: $k'_x \bar{a}(x - x_i) = k'_y \bar{a}(y_i - y)$

$$\Rightarrow 400(x_i - 0.8) = 170(y_i - 0.856) \dots \dots \text{(i)}$$

x_i and y_i are in equilibrium $\Rightarrow y_i = \frac{2x_i}{1+x_i} \dots \dots \text{(ii)}$

Solving (i) and (ii), $x_i = 0.789, y_i = 0.882$.

7.5

(32) At total reflux, $y_{n+1} = x_n \Rightarrow y_6 = x_5 = \underline{0.4}$.

Since y_5 and x_5 are in equilibrium, $y_5 = \frac{\alpha x_5}{1 + (\alpha - 1)x_5} = \frac{(1.8)(0.4)}{1 + (1.8)(0.4)} = \underline{0.545}$

(33) At minimum reflux, the feed line is vertical and meets the equilibrium line at $y^* = \frac{\alpha x}{1 + (\alpha - 1)x} = \frac{(2)(0.6)}{1 + 0.6} = \underline{0.75}$.

Slope of the rectifying section operating line at the minimum reflux $= \frac{0.98 - 0.75}{0.98 - 0.6} = \frac{0.23}{0.38} = 0.605 = \frac{R_m}{R_m + 1} \Rightarrow R_m = \underline{1.53}$

So, a reflux ratio of 1.5 is not practicable.

$$\begin{aligned} (34) \quad y &= \frac{\alpha x}{1 + (\alpha - 1)x} \Rightarrow \frac{dy}{dx} = \frac{\alpha}{1 + (\alpha - 1)x} - \frac{\alpha(\alpha - 1)x}{[1 + (\alpha - 1)x]^2} \\ &\Rightarrow \left[\frac{dy}{dx} \right]_{x=0} = \alpha \quad \text{and} \quad \left[\frac{dy}{dx} \right]_{x=1} = \frac{1}{\alpha}. \quad \underline{\text{Product} = 1} \end{aligned}$$

$$(35) \quad \text{Given: } \frac{dy}{dx} = \frac{\alpha}{1 + (\alpha - 1)x} - \frac{\alpha(\alpha - 1)x}{[1 + (\alpha - 1)x]^2} = 1, \quad \text{and} \quad \alpha = 2.0.$$

$\Rightarrow x = 0.414$ is the required concentration.

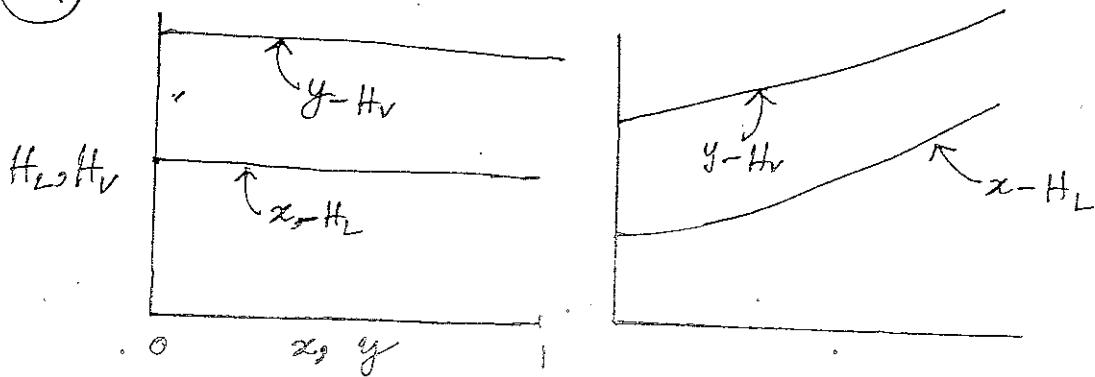
(36) Intersection of the operating lines: $x = 0.5, y = 0.652$.

Feed concentration, $z_F = 0.5 \Rightarrow$ the feed line is a vertical line. The feed is a saturated liquid.

(37) At total reflux, $y_2 = x_1 = \underline{0.95}$.

(38) It is $y=x$ for both the operating lines.

(39)



(40)

Total equilibrium pressure, $P = x_1 \gamma_1 P_1^\infty + x_2 \gamma_2 P_2^\infty$

$$\Rightarrow (0.3)(1.765)(661) + (0.7)(1.195)(289.5) = 592 \text{ mm Hg}$$

$$K_1 = \frac{\gamma_1}{x_1} = \frac{\gamma_1 P_1^\infty}{P} = \frac{(1.765)(661)}{592} = \underline{1.971}$$

$$K_2 = \frac{\gamma_2}{x_2} = \frac{\gamma_2 P_2^\infty}{P} = \frac{(1.195)(289.5)}{592} = \underline{0.584}.$$

Problems

7.1

Vapor pressure equations (Table 7.2):

$$n\text{-Hexane: } P_A^{\circ} = 15.9155 - \frac{2738.42}{\theta + 226.2} ; \theta \text{ in } ^\circ\text{C}; \text{boiling point} = 68.8^\circ\text{C}$$

$$n\text{-Octane: } P_B^{\circ} = 15.9635 - \frac{3128.75}{\theta + 209.35} ; P_A^{\circ} \text{ and } P_B^{\circ} \text{ in mm Hg.}$$

Boiling pt = 125.7 °C

(a) Construction of $T-x-y$ and $x-y^*$ diagrams.

Assume ideal solution behavior. The bubble point remains within T_A and T_B ($T_A \leq T \leq T_B$). Select a set of values of θ (or θ) calculate P_A° and P_B° and then calculate x and y^* .

Sample calculation: $\theta = 90^\circ\text{C} \Rightarrow P_A^{\circ} = 1415, P_B^{\circ} = 252 \text{ mm Hg}$

$$\text{Total pressure} = 760 \text{ mm Hg} = P_A^{\circ}x_A + P_B^{\circ}(1-x_A) = 1415x_A + 252(1-x_A)$$

$$\Rightarrow x_A = 0.437; x_B = 1-x_A = 0.563.$$

$$y_A = \frac{P_A^{\circ}x_A}{P} = \frac{(1415)(0.437)}{760} = 0.813; y_B = 0.187$$

Calculations are repeated for a few other temperatures.

Calculated results and plots (with tie lines)

θ	75 °C	80	85	90	95	100	105	110	115	120
x	0.794	0.656	0.538	0.437	0.350	0.274	0.207	0.148	0.0953	0.048
y^*	0.961	0.921	0.872	0.814	0.745	0.664	0.57	0.460	0.334	0.189

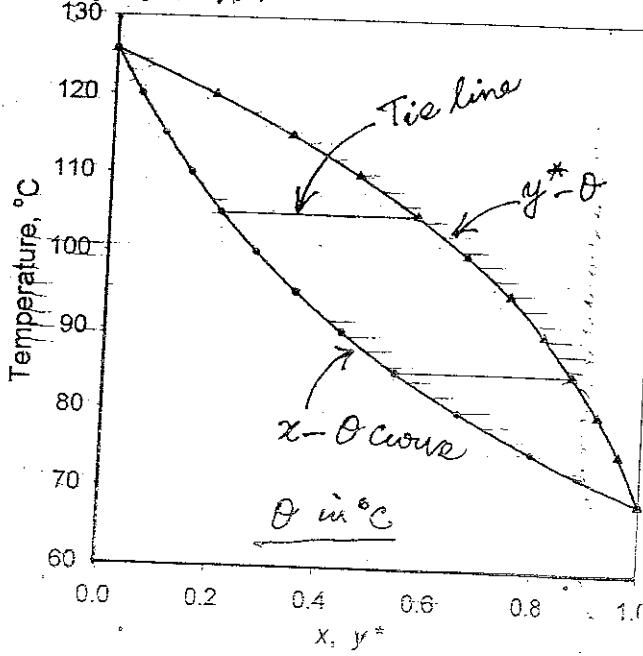


Fig Prob 7.1: $T-x-y^*$ plots

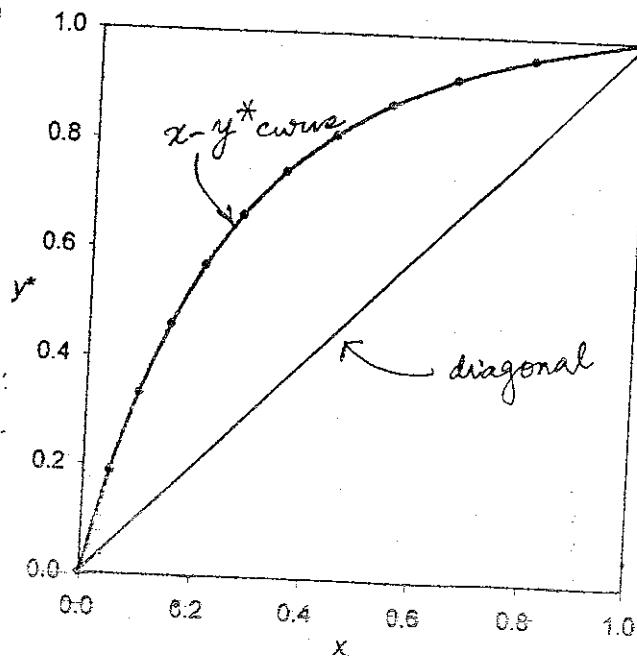


Fig Prob 7.1: $x-y^*$ plot

(78)

(b) A liquid mixture containing 10 mole% n-hexane:

First calculate the bubble point [by trial or by interpolation,
of the data calculated in part (a)]

Try $\theta = 115.5^\circ\text{C}$. $P_A^{\infty} = 2638$ and $P_B^{\infty} = 554 \text{ mm Hg}$

$$\text{Total pressure, } P = x_A P_A^{\infty} + (1-x_A) P_B^{\infty} = (0.1)(2638) + (0.9)(554) \\ = 762 \text{ mm Hg} \approx 1 \text{ atm.}$$

$$\text{Relative volatility, } \alpha_{AB} = \frac{P_A^{\infty}}{P_B^{\infty}} = 2638/554 = \underline{4.76}.$$

For a liquid containing 90 mole% n-hexane, try

$\theta = 71.7^\circ\text{C}$ as the bubble point $\Rightarrow P_A^{\infty} = 831.4$; $P_B^{\infty} = \underline{128 \text{ mm Hg}}$

$$P = (831.4)(0.9) + (128)(0.1) = \underline{761 \text{ mm Hg}}, \text{ acceptable.}$$

$$\alpha_{AB} = 831.4/128 = \underline{6.5}.$$

The relative volatility varies \Rightarrow The solution is not totally ideal.

(c) Initial vapor composition of a liquid ($x_A=0.2$) at its bubble point can be calculated following the procedure given above.

(i) Take $P = 1 \text{ atm} = 760 \text{ mm Hg}$.

Trial temperature, $\theta = 105.5^\circ\text{C} \Rightarrow P_A^{\infty} = 2121$, $P_B^{\infty} = 420.7 \text{ mm}$

$$P = (2121)(0.2) + (420.7)(0.8) = 761 \text{ mm Hg} \text{ (acceptable)}$$

$$y_A = \frac{(2121)(0.2)}{761} = \underline{0.442}.$$

(ii) Take $P = 2 \text{ atm} = 1520 \text{ mm Hg}$.

Trial bubble point, $\theta = 132.5^\circ\text{C} \Rightarrow P_A^{\infty} = 3943$, $P_B^{\infty} = 920 \text{ mm Hg}$

$$P = (3943)(0.2) + (920)(0.8) = 1524 \text{ mm Hg} \approx \underline{2 \text{ atm}}$$

$$y_A = \frac{(3943)(0.2)}{1524} = \underline{0.517}$$

(79)

(d) Consider a condensate of composition $x_A = 0.517$
 [the same as that of the vapor in (ii)].

Trial bubble point, $\theta = 111.7^\circ\text{C}$. $P_A^{12} = 2468$, $P_B^{12} = 509 \text{ mm Hg}$

$$\text{Total pressure} = (2468)(0.517) + (509)(0.483) = 1522 \text{ mm Hg} \approx 2 \text{ atm.}$$

Composition of the initial vapor,

$$y_A^* = \frac{(2468)(0.517)}{1522} = 0.838 \quad (\text{See Note 2 below})$$

Note 1. If we consider a liquid of composition x_A and temperature $\theta^\circ\text{C}$, the total pressure exerted is

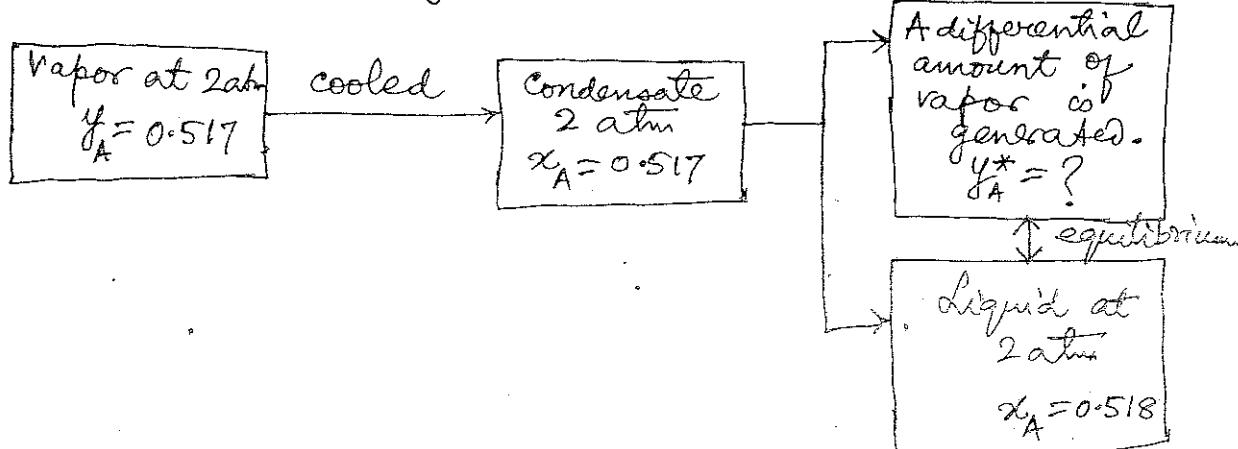
$$\begin{aligned} P &= x_A P_A^{12} + (1-x_A) P_B^{12} \\ &= x_A \cdot \exp \left[15.9155 - \frac{2738.42}{\theta + 226.2} \right] + (1-x_A) \exp \left[15.9635 - \frac{3128.75}{\theta + 209.85} \right] \end{aligned}$$

The above equation has three unknowns — P , x_A and θ .

If the temperature θ and either P or x_A are given, the solution of the above equation for the unknown is straightforward. However, if P and x_A are given, the equation can be solved for θ by trial or by using any numerical method such as Newton-Raphson method.

Note 2:

The condensation and vaporization processes in part (d) may be schematically represented as follows.



(7.10)

- (7.2) Follow the procedures given in Section 7.1.6.

Calculation of the bubble point:

Average molecular weight of the mixture,

$$M_{av} = M_1 x_1 + M_2 x_2 + M_3 x_3 = [\text{methanol-1; ethanol-2; } n\text{-propanol-3}]$$

$$(32)(0.3) + (46)(0.3) + (60)(0.4) = 47.4$$

This is close to the molecular weight of ethanol. Select ethanol as the 'reference compound'. Boiling point of ethanol at 1 atm = 78.3°C = trial bubble point.

Calculate the relative volatilities of the other two compounds at the trial temperature.

$$\text{Vapor pressures at } 78.3^\circ\text{C} : P_1^{\infty} = 1260.2 ; P_2^{\infty} = 760 ; P_3^{\infty} = 349.3 \text{ mm Hg}$$

$$\alpha_{12} = 1260.2 / 760 = 1.658 ; \alpha_{32} = 0.46.$$

$$\text{Eq. (7.14b)} \rightarrow K_r \sum_i \alpha_{i2} x_i = K_r [(1.658)(0.3) + (1)(0.3) + (0.46)(0.4)] = 1.$$

$$\Rightarrow K_r \cdot (0.9814) = 1 \Rightarrow K_r = 1.019.$$

$$\text{Now put } K_r = P_2^{\infty} / P \Rightarrow P_2^{\infty} = (1.019)(760) \text{ mm Hg} = 774.4 \text{ mm Hg}$$

Boiling point of the reference compound (ethanol) at 774.4 mm Hg pressure = 78.8°C , which is the revised estimate of the bubble point of the mixture.

$$\text{At } 78.8^\circ\text{C}, P_1^{\infty} = 1283.2, P_3^{\infty} = 357 \text{ mm Hg}.$$

$$K_1 = P_1^{\infty} / P = 1283.2 / 760 = 1.657 ; K_2 = 1.019 ; K_3 = 0.47$$

$$\sum K_i x_i = (1.657)(0.3) + (1.019)(0.3) + (0.47)(0.4) = 0.9908.$$

This is a bit too low. A third cycle of calculation is needed.

$$\text{Take } K_r = K_2 / 0.9908 = 1.019 / 0.9908 = 1.0285.$$

$$P_2^{\infty} = (1.0285)(760) = 781.6 \text{ mm Hg} ; \text{ corresponding temperature} = \underline{79^\circ\text{C}}$$

$$P_1^{\infty} = 1292 ; P_3^{\infty} = 360 \text{ mm Hg. } K_1 = 1292 / 760 = 1.7 ; K_3 = 0.474.$$

$$\sum K_i x_i = (1.7)(0.3) + (1.0285)(0.3) + (0.474)(0.4) = 1.008.$$

24

This value is pretty close to 1. However, a still better estimate of the bubble point of the mixture may be obtained by interpolation.

$$\text{Bubble point} = 79 - \frac{1.008 - 1}{1.008 - 0.9908} \cdot (79 - 78.8) = \underline{\underline{78.9^\circ\text{C}}}$$

Dew point of a vapor of composition $y_1 = 0.3$, $y_2 = 0.3$ and $y_3 = 0.4$ at 1.5 atm total pressure.

Ethanol is chosen as the reference compound since its molecular weight is nearest to the average molecular weight.

Boiling point of ethanol at 1.5 atm (1140 mm Hg) total pressure = 86.95°C which is taken as the trial 'dew point'.

At 86.95°C , $P_1^v = 1702$ and $P_3^v = 503.9$ mm Hg.

$$\alpha_{12} = 1702/1140 = 1.493; \alpha_{32} = 503.9/1140 = 0.442.$$

$$\text{Eq. (7.15b)} \rightarrow \frac{1}{K_2} \left(\frac{0.3}{1.493} + \frac{0.3}{1.0} + \frac{0.4}{0.442} \right) = 1 \Rightarrow K_2 = 1.406 = \frac{P_2^v}{P}$$

$$P_2^v = (1.406)(1140) = \underline{\underline{1602.8 \text{ mm Hg}}} \text{ Corresponding temperature} = 98.5^\circ\text{C}.$$

This is the second approximation to the dew point.

$$\text{At } 98.5^\circ\text{C}, P_1^v = 2479.3, P_3^v = 796 \text{ mm Hg.}$$

Calculate the K-values at 98.5°C .

$$K_1 = 2479.3/1140 = 2.175; K_2 = 1602.8/1140 = 1.405; K_3 = 0.698.$$

$$\sum_i (y_i/K_i) = 0.3/2.175 + 0.3/1.405 + 0.4/0.698 = \underline{\underline{0.9245}}.$$

This is much smaller than 1. Try next cycle of calculation.

$$\alpha_{12} = 2479.3/1602.8 = 1.547; \alpha_{32} = 796/1602.8 = 0.4969.$$

$$K_2 = K_2 = \sum_i \frac{y_i}{\alpha_{i2}} = \frac{0.3}{1.547} + \frac{0.3}{1.0} + \frac{0.4}{0.4969} = \underline{\underline{1.3}}.$$

$$K_2 = P_2^v/P \Rightarrow P_2^v = (1.3)(1140) = 1482 \text{ mm Hg} \Rightarrow \text{B.P.} = \underline{\underline{94.2^\circ\text{C}}}.$$

Take this as the next revised estimate of the dew point.

(7.13)

At 94.2°C , $P_1^v = 2162$ and $P_3^v = 674 \text{ mm Hg}$.

$$K_1 = 2162/1140 = 1.896; K_2 = 1482/1140 = 1.3; K_3 = 674/1140 = 0.591.$$

$$\sum_i \frac{y_i}{K_i} = \frac{0.3}{1.896} + \frac{0.3}{1.3} + \frac{0.4}{0.591} = 1.066 \rightarrow \underline{\text{not acceptable.}}$$

Let us do another cycle of calculation.

$$\alpha_{12} = 2162/1482 = 1.46; \alpha_{32} = 674/1482 = 0.455$$

$$K_2 = \frac{0.3}{1.46} + \frac{0.3}{1} + \frac{0.4}{0.455} = 1.3846.$$

$$P_2^v = (1.3846)(1140) = 1578.4; \text{ corresponding boiling point} = \underline{96^\circ\text{C}}$$

$$\text{At } 96^\circ\text{C}, P_1^v = \underline{2290}; P_3^v = \underline{723 \text{ mm Hg.}}$$

$$K_1 = 2290/1140 = 2.009; K_2 = 1578.4/1140 = 1.3846; K_3 = \frac{723}{1140} = 0.634.$$

$$\sum_i \frac{y_i}{K_i} = \frac{0.3}{2.009} + \frac{0.3}{1.3846} + \frac{0.4}{0.634} = 0.9969 \simeq 1$$

This value is reasonable. Required dewpoint = 96°C.

(7.4) Use Eq(7.30) to determine the distillate composition for different fractional vaporization. Given: $\alpha = 2.2$; $z_F = 0.4$

$$-\frac{W}{D} = \frac{x_D - z_F}{x_W - z_F} \Rightarrow \frac{W}{D}(0.4 - x_W) + 0.4 = x_D = \frac{2.2x_W}{1 + 1.2x_W}$$

Solve the above equation for different values of W/D

Fraction vaporized, $\frac{D}{W+D}$	$\frac{W}{D}$	x_W	x_D
0.1	9.0	0.3806	0.575
0.2	4.0	0.361	0.5547
0.4	1.5	0.3243	0.5136
0.6	0.6667	0.29	0.4733
0.8	0.25	0.259	0.435
0.9	0.2454	0.2454	0.47

The lowest bottom composition occurs when only a differential amount of liquid remains — i.e. $x_D \approx 0.4$. Corresponding $x_W = 0.232 \Rightarrow$ the bottom composition cannot be 10 mol% ethanol.

(7.5) Two-stage flash distillation

Stage 1: 60 mole% of the feed is vaporized $\Rightarrow \frac{W_1}{D_1} = \frac{0.4}{0.6} = \frac{2}{3}$.

$$Eq(7.30) \rightarrow \frac{2}{3}(0.4 - x_{w_1}) + 0.4 = \frac{2 \cdot 2 x_{w_1}}{1 + 1.2 x_{w_1}} \quad z_{F_1} = 0.4 \\ \alpha = 2.2$$

$$\Rightarrow x_{w_1} = 0.29 = \text{liquid composition}; \quad x_{D_1} = 0.4733.$$

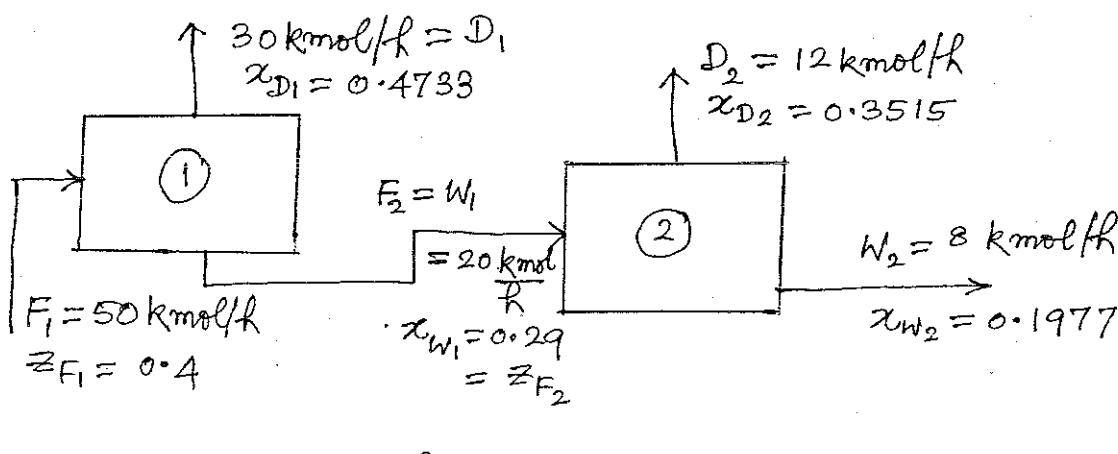
$$\text{Feed, } F_1 = 50 \text{ kmol/h; distillate, } D_1 = (50)(0.6) = 30 \text{ kmol/h}$$

$$W_1 = 20 \text{ kmol/h.}$$

Stage 2: 60 mole% of the bottom liquid ($z_{F_2} = x_{w_1} = 0.29$) is vaporized in the second flash drum.

$$\frac{2}{3}(0.29 - x_{w_2}) + 0.29 = \frac{2 \cdot 2 x_{w_2}}{1 + 1.2 x_{w_2}} \Rightarrow x_{w_2} = 0.1977; \quad x_{D_2} = 0.3515.$$

$$D_2 = (20)(0.6) = 12 \text{ kmol/h; } W_2 = 8 \text{ kmol/h.}$$



(7.6) Since the flowrates and concentrations are given in kg and mass%, respectively, it is convenient to use the equilibrium data after conversion to mass fractions, w_L and w_V , of acetone.

Molecular weight: acetone = 58; phenol = 94.

Sample calculations:

$$x = 0.01 \Rightarrow w_L = \frac{(0.01)(58)}{(0.01)(58) + (0.99)(94)} = 0.0062$$

$$y = 0.67 \Rightarrow w_V = \frac{(0.67)(58)}{(0.67)(58) + (0.33)(94)} = 0.556$$

Calculated equilibrium data

x	0.01	0.04	0.1	0.2	0.3	0.4	0.5
y	0.67	0.776	0.852	0.91	0.94	0.962	0.98
w_L	0.0062	0.025	0.064	0.1336	0.209	0.291	0.3816
w_V	0.556	0.681	0.78	0.862	0.906	0.94	0.968

The equilibrium data are plotted. Graphical solution
to be done

First Stage flash

D_1 = mass of distillate (kg); w_1 = mass of bottom product;

z_{F_1} = feed concentration (mass fraction)

w_{D_1} , w_{w_1} = distillate and bottom concentration, mass fraction.

$F_1 = 3000 \text{ kg/h}$, 50% flashed in stage 1

$$\Rightarrow D_1 = 1500 \text{ kg/h}; w_1 = 1500 \text{ kg/h}; z_{F_1} = 0.4 \text{ mass fraction.}$$

$$-\frac{w_1}{D_1} = \frac{w_{D_1} - z_{F_1}}{w_{w_1} - z_{F_1}} \Rightarrow -\frac{1500}{1500} = \frac{w_{D_1} - 0.4}{w_{w_1} - 0.4} = -1$$

w_{D_1} and w_{w_1} are in equilibrium — to be determined graphically.

Locate the point $(z_F, z_F) \rightarrow (0.4, 0.4)$ on the w_L-w_V diagram. Draw a line of slope $-w_1/D_1 = -1$, which is the first stage operating line that should meet the equilibrium line at $(w_{w_1}, w_{D_1}) \rightarrow (0.0502, 0.75)$.

Second Stage flash

$$F'_2 = w_1 = 1500 \text{ kg/h}; z_{F_2} = w_{w_1} = 0.0502.$$

Locate the point $(z_{F_2}, z_{F_2}) \rightarrow (0.0502, 0.0502)$. The bottom product from this stage must not contain more than 1% acetone, i.e. $w_{w_2} = 0.01$.

Locate the point on the equilibrium line where

Prob 7.6 contd...

7.15

$w_{w_2} = 0.01$. Corresponding equilibrium vapor composition,
 $w_{D_2} = 0.593$ (from the diagram).

The fraction vaporized can be determined algebraically or graphically.

$$-\frac{W_2}{D_2} = \frac{0.593 - 0.0502}{0.01 - 0.0502}$$

$$\text{and } W_2 + D_2 = 1500$$

$$\Rightarrow W_2 = 1396.6 \text{ kg/h}; \\ D_2 = 103.4 \text{ kg/h}$$

Fraction of the second stage feed to be vaporized

$$= \frac{D_2}{W_2 + D_2} = \frac{103.4}{1500} = 6.9\%$$

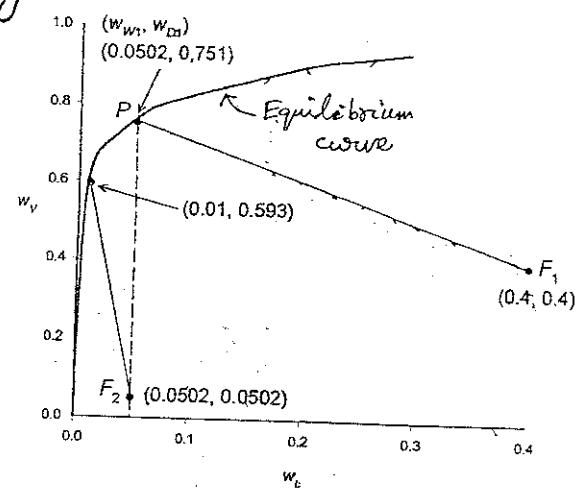


Figure Prob 7.6

(7.7)

Feed, $F = 1 \text{ kmol/h}$; $Z_F = 0.5$; $D = 0.5 \text{ kmol/h}$; $W = 0.5 \frac{\text{kmol}}{\text{h}}$.

Material balance: $F \cdot Z_F = D x_D + W x_W$] solve to get

$$\text{Equilibrium relation: } \frac{x_D(1-x_D)}{x_W/(1-x_W)} = \alpha = 3.0 \quad x_D = 0.634 \\ x_W = 0.366$$

Products: Distillate, $D = 0.5$, $x_D = 0.634$, vapor at 80°C .

Bottom product, $W = 0.5$, $x_W = 0.366$, liquid at 80°C .

Feed temperature = 40°C ; ideal solution $\Rightarrow \Delta H_s = 0$

Take the reference temperature 40°C . (Heat of solution)

Enthalpy of the feed, $H_F = 0$

Enthalpy of the vapor product (use Eq 7.19) at 80°C

$$H_V = (0.634)[(80-40)(20) + 1250] + (1-0.634)[(80-40)(30) + 1700] \\ = 2361.1 \text{ kcal/kmol}$$

$$\text{Liquid} \rightarrow H_L = (0.366)(80-40)(20) + (1-0.366)(80-40)(30) \\ = 1053.6 \text{ kcal/kmol}$$

$$\text{Heat to be supplied, } Q = 0.5 H_V + 0.5 H_L - H_F$$

$$= (0.5)(2361.1) + (0.5)(1053.6) - 0 = \underline{\underline{1707.3 \text{ kcal/h}}}$$

(7-16)

(7.8) Given: $H_L = 9,000 + 1,000x$; $H_v = 35,000 + 8,000y$ kJ/kmol

(a) Heat of condensation of saturated pure vapor of A

$$\text{Put } x=1, y=1 \Rightarrow (\Delta H_v)_A = H_v - H_L$$

$$= [35,000] + 8,000(1) - [9,000] + 1,000(1) = \underline{\underline{33,000 \text{ kJ/kmol}}}$$

For pure vapor of B ($x=0, y=0$), $(\Delta H_v)_B = \underline{\underline{26,000 \text{ kJ/kmol}}}$

(b) Consider a saturated liquid mixture with $x=0.3$.

Enthalpy of the saturated liquid = $9,000 + (1,000)(0.3) = 9,300 \text{ kJ/kmol}$

Actual enthalpy = $8,500 \text{ kJ/kmol} \Rightarrow$ the liquid is subcooled.

Now consider a saturated vapor with $y=0.5$.

Enthalpy = $35,000 + (8,000)(0.5) = 39,000 \text{ kJ/kmol}$.

Actual enthalpy of the vapor = $40,000 \text{ kJ/kmol}$

\Rightarrow The vapor is superheated.

(7.9) Use Eq(7.34) $\rightarrow y_{D_i} = \frac{z_{F_i} (1+W/D)}{(1+W/K_i D)}$

Given: $z_{F_1} = 0.38$, $z_{F_2} = 0.225$, $z_{F_3} = 0.395$, $W/D = 1$.

$$y_{D_1} = \frac{(0.38)(1+1)}{1+(1/1.42)} = \underline{\underline{0.446}}. \text{ Similarly, } y_{D_2} = \underline{\underline{0.208}}; y_{D_3} = \underline{\underline{0.346}}$$

Liquid composition: $x_{W_1} = y_{D_1}/K_1 = 0.446/1.42 = \underline{\underline{0.314}}$.

$$x_{W_1} = 0.208/0.86 = 0.242; x_{W_3} = \underline{\underline{0.444}}$$

7.10

Bubble point of a homogeneous mixture of A and B at 1.5 atm total pressure (assume ideal solution):

$$\text{Total pressure, } P = x_A P_A^{\circ} + x_B P_B^{\circ} = 0.5 (P_A^{\circ} + P_B^{\circ}) \\ = 0.5 (1.75 P_B^{\circ} + P_B^{\circ}) = 1.375 P_B^{\circ} = 1.5 \text{ bar} = \frac{(1.5)(760)}{1.013} = 1125.4 \text{ mm Hg}$$

$$\Rightarrow P_B^{\circ} = 1125.4 / 1.375 = 818.45 \text{ mm Hg.}$$

$$\text{Given: } \ln P_B^{\circ} (\text{mm Hg}) = 14.243 - 2570 / (\theta, {}^{\circ}\text{C} + 232.5)$$

For $P_B^{\circ} = 1125.4 \text{ mm Hg}$, $\theta = 108.8 {}^{\circ}\text{C}$ = required bubble point.

Bubble point of a heterogeneous mixture of the above solution and water at 1.5 bar total pressure:

$$P = 1125.4 \text{ mm Hg} = P_w^{\circ} + 1.375 P_B^{\circ}; \ln P_w^{\circ} = 18.5882 - \frac{3984.92}{\theta {}^{\circ}\text{C} + 233.43}$$

Solving the above equation by trial, $\theta = 95.5 {}^{\circ}\text{C}$

7.11

$$(a) \text{ Use Eq (7.51b)} \rightarrow \ln \frac{F x_F}{W x_W} = \alpha \ln \frac{F(1-x_F)}{W(1-x_W)}$$

75 mole% of the benzene leaves in the vapor product

$$\Rightarrow D \cdot x_D = (0.75) F x_F \Rightarrow W x_W = (0.25) \cdot F x_F \Rightarrow \frac{F x_F}{W x_W} = 4.0$$

$$\ln 4 = 2.51 \ln \left[\frac{F}{W} \cdot \frac{(1-x_F)}{(1-x_W)} \right] = 2.51 \ln \left[\frac{4x_W}{x_F} \cdot \frac{(1-x_F)}{(1-x_W)} \right] \text{ Since } \alpha = 2.51$$

$$\text{Put } x_F = 0.65 \text{ to get } x_W = 0.447.$$

$$\frac{F}{W} = 4 \cdot \frac{x_W}{x_F} = \frac{(4)(0.447)}{0.65} = 2.751. \quad F = 10 \text{ kmol}$$

$$\Rightarrow W = 3.635 \text{ kmol; } D = 6.365 = \text{moles of distillate produced.}$$

(b) The stillpot contains 35 mole% benzene $\Rightarrow x_W = 0.35$.

Put the values of the different quantities in Eq (7.51a).

$$\ln \frac{F}{W} = \frac{1}{\alpha-1} \ln \frac{x_F(1-x_W)}{x_W(1-x_F)} + \ln \frac{1-x_W}{1-x_F}$$

$$\Rightarrow \ln \frac{10}{W} = \frac{1.0}{2.51-1} \ln \frac{0.65(1-0.35)}{0.35(1-0.65)} + \ln \frac{1-0.35}{1-0.65} \Rightarrow W = 2.359 \text{ kmol}$$

$$D = 7.641 \text{ kmol}$$

Prob 7.11 contd.

(7.10)

(c) 50 mole% of the feed is vaporized $\Rightarrow F/W = 2.0$.

Put $F = 10$, $W = 5$, $x_F = 0.65$ and $\alpha = 2.51$ in Eq(7.51a)
to get $x_w = 0.516$, and $D = 5 \text{ kmol}$.

(d) Again use Eq(7.51b) $\rightarrow \ln \frac{F z_F}{W x_w} = \alpha \ln \frac{F (1-x_F)}{W (1-x_w)}$

The distillate has 75 mole% benzene.

Material balance $\rightarrow W x_w = F x_F - D x_D = (10)(0.65) - 0.75D$

$$\Rightarrow \ln \frac{(10)(0.65)}{6.5 - 0.75D} = 2.51 \ln \frac{10(1-0.65)}{W - (6.5 - 0.75D)} = 2.51 \ln \frac{3.5}{(10-D) - (6.5)}$$

Solve to get $D = 7.29 \text{ kmol}$; $W = 2.71 \text{ kmol}$; $x_w = 0.381$.

(7.12)

$F = 70 \text{ kmol/h}$; $z_F = 0.85$.

(a) 70 mole% of the vapor is condensed.

$$W = (70)(0.7) = 49 \text{ kmol/h}; D = 21 \text{ kmol/h}$$

Use Eq(7.30) to calculate x_D and x_w which are at equilibrium.

$$-\frac{W}{D} = \frac{x_D - z_F}{z_F - x_w} \Rightarrow \frac{x_D - 0.85}{0.85 - x_w} = -\frac{49}{21} = -2.33$$

Draw the equilibrium line. Locate the point $(z_F, z_F) - E(0.85, 0.85)$ on the $x-y$ plane and draw a line of slope -2.33 to meet the equilibrium line at the point P. The coordinate of P is $(0.815, 0.93)$ from the plot $\Rightarrow x_w = 0.815$; $x_D = 0.93$. Composition of the uncondensed vapor, $x_D = 0.93$.

(b) The uncondensed vapor leaving the partial condenser has 91.5 mole% methanol $\Rightarrow x_D = 0.915$.

(7.12) contd...

7.17

Locate the point P' for $x_0 = 0.915$. Join FP' . Slope of the line FP' is $-0.929 = -W/D$.

$$W+D = 70, \text{ and } W = 0.929 D.$$

$$\text{Solving } W = 36.3 \text{ kmol/h and } D = 33.7 \text{ kmol/h.}$$

$$\text{Condensate rate} = 36.3 \text{ kmol/h}$$

Note Equilibrium data from Example 7.15 have been used.

x	0.7	0.8	0.9	0.95	1.0
y	0.87	0.915	0.958	0.979	1.0

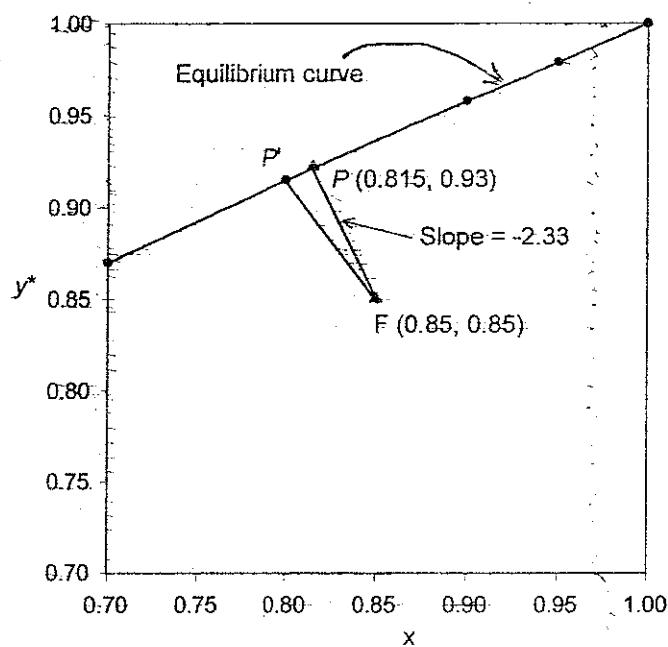


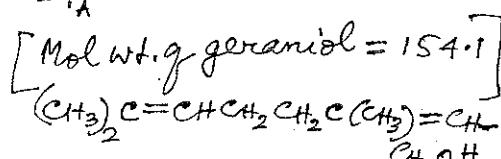
Fig. Prob-7.12

(7.13)

$$\text{Use Eq. (7.42)} \rightarrow m_A = m_B \cdot \frac{EP_A^{\nu}}{P - EP_A^{\nu}}$$

$$m_A = (2 \text{ kg})(0.95) / 154.1 = 0.01233 \quad [\text{Mol wt. g. geraniol} = 154.1]$$

$$P_A^{\nu} = 8.65 \text{ mm Hg} = 0.0115 \text{ bar}$$



$$\text{Vaporization efficiency, } E = 0.9. \quad P - EP_A^{\nu} = P_B^{\nu} = 1.34 \text{ bar}$$

$$\text{Moles of steam required, } m_B = (0.01233) \cdot \frac{1.34}{(0.90)(0.0115)} = 1.596 \text{ kmol}$$

$$\text{Steam rate} = 50 \text{ kg/h} \Rightarrow \text{time required} = \frac{28.73}{50} \text{ h} = 34.5 \text{ min}$$

$$= \frac{28.73}{50} \text{ h} = \underline{\underline{34.5 \text{ min}}}$$

(7.14) Initial moles of the essential oil = $(20)(0.4)/170 = 0.04706$ 7.20
 Moles of impurity (mol wt = 280), $m_c = (20)(0.6)/280 = 0.04255 \text{ kmol}$
 50% of the oil is recovered; moles oil left = $0.04706/2 = 0.02353$

Vapor pressure of water at 108°C (see Prob 7.13), $\frac{m_{Af}}{m_{Ai}} = \frac{P_A^v}{P_B^v}$

$$\frac{P_B^v}{P_A^v} = 1.34 \text{ bar}; \quad \frac{P_A^v}{P_A} = 6.5 \text{ mm Hg} = 8.66 \times 10^{-3} \text{ bar}$$

Vaporization efficiency, $E = 0.85$.

$$P = EP_A^v + P_B^v = 1.347 \text{ bar.}$$

Use Eq. (7.45) to calculate the distillation time.

$$\begin{aligned} \dot{m}_B t &= \left(\frac{P}{EP_A^v} - 1 \right) (m_{Ai} - m_{Af}) + \frac{P m_c}{EP_A^v} \cdot \ln \frac{m_{Ai}}{m_{Af}} \\ &= \left[\frac{1.347}{(0.85)(8.66 \times 10^{-3})} - 1 \right] [0.04706 - 0.02353] \\ &\quad + \frac{(1.347)(0.04255)}{(0.85)(8.66 \times 10^{-3})} \ln 2 = 9.68 \text{ kmol} = 174.2 \text{ kg} \end{aligned}$$

If the steam rate is 50 kg/h , time = $\frac{174.2}{50} = 3.5 \text{ h}$

Rate of distillation when 50% of the oil is vaporized:

Use Eq. (7.44).

$$-\frac{d m_A}{dt} = \dot{m}_B \cdot \frac{E x_A P_A^v}{P - E x_A P_A^v}; \quad x_A = \frac{0.02353}{0.02353 + 0.04255} = 0.356$$

$$\dot{m}_B = 50 \text{ kg/h} = 2.778 \text{ kmol/h} \quad (\text{for } \dot{m}_B = 50 \text{ kg/h mentioned in the question})$$

$$\begin{aligned} -\frac{d m_A}{dt} &= (2.778) \cdot \frac{(0.85)(0.356)(8.66 \times 10^{-3})}{1.34} \\ &= 5.432 \times 10^{-3} \text{ kmol/h} = (5.432 \times 10^{-3})(170) \\ &= 0.923 \text{ kg/h} \end{aligned}$$

7.15

$$\text{Equilibrium relation: } \alpha = 2.8 \Rightarrow y^* = \frac{2.8x}{1+1.8x} \quad \text{--- (i)}$$

7.21

x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
y^*	0	0.237	0.412	0.545	0.651	0.737	0.808	0.867	0.918	0.962	1.0

Given: $z_F = 0.45$; the feed is 50% vapor $\Rightarrow q = 0.5$; $x_D = 0.96$,
slope of the feed line $= \frac{q}{q-1} = \frac{0.5}{0.5-1} = -1$. $x_w = 0.95$

Equation of the feed line:

It passes through $(z_F, z_F) \rightarrow (0.45, 0.45)$ and has a slope $= -1$.

$$\text{Equation} \rightarrow y = -x + c \Rightarrow 0.45 = -0.45 + c \Rightarrow c = 0.9$$

$$\text{Feed line} \rightarrow y = 0.9 - x \quad \text{--- (ii)}$$

Determination of the minimum reflux ratio:

It can be done graphically or algebraically since the equilibrium data are given in the form of an equation.

Algebraic approach: The intersection of the feed line and the equilibrium curve is obtained by solving Eqs. (i) and (ii).

$$y = \frac{2.8x}{1+1.8x} = 0.9 - x \Rightarrow x = 0.325, y = 0.574.$$

The operating line for minimum reflux should pass through $(0.325, 0.574)$ and $(x_D, z_D) \rightarrow (0.96, 0.95)$.

$$\text{Equation} \rightarrow \frac{y - 0.574}{0.95 - 0.574} = \frac{x - 0.325}{0.96 - 0.325} \Rightarrow y = 0.608x + 0.376$$

$$\text{Slope of this line is } 0.608 = R_m/R_m + 1 \Rightarrow R_m = 1.551$$

The minimum reflux ratio is $R_m = 1.551$

Determination of the number of real trays for a reflux ratio, $R = 1.5 R_m = (1.5)(1.551) = 2.326$.

7.15 contd...

(for the rectifying section)

(7.22)

Actual operating line, $y = \frac{R}{R+1}x + \frac{x_D}{R+1}$, Eq. (7.73)

$$\Rightarrow y = \frac{2.326}{1+2.326} \cdot x + \frac{0.96}{1+2.326} = 0.7x + 0.289.$$

Draw the equilibrium line. Locate the point $(x_D, x_D) \rightarrow D(0.96, 0.96)$ on the diagonal. Draw the rectifying section operating line through $(0.96, 0.96)$ with an intercept of 0.289 on the Y-axis; draw the feed line. Locate the point $W(x_W, x_W) \rightarrow (0.95, 0.95)$ and draw the stripping section operating line through W and the intersection of the feed line and the rectifying section operating line.

Draw steps between the equilibrium line and the operating lines. Number of ideal stages = 10

Feed plate \rightarrow 6th

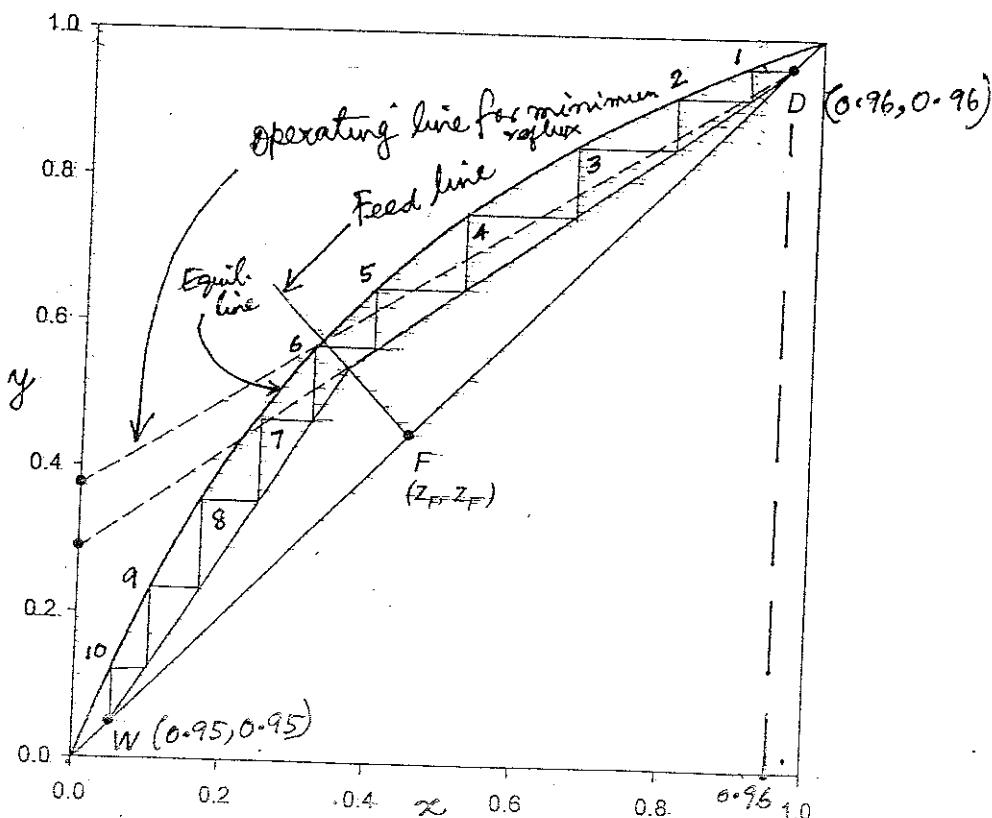


Fig. Prob 7.15. Construction of stages.

(7.16) The rate of product withdrawal (D and W) will be necessary in the calculations. These can be obtained from material balance.

$$F = 40 \text{ kmol/h}; z_F = 0.55; x_0 = 0.975.$$

2% of the dimethylamine leaves at the bottom

$$\Rightarrow Wx_w = (0.02) \cdot Fz_F \Rightarrow Wx_w = (0.02)(40)(0.55) = 0.44$$

Dimethylamine balance: $Fz_F = Dx_D + Wx_w$.

$$\Rightarrow (40)(0.55) = D(0.975) + 0.44 \Rightarrow D = 22.1 \text{ kmol/h}$$

$$x_w = \frac{Wx_w}{W} = \frac{0.44}{17.9} = \underline{\underline{0.0246}} \quad W = 17.9 \text{ kmol/h}$$

(a) Minimum reflux ratio

Locate the points $F(z_F, z_F) \rightarrow (0.55, 0.55)$; $D(x_D, x_0) \rightarrow (0.975, 0.975)$ on the $x-y$ plane; draw the equilibrium curve. Since the feed is a saturated liquid, the feed line is a vertical line through F . The feed line intersects the equilibrium curve at the point P . Join DP and extend it to meet the y -axis at $E_1 \rightarrow (0, 0.5)$.

$$\Rightarrow \frac{x_D}{R_m + 1} = 0.5 \Rightarrow \frac{0.975}{R_m + 1} = 0.5 \Rightarrow R_m = 0.95$$

= minimum reflux

(b) Actual reflux ratio; $R = 1.4 R_m = (1.4)(0.95) = 1.33$.

$$\frac{x_D}{R+1} = \frac{0.975}{1.33+1} = 0.418. \text{ Locate the point } E_2 \rightarrow (0, 0.418)$$

Join DE_2 that meets the feed line at P' . Locate $W(x_w, x_w) \rightarrow (0.0246, 0.0246)$ and join $P'W$.

DP' is the rectifying section operating line and $P'W$ is the stripping section operating line. Conduct stages between the equilibrium curve and the operating lines.

7.24

Prob 7.16 contd...

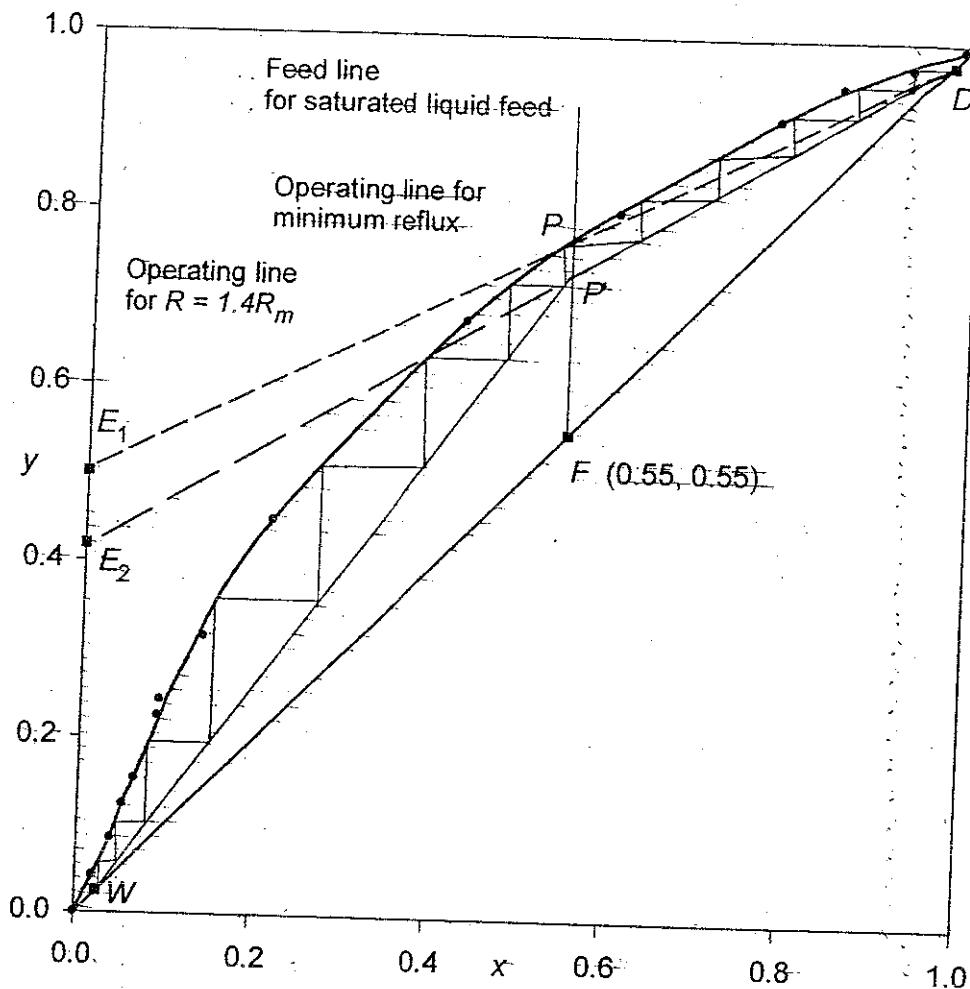


Fig. Prob 7.16. Graphical construction of stages.

The number of theoretical stages = 13.5; feed plate \rightarrow 6

(c) Slopes of the operating lines

$$\text{Rectifying section: Slope} = \frac{R}{R+1} = \frac{1.33}{2.33} = \underline{0.571}$$

$$\text{Stripping section: Slope} = \bar{L}/\bar{V}$$

$\bar{V} = V = D(1+R)$, since the feed is a saturated liquid.

$$\bar{V} = (22.1)(1+1.33) = \underline{51.5} \text{ kmol/h}$$

$$\bar{L} = L + F = R \cdot D + F = (1.33)(22.1) + 40 = \underline{69.4} \text{ kmol/h}$$

$$\text{Slope} = \bar{L}/\bar{V} = 69.4/51.5 = \underline{1.347}$$

7-25

(d) Subcooled reflux [Prob(7.16) contd...]

Product rate = D ; external reflux ratio = R ; vapor flow rate to the condenser = V' ; vapor flow rate in the rectifying section = V . Vapor condensed at the top tray because of the cold reflux = $V - V'$.

$$\Rightarrow V - V' = RD/20 \Rightarrow V = V' + RD/20 = D(R+1) + RD/20$$

Liquid rate in the rectifying section,

$$L = RD + RD/20$$

Slope of the rectifying operating line,

$$\frac{L}{V} = \frac{RD + RD/20}{D(R+1) + RD/20} = \frac{(1+0.05)R}{(1+0.05) \cdot R + 1} = \frac{(1.05)(1.33)}{(1.05)(1.33)+1} = 0.5827.$$

Draw the rectifying section operating line through the point D with a slope 0.5827. The line meets the feed line at P . Join W & P . Draw steps.

Number of theoretical stages = 12.8

Note

The number of plates in this case is less since the internal reflux ratio is higher.

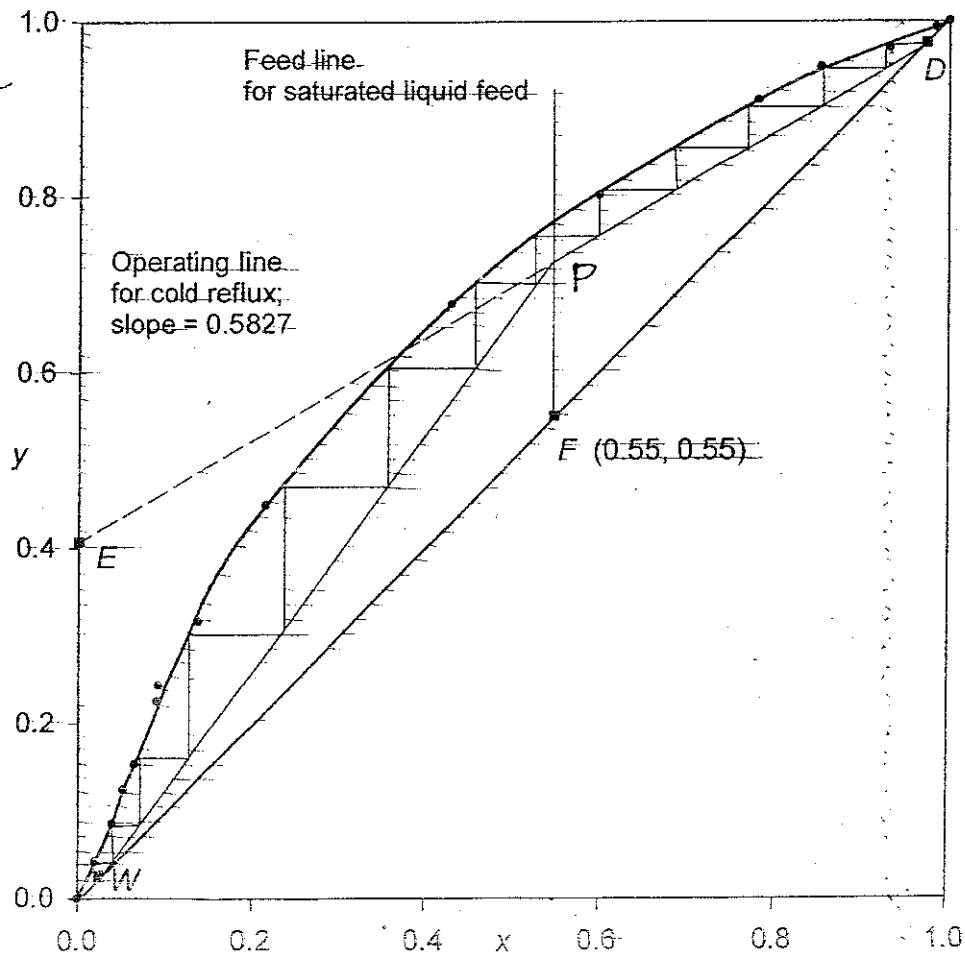
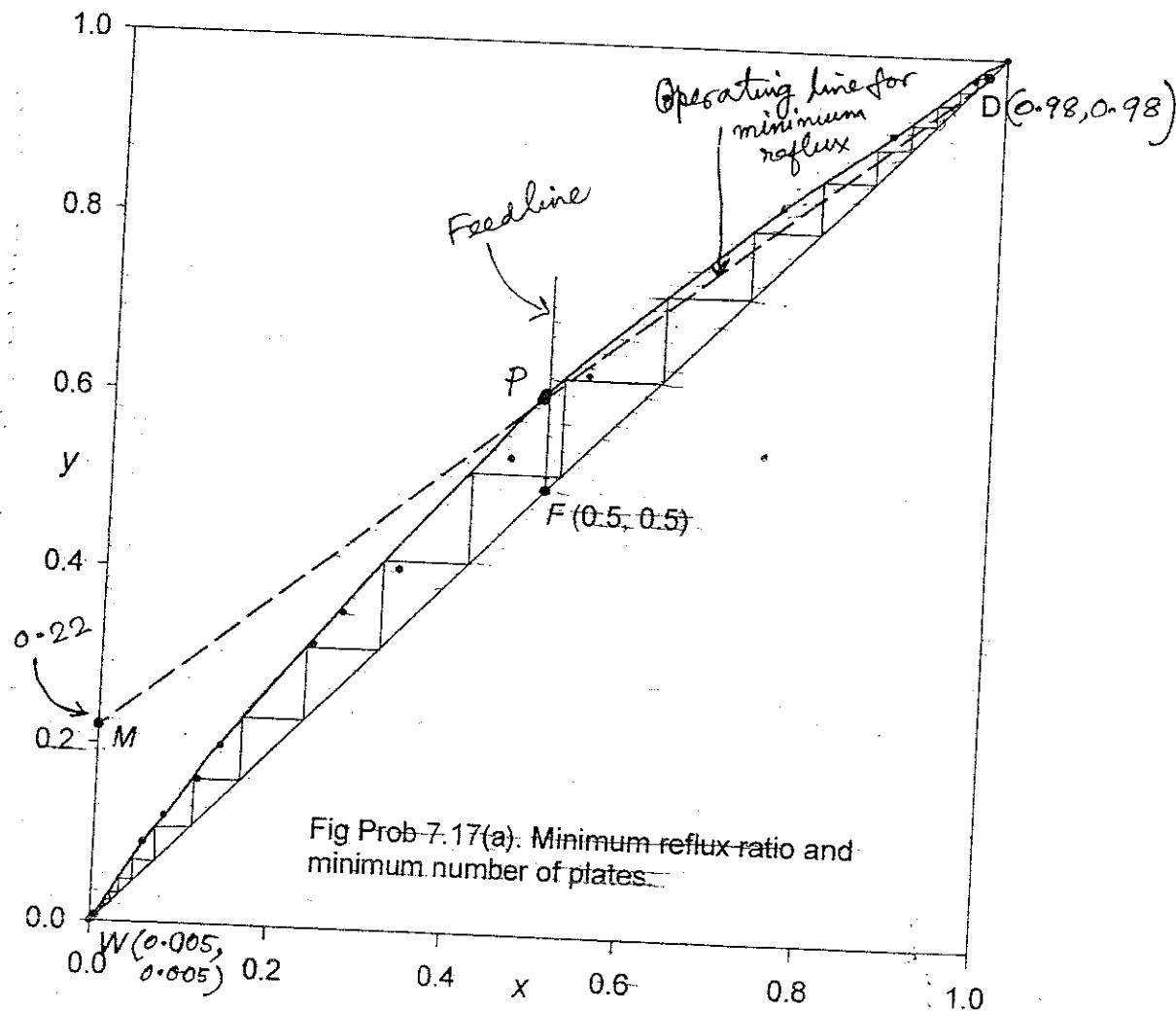


Fig. Prob. 7.16(b). Cold reflux - graphical construction of stages.

7.17 $F = 120 \text{ kmol/h}$; $x_D = 0.98$; $x_w = 0.005$; $Z_F = 0.5$.

(a) The minimum reflux ratio: Since the feed is a saturated liquid, the feed line is vertical. Draw the smoothed equilibrium curve (note that there is a considerable scatter in the data), locate the points $D(0.98, 0.98)$, $W(0.005, 0.005)$ and $F(0.5, 0.5)$. Draw the (vertical) feed line through F that meets the equil.



curve at P which is the pinch point. Join DP and extend it to meet the y -axis at $M(0, 0.22)$.

Then $\frac{x_D}{R_m + 1} = 0.22 \Rightarrow \frac{0.98}{R_m + 1} = 0.22 \Rightarrow R_m = 3.45$

(b) The minimum number of trays (for total reflux) is obtained by drawing steps between the equilibrium curve and the diagonal. Number of trays = 30

[see Fig Prob 7.17(a)]

7.17 contd...

(7.27)

(c) Number of ideal trays for $R = 2 R_m = (2)(3.45) = 6.9$.

Intercept of the rectifying section operating line on the y -axis = $\frac{x_0}{R+1} = \frac{0.98}{6.9+1} = 0.124$. Locate the points D(0.98, 0.98), W(0.005, 0.005), F(0.5, 0.5). Draw the feed

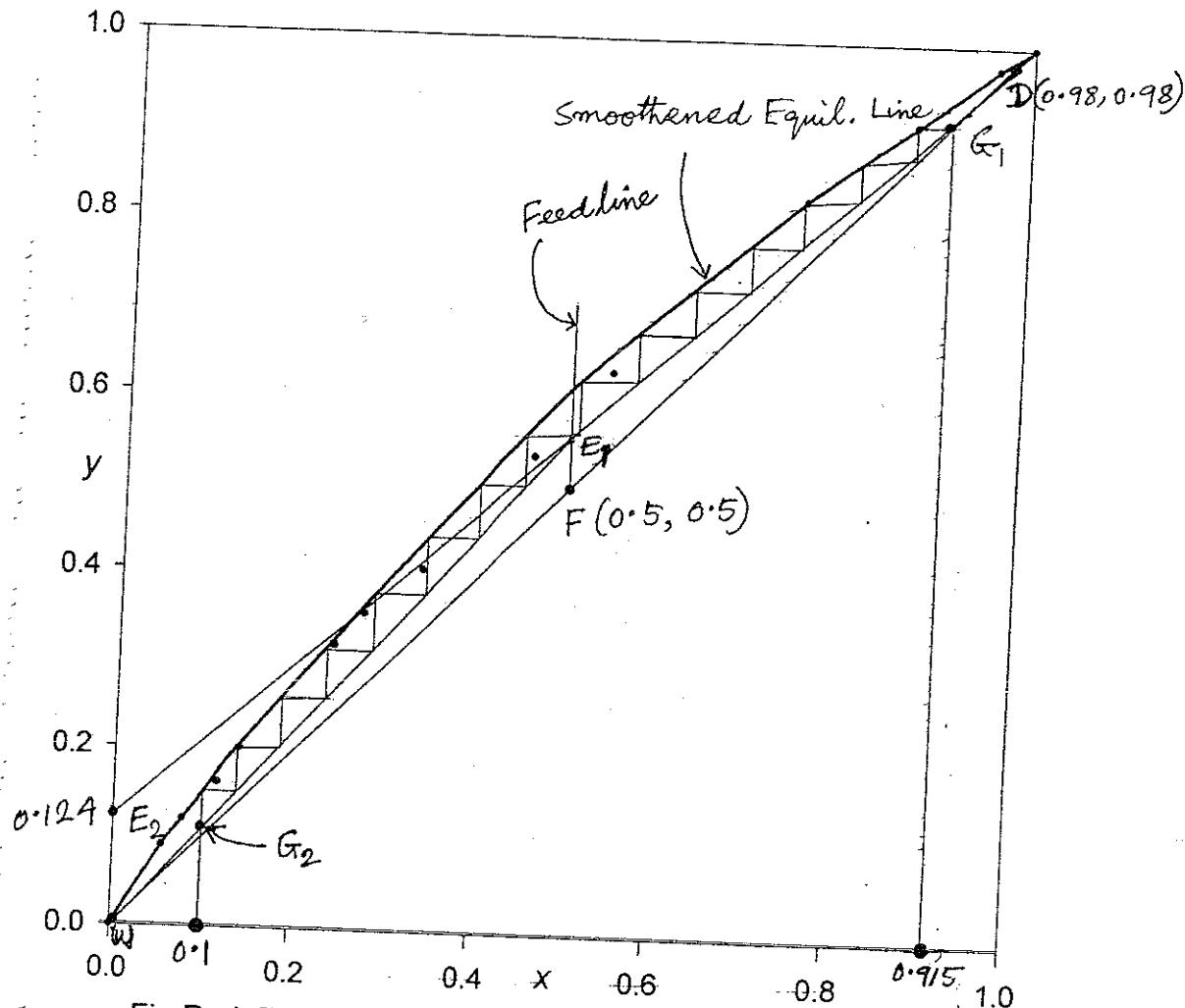


Fig Prob 7.17(b). Number of theoretical plates for $R = 2R_m$.

line through F and join D and $E_2(0, 0.124)$. DE_2 meets the feed line at E_1 . Join E_1W .

The number of ideal trays is to be determined by drawing steps between the equilibrium curve and the operating lines.

It is to be noted that the operating lines and the equilibrium curve are very close in both high and low concentration regions making construction of steps difficult. One approach to tackle this problem

(7.17) contd...

(7.28)

is to draw the high and the low concentration regions of the diagram, fig. Prob 7.17(6), on an enlarged scale and then to construct the stages. The alternative approach is to draw steps in the region where the equilibrium curve and the operating lines are not too close. The number of stages in the remaining regions is determined using the Kremser equation assuming that the equilibrium line is linear in the high and the low concentration regions. We follow the second approach here.

Start drawing steps from $x = 0.1$ (in the stripping section) to $x = 0.915$ (in the rectifying section).

Now consider the region $x = 0.005$ to $x = 0.1$. The slope of the equilibrium line, $m = 1.5$ ($y = 1.5x$ for small x).

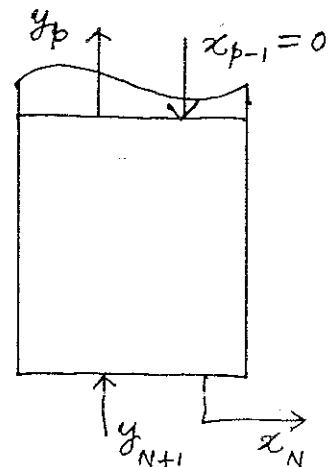
Recall Eq. (4.41). Replace the mole ratio units by mole fraction units (since the total vapor and liquid rates remain constant).

$$N = \frac{\log \left[\frac{(y_{N+1} - \alpha x_0)}{y_1 - \alpha x_0} \right] \left(1 - \frac{1}{A} \right) + \frac{1}{A}}{\log A} \quad (i)$$

Material balance: $D + W = F = 120$

$$Dx_D + Wx_W = Fz_F \Rightarrow D(0.98) + W(0.005) = (120)(0.5)$$

Solving $D = 60.9$, $W = 59.1$.



Rectifying section

Vapor flowrate, $V = (R+1)D = (6.9+1)(60.9) = 481.1 \text{ kmol/h}$

Liquid rate, $L = RD = (6.9)(60.9) = 420.2 \text{ kmol/h}$

Stripping section (saturated liquid feed, $F = 120 \text{ kmol/h}$)

Vapor rate $\bar{V} = V = 481.1$; liquid rate, $\bar{L} = L + F = 440.2 \text{ kmol/h}$

Slope of the stripping section operating line $= \bar{L}/\bar{V} = 1.1228$

It passes through the point $W(0.005, 0.005)$.

$$\Rightarrow \text{Equation: } y = 1.1228x + 6.14 \times 10^{-4} \quad (ii)$$

(7-17) Contd ...

(7-29)

Refer to the sketch of the low part of the column.

Put $x_N = 0.005 \Rightarrow y_{N+1} = 0.005$ (Point W)

$x_{p-1} = 0.1, y_p = 0.1117$ [from the operating line eqn. (ii)]

Over the region $0 \leq x \leq 0.1$, the equilibrium line can be approximated by $y = 1.5x$

$$\Rightarrow m = 1.5 \text{ and } \bar{A} = \frac{\bar{L}}{m\bar{V}} = \frac{1.1228}{1.5} = 0.7487 ; d = m = 1.5$$

$$\text{Put } x_{p-1} = x_0 = 0.1 ; y_p = y_1 = (1.1228)(0.1) + 6.14 \times 10^{-4} = 0.1129$$

$$\text{Eq. (i)} \rightarrow N = \frac{\log \left[\frac{0.005 - (1.5)(0.1)}{0.1129 - (1.5)(0.1)} \cdot \left(1 - \frac{1}{0.7487} \right) + \frac{1}{0.7487} \right]}{\log (0.7487)}$$

$$\Rightarrow \text{Number of plates (for } 0.005 \leq x \leq 0.1) = 13.$$

Number of ideal plates for $0.915 \leq x \leq 0.98$ in the rectifying section:

The equilibrium line can be approximated as

$$y = m'x + c' = 0.7x + 0.3 \quad (\text{iii})$$

$$\text{The operating line eq is } y = \frac{R}{R+1}x + 0.124 = 0.8734x + 0.124$$

$$\Rightarrow y_{n+1} = 0.8734x_n + 0.124$$

Material balance over a section of the column

$$V(y_{n+1} - y_1) = L(x_n - x_0) = L \left(\frac{y_n - c'}{m'} - x_0 \right)$$

$$\Rightarrow y_{n+1} = \bar{A}y_n + \left[y_1 - \frac{L}{V} \left(\frac{c'}{m'} + x_0 \right) \right] ; \bar{A} = \frac{L}{m'V}$$

Solution of the above equation (see Section 4.6.3)

$$y_n = \frac{y_0 - y_1}{1 - \bar{A}} \cdot (\bar{A})^n + \frac{1}{1 - \bar{A}} \left[y_1 - \frac{L}{V} \left(\frac{c'}{m'} + x_0 \right) \right] \quad (\text{iv})$$

(7.17) contd...

(7.30)

To calculate the number of ideal plates, put $n=N$,

$$\bar{A} = \frac{L}{m'V} = \frac{0.8734}{0.7} = \underline{1.2477}; \quad x_N = 0.915; \quad y_N = 0.7x_N + 0.3 \\ = 0.9405; \\ x_0 = 0.98 = y_1; \quad y_0 = 0.7x_0 + C' = (0.7)(0.98) + 0.03 = \underline{0.986}.$$

$$\text{Eq. (iv)} \rightarrow 0.9405 = \frac{0.986 - 0.98}{1 - 1.2477} (1.2477)^N \\ + \frac{1}{1 - 1.2477} \cdot \left[0.98 - 0.8734 \left(\frac{0.3}{0.7} + 0.98 \right) \right]$$

Number of ideal stages = $N = 19$

Number of ideal stages for $0.1 < x < 0.915$ is obtained
by graphical construction (see Fig Prob 7.17b) = 15

Total number of ideal plates = $19 + 15 + 13 = \underline{47}$

Feed plate $\rightarrow (15+8)\text{th}$, i.e. 23rd plate from
the top.

(d) Condenser vapor load, $V = (R+i)D = \underline{481.1 \text{ kmol/h}}$
[see the calculations under (c)]

7.18

(7.31)

Feed: $x_F = 0.3$, Saturated vapor. The feed line is a horizontal through $(0.3, 0.3) \Rightarrow y = 0.3$ — (i)

$$\text{Equilibrium line } (\alpha = 4.12) \rightarrow y = \frac{4.12x}{1 + 3.12x} \quad (\text{ii})$$

Intersection of the feed line and the equilibrium line is obtained by solving (i) and (ii) $\rightarrow x = 0.0942, y = 0.3$

The operating line for minimum reflux passes through $D \rightarrow (x_D, x_D) \rightarrow (0.98, 0.98)$ and $(0.0942, 0.3)$.

$$\text{Slope} = \frac{0.98 - 0.3}{0.98 - 0.0942} = 0.768 = \frac{R_m}{R_m + 1}$$

$$\text{Minimum reflux ratio}, R_m = 3.31$$

Calculation of the boilup ratio (for minimum reflux):

$$\text{Material balance: } F = D + W; \quad F(0.3) = D(0.98) + W(0.01)$$

$$\Rightarrow D = 0.299F; \quad W = 0.701F.$$

Since the feed is a saturated vapor,

$$\bar{V} = V - F = (R_m + 1)D - F = (3.31 + 1)(0.299F) - F = 0.2887F$$

$$\text{Boilup ratio, } R_v = \frac{\bar{V}}{W} = \frac{0.2887F}{0.701F} = 0.412$$

7.19

(a) The purity of the products (both top and bottom products) may be improved by increasing the reflux ratio.

(b) The action will lead to reduction of reflux ratio and less product purity.

(c) Increased steam pressure \rightarrow increased temperature driving force in the reboiler \rightarrow higher reboiler heat duty \rightarrow higher vapor rate \rightarrow higher reflux ratio \Rightarrow larger condenser heat duty and improved product quality.

(7.32)

7.20

Given: $z_F = 0.6$; $x_D = 0.96$; $x_6 = 0.55$; $y_6 = 0.72$; $y_7 = 0.63$.
 Rectifying section operating line $\rightarrow y_{n+1} = ax_n + b$

Passes through $(x_D, x_D) \rightarrow (0.96, 0.96)$. Also $y_7 = 0.63$ and
 $0.96 = a(0.96) + b$, and $0.63 = a(0.55) + b$ $x_6 = 0.55$.

Solve to get $a = 0.805$, and $b = 0.187$.

Rectifying section operating line: $y_{n+1} = 0.805 x_n + 0.187$
 Slope $= \frac{R}{R+1} = 0.805 \Rightarrow R = 4.134$

Relative volatility, $\alpha = \frac{y_6/(1-y_6)}{x_6/(1-x_6)} = \frac{0.72/0.28}{0.55/0.45} = 2.104$

Material balance: $F(0.6) = D(0.96) + W \cdot x_W$; $F = D + W$

97% of A is recovered in the distillate $\Rightarrow (0.97)(F)(0.6) = D(0.96)$

$$0.6 = \frac{D}{F}(0.96) + \left(\frac{F-D}{F}\right) \cdot x_W \quad \Rightarrow \frac{D}{F} = 0.606$$

$$\Rightarrow 0.6 = (0.606)(0.96) + (1-0.606)x_W$$

$$\Rightarrow x_W = 0.0463.$$

Number of ideal plates: Can be obtained by using the Smokers equation or by graphical construction. The feed line is horizontal. The graphical construction is shown below.

$$\alpha = 2.104 \Rightarrow \begin{array}{c|ccccccccc} x & 0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 \\ \hline y & 0 & 0.345 & 0.584 & 0.76 & 0.894 & 1.0 \end{array}$$

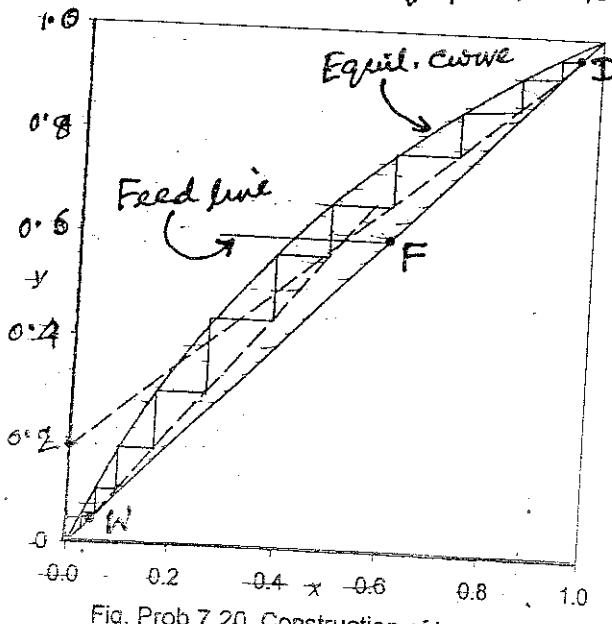


Fig. Prob 7.20. Construction of trays.

Total number of ideal stages = 10.3
 Feed is to be supplied to the sixth tray from the top.

(7-34)

7.22

Given: $x_D = 0.98$; $R = 3.0$; $x_5 = 0.54$; $\alpha = 2.51$.

Rectifying section op. line $\rightarrow y_{n+1} = \frac{3}{3+1}x_n + \frac{0.98}{3+1} = \underline{0.75x_n + 0.245}$

(a) Put $x_n = x_5 = 0.54$ to get $y_6 = (0.75)(0.54) + 0.245 = \underline{0.65}$

The concentration $=$ concentration of vapor entering the 5-th tray.
of the vapor leaving the 5th tray $\rightarrow \frac{y_5(1-y_5)}{x_5/(1-x_5)} = 2.51 \Rightarrow y_5 = \underline{0.747}$

Hint (b) Feed \rightarrow Saturated liquid $\Rightarrow q = 1$; feed line $x = \underline{0.25}$

Point of intersection of the feed line and the rectifying section operating line ($y = 0.75x + 0.245$)

$$\text{is } \underline{x = 0.25}, \underline{y = 0.4325}$$

At the fifth tray, $(x_5, y_5) \rightarrow (0.55, 0.747)$

\Rightarrow The fifth tray belongs to the rectifying section.

(c) Enrichment of the vapor across the 4-th tray:

$$y_5 = 0.75x_4 + 0.245; y_5 = 0.747 \Rightarrow x_4 = \underline{0.67}$$

$$\frac{y_4(1-y_4)}{x_4(1-x_4)} = \alpha = 2.51 \Rightarrow y_4 = \underline{0.836}$$

$$\text{Enrichment across the 4th tray} = y_4 - y_5 = 0.836 - 0.747 \\ = \underline{0.089}$$

(d) 97% of A in the is recovered in the distillate.

$$\Rightarrow (0.97) \cdot F Z_F = D x_D \Rightarrow (0.97) \cdot F \cdot (0.25) = D (0.98) \\ \Rightarrow F = 4.041 D = 4.041 (F-W)$$

$$W = 3.04 D; D = \underline{0.2475 F}$$

Since the feed is a saturated liquid, $\bar{V} = V = (R+1)D = 4D$

$$\text{Boilup ratio; } R_V = \frac{\bar{V}}{W} = \frac{4D}{3.04D} = \underline{1.316}$$

CHAPTER 8: LIQUID-LIQUID EXTRACTION

(8.1)

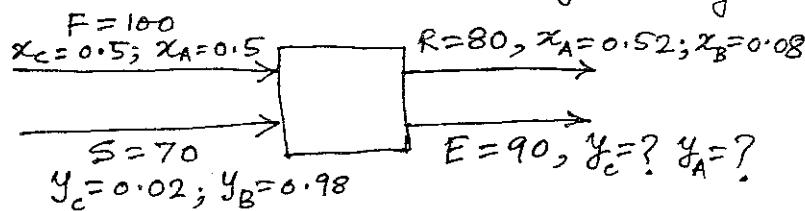
Short and MCQ's

(8.1) (i) 1.0 ; at the plait point, the compositions of the two phases are the same \Rightarrow selectivity is unity.

(8.2)

From total material balance,

$$E = 100 + 70 - 80 \\ = 90 \text{ kg}$$



Solute (c) balance : $(100)(0.5) + (70)(0.02) = (80)(0.4) + (90)(y_c)_E$

$$\Rightarrow (y_c)_E = \frac{0.215}{0.4} = 0.5375$$

Carrier (A) balance : $(100)(0.5) + (70)(0) = (80)(0.52) + (90)(y_A)_E$

Selectivity, $\beta = \frac{(y_c)_E / (y_A)_E}{(x_c)_R / (x_A)_R} = \frac{(0.5375) / 0.0933}{(0.4) / (0.52)} = 3$

(8.3) (iii) 85% ; (8.4) Solvent 3 ; (8.5) (ii) ; (8.6) (i)

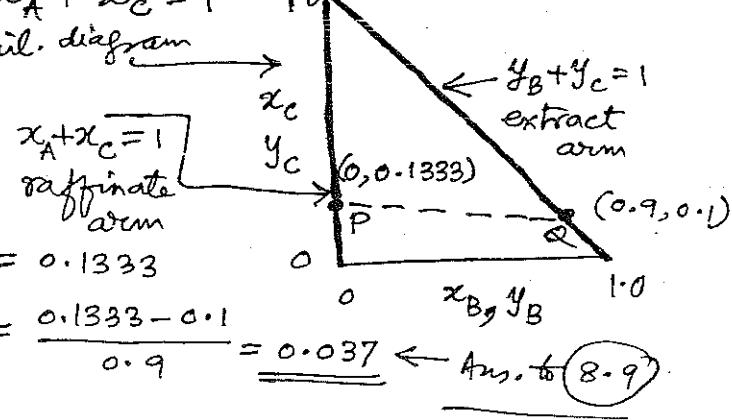
(8.7) $\frac{R_N}{\Delta m} = \frac{S_m \Delta m}{R_N S_m} = \infty \Rightarrow \Delta m = 0 \Rightarrow R_N = S_m \text{ and } F = E_1$

(8.8) A and B are immiscible $\Rightarrow y_A = 0$ and $x_B = 0$.

$$\Rightarrow y_B + y_c = 1, \text{ and } x_A + x_c = 1$$

At the point Q on the extract arm, $y_c = 0.1$, $y_B = 0.9$

Equil. diagram



At the point P on the

raffinate arm, $x_c = y_c / 0.75 = 0.1333$

$$\text{Slope of the tie-line, PQ} = \frac{0.1333 - 0.1}{0.9} = 0.037$$

(8.10) (i) increased.

(8.11) Consider the equil. diagram, fig Ex 8.2

Locate F at $(x_c)_F = 0.2$, S and E at $(y_c)_E = 0.1$ on the extract arm. Join FS. The tie-line through E intersects FS at M.

$$\text{From the graph, } \frac{F}{S} = \frac{MS}{FM} = 1.05 \Rightarrow S = \frac{F}{1.05} = \frac{1}{1.05} = 0.95 \text{ kg}$$

(8.12) No, since the mutual solubility will be more.

8.2

(8.13) By total material balance, $R = 500 + 400 - 550 = \underline{350 \text{ kg}}$

$$\text{Solute balance: } (500)(0.4) + (400)(0) = (500)(0.33) + (350)(x_c)_R$$

$$\text{Distribution coefficient, } K = \frac{x_c}{x_c} = \frac{0.33}{0.053} = \underline{\underline{6.23}}$$

(8.14) Use Table 8.2

Propanoic acid or phosphoric acid \rightarrow belong to Solute class 2
a solvent of class (column) 1 or class 7 or 8 is suitable

Pyridine (solute class 7 matches it better) — use a solvent of
BTX (solute class 11) — no solvent class shows negative
interaction; select one with zero interaction

o-Nitrophenol (solute class 10, intramolecular hydrogen bonding)
use a solvent from class 4 or 1.

(8.15) (ii) Type II ; (8.16) (ii) 5% ; (8.17) No; intersecting tie-

lines mean multiplicity of equilibrium concentrations.

(8.18) (i) Enthalpy ; (8.19) (ii) ; (8.20)

$$(8.21) (i) ; (8.24) (i) ; (8.25) (iii) \bar{a} = \frac{64}{d_{32}} = \frac{(6)(1-0.7)}{500 \times 10^{-6}} = \underline{\underline{3600 \frac{m^2}{m^3}}}$$

(8.26) (ii) 3-15 cm.

(8.27) (ii) 0.4 m

(8.29) (iii) Pod extractor

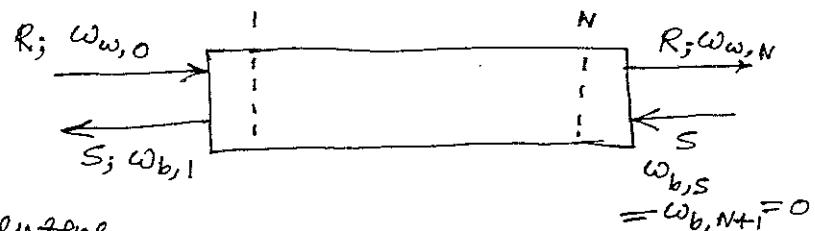
PROBLEMS

(8.1)

Since the solute concentrations are low, the flowrates of the carrier and the solvent phases (immiscible) may be taken to be constant.

(8.3)

$$R = \text{Raffinate (water)} \\ \text{phase flow rate} \\ = 2000 \text{ kg/h}$$



$$w_{w,0} = \text{mass fraction of benzene} \\ \text{in the feed } = 0.02$$

$$S = \text{Solvent flowrate} = ? \quad w_b = \text{mass fraction solute in} \\ \text{the solvent phase.}$$

$$98\% \text{ of the solute is removed} \Rightarrow w_{w,N} = (0.02)(0.02) = 0.0004$$

$$\begin{aligned} &\text{For the minimum solvent rate, the pinch point occurs at} \\ &\text{the feed end of the cascade} \Rightarrow w_{w,0} \text{ and } w_{b,1} \text{ are at equil.} \\ &\Rightarrow w_{w,0}/w_{b,1} = 1.707 \Rightarrow w_{b,1} = w_{w,0}/1.707 = \frac{0.02}{1.707} = 0.0117 \end{aligned}$$

(a) Solute material balance for the minimum solvent rate, S_m :

$$R(w_{w,0} - w_{w,N}) = S_m (w_{b,1} - w_{b,N+1})$$

$$\Rightarrow 2000(0.02 - 0.0004) = S_m (0.0117 - 0) \Rightarrow S_m = 3350 \text{ kg/h.}$$

(b) Actual solvent rate = $1.3 S_m = (1.3)(3350) = 4355 \text{ kg/h}$

Since the flowrates are 'constant' and the equilibrium relation is linear, the number of theoretical stages may be determined by using the Kremser equation. With the above notation, Eq. (4.44) reduces to the form

$$N = \frac{\log \left[\frac{(w_{w,0} - K \cdot w_{b,N+1})}{(w_{w,N} - K \cdot w_{b,N+1})} \right] (1 - \bar{E}) + \bar{E}}{\log (1/\bar{E})} \quad (i)$$

$$\bar{E} = K \cdot R/S = (1.707)(2000)/(4355) = 0.784; \quad K = 1.707; \quad w_{b,N+1} = 0$$

$$N = \frac{\log \left[\left(\frac{0.02 - 0}{0.0004 - 0} \right) (1 - 0.784) + 0.784 \right]}{\log (1/0.784)} = 10 \text{ ideal stages}$$

(c) Given, $N = 4$; the same solvent rate $\Rightarrow \bar{E} = 0.784; w_{w,N} = ?$

(8.1) Contd...

8.4

$$A = \frac{\log \left[\left(\frac{\omega_{w,0} - 0}{\omega_{w,N} - 0} \right) (1 - 0.784) + 0.784 \right]}{\log (1/0.784)} \Rightarrow \omega_{w,0}/\omega_{w,N} = 8.625$$

$$\Rightarrow \omega_{w,N} = 0.02/8.625 = 0.00232$$

$$\Rightarrow \text{Fractional solute removal} = (0.02 - 0.00232)/0.02, i.e. \underline{88.4\%}$$

(d) $N = 5 ; \omega_{w,0} = 0.02 ; \omega_{w,N} = 0.0004 ; S = ?$

$$5 = \frac{\log \left[\left(\frac{0.02 - 0}{0.0004 - 0} \right) (1 - E) + E \right]}{\log (1/E)} \Rightarrow E = 0.528$$

$$\Rightarrow K \cdot R/S = 0.528 \Rightarrow S = K \cdot R / 0.528 = \frac{(1.707)(2000)}{0.528} = \underline{\underline{6466 \text{ kg/h}}}$$

(e) Use the result given in Prob 4.30.

$$A = \frac{\log (0.02 / 0.0004)}{\log (1 + S/KR)} ; R = 2000, K = 1.707$$

$$\Rightarrow S = 5665 \text{ kg/h} \Rightarrow \text{Total Solvent reqd.} = (4)(5665) = \underline{\underline{22,660 \text{ kg/h}}}$$

* * * * *

8.2

Given: $F = 500 \text{ kg} ; (x_c)_F = 0.5 ; (x_B)_F = 0 ; F/S = 1.1$

$$S = 500/1.1 = 454.5 \text{ kg} ; (y_B)_S = 0.98 ; (y_c)_S = 0.02$$

At the 'mixing point', $(x_c)_M = \frac{(500)(0.5) + (454.5)(0.02)}{500 + 454.5} = \underline{0.271}$

The equil. data are plotted on the rectangular coordinates.

Locate the points $F [(x_B)_F = 0, (x_c)_F = 0.5] ; S [(x_B)_S = 0.98, (x_c)_S = 0.02]$;

and the mixing point M on the line FS at $(x_c)_M = 0.271$.

The tie line through the point M is drawn taking help of

the x_c - y_c curve drawn alongside the main figure.

From the terminal points of the tie line, ER ,

$$(x_c)_R = 0.18 ; (y_c)_E = 0.32$$

Solute material balance : $F(x_c)_F + S(y_c)_S = E(y_c)_E + R(x_c)_R$

$$\Rightarrow (500)(0.5) + S(0.02) = E(0.32) + R(0.18) \quad (i)$$

8.2 Contd...

8.5

$$\text{Total material balance: } 500 + 454.5 = E + R \quad (i)$$

$$\text{Solving (i) and (ii), } E = 623.5 \text{ kg; } R = 331 \text{ kg} \quad (ii)$$

Acetone removed from the aqueous phase

$$= (500)(0.5) - (331)(0.18) = 190.4 \text{ kg; i.e., } \frac{190.4}{500} \Rightarrow \underline{\underline{76.2\%}}$$

* * *

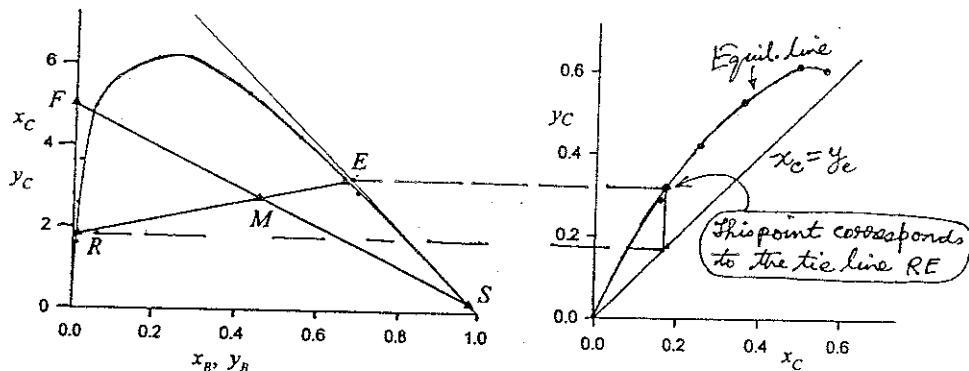


Fig. Prob. 8.2. Single stage extraction.

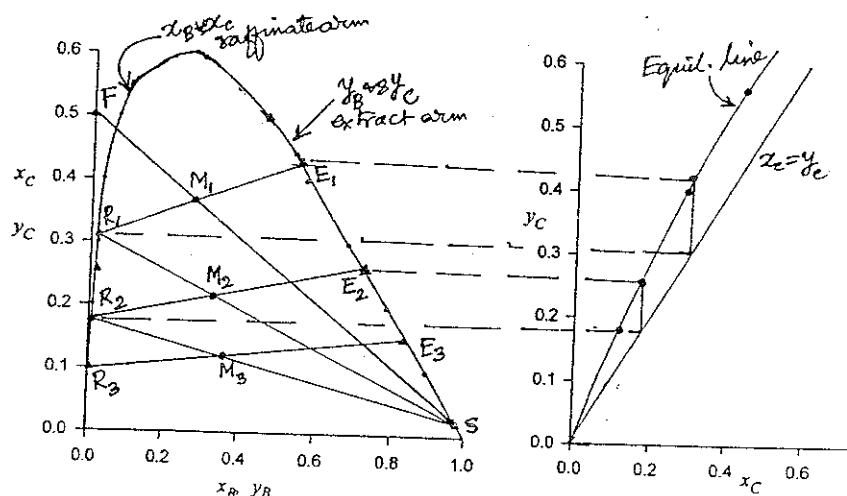


Fig. Prob 8.3. Three stage crosscurrent cascade.
crosscurrent

* * *

8.3 Refer to Fig. 8.8 for three stages. Equal amount of solvent is added to each stage $\Rightarrow S_1 = S_2 = S_3 = S$.

Stage 1 : $F = 500 \text{ kg/h}$; $(x_c)_F = 0.5$; $S_1 = S = 200 \text{ kg/h}$; $(x_c)_S = 0.02$

At the mixing point, M_1 , $(x_c)_{M_1} = \frac{(500)(0.5) + (200)(0.02)}{700} = 0.363$

Draw the LLE curve and locate the points $F(0, 0.5)$ and $S(0.98, 0.02)$ on it; join FS and locate $(x_c)_{M_1} = 0.363$ on the line FS ; draw the tie line $R_1 E_1$ through the point M_1 .

From the figure, $(x_c)_{R_1} = 0.315$, $(y_c)_{E_1} = 0.42$

Material balance : $F(x_c)_F + S(0.02) = E_1(0.42) + R_1(0.315)$

$S = 200$, $F = 500$, $(x_c)_F = 0.5 \Rightarrow E_1 = \underline{319 \text{ kg/h}}$ and $F+S = E_1+R_1$, $R_1 = \underline{381 \text{ kg/h}}$

Stage 2

$R_1 = 381$; $S_2 = S = 200$; $(x_c)_{R_1} = 0.315$; $(y_c)_S = 0.02$

Mixing point, $(x_c)_{M_2} = \frac{(381)(0.315) + (200)(0.02)}{381 + 200} = 0.2135$.

Draw the line $R_1 S$ and locate the point $(x_c)_{M_2}$ on it; draw the tie line $R_2 E_2$ through M_2 .

From the figure, $(x_c)_{R_2} = 0.18$; $(y_c)_{E_2} = 0.26$

By a solute balance, $R_2 = \underline{338.2}$, $E_2 = \underline{242.8}$

Stage 3 For this stage, $(x_c)_{M_3} = 0.1205$ which is located on $R_2 S$. The tie line through $(x_c)_{M_3}$ is $R_3 E_3$

$(x_c)_{R_3} = 0.102$; $(y_c)_{E_3} = 0.15$; by material balance, $R_3 = \underline{330.2}$

Total solute balance over the cascade, $E_3 = \underline{208}$.

$$E_3(y_c)_{E_3} + E_2(y_c)_{E_2} + E_1(y_c)_{E_1} = F(x_c)_F + 3S(y_c)_S - R_3(x_c)_{R_3}$$

which is approximately satisfied. The discrepancy arises out of errors in reading values

Acetone removed from the feed from the plots.

$$\text{Fractional removal} = \frac{216.3}{250} = \underline{0.865}$$

*

*

*

*

(8-7)

8.4 The procedure is similar to that used for Prob. 8.3. The outline of the solution is given. Refer to fig. 8.8.

Stage 1: $F = 1000 \text{ kg/h}$; $(x_c)_F = 0.5$; $S = S_1 = 250 \text{ kg/h}$; $(x_c)_{S_1} = 0$

The LLE diagram is prepared on the rectangular coordinate.

At the mixing point of Stage 1, $(x_c)_{M_1} = \frac{(1000)(0.5) + (250)(0)}{1250} = 0.40$

The points F and S_1 are located and $(x_c)_{M_1}$ is located on the line FS₁. The tie-line through M_1 is drawn having terminals R_1 and E_1 ; $(x_c)_{R_1} = 0.358$; $(y_c)_{E_1} = 0.47$.

Solving the total and solute material balance equations,

$$E_1 = 468.7; R_1 = 781.3$$

Similarly we can proceed for Stages 2, 3 and 4 in which the given amounts of solvent are added.

Results: $(x_c)_{R_2} = 0.22$, $(y_c)_{E_2} = 0.306$, $R_2 = 603.5$, $E_2 = 477.8$

For Stage 3, $(x_c)_{R_3} = 0.11$, $(y_c)_{E_3} = 0.17$, $R_3 = 489.5$, $E_3 = 464$

For Stage 4, $(x_c)_{R_4} = 0.05$, $(y_c)_{E_4} = 0.075$, $R_4 = 516$, $E_4 = 373.5$

Mixing points: $(x_c)_{M_2} = 0.2586$; $(x_c)_{M_3} = 0.1392$; $(x_c)_{M_4} = 0.0605$

See Fig. Prob 8.4. Total amount of solute recovered

$$= \sum_i E_i (y_c)_{E_i} = (468.7)(0.47) + (477.8)(0.306) \\ + (464)(0.17) + (373.5)(0.075) = 473.4 \text{ kg}$$

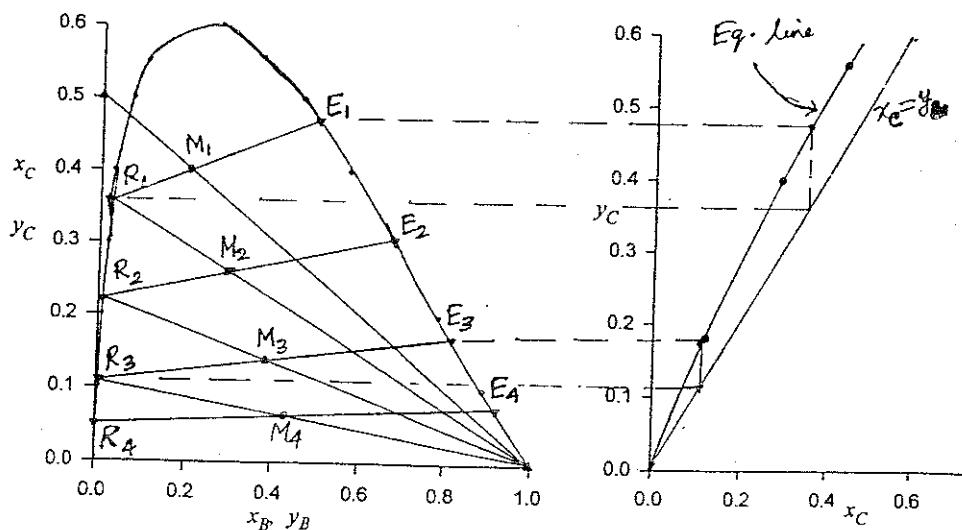
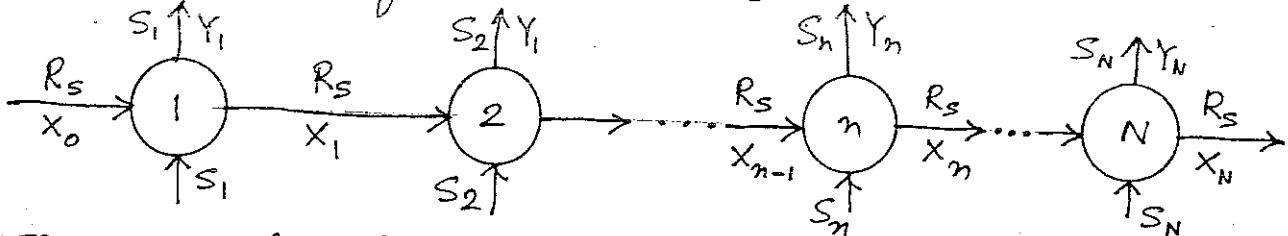


Fig. Prob 8.4. Four stage cross current cascade, quantities of solvent added to the stages are different.

8.5

A sketch of the cascade is given below

8.8



The phase flowrates, R_s and S_i , are on solute-free basis; the concentrations are in the "mass ratio" unit.

Overall mass balance: $R_s(x_0 - x_N) = S_1 Y_1 + S_2 Y_2 + \dots + S_N Y_N = \sum_{n=1}^N S_n Y_n \quad (i)$

Total amount of solvent used $= \sum_{n=1}^N S_n$ = amount of solute recovered.

Profit, \bar{P} = price of extracted solute - cost of processing

$$\Rightarrow \bar{P} = C_1 \cdot R_s(x_0 - x_N) - C_2 \sum_{n=1}^N S_n \quad \dots \quad (ii)$$

For the n -th stage, $R_s(x_{n-1} - x_n) = S_n Y_n$

$$\Rightarrow S_n = \frac{R_s(x_{n-1} - x_n)}{Y_n} = \frac{R_s(x_{n-1} - x_n)}{\psi(x_n)} = \frac{R_s(x_{n-1} - x_n)}{\alpha x_n} \quad (iii)$$

From Eqs. (i), (ii) and (iii), for $N=4$,

$$\bar{P} = C_1 \cdot R_s(x_0 - x_4) - \frac{C_2 R_s}{\alpha} \sum_{n=1}^4 \left(\frac{x_{n-1} - x_n}{x_n} \right)$$

$$\Rightarrow \bar{P} = \frac{\bar{P}}{C_1 \cdot R_s} = (x_0 - x_4) - \gamma \sum_{n=1}^4 \left(\frac{x_{n-1} - 1}{x_n} \right); \quad \gamma = \frac{C_2}{C_1 \alpha}$$

$$\Rightarrow \bar{P} = x_0 - x_4 + 4\gamma - \gamma \left(\frac{x_0}{x_1} + \frac{x_1}{x_2} + \frac{x_2}{x_3} + \frac{x_3}{x_4} \right) \quad (iv)$$

To maximize \bar{P} (i.e. \bar{P}), put $\frac{\partial \bar{P}}{\partial x_1} = \frac{\partial \bar{P}}{\partial x_2} = \frac{\partial \bar{P}}{\partial x_3} = \frac{\partial \bar{P}}{\partial x_4} = 0$

Differentiating \bar{P} , $\frac{\partial \bar{P}}{\partial x_1} = \frac{\gamma x_0}{x_1^2} - \frac{\gamma}{x_2} = 0 \Rightarrow x_1^2 = x_0 x_2 \quad (v)$

$$\frac{\partial \bar{P}}{\partial x_2} = \frac{\gamma x_1}{x_2^2} - \frac{\gamma}{x_3} = 0 \Rightarrow x_2^2 = x_1 x_3 \quad (vi)$$

Similarly, from $\frac{\partial \bar{P}}{\partial x_3} = 0$, $x_3^2 = x_2 x_4 \quad (vii)$ & from $\frac{\partial \bar{P}}{\partial x_4} = 0$, $x_4^2 = x_3 \quad (viii)$

From Eqs (v) to (viii), $x_1 = (\gamma x_0)^{1/5}$; $x_2 = (\gamma^2 x_0^2)^{1/5}$; $x_3 = (\gamma^3 x_0^3)^{1/5}$; $x_4 = (\gamma^4 x_0^4)^{1/5}$

It can be shown that the "sufficient condition" for maximum \bar{P} is also satisfied.

Solvent rate to Stage 1 [Eq.(iii)], $S_1 = \frac{R_s}{\alpha} \left(\frac{x_0}{x_1} - 1 \right) = \frac{R_s}{\alpha} \left[x_0 \left(\frac{x_0}{x_1} \right)^{1/5} - 1 \right]$

8.9

8.6

$$F = 2000 \text{ kg/h}; (x_c)_F = 0.35; S = 2970 \text{ kg/h}; (y_c)_S = 0 \text{ (pure solvent)} \\ \text{raffinate concentration, } (x_c)_{R_N} = 0.1$$

LLE curve is drawn using rectangular coordinates; locate the points F, S and R_N. At the mixing point (M), $x_M = \frac{(2000)(0.35)}{4970} = 0.491$
 Locate M on the line FS, join R_NM and extend to meet the 'extract arm' of the LLE curve at E₁. Join and extend FE₁ and R_NS to get the difference point, Δ. Draw a set of lines from Δ to intersect the LLE curve at a set of points

x_c	0.1	0.12	0.15	0.18	0.21	0.25	0.28	0.32	0.34	[Fig Prob 8.6(a)]
y_c	0.0	0.1	0.2	0.04	0.06	0.082	0.10	0.14	0.16	

The equilibrium curve is drawn as x_c vs y_c on Fig Prob 8.6(b) and the operating line is drawn by joining the above points. Stps are drawn between the eq. and op. lines to find out the number of ideal stages, $N = 5.9$.

To obtain the minimum liquid rate, it is seen that Δ_m can be obtained by drawing the tie line (FE_{1,m}) through F. Join R_N and E_{1,m}. The line intersects FS at M_m. From the figure 8.6(b), FM_m = 5.6 and M_mS = 5.55
 By the Levens Arm Rule, $F/S_m = M_m S_m / F M_m \Rightarrow \frac{2000}{S_m} = \frac{5.55}{5.6} = 0.9911$
 $\Rightarrow S_m = \frac{2018}{2018} \text{ kg/h} = \text{minimum solvent rate}$

The actual solvent rate (S) is $2970 / 2018 = 1.47$ times the minimum.

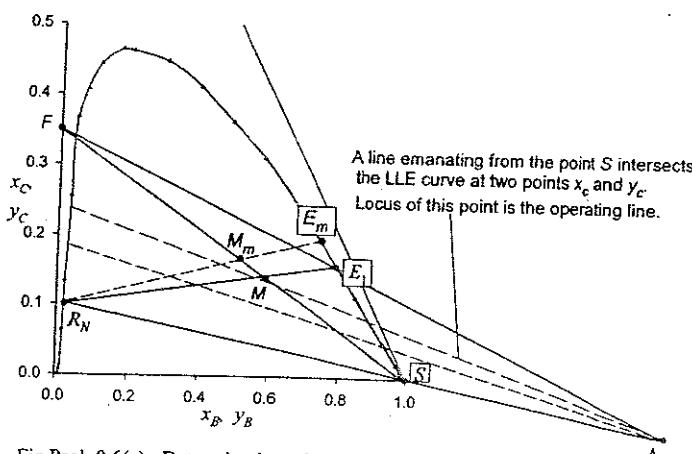
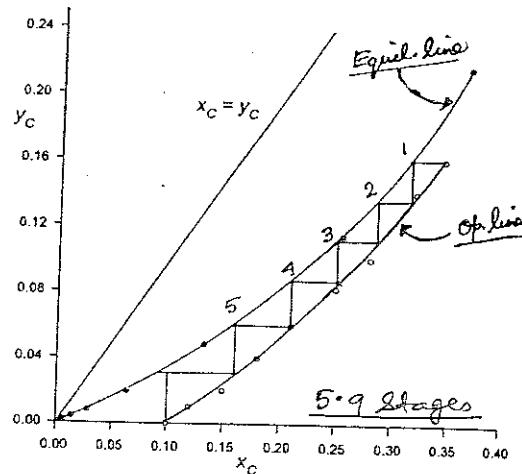


Fig Prob 8.6(a) : Determination of (i) points on the operating line
 (ii) the points E_{1,m} and M_m for the minimum liquid rate.



MT Fig Prob 8.6(b) - determine the number of ideal stages.



8.7

8.10

$$F = 1000 \text{ kg/hr}; (x_c)_F = 0.35; (x_c)_{R_N} = 0.03; S = 1300 \text{ kg/hr}$$

$$\text{At the mixing point (M), } (x_c)_M = \frac{(1000)(0.35)}{2300} = 0.152 \quad (y_c)_S = 0$$

Locate the points F, S, R_N on the LLE plot. Locate M on the line FS, join R_NM to meet the Eq. curve at E₁. Join and extend FE₁ and R_NS to meet at Δ. Determine the points on the operating line by drawing a set of lines from Δ. Find out the number of ideal stages by step construction on the x_c-y_c plane. Number of ideal stages = 4

To determine the minimum solvent rate, draw the tie line through F that meets the extract arm of the LLE curve at (E₁)_m. Join R_N(E₁)_m that intersects the line FS at M_m. Using the Lever Arm Rule,

$$F/S_m = \frac{M_m S_m}{F S_m} = \frac{5.8}{3} = 1.93 \Rightarrow S_m = \frac{1000}{1.93} = 517 \text{ kg/hr}$$

The construction is shown in fig Prob 8.7.

$$\text{Op. line} \rightarrow \begin{array}{c|c|c|c|c|c|c|c} x_c & 0.03 & 0.09 & 0.145 & 0.19 & 0.24 & 0.28 & 0.32 \\ \hline y_c & 0 & 0.03 & 0.06 & 0.08 & 0.12 & 0.15 & 0.19 \end{array}$$

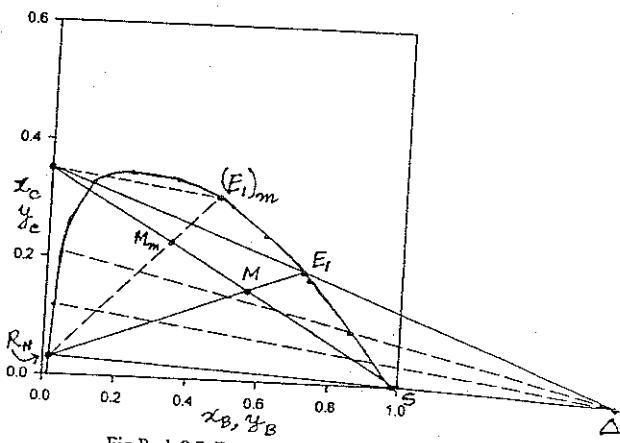
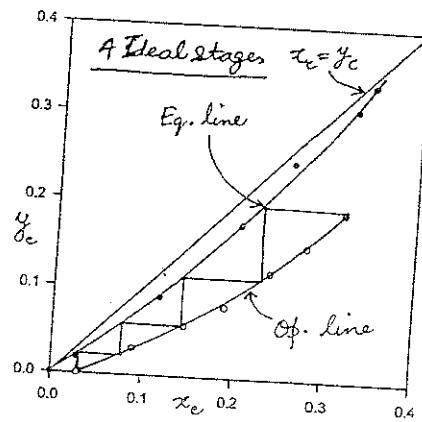


Fig Prob 8.7. Determination of the number of ideal stages.



88

$$F = 1000 \text{ kg/h}; (x_c)_F = 0.5; (x_c)_{R_N} = 0.02; (y_c)_S = 0$$

(8-11)

Draw the LLE curve and locate the points F, R_N and S. Draw the tie line through F, F E_{1,m}. Extend the lines F E_{1,m} and R_NS to get the point Δ_m. The minimum liquid rate can be found from the location of Δ_m, S_m = 647.5 kg/h.

$$\text{Actual liquid rate} = 1.3 S_m = (1.3)(647.5) = \underline{\underline{842}} \text{ kg/h.}$$

Obtain the mixing point (M) on FS where $(x_c)_M = \frac{500}{1842} = 0.271$

Join and extend R_NM to get E₁ on the LLE line. Get the point Δ, the intersection of R_NS and FE₁ extended. Obtain the points on the operating line by drawing a set of lines through Δ. Points on the op. line are

x_c	0.02	0.08	0.12	0.19	0.25	0.305	0.36	0.415	0.45	0.49
y_c	0.0	0.04	0.065	0.113	0.158	0.20	0.242	0.28	0.33	0.365

The number of ideal stages by step construction = 3.4

See the figures below, Fig Prob 8.8.

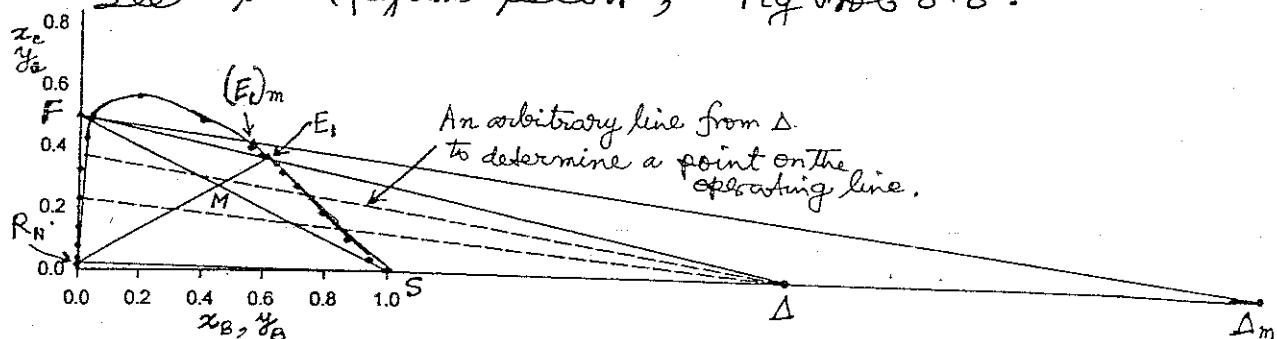
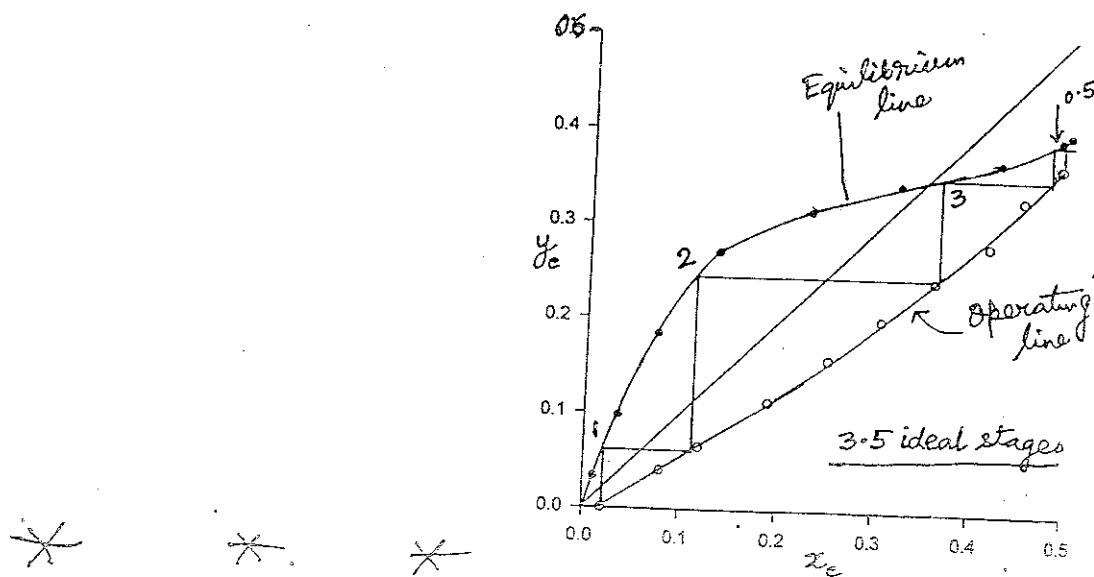


Fig. Prob 8.8(a): Solvent rate and points on the operating line



8.9

(8.12)

The data points under raffinate are to be converted to $X-Z$ and those under extract to $Y-Z$. A sample calculation is shown for the first row of the data table.

$$X = \frac{x_c}{x_A + x_c} = \frac{0.095}{0.697 + 0.095} = 0.12; Z = \frac{x_B}{x_A + x_c} = \frac{0.208}{0.697 + 0.095} = 0.263$$

$$Y = \frac{y_c}{y_A + y_c} = \frac{0.005}{0.111 + 0.005} = 0.0431; \Sigma = \frac{y_B}{y_A + y_c} = \frac{0.884}{0.111 + 0.005} = 7.62$$

Transformed data for preparation of the Maloney-Schubert plot:

X	0.12	0.17	0.252	0.314	0.376	0.59
Z	0.263	0.266	0.284	0.309	0.353	1.0

Y	0.043	0.132	0.252	0.33	0.392	0.59
Z	7.62	6.752	5.62	4.78	4.15	1.0

The data are plotted in Fig Prob 8.9. The lower part represents the raffinate phase and the upper part, the extract phase; P is the plait point. The equilibrium diagram on solvent free basis ($x-y$) is plotted below $X-Z$ and $Y-Z$ curve. Fig Prob 8.9 shows the Jancke plot (the upper figure).

(b) At the mixing point (M')

$$X_{M'} = \frac{(500)(0.3)}{500} = 0.3; Z = \frac{500}{500} = 1.0 \quad [F = 500 \text{ kg/h}, (x_c)_F = 0.30] \quad [(z_A)_F = 0.70]$$

The point M' is located on the Maloney-Schubert plot and the tie-line through M' is drawn ($P'Q'$).

At P' , $X = 0.298$ and $Z = 0.29$ for the raffinate phase

At Q' , $Y = 0.316$ and $Z = 5.0$ " " extract phase.

Let the flowrates (solvent free basis) be extract: E' , raffinate: R' .

Total material balance: $E' + R' = F + S = F + S = 500 + 0$

Solvent balance:

$$E'Z + R'Z = F'Z_F + S'Z_S = 500 \text{ kg solvent}$$

$$\Rightarrow E'(5.0) + (500 - E')(0.29) = 500 \Rightarrow E' = 75.4, R' = 424.6$$

(8.9) contd...

Total phase flowrates :

$$E = E'(1+z) = (75.4)(1+5) = \underline{452.4} ; R = R'(1+z) = (424.6)(1.29)$$

$$= \underline{547.7 \text{ kg/h}}$$

$$\text{Mass of ethylene glycol extracted} = E'Y = (75.4)(0.316)$$

$$\text{Mass of glycol in the raffinate} = \underline{\underline{23.5 \text{ kg}}}$$

$$(\text{Total solute} = 126.5 + 23.5 = 150 \text{ kg/h}) = R'X = (424.6)(0.298)$$

$$\% \text{ Solute recovered} = \frac{23.5}{150} \times 100 = \underline{\underline{15.67\%}}$$

(8.13)

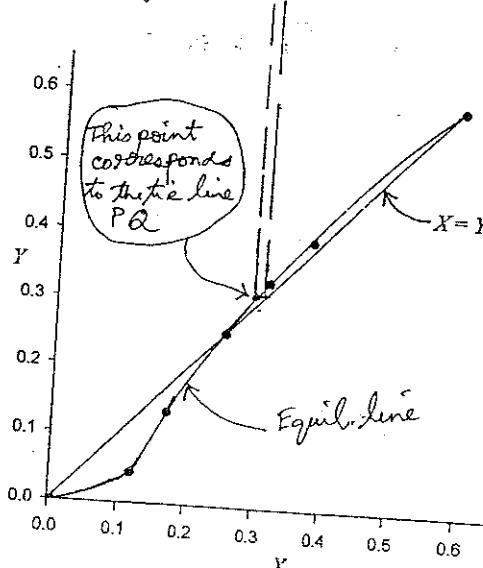
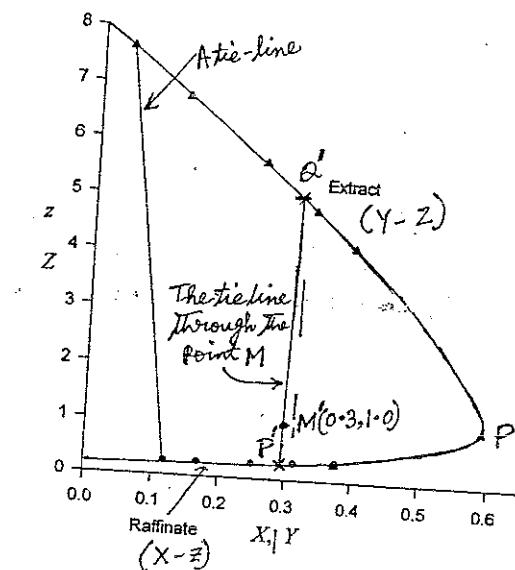


Fig Prob 8.9

(8.14)

(8.10) The final concentration (x_2) of the solute in the dispersed phase (see Example 8.7) if 99% approach to equilibrium is achieved.

$$x_2 = x_{eq} + (x_1 - x_{eq})(1 - 0.99)$$

$$= 0.01504 + (0.3 - 0.01504)(0.01) = 0.01519$$

Calculate the required time from Eq. (8.32)

$$\ln \left[\frac{0.03 - \frac{(3.77)(0.03)(1200)/(4500)}{1 + (3.77)(1200)/(4500)}}{0.01504 - \frac{(3.77)(0.03)(1200)/4500}{1 + (3.77)(1200)/(4500)}} \right] = \frac{(1.940)(7.352)(1.010)(1.577 \times 10^{-5})}{1200} \cdot t$$

$$\Rightarrow 4.6025 = (0.1893)(2.0053) \cdot t$$

$$\Rightarrow t = 12 \text{ second}$$

CHAPTER 9: SOLID-LIQUID EXTRACTION

9.1

Short and MCQ's

- (9.1) (i) and (iii); (9.2) (ii), (iv) and (vi); (9.3) (i) ethanol,
 (ii) water, (iii) ammonium cyanide, (iv) ammoniacal solution,
 (v) hexane, (vi) water; (9.4) (i) solid-phase diffusion,
 (ii) liquid-phase diffusion, (iii) liquid-phase diffusion;
 (9.5) (i); (9.6) (i); (9.7) (i)

- (9.8) NaOH concentrations in the overflow and in the liquid retained in the underflow are equal.

$$\text{Total Solute} = 0.15 \text{ kg}; \text{total solvent (water)} = 1.35 \text{ kg}; \\ \text{fractional solute concentration} = \frac{0.15}{1.35 + 0.15} = 0.1 = \text{solute}$$

$$\text{Mass of solution retained in the underflow} = (0.3 \text{ kg/kg inert})(0.5 \text{ kg inert}) = 0.15 \text{ kg}$$

$$\text{Mass of overflow} = 1.5 - 0.15 = 1.35 \text{ kg}$$

$$\text{Mass of NaOH extracted in the overflow} = (1.35)(0.1) = 0.135 \text{ kg}$$

$$\text{Fractional recovery} = \frac{0.135}{0.15} \Rightarrow 90\% .$$

- (9.9) (i); (9.10) (ii); (9.11) (ii), since the solution of higher density tends to move downward; (9.13) (iii);

- (9.14) (iii); (9.15) (i) Bollman, moving basket, moving belt, (ii) moving belt, (iv) moving belt, (v) Rotocel.

- (9.16) (i), (iii), (v) and (vii).

- (9.19) Overflow: Solution conc. = γ_c , solid free basis.
 1 kg solvent means $\frac{1}{(1-\gamma_c)}$ kg solution.

$$Z_V = 0.1 \gamma_c \text{ kg solid per kg solvent} \Rightarrow \frac{0.1 \gamma_c}{1/(1-\gamma_c)}$$

i.e.

$$\underline{\text{Underflow}} \quad Z_V = 0.1 \gamma_c (1-\gamma_c) \text{ kg solid per kg solution}$$

$$\Rightarrow Z_L = \frac{0.35 - 0.4 X_c^2}{1 - (0.35 - 0.4 X_c^2)} \text{ mass fraction solid in the sludge}$$

$$\text{The curves can now be drawn.}$$

PROBLEMS

(9-2)

- 9.1 Y_c, Z_v values for the overflow, and X_c, Z_L values for the underflow can be calculated from the given data. Take the data set given in the last row of the table, for example. For 1 kg solution, mass of oil, $m_c = 0.5 \text{ kg}$, mass of solvent, $m_B = 0.5 \text{ kg}$, mass of solid in the overflow, $m_A = 0.0$; mass of solids in the underflow, $m'_A = 0.46 \text{ kg}$

$$Y_c = \frac{m_c}{m_B + m_c} = \frac{0.5}{0.5 + 0.5} = 1; Z_v = \frac{m_A}{m_B + m_c} = \frac{0.029}{0.5 + 0.5} = 0.029$$

$$X_c = \frac{m_c}{m_B + m_c} = \frac{0.5}{0.5 + 0.5} = 1; Z_L = \frac{m'_A}{m_B + m_c} = \frac{0.46}{0.5 + 0.5} = 0.46$$

Calculated data for all the data sets are given below.

Y_c	0	0.05	0.20	0.25	0.30	0.35	0.4	0.45	0.50
Z_v	0	0.002	0.005	0.007	0.01	0.013	0.017	0.022	0.029

X_c	0	0.05	0.20	0.25	0.30	0.35	0.4	0.45	0.50
Z_L	0.67	0.66	0.64	0.625	0.6	0.58	0.55	0.51	0.46

For preparing a right triangular diagram, we have to convert the given data to (Y_c, Y_B) for overflow and (X_c, X_B) for underflow. Again consider the last row of the given data table.

$$Y_c = \frac{m_c}{m_A + m_B + m_c} = \frac{0.5}{0.5 + 0.5 + 0.029} = 0.486; Y_B = \frac{0.5}{1.029} = 0.486$$

$$X_c = \frac{m_c}{m'_A + m_B + m_c} = \frac{0.5}{1.46} = 0.342; X_B = \frac{0.5}{1.029} = 0.486$$

Calculated data for all the sets:

Y_c	0	0.05	0.199	0.2482	0.297	0.3446	0.3933	0.4403	0.486
Y_B	1.0	0.95	0.796	0.745	0.693	0.6416	0.59	0.538	0.486

X_c	0	0.0301	0.122	0.1538	0.1875	0.2215	0.258	0.298	0.342
X_B	0.6	0.5723	0.488	0.461	0.4375	0.411	0.387	0.387	0.342

(9.1) contd...

(9.3)

The calculated data are plotted in Fig Prob 9.1(a) in the rectangular coordinates — x_B against x_C for the underflow, and y_B against y_C for the overflow. Three tie lines are shown.

The calculated data are also plotted as the 'Ponchon-Savarit diagram' — Z_L against x_C for the underflow, and Z_V against y_C for the overflow. The tie lines are vertical.

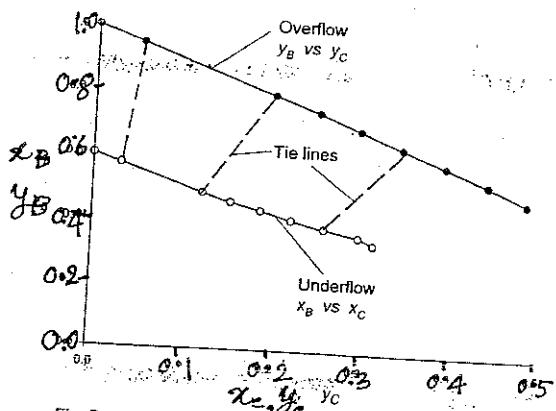


Fig. Prob. 9.1(a): Plot of underflow and overflow concentrations.

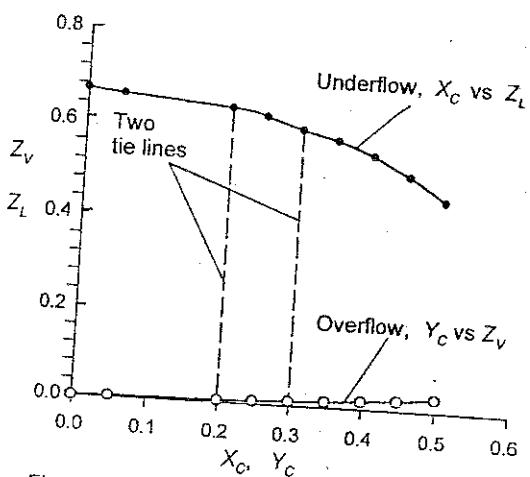


Fig. Prob. 9.1(b): Ponchon-Savarit plots of overflow and underflow concentrations.

(9.2)

* * * * *

The overflow and underflow curves of Fig. Prob 9.1 will be used.

Single stage contact of 1000 kg feed with 1800 kg solvent

(a) Solution using x_B-x_C and y_B-y_C plots, Fig Prob 9.2(a)

$$F = 1000 \text{ kg}; (x_C)_F = 0.25; S = 1800; (y_C)_S = 0$$

$$\text{At the mixing point } M, (x_C)_M = \frac{(0.25)(1000)}{1000 + 1800} = 0.0893$$

The points F and S are located and joined. The mixing point is located on FS and the tie line through M is drawn to meet the overflow line at V and the underflow

(9.2) contd...

line at L. From the figure, $(x_c)_L = 0.07$, $(y_c)_v = 0.13$
By material balance, $L + V = F + S = 2800$

$$\text{and } (L)(0.07) + V(0.13) = (1000)(0.25) \Rightarrow L = 1900 \text{ kg (underflow)}$$

$V = 900 \text{ kg (overflow)}$.

$$\text{Fractional oil removal} = \frac{(900)(0.13)}{(1000)(0.25)} = 0.47$$

Solution using x_c - Z_L and y_c - Z_v plots, Ponchon-Savarit type diagram, Fig Prob 9.2(b)

$$F' = 250 \text{ kg and } S' = 1800 \text{ kg (solid-free basis)}$$

At the mixing point (M'), $Z_{M'} = \frac{750 \text{ kg solid}}{(250+1800) \text{ kg liq}} = 0.366$

$$\text{Also, } (x_c)_{F'} = \frac{250}{250} = 1, (Z)_{F'} = \frac{750 \text{ kg solid}}{250 \text{ kg liq}} = 3.0$$

$$(y_c)_{S'} = 0, (Z)_{S'} = 0 \text{ (since the feed solvent is solid-free)}$$

Locate F' and S' on the graph, join F' - S' and locate M' at $(Z)_{M'} = 0.366$ on the line $F'S'$. Draw the tie-line (it is a vertical line in this case) through M' that meets the overflow curve at V' and the underflow curve at L' .
 $(x_c)_{L'} = 0.12 \Rightarrow (y_c)_{V'}, (Z)_{L'} = 0.655$

$$\text{Total material balance (solid free)} : L' + V' = F' + S' = 250 + 1800$$

$$\text{Solid balance} : (L')(0.655) + (V')(\sim 0.0) = 750 \Rightarrow L' = 1145 \text{ kg}$$

$$\text{Total underflow} = (1145)(1+0.655) = 1895 \text{ kg}$$

$$\text{" overflow} = 905 \text{ kg, Fractional removal of oil} = 45\%$$

Two-stage crosscurrent contact, 1500 kg solvent in each stage

This part of the problem is solved using the x_c - x_B and y_c - y_B plots, see Fig. Prob 9.2(a).

Stage 1 : $F = 1000 \text{ kg}; (x_c)_F = 0.25; S = 1500; (x_c)_S = 0$

$$\text{Mixing point } M_1 \rightarrow (x_c)_{M_1} = \frac{250}{2500} = 0.1$$

Locate F and S and locate M_1 on FS . The tie-line

(9.4)

9.5

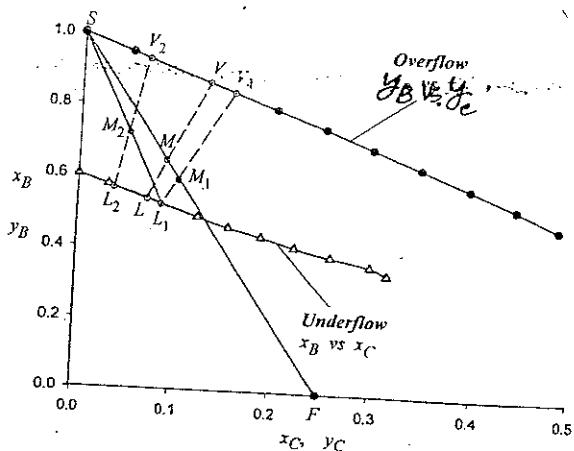


Fig. Prob. 9.2(a): Cross-current leaching, right triangular coordinates

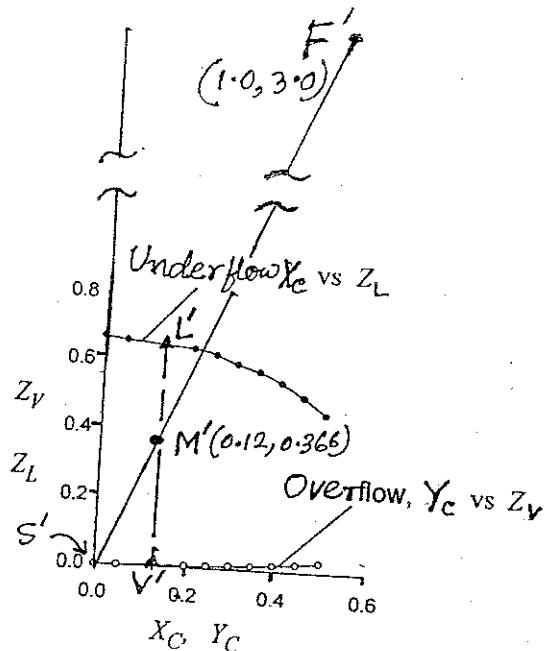


Fig. Prob. 9.2(b): Ponchon-Savarit plot for single-stage leaching.

through M_1 is L_1, V_1 . $(x_c)_{L_1} = \underline{0.085}$, $(y_c)_{V_1} = \underline{0.155}$
By material balance, $L_1 = \underline{1964 \text{ kg}}$, $V_1 = \underline{536 \text{ kg}}$

Stage 2 Underflow from Stage 1: $L_1 = 1964$, $(x_c)_{L_1} = \underline{0.085}$
Solvent, $S = 1500 \text{ kg}$; at the mixing point (M_2), $(x_c)_{M_2} = \underline{0.0482}$
Locate M_2 on the line SL_1 , and draw the tie line L_2V_2 through M_2 . At L_2 , $(x_c)_{L_2} = \underline{0.0355}$; $(y_c)_{V_2} = \underline{0.067}$
Material balance equations: $L_2 + V_2 = L_1 + S = \underline{3464}$
Solute balance: $L_2(0.0355) + V_2(0.067) = (1964)(0.085)$
 $\Rightarrow L_2 = \underline{1996}$; $V_2 = \underline{1468 \text{ kg}}$

Total oil recovered in stages 1 and 2
 $= (V_1)(y_c)_{V_1} + (V_2)(y_c)_{V_2} = (536)(0.155) + (1468)(0.067)$
 $= 181.4 \text{ kg}$

Fractional recovery $= \frac{181.4}{250} = \underline{72.6\%}$



9.3

The points on the underflow and overflow curves (Ponchon-Savart) can be obtained from the given data.

9.6

$$\begin{aligned} Y_C &\rightarrow 0.0 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \\ Z_V &\rightarrow 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \quad 0.0 \\ X_C &\rightarrow 0.0 \quad 0.05 \quad 0.10 \quad 0.15 \quad 0.20 \\ Z_L &\rightarrow \frac{1}{1.39} = 0.72 \quad \frac{1}{1.72} = 0.581 \quad 0.49 \quad 0.37 \quad 0.26 \end{aligned}$$

The clear solution (overflow; it has no solid in it) and the solution retained in sludge (underflow) have the same conc. calculation of wash water rate:

Feed, $F = 1000 \text{ kg}$ having 44.3% solids $\Rightarrow F' = 1000 - 443 = 557 \text{ kg}$ (solid free)

$$Z_{F'} = \frac{443}{557} = 0.795 ; (X_C)_{F'} = \frac{33.5}{33.5 + 22.2} = 0.601$$

Extract leaving the cascade, V'_1 ; $(X_C)_{V'_1} = 0.15$ (given).

Sludge leaving the cascade = L'_N ; mass of water in underflow = $m_w \text{ kg}$.

Alkalienterlip (with 1000 kg feed) = 335 kg, leaving = $(335)(0.05)$

$$(X_C)_{L'_N} = \frac{16.75}{m_w + 16.75} ; (Z)_{L'_N} = \frac{95\% \text{ recovered}}{m_w + 16.75} = 16.75 \text{ kg}$$

The point $[(X_C)_{L'_N}, (Z)_{L'_N}]$ lies on the underflow curve. By trial, $m_w = 660 \text{ kg}$ (so that the point L'_N actually lies on the underflow curve)

$\Rightarrow (X_C)_{L'_N} = 0.0247 ; (Z)_{L'_N} = 0.654$ (the underflow curve)

Now make a total water balance (rate of water supply = $S^1 \text{ kg/h}$)

$$S' + (1000)(0.222) = 660 + (V'_1)(0.85)$$

Alkali balance: $335 - 16.75 = (V'_1)(0.15)$

$$\Rightarrow V'_1 = \underline{\underline{2122 \text{ kg}}} ; S^1 = \underline{\underline{2241 \text{ kg}}} = \text{rate of supply of wash water.}$$

Determination of number of ideal stages (Fig Prob 9.3):

Locate the points F' ($0.601, 0.795$), S' ($0, 0$), V'_1 ($0.15, 0$)

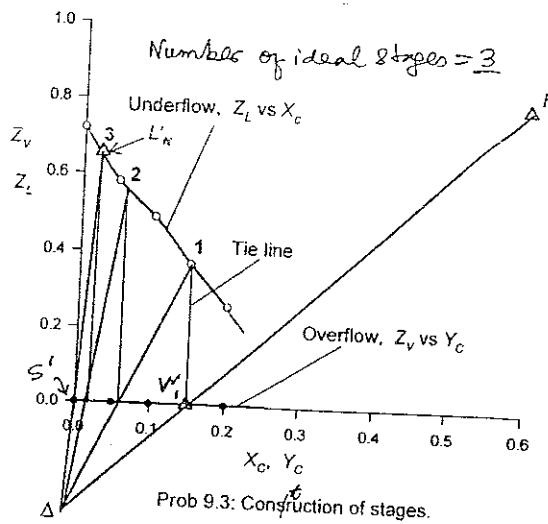
and L'_N ($0.0247, 0.654$) on the 'Ponchon-Savart' diagram.

Join and extend $F'V'_1$ and $L'_N S'$ to meet at A.

Draw stages as usual. Number of ideal stages = 3.

(9.3) Contd...

(9.7)



(9.4) Data points for $X_c(Z)_L$ and $Y_c(Z)_V$ curves are derived from the given data.

$Y_c \rightarrow$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$(Z)_V \rightarrow$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$X_c \rightarrow$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
$(Z)_L \rightarrow$	2.303	1.826	1.94	1.89	1.818	1.752	1.68	1.612

Solid feed; $F' = 250$ kg (solid-free basis)

$$(Z)_F = \frac{750}{250} = 3.0; (X_c)_F = \frac{250}{250} = 1.0$$

$$S' = 2100 \text{ (solid-free)}, (Y_c)_{S'} = 0.005$$

Underflow leaving the battery:

$$(X_c)_{L_N} = \frac{0.015}{0.015 + 0.315} \text{ (extrapolated)}$$

$$= 0.045$$

The points F' , L'_N and S' are located

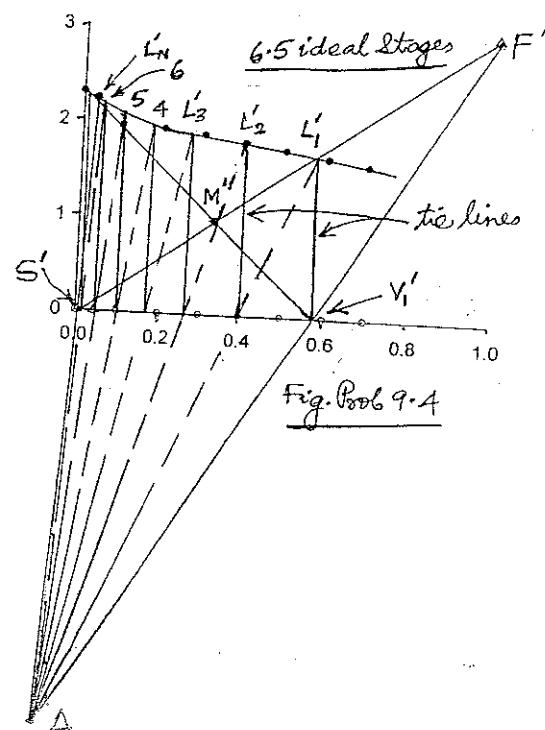
$F'S'$ is joined and M' is located on it at $(Z)_{M'} = \frac{750}{250+2100} = 0.32$

Join $L'_N M'$ and extend to meet the overflow curve at V'_1 .

Join and extend $L'_N S'$ and $F' V'_1$ to meet at Δ . Construct ideal stages as usual.

Number of ideal stages = 6.5

At V'_1 ; $(Y_c)_{V'_1} = 0.56$ = mass fraction of oil in the extract



9.8

- (9.5) The overflow is solid free, the underflow is constant. Eq (9.17) can be used directly; x_0, x_1, x_N and α are to be calculated first. Ref. to Fig. 9.13 b for notations.
- $F = 1000 \text{ kg/h}$; 15% solute, 2% water, 83% inert solid.
- Mass of inert = 830 kg ; mass of water entering with feed = 20 kg/h , mass of solute entering = 150 kg ; mass of solute leaving the cascade with the underflow (0.005 times the solids) = $(830)(0.005) = 4.15$. Mass of solution leaving in the underflow = $(830)(0.4) = 332 \text{ kg/h}$

Concentration of solution leaving the cascade, $\frac{4.15}{332} = 0.0125 = x_1$

To calculate V'_0 , we have to make a solute balance the cascade, $y_N = 0.15 = x_N$.

$$4.15 + [m_w - (332 - 4.15)] \frac{0.15}{0.85} = 150 \Rightarrow m_w = 1154.3 \text{ kg/h}$$

(see Example 9.5)

Wash water entering, $V'_0 = m_w - \text{water entering with feed}$

$$\gamma = \frac{V'_0 / L'}{1134.3 / 332} = \frac{1154.3 - 20}{1134.3} = \frac{3.416}{\underline{\underline{1134.3}}} \quad ; \quad x_0 = y_0 = 0$$

$$\text{From Eq (9.19), } N = \frac{\log \left[1 + (3.416 - 1) \left(\frac{0.15 - 0}{0.0125 - 0} \right) \right]}{\log (3.416)} = 2.8$$

* * * * *

(9.6) Constant underflow ($L'_1 = L'_2 = \dots = L'$); no solids in the overflow,

$$\text{From Eq (9.14), } y_{n-1} = \frac{L'}{V'_0} \cdot x_n + y_0 - \frac{L'}{V'_0} x_1 \quad (i)$$

$$\text{Morphree efficiency, } E_M = \frac{x_n - x_{n-1}}{x_n - x_{n-1}^*} \Rightarrow x_{n-1}^* = x_n - \frac{x_n - x_{n-1}}{E_M} \quad (ii)$$

$$\text{Also } y_{n-1} = \alpha x_{n-1}^* \quad (iii)$$

From Eqs. (i), (ii) and (iii),

$$y_{n-1} = \frac{L'}{V'_0} x_n + y_0 - \frac{L'}{V'_0} x_1 = \alpha x_{n-1}^* = \left[x_n - \frac{x_n - x_{n-1}}{E_M} \right] \alpha$$

(9.6) contd... On Simplification,

$$X_n - \alpha X_{n-1} = [bX_1 - (E_M Y_0 / \alpha)]a ; a = \frac{1}{1 + E_M (\frac{L'}{\alpha V'_0} - 1)} ; b = \frac{L' E_M}{\alpha V'_0}$$

Solving the difference equation:

$$X_n = K a^n + \frac{[bX_1 - (E_M Y_0 / \alpha)]a}{1-a}$$

$$\text{Put } n=0 \text{ to get, } K = X_0 - \frac{bX_1 - (E_M Y_0 / \alpha)a}{1-a}$$

Put $n=N$ to get the number of "real stages".

$$N = \frac{\log \left(\frac{X_N - \gamma}{X_0 - \gamma} \right)}{\log a} ; \gamma = \frac{L' X_1 - \alpha X_0 V'_0}{L' - \alpha V'_0}$$

(9.7) Let $f_n = \frac{\text{kg solution retained}}{\text{kg inert}} \quad * \quad * \quad *$

at the n -th stage

Assume $\frac{1}{f_n} = A + BX_n ; L'_n = W \cdot f_n$, W = constant solid flowrate in the underflow.

Put the above relation in Eq. (9.14) and use $X_{n-1} = Y_{n-1}$

$$V'_0 \left(\frac{A + BX_n}{W} + 1 - \frac{A + BX_n}{A + BX_1} \right) X_{n-1} = X_n + V'_0 \left(\frac{A + BX_n}{W} \right) Y_0$$

On Simplification (see Chen, 1964), $- \frac{A + BX_n}{A + BX_1} \cdot X_1$

$$X_n X_{n-1} + \alpha X_n + b X_{n-1} + c = 0 \quad (\text{this is non-linear Riccati difference equation})$$

$$a = -\frac{A + BY_0 \gamma}{B(\gamma-1)} ; b = \frac{A\gamma + BX_1}{B(\gamma-1)} ; c = \frac{A(X_1 - \gamma Y_0)}{B(\gamma-1)} ; \gamma = \frac{V'_0}{L'}$$

Solution for the number of ideal stages (N) may be

$$\frac{X_N + b + \beta_2}{X_N + b + \beta_1} = \left(\frac{X_0 + b + \beta_2}{X_0 + b + \beta_1} \right) \left(\frac{\beta_2}{\beta_1} \right)^N \quad \text{obtained from}$$

β_1 and β_2 are the unequal real roots

$$\beta = \frac{(a-b) \pm \sqrt{(a+b)^2 - 4c}}{2}$$

Other cases of the roots are discussed in Chen (1964).

(9.8) The problem is similar to Example 9.5 and Prob. 9.5. Take help of those to solve it. Calculated values of the important quantities are given.

$$F = 2 \text{ tons/h} = 2000 \text{ kg/h}; \text{ inert} = 1560 \text{ kg/h}; \text{ water} = 40 \text{ kg}$$

$$L' = (1560)(0.5) = 780 \text{ kg/h}; \text{ solute leaving with the final sludge} = 12 \text{ kg/h}$$

$$X_1 = \frac{12}{780} = 0.0154; X_0 = Y_0 = 0.$$

$$m_w \text{ calculated by Solute balance} = 2966.7; \text{ wash water rate}, V'_0 = m_w - 40 = 2926.7 \text{ kg/h}; r = V'_0/L' = \underline{3.752}$$

From Eq (9.17), no. of ideal stages, $N = \underline{2.5}$

*

*

*

*

(9.9) $Y_c - Z_v$ and $X_c - Z_L$ values are calculated from the given data.

Example of Calculation (see the first row)

$$Y_c = \frac{0.046}{0.046 + 0.952} \approx 0.046; Z_v = \frac{0.002}{0.046 + 0.952} \approx 0.002$$

$$X_c = \frac{0.026}{0.026 + 0.542} = 0.04577; Z_L = \frac{0.432}{0.026 + 0.542} = 0.7606$$

Calculated values:

$Y_c \rightarrow$	0.046	0.032	0.021	0.011	0.006	0.002
$Z_v \rightarrow$	0.002	0.001	0.0	0.0	0.0	0.0
$X_c \rightarrow$	0.04577	0.0326	0.0215	0.01096	0.006	0.002
$Z_L \rightarrow$	0.7606	0.715	0.663	0.6611	0.6584	0.6556

The overflow and underflow curves are drawn in Fig Prob 9.9

The tie-lines are vertical for practical purpose.

$$F = 400 \text{ kg slurry/h}; F' = (400)(0.49 + 0.029) = 207.6 \text{ kg (solid-free).}$$

$$\text{Wash water} = 500 \text{ kg/h}; (X_c)_{F'} = \frac{(400)(0.029)}{207.6} = 0.0559$$

Mixing point, M'

$$(X_c)_{M'} = \frac{(400)(0.029)}{207.6 + 500} = 0.0164$$

$$(Z)_{M'} = \frac{(400)(0.48)}{207.6 + 500} = 0.272$$

The points F' , M' and $S'(0,0)$ on the graph.

(9.9) Contd...

(9.11)

The underflow composition \rightarrow protein concentration
 $6.2\% \quad = 0.002$

The water content in the underflow for a protein concentration of 0.002 mass fraction can be obtained from the given data by interpolation. Use the last two rows of data on underflow composition.

Water content (if protein conc. = 0.002)

$$= 0.5994 + (0.6028 - 0.5994) \left(\frac{0.0036 - 0.002}{0.0036 - 0.0012} \right) = 0.6017$$

For the sludge leaving the last stage,

$$(X_C)_N' = \frac{0.002}{0.6017 + 0.002} = 0.00331$$

The point L'_N is located on the underflow curve for $(X_C)_N' = 0.00331$.

Join $L'_N M'$ and extend to meet the overflow curve at V'_1 .

Join and extend $F'V'_1$ and $L'_N S'$ to meet at the difference point Δ .

Draw stages.

Number of ideal stages

$$= \underline{\underline{4}}$$

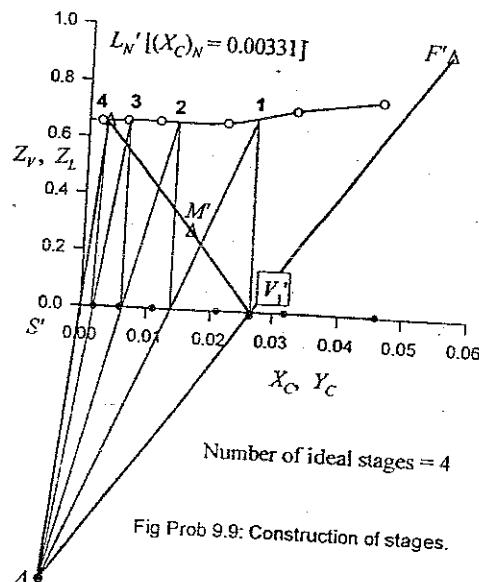


Fig Prob 9.9: Construction of stages.

- (15) Since the makeup water is supplied at a temperature $T_L < T_w$, the steady state water temperature would be a little lower than T_w . The air temperature and humidity would follow the adiabatic saturation line on the psychrometric chart.
- (33) As the 'approach' increases, the driving force becomes larger and the required packed height decreases. If the 'cooling range' increases, the driving force becomes less and the required packed height increases. If there is a wide seasonal variation of the wet-bulb temperature, the packed height should be calculated on the basis of the largest anticipated T_w . However, when the actual wet-bulb temperature drops, the fan r.p.m. can be regulated to put less air into the tower so that the desired cooling may be achieved with a lesser consumption of electrical energy.
- (38) (iii) ; (39) (ii) ; (40) Sketch (i) shows the correct orientation. For (ii) and (iii) water splash will occur, more for (iii). Further, orientation (ii) will allow more light into the tower, promoting algal growth.
- (41) Since air enters into the tower laterally, the cumulative air rate increases along the height. The V-shaped passage allows gradually increasing flow area up the packing in order to

① (ii) ; ② (iii), this is the humidity of saturated air at 30°C and 1.3 atm - can be obtained from the chart or using the vapor pressure equation of water ; ③ (i), it can be calculated as $c_{pL} \cdot \Delta T / \gamma_0$; ④ (ii) ; ⑤ (iii), it can be calculated from the vapor pressure eqn.

$$Y_s' = \frac{P^v}{P - P^v} \times \frac{18.02}{28.97} \Rightarrow 1.6007 \frac{dY_s'}{dT} = \left[\frac{1}{P - P^v} + \frac{P^v}{(P - P^v)^2} \right] \frac{dP^v}{dT}$$

$$\frac{1}{P^v} \frac{dP^v}{dT} = \frac{3984.92}{(T - 39.724)^2}. \text{ Then put the values of } P^v \text{ and } T (= 298\text{K}).$$

⑥ (i) ; ⑦ (ii) ; ⑧ (ii) ; ⑨ (iii) ; ⑩ (iii), see below

$$\frac{h_L}{k_{y1}} = \frac{H'_c - H'}{T_L - T_{Li}} \quad [\text{eq.(10.28)}]; \quad T_{Li} = 39^{\circ}\text{C}, \quad P^v = 0.0692 \text{ bar},$$

$$Y_s' = 0.0456 \text{ kg/kg dry air}, \quad H'_c = 156.5 \text{ at } 39^{\circ}\text{C} \quad [\text{Eq. (10.8)}],$$

$$H' = 135 \text{ kJ/kg dry air (given)}, \quad T_L = 42^{\circ}\text{C} \Rightarrow h_L/k_{y1} = 7.17 \frac{\text{kJ}}{\text{kg}^{\circ}\text{C}}$$

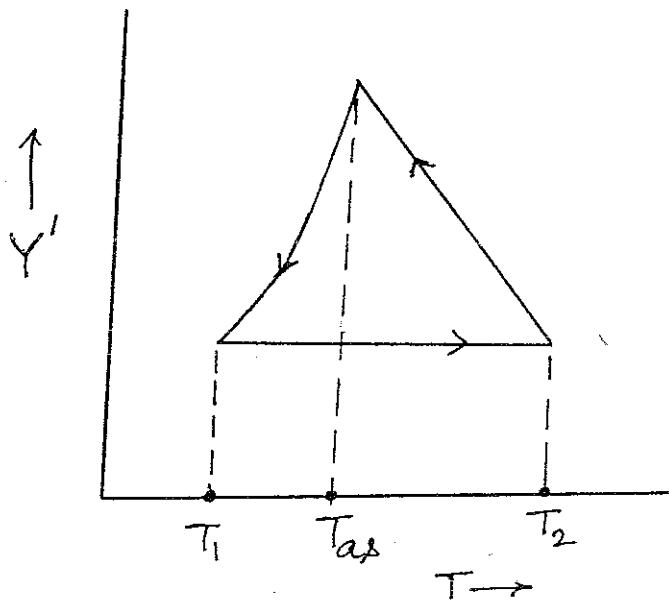
- ⑪ None of these quantities changes ; ⑫ (iii) ; ⑬ (ii) ;
 ⑭ (ii) ; ⑮ given at the end ; ⑯ (i) ; ⑰ (ii) ; ⑱ (iii) ;
 ⑲ (i) and (ii) ; ⑳ (i) ; ㉑ (iii) ; ㉒ (ii) ; ㉓ (iii),
 since the fans are supported on an elevated structure ;
 ㉔ (i) ; ㉖ No ; ㉗ items (ii), (iv) and (vi) will improve
 the capacity ; ㉘ (i) ; ㉙ power plants, metallurgical
 industries, rolling mills, refineries, etc. ; ㉚ (i) ;
 ㉛ (iii) ; ㉞ (ii) and (iii).

- (15) Since the makeup water is supplied at a temperature $T_L < T_w$, the steady state water temperature would be a little lower than T_w . The air temperature and humidity would follow the adiabatic saturation line on the psychrometric chart.
- (33) As the 'approach' increases, the driving force becomes larger and the required packed height decreases. If the 'cooling range' increases, the driving force becomes less and the required packed height increases.
- If there is a wide seasonal variation of the wet-bulb temperature, the packed height should be calculated on the basis of the largest anticipated T_w . However, when the actual wet-bulb temperature drops, the fan r.p.m. can be regulated to put less air into the tower so that the desired cooling may be achieved with a lesser consumption of electrical energy.
- (38) (iii); (39) (ii); (40) Sketch (i) shows the correct orientation. For (ii) and (iii) water splash will occur, more for (iii). Further, orientation (ii) will allow more light into the tower, promoting algal growth.
- (41) Since air enters into the tower laterally, the cumulative air rate increases along the height. The V-shaped passage allows gradually increasing flow area up the packing in order to

10.1(b)

accommodate the increasing air rate. Since the horizontal length of the flow path of air through the packing is less than the vertical flow path in a counterflow tower, the pressure drop is less in a cross-flow tower.

(43)



(44) See the last para of Section 10.5.2.

(45) See Section 10.5.4.

PROBLEMS

(10.2)

(Minor differences from the answers given here are likely owing to 'error' in reading data from the humidity chart)

(10.1) (i) $T_G = 33^\circ\text{C}$, $T_w = 23^\circ\text{C}$. Humidity = $0.014 \text{ kg/kg dry air}$ (from the chart)

(ii) Enthalpy, $H' = (2500)(0.014) + [1.005 + (1.88)(0.014)](33)$
 $= 69 \text{ kJ/kg dry air}$ [Eq.(10.8)]

(iii) Dew point, $T_d = 16^\circ\text{C}$ (follow the horizontal line through $Y' = 0.014$ to the saturation curve and read the temperature)

(iv) Humid volume, $v_H = \left(\frac{1}{28.97} + \frac{0.014}{18.02}\right)(22.4)\left(\frac{33+273}{273}\right)$
 $= 0.886 \text{ m}^3/\text{kg dry air}$ [Eq.(10.6)].

(v) Humid heat, $c_H = 1.005 + 1.88 Y' = 1.005 + (1.88)(0.014)$
 $= 1.031 \text{ kJ/(kg dry air)}(^\circ\text{C})$; Eq.(10.7).

(b) T_w of the air sample if it is heated to 50°C :

Follow the adiabatic saturation line through the point $T_G = 50^\circ\text{C}$ and $Y' = 0.014$ to reach the saturation curve at $T_w = 27^\circ\text{C}$.

(c) First check if some water vapor condenses out when the air is cooled from 33°C to 15°C .

Vapor pressure of water at 15°C , $P^w = 0.0168 \text{ bar}$

Saturation humidity = $\frac{0.0168}{1.013 - 0.0168} \cdot \frac{18.02}{28.97} = 0.0105$

(this can be obtained from the chart also)

The saturation humidity at 15°C is less than 0.014 kg/kg

\Rightarrow condensation of water occurs, $(0.014 - 0.0105) = 0.0035 \text{ kg/kg}$

Heat rejected on cooling of air from 33°C to 15°C

= Sensible heat + latent heat

= $(c_H)(\Delta T) + (\Delta Y')(R) = (1.031)(33-15) + (0.0035)(2500)$

= $27.3 \text{ kJ per kg dry air}$.

(10.3)

(10.1) contd..

(d) First calculate the partial pressure of water vapor in air at 33°C and $\gamma' = 0.014$

$$p_A = \frac{(0.014/18.02)}{(0.014/18.02) + (1/28.97)} \cdot (1.013) \text{ bar} = 0.0223 \text{ bar}$$

As the total pressure is doubled, the partial pressure of the moisture in it is also doubled to $p_A = 0.0446 \text{ bar}$

Vapor pressure of water at $50^{\circ}\text{C} = 0.1221 \text{ bar}$

$$\text{Relative humidity} = \frac{0.0446}{0.1221} \times 100 = 36.5\%$$

Dew point of the air at 50°C and 2 atm: It cannot be obtained from the chart since the chart is for air at 1 atm total pressure.

Obtain the temperature at which $P_v = 0.0446 \text{ bar}$

$$\ln(0.0446) = 11.96481 - \frac{3984.923}{T-39.724} \Rightarrow T = 304 \text{ K}$$

$$= 31^{\circ}\text{C} = \text{dew point of the air}$$

* * * *

(10.2) Inlet air, $T_G = 34^{\circ}\text{C}$, $T_w = 23^{\circ}\text{C} \Rightarrow \gamma' = 0.016/16$

$$\begin{aligned} \text{Enthalpy} &= (0.01)(2500) + [1.005 + (1.88)(0.01)](34) \\ &= 61 \text{ kJ/kg dry air.} \end{aligned}$$

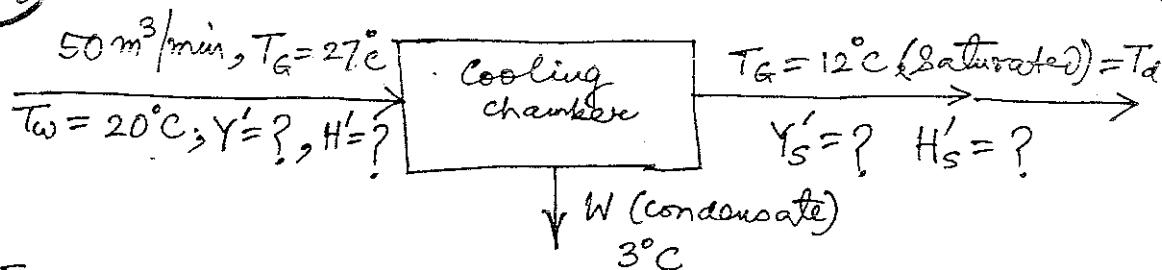
Since the air is contacted with water at the wet bulb temp. (23°C), humidification occurs along the adiabatic saturation line \Rightarrow the enthalpy of air remains constant irrespective of its humidity and temperature.

Exit moisture $\rightarrow 95\%$. Saturated $\Rightarrow \gamma' = 0.0151 \text{ lb/lb}$, temp is pretty close to 23°C .

The relative humidity can be calculated following Prob 10.1.

* * * *

10.3



Feed air: $T_G = 27^\circ\text{C}, T_w = 20^\circ\text{C} \Rightarrow Y' = 0.0137 \text{ kg/kg dry air}$.

Humid volume [Eq.(10.6)] = $0.868 \text{ m}^3/\text{kg dry air}$;

$$\text{mass of dry air} = \frac{50}{0.868} = 57.58 \text{ kg dry air}$$

The exit gas is saturated at $12^\circ\text{C} \Rightarrow Y'_s = 0.01 \text{ kg/kg}$

The rate of water condensation = $(57.58)(0.0137 - 0.01)$,
 $W = 0.213 \text{ kg/min}$

Enthalpy of the feed gas, Eq.(10.8), $H' = (2500)(0.0137)$

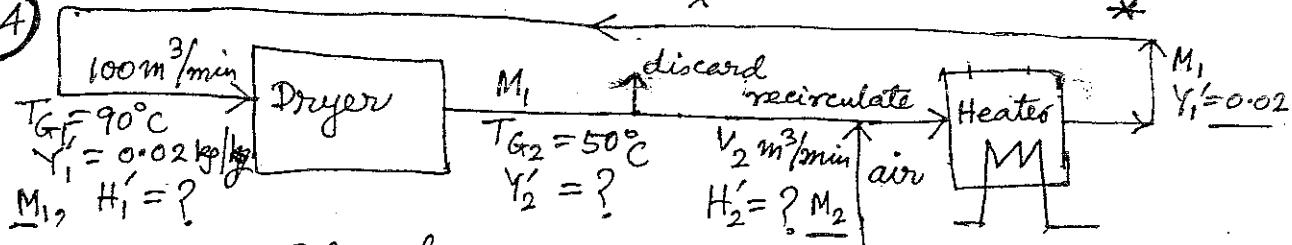
$$+ [1.005 + (1.88)(0.0137)](27) = 62 \text{ kJ/kg}$$

Enthalpy of the exit air ($12^\circ\text{C}, Y'_s = 0.01$), $H'_s = 37.3 \text{ kJ/kg}$

Enthalpy of condensate (3°C) = $(4.187)(3) = 12.6 \text{ kJ/kg}$

$$\begin{aligned} \text{Heat load} &= (62 - 37.3)(57.58) - (12.6)(0.213 \text{ kg/min}) \\ &= 1419.5 \text{ kJ/min} \end{aligned}$$

10.4



Schematic of air flow

First calculate/obtain the temperature, humidity and enthalpy of individual gas streams
 (i) Hot gas to the dryer, $T_G = 90^\circ\text{C}, Y'_1 = 0.02 \text{ kg/kg}$

(10.5)

(10.4) Contd.. Humid heat, Eq (10.6), $\vartheta_H = \left(\frac{1}{28.97} + \frac{0.02}{18.02} \right) (22.4) \cdot \frac{273+90}{273}$
 Air feed to the dryer = $100 \text{ m}^3/\text{h}$, $= 1.0904 \text{ m}^3/\text{kg dry air}$
 \Rightarrow Mass of air fed to the dryer,
 $= \frac{100}{1.0904} \text{ kg} = 91.71 \text{ kg/h (dry basis)} = M_1$

Moisture in with the hot air = $(91.71)(Y'_1) = (91.71)(0.02) = 1.834 \frac{\text{kg}}{\text{min}}$

Air leaves the dryer at 50°C , adiabatic 'humidification' of air occurs, $Y_2 = 0.0362 \text{ kg/kg dry air}$ (obtained by following the adiabatic saturation line)

Enthalpy of the feed air, $H'_1 = (2500)(0.02) + [1.005 + (1.83)(0.02)](90)$
 $= 143.8 \text{ kJ/kg dry air}$

Recirculated air: Enthalpy $H'_2 = 143.8 \text{ kJ/kg}$, the same mass, $M_2 = ?$ as the hot feed air to the dryer.

Fresh air Temp, $T_{G3} = 27^\circ\text{C}$; $T_w = 20^\circ\text{C} \Rightarrow Y'_3 = 0.0137 \text{ kg/kg}$
 Enthalpy, $H'_3 = (2500)(0.0137) + [1.005 + (1.83)(0.0137)](27) = 62 \text{ kJ/kg}$
 $M_3 = ?$

Now the mass of recycled air (M_2) and that of the fresh air (M_3) can be calculated by mass and moisture balance.

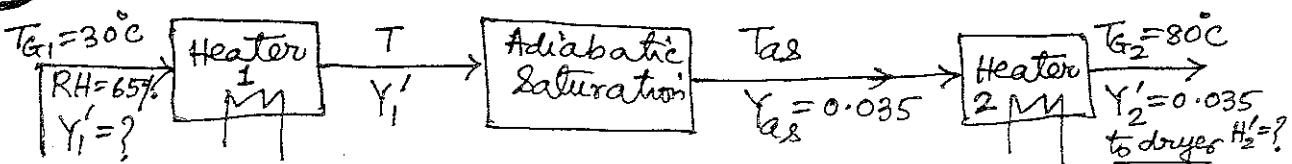
$$M_1 = M_2 + M_3 \Rightarrow M_2 + M_3 = 91.71$$

$$M_1 Y_1 = M_2 Y'_2 + M_3 Y'_3 \Rightarrow (91.71)(0.02) = M_2(0.0362) + M_3(0.0137)$$

$$\text{Solving the above two equations, } M_2 = 25.69 \text{ kg} / \quad M_3 = 66.02 \text{ kg} /$$

$$\begin{aligned} \text{Heat requirement in the heater} &= M_1 H'_1 - M_2 H'_2 - M_3 H'_3 \\ \Rightarrow Q &= (143.8)(91.71 - 25.69) - (66.02)(62) = \underline{\underline{5400 \text{ kJ/min}}} \end{aligned}$$

(10.5)



Humidity of the feed air:

Vapor pressure of water at $30^\circ\text{C} = 0.042 \text{ bar}$; partial pressure of moisture in the air ($RH = 65\%$) = $(0.042)(0.65) = 0.0273 \text{ bar}$

(10.5) Contd.. $Y'_1 = \frac{0.0273}{1.013 - 0.0273} \cdot \frac{18.02}{28.97} = 0.01723 \text{ kg/kg dry air}$ (10.6)

The temperature T (see the sketch) is not known, but on adiabatic saturation, its humidity becomes $Y'_{as} = 0.035$. Start from $Y'_{as} = 0.035$ on the saturation humidity line, move along the adiabatic saturation and reach the point where the humidity is $Y'_1 = 0.01723$. The corresponding temperature is $T = 78^\circ\text{C}$. Also the adiabatic saturation temperature (for $Y'_{as} = 0.035$) is $T_{as} = 34^\circ\text{C} \rightarrow$ Ans. to parts (a) and (b).

Now air at 34°C and $Y' = 0.035$ is heated to 80°C in heater 2. Its enthalpy is

$$H'_2 = (2500)(0.035) + [1.005 + (1.88)(0.035)](80) = 173.2 \text{ kJ/kg}$$

Psychrometric conditioning air fed to the dryer :

$$T = 80^\circ\text{C}, Y' = 0.035, H' = 173.2 \text{ kJ/kg}$$

The total heat load can be found to be $5.837 \times 10^5 \text{ kJ/h}$

*

*

*

*

(10.6)

Moist warm air : $T_G = 70^\circ\text{C}, T_w = 60^\circ\text{C} \Rightarrow H'_i = 0.1525 \text{ kg/kg}$

The dehumidified air is essentially at equilibrium with water at $26^\circ\text{C} \Rightarrow$ its temp = 26°C , Saturated

\Rightarrow its humidity = $H'_o = 0.0282 \text{ kg/kg}$

Moisture removed = $0.1525 - 0.0282 = 0.1243 \text{ kg/kg dry air}$

*

*

*

*

(10.7)

Inlet air at the bottom of the cooling tower,

$$T_{G_1} = 32^\circ\text{C}, T_{w_1} = 22^\circ\text{C}, Y'_1 = 0.013 \text{ kg/kg}, H'_1 = 65.5 \text{ kJ/kg}$$

Water temperature at the bottom, $T_{L_1} = 29^\circ\text{C}$.

Draw the saturation humidity line and locate the lower terminal point of the operating line, Q (29, 65.5) on the $H-T_L, T_G$ plane.

Calculation of the minimum air rate : Draw a tangent

(10.7) contd..

(10.7)

to the 'equilibrium curve' (i.e. the saturation humidity curve) from the point Q [See Fig Prob 10.7(a)]. The slope of the operating line is

$$\frac{L \cdot C_{OL}}{(G_s)_{min}} = \frac{9.37 \frac{\text{kJ}}{\text{kg}^\circ\text{C}}}{\text{kg}} \cdot \text{Given: } L = \frac{2,25,000 \text{ gfh}}{100 \text{ m}^2} = \frac{8505 \text{ kg/m}^2}{\text{kg}}$$

$$(G_s)_{min} = \frac{(8505)(4.187)}{9.37} = \frac{3800 \text{ kg/m}^2}{\text{kg}}$$

$C_{OL} = 4.187 \text{ kJ/kg}^\circ\text{C}; 1 \text{ gal} = 3.78 \text{ liter}$
 $\approx 3.78 \text{ kg}$

Actual air rate, $G_s = \frac{6000}{1+Y_1'} = \frac{6000}{1.013} = \frac{5923 \text{ kg/m}^2}{\text{kg}}$

Actual slope of the operating line = $\frac{(8505)(4.187)}{5923} = 6.01$

(The actual rate of air supply is $\frac{5923}{3800}$, i.e. 1.56 times the minimum)

Draw the actual operating line of slope 6.01 through the point Q. Since warm water enters at 42°C , the upper terminal of the operating line corresponds to $T_L = 42^\circ\text{C}$. Locate P at the intersection of the operating line and the vertical line through $T_L = 42^\circ\text{C}$.

Since the 'overall volumetric coefficient' is given, the enthalpy driving force at any point is $H'^* - H'$; fig 10.7(a).

The driving force line is vertical. (Note that at P, $H' = 147$)

Draw a set of ^{such} vertical lines ^('tie-lines') and read the values of $H'^* - H'$ for a set of values of H' . Such a set of values is given below.

$$H' = 65.5 \quad 83 \quad 102 \quad 115 \quad 127.5 \quad 147$$

$$H'^* = 95.5 \quad 111.5 \quad 129 \quad 129 \quad 159 \quad 186$$

$$\frac{1}{H'^* - H'} = 0.0333 \quad 0.0351 \quad 0.037 \quad 0.037 \quad 0.0317 \quad 0.0256$$

$\frac{1}{H'^* - H'}$ is plotted against H' and area under the curve for $H' = 65.5$ to 147 is determined, fig Prob 10.7(b)

$$\text{Area} = N_{TOG} = 2.83$$

[The data points to draw the 'equilibrium curve' can be easily calculated from the vapor pressure equation for water:

$T_s, ^\circ\text{C}$	20	25	27	32	35	37	40	42	45
H^*	57.4	76.3	85	110.6	129	143	183.5	183.5	213.7

$\frac{\text{kJ}}{\text{kg dry air}}$

(10.8)

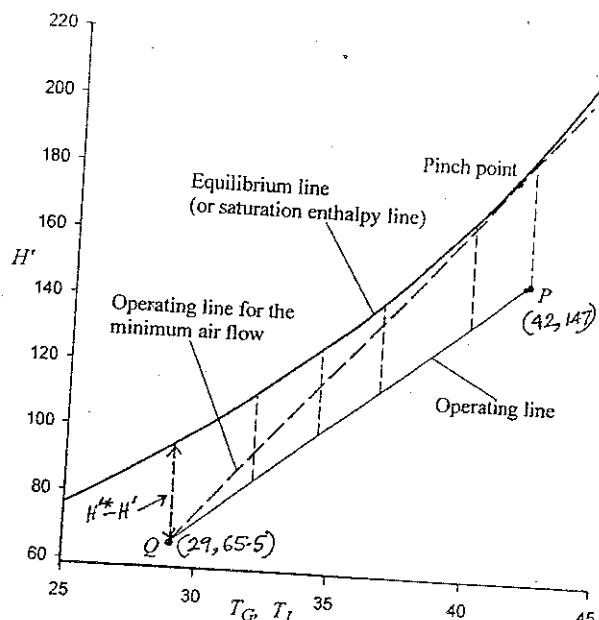


Fig. Prob. 10.7(a)

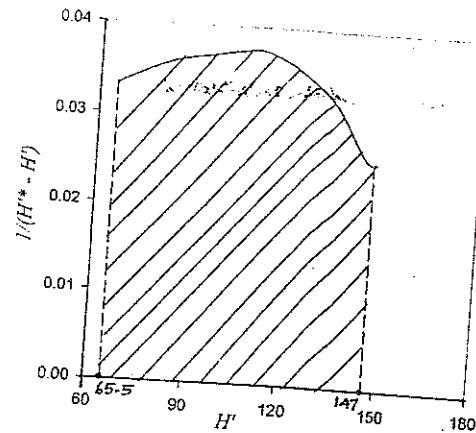


Figure Prob. 10.7(b). Graphical integration for N_{LOG}

(10.8) The problem is similar to the previous one.

Inlet air: $T_{G_1} = 30^\circ\text{C}$, $T_{W_1} = 25^\circ\text{C}$, $\gamma' = 0.019$, $H' = 78.7 \text{ kJ/kg}$
Cooled water leaves at $T_{W_1} = 30^\circ\text{C}$. Locate Q(30, 78.7).

For the minimum air rate, slope of the operating line touching the equilibrium line = $\frac{8.78}{8.78}$,

$$L = 5500 \text{ kg/m}^2 \cdot \text{hr}. (G_s)_{min} = \frac{(5500)(4.187)}{8.78} = 2623 \text{ kg/m}^2 \cdot \text{hr}$$

Actual air rate, $G_s = (1.25)(2623) = 3279 \text{ kg/m}^2 \cdot \text{hr}$

Corresponding slope of the actual operating line = $\frac{(5500)(4.187)}{3279} = 7.02$

Draw the operating line of this slope through the point Q and locate the point P on it corresponding to $T_{L_2} = 45^\circ\text{C}$ (Fig. Prob. 10.8a).

Draw a set of vertical lines between the operating

(10.8) contd...

(10.9)

line and the equilibrium q length $H'^* - H'$.
A set of such values are given below.

H'	78.7	90	102.3	114	135	150	175	186
$H'^* - H'$	18.75	17.25	15	15	15.75	17.25	21.3	31.05
$\frac{1}{H'^* - H'}$	0.0533	0.058	0.0667	0.0667	0.0635	0.058	0.047	0.0317

$\frac{1}{H'^* - H'}$ values are plotted against H' and graphical integration is done as usual (Fig Prob 10.8b); area under the curve = $N_{t0G} = 6.14$

$$H_{t0G} = \frac{\dot{V}_S}{K_Y \bar{a}} = \frac{3279}{2500} = 1.312 \text{ m}$$

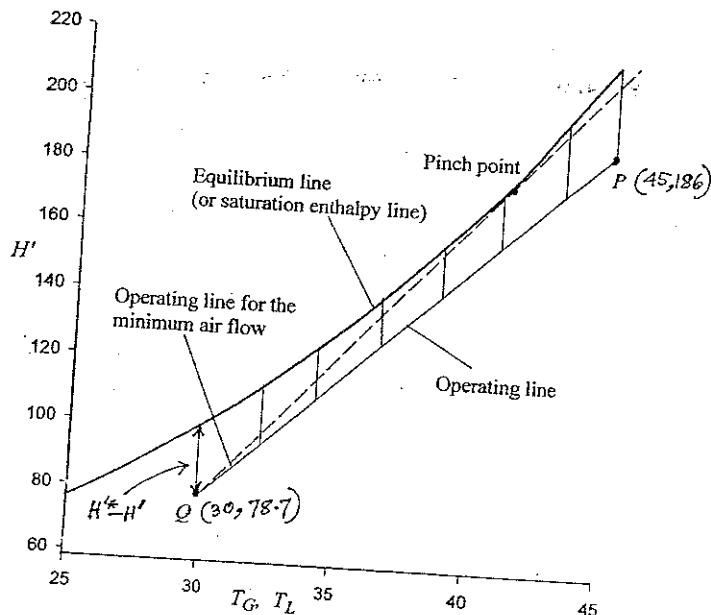


Fig. Prob. 10.8(a)

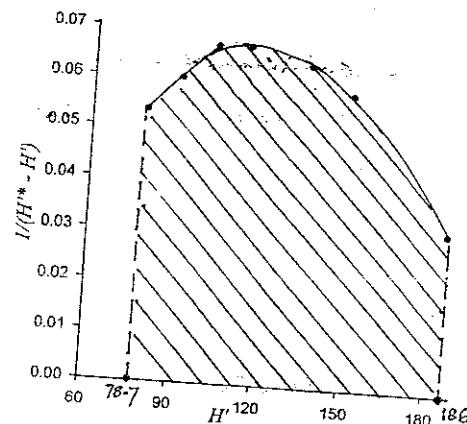


Figure Prob 10.8(b). Graphical integration for N_{t0G}

10-9

Inlet air: $T_{G_1} = 85^\circ\text{C}$, $T_{w_1} = 46^\circ\text{C} \Rightarrow \gamma_1' = 0.0687 \text{ kg/kg dry air}$
Enthalpy, $H_1' = 224 \text{ kJ/kg dry air}$

10-10

Outlet air: wet bulb temp = $31^\circ\text{C} \Rightarrow \gamma_2' = 0.0272$; $H_2' = 100.7$
(The outlet air has a wet bulb temp = 31°C
= its adiabatic saturation temp.)

Inlet water temp, $T_{L_2} = 26^\circ\text{C}$. Locate the point P (26°C ,
 100.7 kJ/kg on the H-T plane)
The upper terminal point of the operating line (that
corresponds to the bottom of the tower in case of dehumidification) is not known. But at the bottom enthalpy
of the feed moist air is $H_1' = 224 \text{ kJ/kg}$.

Now we have to calculate the minimum water rate.
Since the equil. line is convex downward, the pinch point
is obtained by drawing a horizontal line $H' = 224$ to
meet the equilibrium line (see fig Prob 10-9) at Q'.

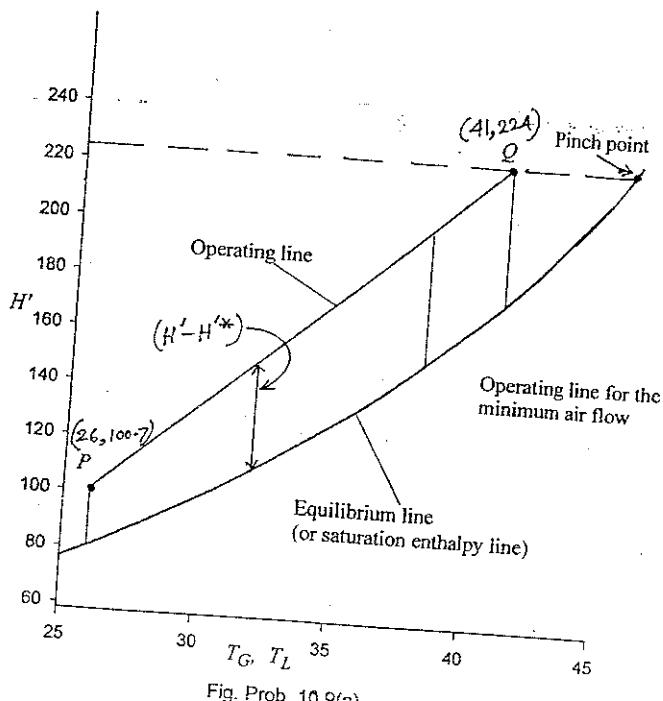
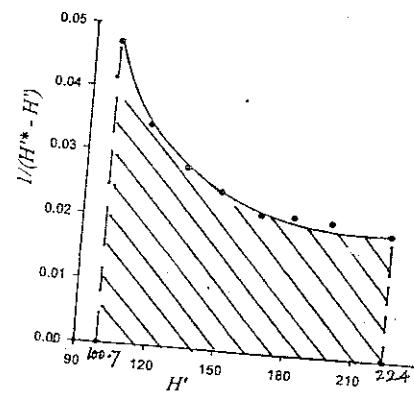


Fig. Prob. 10.9(a)

Fig. Prob 10.9(b). Graphical integration for N_{OG}

Join PQ' (this is the operating line for the minimum water rate). At Q' , the water temp is $T_{L1,\max} = 45.5^\circ\text{C}$. The minimum water rate can be calculated from the following enthalpy balance.

$$\begin{aligned} G_s(H'_1 - H'_2) &= L_{s,\min}(T_{L1,\max} - T_{L2}) \cdot c_w L \\ \Rightarrow 4680(224 - 100.7) &= L_{s,\min}(45.5 - 26)(4.187) \\ \Rightarrow L_{s,\min} &= 7450 \text{ kg/m}^2\text{h} \end{aligned}$$

$$\boxed{\begin{aligned} G_s &= \frac{G_1}{1 + Y_1} \\ &= \frac{5000}{1.0687} \\ &= \frac{4680 \text{ kg dry air}}{\text{air/m}^2\text{h}} \end{aligned}}$$

The minimum water rate can be determined from the slope of the line PQ' as well.

$$\begin{aligned} \text{Actual liquid rate} &= (1.25)L_{s,\min} = (1.25)(7450) \\ &= 9330 \text{ kg/m}^2\text{s} \end{aligned}$$

Actual outlet water temp is given by

$$4680(224 - 100.7) = 9330(T_{L1} - 26)(4.187) \Rightarrow T_{L1} = 41^\circ\text{C}$$

As to part (ii)

(ii) Water condensed per hour Corresponding operating line : PQ
Note that it lies above the equilibrium line.
= moisture removed from air = $G_s(Y'_1 - Y'_2)$

$$= 4680(0.0687 - 0.0272) = 194 \text{ kg/m}^2\text{h}$$

(\because the water rate changes by about 2%)

(iii) The packed height : $N_{tOG} = \int_{H'_2}^{H'_1} \frac{dH'}{(H' - H'^*)}$

As before, draw a set of vertical lines ('tie-lines') between the equilibrium line and the operating line. Obtain the values of $\frac{1}{(H' - H'^*)}$ for different values of H' .

$$\begin{aligned} H' &= 100.7 \quad 116.2 \quad 133.5 \quad 149 \quad 167 \quad 181.5 \quad 198 \quad 224 \\ \frac{1}{H' - H'^*} &= 0.047 \quad 0.0342 \quad 0.0278 \quad 0.0244 \quad 0.0213 \quad 0.0213 \quad 0.0208 \quad 0.0194 \end{aligned}$$

Value of integral obtained graphically = 3.23 = N_{tOG}

$$\frac{H_{tOG}}{k_y \bar{a}} = \frac{G_s}{K_y \bar{a}} = \frac{4680}{2300} = \underline{2.035 \text{ m}}$$

$$\begin{aligned} \text{Height of the packed section} &= (2.035)(3.23) \\ &= \underline{\underline{6.57 \text{ m}}} \end{aligned}$$

* * * *

10.10

Vapor pressure of water at $27^\circ\text{C} = 0.0352 \text{ bar}$
 Saturation humidity at this temperature = 0.0224 kg/kg
 $= Y'_s$ of the feed air.

10.12

Humidity of the inlet air ($T_G = 80^\circ\text{C}$, $T_w = 27^\circ\text{C}$) = 0.001 kg/kg
 Outlet air is 95% saturated air leaving
 the tower = $(0.95)(0.0224) = 0.0213 \text{ kg/kg} = Y'_2$

Since the water is at the adiabatic saturation temperature of the air, use Eq(10.38) to calculate $N_{t0G} \Rightarrow N_{t0G} = \ln \frac{Y'_s - Y'_1}{Y'_s - Y'_2}$
 $= \ln \frac{0.0224 - 0.001}{0.0224 - 0.0213} = 3$

Rate of supply of water required
 $=$ Rate of vaporization loss
 $= G_s(Y'_2 - Y'_1) = (4000)(0.0213 - 0.001) = 88.4 \text{ kg/h}$
 * * * for 1 m^2 of tower cross-section.

10.11

Range of cooling = $45 - 28 = 17^\circ\text{C} = 30.6^\circ\text{F}$

Evaporation loss (Eq 10.40), $E = (10,000)(30.6)(0.0008) = 245 \text{ gph}$

Drift and leakages, $D = 0.1\%$ of the feed water = 10 gph
 Ratio of TDS in the water and in the makeup

$$r = \frac{G_2}{C_1} = \frac{600}{250} = 2.4$$

$$\text{Blowdown rate, } B = \frac{[245 - (10)(2.4 - 1)]}{(2.4 - 1)} \\ = 165 \text{ gph.}$$

$$\text{Make up water} = B + D + E = 165 + 10 + 245 \\ = 420 \text{ gph.}$$

* If we assume that the latent heat of vaporization comes solely from the sensible heat of water, evaporation rate = $(10,000)(3.78) \text{ kg/h} \cdot (17)(4.187)/2500$

= $1076 \text{ kg/h} = 285 \text{ gph}$ which is a little more than that calculated above. This is reasonable.

(10.12) The figure here shows the path followed by the drying air.

We shall calculate — ^{The 'heat duty'}
 of dryer \rightarrow enthalpies ^{and humidities} of air entering and leaving the dryer \rightarrow air flow \rightarrow Temp. T_1 \rightarrow LMTD in the air heater \rightarrow area of the heater.

Heat duty of the dryer : Heat required for evaporation of 300 kg/h water $= (300)(2370) = 7.11 \times 10^5 \text{ kJ/h}$; add 15% to take care of sensible heat supply to the solid and heat losses $\Rightarrow Q = (7.11 \times 10^5)(1.15) = 8.176 \times 10^5 \text{ kJ/h}$.

Feed air : $T_G = 25^\circ\text{C}$, R.H. = 70%. Vapor pressure of water at $25^\circ\text{C} = 0.0313 \text{ bar} \Rightarrow Y = \frac{(0.0313)(0.7)}{1.013 - (0.0313)(0.7)} = 0.01373 = Y_1$

Exhaust air : $T_G = 55^\circ\text{C}$, $T_w = 35^\circ\text{C} \Rightarrow Y_2 = 0.0297 \text{ kg/kg dry air}$.

Rate of supply of drying air $= \frac{\text{moisture removed/h}}{Y_2 - Y_1} = \frac{300 \text{ kg/h}}{0.0297 - 0.01373} = 18785 \text{ kg/h}$
 Total air rate, $G = (18785)(1 + 0.01373) = 19043 \text{ kg/h} \leftarrow G_s = 18785 \text{ kg/h, dry basis}$

Enthalpy of exhaust air $= \frac{19043 \text{ kg/h}}{\text{see Eq (10.8)}} \Rightarrow H_2' = (2500)(0.0297) + [1.005 + (0.0297)x] = 132.6 \text{ kJ/kg dry air}$ (10.88) (55)

Heat duty, $Q = G_s(H_1' - H_2') \Rightarrow 8.176 \times 10^5 = (18785)(H_1' - 132.6)$

$$\Rightarrow H_1' = 176.1 \text{ kJ/kg dry air.}$$

Corresponding temperature is given by

$$176.1 = (2500)(0.01373) + [1.005 + (1.88)(0.01373)](T_1 - 0) \Rightarrow T_1 = 137.5^\circ\text{C}$$

Latent heat of steam at 5 atm, gauge $= 2090 \text{ kJ/kg}$, temp, $T_{st} = 159^\circ\text{C}$

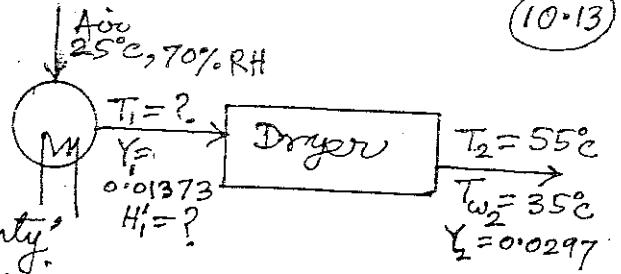
Air heater : Temperature driving force, $(\Delta T)_{in} = 159 - 25 = 134^\circ\text{C}$

$$(\Delta T)_{out} = 159 - 137.5 = 21.5^\circ$$

$$\Rightarrow (\Delta T)_m = \frac{61.5^\circ\text{C}}{32 \text{ W/m}^2\text{C}}, \text{ given} \quad \text{Area of the heater} = \frac{Q}{h \cdot (\Delta T)_m} = \frac{8.176 \times 10^5 \times 1000}{(32)(61.5)(3600)}$$

$$\text{Rate of steam condensation} \Rightarrow A = 115.5 \text{ m}^2$$

$$= \frac{8.176 \times 10^5 \text{ kJ/h}}{2090 \text{ kJ/kg}} = 391 \text{ kg/h.}$$



(10.13)

(11.1)

- ① (ii), since sand is totally nonhygroscopic and all the moisture is "free"; ② (ii) and (iv); ③ (a) ~~→~~ (iii), compared to a 5 cm thick bed, a 10 cm bed of the same material will have a higher critical moisture; ④ (b) \rightarrow (i), the transport resistance to water diffusion is less if the particle size is larger, (c) \rightarrow (i), see Fig. 11.7(a) for these answers; ⑤ (iii) \rightarrow 28°C , which is the wet bulb temperature of the drying air; ⑥ See p. 11.4;

⑦ $k_e/k_{Y_1} C_H = 1$, for

$$Y' = 0.018, C_H = 1040 \text{ J/kg}^\circ\text{C}, k_{Y_1} = 0.144 \text{ kg/m}^2\text{s}(\Delta Y') = 520 \frac{\text{kg}}{\text{m}^2\text{R}(\Delta Y)}; \\ ⑧ (\text{iii}) \rightarrow \text{Since } x^* = 0, t_1 = \left(\frac{W_0}{a} \right) \left(\frac{x_c}{N_c} \right) \ln \frac{x_c}{x_c/10} \text{ for linear falling rate} \\ t_2 = \left(\frac{W_0}{a} \right) \left(\frac{x_c^2}{N_c} \right) \left(\frac{1}{x_c/10} - \frac{1}{x_c} \right) \Rightarrow t_1/t_2 = 0.256$$

- ⑨ (ii) $\rightarrow (N_{tG})_f = \ln \frac{90-33}{70-33} = 0.432$; ⑩ (i) $\rightarrow \ln \frac{T_{Gi}-T_s}{T_{Go}-T_s} = 35$,
 $(\Delta T)_{LM} = (T_{Gi}-T_{Go})/\ln \left(\frac{T_{Gi}-T_s}{T_{Go}-T_s} \right) = 35/3.5 = 10$; ⑪ (ii);
⑫ (ii), since the temperature of the solid remains low; ⑬ (iii);
⑭ yes; ⑮ (ii), (iii) and (vi); ⑯ (i); ⑰ (i); ⑱ (ii);
⑲ (i) spray drier, (ii) drum drier, (iii) direct heat rotary
(hot air, not flue gas, may be used), (iv) rotary, (v) spray,
(vi) belt or conveyor, (vii) belt or conveyor/tray; (viii)
rotary, (ix) tunnel, (x) tunnel, (xi) indirect heat rotary;

- ⑳ If the surface is completely moist (at the wet bulb temp. of the drying gas), constant rate drying flux, $N_c = k_{Y_1} \Delta Y'$
 $= (120)(0.037 - 0.018) = 2.28 \text{ kg/h m}^2$ ($Y'_s = 0.037$) \Rightarrow the
fraction of surface wetted patches $= (1.5 \text{ kg/h m}^2)/(2.28 \text{ kg/h m}^2)$
 $\Rightarrow 65.3\%$ (Note: drying also occurs at the dry patches on the top surface also by vapor diffusion; this has been neglected here.); ⑳ (ii); ㉑ (iii);
㉒ (iii); ㉓ (iii); ㉔ See at the end; ㉕ (c) \rightarrow so long as

(11.2)

The surface of the solid remains moist, the rate of drying, and hence the amount of heat supplied by the gas, remains constant $\Rightarrow T_i - T_0$ remains constant. After this the solid gradually warms up, the rate of heat absorption by the solid decreases, T_0 increases and $T_i - T_0$ decreases; (26) see at the end; (27) (i);

(28) (a) \rightarrow (i), (b) \rightarrow (ii), (c) \rightarrow (ii); (29) (iii), since there is radiant heating, the solid temperature will be above the adiabatic saturation temperature of the gas;

$$(30) (ii) \rightarrow \frac{W_s}{a} \cdot \frac{x_i - x_c}{N_c} = \frac{W_s}{a} \cdot \frac{x_c - x^*}{N_c} \cdot \ln \frac{x_c - x^*}{x_f - x^*}$$

$$\text{given } - x_i = 0.33, x_f = 0.01, x^* \approx 0 \Rightarrow x_c = 0.1$$

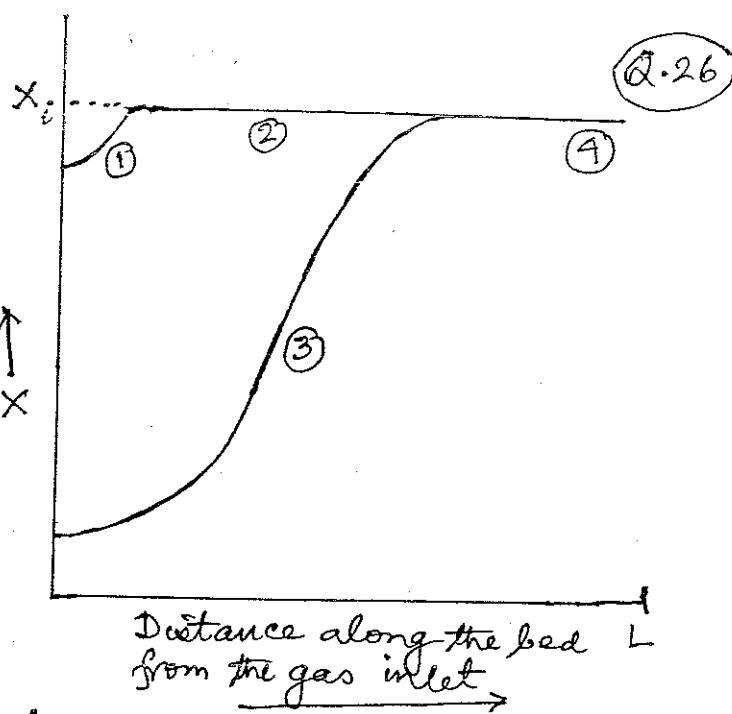
$$(31) (ii) \text{ and } (iii); (32) (ii); (33) (ii)$$

(24) Curve A: There is a single linear falling rate period. A linear falling rate occurs when portions of the surface gets dried up while the rest remains moist. The fractional moist area depends upon the moisture content (see Q.20). This is "unsaturated surface drying". This is observed for a bed of high porosity, big pores, nonhygroscopic solids and a shallow bed.

Curve B: The shape of the drying curve indicates two falling rate zones — one linear and another nonlinear. The solid has both large and small pores. Unsaturated surface drying occurs over a part of the falling rate period (the linear part) because of rapid transport of moisture through the bigger pores. The lower nonlinear part indicates substantial diffusional resistance probably because of diffusion through regions having small pores.

Curve C: The entire falling rate zone is nonlinear. The solid has a fine porous structure creating substantial resistance to liquid diffusion all through the falling rate. This behavior also occurs for a hygroscopic solid.

11.3

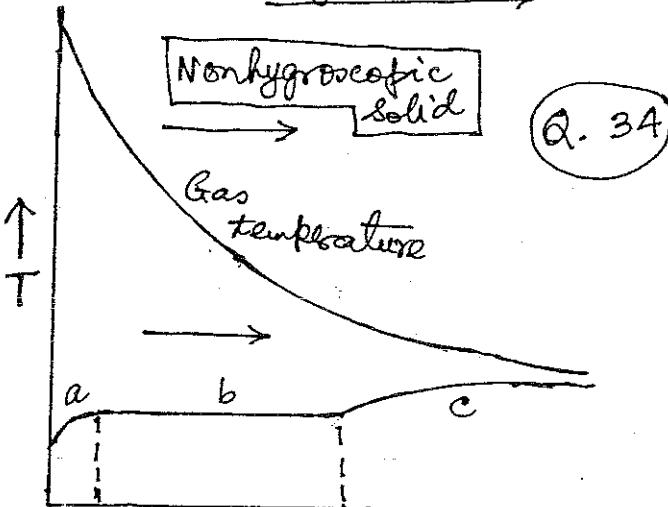


Distance along the bed
from the gas inlet L

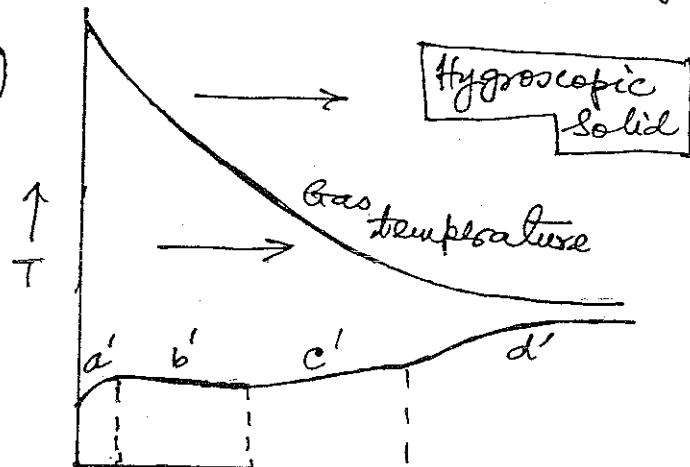
The curve ①-②-④ is representing the moisture profile at a small time after drying starts. Most part of the feed except a small region near the inlet, region ①, has the initial moisture content.

After considerable time has elapsed the concentration profile would look like

③-④ region. There may still be some bound moisture which escapes rather slowly.



- $L \rightarrow$
- Zone a : heating of the solid to the wet bulb temperature,
 - Zone b : loss of moisture while the solid is at the wet bulb temperature.
 - Zone c : heating of the solid with a small moisture content.



- $L \rightarrow$
- Zone a' : initial heating of the solid
 - Zone b' : solid at the wet bulb temperature
 - Zone c' : the solid gets heated while most of the bound moisture is lost.
 - Zone d' : further heating of the solid with a small moisture content.

- (35) In a direct heat rotary dryer, a large quantity of heating gas has to be passed since the source of heat is the hot gas alone. Also the exit gas temperature should be considerably above the solid temperature so that a reasonable temperature driving force is available. Thus the drying gas leaves with a large quantity of heat in it. In an indirect heat unit, on the other hand, a small quantity of inert gas flows through the dryer for carrying away the vapor generated during drying. Its quantity and temperature both remain substantially smaller than in the direct heat unit. As a result the indirect heat dryer is more energy efficient.

(5) $L_s = (H_{tG})_m \cdot (N_{tG})_m = (H_{tG})_R \cdot (N_{tG})_R ;$ $\begin{matrix} R \rightarrow \text{heat transfer} \\ m \rightarrow \text{mass transfer} \end{matrix}$

$$(H_{tG})_m = \frac{G_s}{k_y a} ; \quad (H_{tG})_R = \frac{G_s \cdot c_H}{h_c a}$$

$$\frac{(N_{tG})_m}{(N_{tG})_R} = \frac{L_s / (H_{tG})_m}{L_s / (H_{tG})_R} = \frac{G_s c_H}{h_c a} \cdot \frac{k_y a}{G_s} = \frac{k_y c_H}{h_c} \approx 1 \rightarrow (iii)$$

for the air-water system.

- (6) Since there is some additional heat gain by the solid through the tray floor by conduction, the solid-surface will be above the wet-bulb temperature of the air $\Rightarrow T_s > T_w \rightarrow (iii)$

- (36) We use a prime ('') to indicate the quantities in SI unit.
- $$L_t (\text{feet}) = L'_t (\text{metres}) / 0.3048 ; \quad c_H (\text{Btu}/\text{lb}^{\circ}\text{F}) = \frac{c'_H \text{ kJ/kg}^{\circ}\text{C}}{4.187}$$
- $$d (\text{feet}) = d' (\text{m}) / 0.3048 ; \quad G' (\text{kg}/\text{ft}^2 \cdot \text{h}) = (G')' / 4.8825 \text{ kg/m}^2 \cdot \text{h}$$
- $$\Rightarrow L'_t / 0.3048 = (0.1) \left(\frac{d'}{0.3048} \right) \left(\frac{c'_H}{4.187} \right) \left[\frac{(G')'}{4.8825} \right]^{0.84}$$
- $$\Rightarrow L'_t = 0.0053 c'_H \cdot d' [(G')']^{0.84} \rightarrow \text{Eq. (11-44)}$$
- where the quantities are expressed in the SI unit.

PROBLEMS

11.5

- (11.1) Since the falling rate of drying is linear in the moisture concentration, X , the drying time can be calculated directly from Eq(11.5).

Given: Solid loading, $(W_s/a) = \underline{35 \text{ kg/m}^2 \text{ (dry solid)}}$; $X_i = \text{initial moisture content (dry basis)} = \underline{0.30}$; $X_c = \underline{0.1}$; $X^* = \text{equilibrium moisture} = 0.2\% = \underline{0.002}$; final moisture, $X_f = 1\% = \underline{0.01}$.
 $N_c = \underline{4.5 \text{ kg/m}^2 \text{ h}}$

$$\text{Constant rate drying time}, t_c = \frac{(W_s)(X_i - X_c)}{N_c} = \frac{(35)(0.3 - 0.1)}{4.5} = \underline{1.55 \text{ h}}$$

$$\begin{aligned} \text{Falling rate drying time}, t_f &= \frac{(W_s)(X_c - X^*)}{N_c} \ln \frac{X_c - X^*}{X_f - X^*} \\ &= \frac{(35)(0.1 - 0.002)}{4.5} \ln \frac{0.1 - 0.002}{0.01 - 0.002} = \underline{1.91 \text{ h}} \end{aligned}$$

$$\text{Total time}, t_c + t_f = 1.55 + 1.91 = \underline{3.46 \text{ h}} \leftarrow \text{Ans to (i)}$$

- (ii) The drying rate in the falling rate period is $N = pX + q$

$$\text{At } X = X_c = 0.1, N = N_c = 4.5, \text{ and } X = X^* = 0.002, N_c = 0$$

$$\Rightarrow p = 45.92 \text{ and } q = -0.092$$

$$\text{When } X = 5\% = 0.05, \text{ drying rate } N = (45.92)(0.05) - 0.092$$

$$* * * = \underline{2.2 \text{ kg/m}^2 \text{ h}}$$

- (11.2) We first derive the working equation for the problem.

The initial moisture content = X_i ; equilibrium moisture, $X^* \approx 0$
 $X \geq X_{c_1}$, the drying rate is constant, $N = N_c$.

$X_{c_1} > X > X_{c_2}$, first falling rate period, $N_f = \alpha_1 X + \alpha_2$

$X_{c_2} \geq X \geq 0$, second " " " ", $N_f = \beta X^2$

The constants α_1, α_2 and β are to be determined; N_c is given.

$$\text{At } X = X_{c_1}, \alpha_1 X_{c_1} + \alpha_2 = N_{f_1} = N_c \quad \text{--- (i)}$$

$$\text{At } X = X_{c_2}, \alpha_1 X_{c_2} + \alpha_2 = N_{f_2}; \text{ also at } X = X_{c_2}, N_{f_2} = \beta X_{c_2}^2$$

Also we assume continuity of $\frac{dN}{dx}$ at $X = X_{c_2}$

$$\Rightarrow \frac{d}{dx}(N_{f_1}) = \frac{d}{dx}(N_{f_2}) \text{ at } X = X_{c_2}$$

$$\Rightarrow \underline{\alpha_1 = 2\beta X_{c_2}} \quad \text{--- (ii); equating drying rates at } X = X_{c_2}, \underline{\alpha_1 X_{c_2} + \alpha_2 = \beta X_{c_2}^2} \quad \text{--- (iii)}$$

(11.2) contd...

(11.6)

Solving eqs. (i), (ii) and (iii), $\alpha_1 = \frac{2N_c}{2x_{c_1} - x_{c_2}} \Rightarrow \alpha_2 = -\frac{N_c x_{c_2}}{2x_{c_1} - x_{c_2}}$

Drying time in different periods and $\beta = \frac{N_c}{x_{c_2}(2x_{c_1} - x_{c_2})}$

Constant rate period; $t_c = \left(\frac{W_s}{a}\right) \left(\frac{x_i - x_{c_1}}{N_c}\right)$ (i)

First falling rate period [see Eq.(11.1)]

$$t_{f_1} = \frac{W_s}{a} \int_{x_{c_2}}^{x_{c_1}} \frac{dx}{N} = \frac{W_s}{a} \int_{x_{c_2}}^{x_{c_1}} \frac{dx}{\alpha_1 x + \alpha_2} = \frac{W_s}{a} \cdot \frac{1}{\alpha_1} \ln \frac{\alpha_1 x_{c_1} + \alpha_2}{\alpha_1 x_{c_2} + \alpha_2}$$

$$= \frac{W_s}{a} \cdot \frac{2x_{c_1} - x_{c_2}}{2N_c} \cdot \ln \frac{N_c_1}{N_c_2}$$

$$\boxed{\begin{aligned} N_c_1 &= N_c \\ N_c_2 &= \alpha_1 x_{c_2} + \alpha_2 = \frac{N_c x_{c_2}}{2x_{c_1} - x_{c_2}} \end{aligned}}$$

Second falling rate period:

$$t_{f_2} = \frac{W_s}{a} \int_{x_f}^{x_{c_2}} \frac{dx}{N} = \frac{W_s}{a} \int_{x_f}^{x_{c_2}} \frac{dx}{\beta x^2} = \frac{W_s}{a} \cdot \frac{1}{\beta} \cdot \left(\frac{1}{x_f} - \frac{1}{x_{c_2}} \right)$$

$$= \frac{W_s}{a} \cdot \frac{x_{c_2}(2x_{c_1} - x_{c_2})}{N_c} \cdot \left(\frac{1}{x_f} - \frac{1}{x_{c_2}} \right)$$

Numerical: $x_i = 0.32$, $x_{c_1} = 0.183$, $x_{c_2} = 0.097$, $x_f = 0.01$, $N_c = 4 \frac{\text{kg}}{\text{km}^2}$

From (i), $t_c = (30) \left(\frac{0.32 - 0.183}{4} \right) = 1.027 \text{ hr}; \quad \left(\frac{W_s}{a} \right) = 30 \text{ kg/m}^2$

From (ii), $t_{f_1} = (30) \cdot \frac{(2)(0.183) - 0.097}{(2)(4)} \ln \frac{2x_{c_1} - x_{c_2}}{x_{c_2}} = 1.029 \text{ hr}$

From (iii), $t_{f_2} = (30) \cdot \frac{(0.097)[(2)(0.183) - 0.097]}{4} \left(\frac{1}{0.01} - \frac{1}{0.097} \right)$

$$= 17.552 \text{ hr}$$

Total time, $t = t_c + t_{f_1} + t_{f_2} = 1.027 + 1.029 + 17.552 = 19.6 \text{ hr}$
Assumption made: continuity of dN/dx at $x = x_{c_2}$.

(11.3) Case 1, laboratory test, use Eq.(11.5)

Drying time, $t_1 = \frac{W_s}{a} \cdot \frac{x_i - x_c}{N_c} + \frac{W_s}{a} \cdot \frac{x_c - x^*}{N_c} \ln \frac{x_c - x^*}{x_f - x^*}$

$$\Rightarrow t_1 = \frac{W_s}{a \cdot N_c} \left[(0.28 - 0.1) + (0.1 - 0.005) \ln \left(\frac{0.1 - 0.005}{0.02 - 0.005} \right) \right]$$

$$\Rightarrow 8.5 = \frac{W_s}{a \cdot N_c} (0.18 + 0.175) \Rightarrow \frac{W_s}{a N_c} = 23.92$$

use this value in further calculations

$$\boxed{\begin{aligned} x_i &= 0.28, x_c = 0.1 \\ x^* &= 0.005, x_f = 2\% \\ t &= 8.5 \text{ hr} = 0.02 \end{aligned}}$$

(11.3) contd...

(11.7)

Case 2, drying time to be calculated, use (11.5) again

$$t_2 = \frac{W_s}{a N_c} \left[(0.25 - 0.1) + (0.1 - 0.005) \ln \frac{0.1 - 0.005}{0.015 - 0.005} \right]; \text{ here } x_f = 0.015 \text{ (1.5%)} \\ = (23.92) [0.15 + 0.214] = \underline{\underline{8.7 \text{ hr}}}$$

The highest drying rate : it occurs in the constant rate period.

$$\frac{W_s}{a N_c} = 23.92 \Rightarrow N_c = \left(\frac{W_s}{a} \right) / 23.92 = 20 / 23.92 = 0.836 \frac{\text{kg}}{\text{m}^2 \text{hr}}$$

The lowest drying rate : it occurs when $x = x_f$

$$\text{At } x = x_c = 0.1, N = N_c = p x_c + q \Rightarrow 0.1p + q = 0.836 \quad p = 8.8$$

$$\text{At } x = x^* = 0.005, N = 0 \Rightarrow 0.005p + q = 0 \quad q = -0.044$$

Lowest drying rate (at $x_f = 0.015$) is $N = (8.8)(0.015) - 0.044$

$$* * * = \underline{\underline{0.088 \text{ kg/m}^2 \text{hr}}} *$$

(11.4) Test drying results : $x_i = \frac{0.25}{1 - 0.25} = 0.333; x_c = \frac{0.107}{0.893} = 0.12$

$$x_f = \frac{0.0152}{0.9848} = 0.0154; x^* = \frac{0.005}{0.107} \text{ (5% dry basis); } W_s = 10 \text{ kg; drying time} = 2 \text{ hr}$$

$$\text{Area of drying (consider both sides), } a = 2 \times (0.3 \times 0.18 \text{ m}^2) = 0.108 \text{ m}^2$$

$$\text{Substituting in Eq (11.5), } t = 2 = \frac{W_s}{a N_c} \left[(0.333 - 0.12) + (0.12 - 0.0154)x \right]$$

$$\Rightarrow 2 = \frac{10}{(0.108)N_c} (0.4644) \Rightarrow N_c = 2.15 \frac{\text{kg}}{\text{m}^2 \text{hr}} \quad \ln \frac{0.12 - 0.005}{0.0154 - 0.005}$$

Time of drying of the 1m x 1m board

Since the thickness of the board is the same as that of the above,

$$\frac{W_s}{a} \text{ remains the same} \Rightarrow W_s/a = \frac{10 \text{ kg}}{0.108} = 9.26 \text{ kg/m}^2$$

Drying time to reduce the moisture content from $x_i = \frac{0.35}{0.65}$

$$= \underline{\underline{0.5385}} \text{ to } x_f = \frac{1}{99} = \underline{\underline{0.0101}}$$

$$\text{Time} = \frac{9.26}{2.15} \left[(0.5385 - 0.12) + (0.12 - 0.005) \ln \frac{0.12 - 0.005}{0.0101 - 0.005} \right]$$

$$= \frac{9.26}{2.15} (0.7975) = \underline{\underline{3.346 \text{ hr}}}$$

* * * *

(11.8)

- 11.5 In order to solve the problem, the following quantities will be first calculated in sequence: $Re \rightarrow h_c \rightarrow k_y \rightarrow (H_t G)_m$ and $(N_t G)_m \rightarrow N_c \rightarrow$ drying time.

Drying gas $\rightarrow T_g = 90^\circ C$, $Y_i = 0.018 \Rightarrow T_w = 36.5^\circ C$, $Y_s = 0.0417$

Since the outlet humidity is not yet known, calculation of Re will be done on the basis of inlet gas flow rate.

$$\text{Humid volume, } V_H = \left(\frac{1}{28.97} + \frac{0.018}{18} \right) (22.4) \left(\frac{36.5}{273} \right) = 1.058 \frac{m^3}{\text{kg dry gas}}$$

$$\text{Density of the inlet gas, } \rho_g = \frac{1+0.018}{1.058} = 0.962 \text{ kg/m}^3$$

$$\text{Viscosity, } \mu = 2.1 \times 10^{-5} \text{ N.s/m; } u = 0.8 \text{ m/s;}$$

$$G' = (0.8 \times 3600)(0.962) = 2770 \text{ kg/m}^2\text{h, based on inlet gas flow;}$$

* [Since the gas picks up moisture, the outlet gas flow rate will be larger and an average gas rate effective diameter of a pellet, $d_p = (d_{dc} + 0.5 d_c)^{1/2}$ should be used for better accuracy]

$$= [(5)(5) + 0.5(5)^2]^{1/2} = 6.124 \text{ mm} = \frac{6.124 \times 10^{-3}}{\text{m. sec}} \quad \text{See tab 12-13}$$

$$Re = d_p u \rho_g / \mu = (6.124 \times 10^{-3})(0.8)(0.962) / 2.1 \times 10^{-5} = 224.4$$

$$\text{Calculate } h_c \text{ from Eq. (2-24)} \Rightarrow h_c = 0.214 (2770)^{0.49} / (6.124 \times 10^{-3})^{0.51}$$

$$\text{Calculate } k_y \text{ from the relation } = 140 \text{ W/m}^2\text{C}$$

$$k_y = h_c / C_H ; C_H = \text{humid heat} = [1.005 + (1.88)(0.018)] \text{ kJ/kg°C}$$

$$\Rightarrow k_y = 140 / 1040 = 0.1346 \frac{\text{kg}}{\text{m}^2\text{s(Dry)}} = 1040 \text{ J/kg°C}$$

$$(H_t G)_m = \frac{G'}{k_y \bar{a}} = \frac{2770}{(0.1346)(3600) \cdot (400)} = 0.0143 \text{ m} \quad [\bar{a} = \frac{400 \text{ m}^2\text{/m}^3}{\text{given}}]$$

$$\text{Bed height, } h_s = 30 \text{ mm} = 0.03 \text{ m} \Rightarrow (N_t G)_m = h_s / (H_t G)_m$$

$$\text{Change of humidity of the gas across the bed [Eq. (11-18 b)]} = 0.03 / 0.0143 = 2.1$$

$$Y_o - Y_i = (Y_s - Y_i) [1 - \exp(-N_t G)_m] = (0.0417 - 0.018) [1 - e^{-2.1}] = 0.0208$$

$$\text{Rate of flow of 'dry' gas, } G'_s = G'(1 - Y_i) = (2770)(1 - 0.018)$$

$$\text{Rate of moisture removal (kg/m}^2\text{ s) in the constant rate period, } N_c = G'_s (Y_o - Y_i) = (2720)(0.0208)$$

$$\text{this is the initial drying rate} \leftarrow = 56.58 \text{ kg/m}^2\text{h Ans to (1)}$$

(11.5) contd.. Mass of dry solid per unit bed cross-section,
 $(W_s/a)^0 = (1 \text{ m}^2)(0.03 \text{ m})(600 \text{ kg/m}^3) = 18 \text{ kg (per m}^2\text{ bed)}$
 Moisture contents : $x_i = 0.36$, $x_c = 0.12$, $x^* = 0$, $x_f = 0.002$.

(11.9)

Total drying time [eq(11.5)], $t = \frac{W_s}{aN_c} [(x_i - x_f) + (x_c - x^*) \ln \frac{x_c - x^*}{x_f - x^*}]$
 $= \frac{18}{56.58} \left[(0.36 - 0.002) + (0.12 - 0.0) \ln \frac{0.12 - 0}{0.002 - 0} \right] = 0.233 \text{ h} = 14 \text{ min.}$

Note : R_e has been calculated on the basis of inlet gas velocity. Ans. to (ii)

Once the outlet humidity, Y_o , is known, the outlet gas velocity and density (these depend upon Y_o) can be calculated. An average R_e can now be obtained and used for estimation of h_c and then of k_y . However, this will not change the results appreciably since the change in humidity of the gas is small.

(iii) Time required to heat the solid (from the wet bulb temperature of air, 36.5°C) to 70°C :

The bed being thin, the gas temperature will remain practically constant while heating the dry bed. The following differential heat balance can be written over the bed : $l_s p_s C_{ps} \frac{dT}{dt} = (l_s \cdot i) \bar{a} \cdot h_c (T_G - T)$

$$\Rightarrow \ln \frac{T_G - T_1}{T_G - T_2} = \frac{\bar{a} h_c}{l_s \cdot C_{ps}} \cdot t \quad \text{Given: } p_s = 600 \text{ kg/m}^3, C_{ps} = 400 \text{ J/kg}^\circ\text{C}$$

$$T_1 = 36.5^\circ\text{C}, T_2 = 70^\circ\text{C}$$

$$\Rightarrow t = \frac{(600)(400)}{(400)(140)} \ln \frac{90 - 36.5}{90 - 70} = 4.28$$

(11.6) Drying gas $\rightarrow T_G = 100^\circ\text{C}$, wet bulb temp $= 35^\circ\text{C}$, $Y = 0.0092 \frac{\text{kg}}{\text{kg}}$
 Follow the following sequence of calculation : $v_H \rightarrow P_G \rightarrow G' \rightarrow h_c \rightarrow N_c \rightarrow$ constant and falling rate drying time.

Humid volume of the gas, $v_H = \left(\frac{1}{28.97} + \frac{0.0092}{18} \right) (22.4) \left(\frac{373}{273} \right) = 1.072 \frac{\text{m}^3}{\text{kg}}$

$$P_G = \frac{(1 + 0.0092) \text{ kg}}{1.072 \text{ m}^3} = 0.9413 \text{ kg/m}^3; u = 2 \text{ m/s.}$$

$$G' = (2)(3600)(0.9413) = 6777 \text{ kg/m}^2\text{h}$$

Heat transfer coefficient [eq(11.17)], $h_c = (0.0204)(6777)^{0.8} = 23.7 \frac{\text{W}}{\text{m}^2\text{C}}$
 Solid temperature = wet bulb temp of the gas = $T_w = 35^\circ\text{C}$

(11-10)

(11.6) Contd.. Constant drying rate, $N_c = \frac{h_c(T_g - T_w)}{\lambda_w} = \frac{(23.7)(100-35)}{(2400)(1000)} = 6.42 \times 10^{-4} \text{ kg/m}^2\text{s} = 2.31 \text{ kg/m}^2\text{h}$
 λ_w = latent heat of vaporization

It is assumed that the gas temperature remains constant.

Moisture contents : $x_i = \frac{0.3}{0.7} = 0.4286$, $x_c = 0.1$, $x^* = 0.005$, $x_f = 0.01$

Solid loading = $W_s/a = \frac{(10 \text{ kg wet solid})(0.7)}{0.5 \text{ m}^2} = 14 \frac{\text{kg dry solid}}{\text{m}^2}$

Constant rate drying time, $t_c = 14(0.4286 - 0.1)/N_c$

$$= 4.6/2.31 = 1.991 \text{ hr}$$

Falling rate drying time (see eq 11.5)

$$t_f = \frac{W_s}{a N_c} \cdot (x_c - x^*) \ln \frac{x_c - x^*}{x_f - x^*} = \frac{14}{2.31} (0.1 - 0.005) \ln \frac{0.1 - 0.005}{0.01 - 0.005}$$

Total drying time, $t = 1.991 + 1.695 = 3.7 \text{ hr}$

* * *

11.7

Refer to the expressions for the times of drying in the two falling rate periods derived in Problem 11.2.

$$t_{f1} = \frac{W_s}{a} \cdot \frac{2x_c - x_{c2}}{2N_c} \ln \frac{2x_c - x_{c2}}{x_{c2}}, t_{f2} = \frac{W_s}{a} \cdot x_{c2} \left(\frac{2x_c - x_{c2}}{N_c} \right) \left(\frac{1}{x_f} - \frac{1}{x_{c2}} \right)$$

Given: $x_c = 0.1$, $x_{c2} = 0.05$, $x^* \approx 0$, $x_f = 0.005$.

$$\frac{t_{f1}}{t_{f2}} = \frac{\ln \left(\frac{2x_c - x_{c2}}{x_{c2}} \right)}{2x_{c2} \left(\frac{1}{x_f} - \frac{1}{x_{c2}} \right)} = \frac{\ln \left(\frac{0.2 - 0.05}{0.05} \right)}{(2)(0.05) \left(\frac{1}{0.005} - \frac{1}{0.05} \right)} = 0.061.$$

11.8

Refer to Eq (11.5), put $t_c = t_f$ for part (i)

$$\Rightarrow \frac{W_s}{a} \cdot \frac{x_i - x_c}{N_c} = \frac{W_s}{a} \cdot \frac{(x_c - x^*)}{N_c} \cdot \ln \frac{x_c - x^*}{x_f - x^*} \quad \begin{cases} x_i = 0.3, x^* = 0 \\ x_f = 0.003, x_c = ? \end{cases}$$

$$\Rightarrow (0.3 - x_c) = x_c \ln \frac{x_c}{0.003} \Rightarrow x_c = 0.072$$

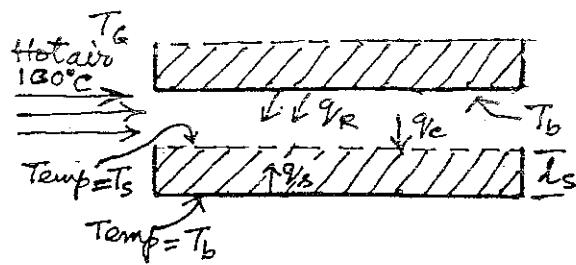
(ii) In this case $(0.3 - x_c) = \frac{1}{2} \left[x_c \ln \frac{x_c}{0.003} \right] \Rightarrow x_c = 0.108$

* * *

11.9

Two consecutive trays are shown in the sketch.

- Heat input through the four vertical edges of a tray is neglected.
- Heat input occurs at the top open surface by convection and radiation. It also occurs from the bottom of a tray by conduction through the layer of solid. (If heat inputs by radiation from above and by conduction from below are neglected, the surface temperature of the moist solid is near the wet bulb temperature.)



Steps to be followed to solve the problem: (i) write down the expressions for heat input both from above and from below; (ii) calculate the heat transfer coefficient and therefrom the mass transfer coefficient, k_y ; (iii) calculate the tray bottom temperature, then the drying rate and drying time (see example 11.3).

Heat input rate to the tray bottom by convection can be equated to the sum of the conduction and radiation heat fluxes at steady state.

$$-h_c(T_G - T_b) = \frac{k_s}{l_s} (T_b - T_s) + \alpha (T_b^4 - T_s^4) = \frac{k_s}{l_s} (T_b - T_s) + h_R (T_b - T_s) \quad (a)$$

$$\text{Heat input to the open surface} = h_c(T_G - T_s) \quad (b)$$

Some of the notations are given in the diagram above; h_R is the heat transfer coefficient for radiation. Since the open solid surface receives heat by convection, we can write the following heat balance at steady state.

$$h_c(T_G - T_b) + h_c(T_G - T_s) = k_y(Y_s - Y_e) A_w \quad (c)$$

The first term on the left has to be evaluated taking help of eq (a) above. Convective heat transfer coefficient, $h_c = 20 \text{ W/m}^2\text{°C}$, given.

Hot gas $\rightarrow T_G = 100^\circ\text{C}$, $T_w = 35^\circ\text{C} \Rightarrow Y_G = 0.0092 \text{ kg/kg dry air}$;

Humid heat, $C_H = 1.005 + (1.88)(0.0092) = 1.022 \text{ kJ/kg} = 1022 \text{ kJ/kg dry air}$

Use the relation, $k_y = h_c/C_H = 20/1022 = 0.01957 \text{ kg/m}^2\text{°C} (\Delta Y)$

(11.9) Contd.. The adiabatic saturation humidity, Y_s is related to T_s through the vapor pressure equation:

$$Y_s = \frac{P^v}{1.013 - P^v} \cdot \frac{18}{29}, \ln P^v = 11.96481 - \frac{3984.923}{T_s + 233.3}, \frac{P^v \text{ in bar}}{T_s \text{ in } ^\circ\text{C}}$$

Eq(c) above has to solved by trial. Assume a value of T_s , calculate T_b from eq(c) and check if eq(a) is satisfied. It is seen that $T_s = 46^\circ\text{C}$ and $T_b = 84^\circ\text{C}$ satisfy the equations with reasonable accuracy. For $T_s = 46^\circ\text{C}$, $Y_s = 0.069$.

The drying rate in the constant rate period

$$= k_r (Y_s - Y_f) = (0.01957)(0.069 - 0.0092) = 1.17 \times 10^{-3} \frac{\text{kg}}{\text{m}^2 \text{s}} \\ = \frac{4.213}{4.213} \frac{\text{kg}}{\text{m}^2 \text{h}} = N_c$$

Moisture concentration terms: $x_i = 0.35$; $x_c = 0.12$; $x^* = 0.005$, $x_f = 0.01$

The constant rate drying time, $t_c = \frac{W_0}{A} \left(\frac{x_i - x_c}{N_c} \right)$ [given: $(0.03)(1200) = W_0$]
 $\Rightarrow t_c = (36) \left(\frac{0.35 - 0.12}{4.213} \right) = 1.965 \text{ hr}$ $= 36 \frac{\text{kg dry solid}}{\text{m}^2}$

The falling rate time, $t_f = (36) \left(\frac{0.12 - 0.005}{4.213} \right) \ln \frac{0.12 - 0.005}{0.01 - 0.005}$

Total time of drying = 3.148 hr .

* * * *

(11.10) Here the rate of drying in the constant rate period is given as kg moisture per kg dry solid per hour, not on the basis of area of drying. A simpler version of Eq (11.5) can be derived for this case.

Given, constant rate of drying, $N' = \frac{\text{kg moisture}}{(\text{kg dry solid})(\text{hr})} = -\frac{dx}{dt}$

Integrating over the constant rate period,

$$t_c = - \int_{x_c}^{x_i} \frac{dx}{N'} = \frac{x_i - x_c}{N'}, \quad (i)$$

In the falling rate period, $-\frac{dx}{dt} = \varphi(x - x^*) \quad (ii)$

Integrating, $t_f = \frac{1}{\varphi} \ln \frac{x_c - x^*}{x_f - x^*} \quad (iii)$

When $x = x_c$, Eq (iii) gives

$$-\frac{dx}{dt} = N' = \varphi(x_c - x^*) \\ \Rightarrow \varphi = N'/(\bar{x}_c - x^*)$$

(11.10) contd...

Substituting for in Eq (iii), $t_f = \frac{x_c - x^*}{\gamma} \ln \frac{x_c - x^*}{x_f - x^*}$ (iv)

(11.13)

Given: $x_i = 1$, $x_c = 0.15$, $x^* = 0.001$, $x_f = 0.003$, $N = 0.5 \frac{kg}{m^2 \cdot hr}$

$$\begin{aligned} \text{Drying time}, t &= \frac{x_i - x_c}{N} + \frac{x_c - x^*}{N} \ln \frac{x_c - x^*}{x_f - x^*} \\ &= \frac{1 - 0.15}{0.5} + \frac{0.15 - 0.001}{0.5} \ln \frac{0.15 - 0.001}{0.003 - 0.001} = \underline{\underline{3 \text{ hours}}} \end{aligned}$$

Note: Since the drying rate is given on the basis of unit mass of dry solid, the area of drying and the total mass of dry solid do not appear in the calculations.

* * * *

(11.11) We derive the working equation first. Let N_c = drying rate in the constant rate period; N_{f_1} = first falling drying rate; N_{f_2} = second falling drying rate. Both N_{f_1} and N_{f_2} are linear in x . Given, equil. moisture, $x^* = 0$

$$x \geq x_{c_1}, N = N_c \dots (i); x_{c_1} \geq x \geq x_{c_2}, N_{f_1} = \alpha_1 x + \beta_1 \dots (ii)$$

$$\text{and } x_{c_2} \geq x \geq x^*, N_{f_2} = \alpha_2 x + \beta_2 \quad (iii)$$

$$\text{At } x = x^* = 0, N_{f_2} = 0 \Rightarrow \beta_2 = 0 \quad (iv)$$

$$\text{At } x = x_{c_2}, N_{f_1} = N_{f_2} \Rightarrow \alpha_1 x_{c_2} + \beta_1 = \alpha_2 x_{c_2} + \beta_2 = \alpha_2 x_{c_2} \dots (v)$$

$$\text{At } x = x_{c_1}, N_{f_1} = N_c \Rightarrow \alpha_1 x_{c_1} + \beta_1 = N_c \quad (vi)$$

Given:
 $x_{c_1} = 0.18, x_{c_2} = 0.1$
 $N_c = 2.5 \frac{kg}{m^2 \cdot hr}$

Time of drying over a concentration range (see Eq 11.2)

$$t = \frac{W_s}{a} \left(\frac{dx}{N} \right); \frac{W_s}{a} = \frac{7 \frac{kg}{hr}}{0.5 \frac{m^2}{hr}} = \frac{14 \frac{kg \text{ dry solid}}{m^2}}{m^2}$$

Given, drying time to reduce the moisture from $x_{c_2} = 0.1$ to $x_f = 0.03$ (in the second falling rate period) = 1.68 hr

$$\Rightarrow t_{f_2} = - \frac{W_s}{a} \int_{x_{c_2}}^{x_f} \frac{dx}{N_{f_2}} = - 14 \int_{x_{c_2}}^{x_f} \frac{dx}{\alpha_2 x} \quad [\text{from (iii), putting } \beta_2 = 0]$$

$$\Rightarrow 1.68 = \frac{14}{\alpha_2} \ln \frac{x_{c_2}}{x_f} = \frac{14}{\alpha_2} \ln \frac{0.1}{0.03} \Rightarrow \underline{\underline{\alpha_2 = 10.03}}$$

$$\text{From Eq (v), } \alpha_1(0.1) + \beta_1 = (10.03)(0.1) \Rightarrow 0.1\alpha_1 + \beta_1 = 1.003 \quad (vii)$$

$$\text{From Eq (vi), } \alpha_1(x_{c_1}) + \beta_1 = N_c \Rightarrow 0.18\alpha_1 + \beta_1 = 2.5 \quad (viii)$$

(11.11) contd.. Solving Eqs (vii) and (viii), $\alpha_1 = 18.71$, $\beta_1 = -0.868$

It is required to calculate the drying time for $x_i = 0.35$

$$\text{Constant rate drying time, } t_c = \frac{W_s/a}{N_c} (x_i - x_{c_1}) \quad \text{to } x_f = 0.01$$

$$= \frac{14}{2.5} (0.35 - 0.18) = 0.952 \text{ hr}$$

First falling rate drying time:

$$t_{f_1} = -\frac{W_s}{a} \int_{x_{c_1}}^{x_{c_2}} \frac{dx}{\alpha_1 x + \beta_1} = \frac{W_s}{a \cdot \alpha_1} \ln \frac{\alpha_1 x_{c_1} + \beta_1}{\alpha_1 x_{c_2} + \beta_1}$$

$$= \frac{14}{18.71} \ln \left[\frac{(18.71)(0.18) - 0.868}{(18.71)(0.10) - 0.868} \right] = 0.683 \text{ hr}$$

Second falling rate drying time:

$$t_{f_2} = -\frac{W_s}{a} \int_{x_{c_2}}^{x_f} \frac{dx}{\alpha_2 x} = \frac{W_s}{a \cdot \alpha_2} \ln \frac{x_{c_2}}{x_f} = \frac{14}{10.03} \ln \frac{0.10}{0.01} = 3.214 \text{ hr}$$

$$\text{Total drying time, } t = t_c + t_{f_1} + t_{f_2} = 0.952 + 0.683 + 3.214$$

$$* \quad * \quad * = 4.85 \text{ hr} \quad *$$

(11.12) The steps to be followed are (see Prob. 11.9): (i) calculate the convective heat transfer coefficient (h_c) using Eq 11.17; (ii) calculate the overall coefficient for heat flow from the bottom, h_b ; (iii) write down the equation balancing heat flow to the top open surface and calculate T_s ; (iv) calculate N_c and then the drying time.

Drying gas : $T_G = 90^\circ C$, $Y_G = 0.015$, $\rho_G = 0.964 \text{ kg/m}^3$
 $u = 3 \text{ m/s}$ (calculation not shown)

$$G' = (u \rho_G)(3600) = (3)(0.964)(3600) = 10411 \text{ kg/m}^2 \text{ s}$$

$$h_c = 0.0204 (10411)^{0.8} = 33.4 \text{ W/m}^2 \text{ }^\circ C$$

Calculation of the coefficient h_b for heat flow through the bottom
given: $k_s = 2 \frac{W}{m \cdot ^\circ C}$, $k_{ls} = 0.03 \text{ m}$
Eq (11.7) $\rightarrow h_b = \frac{h_c k_s}{k_s + h_{ls}} = \frac{(33.4)(2)}{2 + (33.4)(0.03)} = 22.25 \frac{W}{m^2 \text{ }^\circ C}$

Balance of heat flow to the open top surface, Eq (11.8)

$$(h_c + h_b)(T_b - T_s) = k_r(Y_s - Y_G) \cdot 2w \quad (i)$$

11.12) Contd... Humid heat of the drying air, $c_H = 1033 \text{ J/(kg dry air)/}^{\circ}\text{C}$

$$\text{Calculate } k_Y = h_c/c_H = \frac{33.4}{1033} = 0.0323 \frac{\text{kg}}{\text{m}^2 \Delta Y}$$

$$T_G = 90^{\circ}\text{C}, Y_G = 0.015, \lambda_w = (2400)(1000) \text{ J/kg}; T_s = \text{Surface temp of solid}$$

$$\text{From Eq.(i) above: } (33.4 + 22.25)(90 - T_s) = 0.0323 (Y_s - 0.015)$$

$$\Rightarrow T_s = 110.9 - 1393 Y_s \quad (\text{ii})$$

Eq. (ii) can be solved, as in Example 11.3, graphically. It can also be solved by trial — assume T_s , calculate the vapor pressure of water, P_v , using the vapor press. eq. (see Prob 11.9),

$$\text{and corresponding humidity, } Y_s = \left(\frac{P_v}{1.013 - P_v} \right) \left(\frac{18}{28.97} \right)$$

and check if eq. (ii) is satisfied.

$$\text{Solution: } T_s = 40.7^{\circ}\text{C} \text{ and } Y_s = 0.0503$$

$$\text{The constant drying rate} = N_c = k_Y (Y_s - Y_G)$$

$$= (0.0323)(0.0503 - 0.015)(3600) = 4.105 \frac{\text{kg}}{\text{m}^2 \text{h}}$$

$$\text{Given: } X_i = 0.4, X_c = 0.15, X^* = 0.008, X_f = 0.012$$

$$\text{Dry solid loading, } w_s/a = \lambda_g \cdot P_s = (0.03)(700) = 21 \frac{\text{kg dry solid}}{\text{m}^2}$$

$$\begin{aligned} \text{Drying time} &= \frac{w_s}{a N_c} \left[(X_i - X_c) + (X_c - X^*) \ln \frac{X_c - X^*}{X_f - X^*} \right] \\ &= \frac{21}{4.105} \left[(0.4 - 0.15) + (0.15 - 0.008) \ln \frac{0.15 - 0.008}{0.012 - 0.008} \right] = 3.87 \text{ hr} \end{aligned}$$

Part (ii) An outline of the solution is only given. ($u = 1.5 \times 3 \text{ m}/\text{s}$)

$$Q' = (0.964)(1.5 \times 3)(3600) = 15617 \frac{\text{kg}}{\text{m}^2 \text{s}} \Rightarrow h_c = 46.2 \frac{\text{W}}{\text{m}^2 \text{C}},$$

$$h_b = 27.3 \frac{\text{W}}{\text{m}^2 \text{C}}; k_Y = 46.2/1033 = 0.0447 \frac{\text{kg}}{\text{m}^2 \text{s} (\Delta Y)}$$

$$\text{Heat balance: } (46.2 + 27.3)(90 - T_s) = (0.0447)(Y_s - 0.015)(2400)(1000)$$

$$\Rightarrow T_s = 112 - 1459.6 Y_s \Rightarrow T_s = 40.3^{\circ}\text{C} \text{ & } Y_s = 0.04916 \frac{\text{kg}}{\text{kg}}$$

$$N_c = (0.0447)(0.04916 - 0.015)(3600) = 5.5 \frac{\text{kg}}{\text{m}^2 \text{h}}$$

$$\text{Drying time} = 2.9 \text{ hr. Reduction of time} = \frac{3.87 - 2.9}{3.87} = 25\%.$$

Note: (i) The temperature of the solid surface does not change much;

(ii) If there is no heat flow by conduction through the solid, the top surface temp is the wet bulb temp. of air.

(11.13)

Since the solid is in 'plug flow' and the drying gas flows through the bed all along the length of the belt, the time of drying is the same as that of batch drying for the same thickness of the bed for through circulation of the drying gas.

Calculation of batch drying time (see Prob 11.5)

Density of the hot inlet gas, $\rho_{in} = 0.964 \text{ kg/m}^3$ (see Prob 11.12)

The temperature and humidity of the gas at the exit to the bed will be different, but not known. If the gas leaves at its adiabatic saturation temperature, its humidity is $Y_s = 0.038$ (from the humidity chart).

Take the humidity of the drying gas in the bed = $Y = 0.025$

Superficial vel. $= 1 \text{ m/s}$ (this will be used for calculation of

Mass flowrate of the inlet gas = $(\mu \rho_{in})(3600) = (1)(0.964)(3600)$ flowrate and density of the gas only

If the average humidity is taken, $= 3470 \text{ kg/m}^2 \text{ h}$

mass flowrate of the gas, $G' = (3470)(1-0.015)(1+0.025)$
(Inlet gas humidity = 0.015) $= 3503 \text{ kg/m}^2 \text{ h}$

To select the correlation for calculation of the heat transfer coefficient, h , we need to calculate the Reynolds number

$$G' = 3503 \text{ kg/m}^2 \text{ h}, d_p = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}, \mu = 2.13 \times 10^{-5} \text{ Ns/m}$$

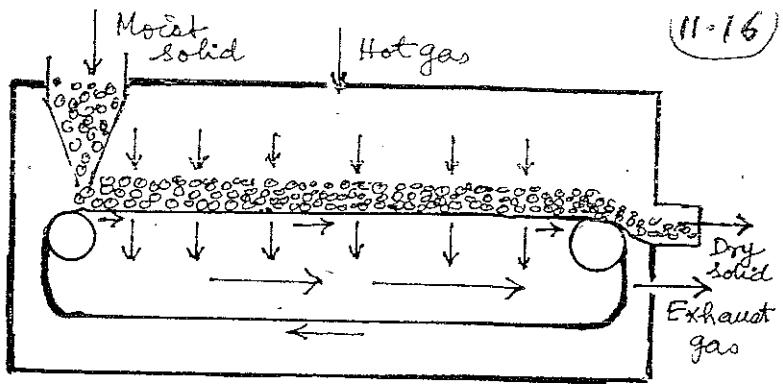
$$\text{Reynolds number, } Re = \frac{d_p G'}{\mu} = \frac{(5 \times 10^{-3})(3503)}{(2.13 \times 10^{-5})(3600)} = 228$$

For this value of Re , use Eq (11.24) to calculate h_c .

$$h_c = 0.214 (3503)^{0.49} / (5 \times 10^{-3})^{0.51} = 174 \text{ W/m}^2 \text{ °C}$$

$$\text{Mass transfer coefficient, } k_f = h_c / C_H = \frac{174}{1052} = 0.165 \text{ kg/m}^2 \text{ s (ΔY)}$$

(The humid heat $C_H = 1052 \text{ J/kg}^\circ\text{C}$)



(11.16)

(11.13) Contd...

The following quantities are given : $x_i = 0.6$, $x_c = 0.2$, $x^* = 0.01$, $x_f = 0.013$; inlet gas humidity, $y_i = 0.015$; specific surface area of the solid, $a = 500 \text{ m}^2/\text{m}^3$; thickness of the bed, $l_s = 3.5 \text{ cm} = 0.035 \text{ m}$; rate of flow of the gas on 'dry basis', $G'_s = 3470 (1 - 0.015) = 3418 \text{ kg/m}^2\text{h}$.

Now shall calculate the 'height of a gas-phase transfer unit', $(H_{tG})_m$ and then $(N_{tG})_m$ in order to calculate the drying rate.

$$(H_{tG})_m = \frac{G'}{k_y a} = \frac{3503}{(0.165)(3600)(500)} = 0.0118 \text{ m}; (N_{tG})_m = l_s / (H_{tG})_m \\ = 0.035 / 0.0118 = 2.97$$

The change in humidity of the gas across

$$\text{the bed [eq (11.18 b)]} = Y_o - Y_i = (Y_s - Y_i) [1 - \exp(-N_{tG})_m]$$

$$= (0.038 - 0.015) [1 - \exp(-2.97)] = 0.0218 \frac{\text{kg}}{\text{kg dry air}}$$

The constant rate of drying, $N_c = G'_s (0.0218) = (3418)(0.0218)$

$$\text{Dry solid loading} = (0.035)(900) = 31.5 \text{ kg/m}^2 = \frac{W_s}{a} = 78.6 \text{ kg/m}^2$$

Time of drying, $t =$

$$\frac{W_s}{N_c} \left[(x_i - x_c) + (x_c - x^*) \ln \frac{x_c - x^*}{x_f - x^*} \right] = \frac{31.5}{78.6} \left[(0.6 - 0.2) + (0.2 - 0.01) \ln \frac{0.2 - 0.01}{0.013 - 0.01} \right] \\ = 0.476 \text{ hour}$$

The belt length, $L = 10 \text{ m}$; belt velocity = $\frac{10 \text{ m}}{0.476 \text{ hr}} = 21 \text{ m/h} = \frac{0.35 \text{ m}}{\text{min}}$

Breadth of the belt = 2 m

$$\text{Production rate} = \frac{\text{rel. of belt}}{\text{bed depth}} \frac{\text{bed breadth}}{\text{bed density}} \frac{\text{density of product}}{\text{(with 1.3% moisture)}} = (21 \text{ m/h})(0.035 \text{ m})(2 \text{ m})(900 \text{ kg/m}^3) = 1323 \text{ kg/hour}$$

$$\text{Production rate (bone dry solid)} = (1323)(1 - 0.013) = 1306 \text{ kg/h}$$

$$\text{Total rate of air supply} = (3470)(2 \times 10 \text{ m}^2) = 69400 \text{ kg/h with a belt area}$$

$$\text{Air supply (dry basis)} = (69400)(1 - 0.015) = 68359 \text{ kg/h}$$

$$\text{Volumetric rate of air supply} = 69400 / 0.964 = 71992 \text{ m}^3/\text{h at } 90^\circ\text{C}$$

Moisture removed from the solid per hour

$$= (1306 \text{ kg/h, bone dry}) (x_i - x_f) = (1306)(0.6 - 0.013) \\ = 766.6 \text{ kg/h}$$

$$\text{Humidity of exhaust air } Y_i + \frac{766.6}{68359} = 0.015 + 0.0112$$

$$= 0.0262 \frac{\text{kg}}{\text{kg dry air}}$$

11.14) The problem is similar to Example 11.5. Refer to Fig Ex 11.5 for notations used.

11.18

$$T_{G_1} = 28^\circ\text{C}; Y_1 = 0.013 \text{ kg/kg dry air}; T_{G_2} = 120^\circ\text{C}; Y_2 = 0.023; T_{G_3} = 65^\circ\text{C}; \\ Y_3 = 0.043; T_{S_i} = 26^\circ\text{C}; T_{S_o} = 45^\circ\text{C}; C_{P8} = 1.2 \text{ kJ/kg°C}; \\ W_s = (1200)(1-0.4) = 720 \text{ kg/h}; X_i = 0.4/0.6 = 0.6667; X_o = \frac{0.01}{0.99} = 0.0101.$$

Calculated enthalpy values (Ref. temp. = 0°C):

$$H'_{S_i} = [C_{P8} + (X_i)(4.18)](T_{S_i} - 0) = [1.2 + (0.6667)(4.18)](26) = \frac{103.7 \text{ kJ}}{\text{kg dry air}}$$

$$H'_{S_o} = [1.2 + (0.0101)(4.18)](45) = \frac{55.9 \text{ kJ}}{\text{kg liquid}}$$

$$H'_{G_1} = [1.005 + (1.88)(0.013)](28) + (0.013)(2500) = \frac{61.3 \text{ kJ/kg}}{\text{Basis: 1-hour}}$$

$$H'_{G_2} = 183.3 \text{ kJ/kg}; H'_{G_3} = \frac{178.1 \text{ kJ/kg}}{\text{Basis: 1-hour}}$$

$$\text{Moisture balance over the dryer: } W_s(X_i - X_o) = G_{S_2}(Y_3 - Y_2)$$

$$\Rightarrow (720)(0.6667 - 0.0101) = G_{S_2}(0.043 - 0.023) \Rightarrow G_{S_2} = \frac{23638 \text{ kg}}{\text{dry basis}}$$

$$\text{Moisture balance at the heater inlet: } G_{S_1}Y_1 + (G_{S_2} - G_{S_1})Y_3 = G_{S_2}Y_2$$

$$\Rightarrow G_{S_1}(0.013) + (23638 - G_{S_1})(0.043) = (23638)(0.023) \Rightarrow G_{S_1} = \frac{15758 \text{ kg}}{\text{dry basis}}$$

$$\text{Air recirculated} = G_{S_2} - G_{S_1} = 23638 - 15758 = 7880 \text{ kg}$$

$$\text{Percentage recirculated} = 7880/23638 \Rightarrow 33.3\%.$$

$$\text{Energy balance over the heater: } G_{S_1}H'_{G_1} + (G_{S_2} - G_{S_1})H'_{G_3} + Q_H = G_{S_2}H'_{G_2}$$

$$\Rightarrow (15758)(61.3) + (7880)(178.1) + Q_H = (23638)(183.3) \Rightarrow Q_H = 1.963 \times 10^6 \text{ kJ/h}$$

$$\text{Energy balance over the dryer: } G_{S_2}H'_{G_2} + W_sH'_{S_i} = G_{S_2}H'_{G_3} + W_sH'_{S_o} + Q'$$

$$\Rightarrow (23638)(183.3) + (720)(103.7) = (23638)(178.1) + (720)(55.9) + Q'$$

$$\Rightarrow Q' = \frac{157333 \text{ kJ/hr}}{\text{rate of}} \rightarrow \text{heat loss from the dryer.}$$

Heater area calculation: We first calculate the temperature of the gas (it is a mixture of fresh air and recirculated gas) at the heater inlet.

Enthalpy balance for the gas mixture:

$$G_{S_1}H'_{G_1} + (G_{S_2} - G_{S_1})H'_{G_3} = G_{S_2} \cdot (H'_{G_2})_{\text{mix}} \Rightarrow \text{calculate } (H'_{G_2})_{\text{mix}} = 100.2 \frac{\text{kJ}}{\text{kg}}$$

$$\text{Temperature of the mixed gas} = (T_G)_{\text{mix}}, Y = Y_2 = 0.023$$

$$(H'_{G_2})_{\text{mix}} = 100.2 = [1.005 + (1.88)(0.023)](T_G)_{\text{mix}} + (2500)(0.023) \Rightarrow (T_G)_{\text{mix}} = 41^\circ\text{C}$$

$$\text{At the heater inlet, } (\Delta T)_{in} = 155 - 41 = 114^\circ\text{C}; \text{ outlet, } (\Delta T)_{out} = 155 - 120 = 35^\circ\text{C}$$

$$(\Delta T)_{LM} = 67^\circ\text{C}. \text{ Area} = Q_H / h_{LM}(\Delta T)_{LM} = (1.963 \times 10^6)(1000) / (20)(3600)(67) = \frac{271 \text{ m}^2}{\text{Steam temp., 4 bar absolute, saturated}} = 155^\circ\text{C}; \text{ heat trans. coeff} = 30 \text{ W/m}^2\text{°C}$$

(11.19)

(11.15) Refer to Fig. Ex 11.6. We have to consider zones I and II only, since no heating of the solid occurs.

Hot gas : temperature, $T_G = T_{G_3} = 150^\circ\text{C}$, $Y = Y_3 = 0.02$. The solid is at the wet bulb temperature of the drying gas, i.e. $T_w = T_s = 45.5^\circ\text{C}$, corresponding saturation humidity, $Y_s = 0.0662$.

We first calculate the temperature and humidity of the gas at boundaries 1 and 2 by moisture and energy balance. Since the dryer is 'adiabatic' the solid remains at $T_s = 45.5^\circ\text{C}$ all through. Given, $X_1 = 0.3/0.7 = 0.4286$; $X_c = 0.12/0.88 = 0.1364$; $X_3 = 0.008/0.992 = 0.00806$; equil. moisture, $X^* = 0$; $C_p = 0.8 \text{ kJ/kgK}$

Enthalpy values of the gas and the solid at the boundaries are:

$$\underline{\text{Boundary 3}} : H'_{G_3} = [1.005 + (1.88)(0.02)](150) + 6.02(2500) = 206.4 \text{ kJ/kg}$$

$$\underline{\text{Solid}} : H'_{S_3} = (C_{ps} + C_{pw} \cdot X_3)(T_s - 0) = [0.8 + (0.00806)(4.187)](45.5) = 38 \text{ kJ/kg dry solid}$$

$$\underline{\text{Boundary 2}} : X = X_c, T = T_s = 45.5^\circ\text{C}; H'_{S_2} = 62.4 \text{ kJ/kg dry solid}$$

$$\underline{\text{Boundary 1}} : X_1 = 0.4286, T_s = 45.5^\circ\text{C}, H'_{S_1} = 118 \text{ kJ/kg}; H'_{G_1} = ?$$

$$\underline{\text{Overall moisture balance}} : L_s(X_1 - X_3) = G_s(Y_1 - Y_3); \frac{L_s}{G_s} = \frac{1500 \text{ kg/hr}}{15000 \text{ "}}$$

$$\Rightarrow 1500(0.4286 - 0.00806) = 15000(Y_1 - 0.02) \Rightarrow Y_1 = 0.06205$$

$$\underline{\text{Overall energy balance}} : L_s(H'_{S_1} - H'_{S_3}) = G_s(H'_{G_1} - H'_{G_3})$$

$$215.6 = [1.005 + (0.06205)(1.88)] T_{G_1} \Rightarrow H'_{G_1} = 215.6 \text{ kJ/kg dry air.}$$

$$+ (0.06205)(2500) \Rightarrow T_{G_1} = 54^\circ\text{C} = \text{temp. of exit gas.}$$

$$\underline{\text{Zone II}} : \underline{\text{Moisture balance}}, L_s(X_2 - X_3) = G_s(Y_2 - Y_3), X_2 = X_c = 0.1364$$

$$\Rightarrow 1500(0.1364 - 0.00806) = 15000(Y_2 - 0.02) \Rightarrow Y_2 = 0.0328$$

$$\underline{\text{Energy balance}} : L_s(H'_{S_2} - H'_{S_3}) = G_s(H'_{G_2} - H'_{G_3}) \Rightarrow H'_{G_2} = 210 \text{ kJ/kg}$$

$$[1.005 + (0.0328)(1.88)](T_{G_2} - 0) + (0.0328)(2500) = 210 \Rightarrow T_{G_2} = 120^\circ\text{C}$$

Use Eq. (11.33) to calculate the drying time in zone I.

$$t_I = \frac{G_s}{k_y(L_s a)} \ln \frac{Y_s - Y_2}{Y_3 - Y_1} = \frac{15000}{(115)(1500 \times 0.08)} \ln \frac{0.0662 - 0.0328}{0.0662 - 0.06205} = 2.267 \text{ hr}$$

Drying time in zone II, Eq. (11.37)

$$t_{II} = \frac{(15,000)(0.1364)}{(115)(1500 \times 0.08)} \cdot \frac{1}{0.00806 + \frac{15000}{1500}(0.0662 - 0.02)} \cdot \ln \frac{(0.0662 - 0.02) \cdot (0.1364)}{(0.0662 - 0.0328)(0.00806)}$$

$$= 0.995 \text{ hr}$$

$$\text{Total drying time} = 2.267 + 0.995 = 3.26 \text{ hr}$$

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(11-20)

11-16 Refer to fig Ex 11-6. Since the dry solid is further heated, all the three zones are to be considered.

Given: $G_s = 12,000 \text{ kg/h}$ (dry basis); $L_s = 1000 \text{ kg/h}$ (dry basis); $X_1 = 0.20$; $X_c = 0.08$; $X_f = 0.01 = X_4$ ($= X_3$, we assume no drying in zone III); $X^* \approx 0$; $T_{G4} = 140^\circ\text{C}$; $Y_4 = 0.017$; $c_{ps} = 1.1 \text{ kJ/kg dry solid \cdot K}$.

The solid temperature at boundary 3 (which is equal to the wet bulb temperature of the gas at that point) is to be calculated by trial. The dry solid leaves after it is heated to 65°C . (this is to be checked)

Zone III: Assume $T_s (= T_w) = 44^\circ\text{C}$, saturation humidity, $Y_s = 0.0625$; $H'_{S3} = 50.2 \text{ kJ/kg dry solid}$ (Calculation not shown); $H'_{G4} = \frac{187.7 \text{ kJ/kg}}{H'_{S4} = 74.2 \text{ kJ/kg}}$

Energy balance: $L_s (H'_{S4} - H'_{S3}) = G_s (H'_{G4} - H'_{G3}) \Rightarrow H'_{G3} = 185.7 \text{ kJ/kg}$

Since $Y_3 = 0.017$ and $H'_{G3} = 185.7 \text{ kJ/kg}$, $T_{G3} = 138^\circ\text{C}$

The wet bulb temp of the gas at boundary 3, $T_w = 44^\circ\text{C}$ which is the assumed value. It matches.

Heat load in zone III, $Q_3 = L_s (H'_{S4} - H'_{S3}) = (1000)(24) = 24000 \text{ kJ/h}$

Temperature driving force: $(\Delta T)_3 = 138 - 44 = 94^\circ\text{C}$, $(\Delta T)_4 = 140 - 65 = 75^\circ\text{C}$

$(\Delta T)_m = (94 - 75)/\ln(94/75) = 84^\circ\text{C}$; $h_c = k_f c_H = (110)(1.037) = 31.7 \text{ W/m}^2\text{C}$

Heating time, $t_{III} = \frac{Q}{(L_s a')(h_c)(\Delta T)_M} = \frac{24000}{(1000 \times 0.0357)(31.7)(84)} = 0.252 \text{ hr}$

Calculation of a' : belt breadth = 2 m; solid depth = 4 cm = 0.04 m; mass of solid per meter length of the belt = $1 \text{ m} \times 2 \text{ m} \times 0.04 \text{ m} \times 700 \text{ kg/m}^3$ (dry solid) = 56 kg; drying area = $1 \times 2 = 2 \text{ m}^2$; drying area per kg = $= 2 \text{ m}^2 / 56 \text{ kg} = 0.0357 \text{ m}^2/\text{kg}$

Zone I: $L_s (X_1 - X_4) = G_s (Y_1 - Y_4) \Rightarrow (1000)(0.2 - 0.01) = 12,000(Y_1 - 0.017)$

$L_s (H'_{S4} - H'_{S1}) = G_s (H'_{G4} - H'_{G1}) \Rightarrow H'_{G1} = 188.6 \text{ kJ/kg} \Rightarrow Y_1 = 0.0328$

$H'_{S1} = [1.1 + (0.2)(4.187)](44 - 0) = 85.2 \text{ kJ/kg dry solid}$; this value is used in the exit gas equation: $H'_{G1} = 188.6 \text{ kJ/kg}$, $Y_1 = 0.0328 \Rightarrow T_{G1} = 100^\circ\text{C}$.

Calculate Y_2 by moisture balance: $L_s (X_1 - X_2) = G_s (Y_1 - Y_2)$; $X_2 = X_c = 0.08$

$$1000(0.2 - 0.08) = 12,000(0.0328 - Y_2) \Rightarrow Y_2 = 0.0228$$

Drying time, $t_I = \frac{12,000}{(110)(1000 \times 0.0357)} \ln \frac{0.0625 - 0.0228}{0.0625 - 0.0328} = 0.887 \text{ h}$ [Eq 11-33]

Drying time, t_{II} from Eq 11-37 $\rightarrow t_{II} = 0.975 \text{ h}$ (calculation not shown)

Total time, $t = t_I + t_{II} + t_{III} = 0.887 + 0.975 + 0.252 = 2.114 \text{ hr}$

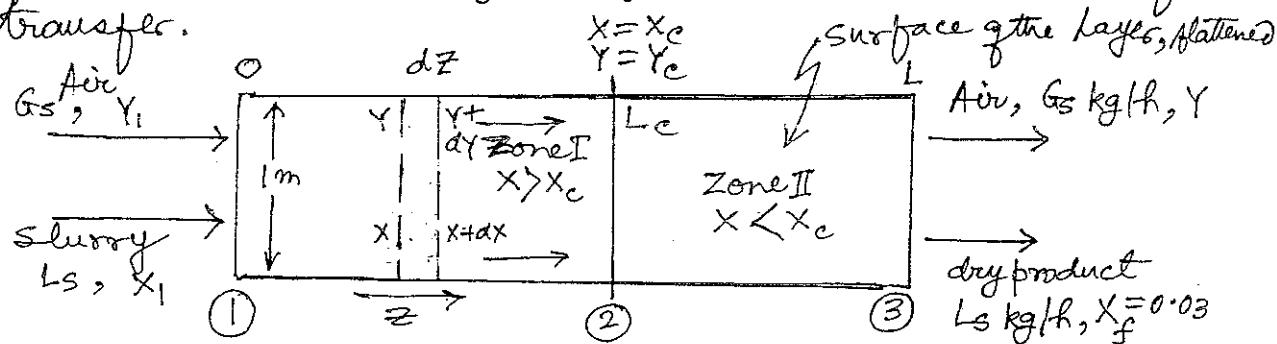
(11.21)

- (11.16) contd... Solid loading per meter belt length = 56 kg/m (dry basis)
 Desired drying rate = 1000 kg/h
 $\text{Belt speed} = (1000 \text{ kg/h}) / (56 \text{ kg/m}) = 17.86 \text{ m/h}$
 $\text{Drying time} = 2.114 \text{ hr.}$
 $\text{Belt length} = (17.86 \text{ m/h})(2.114 \text{ h}) = \underline{\underline{37.75 \text{ m}}}$
- * * * *

(11.17) The slurry and the hot drying air move in the same direction while transport of moisture from the surface of the thin layer of material on the rotating drum to the gas occurs. Thus it is a case of cocurrent continuous drying. As suggested in the problem, we neglect heat transfer from the gas to the slurry layer. It is assumed that the temperature of the material is maintained at 75°C by heat flow from the condensing steam within the drum. It is further assumed that the partial pressure of moisture at the drying surface is the same as vapor pressure of water at 75°C (note that this will not be true for solutions).

The problem can be solved by using suitably modified forms of Eqs. (11.33) and (11.37) for a countercurrent continuous dryer.

However, it will be first solved using alternative equations derived by using the basic principles of mass transfer.



The moving layer of the slurry, flattened

As in Section 11.12.4, we consider two zones of drying of the layer of slurry on the rotating drum. The moisture content reduces from X_1 to X_c (critical moisture) in zone I, and the final moisture content is attained at the end of zone II.

(11.22)

(11.17) contd... Let us consider a thin strip of the layer of width dz over which the moisture concentration in the slurry layer changes by dx (dx is inherently negative) and that in the gas layer changes by dy . Then,

$$-L_s dx = k_y \cdot dz (Y_s - Y) = G_s dy \quad (i)$$

Y_s is the saturation humidity of air at the temperature of the layer (which remains constant at 75°C) and k_y is the mass transfer coefficient. Integrate from $Y = Y_1$ to Y_c

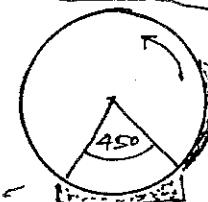
$$\frac{G_s}{k_y} \int_{Y_1}^{Y_c} \frac{dy}{Y_s - Y} = \int_{z=0}^{L_c} dz \Rightarrow L_c = \frac{G_s}{k_y} \ln \frac{Y_s - Y_1}{Y_s - Y_c} \quad \dots \dots \quad (ii)$$

$Y = Y_c$ is the humidity of the bulk gas at $z = L_c$ where the critical moisture content, $X = X_c$, of the slurry layer is reached.

In zone II, the flux is proportional to $(X - X^*)$, besides $Y_s - Y$.

$$-L_s dx = k_y dz \cdot (Y_s - Y) \cdot \left(\frac{X - X^*}{X_c - X^*} \right) = G_s dy$$

$$\therefore X^* = 0, \int_{L_c}^L dz = \frac{G_s X_c}{k_y} \int_{Y_c}^{Y_2} \frac{dy}{(Y_s - Y) X}$$



The local moisture concentrations X and Y are related by the moisture balance : $L_s(X_1 - X) = G_s(Y - Y_1) \Rightarrow X = X_1 - \frac{G_s}{L_s}(Y - Y_1)$ (iii)

$$\text{i.e. } L - L_c = \frac{G_s X_c}{k_y} \int_{Y_c}^{Y_2} \frac{dy}{(Y_s - Y)} \left[X_1 - \frac{G_s}{L_s} (Y - Y_1) \right]$$

$$= \frac{G_s X_c}{k_y} \cdot \frac{1}{\frac{G_s}{L_s} (Y_s - Y_1) - X_1} \cdot \ln \left[\frac{Y_s - Y_c}{Y_s - Y_2} \cdot \frac{X_1 - \frac{G_s}{L_s} (Y_c - Y_1)}{X_1 - \frac{G_s}{L_s} (Y_2 - Y_1)} \right]$$

$$\text{i.e. } L - L_c = \frac{G_s X_c}{k_y} \cdot \frac{1}{\frac{G_s}{L_s} (Y_s - Y_1) - X_1} \cdot \ln \left[\frac{Y_s - Y_c}{Y_s - Y_2} \cdot \frac{X_c}{X_2} \right] \quad (iv)$$

$$\begin{cases} x_c = X_1 - \frac{G_s}{L_s} (Y_c - Y_1) \\ \text{and} \\ x_2 = X_1 - \frac{G_s}{L_s} (Y_2 - Y_1) \end{cases}$$

Then lengths of zone I and zone II can be calculated.

Numerical

from Eqs. (ii) and (iv).

Given : $L_s = 50 \text{ kg/h}$; $G_s = 7 L_s = 350 \text{ kg/h}$; $X_1 = 0.8$; $X_2 = 0.03$; $X_c = 0.15$; $X^* \approx 0$; $Y_1 = 0.015$; $Y_c = 0.108$, $Y_2 = 0.125$ [these can be obtained from the moisture balance eq.(iii)]; $k_y = 160 \text{ kg/m}^2 \text{ (AY)}$; and $Y_s = 0.383$ (saturation humidity at 75°C).

11.17 contd... Putting the values of different quantities,

$$\text{Eq(ii)}, L_c = \frac{350}{160} \cdot \ln \frac{0.383 - 0.015}{0.383 - 0.108} = 0.637 \text{ m} \quad [\text{on the basis of } 1 \text{ m breadth of the layer}]$$

$$\text{Eq(iv)}, L - L_c = \frac{(350)(0.15)}{160} \cdot \frac{1}{\left[\frac{350}{50} (0.383 - 0.015) - 0.8 \right]} \cdot \ln \left[\frac{0.383 - 0.108}{0.383 - 0.125} \cdot \frac{0.15}{0.03} \right]$$

$$\text{Total length} = 0.637 + 0.309 \text{ m} = \underline{\underline{0.946 \text{ m}}} = 0.946 \text{ m}$$

Drum diameter: If the part of the drum dipped in the slurry makes an angle 45° at the axis, fraction of the area dipped $= \frac{45}{360} = 0.125$
 If d is the drum diameter (length of drum = 1 m), $\Rightarrow d = \underline{\underline{0.344 \text{ m}}}, \text{ say } \underline{\underline{0.35 \text{ m}}}$

Drum r.p.m.: It will depend upon the thickness of the film of slurry drawn by the rotating drum. Given: the thickness of the layer is $l_s = 0.002 \text{ m}$; Solids in the slurry is 71.4 kg per m^3 of slurry; the production rate = $50 \text{ kg dry solid/h}$ i.e. $\frac{50}{71.4} \text{ m}^3/\text{h}$.
 $\text{Solid} = \underline{\underline{51.5 \text{ kg product (with 3% moisture, dry basis)}}}$.

$$\text{Speed of the drum} = \frac{50 \text{ kg (dry)/h}}{(0.002)(0.946)(71.4 \text{ kg solid/m}^3)} = \underline{\underline{37 \text{ r.p.h}}} \\ \text{kg/rotation}$$

The same result can be obtained by using modified forms of Eqs. (11.33) and (11.37).

* * *

11.20 Given t - W data can be plotted straightaway [see Fig Prob 11.20(a)]. The drying rate \dot{a} at any time and for the corresponding moisture content can be calculated from the following equation:

$$N = \frac{W_1 - W_2}{t_2 - t_1} \cdot \frac{1}{a} \Rightarrow a = 0.1 \text{ m}^2 \text{ (given)}; \text{ and } X = \frac{W_1 - W_2}{W_s} = \frac{W_1 - W_2}{3 \text{ kg}}$$

The calculated values of X and N are given below.

t (hr)	0	0.25	0.50	0.75	1.0	1.25	1.50	1.75
W (kg)	4.2	4.145	4.091	4.035	3.978	3.92	3.688	3.811
X (kg/kg)		0.382	0.3637	0.345	0.326	0.307	0.29	0.27
N (kg/m ² ·h)		2.2	2.25	2.24	2.36	2.32	2.08	2.28

t (hr)	2.0	2.25	2.50	2.75	3.0	3.25	3.50	3.75	4.0
W (kg)	3.756	3.704	3.653	3.605	3.558	3.516	3.480	3.448	3.421
X (kg/kg)	0.252	0.235	0.218	0.202	0.186	0.172	0.16	0.15	0.1403
N (kg/m ² ·h)	2.2	2.08	2.04	1.92	1.88	1.68	1.44	1.28	1.08

t (hr)	4.25	4.50	4.75	5.0	5.5	6.0	6.5	7.0	7.5	8.0
W (kg)	3.394	3.368	3.345	3.323	3.285	3.253	3.226	3.202	3.183	3.166
X (kg/kg)	0.131	0.1227	0.115	0.1076	0.095	0.0843	0.0753	0.0673	0.061	0.0553
N (kg/m ² ·h)	1.08	1.04	0.92	0.88	0.76	0.64	0.54	0.48	0.38	0.34

The drying rate curve, N against X , is plotted on Fig Prob 11.20(b). The falling rate of drying is found to be linear in moisture content, X .

- (i) From Fig. Prob 11.20(b), critical moisture content, $X_c = 0.224 \frac{\text{kg}}{\text{kg dry solid}}$
 equilibrium moisture content, $X^* = 0.0211 \frac{\text{kg}}{\text{kg dry solid}}$,
 constant drying rate, $N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \text{h}}$;
 falling rate of drying, $N = 10.17X - 0.215$.

11-25

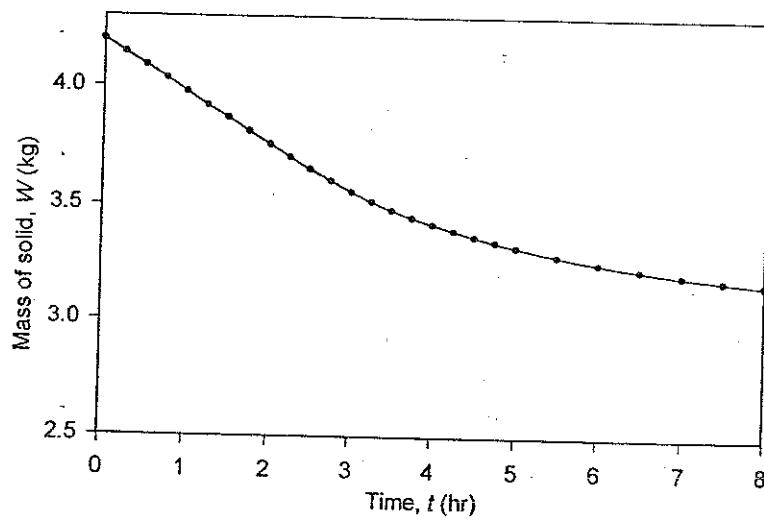


Fig Prob 11.20(a). Plot of the mass of wet solid against time.

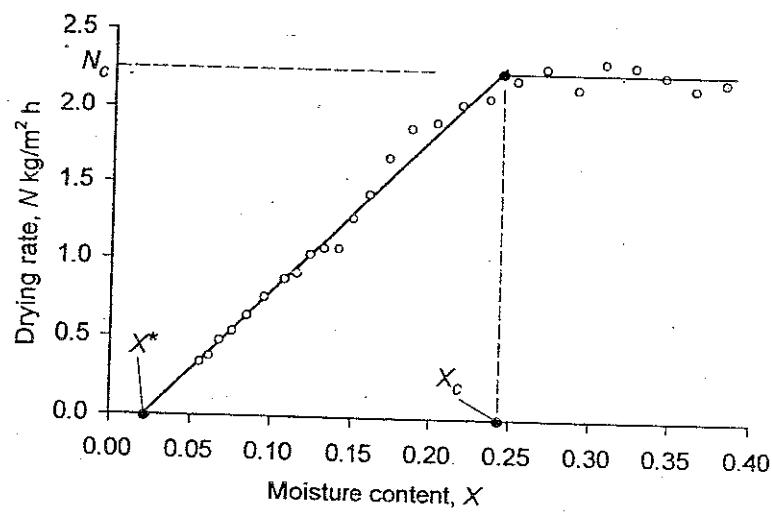


Fig Prob 11.20(b) Drying rate plot, linear falling rate, critical and equilibrium moisture shown.

(ii) Calculation of mass transfer coefficient, k_G

surface temperature of the wet solid = 46°C ; vapor pressure of water at 46°C , $p_s = 0.1 \text{ bar}$; partial pressure of moisture in the drying air (humidity = $0.015 \text{ kg/kg dry air}$, given) is

$$p_b = \frac{(0.015/18)(1.013 \text{ bar})}{(0.015/18)+(0.985/28.92)} = 0.0242 \text{ bar.}$$

Drying rate, $N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}} = 2.27 \frac{\text{kmol}}{18} = 0.126 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h}} = k_G(p_s - p_b) = k_G(0.1 - 0.0242)$

$$\Rightarrow k_G = \frac{0.126}{0.1 - 0.0242} = 1.664 \frac{\text{kmol}}{\text{m}^2 \cdot \text{h} \cdot \text{bar}}$$

(iii) When $X = 0.15$, drying rate, $N = (10.17)(0.15) - 0.215 = 1.31 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}}$
If the solid surface temperature remains as above (46°C), the drying rate from the moist areas of the surface also remains $2.27 \text{ kg/m}^2 \cdot \text{h}$.

Fraction of the surface covered by dry patches

$$= \frac{2.27 - 1.31}{2.27}, \text{ ie: } 42.3\%$$

(iv) Calculation of drying time for $X_i = \frac{0.25}{0.75} = 0.333$,

Use Eq.(11.5). $\frac{W_s}{a} = \frac{3}{0.1} = 30 \frac{\text{kg}}{\text{m}^2}$ $\left| \text{to } X_f = \frac{0.03}{0.97} = 0.0309 \right.$

$$N_c = 2.27 \frac{\text{kg}}{\text{m}^2 \cdot \text{h}}; X_c = 0.244, X^* = 0.0211$$

$$t = t_c + t_f = \frac{W_s}{a \cdot N_c} (X_i - X_c) + \frac{W_s}{a N_c} (X_c - X^*) \ln \frac{X_c - X^*}{X_f - X^*}$$

$$= \frac{30}{2.27} (0.333 - 0.244) + \frac{30}{2.27} (0.244 - 0.0211) \ln \frac{0.244 - 0.0211}{0.0309 - 0.0211}$$

$$= \underline{\underline{10.38 \text{ hr.}}}$$

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11.27

(11.21) The drying time can be calculated directly from Eq.(11.50). We first calculate v_w , which is the volume of material that contains 1 kg of frozen water. No volume change occurs on drying.

Mass of dry material that contains 1 kg water = $v_w \cdot (400 \text{ kg/m}^3)$.

$$\text{Moisture content} = \frac{1 \text{ kg}}{400 \cdot v_w \text{ kg dry solid}} = \frac{130}{100} \quad (\text{given: } 130\% \text{ moisture on dry basis})$$

$$\Rightarrow v_w = 1.923 \times 10^{-3} \text{ m}^3/\text{kg}$$

Other data: $L = 1 \text{ cm} = 10^{-2} \text{ m}$; $\lambda_w = 620 \text{ cal/gm} = 2596 \text{ kJ/kg}$;
 $T_f - T_s = -8 - (-30) = 22^\circ\text{C}$; $k_g = 1.25 \times 10^{-5} \text{ kcal/s.m.K}$
 $= 0.1884 \text{ kJ/s.h.K}$; $h_c = 70 \text{ kJ/m}^2 \cdot \text{h} \cdot \text{K}$.

Substitute in Eq. (11.50).

$$t_f = \frac{2596}{(1.923 \times 10^{-3})(22)} \cdot \left[\frac{(10^{-2})^2}{(8)(0.1884)} + \frac{0.01}{(2)(70)} \right]$$

$$= 6.136 \times 10^4 \cdot (6.635 \times 10^{-5} + 7.143 \times 10^{-5}) \text{ s} = \underline{\underline{8.45 \text{ h}}}$$

CHAPTER 12: ADSORPTION

12.1

Short and MCQ's

$$12.1 \quad q = \frac{6.4 p}{1+1.53 p} = (4.18) \cdot \frac{1.53 p}{1+1.53 p} \Rightarrow q_m = 4.18 \text{ millimol/gm adsorbent}$$

$$\Rightarrow q_m = (4.18)(65) \text{ mg/gm} = \underline{\underline{0.272 \text{ kg/kg adsorbent}}}$$

12.2 Henry's law constant for adsorption equilibrium, Eq. (12.6):

$$K_H = \left[\frac{dq}{dp} \right]_{p \rightarrow 0} = \frac{d}{dp} \left[\frac{14.7 p}{(11.2 + p^{0.424})^{2.36}} \right] \text{ for } p \rightarrow 0$$

$$= \left[\frac{14.7}{(11.2 + p^{0.424})^{2.36}} + \frac{(14.7 p)(-2.36)(0.424) p^{-0.576}}{(11.2 + p^{0.424})^{3.36}} \right]_{p \rightarrow 0} = 0.0491 \frac{\text{mmol}}{\text{gm} \cdot \text{kPa}}$$

12.3 Use Eq.(12.12).

$$q_A = q_{A_m} \frac{K_A p_A}{1 + K_A p_A + K_B p_B}$$

$$= \frac{1.72 p_A}{1 + 0.92 p_A + 0.18 p_B}$$

$$= \frac{(1.72)(50)}{1 + (0.92)(50) + (0.18)(50)}$$

$$= 1.536 \frac{\text{mmol}}{\text{gm}} = \underline{\underline{76.8 \frac{\text{mg}}{\text{gm}}}}$$

$$q_B = q_{B_m} \frac{K_B p_B}{1 + K_A p_A + K_B p_B}$$

$$= \frac{0.33 p_B}{1 + 0.92 p_A + 0.18 p_B}$$

$$= \frac{(0.33)(50)}{1 + (0.92)(50) + (0.18)(50)}$$

$$= 0.295 \frac{\text{mmol}}{\text{gm}} = \underline{\underline{10.6 \frac{\text{mg}}{\text{gm}}}}$$

Total adsorbent loading at equilibrium
 $= 76.8 + 10.6 = \underline{\underline{87.4 \text{ mg/gm adsorbent}}}$

12.4 At $p_A = 60 \text{ kPa}$, $p_B = 40 \text{ kPa}$, $q_A = 1.628 \frac{\text{mmol}}{\text{gm}}$ and $q_B = 0.208 \frac{\text{mmol}}{\text{gm}}$
 $\Rightarrow x_A = \frac{1.628}{1.628 + 0.208} = 0.887$; $x_B = 0.113$; $y_A = 0.6$; $y_B = 0.4$

$$\alpha_{AB} = \frac{x_A/x_B}{y_A/y_B} = \frac{0.887/0.113}{0.6/0.4} = \underline{\underline{5.23}}$$

12.5 (iii) ; 12.6 (i), since the adsorbed phase is more orderly;

12.7 (i); 12.8 (iii); 12.9 (iii); 12.10 (i) molecular sieve,

(ii) silica gel, (iii) active carbon, (iv) active carbon ; 12.11 (i);

12.12 (i); 12.13 (i); 12.14 (iii); 12.16 (i); 12.17 (i);

12.18 (ii); 12.19 (iii); 12.20 (iii); 12.21 (i); 12.22 ;

12.23

Ch 12 MCQ F2

(12.24) (i) ; (12.25) (i) ; (12.26) (ii) ; (12.28) (ii) ;

(12.2)

(12.1) Let C_i = initial concentration of a solute; C_e = equil. concentration in solution; V_L = volume of solution; m_s = mass of the adsorbent used.

Amount of solute removed from solution = $V_L(C_i - C_e)$

Then $\eta_e = \frac{V_L(C_i - C_e)}{m_s} = K'(C_e)^n$. Consistent units are to be used.

(a) Adsorption of 2,4-DCP

$$V_L \rightarrow 1000 \text{ kg} \approx 1000 \text{ liter}; C_i = 200 \text{ ppm} = 200 \text{ gm}/10^6 \text{ gm} \approx 200 \text{ gm}/\text{m}^3$$

$$\Rightarrow \frac{1000 \text{ liter} (200 - C_e) \text{ mg/liter}}{1000 \text{ gm}} = K'(C_e)^n = 502 (C_e)^{17.87}$$

$$\Rightarrow C_e = \frac{7.15 \times 10^{-4} \text{ mg/liter}}{(502)^{\frac{1}{17.87}}}$$

(b) Adsorption of 2,6-DCP

$$\frac{1000 (100 - C_e)}{1000} = 177 (C_e)^{14.83} \Rightarrow C_e = 0.0632 \text{ mg/liter}$$

(12.2) Freundlich isotherm : $\eta_e = K' C_e^{\frac{1}{n}} \Rightarrow \ln \eta_e = \ln K' + \frac{1}{n} \ln C_e$

η_e and C_e values are given in the table. Calculate $\ln \eta_e$ and $\ln C_e$ and make a plot. Fit the data by the least square line. The plots are shown in Fig Prob 12.2.

Results : Glycine - $\eta_e = 5.21 \times 10^{-4} C_e^{0.9736}$

Phenylalanine - $\eta_e = 4.37 \times 10^{-3} C_e^{0.952}$; correlation coeff. $r^2 = 0.999$

The correlation coefficients are very high and the fitting is very good. ; correlation coeff. $r^2 = 0.998$

(12.3)(a) Four sets of adsorption equilibrium data (η vs P) are given. Use Eq. 12.7 to fit the Langmuir isotherm.

Calculate and plot η/P against P [See fig. Prob 12.3]. The 'least square' straight line fitting is shown in figure. In each plot two data points at low pressure deviate.

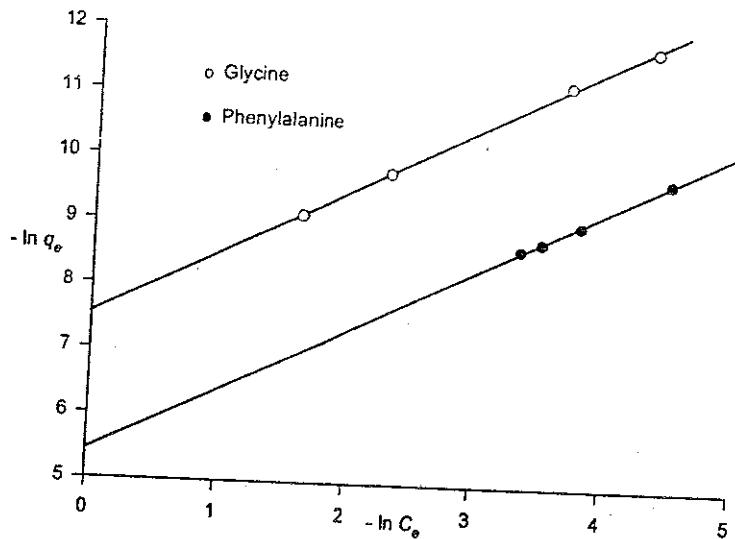


Fig. Prob 12.2. Freundlich isotherm plots - adsorption of glycine and phenylalanine on Amberlite XAD.

12.4

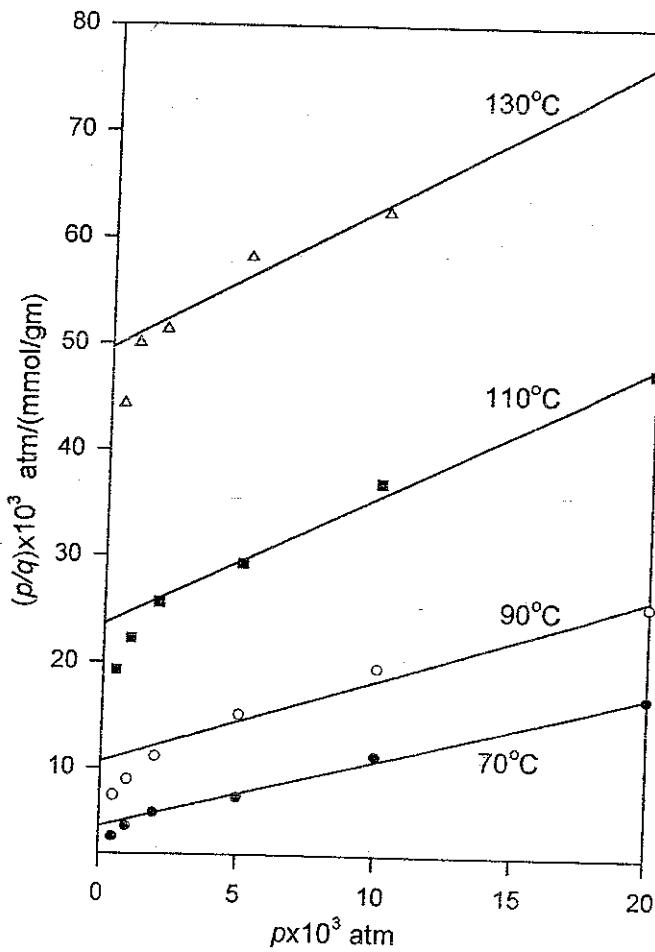


Fig Prob 12.3(a)

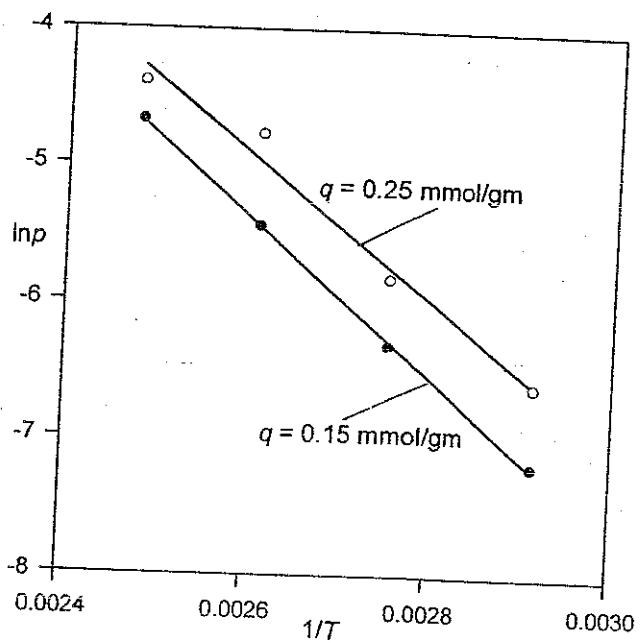


Fig Prob 12.3(b)

(12.5)

Substantially and have been discarded while drawing the least square lines. The fitting is reasonably good for the remaining four data points. The correlation coefficients are also shown on the graph against individual plots. The Langmuir isotherm equations at the four temperatures are given below,

$$T = 70^\circ\text{C} \rightarrow q = (1.591) \cdot \frac{0.137 \times 10^3 p}{1 + 0.137 \times 10^3 p}; T = 90^\circ\text{C} \rightarrow q = (1.284) \cdot \frac{0.073 \times 10^3 p}{1 + 0.073 \times 10^3 p}$$

$$T = 110^\circ\text{C} \rightarrow q = (0.816) \cdot \frac{0.052 \times 10^3 p}{1 + 0.052 \times 10^3 p}; T = 130^\circ\text{C} \rightarrow q = (0.734) \cdot \frac{0.0275 \times 10^3 p}{1 + 0.0275 \times 10^3 p}$$

Fitting of Freundlich isotherm, Eq(12.8) $\rightarrow \ln q = \ln k' + \frac{1}{n} \ln p$

Values of $\ln q$ is plotted against $\ln p$ at each temperature, see Fig Prob 12.3(c). The fitting is much better than for Langmuir isotherm as indicated by the 'correlation coefficient' shown against individual plots. The values of $\ln k'$ and $\frac{1}{n}$ are obtained from the intercept and slope of the 'least square' line at a particular temperature. The isotherm equations are:

$$70^\circ\text{C} \rightarrow q = (17.74) \cdot p^{1/1.578}; 90^\circ\text{C} \rightarrow q = (10.82) \cdot p^{1/1.507};$$

$$110^\circ\text{C} \rightarrow q = (8.893) \cdot p^{1/1.307}; 130^\circ\text{C} \rightarrow q = (8.18) \cdot p^{1/1.242}$$

(b) Isosteres and the heat of adsorption

An isostere is the plot of $\ln p$ against $1/T$ at a constant value of q . We arbitrarily select two q -values, $q = 0.15 \text{ mmol/gm}$, and $q = 0.25 \text{ mmol/gm}$. For each q -value, we calculate the value of p at each of the temperatures. The calculated values are given below (obtained from the Langmuir isotherm)

$$q = 0.15 \text{ mmol/gm}$$

$$q = 0.25 \text{ mmol/gm}$$

$T(K)$	343	363	383	403	343	363	383	403
$10^3 \times P, \text{ atm}$	0.76	1.812	4.33	9.34	1.36	2.96	8.5	12.4
$10^3 \times 1/T$	2.915	2.755	2.611	2.48	2.915	2.755	2.611	2.48
$\ln p$	-7.182	-6.313	-5.442	-4.673	-6.6	-5.822	-4.767	-4.39

The plots of $\ln p$ against $1/T$ are shown in Fig Prob 12.3(b). The goodness of fit is indicated by the correlation coefficients shown on the graph. The heat of adsorption is obtained from the slopes.

$$q = 0.15, \text{ slope} = -5791 = \Delta H/R \rightarrow \Delta H = (-5791)(1.987) = 11.5 \text{ kcal/gmol}$$

$$q = 0.25, \text{ slope} = -5319 = \Delta H/R \rightarrow \Delta H = (-5319)(1.987) = 10.56 \text{ kcal/gmol}$$

Prob 12.3 Contd... The exercise is repeated by calculating p for $q = 0.15$ and 0.25 mmol/gm from the Freundlich isotherm equation at each temperature. The calculated values are given below.

$$q = 0.15 \text{ mmol/gm}$$

T (K)	343	363	383	403
$p \times 10^3, \text{ atm}$	0.536	1.584	4.754	6.967
$1/T \times 10^3$	2.915	2.755	2.611	2.48
$\ln p$	-7.532	-6.448	-5.349	-4.966

$$q = 0.25 \text{ mmol/gm}$$

343	363	383	403
1.2	3.42	9.268	13.1
2.915	2.755	2.611	2.48
-6.726	-5.678	-4.681	-4.332

The least square straight line plots of $\ln p$ against $1/T$ for $q = 0.15$ and $q = 0.25$ are shown in fig Prob 12.3(d). The fitting is good, the correlation coefficients are shown. The heat of adsorption is to be calculated from the slope of a plot.

$$q = 0.15, \text{slope} = -6106 = \Delta H/R \Rightarrow \Delta H = (-6106)(1.987) = -12.13 \text{ kcal/gmol}$$

$$q = 0.25, \text{slope} = -5680 = \Delta H/R \Rightarrow \Delta H = (-5680)(1.987) = -11.3 \text{ " " }$$

Note that The heat of vaporization (or condensation) of benzene in the given temperature range is about 7 kcal/gmol. The estimated heat of adsorption is considerably larger than this for the variety of silica gel.

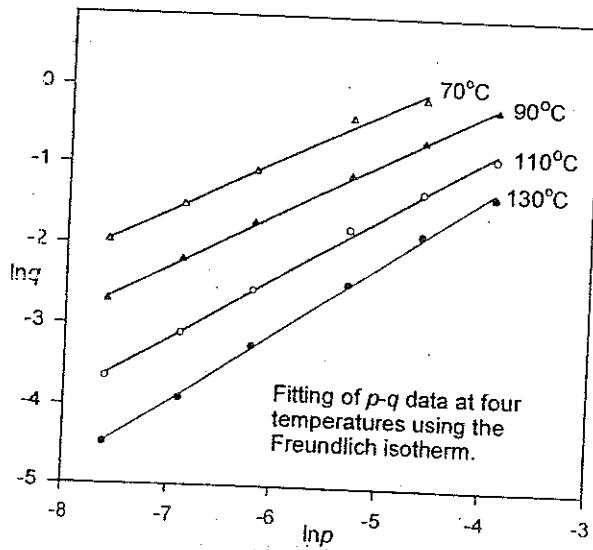


Fig Prob 12.3(c)

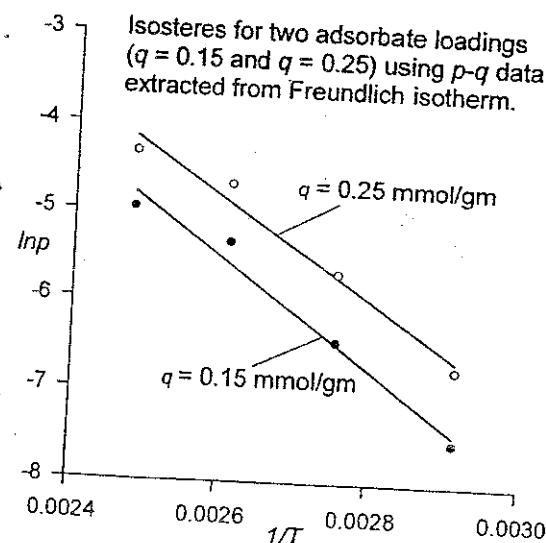


Fig Prob 12.3(d)

12.7

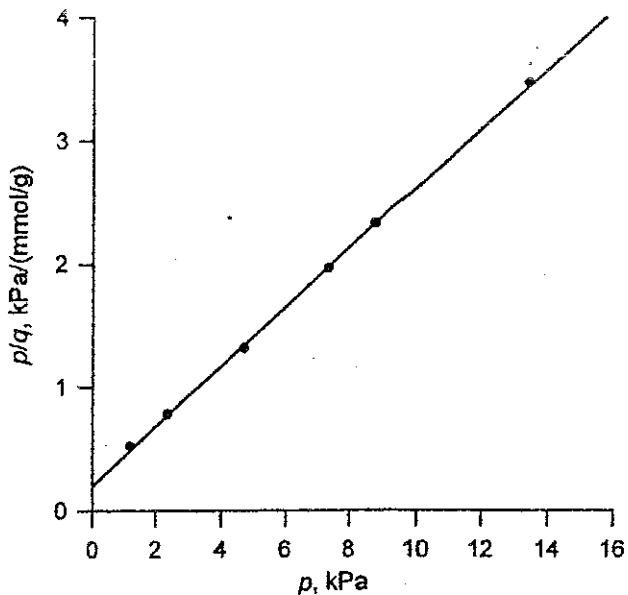


Fig. Prob 12.6. Fitting of the Langmuir Isotherm.

(12.4) The equilibrium adsorption data were fitted in the Langmuir isotherm as before [see figure Prob 12.4(a)]. The results are :

$$T = 310.9 \text{ K} ; q = (5.464) \frac{0.0474p}{1+0.0474p} = \frac{0.259p}{1+0.0474p} ; r^2 = 0.999$$

$$T = 338.7 \text{ K} ; q = (5.32) \frac{0.02p}{1+0.02p} = \frac{0.1064p}{1+0.02p} ; r^2 = 0.992$$

$$T = 366.5 \text{ K} ; q = (4.425) \frac{0.02216p}{1+0.02216p} = \frac{0.098p}{1+0.02216p} ; r^2 = 0.996$$

The fitting is pretty good at each temperature.

Values of p against T for a constant loading ($q = 4 \frac{\text{mmol}}{\text{gm}}$, selected arbitrarily) obtained by interpolation are

$$q = 4 \frac{\text{mmol}}{\text{gm}}$$

p, kPa	65	200	500
T, K	310.9	338.7	366.5

The plot of $\ln p$ against $1/T$ (isotherm) is shown in Fig. Prob 12.4(b).

$$\text{Slope} = -4183 = \Delta H/R \Rightarrow \Delta H = -8.312 \frac{\text{kcal}}{\text{gmol}}$$

(12.8)

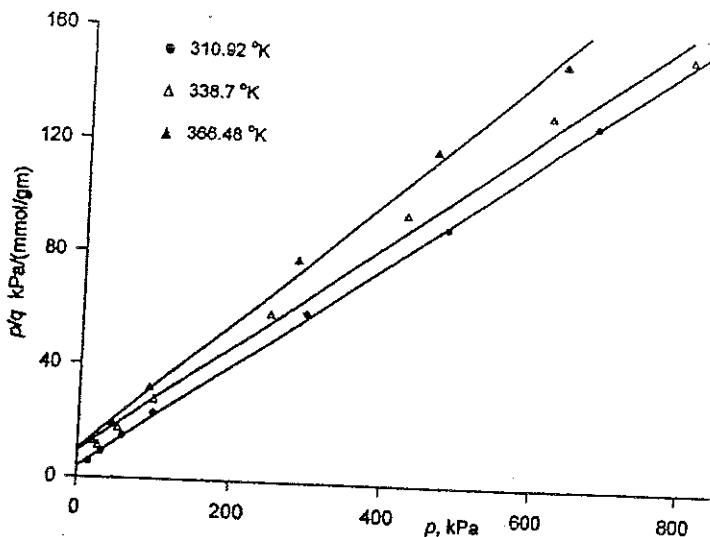


Fig. Prob 12.4(a) Fitting of Langmuir isotherm at three temperatures

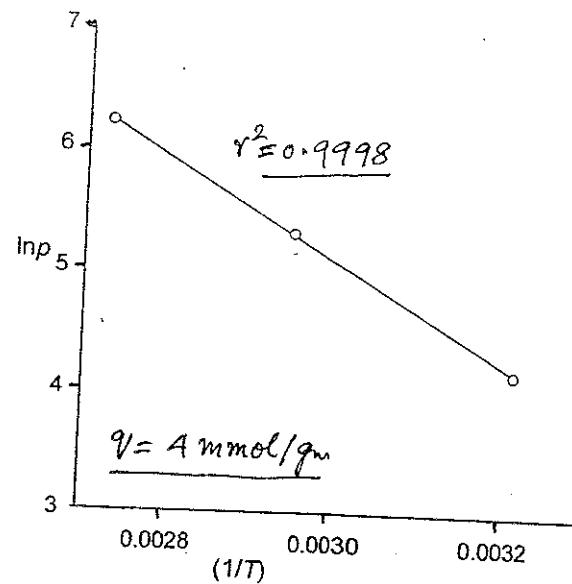


Fig. Prob. 12.4(b). Isosters for the determination of heat of adsorption.

(12.5) The individual adsorption equilibrium data for ethane (1) and propane (2) given in the problem can be fitted in the Langmuir isotherm (details not given)

$$\text{Ethane : } q_1 = \frac{0.342 p_1}{1 + 0.152 p_1} \rightarrow q_1 \text{ in } \frac{\text{mmol}}{\text{gm}}, p_1 \text{ in kPa}$$

$$\text{Propane : } q_2 = \frac{1.57 p_2}{1 + 0.802 p_2} \rightarrow q_2 \text{ and } p_2 \text{ have units as above.}$$

If the Langmuir's isotherm is applied to adsorption of a mixture of the gases [see Eq. 12.12]

$$\text{for ethane, } q_{1,\text{mix}} = \frac{0.342 p_1}{1 + 0.152 p_1 + 0.802 p_2} \quad \text{--- (i)}$$

$$\text{for propane, } q_{2,\text{mix}} = \frac{1.57 p_2}{1 + 0.152 p_1 + 0.802 p_2} \quad \text{--- (ii)}$$

In the given mixture, in equilibrium with the adsorbent, $p_1 = 50 \text{ kPa}$ (for ethane), and $p_2 = 50 \text{ kPa}$ (for propane)

Putting these values in Eqs (i) and (ii)

$$q_{1,\text{mix}} = \frac{(0.342)(50)}{1 + (0.152)(50) + (0.802)(50)} = \frac{0.351 \text{ mmol ethane/gm}}{= 0.351 \text{ mol/kg}}$$

$$\text{and } q_{2,\text{mix}} = \frac{1.57(50)}{1 + 0.152(50) + 0.802(50)} = \frac{1.62 \text{ mmol/gm propane}}{= 1.62 \text{ mol/kg}}$$

(12.9)

12.6 If the adsorption equilibrium data fits the Langmuir isotherm, it is possible to determine the adsorbate loading corresponding to formation of a monomolecular layer. The given data fitted to the Langmuir isotherm (see fig Prob 12.6) yields $\theta_m = \underline{4.14 \text{ mmol/gm}}$ obtained from the slope of the plot (see page 2).

Specific surface area, $S_g = \theta_m N_A$, Eq (12.8)

$$\begin{aligned} \text{Eq. (12.7)} \rightarrow \alpha &= 1.09(M/N_p)^{2/3} = 1.09 \left[\frac{(78)}{(6.023 \times 10^{23})(0.875)} \right]^{2/3} \\ S_g &= \left(4.14 \times 10^{-3} \text{ mmol/gm} \right) \left(\frac{6.023 \times 10^{23} / \text{mmol}}{3.05 \times 10^{-16} \text{ cm}^2} \right) \\ &= \underline{76.05 \text{ m}^2/\text{gm}} \end{aligned}$$

12.7 (a) The minimum quantity of carbon corresponds to attainment of adsorption equilibrium. If C_e , mg/l, is the equilibrium concentration in solution,

$$\frac{(1000 \text{ liter})(C_i - C_e) \text{ mg/l}}{m_s} = 68 C_e^{0.43}$$

Initial concentration,
 $C_i = 10 \text{ ppm}$
 $m_s = \text{mass of carbon, gm} = 10 \text{ mg/liter}$

$$\Rightarrow \frac{(1000)(10 - 0.01)}{m_s} = 68(0.01)^{0.43} \Rightarrow m_s' = 1.064 \text{ kg}$$

Amount of solution = 1 m³
 $C_e = 0.01 \text{ ppm} = 1000 \text{ liter}$

(b) If 1.3 times the minimum quantity (m_s'), mass of carbon used, $m_s = 1.3 \times 1.064 = 1.383 \text{ kg}$. It is required to calculate the time for attaining a concentration 0.01 ppm.

At any time t , let the bulk concentration of the solute be C (ppm or mg/liter).

Loading of the adsorbent at that time, $q = \frac{(1000)(10 - C) \text{ mg}}{1383 \text{ gm}}$

Corresponding equilibrium concentration of the solute in the solution, C_e' , is given by $\frac{(1000)(10 - C)}{1383} = 68(C_e')^{0.43}$

[Note that the quantity C_e' is time-dependent since C is time-dependent] $\Rightarrow C_e' = 2.575 \times 10^{-5} (10 - C)^{2.3255}$

(12.10)

12.7 contd... Instantaneous driving force for transport of the solute from the bulk liquid to the solid surface = $C - C_e'$. An unsteady state solute balance gives

$$-V_L \frac{dC}{dt} = k_L \cdot (m_s \cdot a') (C - C_e'); C \text{ changes from } 10 \text{ to } 0.01 \text{ ppm}$$

Integrating, ^{put C_e' from (1) [Given: $k_L = 1.8 \times 10^{-5} \text{ m/s}$; $a' = 200 \text{ cm}^2/\text{gm adsorbent}$; $V = 1 \text{ m}^3$]}

$$-\int_{10}^{0.01} \frac{dC}{C - 2.575 \times 10^{-5} (10 - C)^{2.3256}} = \int_0^t \frac{(1.8 \times 10^{-5} \frac{\text{m}}{\text{s}})(200 \times 10^{-4} \frac{\text{m}^2}{\text{g}} \times 1383 \text{ gm}) dt}{1 \text{ m}^3}$$

The value of the integral = 7.684 (obtained by using Mathcad Plus 6.0)

$$\Rightarrow 7.684 = 4.98 \times 10^{-4} t \Rightarrow t = 15,4308 = 4.3 \text{ hr}$$

12.8 Follow the procedure of Example 12.3(b). Consider the first stage of contact.

If Y (units/kg) is the color concentration in the oil at any time t , amount of color adsorbed by the clay per kg is

$$x = \frac{(1000 \text{ kg oil})(50 - Y)}{m_s} \frac{\text{unit}}{\text{kg}} = \frac{(1000)(50 - Y)}{12.36} = 80.91(50 - Y) \quad [m_s = 12.36 \text{ kg}]$$

Corresponding 'equilibrium color concentration' in the oil would have been, $Y_e = 4.2 \times 10^{-4} x = (4.2 \times 10^{-4})(80.91)(50 - Y) = 0.034(50 - Y)$

Unsteady color balance : $-\frac{d}{dt}(1000Y) = k_L(m_s \cdot a')(Y - Y_e) \cdot \text{oil}$

$$\Rightarrow -\frac{dY}{dt} = \frac{(5.2 \times 10^{-6} \text{ m/s})(12.36 \text{ kg} \times 25 \frac{\text{m}^2}{\text{kg}})(950 \text{ kg/m}^3)}{1000 \text{ kg}} [Y - 0.034(50 - Y)]$$

$$\Rightarrow -\frac{dY}{dt} = 1.526 \times 10^{-3} (1.034Y - 1.7) \quad \text{contact time} = 30.6 \text{ min.} = 1836 \text{ s}$$

$$-\int_{50}^{Y_1} \frac{dY}{(1.034Y - 1.7)} = 1.526 \times 10^{-3} \int_0^{1836} dt \quad \text{Integrate from } t=0, Y=50 \quad t=1836, Y=Y_1 \quad \Rightarrow \frac{1}{1.034} \ln \frac{(1.034)(50) - 1.7}{(1.034)(Y_1) - 1.7} = 1.526 \times 10^{-3} \frac{-3}{1836}$$

$$\Rightarrow Y_1 = 4.313 \text{ unit/kg oil} \quad [\text{Note that more than 90% of the color gets adsorbed when half the amount of clay of Ex. 12.3 is used}]$$

Second stage of contact

Use the same quantity of clay, 12.36 kg. Starting color concentration in the oil, $Y_1 = 4.313 \text{ units/kg}$

Follow the above procedure, write the unsteady state color balance.

On integration the final color concentration in the oil can be found to be $Y_2 = 0.372 \text{ unit/kg}$.

(12.11)

(12.9) (a) The mass transfer wave front is drawn by plotting the given breakthrough data. For $C/C_0 = 0.025$, $t = 19.5 \text{ min.} = t_b$
 \Rightarrow breakthrough time $= 19.5 \text{ min.}$

The MTZ is not symmetric. The stoichiometric time, t_s , is determined by matching the areas above the breakthrough curve for $t < t_s$, and that above the curve for $t > t_s$ (see Fig. Prob. 12.9).

$$t_s = 46 \text{ min. Velocity of the wave front, } u_s = \frac{L}{t_s} = \frac{0.2 \text{ m}}{46 \text{ min}} = 0.00435 \frac{\text{m}}{\text{min}}$$

The equilibrium time, t_e , is taken as the time for $C/C_0 = 0.095$

$$t_e = 86 \text{ min. Length of the MTZ} = u_s(t_e - t_b) \\ LUB = u_s(t_s - t_b) = (0.00435)(46 - 19.5) = 0.29 \text{ m} = 29 \text{ cm}$$

(b) To design an adsorber bed for treating $3000 \text{ m}^3/\text{h}$ of gas, (i) we use the same superficial velocity as in the experimental feed ($u_0 = 0.29 \text{ m/s}$)

(ii) we assume that the mass transfer front maintains the same shape as it progresses through the bed.

$$\text{Diameter of the bed} \\ = \left[\frac{4}{\pi} \cdot \frac{(3000/3600) \text{ m}^3/\text{s}}{0.29 \text{ m/s}} \right]^{1/2} = 1.91 \text{ m}$$

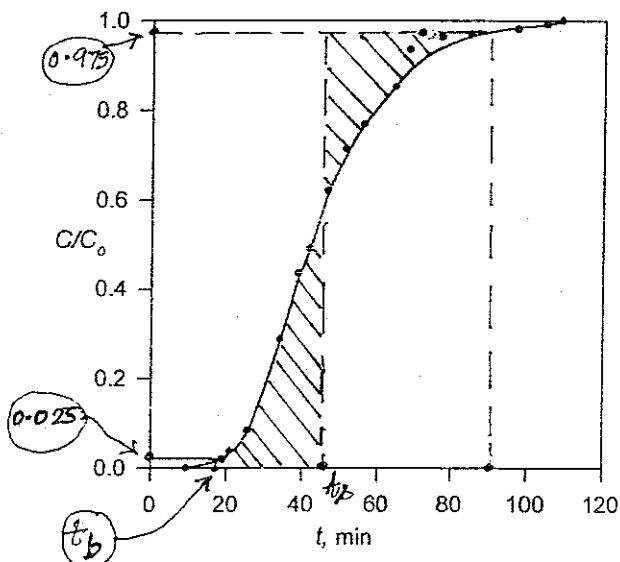


Fig. Prob. 12.9. Mass transfer wave front

Bed height

The adsorption half cycle has a duration of 8 hr.

$$\text{Bed height} = (8 \text{ hr})(u_s) + LUB = (8 \times 60 \text{ min}) (0.00435 \frac{\text{m}}{\text{min}}) + 0.115 \text{ m} \\ = 2.2 \text{ m}$$

Loading of the bed at breakthrough: All the solute entering the column in 8 hr is absorbed. Considering unit area of the bed, moles of MEK entering $= (8 \text{ hr} \times 3600 \frac{\text{sec}}{\text{hr}}) (0.29 \text{ m/s}) (1 \text{ m}^2) (0.11 \text{ g mol/m}^3)$
 $= 919 \text{ g mole}$

12-12

12.9 contd... Total volume of the bed = $(2.2 \text{ m})(1 \text{ m}^2) = 2.2 \text{ m}^3$

Mass of the adsorbent = $(2.2 \text{ m}^3)(P_b) = (2.2 \text{ m}^3)(700 \text{ kg/m}^3) = 1540 \text{ kg}$

Average adsorbent loading = $\frac{919 \text{ g/mol}}{1540 \text{ kg}} = 0.5967 \frac{\text{g/mol}}{\text{kg}}$

Maximum possible adsorbent loading: At the end of 8 hr. of adsorbent cycle time, a bed length = $(8 \text{ hr.})(0.00435 \times 60) = 2.088 \text{ m}$ of the bed gets effectively saturated.

Maximum loading = $\frac{(8 \times 3600)(0.29)(0.11)}{(2.088)(700)} \frac{\text{g/mol}}{\text{kg}} = 0.6287 \frac{\text{g/mol}}{\text{kg}}$

12-10 The breakthrough data are plotted; the mass transfer wave front is reasonably symmetric. Take the breakthrough concentration as $C/C_0 = 0.025$ and the equilibrium concentration as $C/C_0 = 0.97$. (i) Breakthrough time (from the plot), $t_b = 191 \text{ min}$.

Since the wave front is almost symmetric, take the stoichiometric time, $t_m = 232 \text{ min}$ when $C/C_0 = 0.5$.

(ii) Velocity of the MTZ, $U_s = \frac{L}{t_m} = \frac{50 \text{ cm}}{232 \text{ min}} = 0.215 \text{ cm/min}$

$$\text{LUB} = L \left(1 - \frac{t_b}{t_s}\right) = L \left(1 - \frac{t_b}{t_m}\right) = 50 \left(1 - \frac{191}{232}\right) \text{ cm} = 7.6 \text{ cm}$$

(iii) Fraction of the bed utilized = $\frac{L - \text{LUB}}{L} = \frac{50 - 7.6}{50} = 84.5\%$

Evaluation of LUB by the integration technique

Take the breakthrough concentration $C/C_0 = 0.005$ when $t_b = 191 \text{ min}$

Equilibrium time, $t_e = 270 \text{ min}$, when $C/C_0 = 0.992$.

$$\begin{aligned} \text{Eq. (12.24)}, \omega_1 &= U_0 C_0 \int_{t_b=191}^{t_e=270} (1 - \frac{C}{C_0}) dt = U_0 C_0 \int_0^{270} (1 - \frac{C}{C_0}) dt \\ &= U_0 C_0 \int_0^{191} (1 - \frac{C}{C_0}) dt + \int_{191}^{270} (1 - \frac{C}{C_0}) dt = 191 \cdot U_0 C_0 + 41.55 U_0 C_0 \\ \omega_2 &= \int_0^{191} (1 - \frac{C}{C_0}) dt = 191 U_0 S \Rightarrow \text{LUB} = \left(1 - \frac{\omega_2}{\omega_1}\right) L = \left(1 - \frac{191}{232.5}\right)(50 \text{ cm}) \quad (\text{by graphical integration}) \end{aligned}$$

The difference occurred probably due to the error in graphical integration, and a title asymmetry of the wave front. $\Rightarrow 8.9 \text{ cm}$

Fig. \rightarrow p. 13

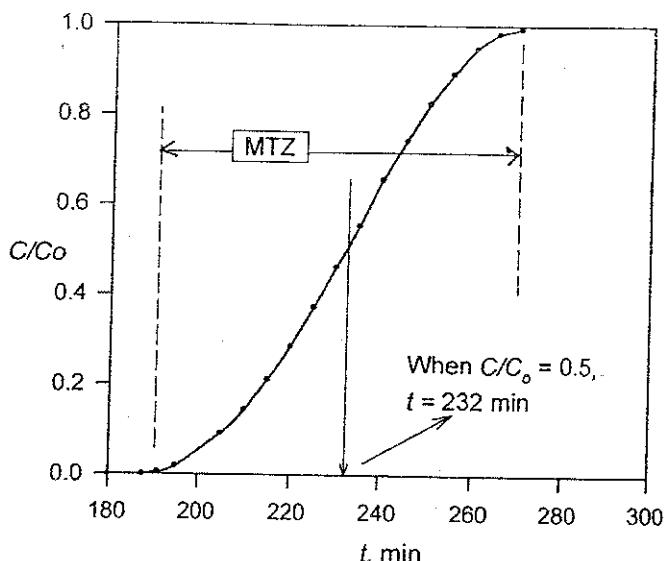


Fig. Prob. 12.10. Mass transfer wave front from experimental breakthrough data.

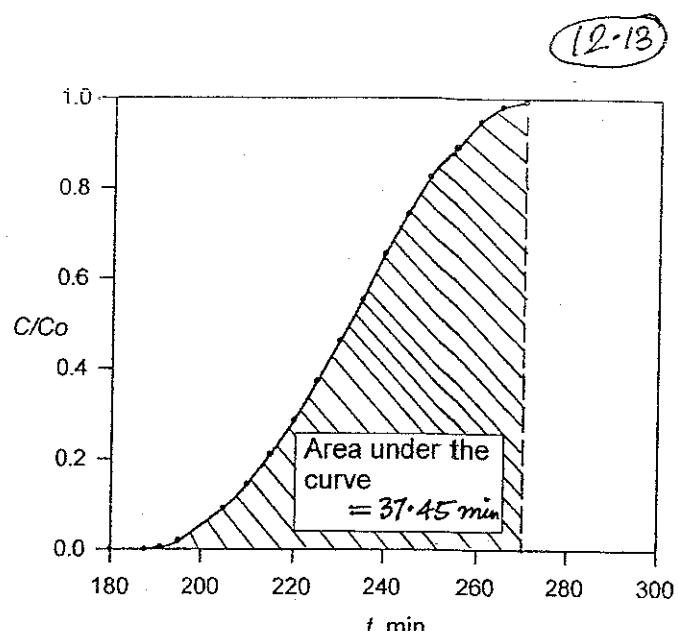


Fig. Prob. 12.10(b). Graphical integration.

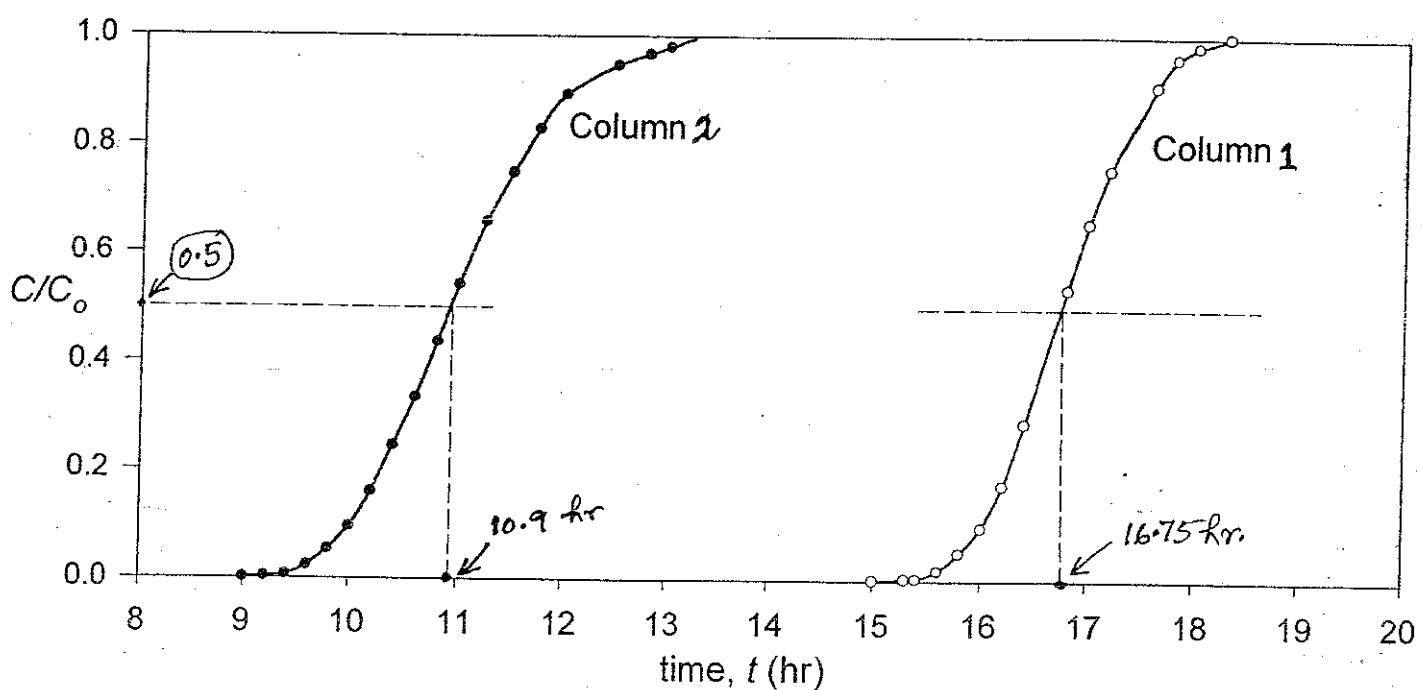


Fig. Prob. 12.11. Breakthrough data for two columns of different lengths.

(12-11) The breakthrough data for both the columns are plotted in Figure Prob 12.11.

For column 1, breakthrough time (for 1 ppm moisture in the effluent)
 $t_{b1} = 15.5 \text{ hr}$

12.11 Contd... The mass transfer wave front is almost symmetric. For $C/C_0 = 0.5$, $t_s = t_m = \underline{16.75 \text{ hr}}$. (12.14)

$$(a) L_{UB} = L \left(1 - \frac{t_b}{t_{s1}}\right) = (44 \text{ cm}) \left(1 - \frac{15.5}{16.75}\right) = \underline{3.3 \text{ cm}}$$

$$(b) \text{ From column-1 data, } u_s = \frac{L_1}{t_{s1}} = \frac{44 \text{ cm}}{16.75 \text{ hr}} = \underline{0.0438 \text{ cm/min}}$$

$$\text{Taking } L_{UB} = 3.3 \text{ for column-2 also, } L_{s2} = L_2 - L_{UB} \\ = 27 - 3.3 = \underline{23.7 \text{ cm}}$$

Breakthrough time for column-2 (calculated from column-1 data)

$$t_{b2} = \frac{L_{s2}}{u_s} = \frac{23.7 \text{ cm}}{0.0438 \text{ cm/min}} = 540 \text{ min} = \underline{9.02 \text{ h}}$$

This appears to be pretty accurate in ^{reflect 98} column-2 data.

(c) Bed-1, saturation concentration:

$L_{s1} = L_1 - L_{UB} = 44 - 3.3 = \underline{40.7 \text{ cm}}$. This length of the bed gets saturated. Consider unit area of the bed.

Total amount of moisture in the bed at

Moisture in the feed = 1490 ppm (v/v) = 0.00149 mole fraction.

Av. Molecular weight of the 'moist' air = 29.9

$$\text{Moles air entering} = \left(\frac{3992}{29.9}\right)(0.00149) = 0.199 \text{ kmol/m}^2 \\ = \underline{3.58 \text{ kg/m}^2}$$

$$\text{Amount of moisture absorbed till the breakpoint time } (t_{b1} = 15.5 \text{ hr}) = (3.58 \text{ kg/m}^2)(15.5 \text{ h}) = \underline{55.52 \text{ kg}}$$

$$\text{Amount of moisture} \\ = (L)(1 \text{ m}^2)(P_b)(0.01) = (0.44)(1)(715)(0.01) = 3.15 \text{ kg}$$

$$\text{Total moisture in the bed at breakthrough} \\ = 3.15 + 55.52 = \underline{58.67 \text{ kg}}$$

(15)

12-11 contd...

(d) Velocity of the MTZ from data from both the columns.

$$\text{Use Eq. (12.) , } u_s = \frac{L_2 - L_1}{t_{S_2} - t_{S_1}} .$$

For $C/C_0 = 0.5$, $t_{S_1} = 16.75 \text{ hr}$; $t_{S_2} = 10.9 \text{ hr}$.

$$u_s = \frac{L_2 - L_1}{t_{S_2} - t_{S_1}} = \frac{0.44 - 0.27}{16.75 - 10.9} = 2.906 \text{ cm/hr} = 0.0484 \frac{\text{cm}}{\text{min}} .$$

which is about 10% larger than the value calculated from column-1 data only.

12-12

$$\text{Toth isotherm [Eq.(12.10)] : } q = q_m \cdot \frac{p}{(b + p^n)^{1/n}}$$

Given: total pressure, $P = 10 \text{ bar}$; CO_2 concentration in the feed gas = 3% $\Rightarrow p_{\text{CO}_2} = (1000 \text{ kPa})(0.03) = 30 \text{ kPa}$.

$b = 22.87$ in kPa unit; $n = 0.628$ $\therefore q_m = 7.935 \text{ gmol/kg}$

\Rightarrow Adsorbent loading at saturation (or equilibrium),

$$q = (7.935) \cdot \frac{30}{[22.87 + (30)^{0.628}]^{1/0.628}} = 0.989 \text{ gmol/kg}$$

Consider 1 m^2 of bed area. $L_s = L - L_{UB} = 2 \text{ m} - 0.15 \text{ m} = 1.85 \text{ m}$

CO_2 loading at breakthrough

$$= L_s \cdot 1 \text{ m}^2 \cdot p_b \cdot q = (1.85 \text{ m})(1 \text{ m}^2)(500 \text{ kg/m}^3)(0.989 \text{ gmol/kg})$$

$$= 914.5 \text{ gmol} = (914.5)(44) \text{ gm} = 40.24 \text{ kg}$$

Residual CO_2 present in the regenerated bed

$$= L \cdot 1 \text{ m}^2 \cdot p_b \cdot q_i = (2 \text{ m})(1 \text{ m}^2)(500 \text{ kg/m}^3)(1 \text{ gm/kg}) = 1000 \text{ gm}$$

CO_2 removed in one adsorption cycle $= 1 \text{ kg}$

$$= 40.24 - 1 = \underline{\underline{39.24 \text{ kg}}}$$

12.13

External mass transfer coefficient

12.16

The benzene concentration in N_2 is low (0.1%). Use the properties of N_2 only in calculations. Temperature = $25^\circ C = 298 K$.

Density of the gas, $\rho_G = \left(\frac{28}{22.4}\right) \left(\frac{273}{298}\right) = 1.145 \text{ kg/m}^3$;

$\mu_G = 1.85 \times 10^{-5} \text{ kg/m.s}$ (given); diffusivity of benzene in

$N_2 = 0.102 \text{ cm}^2/\text{s}$ at $311.3 K \Rightarrow D$ at $298 K = (0.102) \left(\frac{298}{311.3}\right)^{0.75} = 0.0945 \text{ cm}^2/\text{s}$

Particle size, $d_p = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$

Gas velocity through the bed, $u = \frac{u_0}{\epsilon_b} = \frac{0.28 \text{ m/s}}{0.43} = 0.651 \text{ m/s}$

Reynolds number, $Re = \frac{d_p \rho_G u}{\mu_G} = \frac{(3 \times 10^{-3})(1.145)(0.65)}{1.85 \times 10^{-5}} = 121$

Schmidt number, $Sc = \frac{\mu_G}{\rho_G \cdot D} = \frac{1.85 \times 10^{-5}}{(1.145)(0.0945 \times 10^{-4})} = 1.71$

Use Eq (12.74a) $\rightarrow Sh = 2 + 1.1(Re)^{0.6}(Sc)^{1/3}$

$$\Rightarrow \frac{k_c d_p}{D} = 2 + 1.1(121)^{0.6}(1.71)^{1/3} = 25.37$$

External mass transfer coefficient, $k_e = \frac{(25.37)(0.0945 \times 10^{-4})}{3 \times 10^{-3}} = 0.08 \text{ m/s}$

Effective diffusivity in an adsorbent pellet

Use Eq (12.74c). Neglect surface diffusion.

Estimate Knudsen diffusivity using the following equation:

$$D_K = 9700 \gamma_p (T/M)^{1/2}; \gamma_p = 15/2 \text{ nm} = 7.5 \times 10^{-7} \text{ cm} ; M = 78 \quad (\text{see Section 2.8.1})$$

$$D_K = (9700)(7.5 \times 10^{-7})(298/78)^{1/2} = 0.0142 \text{ cm}^2/\text{s}$$

$$\text{Effective diffusivity, } D_e = \frac{\epsilon_p}{\tau} \left[\frac{1}{(1/D) + (1/D_K)} \right]$$

$$\Rightarrow D_e = \frac{0.45}{2.8} \left[\frac{1}{(1/0.0945) + (1/0.0142)} \right] = 0.00198 \text{ cm}^2/\text{s}$$

12.14

Time to reach $C/C_0 = 0.5$ at the bed exit, $t_m = 60 \text{ min}$.

Bed length, $L = 0.5 \text{ m} = 50 \text{ cm} \Rightarrow u_s = 50/60 = 0.833 \text{ cm/min}$

It takes 15 min for the mass transfer wave front to move through its own length (taken as $C_K = 0.02$ to 0.98).

(12.14)

(12.14) contd... Length of the MTZ = $(0.833 \text{ cm/min})(15 \text{ min}) = 12.5 \text{ cm}$
 $\Rightarrow L_{UB} = 12.5/2 = 6.25 \text{ cm}$ (assuming a symmetric wave front).

For 90% bed utilization, $\frac{L - L_{UB}}{L} = 0.9$

$\Rightarrow 1 - \frac{6.25}{L} = 0.9 \Rightarrow L = 62.5 \text{ cm}$, 62.5 cm long bed to be used.

Breakthrough time, $t_b = \frac{L_s}{u_s} = \frac{(62.5 - 6.25) \text{ cm}}{0.833 \text{ cm/min}} = 67.5 \text{ min}$

(12.15) Working equation, Eq. (12.45):

$$\frac{1+KC_0}{KC_0} \ln \frac{C}{C_1} + \frac{1}{KC_0} \ln \frac{1-C_1/C_0}{1-C/C_0} = \frac{k_m \bar{a}}{(1-\epsilon_b) \rho_p} \cdot \frac{C_0}{q_0} (t - t_1)$$

Given: influent concentration, $C_0 = \frac{P_0 / RT}{(1-2)(0.02)} = \frac{(1.2)(0.02)}{(0.08317)(299)} = 9.65 \times 10^{-4} \text{ g/mol/L}$
 (mole fraction, $y_i = 0.02$; total pres. = 1.2 bar)

i.e., $C_0 = 0.965 \text{ g/mol/m}^3$; $\frac{C_0}{q_0} = \frac{1+KC_0}{q_0}$.

$$\Rightarrow \frac{C_0}{q_0} = \frac{1 + (0.12 \text{ m}^3/\text{g mol})(0.965 \text{ g mol/m}^3)}{(6.2 \text{ g mol/kg})(0.12 \text{ m}^3/\text{g mol})} = \frac{1.5 \text{ kg/m}^3}{q_0}; KC_0 = 0.116$$

$$k_m \bar{a} = 35 \text{ sec}^{-1}, \epsilon_b = 0.38$$

$$\Rightarrow (0.116) \ln \frac{C}{C_1} + \ln \frac{1-C_1/C_0}{1-C/C_0} = \frac{(0.116)(35)}{(1-0.38)(700)} \cdot (1.5) \cdot (t - t_1); \rho_p = 700 \text{ kg/m}^3$$

$$\Rightarrow 0.116 \frac{C}{C_1} + \ln \frac{1-C_1/C_0}{1-C/C_0} = 0.014 (t - t_1)$$

Time required for change of the concentration of the effluent from $C/C_0 = 0.025$ to $C/C_0 = 0.975$ is

$$t_{MTZ} = (1/0.014) \left[0.116 \ln \frac{0.975}{0.025} + \ln \frac{1-0.025}{1-0.975} \right] = 5538 = 9.2 \text{ min}$$

Approximate velocity of the wave front, Eq.(12.37)

$$u_s = \frac{u_0}{0.38 + (1-0.38)(700)(\Delta q/\Delta C)_C_0} = \frac{0.32}{(0.38) + (0.62)(700)(0.667)} = 0.0011 \frac{\text{m}}{\text{s}}$$

$$\text{Length of the MTZ} = t_{MTZ} \cdot u_s = (9.2 \times 60)(0.0011) = 0.61 \text{ m}$$

$$L_{UB} = \frac{1}{2} \text{ MTZ} = 30.5 \text{ cm} \quad (\text{for a symmetric wave front})$$

$$L_s = L - L_{UB} = 2 \text{ m} - 0.305 \text{ m} = 1.695 \text{ m}$$

$$t_b = \frac{L_s}{u_s} = \frac{1.695 \text{ m}}{0.0011 \text{ m/s}} = 1541.8 = 25.7 \text{ min}$$

12.18

12.16 Basis of calculation : 1 liter solution
Amount of sodium acetate in solution = 110 gm
(Eq wt = 99)

$$\text{Ca}^{++} \text{ in solution (eq. wt. = 20)} = \frac{110}{99} = 1.111 \text{ eq/liter}$$

$$\text{Ca}^{++} \text{ in the final solution (90% removal)} = 0.1 \text{ eq/liter}$$

$$\text{Equivalent fraction of Ca}^{++} \text{ in solution}, x_D = \frac{0.1}{1.111 + 0.1} = 0.00892$$

$$\text{From Eq (12.56), } \frac{y_D}{(1-y_D)^2} = K_{DB} \frac{\bar{C}_e}{C_e} \cdot \frac{x_D}{(1-x_D)^2}; \frac{\bar{C}_e}{C_e} = 2 \text{ eq/liter}$$

$$\Rightarrow \frac{y_D}{(1-y_D)^2} = (2.6) \cdot \frac{2}{1.111} \cdot \frac{0.00892}{(1-0.00892)^2} \Rightarrow y_D = 0.0035$$

i.e. Concentrating Ca⁺⁺ in the resin phase at equilibrium
= 0.0035 eq/liter

Fraction of ion exchange sites occupied by Ca⁺⁺ ions

$$= \frac{y_D}{\bar{C}_e} = \frac{0.0035}{2} = 0.00175, \text{ i.e., } 0.175\%$$

This is too low a fraction of utilization and the resin does not appear to be commercially attractive for the above propose.

Amount of resin required : Equivalent Ca⁺⁺ absorbed by the resin at equilibrium = (0.1)(0.9) = 0.09 equivalent

Amount of Ca⁺⁺ in the resin, y_D = 0.0035 equivalent

$$\text{Resin required} = \frac{0.09 \text{ eq.}}{0.0035 \text{ eq/liter}} = 25.7 \text{ liter per liter}$$

Resin required for m³ solution = 2.57×10^4 liter

(this is an exceptionally large value and the resin should not be considered for this particular use)

(12.17) Volume of solution = 100 ml; $C_i = 488 \text{ mg/liter}$ (12.19)

$V_e = 0.1 \text{ liter}; C_e = 3.2 \text{ mg/liter}; m_s = 0.9 \text{ gm.}$

$$q_t = V_e (C_i - C)/m_s$$

Time t (min)	0	5	10	15	20	25
$C, \text{mg/L}$	488	328.5	247	175	127	100
$q_t, \text{mg/l}$	0	17.7	26.8	34.8	40.1	43.1
$q_t/(q_t - q_e)$	0	0.536	1.12	2.19	3.8	5.702

Time t, min	30	40	50	60
$C, \text{mg/L}$	79	52	40	32
$q_t, \text{mg/g}$	45.4	48.4	49.8	50.7 = q_e
$q_t/(q_t - q_e)$	8.63	21.4	57.9	57.9

Rate of adsorption, $\frac{dq_t}{dt} = k_2 (q_e - q_t)^2 \Rightarrow \int_0^{q_t} \frac{dq_t}{(q_e - q_t)^2} = k_2 t$

$$\Rightarrow \frac{1}{q_e - q_t} - \frac{1}{q_e} = k_2 t. \text{ Rearranging we get, } \frac{1}{q_t} = \frac{1}{q_e} + \frac{1}{k_2 q_e^2 \cdot t}$$

Make a plot of $1/q_t$ against t

t	5	10	15	20	25	30	40	50
$1/t$	0.2	0.1	0.0667	0.05	0.04	0.0333	0.025	0.02
$1/q_t$	0.0565	0.0373	0.0287	0.025	0.0232	0.022	0.0207	0.02

$$\text{Intercept} = 0.0153 = 1/q_e$$

$$\text{Slope} = 0.2073 = \frac{1}{k_2 q_e^2} \quad (\text{g/mg} \cdot \text{min})$$

$$k_2 = \frac{1}{(0.2073) \cdot q_e^2}$$

It is better to use the experimental value of q_e
 $= 50.7 \frac{\text{mg}}{\text{g}}$

$$k_2 = 1.87 \times 10^{-3} \frac{\text{g}}{\text{mg} \cdot \text{min}}$$

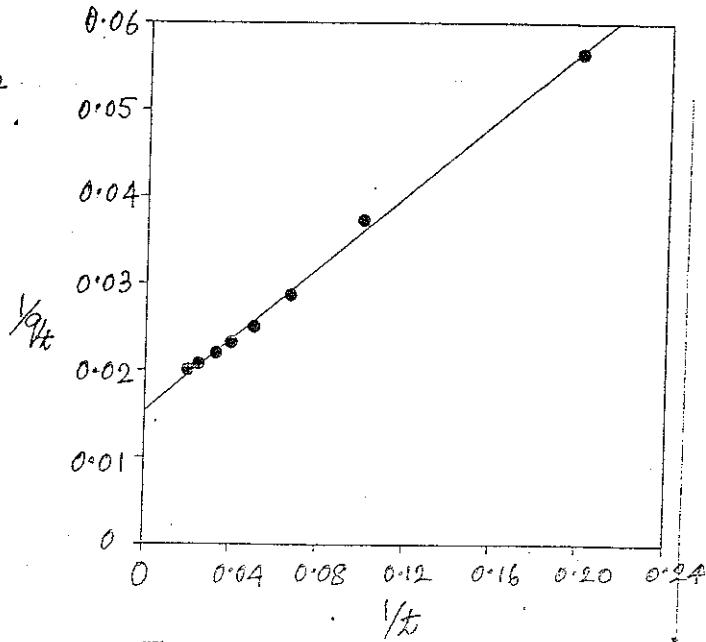


Fig. Prob 12.17. The plot of $1/q_t$ against $1/t$.

(2) See Section 13.3.2. (3) No. Needs a seed crystal or a particle to grow upon; otherwise a very high supersaturation is required. (4) Generally used for organic compounds - p-xylene , dichlorobenzene, naphthalene, bisphenol, etc.

(7) A small crystal has a higher solubility because of a larger surface energy. (8) (ii) $\text{m}^{-3}\text{hr}^{\frac{1}{2}}$, (9) (i) 6.

$$(10) \quad v_p = \Phi_v L^3 \Rightarrow \frac{dv_p}{dt} = 3L^2 \Phi_v \frac{dL}{dt} \Rightarrow \frac{dL}{dt} = G = \frac{(dv_p/dt)}{3L^2 \Phi_v} \\ = \frac{5.2 \times 10^{-4} \text{ m}^3/\text{s}}{(3)(1 \times 10^{-3} \text{ m})^2 (0.42)} = 4.13 \times 10^{-8} \text{ m/s}$$

(11) Φ_v for a sphere $= \frac{\pi}{6} = 0.523 \rightarrow$ (ii). (12) (i) 1/6.

(13) Assume that 'McCabe DL law' is valid.

Case 1: $\frac{L_{t_1} - L_{\infty}}{t_1} = G = \frac{0.1 \text{ mm}}{2R} = 0.05 \frac{\text{mm}}{\text{R}}$; Case 2: $L_{t_2} = L_{\infty} + Gt_2$
 $\Rightarrow L_{t_2} = 0.8 \text{ mm} + (0.05 \text{ mm/R})(4R) = 1 \text{ mm} \rightarrow$ (iii).

(14) (i) -10.3 kcal, heat release occurs.

(15) 1000 kg saturated solution at 80°C. Mass of $\text{CuSO}_4 = \frac{1000 \times 55}{155}$
 $= 355 \text{ kg}$, water $= 1000 - 355 = 645 \text{ kg}$. $\boxed{\text{CuSO}_4 \rightarrow 159.5}$
 $\boxed{\text{CuSO}_4 \cdot 5\text{H}_2\text{O} \rightarrow 249.5}$

Mass of $\text{CuSO}_4 \cdot 5\text{H}_2\text{O}$ produced on cooling = $M_c \text{ kg}$
 CuSO_4 in the product = $M_c \cdot \frac{159.5}{249.5} = 0.64 M_c$; water = $0.36 M_c$

Water left in the cooled solution (30°) = $(645)(0.99) - 0.36 M_c$
 $(1\% \text{ loss})$

CuSO_4 left in the solution at 30°C = $\frac{638.5 - 0.36 M_c}{(638.5 - 0.36 M_c)(25/100)}$

CuSO_4 balance: $355 = 0.64 M_c + \frac{(159.6 - 0.09 M_c)}{(159.6 - 0.09 M_c)}$

$$\Rightarrow M_c = \frac{355}{\text{kg CuSO}_4 \cdot 5\text{H}_2\text{O}}$$

(16) (iii) contact nucleation.

(18) Given: $n = 5.543 \times 10^{12} \times 10^{-4.63L} = 5.543 \times 10^{12} \exp(-10.66L) \text{ m}^{-4}$; L in mm.

(13.2)

- (18) Contd... $\frac{1}{GT} = 10.66 \text{ mm}^{-1}$. (a) Dominant size, $L_D = 3GT = \underline{0.28 \text{ mm}} \rightarrow (\text{ii})$
 (b) Put $T = 200 \text{ min} \Rightarrow G = \frac{1}{(10.66)(200)} = 4.69 \times 10^{-4} \text{ mm/min} = \underline{7.82 \times 10^{-7} \text{ cm/s}} \rightarrow (\text{i})$
 (c) Nucleation rate, $B^o = n^o G = 5.543 \times 10^{12} \text{ m}^{-4} \times 7.82 \times 10^{-7} \frac{\text{cm}}{\text{s}} = 1.56 \times 10^8 \text{ m}^{-3} \text{ s}^{-1} \rightarrow (\text{iii})$

- (19) (ii) $\text{cm}^{-1} \text{m}^{-3}$. (20) $n = 8.23 \times 10^n \exp(-9.58L) \text{ m}^{-4}$; L in mm,
 $GT = \frac{1}{9.58} \text{ mm} = 0.1044 \text{ mm}$. (a) Mass average size, $\bar{L}_m = \sqrt[3]{\frac{1}{G} \int_0^L G(L) dL} = \underline{0.19 \text{ mm}} \rightarrow (\text{iii})$
 (b) Median size, $\bar{L}_M = 3.67 GT = (3.67)(0.1044 \text{ mm}) = \underline{0.38 \text{ mm}} \rightarrow (\text{iii})$
 (c) C.V. = 52%. (iii), see Section 13.5.3.
- (21) (ii) D.T.B. (22) Average growth rate = $\frac{1}{1 \text{ mm}} \int_0^1 G(L) dL$,
 $= (9.85 \times 10^{-7} \text{ m/s}) \cdot \frac{1}{(1 \text{ mm})(1.55)(0.16)} \left[1 + 0.16L \right]_0^1 = 1.03 \times 10^{-6} \text{ m/s} \rightarrow (\text{iii})$

- (23) (i) Yes, since it depends on mass transfer which in turn depends on particle size.
 (24) (ii) Surface reaction. (25) (iii) 10^{-6} m/min ($= 1.67 \times 10^{-8} \text{ m/s}$ lies in the usual range)
 (26) (ii) a low nucleation rate, (27) (i) cooling surface.
 (28) (iii) Steel, since it is the hardest of the three materials.
 (30) (i) for a substance whose solubility does not change much with temperature.

- (31) Put $W(x) = 1 - e^{-x} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6} \right) = 0.1 \Rightarrow x = 1.74 = L/GT$.
 $L = 200 \mu\text{m} \Rightarrow GT = \frac{200 \mu\text{m}}{1.74} = \underline{114.9 \mu\text{m}}$.
 $L_1 = 0.4 \text{ mm} = 400 \mu\text{m} \Rightarrow x_1 = L_1/GT = 3.48 \Rightarrow W(x_1) = 0.46$
 $L_2 = 0.6 \text{ mm} = 600 \mu\text{m} \Rightarrow x_2 = L_2/GT = 5.22 \Rightarrow W(x_2) = 0.764$

Required fraction, $W(x_2) - W(x_1) = 0.764 - 0.46 = \underline{0.304}$

- (32) $m_c = p_c L^3 \Rightarrow \frac{dm_c}{dt} = p_c \cdot 3L^2 \frac{dL}{dt} = k_L \cdot A_c \cdot \Delta C$
 $\Rightarrow p_c \cdot 3L^2 \cdot G = k_L \cdot 6L^2 \cdot \left(\frac{C - Cs}{Cs} \right) Cs \Rightarrow G = \frac{(2)(2.5 \times 10^{-5} \text{ m/s})(0.07)(150 \frac{\text{kg}}{\text{m}^3})}{1800 \text{ kg/m}^3}$
 $\Rightarrow G = \underline{2.9 \times 10^{-7} \text{ m/s}}$ = crystal growth rate
 (33) $T_p = V/Q_p = 50 \text{ gallon}/(1 \text{ gal/min}) = 50 \text{ min} \rightarrow (\text{iii})$

PROBLEMS

(13.3)

(13.1)

$$\text{Use Eq. (13.1)} \rightarrow \ln(C/C_s) = 2\alpha M_w / RT r \cdot p_c$$

Put $C/C_s = 1.05$, $\alpha = 0.03 \text{ J/m}^2$, $M_w = 74.5 \text{ kg/kmol}$, $R = 8.317 \text{ kJ/kmol.K}$
 $T = 293.2 \text{ K}$, $p_c = 1990 \text{ kg/m}^3$, $C/C_s = 1.05$.

$$\Rightarrow \ln(1.05) = \frac{(2)(3 \times 10^{-5} \text{ kJ/m}^2)(74.5 \text{ kg/kmol})}{(8.317 \text{ kJ/kmol.K})(293.2 \text{ K})r \cdot (1990 \text{ kg/m}^3)} \Rightarrow r = 1.9 \times 10^{-8} \text{ m}$$

A spherical particle of radius $r = 19 \text{ nm}$ or less will not grow in a solution of $C = 1.05C_s$.

* * *

(13.2)

The free energy of formation of a nucleus of radius r can be considered to be the sum of its surface free energy and the free energy of phase transformation.

$$\Delta G = 4\pi r^2 \alpha + \frac{4}{3}\pi r^3 \Delta G_{tr} \quad (\text{i})$$

Surface free energy free energy of phase transformation

If ΔG has a critical or minimum value, put $\frac{d(\Delta G)}{dr} = 0$

$$\Rightarrow 8\pi r_c \alpha + 4\pi r_c^2 \Delta G_{tr} = 0 \Rightarrow r_c = -\frac{2\alpha}{\Delta G_{tr}} \quad (\text{ii}) \quad \text{for } r=r_c$$

From eq. (i) and (ii), the free energy change at $r=r_c$ is

$$\Delta G_{cr} = 4\pi r_c^2 \alpha + \frac{4}{3}\pi r_c^3 \left(-\frac{2\alpha}{r_c}\right) = \frac{4}{3}\pi r_c^2 \alpha \quad (\text{iii})$$

Now consider the Gibbs-Thomson eq. (13.1).

$$\ln \frac{C}{C_s} = \ln S = \frac{2\alpha M}{RT r \cdot p_c} = \frac{2\alpha v_m}{RT r}, \quad v_m = \text{volume of a molecule.}$$

$$\text{For } r=r_c, \quad \ln S = \frac{2\alpha v_m}{RT r_c} \Rightarrow r_c^2 = \left(\frac{2\alpha v_m}{RT \ln S}\right)^2$$

$$\text{From Eq. (iii) above, } (\Delta G)_{cr} = \frac{4}{3}\pi \alpha \left(\frac{2\alpha v_m}{RT \ln S}\right)^2 = \frac{16\pi \alpha^3 v_m^2}{3(RT \ln S)^2}$$

Nucleation rate, $B = A' \exp \left[-\frac{\Delta G_{cr}}{kT} \right]$

$$\Rightarrow B^\circ = A' \exp \left[-\frac{16\pi \alpha^3 v_m^2}{3(kT)^3 (\ln S)^2} \right], \quad S = \frac{C}{C_s} = \frac{C - C_s}{C_s} + 1 = \delta + 1.$$

For the change of δ from 1.2 to 1.21,

$$\frac{(B^\circ)_{\delta=1.21}}{(B^\circ)_{\delta=1.20}} = \frac{45,000}{\dots}$$

Take $\alpha = 0.025 \text{ J/m}^2$;
 $p_c = 2200 \text{ kg/m}^3$; $v_m = 4.415 \times 10^{-29} \text{ m}^3$
 $k = 1.38 \times 10^{-23} \text{ J/K}$
 $T = 343 \text{ K}$

(13.2) contd...

$$\text{If } S \text{ changes from } 1.25 \text{ to } 1.26, \frac{(B^o)_{S=1.26}}{(B^o)_{S=1.25}} = \frac{300}{1}$$

(13.4)

Thus, at a lower supersaturation the rate increase in homogeneous nucleation is more.

* * * *

(13.3) Population density data of the crystals are calculated from the given sieve analysis data.

Given: $P_c = 1.4 \text{ gm/cm}^3$
 $t_r = 0.42$

Mesh	Av. size $L, \text{ mm}$	$\Delta L, \text{ mm}$	$\Delta W, \text{ g/liter}$	$n, (\text{cm})^{-4}$	$\ln n$
12/14	1.3	0.22	28.5	211	5.35
14/20	1.015	0.35	29.2	1357	7.21
20/28	0.718	0.246	37.5	7004	8.85
28/35	0.507	0.175	27.0	20194	9.91
35/48	0.358	0.123	24.7	74433	11.22

Smaller than 48 mesh \rightarrow rest

$$n = \frac{0.0285 \text{ gm/cm}^3}{(1.4 \text{ gm/cm}^3)(0.42)(0.13 \text{ cm})^3 (0.022 \text{ cm})} = 211$$

Write L for \bar{L} and make a plot of $\ln(n)$ against L . The least square line is drawn; slope = -6.005 ; intercept on

Crystal size distribution function (CSD)

the ordinate = 13.19

$$\ln(n) = 13.19 - 6.005L \Rightarrow n = 5.35 \times 10^5 \exp(-6.005L)$$

Figure Prob 13.3 below

$$L \text{ is in mm} \Rightarrow GT = \frac{1}{6.005} \text{ mm}$$

$$\Rightarrow GT = 0.1653 \text{ mm}$$

$$\text{Holding time, } T = \frac{V}{Q} = \frac{200 \text{ liters}}{250 \text{ liter/hr}}$$

$$\text{Crystal growth rate, } G = \frac{0.8 \text{ hr}}{0.1653 \text{ mm}} = 5.74 \times 10^{-8} \text{ m/s}$$

$$\text{Zero size population density, } n^o = 5.35 \times 10^5 \text{ (cm)}^{-4}$$

Rate of nucleation, B^o

$$= Gn^o = (5.74 \times 10^{-8} \text{ m/s})(5.35 \times 10^5) \text{ cm}^{-4}$$

$$= 3.09 \times 10^5 \text{ nuclei/(m}^3\text{)} (3)$$

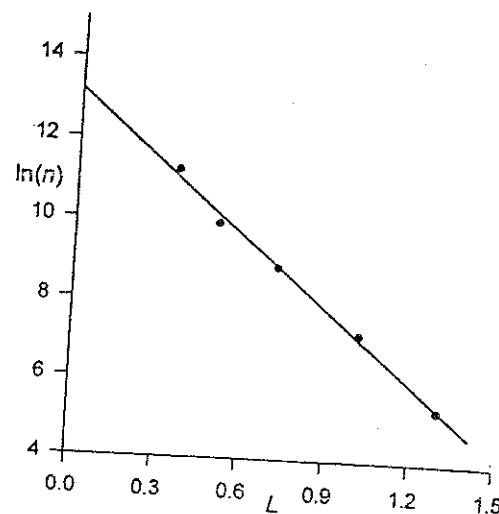


Fig. Prob 13.3. Plot of $\ln(n)$ against $L, \text{ mm}$

*

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13.5

(13.4) Given: $\rho_c = 1.77 \text{ gm/cm}^3$, $\Phi_{\text{re}} = 0.471$. Calculated population data:

<u>Size range</u>	<u>L, mm</u>	<u>$\Delta L, \text{mm}$</u>	<u>$\Delta W, \text{gm}$</u> (out of 100 gm)	<u>$\Delta W, \text{gm/lit}$</u>	<u>n, cm^{-4}</u>	<u>$\ln n$</u>
0.71 to 0.5	0.605	0.21	8.46 gm	12.01 gm/lit.	3098	8.04
0.5 to 0.355	0.427	0.145	19.18	27.23	28933	10.27
0.355 to 0.25	0.302	0.095	30.11	42.76	138030	11.84
0.25 to 0.18	0.215	0.07	22.21	31.54	543818	13.21
0.18 to 0.125	0.152	0.055	11.92	16.93	740267	13.51
0.125 to 0.09	0.107	0.035	5.29	7.51	2101000	14.56
0.09 to 0.063	0.0765	0.027	1.87	2.66	2639608	14.78

Mass of Sample Sieve analyzed = 100 gm. Crystals per liter = 142 lit.

Example: $\Delta W(\text{gm})$, column IV, is converted to $\Delta W, \text{gm/lit}$, column V.
For the first row, $(\Delta W) \text{ gm/lit} = (8.46)(\frac{142}{100}) = 12.01 \text{ gm/lit}$.

Sample Calculation form:

$$\text{Last row, } n = \frac{2.66 \times 10^{-3} \text{ gm/cm}^3}{(1.77 \text{ gm/cm}^3)(0.471)(0.00765 \text{ cm})^3 (0.0027)} = 2639608 \text{ numbers/cm}^4$$

The calculated data are plotted as $\ln(n)$ against $L(\text{mm})$ and the least square line is fitted. Slope = -12.8 , intercept on vertical axis = 15.76

Crystal size distribution (CSD):

$$n = e^{15.76 - 12.8L} = 7 \times 10^6 e^{-12.8L}$$

$$\text{Residence time} = \frac{4 \text{ liters}}{8 \text{ liter/h}} = 0.5 \text{ h.}$$

$$GT = \frac{1}{12.8} \text{ mm; } G = \frac{1}{(12.8)(0.5)} \text{ mm/h}$$

$$\Rightarrow G = 4.34 \times 10^{-8} \text{ m/h} = \text{crystal growth rate}$$

Zero-size population density,

$$n^0 = 7 \times 10^6 (\text{cm})^{-4}$$

Nucleation rate,

$$B^0 = Gn^0 = (4.34 \times 10^{-8} \text{ m/h})(7 \times 10^6 \text{ cm}^{-4})$$

$$= 3.04 \times 10^7 \text{ nuclei} (\text{m}^3) (\text{s})$$

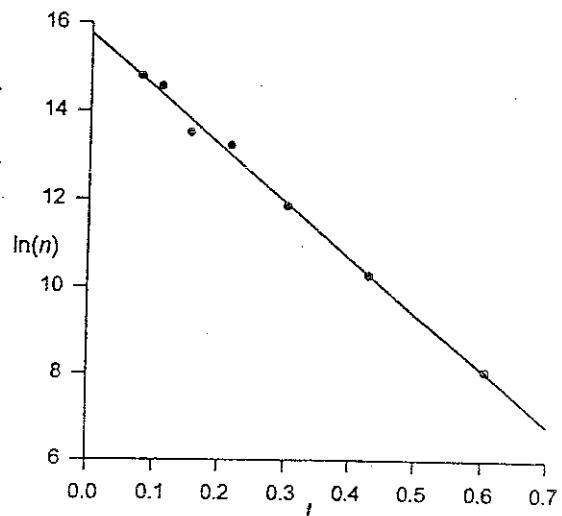


Fig. Prob 13.4.1 Plot of $\ln(n)$ against L, mm

13.6

(13.4) contd...

The nucleation and growth rate data for three set of experiments at three different holding times are tabulated.

$\frac{Q(\text{lit/hr})}{24}$	$\frac{T(h)}{0.167}$	$\frac{B^\circ \text{ number}}{\text{m}^3 \cdot \text{s}}$	$\frac{G, \text{m/s}}{1.3 \times 10^{-7}}$
16	0.25	7.1×10^7	8.6×10^{-8}
8	0.50	3.04×10^7	4.34×10^{-8}

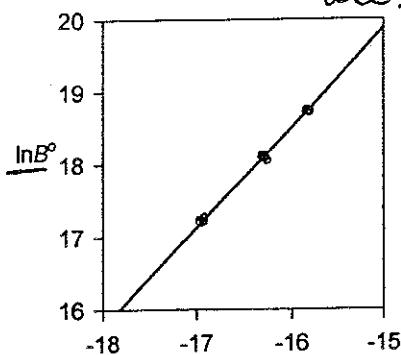
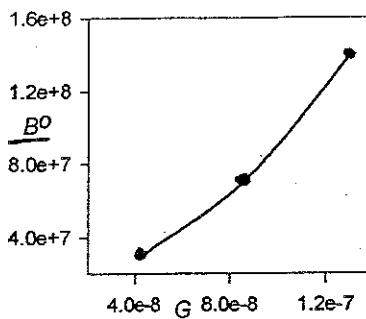


Fig Prob 13.4.3

Fig. Prob 13.4.2. Plot of B° against G .

The plot of B° against G is nonlinear—see Fig. Prob 13.4.2.

However a plot on logarithmic scale (Fig. Prob 13.4.3) is linear with a slope of 1.38 . So the nucleation rate depends upon G as given below

$$B^\circ \propto G^{1.38} \quad \text{in the given case.}$$

(13.5) Mass average crystal size, $L_m = \sqrt[3]{6 \cdot GT}$ [eq.(ix), Example 13.1]

$$\Rightarrow \sqrt[3]{6 \cdot GT} = 1 \text{ mm} \Rightarrow GT = 0.55 \text{ mm}$$

$$\text{Holding time, } T = V/Q = \frac{5 \text{ m}^3}{4 \text{ m}^3/\text{h}} = 1.25 \text{ h}$$

$$\text{Crystal growth rate, } G = 0.55 \text{ mm}/1.25 \text{ h} = 1.222 \times 10^{-7} \text{ m/s}$$

$$\text{Magma density, } M_T = 150 \text{ kg/m}^3 = 6 \text{ g/cc } n^\circ (GT)^4, \text{ eq.(viii), Ex 13.1}$$

$$\Rightarrow 150 = (6)(0.6)(1900) \cdot n^\circ \cdot (5.5 \times 10^{-4})^4$$

$$\Rightarrow n^\circ = \frac{2.4 \times 10^{11} \text{ numbers/m}^3}{\text{zero size population density.}}$$

$$\text{Secondary nucleation rate, } B^\circ = (2.4 \times 10^{11} \text{ m}^{-4})(1.222 \times 10^{-7} \text{ m/s})$$

$$= 2.933 \times 10^4 \text{ numbers/m}^3 \cdot \text{s}$$

(13.6) Working volume of the crystallizer, $V = 5 \text{ m}^3; T = 1.8 \text{ h}$

$$\text{Product withdrawal rate, } Q_p = V/T = 5/1.8 = 2.778 \text{ m}^3/\text{h.}$$

$$\text{Suspension density, } M_T = 150 \text{ kg crystal/m}^3$$

$$\text{Production rate} = Q_p \cdot M_T = (2.778 \frac{\text{m}^3}{\text{h}})(150 \frac{\text{kg}}{\text{m}^3}) = 417 \text{ kg/h}$$

Use Eq(13.28). Median size, $L_M = 0.8 \text{ mm} = 3.67 GT; T = 1.8 \text{ h}$

$$G = \frac{L_M}{3.67 T} = \frac{8 \times 10^{-4} \text{ m}}{(3.67)(1.8 \text{ h})} = 3.364 \times 10^{-8} \text{ m/s}$$

$$\text{Use Eq.(viii), Example 13.1. } M_T = 6 \text{ g/cc } n^\circ (GT)^4$$

$$\Rightarrow 150 \frac{\text{kg}}{\text{m}^3} = (6)(0.65)(2500 \text{ kg/m}^3) \cdot n^\circ \cdot (8 \times 10^{-4} \text{ m/s})^4 \Rightarrow n^\circ = 6.814 \times 10^{12} \text{ m}^{-4}$$

= zero size population density.

(13.6) Contd... Secondary nucleation rate, $B^o = G \cdot n^o$

$$= (3.364 \times 10^{-8} \text{ m/s})(6.814 \times 10^{12} \text{ m}^{-3}) = 2.292 \times 10^5 \frac{\text{nuclei}}{\text{m}^3 \cdot \text{s}}$$

Mass fraction and number fraction of crystals upto $L = 0.2 \text{ mm}$

Dimensionless size, $x = \frac{L}{GT} = \frac{0.2 \text{ mm}}{0.218 \text{ mm}} = 0.917$ $[GT = \frac{0.8 \text{ mm}}{3.67} = 0.218 \text{ mm}]$

Cumulative mass fraction of crystals upto size x , eq(13.25b)

$$W(x) = 1 - e^{-x} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right) \Rightarrow \text{for } x = 0.917, W = 0.014 \text{ or } 1.4\%$$

Number fraction of crystals upto a dimensionless size $x = 0.917$

$$\frac{N}{N_T} = \frac{\int_0^L n^o e^{-L/GT} dL}{\int_0^\infty n^o e^{-L/GT} dL} = \frac{n_0 GT \int_0^{0.917} e^{-x} dx}{n_0 GT \int_0^\infty e^{-x} dx} = 0.6 \Rightarrow 60\%$$

Note that crystals upto a size of 0.2 mm constitute 60% of the population but have a mass of only 1.4% of the total mass.

* * *

(13.7) Given: $G = 2.31 \times 10^{-8} \text{ m/s}$; $T = 2h = 7200s \Rightarrow GT = 1.663 \times 10^{-4} \text{ m}$

(a) Cumulative mass distribution function is given by eq(13.25b)

$$W(x) = 1 - e^{-x} \left(1 + x + \frac{x^2}{2} + \frac{x^3}{6}\right), x = \frac{L}{GT} = \frac{6.013 \times 10^3 L}{1.663 \times 10^{-4} \text{ m}}, \text{ L in meter.}$$

Dominant size, $L_D = 3 \cdot GT = (3)(1.663 \times 10^{-4} \text{ m}) = 0.5 \text{ mm}$

Median size, Eq(13.28), $L_M = 3.67 GT = (3.67)(1.663 \times 10^{-4} \text{ m}) = 0.61 \text{ mm}$

(b) To achieve a dominant size, $L_D = 0.6 \text{ mm}$,

$$GT = \frac{L_D}{3} = 0.6 \text{ mm}/3 = 0.2 \text{ mm}; G = 2.31 \times 10^{-8} \text{ m/s}$$

$$T = \frac{0.2 \text{ mm}}{2.31 \times 10^{-5} \text{ mm/s}} = 8660s = 2.4h$$

Volumetric rate of product withdrawal,

$$Q_p = V/T = \frac{2 \text{ m}^3}{2.4h} = 0.833 \text{ m}^3/\text{h}$$

If we take $L = 0.2 \text{ mm}$, $x = \frac{L}{GT} = \frac{0.2 \text{ mm}}{0.2 \text{ mm}} = 1$

Mass fraction of crystals below $L = 0.2 \text{ mm}$, i.e. $x = 1$ is

$$W(x=1) = 1 - e^{-1} \left(1 + 1 + \frac{1}{2} + \frac{1}{6}\right) = 1 - 0.981 = 0.019 \Rightarrow 1.9\%$$

* * *

(a) Start with the population balance equation (13.18)

$$\frac{dn_G}{dL} + \frac{n}{\tau} = 0; \text{ for } L=0, n=n^o; G=G_0 (1+r_L)^b$$

$$\Rightarrow \frac{dn_G}{dL} + \frac{n_G}{G\tau} = 0 \Rightarrow \frac{dn_G}{n_G} = -\frac{dL}{G\tau} = -\frac{dL}{\tau G_0 (1+r_L)^b}$$

Integrating, $\ln(n_G) = -\frac{1}{\tau G_0} \int \frac{dL}{(1+r_L)^b} + K' = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} (1+r_L)^{1-b} + K'$

Put $L=0, n=n^o, G=G_0$ to get

$$\ln(n^o G_0) = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} + K' \Rightarrow K' = \ln(n^o G_0) + \frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)}$$

The population distribution function becomes

$$\ln\left(\frac{n_G}{n^o G_0}\right) = -\frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} \cdot (1+r_L)^{1-b} + \frac{1}{\tau G_0} \cdot \frac{1}{r(1-b)} = \frac{1-(1+r_L)^{1-b}}{\tau G_0 \cdot r(1-b)}$$

$$\Rightarrow n_G = n^o G_0 \exp\left[\frac{1-(1+r_L)^{1-b}}{\tau G_0 \cdot r(1-b)}\right]$$

$$\Rightarrow n(L) = n^o (1+r_L)^{-b} \exp\left[\frac{1-(1+r_L)^{1-b}}{\tau G_0 \cdot r(1-b)}\right] \quad \dots \dots \dots \quad (i)$$

(b) Given: $G_0 = 2.22 \times 10^9 \text{ m/s}$; $\tau = 1 \text{ h}$; $r = 1.25 \times 10^4 \text{ m}^{-1}$; $b = 0.2$

$$\rho_v = 0.48; \rho_c = 1460 \text{ kg/m}^3; n^o = 8 \times 10^{14} \text{ m}^{-4}$$

Putting the concerned values in eq (i) above, the population density distribution function becomes

$$n(L) = 8 \times 10^{14} (1 + 1.25 \times 10^4 L)^{-0.2} \cdot \exp\left[1.25 \left\{1 - (1 + 1.25 \times 10^4 L)^{0.8}\right\}\right] \text{ kg/m}$$

Magnitude density, $M_T = \int (\rho_v L^3 \rho_c) n(L) dL$

Put the expression for $n(L)$ and numerically integrate

$$\text{to get } M_T = 288 \text{ kg/m}^3$$

(13.9)

(13.9) Given : $G = \frac{5 \times 10^8}{m/s}$; $V = 10 m^3$; $Q_p = 4 m^3/h$; $T = \frac{V}{G} = \frac{2.5 h}{s}$
 $n^o = 5 \times 10^{10} / (\text{cm})(\text{liter}) = \frac{5 \times 10^{15}}{m^{-4}}$; $GT = (5 \times 10^8)(2.5 \times 3600) = 4.5 \times 10^{12} m^{-4}$

(a) Total number of crystals in the vessel at any time,

$$N_T = n^o GT = (5 \times 10^{15} m^{-4})(4.5 \times 10^{-4} m) = \frac{2.25 \times 10^{12} m^{-3}}{\text{Eq. (iii), Ex. 13.1}}$$

(b) For $L = L_1 = 0.5 \text{ mm}$, dimensionless size, $x = \frac{0.5 \times 10^{-3} \text{ m}}{4.5 \times 10^{-4} \text{ m}} = \underline{1.11}$
For $L = L_2 = 1 \text{ mm}$, $x = 2.22$

Mass fraction lying between $x = 1.11$ to $x = 2.22$

$$= W(x_2) - W(x_1) = 0.973 - 0.8154 \Rightarrow \underline{15.8\%}$$

(c) Coefficient of variation = 52%, see Section 13.5.3.

* * * *

(13.10) $B^o = n^o G \Rightarrow 7.33 \times 10^7 M_T^{0.6} G^{0.5} = 9.2 \times 10^{12} G$

Put $M_T = 20 \text{ kg/m}^3$ to get $G = \frac{2.31 \times 10^{-9} m/s}{dt} = \frac{dL}{dt}$

Rate of increase in the mass of a crystal can be

written as $\frac{dm_c}{dt} = \frac{d}{dt} (\rho_0 L^3 \rho_c) = 3 \rho_0 L^2 \rho_c \cdot \frac{dL}{dt}$

Put $\rho_0 = 0.6$; $L = 5 \times 10^{-4} \text{ m}$; $\rho_c = 2700 \text{ kg/m}^3$ to get

$$\frac{dm_c}{dt} = (3)(0.6)(5 \times 10^{-4})^2(2700)(2.31 \times 10^{-9}) = \frac{2.8 \times 10^{-12} \text{ kg/s}}{* * * *}$$

(13.11)

Given : $V = m^3$; $G = 2.31 \times 10^{-8} m/s$; $Q_i = 1 m^3/h$ (since $T = 2 h$).

clear liquid advance rate = $Q_o m^3/h$; underflow = $Q_p m^3/h$.

Assume $Q_i = Q_o + Q_p$.

Use Eq. (13.42b), put $L_f = 0$ since the overflow does not have fines.

$$n = n^o \exp(-L/GT_p) \Rightarrow w(x) = 1 - e^{-x} (1 + x + x^2/2 + x^3/6), \text{ Eq. (13.25b)}$$

so mass % of the crystals are above 0.8 mm $\Rightarrow w(x) = 0.2$

$$\Rightarrow x = 2.31 = \frac{1}{G T_p} = \frac{0.8 \text{ mm}}{(2.31 \times 10^{-8} m/s) T} \Rightarrow T_p = \underline{4.17 \text{ hr}}$$

(13.11) Contd... $Q_p = V/\tau_p = 2m^3/4.17h = 0.48 m^3/h$; i.e. $Q_0 = \underline{0.52 m^3/h}$

Case 2: Fines are removed in the overflow up to a size of $80\mu m$.

Since the fines removed constitute a small mass fraction of the crystals, the distribution function of the former case applies.

The answer remains the same, i.e. $\underline{Q_0 = 0.52 m^3/h}$

(13.12) Given : $V = 3.5 m^3$; $Q_0 = 10 m^3/h$; $Q_p = 3 m^3/h$; $\Phi_{ho} = 0.55$; $G = 1.45 mm/h = \underline{4.03 \times 10^{-7} m/s}$; $M_T = 150 \text{ kg/m}^3$; $P_c = 1993 \frac{\text{kg}}{\text{m}^3}$; $L_F = 7 \times 10^{-5} m$

Use Eq.(i), Example 13.6, to calculate n^o . $\tau_F = 3.5/10 = 0.35 h$

$$M_T = 6 \Phi_{ho} P_c n^o \exp\left[-\frac{L_F}{G\tau_F}\right] (G\tau_p)^4 \quad \tau_p = 3.5/3 = 1.167 h$$

$$\Rightarrow 150 \frac{\text{kg}}{\text{m}^3} = (6)(0.55)(1990 \text{ kg/m}^3) \cdot n^o \cdot \exp\left[-\frac{7 \times 10^{-5} m}{4.03 \times 10^{-7} \frac{m}{s} \times (0.35 \times 3600 s)}\right] \times$$

$$\Rightarrow n^o = \underline{3.192 \times 10^9 \text{ m}^{-4}} \quad (4.03 \times 10^{-7} \frac{m}{s} \times 1.167 \times 3600 s)^4$$

$$\text{Nucleation rate, } B^o = n^o G = (3.192 \times 10^9 \text{ m}^{-4})(4.03 \times 10^{-7} \text{ m/s}) \\ = \underline{1280 \text{ nuclei/m}^3 \cdot s}$$

Mass fraction of crystals removed as fines ($L \leq L_F$) at the top.

Fines per unit volume,

$$M_F = \int_0^{L_F} (\Phi_{ho} L^3 P_c) \cdot n(L) dL; \quad n(L) \text{ is given by Eq.(13.42a)}$$

$$= (\Phi_{ho} P_c n_o) (G\tau_F)^4 \int_0^{\xi_F} \xi^3 e^{-\xi} d\xi; \quad \xi = L/G\tau_F \\ \xi_F = L_F/G\tau_F = 0.138$$

$$\Rightarrow M_F = (0.55)(1990)(3.192 \times 10^9)(5.078 \times 10^{-4})^4 (0.001) = \underline{2.323 \times 10^{-4} \frac{\text{kg}}{\text{m}^3}}$$

Mass fraction of crystals withdrawn as fines at the top

$$= \frac{M_F}{M_T} \cdot \frac{Q_0}{Q_p} = \frac{2.323 \times 10^{-4} \text{ kg/m}^3}{150 \text{ kg/m}^3} \cdot \frac{10 \text{ m}^3/h}{3 \text{ m}^3/h} = \underline{5.16 \times 10^{-6}}, \text{ a very small fraction.}$$

(13.13) Refer to Fig Prob 13.13 (a). If there are no crystals in the feed, the population balance equation can be written as:

$$-VG \frac{dn}{dL} = zQ_p n + (R'-1)Q_p \cdot n \cdot [1 - H(L_F - L)] - (z-1)Q_p \cdot n \cdot [1 - H(L - L_c)].$$

Significances of the terms on the r.h.s.:

$zQ_p n$ = rate of removal of crystals at the bottom

$(R'-1)Q_p \cdot n$ = rate of removal of fines at the top ($L \leq L_F$)

$(z-1)Q_p \cdot n$ = rate of return of the crystals of size $L \leq L_c$ through the elutriation leg.

The equation reduces to the following forms for the three size ranges.

$$\text{For } L \leq L_F, -VG \frac{dn}{dL} = zQ_p n + (R'-1)Q_p \cdot n - (z-1)Q_p \cdot n$$

$$\Rightarrow -VG \frac{dn}{dL} = R'Q_p \cdot n \Rightarrow n = n^* \exp\left(-\frac{R'}{GT} \cdot L\right), T = V/Q_p \dots (i)$$

$$\text{For } L_F \leq L \leq L_c, -VG \frac{dn}{dL} = zQ_p n - (z-1)Q_p \cdot n = Q_p \cdot n$$

$$\Rightarrow n = c' \exp(-L/GT) \dots (ii)$$

$$\text{For } L_c \leq L, -VG \frac{dn}{dL} = zQ_p n \Rightarrow n = c'' \exp\left(-\frac{zL}{GT}\right) \dots (iii)$$

Using the condition of "continuity of population density" at $L = L_F$ and at $L = L_c$, the constants c' and c'' can be determined. The resulting density functions are shown under "Hints" to Prob 13.13.

* * *

(13.14) Feed = 2500 kg solution at $80^\circ C$, 125 kg anhydrous salt per 1000 kg water.

$$\text{Mass of } Na_2CrO_4 = (2500) \left(\frac{125}{1000+125} \right) = \underline{\underline{277.8 \text{ kg}}}; \text{ Water} = \underline{\underline{222.2 \text{ kg}}}$$

3% of the water evaporates $\Rightarrow (2222.2)(0.03) = \underline{\underline{66.7 \text{ kg water lost}}}$

$$\text{Let the mass of } Na_2CrO_4 \cdot 10H_2O \quad \begin{bmatrix} \text{Molar } Na_2CrO_4 = 162 \\ Na_2CrO_4 \cdot 10H_2O = 342 \end{bmatrix}$$

deposited at the end = m kg

$$Na_2CrO_4 \text{ deposited} = \frac{162}{342} \cdot m = \underline{\underline{0.4737 \cdot m \text{ kg}}}; \text{ water} = \underline{\underline{0.5263 \cdot m \text{ kg}}}$$

$$\text{Mass of water in the final slurry} = \frac{2222.2 - 66.7 - 0.5263m}{2155.5 - 0.5263m}$$

(13.14) contd... Solubility of Na_2CrO_4 at 30°C = $88.7 \text{ kg}/1000 \text{ kg water}$.

(13.12)

Mass of Na_2CrO_4 in solution at the end

$$= (2155.5 - 0.5263m) \frac{88.7}{1000} = \underline{\underline{191.2 - 0.0467m \text{ kg}}}$$

Na_2CrO_4 balance: $(277.8 - 0.4737m) = (191.2 - 0.0467) \Rightarrow m = \underline{\underline{203 \text{ kg}}}$

Quantity of Crystals = $203 \text{ kg} + 1.2 \text{ kg (seed)} = \underline{\underline{204.2 \text{ kg}}}$

Crystal size calculation: Seed size = $L_8 = 70 \mu\text{m} = \underline{\underline{7 \times 10^{-5} \text{ m}}}$

Mass of a seed = $\phi_b L_s^3 P_c = (0.45)(70 \times 10^{-6})^3 (1480) = \underline{\underline{2.284 \times 10^{-10} \text{ kg}}}$

Number of seed crystals = $\frac{1.2 \text{ kg}}{2.284 \times 10^{-10} \text{ kg}} = \underline{\underline{5.254 \times 10^9}}$

Mass of a product crystal = $\frac{204.2 \text{ kg}}{5.254 \times 10^9} = \underline{\underline{3.87 \times 10^{-8} \text{ kg}}}$

If L is the average size of a product crystal,

$$\phi_b L^3 P_c = 3.87 \times 10^{-8} \Rightarrow (0.45) \cdot L^3 \cdot (1480) = 3.87 \times 10^{-8} \Rightarrow L = \underline{\underline{3.87 \times 10^{-4} \text{ m}}}$$

$$\text{Residence time} = 6 \text{ h} = \underline{\underline{6 \times 3600 \text{ s}}} = \underline{\underline{0.387 \text{ mm}}}$$

Average growth rate,

$$G = \frac{L - L_8}{t} = \frac{3.87 \times 10^{-4} - 70 \times 10^{-6}}{6 \times 3600} = \underline{\underline{1.47 \times 10^{-8} \text{ m/s}}}$$

* * * *

(13.15)

Calculation of the quantity of feed solution per batch.

Initial solution concentration (80°C) = $0.07 + (0.00132)(80)$

$$= 0.176 \text{ kg } \text{K}_2\text{SO}_4/\text{kg solution} = \frac{0.176}{1 - 0.176} = \underline{\underline{0.213 \text{ kg/kg water}}}$$

Final solution concentration (25°C) = $0.07 + (0.00132)(25)$

$$= 0.103 \text{ kg/kg solution} = \underline{\underline{0.115 \text{ kg per kg water}}}$$

Neglect the mass of seeds added.

Product per kg water = $0.213 - 0.115 = \underline{\underline{0.098 \text{ kg/kg water}}}$

Mass of product required = 400 kg

$$\begin{aligned} \text{Mass of feed solution} &= (400 \text{ kg}) \times \frac{1 \text{ kg water}}{0.098 \text{ kg crystal}} \times \frac{1.213 \text{ kg soln.}}{1 \text{ kg water}} \\ &= \frac{4950}{1200} = \underline{\underline{4.125 \text{ m}^3 \text{ solution}}} \end{aligned}$$

(13.15) contd... Final volume of slurry

(13.13)

$$= \text{volume of crystals} + \text{volume of solution}$$

$$= \frac{400}{2660} + (400 \text{ kg crystal}) \left(\frac{1 \text{ kg water}}{0.98 \text{ kg crystal}} \right) \left(\frac{1.115 \text{ kg slurry}}{1 \text{ kg water}} \right) \left(\frac{1}{1200 \text{ kg/m}^3} \right)$$

$$= 0.15 \text{ m}^3 + 3.79 \text{ m}^3 = 3.94 \text{ m}^3 \text{ slurry.}$$

$$\text{Suspension density} = 400 \text{ kg / } 3.94 \text{ m}^3 \text{ slurry} = \frac{101.5 \text{ kg crystal}}{\text{m}^3 \text{ slurry}}$$

which is less than the allowable limit of 120.

If the crystallization tank is 75% full,

$$\text{volume of crystallizer} = \frac{4.125 \text{ m}^3 (\text{feed})}{0.7} = \underline{5.9 \text{ m}^3, \text{ say } 6 \text{ m}^3}$$

$$\text{Mass of a product crystal } (L = 0.8 \text{ mm} = 8 \times 10^{-4} \text{ m})$$

$$= \rho_w L^3 P_c = (0.9)(8 \times 10^{-4})^3 (2660) = 1.226 \times 10^{-6} \text{ kg}$$

$$\text{Number crystals (in 400 kg product)} = \frac{400 \text{ kg}}{1.226 \times 10^{-6} \text{ kg}} = \underline{3.263 \times 10^8}$$

From this figure it can be shown that the mass of seed crystals is about 0.1% of the product.

$$\text{Average growth rate, } G = 5 \times 10^{-8} \text{ m/s; } L_s = \underline{8 \times 10^{-7} \text{ m}}$$

$$\Rightarrow \text{residence time} = \frac{L - L_s}{G} = \frac{8 \times 10^{-4} - 8 \times 10^{-5}}{5 \times 10^{-8}} = \underline{14,400 \text{ s}}$$

Cooling rate calculation

= 4 hr

$$\text{Solubility, } C = 0.07 + 0.001320$$

Initial cooling rate [See Eq.(13.48)], use Eq.(13.48)

$$-\frac{d\theta}{dt} = \frac{3 M_S}{a_2 V} \cdot \frac{G}{L_s} (z+1)^2 ; M_S = 0.4 \text{ kg} \quad (\text{calculate it!})$$

$$\Rightarrow -\frac{d\theta}{dt}_i = \frac{(3)(0.4)}{(1.32)(4.125)} \cdot \frac{5 \times 10^{-8}}{8 \times 10^{-5}} = \underline{1.3 \times 10^{-3} \text{ }^\circ\text{C/s.}}$$

$$\text{Final cooling rate, } -\frac{d\theta}{dt}_f = 0.13 \text{ }^\circ\text{C/s} \quad (\text{calculate it!})$$

(13.16) Product crystal size = 1.1 mm , mass of product = 500 kg/batch

$$\text{Number of crystals, } N = \frac{500}{\Phi_b L^3 P_c} = \frac{500}{(0.95)(1.1 \times 10^{-3})^3 (1750)} = 2.26 \times 10^8$$

$$\text{Mass of seed crystals} = \Phi_b L^3 P_c N = (0.95)(7 \times 10^{-5})^3 (1750)(2.26 \times 10^8)$$

Batch time (growth time of crystals), $= 0.129 \text{ kg}$

$$t = \frac{L - L_s}{G} = \frac{1.1 \times 10^{-3} - 7 \times 10^{-5} \text{ m}}{6 \times 10^{-8} \text{ m/s}} = 171608 = 4.77 \text{ hr}$$

The evaporation profile can be calculated from Eq.(13.45)

$$-\frac{dV}{dt} = \frac{3 M_s}{C_s} \cdot \frac{G}{L_s} \left(1 + \frac{Gt}{L_s}\right) = \frac{(3)(0.129)}{275} \cdot \frac{6 \times 10^{-8}}{7 \times 10^{-5}} \left(1 + \frac{6 \times 10^{-8}}{7 \times 10^{-5}} t\right)^2$$

$$= 1.206 \times 10^{-6} \left(1 + 8.57 \times 10^{-4} t\right)^2 \rightarrow 1.206 \times 10^{-6} \text{ m}^3/\text{s} \quad \text{at } t = 0$$

$$\text{and } -\frac{dV}{dt} = 1.894 \times 10^{-5} \text{ m}^3/\text{s} \quad \text{at } t = 4.77 \text{ hr.}$$

Volume of slurry: Take maximum allowable slurry density,

$$\text{Final Slurry volume} = \frac{500 \text{ kg crystal}}{1750 \text{ kg/m}^3} + \frac{500 \text{ kg crystal}}{180 \text{ kg crystal/m}^3} \frac{180 \text{ kg/m}^3 \text{ soln.}}{= 3.064 \text{ m}^3}$$

$$\text{Feed solution} \rightarrow 3.064 \text{ m}^3 + 500 \left(1 + \frac{1}{0.275}\right) \times (1150) \text{ m}^3 = 5.08 \text{ m}^3$$

To take into account, bubbling, possible foaming and a guard against entrainment a crystallizer volume of 10 m^3 may be considered.

- (14.1) (iii); (14.2) (i) polysulfone, the other two polymers are not soluble in any solvent; (14.4) (ii); (14.5) (i) cross-linked PVA;
- (14.6) (i) gas separation; (14.7) (iii) pervaporation; (14.8) (i);
 (14.9) (i) very large; (14.10) (i) fully permeable; (14.11) (i) increases;
 (14.12) (iii) track etching; (14.13) (i) water, CA is not soluble in water;
- (14.14) (i) Na^+ , since the membrane has 'fixed' negative charges;
- (14.15) (i) $1 - 100 \text{ nm}$; (14.17) (i) pore mouth adsorption; (14.18) $1 - \frac{0.32}{8} = 0.96$, (iii); (14.19) $\frac{8\%}{4\%} = 2$, (i); (14.20) $R_m + R_g = \frac{\Delta P}{J_w \cdot \mu}$
 $\Rightarrow R_m + R_g = \frac{4 \times 10^5}{(0.052)(9 \times 10^4)} = 8.55 \times 10^{-9} \text{ m}^{-1} \Rightarrow R_g = (8.55 - 2.4) \times 10^{-9}$
 $= 6.15 \times 10^{-9} \text{ m}^{-1}$, (i); (14.21) (iii) $12,000 \text{ m}^2/\text{m}^3$; (14.22) (ii) hemodialysis;
- (14.23) (i) diffusivity increases; (14.24) (iii) pervaporation, since the solvent does not have any affinity for water (water in MIBK has a high fugacity); (14.25) (i) 1.2 Barrer;
- (14.26) (iii) rubbery; (14.27) (i) + (ii); (14.28) (iii) 13.5 Barrer;
- (14.29) Pol. Mod. = $C_m/C_b = \exp(J_w/k_L) = \exp(\frac{0.042/3600 \text{ m}/8}{5 \times 10^{-6} \text{ m}/8}) = 10.3$, (ii);
- (14.30) (ii) CA, polyimide, (iii) PDMS, (iv) PDMPS; (v) asymmetric UF membrane, (vii) crosslinked PVA, (viii) asymmetric UF, polysulfone; (x) PDMPS; (xiii) ceramic membrane, (xiv) CA;
- (14.31) (i) increases; (14.32) (iii) convection; (14.33) (i) 1×10^{16} ;
- (14.34) (iii) m^{-1} ; (14.35) (i) increases; (14.36)
- (14.37) Transport occurs by a combination of convection and diffusion; (14.38) reduced flux because of osmotic pressure, higher salt permeation rate, 'scaling' on the membrane surface; (14.39) 'Apparent' rejection = $1 - C_p/C_b = 0.975$, $C_b = 0.02$
 $\Rightarrow C_p = 5 \times 10^{-4}$, $J_w(C_b - C_p) = k_L(C_m - C_b) \Rightarrow 1.8 \times 10^{-4} (0.02 - 5 \times 10^{-4})$
 $= 4.4 \times 10^{-5} (C_m - 0.02) \Rightarrow C_m = 0.1$, true rejection = $1 - \frac{C_p}{C_m} = 1 - \frac{5 \times 10^{-4}}{0.1} = 0.995$
- (14.40) biological attack, hydrolysis under...; (14.41) $J_A = J_B \Rightarrow$
 $(\hat{P}_A/l_m) \cdot (P_1 x_A - P_2 y_A) = (\hat{P}_B/l_m) [P_1 (1-x_A) - P_2 (1-y_A)]$, given, $y_A = 0$, $P_1 = 12.5$,
 $P_2 = 1.5$, $\hat{P}_A/\hat{P}_B = 3.15$, solve to get $x_A = 0.212$, $x_B = 0.788$.

14.2

- (14.42) (i) $\alpha_{AB} = 1$; (14.43) (ii) decreases; (14.44) The alternative arrangement will not work satisfactorily.

Brine is fed into the cathode compartment and the product NaOH is also withdrawn from this compartment. As a result the product will have a lot of NaCl in it.

(14.45) $\hat{E} = 1 - e^{-N_T}$ [see Eq.(14.52), take $\varsigma \approx 0$] $\Rightarrow 0.85 = 1 - e^{-N_T} \Rightarrow N_T = 1.9$
 Calculate the length of the hollow fibers : $L = a_m / n \cdot B'$; $B' = \frac{\pi d}{4}$
 $\Rightarrow L = 1.2 \text{ m}^2 / n \cdot (\pi \times 300 \times 10^{-6} \text{ m}) = 0.255 \text{ m}$.
 'Height of a transfer unit' $= L/N_T = 0.255 \text{ m} / 1.9 = \underline{\underline{0.134 \text{ m}}}$

PROBLEMS

- (14.1) Use Eq.(14.4). Plot $J_w (\text{kg/m}^2\text{h})$ against $\Delta P (\text{kPa})$, Fig Prob 14.1.
 Slope = $12.5 \text{ kg/hm}^2\text{kPa}$

$$= \underline{\underline{0.0125 \text{ m}^3/\text{hm}^2\text{kPa}}}$$

$$\Rightarrow \frac{\epsilon d^2}{32\mu l_m T} = 0.0125 \frac{\text{m}^3}{\text{hm}^2\text{kPa}}$$

$$\text{Given: } \epsilon = 0.32, l_m = 100 \mu\text{m} = \underline{\underline{10^{-4} \text{ m}}}$$

$$d = 1.0 \mu\text{m} = \underline{\underline{1.0 \times 10^{-6} \text{ m}}}$$

$$\mu = 0.9 \text{ cP} = 9 \times 10^{-4} \text{ kg/m s}$$

$$T = \frac{(0.32)(1.0 \times 10^{-6})^2 (3600)(100)}{(32)(9 \times 10^{-4})(10^{-4})(0.0125)}$$

$$= \underline{\underline{3.2}}$$

* * *

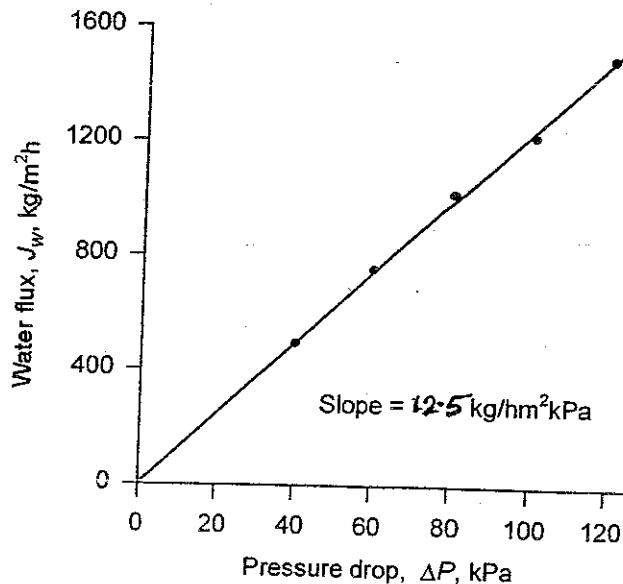


Figure Prob 14.1: Plot of J_w against ΔP

- (14.2) Solute rejection, $R' = 0.96$; no concentration polarization

$$\Rightarrow C_m = C_b = 0.015 \text{ molar}; C_p = (0.015)(1-0.96) = 6 \times 10^{-4} \text{ molar}$$

'Effective' pressure driving force = $\Delta P - \Delta T T = 2.9 - 0.357 = \underline{\underline{2.543 \text{ bar}}}$

$$[\Delta P = 4 - 1.1 = 2.9 \text{ bar}; \Delta T T = \Delta C \cdot RT = (0.015 - 6 \times 10^{-4})(0.082)(298) = \underline{\underline{0.357 \text{ bar}}}]$$

(14.3)

(14.2) contd... 2nd part: Polarization modulus = $C_m/C_b = 5.5$

$$\Rightarrow C_m = (5.5)(0.015) = 0.0825 \text{ molar. } C_p = (0.0825)(1-0.96) = 0.0033.$$

$$\Delta \Pi = (0.0825 - 0.0033)(0.0821)(298)(1.013) = 1.963 \text{ bar}$$

'Effective' pressure driving force = $\Delta P - \Delta \Pi = 2.9 - 1.963 = 0.937 \text{ bar}$

(14.3) Follow the procedure of Example 14.4.

Initial volume of solution = V_0 , number of moles = n_0 ; at time t , volume = V , number of moles = n (since the rejection is not 100%); change of solution volume = dV in time dt .

Moles of solute leaving with the permeate in time dt

$$= -\left(\frac{n}{V}\right) \cdot dV(1-R') = -dn \Rightarrow (1-R') \ln \frac{V}{V_0} = \ln \frac{n}{n_0} \Rightarrow n = n_0 \left(\frac{V}{V_0}\right)^{1-R'}$$

'Effective' driving force at any time, t ,

$$= \Delta P - \frac{RT \cdot n}{V} \cdot R' = \Delta P - \frac{RT \cdot R' \cdot n_0 (V/V_0)^{1-R'}}{V} = \Delta P - RT \cdot \frac{R' n_0}{V^{(1-R')}} \cdot \frac{1}{V R'}$$

$$J_w = -\frac{1}{A_m} \frac{dV}{dt} = L_p \left[\Delta P - \frac{\beta \cdot \Delta P}{V^{R'}} \right]; \quad t=0, V=V_0 \quad = \Delta P - \frac{\beta \cdot \Delta P}{V^{R'}} \\ - \int_{V_0}^V \frac{V^{R'}}{V^{R'} - \beta} \cdot dV = L_p \cdot \Delta P \cdot A_m \cdot t = (V_0 - V) + \beta \ln \frac{V_0 - \beta}{V - \beta} \dots (i) \quad \beta = \frac{RT \cdot R' n_0}{\Delta P \cdot V_0^{(1-R')}}$$

$$\text{Given: } V_0 = 800 \text{ gal} = (800)(3.78)/1000 = 3.024 \text{ m}^3; \quad \begin{matrix} \text{Molar of} \\ = 1213 \end{matrix}$$

$$n_0 = (3.024)(14/1213) = 0.0349 \text{ kmol}; \quad \text{final concentration} =$$

$$n/V = 84 \text{ kg/m}^3 = 84/1213 = 0.0692 \text{ kmol/m}^3 = \left(\frac{n_0}{V}\right) \left(\frac{V}{V_0}\right)^{1-R'}$$

$$\Rightarrow 0.0692 = \frac{0.0349}{V} \cdot \frac{V^{1-R'}}{(3.024)^{1-R'}} \Rightarrow V = 0.477 \text{ m}^3 \quad (\text{putting } R' = 0.97)$$

Now consider Eq (i) to calculate the membrane area, A_m , after putting the values of the different quantities.

$$\text{Pure water flux (given)} = L_p \cdot \Delta P = 11.5 \text{ gal/ft}^2 \cdot h = \frac{(11.5)(3.78)(10^{-3})}{(0.3048)^2 (3600)}$$

$$= \frac{1.3 \times 10^{-4}}{m^3/m^2 \cdot s} ; \quad \beta = \frac{RT \cdot R' n_0}{\Delta P \cdot V_0^{1-R'}} = \frac{(0.0821)(298)(0.97)(0.0349)}{(3/1.013)(3.024)^{0.03}}$$

$$\Rightarrow \beta = 0.2706 ; \quad t = 2 \text{ hr} = 7200 \text{ s};$$

$$\text{Put in Eq (i) to get } A_m = 3.47 \text{ m}^2$$

A more accurate calculation may be done by putting $R' = 0.97$ and evaluating the integral in Eq (i) numerically.

* * * *

14.4

(14.4) From Eq (14.4), membrane resistance, $R_m = \frac{32\tau h_m}{\epsilon d^2}$
 Given: $\tau = 2.7$; $h_m = 0.3 \times 10^{-6} \text{ m}$; $\epsilon = 0.35$; $d = 15 \times 10^{-9} \text{ m}$.
 $R_m = \frac{(32)(2.7)(0.3 \times 10^{-6})}{(0.35)(15 \times 10^{-9})^2} = \underline{\underline{3.3 \times 10^{11} \text{ m}^{-1}}}$

$$\Delta P = 4 \text{ bar} = 4 \times 10^5 \text{ N/m}^2; \mu = 9 \times 10^{-4} \text{ kg/m.s}; J_w = 1.2 / 3600 = \frac{3.333 \times 10^{-4} \text{ m}^3}{\text{m}^2 \cdot \text{s}}$$

$$J_w = \frac{\Delta P}{(R_m + R_g)\mu} \Rightarrow R_m + R_g = \frac{4 \times 10^5}{(3.333 \times 10^{-4})(9 \times 10^{-4})} = \underline{\underline{1.333 \times 10^{12} \text{ m}^{-1}}}$$

$$R_g = 1.333 \times 10^{12} - 3.3 \times 10^{11} = 1.003 \times 10^{12} = \frac{R_g = \text{gel resistance}}{180(1 - \epsilon_g)^2 \cdot l_g}$$

Put gel porosity, $\epsilon_g = 0.5$; $d_g = 30 \text{ nm} = 30 \times 10^{-9} \text{ m}$; $l_g = \frac{d_g^2 \cdot \epsilon^3}{(180)(1 - 0.5)^2} = 2.507 \times 10^{-6} \text{ m}$

Gel layer thickness, $l_g = \frac{(1.003 \times 10^{12})(30 \times 10^{-9})^2 (0.5)^3}{(180)(1 - 0.5)^2} = \underline{\underline{2.507 \mu\text{m}}}$

(14.5) Given: $C_b = 0.0152$ mass fraction; $C_m = C_g = 0.15$

(a) $J_w = 0.032 \text{ m/h}$, $R' = 0.98$

Use Eq (14.10). $\frac{0.15}{0.0152} = \frac{\exp(0.032/k_L)}{0.98 + (1 - 0.98) \cdot \exp(0.032/k_L)}$

$$\Rightarrow k_L = 1.285 \times 10^{-2} \text{ m/h} = 3.57 \times 10^{-6} \text{ m/s}$$

Calculation of the permeability of the gel layer - first calculate R_g

Pure water flux, $J_w = \frac{\Delta P}{\mu R_m} \Rightarrow 0.8 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}} = \frac{6 \text{ bar}}{\mu R_m}$

$$\Rightarrow \mu R_m = 7.5 \frac{\text{bar} \cdot \text{m}^2 \cdot \text{R}}{\text{m}^3} \dots (i)$$

For ultrafiltration, flux = $0.032 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}} = \frac{6 \text{ bar}}{\mu(R_m + R_g)} \dots (ii)$

From (i) and (ii), $\mu R_g = 180 \frac{\text{bar} \cdot \text{m}^2 \cdot \text{R}}{\text{m}^3} \dots (iii)$

Permeability of the gel layer, $L_{P,g} = \frac{1}{\mu R_g} = \frac{1}{180} = 5.55 \times 10^{-3} \frac{\text{m}^3}{\text{m}^2 \cdot \text{bar}}$

(The effect of osmotic pressure has been neglected.)

(b) If there is a sudden increase in the pressure difference, there will be sudden rise in flux that will decline with time. Instantaneous flux is

$$J_w = (0.032) \left(\frac{8 \text{ bar}}{6 \text{ bar}} \right) = \underline{\underline{0.0427 \text{ m}^3/\text{m}^2 \cdot \text{h}}}$$

(14.5)

(14.5) Contd... Since the resistance to flow offered by the membrane is pretty small, we assume that the UF flux of $0.032 \text{ m}^3/\text{m}^2\text{h}$ is nearly the 'limiting flux'.

→ The steady state flux at $\Delta P = 8 \text{ bar}$ remains the same.

Fractional increase in the gel layer

$$\frac{\text{thickness}}{*} = \frac{8-6}{6} = \frac{2}{6} = \underline{\underline{33.3\%}}$$

(14.6) First we relate the bulk concentration, C_b , with the concentration at the membrane surface, C_m , considering both concentration polarization and effect of osmotic pressure on effective pressure driving force.

$$J_w = k_L \ln \frac{C_m}{C_b} \quad [\text{Eq. (4.11)}] ; \text{ and } J_w = L_p \Delta P \left(1 - \frac{\Delta \Pi}{\Delta P}\right) = L_p \Delta P \left[1 - \frac{C_m RT}{\Delta P}\right]$$

$$\Rightarrow J_w = k_L \ln \frac{C_m}{C_b} = L_p \cdot \Delta P \cdot \left[1 - \frac{C_m RT}{\Delta P}\right] \xrightarrow{(i)} \text{i.e., } C_m \text{ can be expressed in terms of } C_b.$$

Now consider the change in volume and concentration of the solution as filtration proceeds. $V_0, C_0 \rightarrow$ initial volume and concentr.

$V, C_b \rightarrow$ volume and concentration at time t .

$$\Rightarrow V_0 C_0 = V C_b \Rightarrow V = \frac{V_0 C_0}{C_b} \Rightarrow \frac{dV}{dt} = \frac{V_0 C_0}{C_b^2} \cdot \frac{dC_b}{dt}.$$

$$\text{Instantaneous flux, } J_w = - \frac{1}{A_m} \frac{dV}{dt} = - \frac{V_0 C_0}{C_b^2 A_m} \cdot \frac{dC_b}{dt} \dots (ii)$$

$$\text{From (i) and (ii)} \quad - \frac{V_0 C_0}{C_b^2 A_m} \cdot \frac{dC_b}{dt} = k_L \ln \frac{C_m}{C_b}; \quad t=0, C_b = C_0$$

The above equation can be integrated to obtain the membrane area to get a specified solution concentration in the retentate within a specified time. The concentration C_m has to be expressed in terms of C_b (before integration can be done) from Eq (i).

A numerical strategy has to be adopted.

* * * *

(14.7) To determine R_m (membrane resistance) and r_c (specific cake resistance) from the experimental flux data, plot

(14.6)

(14.7) Contd... Plot $\frac{1}{J_w^2}$ against time (t) and determine the intercept and slope of the 'least square' line; Eq.(14.21).

t, s	10	20	30	40	50	60	70	80
$\frac{1}{J_w^2} \times 10^{-2}$	2.31	3.73	5.05	6.61	8.07	9.35	10.6	12.22

J_w in liter/m².s

$$\text{Intercept of the line} = 90.7 \left(\frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2$$

$$\text{Slope} = 14.03 \left(\frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2 \cdot \frac{1}{\text{s}}$$

$$\text{Given: } \mu = 9 \times 10^{-4} \text{ kg/m.s}$$

$$C_b = 20 \text{ kg/m}^3; \Delta P = 30 \times 10^3 \text{ N/m}^2; R' = 1$$

$$\frac{R_m^2 \cdot \mu^2}{(\Delta P)^2} = 90.7 \left(\frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2$$

$$\Rightarrow R_m^2 = \frac{90.7 \times 10^6 \cdot \frac{\text{m}^4 \cdot \text{s}^2}{\text{m}^6}}{(9 \times 10^{-4} \text{ kg/m.s})^2} \cdot (30 \times 10^3 \text{ kg.m.s}^2 \cdot \frac{1}{\text{m}^2})^2 \Rightarrow R_m = 3.17 \times 10^{14} \text{ m}^{-1}$$

$$\text{Slope, } \frac{2\mu C_b \cdot r_c \cdot R'}{P_c \cdot \Delta P} = 14.03 \left(\frac{\text{m}^2 \cdot \text{s}}{\text{liter}} \right)^2 \cdot \frac{1}{\text{s}}$$

$$\Rightarrow \left(\frac{r_c}{P_c} \right) = \frac{14.03 \times 10^6 \left(\frac{\text{m}^2 \cdot \text{s}}{\text{m}^3} \right)^2 \cdot \frac{1}{\text{s}} \cdot 30 \times 10^3 \frac{\text{kg.m}}{\text{s}^2} \cdot \frac{1}{\text{m}^2}}{(2)(9 \times 10^{-4} \text{ kg/m.s})(20 \text{ kg/m}^3)(1)} = 1.17 \times 10^{13} \text{ m/kg}$$

(14.8) V_p = Accumulated volume of permeate $\Rightarrow \frac{dV_p}{dt} = a_m \cdot J_w$,

$$\Rightarrow \frac{dV_p}{dt} = a_m \cdot (2.5 \times 10^{-5}) V_p^{-0.2}; t = 0, V_p = 0$$

Feed = 1 m³, $V_p = 0.5$, Permeate = 0.5 m³, Time $t = 1 \text{ hr} = 3600 \text{ s}$.

Integrate to get $\int V_p^{0.2} dV_p = a_m \cdot (2.5 \times 10^{-5}) \int dt$

$$\Rightarrow \frac{1}{1.2} [V_p^{1.2}]_0^a = a_m \cdot (2.5 \times 10^{-5}) \cdot 3600 \Rightarrow a_m = \frac{(0.5)^{1.2}}{(1.2)(2.5 \times 10^{-5})(3600)}$$

$$\Rightarrow a_m = 4 \text{ m}^2$$

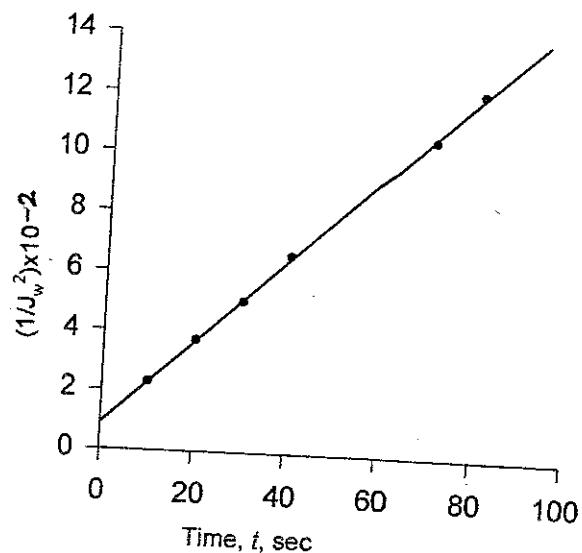


Figure Prob 14.7.

(14.7)

(14.9) Eq. (14.25) for the limiting flux, osmotic pressure model:

$$J_w = \frac{\Delta P - 5C_b^n \exp(n \cdot J_w / k_L)}{\mu R_m}$$

Given: $5 = 120$; $C_b = 0.006 \text{ kmol m}^{-3}$

$n = 2$; $\mu R_m = 6 \times 10^5 \text{ atm}$

$$k_L = 3 \times 10^{-6} \text{ m/s}$$

Calculated values, J_w against ΔP

$J_w, \frac{m^3}{m^2 s} \times 10^5$	0.2	0.5	0.7	1.0
$\Delta P, \text{atm}$	1.216	3.121	4.66	9.39

$J_w \times 10^5$	1.1	1.2	1.25	1.3	1.35	1.4	1.5	1.55
ΔP	13.21	20.1	25.5	32.8	43.1	57.25	104.1	142.1

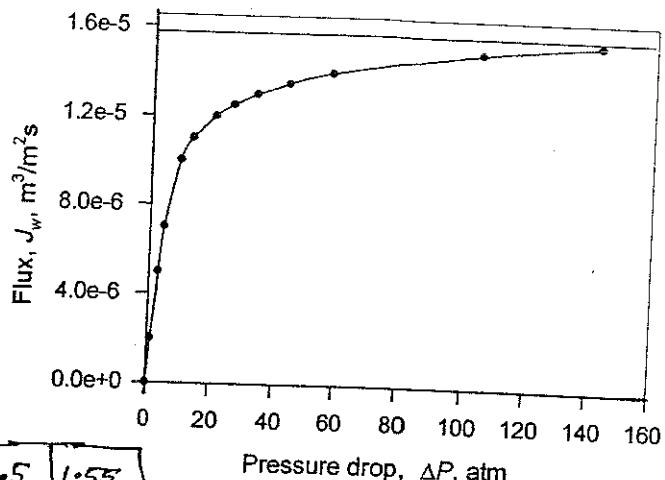


Figure Prob 14.9

The values are plotted in Fig Prob 14.8. The limiting flux according to the osmotic pressure model is about $1.6 \times 10^{-5} \frac{\text{m}^3}{\text{m}^2 \cdot \text{s}}$

(14.10) Membrane resistance to flow, $R_m = \frac{32 T \ln}{\epsilon d^2}$.

Put $T = 1.7$; $d_m = 0.45 \times 10^{-6} \text{ m}$; $\epsilon = 0.35$; and $d = 20 \times 10^{-9} \text{ m}$.

$$R_m = \frac{(32)(1.7)(0.45 \times 10^{-6})}{(0.35)(20 \times 10^{-9})^2} = 1.748 \times 10^{11} \text{ m}^{-1}$$

Calculation of k_L : Use the correlation $Sh = 0.025 Re^{0.75} Sc^{1/3}$.

$$Re = 6000 \text{ (given)}; Sc = \mu/PD = \frac{9.5 \times 10^3}{(1050)(3.5 \times 10^{-11})} = 2.635 \times 10^5$$

$$\Rightarrow Sh = (0.025)(6000)^{0.75} (2.635 \times 10^5)^{1/3} = 1150 = \frac{k_L \cdot L}{D}; L = 7 \times 10^{-3} \text{ given}$$

$$k_L = (1150)(3.5 \times 10^{-11}) / 7 \times 10^{-3} = 5.8 \times 10^{-6} \text{ m/s}$$

$$\text{Use (14.8). } k_L \cdot \ln \frac{C_g - C_p}{C_g + C_p} = \frac{\Delta P - \Delta \Pi}{\mu(R_m + R_g)} \quad \dots (i)$$

$$C_g - C_p = 0.99 C_g = (0.99)(0.12) \Rightarrow C_p = 1.2 \times 10^{-3}; C_g - C_p = \frac{120 - 1.2}{5000} \frac{\text{kmol}}{\text{m}^3}$$

$$C_g - C_p = 0.013 - 0.0012 = 0.0118 \text{ kg/m}^3$$

$$\Delta \Pi = (C_g - C_p)RT = (0.02376)(0.0821)(300 \text{ K}) = 0.585 \text{ atm} = 0.593 \text{ bar}$$

(14.8)

$$(14.10) \text{ Contd... Put in Eq(i)} \quad 5.8 \times 10^{-6} \ln \frac{(0.12)(0.99)}{0.0118} = \frac{(3 - 0.593) \times 10^5}{\mu (R_m + R_g)}$$

Put $\mu = 8.8 \times 10^{-4} \text{ N.S/m}^2$ for the solvent $\Rightarrow R_m + R_g = 2.042 \times 10^{13} \text{ m}^{-1}$

$$R_g = 2.042 \times 10^{13} - R_m = 2.042 \times 10^{13} - 1.748 \times 10^{11} = 2.022 \times 10^{13} \text{ m}^{-1}$$

The membrane resistance is about 1% of the total resistance. Calculate the gel layer thickness from

$$R_g = \frac{(180)(1-\epsilon_g)^2 d_g}{d_s^2 \epsilon_g^3} = 2.022 \times 10^{13}$$

Put $\epsilon_g = 0.5$, and $d_s = 40 \text{ nm} = 40 \times 10^{-9} \text{ m}$ to get $d_g = 90 \mu\text{m}$

(b) The pressure drop (and corresponding flux) at which the gel layer starts forming, the solute concentration at the membrane surface reaches $c_g = 0.12$ mass fraction. There is no gel layer resistance ($R_g = 0$) since the gel is yet to form. Put the relevant values in Eq(i) and take $R_g = 0$.

$$5.8 \times 10^{-6} \ln \frac{(0.12)(0.99)}{0.0118} = \frac{(3 - 0.593) \times 10^5}{(8.8 \times 10^{-4})(1.748 \times 10^{11})} \Rightarrow \Delta P = 0.62 \text{ bar}$$

Thus gel formation starts when ΔP is only slightly larger than 0.62 bar.

$$(14.11) \text{ Rejection} = 100\% \Rightarrow V_0 C_0 = V_f C_f \quad (f = \text{final}) ; V_0 = 600 \text{ gal} \\ \text{Use Eq(i) of Example 14.4} \quad V_f = (600) \left(\frac{1.4}{10.5} \right) \text{ gal} = \frac{2.268 \text{ m}^3}{0.302 \text{ m}^3}$$

$$\text{Put } L_p \cdot \Delta P = \text{pure water flux} = 9.5 \text{ gal/h ft}^2 = \frac{(9.5)(0.00378)}{(3600)(0.3048)^2}$$

$$a_m = 50 \text{ ft}^2 = 4.645 \text{ m}^2 ;$$

$$n_0 = \frac{(2.268 \text{ m}^3)(14 \text{ kg/m}^3)}{2020 \text{ kg/kmol}} = 0.0157 \text{ kmol} ; \quad \Delta P = 3 \text{ bar} = \frac{1.074 \times 10^{-4} \text{ m}^3}{8 \text{ m}^2} = 2.961 \text{ atm}$$

$$RT n_0 / \Delta P = (0.0821)(300)(0.0157) / 2.961 = \frac{0.1306 \text{ m}^3}{}$$

$$t = \frac{1}{a_m L_p \Delta P} \left[(V_0 - V_f) + \frac{RT n_0}{\Delta P} \ln \frac{V_0 - (RT n_0 / \Delta P)}{V_f - (RT n_0 / \Delta P)} \right]$$

$$\Rightarrow t = \frac{1}{(4.645)(1.074 \times 10^{-4})} \left[(2.268 - 0.302) + (6.1306) \ln \frac{2.268 - 0.1306}{0.302 - 0.1306} \right]$$

$$= 4600 \text{ s} = \underline{\underline{1.28 \text{ h}}} \quad * \quad * \quad *$$

(14.9)

14.12

Rate of production of water = $a_m \cdot L_p (\Delta P - \Delta \Pi)$; $a_m = 20 \text{ m}^2$.
 $\Delta P = 70 - 1 = 69 \text{ atm}$; $\Delta \Pi = (5.2)(39.5/5) = 41.1 \text{ atm}$ [at 5% salt]
Theoretical production rate = $(20 \text{ m}^2)(7.5 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h} \cdot \text{atm}} \times 10^4) (69 - 41.1) \text{ atm}$
 $= 0.4185 \text{ m}^3/\text{h} = 418.5 \text{ liter/h}$.

The actual production rate (300 liter/h) is less than this.
So there is concentration polarization.

Put $L_p \cdot a_m (\Delta P)_{\text{eff.}} = 300 \text{ liter/h} = 0.3 \text{ m}^3/\text{h}$

$$\Rightarrow (\Delta P)_{\text{eff.}} = \frac{0.3 \text{ m}^3/\text{h}}{(20 \text{ m}^2)(7.5 \times 10^4 \frac{\text{m}^3}{\text{m}^2 \cdot \text{h} \cdot \text{atm}})} = 20 \text{ atm} = \Delta P - \Delta \Pi$$

Actual $\Delta \Pi = \Delta P - 20 = 69 - 20 = 49 \text{ atm}$.

Corresponding salt concentration at the membrane surface,
 $C_m = (49)(5/39.5) = 6.2\%$.

Concentration polarization modulus = $\frac{C_m}{C_b} = \frac{6.2}{5.2} = 1.2$

14.13 Feed-side concentration = 3.2%; no concentration polarization.

Osmotic pressure on the feed side = $(3.2)(39.5/5) = 25.3 \text{ atm} = \Delta \Pi$

Effective pressure differential
 $= \Delta P - \Delta \Pi = (40 - 1.5) - 25.3 = 13.2 \text{ atm} = (\Delta P)_{\text{eff.}}$

Water flux, $J_w = L_p \cdot (\Delta P)_{\text{eff.}} = 9 \times 10^{-4} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h} \cdot \text{atm}} \times 13.2 \text{ atm} = 1.188 \times 10^{-2} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}}$

Salt flux [Eq 14.29(a) and (b); take $\alpha = 1$]

$$J_s = P_s (C_b - C_p) = 3.2 \times 10^{-2} \frac{\text{m}}{\text{s}} (32 \text{ kg/m}^3 - 0) = 1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

Salt concentration in the permeate = $\frac{J_s}{J_w} = \frac{1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{3600} = \frac{1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{3600} = 2.844 \times 10^{-10} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$

This is above the allowable limit, 200 ppm. $\frac{1.024 \times 10^{-6} \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}}{200 \text{ ppm}} = 5.12 \text{ ppm}$

If the salt flux remains as above (assume that the salt flux is pressure independent), required water flux to maintain the quality is $J_w = (1.188 \times 10^{-2}) (310/200) = 1.841 \times 10^{-2} \frac{\text{m}^3}{\text{m}^2 \cdot \text{h}}$

$$L_p (\Delta P - 25.3) = 1.841 \times 10^{-2} \Rightarrow \Delta P = 20.5 + 25.3 = 45.8 \text{ atm}$$

Feed side pressure required = $45.8 + 1.5 = 47.3 \text{ atm}$

(14-10)

(14.14) Yield of permeate, $\frac{Q_p}{Q_{fi}} = 0.3 = \theta_{wo}$; rejection, $R' = 0.98$

$$\text{Eq. (14.45)} \rightarrow C_{fo} = C_{fi} (1 - \theta_{wo})^{-R'} = (3.3) (1 - 0.3)^{-0.98} = 4.68\%$$

Take the exit concentration of the mixed permeate $= 300 \text{ ppm}$

$$\text{Polarization modulus} = 1.2 \quad = 0.03\%$$

Feed side concentration: Inlet $\rightarrow C_{mi} = (1.2)C_{fi} = (1.2)(3.3)$

$$\text{Outlet} \rightarrow C_{mo} = (4.68)(1.2) = 5.62\% \quad = 3.96\%$$

$$(\Delta C)_i = 3.96 - 0.03 = 3.93\% ; (\Delta C)_o = 5.62 - 0.03 = 5.59\%$$

Osmotic pressure difference at the

$$\text{Feed end}, (\Delta \pi)_i = \left(\frac{39.5}{5}\right)(3.93) = 31.05 \text{ atm} ; (\Delta \pi)_o = \left(\frac{39.5}{5}\right)(5.59) = 44.16 \text{ atm}$$

$$\text{Take the average value, } \Delta \pi = \frac{31.05 + 44.16}{2} = 37.6 \text{ atm}$$

$$\text{Effective pressure driving force, } (\Delta P)_{eff} = (75 - 1) - (37.6) = 36.4 \text{ atm}$$

$$\text{Water flux} = L_p \cdot (\Delta P)_{eff} = \frac{(36.4)(1.013)}{10} = 3.687 \text{ MPa}$$

$$= (70 \text{ liters/m}^2 \text{ day})(3.687 \text{ MPa}) = 0.258 \text{ m}^3/\text{m}^2 \text{ day}$$

$$\text{Required capacity} = 20,000 \text{ m}^3/\text{day}$$

$$\text{Membrane area} = \frac{20,000}{(24)(0.258)} = \frac{3230 \text{ m}^2}{*} \quad *$$

(14.15) Inlet concentration of Na-acetate ($Molar = 82$) = 3% $\rightarrow \frac{30 \text{ kg}}{\text{m}^3}$

Water (dialysate)-side concentration ≈ 0 (high flow rate).

$$(\Delta C)_{inlet} = \frac{30}{82} = 0.366 \text{ kmol/m}^3$$

$$\text{Exit concentration} = 500 \text{ ppm} = \frac{0.05 \text{ kg}}{\text{m}^3} = 0.0061 \text{ kmol/m}^3 = (\Delta C)_o$$

$$\text{Log mean driving force is taken; } (\Delta C)_m = \frac{0.366 - 0.0061}{\ln \frac{0.366}{0.0061}} = 0.088 \text{ kmol/m}^3$$

Total resistance to mass transfer in the dialyzer

= fluid phase film resistance + resistance for diffusion

$$= 2.5 \times 10^5 \text{ s/m} + \frac{lm}{D \cdot \epsilon} ; lm = \text{membrane thickness} = 100 \times 10^{-6} \text{ m}$$

$$= 2.5 \times 10^5 + \frac{100 \times 10^{-6}}{(1.2 \times 10^{-9})(0.4)}$$

$$= 4.58 \times 10^5 \text{ s/m}$$

(14.15) Contd...

(14.11)

$$\text{Overall coefficient, } K_L = 1/(4.58 \times 10^5) \text{ m}/\text{s} = 2.18 \times 10^{-6} \text{ m/s.}$$

Rate of flow of the feed, $Q_f = 200 \text{ litres/h} = 0.2 \text{ m}^3/\text{h}$

$$\text{Then } Q_f(C_f - C_0) = K_L a_m (\Delta C)_m$$

$$\Rightarrow 0.2(0.366 - 0.0061) = (2.18 \times 10^{-6})(3600) \cdot a_m \cdot (0.088) \Rightarrow a_m = 104 \text{ m}^2$$

Loss of the target substance (Mol. wt. = 520)

$$\text{Inlet concentration (8%), } C_{in} = \frac{80}{520} = 0.1538 \text{ kmol/m}^3$$

$$\text{Outlet conc.} = C_{out} \cdot \text{Mean driving force} = \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}}$$

$$\text{Rate of transfer: } Q_f (0.1538 - C_{out}) = K'_L a_m \cdot \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}} \quad \dots (i)$$

$$K'_L \text{ for the target substance} = K_L/10 = 2.18 \times 10^{-7} \text{ m/s}$$

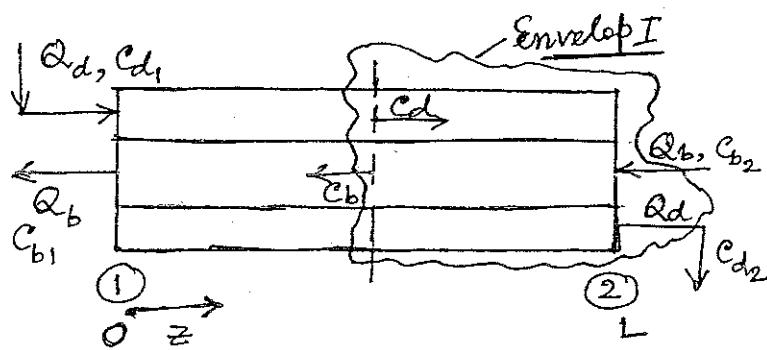
Since transport through the film or through a membrane and the diffusivity of the substance is $\frac{1}{10}$ times that of Na-acetate. Put the values in Eq (i) above.

$$(0.2)(0.1538 - C_{out}) = (2.18 \times 10^{-7})(3600)(104) \cdot \frac{0.1538 - C_{out}}{\ln \frac{0.1538}{C_{out}}}$$

$$\Rightarrow C_{out} = 0.102$$

$$\text{Loss of the substance} = \frac{0.1538 - 0.102}{0.1538} = 33.7\%$$

(14.16) A hemodialyzer with countercurrent flow of blood and the dialyzate is shown in the sketch. The flow rates and concentrations are also shown.



Equations for axial concentration distribution:

$$\text{Blood: } Q_b \frac{dc_b}{dz} = K_L B' (C_b - C_d) \quad \dots (i) \quad | K_L = \text{overall mass transfer coeff.}$$

$$\text{Dialyzate: } Q_d \frac{dc_d}{dz} = K_L B' (C_d - C_b) \quad \dots (ii) \quad | B' = \text{total perimeter of the tubes.}$$

(14-16) contd...

(14-12)

Total Solute balance over the dialyzer : $Q_b (C_{b_2} - C_{b_1}) = Q_d (C_{d_2} - C_{d_1})$

Solute balance over envelop I :

$$Q_b (C_{b_2} - C_b) = Q_d (C_{d_2} - C_d)$$

$$\Rightarrow C_d (\bar{z}) = C_{d_2} + \xi (C_{b_2} - C_b) \dots (iv)$$

Consider Eq (i) above and use Eq (iv). $Q_b \frac{dC_b}{dz} = K_L B' [C_b - C_{d_2} + \xi C_{b_2} - \xi C_b]$

$$\Rightarrow \int_{C_{b_1}}^{C_{b_2}} \frac{dC_b}{C_b(1-\xi) + (\xi C_{b_2} - C_{d_2})} = \int_0^L \frac{K_L B'}{Q_b} dz = \frac{K_L (B' L)}{Q_b} = \frac{K_L A_m}{Q_b} = N_T$$

$$\Rightarrow \frac{1}{(1-\xi)} \ln \frac{C_{b_2}(1-\xi) + (\xi C_{b_2} - C_{d_2})}{C_{b_1}(1-\xi) + (\xi C_{b_2} - C_{d_2})} = N_T$$

$$\Rightarrow \ln \left[\frac{C_{b_2} - C_{d_2}}{(C_{b_1} - C_{d_2}) + \xi (C_{b_2} - C_{b_1})} \right] = (1-\xi) N_T ; \quad \begin{aligned} & \text{Using Eq (iii),} \\ & (C_{b_1} - C_{d_2}) + \xi (C_{b_2} - C_{b_1}) \\ & = C_{b_1} - C_{d_2} + C_{d_2} - C_{d_1} = C_{b_1} - C_{d_1} \end{aligned}$$

$$\Rightarrow \ln \left[\frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} \right] = (1-\xi) N_T \Rightarrow \frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} = \exp[(1-\xi) N_T] \dots (v)$$

The 'extraction ratio' — see Eq.(14.52) — can be shown to be given as

$$\hat{E} = \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}} = \frac{\exp[(1-\xi) N_T] - 1}{\exp[(1-\xi) N_T] - \xi}$$

$$\begin{aligned} \text{Using (v), } \frac{\exp[(1-\xi) N_T] - 1}{\exp[(1-\xi) N_T] - \xi} &= \frac{\frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} - 1}{\frac{C_{b_2} - C_{d_2}}{C_{b_1} - C_{d_1}} - \xi} = \frac{C_{b_2} - C_{d_2} - C_{b_1} + C_{d_1}}{C_{b_2} - C_{d_2} - \xi C_{b_1} + \xi C_{d_1}} \\ &= \frac{(C_{b_2} - C_{b_1}) - (C_{d_2} - C_{d_1})}{C_{b_2} - C_{d_2} - \xi C_{b_1} + \xi C_{d_1} + C_{d_1} - C_{d_1}} = \frac{(C_{b_2} - C_{b_1}) - \xi (C_{b_2} - C_{b_1})}{C_{b_2} + \xi C_{d_1} - C_{d_1} - [(C_{d_2} - C_{d_1}) + \xi C_{b_1}]} \\ &= \frac{(C_{b_2} - C_{b_1})(\xi - 1)}{C_{b_2} + \xi C_{d_1} - C_{d_1} - \xi C_{b_2}} = \frac{(C_{b_2} - C_{b_1})(\xi - 1)}{C_{b_2}(1-\xi) - C_{d_1}(1-\xi)} = \frac{(C_{b_2} - C_{b_1})(\xi - 1)}{(C_{b_2} - C_{d_1})(\xi - 1)} \\ &= \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}} = \hat{E} \end{aligned}$$

*

*

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(14.13)

14.17 (a) Extraction ratio, $\hat{E} = \frac{\exp[(1-\zeta)N_T] - 1}{\exp[(1-\zeta)N_T] - \zeta} = \frac{C_{b_2} - C_{b_1}}{C_{b_2} - C_{d_1}}$
 for countercurrent dialysis, Eq(14.53)

Given: $Q_d = 10 Q_b \Rightarrow \zeta = Q_b / Q_d = 0.1$; $Q_b = 300 \text{ ml/min} = 5 \times 10^{-6} \text{ m}^3/\text{s}$;
 $a_m = 1.2 \text{ m}^2$; $K_L = 1.05 \times 10^6 \text{ m/s}$.

$$N_T = \frac{K_L a_m}{Q_b} = \frac{(1.05 \times 10^6)(1.2)}{5 \times 10^{-6}} = 0.252$$

$$\hat{E} = \frac{\exp[(1-0.1)(0.252)] - 1}{\exp[(1-0.1)(0.252)] - 0.1} = 0.2205$$

(b) If the urea concentration in blood at any time is C_{b_2} , the change in concentration within a small time dt is given by [assume: $C_{d_1} = \text{inlet conc. of dialyzate} = 0$]

$$-V_L \cdot dC_{b_2} = Q_b(C_{b_2} - C_{b_1}) \cdot dt = Q_b \cdot \hat{E} \cdot C_{b_2} \cdot dt$$

$$\text{Integrating, } -\int_{C_i}^{C_f} \frac{dC_{b_2}}{C_{b_2}} = \frac{Q_b \hat{E}}{V_L} \int_0^t dt \Rightarrow \ln \frac{C_i}{C_f} = \frac{Q_b \hat{E}}{V_L} \cdot t$$

The urea content in blood should change from 220 to 20 (mg%)

$$\Rightarrow \ln \frac{220}{20} = \frac{(5 \times 10^{-6} \text{ m}^3/\text{s})(0.2205)}{5.5 \times 10^{-3} \text{ m}^3} \cdot t; \text{ blood volume, } V_L = 5.5 \text{ liters} = 5.5 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow t = 119608 = \underline{\underline{200 \text{ min}}}$$

Time of dialysis = 200 min *

14.18 Rate of transport of a gas through a film [Eq(14.70)]

$$= a_m \cdot \frac{P_A}{l_m} (P_1 x_A - P_2 y_A) \cdot \begin{cases} P x_A = \text{vapor pressure of water at } 25^\circ C \\ = 24 \text{ mm Hg} \end{cases} \\ = (1000) \left(\frac{1.4 \times 10^{-10} \text{ cm}^3 (\text{STP}) \cdot \text{cm}}{0.003 \text{ cm}} \right) \times \begin{cases} P_2 y_A = (0.70)(24 \text{ mm Hg}); \frac{70\% \text{ RH}}{\text{air}} \\ P_A = 1.4 \text{ Barriers}; l_m = 0.003 \text{ cm} \end{cases} \\ = 6.48 \times 10^{-5} \frac{\text{cm}^3 (\text{STP})}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}} (2.4)(1-0.7) \text{ cm Hg}$$

$$= (6.48 \times 10^{-5} / 22414)(18)(3600) \text{ gm moisture/h} = \underline{\underline{1.873 \times 10^{-4} \text{ gm/h}}}$$

Moisture to be lost for reducing the moisture content from 20% to 12% (wet basis) = $(1 \text{ kg})(0.2) - \frac{(0.8)(0.12)}{0.88} \text{ kg} = \underline{\underline{91 \text{ gm}}}$

(14.18) contd... Time for loss of 91 gm moisture

$$= \frac{91 \text{ gm}}{1.873 \times 10^{-4} \text{ gm/hr}} = 485850 \text{ hr.} = 55.5 \text{ years}$$

(14.19) $\text{CO}_2 \rightarrow A ; \text{N}_2 \rightarrow B$; membrane thickness: PDMS, $l_m' = 5 \times 10^{-4} \text{ cm}$
Given permeability values: CA, $l_m'' = 2 \times 10^{-5} \text{ cm}$

$$\text{PDMS} - \hat{P}_A' = 4550, \hat{P}_B' = 351; \text{CA} - \hat{P}_A'' = 4.75, \hat{P}_B'' = 0.15 \text{ Barres}$$

Resistance to transport, Eq. 14.74 (Use resistance-in-series model)

$$R_A = R_A' + R_A'' = \frac{l_m'}{\hat{P}_A'} + \frac{l_m''}{\hat{P}_A''} = \frac{5 \times 10^{-4}}{4550 \times 10^{-10}} + \frac{2 \times 10^{-5}}{4.75 \times 10^{-10}} = 4.32 \times 10^4$$

$$\text{Total thickness of the composite membrane, } l_m = l_m' + l_m'' = 5 \times 10^{-4} + 2 \times 10^{-5} = 5.2 \times 10^{-4} \text{ cm}$$

$$\text{Permeability of CO}_2 \text{ in the composite membrane, } \hat{P}_A = \frac{l_m}{R_A} = 120.4 \text{ Barres}$$

Similarly for B,

$$R_B = R_B' + R_B'' = \frac{l_m}{\hat{P}_B'} + \frac{l_m''}{\hat{P}_B''} = \frac{5 \times 10^{-4}}{351 \times 10^{-10}} + \frac{2 \times 10^{-5}}{1.5 \times 10^{-11}} = 1.3476 \times 10^6 \frac{\text{cm}^2 \cdot \text{S} \cdot \text{cm Hg}}{\text{cm}^3 (\text{STP})}$$

$$\hat{P}_B = \frac{l_m}{R_B} = \frac{5.2 \times 10^{-4} \text{ cm}}{1.3476 \times 10^6} = 3.86 \times 10^{-10} \frac{\text{cm}^3 (\text{STP}) \text{ cm}}{\text{cm}^2 \cdot \text{S} \cdot \text{cm Hg}} \text{ or } 3.86 \text{ Barres}$$

$$\text{Ideal separation factor, } \alpha_{AB}^{*} = \hat{P}_A / \hat{P}_B = \frac{120.4}{3.86} = 31.2$$

(14.20) Flux equation for concentration-dependent diffusivity:

$$\frac{dJ_A}{dz} = 0 \text{ at steady state} \Rightarrow -D \frac{dc}{dz} = \text{constant} = K_1 \dots \text{(i)}$$

$$\text{Given: } D = D_0 e^{\beta C} \Rightarrow -D_0 e^{\beta C} \cdot \frac{dc}{dz} = K_1.$$

$$\text{Integrate to get } - \int_{C_1}^{C_2} e^{\beta C} dc = \frac{K_1}{D_0} \int_{l_m}^0 dz \Rightarrow K_1 = \frac{D_0}{\beta l_m} [e^{\beta C_1} - e^{\beta C_2}] \text{ (i)}$$

$$\text{Calculation of } \beta C_1 \text{ and } \beta C_2 \mid C = 8.33 \frac{m^3 (\text{STP})}{m^3}, p \text{ in MPa} \quad \underbrace{= \text{flux, see Eq. (i)}}_{\text{flux}}$$

$$\text{Upstream side, } p_1 = 2.3 \text{ MPa} \Rightarrow \beta C_1 = (0.07)(8.33 \times 2.3) = 1.341$$

$$\text{Downstream side, } p_2 = 1.5 \text{ MPa} \Rightarrow \beta C_2 = (0.07)(8.33 \times 1.5) = 0.8746$$

$$\text{Steady state flux, } J_A (= K_1) = \frac{4.53 \times 10^{-17} \text{ m}^2/\text{s}}{\left(0.07 \frac{m^3}{m^3 \text{ STP}}\right) (20 \times 10^{-6} \text{ m})} \left[e^{1.341} - e^{0.8746} \right]$$

$$\Rightarrow J_A = 4.611 \times 10^{-11} \frac{m^3 (\text{STP})}{m^2 \text{s}} = 2.057 \times 10^{-12} \frac{\text{kmol}}{m^2 \text{s}}$$

14.15

(14.20) Contd... Eq(4.70) \rightarrow flux = $\frac{\hat{P}_A}{l_m} (P_1 - P_2) \Rightarrow \hat{P}_A = \frac{J_A \cdot l_m}{P_1 - P_2}$

$$P_1 - P_2 = 2.3 - 1.5 \text{ MPa} = .8 \text{ bar} = \frac{8}{1.013} \times 76 = 600 \text{ cm Hg}$$

Average permeability, $\hat{P}_A = \frac{4.611 \times 10^{-11} \text{ m}^3/\text{m}^2\text{s} \cdot 20 \times 10^{-4} \text{ cm}}{600 \text{ cm Hg}}$

$$\Rightarrow \hat{P}_A = 1.537 \times 10^{-14} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}} = 1.537 \times 10^{-4} \text{ Barrer}$$

(14.21) $O_2 \rightarrow A, N_2 \rightarrow B. R'_A, R'_B \rightarrow$ resistance to permeation; PDMS layer
 R''_A, R''_B = resistance to permeation; polysulfone backing.

The feed-side mixture is equimolar, downstream pressure is small.

$$\Rightarrow J_A = \frac{\Delta P_A}{R'_A + R''_B} ; J_B = \frac{\Delta P_B}{R'_B + R''_A} ; \Delta P_A = \Delta P_B \Rightarrow R''_A = R''_B \text{ (given)}$$

and $J_A = 2 J_B$

$$\hat{P}'_A = \hat{P}'_B \cdot \alpha_{AB}^{*} = (270)(2.2) = 594 \text{ Barrers}$$

$$\text{If } l_m = \text{thickness of the PDMS layer}, R'_A = \frac{l_m}{594} ; R'_B = \frac{l_m}{270}$$

$$\text{Since } J_A = 2 J_B \Rightarrow \frac{\Delta P_A}{R'_A + R''_B} = 2 \cdot \frac{\Delta P_B}{R'_B + R''_A} \Rightarrow R'_B + R''_B = 2(R'_A + R''_A)$$

$$\Rightarrow \frac{l_m}{270} + 10^3 = \frac{2 \times l_m}{594} + 2 \times 10^3 \Rightarrow l_m \left(\frac{1}{270} - \frac{2}{594} \right) = 10^3 \frac{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}{\text{cm}^3(\text{STP}) \cdot \text{cm}}$$

$$\Rightarrow l_m = 10^3 \frac{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}{\text{cm}^3(\text{STP}) \cdot \text{cm}} \times 2970 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cm Hg}}$$

$$= 2970 \times 10^{-7} \text{ cm} = 3 \text{ micron}$$

A 3 μm layer of PDMS has to be applied on the porous polysulfone backing to achieve $J_A = 2 J_B$.

(14.22) Given: $\hat{P}_A = 30 \text{ Barrer}; \hat{P}_B = 0.12 \text{ Barrer}; P_1 = 30 \text{ bar}; P_2 = 1.1 \text{ bar}; x_A = 0.5 = x_B; l_m = 0.2 \mu\text{m} = 2 \times 10^{-5} \text{ cm}.$

(a) Flux of A, $J_A = \frac{\hat{P}_A}{l_m} \cdot (P_1 x_A - P_2 y_A)$ Assume $P_2 \ll P_1$

$$= \left(\frac{30 \times 10^{-10}}{2 \times 10^{-5}} \right) (30 \times 0.5) \left(\frac{76 \text{ cm Hg}}{1.013} \right) = 0.1688 \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}}$$

$$= 7.53 \times 10^{-6} \frac{\text{gmol}}{\text{cm}^2 \cdot \text{s}}$$

(14.22) Contd... $J_B = J_A (0.12/30)$ since $x_A = x_B$

$$= (7.53 \times 10^{-6}) (0.12/30) = \frac{3.012 \times 10^{-8}}{\text{cm}^2/\text{s}} \frac{\text{gmol}}{\text{cm}^2/\text{s}}$$

Composition of the permeate: $y_A = \frac{J_A}{J_A + J_B} = \frac{7.53}{7.53 + 0.03012} = 0.996$

(b) The 'ideal separation factor', $\alpha_{AB}^* = \hat{P}_A / \hat{P}_B = 30/0.12 = 250$

The permeateside pressure is not neglected now.

Pressure ratio: $r = 1/30 = 0.0367$. Use Eq.(4.83) to calculate the actual separation factor.

$$\alpha = \alpha_{AB}^* \cdot \frac{y_A(\alpha-1) + 1 - r\alpha}{y_A(\alpha-1) + 1 - r} \Rightarrow \alpha = (250) \cdot \frac{(0.5)(\alpha-1) + 1 - 0.0367\alpha}{0.5(\alpha-1) + 1 - 0.0367}$$

Solving the quadratic equation, $\alpha^2 - 230.7\alpha - 250 = 0$

we get, $\alpha = 232$ which is less than the ideal

* * * separation factor, $\alpha_{AB}^* = 250$.

(14.23) Refer to Fig 14.36(a), a 'well-mixed reactor'.

Product cut = 0, $Q_p = Q_f \cdot \theta$; $Q_f = 1 \text{ m}^3(\text{STP})/\text{h}$; $x_{Af} = 0.21$

Component A balance: $Q_f x_{Af} = \theta Q_f y_A + (1-\theta)Q_f x_A \quad A \rightarrow \text{oxygen}$

$$\Rightarrow 0.21 = 0.25 y_A + 0.75 x_A \quad (\text{putting } \theta = 0.25),$$

$$\Rightarrow y_A = 0.84 - 3x_A \quad \dots \dots (i) \quad \text{and} \quad y_B = 1 - y_A$$

Fluxes: $J_A = (\hat{P}_A/\text{lm})(P_1 x_A - P_2 y_A)$; $J_B = (\hat{P}_B/\text{lm})(P_1 x_B - P_2 y_B)$

$$\Rightarrow \frac{J_A}{J_B} = \frac{\hat{P}_A}{\hat{P}_B} \cdot \frac{(P_1 x_A - P_2 y_A)}{(P_1 x_B - P_2 y_B)} = \frac{Q_p \cdot y_A}{Q_p \cdot y_B} = \frac{y_A}{y_B}; \quad \frac{\hat{P}_A}{\hat{P}_B} = \alpha_{AB}^* = 5.2$$

Put $P_1 = 10 \text{ bar}$, $P_2 = 1 \text{ bar}$. Note that x_A and y_A are related by

$$\frac{0.84 - 3x_A}{1 - (0.84 - 3x_A)} = (5.2) \cdot \frac{10x_A - (1)(0.84 - 3x_A)}{10(1-x_A) - (1)(1 - 0.84 + 3x_A)} \quad \text{Eq. (i) above.} \Rightarrow x_A = 0.1448$$

Feed rate, $Q_f = 100 \text{ m}^3(\text{STP})/\text{h}$; $Q_p = \theta \cdot Q_f = (0.25)(100) = 25 \text{ m}^3/\text{h}$.

Rate of permeation of A = $y_A Q_p = a_m (\hat{P}_A/\text{lm})(P_1 x_A - P_2 y_A); \text{ lm} = 0.15 \times 10^{-4} \text{ cm}$

$$\Rightarrow \frac{(0.4057)(25) \times 10^6}{3600} \frac{\text{cm}^3(\text{STP})}{\text{s}} = a_m \left(\frac{1.7 \times 10^{-10}}{0.15 \times 10^{-4}} \right) [(10)(0.1448) - (1)(0.4057)]$$

$$\Rightarrow \text{Membrane area, } a_m = 3.18 \times 10^6 \text{ cm}^2 = 318 \text{ m}^2 \quad \times (76/1013) \text{ cm Hg}$$

(14-17)

(14.24) $\text{CO}_2 \rightarrow A$, $\text{CH}_4 \rightarrow B$. Given: $\hat{P}_A = 4570 \text{ Barres}$, $\alpha_{AB}^* = 3.15$
 $x_A = 0.2$; $x_B = 0.8$; $y_A = 0$, since $\Rightarrow \hat{P}_B = 4570/3.15 = 1451 \text{ Barres}$
the permeate compartment is continuously swept with CH_4 ; $y_B = 1.0$.
 $P_1 = 12.5 \text{ atm}$; $P_2 = 1.5 \text{ atm}$; $l_m = 300 \mu\text{m} = 300 \times 10^{-4} \text{ cm}$.

(a) Flux of CO_2 , $J_A = (\hat{P}_A/l_m)(P_1 x_A - P_2 y_A) = \frac{4570 \times 10^{-10}}{300 \times 10^{-4}} \cdot [(12.5)(0.2) - 0] \quad (76)$

$$\Rightarrow J_A = 2.894 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 1.27 \times 10^{-7} \text{ gmol/cm}^2 \cdot \text{s}$$

Flux of CH_4 , $J_B = (\hat{P}_B/l_m)(P_1 x_B - P_2 y_B) = \left(\frac{1451 \times 10^{-10}}{300 \times 10^{-4}} \right) (0.8 \times 12.5 - 1.5 \times 1) \quad (76)$

$$= 3.124 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 1.394 \times 10^{-7} \text{ gmol/cm}^2 \cdot \text{s}$$

(b) Permeance of CO_2 , $\tilde{P}_A = \hat{P}_A/l_m = \frac{4570 \times 10^{-10}}{300 \times 10^{-4}} = 1.523 \times 10^{-5} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s} \cdot \text{cmHg}}$

(c) Diffusivity of CH_4 in PDMS, Use Eq.(14-71)

$$D_B = \hat{P}_B/S_B = \frac{1451 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cmHg}}}{(0.0059 \times 10) \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s} \cdot \text{cmHg}}} = 2.46 \times 10^{-6} \text{ cm}^2/\text{s}$$

(14.25) Using the given flux values of A and B,

$$\frac{J_A}{J_B} = \frac{1.245 \times 10^{-3}}{1.06 \times 10^{-3}} = 1.1745 \Rightarrow \frac{y_A}{y_B} = 1.1745 = \frac{y_A}{1-y_A} \Rightarrow y_A = 0.54$$

The feed compartment receives a large rate of air $\Rightarrow x_A = 0.21$

$$\text{Separation factor, } \alpha = \frac{x_A/(1-y_A)}{x_A/(1-x_A)} = \frac{0.54/0.46}{0.21/0.79} = 4.42$$

Permeability of A is given by $l_m = 2 \times 10^{-5} \text{ cm}$

$$J_A = (\hat{P}_A/l_m) \cdot (P_1 x_A - P_2 y_A) \Rightarrow 1.245 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = \frac{\hat{P}_A}{2 \times 10^{-5}} \left[(1.5)(0.21) - (1.5) \times (0.54) \right] \quad (76)$$

$$\Rightarrow \hat{P}_A = 1.4 \times 10^{-10} \frac{\text{cm}^3(\text{STP}) \cdot \text{cm}}{\text{cm}^2 \cdot \text{s} \cdot \text{cmHg}} = 1.4 \text{ Barres}$$

Calculate

$$\alpha_{AB}^* = 5.6 \Rightarrow \hat{P}_B = 0.25 \text{ Barres}$$

Henry's law constant

$$\text{Equilibrium solubility of the gas, } G_A = \frac{10.4 \text{ cm}^3(\text{STP})}{10.5 \text{ gm}/1.02 \text{ gm/cm}^3} = \frac{1.01}{\text{cm}^3 \text{ polymer}}$$

$$\hat{P}_A = (5 \text{ atm})(0.21) = 1.05 \text{ atm. Solubility coeff., } S_A = \frac{G_A/p}{\hat{P}_A} = \frac{1.01}{1.05}$$

$$\text{Average diffusivity, } D_A = \hat{P}_A/S_A = 1.4 \times 10^{-8} \text{ cm}^2/\text{s} = 0.962 \frac{\text{cm}^3 \text{ STP}}{(\text{cm}^2 \text{ polymer})(\text{atm})}$$

14.26 Refer to Fig Prob 14.26. $x, y \rightarrow$ mole fractions of oxygen. Pressure ratio (equal in the two stages), $r = \frac{1.103}{1.15} = 0.96$ (more permeating). Retentate from the second stage (96% N₂) = $x_{2,0} = 1 - 0.96 = 0.04$. Write down Eq (14.80) for the second stage to calculate y_2 .

$$\frac{y_2}{1-y_2} = \alpha^* \cdot \frac{x_{2,0} - r y_2}{(1-x_{2,0}) - r(1-y_2)} \Rightarrow \frac{y_2}{1-y_2} = (4.3) \cdot \frac{(0.04) - (0.0881)y_2}{(1-0.04) - (0.0881)(1-y_2)} \Rightarrow y_2 = 0.1244$$

Now make a material balance on stage 2 to relate $x_{2,0}$ with θ .

$$Q_f(1-\theta) \cdot x_{2,0} = [Q_f(1-\theta) - 100] y_2 + (100)(0.04)$$

Product from the second stage = $100 \text{ m}^3/\text{h} = Q_f(1-\theta)^2$

$$\Rightarrow x_{1,0} = \theta y_2 + (1-\theta)(0.04) = 0.1244\theta + (1-\theta)(0.04)$$

$$\Rightarrow x_{1,0} = 0.04 + 0.0844\theta \quad \dots \quad (i)$$

Write down Eq (14.84) for stage 1 (this is also the A-balance equation).

$$x_{1,0} = \frac{0.21 - \theta y_1}{1-\theta} \Rightarrow \theta y_1 = 0.21 - (1-\theta)x_{1,0}; x_{1,0} \text{ given by Eq (i).}$$

$$\Rightarrow \theta y_1 = 0.21 - (1-\theta)(0.04 + 0.0844\theta) = 0.17 - 0.0444\theta + 0.0844\theta^2$$

Now write down Eq (14.80) for stage 1.

$$\frac{y_1}{1-y_1} = (4.3) \frac{x_{1,0} - 0.088y_1}{(1-x_{1,0}) - 0.088(1-y_1)} \Rightarrow y_1 = \frac{4.3x_{1,0} - 0.3784y_1}{0.912 + 3.3x_{1,0} - 0.2904y_1} \dots (iii)$$

Eliminate $x_{1,0}$ and y_1 from (iii) using their expressions for θ from Eqs. (i) and (ii). We get a biquadratic equation in θ .

$$\theta^4 - 11.84\theta^3 - 9.212\theta^2 + 11.52\theta - 0.393 = 0 \Rightarrow \text{solution, } \theta = 0.66$$

Putting the values of θ in Eq (i) and (ii), $x_{1,0} = 0.0957; y_1 = 0.269$

Stage 2 : Oxygen flux, $J_{A2} = (\hat{P}_A / l_m) (P_1'' x_{2,0} P_2'' y_2) \quad \hat{P}_A = 48 \text{ Bar} \text{es}$

$$\Rightarrow J_{A2} = \frac{48 \times 10^{-10}}{5 \times 10^{-5}} \cdot \left[\frac{(1.15)(0.04)}{1.013} - (1)(0.1244) \right] (76) \quad l_m = 5 \times 10^{-5} \text{ cm}$$

$$= 2.405 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}} = 0.0866 \text{ m}^3(\text{STP}) / \text{m}^2 \cdot \text{h}$$

$$\text{Permeation rate of A} = (100) \left(\frac{\theta}{1-\theta} \right) \cdot y_2 = (100) \left(\frac{0.66}{0.34} \right) (0.1244) = 24.15 \frac{\text{m}^3}{\text{h}}$$

$$\text{Membrane area, } A_{m_2} = \frac{24.15 \text{ m}^3/\text{h}}{0.0866 \text{ m}^3/\text{h m}^2} = 280 \text{ m}^2$$

14.19

14.26 Contd... Membrane area in Stage 1

See Fig Prob 14.26. Retentate from Stage 2, $100 \text{ m}^3/\text{h} = Q_f (1-\theta)^2$

$$\Rightarrow \text{Rate of Supply of feed air to Stage 1}, Q_f = \frac{100}{(1-0.66)^2} = 865 \text{ m}^3/\text{h}$$

$$\text{Permeate from Stage 1} = Q_f \cdot \theta = (865)(0.66) = 571 \text{ m}^3/\text{h}$$

$$\text{Flux of A in Stage 1}, J_{A1} = (\hat{P}_A / l_m) [P_1' x_{1,0} - P_2' y_1]$$

$$\Rightarrow J_{A1} = \frac{48 \times 10^{-10}}{5 \times 10^{-5}} \left[\frac{11.5}{5.013} (0.095) - (1)(0.269) \right] (76) = 5.964 \times 10^{-3} \frac{\text{cm}^3(\text{STP})}{\text{cm}^2 \cdot \text{s}}$$

$$\text{Membrane area, } Q_m = \frac{Q_f \theta \cdot y_1}{J_{A1}} = \frac{(571)(0.269)}{0.215} = 715 \text{ m}^2$$

* * * *

14.27 The problem can be solved following Example 14.15.

A reasonable average value of α has to be selected first.

At the inlet to the permeator, $x = 0.21$ for oxygen.

Pressure ratio, $r = 1/8 = 0.0555$; $\alpha_{AB}^* = 4.5$. y is given by

$$\frac{y}{1-y} = (4.5) \cdot \frac{0.21(\alpha-1) + 1 - 0.0555\alpha}{0.21(\alpha-1) + 1 - 0.0555} \Rightarrow \alpha = 4.02$$

Take a guess value $x_R = 0.08$ to have a value at the exit.

$$\frac{y}{1-y} = (4.5) \cdot \frac{0.08(\alpha-1) + 1 - 0.0555\alpha}{0.08(\alpha-1) + 1 - 0.0555} \Rightarrow \alpha = 3.88$$

Take the average $\alpha = 3.93$ when $x_R = 0.112$

After another we select an average $\alpha = 3.97$ and $x_R = 0.112$

Use Eq (i) of Example 14.15 to determine the membrane area.

Use the values $- x_R = 0.112$; $\theta = 0.3$; $l_m = 10^{-5} \text{ cm}$; $P_1 = 18 \text{ atm}$;

$r = 0.0555$, $\hat{P}_A = 12 \text{ Barres}$; $Q_f = (300 \times 10^6 / 3600) \text{ cm}^3/\text{sec}$

$$Q_m = \frac{10^{-5}}{12 \times 10^{-10}} \cdot \frac{\frac{(300 \times 10^6)}{3600} (1-0.3) (3.97)}{(18 \times 76) (3.97-1)} (0.112)^{-0.337} (0.888)^{1.346} \cdot I$$

$I = \text{integral.} = 0.6077$

(14.27) contd...

Membrane area, $A_m = 51.5 \text{ m}^2$

(14.20)

The mixed permeate concentration can be obtained by material balance.

$$(Q_f)(0.2) = Q_f(1-\theta)(0.112) + Q_f\cdot\theta(y_p), \theta = 0.3$$

$$\Rightarrow y_p = \underline{\underline{0.44}} ; Q_p = \text{permeate rate} = 0.2Q_f = \underline{\underline{90 \text{ m}^3/\text{h}}} \\ * * * *$$

(14.28) Check for any resistance to oxygen diffusion through the wall/ga fibers.

Refer to Fig Prob 14.28.

Diffusion of oxygen through the wall occurs through

Following equation applies at steady state

$$A_{LM} \frac{\epsilon D_K P}{RT \tau L_m P_{BM}} (P_{A_i} - P_{A_0}) = k_L A_0 (C_0 - C_b) \dots \dots (i)$$

The left side expression is the rate of transport through the porous wall; $A_{LM} = 2\pi r_{LM} L$ = logmean area of the fiber of length L, $r_{LM} = (r_o - r_i)/\ln(r_o/r_i)$, r_o and r_i are outside and inside radius of the fiber wall; ϵ = porosity of the wall; D_K = Knudsen diffusivity of O_2 through the pores of the wall; ϵ = wall porosity; P = total pressure; P_{A_i} , P_{A_0} are the partial pressure of O_2 at the inside and outside of the wall; P_{BM} = logmean partial pressure of N_2 (non-diffusing species); k_L = liquid film mass transfer coefficient, outside the wall; A_0 = outside wall area, $2\pi r_o L$; C_0 = oxygen concentration in the liquid in equilibrium with oxygen at partial pressure P_{A_0} at the outside of the wall ($r=r_o$); C_b = oxygen concentration in the bulk liquid on the shell side. Take $C_b=0$ to check for gas phase resistance. [τ = tortuosity of the pores on the wall]

(14.28) Contd..

14.21

Solubility relation: $C = 37.4 \text{ ppm} = 37.4 \text{ g/m}^3 = 1.17 \times 10^{-3} \frac{\text{kmol}}{\text{m}^3}$

Henry's law constant:

$$H = 1.17 \times 10^{-3} \frac{\text{kmol}}{\text{m}^3 \text{ atm}}$$

for $P_{O_2} = 1 \text{ atm}$.

$$C_{O_2} = H P_{O_2}$$

Now consider Eq. (i) above.

$$\gamma_o = 8.75 \times 10^{-5} \text{ m}, \gamma_i = 4.75 \times 10^{-5} \text{ m}, \gamma_{LM} = 6.548 \times 10^{-5} \text{ m};$$

$$D_L = 5 \times 10^{-6} \text{ m}^2/\text{s}; P = 1 \text{ atm}; P_i = 0.21 \text{ atm}; k_L = 5 \times 10^{-6} \text{ m/s};$$

$$R = 0.0821 \text{ m}^3 \cdot \text{atm}/\text{K} \cdot \text{kmol}; T = 303 \text{ K}; \epsilon = 0.4; \tau = 2.55$$

Put the values in Eq. (i)

$$\frac{(2\pi L)(6.548 \times 10^{-5})(0.4)(5 \times 10^{-6})(1)}{(0.0821)(303)(2.55)(4 \times 10^{-5})} \cdot \ln \frac{1 - P_o}{1 - 0.21}$$

$$= (5 \times 10^{-6})(8.75 \times 10^{-5})(1.17 \times 10^{-3}) \cdot P_o$$

The above equation can be solved by trial for P_o , the partial pressure of oxygen at $r = r_o$.

Solution: $P_o \approx 0.21 \text{ atm} \Rightarrow$ there is practically no drop of oxygen partial pressure across the wall of the tube \Rightarrow there is virtually no gas-phase mass transfer resistance \Rightarrow the liquid phase diffusion controls.

Mass transfer calculation is now greatly simplified.

If C_b is the concentration of dissolved oxygen in the liquid at any time t ,

$$V_L \frac{dC_b}{dt} = k_L a_m (C^* - C_b); a_m = \text{area of the module.}$$

$$\text{Integrating, } \ln \frac{C^* - C_{bi}}{C^* - C_{bf}} = \frac{k_L a_m}{V_L} \cdot t \quad \dots \dots \text{(ii)}$$

Assume that oxygen partial pressure remains uniform at $P_{O_2} = 0.21 \text{ atm} \Rightarrow C^* = H P_{O_2} = (1.17 \times 10^{-3})(0.21)$

$$C_{bi} = 0 \text{ (initially the water was oxygen-free)} \Rightarrow C^* = 2.457 \times 10^{-4} \frac{\text{kmol}}{\text{m}^3} = 7.85 \text{ ppm}$$

$$C_{bf} = 5 \text{ ppm}; V_L = 200 \text{ ml} = 2 \times 10^{-4} \text{ m}^3$$

(14.22)

(14.28) Contd...

$$\text{Area, } a_m = n \cdot \pi d_0 L = (500) \cdot \pi (1.75 \times 10^4) (0.25) = \underline{\underline{0.0687 m^2}}$$

Putting the values in Eq (ii),

$$t = \frac{2 \times 10^{-4} m^3}{(5 \times 10^6 m/s)(0.0687 m^2)} \cdot \ln \frac{7.85}{7.85 - 5} = \underline{\underline{5908}}, \text{ i.e. about } \underline{\underline{10 \text{ mins.}}}$$

(14.29)