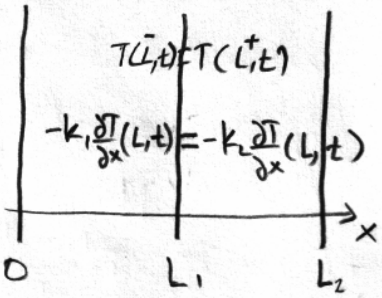
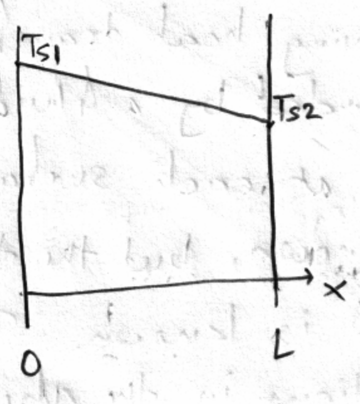


- Interface boundary Condition
  - two layers in contact
  - Same temperature at contact surface
  - matching flux at contact surface

- Combined/Generalized boundary conditions
  - heat transfer to the surface in all modes = heat transfer from the surface in all modes

Steady State Conduction: One Dimensional

Consider steady heat transfer through a wall. If temperature at different locations of the same surface of the wall is a constant, heat transfer can be assumed to be 1-D (one dimensional) in the direction perpendicular to the surface, because heat transfer is governed by a temperature gradient.



Choice: Cartesian coordinate system

Assumptions:

- Steady State
- No heat source or sink
- One dimensional heat conduction

Simplified Heat conduction equation:

$$0 = \frac{d}{dx}\left(k\frac{dT}{dx}\right)$$

Farther assuming $k \equiv$ constant

$$T = C_1 x + C_2$$

Boundary conditions:

$$T = T_{s1} \quad \text{at } x = 0$$

$$T = T_{s2} \quad \text{at } x = L$$

The condition at $x=0$ gives $C_2 = T_{s1}$

The condition at $x=L$ gives $C_1 = \frac{T_{s2} - T_{s1}}{L}$

The temperature profile in the wall is given as

$$T(x) = \frac{T_{s2} - T_{s1}}{L} x + T_{s1}$$ ---- Linearly varying temperature

Once the temperature profile is known, quantities such as rate of heat transfer, heat flux can be calculated.

Flux: $$q = -k \frac{dT}{dx}$$ Fourier's Law

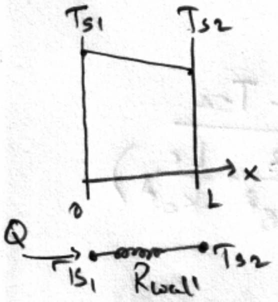
$$= -k \frac{T_{s2} - T_{s1}}{L}$$

Rate of heat transfer across an area A
$$Q = -kA \frac{dT}{dx} = -kA \frac{T_{s2} - T_{s1}}{L}$$

Note: In many problems involving heat transfer through a wall, the wall is surrounded by a fluid (liquid or gas). Further, temperature at each surface of the wall is often not known, but the temperature of the surrounding fluid is known. Thus, dirichlet boundary conditions in the above example are replaced by Neumann boundary condition

Revisiting Thermal Resistance

Rearranging the heat transfer rate equation



$$Q = \frac{T_{s1} - T_{s2}}{R_{wall}}$$

where

$$R_{wall} = \frac{L}{kA}$$

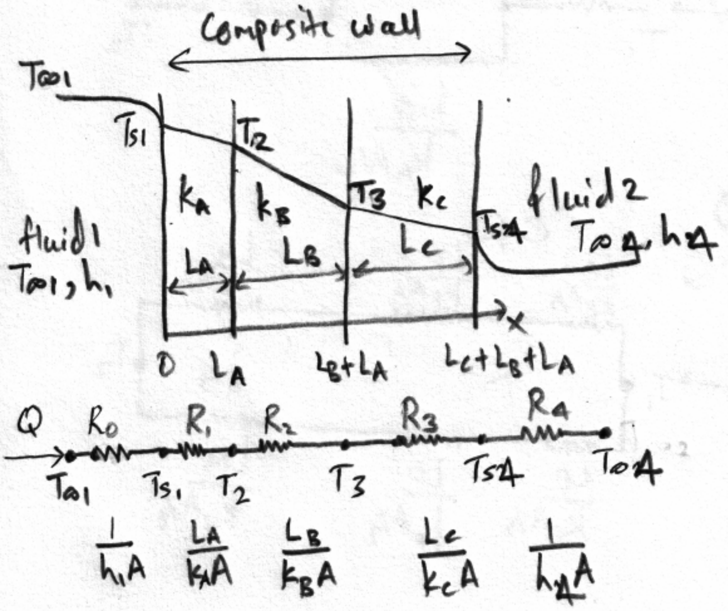
Analogy:

$$I = \frac{V_1 - V_2}{R_e}$$

$$\text{Flow rate of charge} = \frac{\text{Driving potential difference}}{\text{resistance to flow of charge}}$$

Trivial result for this problem, but a powerful concept in solving complicated problems. Resistances (thermal) can be in series or parallel arrangements, and an equivalent resistance can be computed.

Example: Composite Wall with convection boundary conditions.



- 1D heat transfer rate

$$Q = \frac{T_{\infty 1} - T_{\infty 4}}{R_{tot}}$$

where

$$R_{tot} = \frac{1}{h_1A} + \frac{L_A}{k_AA} + \frac{L_B}{k_BA} + \frac{L_C}{k_CA} + \frac{1}{h_4A}$$

$$= R_0 + R_1 + R_2 + R_3 + R_4$$

Overall heat transfer coefficient

With composite systems, a "blackbox" approach would be to use an expression analogous to Newton's Law of cooling

$$Q = UA\Delta T$$

where

$U \equiv$ overall heat transfer coefficient

$$U = \frac{1}{R_{tot} A}$$

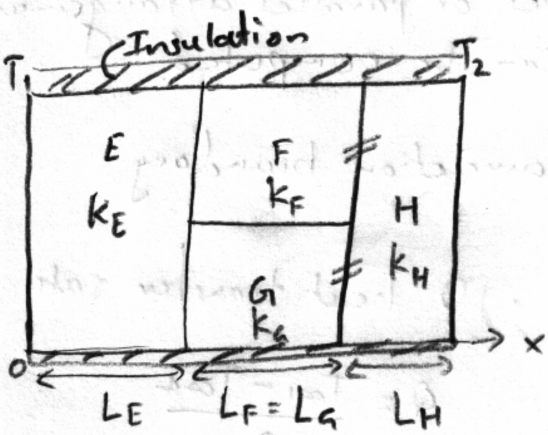
Note: At steady state, heat is not accumulated in the composite. Thus, all of the expressions below are equivalent.

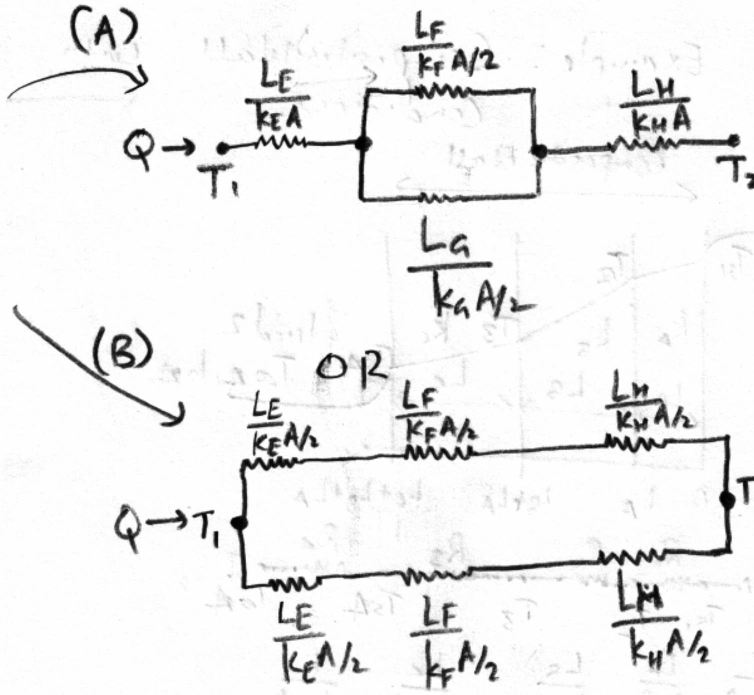
$$Q = \frac{T_{\infty 1} - T_{S1}}{(1/h_1 A)} = \frac{T_{S1} - T_2}{(L_A / k_A A)} = \frac{T_2 - T_{S4}}{\left(\frac{L_B}{k_B A} + \frac{L_C}{k_C A}\right)}$$

– Generally true as long as Q is a constant

Example: A series-parallel composite wall

Equivalent thermal





Case A) Temperature gradient is only in the x-direction
i.e.
any plane wall normal to x-axis is isothermal

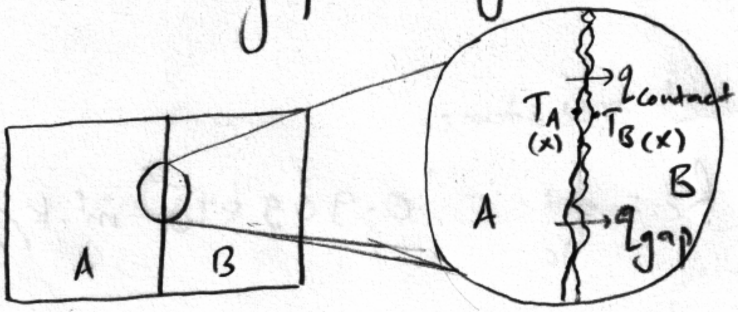
Case B) Heat transfer is only in the x-direction
i.e.
any plane wall parallel to x-axis is adiabatic

Note: Although heat flow is multidimensional, it is reasonable to assume 1D heat conduction in many cases.

Contact Resistance

In composite systems, lack of perfect contact at the interface leads to "contact resistance".

A practical interface that is "smooth" is microscopically rough. Thus, there are contact spots along with gaps that are typically filled with air. Conduction heat transfer takes place across actual contacts, and Conduction and/or radiation heat transfer takes place across the gaps through the filled fluid (air) or vacuum.



If the heat transfer rate is

$$Q = Q_{contact} + Q_{gap}$$

and the effective temperature difference across the interface is ΔT, with A being the apparent contact area;

$$h_{contact} = \frac{Q/A}{\Delta T} \quad \text{and} \quad R_c = \frac{1}{h_c} = \frac{\Delta T}{Q/A}$$

Note:

- Contact resistance can be decreased
  - → by selecting an interfacial fluid of large thermal conductivity
    - eg: thermal grease used at interfaces involving electronic components "thermal paste (cpu heat sink)
  - → by increasing the contact area

Example: Thermal contact conductance at the interface of two 1cm thick aluminium plates is measured to be 11,000 $W/m^2 \cdot K$. Determine the thickness of an aluminium plate whose thermal resistance is equal to the thermal resistance of the interface between the plates. Note that thermal conductivity of aluminium at the temperature of the problem is $k = 237\ W/m \cdot K$.

Thermal contact resistance

$$R_c = \frac{1}{h_c} = 0.909 \times 10^{-4}\ m^2 \cdot K/W$$

For a unit surface area, an equivalent resistance due to conduction across an aluminium plate of length L is

$$R = \frac{L}{k}$$

Equating the two,

Equivalent Thickness for Contact Resistance

$$L = kR_c = 0.0215\ m = 2.15\ cm$$

Note: In this example, the thermal contact resistance is greater than the sum of thermal resistances of the two plates.

Thus, ignoring contact resistance would have resulted in a significant error.