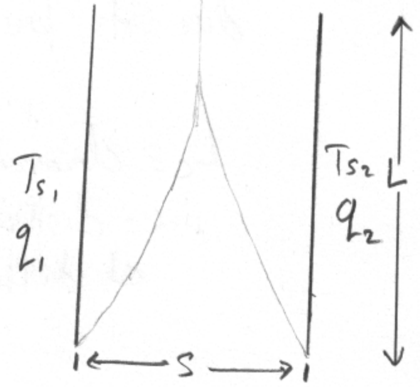
Enclosures and Finned Surfaces

Two isothermal vertical plates

· Examples: Rectangular vertical finned surfaces (heat sink).



Ambient fluid T_∞

· Boundary layers from each plate eventually merge (for $L/s >> 1$)

· Can use results for two isolated plates if $L/s << 1$, since boundary layers will not merge.

$$Nu_s = \left[\frac{C_1}{\left(Ra_s \frac{s}{L}\right)^2} + \frac{C_2}{\left(Ra_s \frac{s}{L}\right)^{1/2}}\right]^{-1/2}$$ for isothermal plates

$$Nu_s = \left[\frac{C_1}{Ra_s^* \frac{s}{L}} + \frac{C_2}{\left(Ra_s^* \frac{s}{L}\right)^{2/5}}\right]^{-1/2}$$ for isoflux plates

with $Ra_s = \frac{g\beta(T_s - T_\infty)s^3}{\nu\alpha}$ being the Rayleigh number

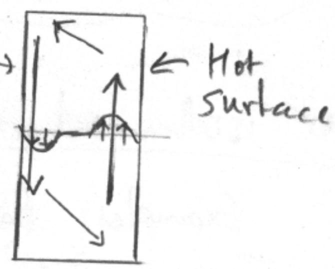
and $Ra_s^* = \frac{g\beta q_s s^4}{k\nu\alpha}$ being the modified Rayleigh number

Table 9.A (Incropera's) or Section 9-A (Cengel's)

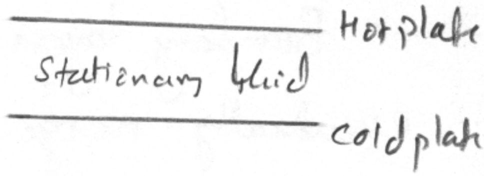
Note: Rate of mass flow between neighboring fins is a balance between buoyancy forces and viscous forces $\Rightarrow S_{optimal}$

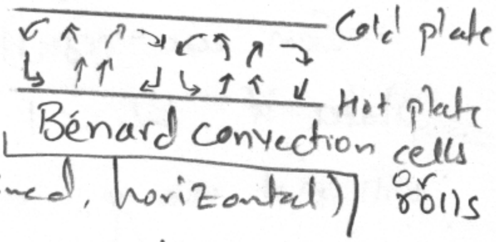
Enclosures

- Fluid "circulates" in enclosures with surfaces at different temperatures, due to buoyancy



$L_c \equiv$ characteristic length
i.e. distance between plates at different temperatures





Correlations are available for
- rectangular enclosures (vertical, inclined, horizontal)
- Concentric cylinders and concentric spheres

Section 9.8 (Incropera's) or Section 9-5 (Cengel's)

Example: Verticle plate

A glass door fire screen, used to reduce exfiltration of room air through a chimney has a height of 0.71 m and a width of 1.02 m, and reaches a temperature of 232°C. If the room temperature is 23°C, estimate the convection heat rate from the fireplace to the room.

Properties of air at $T_f \sim \frac{232+23}{2} \approx 127°C$ (= 400 K) and 1 atm

$k = 33.8 \times 10^{-3}$ W/m.K $\qquad \nu = 26.4 \times 10^{-6}$ m²/s

$\alpha = 38.3 \times 10^{-6}$ m²/s $\qquad Pr = 0.69 \qquad \beta = 0.0025\ K^{-1}$

(assuming air is an ideal gas) $= 1/T_f$

$$Q = hA_s(T_s - T_\infty)$$

where

$$A_s = 0.71 \times 1.02 = 0.7242 \text{ m}^2$$

$$T_s - T_\infty = 232 - 23 = 209 \text{ K}$$

To determine h,

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\alpha\nu} = 1.813 \times 10^9$$

For this value of Ra_L, Churchill and Chu's correlation for a vertical isothermal plate gives

$$Nu = \left\{0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1+\left(\frac{0.492}{Pr}\right)^{9/16}\right]^{8/27}}\right\}^2 = 147$$

$$h = \frac{Nu\, k}{L} = 7 \text{ W/m}^2\cdot\text{K}$$

$$\Rightarrow \quad Q = 1060 \text{ W}$$

Note: Assuming $\varepsilon = 1$, the rate of radiation heat transfer

$$Q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_\infty^4) = 2355 \text{ W}$$

which is significantly larger than that by convection alone, and should have been included.

Example: Horizontal Cylinder

A 6-m long section of an 8 cm diameter horizontal hot water pipe passes through a large room whose temperature is 20°C. If the outer surface temperature of the pipe is 70°C, determine the rate of heat loss from the pipe by natural

Convection.

Properties of air at $T_f \approx \left(\frac{70+20}{2}\right) = 45^\circ C$ and 1 atm

$k = 0.02699 \ W/m.K$ $\qquad$ $Pr = 0.7241$

$\nu = 1.750 \times 10^{-5} \ m^2/s$ $\qquad$ $\beta = \frac{1}{273+45} K^{-1}$

(assuming air is an ideal gas)

$$Q = h A_s (T_s - T_\infty)$$

$$A_s = \pi D L = 1.508 \ m^2$$

$$(T_s - T_\infty) = 50^\circ C$$

To determine h,

$$Ra_D = \frac{g \beta (T_s - T_\infty) D^3}{\nu} Pr = 1.867 \times 10^6$$

Using the correlation

$$Nu = \left[0.6 + \frac{0.387 \, Ra_D^{1/6}}{\left[1 + \left(\frac{0.559}{Pr}\right)^{9/16}\right]^{8/27}}\right]^2 \quad \text{for } Ra_D \leq 10^{12}$$

$$Nu = 17.39$$

$$h = \frac{Nu \, k}{D} = 5.867 \ W/m^2 K$$

$$Q = 442 \ W$$

Note: Again, assuming $\varepsilon = 1$,

$$Q_{rad} = \varepsilon A_s \sigma (T_s^4 - T_0^4) = 553 \ W \quad \text{(significant)}$$

Thus, unless $\varepsilon \to 0$, radiation heat transfer must be accounted for.