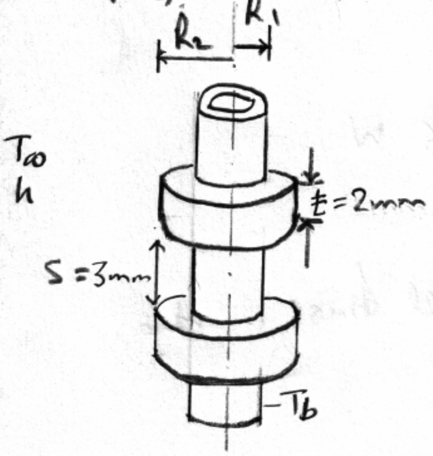
Example: Heat transfer from steam pipes

In a heating system, steam flows through tubes whose outer diameter is $D_1 = 3cm$ and whose walls are maintained at a temperature of $120°C$. Circular aluminium fins ($k = 180\ W/m.K$) of outer diameter $D_2 = 6cm$ and constant thickness $t = 2mm$ are attached to the tube. The space between fins is 3mm. Thus there are 200 fins per meter length of tube. Heat is transferred to surrounding air at $T_\infty = 25°C$, with $h = 60\ W/m^2$. Determine the increase in heat transfer from the tube per meter length as a result of adding fins.



Assumptions

- Steady state
- Uniform h, k
- Negligible radiation heat transfer

Given

- $k = 180\ W/mk$
  $h = 60\ W/m^2$
  $T_\infty = 25°C$
  $T_b = 120°C$
  $L_t = 1\ m$ ....."per meter"
  $N = 200$

In case of no fins

$$A_{nofin} = \pi D_1 L_t = 0.0942\ m^2$$

$$Q_{nofin} = h A_{nofin}(T_b - T_\infty) = 537\ W$$

In case of fins,

$$L = \frac{D_2 - D_1}{2} = R_2 - R_1 = 0.015\ m$$

$$L_c = L + \tfrac{1}{2}t = 0.016\ m \quad ; R_{2c} = R_2 + \tfrac{1}{2}t = 0.031m$$

$$\sqrt{\frac{h}{kt}}\,L_c = 0.207 \quad ; \quad \frac{R_{2c}}{R_1} = 2.07$$

From efficiency curves for these fins. (Figure 3.20 - Incropera or Figure 3-43 - Cengel)

$$\eta_f \approx 0.95$$

$$A_{fin} = 2\pi(R_2^2 - R_1^2) + 2\pi R_2 t = 0.00462 \text{ m}^2$$

$$Q_{fin} = \eta_f \, h A_{fin} (T_b - T_\infty) = 25 \text{ W}$$

From the prime surface (unfinned portion of the tube between two consecutive fins

$$A_{unfin} = 2\pi R_1 S = 0.000283 \text{ m}^2$$

$$Q_{unfin} = h A_{unfin} (T_b - T_\infty) = 1.6 \text{ W}$$

Thus, the total rate of heat transfer from all fins in the array and the entire prime surface

$$Q_{total,fin} = N(Q_{fin} + Q_{unfin}) = 5320 \text{ W}$$

Thus, the increase in heat transfer due to fins is

$$5320 - 537 = 4783 \text{ W}$$

i.e. the overall effectiveness of the fin array is

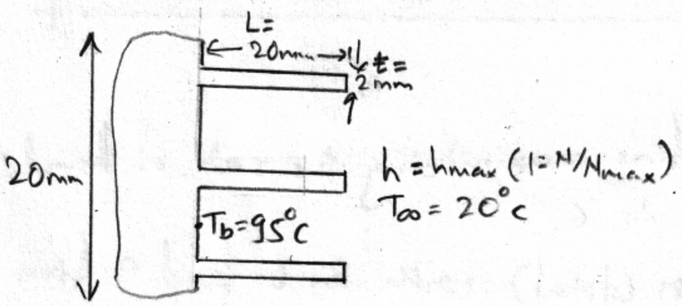
$$\varepsilon_{f,overall} = \frac{Q_{total,fin}}{Q_{total,nofin}} = \frac{5320}{537} = 9.9$$

Notes:

- The assumption that h is unchanged upon adding fins needs verification
- Fin efficiency can help decide the length of the fins. $\eta_f = 0.95$ is good efficiency.

Example: Convection heat transfer coefficient and fins

Due to blockage of fluid flow, the convection heat transfer coefficient for the finned and prime surfaces of a fin array in general decreases as the number of fins increases. For a fin array shown in the figure below, wall height is 20mm, w = 20 mm, t = 2 mm and L = 20mm. The fins and wall are aluminium and the base and environment temperatures are $T_b = 95°C$ and $T_\infty = 20°C$. As a first approximation, assume $h = h_{max}(1 - N/N_{max})$, where N_{max} is the maximum number of fins that can be placed on a surface, so that $N_{max} = 20mm/2mm = 10$. For $h_{max} = 50\ W/m^2K$, determine the total rate of heat transfer for N = 0, 3, 6, 9.



Assumptions:
- Steady state
- Uniform h, k
- Negligible radiation heat transfer

Given:

$k = 237\ W/m.K$ $\qquad$ $h_{max} = 50\ W/m^2\ K$

$T_b = 95°C$ $\qquad$ $T_\infty = 20°C$

$L = 20mm$ $\qquad$ $t = 2mm$

$W = 20\ mm$

For each fin,

$$L_c = L + t/2 = 0.021\ m$$

$$A_{fin} = PL + Wt = 2(W+t)L + Wt = 920 \times 10^{-6}\ m^2$$

$$A_{unfin,\,total} = A_{nofin} - NA_{fin} = 20 \times 20 \times 10^{-6}\ m^2 - N\,Wt$$

$$Q_{fin} = \eta_f h(N) A_{fin} (T_b - T_\infty)\ ;\ Q_{unfin} = h(N) A_{unfin} (T_b - T_\infty)$$

N	0	3	6	9
h (W/m^2·K)	50	35	20	5
$\sqrt{\frac{h}{kt}} L_c$	0.22	0.18	0.14	0.07
η_f	0.94	0.95	0.96	0.98
Q_{fin} (W)	3.24	2.29	1.32	0.34
A_{unfin} (m^2) (total)	0.0004	0.00028	0.00016	0.00004
Q_{unfin} (W) (total)	1.5	0.735	0.24	0.015
Q_{total} (W) $= NQ_{fin} + Q_{unfin}$ (total)	1.5	7.61	8.16	3.08

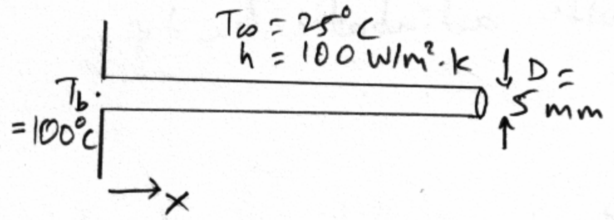
Notes:

- The number of fins maximizing the rate of heat transfer is close to 6.
- The effectiveness (total) with 3, 6 and 9 fins is 5.07, 5.44 and 2.05, respectively
- The efficiency (total) with 3, 6 and 9 fins is 0.957, 0.961, 0.980. However, it is misleading to compare efficiency in this case, since $Q_{total,f} = h A_{total} \eta_{f,overall}$; and all three quantities on the R.H.S. depend on N.

Example: Pin fins and efficiency

A very long rod 5mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 W/m²·K.

1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminium alloy, and type AISI 316 stainless steel. What are the corresponding rates of heat loss from the rods?
2. Estimate how long the rods must be for the assumption of infinite length to yield an accurate estimate of the rate of heat loss.



Assumptions:

- Steady state
- Uniform k, h
- Negligible radiation heat transfer
- Infinitely long rod

Known:

$k = 398\ W/m\cdot K$ for Copper
$k = 180\ W/mK$ for 2024 aluminium
$k = 14\ W/mK$ for AISI 316 stainless steel

$D = 5mm$
$T_b = 100°C$ $\quad T_\infty = 25°C \quad \Rightarrow \quad \theta_b = T_b - T_\infty = 75°C$
$h = 100\ W/m^2\cdot K$

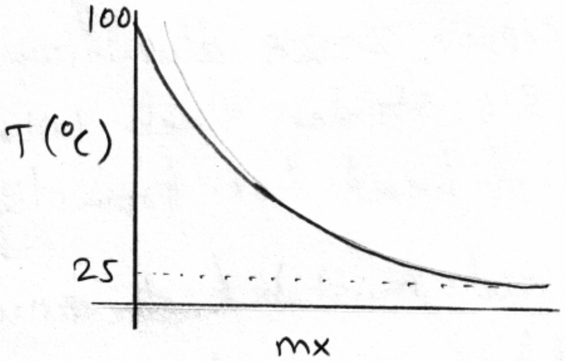
1. Assuming infinitely long fin, the temperature distribution along the length of the rod

$$\frac{\theta}{\theta_b} = e^{-mx} \Rightarrow T(x) = T_\infty + (T_b - T_\infty)\, e^{-\sqrt{\frac{hP}{kA_c}}\,x}$$

For the geometry of the rod, $P = \pi D \quad A_c = \pi D^2/4$

Thus, $m = \sqrt{\frac{hP}{kA_c}} = \sqrt{\frac{4h}{kD}}$ and $Q_f = M = \sqrt{hPkA_c}\,\theta_b = \left(\frac{\pi^2 D^3}{4} kh\right)^{1/2} \theta_b$ (5A)

Material →	Copper	2024 Aluminium alloy	AISI 316 Stainless Steel
m (m^{-1})	14.2	21.2	75.6
Q_f (W)	8.3	5.6	1.6



As m increases, the length-scale at which temperature drops decreases.

Also evident from

$$\frac{\theta}{\theta_b} = e^{-x/m^{-1}}$$

2.

For a finite length rod, with adiabatic fin tip,

$$Q_f = M \tanh mL$$

A rod that is long enough to qualify as an infinite length rod will have the same rate of heat transfer as an infinite length rod

Thus $Q_f = M \tanh mL^* \to M$

$\Rightarrow \tanh mL^* \to 1$

If we use 99% of the value for an infinitely long rod's rate of heat transfer as "nearly there", then our condition is

$$\tanh mL^* = 0.99$$

Thus $mL^* = 2.65$. or $L^* = 2.65/m$

For copper $L^* = 0.19$ m
aluminium $L^* = 0.13$ m
steel $L^* = 0.04$ m