Convection Heat Transfer over a Flat Plate (General)

Laminar flow: We had seen that (Blasius Solution)

$$\delta = \frac{4.91x}{\sqrt{Re_x}}$$

$$Re_x < 5\times10^5$$

$$C_{f,x} = 0.664\, Re_x^{-1/2}$$

$$\delta_t = \frac{4.91x}{\sqrt{Re_x}\, Pr^{1/3}}$$

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re^{1/2}\, Pr^{1/3} \qquad Pr > 0.6$$

Averages: $C_f = 1.33\, Re_L^{-1/2}$; $Nu = 0.664\, Re_L^{1/2}\, Pr^{1/3}$

Turbulent flow: Experiments indicate that

$$\delta = \frac{0.38x}{Re_x^{1/5}}$$

$$5\times10^5 < Re_x < 10^7$$

$$C_{f,x} = \frac{0.059}{Re_x^{1/5}}$$

$$Nu_x = 0.0296\, Re^{0.8}\, Pr^{1/3} \qquad 0.6 < Pr < 60$$

Averages: $C_f = 0.074\, Re_L^{-1/5}$; $Nu = 0.037\, Re^{0.8}\, Pr^{1/3}$

Flow over a flat plate

- Although flow starts out as laminar, $Re_x = \frac{\rho u_\infty x}{\mu}$. Thus, flow becomes turbulent for a sufficiently long plate. The critical value for transition is $Re_{cr} = \frac{\rho u_\infty x_{cr}}{\mu} = 5\times10^5$

For such a long plate, part of the plate experiences laminar fluid flow, while the flow transitions to turbulent over the rest of the plate.

Then, average friction coefficient

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,\text{Laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x,\text{Turbulent}}\,dx\right)$$

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5\times10^5 \le Re_L \le 10^7$$

If the plate is very long, then $x_{cr} \ll L$, and turbulent flow expression may be used.

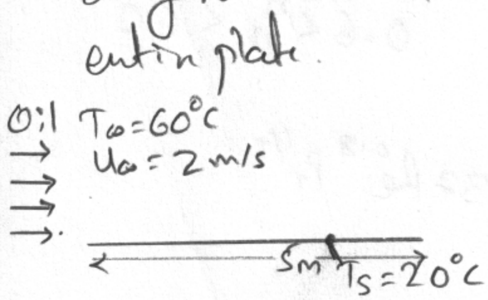
Similarly, average Nusselt number

$$Nu = (0.037\,Re_L^{0.8} - 871)\,Pr^{1/3} \qquad 5\times10^5 \le Re_L \le 10^7, \quad 0.6 \le Pr \le 60$$

$$\ldots\ldots\ h = \frac{1}{L}\left(\int_0^{x_c} h_{x,\text{laminar}}\,dx + \int_{x_c}^{L} h_{x,\text{turbulent}}\,dx\right)$$

Example: Flow over a Flat Plate

Engine oil at 60°C flows over the surface of a 5 m long flat plate, whose temperature is 20°C with a velocity of 2 m/s. Determine the total drag force and the rate of heat transfer per unit width of the entire plate.



Assumptions:
- steady, incompressible flow
- $Re_{cr} = 5\times10^5$ (marking laminar → turbulent transition)

Given: Engine Oil at $T_\infty = 60°C$
properties at 40°C (mean temperature)
$\rho = 876\ kg/m^3$ $\quad Pr = 2962$
$k = 0.1444\ W/m\,K$ $\quad \nu = 2.485\times10^{-4}\ m^2/s$

$Re_L = \frac{u_\infty L}{\nu} = 4.024\times10^4 < Re_{cr} \Rightarrow$ Laminar flow throughout the plate

$C_f = 1.33\,Re_L^{-1/2} = 0.00663 \Rightarrow$ drag force per width, $\frac{F_D}{W} = C_f \frac{A}{W}\frac{\rho u_\infty^2}{2} = 58.1\ N/m$ with $A = LW$

$$Nu = \frac{hL}{k} = 0.664\, Re^{1/2}\, Pr^{1/3} = 1913$$

$$\Rightarrow h = 55.25 \ W/m^2\text{-}K$$

Once h is known, Q/W can be calculated

$$\frac{Q}{W} = h \frac{A_s}{W}(T_\infty - T_s) = 11050 \ \frac{W}{m} \quad \text{from oil to the surface.}$$

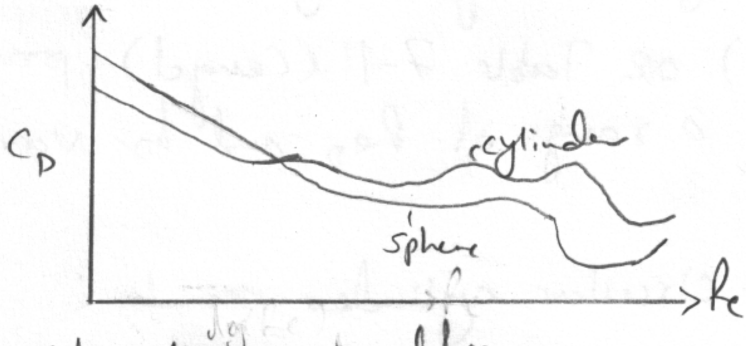
Convective Heat Transfer in Flow across Cylinders/Spheres

Characteristic lengthscale, diameter of the cylinder/sphere

$$Re_D = \frac{U_\infty D}{\nu} \quad \text{with } Re_{cr} \approx 2 \times 10^5$$

Drag coefficient

$$C_D = \frac{F_D}{A_f(\rho U_\infty^2/2)} \quad \text{with } A_f \equiv \text{projected area}$$



Schematic reproduced from Fig. 7.9 (Incropera) or Fig 7-17 (Cengel), originally from Schlichting

Flow typically involves separation at rear of the cylinder/sphere. Hence, heat transfer is lot more complicated.

Fig 7.10 (Incropera) or Fig 7-22 (Cengel) show dependence of local Nusselt number.

Average Nusselt Number

$$Nu_{cyl} = \frac{hD}{k} = 0.3 + \frac{0.62\, Re^{1/2}\, Pr^{1/3}}{\left[1 + \left(\frac{0.4}{Pr}\right)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

with properties evaluated at $\frac{1}{2}(T_s + T_\infty)$

– Churchill and Bernstein

$$Nu_{sph} = \frac{hD}{k} = 2 + \left[0.4\, Re^{1/2} + 0.06\, Re^{2/3}\right] Pr^{0.4} \left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

with properties evaluated at T_∞, except for μ_s, evaluated at T_s.

– Whitaker

For non-circular cylinders, a general form is

$$Nu_{cyl} = C\, Re_D^{m}\, Pr^{1/3}$$

with C, m depending on the geometry

Tables 7.2 + 7.3 (Incropera) OR Table 7-1 (Cengel) provide values of C, m for a range of Re_D and for various geometries.

For example, for a circular cylinder

Re_D	C	m
0.4 – 4	0.989	0.33
4 – 40	0.911	0.385
40 – 4000	0.683	0.466
4000 – 40,000	0.193	0.618
40,000 – 400,000	0.027	0.805