$$G_p = \frac{2e^{-s}}{5s+1}$$

PI Controller

- Choose τ_I s.t. the $\angle$ contribution at ω_c due to I action is $5^\circ - 10^\circ$
- K_u & P_u & set $\tau_I \approx P_u$ (ZN)
- Then adjust K_c for GM/PM or other criterion.

PI Controller design for GM = 2

$$G_{oL} = \frac{2K_c(\tau_I s+1)e^{-s}}{\tau_I s(5s+1)}$$

Take $\tau_I = 4$ mins

$$\angle G_{oL} = -\omega - \tan^{-1}5\omega + \underbrace{\tan^{-1}\tau_I\omega - \frac{\pi}{2}}$$

contribution of I action to $\angle G_{oL}$

$\angle G_{oL} = -\pi \Rightarrow \omega_c = 1.539$ rad/min

$\angle$ I action $= 9.2^\circ$

$\tau_I = 3.7$ mins $\qquad \omega_c = 1.526$ rad/min $\qquad \angle$ action $= 10.05°$

$$GM = 2 \;\Rightarrow\; \left|G_{OL}\right|_{\omega_c} = \frac{1}{2}$$

$$\Rightarrow \left| \frac{2K_c e^{-j\omega}(4\omega j + 1)}{4\omega j(5\omega j + 1)} \right| = \frac{1}{2} \Rightarrow K_c^{GM} = 1.89$$

$K_c = 1.89 \qquad \tau_I = 3.7$ mins

PID Controller

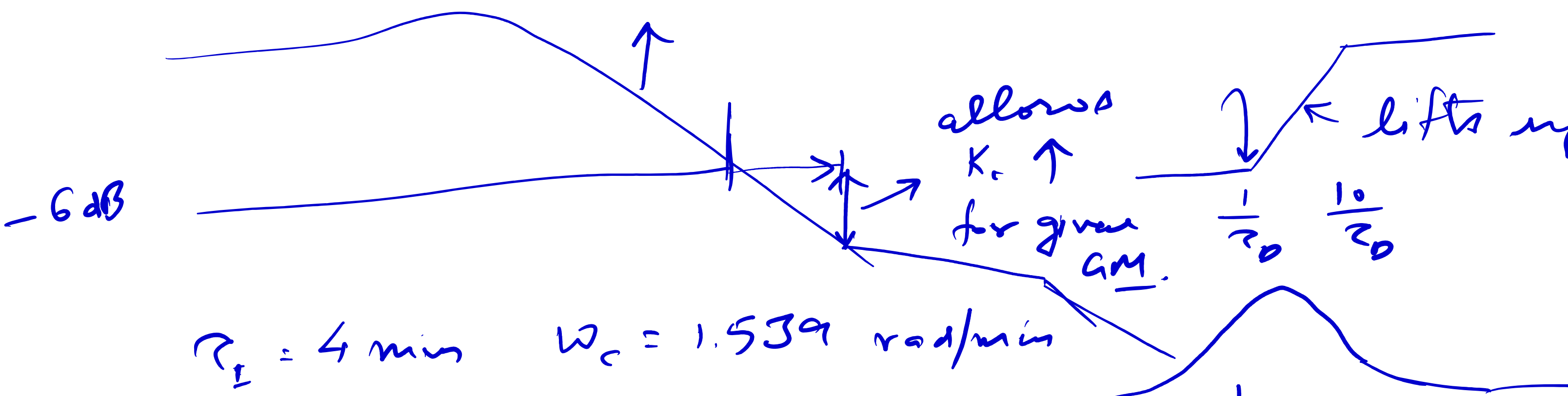
Choose a reasonable τ_I value ($\tau_I = 4$ min)

$$G_{OL} = \frac{2K_c(4s+1)(\tau_D s+1)\,e^{-s}}{4s\,(0.1\tau_D s+1)(5s+1)}$$

$$\angle G_{OL} = -\omega - \tan^{-1}5\omega \underbrace{- \frac{\pi}{2} + \tan^{-1}4\omega}_{\text{I action}} + \underbrace{\tan^{-1}\tau_D\omega - \tan^{-1}0.1\tau_D\omega}_{\text{D action}} \quad \leftarrow \omega_c$$

$20\log_{10}\frac{1}{2}$ $\quad -6$ dB

$\left[\frac{1}{\tau_D} - \frac{10}{\tau_D}\right]$ ϕ lead



Try $\tau_D = 0.3$ min

$\tau_I = 4$ min

$\tau_D = 0$ min

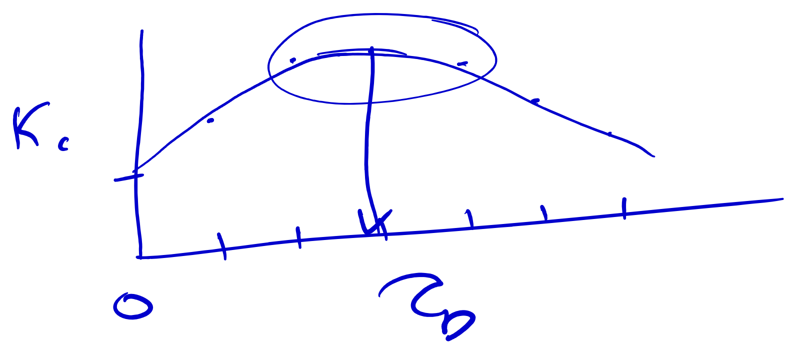
$\omega_c = 1.539$ rad/min

$$\frac{1}{\tau_D} = \frac{1}{2\times\omega_c} \quad \text{or} \quad \frac{1}{3\times\omega_c}$$

$\angle G_{OL} = -\pi \Rightarrow \omega_c = 2.033$ rad/min

$|G_{OL}| = \frac{1}{2} \Rightarrow K_c = \underline{\underline{2.168}}$

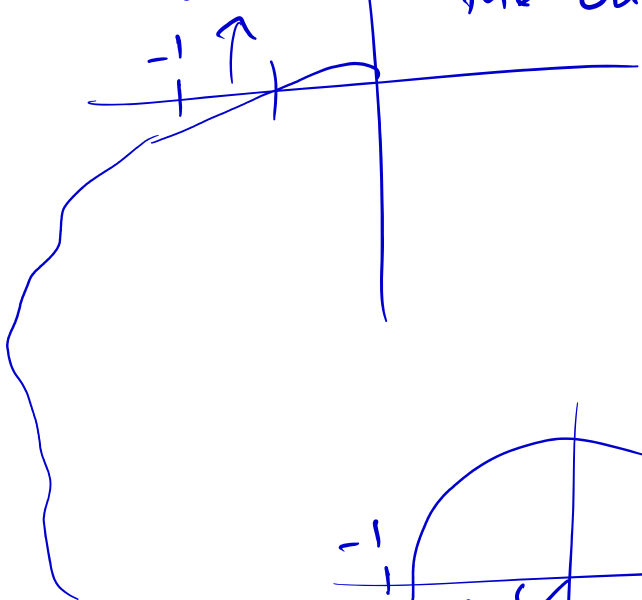
$\underline{1.9}$



$\tau_D = 0.25$

$K_c = 2.188$

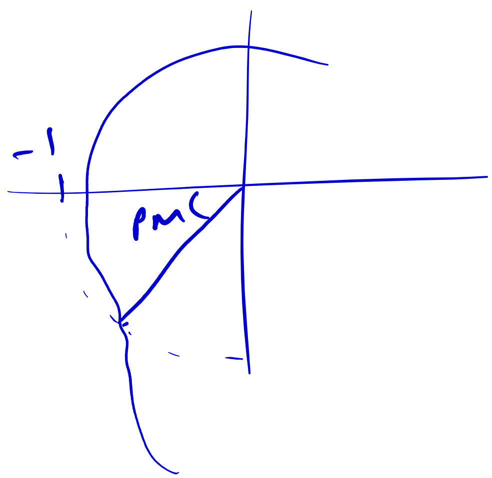
good GM

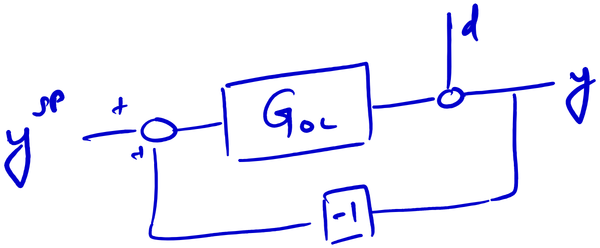


yet G_{OL} polar plot passes very close to the critical (-1, 0) point

$\Rightarrow$ Too oscillatory a CL response despite good GM

$\underline{\underline{G_{OL}}}$





$$G_{CL}^{reg} = \frac{1}{1+G_{OL}} \qquad G_{CL}^{servo} = \frac{G_{OL}}{1+G_{OL}}$$

$$S = \frac{1}{1+G_{OL}} \qquad T = \frac{G_{OL}}{1+G_{OL}}$$

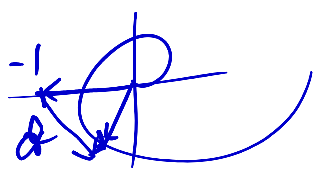
$$S + T = 1$$

$$M_S = \max |S_{j\omega}| \qquad M_T = \max |T_{j\omega}|$$

$$\vec{a} - \vec{b} = \vec{c}$$

$$\vec{a} = \vec{c} + \vec{b}$$

G_{OL}



$$G_{OL} - (-1) = \underline{1 + G_{OL}}$$

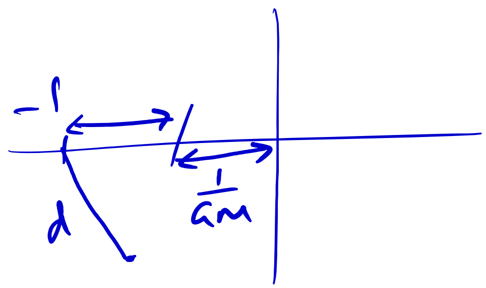
$$\vec{G}_{OL} + \vec{r} = -\vec{1}$$

$$1 + G_{OL} = -\vec{r}$$

$S = \underline{\underline{\frac{1}{1+G_{OL}}}} \equiv$ inv. of distance of G_{OL} from $(-1, 0)$

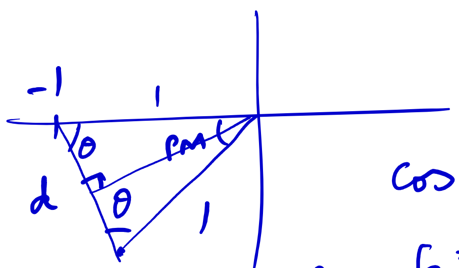
$\underline{\underline{M_S}} \equiv$ min distance of G_{OL} polar plot from $(-1, 0)$

$M_s \overset{?}{\Longrightarrow}$ Guarantees on min GM or PM.



$$2\theta + PM = 180^\circ$$

$$\theta = 90 - \frac{PM}{2}$$



$$\cos\theta = \frac{d}{2}$$

$$2\cos\left[90^\circ - \frac{PM}{2}\right] = d$$

$$d > \frac{1}{M_s}$$

$$2\sin\frac{PM}{2} = d > \frac{1}{M_s}$$

$$d = \frac{1}{M_s}$$

$$1 - \frac{1}{GM} > d$$

$$1 - \frac{1}{GM} > \frac{1}{M_s}$$

$$1 - \frac{1}{M_s} > \frac{1}{GM}$$

$$\frac{M_s - 1}{M_s} > \frac{1}{GM}$$

$$\boxed{GM > \frac{M_s}{M_s - 1}}$$

$$\frac{1}{2M_s} < \sin\frac{PM}{2}$$

$$\boxed{PM > \left[2\sin^{-1}\frac{1}{2M_s}\right]}$$

$$T = \frac{G_{OL}}{1+G_{OL}}$$

$$\underline{\underline{S}}$$

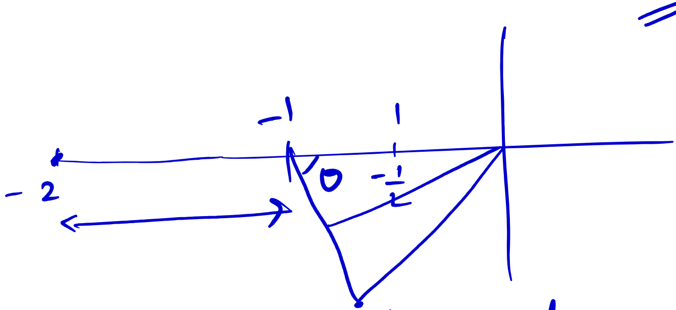
$$\underline{\underline{T}} = \frac{1}{1+\frac{1}{G_{OL}}}$$

G

dist of $\frac{1}{G_{OL}}$ $(-1,0)$

$T \equiv$ inv of

$M_T \equiv$ max



$$GM - 1 > \frac{1}{M_T}$$

$$GM > 1 + \frac{1}{M_T}$$

$$|G_{OL}| = 1 \qquad \sin\theta = \frac{d}{2}$$

$$\left|\frac{1}{G_{OL}}\right| = 1 \qquad d > \frac{1}{M_T}$$

$$2\sin\frac{PM}{2} > \frac{1}{M_T} \qquad \boxed{PM > 2\sin^{-1}\frac{1}{2M_T}}$$

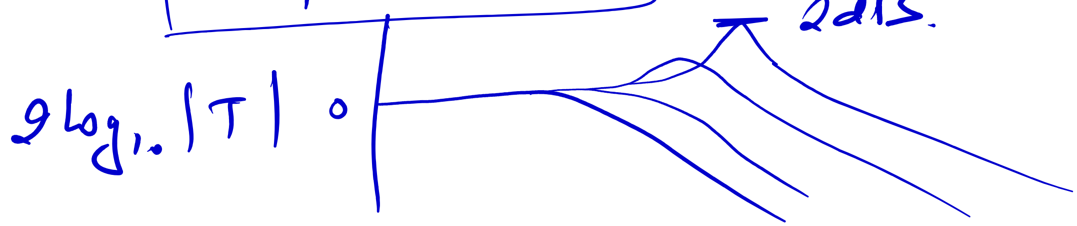
$$T = \frac{G_{OL}}{1 + G_{OL}}$$

$$T = \frac{\dfrac{K K_c}{(\tau s+1)^3}\,\dfrac{\tau_I s+1}{\tau_I s}}{1 + \dfrac{K_c K}{(\tau s+1)^3}\,\dfrac{(\tau_I s+1)}{\tau_I s}}$$

$$T = \frac{K K_c (\cancel{\tau_I s}+1)}{\underbrace{\tau_I s(\tau s+1)^3}_{0} + K K_c(\underbrace{\tau_I s}_{0}+1)}$$

Tune K_c st

$$\boxed{M_T = 2\,dB}$$



$|T| \to 1$ for small ωs

Maximum Closed Loop Log Modulus Tuning

$$G_p = \frac{2e^{-s}}{5s+1} \qquad G_c = K_c$$

$$M_T = 2\,dB$$

$$G_{OL} = \frac{2K_c e^{-s}}{5s+1} \qquad T = \frac{G_{OL}}{1+G_{OL}} \Rightarrow T = \frac{2K_c e^{-s}}{5s+1+2K_c e^{-s}}$$

$$T_{j\omega} = \frac{2K_c e^{-j\omega}}{5\omega j + 1 + 2K_c e^{-\omega j}} \qquad e^{-\omega j} = \cos\omega - j\sin\omega$$

$$|T_{j\omega}| = \frac{2K_c}{\left\{\left[5\omega - 2K_c\sin\omega\right]^2 + \left[1+2K_c\cos\omega\right]^2\right\}^{1/2}}$$

$$\frac{2K_c}{\sqrt{(5\omega-2K_c\sin\omega)^2+(1+2K_c\cos\omega)^2}} = 1.2589 \quad \text{— (B)}$$

At max $|T|$, $\frac{d}{d\omega}[\text{den}] = 0 \Rightarrow \cancel{2}[5\omega - 2K_c\sin\omega][5-2K_c\cos\omega] - \cancel{2}[1+2K_c\cos\omega]2K_c\sin\omega = 0$

At M_T $[5\omega - 2K_c\sin\omega][5-2K_c\cos\omega] - 2K_c\sin\omega[1+2K_c\cos\omega] = 0$ — (A)

→ Assume K_c
Get ω from Ⓐ
Check if $LHS_{Ⓑ} = 1.2589$

$$\boxed{K_c^{2dB} = 2.063}$$

K_c guess	$\omega_{Ⓐ}$	$LHS_{Ⓑ}$ (1.2589)
2	1.2327 rad/min	1.1992
2.1	1.2760	1.2958
2.05	1.2550	1.2463
2.06		1.2550
<u>2.063</u>		1.2590