FINANCIAL ENGINEERING

IME611A

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SESSION OBJECTIVES

- The forward rates
- Term structure explanation
- Spot rate
- Relationship between spot rate and forward rate

FORWARD RATES

- **Forward rates:** Interest rates for money to be borrowed between ***two dates in future***, *but under terms* ***agreed upon today****.*

- For example, knowing the spot rates for two years s_1 and s_2, one can obtain the forward rate f for the borrowing between year 1 and 2.

$$(1 + s_2)^2 = (1 + s_1)(1 + f)$$

$$f = \frac{(1+s_2)^2}{(1+s_1)} - 1$$

- Practice Example 4.4

IMPORTANT RESULT

- ***Forward rate definition:*** The forward rate between times t_1 and t_2 with $t_1 < t_2$ is denoted by $f_{t1, t2.}$ It is the rate of interest charged for borrowing money at time t_1 which is to be repaid with interest at time t_2.

- Two types of forward rates
  - Implied forward rates
  - Market forward rates

$$(1 + s_j)^j = (1 + s_i)^i \left(1 + f_{i,j}\right)^{j-i}$$

COMPOUNDING FREQUENCIES

- **Forward rate formulas:** The implied forward rate between times t_i and $t_i > t_i$ is the rate of interest between those times that is consistent with a given spot rate curve.

Frequency	Spot-forward relationship	Forward rate formula
Yearly	$(1+s_j)^j = (1+s_i)^i (1+f_{i,j})^{j-i}$	$f_{i,j} = \left[\frac{(1+s_j)^j}{(1+s_i)^i}\right]^{1/(j-i)} - 1$
m-period	$(1+s_j/m)^j = (1+s_i/m)^i (1+f_{i,j}/m)^{j-i}$	$f_{i,j} = \left[\frac{(1+s_j/m)^j}{(1+s_i/m)^i}\right]^{1/(j-i)} - m$
Continuous	$e^{(t_2 * s_{t_2})} = e^{(t_1 * s_{t_1})}\, e^{(t_2 - t_1) * f_{t_1,t_2}}$	$f_{t_1,t_2} = \frac{t_2 * s_{t_2} - t_1 * s_{t_1}}{t_2 - t_1}$

TERM STRUCTURE EXPLANATIONS

1. Expectations theory

2. Liquidity preference

3. Market segmentation

EXPECTATIONS DYNAMICS

- Spot rate forecasts

$$s'_{j-1} = f_{1,j} = \left[\frac{(1+s_j)^j}{(1+s_1)}\right]^{1/(j-1)} - 1$$

SPOT RATE FORECASTS: AN EXAMPLE

- Given current spot rate curve, **construct forecast** curve for next year.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Current	6.00	6.45	6.80	7.10	7.36	7.56	7.77
Forecast (+1)							

SPOT RATE FORECASTS: AN EXAMPLE

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	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Current	6.00	6.45	6.80	7.10	7.36	7.56	7.77
Forecast (+1)	6.90	7.20	7.47	7.70	7.88	8.06	

SPOT RATE FORECASTS: TRIANGULAR ARRAY STRUCTURE

	s_1	s_2	s_3	…	s_{n-2}	s_{n-1}	s_n
Current	$f_{0,1}$	$f_{0,2}$	$f_{0,3}$	---	$f_{0,n-2}$	$f_{0,n-1}$	$f_{0,n}$
Forecast (+1)	$f_{1,2}$	$f_{1,3}$	$f_{1,4}$	---	$f_{1,n-1}$	$f_{1,n}$	
Forecast (+2)	$f_{2,3}$	$f_{2,4}$	$f_{2,5}$	---	$f_{2,n}$		
. . .	. .		.				
Forecast (+n-2)	$f_{n-2,n-1}$	$f_{n-2,n}$					
Forecast (+n-1)	$f_{n-1,n}$						

DISCOUNT FACTORS

- **Discount factor $d_{j,k}$** denotes the discount factor used to discount cash received at time *k* back to an equivalent amount of cash at time *j*.

$$d_{j,k} = \left[\frac{1}{1 + f_{j,k}}\right]^{k-j}$$

- **Discount factor relation**

$$d_{i,k} = d_{i,j}\, d_{j,k}$$

DISCLAIMER

- The information in this presentation has been compiled from the following textbook which has been mentioned as a reference text for this course on **Financial Engineering**.

- Reference Text:
  - **Investment Science**, 2nd Edition, Oxford University Press, David G. Luenberger