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Q 2) The following linear differential equation describe the dynamic behavior of MIMO process.

$$\frac{d^2 y_1}{dt^2} + 2\frac{dy_1}{dt} + y_1 = m_1 \; ; \quad y_1'(0) = y_1(0) = 0$$

$$\frac{dy_2}{dt} - 3\frac{dy_1}{dt} - y_1 - 2y_2 = m_2 \; ; \quad y_2(0) = 0$$

assume manipulated variable m_i is coupled with the controlled output y_i through proportional controller.

a) For the process do the control loops interact.

b.) How would you tune the proportional controllers of the interacting loops so that the overall process is stable.

Ans 2) a) Laplace transform

$$s^2 \bar{y}_1(s) + 2s\bar{y}_1(s) + \bar{y}_1(s) = \bar{m}_1(s)$$
$$s\bar{y}_2(s) - 3s\bar{y}_1(s) - \bar{y}_1(s) - 2\bar{y}_2(s) = \bar{m}_2(s)$$

$$\begin{bmatrix} s^2+2s+1 & 0 \\ -3s-1 & s+2 \end{bmatrix} \begin{bmatrix} \bar{y}_1(s) \\ \bar{y}_2(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{m}_1(s) \\ \bar{m}_2(s) \end{bmatrix}$$

$$\begin{pmatrix} \bar{y}_1(s) \\ \bar{y}_2(s) \end{pmatrix} = \frac{1}{(s+1)^2(s-2)} \begin{bmatrix} s-2 & 0 \\ 3s+1 & (s+1)^2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{m}_1(s) \\ \bar{m}_2(s) \end{bmatrix}$$

$$\begin{pmatrix} \bar{y}_1(s) \\ \bar{y}_2(s) \end{pmatrix} = \begin{bmatrix} \frac{1}{(s+1)^2} & 0 \\ \frac{3s+1}{(s+1)^2(s-2)} & \frac{1}{(s-2)} \end{bmatrix} \begin{bmatrix} \bar{m}_1(s) \\ \bar{m}_2(s) \end{bmatrix}$$

There is one-way interaction b/w the loop 1 (m_1, y_1) affects look 2 (m_2, y_2), while the reverse does not happen

b.) characteristic eq^n of loop 1 (m_1, y_1)

$$1 + G_{11} K_{c_1} = 0 \Rightarrow 1 + \frac{1}{(s+1)^2} K_{c_1} = 0.$$

$$s^2 + 2s + (1 + K_c) = 0 \Rightarrow \text{stable } \forall\ K_c > 0$$

characteristic eq^n of loop 2 (m_2, y_2)

$$1 + G_{22} K_{c_2} = 0 \Rightarrow 1 + \frac{1}{(s-2)} K_{c_2} = 0.$$

$$s + K_{c_2} - 2 = 0.$$

loop 2 is stable if $K_{c_2} - 2 > 0 \Rightarrow \boxed{K_{c_2} > 2}$ --- (4)

tuning with both loops closed results in the following characteristic equation

$$\left[1 + \frac{K_{c_1}}{(s+1)^2}\right]\left[1 + \frac{K_{c_2}}{(s-2)}\right] = 0$$

$$s^3 + K_{c_2} s^2 + (K_{c_1} + K_{c_2} - 1) s + (K_{c_2} - 2K_{c_1} + K_{c_1}K_{c_2} - 2) = 0$$

since the resulting eq^n is of third order both Routh array test should be performed.

The first test gives (necessary cond^n)

$$\boxed{K_{c_1} + K_{c_2} - 1 > 0} \quad (1)$$

$$\boxed{K_{c_2} - 2K_{c_1} + K_{c_1}K_{c_2} - 2 > 0} \quad (2)$$

for 2nd test an additional test that needs to be satisfied is $\boxed{K_{c_2}^2 + 2K_{c_1} - 2K_{c_2} + 2 > 0}$

--- (3)

∴, the 4 inequalities valid simultaneously are

$$K_{c_2} > 2 \quad \text{--- (1)}$$

$$K_{c_1} + K_{c_2} > 1 \quad \text{--- (2)}$$

$$K_{c_2} - 2K_{c_1} + K_{c_1}K_{c_2} - 2 > 0 \quad \text{--- (3)}$$

$$K_{c_2}^2 + 2K_{c_1} - 2K_{c_2} + 2 > 0 \quad \text{--- (4)}$$

if (1) is true.
then (2) (3) (4) all are true.

∴ the tuning for the controller must be such that

$\boxed{K_{c_2} > 2}$ $\boxed{K_{c_1} \text{ any value}}$