SHUBHAM GUPTA 180749.

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(Q 3) Consider a liquid mixture of A & B which flows through a cylindrical tube. Assume that the flow is turbulent and the plug flow approximation i.e. the concentration of A & B is a function of time and the position along the length of the tube only (i.e. there is no radial concⁿ gradient) is valid

a) Show that the material balance around a cylindrical element of length dz leads to:

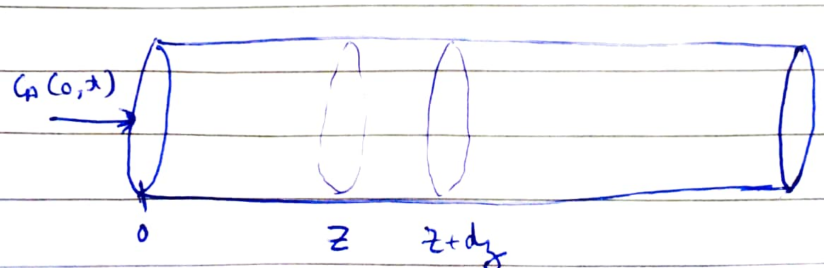
$$\frac{d(v\,C_A)}{dz} + \frac{d(C_A)}{dt} = 0$$

where $C_A = C_A(z,t)$ is concⁿ of A at time t and a cross sectional area at distance z from the inlet of the tube, which is assumed to be constant.

b.) ~~[illegible]~~ Find the transfer funcⁿ b/w $C_A(z,t)$ and the inlet concⁿ $C_A(0,t)$.

c.) Find the transient responce of the concⁿ at $z=20$ when a unit impulse in the inlet concⁿ is applied at time $t=0$

d) Find the transient responce of the concⁿ along the tube length when the inlet concⁿ changes by a unit step at $t=0$



Ans 3) a) Taking balance of component A

$$A\,\Delta z\left[(C_A)_{t+\Delta t} - (C_A)_t\right] = v\,A\,(C_A)_z\,\Delta t$$

{accumulation of component A during Δt} { flow in of A during Δt }

{flow out of A during Δt} $\left(-\,v\,A\,(C_A)_{z+\Delta z}\,\Delta t\right)$

A → cross sectional area.
divide both sides by $\Delta z\,t$.

$$\frac{\left[(C_A)_{t+\Delta t} - (C_A)_t\right]}{\Delta t} = \left[\frac{v(C_A)_z}{\Delta z} - \frac{v\,(C_A)_{z+\Delta z}}{\Delta z}\right]$$

let $\Delta z \to 0$, $\Delta t \to 0$

$$\boxed{\frac{dC_A}{dt} + \frac{d\,vC_A}{dz} = 0}$$

initial condition : $C_A(z,0) = 0$.
boundary condition : $C_A(0,t) = C_A$ (given)

$\boxed{\text{Hence proved}}$

b.) laplace transform w.r.t t

$$v\frac{d\bar{C}_A(z,s)}{dz} + s\,\bar{C}_A(z,s) - \bar{C}_A(z,0) = 0.$$

laplace transform again w.r.t z

$$v\left(w\,\bar{\bar{C}}_A(w,s) - \bar{C}_A(0,s)\right) + s\,\bar{\bar{C}}_A(w,s) - \bar{\bar{C}}_A(w,0) = 0$$

assuming deviation variables from steady state

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$C_A(z,0) = 0$ $\bar{C}_A(\omega,0) = 0$

$$\bar{C}_A(\omega,s) = \frac{\bar{C}_A(0,s)}{\omega + \frac{s}{v}}$$

$$\bar{C}_A(z,s) = e^{-(s/v)z}\,\bar{C}_A(0,s) \quad \text{(laplace inversion)}$$

$$\bar{C}_A(z,s) = e^{-z_d s}\cdot\bar{C}_A(0,s), \quad \text{where } \boxed{z_d = z/v}$$

c.) $C_A(0,t) =$ unit impulse
$\bar{C}_A(0,s) = 1$

$$\therefore \bar{C}_A(z,s)\Big|_{z=20} = e^{-\frac{20}{v}s}$$

invert $C_A(20,t) =$ unit impulse delayed by $-20/v$

d) $C_A(0,t) = u(t)$ unit step at $t=0$. Then, $\bar{C}_A(0,s) =$

and $$\bar{C}_A(z,s) = \frac{e^{-(z/v)s}}{s}$$

invert and find

$$C_A(z,t) = u\left(t - \frac{z}{v}\right)$$ unit step delayed by (z/v).