Module 4.8.2

Advanced Controller Structures

Cascade Control

Lectures on

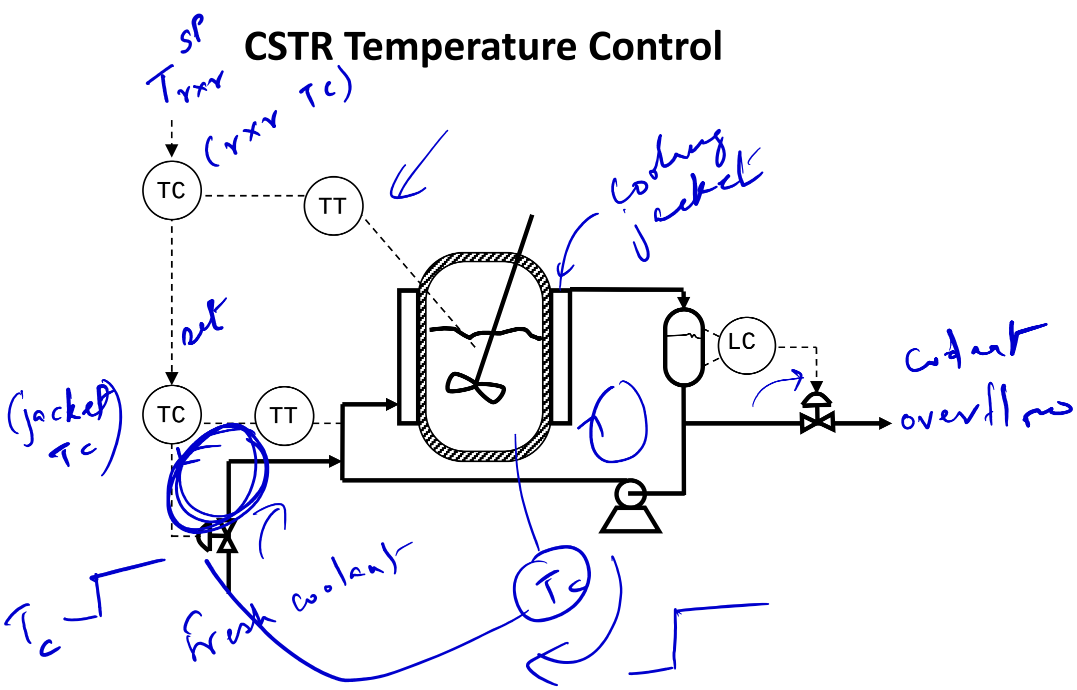
CHEMICAL PROCESS CONTROL

Theory and Practice

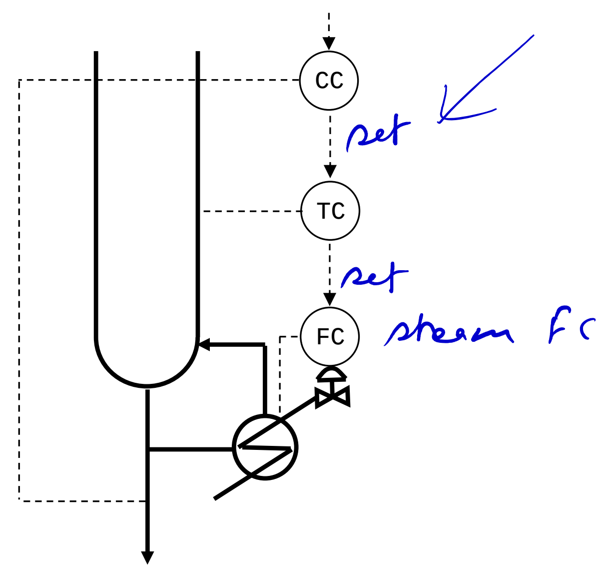
Cascade Control

Master (primary) controller sets setpoint of a slave (secondary) controller.
Slave output PV is strongly related to the master output PV

CSTR Temperature Control

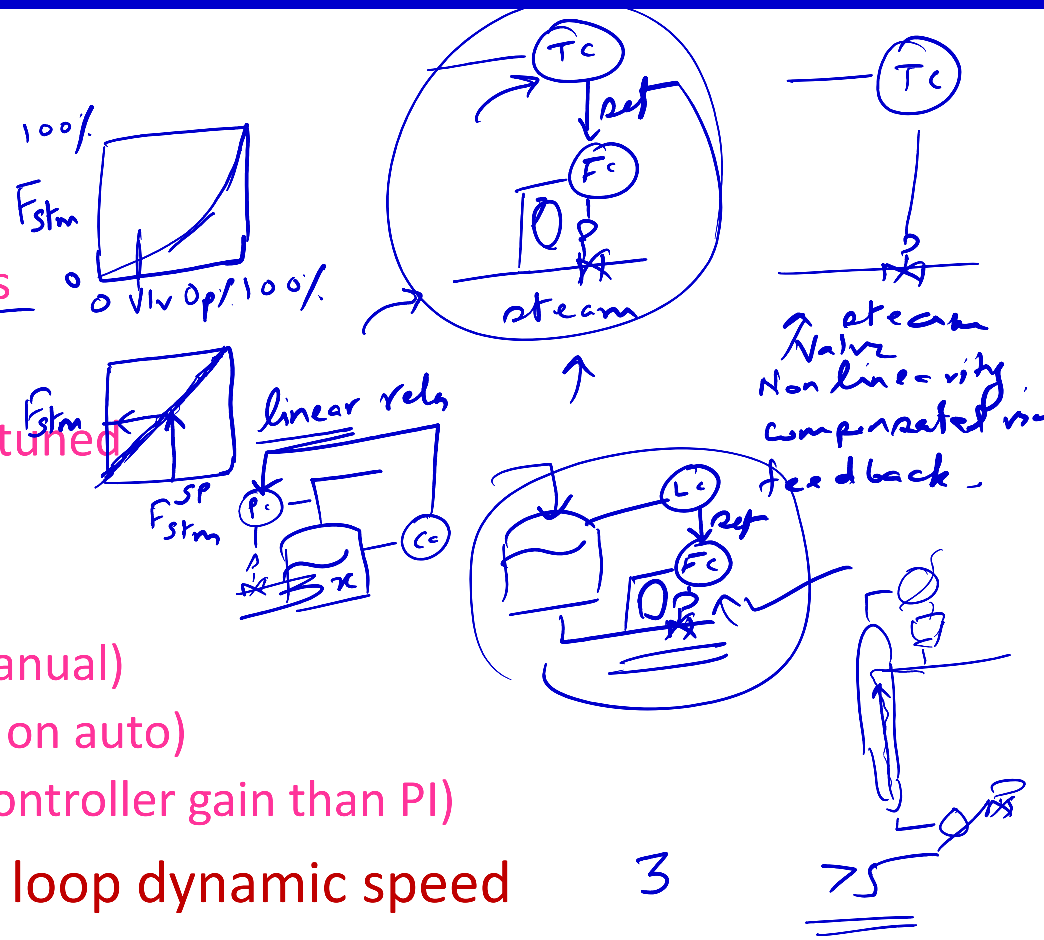


Column Composition Control

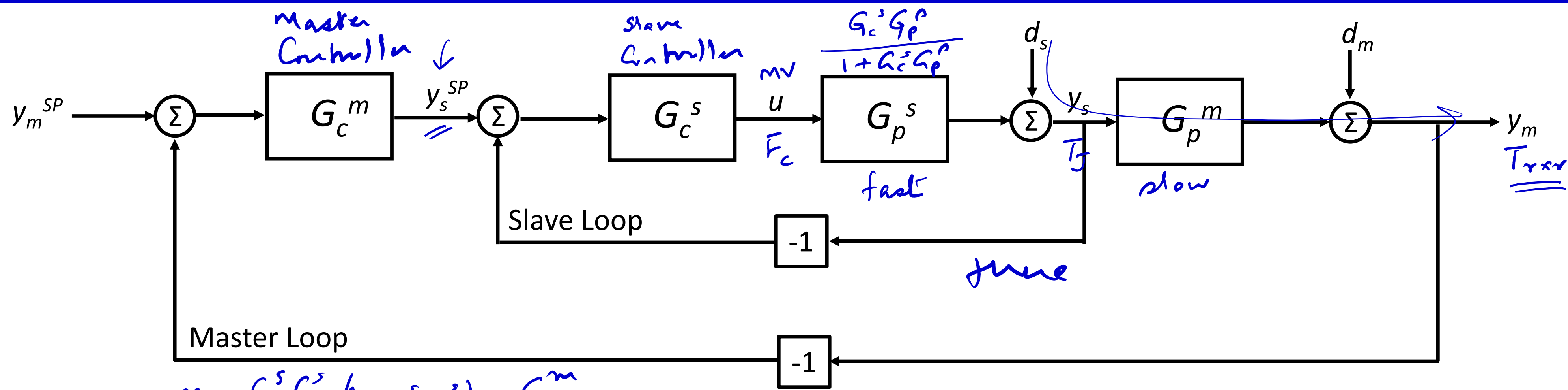


Why and Tuning

- Pros
  - Speeds up primary PV control
  - Slave loop removes local disturbances
  - Slave loop removes local non-linearities
- Cons
  - More instrumentation and loops to be tuned
- Tuning
  - Inside-out
  - Tune innermost first (outer loops on manual)
  - Then tune next outer loop (inner loops on auto)
  - Inner loops may be P only (for higher controller gain than PI)
- Slave loop dynamic speed >> Master loop dynamic speed



Series Cascade

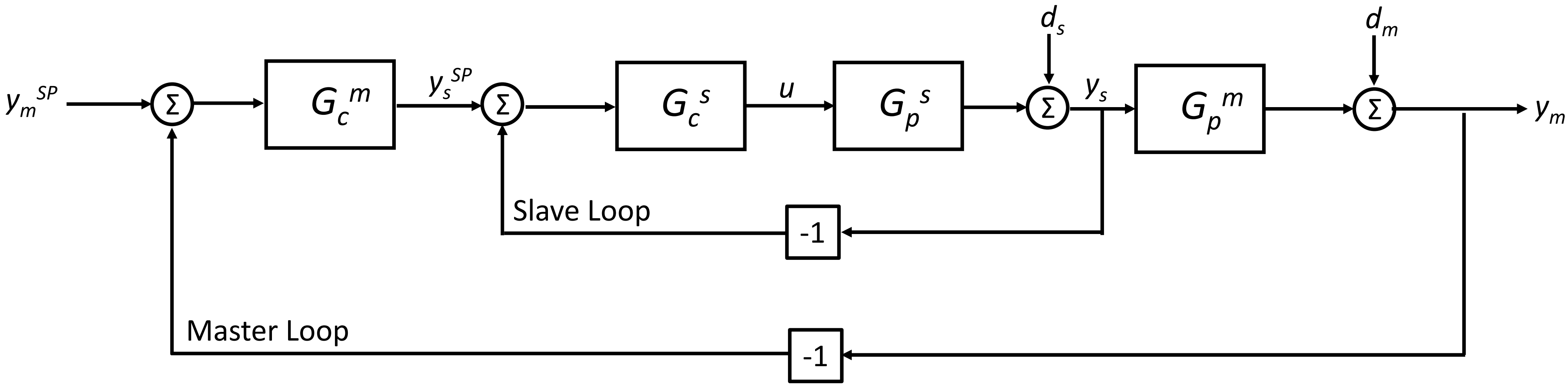


$$\frac{y_m}{y_m^{sp}} = \frac{G_c^m \cdot G_c^s G_p^s / (1+G_c^s G_p^s) \cdot G_p^m}{1 + G_c^m G_c^s G_p^s G_p^m / 1+G_c^s G_p^s}$$

$$\frac{y_m}{d_s} = \frac{G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^s G_p^m}$$

$$\frac{y_m}{y_m^{sp}} = \frac{G_c^m G_c^s G_p^s G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^s G_p^m}$$

Series Cascade

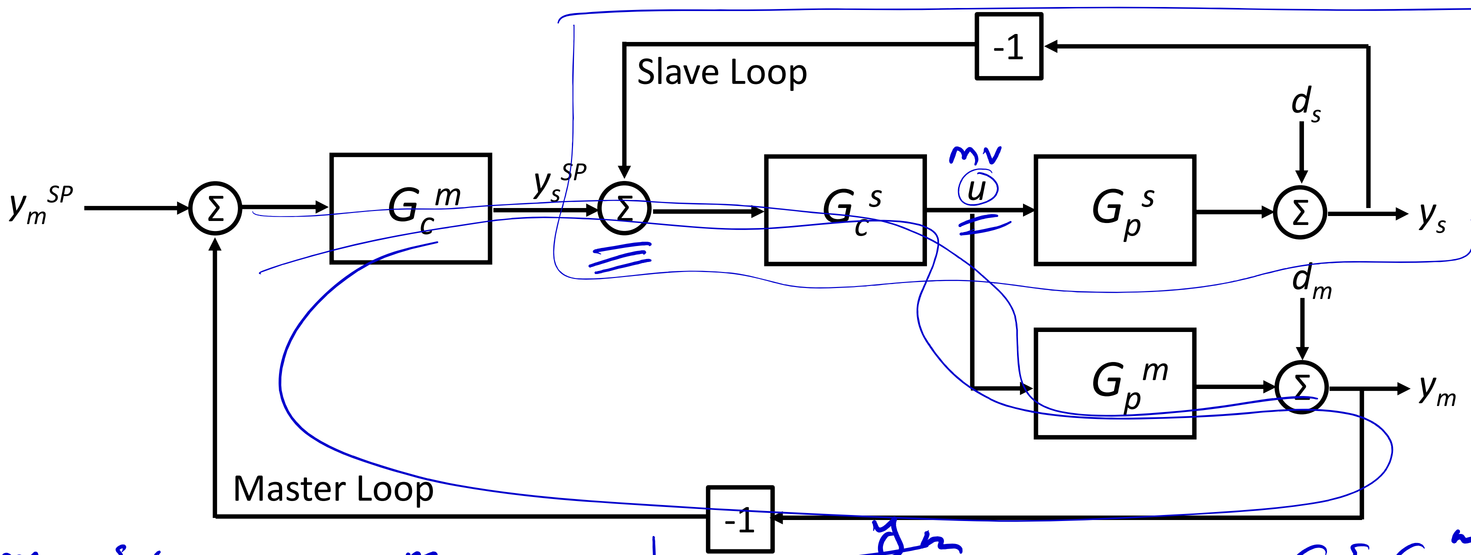


$$\frac{y_m}{y_m^{sp}} = \frac{G_c^m G_c^s G_p^s G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^s G_p^m}$$

$$\frac{y_m}{d_s} = \frac{G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^s G_p^m}$$

$$\frac{y_m}{d_m} = \frac{1}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^s G_p^m}$$

Parallel Cascade



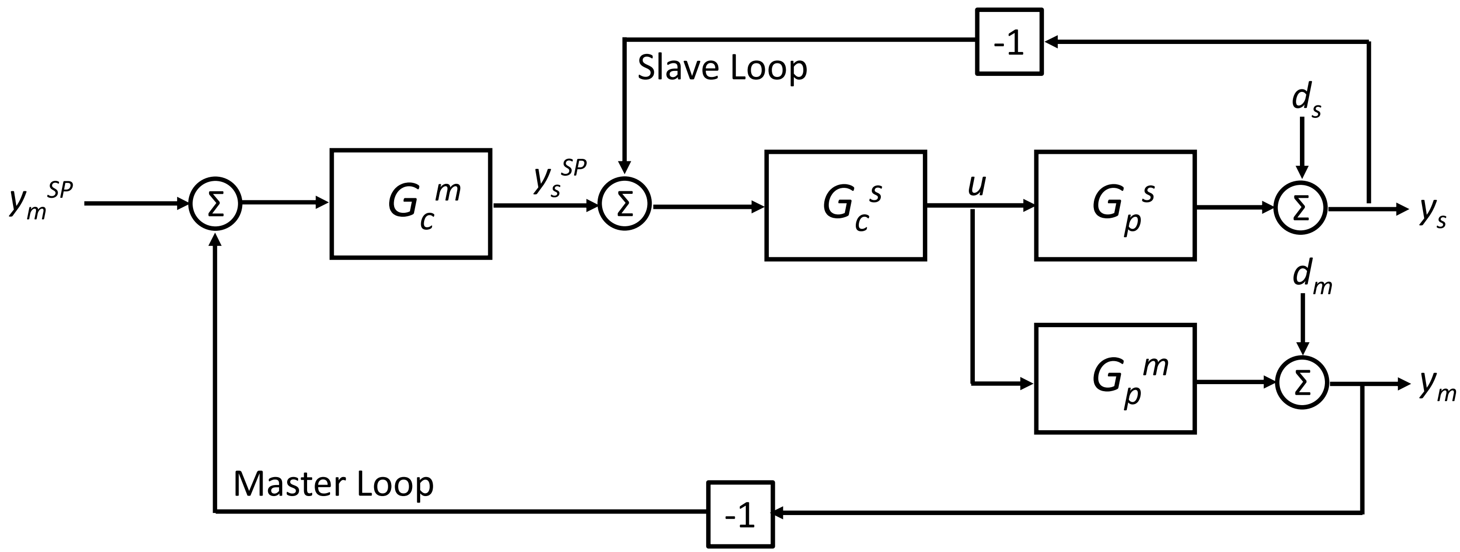
$$\frac{u}{y_s^{sp}} = \frac{G_c^s}{1+G_c^s G_p^s}$$

$$\frac{y_m}{d_s} = \frac{-G_c^s G_p^m}{1+G_c^s G_p^s + G_c^s G_c^m G_p^m}$$

$$\frac{y_m}{y_m^{sp}} = \frac{G_c^m G_c^s (1+G_c^s G_p^s) \cdot G_p^m}{1 + G_c^m G_c^s / (1+G_c^s G_p^s) \cdot G_p^m}$$

$$= \frac{G_c^m G_c^s G_p^m}{1 + G_c^s G_p^s + G_c^s G_c^m G_p^m}$$

Parallel Cascade



$$\frac{y_m}{y_m^{sp}} = \frac{G_c^m G_c^s G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^m} \qquad \frac{y_m}{d_s} = \frac{-G_c^s G_p^m}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^m} \qquad \frac{y_m}{d_m} = \frac{1}{1 + G_c^s G_p^s + G_c^m G_c^s G_p^m}$$

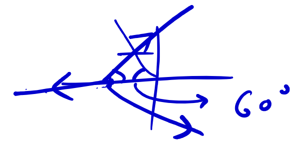
Example

SERIES CASCADE CONTROLLER DESIGN

$$G_p^s = \frac{2}{(s+1)^3} \qquad G_p^m = \frac{1}{(5s+1)^3}$$

Slave controller is P only. Master controller is PI.
Slave and master controllers tuned for $\xi = 0.5$.
Master controller τ_I chosen to cancel dominant open loop pole.

Compare performance with a simple PI controller tuned for $\xi = 0.5$.



$$G_{OL} = \frac{2}{(s+1)^3}$$

$$s^3 + 3s^2 + 3s + 1 + 2K_c = 0 \quad \text{CLCE slave}$$

$$s_{1,2} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j \Rightarrow \xi = 0.5$$

$$s_3 = -2$$

$$K_c = \left| \frac{2}{(s+1)^3} \right|^{-1}_{s=s_1}$$

$$K_c = \frac{1}{2}$$

$$s = -\frac{1}{2} + \frac{\sqrt{3}}{2}j$$

$$\sum p_i = \sum r_i \qquad n - m \geq 2$$

$$s = -a + \sqrt{3}aj$$

$$8a^3 \cancel{7}\times 2a^2[1+\sqrt{3}j] + 3a[-1+\sqrt{3}j] + 1 + 2K_c = 0$$

$$[8a^3 - 6a^2 - 3a + 1 + 2K_c] + 3\sqrt{3}aj[1 - 2a] = 0$$

$$K_c = \frac{1}{2}(3a + 6a^2 - 8a^3 - 1) \qquad a = 1/2$$

$$= \frac{1}{2}\left(\frac{3}{2} + \frac{6}{4} - \frac{8}{8} - 1\right) = \frac{1}{2}$$

Example

Series Cascade Controller Design

$$G_p^s = \frac{2}{(s+1)^3} \qquad G_p^m = \frac{1}{(5s+1)^3}$$

Slave controller is P only. Master controller is PI.
Slave and master controllers tuned for *ξ = 0.5*.
Master controller τ_I chosen to cancel dominant open loop pole.

Compare performance with a simple PI controller tuned for *ξ = 0.5*.

SLAVE CONTROLLER DESIGN

CLCE: $s^3 + 3s^2 + 3s + 1 + 2K_c = 0$

For *ξ = 0.5* (ie *φ* = cos^{-1} *ξ* = 60°), *s = -a + √3aj* satisfies CLCE

$\therefore\ 8a^3 - 6a^2\left(1+\sqrt{3}j\right) + 3a\left(-1+\sqrt{3}j\right) + 1 + 2K_c = 0$

$[8a^3 - 6a^2 - 3a + 1 + 2K_c] + 3\sqrt{3}aj[1-2a] = 0$

$\therefore\ a = 0.5$ and $K_c = 0.5$

Conjugate pair CLCE roots are $s = -1/2 \pm \sqrt{3}/2\,j$

By conservation of roots, third root is $s = -2$

$$\therefore\ G_{CL}^s = \frac{1}{s^3+3s^2+3s+2} = \frac{1}{(s+1/2-\sqrt{3}/2j)(s+1/2+\sqrt{3}/2j)(s+2)}$$

Example$_{\text{continued}}$

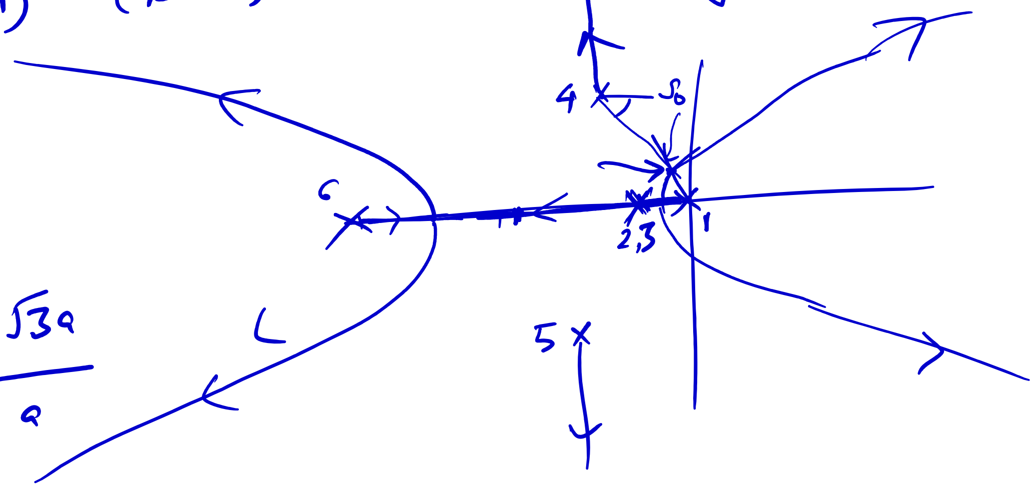
$$G_{CL}^{s} = \frac{1}{s^3+3s^2+3s+2} \qquad \tau_I = 5\,\text{min} \qquad G_c^m = K_c\,\frac{5s+1}{5s}$$

$$G_{OL}^{m} = \frac{\cancel{5s+1}}{5s}\cdot\frac{1}{s^3+3s^2+3s+2}\cdot\frac{1}{(5s+1)^{2}}$$

$s = 0, -0.2$ (twice)

$-\frac{1}{2} \pm \frac{\sqrt{3}}{2}j, \; -2$

$$G_{OL}^{m} = \frac{1}{5s\left(s+\frac{1}{2}-\frac{\sqrt{3}}{2}j\right)\left(s+\frac{1}{2}+\frac{\sqrt{3}}{2}j\right)(5s+1)^2(s+2)}$$



$$\phi_1 = \pi - \tan^{-1}\sqrt{3} \qquad \underline{s_0 = -a + \sqrt{3}a\,j}$$

$$\phi_{2,3} = \tan^{-1}\frac{\sqrt{3}a}{0.2-a}$$

$$\phi_4 = -\tan^{-1}\frac{\frac{\sqrt{3}}{2}-\sqrt{3}a}{\frac{1}{2}-a} \qquad \phi_5 = \tan^{-1}\frac{\frac{\sqrt{3}}{2}+\sqrt{3}a}{\frac{1}{2}-a}$$

$$\phi_6 = \tan^{-1}\frac{\sqrt{3}a}{2-a}$$

$$-\tan^{-1}\sqrt{3} + 2\tan^{-1}\frac{\sqrt{3}a}{0.2-a} - \tan^{-1}\frac{\frac{\sqrt{3}}{2}-\sqrt{3}a}{\frac{1}{2}-a} + \tan^{-1}\frac{\frac{\sqrt{3}}{2}+\sqrt{3}a}{\frac{1}{2}-a} + \tan^{-1}\frac{\sqrt{3}a}{2-a} = 0$$

Solve to get a

Example$_{\text{continued}}$

Once s_0 is known

$$K_c = \left| \frac{1}{G_{OL}} \right|_{s=s_0}$$

$$\Rightarrow \quad K_c = \underline{\underline{\qquad}} \qquad\qquad K_c \qquad \tau_I$$

Example$_{\text{continued}}$

MASTER CONTROLLER DESIGN

Use $\tau_I = 5\text{min}$ $\quad \therefore G_{OL} = \frac{1}{5s(5s+1)^2(s^3+3s^2+3s+2)}$

Open loop roots: $s = -1/2 \pm \sqrt{3}/2\,j,\ -2, -1/5\ \text{(twice)}, 0$

CLCE: $125s^6 + 425s^5 + 530s^4 + 415s^3 + 115s^2 + 10s + K_c = 0$

Let $s = -a + bj$ satisfy CLCE. From root locus, $a < 0.2$

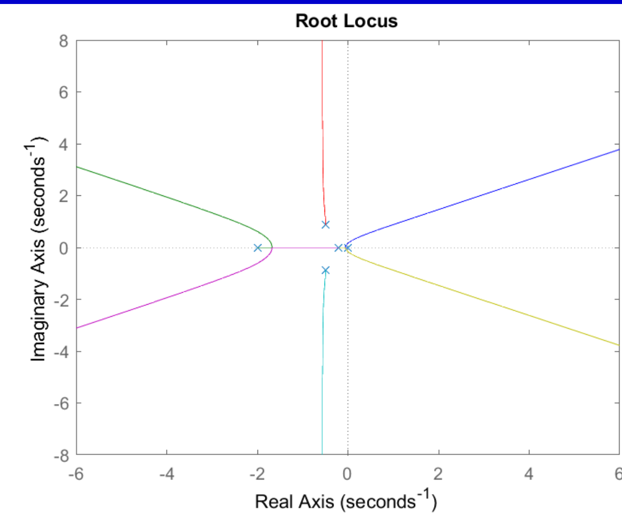
Angle condition must be satisfied for s to lie on root locus

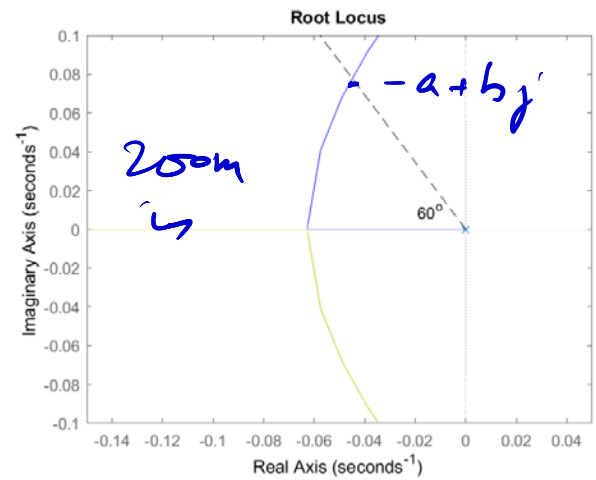
$$-\tan^{-1}\frac{b}{a} + 2\tan^{-1}\frac{b}{0.2-a} + \tan^{-1}\frac{b}{2-a} + \tan^{-1}\frac{\sqrt{3}/2+b}{0.5-a} - \tan^{-1}\frac{\sqrt{3}/2-b}{0.5-a} = 0^\circ$$

Put $b=\sqrt{3}a$ (for $\xi = 0.5$) and solve for a iteratively to get $a = 0.04504$

CLCE dominant root pair is $s_{1,2} = -0.04504 \pm 0.07801j$

By magnitude condition $K_c = |1/G_{OL}|_{s=s_1} = 0.6323$





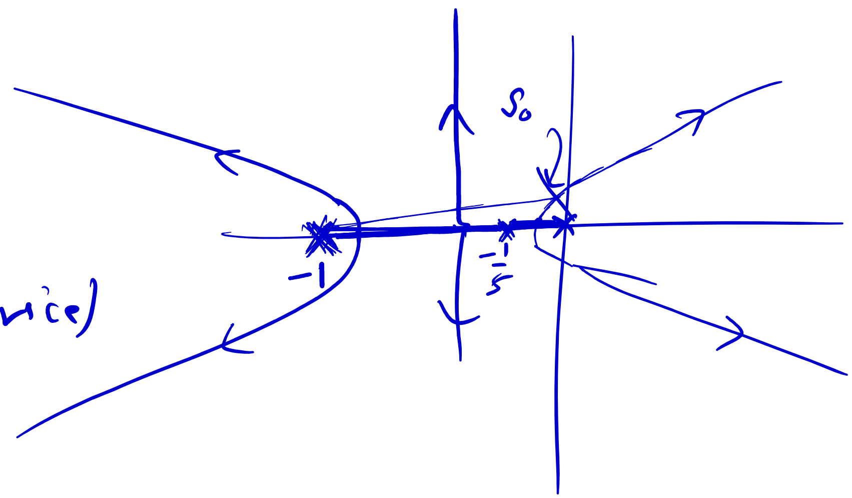
CASCADE CONTROLLER DESIGN

Slave Loop:	K_c = 0.5 (P only)	K_c = 0.1875 τ_I = 1 min
Master Loop:	K_c = 0.6323 τ_I = 5 min	K_c = 0.2815 τ_I = 5 min

Example$_{\text{continued}}$

$$G_{OL} = \frac{2}{(s+1)^3\,(5s+1)^3}\;\frac{(5s+1)}{5s}$$

$$G_{OL} = \frac{2}{5s\,(5s+1)^2\,(s+1)^3}$$



OL poles at $s = 0, -\frac{1}{5}$ (twice), -1 (thrice)

$$\cancel{\pi} - \tan^{-1}\sqrt{3} + 2\tan^{-1}\frac{\sqrt{3}\,a}{0.2-a} + 3\tan^{-1}\frac{\sqrt{3}\,a}{1-a} = \cancel{\pi}\;0$$

$$a = ?$$

Example$_{\text{continued}}$

$s_o = -a + \sqrt{3}aj$ lies on root locus

$$K_c = \left| \frac{1}{G_{OL}} \right|_{s=s_o}$$

$K_c =$

PI

$\tau_I = 5$ min

$K_c =$ ______

Example$_{\text{continued}}$

FEEDBACK CONTROLLER DESIGN

Use $\tau_I = 5\text{min}$ $\quad \therefore G_{OL} = \frac{2}{5s(5s+1)^2(s+1)^3}$

Open loop roots: $s = -1$ (thrice), $-1/5$ (twice), 0

CLCE: $125s^6 + 425s^5 + 530s^4 + 290s^3 + 65s^2 + 5s + 2K_c = 0$

Let *s = -a + bj* satisfy CLCE. From root locus, *a < 0.2*

Angle condition must be satisfied for *s* to lie on root locus

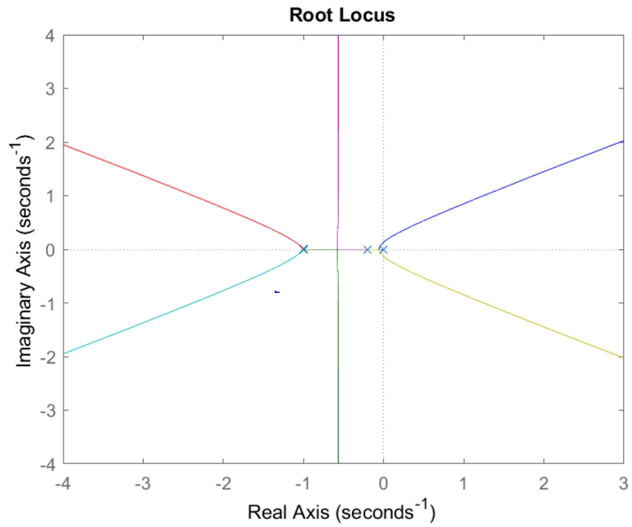
$$-\tan^{-1}\frac{b}{a} + 2\tan^{-1}\frac{b}{0.2-a} + 3\tan^{-1}\frac{b}{1-a} = 0^\circ$$

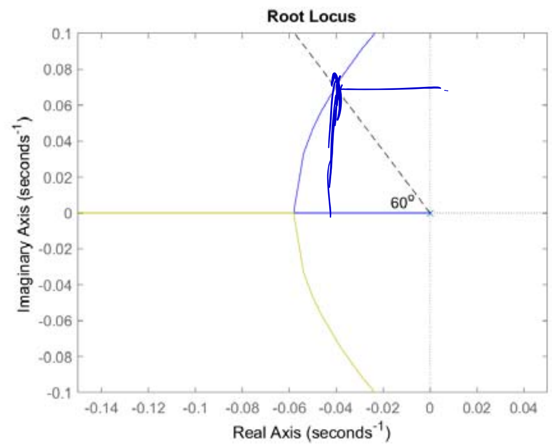
$s_o = -a + bj$

Put *b=√3a* (for *ξ = 0.5*) and solve for *a* iteratively to get $a = 0.04049$

CLCE dominant root pair is $s_{1,2} = -0.04049 \pm 0.07012j$

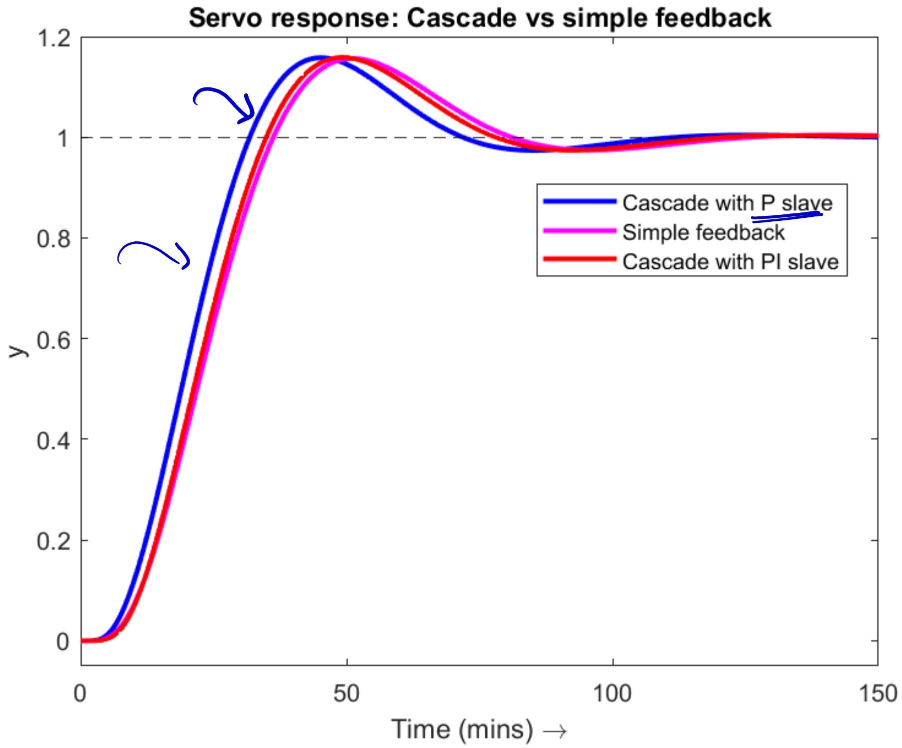
By magnitude condition $K_c = |1/G_{OL}|_{s=s_1} = 0.1368$





Example: Dynamic Results

Servo



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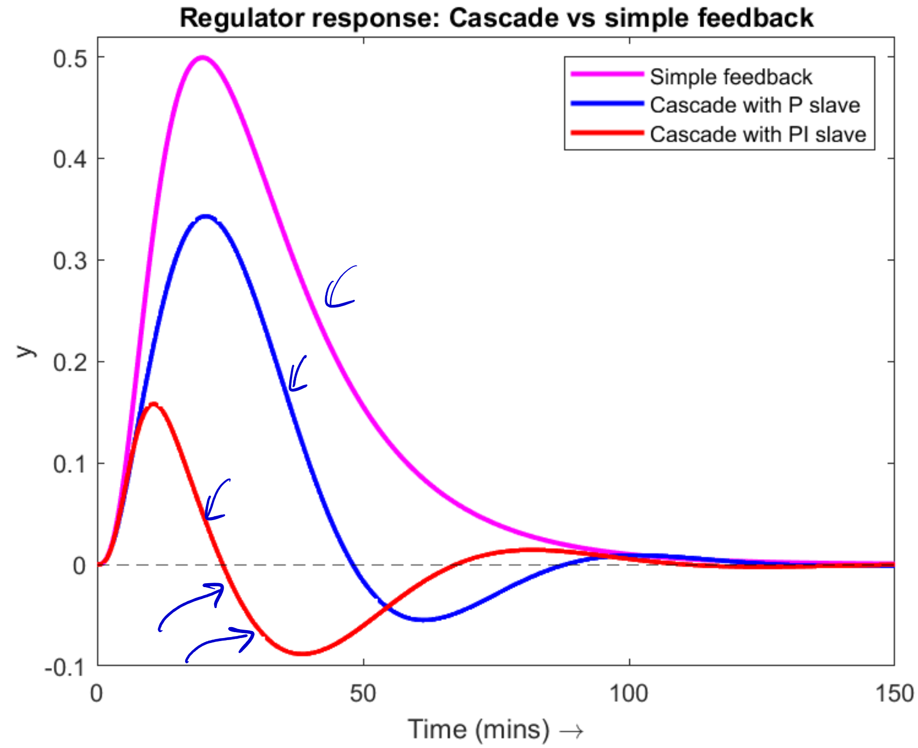

Summary

- Cascade control
  - Used when controlling a secondary (fast) PV helps regulate a primary (slow) PV
  - Set point of slave (fast) loop remotely set by master (slow) loop
  - Inside-out tuning
- Slave loop
  - Removes local disturbances
  - Removes local non-linearities
    - Flow controller mitigates valve non-linear characteristic
  - Somewhat speeds up the master loop response
- Slave loop dynamic speed >> Master loop dynamic speed
- Very commonly employed in process industry due to time scale separation