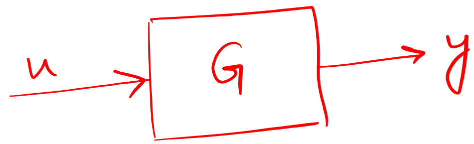
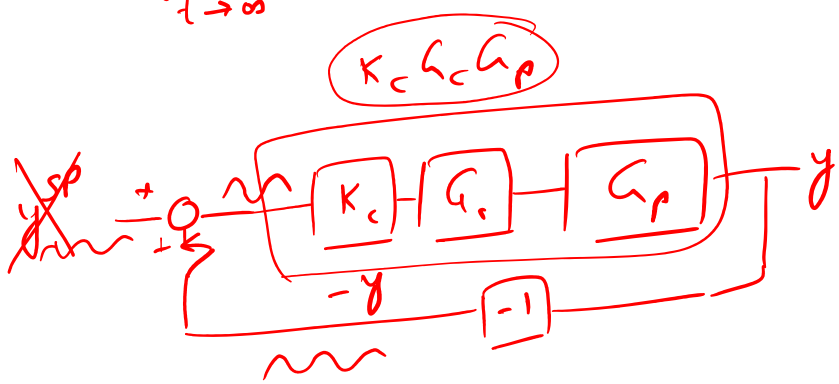


$u = a \sin \omega t$

$AR_{(\omega)} = \frac{b}{a} = |G_{j\omega}|$

$y_{t \to \infty} = b \sin(\omega t + \phi)$

$\phi = \angle G_{j\omega}$



$\underline{\underline{\omega_c}}$ for which $\underline{\underline{\phi = -180^\circ}}$

Then $-y$ is in phase with

$y^{sp} = \sin \omega_c t$

Adjusted $\underline{\underline{K_c}}$ s.t.

amplitude of $(-y)$ sin wave

is same as of y^{sp} sinwave

$\omega_c = \omega$ at which $\phi_{G_{OL}} = -180°$

If $AR_{\omega_c} = 1$ sustained oscillations

> 1 oscillations that blow up

< 1 oscillations that die out / decay

If $\underline{\underline{AR_{G_{OL}}}}(\omega = \omega_c) < 1$ at ω_c

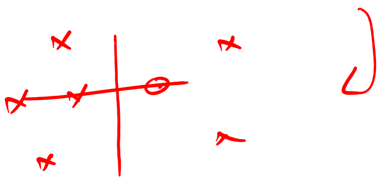
then closed loop system is stable

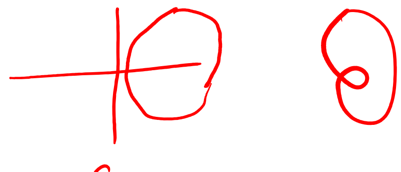
BODE STABILITY CRITERION.

ω_c – phase crossover frequency

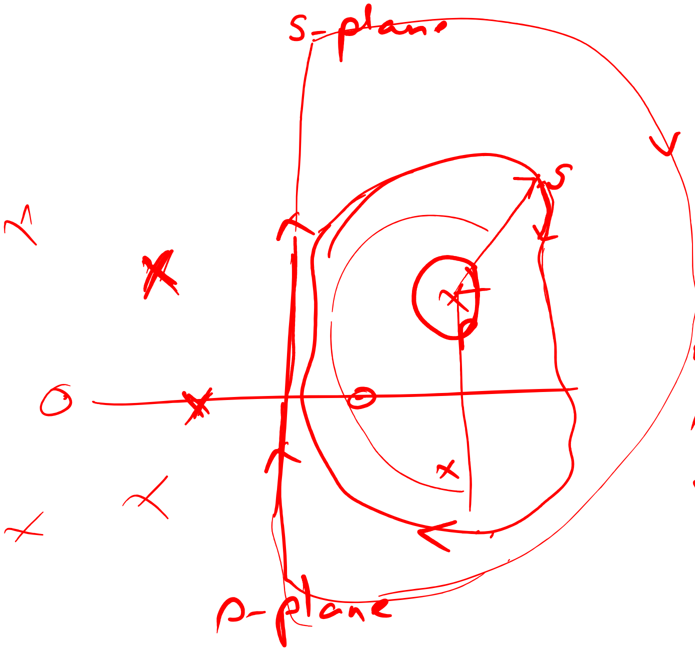
$\boxed{1+G_{OL}=0}$ CLCE

Does CLCE have any RHP roots





$$G_{OL} = K\frac{\prod_{j=1}^{m}(s-z_j)}{\prod_{i=1}^{n}(s-p_i)} \qquad m \le n$$

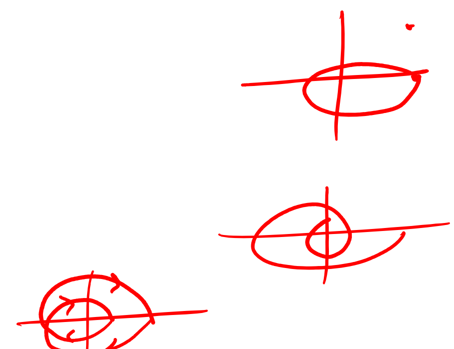


Does G encircle origin? How many times?

(zero)

A pole inside the contour sweeps $+2\pi$ (-2π) angle in the G plane as s moves around the contour once

* P or Z outside contour sweep zero angle as s moves around closed contour once.

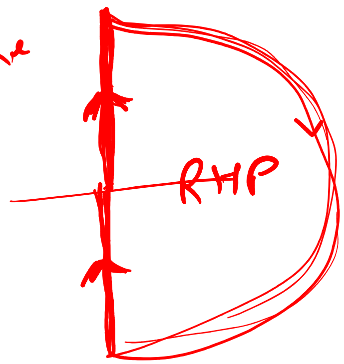


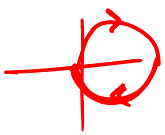
# of clockwise Encirclements of origin in G plane = [# Z − # P] inside closed contour of s

$1 + G_{OL} = 0$

s-plane

$\frac{1}{\tau s+1}$





G-plane

$G = \boxed{\frac{1}{\tau s+1}}$

$s = Re^{j\theta}$

$\theta = +\frac{\pi}{2}$ to $-\frac{\pi}{2}$

$G^{OL}_{s=j\omega}$

$1 + G_{OL}$

G_{OL} $\boxed{G_{OL}}$

$m > n$

$G = \frac{1}{\tau Re^{j\theta}+1}$

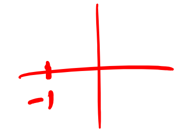
$R \to \infty \quad |G| \to 0$

$1 + \frac{N_s}{D_s} = \frac{D_s + N_s}{D_s}$

origin

$G_{OL} = \frac{N_s}{D_s}$

$(-1, 0)$



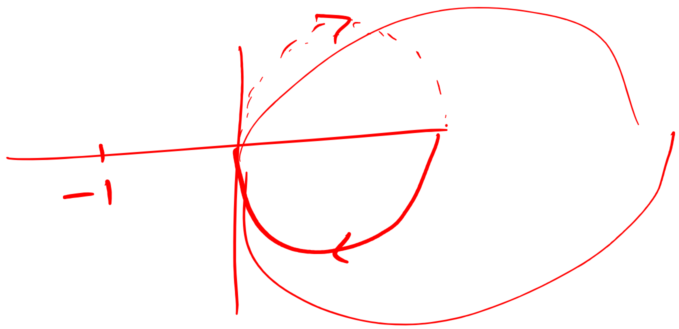
# of clockwise encircle-ments of $(-1,0)$ by G_{OL} = [# of ~~roots~~ zeros of $1+G_{OL}$ − # poles of G_{OL} in RHP]

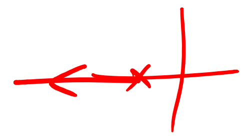
0 for G_{OL} stable

If G_{OL} is stable

# of encirclements of $(-1,0)$ by G_{OL} = # of CLCE roots in RHP.

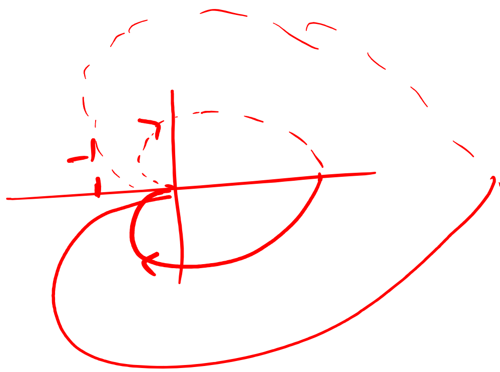
$\boxed{G_{OL,j\omega}}$ = Nyquist plot of G_{OL}

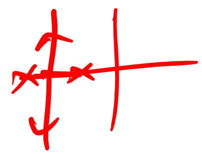




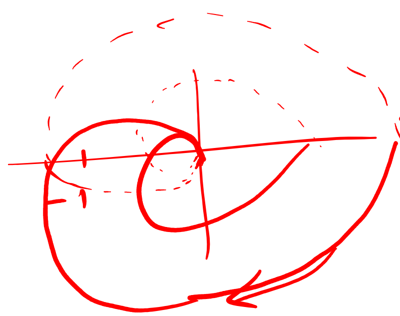
1st order never goes unstable (under P control)

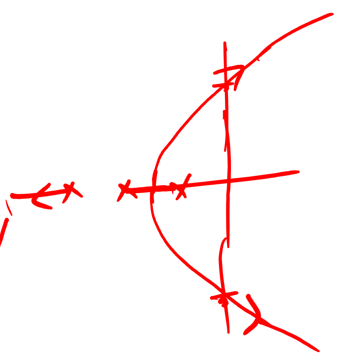
2nd order system under P control never goes unstable

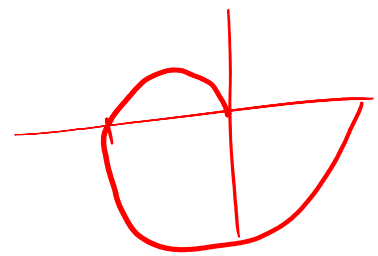


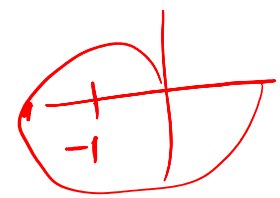


$\boxed{1 + G_{OL}}$







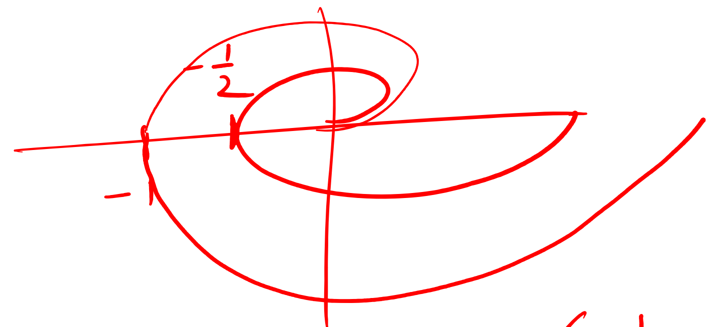


$\boxed{(-1, 0)}$

Tune controller st G_{OL} nyquist plot remains sufficiently away from the critical point $(-1, 0)$

1. GAIN MARGIN

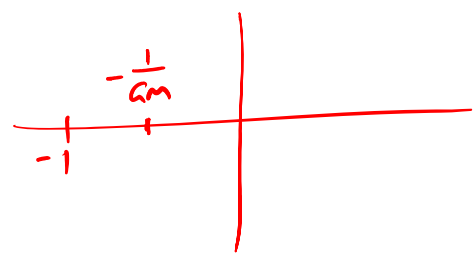
2. PHASE MARGIN

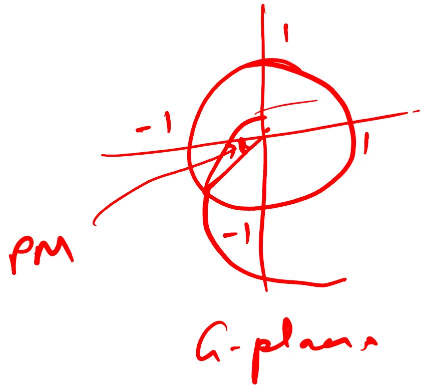


G $\quad (-\frac{1}{GM}, 0)$

$$AR_{\omega_c} = \frac{1}{GM}$$

Eg $\underline{GM = 2} \Rightarrow K_c$ chosen s.t





Phase is independent of K_c

PM 45°

$\omega = ?$ $\underline{\underline{\phi}} = -(180° - PM°)$

$\angle G_{j\omega} = \phi \leftarrow$

$$G_{p_s} = \frac{2e^{-s}}{5s+1}$$

$$G_c = K_c$$

$$G_{OL_s} = \frac{2K_c e^{-s}}{5s+1}$$

$$G_{OL_{j\omega}} = \frac{2K_c e^{-j\omega}}{5\omega j + 1}$$

$$\angle G_{OL} = -\omega - \tan^{-1} 5\omega$$

Find K_c for $GM = 2$

$$-\omega_c - \tan^{-1}\omega_c = -180^\circ$$

$$\omega_c = 1.69 \text{ rad/min}$$

$$\left| G_{OL_{j\omega}} \right|_{\omega_c} = \frac{1}{GM} = \frac{1}{2}$$

$$\frac{2K_c \cdot 1}{\sqrt{25\omega_c^2 + 1}} = \frac{1}{2}$$

$$K_c = \frac{\sqrt{25\omega_c^2 + 1}}{4} = \underline{2.123}$$

PM: $\underline{30^\circ - 50^\circ}$

$K_c = ?$

$PM = 45^\circ$

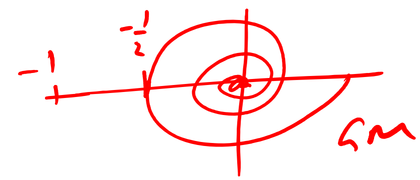
$\phi = -[180^\circ + PM]$

$= -135^\circ$

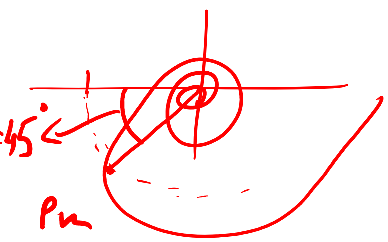
$-\frac{3\pi}{4}$ ✓

$$-\omega^* - \tan^{-1} 5\omega^* = -\frac{3\pi}{4}$$

$$\omega^* = 0.99 \text{ rad/min}$$



$$|G_{OL}(j\omega)|_{\omega^*} = 1 \Rightarrow \frac{2K_c \cdot 1}{\sqrt{25\omega^{*2}+1}} = 1$$



$$\Rightarrow K_c = \frac{\sqrt{25\omega^{*2}+1}}{2} \Rightarrow \underline{K_c = 2.51}$$

$$G_p = \frac{1}{(s+1)^3} \qquad \underline{\underline{G_c = K_c}}$$

$$GM = 1$$

$$G_{OL} = \frac{K_c}{(s+1)^3}$$

$$\angle G_{OL} = -3\tan^{-1}\omega$$

$$G_{OL_{j\omega}} = \frac{K_c}{(j\omega+1)^3}$$

$$-3\tan^{-1}\omega_c = -180^\circ$$

$$\tan^{-1}\omega_c = 60^\circ$$

$$\omega_c = \sqrt{3} \text{ rad/min}$$

$$\left| G_{OL_{j\omega}} \right|_{\omega_c} = 1$$

$$\Rightarrow \frac{K_c}{\left[\sqrt{\omega^2+1}\right]^3} = 1 \Rightarrow K_c = (\omega^2+1)^{3/2}$$

$$K_c = 8 \leftarrow \text{ultimate gain}$$

$$G_p = \frac{1}{(s-1)\left(\frac{1}{10}s+1\right)^2}$$

$$G_{OL} = \frac{K_c}{(s-1)\left(\frac{1}{10}s+1\right)^2}$$

$$G_{OL_{j\omega}} = \frac{K_c}{(j\omega-1)\left(\frac{\omega}{10}j+1\right)^2}$$

$$\angle G_{OL} = -2\tan^{-1}\frac{\omega}{10} - \pi + \tan^{-1}\omega$$

$$\angle G_{OL} = -\pi + \tan^{-1}\omega - 2\tan^{-1}\frac{\omega}{10}$$

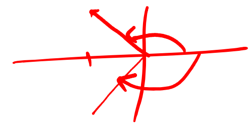
$\omega \to 0$ $\quad \angle G_{OL} = -180^\circ$

$\omega \to \infty$ $\quad \angle G_{OL} = -270^\circ \quad |G_{OL}| \to 0$

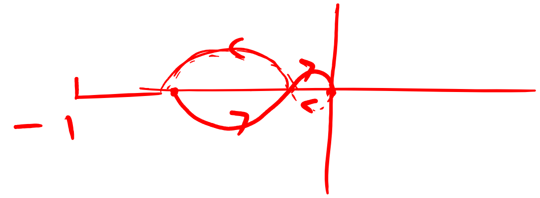
$G_c = K_c$

$\# O = \# Z - \# P$

I $\quad 0 = \# Z - 1 \Rightarrow \underline{\underline{\# Z = 1}}$

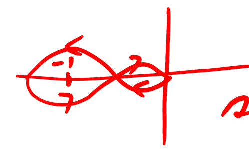


unstable



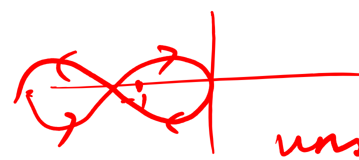
I

II $\quad -1 = \# Z - 1 \qquad \# Z = 0$



stable

II



unstable

III

III $\quad 1 = \# Z - 1$

$\# Z = \underline{\underline{2}}$