SHUBHAM GUPTA

180749

Theo Ass - 1

5) a) $\min(n^2, 10^{12}) = O(1)$

for $n > 10^6$ $\quad 10^{12} < n^2$

we see asymptotically large values

$\therefore \min(n^2, 10^{12}) = 10^{12}$

to proove $10^{12} = O(1)$

$\frac{f(n)}{g(n)} = \frac{10^{12}}{1} = 10^{12}$

$\therefore 10^{12}$ is $O(1)$ as $10^{12} \le 10^{12}$

HP

b) $n^2 + n \log n = O(n^2)$

$\frac{f(n)}{g(n)} = \frac{n^2 + n \log n}{n^2} < \frac{n^2 + n^2}{n^2} = 2$

$\therefore \log n < n$

let $c = 2$

$\therefore n^2 + n \log n$ is $O(n^2)$ since

$n^2 + n \log n \le 2n^2$

HP

c) $n^3 + 3n^2 + 8 \neq O(n^2)$

$\frac{f(n)}{g(n)} = \frac{n^3 + 3n^2 + 8}{n^2} > \frac{n^3 + 3n^2}{n^2} = (n+3)$

now $n + 3 > C$

$f(n) > C n^2$

$\therefore \boxed{f(n) \neq O(n^2)}$

HP

d) $4^n \neq O(2^n)$

$$\frac{f(n)}{g(n)} = \frac{4^n}{2^n} = 2^n$$

since, $\frac{f(n)}{g(n)} \neq$ const.

$\therefore \quad 4^n \neq O(2^n)$

e) $\log(n!) = O(n \log n)$

$$n! = n(n-1) \dots 1$$

$$n! = n\left(1-\frac{0}{n}\right) n\cdot\left(1-\frac{1}{n}\right) n\left(1-\frac{2}{n}\right) \dots n\left(1-\frac{n-1}{n}\right)$$

$$n! = n^n \prod_{k=0}^{n-1}\left(1-\frac{k}{n}\right)$$

take log both sides

$$\log n! = n\log(n) + \log\left[\prod_{k=0}^{n-1}\left\{\left(1-\frac{k}{n}\right)\right\}\right]$$

$$\log n! \leq n \log n$$

$$\therefore \log(n!) = O(n \log n)$$