Module 4.8.1

Advanced Controller Structures
Feedforward and Ratio Control

Lectures on

CHEMICAL PROCESS CONTROL
Theory and Practice

Feedforward Control

Adjust MV to counter ‘expected’ effect of measured disturbance

Gain + Lead-lag + deadtime

$e^{+\theta s}$ unrealizable

$$G_{ff} = -\frac{\hat{G}_d}{\hat{G}_p}$$

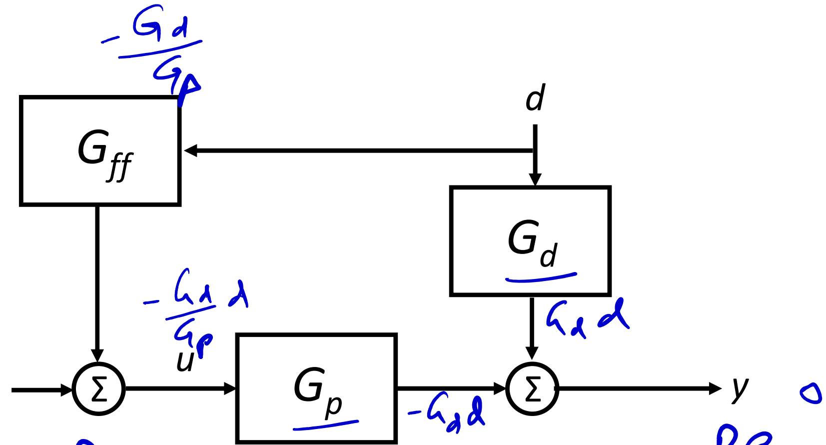
$$\hat{G}_p = \frac{K_p e^{-\theta_p s}}{\tau_p s+1} \qquad G_d = \frac{K_d e^{-\theta_d s}}{\tau_d s+1}$$

$$G_{ff} = -\frac{K_d}{K_p} \frac{\tau_p s+1}{\tau_d s+1} e^{-(\theta_d-\theta_p)s}$$

Gain lead-lag dead-time

-ve if $\theta_p > \theta_d$

from feedback controlller



$e^{+\theta_p s}$ ← unrealizable

$e^{-\theta_p s}$

Example

$$G_p = \frac{2}{(s+1)^3} \qquad G_d = \frac{1}{(4s+1)^2(2s+1)}$$

Fit FOPDT models to G_p and G_d
Obtain feedforward compensator
Use analytical methods only (no simulation)

$$y_u = 2\left[\frac{1}{s(s+1)^3}\right] = 2f$$

$$= 2\left[\frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2} + \frac{D}{(s+1)^3}\right]$$

$$A = [sf]_{s=0} \Rightarrow A = 1$$

$$D = [(s+1)^3 f]_{s=-1} \Rightarrow D = -1$$

$$C = \left[\frac{d(s+1)^3 f}{ds}\right]_{s=-1} \Rightarrow C = -1$$

$$B = \frac{1}{2}\frac{d^2(s+1)^3 f}{ds^2}\Big|_{s=-1} = \frac{1}{2}\frac{2}{s^3}\Big|_{s=-1} = -1$$

$$\mathcal{L}^{-1}\left[\frac{1}{(s+1)^n}\right] = \frac{1}{(n-1)!}t^{n-1}e^{-t}$$

$$y_d = \frac{1}{s(2s+1)(4s+1)^2}$$

$$(s+1)^3 f = \frac{A}{s}(s+1)^3 + B(s+1)^2 + C(s+1) + D$$

$$\frac{d}{ds}(s+1)^3 f = A\left[\frac{-(s+1)^3}{s^2} + \frac{3(s+1)^2}{s}\right] + 2B(s+1) + C + 0$$

$$\frac{d^2}{ds^2}(s+1)^3 f\Big|_{s=-1} = A(s+1)[\quad] + 2B\Big|_{s=-1} \Rightarrow B = \frac{1}{2}\frac{d^2(s+1)^3 f}{ds^2}\Big|_{s=-1}$$

$$y_u = 2\left[\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3}\right]$$

$$y_u = 2\left[1 - e^{-t} - te^{-t} - \frac{1}{2}t^2 e^{-t}\right]$$

0.2835
0.5620
1.264

$t_{63.2} = 3.26$ min

$t_{28.3} = 1.85$ min

$$\tau = \frac{3}{2}[t_{63.2} - t_{28.3}] \Rightarrow \tau = 2.11 \text{ min}, \quad \theta = 1.15 \text{ min}$$

$$\theta = \frac{1}{2}[3t_{28.3} - t_{63.2}]$$

$$\hat{G}_p = \frac{2e^{-1.15s}}{2.11s+1}$$

Example

Example

$$G_p = \frac{2}{(s+1)^3} \qquad G_d = \frac{1}{(4s+1)^2(2s+1)}$$

Fit FOPDT models to G_p and G_d
Obtain feedforward compensator
Use analytical methods only (no simulation)

Unit step response to *u* (using partial fractions)

$$y_u = \frac{2}{s(s+1)^3} = 2\left[\frac{1}{s} - \frac{1}{s+1} - \frac{1}{(s+1)^2} - \frac{1}{(s+1)^3}\right]$$

Inverting to time domain gives

$$y_u = 2\left[1 - e^{-t} - te^{-t} - \frac{1}{2}t^2e^{-t}\right]$$

Obtain $t_{28.3}$ and $t_{63.2}$ iteratively

$$t_{28.3} = 1.85 \text{ min} \qquad t_{63.2} = 3.26 \text{ min}$$

$$\hat{G}_p = \frac{2e^{-1.15s}}{2.11s+1}$$

Unit step response to *d* (using partial fractions)

$$y_d = \frac{1}{s(4s+1)^2(2s+1)} = \frac{1}{s} - \frac{1}{s+1/2} - \frac{1/2}{(s+1/4)^2}$$

Inverting to time domain gives

$$y_d = 1 - e^{-t/2} - \frac{1}{2}te^{-t/4}$$

Obtain $t_{28.3}$ and $t_{63.2}$ iteratively

$$t_{28.3} = 6.03 \text{ min} \qquad t_{63.2} = 10.79 \text{ min}$$

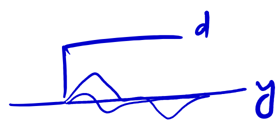
$$\hat{G}_d = \frac{e^{-3.65s}}{7.14s+1}$$

$$\boxed{G_{ff} = -\frac{\hat{G}_d}{\hat{G}_p} = \frac{1}{2}\left(\frac{2.11s+1}{7.14s+1}\right)e^{-2.5s}}$$

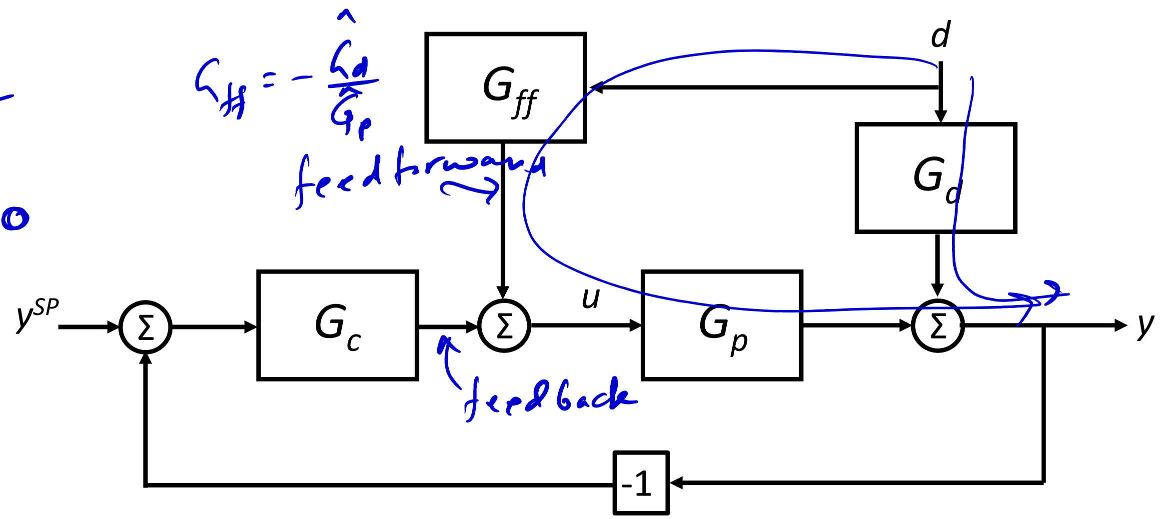
Feedforward-Feedback Control

$$\frac{y}{y^{sp}} = \frac{G_c G_p}{1 + G_c G_p}$$

$$\frac{y}{d} = \frac{G_d - \frac{\hat{G}_d}{\hat{G}_p} G_p}{1 + G_c G_p} \approx 0$$



$$G_{ff} = -\frac{\hat{G}_d}{\hat{G}_p}$$



$$\frac{y}{y^{SP}} = \frac{G_c G_p}{1 + G_c G_p} \qquad \frac{y}{d} = \frac{G_d - \frac{\hat{G}_d}{\hat{G}_p} G_p}{1 + G_c G_p}$$

Tune feedback loop using standard methods

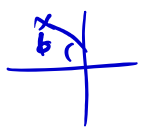
Example

$$G_p = \frac{2}{(s+1)^3} \qquad G_d = \frac{1}{(4s+1)^2(2s+1)}$$

Tune PID controller with τ_I and τ_D chosen to cancel appropriate open loop poles and K_C chosen for $\xi = 0.5$

$$G_c = K_c \frac{(s+1)}{s} \frac{(s+1)}{(0.1s+1)}$$

$$G_c G_p = \frac{2K_c}{s(s+1)(0.1s+1)}$$



CLCE $\quad 1 + G_c G_p = 0 \Rightarrow s(s+1)(0.1s+1) + 2K_c = 0$

$\Rightarrow 0.1s^3 + 1.1s^2 + s + 2K_c = 0 \quad$ — (1)

$\xi = 0.5 \Rightarrow \phi = \cos^{-1}\xi = 60^\circ \Rightarrow s = -a + \sqrt{3}aj$ satisfies CLCE

$$0.1 \times 8a^3 + 1.1[-2 - 2\sqrt{3}j]a^2 - a + \sqrt{3}aj + 2K_c = 0$$

$$\underbrace{[0.8a^3 - 2.2a^2 - a + 2K_c]}_{0} + \sqrt{3}aj\,\underbrace{[1 - 2.2a]}_{0} = 0$$

$$a = \frac{1}{2.2} = 0.4545$$

$$K_c = \frac{1}{2}[a + 2.2a^2 - 0.8a^3]$$

$\Rightarrow K_c = 0.417$

$K_c = 0.417$

$\tau_I = 1$ min

$\tau_D = 1$ min

Example

$$G_p = \frac{2}{(s+1)^3} \qquad G_d = \frac{1}{(4s+1)^2(2s+1)}$$

Tune PID controller with τ_I and τ_D chosen to cancel appropriate open loop poles and K_c chosen for $\xi = 0.5$

Choose τ_I = 1 min and τ_D = 1 min to cancel open loop poles at $s = -1$

$$G_c = K_c\left(\frac{s+1}{s}\right)\left(\frac{s+1}{0.1s+1}\right) \qquad G_cG_p = \frac{2K_c}{s(s+1)(0.1s+1)}$$

Closed loop characteristic equation then becomes

$$0.1s^3 + 1.1s^2 + s + 2K_c = 0$$

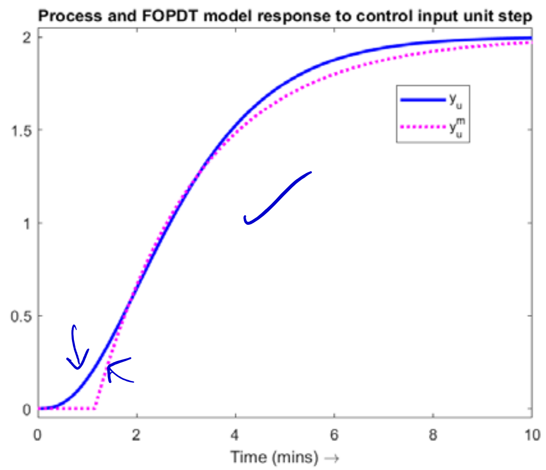
For $\xi = 0.5$, $s = -a + \sqrt{3}aj$ must satisfy CLCE. Substituting and collecting real and imaginary terms

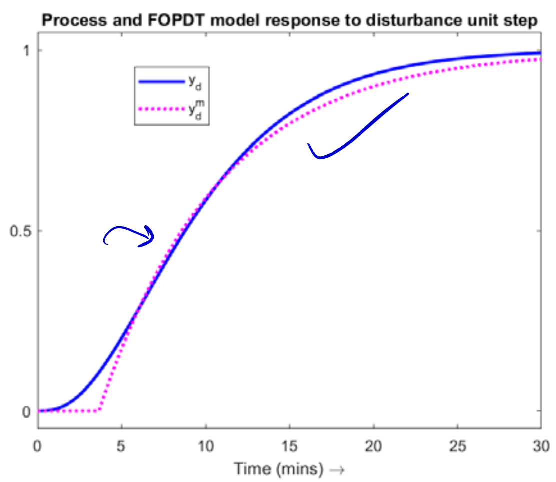
$$[0.8a^3 - 2.2a^2 - a + 2K_c] + \sqrt{3}aj[1 - 2.2a] = 0$$

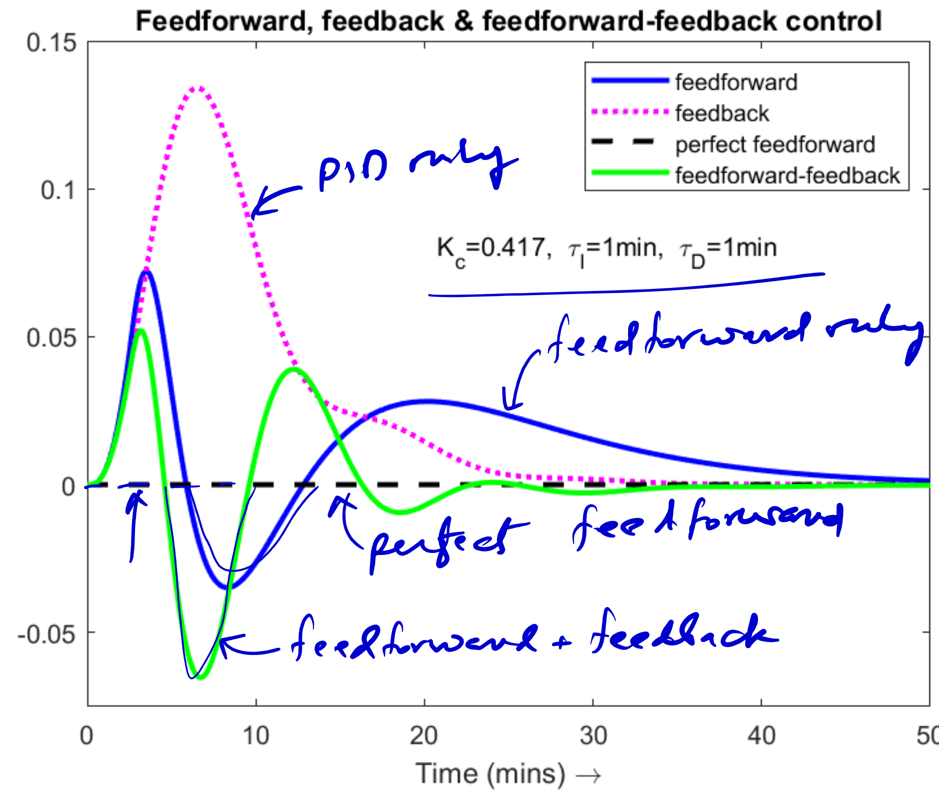
Thus, $a = 0.4545$ and $s = -0.4545 + 0.7873j$ is the CLCE root

$$K_c = \frac{1}{2}|s(s+1)(0.1s+1)|_{s=-0.4545+0.7873j} = 0.417$$

Dynamic Results







$$G_p = \frac{2}{(s+1)^3}$$

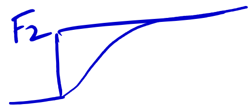
$$G_d = \frac{1}{(4s+1)^2(2s+1)}$$

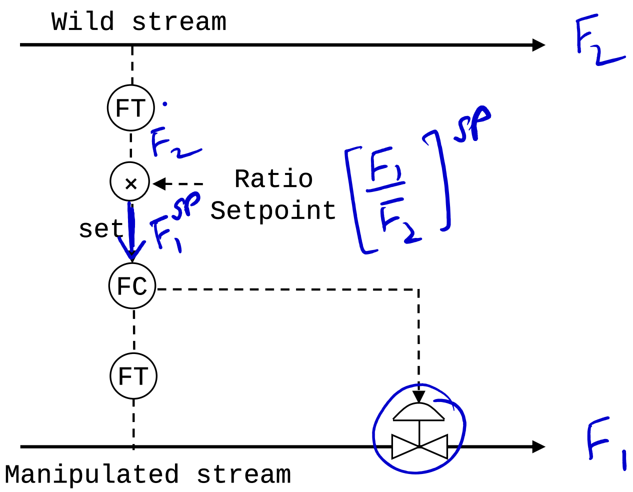
$$-\frac{G_d}{G_p} = -\frac{1}{2}\frac{(s+1)^3}{(4s+1)^2(2s+1)}$$

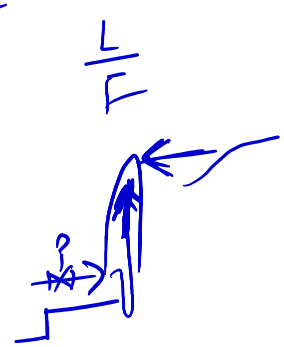
Need highly accurate models for feedforward to be better than simple feedback

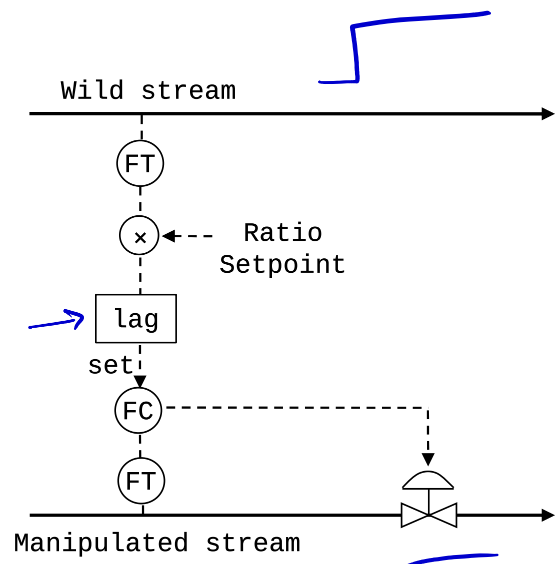
Ratio Control

Maintains a process stream in ratio with another (wild) stream

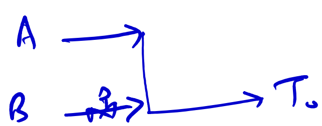


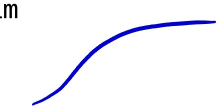






$A + B \rightarrow C$





Summary

- Feedforward control
  - Adjusts MV to counteract 'expected' effect of a **measured** disturbance

- $G_{ff} = -\frac{\hat{G}_d}{\hat{G}_p}$ ✓
  - Requires model of disturbance and MV effect on output ($\hat{G}_d$ and $\hat{G}_p$)

- Feedforward effective only with high fidelity models ($\hat{G}_d$ and $\hat{G}_p$)
  - Otherwise simple feedback outperforms feedforward

- Ratio control
  - Maintains a process stream in ratio with a 'wild' stream
  - Used for moving flows in tandem