Ch. 14: Basic Options Theory

IME 611

Options

- A right, not obligation to buy (sell) an asset under specified terms
  - Call option: the right to purchase at a specified price and date
  - Put option: the right to sell
  - An option itself has a price; referred as the option premium
  - Example: pay (premium) ₹15,000 for the option to purchase the house at ₹200,000
  - Holder buys or sells the asset according to the terms of the option - Exercise

Options

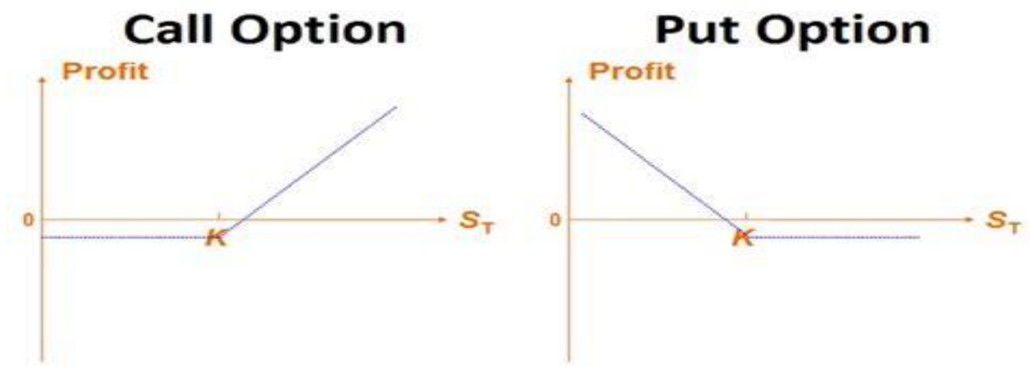
- Specifications of options
  - What can be bought or sold (underlying asset)
  - Call or put
  - Strike price – price at which the asset can be purchased upon exercise of the option
  - Price of the option – premium
  - The period to time for which the option is valid
    - European option: allows exercise only on the expiration date
    - American option: allows exercise at any time before and including the expiration date

Mechanisms of option trading

- Writing: the party that grants the option
  - The purchasing party faces no risk, except losing premium
- Options on many stocks are traded on an exchange through brokers
- An option writer must maintain a margin account with exchange
- In general, options are rarely exercised, with the underlying security being bought or sold. Instead, if the price of the security moves in a favorable direction, the option price will increase accordingly, and most option holders choose to sell their options

Option values

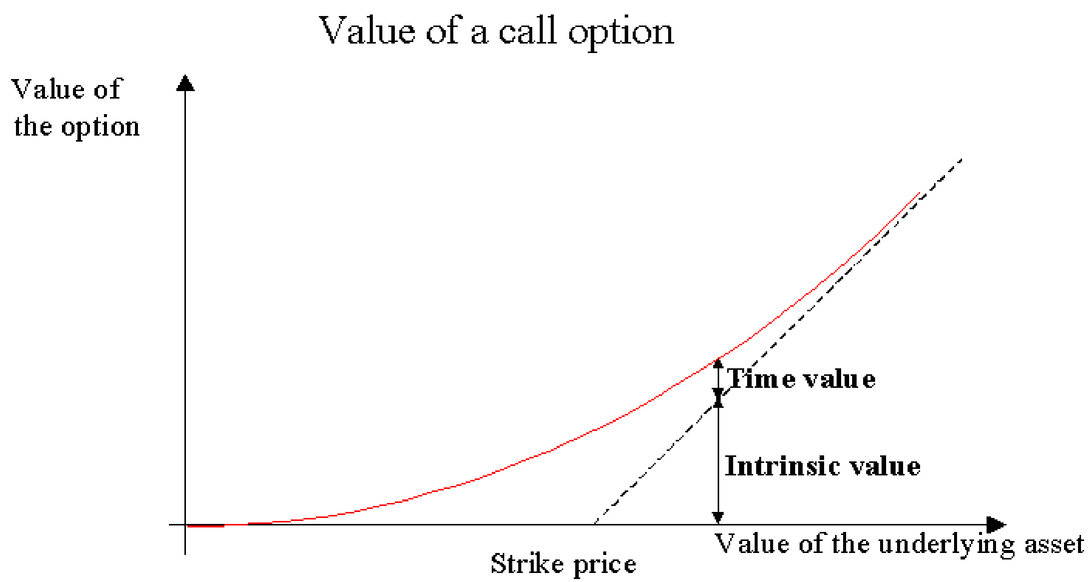
- Call option with strike price K
  - On the expiration date the price of the underlying stock is S
  - Value of option at expiration, $C = \max(0, S - K)$
- Put option with strike price K
  - Right to sell an asset at a given strike price
  - Value of option at expiration, $P = \max(0, K - S)$



Option values..

- A call option is in the money, at the money, or out of the money: $S > K, S = K$ or $S < K$
- Puts have the reverse terminology
- Time value of Options

- Intrinsic = value at expiration
- Volatility, interest rate



Combination of Options

- Collar: limits both gains and losses
- Butterfly
  - Long two calls with strike price K_1 and K_3, and short two units of a call with strike price K_2 $(K_1 < K_2 < K_3)$

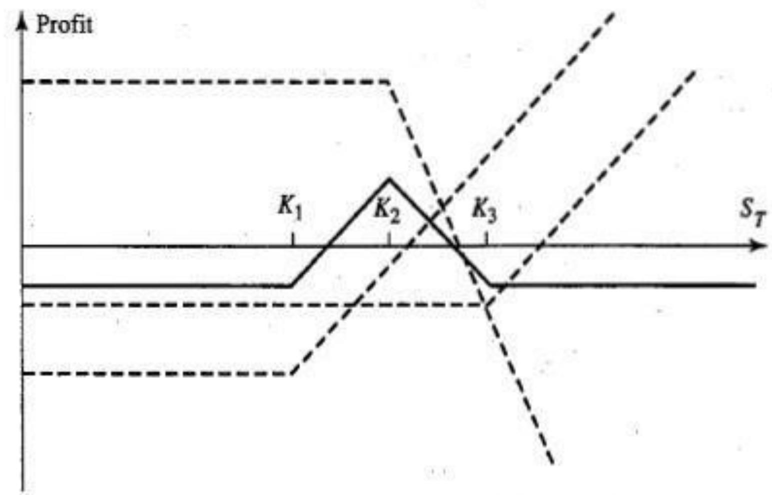
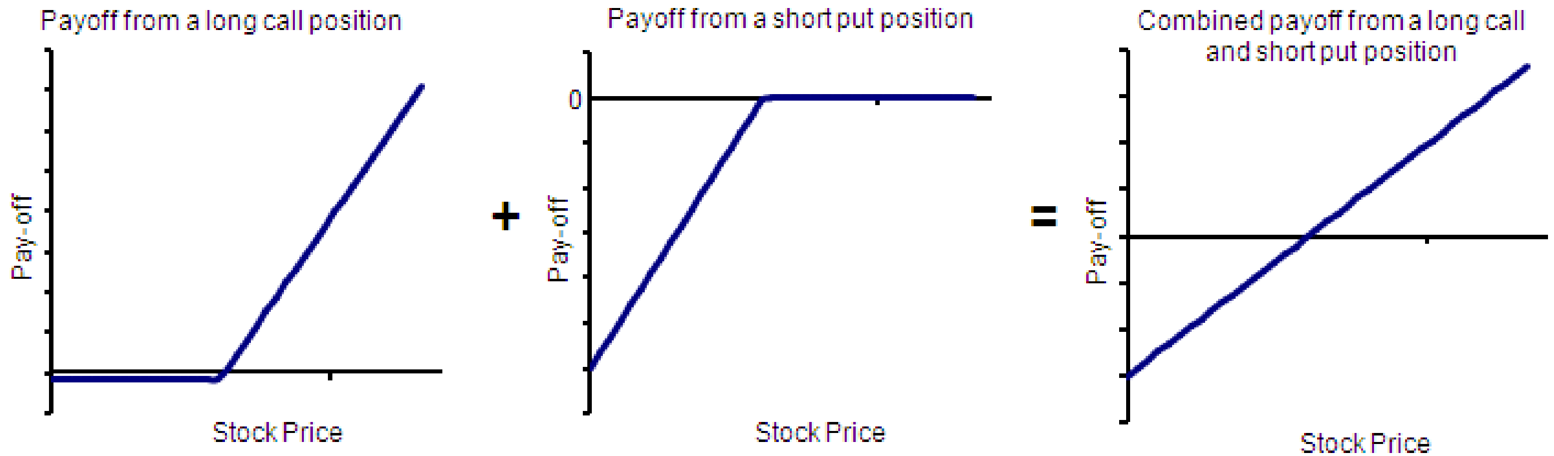


Figure 4 Profit from butterfly spread using call options.

  - Usually K_2 is close to current stock price
  - Yields a positive profit if S_T is close to K_2

Put-call parity

- For European options between the prices of corresponding puts and calls
  - Call - put + risk-free loan = underlying stock
  - Buy 1 call, sell 1 put, and lend an amount dK, d is the discount factor for the period
  - $C - P + dK = S$



Early exercise

- An American option offers the possibility of early exercise
- Result: *For American call options on a stock that pays no dividends prior to expiration, early exercise is never optimal.*

- Intuition
  - Suppose we are holding a call option at time t and expiration is at time $T > t$. Current stock price is $S(t)$.
    - If $S(t) < K$, no exercise
    - If $S(t) > K$, for immediate exercise we pay K
    - If we hold option until $t'(t' > t)$, and exercise we will obtain the stock for the strike price K, but we will also earn additional interest, $Ke^{r(t'-t)}$
    - If the stock declines below K, in this waiting period, we will not exercise and be happy that we did not do so earlier!

Single-period binomial options theory

- Calculating the theoretical value of an option
  - No arbitrage principal
  - Simplest is binomial model of stock price – widely used in practice

- Single period binomial pricing
  - Initial price of stock is S
  - At the end of the period the price will either be uS with probability p or dS with probability $(1-p)$
  - Assumptions: (a) $0 < d < u$, and (b) at every period it is possible to borrow or lend at a common risk-free interest rate r, $R = 1 + r$

Single period binomial pricing

- To avoid arbitrage we must have $d < R < u$
- Case-1: $d < u < R$, and $0 < p < 1$
  - Stock performs worse than the risk-free asset, even in the "up" branch of the lattice
  - Say, we could short ₹1 of the stock and loan the proceeds, thereby obtaining a profit of, $R - u$ or $R - d$
- Case-2: $u > d \geq R$
  - Similar argument, borrow money from the bank and invest in stock
- Additional notes on binomial options pricing

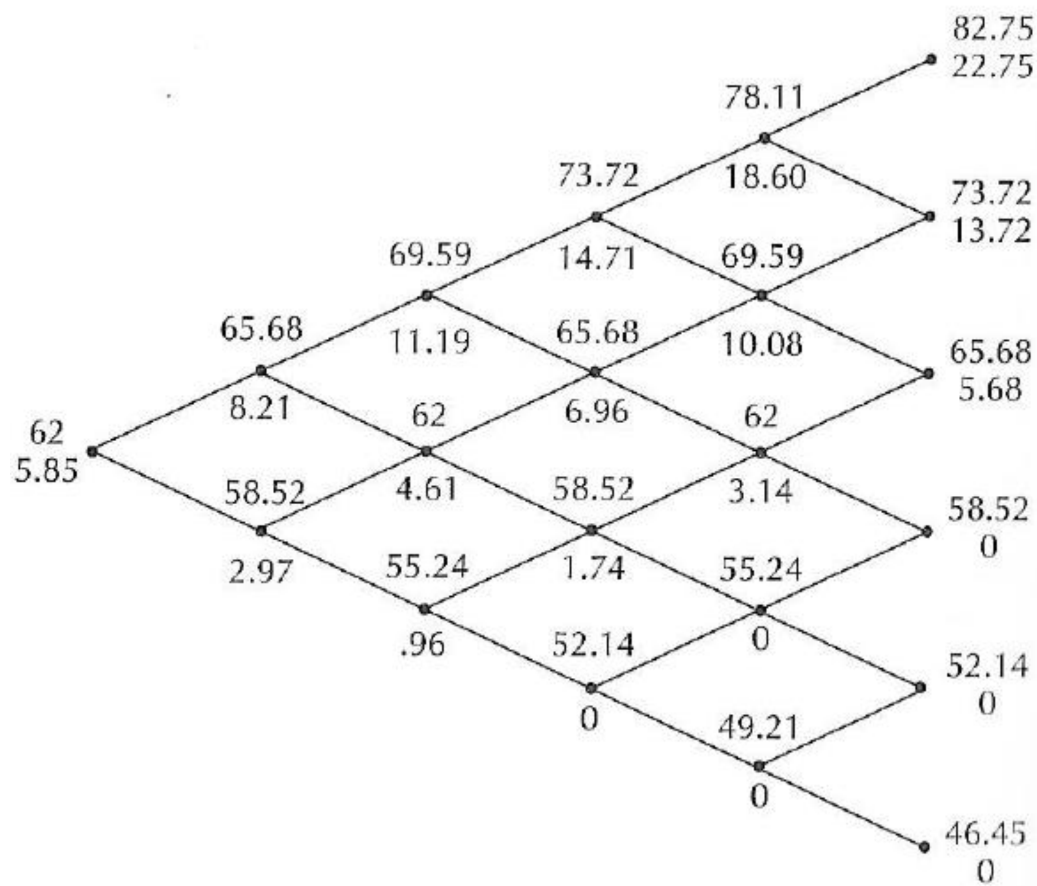
A 5-month call

- $S = ₹62$, pays no dividend, call option has an expiration date 5 months, with $K = ₹60$, $r = 10\%$, compounded monthly, yearly standard deviation of its logarithm $\sigma = 0.2$
  - Determine the theoretical price of this call.

- Solution
  - Period length, $\Delta t = \frac{1}{12}$
  - $u = e^{\sigma\sqrt{\Delta t}} = 1.0594$, $d = e^{-\sigma\sqrt{\Delta t}} = 0.9439$
  - $R = 1 + r = 1 + \frac{0.1}{12} = 1.00833$
  - $q = \frac{R-d}{u-d} = 0.5577$

5-month call using binomial lattice

- Number above the node is stock price, below is the call option price
- Work backwards
  - $\max(0, S-K)$
  - Value at previous time
  - $\frac{1}{1.0083}[.557 \times 22.75 + (1-.557) \times 13.72]$



No early exercise (2 period illustration)

- Note that
  - $C_{uu} \geq u^2S - K$
  - $C_{ud} \geq udS - K$
  - $C_{dd} \geq d^2S - K$
- Hence, $C_u = \frac{1}{R}[q \times (u^2S - K\ ) + (1-q) \times (udS - K\ )]$
  - $\geq \frac{1}{R}[uS - K] > uS - K$
- Similarly, $C_d > dS - K$
- The value of the option at the end of one period is greater than the amount that would be obtained by exercise at that period
  - These inequalities extend to the next period as well in a lattice with multiple periods
- It is not optimal to exercise the option early
  - However, this argument does not hold for all options

Put options

- European put options
  - The terminal values for the option are different – but put-call parity $C - P + dK = S$ could be used
  - Then work backwards recursively

- American put option
  - Early exercise may be optimal
  - At each node calculate: (a) put option value, (b) value of immediate exercise
    - Select the larger of these two values

5-month American put option

FIGURE 14.9 Calculation of a 5-month put option price. The put values in the lower portion of the figure are found by working backward. Boldface entries indicate points where it is optimal to exercise the option.

62.00	65.68	69.59	73.72	78.11	82.75
	58.52	62.00	65.68	69.59	73.72
		55.24	58.52	62.00	65.68
	Stock price		52.14	55.24	58.52
				49.21	52.14
					46.45
1.56	0.61	0.12	0.00	0.00	0.00
	2.79	1.23	0.28	0.00	0.00
		4.80	2.45	0.65	0.00
	Put option		**7.86**	**4.76**	**1.48**
				10.79	**7.86**
					13.55

- $K = 60, R = 1.0083, q = 0.5577, u = 1.059, d = 0.944$
- Discounted put option is compared with the value of immediate exercise $K - S$
  - Larger of the two values is assigned to the current node
  - Illustration: fourth entry in the second to last column, $\frac{1}{1.0083}[.557 \times 1.48 + (1 - .557) \times 7.86] = 4.26$, and the exercise value is $60 - 55.24 = 4.76$ (indicated by bold)
- Early exercise may be optimal – bounded upside profit

Modifications of binomial lattice

- Call option that pays a dividend
- Interest rate is not constant

Real investment opportunities

- Gold mine XX, can extract gold at a rate of up to 10,000 kg per year at a cost of ₹200 per kg. The gold price fluctuates randomly, the interest rate is constant at 10%. Cash flows occur at the end of the year. Determine the value of a 10-year lease of this mine.
- Gold price
  - Represented as a binomial lattice with $u = 1.2, q = 0.75$ and $d = .9, 1 - q = 0.25$
- Problem: find the lease value by the methods developed for options pricing
  - Lease can be treated as a financial instrument. It has a value that fluctuates in time as the price of gold fluctuates.
  - Lease on the gold mine is a derivative whose underlying security is gold

0	1	2	3	4	5	6	7	8	9	10
400.0	480.0	576.0	691.2	829.4	995.3	1194.4	1433.3	1719.9	2063.9	2476.7
	360.0	432.0	518.4	622.1	746.5	895.8	1075.0	1289.9	1547.9	1857.5
		324.0	388.8	466.6	559.9	671.8	806.2	967.5	1161.0	1393.1
			291.6	349.9	419.9	503.9	604.7	725.6	870.7	1044.9
				262.4	314.9	377.9	453.5	544.2	653.0	783.6
Gold price (dollars)					236.2	283.4	340.1	408.1	489.8	587.7
						212.6	255.1	306.1	367.3	440.8
							191.3	229.6	275.5	330.6
								172.2	206.6	247.9
									155.0	186.0
										139.5

0	1	2	3	4	5	6	7	8	9	10
24.1	27.8	31.2	34.2	36.5	37.7	37.1	34.1	27.8	16.9	0.0
	17.9	20.7	23.3	25.2	26.4	26.2	24.3	20.0	12.3	0.0
		12.9	15.0	16.7	17.9	18.1	17.0	14.1	8.7	0.0
			8.8	10.4	11.5	12.0	11.5	9.7	6.1	0.0
				5.6	6.7	7.4	7.4	6.4	4.1	0.0
Lease value (millions)					3.2	4.0	4.3	3.9	2.6	0.0
						1.4	2.0	2.1	1.5	0.0
							0.4	0.7	0.7	0.0
								0.0	0.1	0.0
									0.0	0.0
										0.0

Lease value

- For the final nodes, at the end of 10 years the value is zero
- At a node representing 1 year to go, the value of the lease = profit that can be made from mining in that year
  - For example, top node for year 9 is $\frac{10{,}000(2063.9-200)}{1.1} =$ 16.94 million
  - For an earlier node, the value is the sum of,
    - The profit that can be made that year
    - The risk-neutral expected value of the lease for the next period, discounted back one period
    - $q = \frac{R-d}{u-d} = \frac{2}{3}$

Real options

- Options associated with investment opportunities that are not financial instruments
- Example: when operating a factory, a manager may have the option of hiring additional employees or buying new equipment

- Manufacturing plants could be described by
  - Fixed cost (F): equipment, management and rent
  - Variable cost (V): material, labor, and utilities
  - Total cost, $T = F + Vx$, x is the amount of production
  - Plant profit, $T = px - F - Vx$
    - If $p > V$, the firm operate at max. capacity
    - If $p \leq V$, it will not operate
    - Hence the firm has a continuing option to operate, with a strike price equal to the rate of variable cost.

The end!