FINANCIAL ENGINEERING

IME611A

Suman Saurabh, IIT Kanpur

SESSION OBJECTIVES

- The Short rate
- Relationship between Short rate and forward rate
- Useful formulation for present value

SHORT RATES

- **Short rates:** these are forward rates spanning **a single time period**.

$$(1 + s_k)^k = (1 + r_0)(1 + r_1)(1 + r_2) \dots (1 + r_{k-1})$$

- All forward rates can be found from the short rates in similar way,

$$(1 + f_{i,j})^{j-i} = (1 + r_i)(1 + r_{i+1}) \dots \left(1 + r_{j-1}\right)$$

FORWARD RATES, SHORT RATES: AN EXAMPLE

- Given current spot rate curve, **construct forecast** curve for next year.

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Current	6.00	6.45	6.80	7.10	7.36	7.56	7.77

FORWARD RATES

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Current	6.00	6.45	6.80	7.10	7.36	7.56	7.77

t_1	t_2	t_3	t_4	t_5	t_6	t_7
6.00	6.45	6.80	7.10	7.36	7.56	7.77
6.90	7.20	7.47	7.70	7.88	8.06	
7.50	7.75	7.97	8.12			
8.00	8.20	8.33				
8.40	8.50	8.67				
8.60	8.80					
9.00						

SHORT RATES

	s_1	s_2	s_3	s_4	s_5	s_6	s_7
Current	6.00	6.45	6.80	7.10	7.36	7.56	7.77

t_1	t_2	t_3	t_4	t_5	t_6	t_7
6.00	6.90	7.50	8.00	8.40	8.60	9.00
6.90	7.50	8.00	8.40	8.60	9.00	
7.50	8.00	8.40	8.60	9.00		
8.00	8.40	8.60	9.00			
8.40	8.60	9.00				
8.60	9.00					
9.00						

PRESENT VALUE IN TERM STRUCTURE FRAMEWORK

- We know, for a cashflow stream, $(x_0, x_1, \ldots, x_n)$

$$PV(0) = x_0 + d_1 x_1 + d_2 x_2 + \ldots + d_n x_n$$

- Alternatively,

$$PV(0) = x_0 + d_1[x_1 + (d_2/d_1)x_2 + \ldots + (d_n/d_1)\, x_n]$$

- Now, $\left(\frac{d_k}{d_1}\right)$ for k = 2, 3, …, n are **discount factors 1 year from now** under an assumption of expectation dynamics

$$PV(0) = x_0 + d_1 PV(1)$$

IMPORTANT RESULT

- **Present value updating:** The running present values satisfy the recursion

$$PV(k) = x_k + d_{k,k+1} PV(k+1)$$

where , $d_{k,k+1} = 1/(1 + f_{k,k+1})$ is the discount factor for the short rate at k.

- Practice Example 4.6, 4.7

DISCLAIMER

- The information in this presentation has been compiled from the following textbook which has been mentioned as a reference text for this course on **Financial Engineering**.

- Reference Text:
  - **Investment Science**, 2nd Edition, Oxford University Press, David G. Luenberger