

## END SEM

$$1) a) \quad G_p = \frac{e^{-\Delta}}{\Delta} \quad G_c = \frac{k(z_1 \Delta + 1)}{z_1 \Delta}$$

$$G_p G_c = \frac{k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2}$$

Nyquist plot

Im

Re

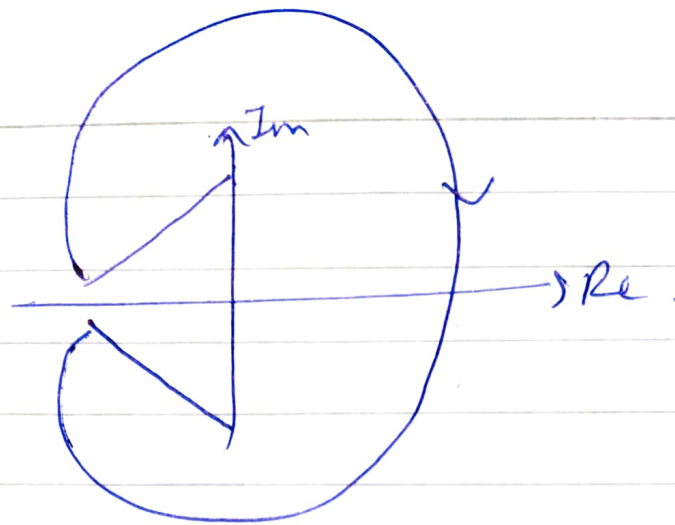
Bode plot

 $|G_p G_c|$  $\angle G_p G_c$ 

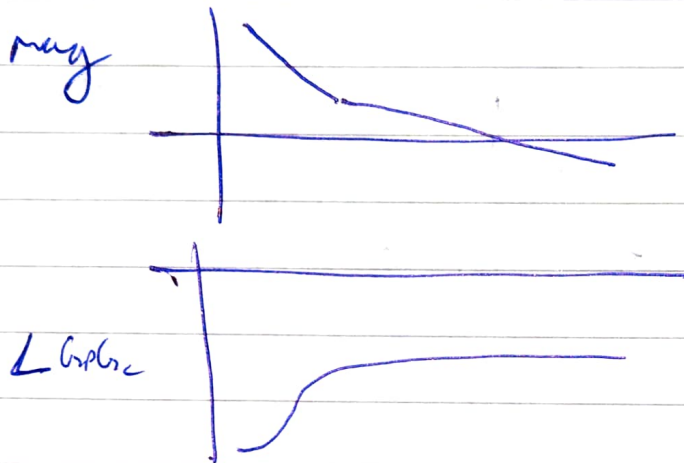
b)

$$-\omega + \tan^{-1} z_1$$

★ Nyquist



★ Bode



b)  $-\omega + \tan^{-1}(z_I \omega) = -\pi = \angle G_{OL}$

$$\frac{d\angle G_{OL}}{d\omega} = -1 + \frac{z_I}{z_I^2 \omega^2 + 1} = 0$$

$$z_I = z_I^2 \omega^2 + 1$$

$$z_I^2 \omega^2 - z_I + 1 = 0$$

$$\omega^2 = \frac{z_I - 1}{z_I^2}$$

$$-\left(\frac{z_I - 1}{z_I^2}\right) + \tan^{-1}\left(\frac{z_I - 1}{z_I}\right) = -\pi$$

$$c) \quad \frac{G_{OL}}{1+G_{OL}} = \frac{\frac{k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2}}{\frac{z_1 \Delta^2 + k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2}}$$

$$G_{CL} = \frac{k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2 + k e^{-\Delta} (z_1 \Delta + 1)}$$

$$|G_{CL}| = 1.258$$

d)

$$1 + \frac{k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2} = 0$$

$$G_p G_c = \frac{k e^{-\Delta} (z_1 \Delta + 1)}{z_1 \Delta^2}$$

$$\angle G_{OL} = -\omega + \tan^{-1}(z_1 \omega) - \frac{\pi}{2} - \frac{\pi}{2} = -\pi$$

$$|G_{OL}| = \frac{k \sqrt{z_1^2 \omega^2 + 1}}{z_1 \omega^2} = 1$$

$$+\tan^{-1}(z_1 \omega) - \frac{\pi}{2} = +\frac{\pi}{10}$$

~~max~~

solving ① ③

$$z_1 = 3.33$$

$$\omega = 1.55$$

$$K_v = 1.317$$

2.)  $z_1 = 1.88$

$$\angle G_{OL} = -\omega + \tan^{-1}(1.88\omega) = 0$$

$$\boxed{\omega = 1.13}$$

$$|G_{OL}| = \frac{1}{2} = \frac{k \sqrt{\omega^2 z_1^2 + 1}}{z_1 \omega^2}$$

$$\boxed{k = 0.511}$$

(A)

1.)  $z_1 = 1.88$

$$\angle G_{OL} = -\omega + \tan^{-1}(1.88\omega) = \frac{\pi}{4}$$

$$\boxed{\omega = 2.1}$$

~~(A)~~

$$|G_{OL}| = 1 = \frac{k \sqrt{\omega^2 z_1^2 + 1}}{z_1 \omega^2}$$

$$\boxed{k = 2.035}$$

(A)

2)  $k_p = G_m G_c = \begin{bmatrix} 1 & 2 \\ 1.5 & 0.5 \end{bmatrix}$

a, b

$$NI_1 = \frac{0.5 - 3}{0.5} = -5$$

$$RGA_1 = [k^{-1}]^T = \begin{bmatrix} -0.2 & 0.6 \\ 0.8 & -0.4 \end{bmatrix}$$

$$RGA_1 = \begin{bmatrix} -0.2 & 1.2 \\ 1.2 & -0.2 \end{bmatrix}$$

$$k_p = \begin{bmatrix} 2 & 1 \\ 0.5 & 1.5 \end{bmatrix}$$

$$NI_2 = \frac{3 - 0.5}{0.5} = 5 \quad \underline{\text{preferred}}$$

$$RGA_2 = [k^{-1}]^T = \begin{bmatrix} 0.6 & -0.2 \\ -0.4 & 0.8 \end{bmatrix}$$

$$RGA_2 = \begin{bmatrix} 1.2 & -0.2 \\ -0.2 & 1.2 \end{bmatrix} \quad \underline{\text{preferred}}$$

Both RGAs are good  
2nd NI is good.

c.)  $G_p = \frac{2 e^{-0.5s}}{3s+1}$

$$G_c = \frac{k(z-1)}{z-1}$$

$$z_T = 3 \text{ min}$$

$$G_p G_c = \frac{2k e^{-0.5s}}{3s+1}$$

$$G_{OL} = \frac{2k e^{-0.5\omega j}}{2\omega j}$$

$$\angle G_{OL} = -0.5\omega_c - \pi/2 = -\pi$$

$$\omega_c = \pi$$



$$|G_{OL}|_{\omega_c} = \frac{1}{G_M} = 1 = \frac{2K_c}{3\pi}$$

$$K_c = \frac{3\pi}{2} \approx 4.712$$

$$P \approx \frac{2\pi}{\omega} = 2$$

$$K_U = \frac{3\pi}{2} \quad P_U = 2$$

2N settings

$$PI \quad K_c = 2.14$$

$$Z_I = 1.66$$

$$d) \quad G_P = \frac{1.5e^{-2s}}{8s+1} \times \frac{0.5e^{-s}}{5s+1} \quad G_C = \frac{K(Z_I s+1)}{Z_I s}$$

$$Z_I = 2$$

$$G_P G_C = \frac{0.75 K e^{-3s} (2s+1)}{(8s+1)(5s+1)(2s)}$$

$$\angle G_{OL} = -\tan^{-1}(8\omega) - \tan^{-1}(5\omega) - \pi/2 + \tan^{-1}(2\omega) - 3\omega_c = -\pi$$

$$\omega_c = 1.36$$

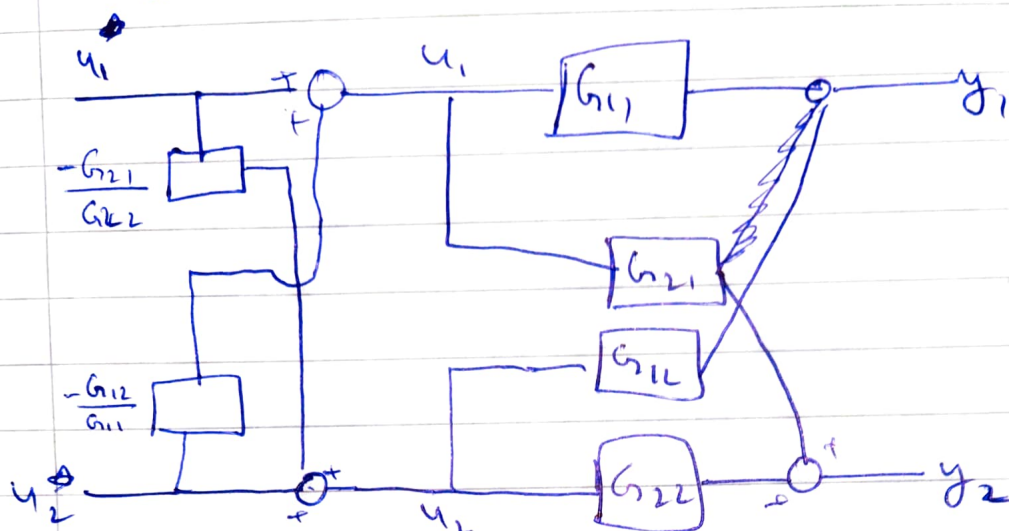
$$|G_{OL}| = \frac{1}{G_M} = \frac{1}{2} = \frac{0.375 K \sqrt{4\omega^2+1}}{\sqrt{64\omega^2+1} \sqrt{25\omega^2+1}}$$

$$k = \frac{\omega \cdot \sqrt{64\omega^2 + 1} \cdot \sqrt{25\omega^2 + 1}}{0.75 \cdot \sqrt{4\omega^2 + 1}}$$

$$k = 46.9$$

$$\boxed{k = 47.}$$

2.) dynamic decoupler



$$G_{11} = \frac{e^{-2s}}{10s+1}$$

$$G_{22} = \frac{0.5 e^{-s}}{5s+1}$$

$$G_{12} = \frac{2 e^{-0.5s}}{3s+1}$$

$$G_{21} = \frac{1.5 e^{-2s}}{8s+1}$$

$$\frac{-G_{21}}{G_{22}} = \frac{3 \cdot e^{-s} (5s+1)}{(8s+1)}$$

$$\frac{-G_{12}}{G_{11}} = \frac{2 \cdot e^{+1-5s} (10s+1)}{(3s+1)}$$

$$G_{11} = \frac{2 e^{-0.5s}}{3s+1}$$

$$G_{12} = \frac{e^{-2s}}{10s+1}$$

$$G_{21} = \frac{0.5 e^{-s}}{5s+1}$$

$$G_{22} = \frac{1.5 e^{-2s}}{8s+1}$$

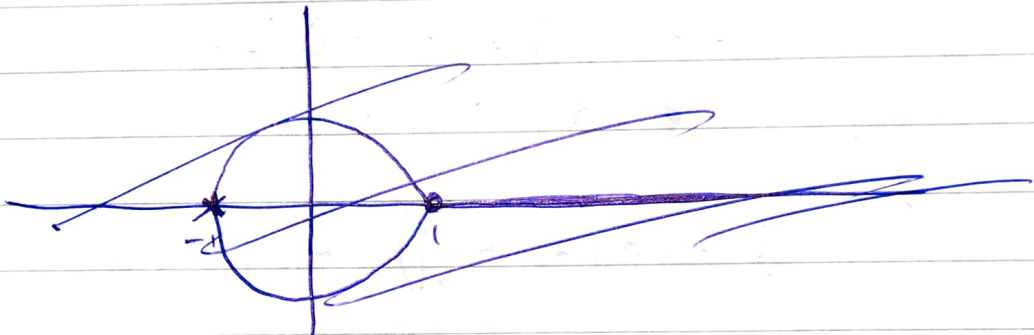


3 a)  $G_p = \frac{2(-s+1)}{(s+1)^2}$   $G_c = k$

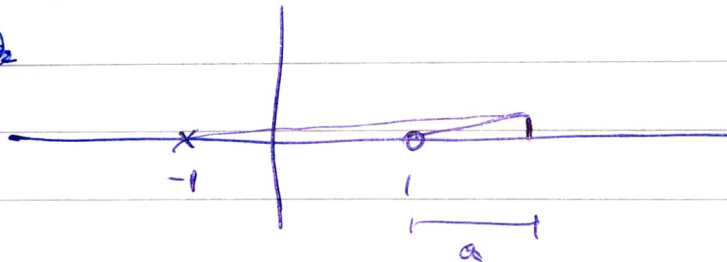
$G_p G_c = \frac{2k(-s+1)}{(s+1)^2}$

poles =  $-1, -1$

zeros =  $+1$



~~break~~



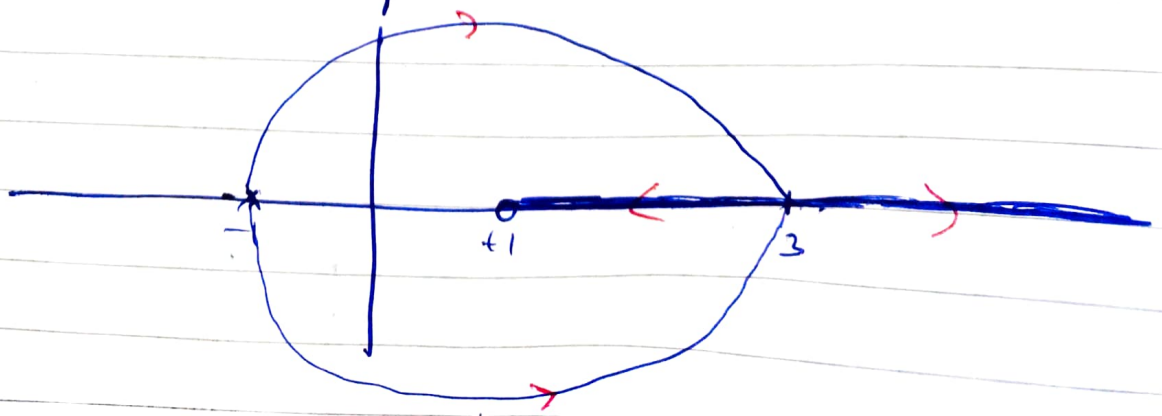
$2 \tan^{-1}\left(\frac{s}{a+2}\right) - \tan^{-1}\frac{s}{a} = 0$

$\frac{2}{a+2} - \frac{1}{a} = 0$

$2a - a - 2 = 0$

$a = 2$

break in pt at 3



## END SEM

b)

$$1 + G_p G_c = 0$$

$$(\Delta + 1)^2 + 2k(1 - \Delta) = 0$$

$$\Delta^2 + 2\Delta + 1 + 2k - 2k\Delta = 0$$

$$\Delta^2 + (2 - 2k)\Delta + 1 + 2k = 0$$

$$\xi = 1/\sqrt{2} \Rightarrow \Delta = -\alpha + \alpha j$$

$$(-\alpha + \alpha j)(-\alpha + \alpha j) + (2 - 2k)(-\alpha + \alpha j) + 1 + 2k = 0$$

$$(\alpha^2 - \alpha^2 j - \alpha^2 j - \alpha^2) + (-2\alpha + 2\alpha j + 2k\alpha - 2k\alpha j) + 1 + 2k = 0$$

$$(-2\alpha^2 j + 2\alpha j - 2k\alpha j) + (-2\alpha + 2k\alpha + 1 + 2k) = 0$$

$$-2\alpha^2 + 2\alpha - 2k\alpha = 0$$

$$-2\alpha + 2 - 2k = 0 \quad \text{--- (1)}$$

$$-2\alpha + 2k\alpha + 1 + 2k = 0 \quad \text{--- (2)}$$

using MATLAB

solving  $\boxed{k = 0.1771}$

$$c.) \quad G_p = \frac{2(-\Delta + 1)}{(\Delta + 1)^2}$$

$$G_c = \frac{k(z_0 \Delta + 1)}{(0.12 z_0 \Delta + 1)}$$

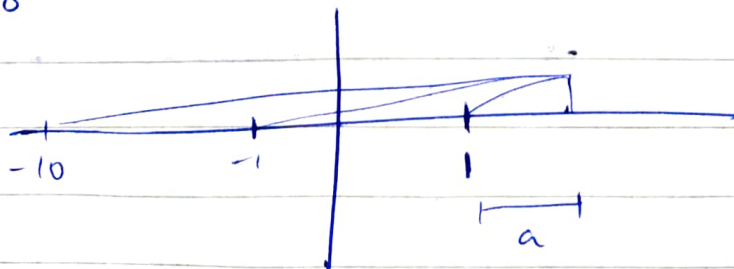
$$z_0 = 1 \text{ m}$$

$$G_p G_c = \frac{2k(-\Delta + 1)}{(\Delta + 1)(0.12\Delta + 1)}$$

zeros = -1, -10

poles = 1

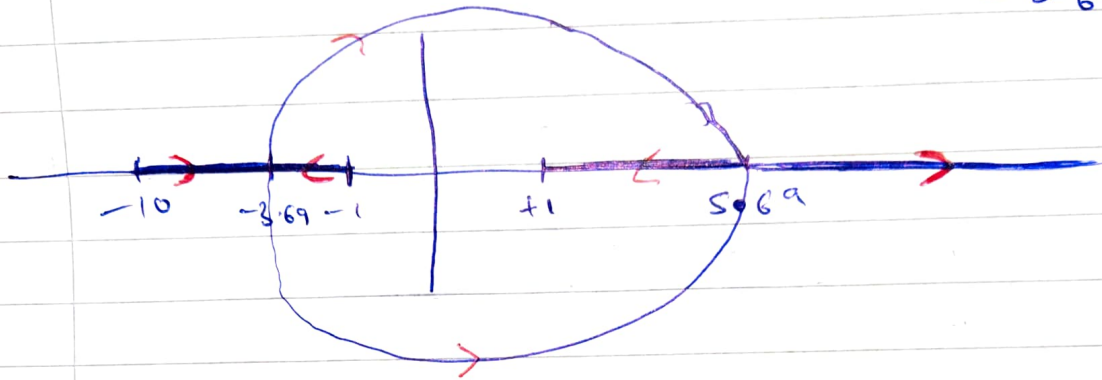
$$\tan^{-1} \frac{s}{a+1} + \tan^{-1} \frac{s}{a+2} - \tan^{-1} \frac{s}{a}$$



$$\frac{1}{a+1} + \frac{1}{a+2} + \frac{1}{a} = 0$$

$$a = \sqrt{22}$$

$$= 5.69, -3.69$$



d)  $1 + G_p G_c = 0$

$$(s+1)(0.1s+1) + 2k(1-s) = 0$$

$$s = 1/\sqrt{2} \Rightarrow s = -a + aj$$

$$0.1s^2 + \cancel{0.1} \cdot 1 \cdot s + 1 + 2k - 2ks = 0$$

$$0.1(-a^2j) - 1 \cdot 1a + 1 \cdot 1aj + 1 + 2k + 2ka - 2kaj = 0$$

$$\underbrace{(-0.1a^2j + 1 \cdot 1aj - 2kaj)}_{(1)} + \underbrace{(-1 \cdot 1a + 1 + 2k + 2ka)}_{(2)} = 0$$

solving for (1) & (2) using MATLAB we get

$$k = 0.2645$$

$$4.) a) G_p = \frac{2}{(2s+1)(s+1)^2} \quad G_D = \frac{-1}{(4s+1)(2s+1)(s+1)}$$

$$2 \left[ \frac{A}{(2s+1)} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2} \right] + \frac{D}{s}$$

solving for A, B, C

$$A = -8 \quad B = +3 \quad C = +1 \quad D = 1$$

$$2 \left[ \frac{-8}{2s+1} + \frac{3}{s+1} + \frac{1}{(s+1)^2} \right] + \frac{1}{s}$$

inverse laplace.

$$y = 2 \left[ 4e^{-t/2} + 3e^{-t} + te^{-t} + 1 \right]$$

$$t_{28.3} \rightarrow \text{put } y = 0.5670$$

$$t_{63.2} \rightarrow \text{put } y = 1.2640$$

$$t_{28.3} = 2.4$$

$$t_{63.2} = 4.3$$

$$z = \frac{3}{2} (4.3 - 2.4) = 2.85$$

$$\theta = \frac{1}{2} (3 \times 2.4 - 4.3) = 1.45$$

$$\hat{G}_p = \frac{2e^{-1.45s}}{2.85s+1}$$

similarly for  $G_D$

$$-1 \left[ \frac{A}{4s+1} + \frac{B}{(2s+1)} + \frac{C}{(s+1)} + \frac{D}{s} \right]$$



solving for A, B, C, D

$$A = \frac{32}{3} \quad B = \frac{+4}{3} \quad C = -\frac{1}{3} \quad D = 1$$

$$-1 \left[ \frac{8}{s(s+1)} - \frac{2}{2s+1} + \frac{1}{3(s+1)} \right]$$

inverse Laplace

$$y = -1 \left( -\frac{8}{3} e^{-x/1} + 2e^{-x/2} + \frac{1}{3} e^{-x/1} + 1 \right)$$

$$t_{20.3} \rightarrow \text{put } y = 0.2835$$

$$t_{63.2} \rightarrow \text{put } y = 0.632$$

$$t_{0.5} = 4.03$$

$$t_{63.2} = 7.40$$

$$\tau = \frac{3}{2} (7.40 - 4.03) = 5.050$$

$$\sigma = \frac{1}{2} (3 \times 4.03 - 7.4) = 2.34$$

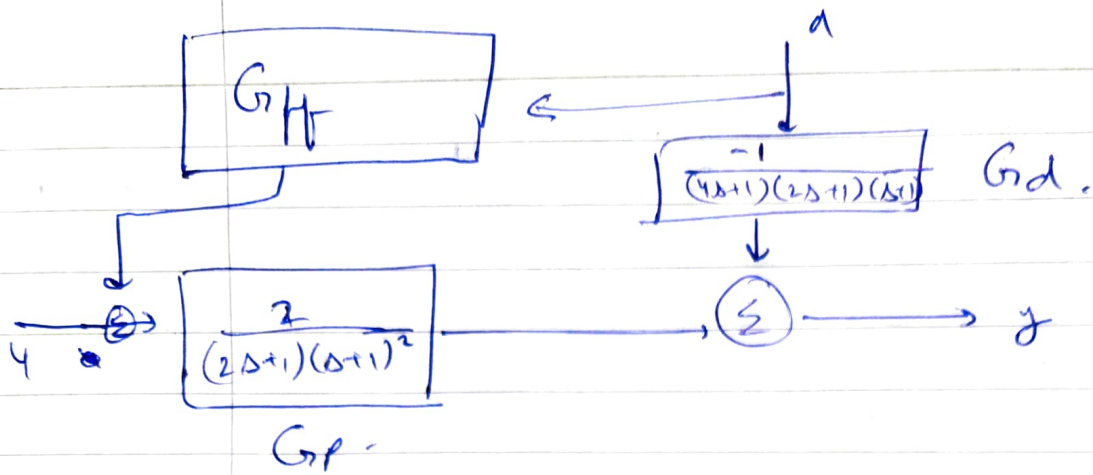
$$\hat{G}_D = \frac{-1 e^{-2.34s}}{(5.05s+1)}$$

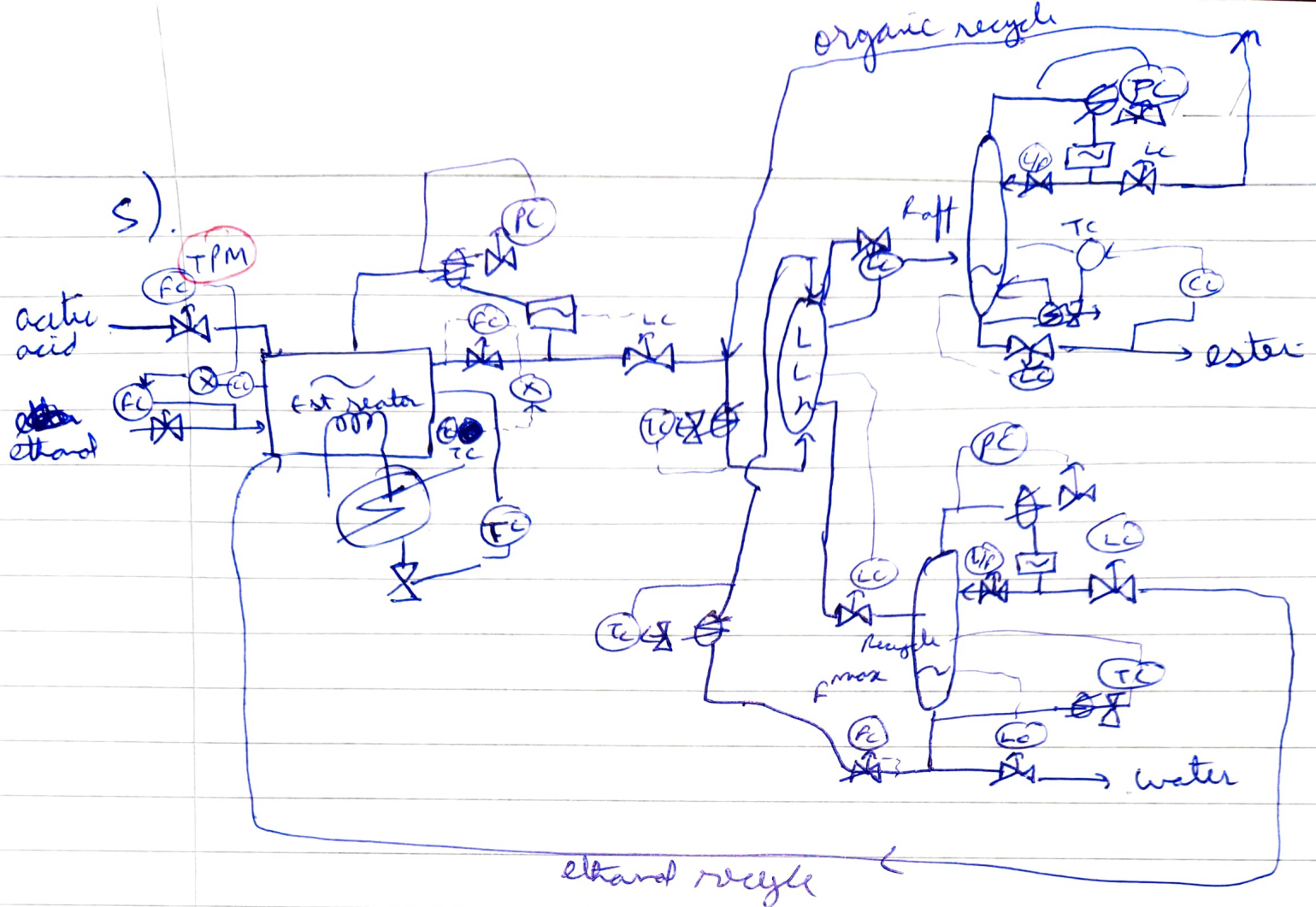
$$b.) \quad G_H = - \frac{\hat{G}_D}{\hat{G}_P}$$

$$G_H = \frac{e^{-2.34s} (2.85s+1)}{2(5.05s+1) e^{-1.45s}}$$

$$G_H = \frac{1}{2} \frac{e^{-0.89s} (2.85s+1)}{(5.05s+1)}$$







$$\text{Control DOF} = 2 + 4 + 2 + 1 + 2 + 5 + 5 \\ = 21$$

$$\text{SS DOF} = 2 + 2 + 2 + 1 + 2 + 2 \\ = 11$$

$$\left[ \begin{array}{l} F_{AA}, F_{ETH}, T_{\text{est reactor}}, \text{L/F est reactor}, T_{LHX \text{ bottom}} \\ T_{LHX \text{ top}}, F_{\text{water}}, T_{\text{recycle}}, \text{L/F recycle} \\ T_{\text{raff.}}, (\text{L/F})_{\text{raffinater}} \end{array} \right]$$