FINANCIAL ENGINEERING
IME611A

Suman Saurabh, IIT Kanpur

SESSION OBJECTIVES

- Duration of a portfolio of fixed income instruments
- Immunization
- Convexity and interest rate sensitivity

DURATION OF A PORTFOLIO

- Consider a portfolio of several bonds of different maturities.

- **Suppose** that all bonds have same yield.

$$D = \frac{P^A D^A}{P} + \frac{P^B D^B}{P}$$

- Proof of the above.
- [Hint: $D^A = \frac{\sum t_k PV_k^A}{P^A}$,and P = P^A + P^B]

IMPORTANT RESULT

- **<u>Duration of a portfolio</u>:** Suppose there are ***m*** fixed income securities with prices and durations of P_i ***and*** D_i, respectively, ***i = 1, 2, … , m,*** all computed at a common yield. The portfolio consisting of the aggregate of these securities has price ***P*** and duration ***D*** given by

$$P = P_1 + P_2 + \ldots + Pm$$

$$D = w_1 D_1 + w_2 D_2 + \ldots + w_m D_m$$

where, $w_i = Pi/P$

IMMUNIZATION

- **Immunization:** Structuring of a bond portfolio to **protect** against the interest rate risk.

1. Present values are matched. $V_1 + V_2 = PV$
2. Durations are matched. $V_1D_1 + V_2D_2 = PV*D$

- An example and solution approach [Refer illustration excel sheet].

CONVEXITY

- Price yield relationship is inverse.

- Bonds are, therefore, subject to interest rate risk.

- **Modified duration** measures relative **slope** of the price-yield curve at ***a given point***. [A linear approximation!]

- **Convexity** measures the relative **curvature** of the price-yield curve. [Non-linear approximation!]

FORMULA FOR CONVEXITY

$$C = \frac{1}{P}\frac{d^2P}{d\lambda^2}$$

$$C = \frac{1}{P}\sum_{k=1}^{n}\frac{d^2PV_k}{d\lambda^2}$$

- Assuming m coupons (and m compounding periods per year)

$$C = \frac{1}{P[1+(\lambda/m)]^2}\sum_{k=1}^{n}\frac{k(k+1)}{m^2}\frac{c_k}{[1+(\lambda/m)]^k}$$

$$\boldsymbol{\Delta P \approx -D_M P\,\Delta\lambda + \frac{PC}{2}(\Delta\lambda)^2}$$

DISCLAIMER

- The information in this presentation has been compiled from the following textbook which has been mentioned as a reference text for this course on **Financial Engineering.**

- **Reference Text:**
  - Investment Science, 2nd Edition, Oxford University Press, David G. Luenberger