Module 4.7.1

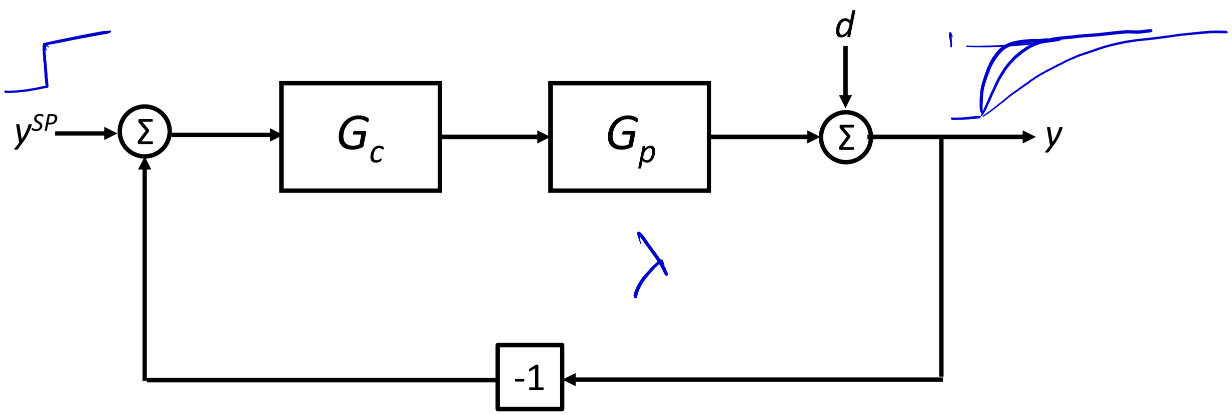
Laplace Domain Analysis
Controller Direct Synthesis

Lectures on

CHEMICAL PROCESS CONTROL
Theory and Practice

The Basic Idea

- Specify unit step servo response transfer function G_{CL}^{spec} for given G_p
- Back-calculate G_c from feedback loop transfer function equation



$$\left[\frac{y}{y^{SP}}\right]_{spec} = G_{CL}^{spec} = \frac{G_c G_p}{1 + G_c G_p}$$

G_p and G_{CL}^{spec} are known

So G_c can be back-calculated as

$$G_c = \frac{1}{G_p}\left[\frac{G_{CL}^{spec}}{1 - G_{CL}^{spec}}\right]$$

$$G_{CL}^{spec} = \left[\frac{y}{y^{SP}}\right] = \frac{G_c G_p}{1 + G_c G_p}$$

$G_c = ?$

$$G_{CL}[1 + G_c G_p] = G_c G_p$$

$$G_c[G_p - G_p G_{CL}] = G_{CL}$$

$$\boxed{G_c = \frac{1}{G_p}\frac{G_{CL}^{spec}}{[1 - G_{CL}^{spec}]}}$$

Controller Direct Synthesis: Pure Integrator

$$G_p = \frac{K}{s} \qquad G_{CL}^{spec} = \frac{1}{\lambda s + 1} \qquad G_c = \frac{1}{K\lambda}$$

P only controller with

$$K_c = \frac{1}{K\lambda}$$

$$G_c = \frac{1}{K/s}\left[\frac{1/(\lambda s+1)}{1 - \frac{1}{\lambda s+1}}\right]$$

$$= \frac{s}{K}\,\frac{1}{\lambda s + \cancel{1} - \cancel{1}}$$

$$G_c = \frac{1}{K\lambda} \leftarrow \text{P only controller}$$

$\lambda \downarrow \quad K_c \uparrow$

Parameter that determines the tuning

$$G_{CL}^{spec} = \frac{1}{\lambda s + 1}$$

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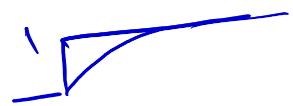

Controller Direct Synthesis: First Order Lag

$$G_p = \frac{K}{\tau s + 1}$$

$$G_{CL}^{spec} = \frac{1}{\lambda s + 1} \qquad G_c = \frac{1}{K}\frac{\tau}{\lambda}\left[1 + \frac{1}{\tau s}\right]$$

PI controller with

$K_c = \frac{1}{K}\frac{\tau}{\lambda} \qquad \tau_I = \tau$

$$G_{CL}^{spec} = \frac{1}{\lambda s + 1}$$

$$G_c = \frac{1}{K/(\tau s + 1)} \frac{1/(\lambda s + 1)}{\left[1 - \frac{1}{(\lambda s + 1)}\right]}$$

$$= \frac{1}{K}\frac{(\tau s + 1)}{\lambda s} = \frac{1}{K} \cdot \frac{\tau}{\lambda}\left[1 + \frac{1}{\tau s}\right]$$

PI Controller $\quad K_c = \frac{1}{K}\frac{\tau}{\lambda} \qquad \tau_I = \tau$

$\lambda \downarrow \quad K_c \uparrow$

Controller Direct Synthesis: Second Order Lag

$$G_p = \frac{K}{(\tau_1 s+1)(\tau_2 s+1)} \qquad G_{CL}^{spec} = \frac{1}{\lambda s+1} \qquad G_c = \frac{1}{K}\frac{(\tau_1+\tau_2)}{\lambda}\left[1+\frac{1}{(\tau_1+\tau_2)s}+\frac{\tau_1\tau_2}{(\tau_1+\tau_2)}s\right]$$

$$G_{CL}^{spec} = \frac{1}{\lambda s+1}$$

$$G_c = \frac{(\tau_1 s+1)(\tau_2 s+1)\cdot 1/(\lambda s+1)}{K\left[1-\frac{1}{\lambda s+1}\right]}$$

$$= \frac{1}{K}\left[\frac{\tau_1\tau_2 s^2+(\tau_1+\tau_2)s+1}{\lambda s}\right] \Rightarrow G_c = \frac{1}{K}\frac{(\tau_1+\tau_2)}{\lambda}\left[1+\frac{1}{(\tau_1+\tau_2)}\cdot\frac{1}{s}+\frac{\tau_1\tau_2}{\tau_1+\tau_2}s\right]$$

PID controller with

$$K_c = \frac{1}{K}\frac{(\tau_1+\tau_2)}{\lambda} \qquad \tau_I = \tau_1+\tau_2 \qquad \tau_D = \frac{\tau_1\tau_2}{(\tau_1+\tau_2)}$$

PID $K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\lambda}$ $\quad \tau_I = \tau_1+\tau_2$ $\quad \tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$

PID $u = K_c\left[e+\frac{1}{\tau_I}\int e\,dt+\tau_D\frac{de}{dt}\right] \Rightarrow G_c^{PID} = \frac{u}{e} = K_c\left[1+\frac{1}{\tau_I s}+\tau_D s\right]$

Exercise

First Order Lag Plus Dead Time

$$G_p = \frac{Ke^{-\theta s}}{(\tau s + 1)}$$

$$G_{CL}^{spec} = \frac{e^{-\theta s}}{\lambda s + 1}$$

Obtain controller using Direct Synthesis Method

PI

Second Order Lag Plus Dead Time

$$G_p = \frac{Ke^{-\theta s}}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

$$G_{CL}^{spec} = \frac{e^{-\theta s}}{\lambda s + 1}$$

Obtain controller using Direct Synthesis Method

PID

$K_c = ?$

$\tau_I = ?$

$\tau_D = ?$

Summary

- Controller Direct Synthesis
  - Gives P, PI and PID control algorithms for simple G_p

$$\rightarrow G_{CL}^{servo} = \frac{G_c G_p}{1+G_c G_p}$$

$$\rightarrow G_{CL}^{reg} = \frac{G_d}{1+G_c G_p}$$

- Gives tuning parameters in terms of G_p parameters and λ
  - λ is a tuning parameter that determines aggressiveness of control

- Also known as Lambda Tuning Method

- The method is tailored towards desired servo response
  - Regulator response may be quite sluggish