Assignment 3

Solve question using Laplace transform.

1. Consider the following differential equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = 5t$$

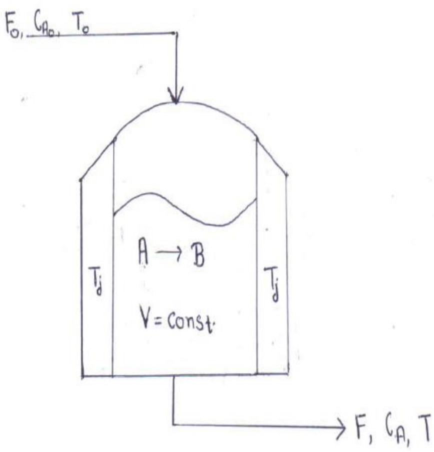
Find $\hat{y}(t)$ in terms of deviation variable.

2. Consider a non-isothermal jacketed CSTR. The jacket temperature is T_j, the overall heat transfer coefficient is U and the heat transfer area is A. The exothermic reaction $A \rightarrow B$ occurs in the CSTR. The reaction kinetic expression is $r=k_0 \exp(-E_A/RT)\, C_A$. Assume constant C_p, $\Delta H_{rxn.}$, density etc as well as perfect CSTR level control.

Find transfer function for using:

a. $\dfrac{\hat{C}_A}{\hat{C}_{A_o}}, \dfrac{\hat{C}_A}{\hat{F}_{A_o}}, \dfrac{\hat{C}_A}{\hat{T}_o}$

b. $\dfrac{\hat{T}}{\hat{T}_o}, \dfrac{\hat{T}}{\hat{T}_j}, \dfrac{\hat{T}}{\hat{F}}, \dfrac{\hat{T}}{\hat{C}_A}$



3. A continuous model for the earnings of a company y(t) in dollars) is given by

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5u(t)$$

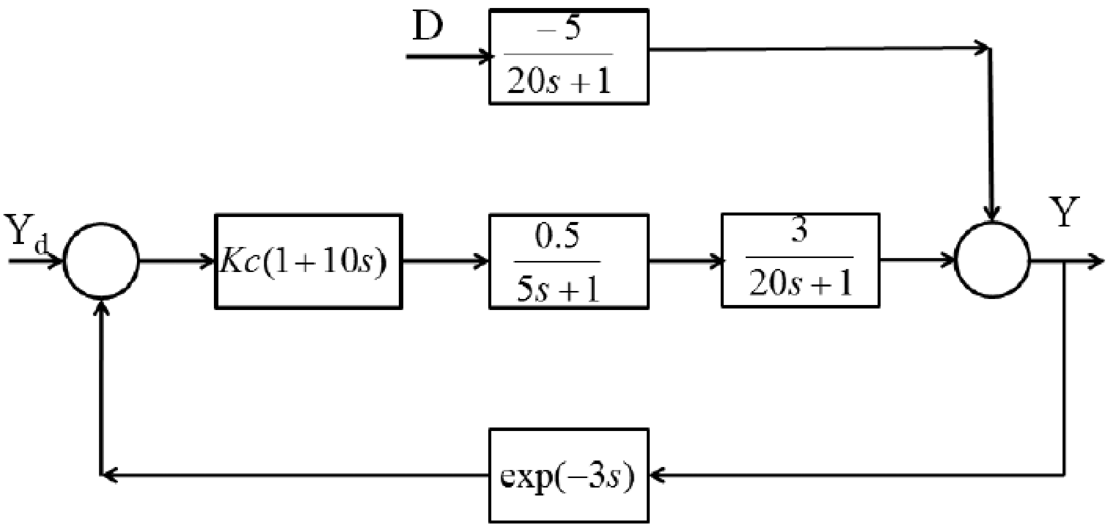
where u(t) is the total number of new versions of the product in the market, and t is time in months. If the total number of new products in the market at any time decreases exponentially with the time elapsed (u(t)=exp(-t)) find the earning function.

4. A dynamic mathematical model for the height of the liquid from its equilibrium position (or steady value) inside a U-tube manometer is given by

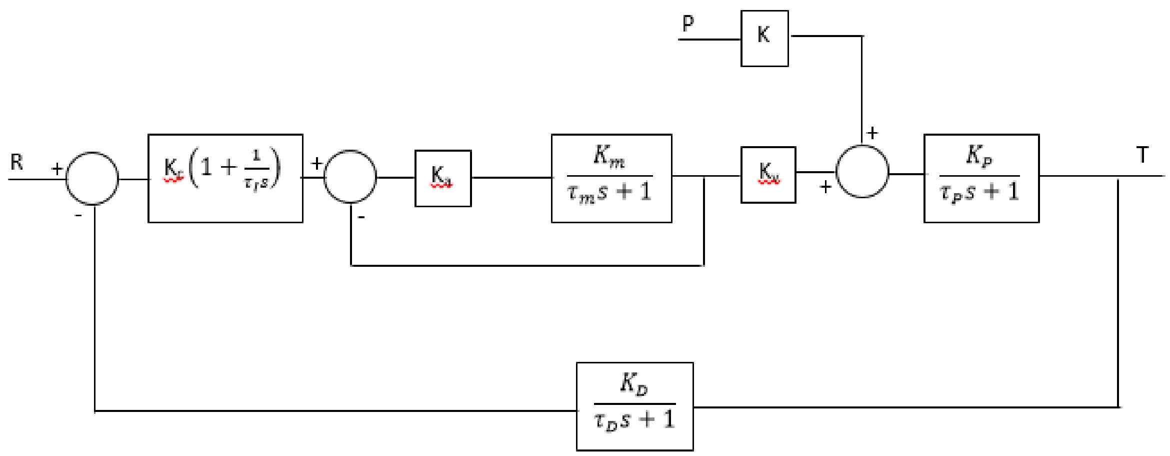
$$\frac{d^2h}{dt^2} + \frac{6\mu}{\rho R^2}\frac{dh}{dt} + \frac{3gh}{2L} = \frac{3}{4\rho L}\Delta P$$

If the pressure is ramped up ($\Delta P = At$) how would liquid height vary as a function of time.

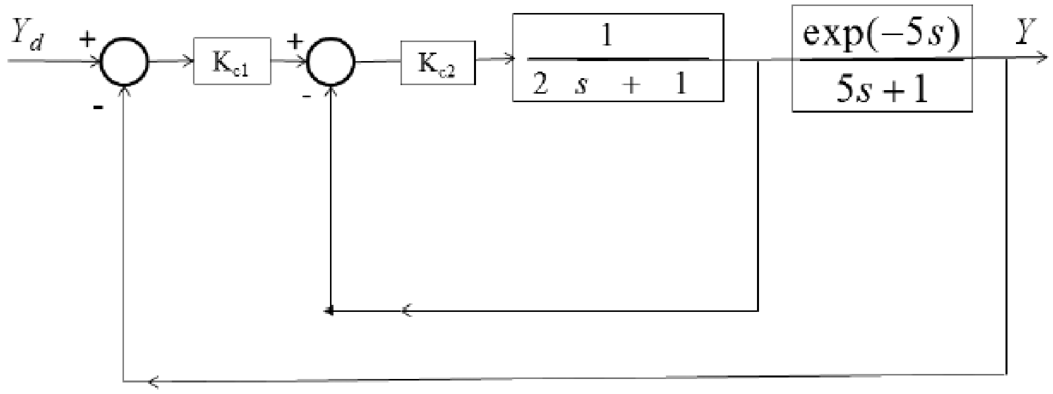
5. Simplify the following block diagram and find the respective TF wrt to servo response as well as regulatory response



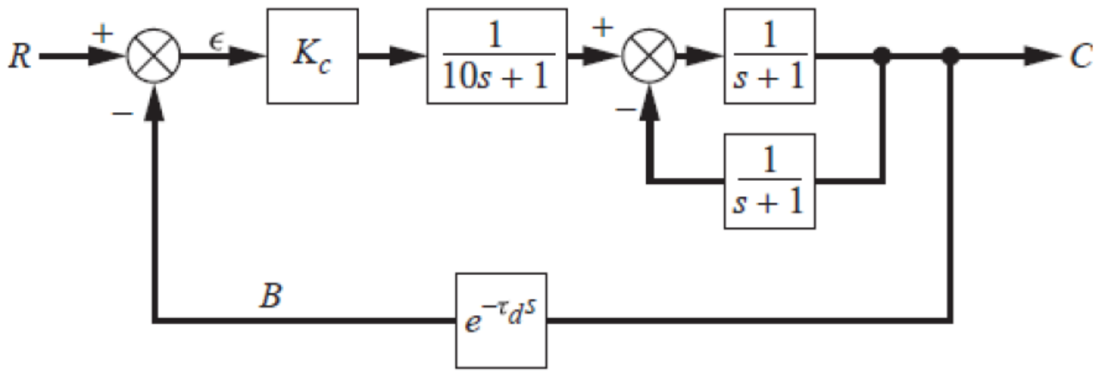
6. Simplify the following block diagram and find the respective TF w.r.t to servo response as well as regulatory response



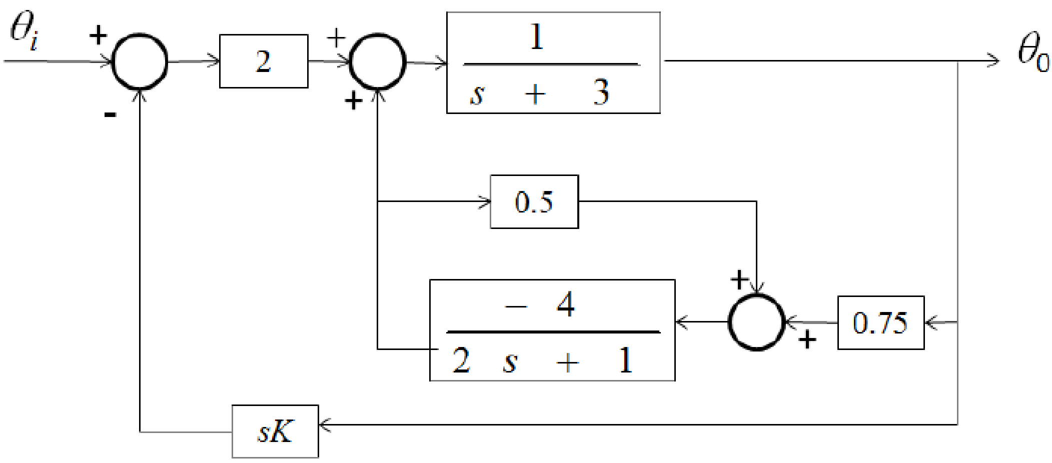
7. Find the closed loop transfer function relating the output to the input for the block diagram shown in Fig.



8. Find the closed loop transfer function relating the output to the input for the block diagram shown in Fig.



9.
   a. Find the closed loop transfer function relating the output to the input for the block diagram shown in Fig.
   b. For what values of *K* the closed-loop response will be critically damped?



10. Consider the second order ODE describing the open loop dynamics of a SISO process

$$24\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + y = 2u(t)$$

a. A P controller is used to control the process. Obtain K_c for a critically damped closed loop response; and its value for closed loop damping coefficient of 0.5.
b. A PI controller is now used. For τ_I values of 6 mins. And 2 mins., find the corresponding controller gain at which the closed loop response exhibits sustained oscillations.

11. Consider the following transfer function

$$G_P = \frac{8}{(4S+1)(6S+1)(12S+1)}$$

a. Design a P and PI controller for a gain margin of 2. Set reset time to approximately equal the ultimate period.
b. Design a P and PI controller for a phase margin of 45°. Use a same reset time as in part 1
c. Design a PID controller for a gain margin of 3 and a phase margin > 30° use a same reset time as in part a.

12. Consider the ODEs describing a process.

$$4\frac{dx}{dt} + x = 2u(t)$$

$$6\frac{dy}{dt} + y = 2x(t)$$

$$12\frac{dz}{dt} + z = 2y(t)$$

The output Z is controlled by manipulating u.

a. Obtain the third order ODE describing the transient in z (output) for a change in u (input)
b. Tune a P controller for a critically damped response.
c. Obtain the K_U and P_U and corresponding Ziegler Nicholas controller tuning parameters for a P, PI, and PID controller.

13. Consider the following differential equation

$$\frac{5d^2y}{dt^2} + 6\frac{dy}{dt} + y = u$$

a. Control using a P controller and also find gain for critical damping, gain for damping coefficient of $\frac{1}{\sqrt{2}}$ and K_U andP_U.
b. Control using a PI controller with τ_i = 0.5 min and also find gain for critical damping, gain for damping coefficient of $\frac{1}{\sqrt{2}}$ and K_U andP_U.

14. Consider the inverse response process

$$G_P = \frac{-2s+1}{(8s+1)(5s+1)}$$

Design P, PI and PID controller using root locus diagram.

15. Consider the first order surge tank system having transfer function has been given as

$$G(s) = \frac{1}{5s+1}$$

Find the root locus for a PI controller
a. $\tau > \tau_i$
b. $\tau = \tau_i$
c. $\tau < \tau_i$

16. Consider a cylindrical vessel storing liquid with inflow and outflow being independently set. The tank level then behaves like an integrator w.r.t the difference between the inflow and the outflow. To stabilize the system and ensure the tank level is well regulated, a PI controller is installed that manipulates the outflow to hold the level. When tuning the PI controller, it is observed that for a chosen value of the integral time, increasing the controller gain causes a reduction in the oscillatoriness of the closed loop system, which is counter-institutive. Justify the observation using the root locus technique.

17. Consider the unstable transfer function $G_P = \frac{2}{(s-1)(2s+1)(0.5s+1)}$. A control engineer proposes to stabilize the process by implementing a P controller. Is stabilization feasible? Justify, why or why not using root locus.