

Q 1 A first order process with transfer function
 $G_p(s) = \frac{5}{0.1s+1}$ is controlled by a PI controller

Transfer function of measuring device is

$$G_m(s) = \frac{K_m}{Z_m s + 1}$$

- Set $K_m=1$, $Z_m=1$ and using Routh array criterion find a pair of values K_c & Z_I which yield stable close loop response
- Using the values of K_c & Z_I found in (a) examine the effect of changing K_m on the stability of close loop response
- Do the same with Z_m
- Based on above results, discuss the effect that measurement dynamics have on the stability of the close-loop response.

Ans. 1.) a) CLCE: $1 + \frac{s}{(0.1s+1)} \frac{K_c(Z_I s+1)}{Z_I s} \frac{K_m}{(Z_m s+1)} = 0$

$$\frac{(0.1 Z_I Z_m) s^3 + Z_I (Z_m + 0.1) s^2 + Z_I (1 + s K_c K_m) s + s K_c K_m}{(0.1s+1)(Z_I s)(Z_m s+1)} = 0$$

$$Z_m = 1 = K_m$$

$$0.1 Z_I s^3 + 1.1 Z_I s^2 + Z_I (1 + s K_c) s + s K_c = 0$$

Routh array

$$\begin{array}{l} 1 \quad 0.01 Z_I \quad Z_I(1+K_c) \\ 2 \quad 1.1 Z_I \quad s K_c \\ 3 \quad \frac{1.1 Z_I^2 (1+s K_c) - 0.05 Z_I K_c}{1.1 Z_I} = 0 \\ 4 \quad s K_c \end{array}$$

$$K_c > 0, \quad Z_I > 0$$

$$1.1 Z_I^2 (1+s K_c) - 0.05 Z_I K_c > 0$$

satisfied for $K_c \geq 1$ & $Z_I \geq 0.1$

b.) $K_c = 1, Z_I = 0.1$

$$0.01 Z_m s^3 + 0.1 (Z_m + 0.1) s^2 + 0.1 (1 + s K_m) s + s K_m = 0$$

Routh array

$$\begin{array}{l} 1 \quad 0.01 Z_m \quad 0.1 (1 + s K_m) \\ 2 \quad 0.1 (Z_m + 0.1) \quad s K_m \\ 3 \quad \frac{(0.1)^2 (Z_m + 0.1) (1 + s K_m) - 0.05 Z_m K_m}{0.1 (Z_m + 0.1)} = 0 \\ 4 \quad s K_m \end{array}$$

$$K_m > 0, \quad Z_m > 0$$

$$(Z_m + 0.1) (1 + s K_m) - s Z_m K_m > 0$$

for $Z_m = 1 \Rightarrow K_m > -2.2$

i.e. for all value of K_m

c.) set $K_c = 1$, $Z_E = 0.1$ & $K_m = 1$

we get the inequality

$$(Z_m + 0.1)(6) - 5Z_m > 0.$$

$$\boxed{Z_m > 0.6}$$

the system is stable for all values of Z_m

$$\text{if } \underline{Z_m > 0.6}$$

d) K_m & Z_m have no effect on the stability of the closed loop response.