FINANCIAL ENGINEERING
IME611A

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SESSION OBJECTIVES

- Fixed income instrument: Interest rate risk
- Duration
- Duration and interest rate sensitivity

DURATION

- **Duration:**
  - <u>Weighted average</u> of the **times at which cashflows occur**, where the **weights** are the present values at each time.
  - Measure of <u>interest rate sensitivity</u>

- For a cashflow occurring at t_0, t_1, t_2, t_3, t_4, t_5, … , t_n **duration** can be calculated as

$$D = \frac{PV(t_0)t_0 + PV(t_1)t_1 + PV(t_2)t_2 + PV(t_3)t_3 + \ldots + PV(t_n)t_n}{PV}$$

$$PV = \sum_{k=0}^{n} PV(t_k)$$

DURATION: AN EXAMPLE

- **Question:** A **bond** issued by L&T Infrastructure offers 5% coupon payment paid annually on a face value of ₹100 and a yield of 6% per annum. The term to maturity is 3 years. Calculate the duration of the bond.

$$D = \frac{1 * \frac{5}{(1+0.06)^1} + 2 * \frac{5}{(1+0.06)^2} + 3 * \frac{105}{(1+0.06)^3}}{\frac{5}{(1+0.06)^1} + \frac{5}{(1+0.06)^2} + \frac{105}{(1+0.06)^3}}$$

D = 2.857 years

MACAULAY DURATION

- Suppose a financial instrument makes payments **m** times per year, with the payment in period **k** being **c_k**, and there are **n** periods remaining.

- **Macaulay duration D** is given by

$$D = \frac{\sum_{k=1}^{n} (k/m)\, c_k/[1 + (\lambda/m)]^k}{PV}$$

  - Where, λ is the yield to maturity and

$$PV = \sum_{k=1}^{n} \frac{c_k}{[1 + (\lambda/m)]^k}$$

MACAULAY DURATION FORMULA

- The Macaulay duration for a bond with a coupon rate ***c*** per period, yield ***y*** per period, ***m*** periods per year, and exactly ***n*** periods remaining, is

$$D = \frac{1 + y}{my} - \frac{1 + y + n(c - y)}{mc[(1 + y)^n - 1] + my}$$

- Practice problem:
  - Example 3.7: *Calculate the duration for a 30-year bond paying 10% coupon where coupons are paid semiannually. The yield is 10%. Assume that the bond is **trading at par**.*

DURATION: SOME PROPERTIES

- Duration of a coupon paying bond is always **less than the maturity**, and often surprisingly short.

- As time to maturity increases to infinity, duration does not rise to infinity, but instead **tend to a finite limit** that is **independent of coupon rate**.

- Duration of a zero-coupon bond is **equal to maturity**.

- For a finite maturity bond, higher the coupon rate **lower is the duration**.

DURATION AND INTEREST RATE SENSITIVITY

- Case when payments are **m** times per year.

$$PV_k = \frac{c_k}{[1 + (\lambda/m)]^k}$$

$$\frac{dPV_k}{d\lambda} = \frac{-(k/m)c_k}{[1 + (\lambda/m)]^{k+1}} = -\frac{k/m}{1 + (\lambda/m)} PV_k$$

$$P = \sum_{k=1}^{n} PV_k$$

$$\frac{dP}{d\lambda} = \sum_{k=1}^{n} \frac{dPV_k}{d\lambda} = -\sum_{k=1}^{n} \frac{k/m}{1 + (\lambda/m)} PV_k = -\frac{1}{1 + (\lambda/m)} DP = -\boldsymbol{D_M P}$$

$\boldsymbol{D_M = Modified\ Duration = {}^{D}\!/_{(1+{}^{\lambda}\!/_{m})}}$

IMPORTANT RESULT

- **Price-sensitivity formula:** The derivative of price P with respect to yield λ of a fixed income security is

$$\frac{dP}{d\lambda} = -D_M P$$

Where $D_M = D/(1+\frac{\lambda}{m})$ is the modified duration.

- D_M measures the **relative change in a bond's price directly as λ changes**.

$$\frac{1}{P}\frac{dP}{d\lambda} = -D_M$$

$$\Delta P = -D_M P \Delta\lambda$$

- Practice Example 3.8

DISCLAIMER

- The information in this presentation has been compiled from the following textbook which has been mentioned as a reference text for this course on **Financial Engineering.**

- **Reference Text**:
  - Investment Science, 2nd Edition, Oxford University Press, David G. Luenberger