Module 4.7.2

Laplace Domain Analysis
Internal Model Control

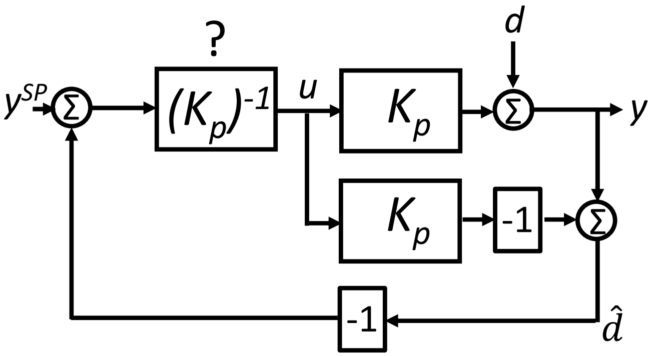
Lectures on

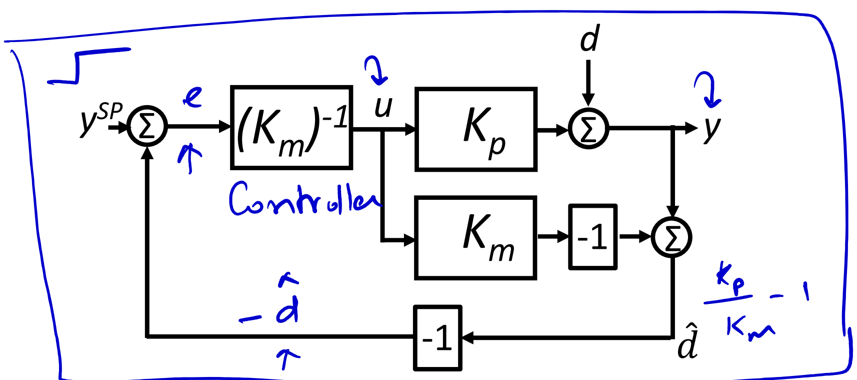
CHEMICAL PROCESS CONTROL
Theory and Practice

The Basic Idea

- Use model of process to make changes to control input
- Combine with feedback for further corrections

Motivating Example





i	Δy_i^{SP}	Δu_i	Δy_i	$-\Delta \hat{d}_i$
1	1	$\frac{1}{K_m}$	$\frac{K_p}{K_m}$	$1-\frac{K_p}{K_m}$
2	0	$\frac{1}{K_m}\left[1-\frac{K_p}{K_m}\right]$	$\frac{K_p}{K_m}\left(1-\frac{K_p}{K_m}\right)$	$\left(1-\frac{K_p}{K_m}\right)^2$
3		$\vdots$	$\vdots$	$\left(1-\frac{K_p}{K_m}\right)^3$
$\vdots$				$\vdots$

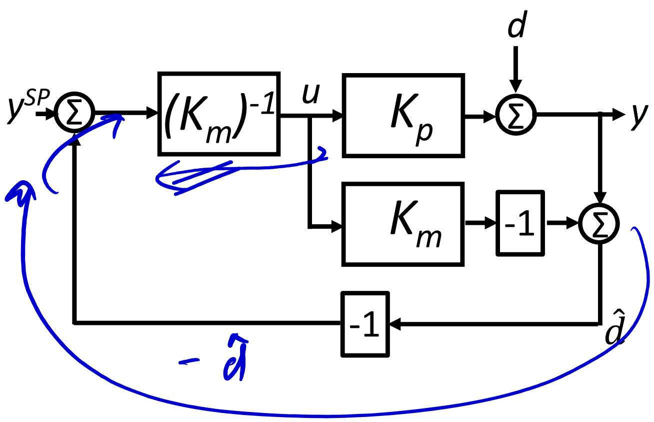
$$\Delta y = \sum_i \Delta y_i = \frac{K_p}{K_m}\left[1 + \left(1-\frac{K_p}{K_m}\right) + \left(1-\frac{K_p}{K_m}\right)^2 + \dots + \left(1-\frac{K_p}{K_m}\right)^{n-1}\right]$$

$$\Delta y = \frac{\cancel{K_p}}{K_m}\left[\frac{1-\left(1-\frac{K_p}{K_m}\right)^n}{\cancel{1-\left(1-\frac{K_p}{K_m}\right)}}\right] = 1 - \underbrace{\left[1-\frac{K_p}{K_m}\right]^n}_{0}$$

$n \to \infty$

$\Delta y \to 1$

The Basic Idea



i	Δy_i^{SP}	Δu_i	Δy_i	$\Delta \hat{d}_i$
1	1	$\frac{1}{K_m}$	$\frac{K_p}{K_m}$	$1-\frac{K_p}{K_m}$
2	0	$\frac{1}{K_m}\left(1-\frac{K_p}{K_m}\right)$	$\frac{K_p}{K_m}\left(1-\frac{K_p}{K_m}\right)$	$\left(1-\frac{K_p}{K_m}\right)^2$
3	0	$\frac{1}{K_m}\left(1-\frac{K_p}{K_m}\right)^2$	$\frac{K_p}{K_m}\left(1-\frac{K_p}{K_m}\right)^2$	$\left(1-\frac{K_p}{K_m}\right)^3$
⋮	⋮	⋮	⋮	⋮
n	0	$\frac{1}{K_m}\left(1-\frac{K_p}{K_m}\right)^{n-1}$	$\frac{K_p}{K_m}\left(1-\frac{K_p}{K_m}\right)^{n-1}$	$\left(1-\frac{K_p}{K_m}\right)^n$

Robust to plant-model-mismatch

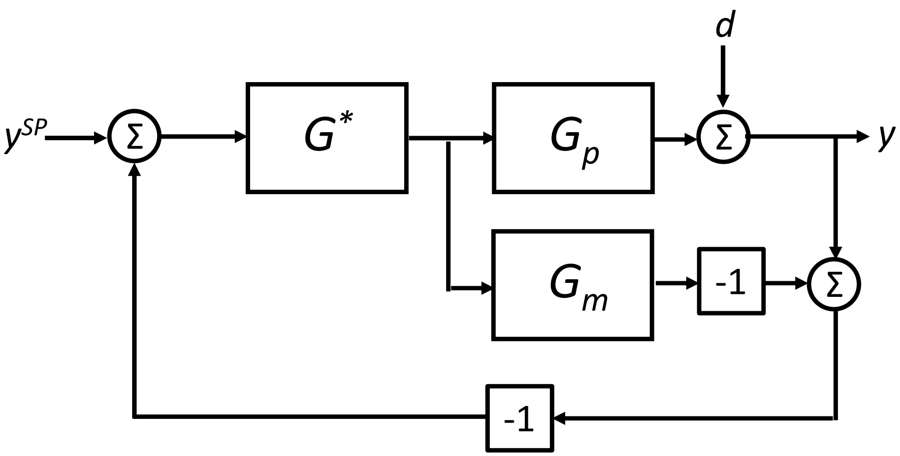
$$\Delta y = \sum_{i=1}^{\infty} y_i = \frac{K_p}{K_m}\left[1+\left(1-\frac{K_p}{K_m}\right)+\left(1-\frac{K_p}{K_m}\right)^2+\cdots+\left(1-\frac{K_p}{K_m}\right)^{n-1}\right] = 1-\left(1-\frac{K_p}{K_m}\right)^n$$

0.1

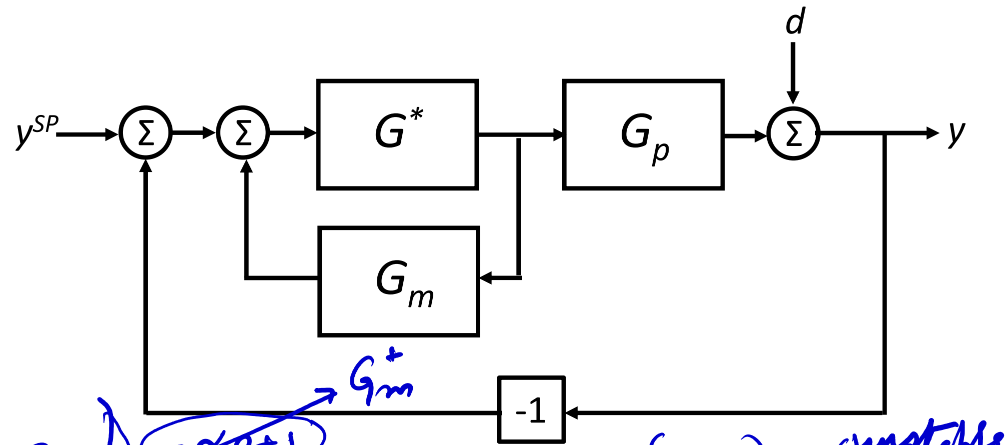
$$\left|\frac{K_p}{K_m}\right| < 2$$

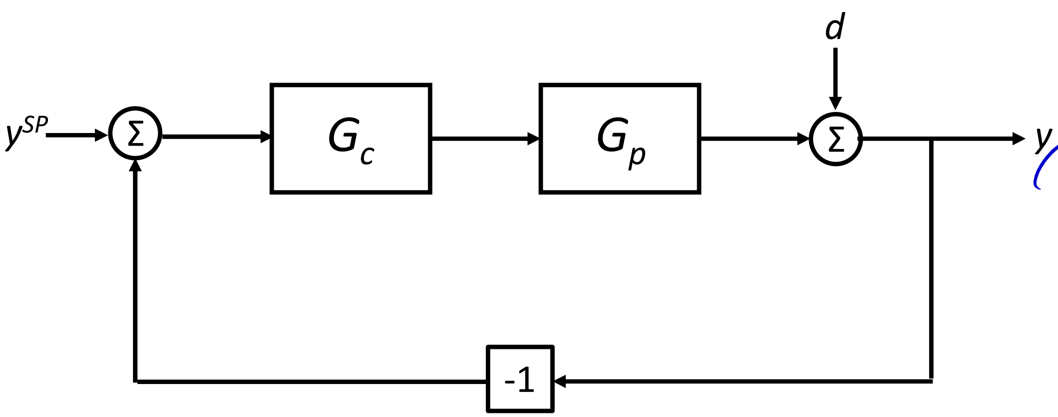
$$\Delta y \to 1 \text{ as } n \to \infty \text{ for } \left|1-\frac{K_p}{K_m}\right| < 1$$

The IMC Structure



Equivalent Simple Feedback Loop?





$$G_m \frac{K(-\alpha s+1)}{(\tau s+1)(\tau_2 s+1)} \quad \frac{1}{G_m} =$$

G_m^+, G_m^-

$$\boxed{G_c = \frac{G^*}{1 - G^* G_m}}$$

$$G^* = \frac{1}{G_m^-} f \qquad (-\alpha s+1) \text{ unstable}$$

f for physical realizability

$G_m = G_m^+ G_m^-$ ← invertible

non-invertible

Use for deriving feedback control laws

G_c ← PID

$$f = \frac{1}{(\lambda s+1)^n}$$

$$f = \frac{\gamma s+1}{(\lambda s+1)^n}$$

IMC Design Procedure

Split $G_m = G_m^+ G_m^-$

G_m^+: not invertible & G_m^-: invertible

Set $G^* = \frac{1}{G_m^-} f$ with filter f chosen for realizable G^* and appropriate algebra for feedback controller G_c in standard PID form

Usually $f = \frac{1}{(\lambda s+1)^n}$ or $f = \frac{\gamma s+1}{(\lambda s+1)^n}$

Obtain equivalent feedback controller G_c

$$G_c = \frac{G^*}{1-G^* G_m}$$

Express G_c in standard PID form to obtain tuning parameters (K_C, τ_I and τ_D) in terms of model parameters & λ (γ)

IMC Design: First order lag

$$G_p = \frac{K}{\tau s + 1}$$

$$G^* = \frac{1}{K}\left(\frac{\tau s + 1}{\lambda s + 1}\right) \quad G_c = \frac{1}{K}\frac{\tau}{\lambda}\left(1 + \frac{1}{\tau s}\right)$$

PI controller with

$K_c = \frac{1}{K}\frac{\tau}{\lambda} \quad \tau_I = \tau$

$G_m^+ = 1 \quad G_m^- = \frac{K}{\tau s + 1}$

$G_m = G_m^+ \cdot G_m^- = \frac{K}{\tau s + 1}$

$G^* = \frac{1}{K}\frac{(\tau s + 1)}{(\lambda s + 1)}$

$G_c = \frac{G^*}{1 - G^* G_m}$

$$G_c = \frac{1}{K}\frac{(\tau s + 1)/(\lambda s + 1)}{1 - \frac{1}{\cancel{K}}\frac{\cancel{(\tau s + 1)}}{(\lambda s + 1)} \cdot \frac{\cancel{K}}{\cancel{(\tau s + 1)}}}$$

$$G_c = \frac{1}{K}\frac{(\tau s + 1)}{\lambda s + \cancel{1} - \cancel{1}} = \frac{1}{K}\left[\frac{\tau}{\lambda} + \frac{1}{\lambda s}\right]$$

$$G_c = \frac{1}{K}\frac{\tau}{\lambda}\left[1 + \frac{1}{\tau s}\right]$$

$\equiv$ PI controller $\quad K_c = \frac{1}{K}\frac{\tau}{\lambda} \quad \tau_I = \tau.$

IMC Design: Second order lag

$$G_p = \frac{K}{(\tau_1 s + 1)(\tau_1 s + 1)}$$ 2

$$G^* = \frac{1}{K}\frac{(\tau_1 s + 1)(\tau_1 s + 1)}{\lambda s + 1}$$

$$G_c = \frac{1}{K}\frac{(\tau_1 + \tau_2)}{\lambda}\left[1 + \frac{1}{(\tau_1 + \tau_2)s} + \frac{\tau_1\tau_2}{(\tau_1 + \tau_2)}s\right]$$

PID controller with

$$K_c = \frac{1}{K}\frac{(\tau_1+\tau_2)}{\lambda} \qquad \tau_I = \tau_1 + \tau_2 \qquad \tau_D = \frac{\tau_1\tau_2}{(\tau_1+\tau_2)}$$

$$G^* = \frac{1}{K}\frac{(\tau_1 s+1)(\tau_2 s+1)}{(\lambda s+1)}$$

$$G_c = \frac{G^*}{1 - G^* G_m}$$

$$= \frac{1}{K}\frac{(\tau_1 s+1)(\tau_2 s+1)}{(\lambda s+1) - 1}$$

$$= \frac{1}{K}\frac{\tau_1\tau_2 s^2 + (\tau_1+\tau_2)s + 1}{\lambda s}$$

$$K_c = \frac{1}{K}\frac{\tau_1+\tau_2}{\lambda} \qquad \tau_I = \tau_1 + \tau_2 \qquad \tau_D = \frac{\tau_1\tau_2}{\tau_1+\tau_2}$$

$$G_c = \frac{1}{K}\frac{(\tau_1+\tau_2)}{\lambda}\left[1 + \frac{1}{(\tau_1+\tau_2)}\cdot\frac{1}{s} + \frac{\tau_1\tau_2}{(\tau_1+\tau_2)}s\right]$$

$$\Rightarrow G_c = \frac{1}{K}\left[\frac{\tau_1+\tau_2}{\lambda} + \frac{1}{\lambda s} + \frac{\tau_1\tau_2}{\lambda}s\right]$$

IMC Design: FOPDT$_I$

$$G_p = \frac{Ke^{-\theta s}}{\tau s + 1} \approx \frac{K(1-\theta s)}{\tau s + 1}$$

$$G^* = \frac{1}{K}\left(\frac{\tau s + 1}{\lambda s + 1}\right)$$

$$G_c = \frac{1}{K}\frac{\tau}{(\lambda + \theta)}\left(1 + \frac{1}{\tau s}\right)$$

PI controller with

$K_c = \frac{1}{K}\frac{\tau}{\lambda+\theta}$ $\tau_I = \tau$

$$G^* = \frac{1}{K}\frac{(\tau s+1)}{(\lambda s+1)}$$

$$G_c = \frac{1}{K}\frac{\tau s+1}{(\lambda s+1) - (1-\theta s)}$$

Lag dominant $\theta = 1$ min, $\tau = 50$ min, $\tau >> \theta$

$$= \frac{1}{K}\frac{\tau s+1}{(\lambda+\theta)s}$$

PI w/ $K_c = \frac{1}{K}\frac{\tau}{\lambda+\theta}$ $\tau_I = \tau$

$$= \frac{1}{K}\left[\frac{\tau}{\lambda+\theta} + \frac{1}{(\lambda+\theta)}\cdot\frac{1}{s}\right] \Rightarrow G_c = \frac{1}{K}\frac{\tau}{(\lambda+\theta)}\left[1 + \frac{1}{\tau}\cdot\frac{1}{s}\right]$$

IMC Design: FOPDT_{II}

$$G_p = \frac{Ke^{-\theta s}}{\tau s+1} \approx \frac{K}{\tau s+1}\left(\frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}\right)$$

$$G^* = \frac{1}{K}\frac{(\tau s+1)\left(\frac{\theta}{2}s+1\right)}{\underline{(\lambda s+1)}}$$

$$G_c = \frac{1}{K}\frac{\left(\tau+\frac{\theta}{2}\right)}{\left(\lambda+\frac{\theta}{2}\right)}\left[1+\frac{1}{\left(\tau+\frac{\theta}{2}\right)s}+\frac{\tau\theta}{(2\tau+\theta)}s\right]$$

PID controller with

$$K_c = \frac{1}{K}\frac{\left(\tau+\frac{\theta}{2}\right)}{\left(\lambda+\frac{\theta}{2}\right)} \qquad \tau_I = \tau+\frac{\theta}{2} \qquad \tau_D = \frac{\tau\theta}{(2\tau+\theta)}$$

$$G^* = \frac{1}{K}\frac{(\tau s+1)\left(1+\frac{\theta}{2}s\right)}{(\lambda s+1)}$$

$$G_c = \frac{1}{K}\frac{(\tau s+1)\left(1+\frac{\theta}{2}s\right)/(\lambda s+1)}{1-\frac{1}{\cancel{K}}\frac{\cancel{(\tau s+1)}\left(1+\frac{\theta}{2}s\right)}{(\lambda s+1)}\frac{\cancel{K}\left(1-\frac{\theta}{2}s\right)}{\cancel{(\tau s+1)\left(1+\frac{\theta}{2}s\right)}}}$$

$$\boxed{G_c = \frac{1}{K}\underline{\frac{\tau+\theta/2}{\lambda+\theta/2}}\left[1+\frac{1}{\left(\tau+\frac{\theta}{2}\right)}\cdot\frac{1}{s}+\underline{\frac{\tau\theta}{2\lambda+\theta}}s\right]}$$

$$G_c = \frac{1}{K}\frac{(\tau s+1)\left(\frac{\theta}{2}s+1\right)}{\lambda s+1-\left(1-\frac{\theta}{2}s\right)} = \frac{1}{K}\frac{\frac{\tau\theta}{2}s^2+\left(\tau+\frac{\theta}{2}\right)s+1}{\left(\lambda+\frac{\theta}{2}\right)s}$$

$$\Rightarrow G_c = \frac{1}{K}\left[\frac{\tau+\theta/2}{\lambda+\theta/2}+\frac{1}{\lambda+\frac{\theta}{2}}\cdot\frac{1}{s}+\frac{\tau\theta/2}{\lambda+\theta/2}s\right]$$

IMC Design: Integrator + Dead Time

$$G_p = \frac{Ke^{-\theta s}}{s} \approx \frac{K(1-\theta s)}{s} \qquad G^* = \frac{1}{K}\frac{s(\gamma s+1)}{(\lambda s+1)^2} \qquad G_c = \frac{1}{K}\frac{(2\lambda+\theta)}{(\lambda+\theta)^2}\left[1+\frac{1}{(2\lambda+\theta)s}\right]$$

PI controller with

$$K_c = \frac{1}{K}\frac{(2\lambda+\theta)}{(\lambda+\theta)^2} \qquad \tau_I = 2\lambda+\theta$$

$$G^* = \frac{s}{K}\cdot\frac{(\gamma s+1)}{(\lambda s+1)^2}$$

$$G_c = \frac{1}{K}\frac{s(\gamma s+1)}{(\lambda s+1)^2-(\gamma s+1)(1-\theta s)}$$

$$= \frac{1}{K}\frac{s(\gamma s+1)}{(\lambda^2+\gamma\theta)s^2+(2\lambda+\theta-\gamma)s}$$

$$= \frac{1}{K}\frac{\gamma s+1}{(\lambda^2+\gamma\theta)s+\underline{(2\lambda+\theta-\gamma)}}$$

Put $\gamma = 2\lambda+\theta$

$$G_c = \frac{1}{K}\frac{(2\lambda+\theta)s+1}{(\lambda^2+2\lambda\theta+\theta^2)s} = \frac{1}{K}\frac{(2\lambda+\theta)s+1}{(\lambda+\theta)^2 s}$$

$$= \frac{1}{K}\frac{(2\lambda+\theta)}{(\lambda+\theta)^2}\left[1+\frac{1}{2\lambda+\theta}\cdot\frac{1}{s}\right]$$

Exercise

$$G_p = \frac{Ke^{-\theta s}}{s} \approx K\left(\frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}\right)$$

Derive PID controller tuning using IMC design

IMC Design: Unstable Process

$$G_p = \frac{K}{\tau s - 1} \qquad G^* = \frac{1}{K}\frac{(\tau s - 1)(\gamma s + 1)}{(\lambda s + 1)^2} \qquad G_c = \frac{1}{K}\frac{(\lambda + 2\tau)}{\lambda}\left[1 + \frac{1}{\lambda\left(\frac{\lambda}{\tau} + 2\right)s}\right]$$

PI controller with

$$K_c = \frac{1}{K}\frac{(\lambda+2\tau)}{\lambda} \qquad \tau_I = \lambda\left(\frac{\lambda}{\tau} + 2\right)$$

$$G^* = \frac{\tau s - 1}{K}\frac{(\gamma s+1)}{(\lambda s+1)^2}$$

$$\boxed{\lambda}$$

$$G_c = \frac{1}{K}\frac{(\tau s-1)(\gamma s+1)/(\lambda s+1)^2}{1 - \frac{\cancel{(\tau s-1)}(\gamma s+1)}{\cancel{K}(\lambda s+1)^2}\cdot\frac{\cancel{K}}{\cancel{(\tau s-1)}}}$$

$$= \frac{1}{K}\frac{\lambda + 2\tau}{\lambda}\left[1 + \frac{1}{\lambda\left(\frac{\lambda}{\tau}+2\right)}\cdot\frac{1}{s}\right]$$

$$G_c = \frac{1}{K}\frac{\left(\frac{\lambda^2}{\tau}+2\lambda\right)s + 1}{\frac{\lambda^2}{\tau}s} = \frac{1}{K}\frac{\frac{\lambda^2}{\tau}+2\lambda}{\frac{\lambda^2}{\tau}}\left[1 + \frac{1}{\frac{\lambda^2}{\tau}+2\lambda}\cdot\frac{1}{s}\right]$$

$$G_c = \frac{1}{K}\frac{(\tau s-1)(\gamma s+1)}{(\lambda s+1)^2 - (\gamma s+1)}$$

$$= \frac{1}{K}\frac{(\tau s-1)(\gamma s+1)}{\lambda^2 s^2 - (\gamma - 2\lambda)s} = \frac{1}{K}\frac{\cancel{(\tau s-1)}(\gamma s+1)}{(\gamma - 2\lambda)s\cancel{\left[\frac{\lambda^2}{\gamma - 2\lambda}s - 1\right]}}$$

$$\tau = \frac{\lambda^2}{\gamma - 2\lambda}$$

$$\gamma = \frac{\lambda^2}{\tau} + 2\lambda$$

Summary

- IMC design suggests
  - Standard feedback control is equivalent to internal model based control with feedback correction for plant-model mismatch or disturbances
- The IMC structure is robust to plant-model mismatch
- Provides a systematic method for deriving tuning rules for simple process transfer functions
- IMC based tuning results in
  - Good servo response
  - Sluggish regulator response for lag-dominant processes