FINANCIAL ENGINEERING

IME611A

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SESSION OBJECTIVES

- The yield curve
- Term structure of interest rates
- Spot rate curve
- Use of spot rates in valuation

TERM STRUCTURE OF INTEREST RATES

and other advanced issues in modeling of Fixed-Income securities

THE YIELD CURVE

- **Yield curve:** relationship between yield and time to maturity

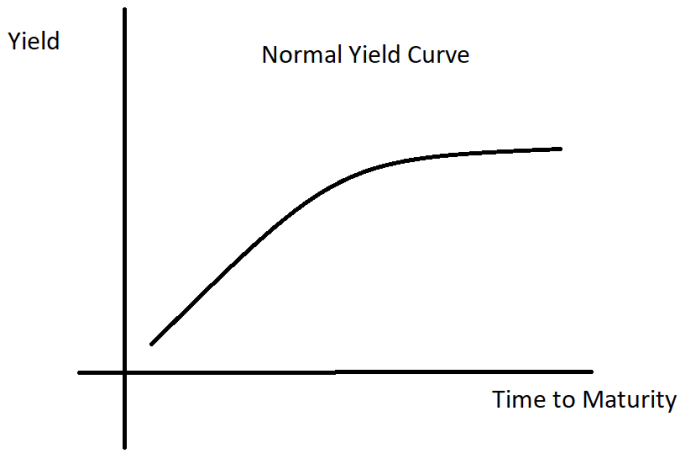
- **Normal yield curve:** A rising curve
  - *Long maturity bonds* have **higher yields** than *short maturity bonds*.

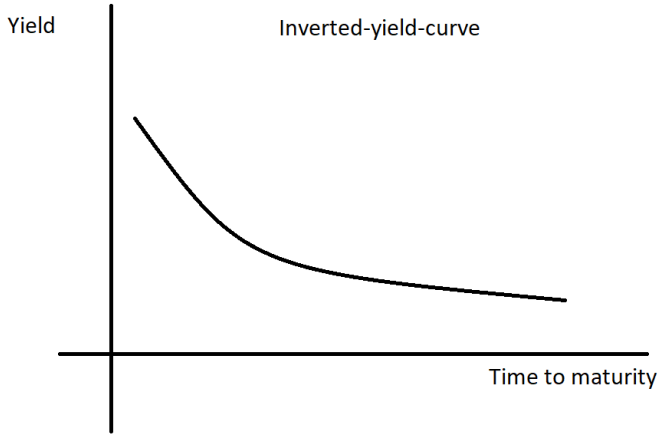
- **Flat yield curve**

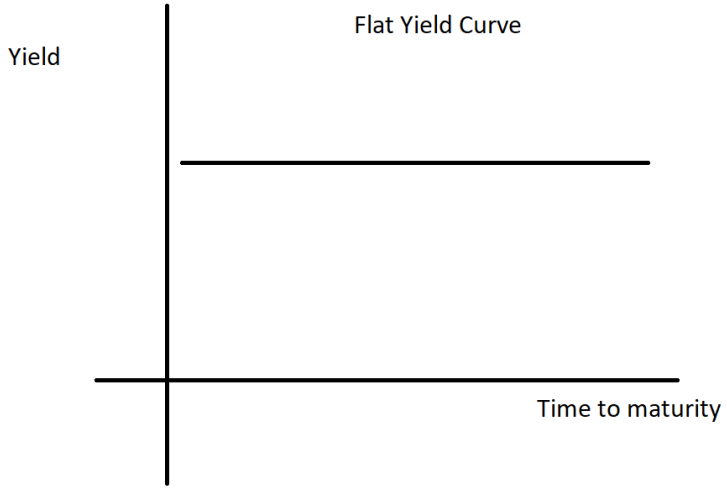
- **Inverted yield curve:** A declining curve
  - *Long maturity bonds* have **lower yields** than *short maturity bonds*.

Additional reading: https://www.cnbc.com/2019/03/25/the-us-bond-yield-curve-has-inverted-heres-what-it-means.html

TYPES OF YIELD CURVE







THE TERM STRUCTURE

- **Spot rates**: basic interest rates defining the term structure

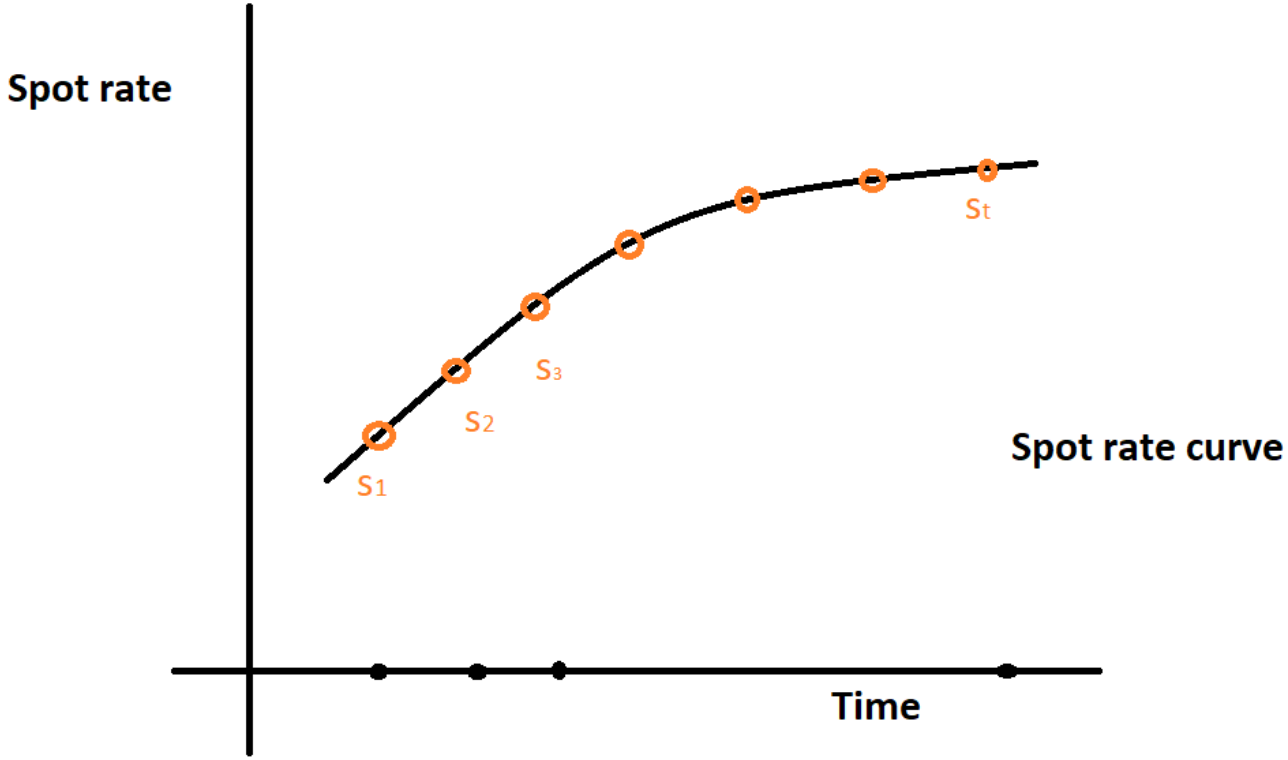
- Spot rate S_t : **interest rate**, expressed in yearly terms, charged for money held from the present time (t = 0) until time t.

- Under different conventions,

Frequency of compounding	Formula
Annual	$(1 + S_t)^t$
m periods per year	$(1 + S_t/m)^{mt}$
Continuous	$e^{(t * St)}$

SPOT RATE CURVE

- **Spot rate curve:** Relationship between spot rate and the time (years) to maturity



DISCOUNT FACTOR AND PRESENT VALUE

- For a cashflow stream, $(x_0, x_1, \ldots, x_n)$

$$PV = x_0 + d_1 x_1 + d_2 x_2 + \ldots + d_n x_n$$

- Using the spot rates S_t, one can obtain the set of discount factors.

Frequency of compounding	Formula
Annual	$d_k = \frac{1}{(1+s_k)^k}$
m periods per year	$d_k = \frac{1}{(1+s_k/m)^{mk}}$
Continuous	$d_t = e^{-(t * s_t)}$

Practice Problem: Example 4.1 and 4.2

DETERMINATION OF SPOT RATE

- **Approach 1:** Price of a series of zero-coupon bonds with various maturities

- **Approach 2:** Using coupon paying bonds step-by-step along with a 1-year treasury bill

$$P = \frac{C}{1 + s_1} + \frac{C + F}{(1 + s_2)^2}$$

- **Approach 3:** Using subtraction process. Two bonds of different coupon rates but identical maturity dates

ILLUSTRATIONS

1. Three ZCB of maturity period, 1 year, 2 year and 3 year have yield to maturity of 3.5%, 4% and 5%. Calculate the spot rates for the three years.

2. A 2-year bond pays 5% coupon rate and has a face value of $100. Also, 1-year treasury offers a yield of 4%. Calculate the spot rates for the two years.

3. Bond A is a 10-year bond with 10% coupon and is priced at P_A = $98.72. Bond B is a 10-year bond with 8% coupon and is priced at P_B = $85.89. Both bonds have same face value of $100. Calculate the 10-year spot rate.

DISCLAIMER

- The information in this presentation has been compiled from the following textbook which has been mentioned as a reference text for this course on **Financial Engineering.**

- Reference Text:
  - **Investment Science**, 2nd Edition, Oxford University Press, David G. Luenberger