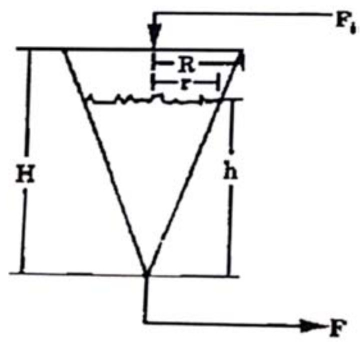
Assignment 2

1. Consider a conical tank of height H and radius R (as shown in figure) in which the liquid height is h. The outflow F = kh. Develop a linearized dynamic material balance equation for given system and find the steady-state gain and the time-constant in terms of known variables.



2. Use method of undetermined coefficients to solve $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y = f_{(t)}$ when the forcing function $f_{(t)}$ is
   a) Unit step
   b) Ramp $f_t = t$

3. Solve for zero initial conditions
   a) $\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 1$
   b) $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 1$
   c) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 4y = 1$
   d) $\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 11\frac{dy}{dt} + 6y = 1$

4. Develop a simple pandemic dynamics model using your knowledge of reaction kinetics. Assume elementary kinetics and make reasonable assumptions so that an analytical solution is obtained for the time variation in the number of infected individuals. A possible simple reaction kinetic scheme is

   Infected + Uninfected $\xrightarrow{k_f}$ 2 Infected (rate constant k_i)

   Infected $\xrightarrow{k_r}$ Recover (rate constant k_r)

   Infected $\xrightarrow{k_d}$ Dead (rate constant k_d)

5. A series reaction A $\xrightarrow{k_1}$ R $\xrightarrow{k_2}$ S occurs in a batch reactor, R being the desirable product. The reactor is initially loaded with pure A. Obtain the time trajectory of the component concentrations (c_A, c_R, c_S). Find the time at which c_R is maximum.

6. Consider the unstable first order processes whose eqn. is as follow

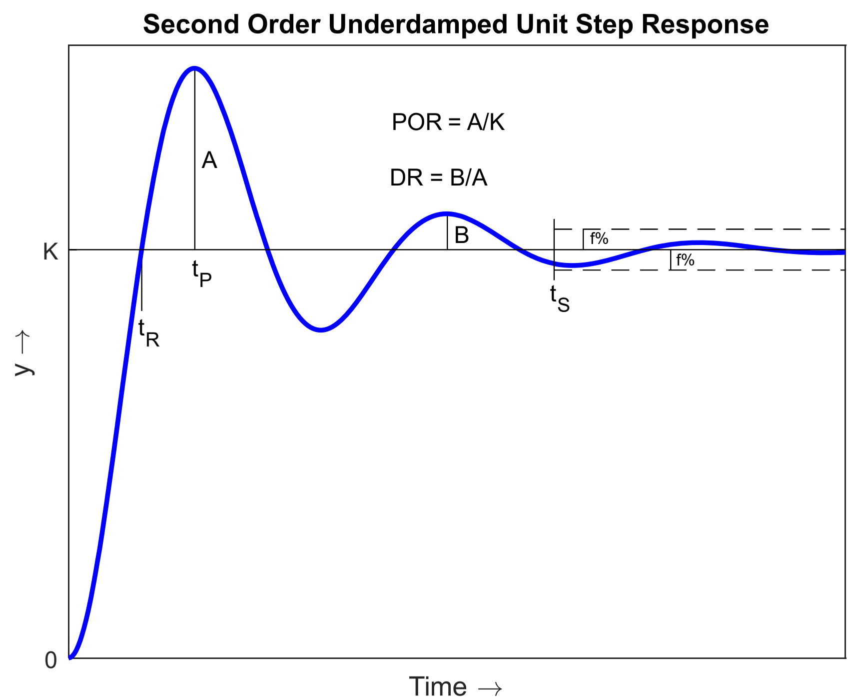
$$\tau \frac{dy}{dt} - y = Ku$$

Determine whether the proportional controller can stabilize this process. If yes then find the condition on K_C (controller gain) for stabilization.

7. For the second order underdamped system ODE

$$\tau^2 \frac{d^2y}{dt^2} + 2\xi\tau \frac{dy}{dt} + y = Ku$$

where $0 < \xi < 1$. The response is characterized by the rise time (t_R), peak time (t_P), peak overshoot ratio (POR) and the decay ratio (DR), and the $f_{\%}$ response settling time, as illustrated in the Figure.



Show that

a) The unit step response is given by

$$y = K\left\{1 - \frac{e^{-\frac{\xi}{\tau}t}}{\sqrt{1-\xi^2}} \sin\left(\frac{\sqrt{1-\xi^2}}{\tau}t + \phi\right)\right\} \text{ with } \phi = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

b) $\frac{t_R}{\tau} = \frac{\pi - \phi}{\sin\phi}$

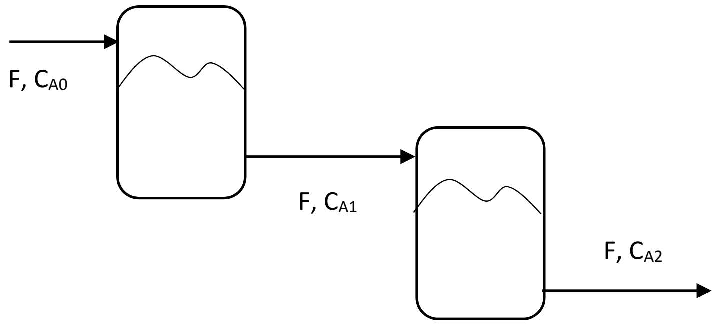
c) $\frac{t_P}{\tau} = \frac{\pi}{\sin\phi}$

d) $POR = e^{-\pi\cot\phi}$

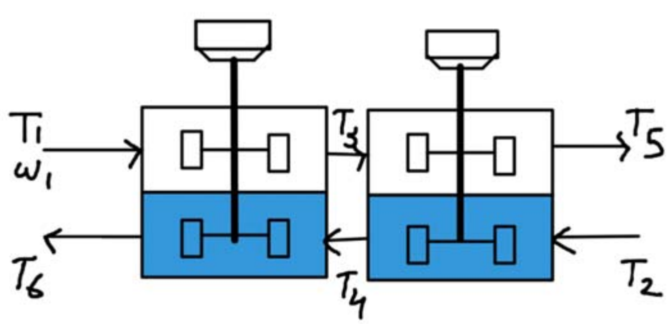
e) $DR = e^{-2\pi\cot\phi}$

f) $\frac{t_S}{\tau} = \frac{\ln[1/(f\sin\phi)]}{\cos\phi}$

8. Develop a dynamic model for two *identical* tanks in series as shown in Figure, relating c_{A2} to a change in the inlet concentration c_{A0}. Assume constant F and V. Use the integration factor approach to solve the ODE system to obtain an analytical expression for $c_{A2(t)}$.



9. Distributed parameter systems such as tubular reactors and heat exchangers often can be modelled as a set of lumped parameter equations. In this case an alternative physical interpretation of the process is used to obtain an ODE model directly rather than by converting a PDE to ODE form by means of a lumping method such as finite differences. As an example, consider a single centric-tube exchanger with energy exchange between two liquid streams flowing in opposite directions as shown in fig. below. We might model this process as if it were two small, perfectly stirred tanks with heat exchange. If the mass flow rate w_1 and w_2 and the inlet temperature T_1 & T_2 are known functions of time, derive the governing system of ODEs that need to be solved simultaneously. Assume that all liquid properties (ρ_1, ρ_2, C_{p1},C_{p2}) are constant, that the area for the heat exchanger in each stage is A, that the overall heat transfer coefficient U is the same in each stage, and the wall between the two liquids has negligible thermal capacitance.



10. Consider PI temperature control of the heated tank, that has been considered in class. Draw a rough sketch for the locus of the two roots of the closed loop characteristic equation as K_C is increased from 0 to ∞ for the case when $\tau_I < \tau$ (ie ($R > 1$)

11. Consider P control of a process with the governing ODE
    a) $\tau \frac{dy}{dt} + y = Ku$
    b) $\tau_1 \tau_2 \frac{d^2 y}{dt^2} + (\tau_1 + \tau_2)\frac{dy}{dt} + y = Ku$

    Both y and u are deviation variables so that for a P controller, $u = K_C(y^{SP} - y)$

    For the characteristic equation of the controlled system, as K_C is changed, the roots will change. Sketch the locus of the characteristic equation roots as $|K_C|$ is increased from 0 to ∞.