

$$G_p = \frac{K e^{-\theta s}}{\tau s + 1}, \quad G_d = \frac{K_d e^{-\theta_d s}}{\tau_d s + 1}$$

We need to design a PID controller with the integral time  $\tau_I = \tau$  and the derivative time,  $\tau_D$ , adjusted to maximise  $K_C$ , where we have to choose  $K_C$  for a maximum closed-loop log modulus ( $L_{CL}^{MAX}$ ) of 2dB.

Given:

- $K = 2$
- $\tau = 1 \text{ min}$
- $\theta = 5 \text{ min}$
- $K_d = -2$
- $\tau_d = 2 \text{ min}$
- $\theta_d = 8 \text{ min}$

Then we need to obtain  $K_C$  and  $\tau_D$  and Plot the variation in  $K_C$  with  $\tau_D$  to clearly show the maximum in  $K_C$  and the corresponding  $\tau_D$  value. We also need to plot  $L_{CL}$  vs  $\log(\omega)$  to show the LCLMAX peak clearly.

Finally, we need to plot the servo and regulator unit step responses using the PID controller and compare them with a PI controller with  $\tau_I = \tau$  and  $K_C$  adjusted for  $L_{CL}^{MAX} = 2\text{dB}$ .

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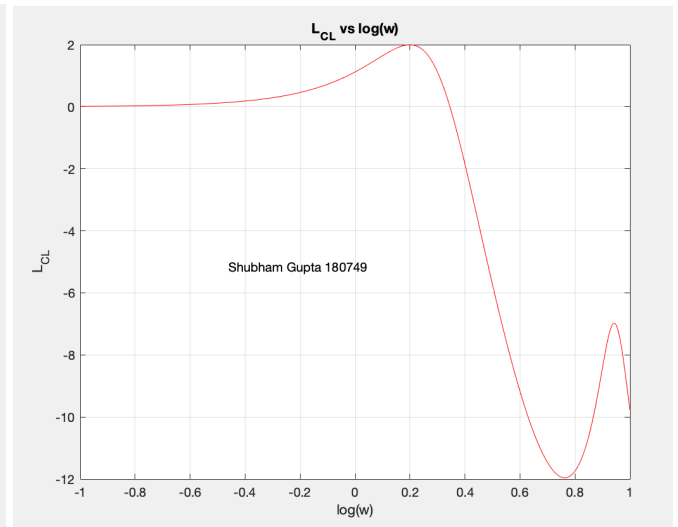
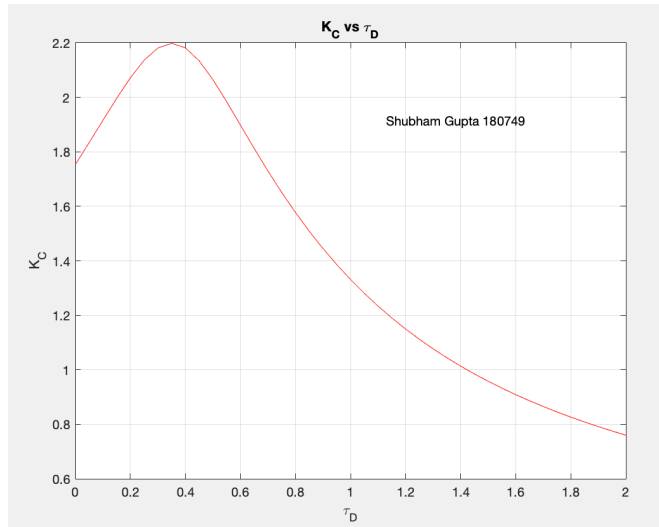
### For PID control

We first need to assume  $K_C$  and then go about iterating in a loop with varying  $\omega$ . For each iteration, we check if the value of  $20\log_{10}(G_{CL})$  is equal to two. If yes, we break the loop and report the corresponding  $K_C$ . If no, we shift our  $K_C$  to positive/negative according to the value of  $L_{CL}$  obtained and run another iteration and do as done above.

The optimal  $K_C$  obtained can be checked while drawing  $K_C$  vs  $\tau_D$ , and  $\tau_D$  will be at maximum  $K_C$ .

After running the iterations, we obtain  $K_C = 2.1981$  and  $\tau_D = 0.35$ . We will use these results for further simulations.

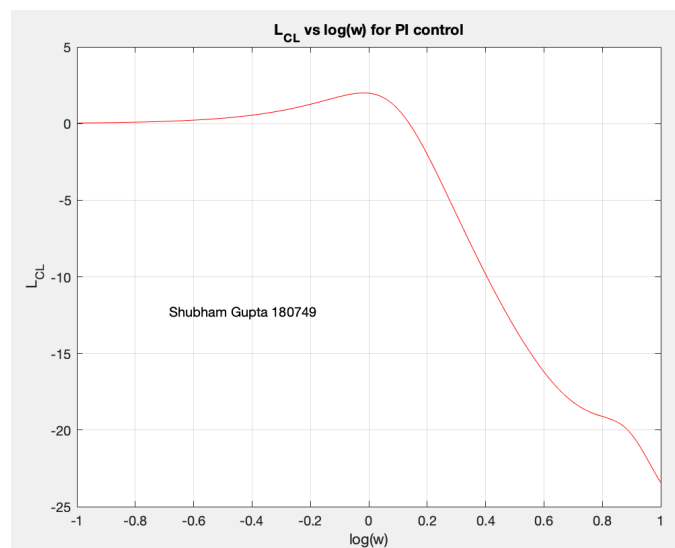
Following are the plots of  $K_C$  vs  $\tau_D$  and  $L_{CL}$  vs  $\log(\omega)$  for PID control.



## For PI control

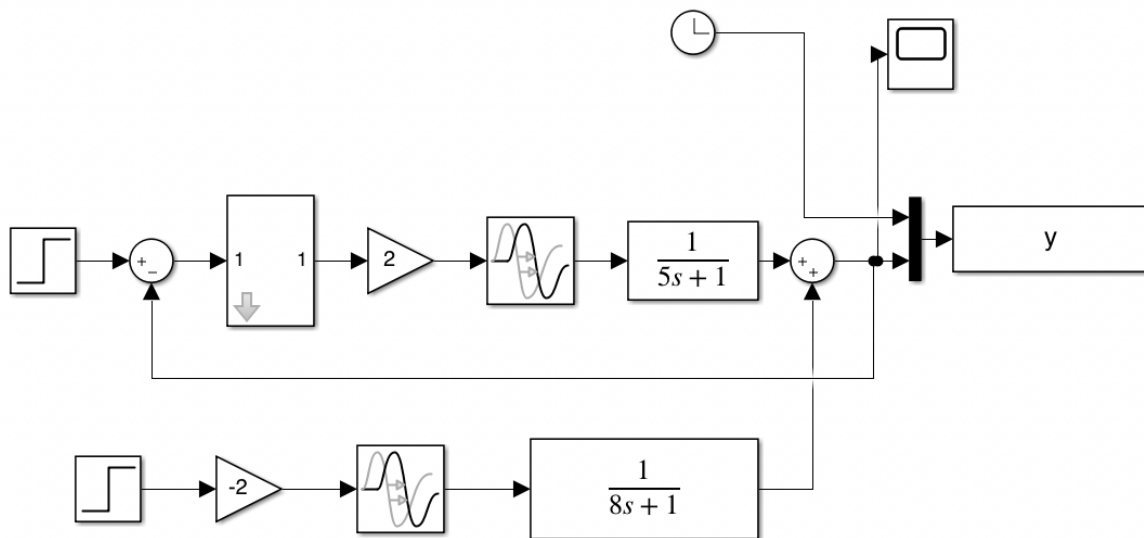
We use the same algorithm as described above. The only difference will be in  $G_{OL}$  as we don't have to account for  $\tau_D$ . After running the iterations, we obtain  $K_C = 1.7527$ . We will use this result for further simulations.

Following is the plot of  $L_{CL}$  vs  $\log(\omega)$  for PI control.



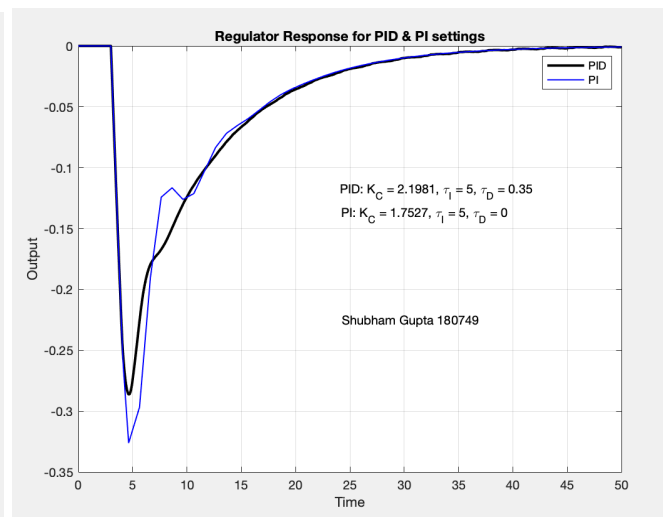
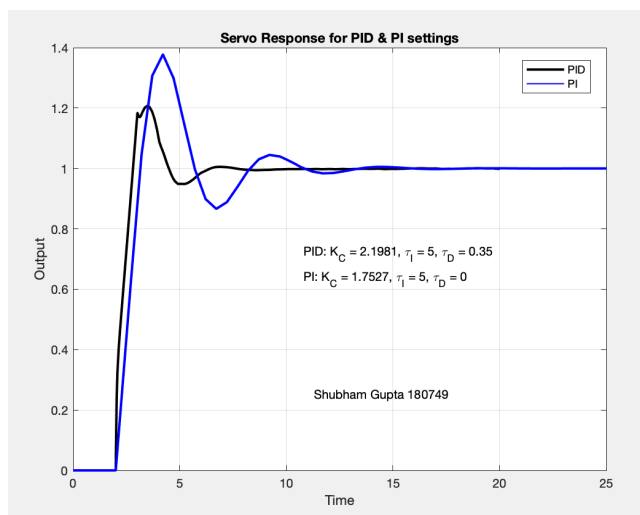
## Servo & Regulator response for PID & PI control

We first need to design our simulation model.



We run this simulation for servo and regulator response using the values in the following table,

	PID	PI
$K_C$	2.1981	1.7527
$\tau_I$	5 min	5 min
$\tau_D$	0.35 min	0 min



## CONCLUSION

- For this particular setup, servo response works better than regulator response, and it achieves its setpoint faster than regulator response.
- We find a maximum in the plot of  $K_C$  vs  $\tau_D$ , which indicate that there is only one optimal value of  $K_C$  for the PID setup. We use this  $K_C$  and corresponding  $\tau_D$  for our simulations.